Sports Betting - A quantitative approach

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August 5, 2023

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1.1 3 outcomes events - Soccer 1X2

In Team A vs Team B match it is possible to bet on 1X2 i.e. A wins, draw, B wins.

The Betting Agency (BA) provides odds for the 3 events, i.e. how much a better can win vs how much they bet, we call them o_1, o_2, o_3 respectively

From them, we can compute the "quotes" defining the actual in pocket money, assuming unitary denominator, simply as $q_i = o_i - 1$

So we can write the matrix defining the better payout for the 3 events assuming the desired payout is equal for the 3 events

$$\begin{cases} ax_1 - x_2 - x_3 = 1 \\ -x_1 + bx_2 - x_3 = 1 \\ -x_1 - x_2 + cx_3 = 1 \end{cases}$$

The $AX = B$ format with

$$A = \begin{pmatrix} q_1 & -1 & -1 \\ -1 & q_2 & -1 \\ -1 & -1 & q_3 \end{pmatrix}$$

with
- $X = (x_1, x_2, x_3)^T$ - $B = (1, 1, 1)^T$

Solving this system returns the $\{x_i\}_{i=1,2,3}$ betting allocation for the 3 events

The general solution is

$$x_i = \frac{-(q_j + q_k + 1)}{\sum_{s=\{1,2,3\}} q_s + 2 - \prod_{s=\{1,2,3\}} q_s}$$

with all the possible permutations of i, j, k for $1, 2, 3$ here

For the solution to be admissible, it is required $x_i \leq 0 \quad \forall i = \{1, 2, 3\}$ and since $q_i > 1 \forall i = \{1, 2, 3\}$ by construction then it means the numerator is always negative so the denominator has to be as well and therefore

$$\prod_{s=\{1,2,3\}} q_s > \sum_{s=\{1,2,3\}} q_s + 2$$

This is a simple general criteria to identify arbitrage opportunities in 1x2 bets

1.1.1 Examples

Real live quotes

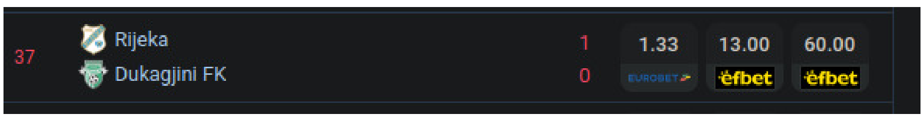

Figure 1: Alt text

We check $1037, 4 > 76, 33$ so there is arbitrage and the optimal allocation is $X = \{0.889, 0.090, 0.019\}$ resulting in a 0.182 i.e. approx 18% arb gain

The problem of this approach is in the actual execution and sclaing of this strategy since

- terms of agreements: the BAs require KYC and by communicating among them, they can detect Arbing and block the accounts since they usually do not allow it in their terms
- liquidity: the odds in reality are $o_i(x_i)$ i.e. function of the amount bet on that specific event, therefore there is some $x_{i,th}$ threshold amount for which the $o_i(x) < o_i(x_{th}) \quad x > x_{th}$ i.e. odds change