

The Effects of Tax Enforcement on the Firm Size Distribution and Aggregate Productivity *

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Abstract

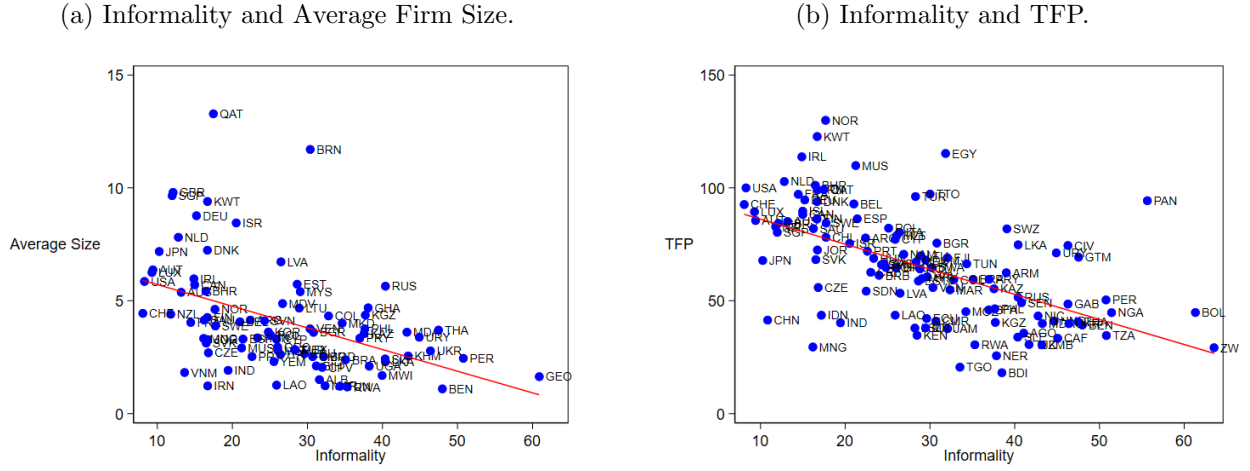
How does tax enforcement affect the firm size distribution and total factor productivity (TFP)? To answer this question, I develop a quantitative model characterized by heterogeneous agents who choose whether to be workers, entrepreneurs in the formal sector, or entrepreneurs in the informal sector. Informal entrepreneurs do not pay taxes, but face a probability of detection that is increasing in firm size. In the model, stricter tax enforcement results in lower informality and affects the firm size distribution and TFP through two mechanisms. First, some relatively unproductive agents choose to be workers rather than entrepreneurs, leading to higher average productivity and firm size through a composition effect. Second, some entrepreneurs switch to the formal sector and expand their scale compared to informal entrepreneurs of the same productivity. Using data from Brazil, I calibrate the model and estimate that a counterfactual tax enforcement that reduces the informality rate from 36% to 30% of total output - the value measured in a weighted average of selected Latin American countries - would account for about 9% and 28% of the observed differences in TFP and average firm size.

1 Introduction

Developing countries are characterized by the presence of a firm size distribution that is highly concentrated on businesses with a low number of employees. In fact, an overwhelming majority of firms and a relatively large fraction of value added are accounted for by micro firms with 5 employed persons at most. Informality rates among small firms are substantial, reflecting the government's inability to detect and tax these entities. For instance, the informality rate among firms smaller than

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Figure 1: Correlation between Informality, Average Firm Size, and TFP across Countries



Data sources: [Elgin et al. \(2021\)](#) for informality, [Bento and Restuccia \(2021\)](#) for average size, [Feenstra et al. \(2015\)](#) for TFP.

Informality and TFP data are country averages over the period 2005-2015.

5 employees is 86% in Brazil and 94% in Mexico¹. In addition, compared to advanced economies, developing countries display lower measures of total factor productivity (TFP), which is generally deemed as the main source of cross-country differences in per-capita income. Figure 1 shows cross-country correlations between informality rates, average firm size (panel a), and TFP (panel b). To sum up, developing countries display coexisting high informality and tax evasion rates, low average firm size, and low aggregate productivity.

How much does tax enforcement account for the observed differences in size and productivity across countries? This paper investigates the impact of tax enforcement on the firm size distribution and TFP. To assess the magnitude of the impact, I build a dynamic quantitative model characterized by heterogeneous agents, occupational choice, and the presence of an informal sector. Agents can choose whether to be workers, entrepreneurs in the formal sector, or entrepreneurs in the informal sector. Formal entrepreneurs pay sales and payroll taxes, while informal entrepreneurs do not. However, the latter are detected with a probability that increases with the size of their business. In case of detection, informal entrepreneurs pay the due amount of taxes plus a proportional fine. Tax enforcement is then summarized by the combination of tax rates, penalty rates, and a probability of detection that is an increasing function of the number of employees in the business.

In the model, stricter tax enforcement affects the firm size distribution and TFP through two mechanisms. First, the profitability to operate in the informal sector declines, inducing some relatively unproductive agents to be workers. These agents would operate as informal entrepreneurs with looser tax enforcement, because they would be able to circumvent tax payments. Moreover, these entrepreneurs would operate at a low scale, given the positive relationship between productivity

¹Data for Brazil are presented in Section 3.1. See [Alvarez and Ruane \(2023\)](#) for Mexican data.

and size. Consequently, average size and productivity increase thanks to a composition effect (*selection channel*). Second, to avoid detection by the tax authority, informal entrepreneurs operate at a lower scale than formal entrepreneurs of the same productivity. Therefore, as the rate of informality decreases due to stricter tax enforcement, fewer entrepreneurs decide to keep their scale at suboptimally low levels. This *detection channel* has a positive effect on average firm size and allocative efficiency.

To calibrate the model, I use data from Brazil. More specifically, I substantially rely on the *Economia Informal Urbana* (ECINF), a detailed firm-level survey lastly conducted in 2003, which contains information on informality status and number of employees. The calibrated model is able to replicate several features of the Brazilian economy, such as the high share of micro and small firms, high informality (36%) and tax evasion rates, high entrepreneurship (32%) and self-employment rates, and the fact that formal entrepreneurs are on average larger and more productive than informal entrepreneurs.

I then perform a counterfactual analysis to measure the impact of tax enforcement. To simulate stricter tax enforcement in Brazil, I change the parametrization of the detection probability function to match an informality rate of 30%, which is a GDP-weighted average of selected Latin American countries. The counterfactual calibrated economy generates an increase in output and TFP by approximately 4.3% and 0.5%, respectively, and a rise in average firm size from 2.08 in the baseline to 2.37. These changes account for about 13%, 9%, and 28% of the observed differences in GDP per capita, TFP, and average firm size between Brazil and the weighted averages of the countries in the group. These results are consistent across several exercises in which different countries or group of countries are taken as counterfactuals. Overall, tax enforcement can account for about 25-30% of the estimated differences in average firm size and for about 8-15% of the estimated differences in GDP per capita and TFP.

What are the mechanisms through which stricter tax enforcement results in higher TFP and average firm size? The counterfactual economy displays lower rate of entrepreneurship (3 percentage points lower than in the baseline) and lower rates of informality for given values of firm-level productivity. Overall, there is a reallocation of resources from low-productivity informal entrepreneurs to high-productivity formal entrepreneurs. From a quantitative point of view, decomposing the impact between the selection and detection channel highlights that both channels have substantial positive impacts on average firm size (they both account for about 50% of the overall impact), while only the detection channel has a significant positive effect on TFP.

This paper proceeds as follows. Section 2 introduces the quantitative model. Section 3 presents the data and the baseline calibration of the model. Section 4 describes the counterfactual experiments and presents the quantitative results. Section 5 concludes.

Related literature. This paper relates to three main strands of literature. First, it relates to the literature studying the aggregate effects of tax enforcement, either in developing countries or

in advanced economies. My framework shows some similarities with the work of [Leal Ordóñez \(2014\)](#), who focuses on Mexico, and [Di Nola et al. \(2021\)](#), who focus on the US, for the presence of selection and detection channels as the main mechanisms affecting aggregate outcomes. With regard to cross-country studies, [Bachas et al. \(2019\)](#) analyze how size-dependent taxation affects TFP across several countries and sectors. Differently from my paper, which relies on informal businesses' statistics for the quantification, [Bachas et al. \(2019\)](#) use World Bank data from registered formal firms to estimate the model and perform counterfactual analysis.

The paper is also related to the literature that analyzes the relationship between informality and aggregate outcomes. These studies often focus on a single country such as Brazil ([Ulyssea \(2018\)](#), [Erosa et al. \(2023\)](#)) or Mexico ([Leal Ordóñez \(2014\)](#)). [Ulyssea \(2018\)](#) studies the effects of policy reforms that reduce informality on output, TFP, and welfare. The author develops a quantitative model that features two margins of informality: an *extensive margin*, associated with the business side, and an *intensive margin*, which reflects the hiring of “off-the-books” workers by formal firms. He then shows that policies affecting the intensive margin do not necessarily increase output and welfare. [Erosa et al. \(2023\)](#) extend Ulyssea’s framework to assess the interaction between informality and financial frictions. While getting similar qualitative results as in [Ulyssea \(2018\)](#), they show that tight borrowing constraints amplify the negative effects of informality on the allocation of resources. My work distinguishes from the above papers in three dimensions. First, my model introduces a microfoundation of the relationship between size and the cost of informality that depends on tax enforcement and is disciplined by statutory tax rates and moments from the informal firm size distribution. Second, I focus my analysis on the effects on average firm size and TFP. Third, I estimate how tax enforcement can account for the differences in average firm size and TFP between Brazil and other countries displaying lower informality rates.

Finally, the paper is connected with the literature studying the effects of correlated distortions and size-dependent policies on TFP ([Restuccia and Rogerson \(2008\)](#); [Guner et al. \(2008\)](#); [Hsieh and Klenow \(2009\)](#); [Buera et al. \(2013\)](#)) and the firm size distribution ([Garicano et al. \(2016\)](#); [Bento and Restuccia \(2017\)](#); [Bento and Restuccia \(2021\)](#)). For example, [Bento and Restuccia \(2021\)](#) show how different levels of elasticity between distortions and size can account for cross-country differences in average establishment size. My contribution to this literature is to highlight how tax enforcement could represent one of the underlying sources of the measured differences in correlated distortions across countries and constitute a relevant factor in accounting for differences in aggregate outcomes.

2 Model

Time is discrete. A small open economy is populated by a continuum of infinitely-lived individuals who are heterogeneous in their ability to run a business. Individuals choose whether to be entrepreneurs or workers. Each worker inelastically supplies one unit of labor. Entrepreneurs ad-

ditionally choose whether to operate the firm in the formal or informal sector.

Entrepreneurs. Both formal and informal entrepreneurs adopt the same technology in producing a homogeneous final good. Entrepreneur i 's production function is

$$y_{it} = z_{it}(k_{it}^\alpha(\bar{\ell} + n_{it})^{1-\alpha})^\gamma$$

Where y denotes output, z is managerial ability (i.e., firm-level productivity), k is capital, $\bar{\ell}$ is the owner's labor supply entering the production function, and n is hired labor. $\alpha \in (0, 1)$ denotes the capital share, while $\gamma \in (0, 1)$ denotes the span-of-control parameter. The value of $\bar{\ell}$ is common across entrepreneurs. Entrepreneurial ability z_{it} evolves over time according to an $AR(1)$ process. Capital is borrowed at a rate $(r + \delta)$, where δ denotes depreciation, while labor is provided by workers at a hourly wage rate w . Entrepreneurs also pay a variable cost proportional to the number of employees $\theta \times n$, which reflects the cost of managing employees.² The baseline economy is characterized by perfect financial markets, so entrepreneurs can rent any desired amount of capital for production.

Each entrepreneur decides whether to keep the firm formal or informal. Formal firms pay an output (sales) tax τ_y and a payroll tax τ_n , while informal firms do not. However, informal firms are detected with probability $p(n)$, with $p'(n) > 0$, that is, the detection probability is increasing in the amount of hired labor used for production. A detected firm has to pay a surcharge $(1 + \kappa)$ of its due taxes to the government, with $\kappa > 0$. Therefore, tax enforcement is defined by the combination of tax rates τ_y and τ_n , a surcharge rate κ , and a probability of detection function $p(n)$.

Preferences. Each individual maximizes the expected present discounted value of the following streams of utilities subject to an infinite sequence of budget constraints³

$$\begin{aligned} \max_{\{c_t\}_{t=0}^{\infty}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t) \\ & c_t + a_{t+1} \leq e_t(z_t, a_t) + (1 + r)a_t \\ & a_{t+1} \geq 0 \end{aligned}$$

Where c_t denotes consumption, a_t denotes assets (savings), e_t denotes earnings from either working or running a formal or informal firm. The utility function $u(\cdot)$ satisfies the usual properties.

Individuals are subject to taste shocks that affect their occupational choices. These shocks are (i) distributed according to a Gumbel distribution with scale parameter σ_ϵ , (ii) iid across occupational choices within individuals, and (iii) iid across individuals and across time.

Government. The government balances the budget in every period. It collects taxes from entrepreneurs in the formal sector and ex-post punishment amounts from entrepreneurs in the informal

²Adding the variable cost $\theta \times n$ helps capturing the very high level of self-employment among Brazilian informal firms.

³I removed the i subscripts for ease of notation.

sector. The collected revenues are used to finance an exogenous amount of public expenditures G , which does not enter the agents' utility function.

Timing. Within each period, timing is the following:

- Productivity and taste shocks realize.
- Agents decide whether to be workers, formal, or informal entrepreneurs.
- Capital and labor decisions are made. Production occurs.
- Formal firms pay taxes to the government. Detected informal firms pay the ex-post punishment amount.
- Individuals make consumption and saving decisions given after-tax (or punishment) earnings.

Recursive Problems. The worker's problem can be written in the following recursive form:

$$\begin{aligned} V_W(z, a) &= \max_{c, a'} \{u(c) + \beta \mathbb{E}_{z'} V(z', a')\} \\ \text{s.t. } c + a' &\leq w + (1 + r)a \\ a' &\geq 0 \end{aligned} \quad (1)$$

The formal entrepreneur's problem can be written in the following recursive form:

$$\begin{aligned} V_F(z, a) &= \max_{c, a', k, n} \{u(c) + \beta \mathbb{E}_{z'} V(z', a')\} \\ \text{s.t. } c + a' &\leq \pi_F(z) + (1 + r)a \\ \pi_F(z) &= (1 - \tau_y)z(k^\alpha(\bar{\ell} + n)^{1-\alpha})^\gamma - (r + \delta)k - [w(1 + \tau_n) + \theta]n \\ a' &\geq 0 \end{aligned} \quad (2)$$

The informal entrepreneur's value function is

$$V_I(z, a) = \max_{k, n} \{(1 - p(n))V_I^{nd}(z, a, k, n) + p(n)V_I^d(z, a, k, n)\} \quad (3)$$

Where $V_I^d(z, a, k, n)$ and $V_I^{nd}(z, a, k, n)$ denote the value functions of detected and not detected entrepreneurs, respectively.

For $j = \{d, nd\}$, the recursive problem can be written as

$$\begin{aligned} V_I^j(z, a, k, n) &= \max_{c, a'} \{u(c) + \beta \mathbb{E}_{z'} V(z', a')\} \\ \text{s.t. } c + a' &\leq \pi_I^j(z) + (1 + r)a \\ \pi_I^j(z) &= z(k^\alpha(\bar{\ell} + n)^{1-\alpha})^\gamma - (r + \delta)k - (w + \theta)n - \\ &\quad \mathbb{1}_{j=d} \times [(1 + \kappa)(\tau_y z(k^\alpha(\bar{\ell} + n)^{1-\alpha})^\gamma + \tau_n wn)] \\ a' &\geq 0 \end{aligned}$$

Finally, an agent with productivity z , assets a , and taste shocks $\epsilon_W, \epsilon_I, \epsilon_F$ chooses an occupation according to

$$V(z, a) = \max\{V_W(z, a) + \epsilon_W, V_I(z, a) + \epsilon_I, V_F(z, a) + \epsilon_F\} \quad (4)$$

Since the shocks are iid and follow a Gumbel distribution with scale parameter σ_ϵ , the (ex-ante) probability of choosing occupation $o \in \{W, I, F\}$, given (z, a) is

$$q(o|z, a) = \exp\left(\frac{V_o(z, a)}{\sigma_\epsilon}\right) / \sum_{o' \in \{W, I, F\}} \exp\left(\frac{V_{o'}(z, a)}{\sigma_\epsilon}\right) \quad (5)$$

Equilibrium. Let $x = (z, a)$. Given a government policy $\{G, \tau_y, \tau_n, \kappa\}$, a detection probability function $p(\cdot)$, and a interest rate r , a *stationary competitive equilibrium* is wage $\{w\}$, allocations $c_W(x), a_W(x), c_F(x), a_F(x), k_F(x), n_F(x), k_I(x), n_I(x), c_{I,d}(x), c_{I,nd}(x), a_{I,d}(x), a_{I,nd}(x)$, occupation probabilities $q(o|x)$, and a distribution $\mu(x)$ such that:

- Given w , worker's value function $V_W(x)$ and allocations $c_W(x), a_W(x)$ solve problem (1).
- Given w, τ_y , and τ_n , formal entrepreneur's value function $V_F(x)$ and allocations $c_F(x), a_F(x), k_F(x), n_F(x)$ solve problem (2).
- Given w and $\{\tau_y, \tau_n, \kappa, p(\cdot)\}$, informal entrepreneur's value function $V_I(x)$ and allocations $c_{I,d}(x), c_{I,nd}(x), a_{I,d}(x), a_{I,nd}(x), k_I(x), n_I(x)$ solve problem (3).
- Given w and $\{\tau_y, \tau_n, \kappa, p(\cdot)\}$, occupation probabilities $q(o|x)$ are given by (5).
- Labor Market clears.

$$L = \int q(W|x) d\mu(x) = \int [n_F(x)q(F|x) + n_I(x)q(I|x)]d\mu(x)$$

- Government balances budget

$$G = \int \left[(\tau_y y_F(x) + \tau_n w n_F(x)) \times q(F|x) + p(n_I(x)) \times (1 + \kappa) (\tau_y y_I(x) + \tau_n w n_I(x)) \times q(I|x) \right] d\mu(x)$$

- The net foreign asset position is stationary

$$NFA = A - K$$

$$\begin{aligned} &= \int \left[a_W(x)q(W|x) + a_F(x)q(F|x) + [a_{I,d}(x) \times p(n_I(x)) + a_{I,nd}(x) \times (1 - p(n_I(x)))]q(I|x) \right] d\mu(x) \\ &\quad - \int [k_F(x)q(F|x) + k_I(x)q(I|x)]d\mu(x) \end{aligned}$$

- The current account surplus/deficit is stationary

$$CA = \int [y_F(x)q(F|x) + y_I(x)q(I|x)]d\mu(x) - \delta K - G - \theta L \\ - \int \left[c_W(x)q(W|x) + c_F(x)q(F|x) + [c_{I,d}(x) \times p(n_I(x)) + c_{I,nd}(x) \times (1 - p(n_I(x)))]q(I|x) \right] d\mu(x)$$

- The distribution $\mu(x)$ is stationary. Given a one-period ahead transition operator \mathcal{M}

$$\mu = \mathcal{M}(\mu)$$

Static version of the model. Appendix A.1 introduces a simplified version of the model characterized by a discrete probability of detection, which is equal to 0 if the number of employees is less than a given threshold and to 1 otherwise. This eliminates the uncertainty associated with the detection by the tax authority for informal entrepreneurs. The main mechanisms affecting average size and TFP carry over into the full model presented above.

3 Calibration

In this section, I first describe the Brazilian data used for the calibration (section 3.1). Then, I introduce the functional forms (section 3.2) and I separate model parameters in two groups: those who are externally set based on conventional or statutory values, and those who are internally calibrated. For the latter, I choose the parametrization that minimizes the distance between model and data moments (section 3.3). Finally, I show how the calibrated model performs along targeted and non-targeted dimensions (section 3.4).

3.1 Data

Data for Brazil are collected from different sources.⁴ Moments of informal firms are collected from the *Economia Informal Urbana* (ECINF), a representative survey that was lastly conducted in 2003 to collect information on the informal sector in Brazil. It contains characteristics of self-employed workers and employers running a non-agricultural business with up to five employees. Respondents are directly asked whether they possess a tax identification number for firms (*Cadastro Nacional de Pessoa Juridica* (CNPJ)). Owners who are not registered with the tax authority are classified as informal. ECINF also provides information on number of employees, total sales, capital, and debt. While ECINF displays a comprehensive picture of small formal and informal firms, the size threshold of 5 employees makes the sample not representative of the universe of formal firms. Therefore, I

⁴Appendix A.5 provides more details on how I obtain the samples from which I compute the moments used in the calibration.

integrate ECINF with statistics on formal firms taken from the *Relação Anual de Informações Sociais* (RAIS), an administrative dataset containing number and characteristics of workers as reported by formal employers.⁵

The entrepreneurship rate is the sum of self-employed and employers out of the labor force and is taken from the *Pesquisa Nacional por Amostra de Domicílios* (PNAD), an annual survey containing several characteristics of the population such as employment status, education, and income.

Finally, the share of informal output in the Brazilian economy is taken from [Elgin et al. \(2021\)](#), which provides estimates of informality rates for more than 160 countries over the period 1990-2019.⁶ The value for Brazil in 2003 is 36%.

Table 1: Data for Brazil in 2003.

Moments	Source	Data
<i>Informality</i>		
Informality/GDP	Elgin et al. (2021)	0.36
Share of informal firms	Erosa et al. (2023)	0.76 ⁷
Entrepreneurship rate	PNAD	0.32
<i>Size distribution (formal) : firms shares</i>		
1-5	RAIS	0.70
6-10	RAIS	0.14
11-20	RAIS	0.08
21-50	RAIS	0.05
<i>Size distribution (informal) : firms shares</i>		
≤ 1	ECINF	0.82
≤ 2	ECINF	0.95
≤ 5	ECINF	0.998

Source: Own calculation from ECINF and PNAD (2003), tabular data from RAIS.

⁵I do not have direct access to the RAIS micro data since it is a restricted administrative dataset. However, I can get firm size distribution moments from tabular data and from other papers' statistics (e.g., [Ulyssea \(2018\)](#), [Erosa et al. \(2023\)](#)).

⁶These estimates are obtained through national account statistics and the use of a dynamic general equilibrium model, which determines how optimizing households will allocate labor between formal and informal sectors. See Appendix A.3 for more details.

⁷This number is obtained by merging sample data from ECINF with administrative data from RAIS.

Table 1 summarizes the main data. Of particular relevance are the high rate of entrepreneurship (32%), the high share of informal firms (76%), and the fact that 95% of informal firms have two employees (including the owner) at most.

3.2 Functional Forms

I set the utility function to be $u(c_t) = \log(c_t)$.

Managerial productivity evolves according to the following AR(1) process

$$\log(z_{t+1}) = \rho \log(z_t) + \nu_{t+1}, \quad \nu \sim N(0, \sigma_\nu^2)$$

Where ρ and σ_ν denote the persistence and the standard deviation of the productivity process.

Regarding the detection probability function, I follow [Di Nola et al. \(2021\)](#) in adopting a logistic function

$$p(n) = \frac{1}{1 + p_1 \exp(-p_2 n)}$$

The two parameters to be estimated are p_1 and p_2 , which control the inflection point and the slope of the detection probability function, respectively.

3.3 Choice of Parameters

Externally set parameters. Table 2 shows the parameters that are calibrated outside the model. I set the discount factor β to .96, the capital depreciation rate δ to .05, the capital share α to .33, and the span-of-control parameter ν to .9. Tax and surcharge rates are set according to their statutory values. The sales tax τ_y is set to .293, while the payroll tax is set to .375. The output tax includes two VAT federal taxes of 20 and 9 percent, respectively, while the payroll tax includes social security contributions (20 percent), direct payroll tax (9 percent), and severance contributions (8.5 percent).⁸ Finally, the surcharge rate κ is set to the regular penalty, which is 75% of the due taxes.⁹

Internally calibrated parameters. There are 6 parameters left to be estimated: the persistence and the standard deviation of the productivity shock ρ and σ_ν , the detection probability parameters p_1 and p_2 , the entrepreneurial amount of labor entering the production function $\bar{\ell}$, and the variable cost parameter θ . Model parameters are jointly calibrated and are chosen to minimize the distance between the following model statistics and their corresponding moments in the data:

1. Informality rate (output): share of output produced by informal entrepreneurs.
2. Informality rate (firms): share of informal firms (out of all firms).
3. Share of informal self-employed (out of informal firms).
4. Share of informal firms with less than 2 employees (out of informal firms).

⁸See [Ulyssea \(2018\)](#) for more details.

⁹See [Franjo et al. \(2022\)](#).

Table 2: Externally Set Parameters.

Parameter	Description	Value	Source
β	Discount factor	0.96	Standard value
δ	Capital depreciation rate	0.05	Standard value
α	Capital share	0.33	Standard value
γ	Span-of-control	0.9	Standard value
τ_y	Sales tax	29.3%	Statutory rate
τ_n	Payroll tax	37.5%	Statutory rate
κ	Surcharge rate	75%	Statutory rate

5. Share of formal firms with less than 5 employees (out of formal firms).

6. Entrepreneurship rate: sum of self-employed and employers (out of the labor force).

It is important to highlight that parameters p_1 and p_2 are mostly informative of the share of informal output and informal firms in the economy, while ρ and σ_ν mainly affect the firm size distribution moments. On the other hand, $\bar{\ell}$ and θ mainly determine the entrepreneurship and self-employment rates. Table 3 shows the most informative moment for each internally calibrated parameter.

Table 3: Internally Calibrated Parameters.

Parameter	Description	Moment
p_1	Detection probability (intercept)	Informality rate (output)
p_2	Detection probability (slope)	Informality rate (firms)
$\bar{\ell}$	Owner labor	Entrepreneurship rate
θ	Variable cost of employees	Share informal firms ≤ 1
ρ	Persistence shock	Share informal firms ≤ 2
σ_ν	Standard deviation shock	Share formal firms ≤ 5

3.4 Calibration Results

Model fit. Table 4 shows the model fit over targeted dimensions and the associated parameter values. The model replicates well the share of informal output, the entrepreneurship rate and the informal firm size distribution moments, while it slightly underestimates the share of small (≤ 5 employees) formal firms (62% in the model compared to 70% in the data) and it overestimates the share of informal entrepreneurs (84% in the model compared to 76% in the data).

Table 4: Data and Model Statistics over Targeted Dimensions.

Parameter	Value	Target	Model	Data
p_1	244.69	Informality rate (output)	0.36	0.36
p_2	3.55	Informality rate (firms)	0.84	0.76
$\bar{\ell}$	0.17	Entrepreneurship rate	0.34	0.32
θ	0.14	Share informal firms ≤ 1	0.84	0.82
ρ	0.97	Share informal firms ≤ 2	1.00	0.95
σ_ν	0.06	Share formal firms ≤ 5	0.62	0.70

Table 5 displays how the model performs over some dimensions that are not targeted. These variables are the informality rates for self-employed and small firms (≤ 5 employees), additional moments of the formal firm size distribution, including average firm size, and some public finance statistics.¹⁰ Despite these variables are not targeted in the calibration, the model replicates them fairly well.

Table 5: Data and Model Statistics over Untargeted Dimensions.

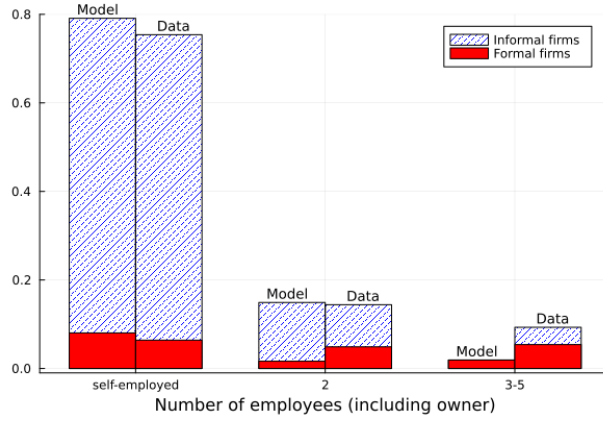
Moment	Model	Data
Informality rate among self-employed	0.90	0.91
Informality rate among firms ≤ 5	0.90	0.86
Share formal firms 6 – 10	0.12	0.14
Share formal firms 11 – 20	0.16	0.08
Share formal firms 21 – 50	0.08	0.05
Share formal firms 50+	0.02	0.03
Average firm size	2.08	2.39
Sales tax/GDP	0.20	0.17
(Sales tax + Income tax)/GDP	0.20	0.24
Payroll tax/GDP	0.07	0.07

How does the model replicate the informality rates and the shares of small firms across different size categories? Figure 2 depicts the shares of formal and informal firms for self-employed, firms with 2 employees (including the owner), and firms with 3-5 employees in the model (left columns) and in the data (right columns). While the model replicates well the informality rate for self-employed (90% v. 91% in the data), it overestimates it for firms with 2 employees (89% v. 66%) and it

¹⁰As pointed out by [Erosa et al. \(2023\)](#), it seems more correct to compare sales tax revenues in the model to the sum of sales and income tax revenues in the data, since there is no income tax in the model.

underestimates it for firms with 3-5 employees (0% v. 42%).

Figure 2: Informality Rates for Small Firms.



Source: Data from ECINF.

Further model implications. To conclude the model fit analysis, I present further statistics that do not have an exact analogous in the data but that replicate well-known patterns of Brazilian and other developing countries' economies.

Figure 3: Probability of Detection for Different Levels of Employment.

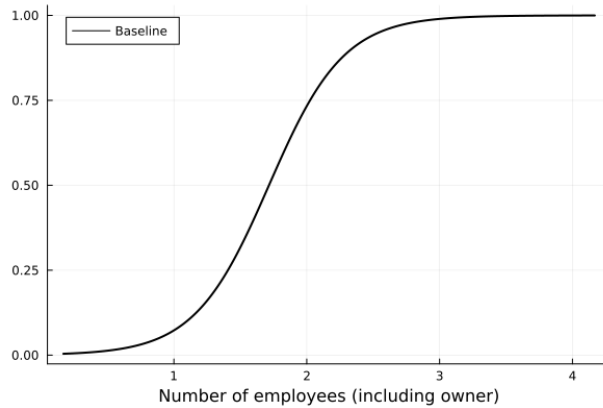


Figure 3 shows the relationship between the number of employees and the probability of detection from the tax authority. The detection probability for a self-employed agent is between 1% and 7%¹¹, whereas it dramatically increases as employment goes up. In fact, the detection probability increases up to 73% for a firm with 2 employees and to 100% for a firm with 5 employees. This is consistent with the fact that about 95% of informal firms have two employees (including the owner)

¹¹Notice that number of employees is a continuous variable in the model, therefore the detection probability of a self-employed agent is not constant. In the model, a self employed agent is defined as such if $\bar{\ell} + n \leq 1$.

or less in the Brazilian data.

Figure 4 shows the share of formal (red solid line) and informal (blue dashed line) entrepreneurs over different levels of the productivity distribution. The dotted black line represents the overall share of entrepreneurs. For each share, the denominator is the total number of agents in the economy. The graph points out that informality is the preferred choice for entrepreneurs with low levels of productivity, but this preference is reversed as we move to the right of the distribution.

Figure 4: Share of Formal, Informal, and Entrepreneurs over the Productivity Distribution.

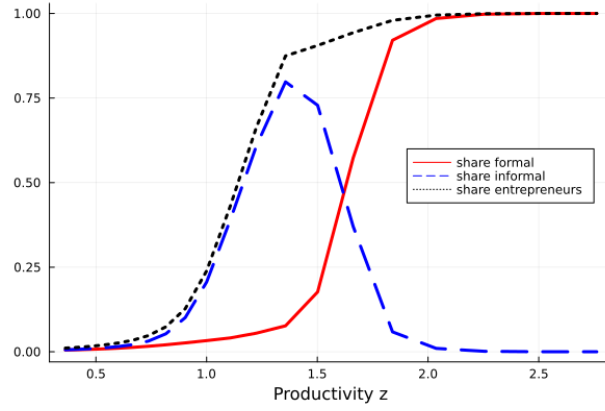
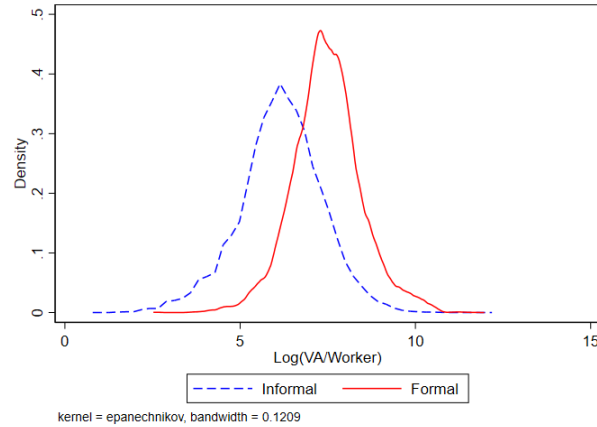


Figure 5: Kernel Distribution of Productivity for Small (≤ 5) Formal and Informal Firms.



Productivity is measured as value added per worker (in logs). Source: Own calculation from ECINF.

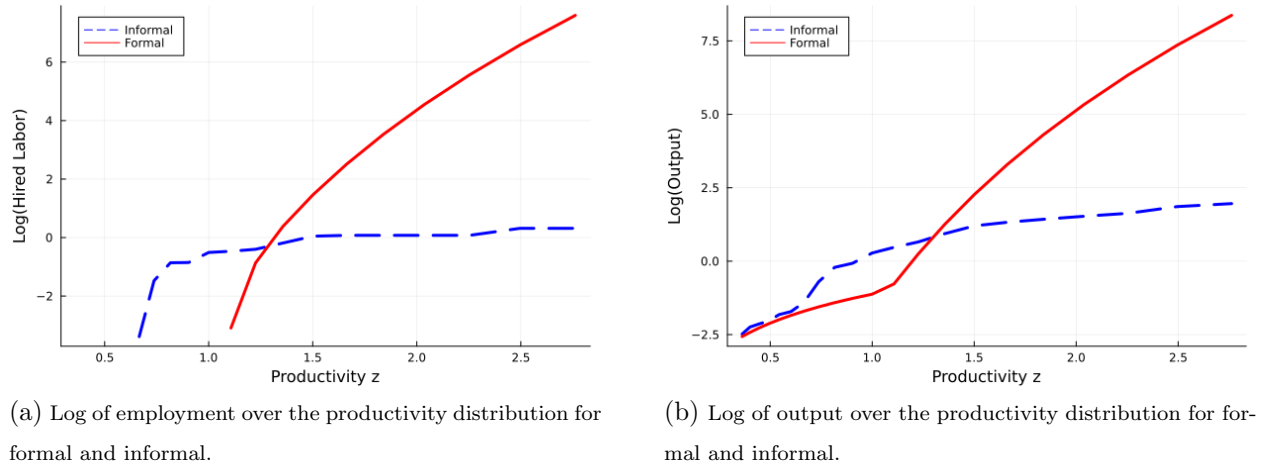
The graph is consistent with two patterns observed in the data: (i) more productive agents tend to be entrepreneurs rather than workers, and (ii) formal entrepreneurs are on average more productive than informal entrepreneurs. The latter pattern is observed in cross-country data¹² and in Brazilian

¹²For example, [La Porta and Shleifer \(2014\)](#) highlight the relationship between formality status and productivity across several countries using World Bank Enterprise Surveys data.

data as well. In fact, Figure 5 depicts the productivity distributions of formal (red solid line) and informal (blue dashed line) firms with at most 5 employees in the ECINF data. Productivity is approximated as value added per worker.

Finally, Figure 6 shows optimal labor and output policies for formal and informal entrepreneurs. In addition to the composition effect in productivity already described, informal entrepreneurs hire fewer employees compared to formal entrepreneurs with the same level of productivity (panel a). The reason for this behavior is to avoid detection from the tax authority. Consequently, the optimal choice of output is also lower for a given level of productivity (panel b).¹³

Figure 6: Employment and Output for Different Productivity Levels.



4 Quantitative Experiments

In this section, I perform some counterfactual experiments to assess the effects of tax enforcement on the variables of interest. The purpose of this exercises is to estimate how much of the variation in tax enforcement, measured indirectly through the variation in informality, can account for the observed differences in output, TFP, and average firm size. In the experiments, I change the parameter p_1 of the detection probability function to match other countries' share of informality. I also adjust the sales tax rate τ_y to keep total revenues as close as possible to the baseline amount. All the remaining parameters are left unchanged. Therefore, we can interpret the counterfactual economy as a version of the Brazilian economy characterized by a counterfactual tax enforcement.

¹³This is not true for a few low productivity values. At these values, formal firms have to pay (distortionary) taxes, while informal firms do not and face a probability of detection that is close to 0. However, this effect tends to be quantitatively small in the aggregate, given the low mass of formal firms at these productivity levels.

4.1 Main Counterfactual

The main counterfactual matches the informality rate of a GDP-weighted average of selected Latin American countries, namely, Argentina, Chile, Colombia, Mexico, Peru, and Venezuela. The last column in Table 6 shows the averages statistics for the counterfactual economy. The informality rate stands at around 30%, 6 percentage points lower than in Brazil. GDP per capita and TFP are about 32% and 6% higher than in Brazil. Finally, average firm size is 3.45, compared to 2.39 in the baseline.

In order to match the lower informality rate, the parameter p_1 , which controls the inflection point of the detection probability function, goes down from a value of 245 to a value of 184. Figure 7 shows the probability of detection function as a function of number of employees in the baseline (black solid line) and in the counterfactual economy (gray dashed line). The probability of detection for a self-employed and for a firm with two employees are as high as 9% and 79%, respectively, compared to 7% and 73% in the baseline.

Figure 7: Probability of Detection for Different Levels of Employment, Baseline and Counterfactual.

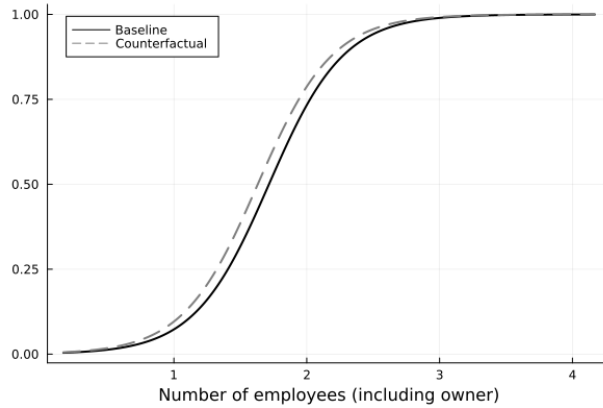


Table 6 points out that a stricter tax enforcement results in increased output, TFP, and average firm size.

Compared to the baseline, output increases by 4.3 percentage points, which accounts for 13% of the measured difference in GDP per capita in 2003. TFP also goes up by about 0.5 percentage points, which is approximately 9% of the measured difference in TFP in 2003.¹⁴ Average firm size increases from 2.08 in the baseline to 2.37 in the counterfactual. [Bento and Restuccia \(2021\)](#) estimate average firm size to be 2.39 in Brazil, while the figure for the GDP-weighted average of the six countries considered is 3.45. Therefore, the model with counterfactual stricter tax enforcement accounts for almost 30% of the estimated differences in average firm size.

In this experiment, I change the sales tax rate to $\tau_y = 0.268$ ($\tau_y = 0.293$ in the baseline) to maintain total revenues close to the baseline amount. Therefore, the experiment suggests that a stricter tax

¹⁴GDP and TFP data are from [Feenstra et al. \(2015\)](#). In the model, TFP is measured according to the formula $TFP = \frac{Y}{(K^\alpha (L + s_e \ell)^{1-\alpha})^\gamma}$ where s_e is the share of entrepreneurs in the population.

Table 6: Matching Counterfactual’s Weighted Average Informality Rate - Counterfactual Moments. Share Accounted for by the Model in Brackets.

Variable	Brazil		Counterfactual	
	Model	Data	Model	Data
Informality rate	0.365	0.360	0.296	0.298
GDP per capita	1.000	1.000	1.043 [13.2%]	1.325
TFP	1.000	1.000	1.005 [8.8%]	1.062
Average firm size	2.080	2.390	2.374 [27.5%]	3.450

The counterfactual economy’s data are computed as GDP-weighted averages of Argentina, Chile, Colombia, Mexico, Peru, and Venezuela. Data sources: own calculations based on [Elgin et al. \(2021\)](#) (informality), [Feenstra et al. \(2015\)](#) (GDP per capita and TFP), and [Bento and Restuccia \(2021\)](#) (average firm size). Data are for 2003 except for average firm size.

enforcement would not only reduce the level of informality, but also allow to lower taxes that are inefficiently high for the most productive agents in the economy.¹⁵

4.2 Other Countries

This section describes additional quantitative experiments taking different countries or group of countries as counterfactuals. Besides considering single countries such as Argentina and Chile, I also perform an exercises that takes GDP-weighted average informality of a larger group of Latin American countries as counterfactual.¹⁶ As for the previous exercise, the detection probability parameter p_1 is changed to match the correspondent level of informality for each country or group. I then compare model-implied GDP-weighted averages of GDP per capita, TFP, and average firm size with their measured counterpart.

Results of the counterfactual experiments are shown in Table 7. The top half of the table displays measures of informality, GDP per capita, TFP, and average firm size for each country and group of countries. The bottom half of the table presents the corresponding model-implied statistics. The percentages in brackets denote the shares of the measured differences in the variable of interest that can be accounted for by the model. The table points out that tax enforcement differences can account for a relatively large fraction of the observed differences in average firm size, from a minimum of 21.0% (Group 2) to a maximum of 71.5% (Mexico). On the other hand, the effects on

¹⁵It has been documented that several developing economies are characterized by the coexistence of high informality rates and high tax rates (for the minority of firms who pay taxes).

¹⁶The group includes 17 countries: Argentina, Bolivia, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, Guatemala, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Uruguay, Venezuela.

Table 7: Effects of Tax Enforcement. Data and Model Statistics, Share Accounted for by the Model in Brackets.

Country	Informality (1)	GDP pc (2)	TFP (3)	Average Size (4)
<i>Data</i>				
Brazil (<i>Baseline</i>)	0.36	1.000	1.000	2.39
Group 1 (<i>Main</i>)	0.30	1.325	1.062	3.45
Group 2	0.31	1.248	1.051	3.42
Argentina	0.23	1.334	1.104	4.14
Chile	0.19	1.307	1.050	5.67
Mexico	0.29	1.567	1.187	2.83
Peru	0.55	0.633	0.675	2.45
<i>Model</i>				
Brazil (<i>Baseline</i>)	0.36	1.000	1.000	2.08
Group 1 (<i>Main</i>)	0.30	1.043[13.2%]	1.005[8.8%]	2.37[27.5%]
Group 2	0.31	1.034[13.9%]	1.004[7.9%]	2.30[21.0%]
Argentina	0.23	1.097[29.1%]	1.012[11.1%]	2.82[42.1%]
Chile	0.19	1.115[37.6%]	1.013[26.4%]	3.04[29.2%]
Mexico	0.29	1.047[8.3%]	1.006[3.3%]	2.40[71.5%]
Peru	0.55	0.919[22.1%]	0.975[7.8%]	1.69[-]

Data sources: Informality rates are from [Elgin et al. \(2021\)](#). GDP per capita and TFP are from [Feenstra et al. \(2015\)](#). Average firm size are from [Bento and Restuccia \(2021\)](#). Data are for 2003 except for average firm size.

Group 1: GDP-weighted average of Argentina, Chile, Colombia, Mexico, Peru, Venezuela. Group 2: GDP-weighted average of Argentina, Bolivia, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, Guatemala, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Uruguay, Venezuela.

GDP per capita and TFP appear to account for a smaller fraction, spanning from a minimum of 8.3% and 3.3% (Mexico) to a maximum of 37.6% and 26.4% (Chile).

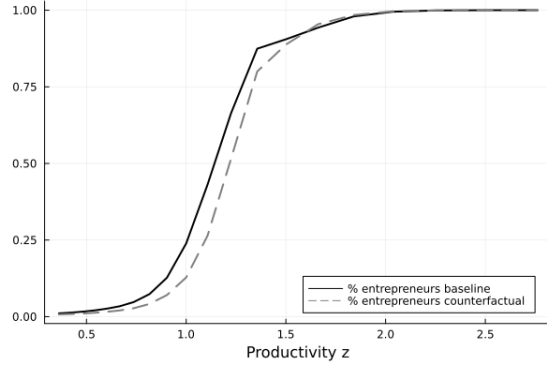
The low effect on TFP requires further discussion. It is important to remember that the only parameters that differ across experiments are the detection probability parameter p_1 and the sales tax rate τ_y , while all the other parameters have the same value as in the baseline Brazilian economy. Importantly, the invariant distribution of the productivity process is kept constant across experiments. Therefore, differences in aggregate productivity can only stem from heterogeneity in occupational choices and in capital and labor policy decisions. It can be argued that at least part of the observed differences in TFP is due to differences in the distribution of skills that cannot be accounted for by the model.

4.3 Economic Mechanisms

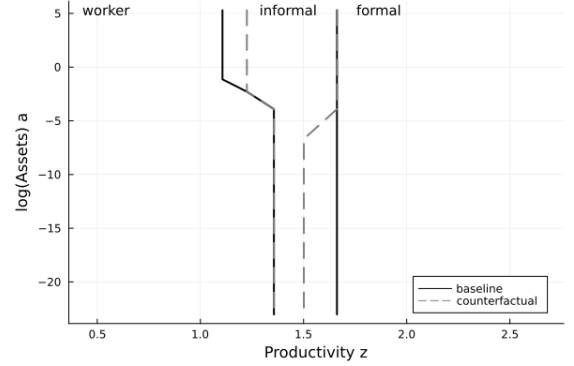
What are the underlying sources of these differences between the baseline and the counterfactual economies? To answer this question, it is useful to compare the occupational choices and the policy functions in the two economies. For illustration purposes, I will compare occupational and policy functions in the baseline and in the counterfactual where tax enforcement is such that it matches the Chilean informality rate (19%).¹⁷ In the following graphs, the solid lines represent the values for the baseline economy, while the dashed lines represent the counterfactual values.

Figure 8 panel (a) shows that the share of entrepreneurs in the counterfactual economy is significantly lower for low values of productivity.

Figure 8: Share of Total and Informal Entrepreneurs for Different Productivity Levels.



(a) Share of entrepreneurs (formal and informal) over the productivity distribution.



(b) Occupational choices for different levels of productivity z (x-axis) and log-assets a (y-axis).

Therefore, stricter tax enforcement leads to higher average entrepreneurial ability, inducing in turn a positive effect on average firm size given the positive relationship between firm-level productivity and optimal scale. Panel (b) displays occupational choices for different levels of productivity z (x-axis) and log-assets a (y-axis). The lines demarcate three areas, which correspond to combinations of productivity and assets for which it is optimal to either be workers, informal entrepreneurs, or formal entrepreneurs.¹⁸ Notice that, for some values of productivity, agents with sufficient assets prefer to become informal entrepreneurs, while agents with low assets choose to be workers or formal entrepreneurs. This is a consequence of the uncertainty in the detection by the tax authority. In other words, for some productivity values, the expected earnings are higher if agents decide to be informal entrepreneurs, but they only choose so if they have enough assets. The graph shows not only that the share of entrepreneurs is lower in the counterfactual, but also that the formality rate among entrepreneurs is higher. This is caused by a combination of stricter detection probability and lower tax rates. Taken together, the two panels suggest that informal entrepreneurs reallocate

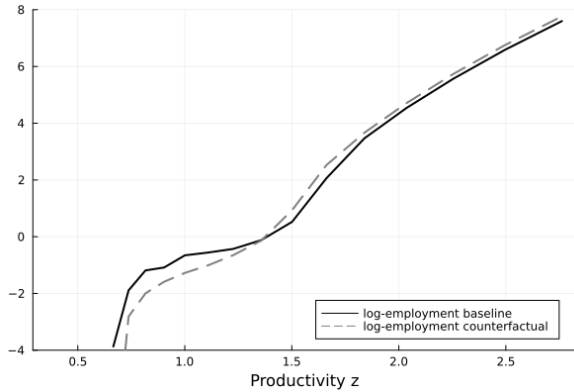
¹⁷In this calibrated economy, $p_1 \approx 90$ and $\tau_y = 0.228$.

¹⁸In reality, the presence of occupational shocks leads to a probability distribution over occupational choices for each combination of productivity and assets. Therefore, the graph represents the choice with the highest probability.

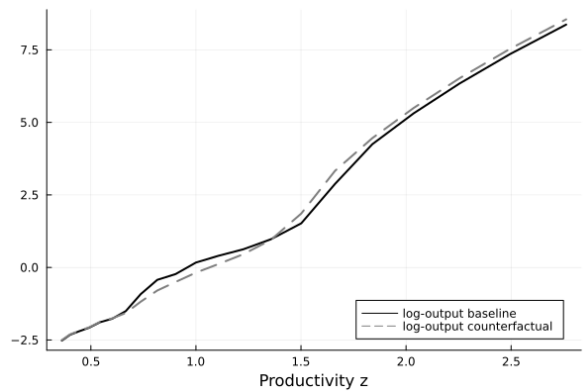
either as workers or as entrepreneurs in the formal sector.

The reallocation effect can also be observed in Figure 9, which depicts the average log-employment and log-output (conditional on being entrepreneurs) as a function of productivity in the two economies. Notice that employment and output are lower in the counterfactual economy for low values of productivity. There are two reasons for this pattern. First, for very low levels of productivity, formal firms tend to be smaller than informal firms. This is a consequence of the fact that small formal firms have to pay taxes, while small informal firms do not and face a detection probability which is close to 0. Second, as pointed out in Figure 7, for a given level of employment informal firms face a higher detection probability in the counterfactual economy. Therefore, if they choose informality in the counterfactual economy they might choose a lower scale to reduce the detection probability. However, these effects are quantitatively small in the aggregate and are more than compensated by higher values of labor and output for higher levels of productivity, as shown by the dashed line being on top of the solid line for both employment and output. The graphs point out to a reallocation of labor and output from low-productivity agents to high-productivity agents, which in turn leads to an increase in TFP and average firm size.

Figure 9: Employment and Output for Different Productivity Levels.



(a) Log of employment over the productivity distribution.



(b) Log of output over the productivity distribution.

4.4 Decomposing Selection and Detection Channels

While in section 4.3 I presented some qualitative analysis of the channels through which tax enforcement affects the outcomes of interest, in this section I quantitatively assess how much the two main channels - selection and detection channel - contribute to the overall estimated impact. To decompose the effect, I shut down one channel at a time, and then compare TFP, output, average firm size, and other statistics when only some decisions are allowed to change. The counterfactual considered is the GDP-weighted average of six Latin American countries (*group 1*).

Table 8 displays the decomposition. Since the invariant distribution $\mu(z, a)$ is different in the

Table 8: Decomposition of Selection and Detection Channels.

	Baseline				Counterfactual
	(1)	(2)	(3)	(4)	(5)
Fixed decisions	-	ℓ, k, sector	% entrepr.	-	-
Distribution $\mu(z, a)$	-	Baseline	Baseline	Baseline	Counterfactual
Channels	-	Selection	Detection	Sel.+Det.	Sel.+Det.+Distr.
<i>Outcome</i>					
Output	1.000	0.982	1.078	1.061	1.043
Capital	4.54	4.44	4.98	4.88	4.78
Labor	0.66	0.65	0.71	0.70	0.69
TFP	1.000	1.000	1.004	1.007	1.005
Average firm size	2.08	2.16	2.24	2.33	2.37
Entrepreneurship	0.34	0.32	0.34	0.32	0.31
Informality (Firms)	0.84	0.84	0.82	0.81	0.82
Informality (Output)	0.36	0.35	0.31	0.30	0.30

Column 1: baseline. Column 2: fixed capital and labor decisions, change in occupational choices allowed (*selection*). Column 3: fixed occupational choices, changes in capital and labor decisions allowed (*detection*). Column 4: sum of both effects keeping the same distribution $\mu(z, a)$ as in the baseline. Column 5: distribution adjusted (main counterfactual).

counterfactual, I first fix the distribution as in baseline in columns (2) to (4), while I adjust it to the counterfactual distribution in column (5). The selection effect in column (2) is obtained by maintaining the capital and labor decisions and the share of informality for every productivity level as in the baseline while allowing the share of entrepreneurs to change as in the counterfactual. Since the number of entrepreneurs goes down and production for the remaining entrepreneurs remains constant, output mechanically decreases. Despite an increase in average entrepreneurial ability, TFP does not seem to be affected by the selection channel. On the other hand, average firm size increases from 2.08 to 2.16.

Column (3) displays the contribution of the detection channel, which is obtained by maintaining the same share of entrepreneurs as in the baseline while allowing changes in the decisions of capital, labor, and in which sector to operate. The decrease in informality over the firm-level productivity distribution induces an overall increase in aggregate capital, labor, and output. Moreover, the reallocation of resources towards more productive and larger firms implies an increase in TFP.

Column (4) shows the sum of the two effects while keeping the same invariant distribution as in the baseline. Two properties stand out. First, the rise in output due to the detection channel more than compensate the negative impact of the selection channel. Second, the interaction between the two channels seems to produce a positive effect on TFP and average firm size that is larger than

the sum of the two channels taken in isolation.

Finally, in column (5) I let the distribution to vary to get the counterfactual statistics already discussed in Table 6.

5 Conclusion

This paper develops a quantitative framework to assess the impact of tax enforcement on the firm size distribution and TFP. As tax enforcement becomes stricter, fewer agents choose to be entrepreneurs, and fewer entrepreneurs choose to operate in the informal sector. These mechanisms lead to a rise in average firm size and an improvement in allocative efficiency. Counterfactual analysis suggests that stricter tax enforcement in Brazil would result in sizable gains in average firm size, GDP per capita, and TFP. Considering several counterfactual experiments, the gains account for about 25-30% of the estimated differences in average firm size and for about 8-15% of the estimated differences in GDP per capita and TFP.

Therefore, this paper identifies tax enforcement as a relevant source of the observed cross-country differences in average firm size, TFP and GDP per capita.

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A Appendix

A.1 Simple Version of the Model

It is useful to consider a simplified static version of the model to highlight the main mechanisms that carry over into the dynamic model presented in section 2. In this version, owner's labor $\bar{\ell}$ is set equal to 0 and the occupational shocks are shut down. Moreover, the probability of detection is a discrete function: it jumps from 0 to 1 if the number of employees is larger than a threshold \bar{n} .

$$p(n) = \begin{cases} 0 & \text{if } n \leq \bar{n} \\ 1 & \text{if } n > \bar{n} \end{cases}$$

This last assumption eliminates the uncertainty regarding detection from the tax authority. As a consequence, agents choose the occupation that gives them the highest earnings. As long as \bar{n} is high enough, there will be two productivity thresholds z_1 and z_2 such that:

- (a) Each agent with $z \leq z_1$ is a worker.
- (b) Each agent with $z_1 < z \leq z_2$ is an informal entrepreneur.
- (c) Each agent with $z > z_2$ is a formal entrepreneur.

The intuition behind this result is the following. First, notice that labor earnings w do not depend on productivity z . On the other hand, informal entrepreneur's profits $\pi_I(z)$ are increasing in z and equal to:

$$\pi_I(z) = \begin{cases} \gamma^{\frac{\gamma}{1-\gamma}} (1-\gamma) z^{\frac{1}{1-\gamma}} \left(\frac{\alpha}{r+\delta} \right)^{\frac{\gamma\alpha}{1-\gamma}} \left(\frac{1-\alpha}{w+\theta} \right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} & \text{if } n_I(z) < \bar{n} \\ \gamma^{\frac{\gamma\alpha}{1-\gamma\alpha}} (1-\gamma\alpha) z^{\frac{1}{1-\gamma\alpha}} \left(\frac{\alpha}{r+\delta} \right)^{\frac{\gamma\alpha}{1-\gamma\alpha}} \bar{n}^{\frac{\gamma(1-\alpha)}{1-\gamma\alpha}} - (w+\theta)\bar{n} & \text{if } n_I(z) = \bar{n} \end{cases}$$

where

$$n_I(z) = \min \left\{ \bar{n}, (\gamma z)^{\frac{1}{1-\gamma}} \left(\frac{\alpha}{r+\delta} \right)^{\frac{\gamma\alpha}{1-\gamma}} \left(\frac{1-\alpha}{w+\theta} \right)^{\frac{1-\gamma\alpha}{1-\gamma}} \right\}$$

Therefore, z_1 is the productivity value such that $\pi_I(z_1) = w$.

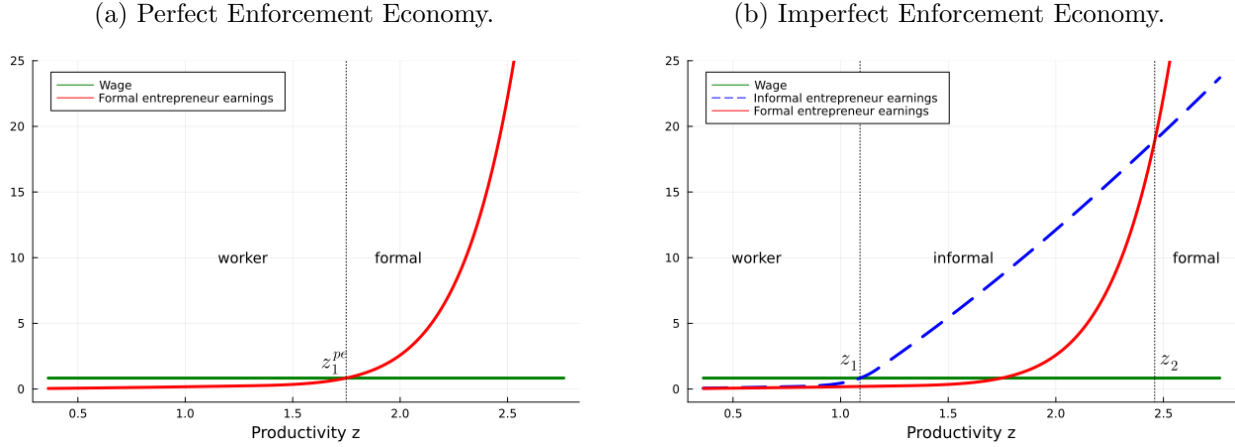
Formal entrepreneur's profits $\pi_F(z)$ are also increasing in z and equal to:

$$\pi_F(z) = \gamma^{\frac{\gamma}{1-\gamma}} (1-\gamma) ((1-\tau_y)z)^{\frac{1}{1-\gamma}} \left(\frac{\alpha}{r+\delta} \right)^{\frac{\gamma\alpha}{1-\gamma}} \left(\frac{1-\alpha}{w(1+\tau_n)+\theta} \right)^{\frac{\gamma(1-\alpha)}{1-\gamma}}$$

The profits equations point out that $\pi_F(z) < \pi_I(z)$ for each value of z such that $n_I(z) \leq \bar{n}$, because informal entrepreneurs do not pay taxes. However, since $\frac{1}{1-\gamma} > \frac{1}{1-\gamma\alpha}$, as productivity z grows, the profit function for formal entrepreneurs increases at a faster rate than the profit function for informal entrepreneurs with $n_I(z) = \bar{n}$. In fact, formal entrepreneurs can expand their scale and choose the optimal level of capital and labor. Therefore, there exists z_2 such that $\pi_I(z_2) < \pi_F(z_2)$.

I now perform a comparative statics exercise between two economies that differ in their detection probability parameter \bar{n} . First, I consider a *perfect enforcement* economy characterized by the absence of an informal sector since $\bar{n} = 0$. In this economy, there is a productivity threshold z_1^{PE} such that all agents with $z \leq z_1^{PE}$ are workers and all agents with $z > z_1^{PE}$ are entrepreneurs.

Figure 10: Earnings and Occupational Choices for Different Productivity Levels.



Labor earnings (green line), formal entrepreneurs profits (red solid line) and informal entrepreneurs profits (blue dashed line).

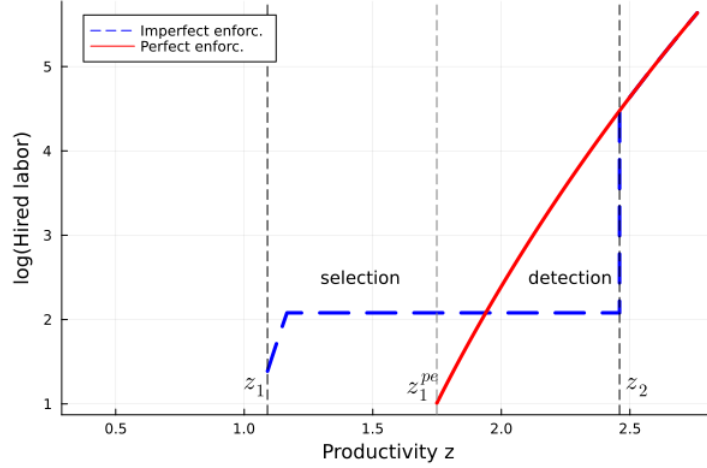
Figure 10 panel (a) depicts labor earnings (green line) and profits (red line) over the productivity distribution. The threshold z_1^{PE} is the value at which entrepreneurial profits are equal to the wage rate w . All the agents above the threshold choose to be (formal) entrepreneurs.

In an *imperfect enforcement* economy with $\bar{n} > 0$ high enough, however, an equilibrium is characterized by two productivity thresholds. Figure 10 panel (b) depicts labor earnings (green solid line), profits for formal (red solid line) and informal entrepreneurs (blue dashed line) in the imperfect enforcement economy. The threshold z_2 is the threshold above which agents decide to be formal entrepreneurs.

How is labor allocated among agents in the two economies? Figure 11 plots the log of labor hired by each agent in the perfect enforcement economy (red solid line) and in the economy with informality (blue dashed line). The graph highlights the two main mechanisms. First, in the economy with informality more agents decide to become entrepreneurs, since the possibility of not paying taxes increases the value of entrepreneurship. Given the positive relationship between firm-level productivity and size, this *selection channel* leads to a decrease in both average firm size and average productivity.¹⁹ Second, compared to the perfect enforcement economy, some entrepreneurs operate at lower scale and choose to be small enough not to pay taxes (*detection channel*). This induces a drop in average firm size, aggregate labor demand, and output.

¹⁹However, the selection channel partly offsets the negative effect on output stemming from the detection channel.

Figure 11: Hired Labor over the Productivity Distribution in the Two Economies.



Log of hired labor over productivity under perfect enforcement (red solid line) and imperfect enforcement (blue dashed line). The wage rate w is kept constant in the two economies (partial equilibrium).

A.2 Solution Method

This section describes the method used to get the value and policy functions introduced in Section 2. The recursive problem of a worker with productivity z and asset a is

$$\begin{aligned} V_W(z, a) &= \max_{c, a'} u(c) + \beta \mathbb{E}V(z', a') \\ \text{s.t. } c + a' &= (1 + r)a + w \quad a' \geq 0 \end{aligned}$$

Where V is the value function of an agent before the employment decision is made.

First order conditions imply

$$u'(c) = \beta \mathbb{E}V_a(z', a') \quad (6)$$

I can get an update of the value function V_W by using the endogenous grid method: (i) start with an exogenous asset grid over a' , (ii) invert the Euler equation (6) and get an endogenous consumption grid, (iii) use the budget constraint to get an endogenous asset grid a_e , (iv) interpolate over a' values to get the asset policy function a_p and the expected value function $\mathbb{E}V(z', a_p)$.

However, because of the discrete employment decision, V is in general not concave and the dynamic programming is non-convex. The Euler equation (6) is therefore necessary but not sufficient. As a consequence, the endogenous asset grid a_e might not be monotone, and there might be regions in which for one value of a_e there are multiple values of a' . To solve this issue, an additional step is needed compared to the standard endogenous grid method. This step is called *upper envelope*, which consists in computing the value function in the region when there are multiple a' for a given a_e , pick the (a_e, a') pair that returns the highest value function, and discard the other pairs (Fella (2014), Iskhakov et al. (2017)). I can get $V_F(z, a)$ and $V_I(z, a)$ in a similar fashion.

Now that I have $V_o(z, a)$ for $o = \{W, F, I\}$, and since the taste shock is iid EV1, I can compute $V(z, a)$ using the logsum formula:

$$V(z, a) = \sigma \log \left(\sum_{o=\{W, F, I\}} \exp \left(\frac{V_o(z, a)}{\sigma} \right) \right) \quad (7)$$

The algorithm can be summarized through the following steps:

1. Start with a guess $V(z, a')$ over an exogenous grid on z and a' .
2. Compute $\beta \mathbb{E}V_a(z', a')$ (using approximate derivatives in the first iteration, or the envelope condition in subsequent iterations).
3. Get $a_e(z, a)$ and $c_e(z, a)$ using the worker's first order conditions and budget constraint.
4. Interpolate the asset policy function, which is currently on the endogenous grid a_e , on the exogenous asset grid a' and get the asset policy a_p . Interpolate the next period expected value function on the optimal asset policy as well and get $\beta \mathbb{E}V(z', a_p)$.
5. *Upper envelope step*: For a given a' , there might be multiple a_e and, therefore, multiple a_p and $V_W(z, a')$. Keep the maximum $V_W(z, a')$ and the related asset/consumption policies, discard the others.
6. get $V_F(z, a')$ and $V_I(z, a')$ in a similar fashion as in steps 3-5.
7. Use the logsum formula (7) to get $V(z, a')$.
8. Check convergence. If convergence is not satisfied, go back to step 2 using the updated $V(z, a')$.

A.3 Cross-Country Measures of Informality

To use measures of informality that are consistent across countries and over time, I rely on the database in [Elgin et al. \(2021\)](#), which provides estimates of informality rates for more than 160 countries over the period 1990-2019.

[Elgin et al. \(2021\)](#) introduce two time series who stem from different methods and assumptions. The first method, whose associated database I employ in the paper for cross-country comparisons, is based on a *dynamic general equilibrium* (DGE) model developed by [Elgin and Öztunali \(2012\)](#). In the model, a representative household decides how much labor to allocate to the formal and to

the informal sector.

$$\begin{aligned}
& \max_{\{C_t, X_t, N_{St}, N_{Ft}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t) \\
& \text{s.t. } C_t + X_t = (1 - \tau_t) \theta_{Ft} K_t^\alpha N_{Ft}^{1-\alpha} + \theta_{St} N_{St}^\gamma \\
& K_{t+1} = X_t + (1 - \delta) K_t \\
& N_{St} + N_{Ft} = H_t
\end{aligned}$$

where N_{Ft} , N_{St} are the amounts of hours the household devotes to the formal and informal sector, respectively.

The various parameters and variables are either externally set according to conventional values, or retrieved by using first order conditions and national accounts data, or calibrated. An additional assumption is needed to derive θ_{St} , the productivity growth rate in the informal sector: it is assumed to be the average between the productivity growth in the formal sector θ_{Ft} , and the growth rate of capital K_t . Notice that the procedure allows to determine the growth rate of the informal sector, not its absolute value. Therefore, for each country the share of informal output over GDP in 2007 is set to be equal to the shadow economy size as estimated in [Schneider et al. \(2010\)](#) for the same year.

The *Multiple Indicators Multiple Causes* (MIMIC) in [Schneider et al. \(2010\)](#) constitutes the source of the second database in [Elgin et al. \(2021\)](#). The method consists of a simultaneous specification of a factor (measurement) model and a structural model. The main idea is to retrieve an unobserved variable using some structural equations and the sample covariances between observed variables.

The structural model can be written as

$$\eta = \gamma' X + \xi$$

where η is the unobserved informal economy over GDP ratio, and X is a vector of explanatory variables (causes) such as the share of direct taxes over overall taxes, government expenditures over GDP, and fiscal freedom, business freedom, and regulatory indices.

The measurement model can be written as

$$y = \lambda \eta + \epsilon$$

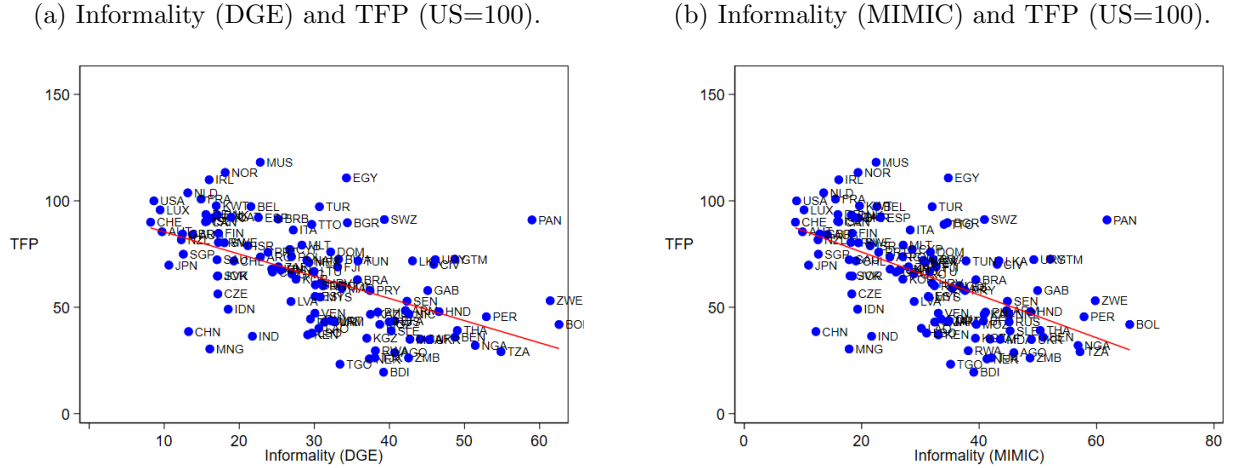
where y is a vector of indicator variables such as currency (M0 or M1), GDP per capita growth, and labor force participation. By combining the two equations, one can get a relationship between causes X and indicators y and the related covariance matrix. To derive the coefficients needed to infer the size of the informal economy η , the authors use maximum likelihood in order to minimize the distance between theoretical and sample covariances. This second method also allows to compute the relative size of the shadow economy, not the absolute size. Therefore, estimates based on a currency approach for the year 2000 are used as base year.

A.4 Cross-Country Relationship between Informality and TFP

Table 9 shows the world averages of informal share of GDP and TFP from 1990 to 2018. Using the DGE based measure of informality as a benchmark (column 1), the table highlights that the informal share of GDP has decreased by about 8 percentage points from 34.8 to 26.8 in the period considered.²⁰ In the same period, TFP has increased by about 6 percentage points on average (column 4). Column (3) displays the average estimate of TFP as a percentage of US TFP. This is a measure used for comparisons of TFP across countries at a given point in time.

Figure 12 depicts the cross-country correlation between informality and TFP. The left panel shows the DGE-based measure, while the right panel displays the MIMIC-based measure. TFP is measured as a percentage of US TFP. The two panels point out a negative correlation between TFP on the one hand and both measures of informality on the other hand.

Figure 12: Correlation between Informality and TFP across Countries.



Data sources: [Elgin et al. \(2021\)](#) for informality, [Feenstra et al. \(2015\)](#) for TFP. Each data point represents the country's averages of informality and TFP over the period between 1990 and 2018.

To further assess the relationship between informality and TFP, I perform a series of cross-country regressions. First, I run an OLS regression according to the following specification:

$$TFP_{it} = \alpha_0 + \alpha_1 INF_{it} + \gamma GDP_{it} + Year_t + \epsilon_{it}$$

where TFP_{it} is TFP in country i in year t (relative to US TFP in year t), INF_{it} is the share of informal output, GDP_{it} is GDP per capita, $Year_t$ denotes year fixed effects, and ϵ_{it} is the error term. α_1 is the coefficient of interest in the relationship between informality and TFP. Since the two series of informality are highly correlated, I adopt the DGE-based measure, which is the one used in the quantitative experiments in the paper.

²⁰Informality has also decreased according to the MIMIC measure of informality, but by about 3 percentage points (column 2).

Table 9: Informality and TFP. Simple Averages.

Year	% Informal GDP (DGE) (1)	% Informal GDP (MIMIC) (2)	TFP (US=100) (3)	TFP (2017=100) (4)
1990	34.8	.	72.4	94.4
1991	34.7	.	71.0	93.1
1992	34.3	.	69.9	93.8
1993	34.3	34.7	68.7	93.6
1994	34.1	34.6	65.4	88.8
1995	34.0	34.5	65.5	88.8
1996	33.8	34.3	65.7	89.5
1997	33.6	34.2	65.2	90.4
1998	33.4	34.2	63.5	90.3
1999	33.2	34.2	63.3	90.3
2000	33.1	34.0	65.3	91.1
2001	32.9	34.0	65.0	91.6
2002	32.7	34.1	65.3	92.6
2003	32.5	34.0	64.8	93.7
2004	32.3	33.7	65.1	96.2
2005	32.2	33.6	67.2	97.5
2006	31.9	33.2	68.0	99.2
2007	31.6	32.9	68.4	100.4
2008	31.3	32.8	68.0	99.9
2009	30.9	33.4	64.8	97.8
2010	30.7	33.0	64.7	99.3
2011	30.4	32.9	66.8	100.1
2012	30.0	32.8	67.4	100.4
2013	29.7	32.7	66.9	100.4
2014	29.4	32.6	65.9	100.3
2015	29.1	32.5	63.8	99.7
2016	28.9	32.4	63.7	99.6
2017	28.7	32.1	64.4	100.0
2018	26.8	31.9	64.0	99.9

World averages. Sources: Informality data are from [Elgin et al. \(2021\)](#). TFP data are from [Feenstra et al. \(2015\)](#).

Second, I exploit the time series dimension of the data to perform a panel regression:

$$TFP_{it} = \alpha_0 + \alpha_1 INF_{it} + \gamma GDP_{it} + c_i + \epsilon_{it}$$

where c_i denotes country-specific fixed effects. Since this type of panel regression captures the effect within a country, the correct measure of TFP to use is the one relative to a base year (2017) for that country, that is, the series whose cross-country averages are shown in column (4) of Table 9. Table 10 shows the results of the regressions described above. Column (1) displays the coefficient of the OLS specification without GDP per capita, while column (2) includes it. The inclusion of GDP per capita reduces the magnitude of the coefficient of interest, which remains negative and statistically significant. A coefficient of 0.26 can be interpreted in the following way: a decrease in informality by one percentage point is associated with an increase in TFP (relative to US TFP) by 0.25 percentage points. Column (3) and column (4) display the results of the panel regressions

Table 10: Relationship between TFP and Informality.

	(1)	(2)	(3)	(4)
	OLS	OLS	Panel	Panel
	TFP (US=100)	TFP (US=100)	TFP (2017=1)	TFP (2007=1)
Informality (DGE)	-1.026*** (0.0313)	-0.261*** (0.0344)	-0.879*** (0.321)	-0.852** (0.339)
GDP per capita (th)		0.828*** (0.0237)		0.0271 (0.227)
Year fixed effects	YES	YES	NO	NO
Country fixed effects	NO	NO	YES	YES
Observations	3198	3198	3198	3198

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

excluding and including GDP per capita as control, respectively. The coefficients on informality are negative and statistically significant. In this case, a coefficient of -0.85 means that a reduction in informality by one percentage point is associated with an increase in TFP by 0.85 percentage points.

A.5 Brazilian Data

ECINF. *Pesquisa de Economia Informal Urbana* (ECINF) is a survey conducted by the Brazilian Bureau of Statistics (IBGE) in 1997 and 2003 to collect information about the informal sector in Brazil. Owners are classified as informal if they do not possess a tax identification number (*Cadastro Nacional de Pessoa Juridica*(CNPJ)).²¹ By matching owners and businesses with employees it is possible to obtain the number of employees for each business. Additional information such as sector and financial variables is also available.

The survey is representative at the national level for firms with at most 5 employees. From the original dataset, which is publicly available, I exclude the following observations:

- (i) Owners operating in the agricultural sector.
- (ii) Owners who do not have a facility exclusively dedicated to the business outside their house.
- (iii) Owners who have another job.
- (iv) Businesses who have no owners or more than 4 owners.
- (v) Businesses who have more than 7 employees.

²¹Ulyssea (2018) points out that strict confidentiality clauses and IBGE's reputation induce respondents to report fairly accurately.

(ii) and (iii) directly address the concern that the data might measure home production rather than entrepreneurship.

The final sample from which I compute moments used in the calibration contains about 30,000 firms. Table 11 summarizes sector composition and size distribution by formality status.

PNAD. I use data from the 2003 *Pesquisa Nacional por Amostra de Domicílios* (PNAD), a repeated cross-section representative at the national level, to compute the rate of entrepreneurship. Individuals who are either self-employed or employers are classified as entrepreneurs.²²

Table 11: ECINF. Sector and Size Statistics.

	Formal Shares	Informal Shares	Total Shares
<i>Industry</i>			
Manufacturing	8.69	11.27	10.82
Retail	50.50	37.70	39.93
Services	40.81	51.03	49.25
<i>Number of Employees</i>			
1	28.95	79.30	70.53
2	27.39	14.14	16.45
3	16.35	3.68	5.89
4	11.71	1.72	3.46
5	8.65	0.79	2.16
6	4.49	0.30	1.03
7	2.47	0.07	0.49
<i>Observations</i>	5,261	24,924	30,185

Source: Own calculation from ECINF (2003). Note that the shares reflect the raw unweighted sample.

²²See question v9029 in the individual's section of the survey.