PHYS 512 - Computation Physics with Applications

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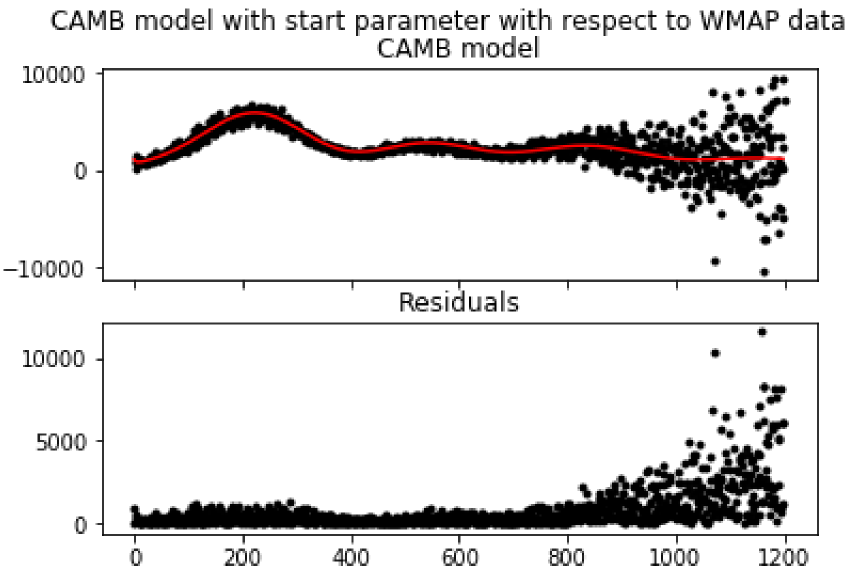
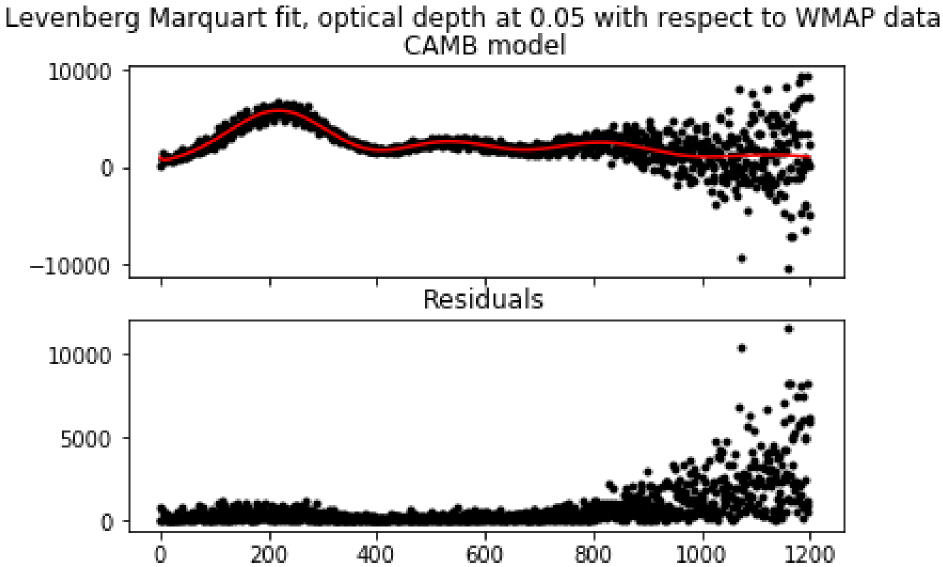
1. Using the given parameters: $H_0 = 65$ km/s, $w_b h^2 = 0.02$, $w_c h^2 = 0.1$, $\tau = 0.05$, $A_s = 2 \cdot 10^{-9}$, $n_s = 0.98$, evaluating the χ^2 of the CAMB model with the above parameters with respect to the WMAP data gives $\chi^2 = 1588.238(0.001)$.

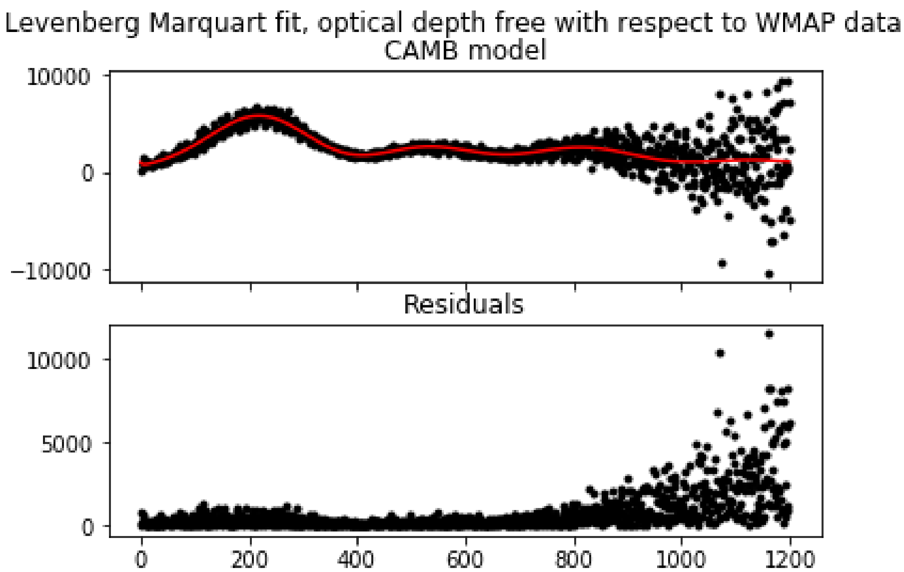


2. The CAMB library does not provide derivative with respect to each parameter, but we can evaluate them numerically using four points derivatives:

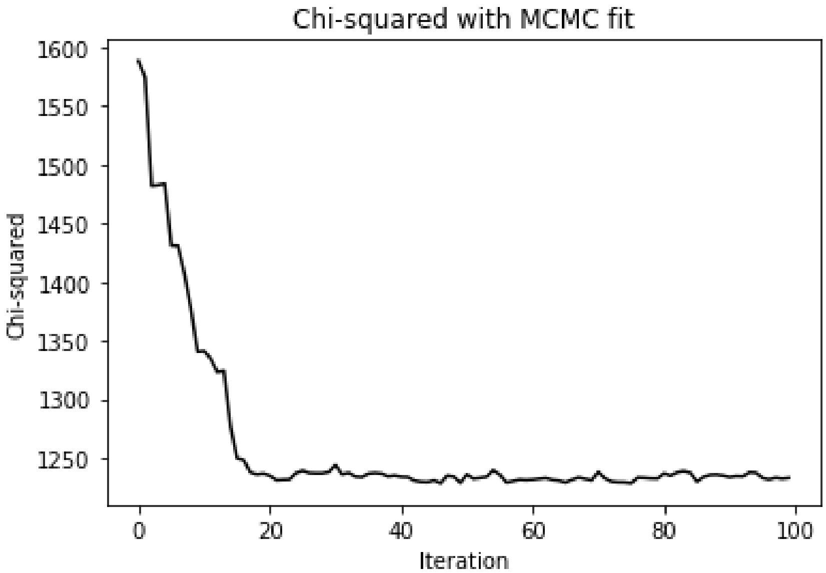
$$\frac{\partial f(x)}{\partial x_i} = \frac{8[f(x + x_i dx) - f(x - x_i dx)] - [f(x + 2x_i dx) - f(x - 2x_i dx)]}{12 x_i dx} + O(dx^4)$$

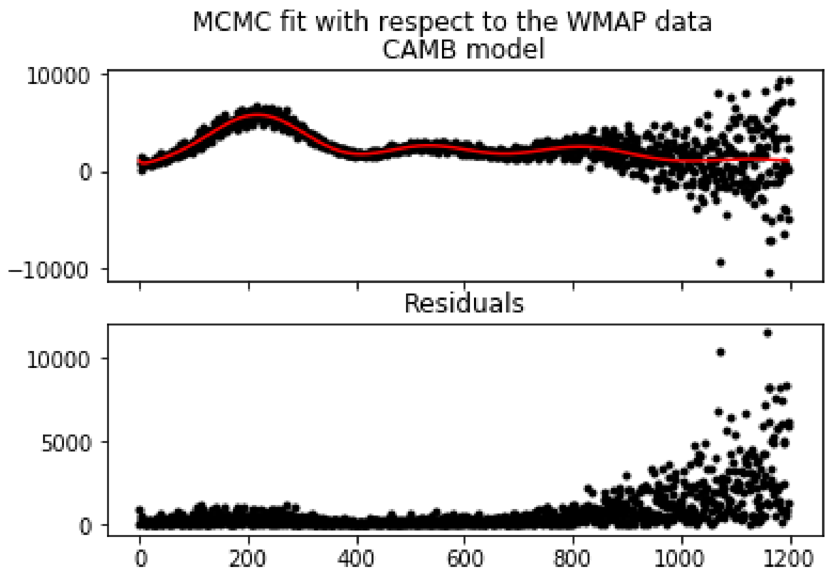
   where we generate the new results by creating a new CAMB model.
   Since we assume that our errors are uncorrelated, the covariance matrix N is diagonal with the errors given in the WMAP. The following non-linear fit of parameters were done using the Newton-Raphson Levenberg-Marquart method.
   If we keep the optical depth $\tau = 0.05$ fixed, after 10 iterations, the parameters converge to $H_0 = 7.462 \cdot 10^1, w_b h^2 = 2.344 \cdot 10^{-2}, w_c h^2 = 1.046 \cdot 10^{-1}, A_s = 2.000 \cdot 10^{-9}, n_s = 1.001 \cdot 10^0$ with $\chi^2 = 1233.604$ (keeping 3 decimal digits).
   However, if we also fit τ, after 10 iterations, the parameters converge to $H_0 = 6.962 \cdot 10^1, w_b h^2 = 2.270 \cdot 10^{-2}, w_c h^2 = 1.131 \cdot 10^{-1}, \tau = 3.930 \cdot 10^{-2}, A_s = 2.000 \cdot 10^{-9}, n_s = 9.763 \cdot 10^{-1}$ with $\chi^2 = 1228.629$ (keeping 3 decimal digits).



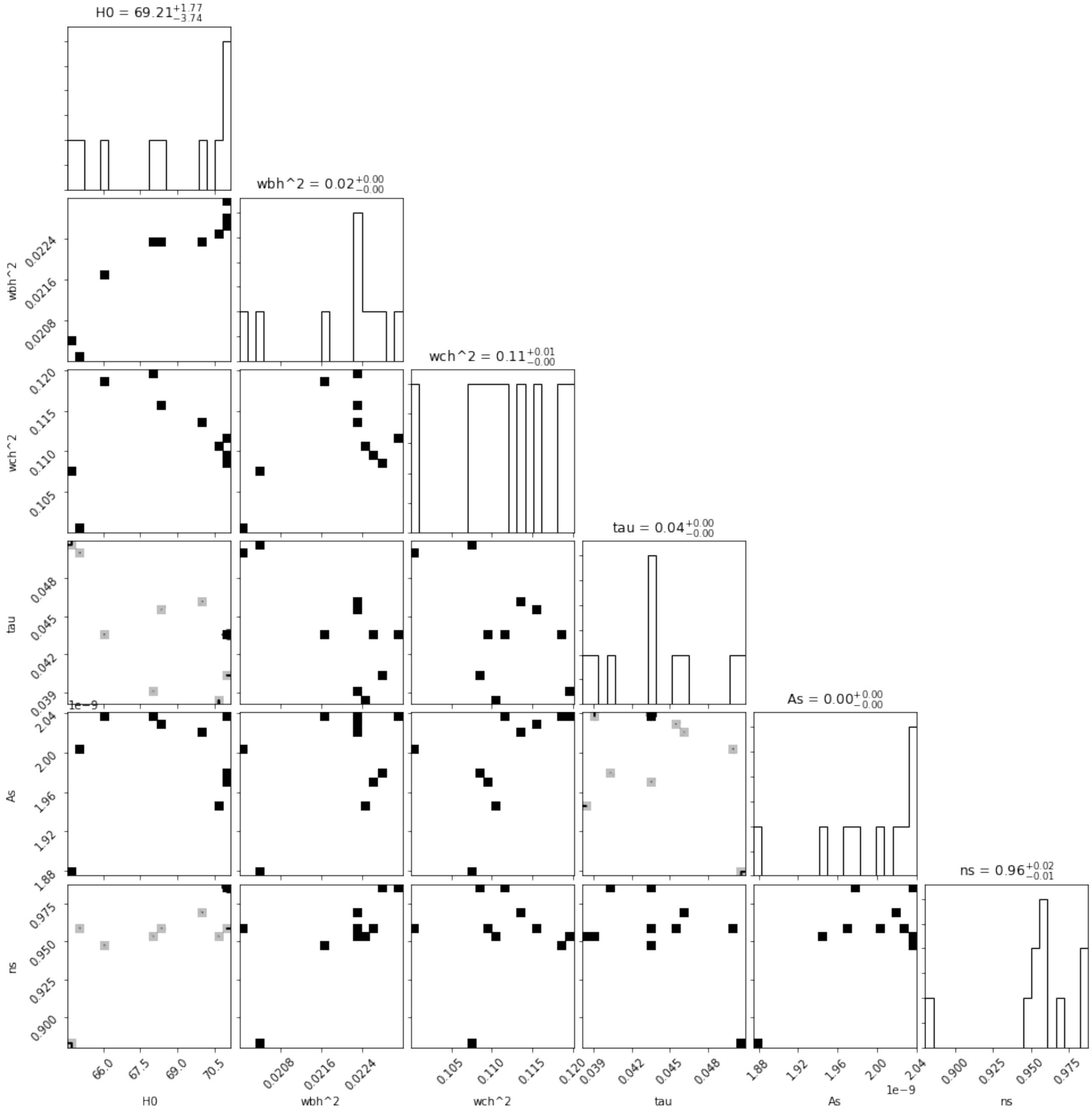


3. For the Markov chain, we use as probability of acceptance $e^{-0.5\delta\chi^2}$ if the chi-squared value is not reduced. The step we try before checking the probability acceptance is: param = param + gaussian(standard derivation = 0.02·param). After 100 iterations, the best parameters obtained where: $H_0 = 6.994 \cdot 10^1, w_b h^2 = 2.233 \cdot 10^{-2}, w_c h^2 = 1.111 \cdot 10^{-1}, \tau = 4.506 \cdot 10^{-2}, A_s = 1.996 \cdot 10^{-9}, n_s = 9.666 \cdot 10^{-1}$ with $\chi^2 = 1228.515$ (keeping 3 decimal digits).

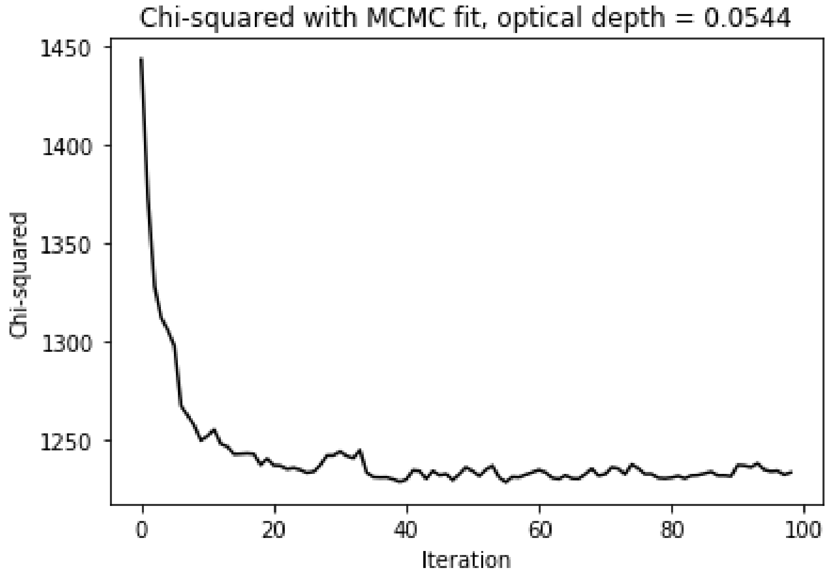


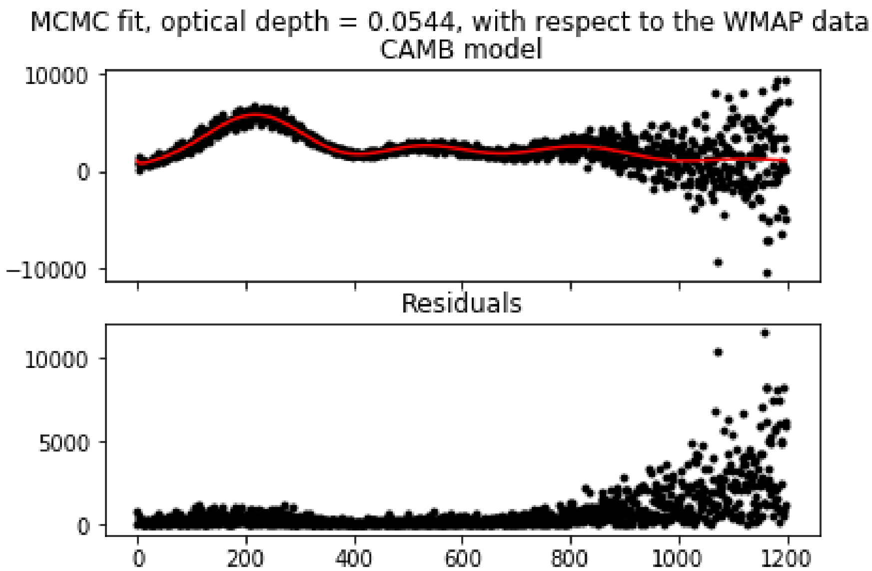


The chain may have reached a local convergent point. To verify, we would need to plot more points and do a contour plot to see if all the regions have been sampled. Unfortunately, with 100 iterations, there is not enough points to determine that.



4. We used the same process but now kept the optical depth fixed at $\tau = 0.054$. After 100 iterations, the best parameters obtained where: $H_0 = 6.964 \cdot 10^1, w_b h^2 = 2.272 \cdot 10^{-2}, w_c h^2 = 1.139 \cdot 10^{-1}, \tau = 5.4 \cdot 10^{-2}, A_s = 2.068 \cdot 10^{-9}, n_s = 9.746 \cdot 10^{-1}$ with $\chi^2 = 1228.235$ (keeping 3 decimal digits).





The chi-squared value is lower relative to the other, . Since the optical depth values are different, we probably found a better local convergence point meaning the previous Markov chain was not converging at the optimal point