PHYS 512 - Computation Physics with Applications

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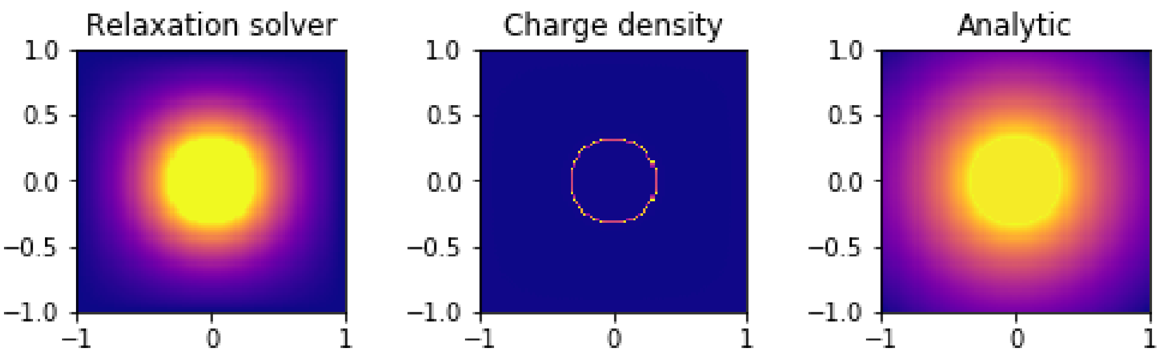
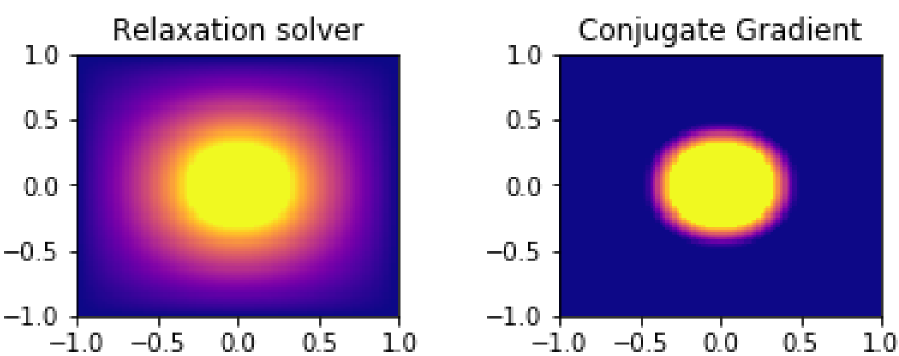
1. The relaxation solves for the potential by assuming that Laplace's equation is satisfied outside of the cylinder and the boundaries and iterating through $v_{i,j} = 0.25(v_{i+\delta i,j} + v_{i-\delta i,j} + v_{i,j+\delta i} + v_{i,j-\delta i})$.

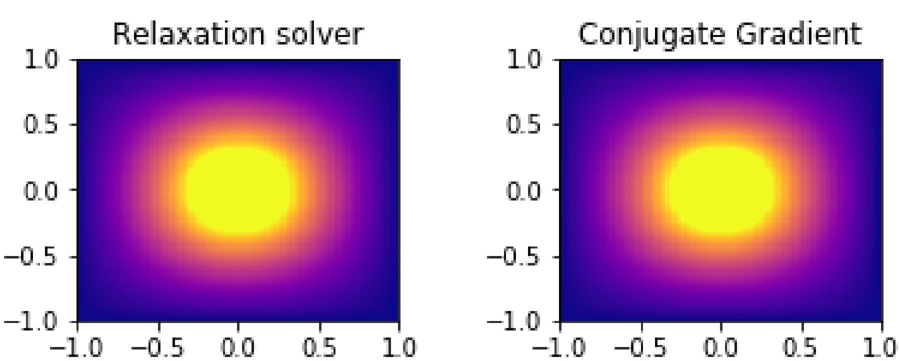
   The charge density is only on the surface of the cylinder as expected since there is no electric field inside a conductor. The analytic solution is different from the solver since we set the potential at 0 on the walls.



2. Using the conjugate gradient, we can obtain a solution using less iterations. If our threshold is set to 10^{-6} and we evaluate $err = max\{|(x_{new\ i,j} - x_{old\ i,j}|\} < threshold$ to determine if we converged. We get that the relaxation solver takes 4080 ± 5 steps to converge whilst the conjugate gradient solver takes 10 steps (with a resolution of 100x100).

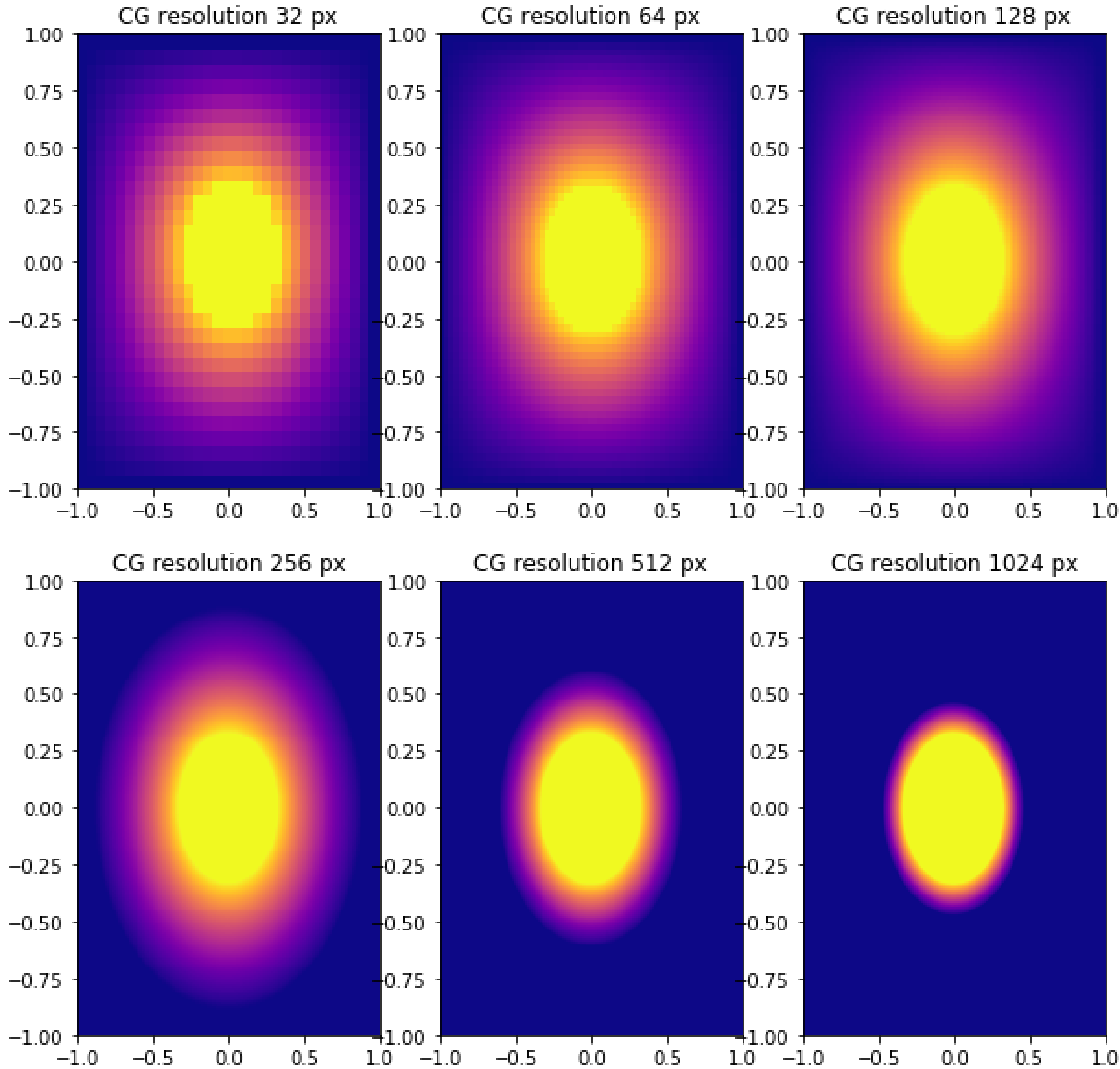


   We can see that the conjugate gradient did not yield a good result. A better approach would be to use the root-mean-square error. Using that criterion, the relaxation solver takes 9050 ± 5 steps to converge takes and the conjugate gradient takes 100 steps to achieve the same threshold (with a resolution of 100x100).

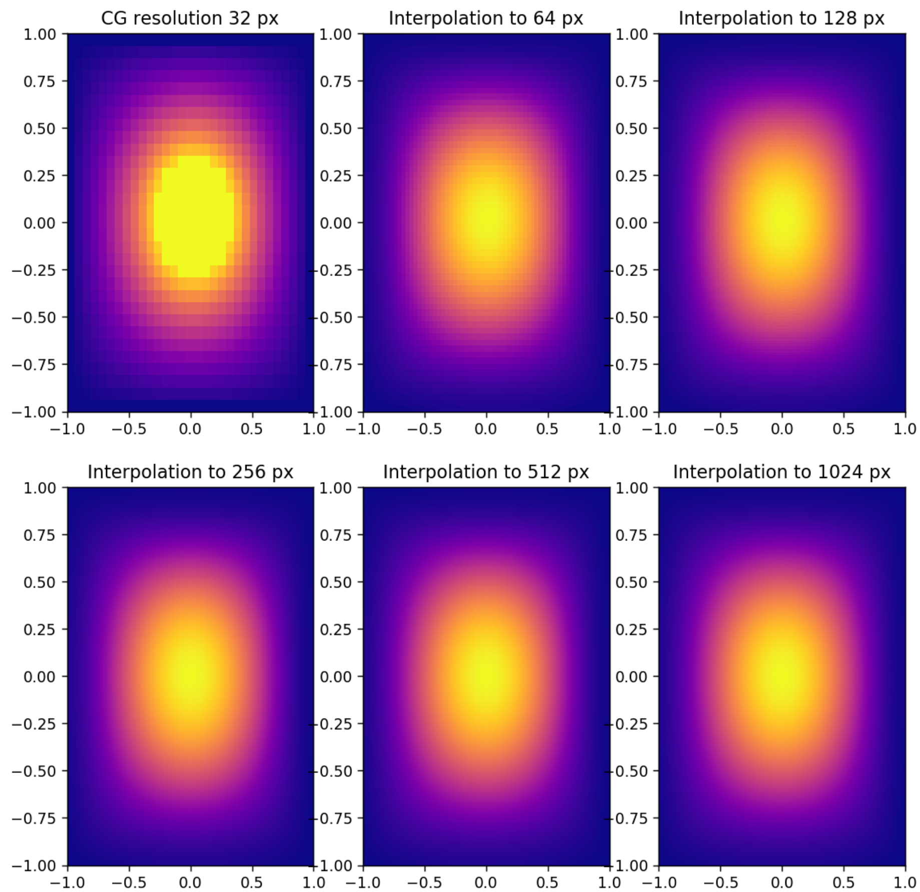


3. We used the conjugate gradient to evaluate the resolution of the cylindrical conductor at 6 different resolutions. The conjugate gradient solver takes 0.003(1) s to solve for 32 px, 0.01(1) s to solve for 64

px, 0.03(1) s to solve for 128 px, 0.4(1) s to solve for 256 px, 1(1) s to solve for 512 px and 77(1) s to solve for 1024 px.



Whilst the interpolation method starting at 32 px is much faster (less then 10ms at 1024 px), information seems to be lost.



4. Adding a bump on a cylindrical conductor amplifies the electric field near that bump. This would create an energy leak in a electric wire, thus is to avoid for a power company.