# LAB 4: Contact consistent (fixed) base dynamics

1) *Simulation of contact consistent dynamics:* Generate a sinusoidal reference for Shoulder Lift joint (see *ex_4_conf.py*) with amplitude 0.6 rad and frequency 1 $Hz$.

2) *Computation of the Constraint Consistent Dynamics:* Compute the constraint consistent accelerations $\ddot{q}_c$ where a point contact (i.e. 3D) is possible only at the end-effector (*ee_link* frame).

$$\ddot{q}_c = M^{-1}[N_{\bar{M}}(S^T\tau - h) - J^T\Lambda\dot{J}\dot{q}] \tag{1}$$

Where $N_{\bar{M}}$ is the null-space projector for $J^{T\#}$. What is the best way to compute the term $\dot{J}\dot{q}$? (hint: since $\ddot{x} = J\ddot{q} + \dot{J}\dot{q}$ compute the acceleration at the end-effector while setting $\ddot{q} = 0$).

3) *Update of joint velocities after inelastic impact:* After an unelastic impact the end-effector velocity is going instantaneously to zero $\dot{x}_e = 0$. An impulse is applied to achieve this, that creates a discontinuity in the joint velocities. Compute the new value of joint velocities that are consistent with the constraint $\dot{x}_e = 0$ by using the null-space projector $N$ for $J$.

$$\ddot{q}^+ = N\dot{q}^- \tag{2}$$

4) *Simulate the Constraint Consistent Dynamics:* Use the pre-implemented PD controller (with gravity compensation) and the logic to deal with the contact. At the contact consider the appropriate projection of the dynamics (1) before integrating the accelerations and the velocities (with forward Euler).

5) *Contact forces disappear when projecting the dynamics:* Check contact force disappears when after the projection in the null-space of $J^{T\#}$ (i.e. $N_{\bar{M}}J^Tf = 0$) What happens if you use the Moore-penrose pseudo-inverse to compute the projector? Plot also the torques in the null-space of $J^{T\#}$ during the contact, check that they are barely zero because there is almost no internal joint motion. Compare them with a plot of the joint torques to see they are much bigger. The motion during the contact depends mainly only on the null-space projector $N_{\bar{M}}$ that "cuts-out" the torques that generate contact forces, leaving only the ones that generate internal motions (in this case very small).

6) *Constraint consistent joint reference:* Try to double the amplitude of the reference of Shoulder Lift joint to 1.2 rad. Design a reference trajectory that is consistent with the contact (hint: compute $\dot{q}^d$ project with $N$ and integrate to get $q^d$).Rember to comment the previous reference generation. How the reference of the joint changes? it the robot able to move out from the contact?

7) *Gauss principle of least effort*: Verify that the Gauss principle of least constraint is satisfied (e.g. solve the QP where you minimize the 2-norm distance of the constraint acceleration vector from the unconstraint acceleration one, under the contact constraint).

8) *Change in the contact location*: Modify the code in order to allow the contact at a different location (e.g. origin of *wrist_3_link* frame). Verify that the end-effector now penetrates the ground. By activating the **TF** function in *rviz* you can check the relative location of the frames.

9) *Check the shifting law (Optional):* The twist at *ee_link* is $v_e$, twist at origin of *wrist_3_joint* is: $v_o$. Since they belong to the same rigid body (wrist_3_link) they are linked the "shifting law", by a time-invariant *motion* transform ${}_eX_o \in \mathbb{R}^{6\times6}$:

$$v_e = {}_{ee}X_o v_o \tag{3}$$

$$J_e\dot{q} = {}_{ee}X_o J_o \dot{q} \tag{4}$$

therefore, also for the Jacobians holds:

$$J_e = {}_eX_o J_o \tag{5}$$

where:

$${}_oX_e = \begin{bmatrix} {}_oR_e & -{}_oR_e[{}_et]_\times \\ 0 & {}_oR_e \end{bmatrix} \tag{6}$$

${}_eX_o = {}_oX_e^{-1}$ and ${}_et$ is the relative position of the origin of frame $o$ w.r.t. frame $e$ expressed in frame $e$.