UCLA Anderson School of Management

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Problem Set #7

You may work on this problem set in groups . Hand in one solution per group. You may discuss the problems only with members of your group. Answers should be typed (or printed legibly) and are due **at the beginning of the week 10 class. No late assignments will be accepted.**

Problem 1. In this problem, you have to use MATLAB. Suppose that the dynamics of the stock follow the dynamics

$$dS_t = \mu S_t dt + \sigma S_t d\overline{W}_t$$

where $S_0 = 100, \sigma = 0.2, \mu = 0.05$. The interest rate is constant and given by $r = 0$.

Before starting calculations, please reply to the following question: What are the dynamics of S_t under the risk neutral probability measure Q?

Now fix $\Delta t = 0.01$ and simulate $1,000$ paths for S_t (all starting at $S_0 = 100$) under the probability measure Q and price the following claims:

a) A European Call option with strike price of $K = 100$ and $T = 1$. Compare that price with the analytical price provided by Black Sholes.

b) A European "Asian" option with string price of $K = 100$ and $T = 1$. The payoff of an Asian option is

$$\max\left\{\left(\int_0^1 S_s ds\right) - K, 0\right\}$$

Problem 2. Exercise 15.1 in ATC

Problem 3. Weather is very important for farmers. Suppose that you are approached by a farmer who is worried about warm temperatures and wishes to purchase a call option on the average weather temperature over the next year. Specifically, the weather (in Fahrenheit) follows the process

$$dx_t = -\lambda\left(x_t - \overline{x}\right)dt + \sigma d\overline{W}_t$$

and the farmer wants a payoff equal to

$$\max\left(X_T - K, 0\right) \tag{1}$$

You observe that call options on the weather follow the following process

$$dC_t = rC_t dt + \sigma_C\left(t\right) C_t d\overline{W}_t$$

a) Use Exercise 15.1 in ATC to prove that $\lambda\left(x_t\right) = 0$ for all weather derivatives. (Hint: Observe that the average return from investing in weather derivatives is just the riskless rate)

b) Provide an analytical price for the option with payoff (1). (Hint: Observe that x_T is normally distributed, since x_t is an Ornstein Uhlenbeck process).