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DIPARTIMENTO DI MECCANICA

Enhanced Vibration Control on simple Electro-Mechanical System using Piezoelectric Patches

Barutta Edoardo	10726132
Bauce Lorenzo	10743634
Meneghini Davide	10715030
Milic Kristjan	11012938
Visentin Nicola	10797203

Data Analysis and Experimental Characterization for Robotic and Mechatronic Systems
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Lecturers: Manzoni S., Lucà F.

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Summary

1. Introduction
2. Analytical model
3. Experimental testing
4. Comparison and conclusions

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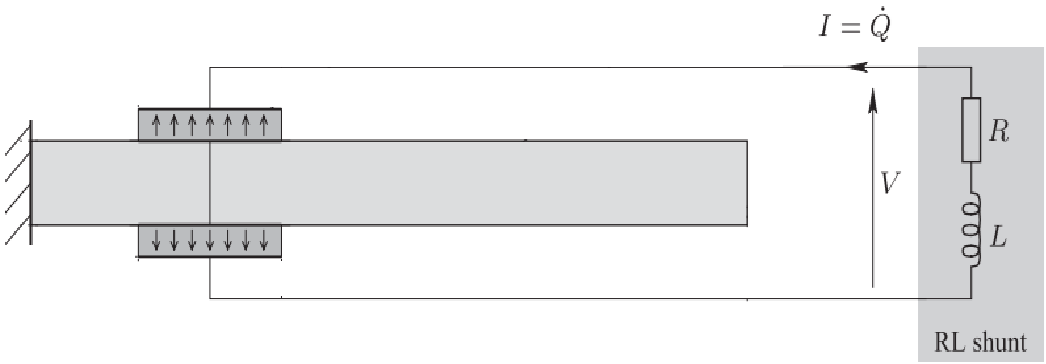
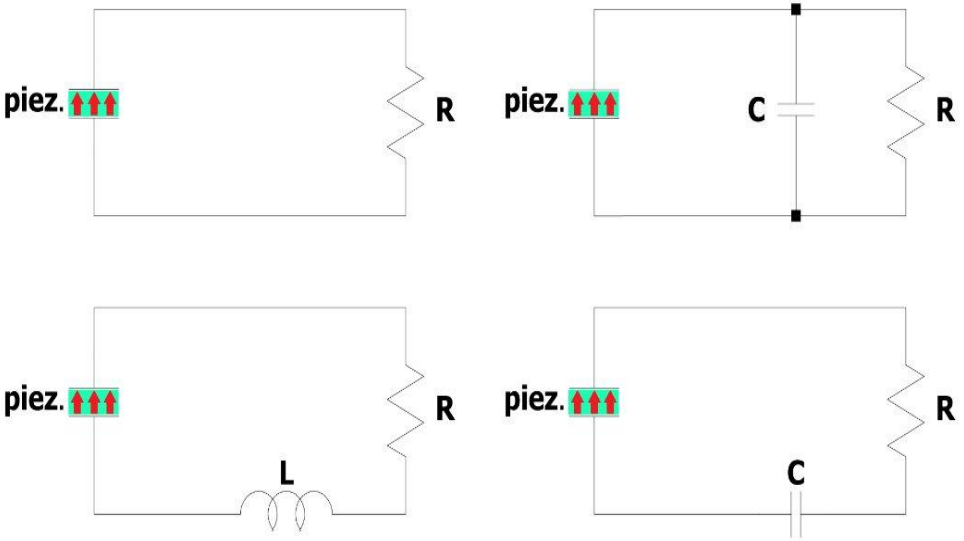
1. INTRODUCTION

VIBRATION ATTENUATION THROUGH PIEZOELECTRIC PATCHES

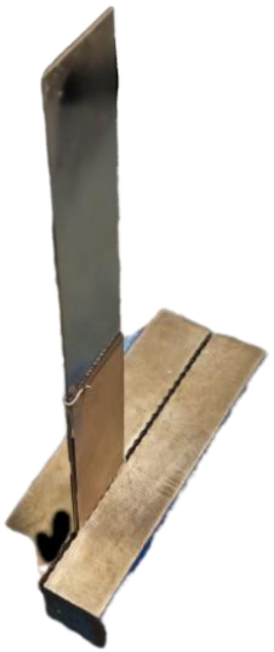
- Direct (sensor) and inverse (actuator) piezoelectric effect ➔ Electro-mechanical system (EMS)
- Small size and weight, low or null energy consumption ➔ Small-medium size structures
- Active or passive circuits
- Single or multi-mode attenuation

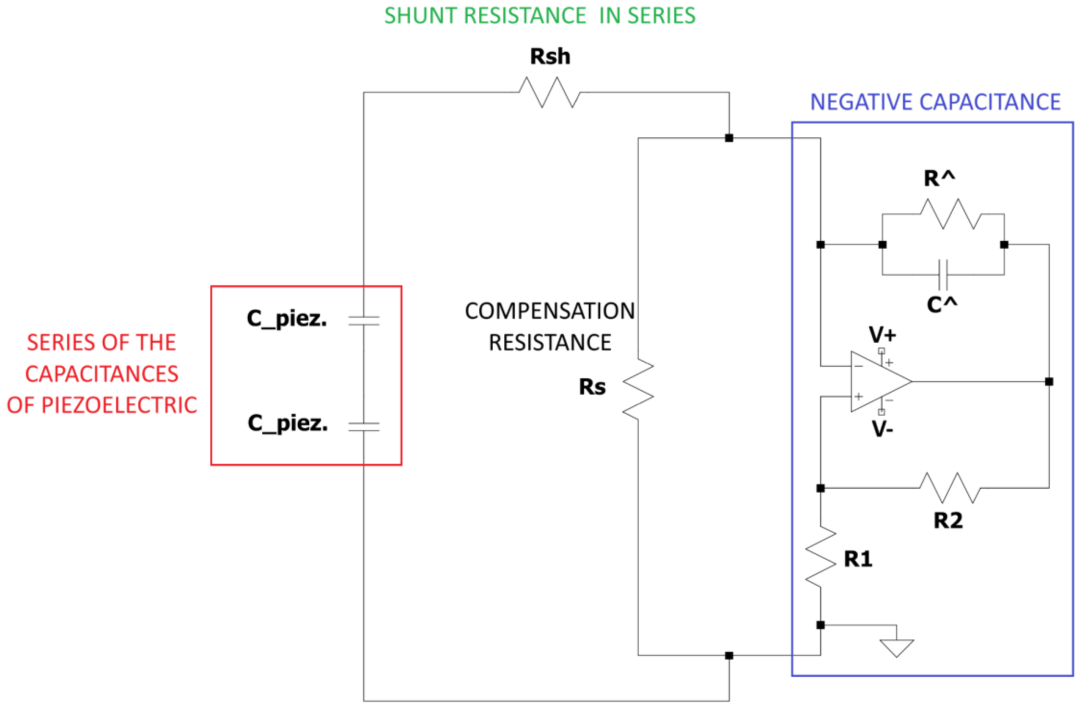




OUR CASE: CANTILEVER BEAM, TWO PATCHES AND R.C.

- Stainless steel cantilever beam vibrations bending modes
- **Two piezoelectric patches** wired in series (PIC151)
- Active Circuit: real circuit (**RC**) with **shunt resistance** in **series** with a virtual **negative capacitance**
- Single mode attenuation on **1st mode**





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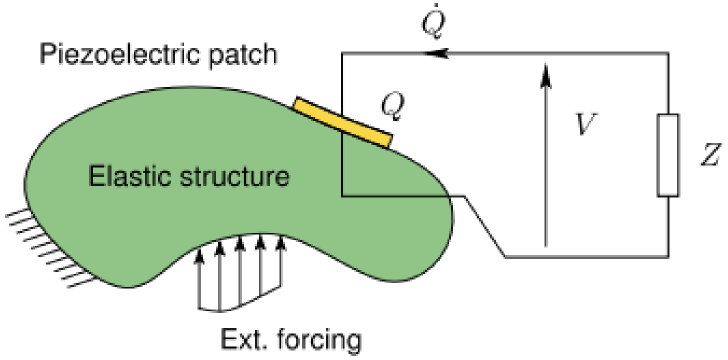
2. ANALYTICAL MODEL

ELECTRO-MECHANICAL SYSTEM (EMS)

- Elastic structure coupled with piezoelectric patch.
- External force F_{ext}.
- Shunt impedance Z_{sh}.
- Voltage V and charge Q at the piezoelectric actuator.



EMS dynamics is derived by means of the modal coordinates $q_i(t)$, while the electrical behaviour is described by the balance of electric charges within the piezoelectric patches. Considering *i*th mode:

$$\begin{cases} \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - x_i V = F_i \\ \displaystyle\sum_{i=1}^{N} x_i q_i + C_\infty V = Q \end{cases}$$

PIEZOELECTRIC CAPACITANCE ESTIMATION

The piezoelectric capacitance is influenced by the dynamics of the structure:

$$C(\Omega) \cong C_\infty + \sum_{i=1}^{N} \frac{x_i^2}{\omega_i^2 + 2j\xi_i\omega_i\Omega \; - \Omega^2}$$

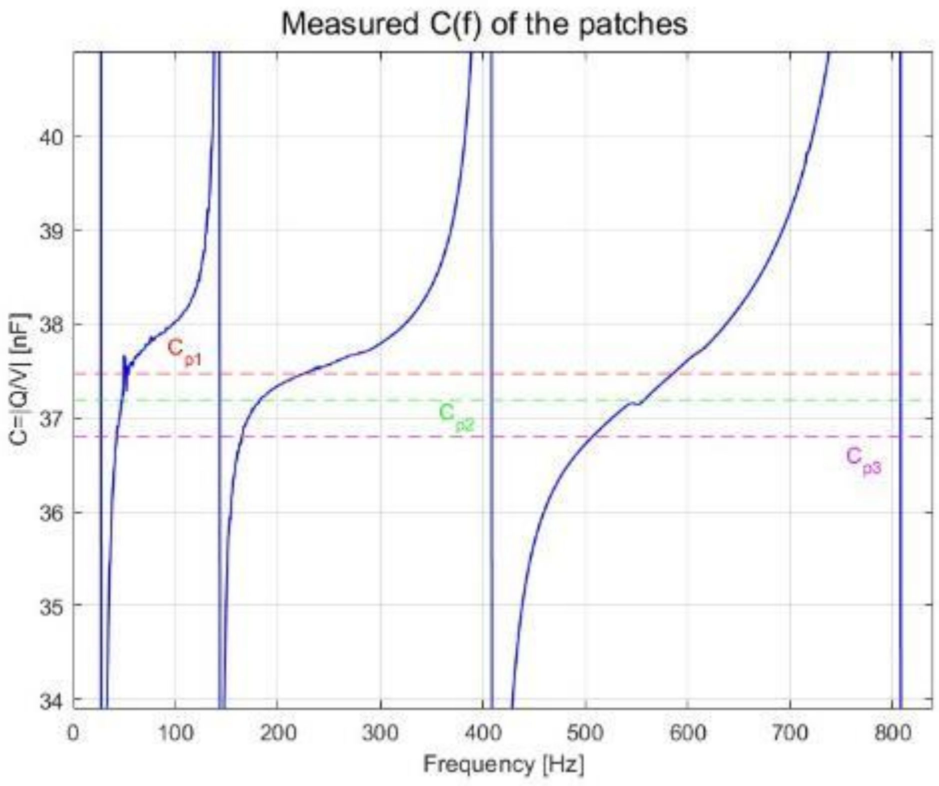
For $\Omega \cong \omega_i$:

$$C(\Omega) \cong C_{pi} + \frac{x_i^2}{\omega_i^2 + 2j\xi_i\omega_i\Omega - \Omega^2}$$

$$C_{pi} = C_\infty + \sum_{n=i+1}^{N} \frac{x_n^2}{\omega_n^2}$$

The simplified EMS model becomes:

$$\begin{cases} \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - x_i V = F_i \\ x_i q_i + C_{pi} V = Q \end{cases}$$



MODAL COUPLING FACTOR

Changing coordinates: $\bar{V} = V\sqrt{C_{pi}}$ $\bar{Q} = \frac{Q}{\sqrt{C_{pi}}}$ $\Rightarrow$ $\begin{cases} \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - k_i\omega_i\bar{V} = F_i \\ \bar{V} - \bar{Q} - k_i q_i \omega_i = 0 \end{cases}$

k_i is defined as the Modal Electro-Mechanical Coupling Factor (MEMCF):

$$k_i = \frac{x_i}{\omega_i\sqrt{C_{pi}}} \qquad or \qquad |k_i| = \sqrt{\frac{\widehat{\omega}_i^2 - \omega_i^2}{\omega_i^2}}$$

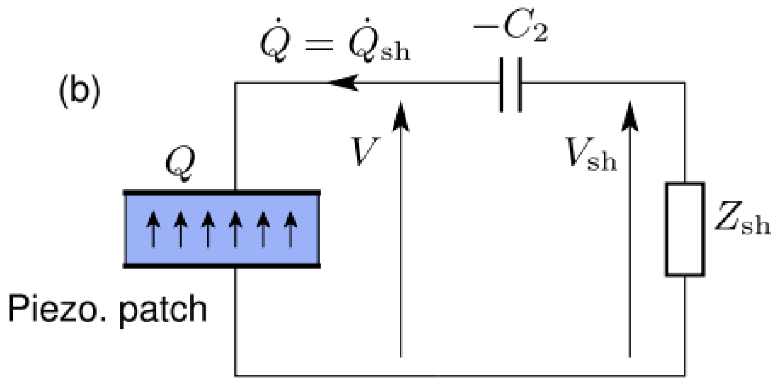
ω_i and $\widehat{\omega}_i$ are the short and open natural frequencies computed experimentally.

NEGATIVE CAPACITANCE ACTIVE CIRCUIT

Enhanced vibration control is performed by a negative capacitance (NC) circuit in **series configuration:**

- Negative capacitance $-C_2$ is connected to the shunt impedance Z_{sh}.
- Series configuration provides better attenuation at low ω.
- New equivalent capacitance leads to the Enhanced Modal Electro-Mechanical Coupling Factor (EMEMCF) which depends on the natural frequency of the EMS with the shunt Z_{sh} short-circuited and in open-circuit.

$$|\tilde{k}_i| = \sqrt{\frac{(\omega_i^{oc})^2 - (\omega_i^{sc})^2}{\omega_i^2}}$$



- $\tilde{k}_i / k_i$ ratio must be maximized by selecting β_2 index, without reaching instability condition.

$$\beta_2 = \frac{C_{pi}}{C_2} \qquad \frac{\tilde{k}_i}{k_i} = \frac{1}{\sqrt{1-\beta_2}}$$

$$\omega_i^{sc} = \omega_i \sqrt{1 - \frac{\beta_2 k_i^2}{1-\beta_2}} \qquad \omega_i^{oc} = \widehat{\omega}_i$$

OPTIMAL TUNING FOR SHUNT RESISTANCE

By neglecting the structural damping at this step, when R_{sh} is varied there exists a point F common to the amplitudes of all the FRFs.

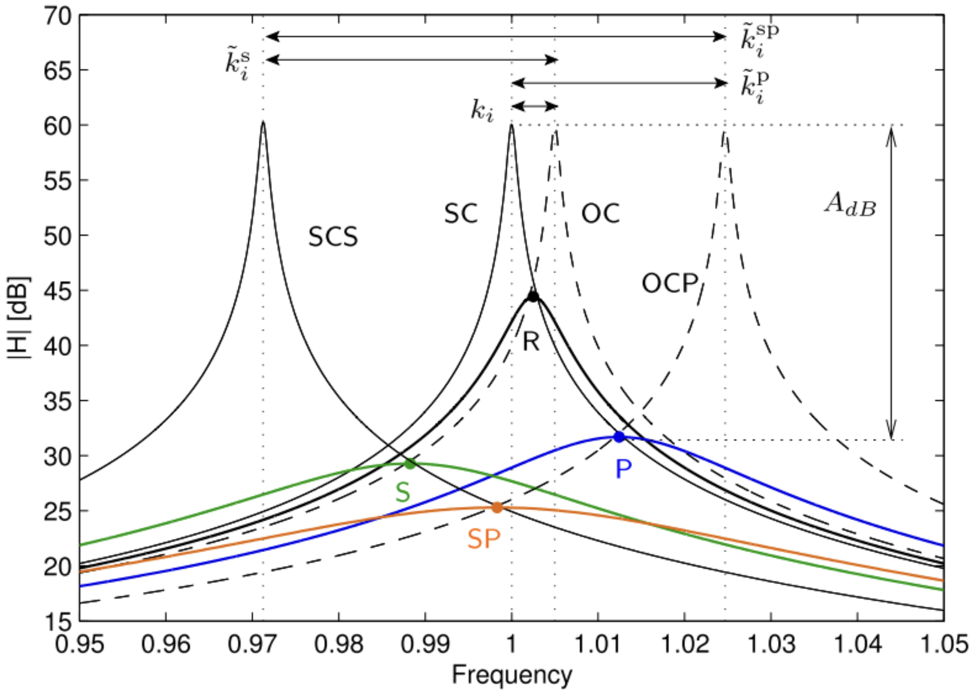
The optimum FRF corresponding to the best attenuation is that with its maximum at point F which frequency value ω_F is:

$$\omega_F = \sqrt{\frac{(\omega_i^{oc})^2 + (\omega_i^{sc})^2}{2}}$$

τ_e denotes the optimal electric time constant for RC circuits:

$$\tau_e = R_{sh} C_{eqs} = R_{sh} \frac{C_{pi}}{1 - \beta_2}$$

$$\tau_e^{opt} = \frac{1}{\omega_F}$$



τ_e^{opt} depends on the considered mode as depicted from C_{pi} and by the tuning of the NC C_2 and the shunt resistance R_{sh}

ANALYTICAL ATTENUATION FOR SINGLE-MODE CONTROL

The EMS with the shunt resistance will show a new frequency response function $H_i(\Omega)$ which depends on the electric time constant and the new modal parameters:

$$H_i(\Omega) = \frac{q_i}{F_i} = \frac{1 + j\tau_e\Omega}{(\omega_i^{sc})^2 - (1 + 2j\xi_i\omega_i\tau_e)\Omega^2 + j\Omega[\tau_e\left(\omega_i^{oc}\right)^2 + 2\xi_i\omega_i - \tau_e\Omega^2]}$$

The performance of the shunts is then evaluated by defining the vibration attenuation parameter A_{dB}:

$$A_{dB} = 20\log_{10}\frac{H_{sc}}{H_{sh}} = 20\log_{10}\frac{\tilde{k}_i^2 + 2\sqrt{2}\xi_i\sqrt{2 + \tilde{k}_i^2 - 2K^2}}{4\xi_i\sqrt{1 - \xi_i^2}}$$

- $K = \tilde{k}_1\sqrt{\beta_2}$
- H_{sc} is the FRF peak amplitude in $\Omega = \omega_i$ evaluated for the short circuit not controlled.
- H_{sh} is the FRF peak amplitude in $\Omega = \omega_F$ evaluated for the active circuit with the optimal shunt resistance.

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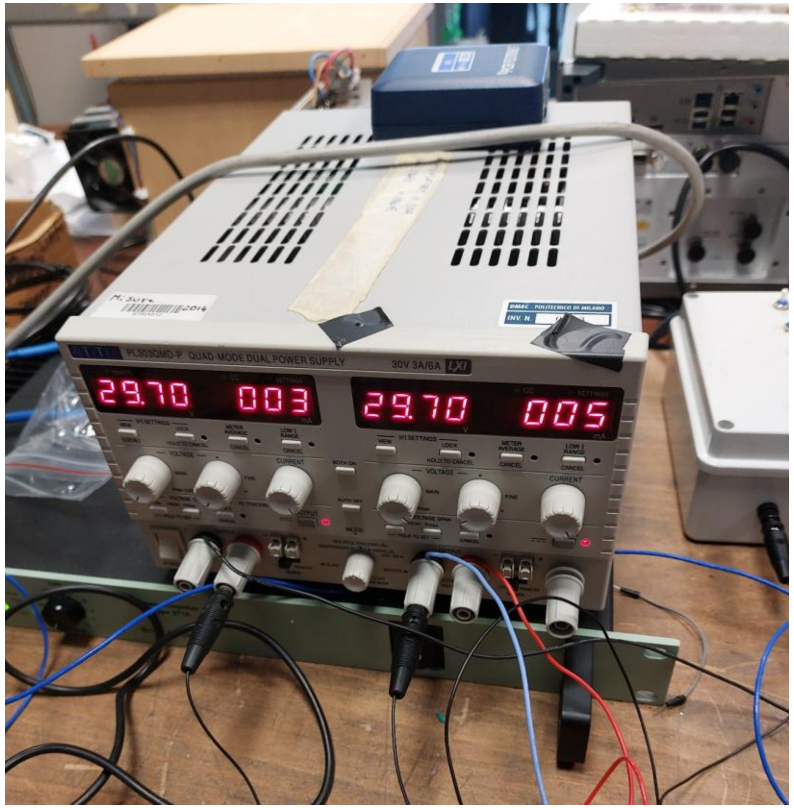
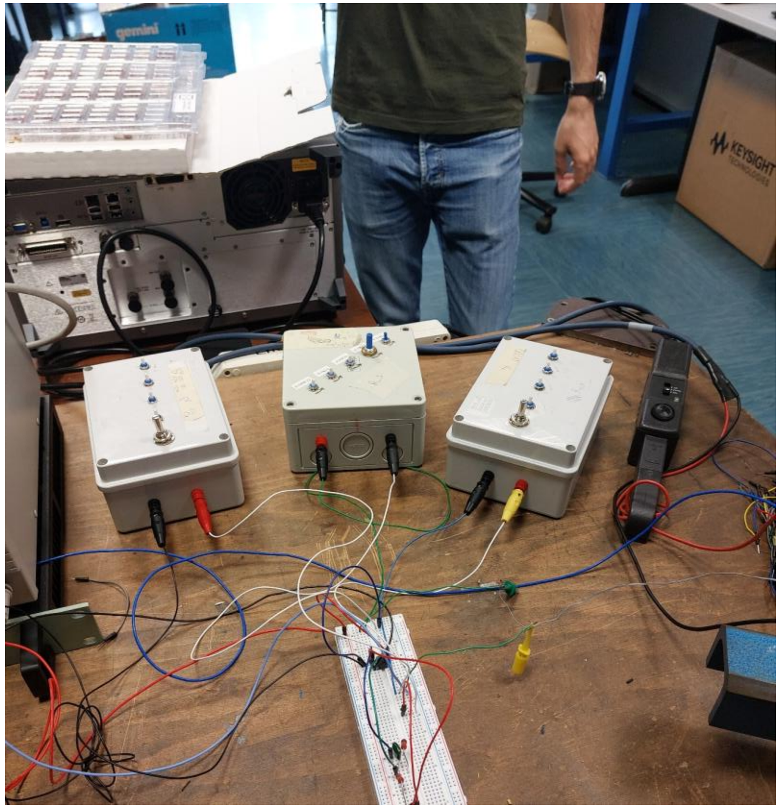
3. EXPERIMENTAL TESTING

EXPERIMENTAL SETUP

Accelerometer sensitivity: $0.0104\ \frac{V}{m/s^2}$

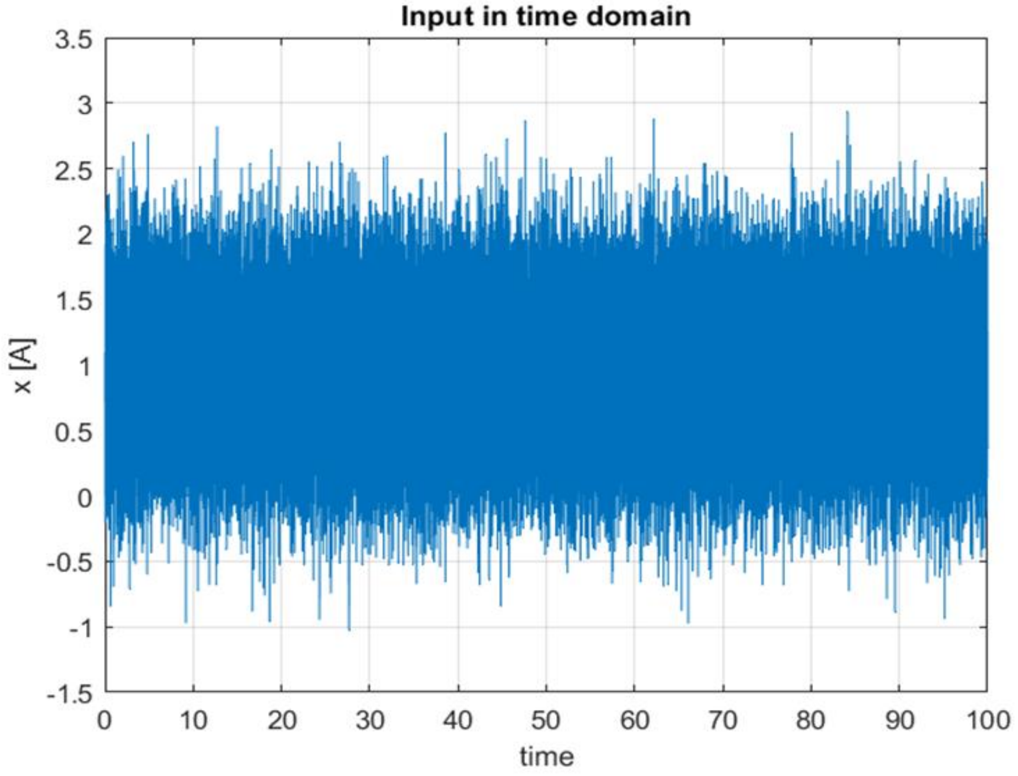
Input sensitivity: $1\ \frac{V}{A}$

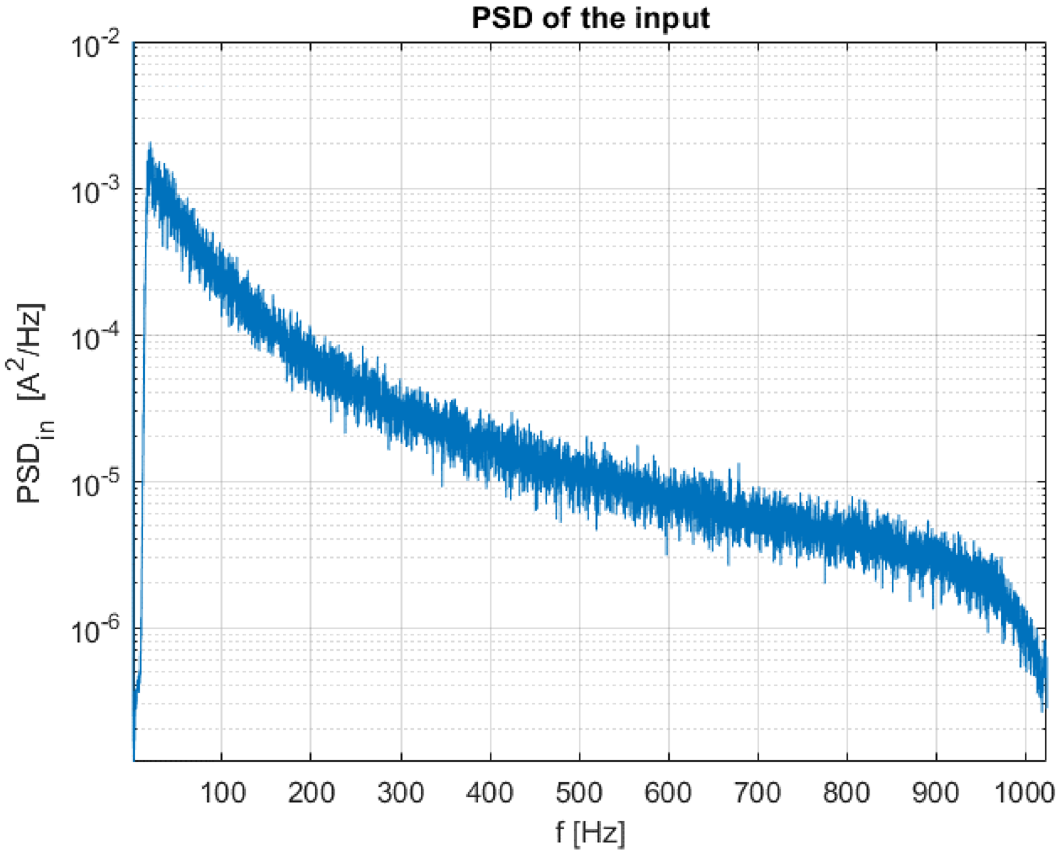




INPUT SIGNAL DEFINITION

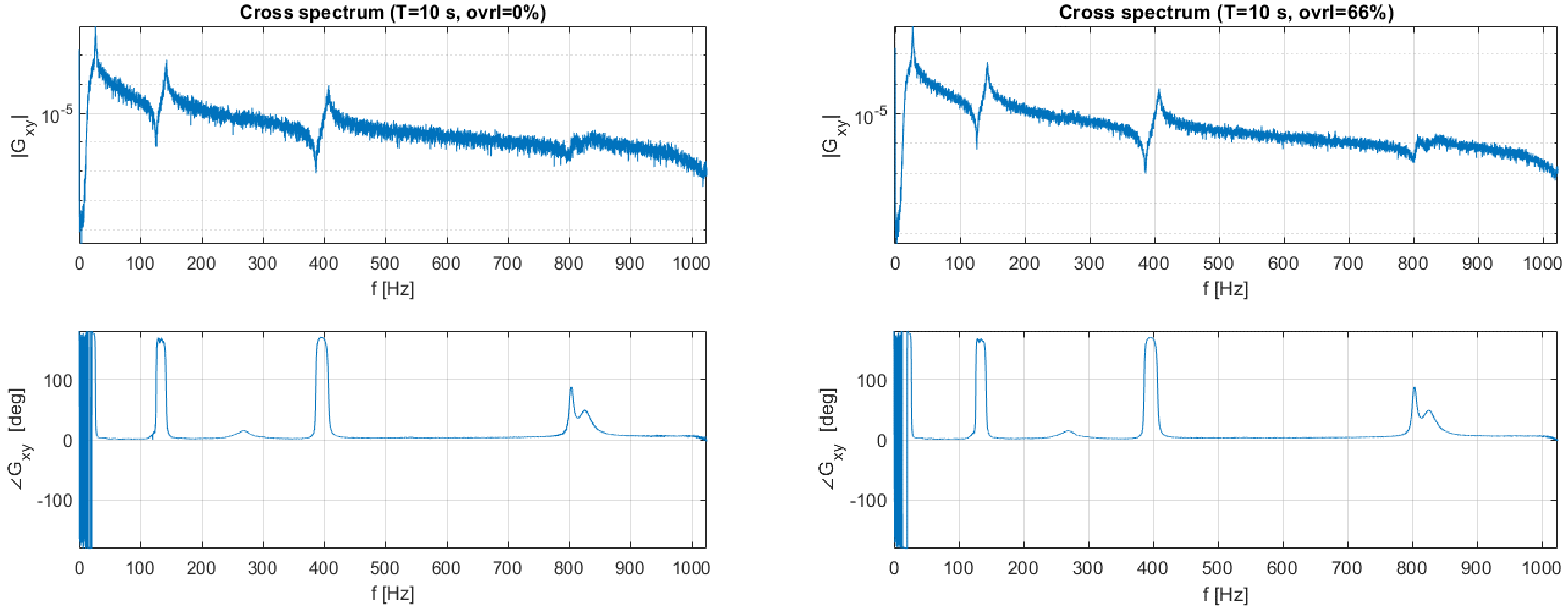
- Band limited random noise ($f_{cut,in} = 15\ Hz$, $f_{cut,out} = 1000\ Hz$)
- Amplitude limited to avoid saturation
- Single test with $T = 100\ s$, all frequency of interest are excited





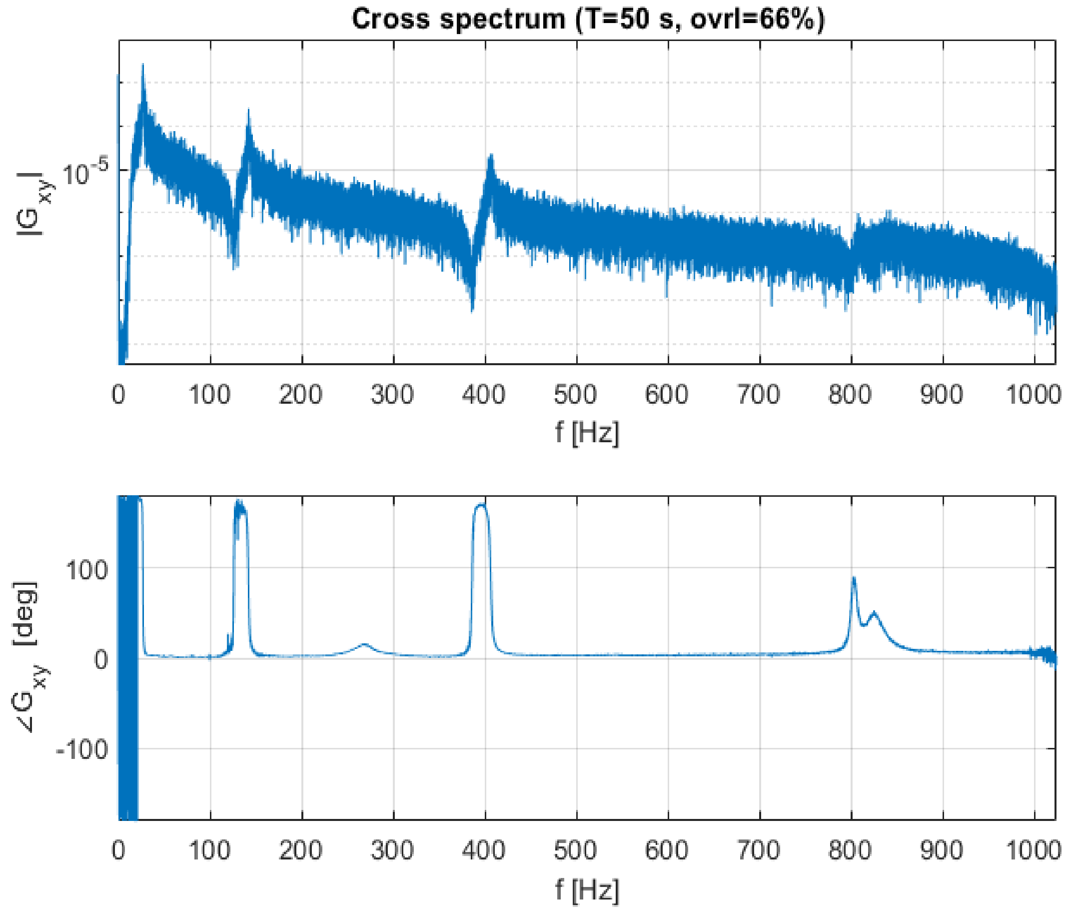
DATA PROCESSING – WELCH APPROACH

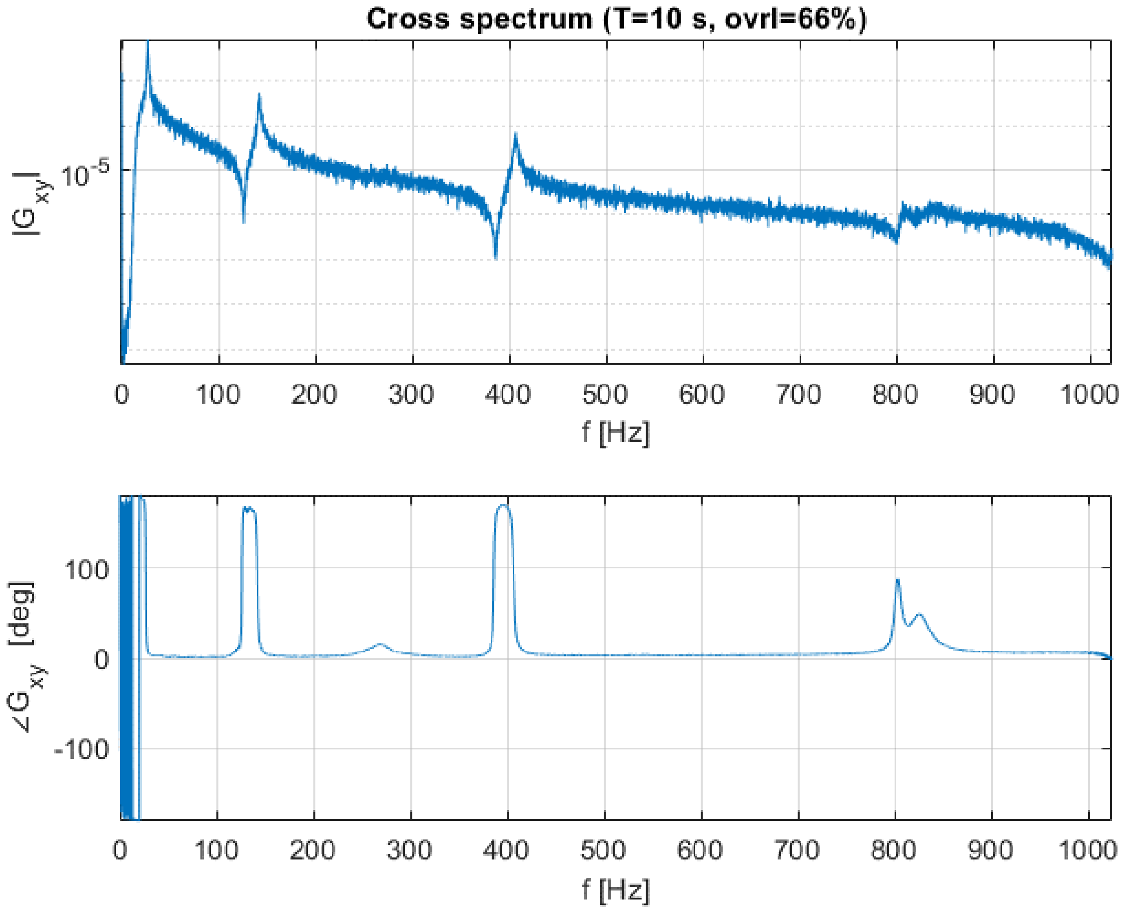
Varying overlap



DATA PROCESSING – WELCH APPROACH

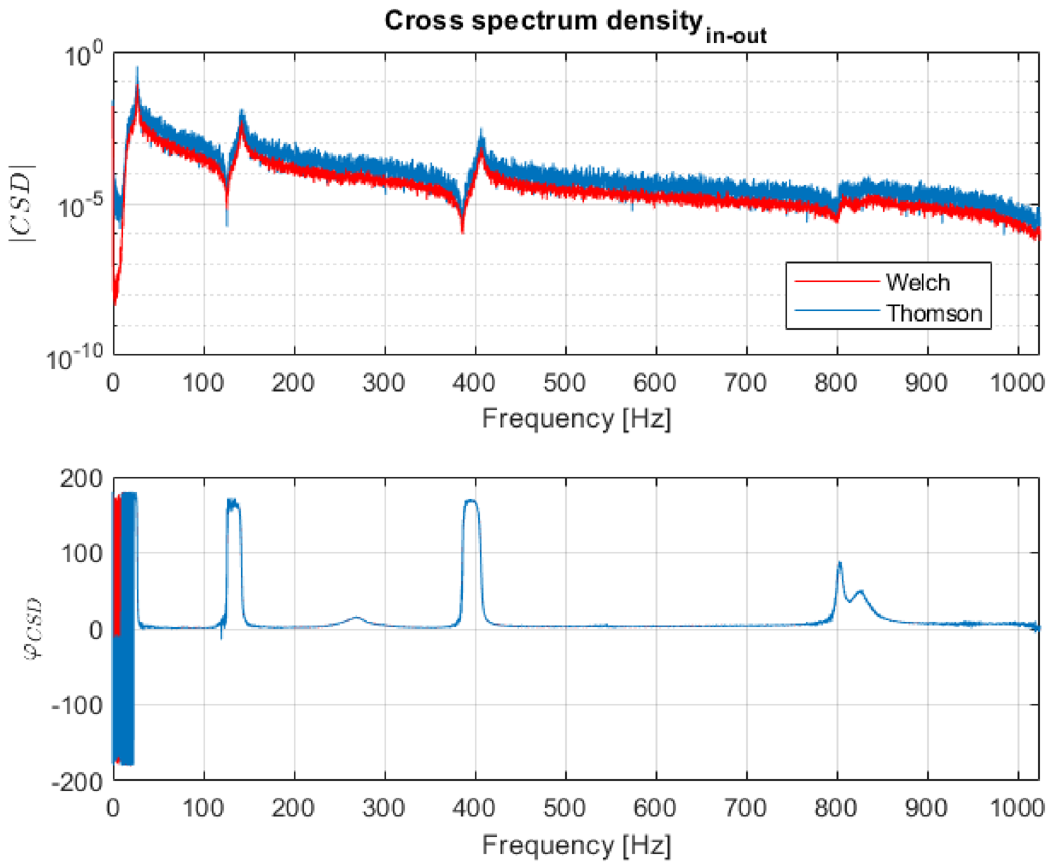
Varying acquisition duration



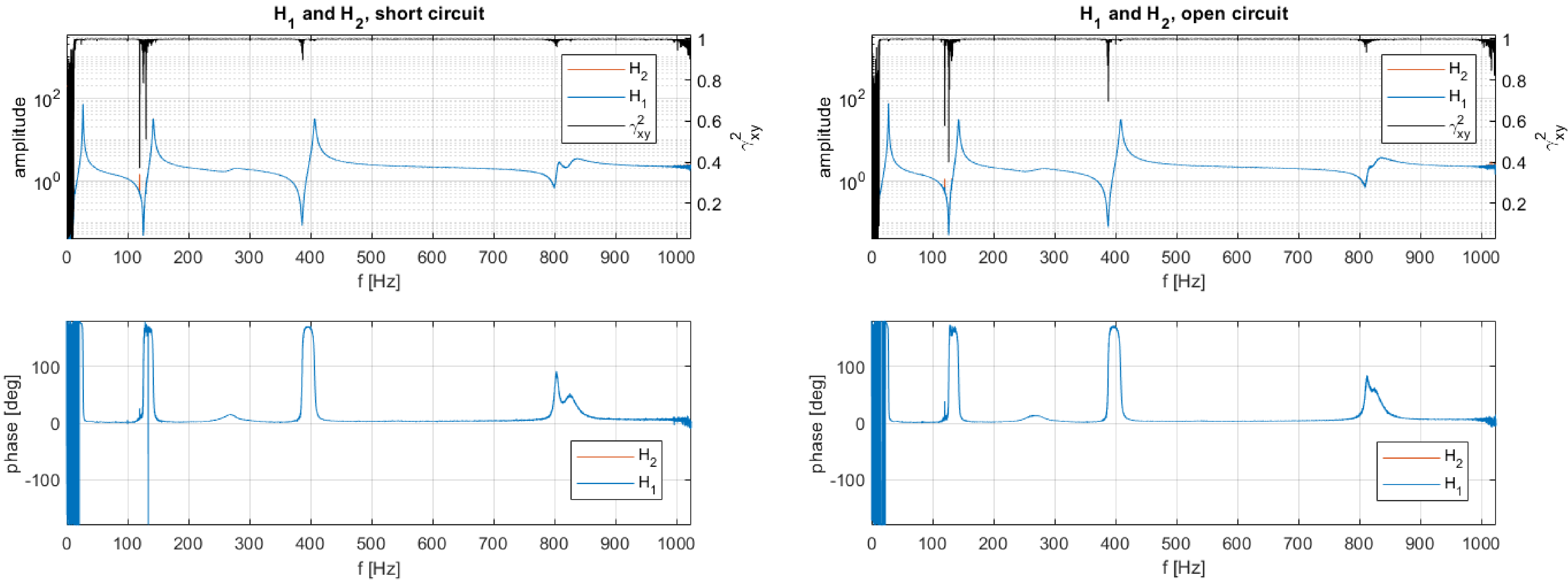


DATA PROCESSING – THOMSON APPROACH

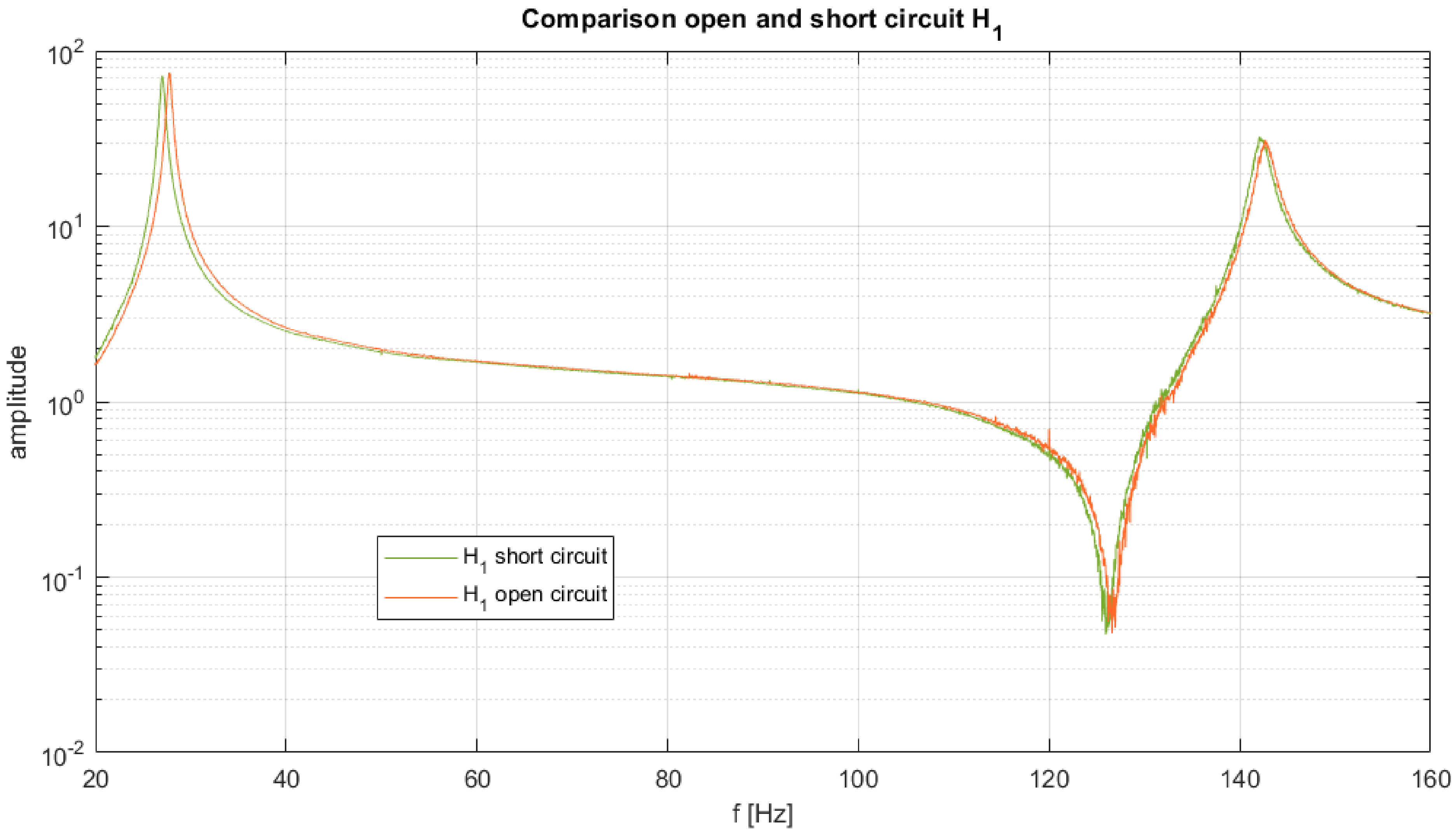
- Used to avoid the problem with resolution reduction when the acquisition time is too short
- Slepian windows to reduce leakage (k=6)



TRANSFER FUNCTION ESTIMATION

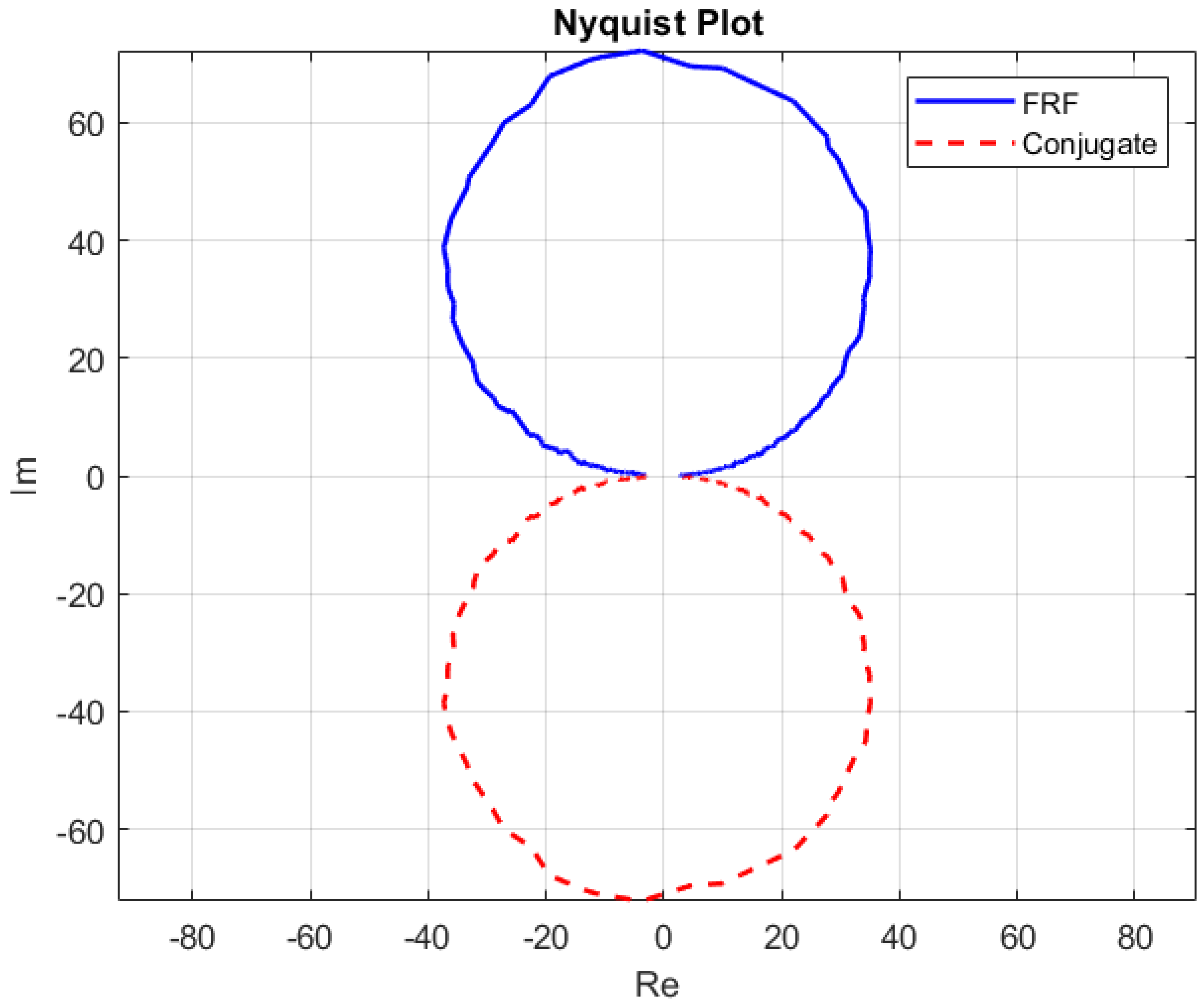


TRANSFER FUNCTION ESTIMATION



SDOF APPROXIMATION – NYQUIST PLOT

Very separate peaks
Low damping



SDOF APPROXIMATION – BEST FITTING

Least square minimization using as model the single DOF approximated TF:

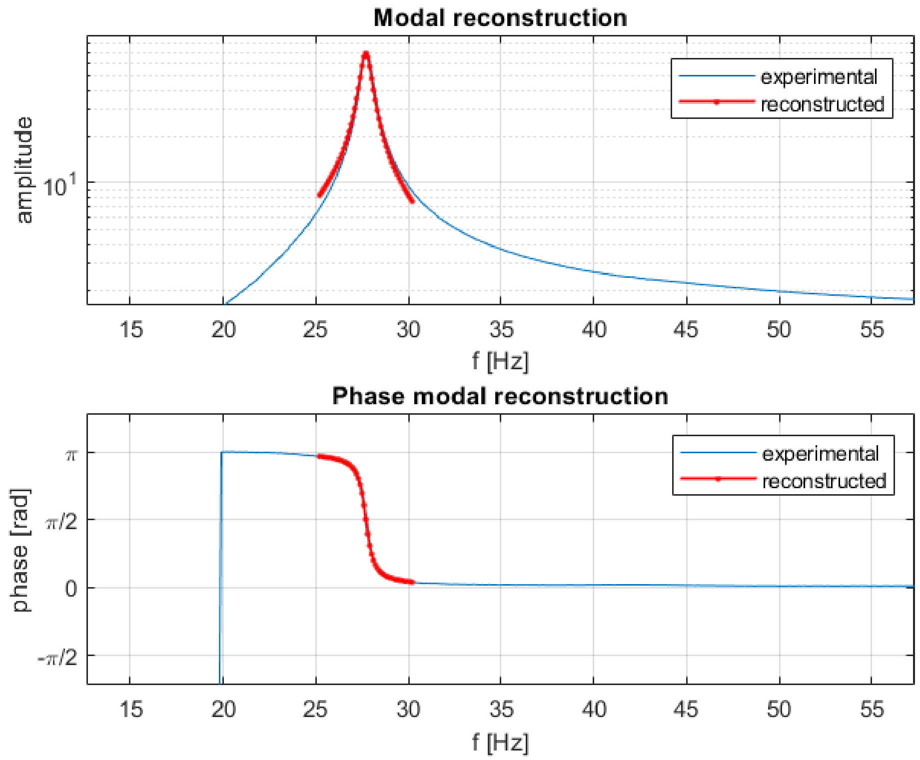
$$H^{mod}(\omega_k) = \frac{-\omega^2 A_k}{-\omega^2+2j\xi_k\omega_k\omega+\omega_k^2} + R_{k,l} + R_{k,h}\ \omega^2 \quad \text{with} \quad A_k = \frac{\psi_{ki}\cdot\psi_{kj}}{m_k}$$

First guesses:

- Natural frequencies: peak of the FRF magnitude
- Damping: phase derivative $\xi_k = -\frac{1}{\omega_k \cdot \frac{\partial\varphi}{\partial\omega}\Big|_{\omega_k}}$
- $A_k = -2\ Im[H(\omega_k)]\ \xi_k$

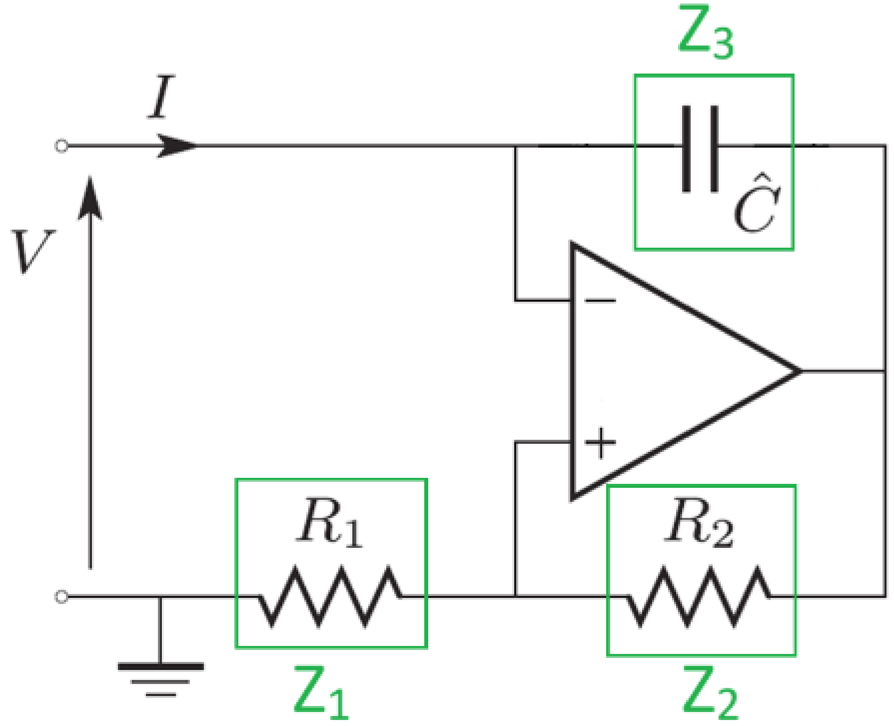
Results:

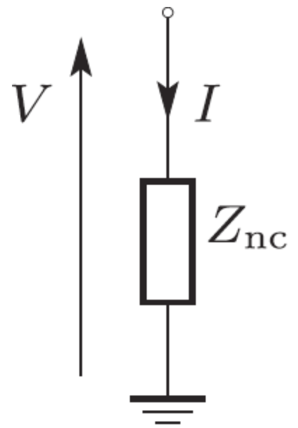
	f_1	ξ_1
Short circuit	27.00 Hz	0.0087
Open circuit	27.71 Hz	0.0132



BUILDING THE NEGATIVE CAPACITANCE

Considering an ideal OP-AMP





$$Z_{nc} = -\frac{Z_1 Z_3}{Z_2} = \frac{1}{j\,\Omega\,C_n}$$

$$C_n = -\frac{R_2 \hat{C}}{R_1}$$

This is called «ideal circuit» (IC)

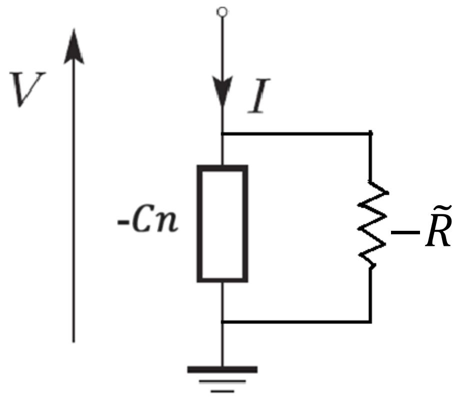
BUILDING THE REAL CIRCUIT

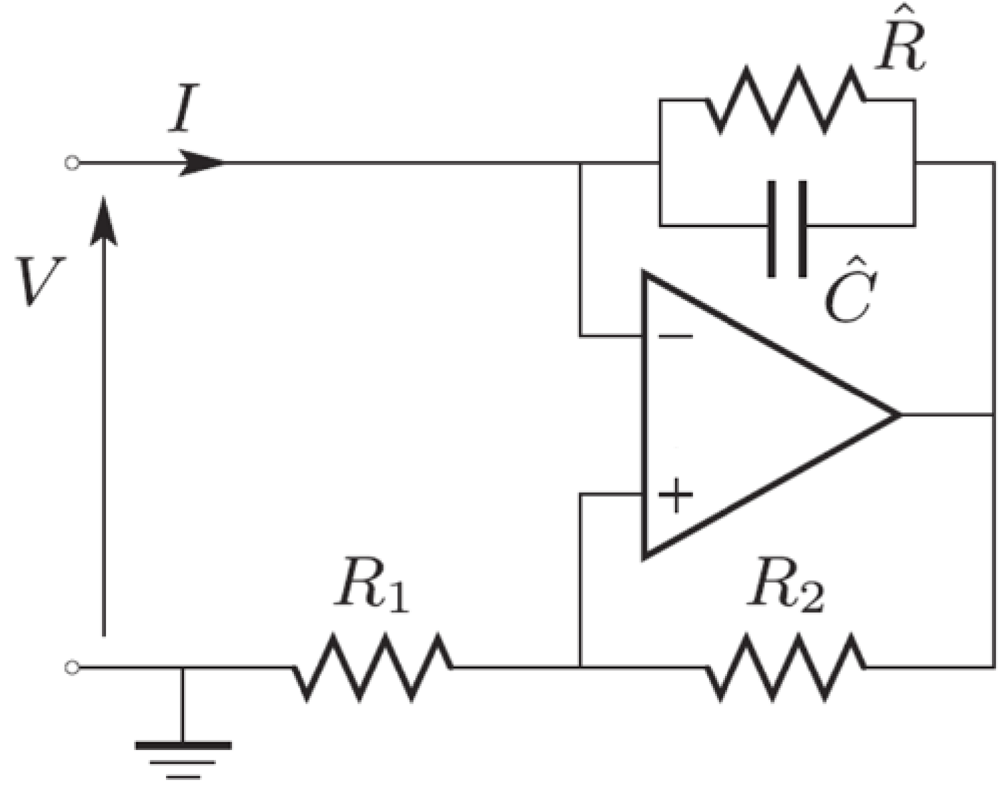
- Add $\hat{R}$ in parallel, that acts as a high-pass filter:

$$Z_{nc} = -\tilde{R} \,//\, C_n = -\frac{1}{\frac{1}{\tilde{R}} + j\omega C_n}$$

with $\tilde{R} = \frac{R_1 \hat{R}}{R_2}$

- Not anymore a negative pure capacitance!





This is called «real circuit» (RC)

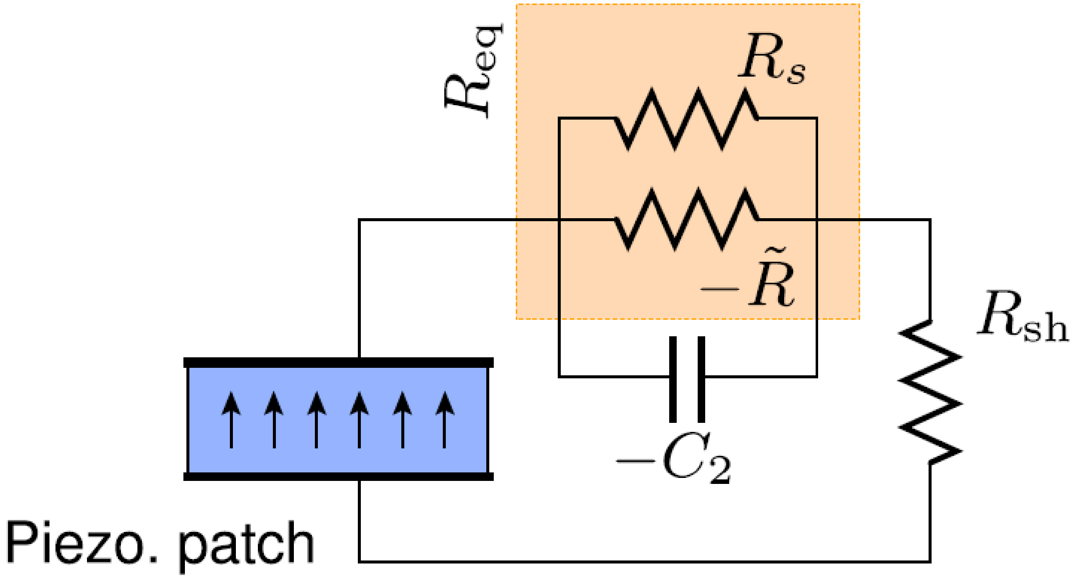
COMPENSATION RESISTANCE

- Add compensation resistance R_s in parallel with -$\tilde{R}$ and -C_2
- Equivalent resistance:

$$R_{eq} = -\tilde{R} \,//\, R_s = -\frac{\tilde{R}R_s}{R_s - \tilde{R}}$$

- Choose R_s to get $R_{eq} \rightarrow -\infty$

RC' ≈ IC



ELECTRICAL CIRCUIT SET-UP

- Real circuit

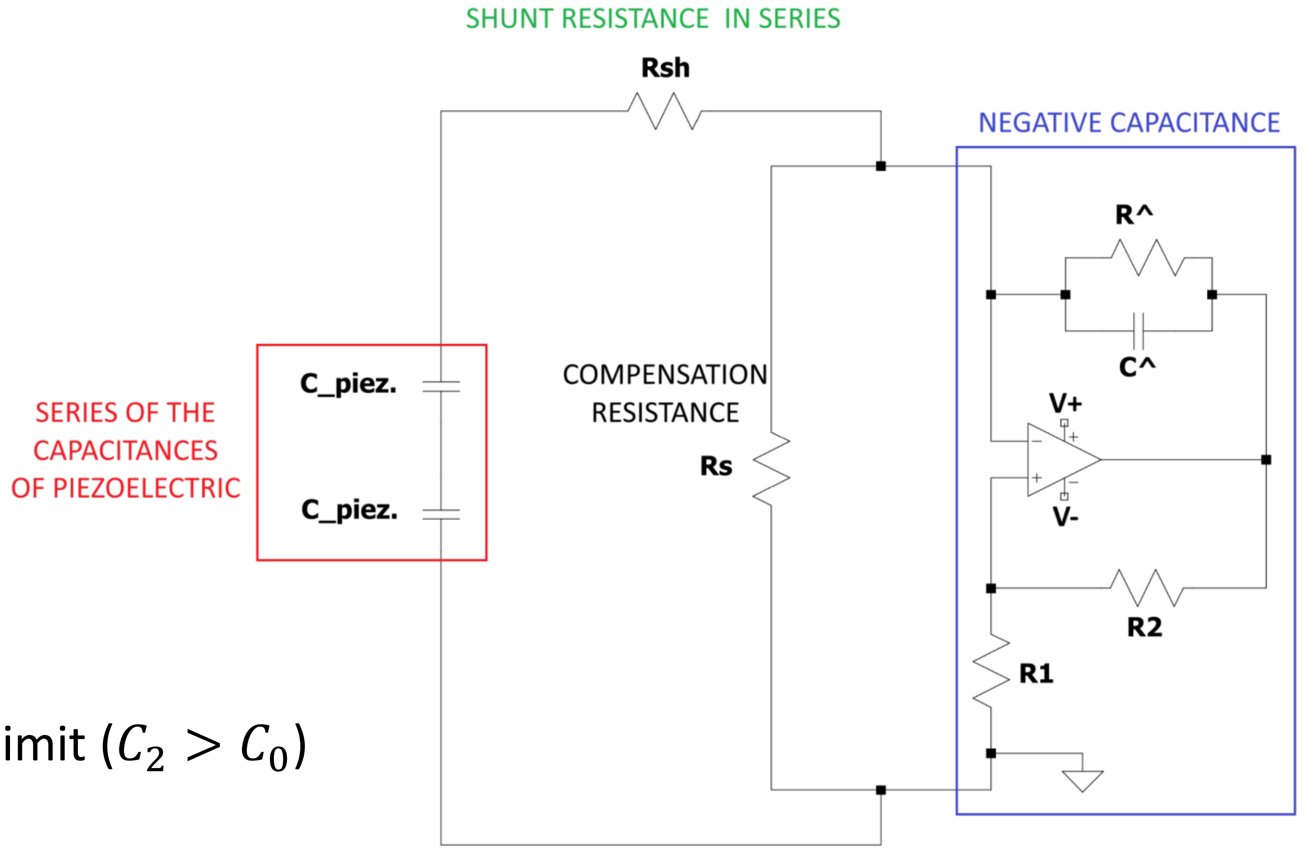
- Series configuration

- Higher performance at lowest mode

$$\text{highest } C_{pi} \;\; and \;\; \beta_2 = \frac{C_{pi}}{C_2}$$

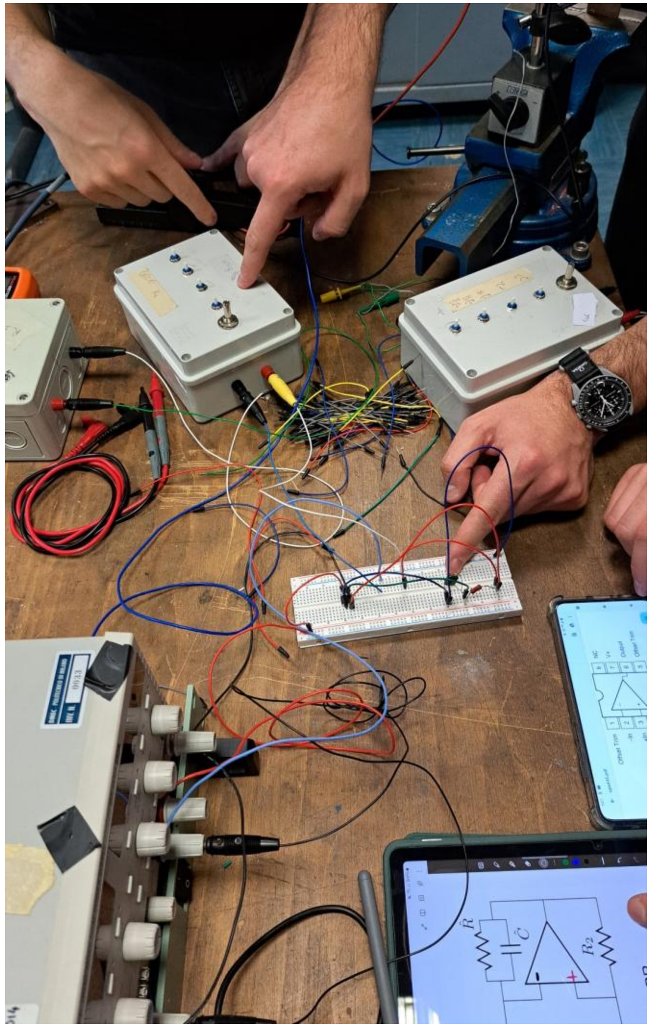
- Optimised for 1^{st} mode far from instability limit ($C_2 > C_0$)

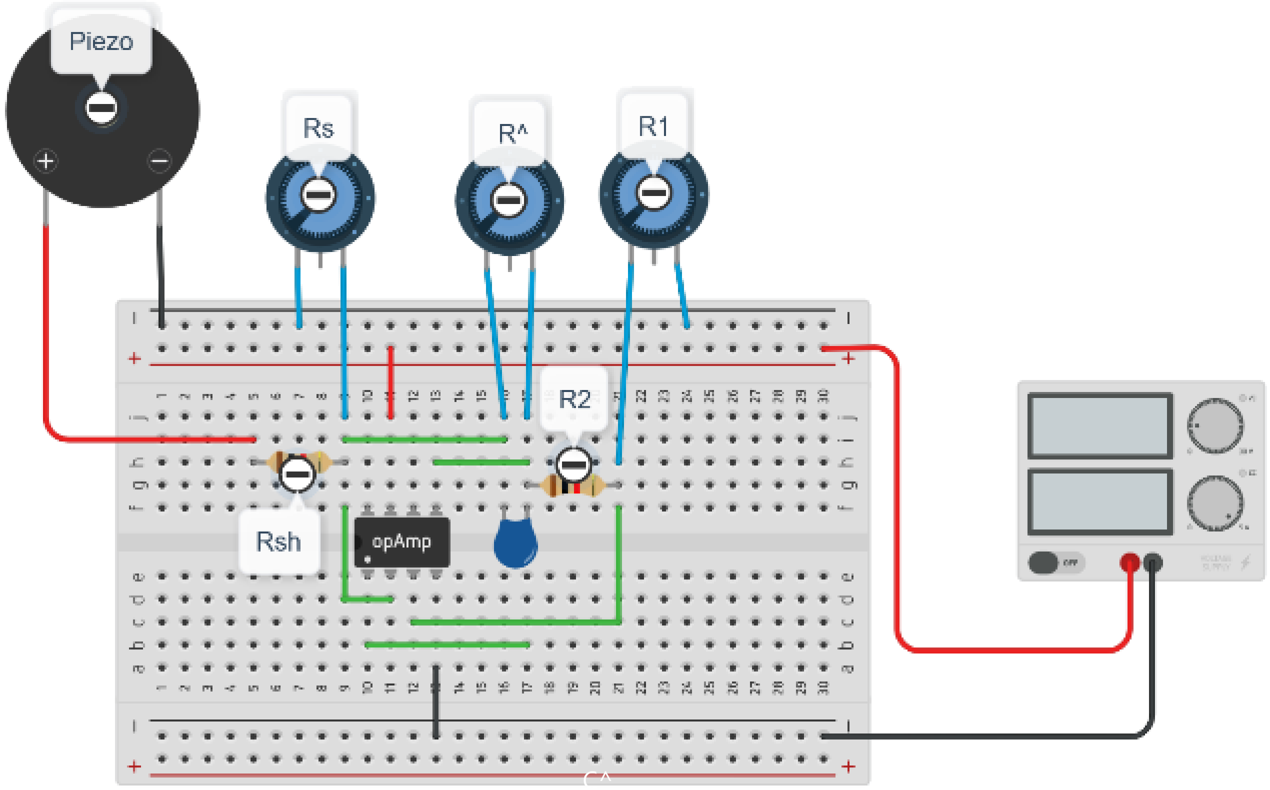
$$\beta_2 = \frac{C_{p1}}{C_2} = 0.70$$



Real circuit – Series Configuration

ELECTRICAL CIRCUIT SET-UP

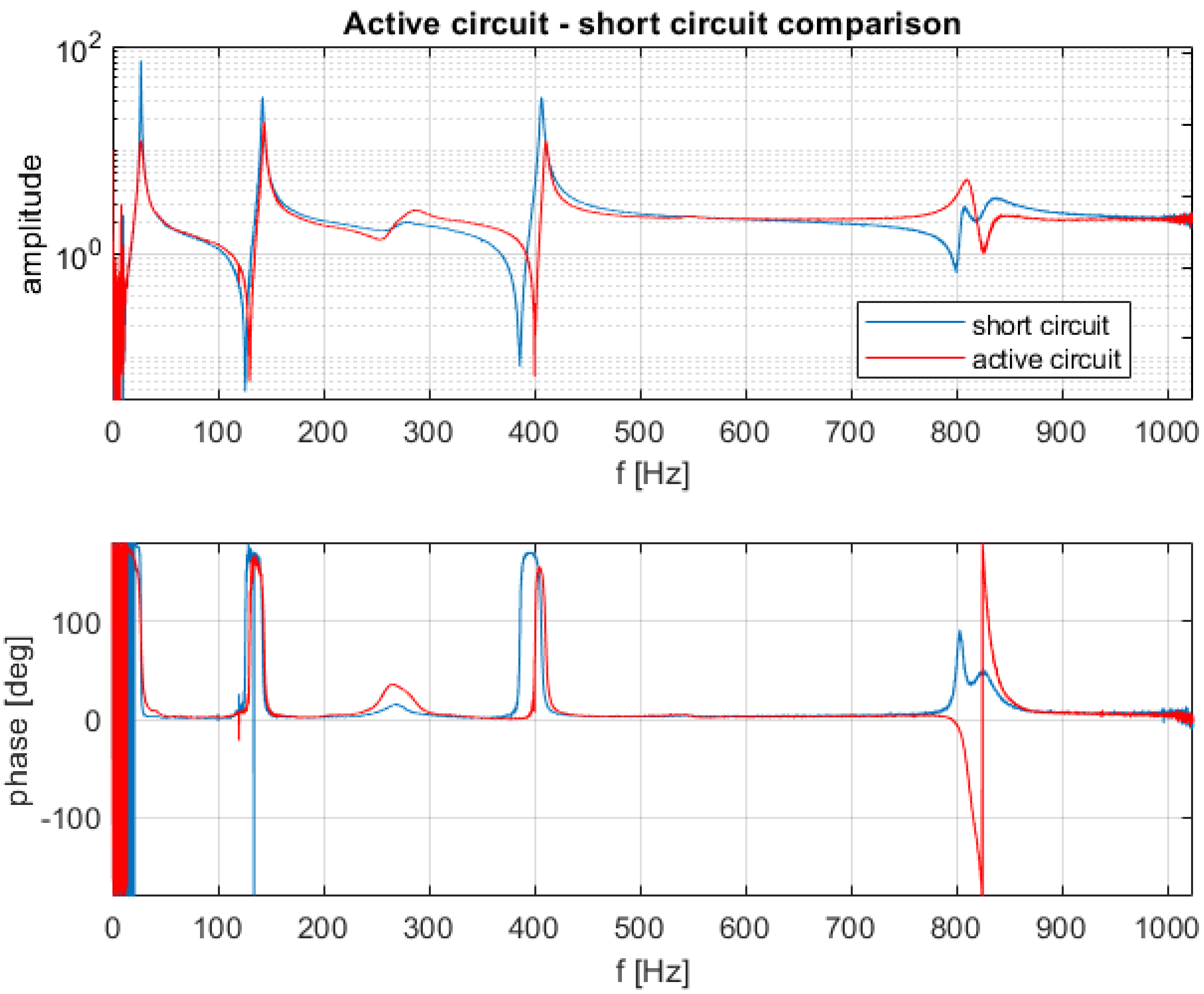




C_{p1}	C_2	$\hat{C}$	R_{sh}
37.75 nF	53.60 nF	67.80 nF	47.58 kΩ

R_2	R_1	$\hat{R}$	R_s
8.2 kΩ	10.4 kΩ	1.0 MΩ	1.30 MΩ

ACTIVE CIRCUIT EFFECT



	f_1	ξ_1
Short circuit	27.00 Hz	0.0087
Open circuit	27.71 Hz	0.0132
Active circuit	27.15 Hz	0.0848

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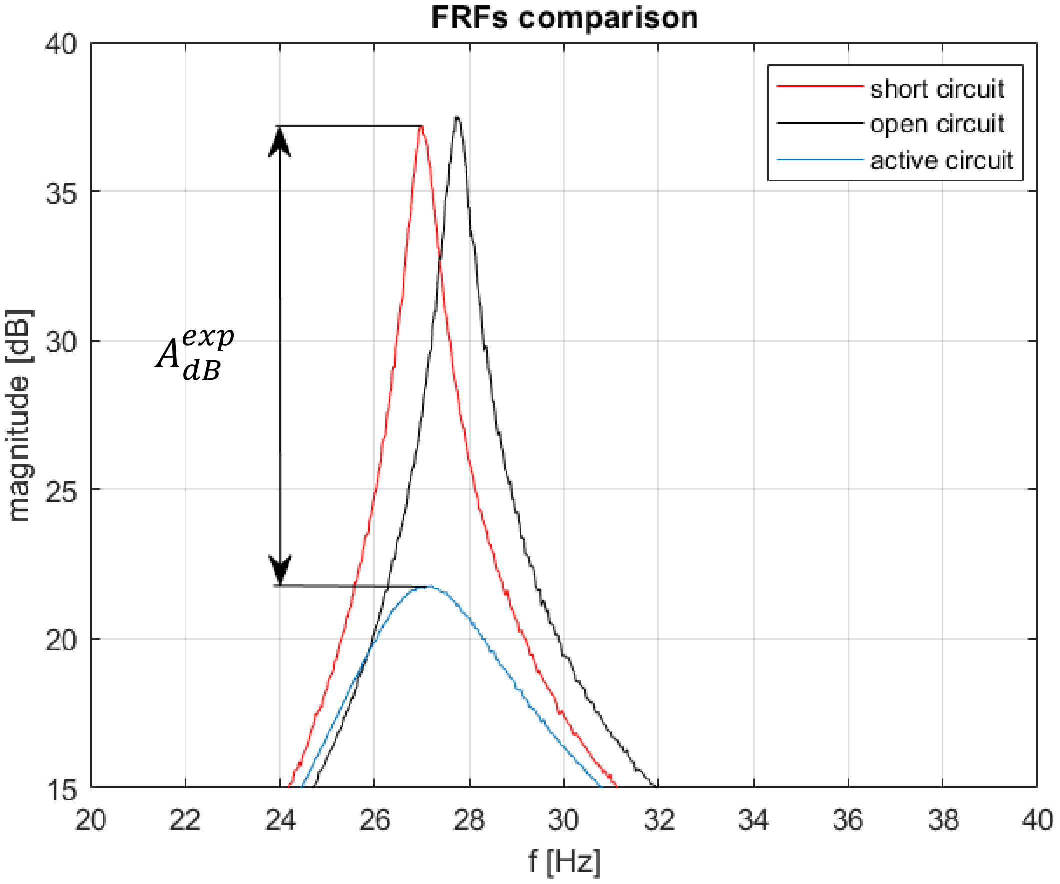
4. COMPARISON AND CONCLUSIONS

ATTENUATION EVALUATION - FREQUENCY



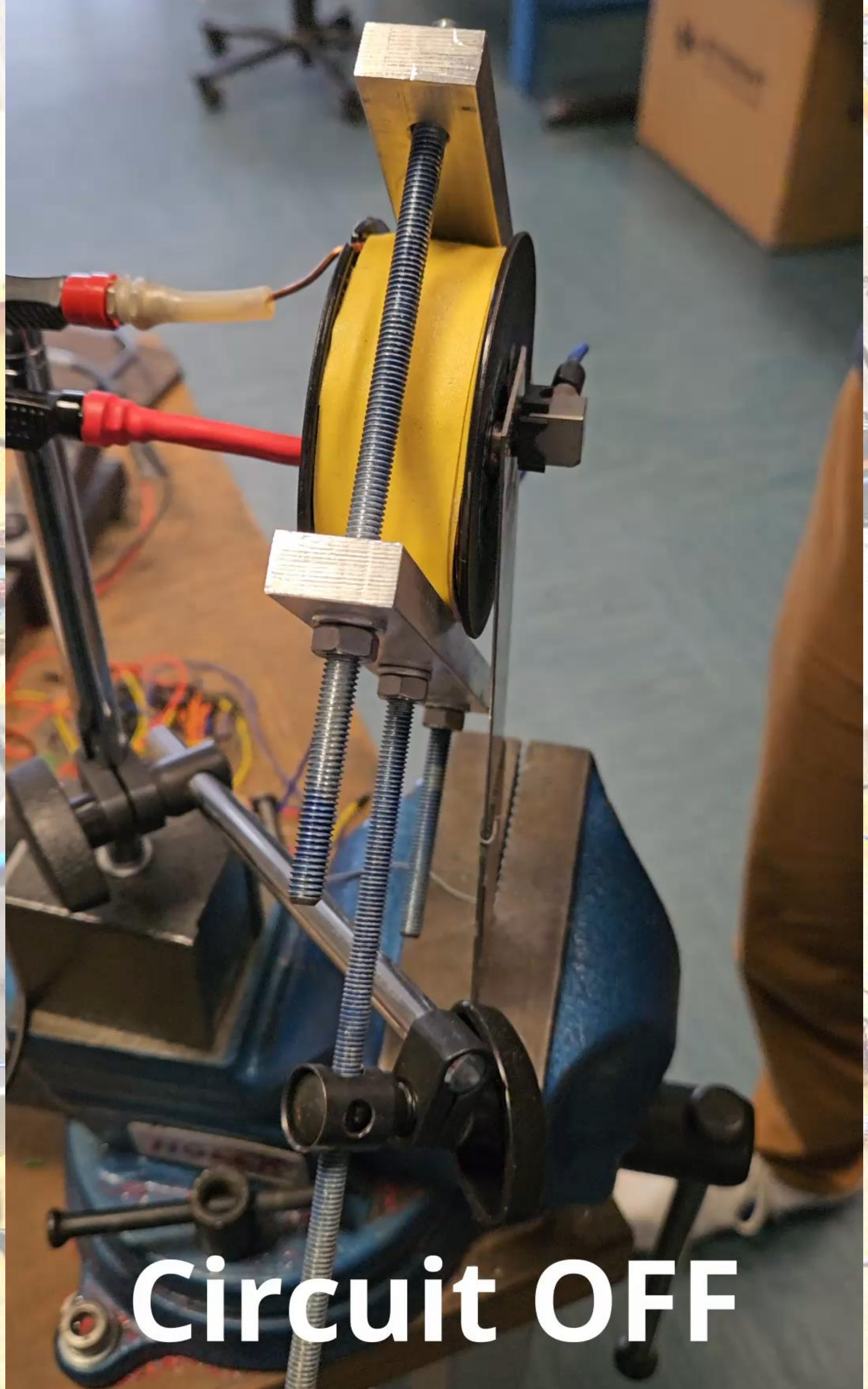
β_2	C_{p1} [nF]	k_1	$\tilde{k}_1$
0,704	37.75	0,24	0,44
τ_e^{opt}	R_{sh} [$k\Omega$]	$\tilde{k}_1/\mathrm{k}_1$	ξ_1^{active}
0,006	47,58	1,84	0.0848

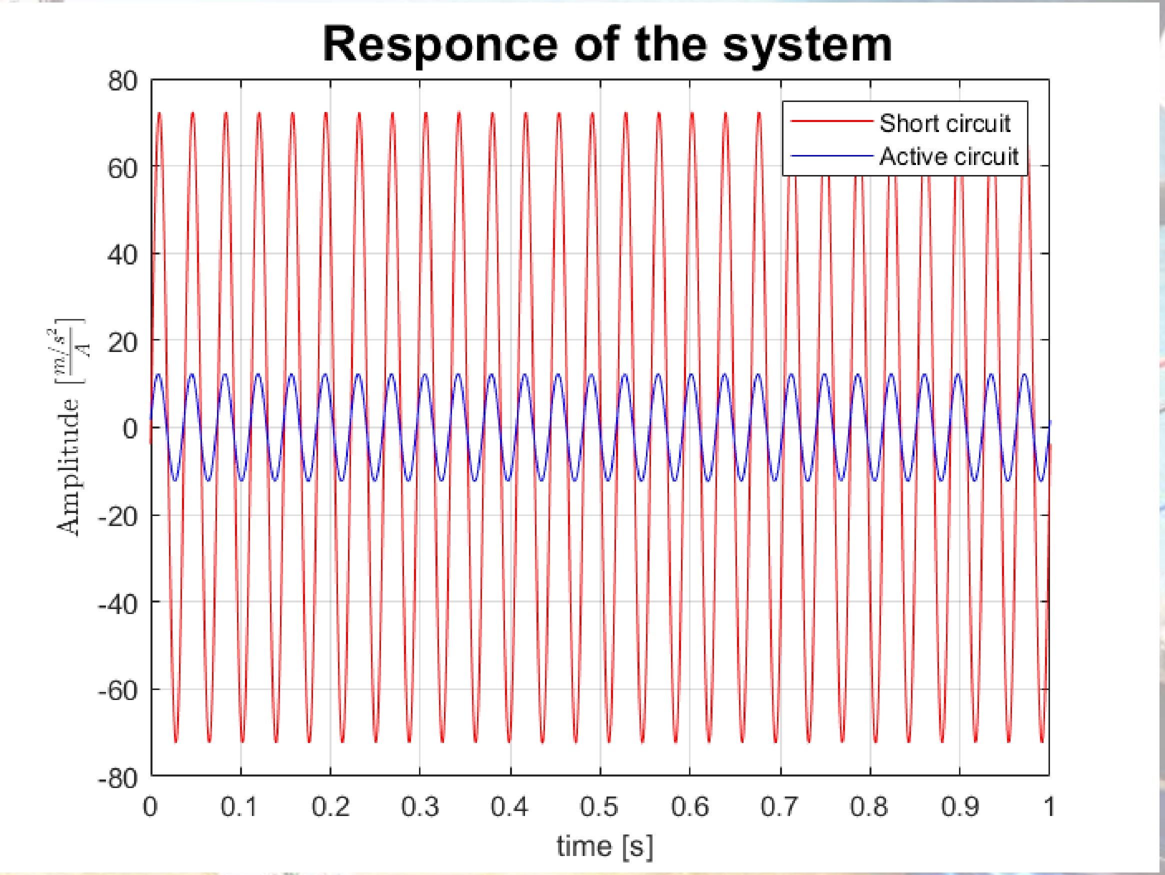
The resonance amplitude attenuation is:

$$A_{dB}^{exp} = 15.42\ dB$$

$$A_{dB}^{th} = 15.71\ dB$$

ATTENUATION EVALUATION - TIME





COSIDERATIONS

- Longer acqusition time
- Higher β_2
- Real components available are different from the ideal ones
- Changing circuit configuration
- Use of the better estimation of C_{p1}

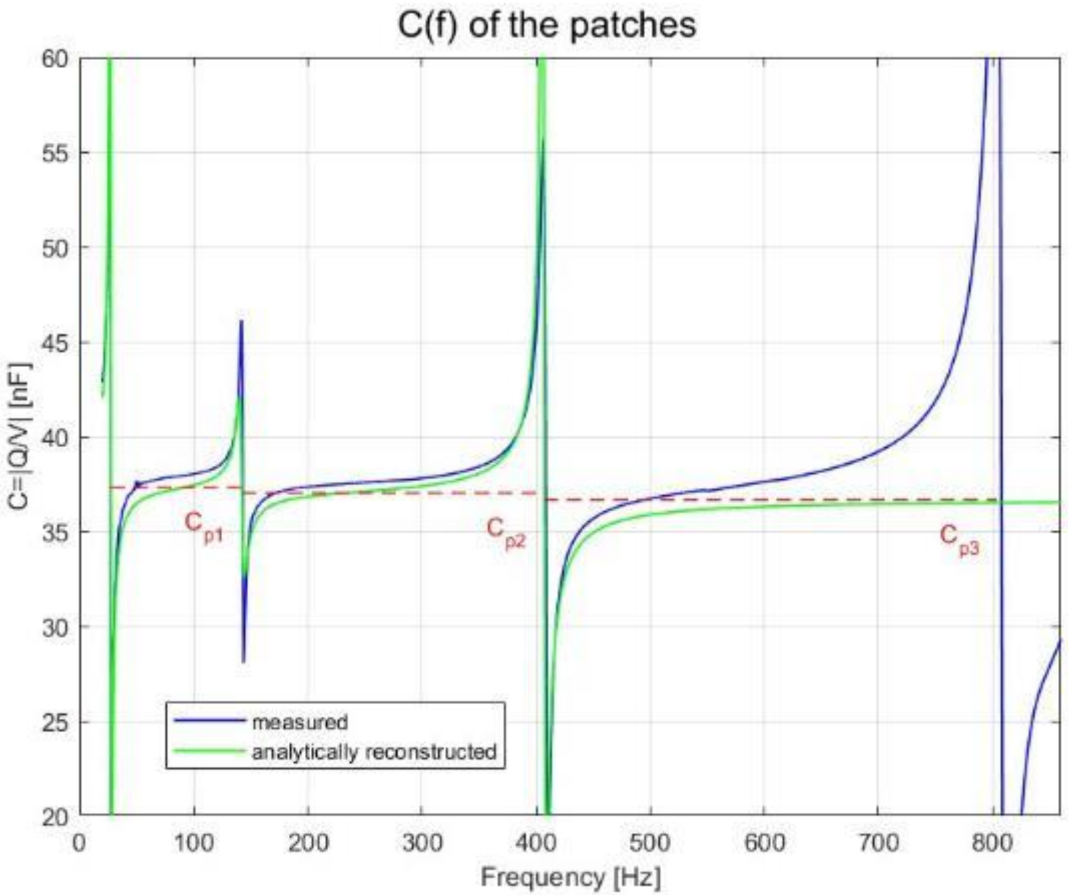
RECONSTRUCTED C(f) AND C_{PI}

C_{pi} values estimated by a fitting between analytical model and data provided.

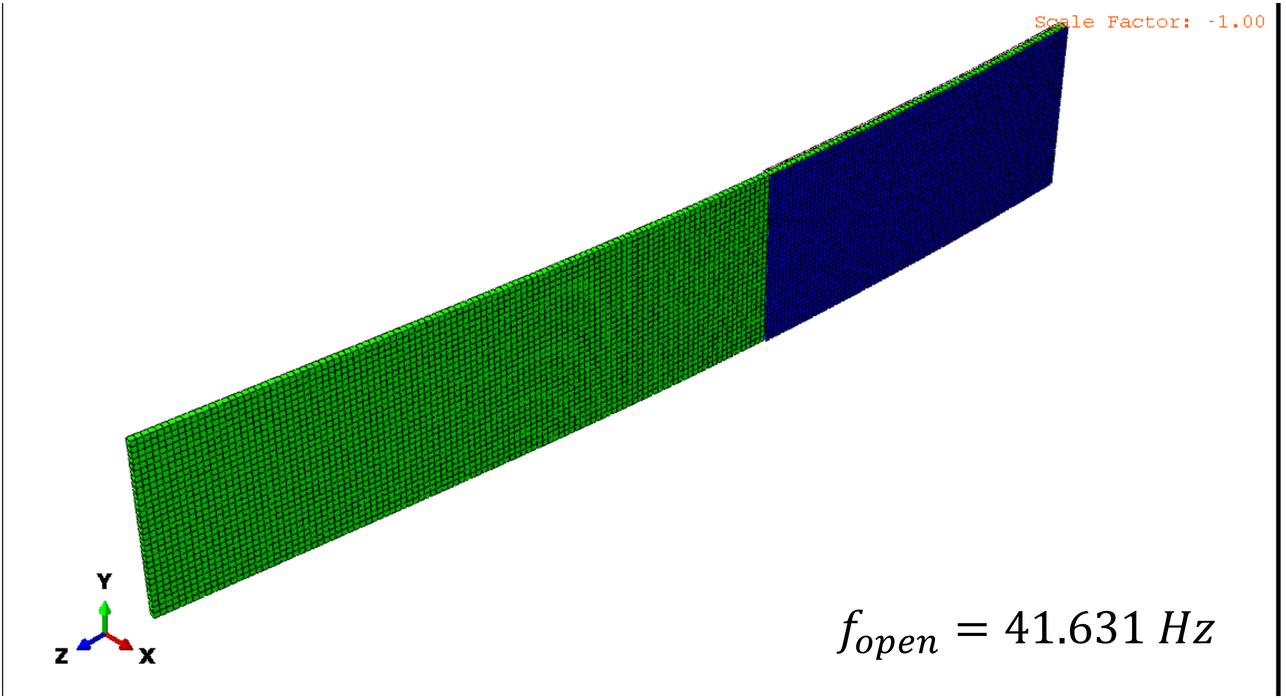
$$C(\Omega) \cong \sum_{i=1}^{3} \frac{C_{pi}\, k_i^2}{-\frac{\Omega^2}{\omega_i^2} + 2j\frac{\xi_i}{\omega_i}\Omega + 1} + C_{p4}$$

$$\text{with } C_{pi} = C_{pi+1}\,(1 + k_{i+1}^2)$$

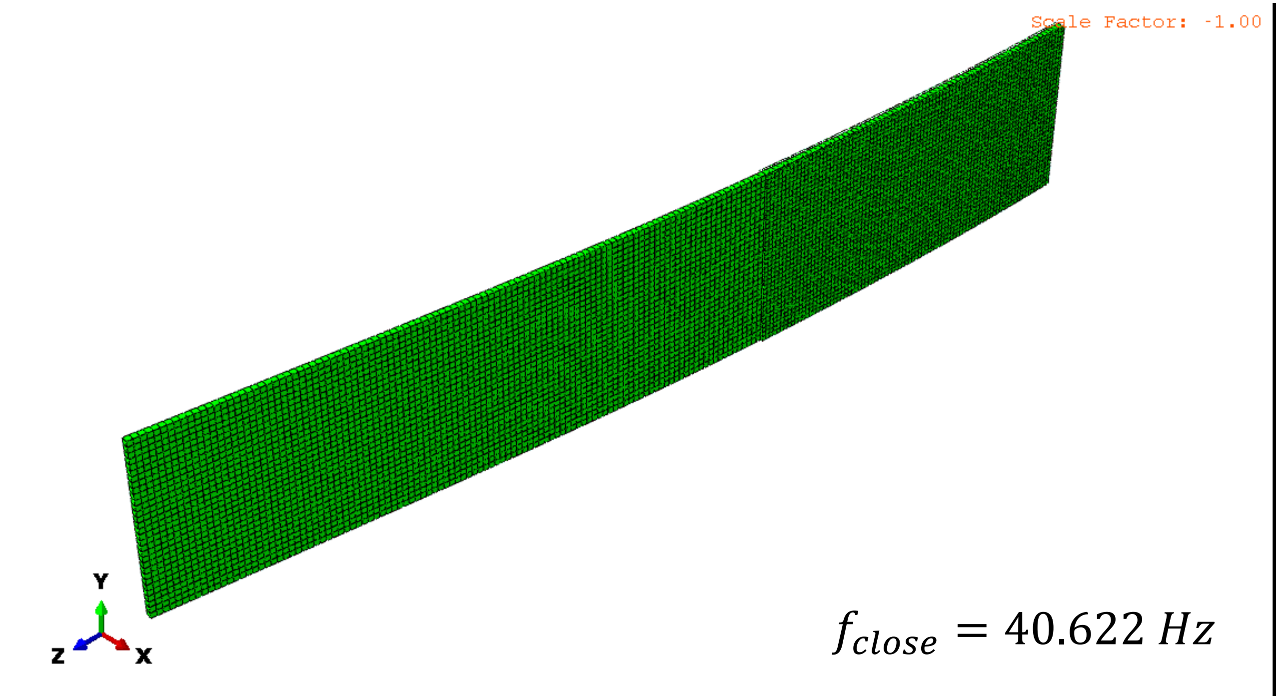
$\mathbf{C_{p0}}$	$\mathbf{C_{p1}}$	$\mathbf{C_{p2}}$	$\mathbf{C_{p3}}$
39.45 nF	37.30 nF	37.03 nF	36.65 kΩ



FEM MODEL



$f_{open} = 41.631\ Hz$



$f_{close} = 40.622\ Hz$

Note: the values shown should be compared to the ones of the paper as we have used the dimensions reported there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CONTACTS

edoardo.barutta@mail.polimi.it
lorenzo.bauce@mail.polimi.it
davide.meneghini@mail.polimi.it
kristjan.milic@mail.polimi.it
nicola.visentin@mail.polimi.it

www.mecc.polimi.it

@meccpolimi

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