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DEPARTMENT OF MECHANICAL ENGINEERING

2D Ball Balancing

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Summary

1. Bench description and calibration
2. Mathematical model
3. Control logics and experimental testing
4. Conclusions

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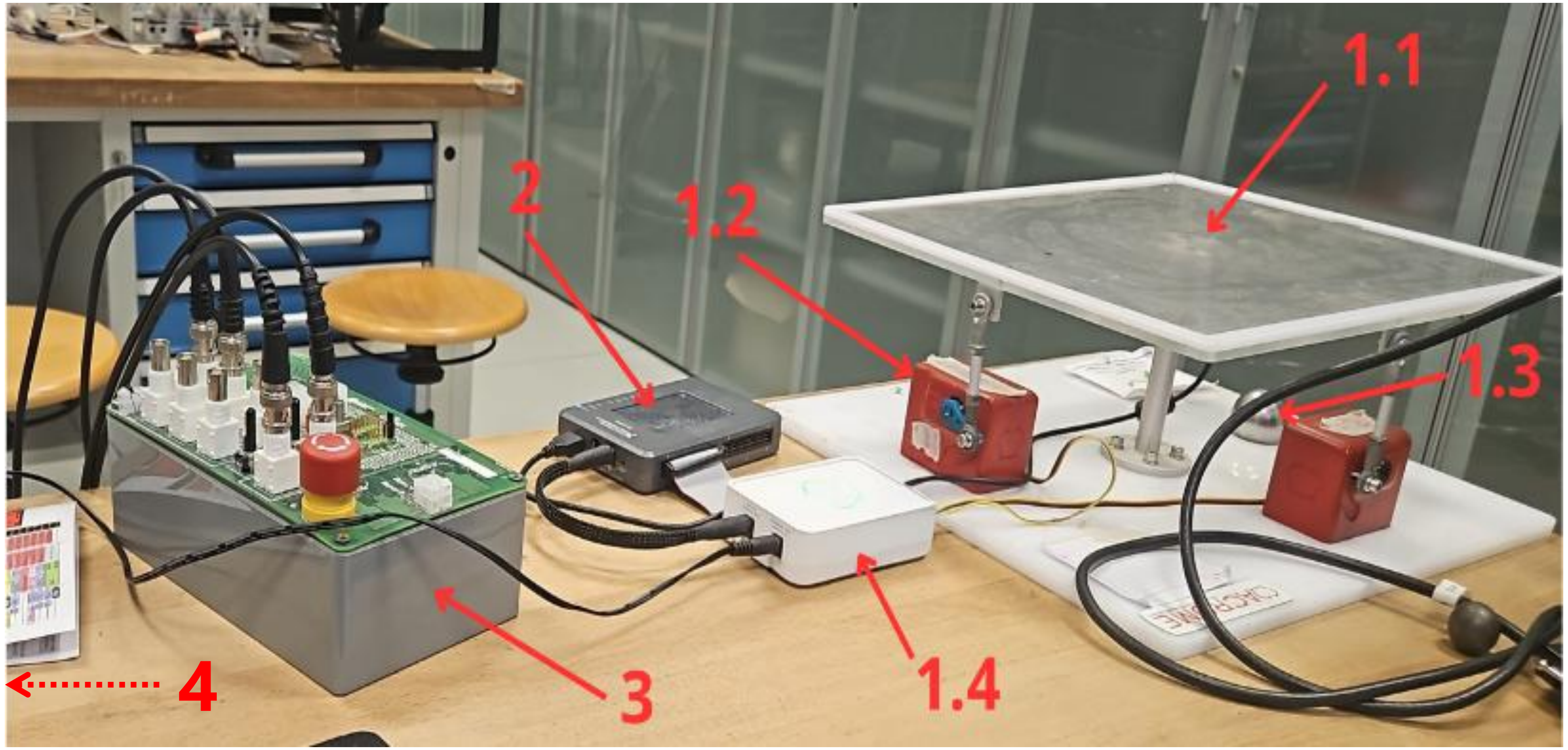
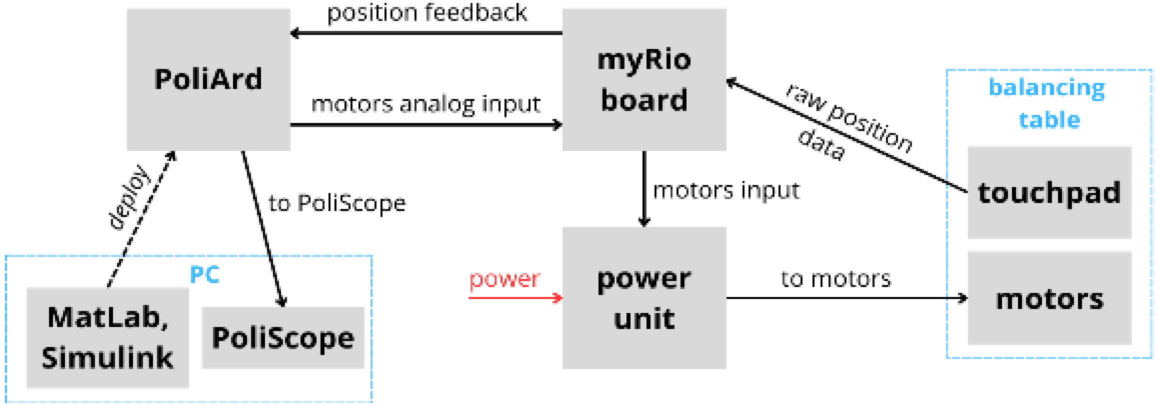
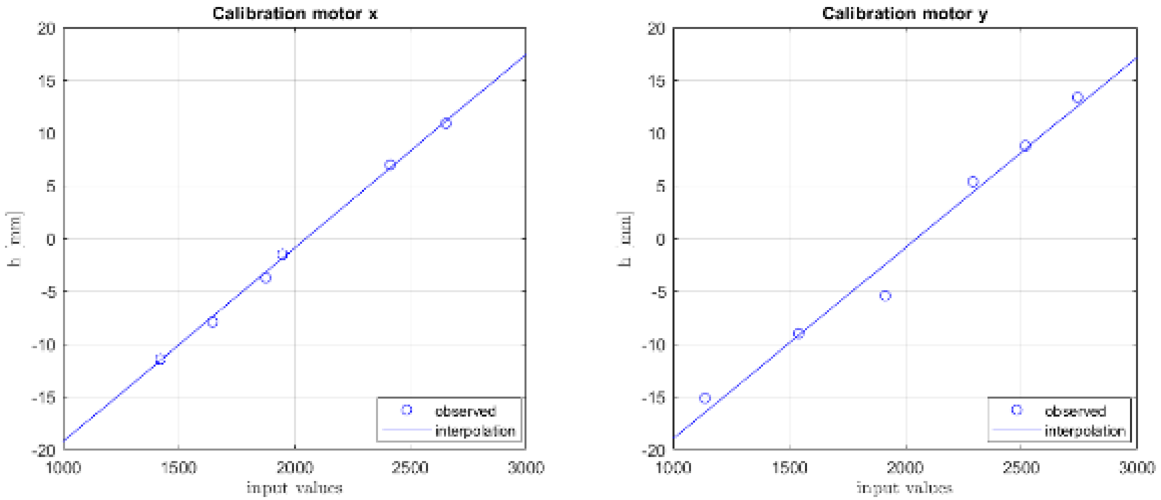
1. BENCH DESCRIPTION AND CALIBRATION

ORIGINAL BENCH

1. Acrome Ball Balancing Table set:
   1.1. touchpad panel
   1.2. servomotors
   1.3. steel ball
   1.4. power unit
2. MyRio microprocessor board
3. PoliArd unit (Arduino)
4. PC (Matlab/Simulink and PoliScope)

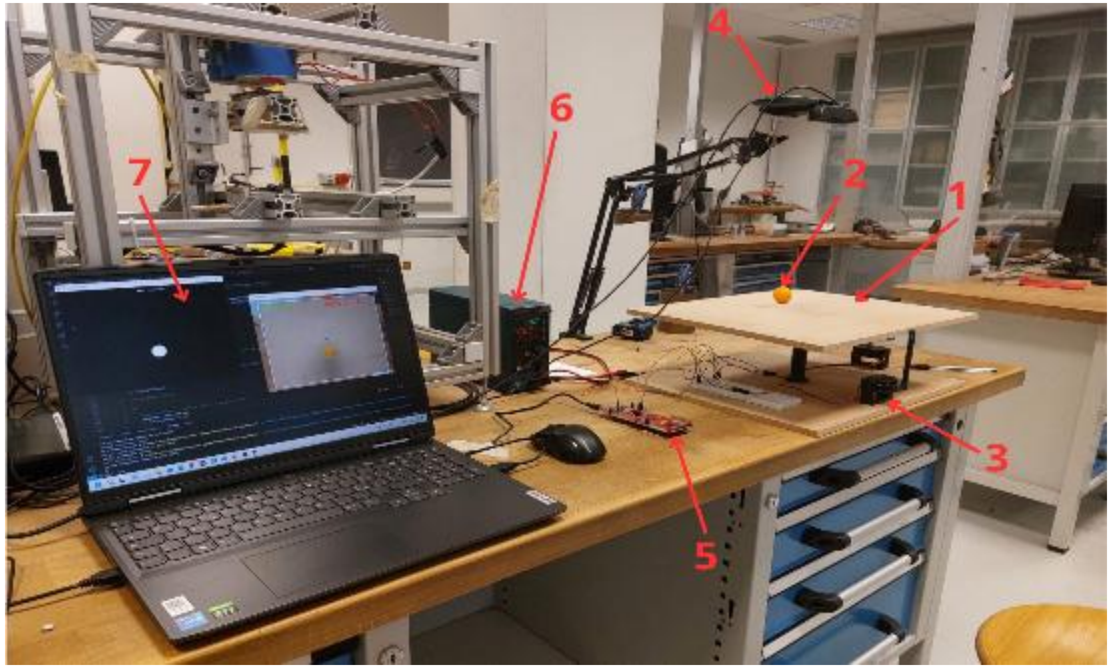


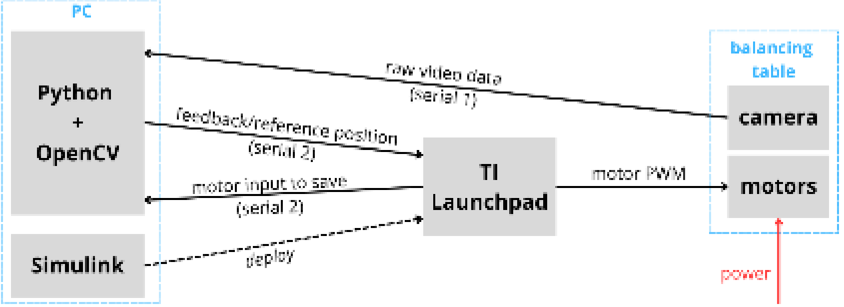


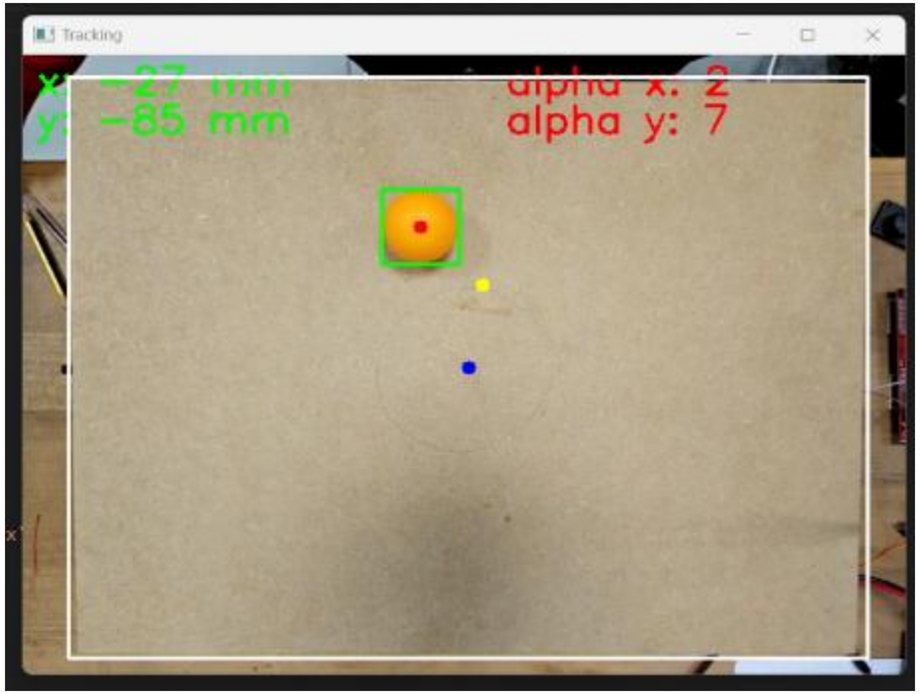


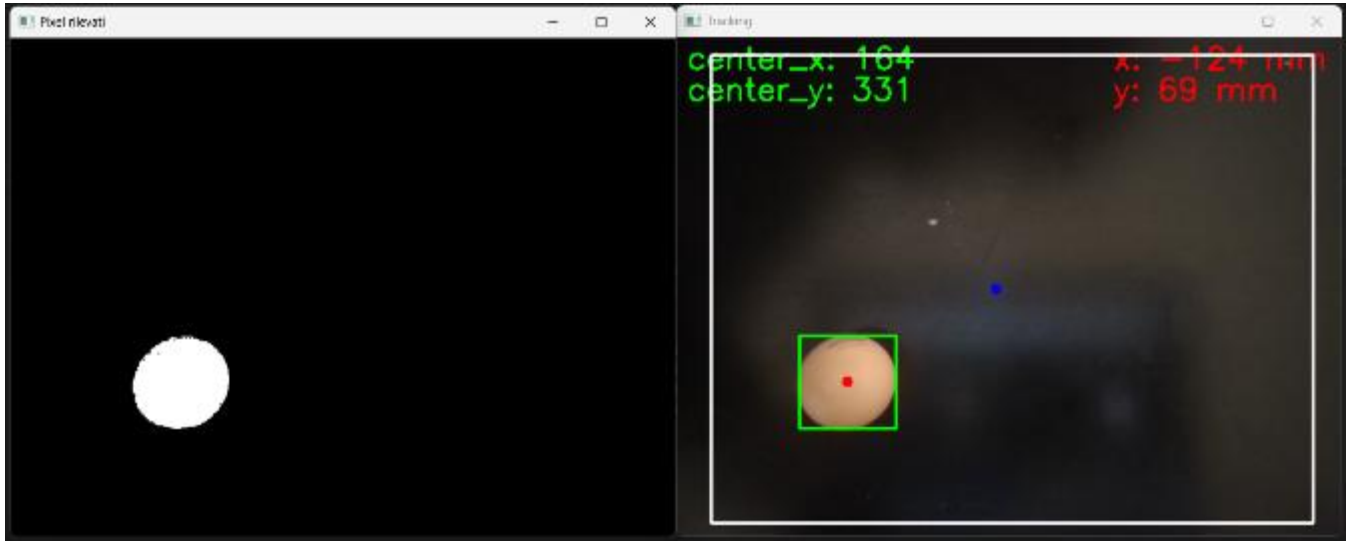
NEW BENCH

1. MDF panel
2. ping pong ball
3. servomotors
4. camera (smartphone)
5. TI Launchpad microcontroller board
6. Power supply
7. PC (Phyton/OpenCV and Matlab/Simulink)









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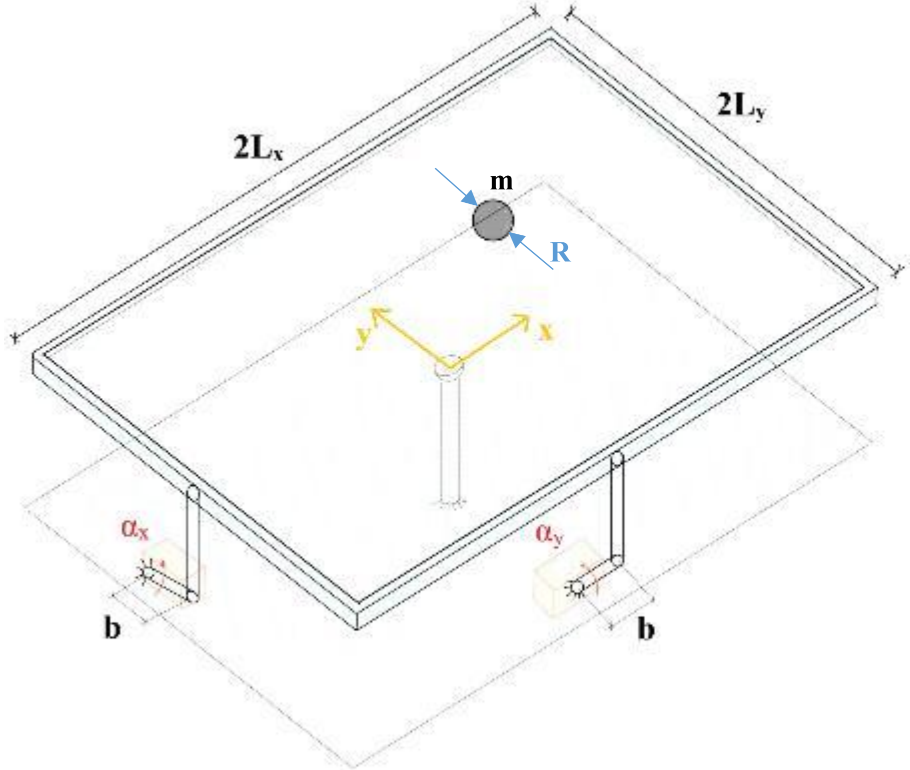
2. MATHEMATICAL MODEL

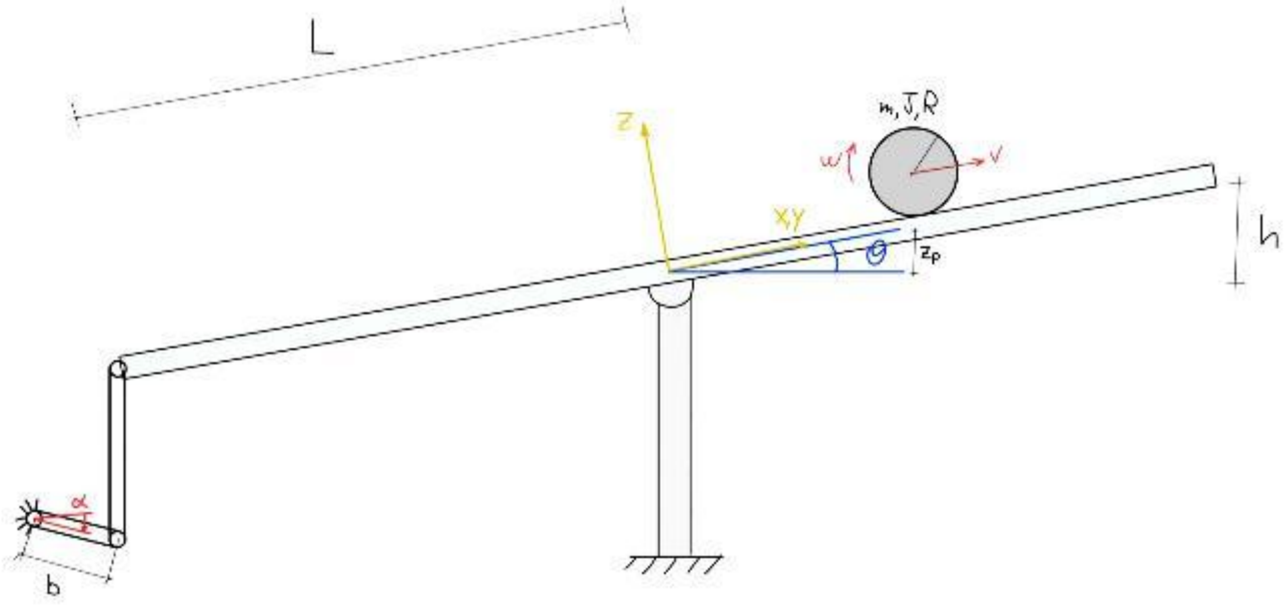
ASSUMPTIONS AND DATA

- Ball rolls without slipping
- Massless table and rods
- Motions in x and y are decoupled
- Small θ angles
- Negligible dissipations of the rolling ball
- Negligible centrifugal force due to platform rotation



	Original bench	New bench
L_x	$182\ mm$	$225\ mm$
L_y	$150\ mm$	$175\ mm$
b	$24.5\ mm$	$30\ mm$
m	$0.264\ kg$	$0.004\ kg$
R	$20\ mm$	$20\ mm$
J	$4.22 \cdot 10^{-5}\ kg \cdot m^2$	$2.56 \cdot 10^{-6}\ kg \cdot m^2$

DERIVATION



Lagrange equations approach:

Kinetic energy $\rightarrow$ $E_{kin} = \frac{1}{2}\left(m + \frac{J}{R^2}\right)\dot{x}^2$

Potential energy $\rightarrow$ $E_{pot} = mgx\, sin\theta$

$L\, sin\theta = -b\, sin\alpha$

$$\frac{d}{dt}\frac{\partial E_{kin}}{\partial \dot{x}} - \frac{\partial E_{kin}}{\partial x} + \frac{\partial D}{\partial \dot{x}} + \frac{\partial E_{pot}}{\partial x} = Q$$

Equation of motion:

$$\ddot{x} = \frac{mg}{\left(m+\frac{J}{R^2}\right)}\frac{b}{L}\sin\alpha \rightarrow \begin{cases} \dot{x} = \dot{z}_1 = z_2 \\ \ddot{x} = \dot{z}_2 = \frac{mg}{\left(m+\frac{J}{R^2}\right)}\frac{b}{L}\sin\alpha = C\, sin\alpha \end{cases} \rightarrow \quad linearization: \begin{bmatrix}\dot{z}_1 \\ \dot{z}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix} \cdot \begin{bmatrix}z_1 \\ z_2\end{bmatrix} + \begin{bmatrix}0 \\ C\end{bmatrix}\alpha$$

[same for x and y directions]

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3. CONTROL LOGICS AND EXPERIMENTAL TESTING

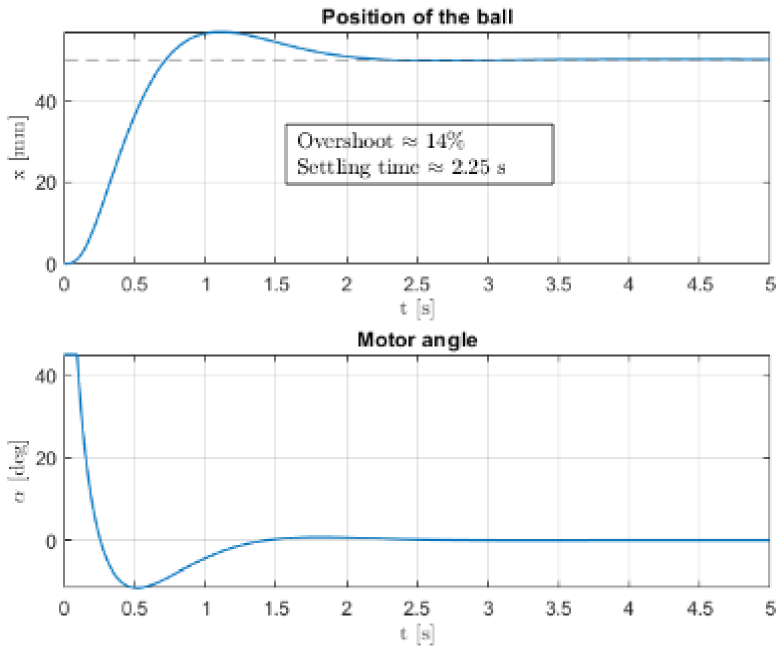
PD - ANALITICAL TUNING

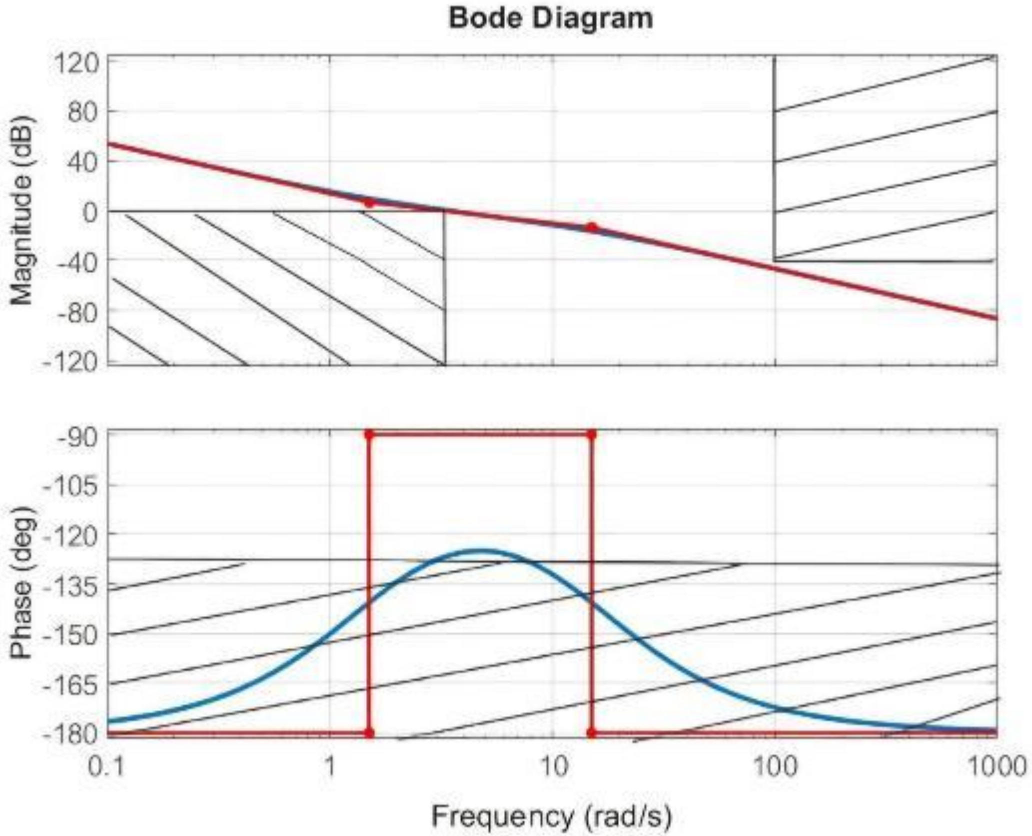
Overshoot: $S\% < 20 \rightarrow \Psi_m > 54°$

Settling time: $T_{s,\epsilon} = \dfrac{4.6}{\omega_c\,\xi} < 3\ s \rightarrow \omega_c > 3.4\ \dfrac{rad}{s}$

Noise reduction: ($40\ dB$ attenuation after $100\ \dfrac{rad}{s}$)

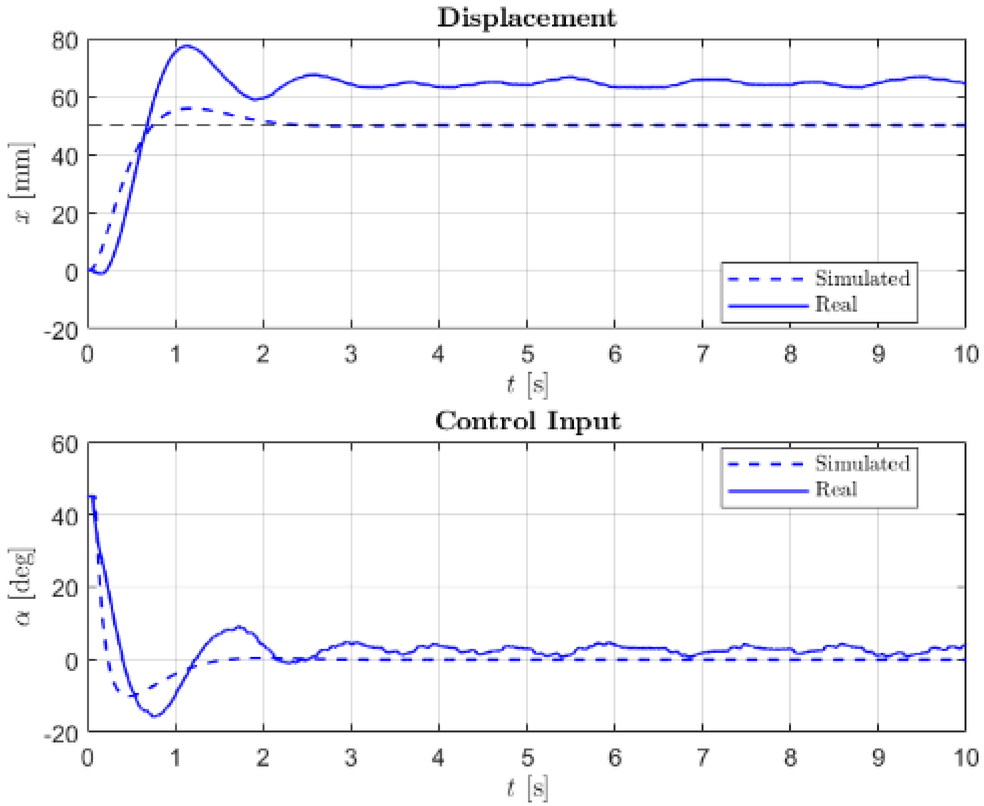
$\rightarrow |F| \approx |L| < -40\ dB$



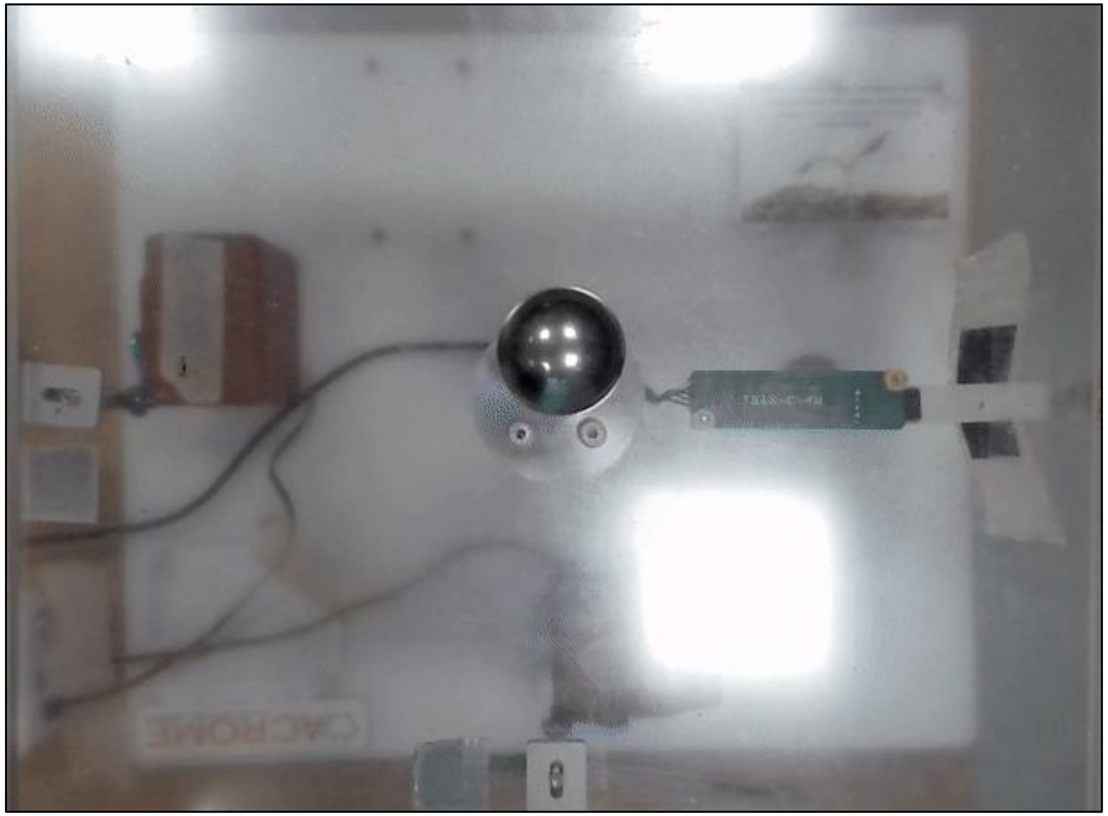


Resulting gains $\rightarrow K_p = 5,\ K_d = 3$

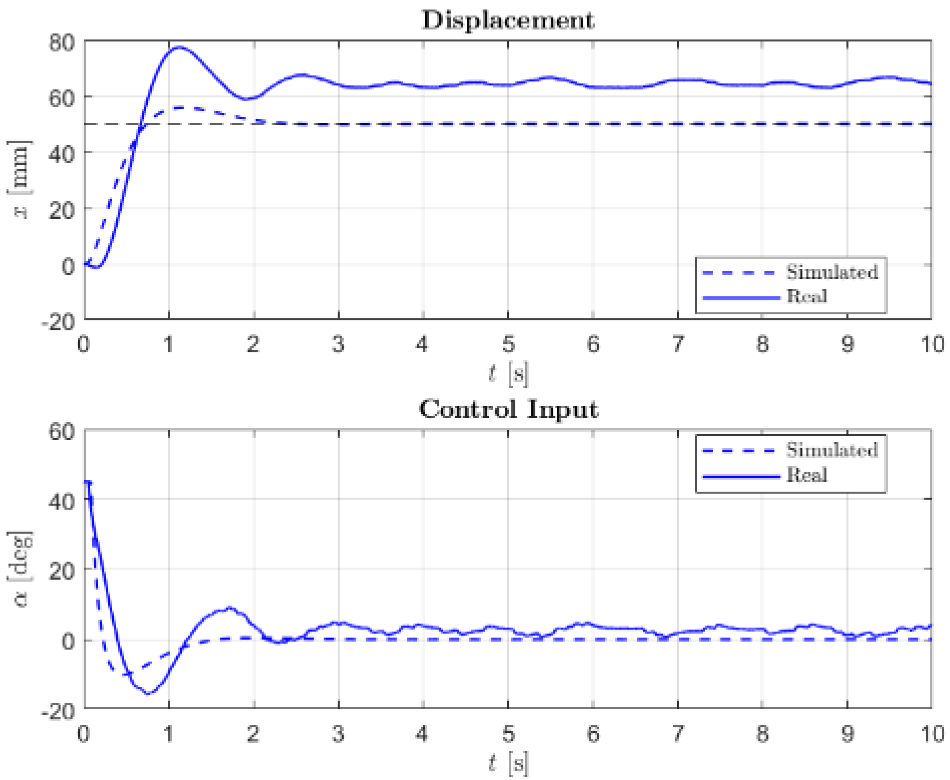
PD - STEP RESPONSE



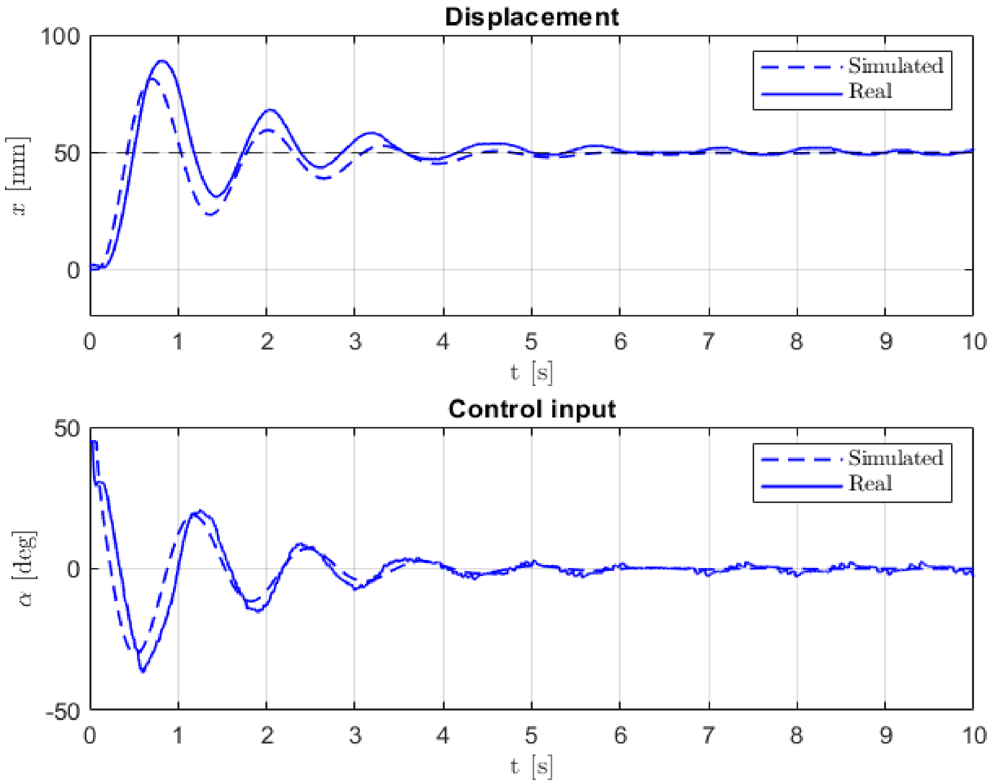
PD: $K_p = 5, \quad K_d = 3$



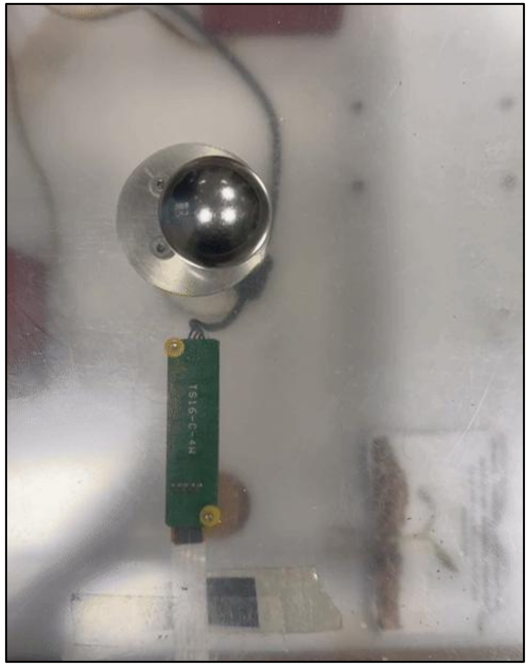
PD AND PID - STEP RESPONSE



PD: $K_p = 5, \;\; K_d = 3$

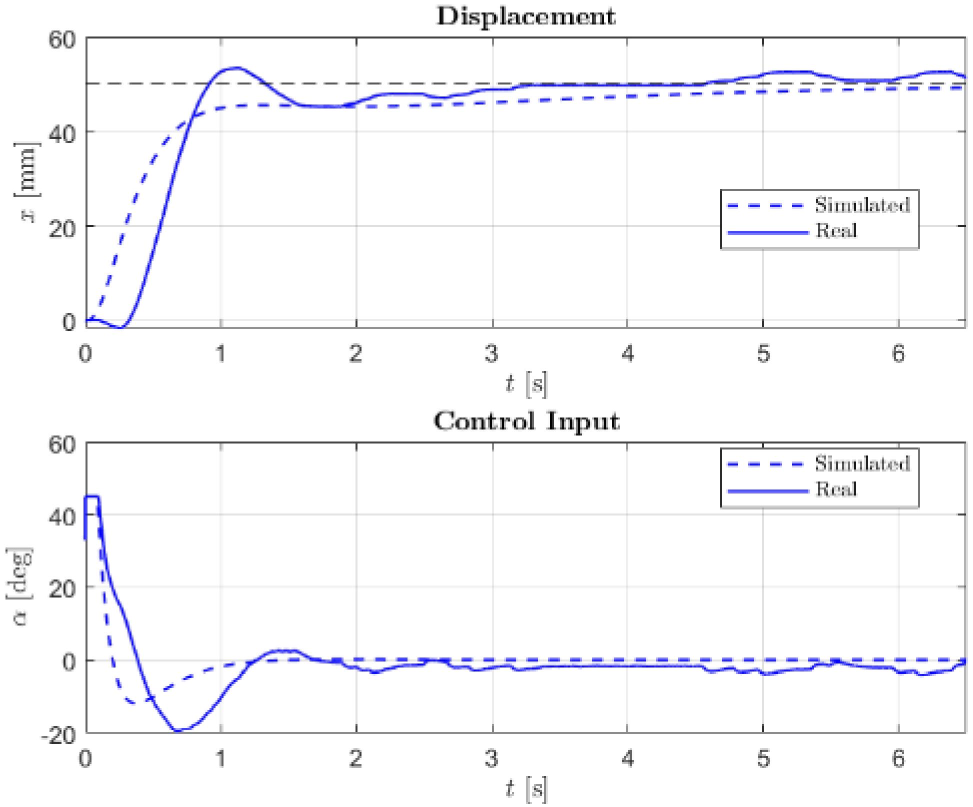


PID: $K_p = 6, \;\; K_i = 3, \;\; K_d = 3$

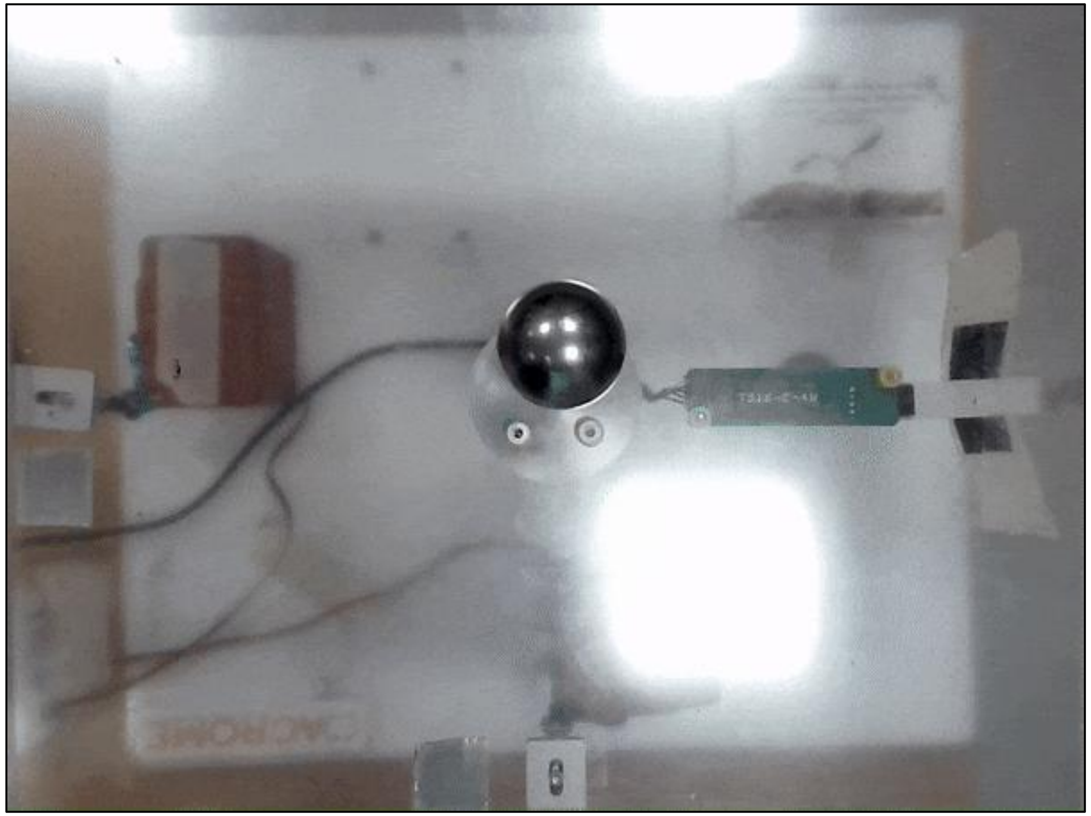


Integral action: no steady-state error, more overshoot and longer settling time → slower response.

PID - STEP RESPONSE

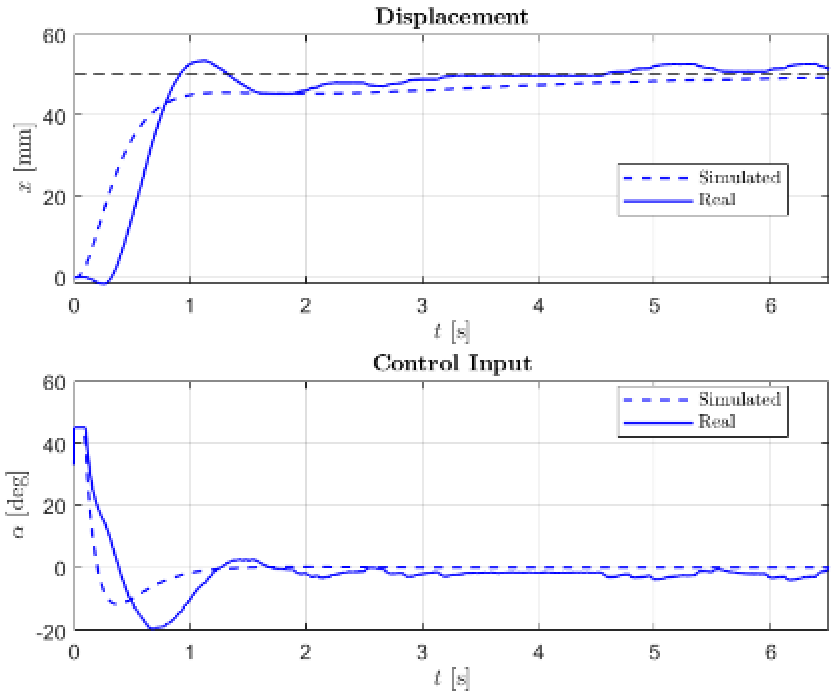


PID: $K_p = 4.5, \;\; K_i = 1.5, \;\; K_d = 3.5$

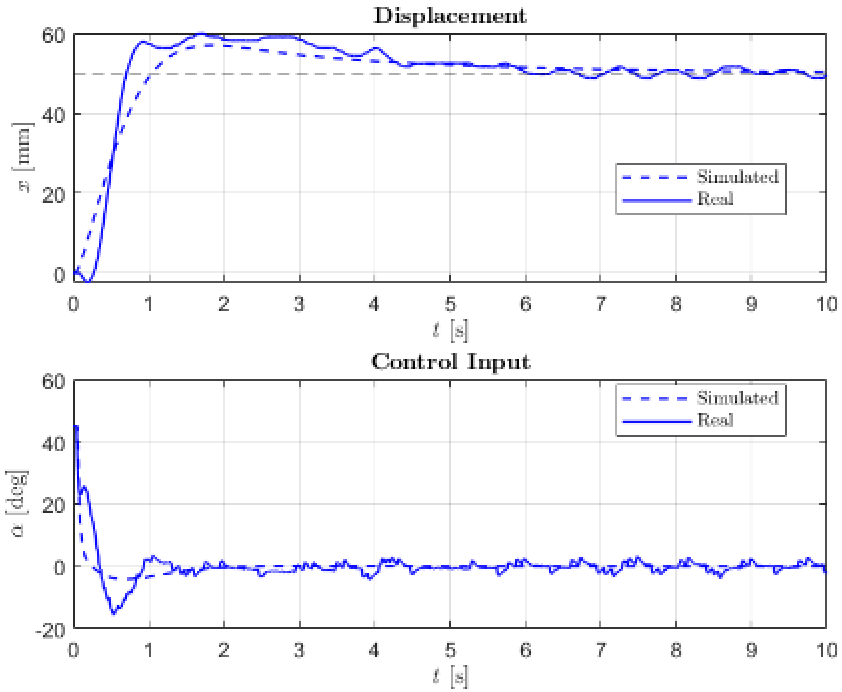


- Reducing K_p and K_i → lower overshoot and settling time
- Anti-windup for the integral action

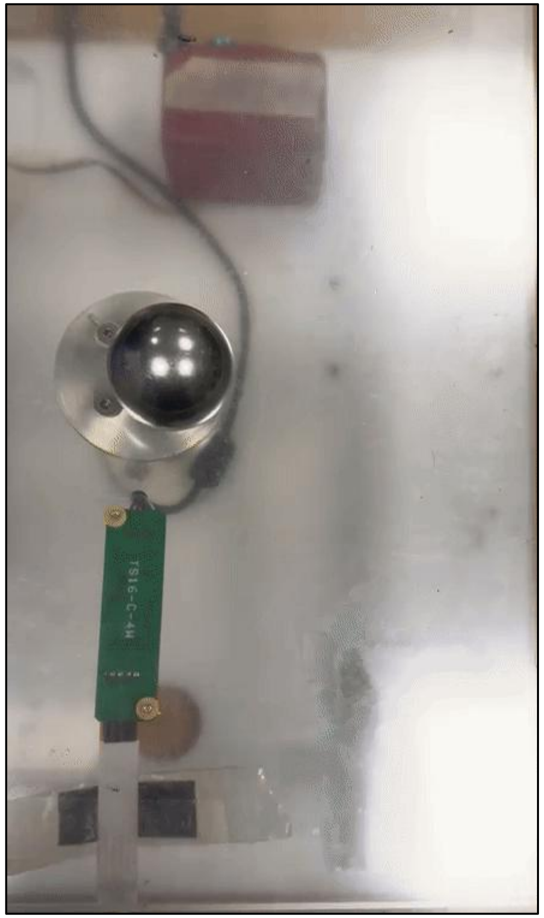
PID - STEP RESPONSE



PID: $K_p = 4.5, \ K_i = 1.5, \ K_d = 3.5$

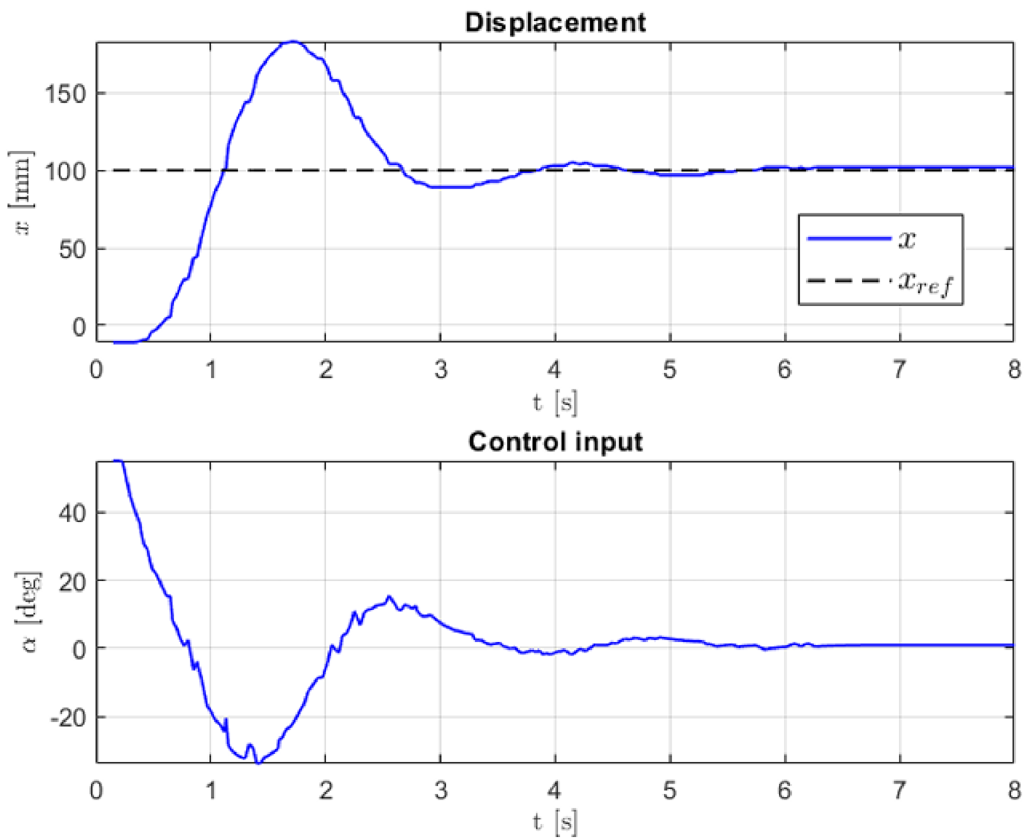


PID: $K_p = 6.4, \ K_i = 1.8, \ K_d = 4$

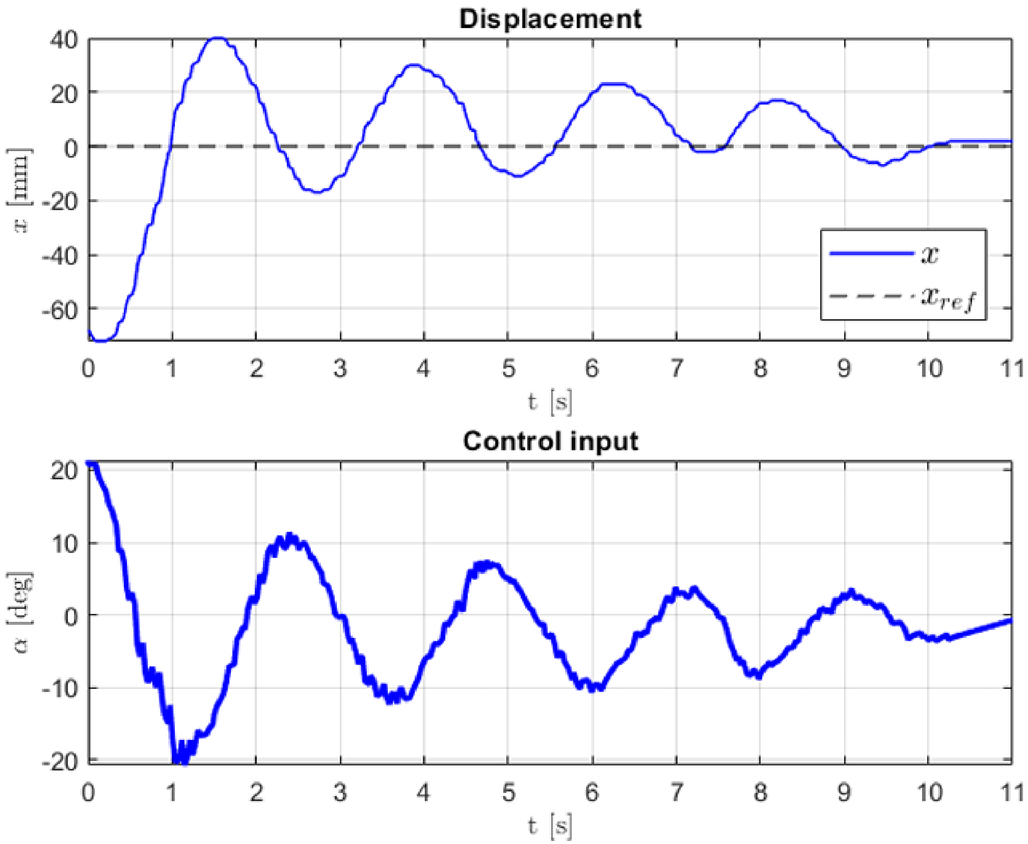


- Increasing K_p and K_i → higher overshoot and settling time
- Increasing K_d → reduction of oscillation but more noise

PID - STEP RESPONSE (NEW BENCH)



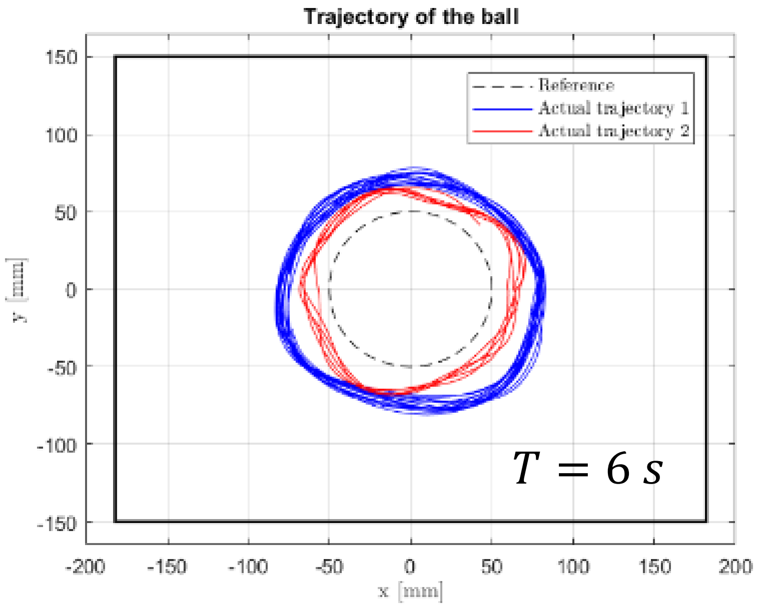
PID: $K_p = 4.5, \;\; K_i = 1.5, \;\; K_d = 3.5$

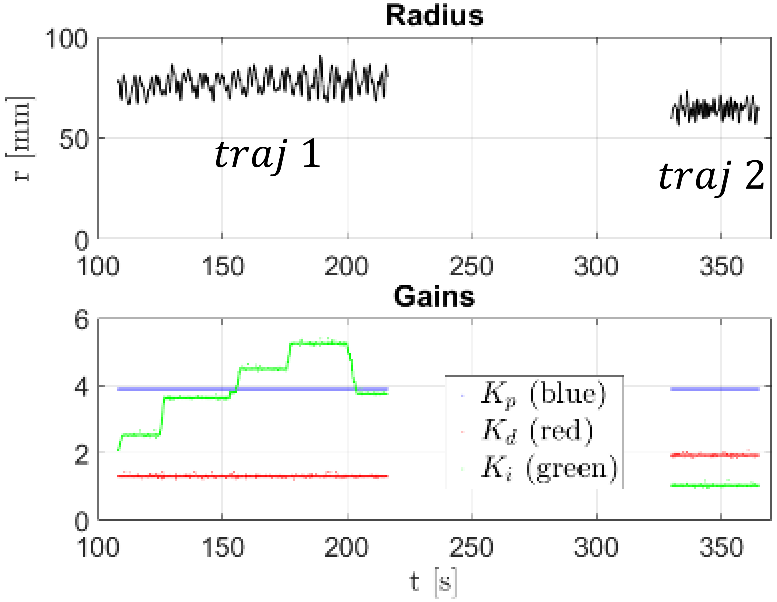


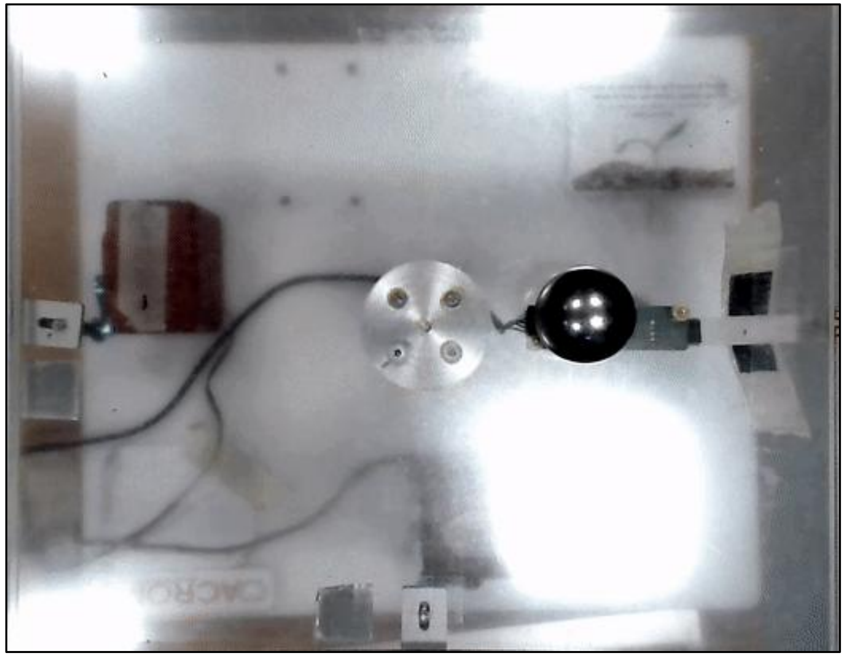
PID: $K_p = 6.4, \;\; K_i = 1.8, \;\; K_d = 4$

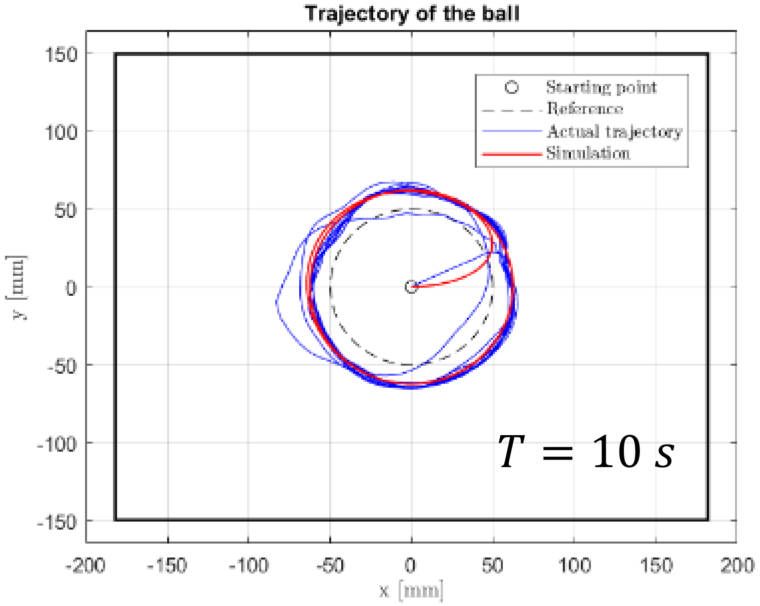
- Same considerations of the original bench
- Further observations at the end of the presentation

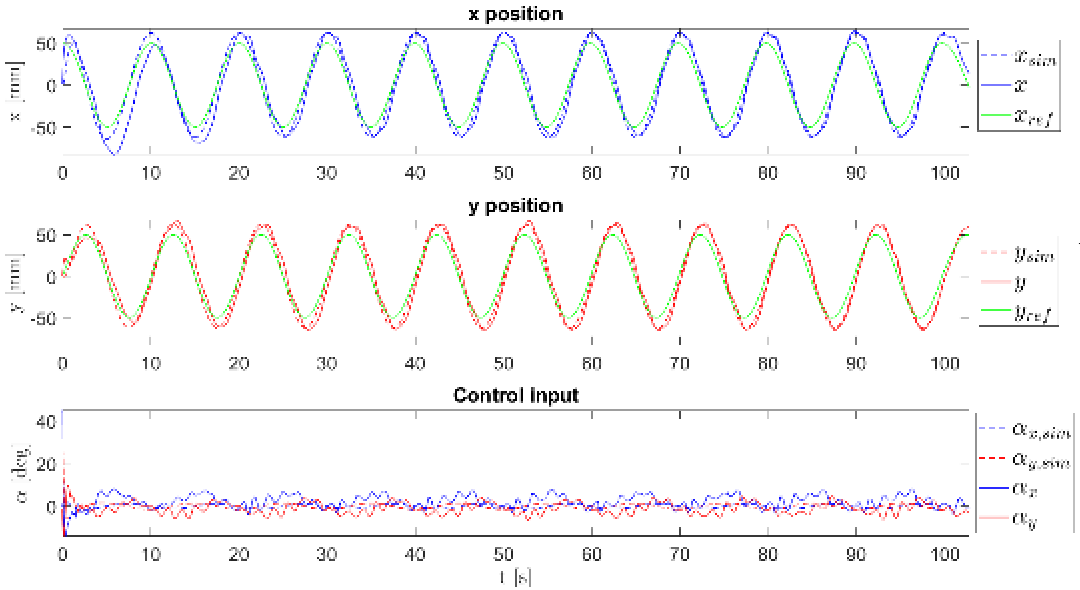
PID - TRAJECTORY TRACKING





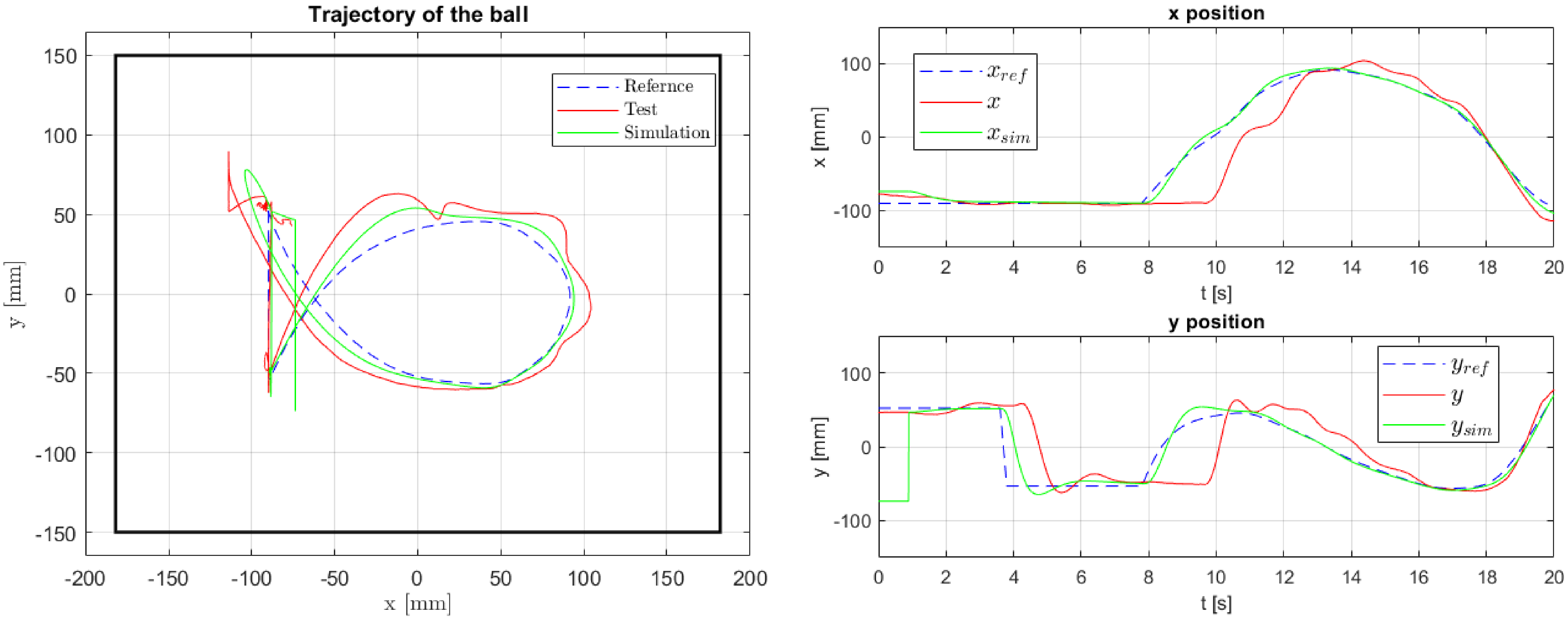






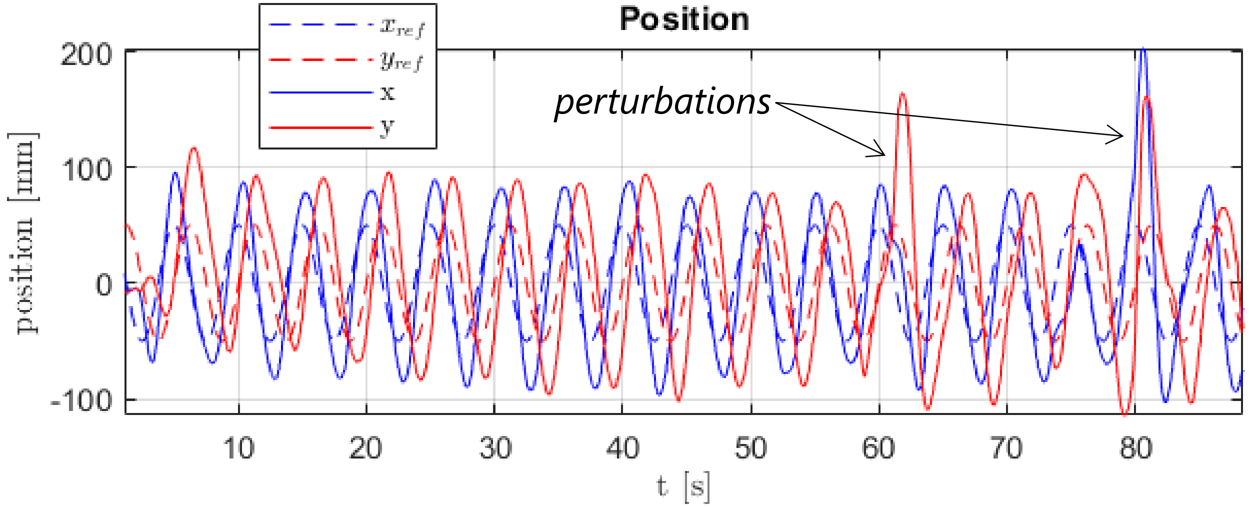
Period $T = 10\ s$.
PID controller $K_p = 2$, $K_i = 0.5$, $K_d = 1.8$

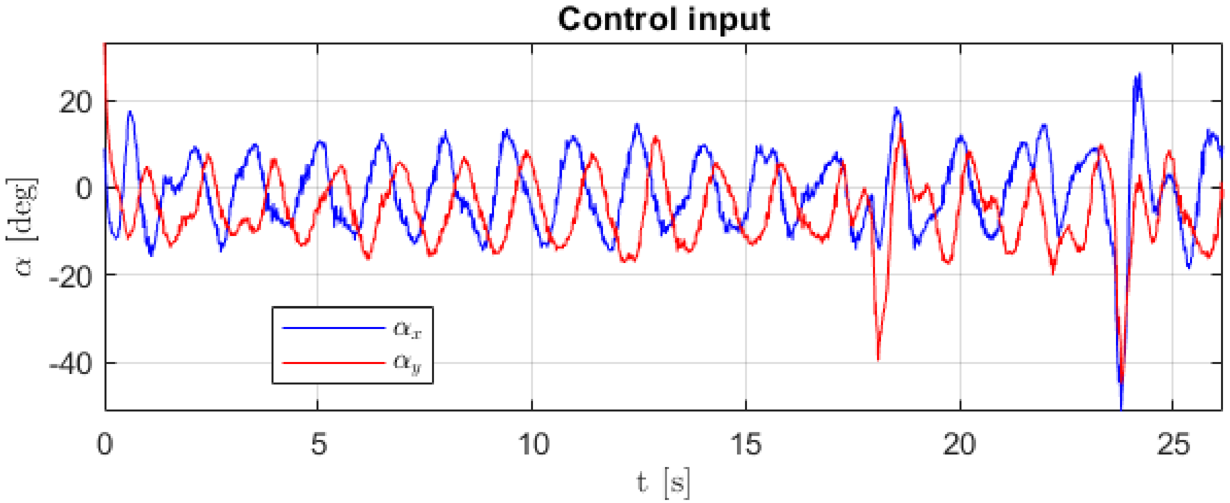
PID - TRAJECTORY TRACKING

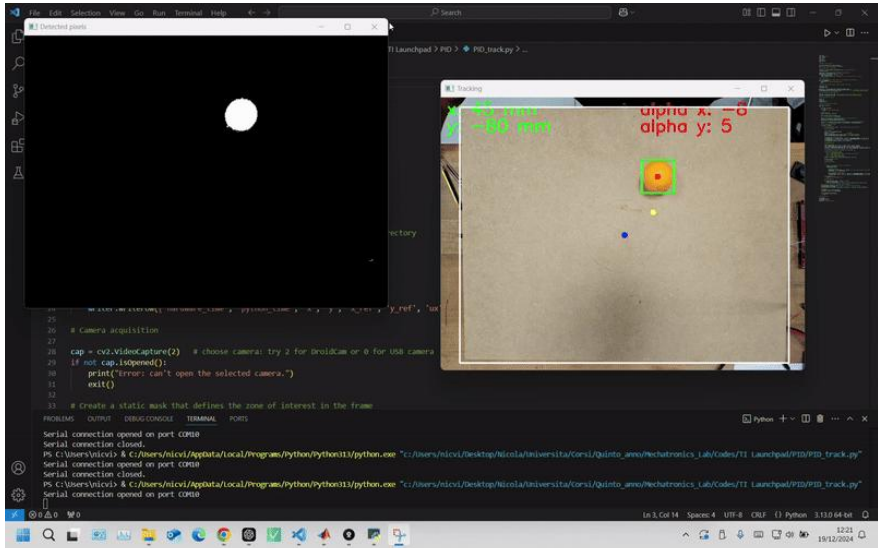


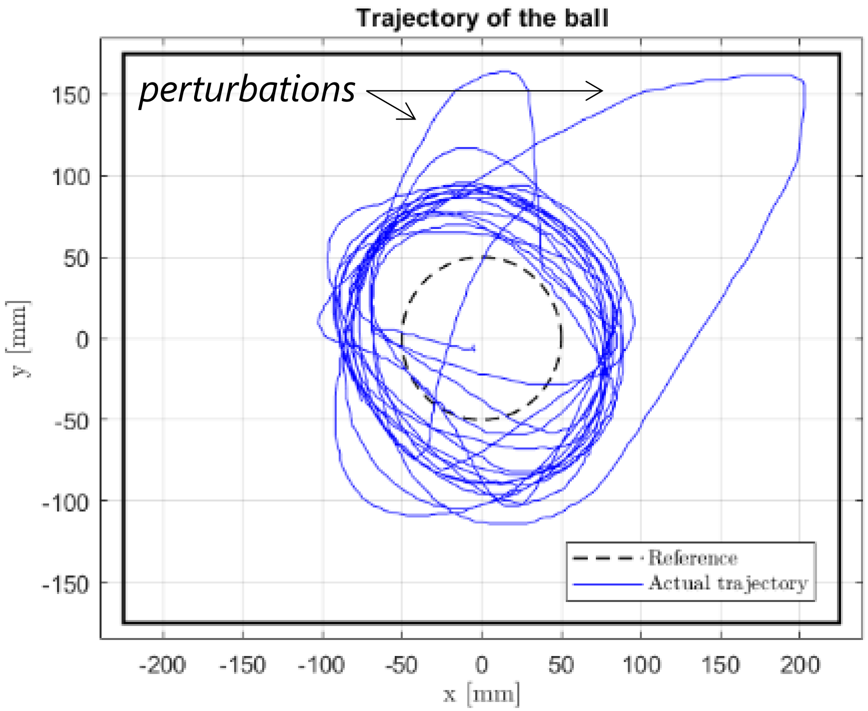
User-defined trajectory tracking with original bench. Duration $T = 20\ s$. PID controller $K_p = 2$, $K_i = 0.5$, $K_d = 1.8$

PID - TRAJECTORY TRACKING (NEW BENCH)









Circle trajectory tracking with new bench.
Period $T = 5\ s$. PID controller $K_p = 2.6$, $\mathrm{K_i} = 0.7$, $\mathrm{K_d} = 0.9$

POLE PLACEMENT

Extended system to eliminate steady state error:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_{ex}\boldsymbol{x} + \boldsymbol{B}_{ex}\boldsymbol{u}$$

Poles of the uncontrolled system: $|s\boldsymbol{I} - \boldsymbol{A}| = 0 \rightarrow s^3 = 0$

Poles of the controlled system:

$$|s\boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{BK})| = 0 \ \rightarrow s^3 + \frac{981}{980}k_2 s^2 + \frac{981}{980}k_1 s + \frac{981}{980}k_3 = 0$$

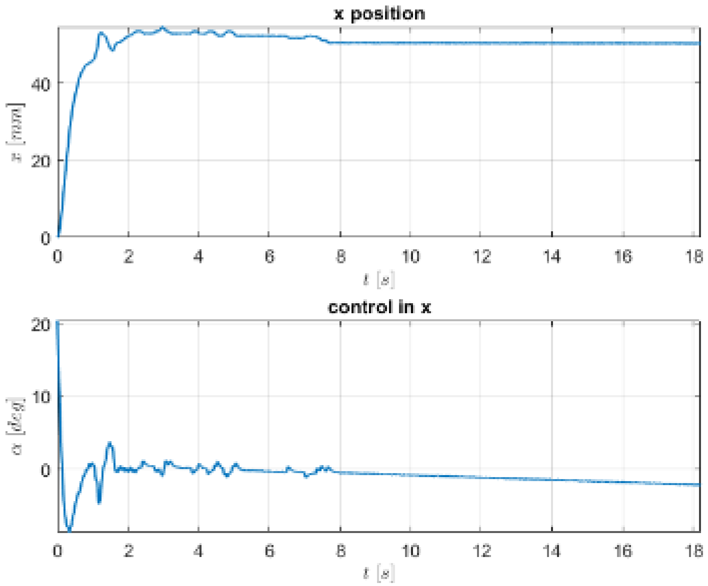
Desired poles of the controlled system:

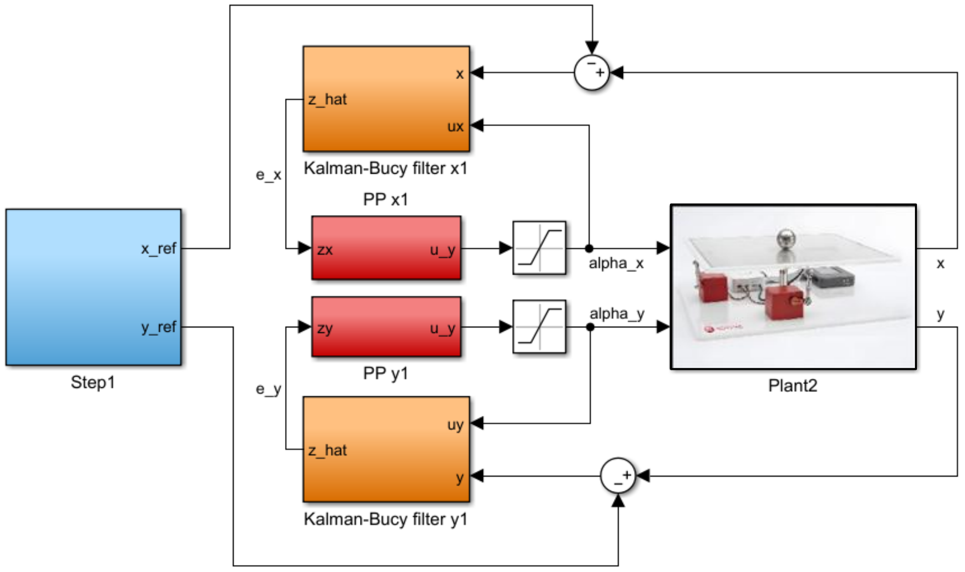
$$s = [-0.5;\ -0.1;\ -5] \ \rightarrow s^3 + \frac{28}{5}s^2 \ + \frac{61}{20}s \ + \frac{1}{4} = 0$$

Equating the coefficients of the last two expressions the controller gains are obtained:

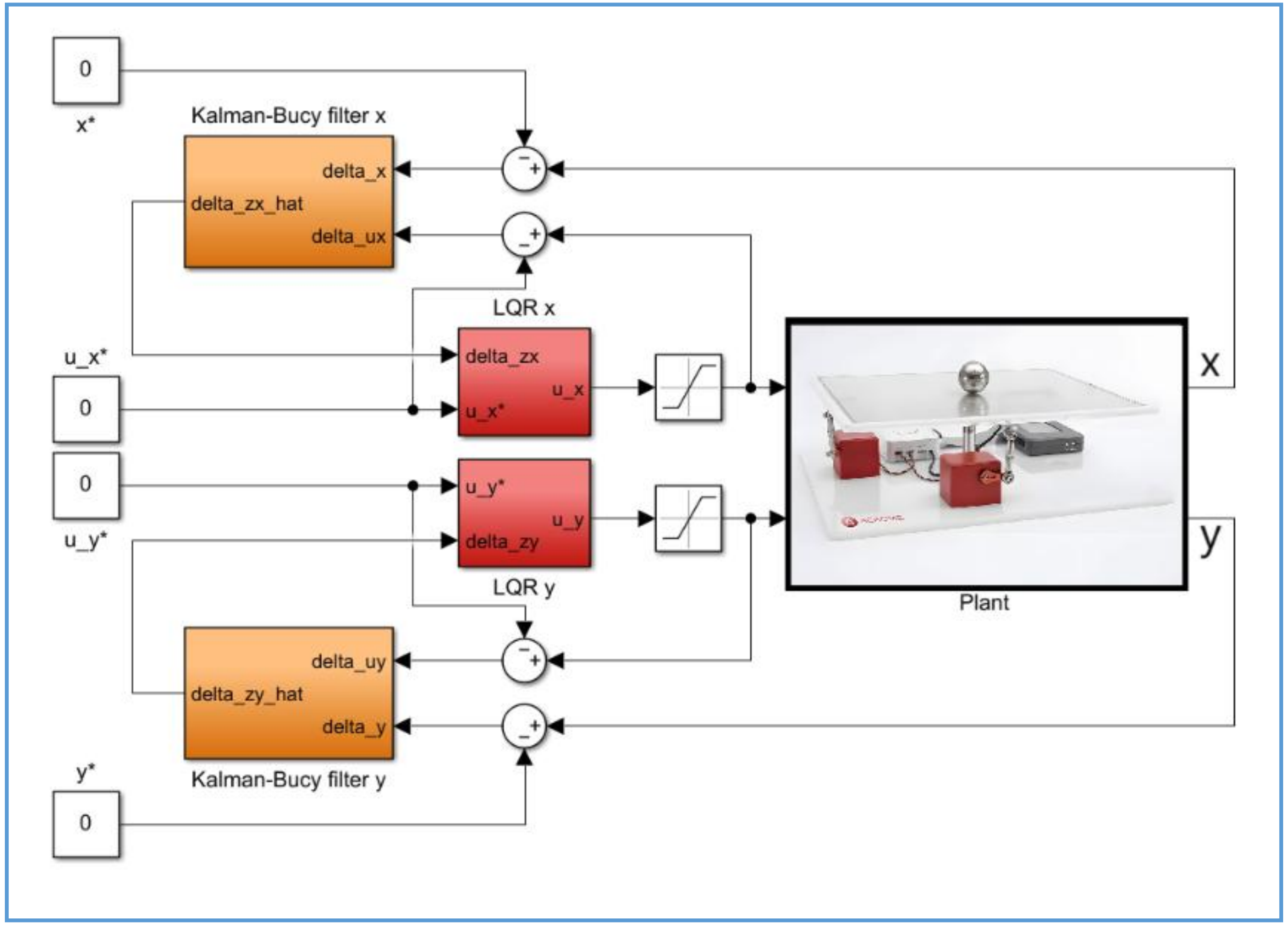
$$k_1 = \frac{2989}{981} \approx 3.05 \quad k_2 = \frac{5488}{981} \approx 5.59 \quad k_3 = \frac{245}{981} \approx 0.25$$

- Almost no overshoot
- Long settling time
- Less control effort compared with PID controllers





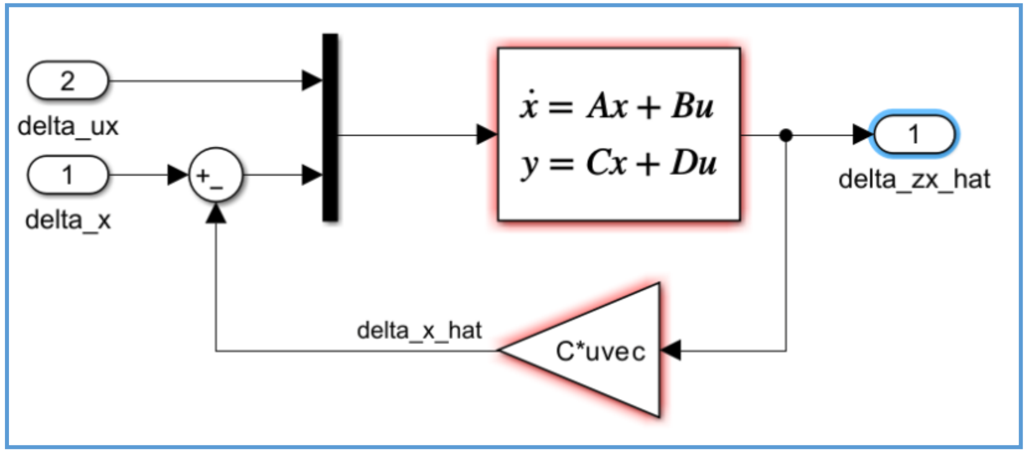
KALMAN FILTER



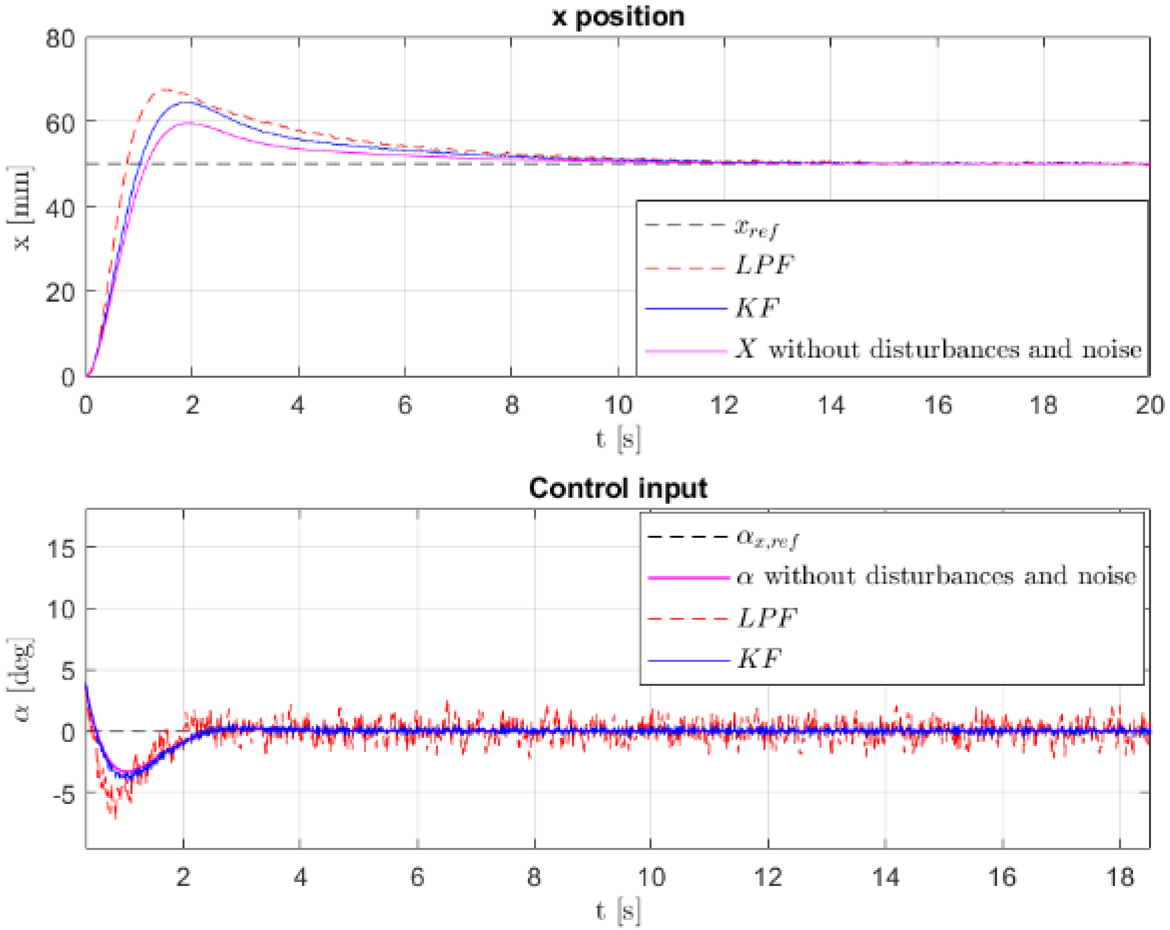
Steady state **Kalman Bucy** estimator

$$\mathbf{Q_{obs}} = \begin{bmatrix} 100 & 0 \\ 0 & 1000 \end{bmatrix} \qquad \mathbf{R_{obs}} = [0.5]$$

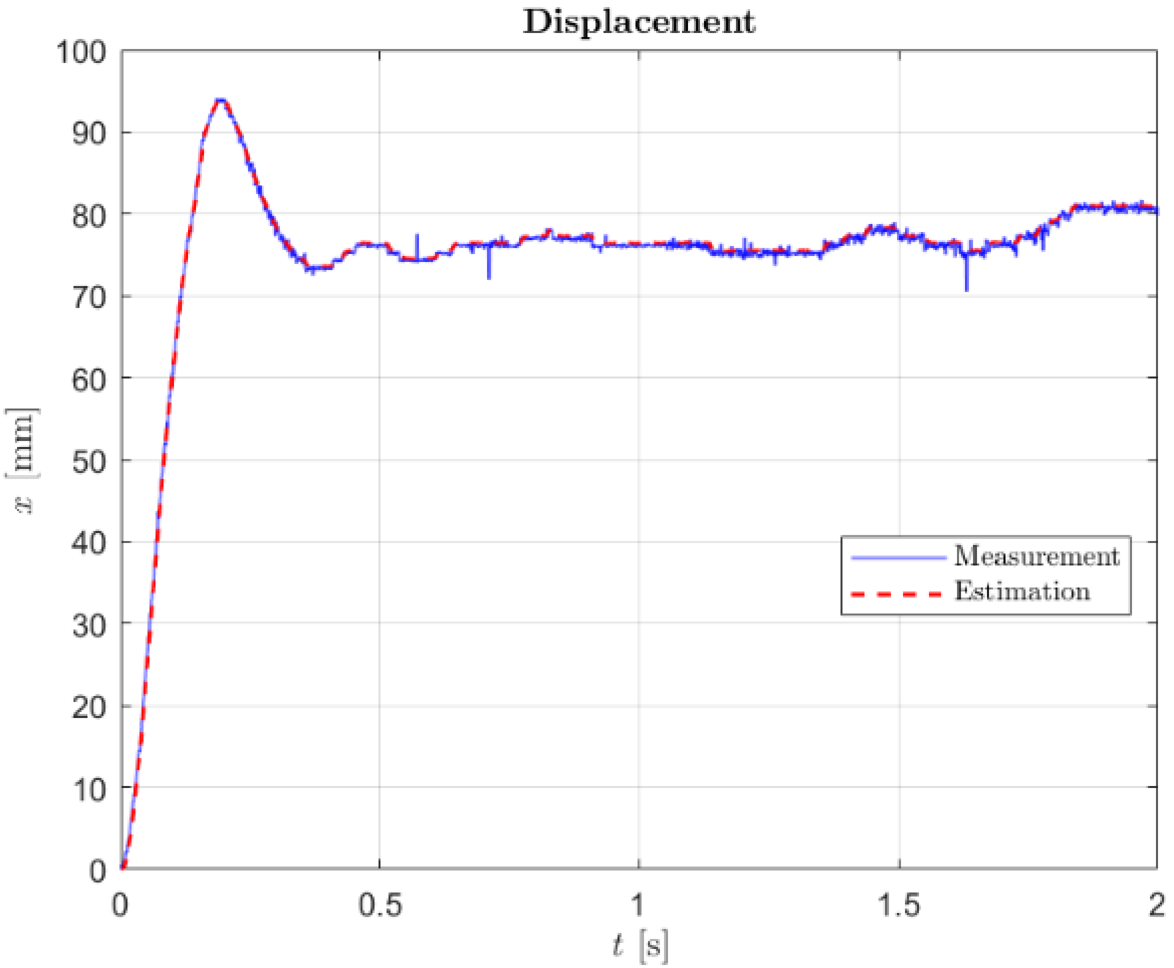
$$\dot{\hat{\boldsymbol{z}}} = \boldsymbol{A}\hat{\boldsymbol{z}} + \boldsymbol{B}u + \boldsymbol{K_{obs}}(x - \boldsymbol{C}\hat{\boldsymbol{z}})$$



KALMAN FILTER



Simulation: Low Pass Filter vs Kalman Filter



Testing Kalman Filter

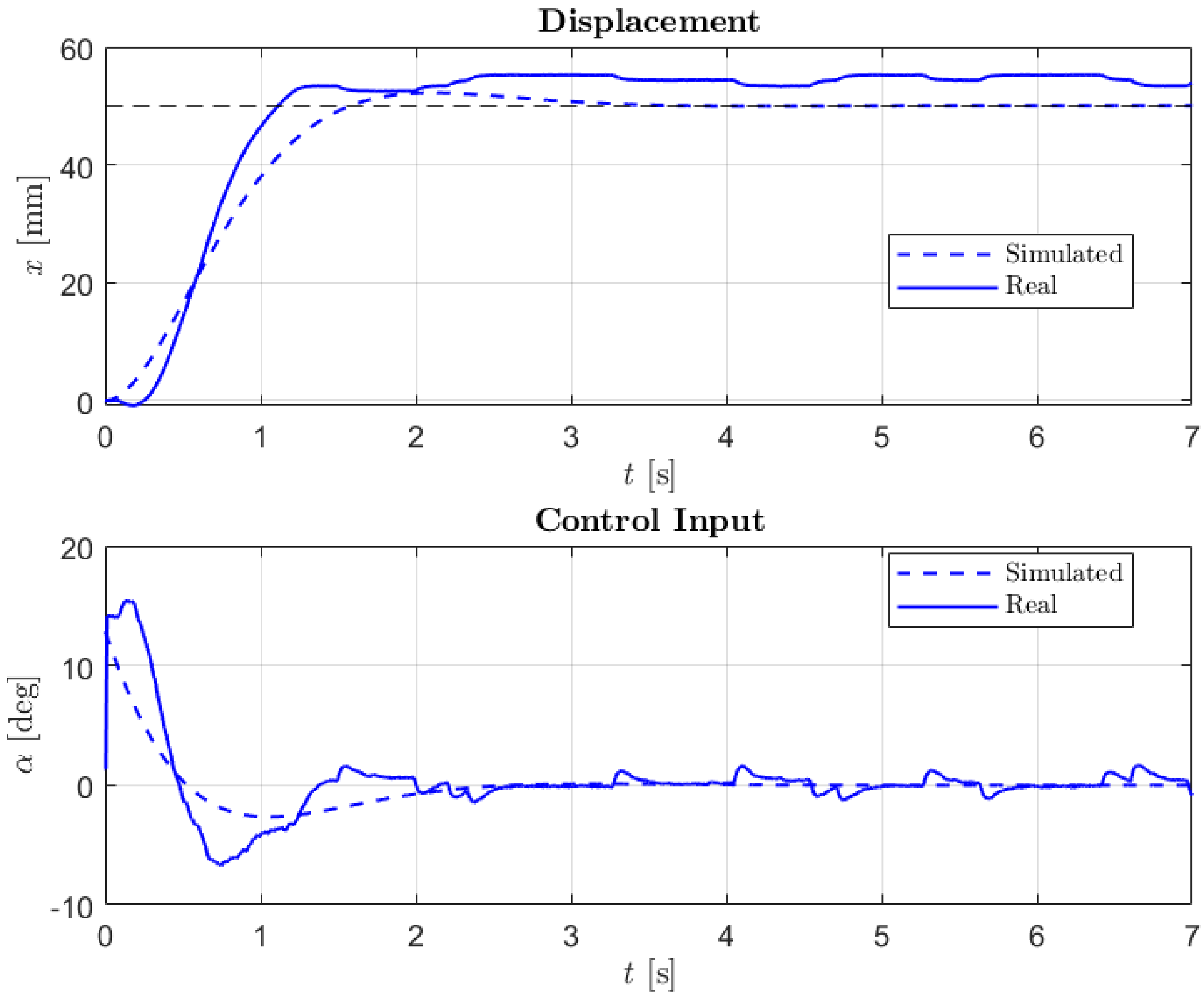
LQR - STEP RESPONSE

Infinite time horizon LQR + Kalman Filter

$$J = \int_0^\infty \frac{1}{2}\left[\mathbf{z}^T(t)\ \mathbf{Q}\ \mathbf{z}(t) + \mathbf{u}^T(t)\ \mathbf{R}\ \mathbf{u}(t)\right] dt$$

subject to: $\dot{\mathbf{z}}(t) = \mathbf{A}\ \mathbf{z}(t) + \mathbf{B}\ \mathbf{u}(t)$

Weights: $\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\mathbf{R} = [0.05]$



LQRI

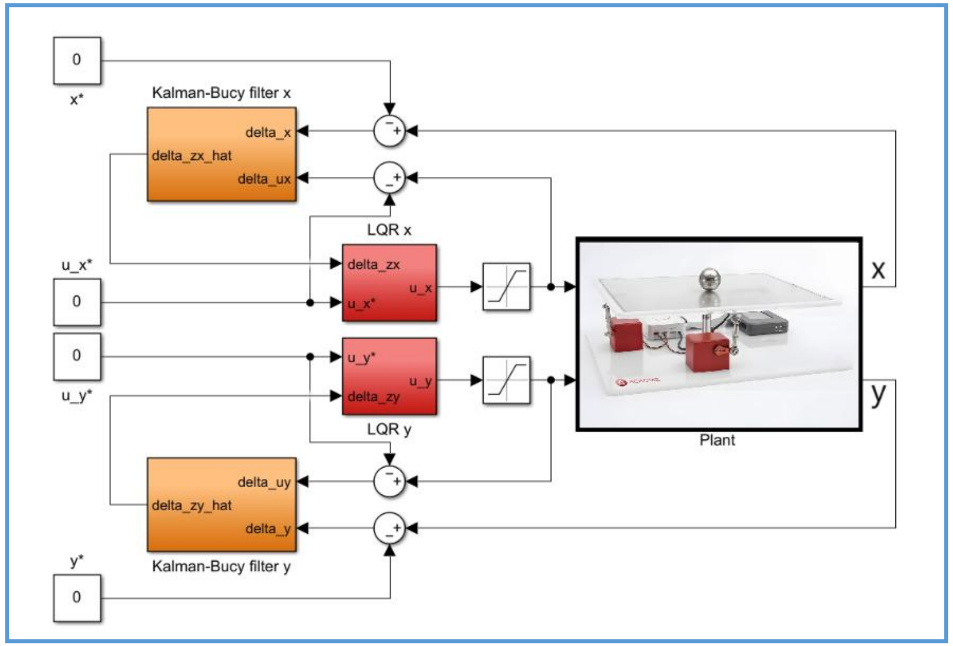
Addition of the **integral action**

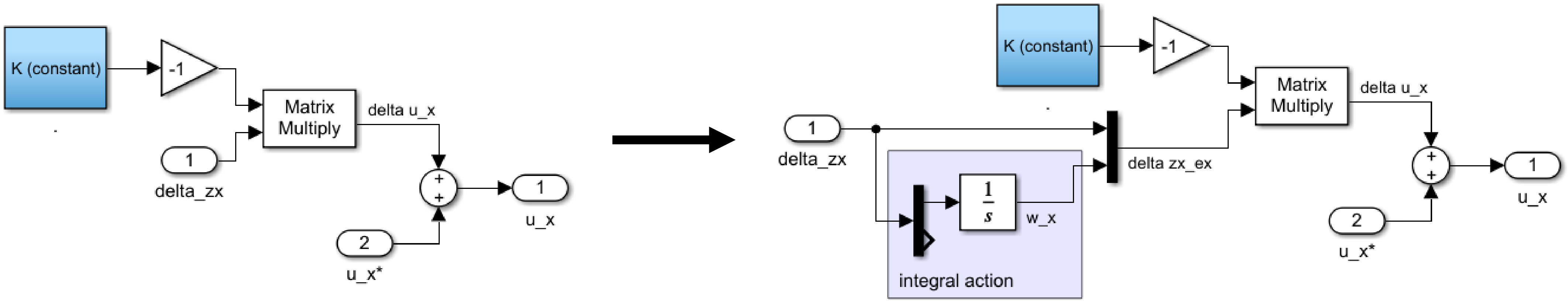
$$w = \int (x - x_{ref})\, dt$$

Augmented system and weighting matrix Q

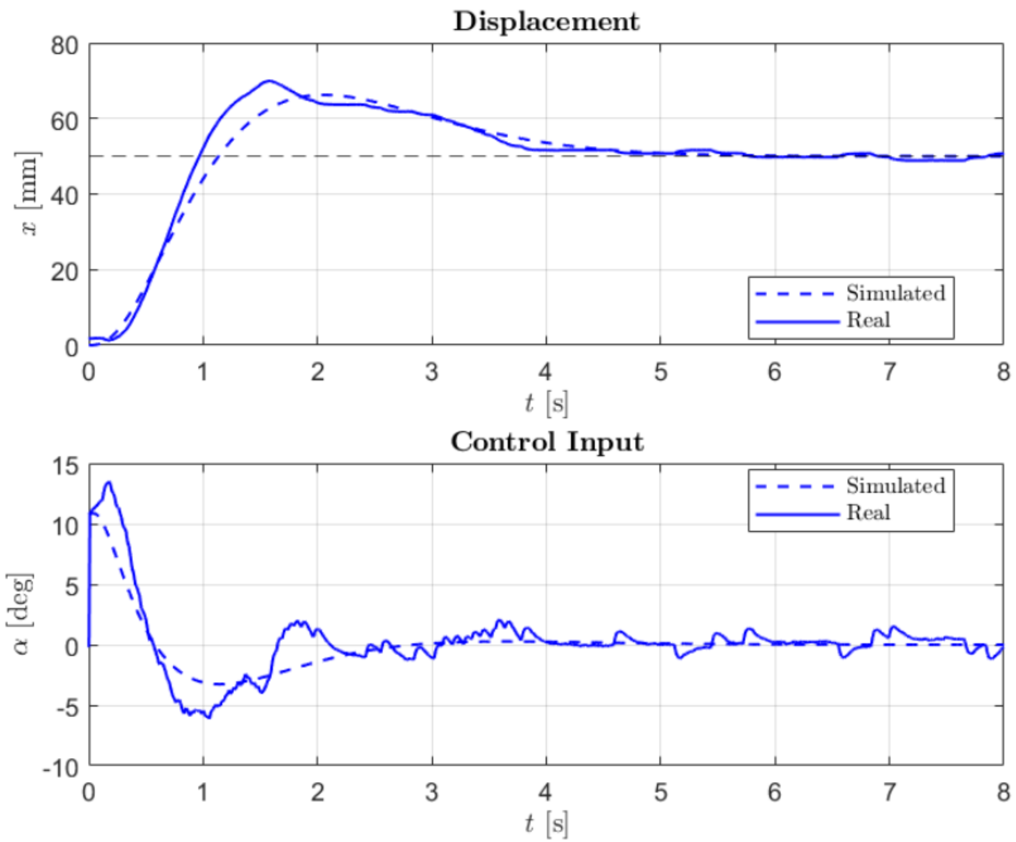
$$u = -\mathbf{K}\mathbf{z} - k_i \cdot w = -\mathbf{K}_{\mathbf{ext}} \cdot \mathbf{z}_{\mathbf{ext}}$$

$$\mathbf{Q}_{\mathbf{ext}} = \begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & Q_w \end{bmatrix}$$

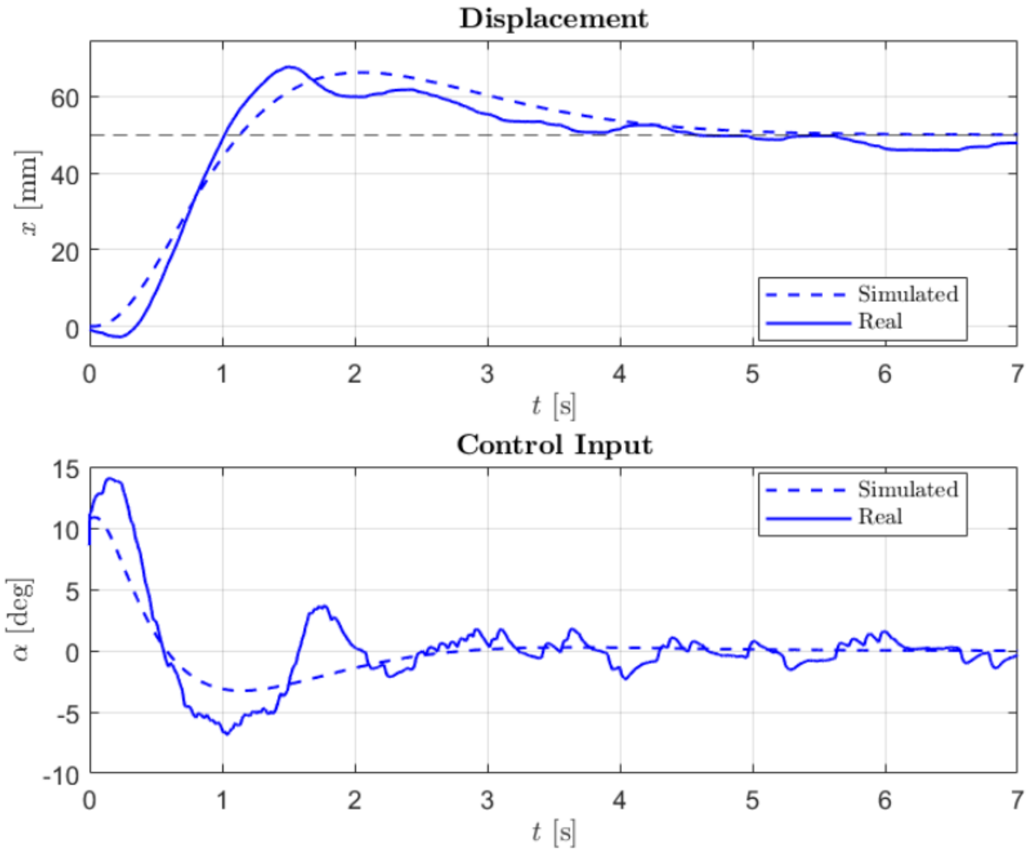




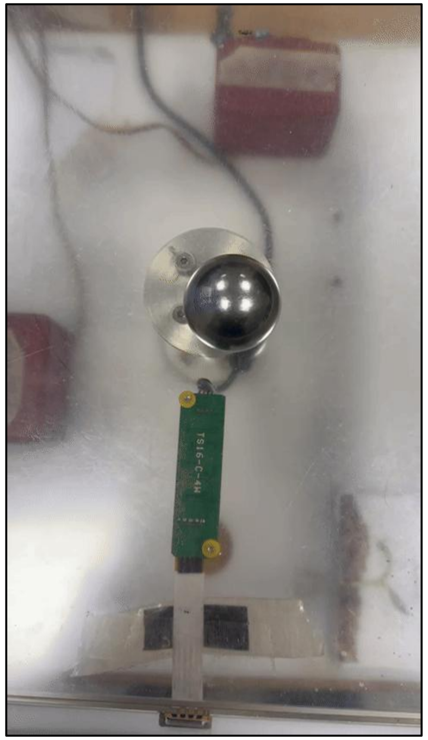
LQRI – STEP RESPONSE

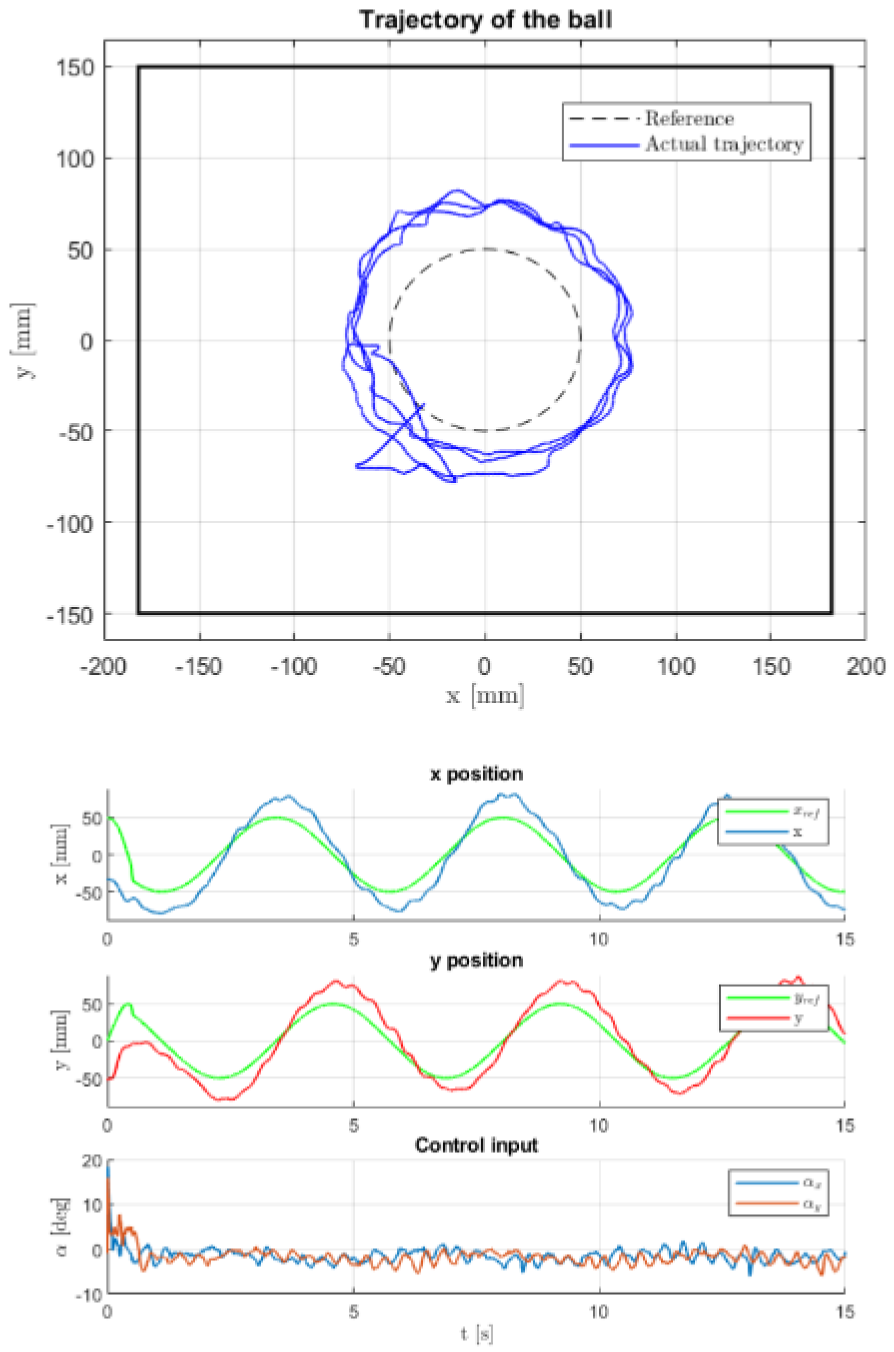


$$R = 0.11, \quad Q_x = 1, \quad Q_{\dot{x}} = 0, \quad Q_w = 0.1$$
$$K_{ext} = [3.64, 2.70, 0.91]$$

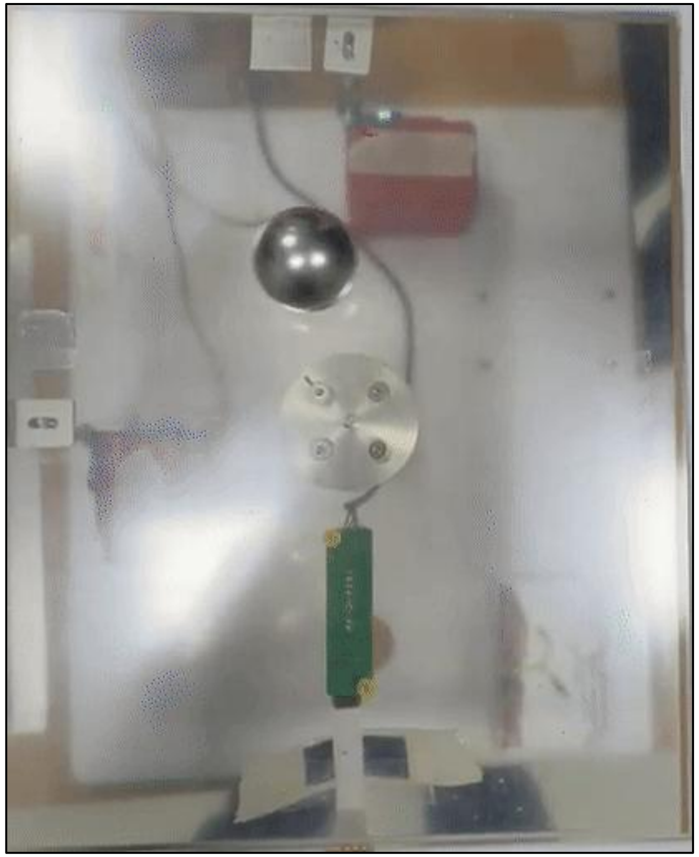


$$R = 0.275, \quad Q_x = 1, \quad Q_{\dot{x}} = 0, \quad Q_w = 1$$
$$K_{ext} = [5.19, 3.22, 2.89]$$





LQR - TRACKING



$$Q_x = 1,\ Q_{\dot{x}} = 0,\ R = 0.05,\ T = 5s$$

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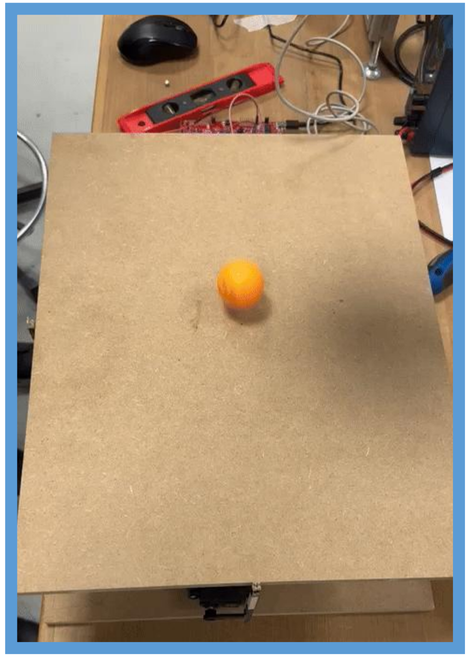
4. CONCLUSIONS

LIMITATIONS AND IMPROVEMENTS

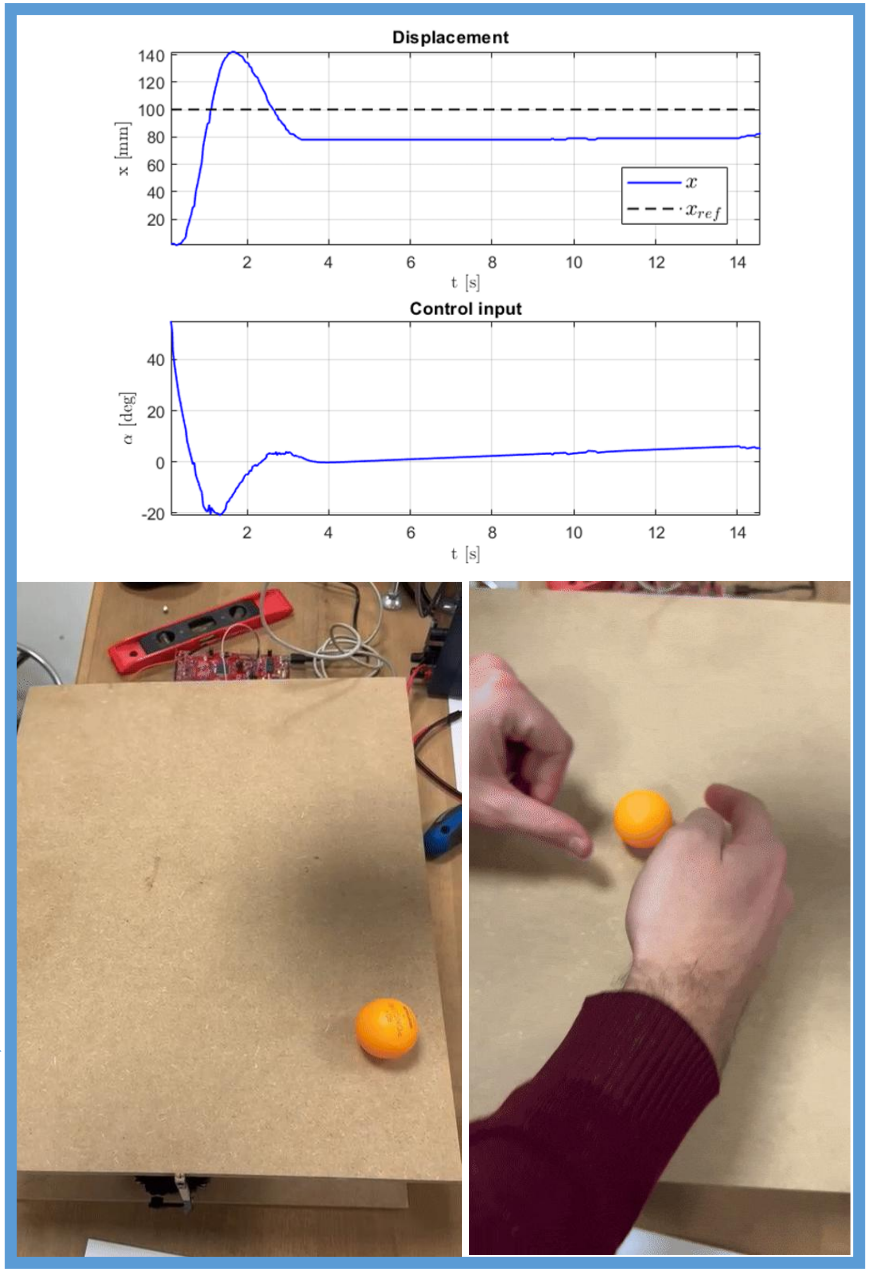
Original bench:

- Slipping motor's rod
- Limited operative area on touchpad panel
- Lower frequency of the microcontroller
- Metal ball, touching the panel



New bench:

- Changing ball and platform colours
- Fixing camera to the bench
- Smoother platform to reduce friction
- Improve data handling and transmission



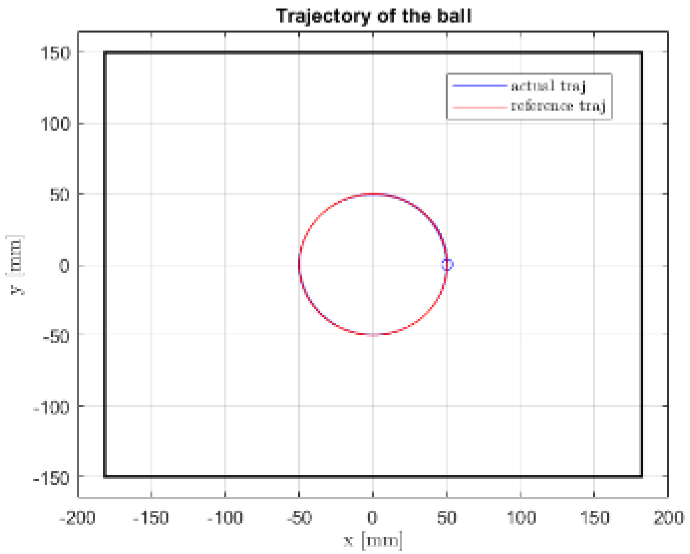
Mathematical model:

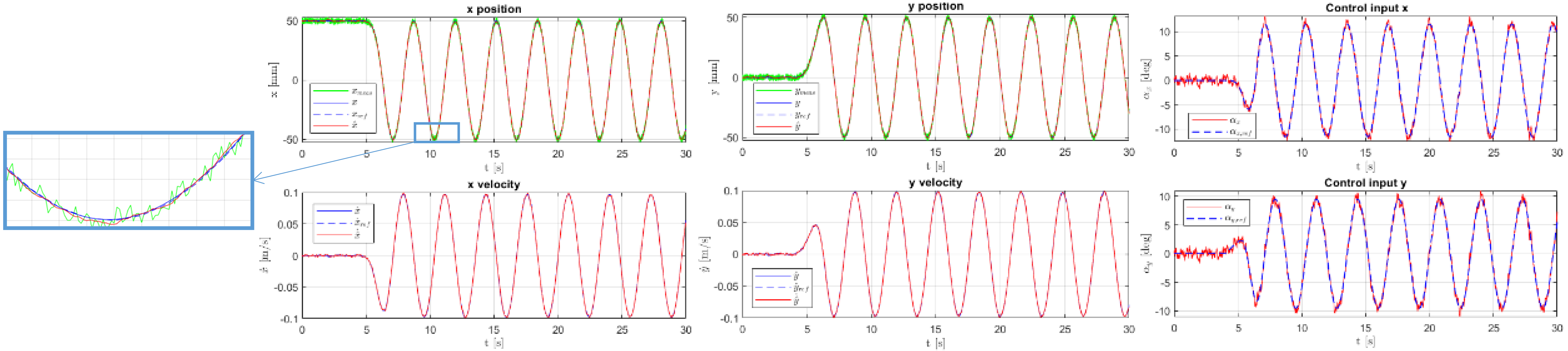
- Not accounting for the moving platform

LIMITATIONS AND IMPROVEMENTS

Control:

- Additional control loop on the motors → encoder
- Improving filtering (EKF, disturbances estimation, etc) → computational power
- Better tracking control strategies (ex.: finite time LQR) → computational power





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