

# Analog control of switched mode converters

## Topic 5.

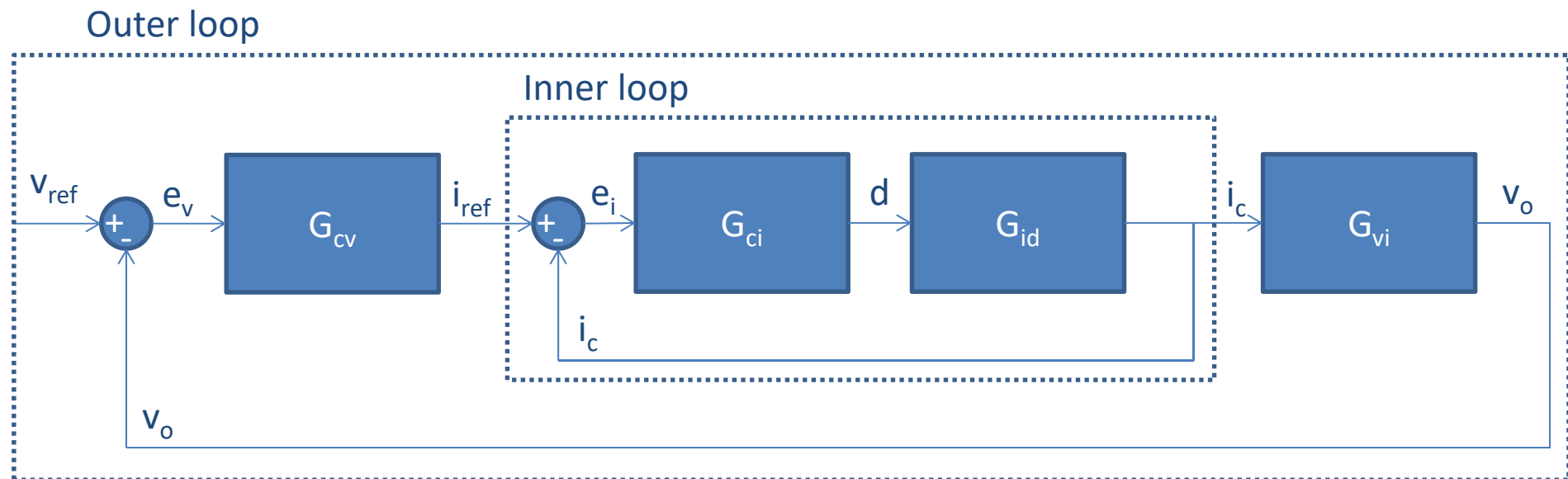
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# Agenda

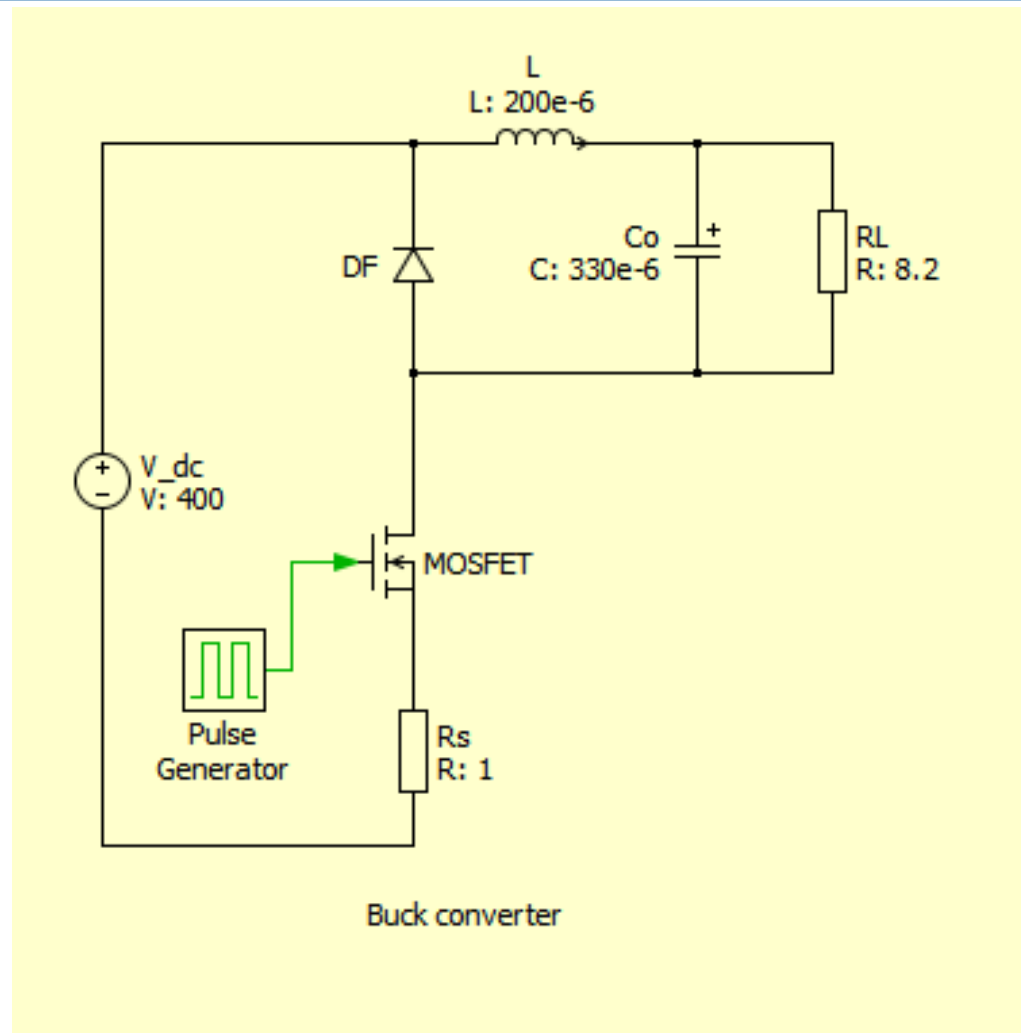
- Introduction
- Buck – Forward converter
  - Switching waveforms
  - Voltage mode control
  - Current mode control
  - Control-to-output function  $G_{vc}$ : using  $i_c = i_L$  approximation
  - Control-to-output function  $G_{vFB}$  (including transformer and shunt resistor)
  - Controller function: Type II
- Implementation
- Controller design with Operational Amplifier
- Controller design with TL431 Shunt regulator
- Exercise

# Introduction

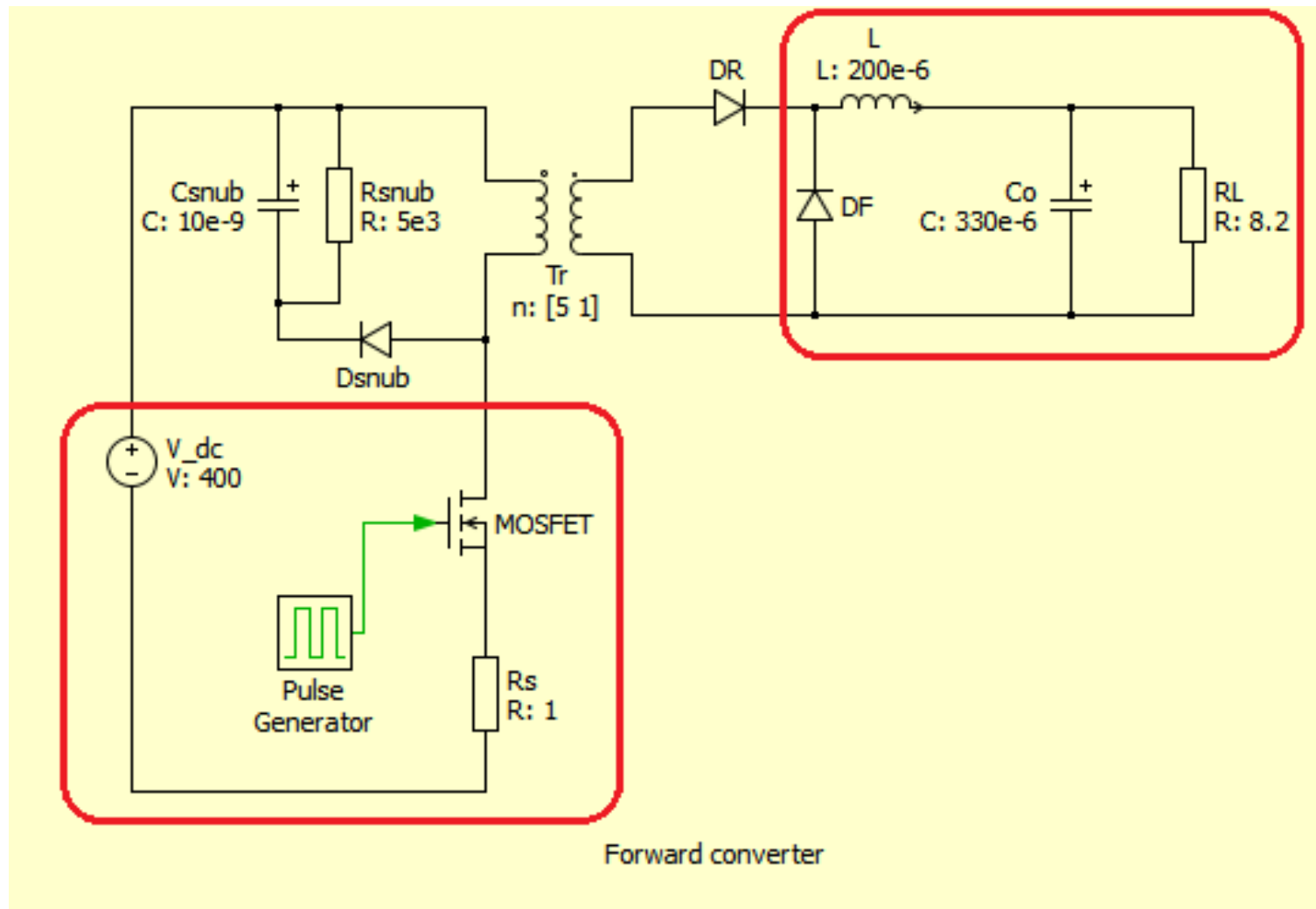


Control structure for a switched mode power converter

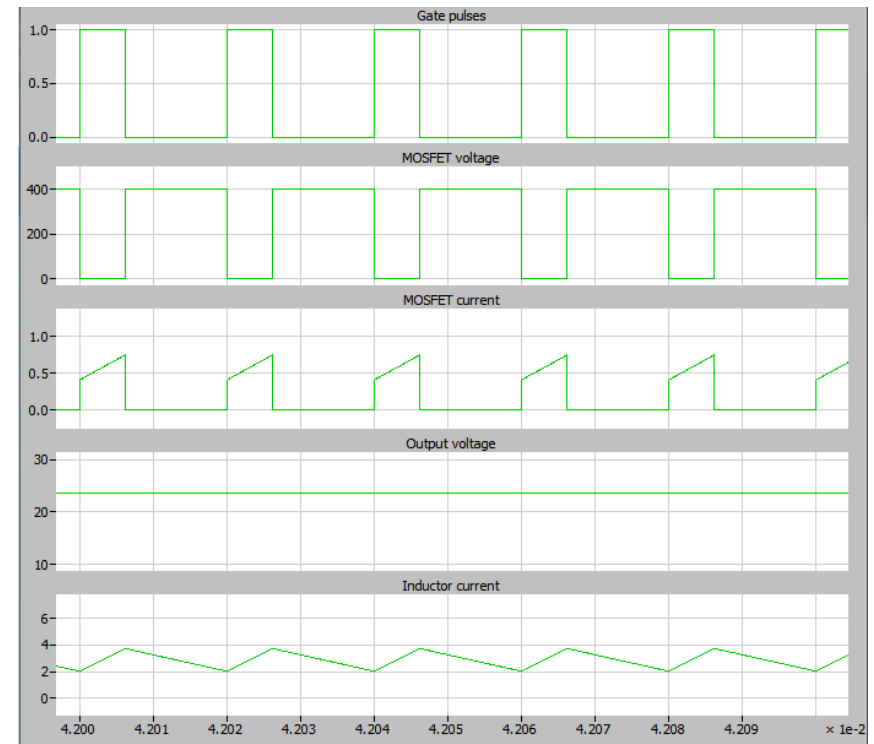
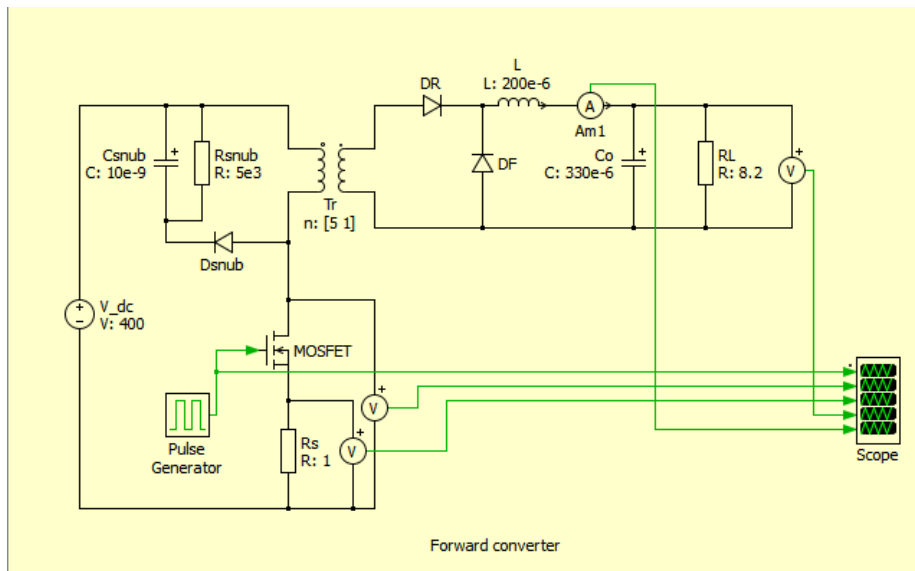
# Buck converter



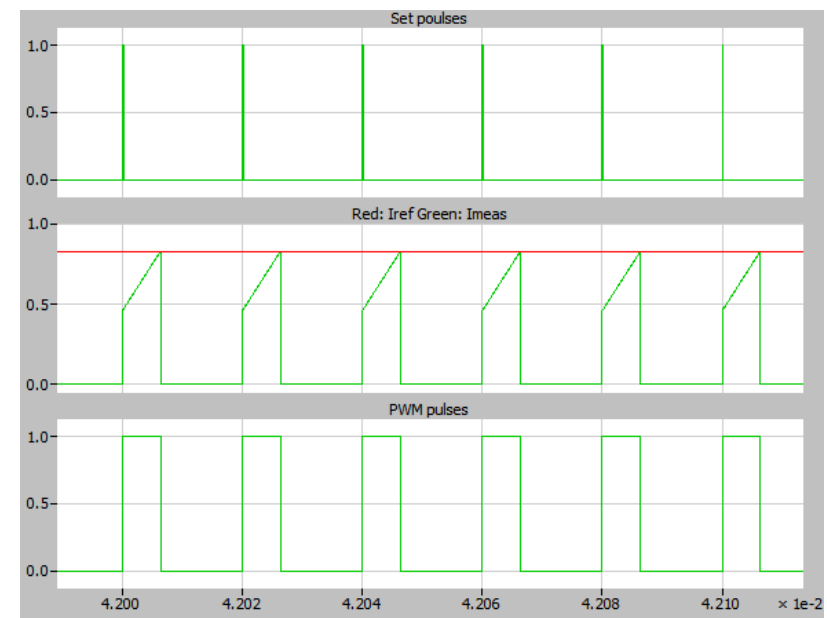
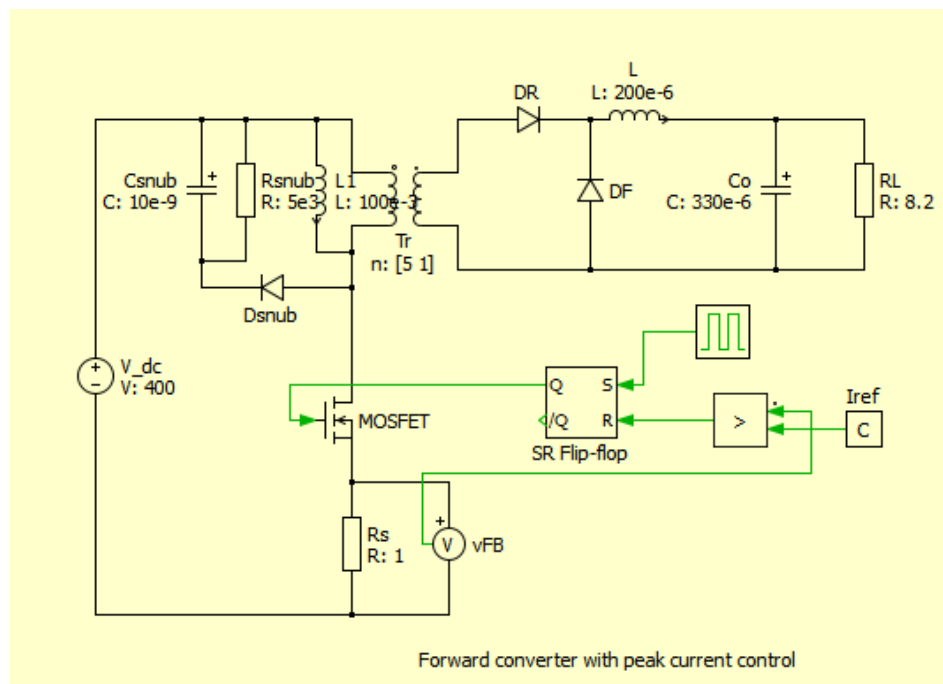
# Forward converter



# Forward converter switching waveforms



# Inner loop - peak current controller



Now the current is controlled, we focus on  
the output voltage!



# Voltage control: Control-to-Output function

$$\begin{cases} Ls\hat{i}_L = V_g\hat{d} + D\hat{v}_g - \hat{v}_o \\ Cs\hat{v}_o = \hat{i}_L - \frac{\hat{v}_o}{R} \end{cases}$$



$$G_{vd} = \left. \frac{\hat{v}_o}{\hat{d}} \right|_{\hat{v}_g=0}$$



$$G_{vd} = \frac{V_g}{D} \frac{\frac{1}{LC}}{s^2 + \frac{1}{CR}s + \frac{1}{LC}}$$

# Current control: Control-to-Output function

- Assumption:  $i_L = i_c$

$$\begin{cases} Ls\hat{i}_c = V_g\hat{d} + D\hat{v}_g - \hat{v}_o \\ Cs\hat{v}_o = \hat{i}_c - \frac{\hat{v}_o}{R} \end{cases}$$



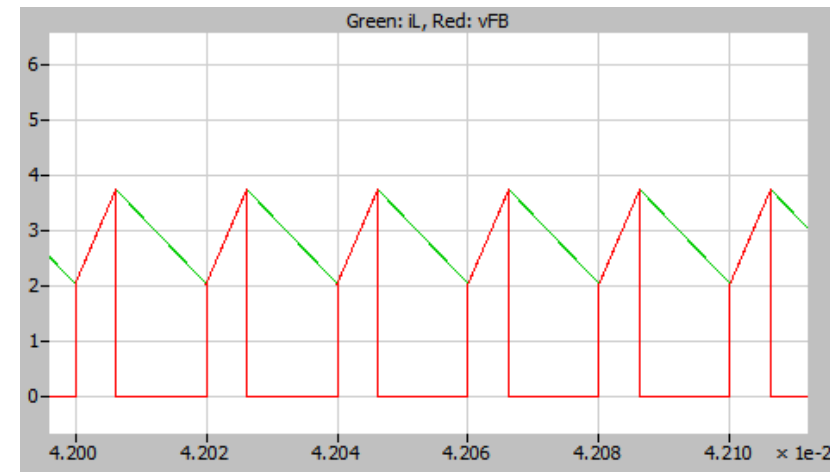
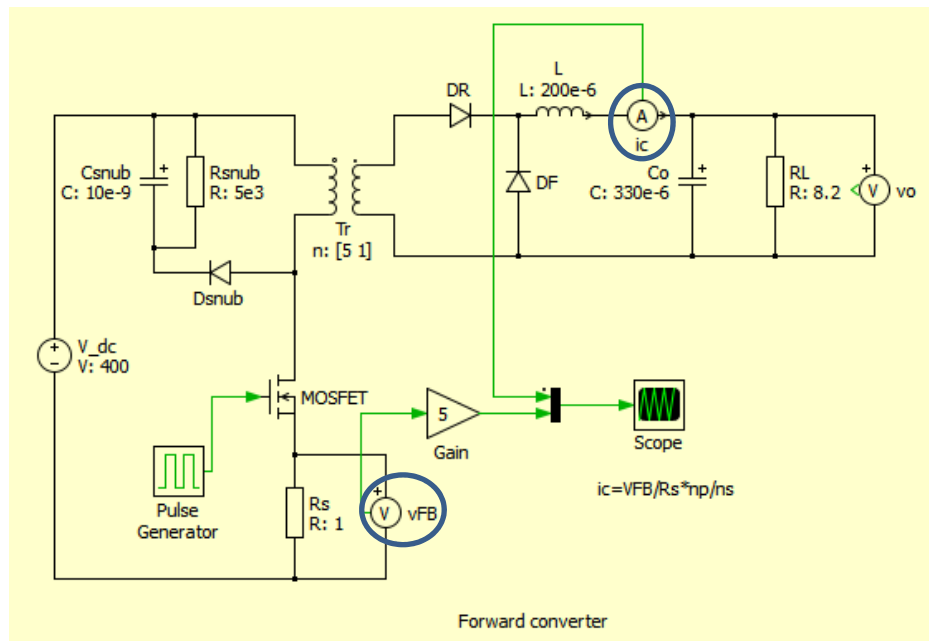
$$G_{vd} = \left. \frac{\hat{v}_o}{\hat{i}_c} \right|_{\hat{v}_g=0}$$

$$G_{vc} = \frac{R}{RCs + 1}$$

# Control-to-Output function including:

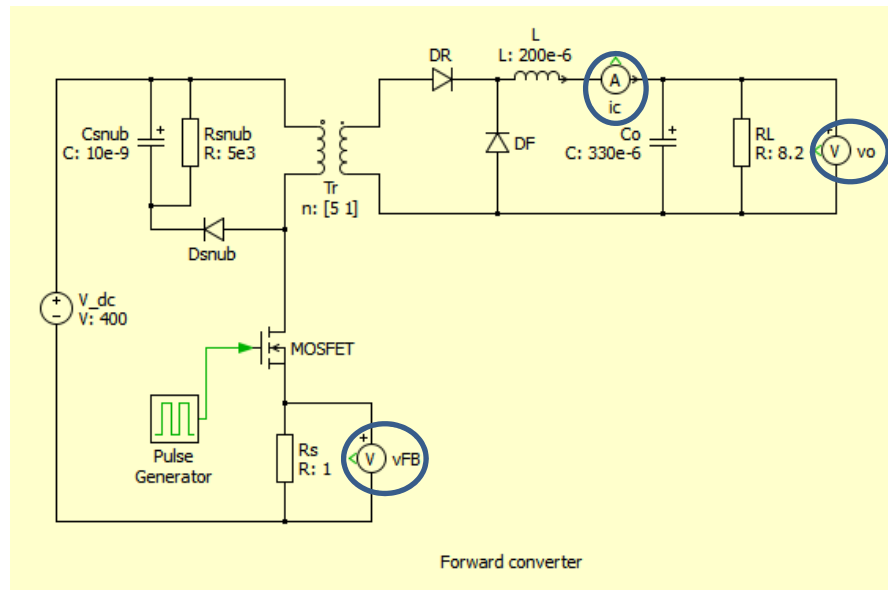
- Transformer turns ratio:  $n_p/n_s$
- Shunt resistor:  $R_s$

$$G_{cFB} = \frac{\hat{i}_c}{\hat{v}_{FB}} = \frac{1}{R_s} \frac{n_p}{n_s}$$



# Control-to-Output function including:

- Transformer turns ratio:  $n_p/n_s$
- Shunt resistor:  $R_s$



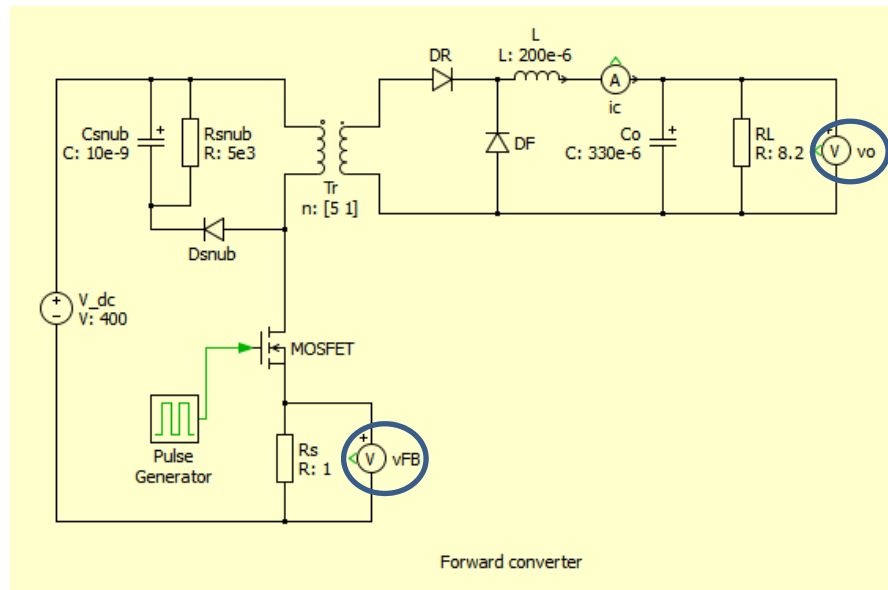
$$G_{vc} = \frac{\hat{v}_o}{\hat{i}_c} = \frac{R}{RCs + 1}$$

$$G_{cFB} = \frac{\hat{i}_c}{\hat{v}_{FB}} = \frac{1}{R_s} \frac{n_p}{n_s}$$

$$G_{vFB} = G_{vc} G_{cFB}$$

# Control-to-Output function including:

- Transformer turns ratio:  $n_p/n_s$
- Shunt resistor:  $R_s$

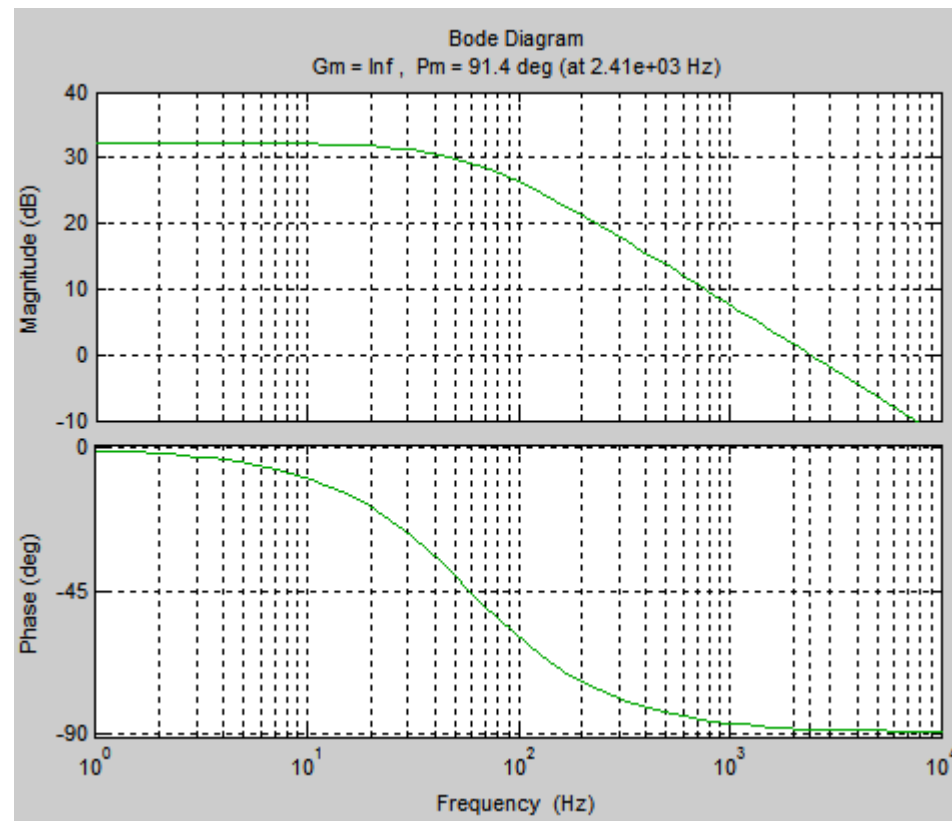


$$G_{vFB} = G_{vC} G_{CFB}$$

$$G_{vFB} = \frac{\hat{v}_o}{\hat{v}_{FB}} = \frac{1}{R_s} \frac{n_p}{n_s} \frac{R}{RCs+1}$$

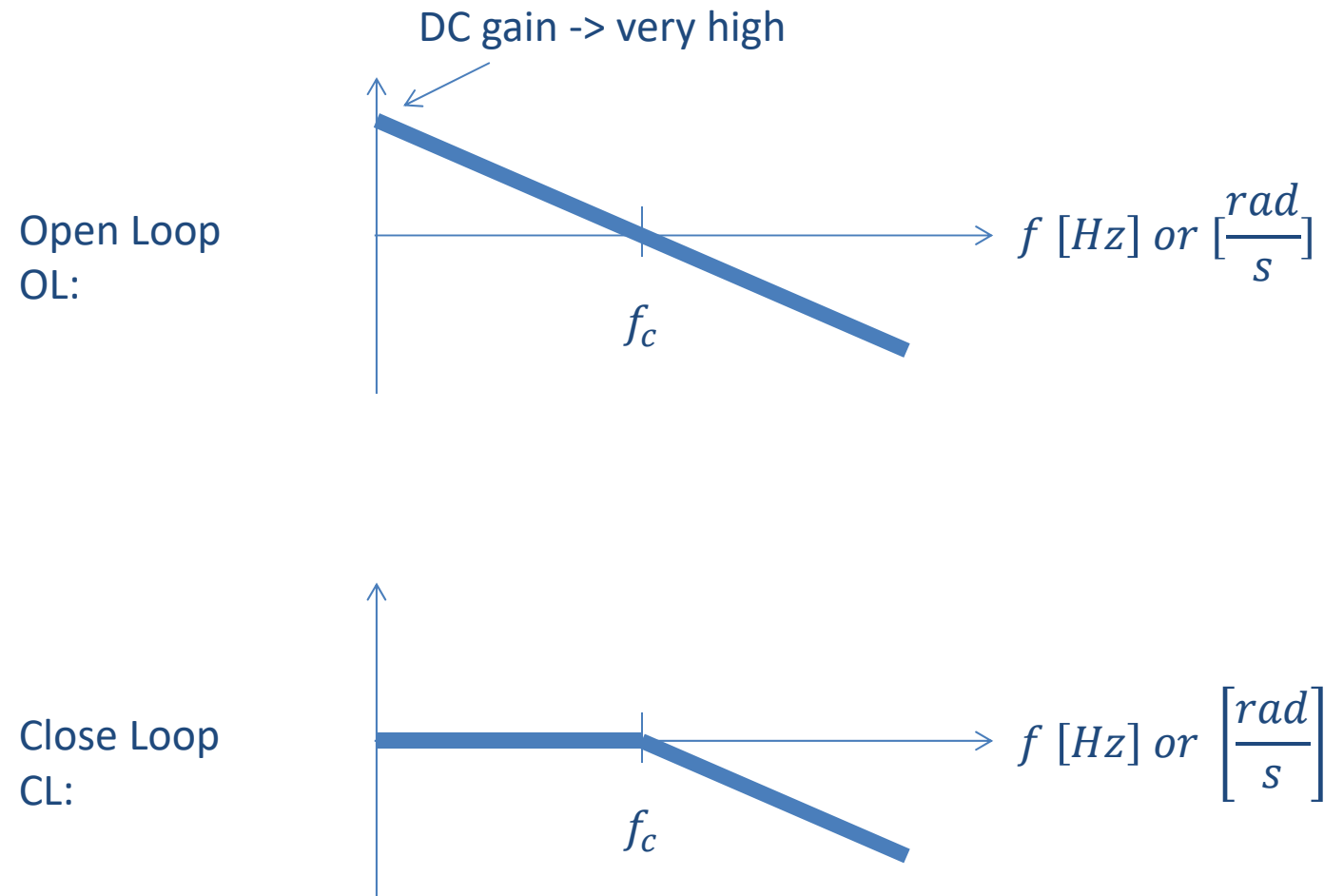
# Control-to-Output function:

$$G_{vFB} = \frac{\hat{v}_o}{\hat{v}_{FB}} = \frac{1}{R_s} \frac{n_p}{n_s} \frac{R}{RCs+1} = K \frac{1}{\frac{s}{\omega_p} + 1}$$



We need a controller!

# Expected behavior of the controlled system

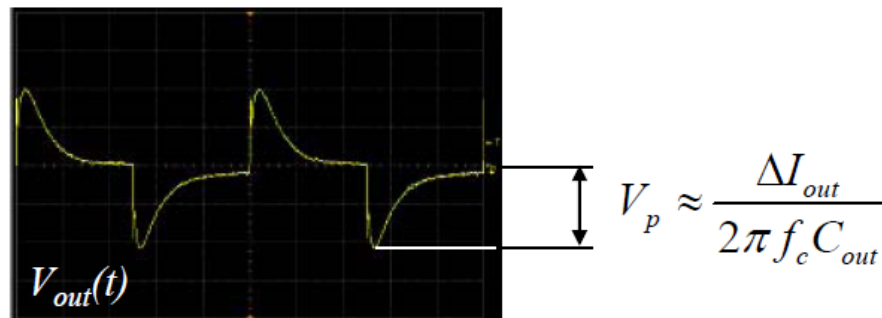




# Expected behavior of the controlled system

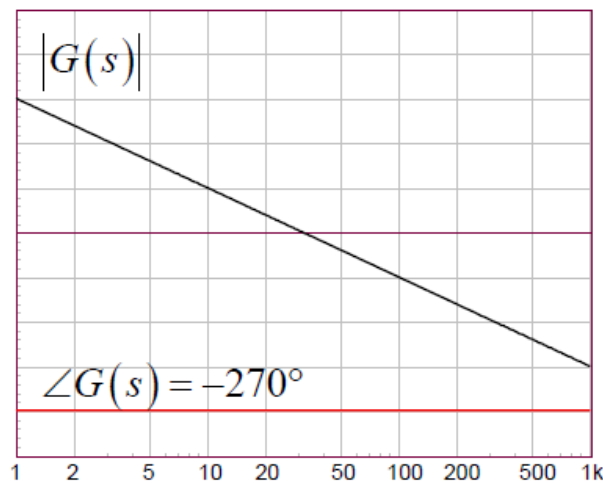
## Which Crossover Frequency to Select?

- Switching frequency: theoretical limit is  $F_{sw} / 2$ 
  - in practice, stay below  $1/5$  of  $F_{sw}$  for noise concerns
- Presence of a Right-Half Plane Zero (RHPZ):
  - You cannot cross over beyond 30% of the lowest RHPZ position
- Output undershoot specification:
  - Select crossover frequency based on undershoot specs

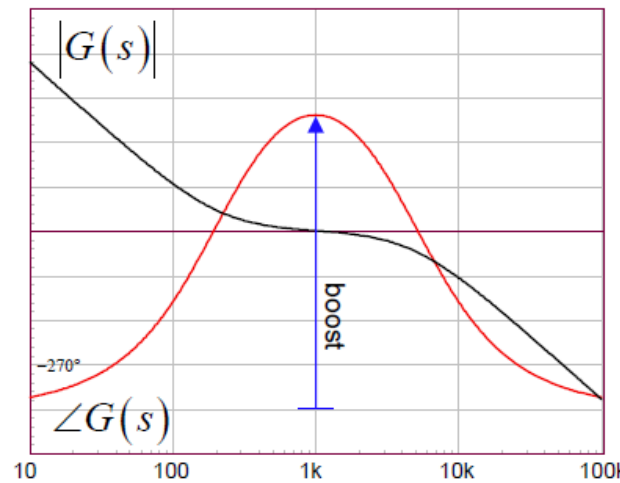


# Controller types

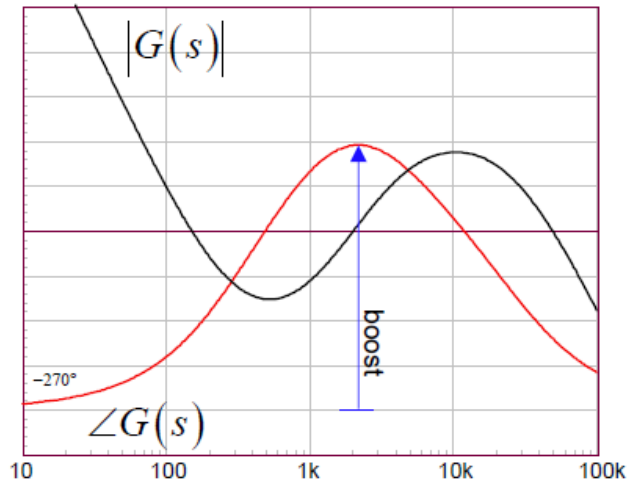
- type 1, 1 pole at the origin, no phase boost
- type 2, 1 pole at the origin, 1 zero, 1 pole. Phase boost up to  $90^\circ$
- type 3, 1 pole at the origin, 1 zero pair, 1 pole pair. Boost up to  $180^\circ$



Type 1



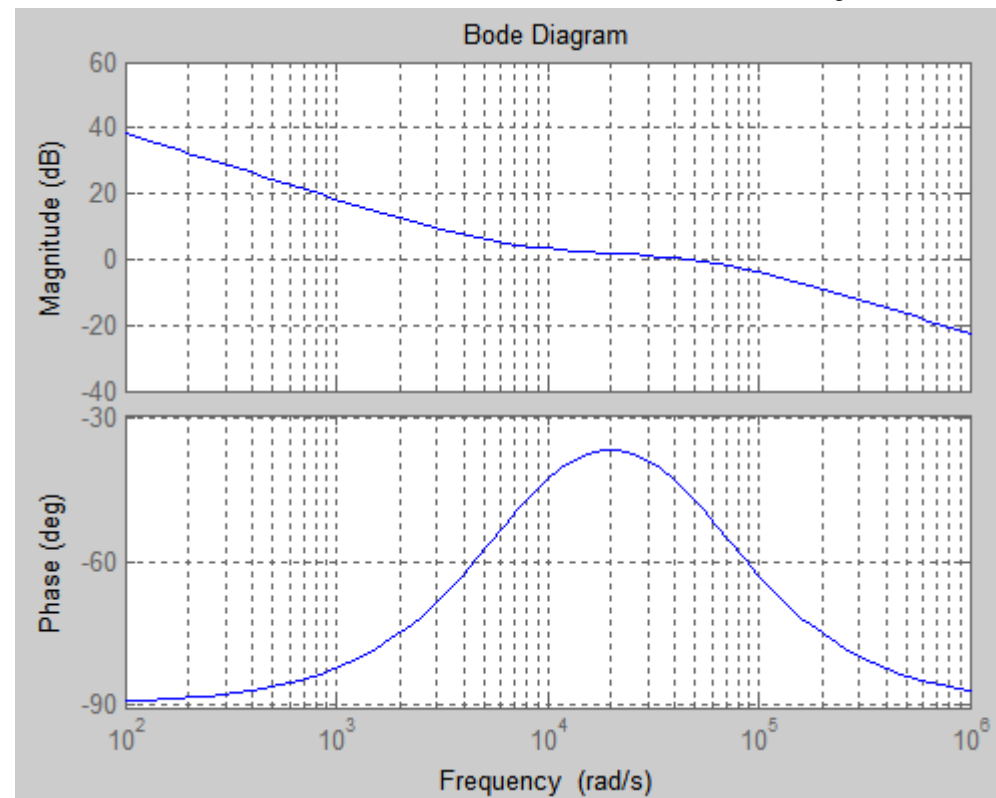
Type 2



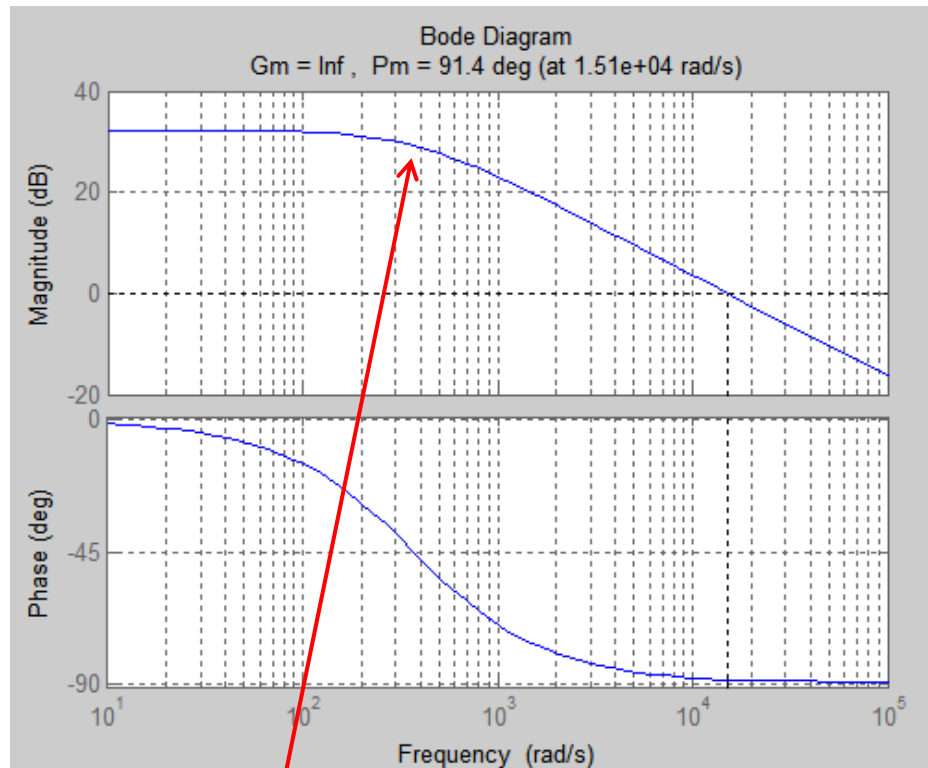
Type 3

# Type II controller

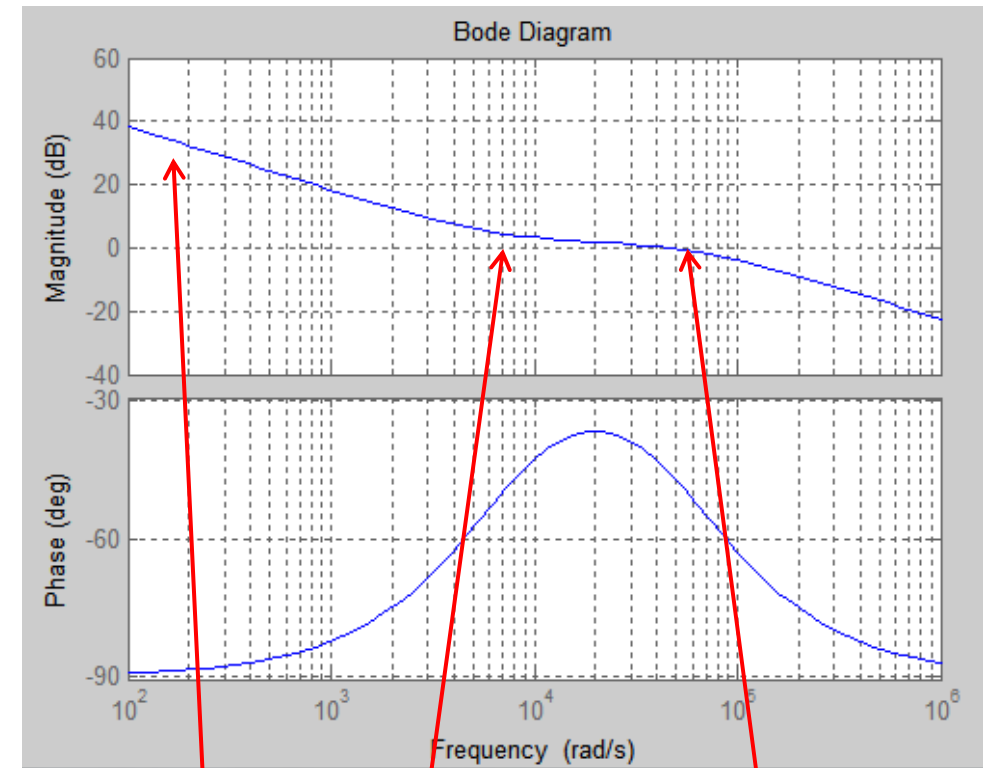
$$G_c = G_0 \cdot \frac{1}{\tau_0 s} \cdot \frac{\tau_{zc} s + 1}{\tau_{pc} s + 1} = G_0 \cdot \frac{1}{\frac{s}{\omega_0}} \cdot \frac{\frac{s}{\omega_{zc}} + 1}{\frac{s}{\omega_{pc}} + 1}$$



# Why Type II controller?



$$\frac{1}{R_s} \frac{n_p}{n_s} \frac{1}{\frac{s}{\omega_p} + 1}$$

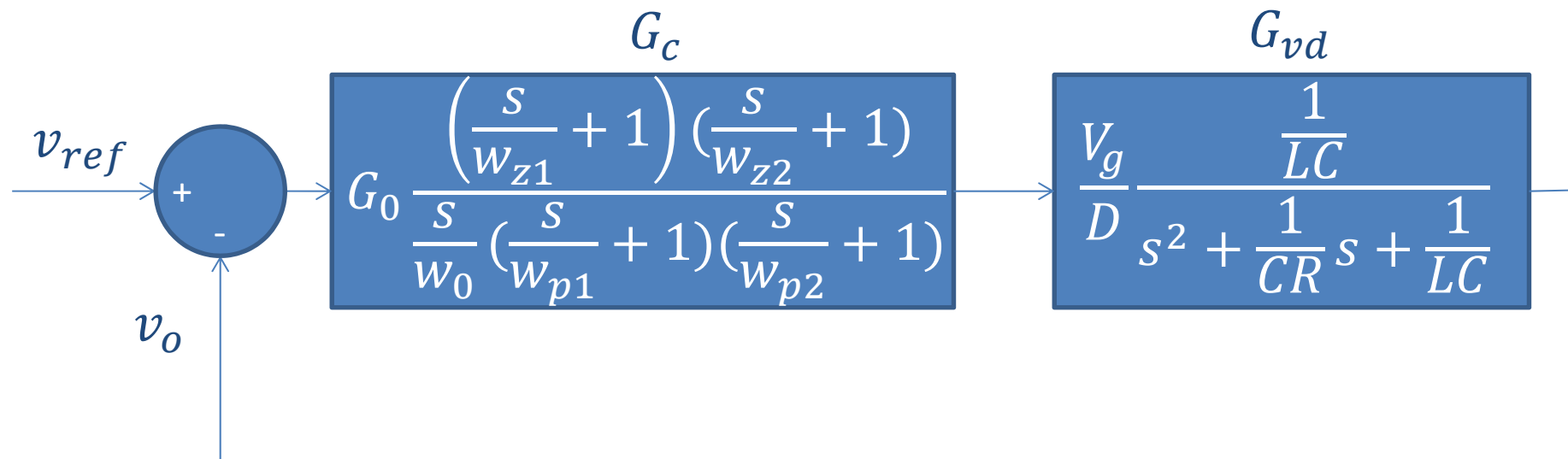


$$\frac{1}{\frac{s}{\omega_0}}$$

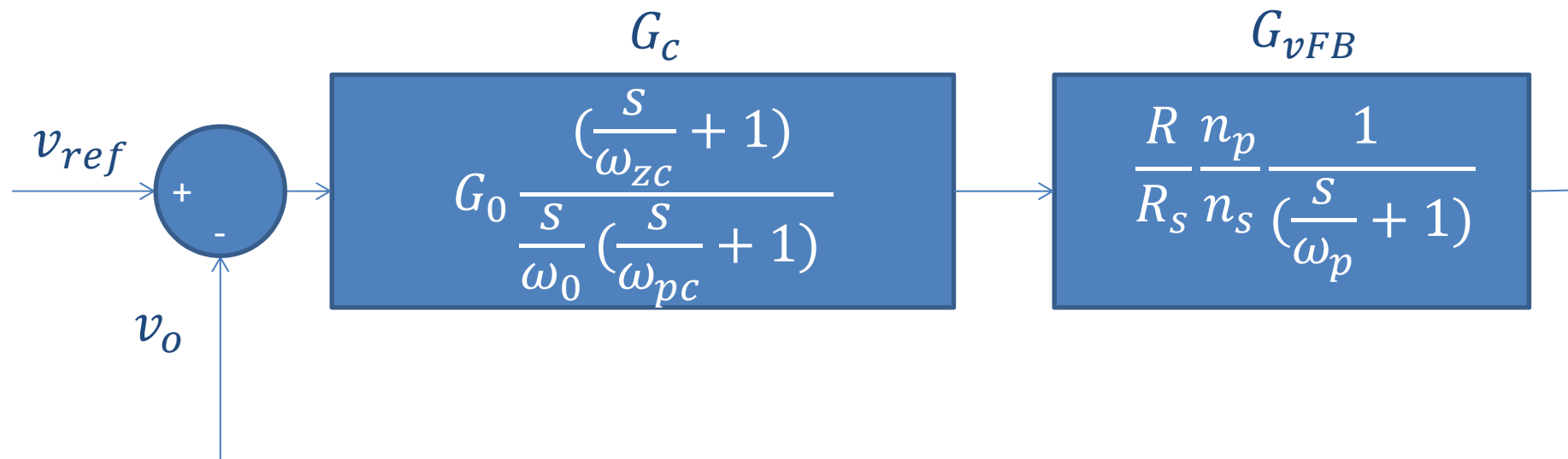
$$\frac{s}{\omega_{zc}} + 1$$

$$\frac{s}{\omega_{pc}} + 1$$

# Control structure – voltage mode



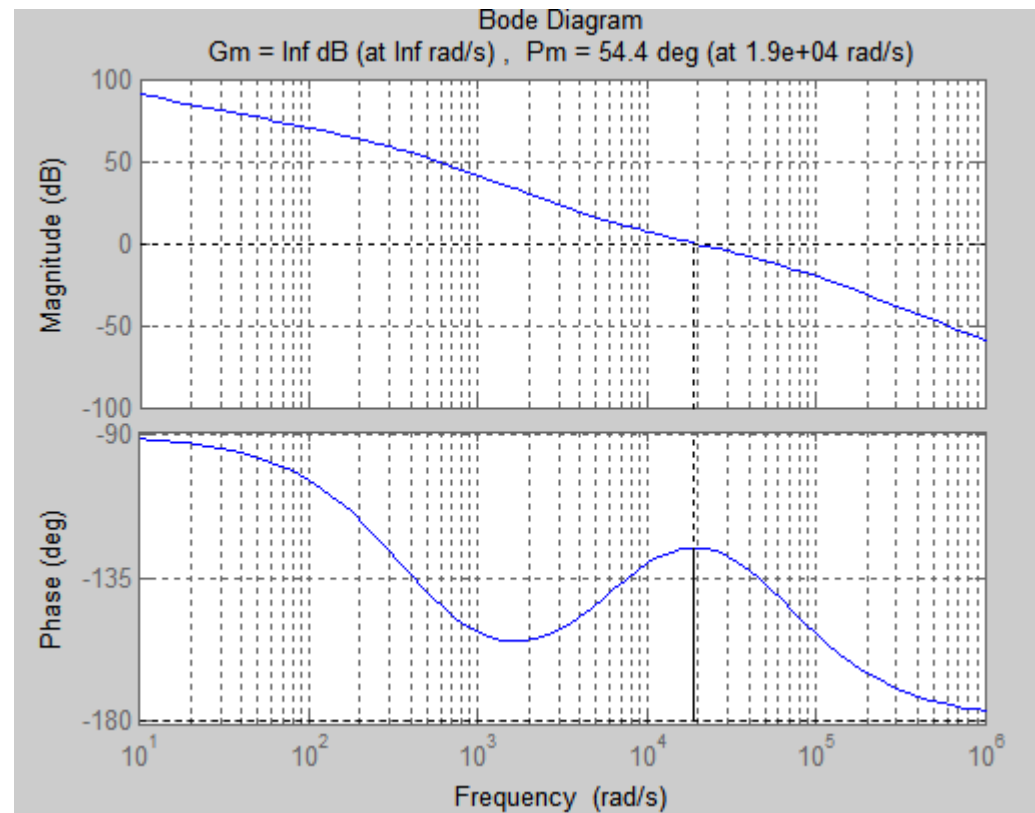
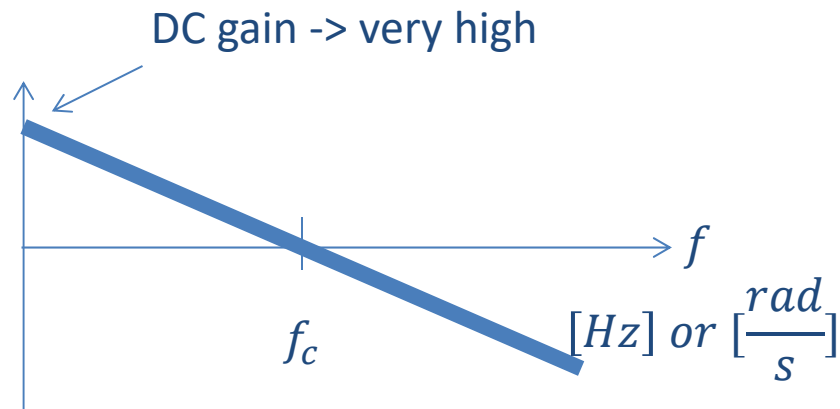
# Control structure – current mode



# Open loop gain – $T_{OL}(s)$

$$T_{OL}(s) = G_C(s) \cdot G_{vFB}(s)$$

$$G_{CL}(s) = \frac{T_{OL}(s)}{1 + T_{OL}(s)}$$

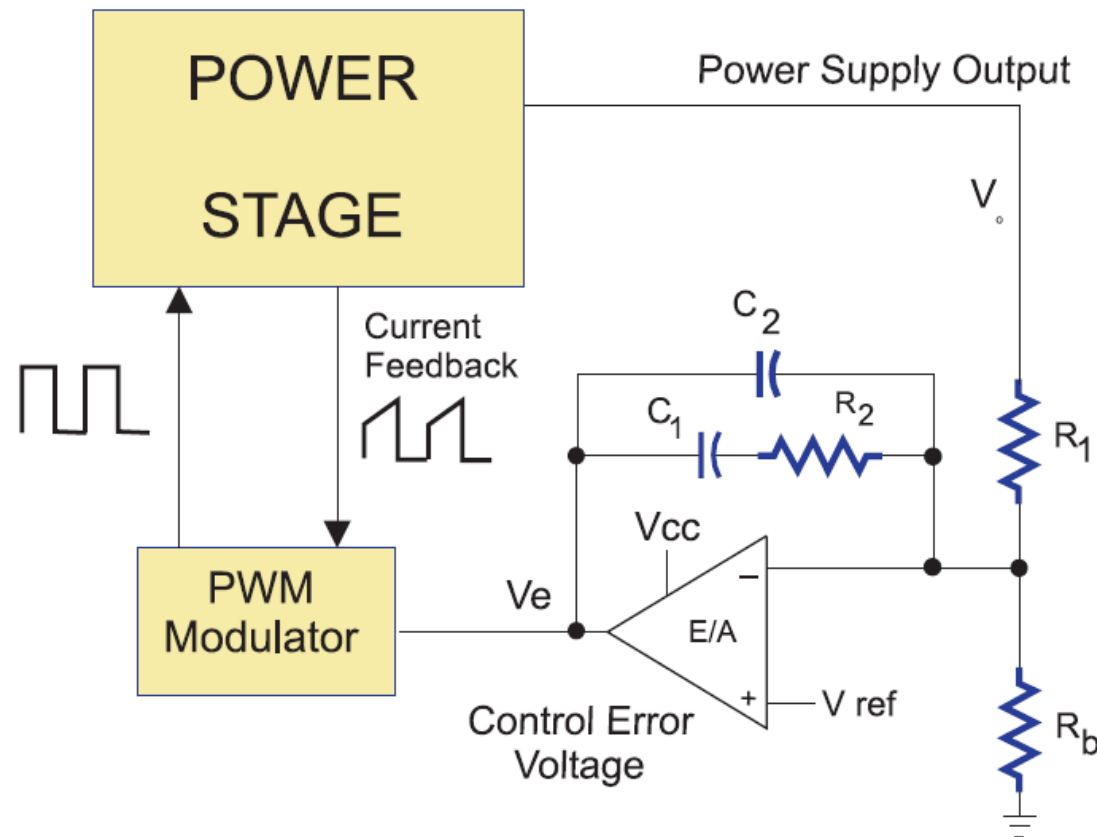


Notice :  $T_{ol}$  – strongly depend on load - R

# Implementation

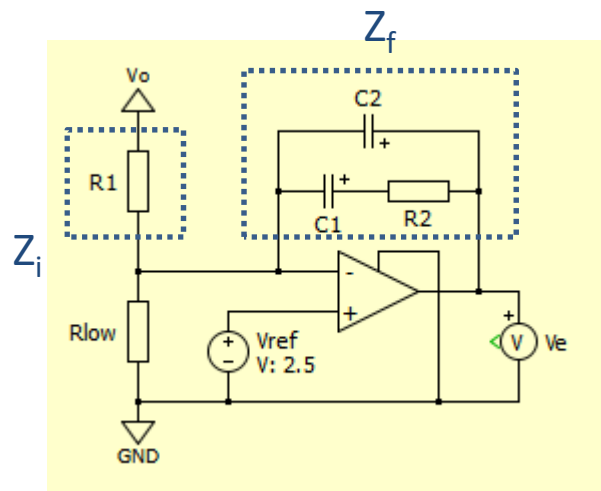


# Implementation with operational amplifier



*Type II Compensation Feedback using Operational Amplifier\**

# Implementation with operational amplifier

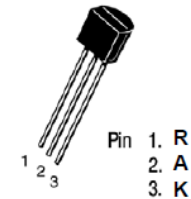
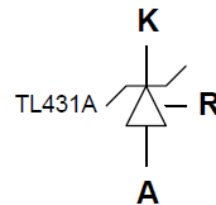
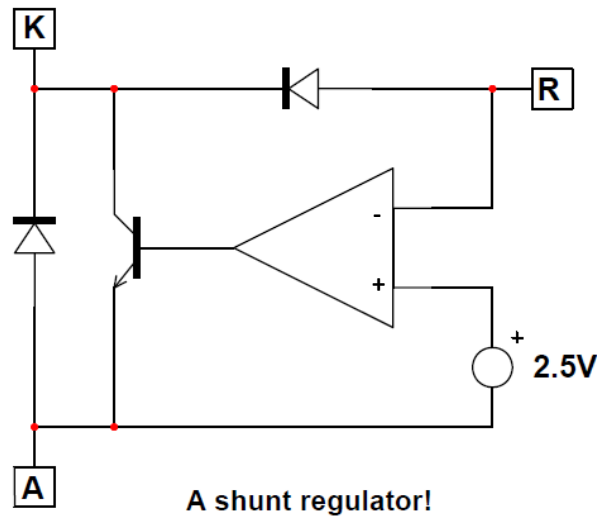


$$G_C(s) = \frac{v_e}{v_o} = \frac{Z_f}{Z_i} = G_0 \cdot \frac{1}{\frac{s}{\omega_0}} \cdot \frac{\frac{s}{\omega_{zc}} + 1}{\frac{s}{\omega_{pc}} + 1}$$

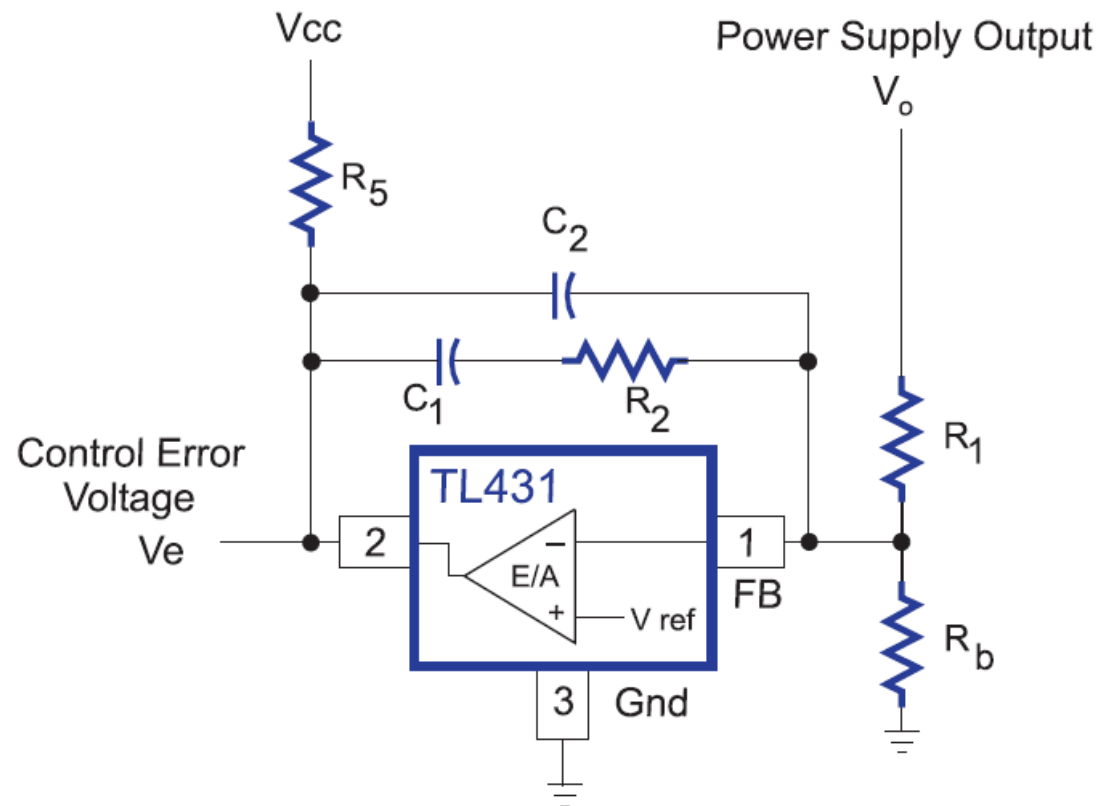
$$\omega_0 = \frac{1}{R_1 C_1}; \omega_{zc} = \frac{1}{R_2 C_1}; \omega_{pc} = \frac{1}{R_2 C_2}$$

- Valid if  $C_2 \ll C_1$

# Implementation with TL431

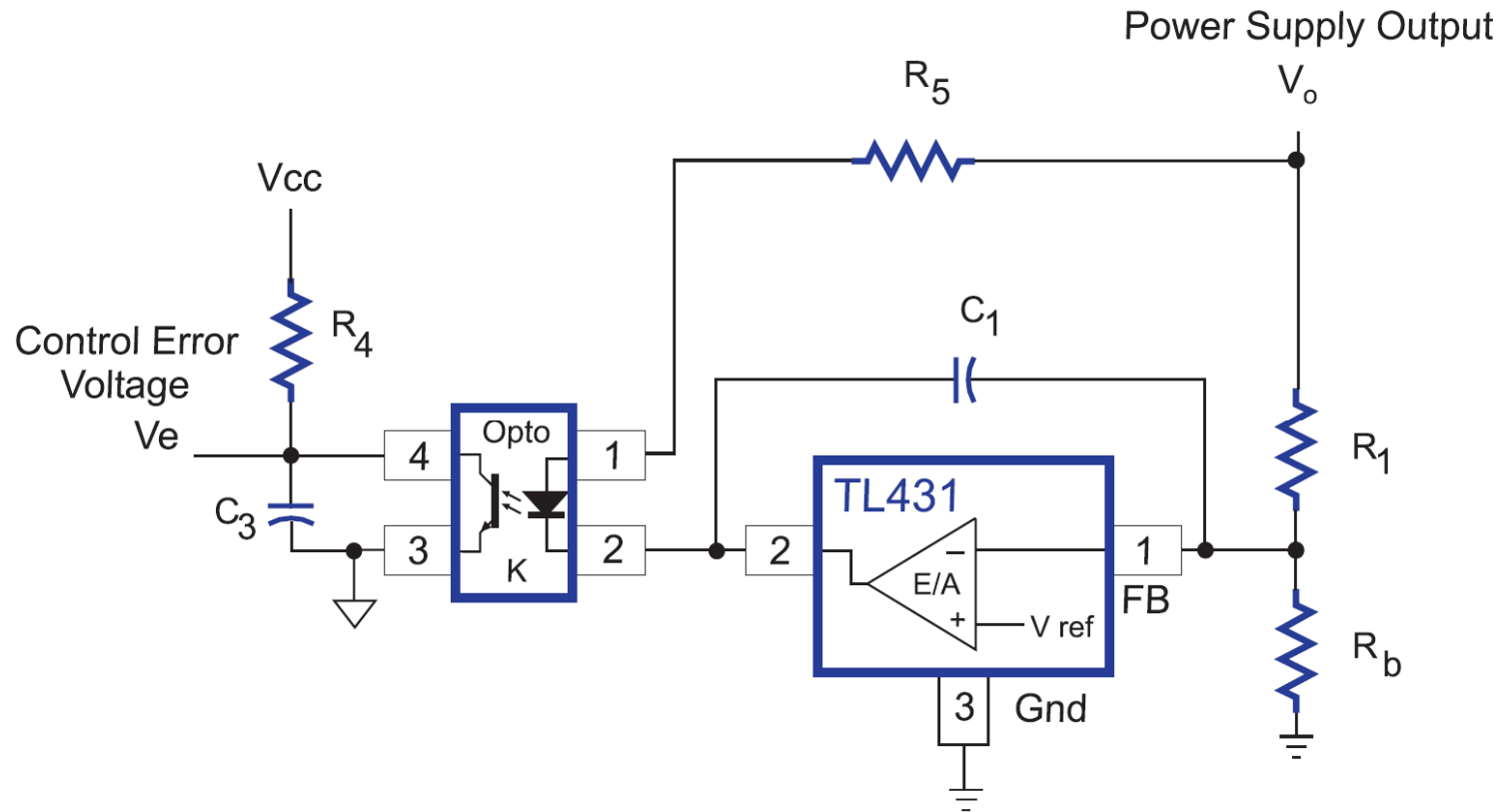


# Implementation with TL431



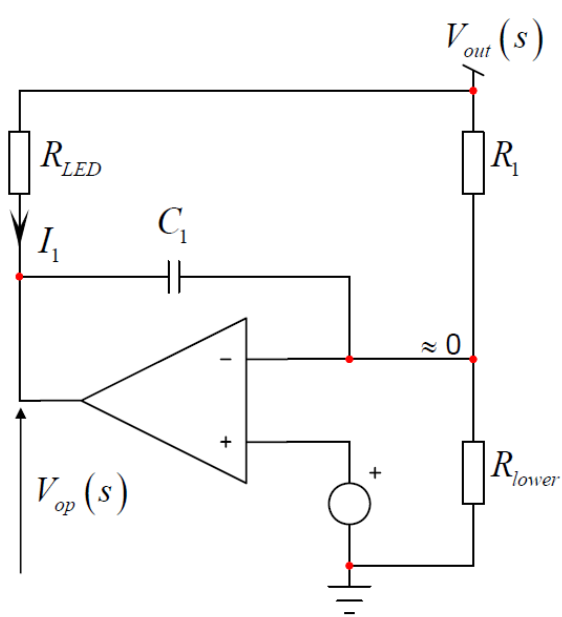
*Type II Compensation Feedback using TL431\**

# Implementation with TL431 and optocoupler



*Type II Compensation Feedback using TL431 and optocoupler\**

# Implementation with TL431 and optocoupler



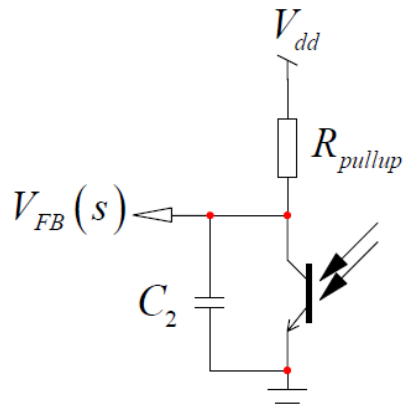
$$I_1(s) = \frac{V_{out}(s) - V_{op}(s)}{R_{LED}}$$

$$V_{op}(s) = -V_{out}(s) \frac{sC_1}{R_{upper}} = -V_{out}(s) \frac{1}{sR_{upper}C_1}$$

$$I_1(s) = V_{out}(s) \frac{1}{R_{LED}} \left[ 1 + \frac{1}{sR_{upper}C_1} \right]$$

We know that:  $V_{FB}(s) = -CTR \cdot R_{pullup} \cdot I_1$

$$\frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup} CTR}{R_{LED}} \left[ \frac{1 + sR_{upper}C_1}{sR_{upper}C_1} \right]$$



Add a cap. from collector to ground

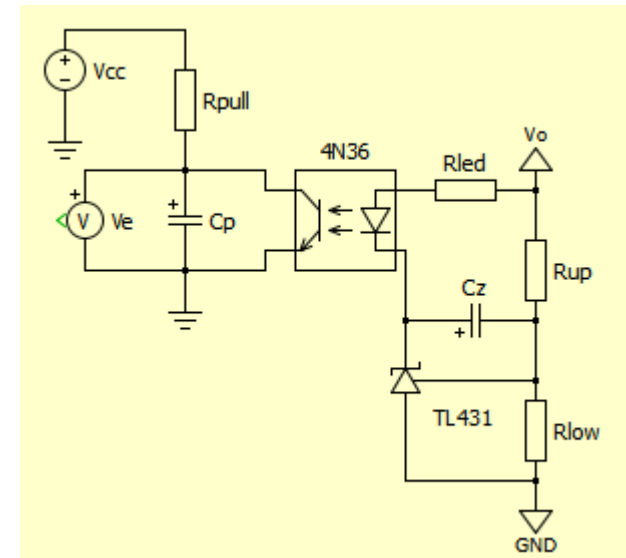
Type 2 transfer function

$$\frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup} CTR}{R_{LED}} \left[ \frac{1 + sR_{upper}C_1}{sR_{upper}C_1 (1 + sR_{pullup}C_2)} \right]$$

# Implementation with TL431 and optocoupler

$$G_C(s) = \frac{v_e}{v_o} = G_0 \cdot \frac{1}{\frac{s}{\omega_0} + 1} \cdot \frac{\frac{s}{\omega_{zc}} + 1}{\frac{s}{\omega_{pc}} + 1}$$

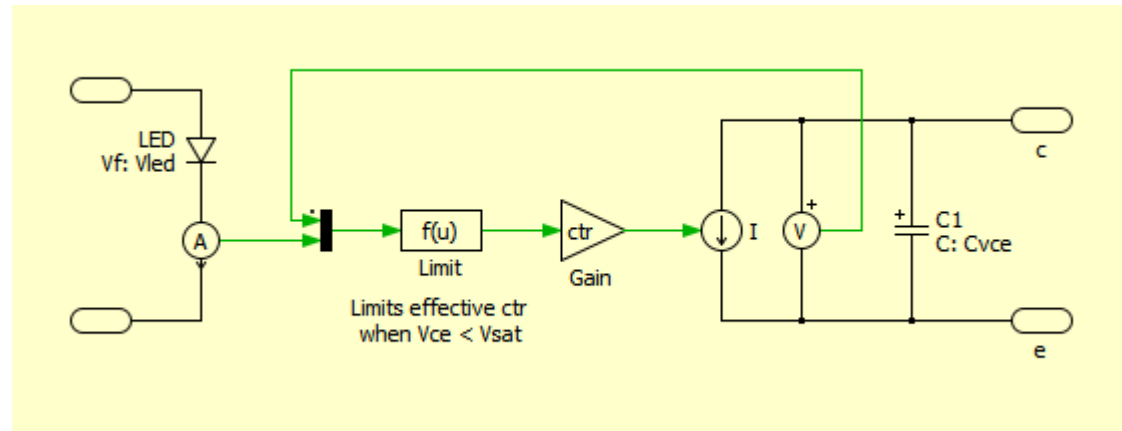
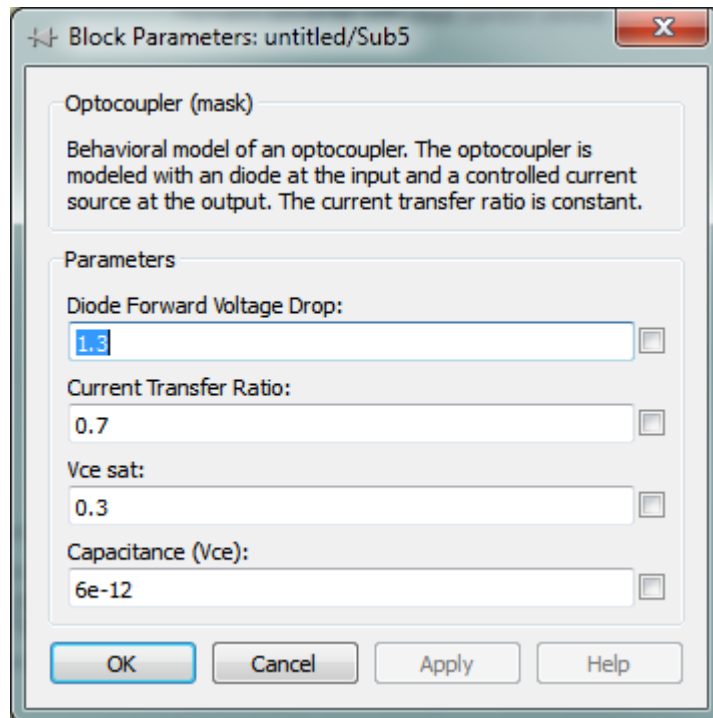
$$G_0 = \frac{R_{pull} CTR}{R_{led}}; \omega_0 = \frac{1}{R_{up} C_z}; \omega_{zc} = \frac{1}{R_{up} C_z}; \omega_{pc} = \frac{1}{R_{pull} C_p}$$



Notice : CTR – Current Transfer Ratio of the optocoupler, it can be found in a datasheet:  
Ex. <http://www.vishay.com/docs/81181/4n35.pdf>

Also useful information about a real TL431 you can find from a datasheet:  
Ex. <http://www.ti.com/lit/ds/symlink/tl431.pdf>

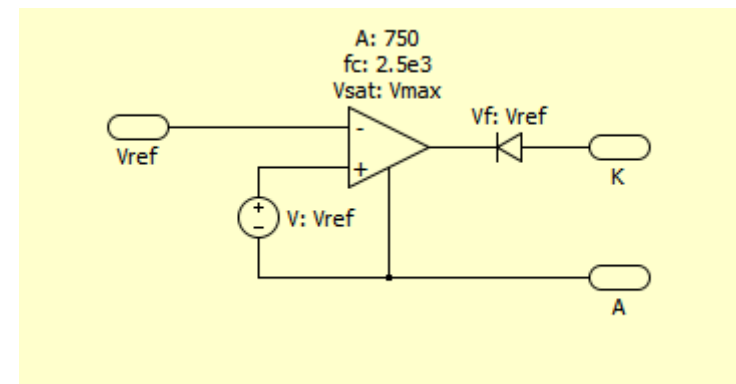
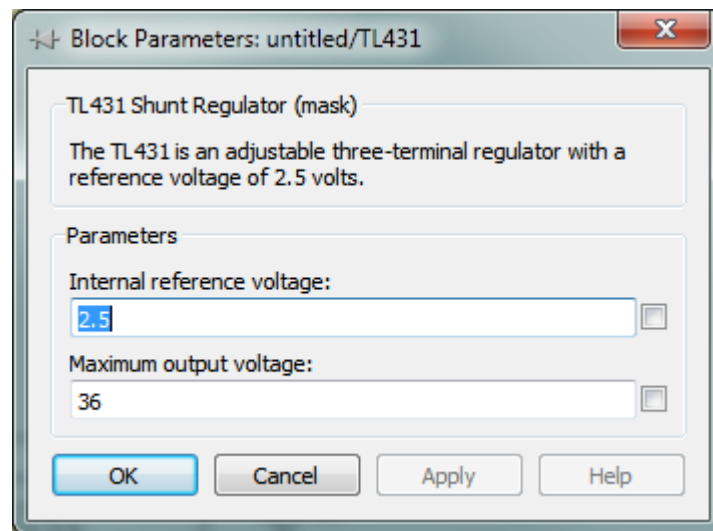
# Optocoupler model



*Model of the optocoupler in PLECS*



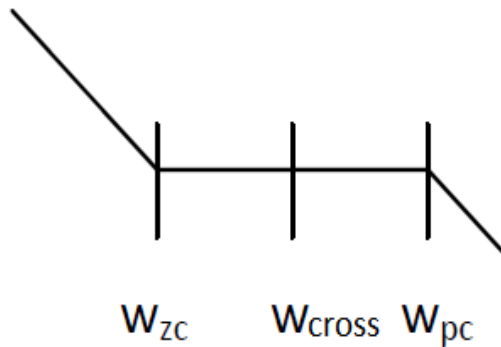
# TL431 model



*Model of the TL431 in PLECS*

# Technical issues

Determine the  $\omega_z$  and the  $\omega_n$



$$\omega_{cross} = \sqrt{\omega_{cz} \omega_{cp}}$$

Determine the  $\|G_C(\omega_{cross})\|$  ( $s = j\omega_{cross}$ )

$$G_C(\omega_{cross}) = G_0 \frac{\omega_0}{j\sqrt{\omega_{cz}\omega_{cp}}} \frac{1 + j\sqrt{\frac{\omega_{cp}}{\omega_{cz}}}}{1 + j\sqrt{\frac{\omega_{cz}}{\omega_{cp}}}}$$

$$\|G_C(\omega_{cross})\| = G_0 \frac{\omega_0}{\sqrt{\omega_{cz}\omega_{cp}}} \frac{\sqrt{1 + \frac{\omega_{cp}}{\omega_{cz}}}}{\sqrt{1 + \frac{\omega_{cz}}{\omega_{cp}}}}$$

$$\|G_C(\omega_{cross})\|_{dB} = -\|G_{vFB}(\omega_{cross})\|_{dB}$$

$$\|G_C(\omega_{cross})\|_{mag} = G_0 = \frac{R_{pull} CTR}{R_{led}}$$

# Technical issues

*Calculating  $R_{led}$*

$$R_{led} = (V_o - V_F - V_K) / i_F$$

*Obtaining CTR*

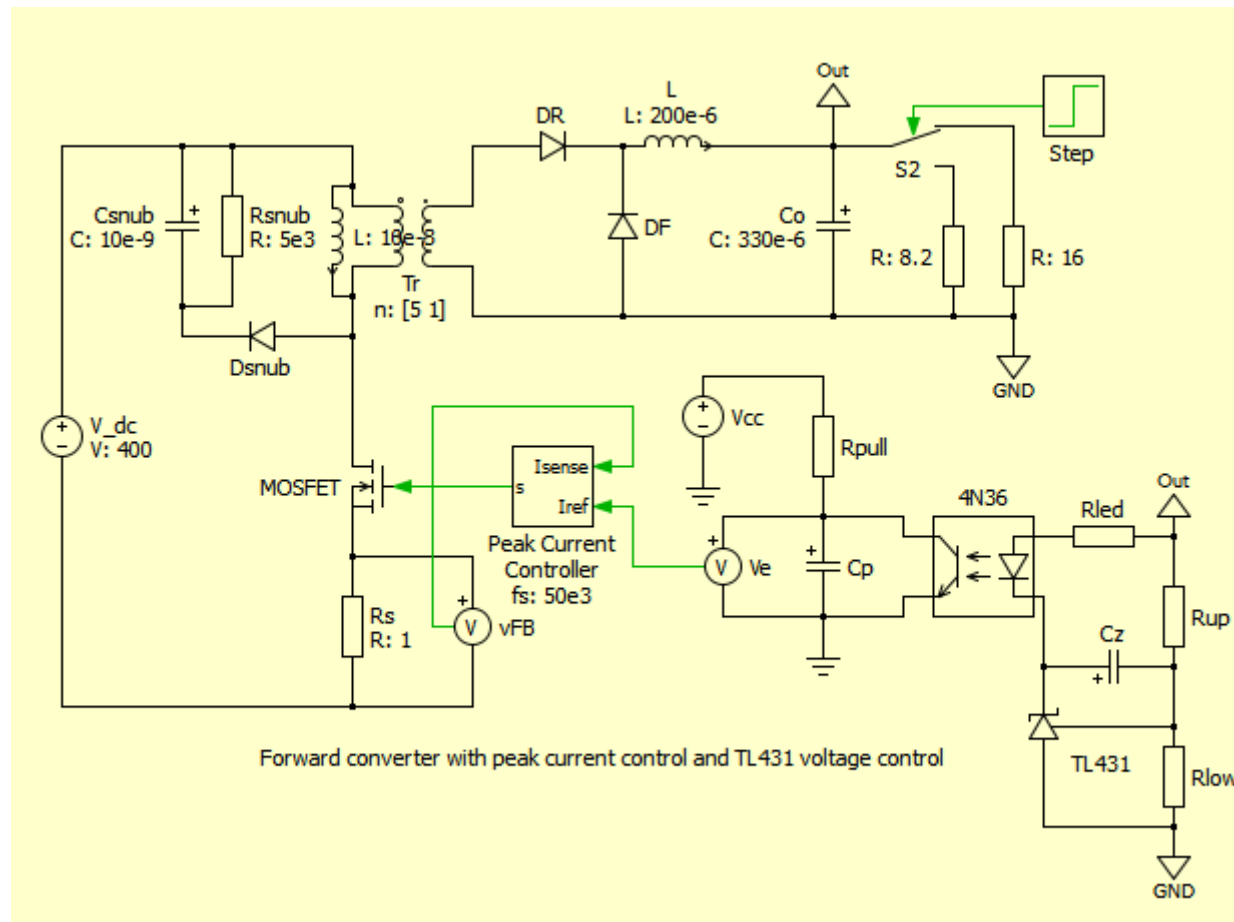
$$CTR = Gain = \frac{i_C}{i_F}$$

*$i_c$  = transistor collector current and  $i_F$  is diode current  
(see datasheet)*

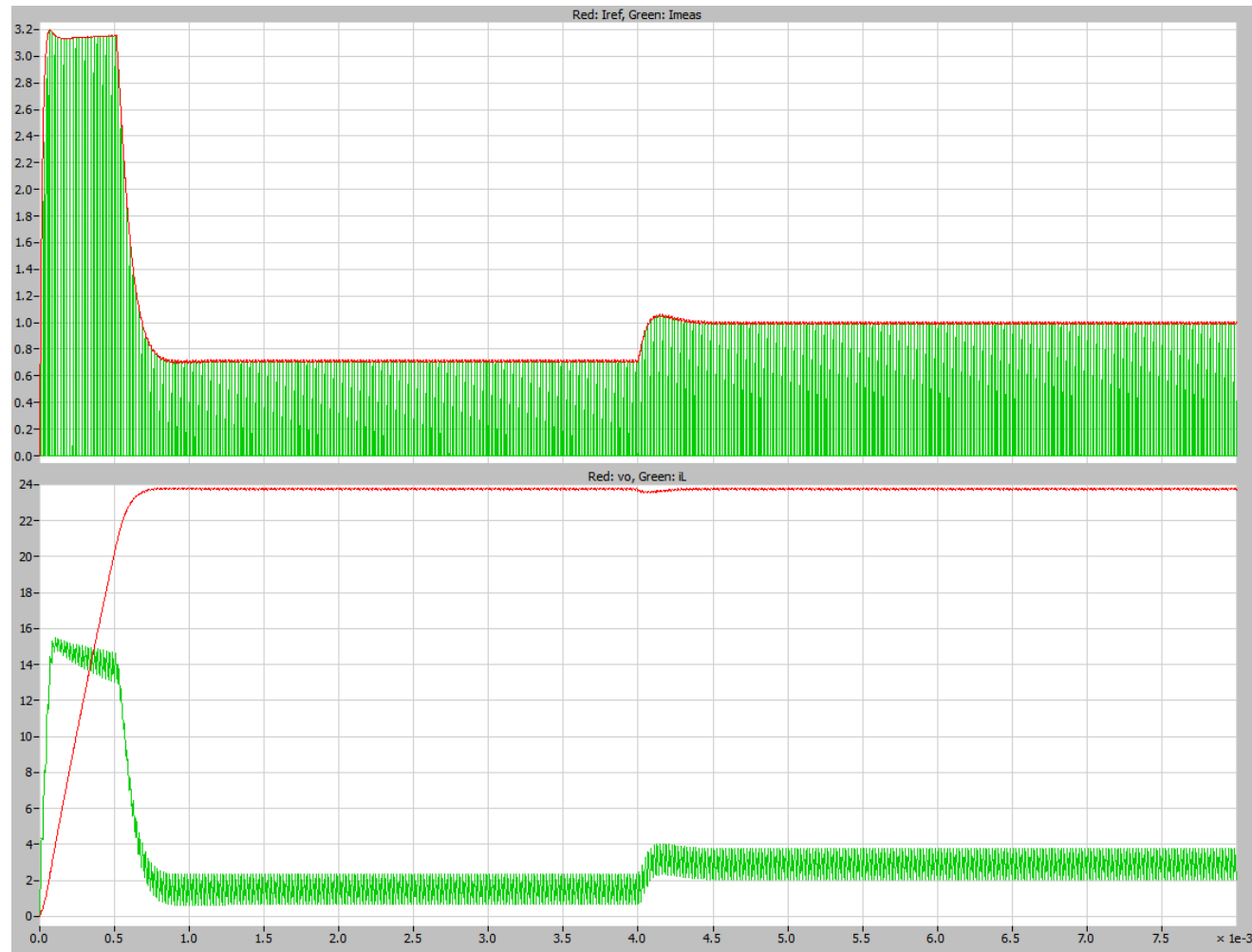
*Finding  $R_{up}$  and  $R_{low}$*

$$2.5V = 24V \frac{R_{low}}{R_{up} + R_{low}}$$

# Simulations



# Simulations



# Exercise:

Given is a forward converter model in PLECS (see Starting Model.plecs file)

- The converter has  $v_o=24V$  and it delivers 70W power.
- Switching frequency should be 50 kHz
- $n_p/n_s = 5$
- Desired crossover frequency:  $\omega_{cross} = 20000 \frac{rad}{s}$

Task: Design a control structure for the converter using the peak current controller, the TL431 and the optocoupler blocks – similarly to the one presented during the course.

0: Using the formulas given in the lecture, sketch the bode plot of  $G_{vFB}(s)$  in Matlab. Also use Matlab for checking your controller's behavior on its bode plots.

- 1: Select the values for  $R_{up}$  and  $R_{low}$
- 2: Find the desired gain at  $\omega_{cross}$  ( $G_0 = CTR * R_{pull} / R_{led}$ )
- 3: Find  $\omega_{zc} = \omega_{cross} / 3$ ; select  $C_z$
- 4: Find  $\omega_{pc} = \omega_{cross} * 3$ ; select  $C_p$ ,  $R_{pull}$  and  $R_{led}$

See next page for 5: 😊

# Exercise:

5: Simulate in PLECS:

- a. Test power circuit in open loop and see waveforms
- b. Build current controller
- c. Build TL431 control circuit
- d. Simulate the converter in closed loop and plot the  $T_{oi}$  as well.