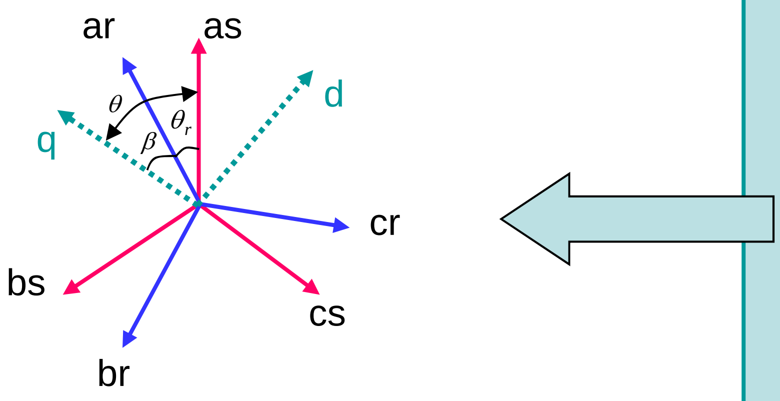
Lecture 6 - contents

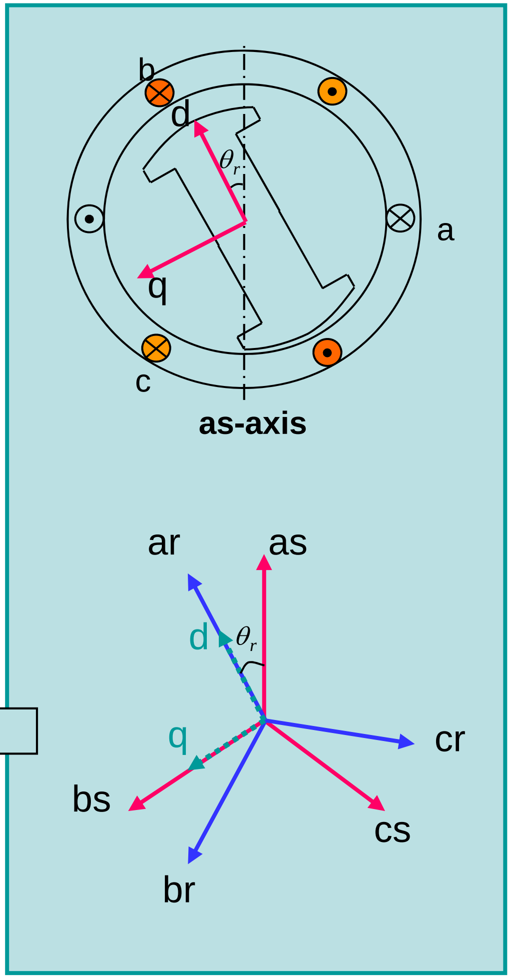
- Modeling of IM in an arbitrary reference-frame

Modeling of IM in an arbitrary reference-frame

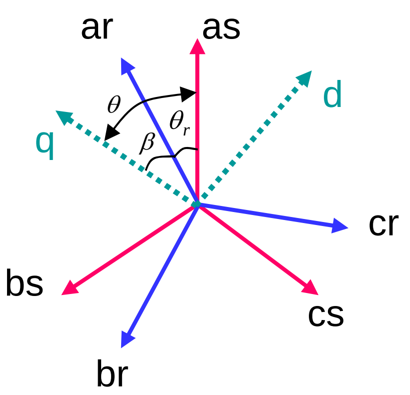
Reference frame definition - mapping the synchronous machine to the induction machine

An arbitrary rotating reference frame (qd0):





Stator self-inductance matrix

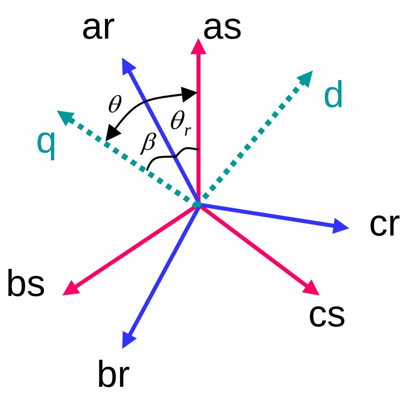


$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \underline{L_s} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & L_{ls} + L_{ms} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & \boxed{-\frac{L_{ms}}{2}} & L_{ls} + \boxed{L_{ms}} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

- Learned from lecture 3 – how the inductances are determined

Observation of these two values ……

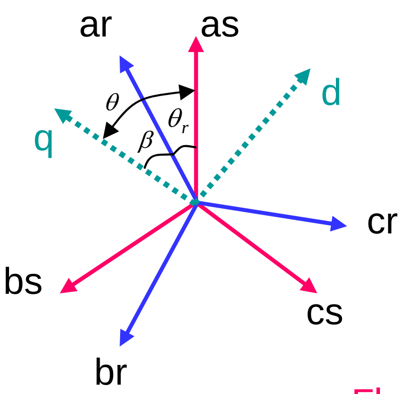
Rotor self-inductance matrix



$$\begin{bmatrix} \lambda_{ar} \\ \lambda_{br} \\ \lambda_{cr} \end{bmatrix} = \underline{L_r} \cdot \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} = \begin{bmatrix} L_{lr} + L_{mr} & -\frac{L_{mr}}{2} & -\frac{L_{mr}}{2} \\ -\frac{L_{mr}}{2} & L_{lr} + L_{mr} & -\frac{L_{mr}}{2} \\ -\frac{L_{mr}}{2} & -\frac{L_{mr}}{2} & L_{lr} + L_{mr} \end{bmatrix} \cdot \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

- Stator and rotor self-inductances are position independent
- Stator and rotor self-inductance matrices have the similar form
- It could be expected that $L_{mr}\left(\frac{N_s}{N_r}\right)^2 = L_{ms}$

Rotor to stator mutual inductance matrix



$$\begin{bmatrix} \lambda_{asr} \\ \lambda_{bsr} \\ \lambda_{csr} \end{bmatrix} = \underline{L_{sr}} \cdot \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} = M_{sr} \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r \end{bmatrix} \cdot \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

Flux-axis, the phase with current for producing that flux linkage

$$M_{sr}\,\mathrm{Re}\left(\frac{e^{j\left(\theta_r + \frac{2\pi}{3}\right)}}{e^{j\left(-\frac{2\pi}{3}\right)}}\right) \mathrm{Re}\left(\frac{e^{j\left(\theta_r + \frac{2\pi}{3}\right)}}{e^{j\left(\theta_r + \frac{2\pi}{3}\right)}}\right) = M_{sr} \cos\left(\theta_r - \frac{2\pi}{3}\right)$$

Rotor phase b axis

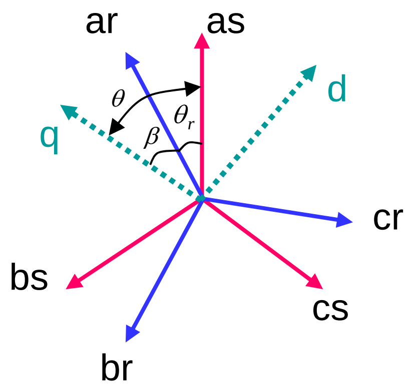
Stator phase c axis

From stator to rotor:

$$M_{sr}\,\mathrm{Re}\left(\frac{e^{j\left(\frac{2\pi}{3}\right)}}{e^{j\left(\theta_r - \frac{2\pi}{3}\right)}}\right) \mathrm{Re}\left(\frac{e^{j\left(\frac{2\pi}{3}\right)}}{e^{j\left(\frac{2\pi}{3}\right)}}\right) = M_{sr} \cos\left(\theta_r + \frac{2\pi}{3}\right)$$

$$\underline{\lambda_{rs}} = \underline{L_{rs}} \cdot \underline{i_{abcs}} = \underline{L_{sr}}^{T} \cdot \underline{i_{abcs}}$$

Transform the stator flux linkage to an arbitrary qd0 system



$$\underline{\lambda}_{abcs} = \underline{L}_s \cdot \underline{i}_{abcs} + \underline{L}_{sr} \cdot \underline{i}_{abcr}$$

$$\underline{\lambda}_{qd0s} = \underline{K}_s \underline{L}_s \underline{K}_s^{-1} \cdot \underline{i}_{qd0s} + \underline{K}_s \underline{L}_{sr} \underline{K}_r^{-1} \cdot \underline{i}_{qd0r}$$

$$K_s = \frac{2}{3}\begin{bmatrix} \cos\boxed{\theta} & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$K^{-1}{}_s = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

$$K_r = \frac{2}{3}\begin{bmatrix} \cos\boxed{\beta} & \cos\left(\beta - \frac{2\pi}{3}\right) & \cos\left(\beta + \frac{2\pi}{3}\right) \\ \sin\beta & \sin\left(\beta - \frac{2\pi}{3}\right) & \sin\left(\beta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$K^{-1}{}_r = \begin{bmatrix} \cos\beta & \sin\beta & 1 \\ \cos\left(\beta - \frac{2\pi}{3}\right) & \sin\left(\beta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\beta + \frac{2\pi}{3}\right) & \sin\left(\beta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

Transform the stator flux linkage to an arbitrary qd0 system

$$\underline{L}_{sqd0} = \underline{K}_s \underline{L}_s \underline{K}_s^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2}L_{ms} & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2}L_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

$$\underline{L}_{srqd0} = \underline{K}_s \underline{L}_{sr} \underline{K}_r^{-1} = \begin{bmatrix} \frac{3}{2}M_{sr} & 0 & 0 \\ 0 & \frac{3}{2}M_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{Define} \quad \frac{3}{2}L_{ms} = L_m \quad \Longrightarrow \quad M_{sr}\frac{N_s}{N_r} = L_{ms} = \frac{2}{3}L_m}$$

$$\underline{\lambda}_{qd0s} = \begin{bmatrix} L_{ls} + L_m & 0 & 0 \\ 0 & L_{ls} + L_m & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \cdot \underline{i}_{qd0s} + \begin{bmatrix} L_m & 0 & 0 \\ 0 & L_m & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \boxed{\begin{matrix} \frac{N_r}{N_s} i_{qr} \\ \frac{N_r}{N_s} i_{dr} \\ \frac{N_r}{N_s} i_{0r} \end{matrix}} \end{bmatrix}$$

$$\boxed{i'_j = \frac{N_r}{N_s} i_j, \quad j = qr, dr, 0r, ar, br, cr}$$

Transform the rotor flux linkage to an arbitrary qd0 system

$$\underline{\lambda}_{abcr} = \underline{L}_r \cdot \underline{i}_{abcr} + \underline{L}_{sr}^T \cdot \underline{i}_{abcs}$$

$$\underline{\lambda}_{qd0r} = \underline{K}_r \underline{L}_r \underline{K}_r^{-1} \cdot \underline{i}_{qd0r} + \underline{K}_r \underline{L}_{sr}^T \underline{K}_s^{-1} \cdot \underline{i}_{qd0s}$$

$$\underline{\lambda}_{qd0r} = \begin{bmatrix} L_{lr} + \frac{3}{2}L_{mr} & 0 & 0 \\ 0 & L_{lr} + \frac{3}{2}L_{mr} & 0 \\ 0 & 0 & L_{lr} \end{bmatrix} \cdot \underline{i}_{qd0r} + \begin{bmatrix} \frac{3}{2}M_{sr} & 0 & 0 \\ 0 & \frac{3}{2}M_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \underline{i}_{qd0s}$$

Define $\lambda'_j = \frac{N_s}{N_r}\lambda_j \Rightarrow u'_j = \frac{N_s}{N_r}u_j \qquad j = qr, dr, 0r, ar, br, cr$

$$L_{mr}\left(\frac{N_s}{N_r}\right)^2 = L_{ms} = \frac{2}{3}L_m\,, \qquad M_{sr}\frac{N_s}{N_r} = L_{ms} = \frac{2}{3}L_m$$

$$i'_j = \frac{N_r}{N_s}i_j$$

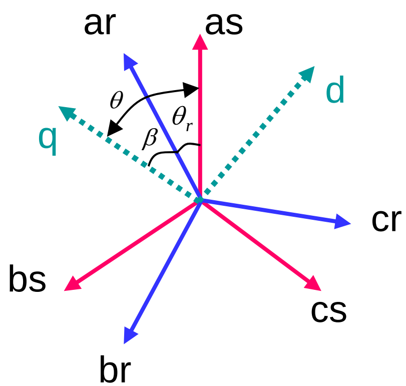
$$L'_{lr} = L_{lr}\left(\frac{N_s}{N_r}\right)^2$$

$$\underline{\lambda}'_{qd0r} = \begin{bmatrix} L'_{lr} + L_m & 0 & 0 \\ 0 & L'_{lr} + L_m & 0 \\ 0 & 0 & L'_{lr} \end{bmatrix} \cdot \underline{i}'_{qd0r} + \begin{bmatrix} L_m & 0 & 0 \\ 0 & L_m & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \underline{i}_{qd0s}$$

Transform the voltage equations to an arbitrary qd0 system

- According to the slide P11-12, lecture 4, the stator voltage equations

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{0s} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} - \omega_\theta \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$



- Similarly, the rotor voltage equations would be

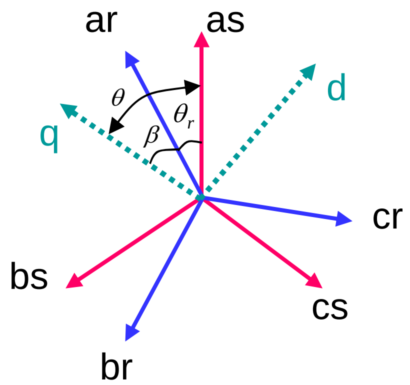
$$\begin{bmatrix} u'_{qr} \\ u'_{dr} \\ u'_{0r} \end{bmatrix} = \begin{bmatrix} R'_r & 0 & 0 \\ 0 & R'_r & 0 \\ 0 & 0 & R'_r \end{bmatrix} \cdot \begin{bmatrix} i'_{qr} \\ i'_{dr} \\ i'_{0r} \end{bmatrix} + p \begin{bmatrix} \lambda'_{qr} \\ \lambda'_{dr} \\ \lambda'_{0r} \end{bmatrix} - \boxed{(\omega_\theta - \omega_r)} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda'_{qr} \\ \lambda'_{dr} \\ \lambda'_{0r} \end{bmatrix}$$

Why?

Advanced notes

Another method - Using the vectors instead of the matrices

- Please refer to 'Vector control and dynamics of AC machines' By D.W. Novotny, and T.A. Lipo, pp43~61



$$\bar{u}_{abcs} = \frac{2}{3}\begin{bmatrix}1 & \alpha & \alpha^2\end{bmatrix}\begin{bmatrix}u_{as}\\ u_{bs}\\ u_{cs}\end{bmatrix} = R_s \bar{i}_{abcs} + \frac{d}{dt}\left[(L_{ls}+L_m)\bar{i}_{abcs} + L_m \bar{i}_{abcr} e^{j\boxed{\theta_r}}\right]$$

$$\bar{\lambda}_{abcs} = (L_{ls}+L_m)\bar{i}_{abcs} + L_m \bar{i}_{abcr} e^{j\theta_r}$$

$$\boxed{\begin{aligned} \bar{u}_{abcs} &= \bar{u}_{qds}\cdot e^{j\theta}, & \bar{i}_{abcs} &= \bar{i}_{qds}\cdot e^{j\boxed{\theta}}, \\ \bar{\lambda}_{abcs} &= \bar{\lambda}_{qds}\cdot e^{j\theta}, & \bar{i}_{abcr} &= \bar{i}_{qdr}\cdot e^{j\boxed{\beta}} \end{aligned}}$$

$$\bar{f}_{qd} = f_q - jf_d$$

$$\bar{\lambda}_{qds} = (L_{ls}+L_m)\bar{i}_{qds} + L_m \bar{i}_{abcr} e^{-j\beta} = (L_{ls}+L_m)\bar{i}_{qds} + L_m \bar{i}_{qdr}$$

No zero component involved in the qd vectors!

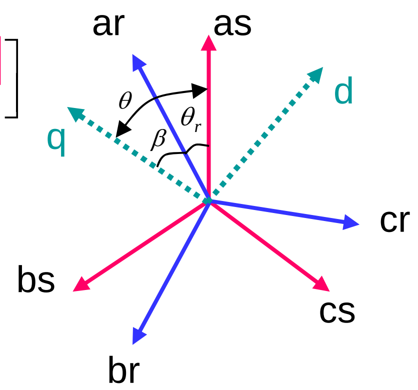
$$\begin{aligned} \bar{u}_{qds} &= R_s \bar{i}_{qds} + e^{-j\theta}\frac{d}{dt}\left[(L_{ls}+L_m)\bar{i}_{abcs} + L_m \bar{i}_{abcr} e^{j\theta_r}\right] \\ &= R_s \bar{i}_{qds} + \frac{d}{dt}\left[(L_{ls}+L_m)\bar{i}_{qds} + L_m \bar{i}_{qdr}\right] - \frac{d}{dt}e^{-j\theta}\cdot\left[(L_{ls}+L_m)\bar{i}_{abcs} + L_m \bar{i}_{abcr} e^{j\theta_r}\right] \\ &= R_s \bar{i}_{qds} + \frac{d}{dt}\lambda_{qds} + j\omega\cdot\lambda_{qds} \end{aligned}$$

Advanced notes

Another method - Using the vectors instead of the matrices

- For the rotor side equations

$$\bar{u}'_{abcr} = \frac{2}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix}\begin{bmatrix} u'_{ar} \\ u'_{br} \\ u'_{cr} \end{bmatrix} = R'_r \bar{i}'_{abcr} + \frac{d}{dt}\left[\left(L'_{ls} + L_m\right)\bar{i}'_{abcr} + L_m \bar{i}_{abcs} e^{j\boxed{(-\theta_r)}}\right]$$



$$\bar{\lambda}'_{abcr} = \left(L'_{lr} + L_m\right)\cdot \bar{i}'_{abcr} + L_m \bar{i}_{abcs} e^{-j\theta_r}$$

$$\bar{\lambda}'_{qdr} = \left(L'_{lr} + L_m\right)\cdot \bar{i}'_{qdr} + L_m \bar{i}_{qds}$$

$$\begin{aligned}
\bar{u}'_{qdr} &= R'_r \bar{i}'_{qdr} + e^{-j\beta}\frac{d}{dt}\left[\left(L'_{ls} + L_m\right)\bar{i}'_{abcr} + L_m \bar{i}_{abcs} e^{j(-\theta_r)}\right] \\
&= R'_r \bar{i}'_{qdr} + \frac{d}{dt}\left[\left(L'_{ls} + L_m\right)\bar{i}'_{qdr} + L_m \bar{i}_{qds}\right] - \frac{de^{-j\beta}}{dt}\left[\left(L'_{ls} + L_m\right)\bar{i}'_{abcr} + L_m \bar{i}_{abcs} e^{j(-\theta_r)}\right] \\
&= R'_r \bar{i}'_{qdr} + \frac{d}{dt}\bar{\lambda}'_{qdr} + j\left(\omega_\theta - \omega_r\right)\cdot \bar{\lambda}'_{qdr}
\end{aligned}$$

- It is not recommended to apply this method to the synchronous motor!

Exercises

Use the obtained induction machine qd model, please give the machine equations when the q-axis of the rotating qd reference frame is aligned with the rotor flux vector.

What the machine equations will be when in afa-beta reference frame?