Self and mutual inductance matrix of the Induction Machine

Stator self-inductance $\boxed{L_{aaq} \equiv L_{aad} = L_{ms}}$ ← Stator main or called magnetization inductance *defined variable!*

$$L_{asasm} = L_1 - L_2 \cos(2\theta_r) = L_{aaq} \cos^2 \theta_r + L_{aad} \sin^2 \theta_r = L_{ms}$$

Following the same principle, it may be easily found that:

$$L_{bsbsm} = L_{\mathrm{csc}\,sm} = L_{ms}$$

Mutual-inductance

$$\boxed{L_{aaq} \equiv L_{aad} = L_{ms}}$$

$$M_{asbsm} = L_{aaq}\cos\theta_r \cos\left(\theta_r - \frac{2\pi}{3}\right) + L_{aad}\sin\theta_r \sin\left(\theta_r - \frac{2\pi}{3}\right) = L_{ms}\cos\frac{2\pi}{3} = -\frac{1}{2}L_{ms}$$

$$M_{csasm} = L_{aaq}\cos\theta_r \cos\left(\theta_r + \frac{2\pi}{3}\right) + L_{aad}\sin\theta_r \sin\left(\theta_r + \frac{2\pi}{3}\right) = -\frac{1}{2}L_{ms}$$

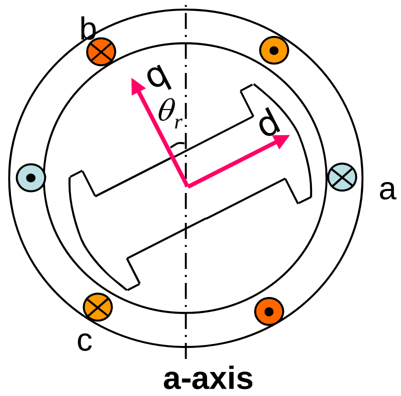
$$M_{bscsm} = L_{aaq}\cos\left(\theta_r - \frac{2\pi}{3}\right)\cos\left(\theta_r + \frac{2\pi}{3}\right) + L_{aad}\sin\left(\theta_r - \frac{2\pi}{3}\right)\sin\left(\theta_r + \frac{2\pi}{3}\right) = -\frac{1}{2}L_{ms}$$

It may be arranged to:

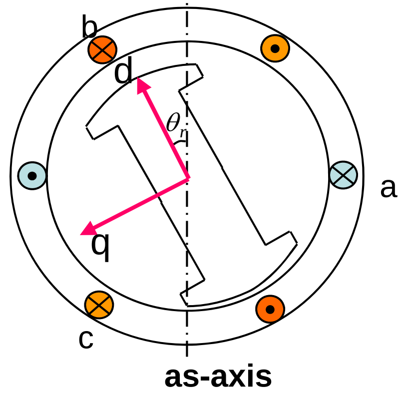
$$\lambda_{ms} = \begin{bmatrix} L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ms} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Mutual inductance between stator and rotor

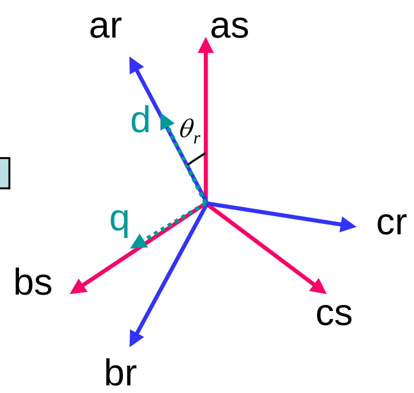
First to image the reference frame changed from qd0 to dq0 like



qd0 system

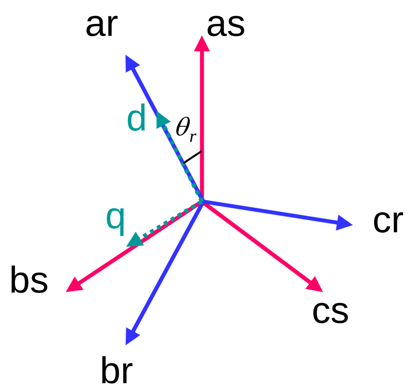


dq0 system



We may see that for induction machine,
If d-axis is aligned with rotor phase a-axis,
It is naturally in dq0 reference frame!

Mutual inductance between stator and rotor



Flux-axis, q-axis

Flux-axis, d-axis

$$M_{asbrm} = L_{aaq} \mathrm{Re}\left(\frac{e^{j\left(\theta_r + \frac{\pi}{2}\right)}}{e^{j\left(\theta_r + \frac{2\pi}{3}\right)}} \right) \mathrm{Re}\left(\frac{e^{j\left(\theta_r + \frac{\pi}{2}\right)}}{e^{j0}} \right) + L_{aad} \mathrm{Re}\left(\frac{e^{j\theta_r}}{e^{j\left(\theta_r + \frac{2\pi}{3}\right)}} \right) \mathrm{Re}\left(\frac{e^{j\theta_r}}{e^{j0}} \right)$$

$$= -L_{aaq} \sin\frac{2\pi}{3} \sin\theta_r + L_{aad} \cos\frac{2\pi}{3} \cos\theta_r = L_{ms} \cos\left(\theta_r + \frac{2\pi}{3}\right)$$

$$\boxed{L_{aaq} \equiv L_{aad} = L_{ms}}$$

Rotor phase b-axis

Stator phase a-axis

Or there is another way! suppose rotor phase-b has current – the flux is on the rotor phase b-axis. Then following the example given on page 14, we will have:

Flux-axis, rotor phase b

$$M_{asbrm} = L_{ms} \mathrm{Re}\left(\frac{e^{j\left(\theta_r + \frac{2\pi}{3}\right)}}{e^{j\left(\theta_r + \frac{2\pi}{3}\right)}} \right) \mathrm{Re}\left(\frac{e^{j\left(\theta_r + \frac{2\pi}{3}\right)}}{e^{j0}} \right) = L_{ms} \cos\left(\theta_r + \frac{2\pi}{3}\right)$$

Rotor phase b-axis

Stator phase a-axis

The same results obtained!

Mutual inductance between stator and rotor

The final result – from rotor currents to stator flux linkage

$$\begin{bmatrix}\lambda_{as}\\ \lambda_{bsr}\\ \lambda_{cs}\end{bmatrix}=\begin{bmatrix}M_{asarm} & M_{asbrm} & M_{ascrm}\\ M_{bsarm} & M_{bsbrm} & M_{bscrm}\\ M_{csarm} & M_{csbrm} & M_{cscrm}\end{bmatrix}\cdot\begin{bmatrix}i_{ar}\\ i_{br}\\ i_{cr}\end{bmatrix}=L_{sm}\begin{bmatrix}\cos\theta_r & \cos\left(\theta_r+\frac{2\pi}{3}\right) & \cos\left(\theta_r-\frac{2\pi}{3}\right)\\ \cos\left(\theta_r-\frac{2\pi}{3}\right) & \cos\theta_r & \cos\left(\theta_r+\frac{2\pi}{3}\right)\\ \cos\left(\theta_r+\frac{2\pi}{3}\right) & \cos\left(\theta_r-\frac{2\pi}{3}\right) & \cos\theta_r\end{bmatrix}\cdot\begin{bmatrix}i_{ar}\\ i_{br}\\ i_{cr}\end{bmatrix}$$

$$\boxed{\begin{aligned}M_{ascrm} &= L_{aaq}\,\mathrm{Re}\left(\frac{e^{j\theta_r}}{e^{j\left(\theta_r-\frac{2\pi}{3}\right)}}\right)\mathrm{Re}\left(\frac{e^{j\theta_r}}{e^{j0}}\right)+L_{aad}\,\mathrm{Re}\left(\frac{e^{j\left(\theta_r-\frac{\pi}{2}\right)}}{e^{j\left(\theta_r-\frac{2\pi}{3}\right)}}\right)\mathrm{Re}\left(\frac{e^{j\left(\theta_r-\frac{\pi}{2}\right)}}{e^{j0}}\right)\\ &= L_{aad}\cos\frac{2\pi}{3}\cos\theta_r+L_{aaq}\sin\frac{2\pi}{3}\sin\theta_r=L_{ms}\cos\left(\theta_r-\frac{2\pi}{3}\right)\end{aligned}}$$