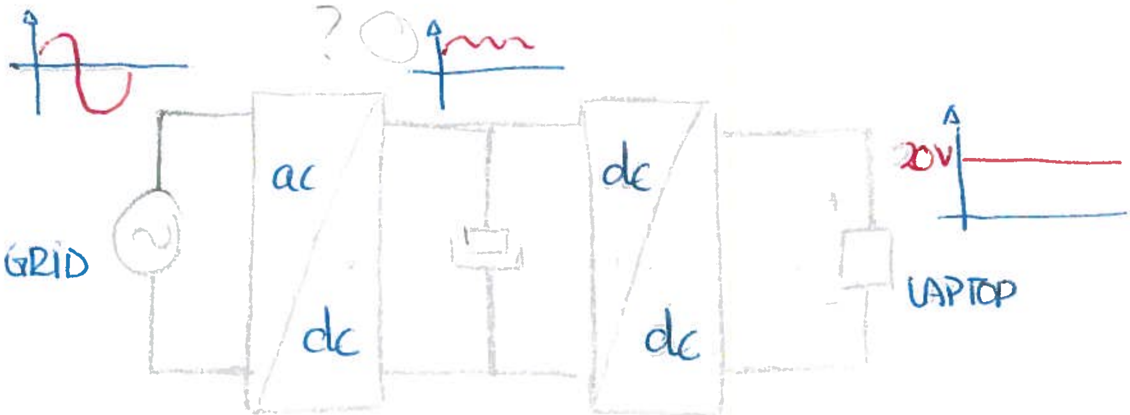
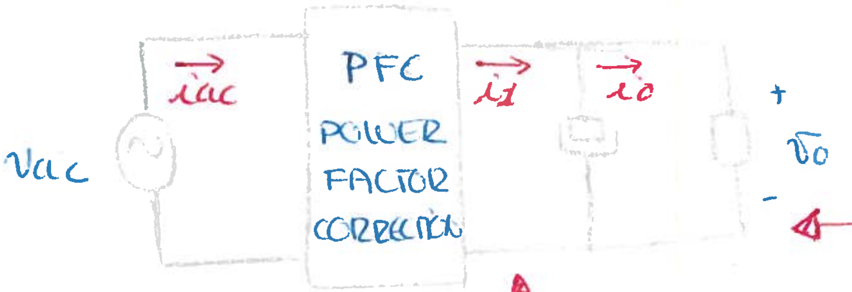
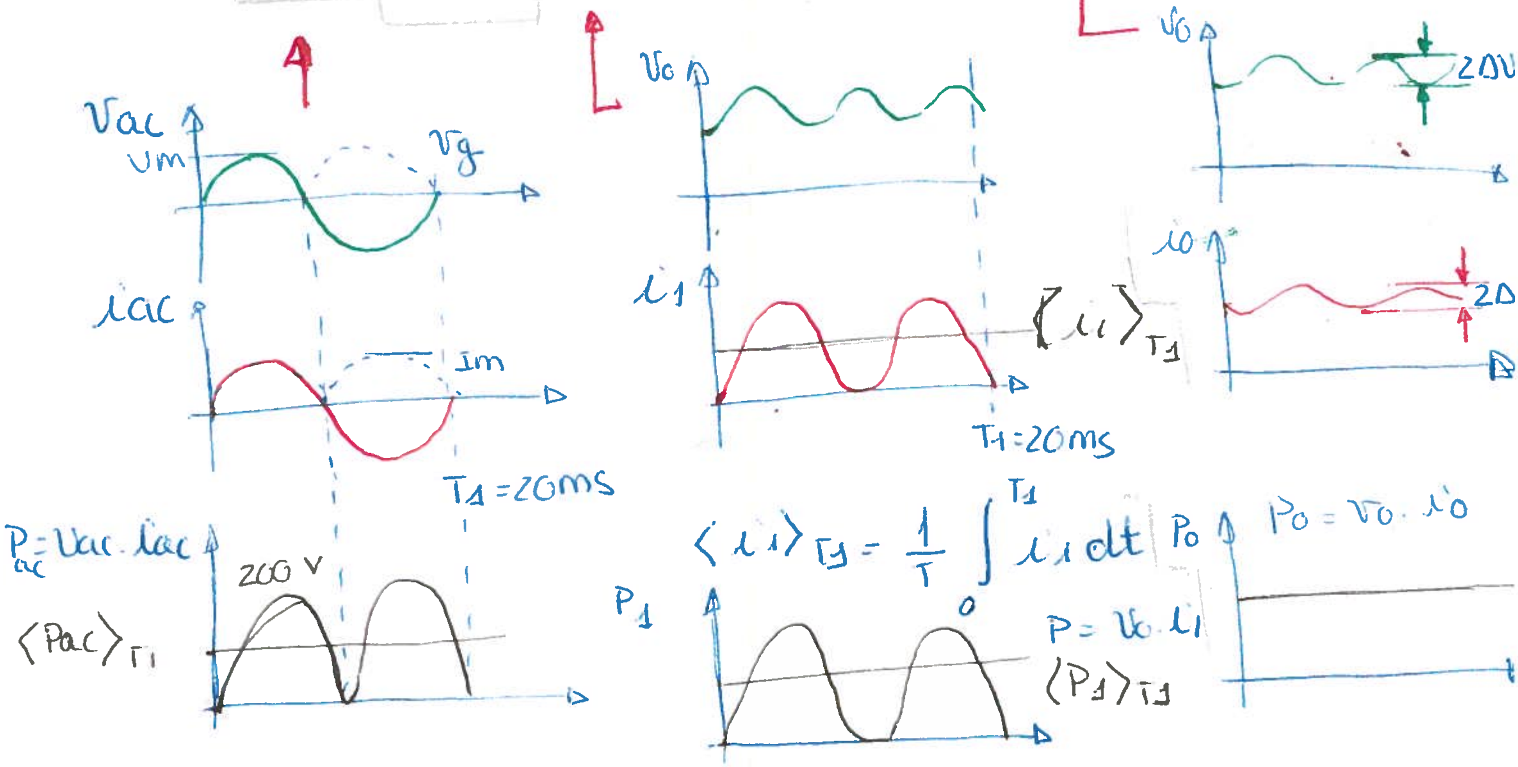
THE IDEAL RECTIFIER



THE IDEAL RECTIFIER:



$$R_e = \frac{v_{ac}}{i_{ac}} = \frac{V_{ac}}{I_{ac}}$$



$$\langle i_1 \rangle_{T_1} = \frac{1}{T} \int_0^{T_1} i_1 \, dt$$

IDEAL RECTIFIER $\Longrightarrow$ $\boxed{\langle P_{ac} \rangle_{T_1} = \langle P_1 \rangle_{T_1}}$

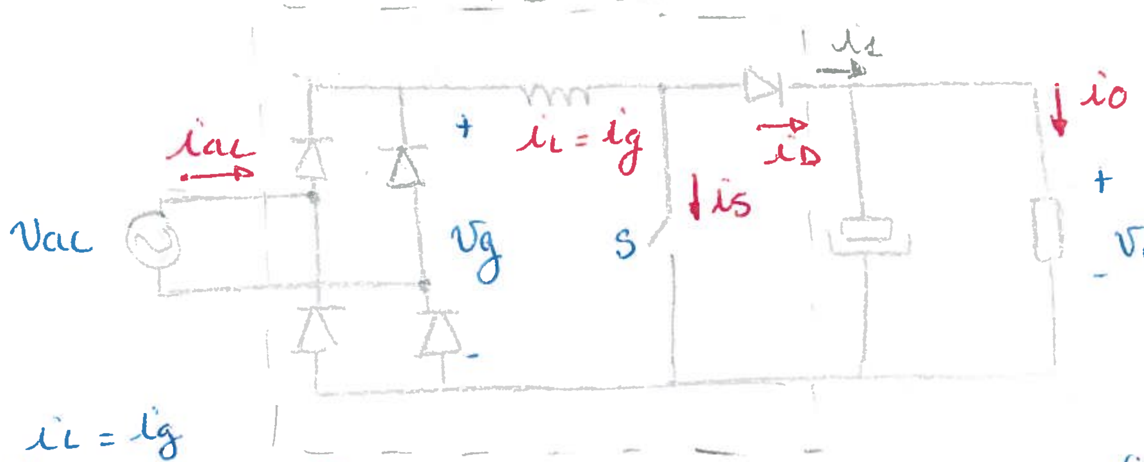
$$\boxed{P_{ac} = P_1 = P_o}$$

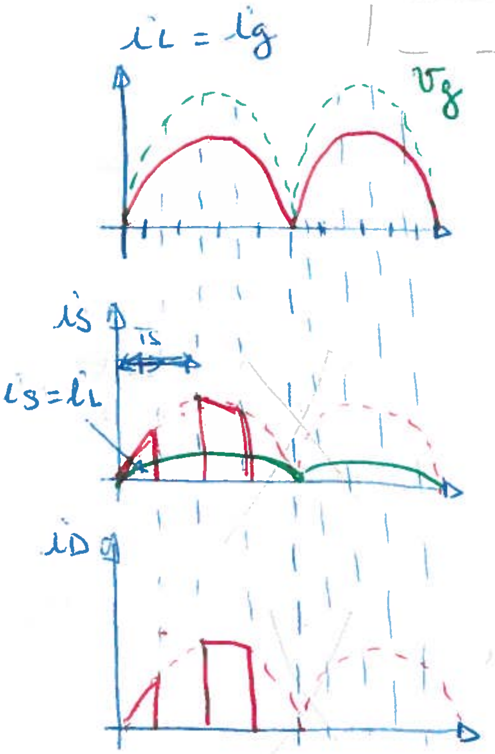
$$\boxed{\langle P_{ac} \rangle_{T_1} = \langle P_1 \rangle_{T_1} = \langle P_o \rangle_{T_S}}$$

→ For a 3-phase rectifier (remembering that the phases are de-phased 120°), when you "sum" each phase, you also get a DC power

→ Single phase is just for low power app.

PFC - BOOST

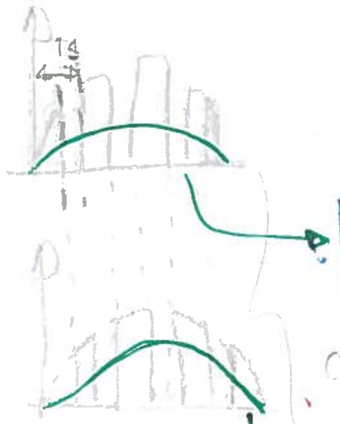




$$\langle i_s\rangle_{T_s} = \frac{1}{T_s}\int_0^{T_s} i_s\,dt$$

$$\cancel{= \frac{1}{T_s}\int_0^{dt_s}\langle i_L\rangle_{T_s}\,dt}$$

$$= \frac{1}{T_s}\int_0^{T_s}\langle i_L\rangle_{T_s}\,dt + \frac{1}{T_s}\int_{dt_s}^{T_s} 0\,dt$$



$$\boxed{\langle i_s\rangle_{T_s} = \langle i_L\rangle_{T_s}\cdot d}$$

COMPLEMENTARIOS

$$\boxed{\langle i_D\rangle_{T_s}}$$

Ex

$f_1 = 50\ Hz \rightarrow T_1 = 20000\ \mu s$

$f_s = 30\ kHz \rightarrow T_s = 33\ \mu s$

$$\langle i_D\rangle_{T_s} = \frac{1}{T_s}\int_0^{dT_s} 0\cdot dt + \frac{1}{T_s}\int_{dt_s}^{T_s}\langle i_L\rangle_{T_s}\,dt$$

$$\boxed{\langle i_D\rangle_{T_s} = \langle i_L\rangle_{T_s}(1-d)} = i_1$$

↳ COMPLEMENTARIOS DE i_s

$$\langle i_L \rangle_T = | I_m \cdot \sin \omega_1 t |$$

$$1 - d = \frac{\langle v_g \rangle_{Ts}}{\langle v_g \rangle_{Ts}}$$

CCM

$$v_o = v_g \, \frac{1}{1-d}$$

$$1 - d = \frac{v_g}{v_o}$$

$$\frac{\langle v_g \rangle_{Ts}}{\langle v_o \rangle_{Ts}} = 1 - d$$

$$\langle i_D \rangle_{Ts} = | I_m \cdot \sin \omega_1 t | \, \frac{| V_m \cdot \sin \omega t |}{\langle v_o \rangle_{Ts}}$$

$$\langle i_D \rangle_{Ts} = \frac{I_m \cdot V_m \cdot \sin^2 \omega t}{\langle v_o \rangle_{Ts}} = \frac{I_m \cdot V_m}{\langle v_o \rangle_{Ts}} \cdot \frac{1}{2} \cdot (1 - \cos 2\omega_1 t)$$

$$\langle i_D \rangle_{Ts} = \frac{I_m \cdot V_m}{\langle v_o \rangle_{Ts}} \cdot \frac{1}{2} \Longrightarrow \boxed{\langle i_D \rangle_{Ts} \cdot \langle v_o \rangle_{Ts} = \frac{I_m \cdot V_m}{2}}$$

$$P_a = I_{ac\_rms} \cdot V_{ac\_rms} = \frac{I_m}{\sqrt{2}} \cdot \frac{V_m}{\sqrt{2}}$$

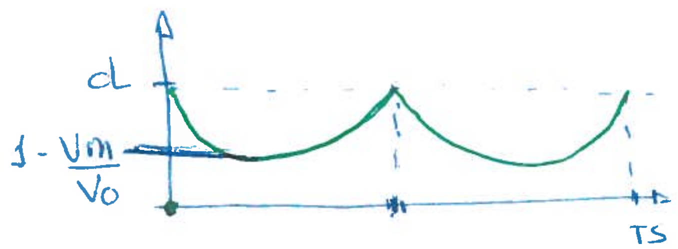
$$R_e = \frac{V_{ac\_rms}}{I_{ac\_rms}} = \frac{V_m}{I_m} \Rightarrow I_m = \frac{V_m}{R_e}$$

WHAT IS THE DUTY CYCLE (IDEAL RECTIFIER)

$$d = 1 - \frac{\langle v_g \rangle_{Ts}}{\langle v_o \rangle_{Ts}} \simeq 1 - \frac{| V_m \cdot \sin \omega_1 t |}{V_o}$$

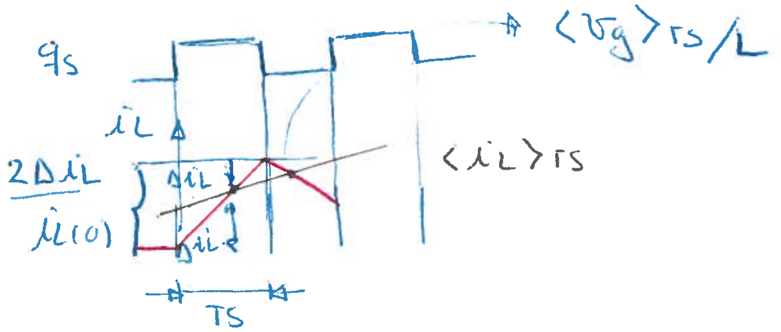
$\to V_o < V_m \Rightarrow d < 0$

IMPOSIBLE



$$1 - \frac{V_m}{V_o} < d < 1$$

OPERATION MODES



$$2 \Delta i_L = \frac{\langle v_g \rangle_{Ts}}{L} \, dT_s$$

$$\boxed{\Delta i_L < \langle i_L \rangle_{Ts}}$$

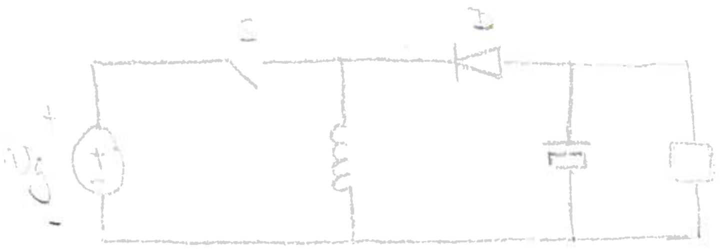
CONTINUOUS CONDUCTION MODE

(CCM)

$$\frac{\langle v_g \rangle_{Ts}}{2L} \cdot dTs < \frac{\langle v_g \rangle_{Ts}}{Re} \Rightarrow \boxed{d < \frac{2L}{Re \cdot Ts}} \quad \text{CCM}$$

If we want to have the rectifier working in CCM, this relations NEED to be respected.

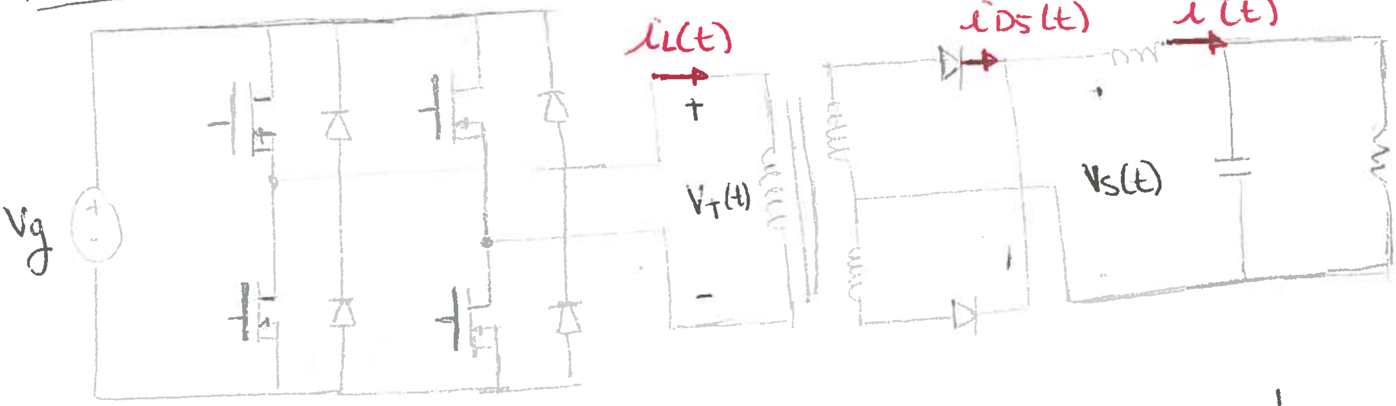
BUCK - BOOST



$$\langle v_o \rangle_{Ts} = -v_g \cdot \frac{d}{1-d}$$

EXERCISES: CURRENT PROGRAMMED CONTROL

LECTURE



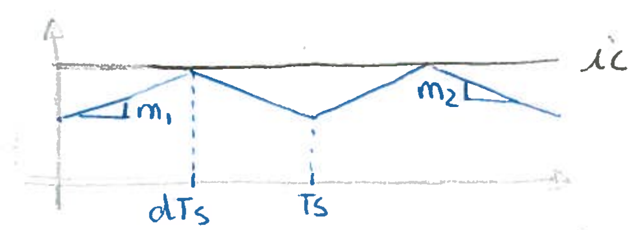
$V_g = 320V$
$P = 1000\,W$
$V_o = 42V$

$DC = 0'7$
$T_S = 10\mu s$
$L = 50\mu H$
$C = 100\mu F$

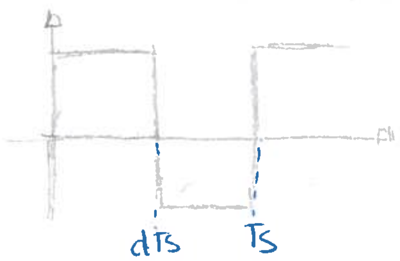
Current controller whose waveforms are refered to the secondary side of the transformer

a) Sketch the waveforms $V_S(t)$ and $i(t)$ for T_S. Calculate m_1 and m_2.

$i(t) = i_L(t)$



$V_S(t)$ (INPUT TO THE BUCK CONVERTER)



BUCK CONVERTER

$V_o = V_S \cdot D \rightarrow V_S = \frac{V_o}{D} = \frac{42}{0'7} = 60V$

$m_1\,(BUCK) = \frac{V_S - V_o}{L} = \frac{60-42\ [V]}{50 \cdot 10^{-6}\ [\mu H]} = 0'36\ A/\mu s$

$m_2\,(BUCK) = -\frac{V_o}{L} = \frac{42}{50 \cdot 10^{-6}} = 0'84\ A/\mu s$

b) What is the minimum artificial ramp slope m_a, that will stabilize the controller at the given operating point? Express your result in terms of m_2 and D.

$\alpha = \frac{m_2 - m_a}{m_1 + m_a} = 1 \rightarrow m_2 - m_a = m_a + m_1$

$2m_a = m_2 - m_1$

$m_a = \frac{m_2 - m_1}{2}$

$m_a = \frac{m_2 - m_2 \cdot \frac{D'}{D}}{2} \leftarrow \frac{m_2}{m_1} = \frac{D}{D'} \leftarrow$

c) Calculate the value of ma and R

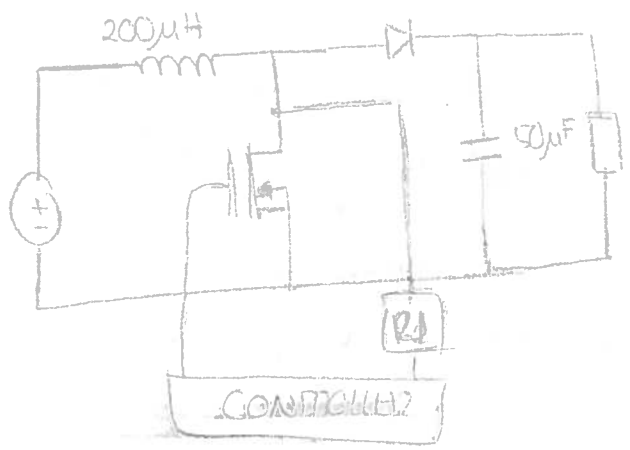
$m_2 = 0'84\ A/\mu s$

$ma = 0'84 \cdot \frac{1 - D'/D}{2} = 0'84 \cdot \frac{(1 - 0'3/0'7)}{2} = 0'24\ A/\mu s$

$P = V\,I \longrightarrow I = P/V = 1000/42 = 23'80\ A$

$V = R \cdot I \rightarrow \longrightarrow R = \frac{V}{I} = \frac{42}{23'8} = 1'764\,\Omega$

EXAM JANUARY 2011



$T_s = 10\mu s$

$R = 20\,\Omega$

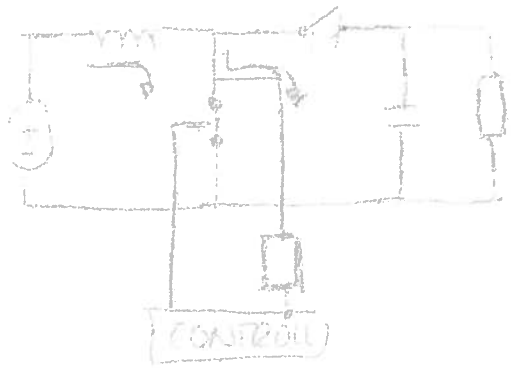
$V_S = 20V$

$V = 100V$

$Rf = 1\,\Omega$

a) Calculate the average current of the switch.

- Current in the switch → just in switches
- SWITCH ON CIRCUIT : $0 < t < dT_s$



The current in the switch will be the same that in the inductance.

$i_L = i_L(0) + m_1\, d\, T_s$

BOOST CONVERTER: $m_1 = \frac{V_S}{L}$

$m_2 = \frac{V_S - V}{L}$

$m_1 = \frac{20}{200 \cdot 10^{-6}} = 0'1\ A/\mu s$

$m_2 = \frac{20 - 100}{200 \cdot 10^{-6}} = -0'4\ A/\mu s$

STEADY STATE $\rightarrow \frac{M_2}{M_1} = \frac{D}{D'} = \frac{D}{1-D}$

$\frac{0'4}{0'1} \cdot (1 - D) = D$

$4 - 4D = D \rightarrow 5D = 4$

$D = 4/5 = 0'8$

STEADY STATE $\rightarrow D = d = 0'8$

$i_L = i_L(0) + \frac{0'1\,A}{\mu s} \cdot 0'8 \cdot 10\,\mu s$

$i_L = i_L(0) + 0'8$

b) Add a stabilazing ramp and calculate the minimum slape m_a that will give stability.

$$\alpha = \frac{m_2 - m_a}{m_1 + m_a} \longrightarrow |\tilde{x}_L(nT_s)| \begin{cases} 0 & \text{when } |\alpha| < 1 \\ \infty & \text{when } |\alpha| > 1 \end{cases}$$

STABILAZING : $\alpha = 1$

¡SIGNOS!?

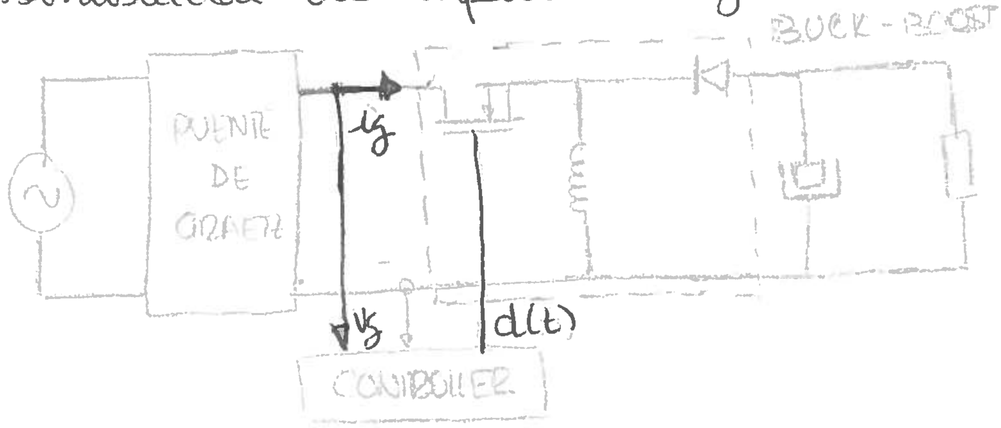
$$\frac{m_2 - m_a}{m_1 + m_a} = 1 \Longrightarrow m_2 - m_a = m_1 + m_a \rightarrow 2m_a = m_2 - m_1$$

$$m_a = \frac{m_2 - m_1}{2} = \frac{-0'4 - 0'1}{2} = \frac{-0'5}{2} = \underline{\underline{-0'25}}$$

EXERCISE: PULSE WIDTH MODULATED RECTIFIERS

18.1

The BUCK of figure 18.5 (p. 642) is replaced by a BUCK-BOOST. The inductor energy storage has neglible influence on the low frequency components of the converter waveforms. The average load power is Pload. The dc output voltage is V and the sinusoidal ac input voltage has a peak ~~value~~ amplit VM.



BUCK - BOOST

$$d = \frac{V_o}{V_o - V_g}$$

a) Determine expressions for the duty cycle variation d(t) and the inductor current i(t) assuming that the converter is operating in CCM.

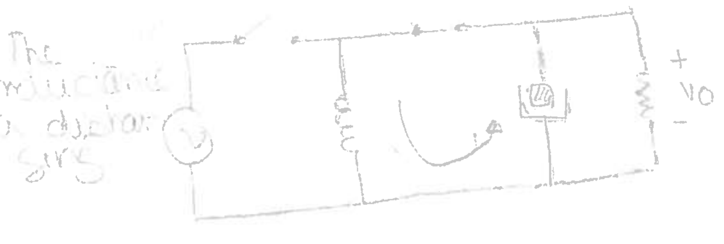
$$V_o = V_{ac} \cdot \frac{(-\delta)}{1-\delta}$$

$$\delta = \frac{t_1}{T}$$

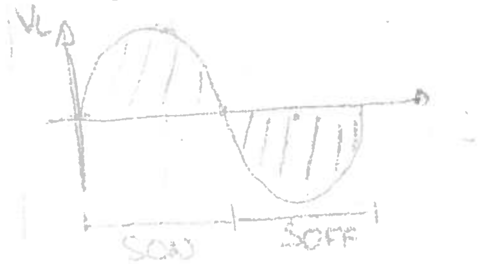
· DEDUCE THE EQUATIONS FOR A BUCK-BOOST

S ON: $0 < t < DT$

S OFF: $DT < t < T$



STUDYING THE INDUCTANCE



STEADY STATE: $u_L = 0$

$u_L(t_{ON}) = V_{ac} \cdot dt$

$u_L(t_{OFF}) = -V_o \cdot (1 - dt)$

$u_L(t)_{ON} = u_L(t)_{OFF}$

$V_{ac} \cdot dt = -V_o \cdot (1 - dt)$

$V_{ac} \cdot dt + V_o(1 - dt) = 0$

$d(t)(V_{ac} - V_o) + V_o = 0$

$$\boxed{d(t)} = \frac{-V_o}{(V_{ac} - V_o)} = \frac{-V_o}{V_o - V_{ac}}$$

Knowing: $V_{ac} = V_{ac} \cdot \sin(\omega t)$

$$\boxed{d(t) = \frac{V_o}{V_o - V_{ac}|\sin(\omega t)|}} = \boxed{\frac{1}{1 - \frac{V_{ac}}{V_o}|\sin(\omega t)|}}$$

STEADY STATE EQUATION

FOR NON-STEADY STATE:

$$\langle v_L \rangle_{Ts} = \langle v_o \rangle_{Ts} + (\langle v_g \rangle_{Ts} - \langle v_o \rangle_{Ts}) d$$

CCM

$$0 = \langle v_o \rangle_{Ts} + (\langle v_g \rangle_{Ts} - \langle v_o \rangle_{Ts}) \cdot d$$

$$\boxed{d = - \frac{\langle v_o \rangle_{Ts}}{\langle v_g \rangle_{Ts} - \langle v_g \rangle_{Ts}}}$$

CURRENT $i(t)$:

$$\left.\begin{aligned} \langle i_g \rangle_{Ts} &= d \cdot \langle i_L \rangle_{Ts} \\ R_e &= \frac{\langle v_g \rangle_{Ts}}{\langle i_g \rangle_{Ts}} \end{aligned}\right\} \quad \langle i_L \rangle_{Ts} = \frac{\langle i_g \rangle_{Ts}}{d} \qquad \langle i_L \rangle_{Ts} = \frac{\langle v_g \rangle_{Ts}}{R_e \cdot d}$$

$$\langle i_L \rangle_{Ts} = \frac{\hat{V}_{ac} |\sin \omega t|}{R_e} \cdot \frac{1}{\left(1 - \frac{\hat{V}_{ac}}{V_o} |\sin \omega t|\right)^{-1}} \; ? \; ?$$

$$\langle i_L \rangle_{Ts} = \hat{V}_{ac} \cdot |\sin \omega t| \cdot \left(1 - \frac{\hat{V}_{ac}}{V_o} \cdot |\sin \omega t|\right) \cdot \frac{1}{R_e}$$

b) Derive the conditions for operation in the continuous mode. Manipulate your result to show that the converter operates in CCM when Re is less than Re,crit (L, Ts, Vg(t), V) and determine Re,crit.

In CCM it is required:

$$\langle i_L \rangle_{Ts} > \Delta i_L$$

$$\Delta i_L = \frac{\langle v_g \rangle_{Ts} \cdot T_s \cdot d}{2L} \, , \qquad \langle i_L \rangle_{Ts} = \frac{\langle v_g \rangle_{Ts}}{R_e \cdot d}$$

$$\frac{\cancel{\langle v_g \rangle_{Ts}}}{R_e \cdot d} > \frac{\cancel{\langle v_g \rangle_{Ts}} \cdot T_s \cdot d}{2L} \Rightarrow \frac{1}{R_e \cdot d} > \frac{d \cdot T_s}{2L}$$

$$R_e < \frac{1}{d^2} \cdot \frac{1}{T_s} \cdot 2L \Rightarrow \boxed{R_e < \left(\frac{V_o}{V_o - V_g}\right)^2 \cdot \frac{2L}{T_s}} \; \times$$

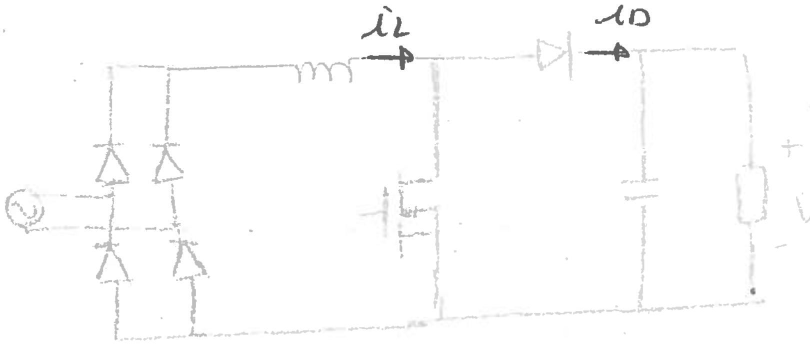
BUCK-BOOST EQ:

$$\frac{V_o}{V_g} = \frac{-d}{1-d} \rightarrow \frac{V_o}{V_g}(1-d) = -d \Rightarrow \frac{V_o}{V_g} - \frac{V_o}{V_g} \cdot d = -d$$

$$\left(\frac{V_o}{V_g} - 1\right) d = \frac{V_o}{V_g} \rightarrow \boxed{d = \frac{V_o - V_g}{V_o}}$$

EXERCISE 3. EXAM JANUARY 2011

Boost converter



$V_o = 390\,V$

$P_o = 500\,W$

$Rms = 230\,V$

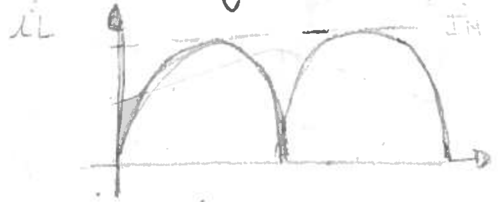
$\eta = 0'95$

$f = 50\,Hz$

$f_s = 100\,kHz$

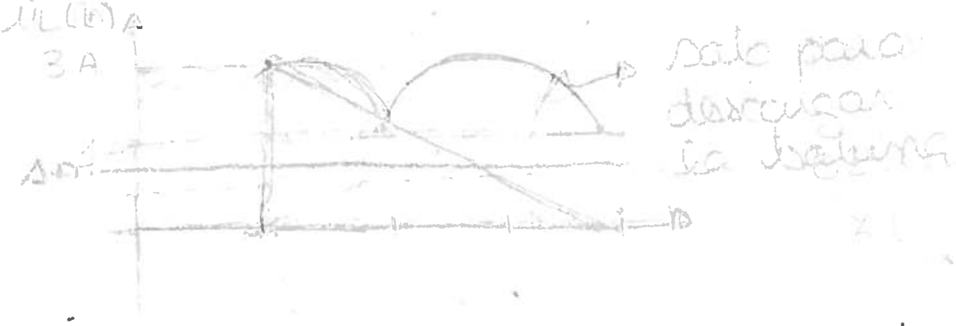
a) and b) converter is operating in CCM ...

a) For one fundamental period sketch the current of $i_L(t)$



$$I_L = \frac{V_M \cdot |\sin(\omega t)|}{R_e}$$

b) For one switching period sketch $i_D(t)$ assume the average current is 1A and the duty cycle 25%.



c) Calculate the value of the emulated resistance

$$P_{INPUT} = \frac{P_{OUTPUT}}{\eta} = \frac{500}{0'95} = 526'32\,W \qquad I_L = \frac{P}{V} = 2'287\,A$$

$V_i = 230$

$$R_e = \frac{V_M\,|\sin(\omega t)|}{I_L} = \frac{230\cdot\sqrt{2}\,|\sin(2\pi 50 \cdot 10\cdot 10^{6})|}{2'287}$$

c) For what values of Re does the converter alway operates in CCM? and in DCM?

MINIMUM: R_{CRIT} at $V_g = 0 \rightarrow V_L = 0$

$$R_{CRIT} < \frac{2L}{T_S}$$

MAXIMUM: R_{CRIT} at $V_g = V_{ac}$

$$R_{CRIT} = \left(\frac{V_0 - V_{ac}}{V_{ac}}\right)^2 \cdot \frac{2L}{T_S}$$

??

d) The ac input voltage has rms amplitud in the range 108V - ~~1~~132V. The maximum load power is 100W, and the minimum load power is 10W. The dc output voltage is 120V. The switching frequency is 75 KHz. What value of L guarantees that the converter always operates in CCM? in DCM?

$$R_e\ MAX = \left(\frac{132}{10}\right)^2 = 1742\,\Omega$$

$$R_e\ MIN = \left(\frac{108}{100}\right)^2 = 117\,\Omega$$

$$1 < \frac{2L}{R_e T_S}$$

$$L > \frac{R_e \cdot T_S}{2}$$

So CCM is obtained for:

$$L > \frac{R_0 \cdot T_S}{2}$$

$$L > \frac{1742 \cdot \frac{1}{75\,10^3}}{2} \longrightarrow \boxed{L > 12mH}$$

Whereas in DCM:

$$L < \frac{R_0 \cdot T_S}{2} \cdot \left(\frac{V_{ac}}{V_0 - V_{ac}}\right)^2$$

$$L < \frac{117 \cdot \frac{1}{75\,10^3}}{2} \cdot \left(\frac{108}{120 - 108}\right)^2$$

$$\boxed{L < 125\,\mu H}$$