Lecture 8 - contents

- Modeling of Permanent Magnet Sync. Machines
  - Machine structure analysis
  - Voltage equations
  - torque equations
- Steady-state analysis

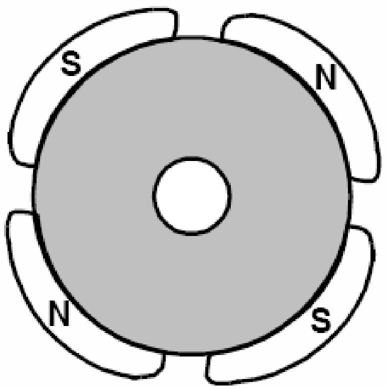
Machine Structure Analysis

Purpose:

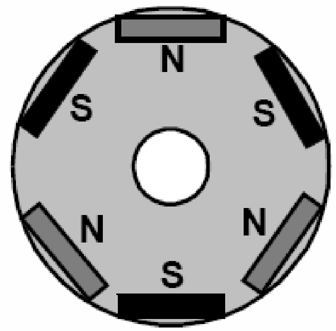
- Observe/understand how the rotor flux links the stator side
- d-axis, and q-axis inductance determination
- How the machine may be related to an known machine

Machine Structure Analysis

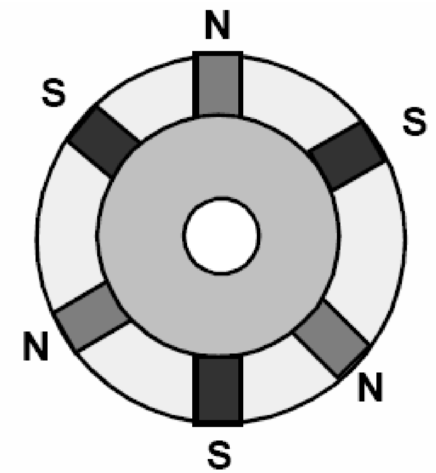
Surface mounted or buried PM machines



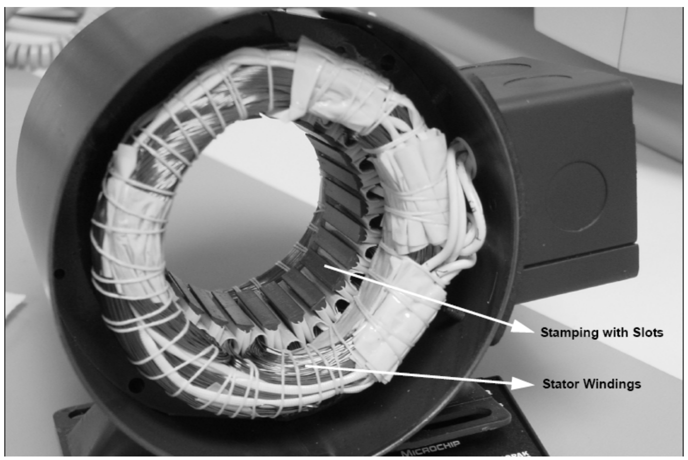
Circular core with magnets on the periphery

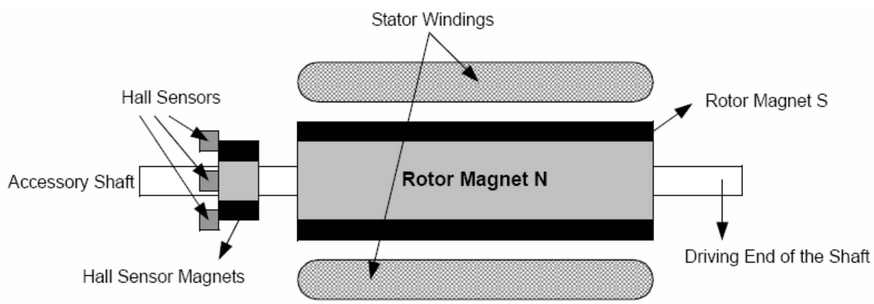


Circular core with rectangular magnets embedded in the rotor

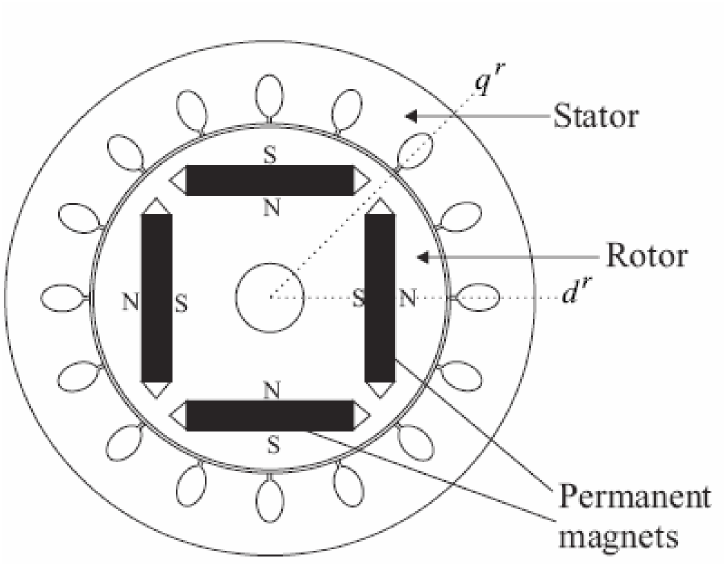


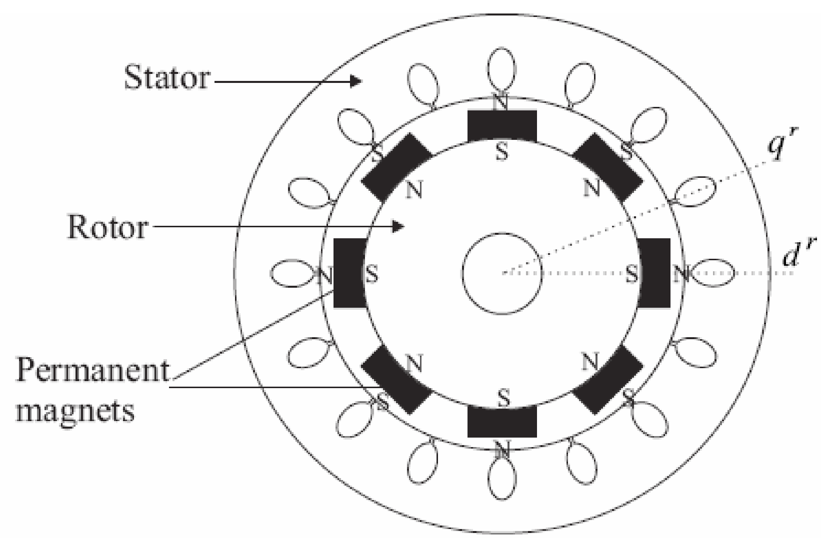
Circular core with rectangular magnets inserted into the rotor core

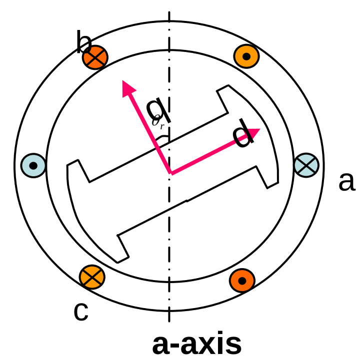




How the machine may be related to an known machine?

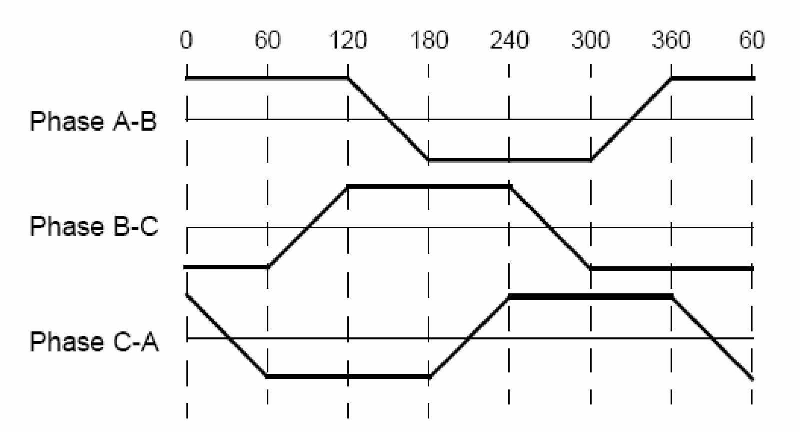


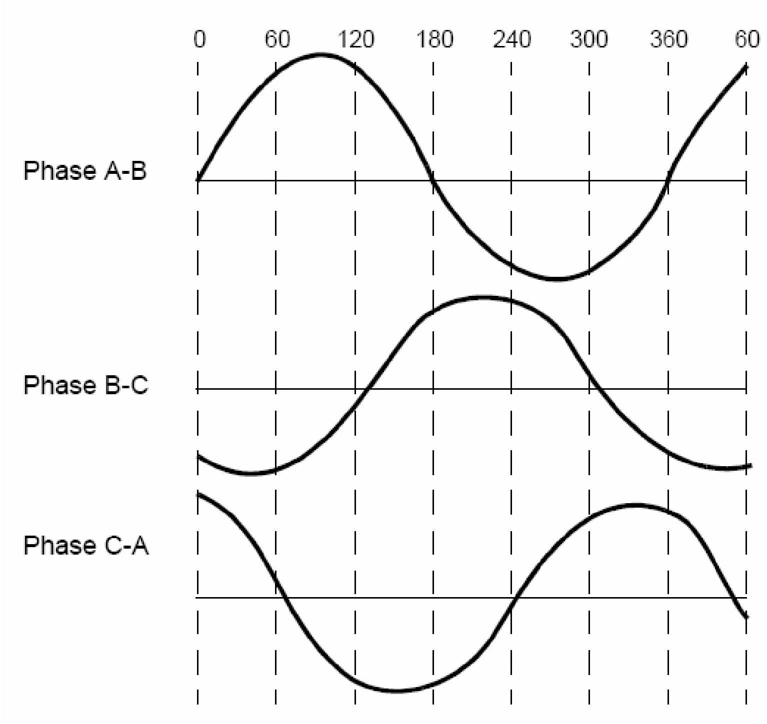




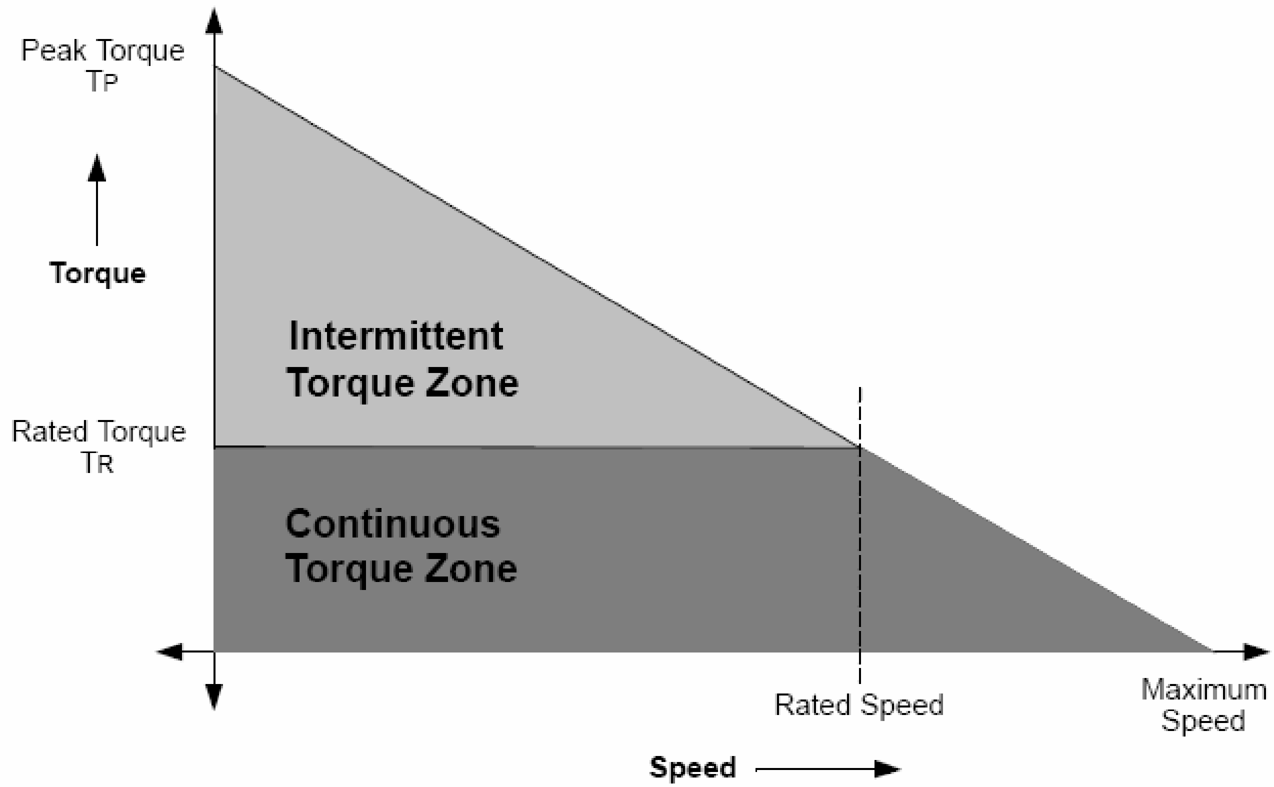
Machine may also be characterized by their back EMF shape

Sinusoidal or trapezoidal back EMF

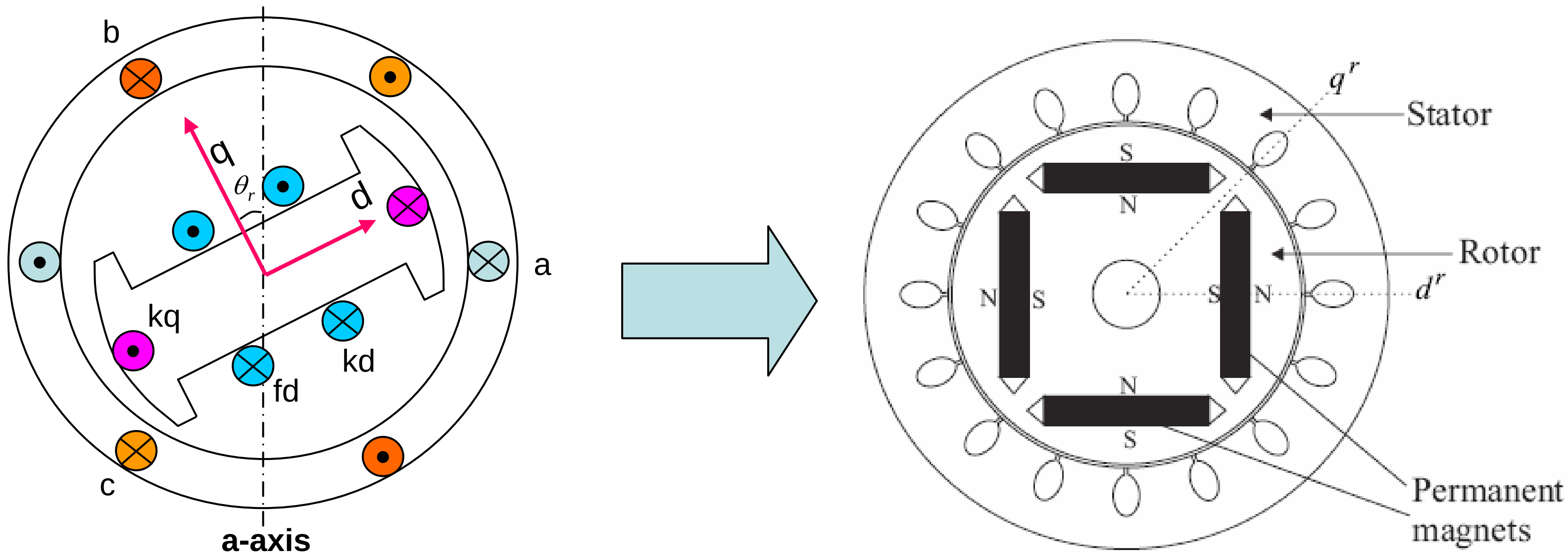




General torque vs. speed characteristics



Voltage equations



No rotor winding equations! No need for turns ratio transformation!

How to represent the rotor magnet field?

Voltage equations

Synchronous Motor
Stator winding equations

$$u_q = Ri_q + p\lambda_q + \omega_r\lambda_d \qquad \lambda_q = L_{ls}i_q + L_{mq}\left(i_q + \cancel{i'_{kq}}\right)$$
$$u_d = Ri_d + p\lambda_d - \omega_r\lambda_q \qquad \lambda_d = L_{ls}i_d + L_{md}\left(i_d + i'_{fd} + \cancel{i'_{kd}}\right)$$

PMSM motor
Stator winding equations

$$u_q = Ri_q + p\lambda_q + \omega_r\lambda_d \qquad \lambda_q = \left(L_{ls} + L_{mq}\right)i_q = L_q i_q$$
$$u_d = Ri_d + p\lambda_d - \omega_r\lambda_q \qquad \lambda_d = \left(L_{ls} + L_{md}\right)i_d + \lambda_{mpm} = L_d i_d + \lambda_{mpm}$$

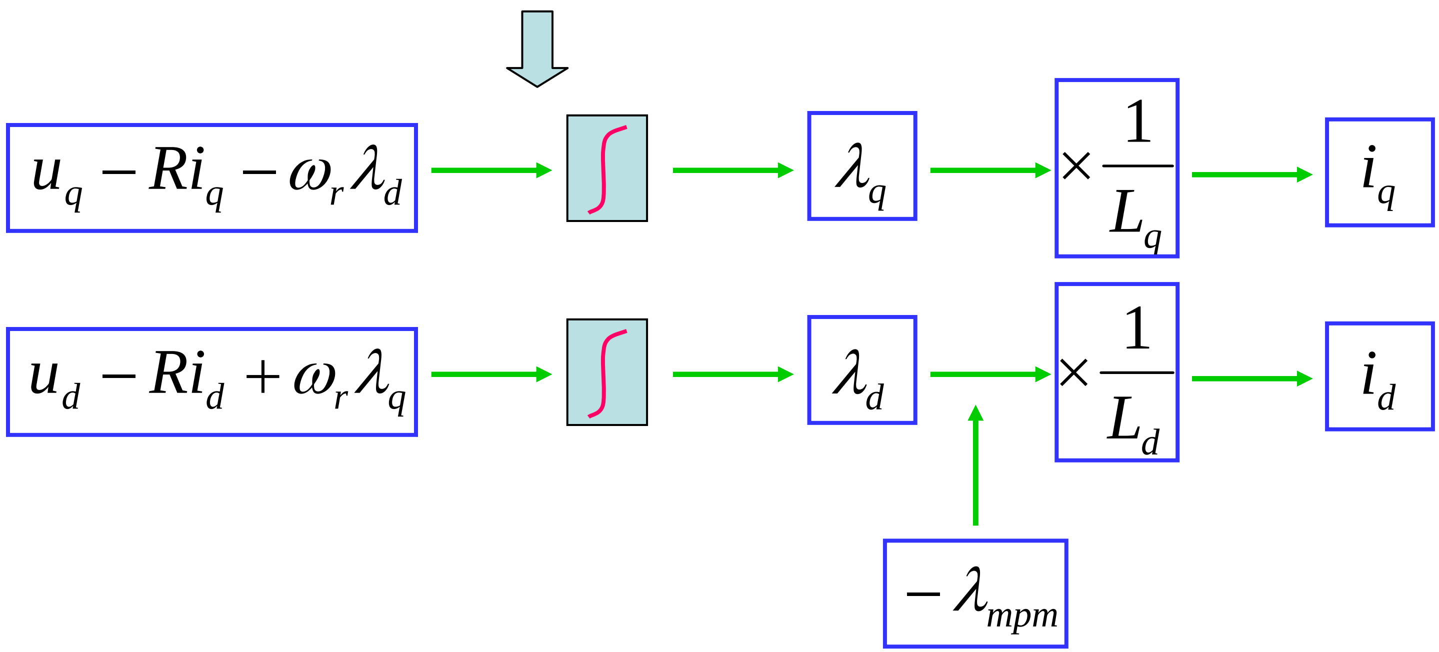
Peak value

Why is the 'peak value' of rotor magnet field should be used?

Dynamic modeling

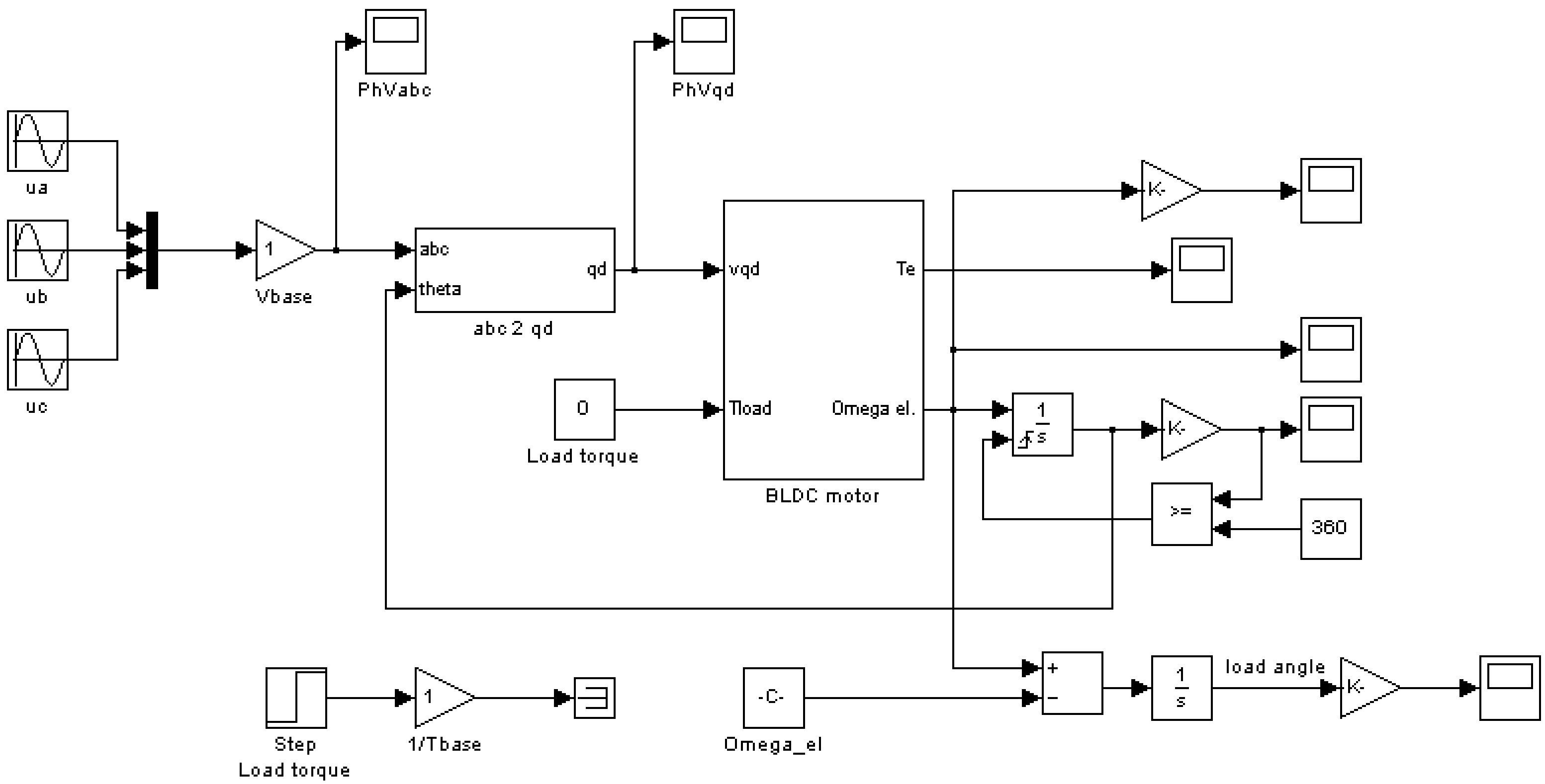
$$u_q = Ri_q + p\lambda_q + \omega_r \lambda_d \qquad \lambda_q = (L_{ls} + L_{mq}) i_q = L_q i_q$$

$$u_d = Ri_d + p\lambda_d - \omega_r \lambda_q \qquad \lambda_d = (L_{ls} + L_{md}) i_d + \lambda_{mpm} = L_d i_d + \lambda_{mpm}$$



All the integrators will potentially require initial conditions

Dynamic modeling



The torque equation

The instantaneous torque equation

$$T_e = \frac{3}{2} p\left(\lambda_d i_q - \lambda_q i_d\right)$$

The same expression as the synchronous motor that has been discussed. Why?

$$T_e = \frac{3}{2} p\left[\lambda_{mpm} i_q + \left(L_d - L_q\right) i_d i_q\right]$$

If d and q axes inductances are equal

$$T_e = \frac{3}{2} p\left(\lambda_{mpm} i_q\right)$$

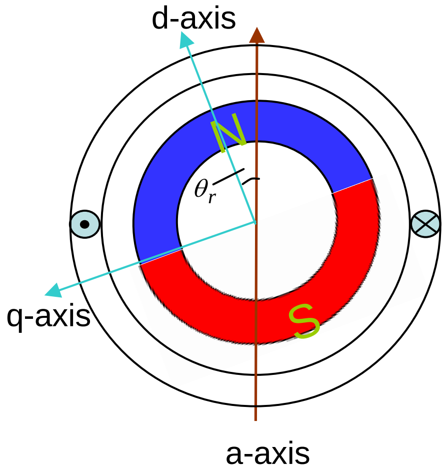
Compare this with the average interaction torque equation given on lecture 7 P7, which is

$$T_{pm,ave} = \frac{1}{2} I_m \lambda_{mpm} \cos\theta_t = \frac{1}{2} I_q \lambda_{mpm}$$

(assuming in steady-state, for single-phase)

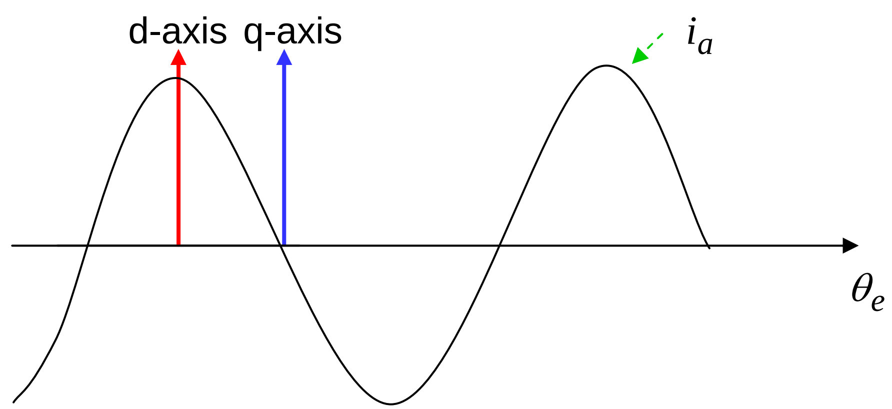
The torque equation

It was mentioned in lecture 7 that . . . .



dq0 reference frame!

$$i_a = -I_m \sin(\theta_{ev} + \theta_t) \qquad -\frac{\pi}{2} < \theta_t < \frac{\pi}{2}$$



This required current waveform corresponds to

$$i_a = I_m \cos(\theta_{ev} + \theta_t)$$

In qd0 reference frame!

So we have

$$\begin{bmatrix} I_q \\ I_d \\ I_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} I_m \cos(\theta_{ev} + \theta_t) \\ I_m \cos\left(\theta_{ev} + \theta_t - \frac{2\pi}{3}\right) \\ I_m \cos\left(\theta_{ev} + \theta_t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} I_q \\ I_d \\ I_0 \end{bmatrix} = \begin{bmatrix} +I_m \cos(\theta_{ev} + \theta_t - \theta_r) \\ -I_m \sin(\theta_{ev} + \theta_t - \theta_r) \\ 0 \end{bmatrix} \xrightarrow[\theta_{ev} = \omega_e t = \omega_r t = \theta_r]{} \begin{bmatrix} I_q \\ I_d \\ I_0 \end{bmatrix} = \begin{bmatrix} +I_m \cos\theta_t \\ -I_m \sin(\theta_t) \\ 0 \end{bmatrix}$$

That is why we have the same torque expressions on P10!

Some definitions regarding the initial angles

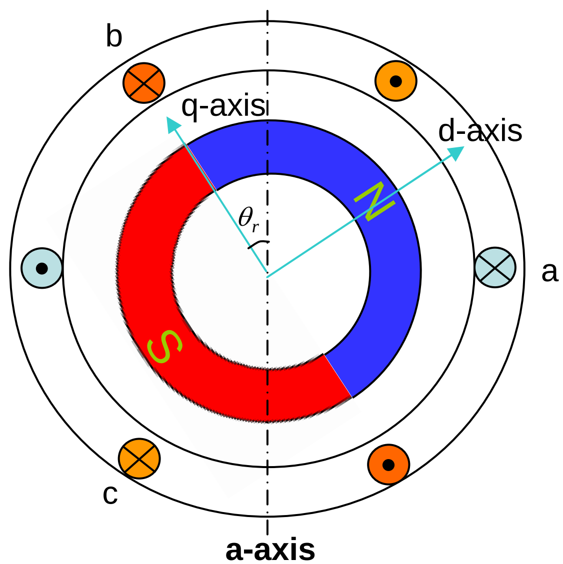
Normally it is defined that (in the book)

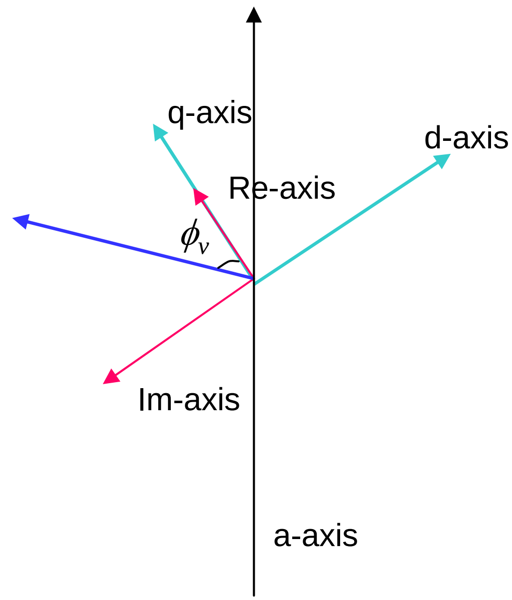
$$\theta_{ev} = \omega_e t + \theta_{ev}(0)$$
$$\theta_r = \omega_r t + \theta_r(0)$$

If we let $\theta_r(0) = 0$ so we have

Steady-state

$$\phi_v = \theta_{ev} - \theta_r = \theta_{ev}(0)$$





Analysis of steady-state operation

Steady-state equations

$$u_q = Ri_q + \omega_r \lambda_d \qquad \lambda_q = L_q i_q$$

$$u_d = Ri_d - \omega_r \lambda_q \qquad \lambda_d = L_d i_d + \lambda_{mpm}$$

Why the differential item can be eliminated?

$$\overline{F}_{qd} = F_q - jF_d$$

$$(u_q = Ri_q + \omega_r \lambda_d) \; - \; j \times (u_d = Ri_d - \omega_r \lambda_q)$$

$$\boxed{\overline{U}_{qd} = R\overline{I}_{qd} + j\omega_r L_q \overline{I}_{qd} + \omega_r \left[(L_d - Lq) I_d + \lambda_{mpm} \right]}$$

Denoted as $\overline{V}_{as}$ in the book

Analysis of steady-state operation

Regarding the voltage vector

$$\overline{U}_{abc}=\frac{2}{3}\left(\sqrt{2}V_s\cos\theta_{ev}+\sqrt{2}V_s\cos\left(\theta_{ev}-\frac{2\pi}{3}\right)e^{j\frac{2}{3}\pi}+\sqrt{2}V_s\cos\left(\theta_{ev}+\frac{2\pi}{3}\right)e^{-j\frac{2}{3}\pi}\right)$$

$$\cos\theta=\frac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right)$$

$$\overline{U}_{abc}=\sqrt{2}V_s e^{j\theta_{ev}}=\sqrt{2}V_s e^{j\theta_r}e^{j\phi_v} \qquad \overline{U}_{qd}=\sqrt{2}V_s e^{j\phi_v}=\overline{U}_{abc}e^{-j\theta_r}$$

Torque vs. speed curve in steady-state

Assuming $\phi_v = 0 \implies U_q = \sqrt{2}V_s \cos\phi_v = \sqrt{2}V_s$

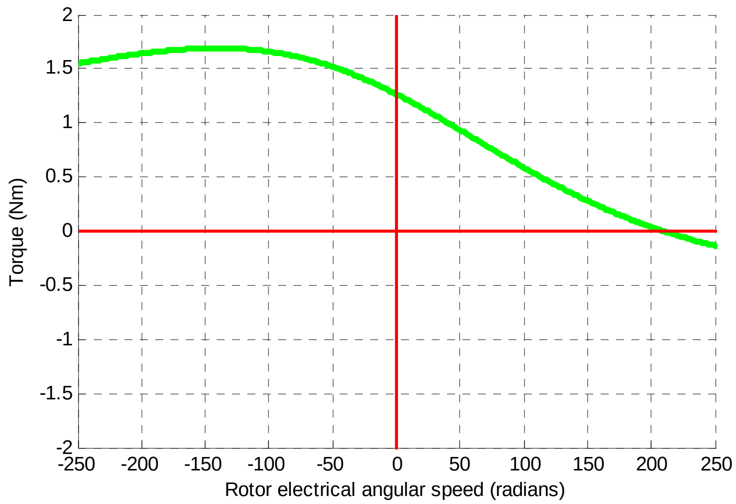
$$U_d = -\sqrt{2}V_s \sin\phi_v = 0$$

$$u_q = Ri_q + \omega_r \lambda_d \qquad \lambda_q = L_q i_q$$
$$u_d = Ri_d - \omega_r \lambda_q \qquad \lambda_d = L_d i_d + \lambda_{mpm}$$

Assuming $L_d = L_q = L_s$

$$I_d = \frac{\omega_r L_q}{R} I_q \qquad I_q = \frac{R}{R^2 + (\omega_r L_s)^2}\left(U_q - \omega_r \lambda_{mpm}\right)$$

$$T = \frac{3}{2} p \frac{R\lambda_{mpm}}{R^2 + (\omega_r L_s)^2}\left(U_q - \omega_r \lambda_{mpm}\right)$$



Torque vs. speed curve in steady-state

Assuming $\phi_v \neq 0 \Longrightarrow$

$$\boxed{\begin{aligned} U_q &= \sqrt{2}V_s \cos\phi_v \\ U_d &= -\sqrt{2}V_s \sin\phi_v \end{aligned}}$$

$$\boxed{\begin{aligned} u_q &= Ri_q + \omega_r \lambda_d & \lambda_q &= L_q i_q \\ u_d &= Ri_d - \omega_r \lambda_q & \lambda_d &= L_d i_d + \lambda_{mpm} \end{aligned}}$$

Assuming $L_d = L_q = L_s$

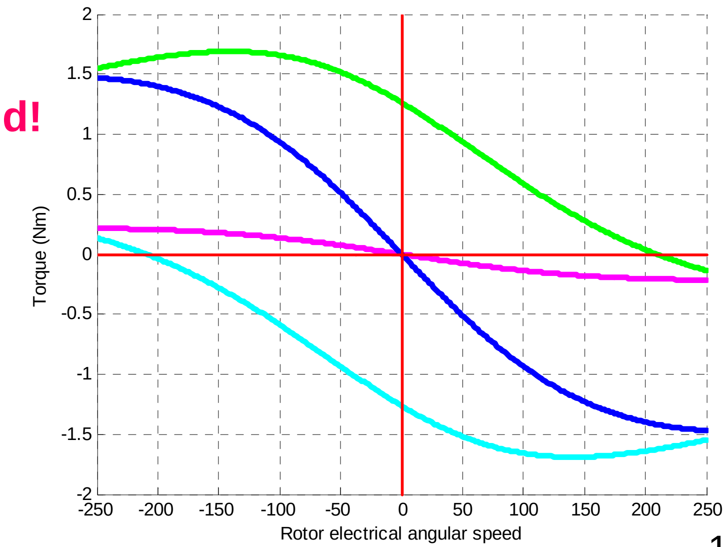
$$I_d = \frac{U_d + \omega_r L_q I_q}{R} \qquad I_q = \frac{\sqrt{2}V_s R}{R^2 + (\omega_r L_s)^2}\left(\cos\phi_v + \omega_r \frac{L_s}{R}\sin\phi_v - \omega_r \frac{\lambda_{mpm}}{\sqrt{2}V_s}\right)$$

$$\boxed{\tau_s = \frac{L_s}{R_s} \qquad \tau_v = \frac{\lambda_{mpm}}{\sqrt{2}V_s}}$$

→ **Unit is second!**

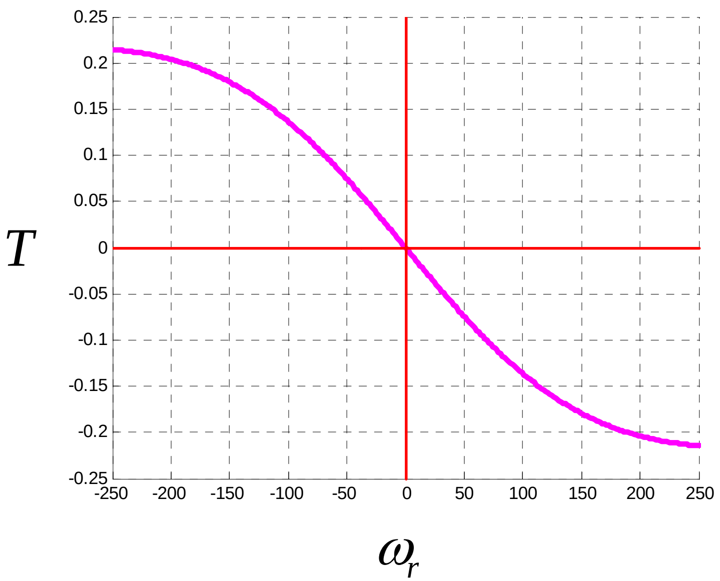
$$I_q = \frac{\sqrt{2}V_s R}{R^2 + (\omega_r L_s)^2}\left[\cos\phi_v + \omega_r(\tau_s \sin\phi_v - \tau_v)\right]$$

$$T = \frac{3}{2} p \lambda_{mpm} I_q$$

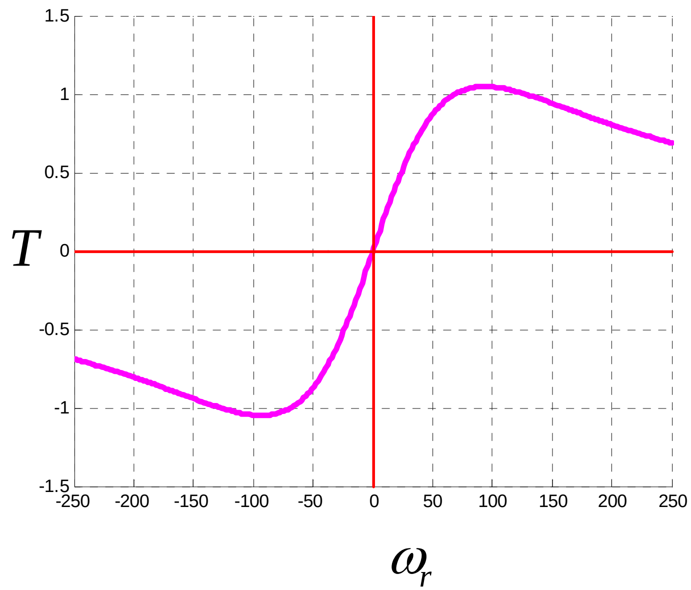


If $\phi_v = \frac{\pi}{2}$ $\Longrightarrow$ $T = K\omega_r(\tau_s - \tau_v)$

$\tau_s < \tau_v \leftarrow T < 0$



$\tau_s > \tau_v \rightarrow T > 0$



Exercises

1. For a surface mounted motor, if the motor is required to operate above the rated speed, how the d-axis current should be controlled? Any e effects to the motor torque generated?

2. For a motor with buried magnets, and Ld > Lq, repeat questions No. 1. What if the motor has d, q-axes inductace like Ld < Lq.

3. What is the initial conditions that should be set for the two integrators in the dynamic model of the machine, as described on P9.

4. Suppose the PM machine is driven by the load motor, and rotating at the rated speed with stator windings open-circuited, what will be the winding terminal voltages measured on the q,d –frame?

5. Play with the Simulink model supplied. To check at what Uq and Ud voltages the motor will run with Id=0, at rated speed and rated output torque.