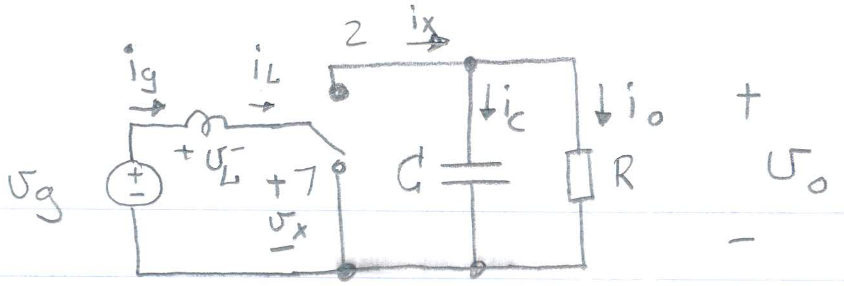
7.1.



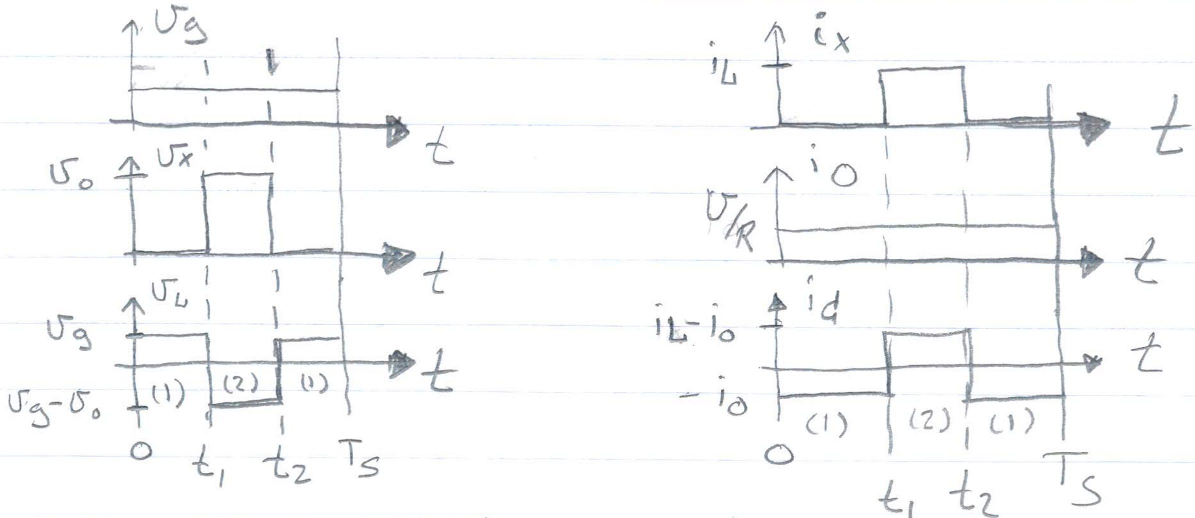
7/

Equations :

$$v_g - v_L - v_x = 0 \qquad i_x - i_c - i_o = 0$$

$$v_x = \begin{cases} v_o & (2) \\ 0 & (1) \end{cases} \qquad i_x = \begin{cases} i_L & (2) \\ 0 & (1) \end{cases}$$

waveforms :



Averaged inductor equation:

$$\langle v_L \rangle_{T_s} = L \frac{d \langle i_L \rangle_{T_s}}{dt} \qquad \langle v_L \rangle_{T_s} = \frac{1}{T_s} \int_0^{T_s} v_L \, dt$$

$$\langle v_L \rangle_{T_s} = \frac{1}{T_s} \int_0^{t_1} \langle v_g \rangle_{T_s} dt + \frac{1}{T_s} \int_{t_1}^{t_2} \langle v_g \rangle_{T_s} - \langle v_o \rangle_{T_s} dt$$
$$+ \frac{1}{T_s} \int_{t_2}^{T_s} \langle v_g \rangle_{T_s} dt$$

$$\langle v_L \rangle_{T_s} = \frac{1}{T_s} \left[-(t_2 - t_1) \langle v_o \rangle_{T_s} + T_s \langle v_g \rangle_{T_s} \right]$$

$$\langle v_L \rangle_{T_s} = \langle v_g \rangle_{T_s} - \frac{(t_2 - t_1)}{T_s} \langle v_o \rangle_{T_s}$$

Now be care full regarding duty-cycle.

Normally the duty-cycle represent $\frac{t_{on}}{T_s}$ for the transistor.

Here $\frac{t_2 - t_1}{T_s}$ represent the on-time for the diode.

So we know

$$\underbrace{t_2 - t_1}_{\text{on time for diode}} + \underbrace{t_1 - 0 + T_s - t_2}_{\text{on time for transistor}} = T_s$$

$$t_2 - t_1 + t_{on} = T_s$$

$$\frac{t_2 - t_1}{T_s} + \frac{t_{on}}{T_s} = 1$$

$$\frac{t_{on}}{T_s} = 1 - \frac{t_2 - t_1}{T_s}$$

or

$$\frac{t_2 - t_1}{T_s} = 1 - \frac{t_{on}}{T_s}$$

$$= 1 - d$$

So:

$$\langle v_L \rangle_{T_s} = \langle v_g \rangle_{T_s} - (1-d) \langle v_o \rangle_{T_s}$$

$$\boxed{L \frac{d \langle i_L \rangle_{T_s}}{dt} = \langle v_g \rangle_{T_s} - (1-d) \langle v_o \rangle_{T_s}}$$

capacitor:

$$\langle i_c \rangle_{T_s} = \frac{1}{T_s} \int_0^{T_s} i_x - i_o \, dt$$

$$\langle i_c \rangle_{T_s} = \frac{1}{T_s} \int_0^{t_1} -\langle i_o \rangle_{T_s} dt + \frac{1}{T_s} \int_{t_2}^{T_s} -\langle i_o \rangle_{T_s} dt$$

$$+ \frac{1}{T_s} \int_{t_1}^{t_2} \langle i_L \rangle_{T_s} - \langle i_o \rangle_{T_s} dt$$

$$\langle i_c \rangle_{T_s} = \frac{+\langle i_o \rangle_{T_s}}{T_s} \left[-t_1 - T_s + t_2 \right]$$

$$+ \frac{t_2 - t_1}{T_s} \left[\langle i_L \rangle_{T_s} - \langle i_o \rangle_{T_s} \right]$$

$$= -\langle i_o \rangle_{T_s} + \frac{t_2 - t_1}{T_s} \langle i_L \rangle_{T_s}$$

$$\boxed{\begin{aligned} \langle i_c \rangle_{T_s} &= -\langle i_o \rangle_{T_s} + (1-d) \langle i_L \rangle_{T_s} \\ &= C \frac{d \langle v_o \rangle_{T_s}}{dt} \end{aligned}}$$

Averaging result

$$L\frac{d\langle i_L\rangle_{T_s}}{dt} = \langle v_g\rangle_{T_s} - (1-d)\langle v_o\rangle_{T_s}$$

$$C\frac{d\langle v_o\rangle_{T_s}}{dt} = -\langle i_o\rangle_{T_s} + (1-d)\langle i_L\rangle_{T_s}$$

Small signal model

DC + ac: $L\frac{d\hat{i}_L}{dt} = V_g + \hat{v}_g - (1-(D+\hat{d}))(V_o+\hat{v}_o)$

$$= V_g + \hat{v}_g - (-DV_o - D\hat{v}_o - V_o\hat{d} + V_o + \hat{v}_o)$$

DC: $\underline{\underline{0 = V_g - (1-D)V_o}}$

ac: $L\frac{d\hat{i}_L}{dt} = \hat{v}_g - (-D\hat{v}_o - V_o\hat{d} + \hat{v}_o)$

$$\underline{\underline{L\frac{d\hat{i}_L}{dt} = \hat{v}_g + V_o\hat{d} - (1-D)\hat{v}_o}}$$

dc+ac: $C\frac{d\hat{v}_o}{dt} = -I_o - \hat{i}_o + ((1-D)-\hat{d})(I_L + \hat{i}_L)$

DC: $0 = -I_o + (1-D)I_L$

ac: $C\frac{d\hat{v}_o}{dt} = -\hat{i}_o + (1-D)\hat{i}_L - \hat{d}\, I_L$