Written exam in Probability & Statistics

PM6 & ET6
Lecturer: Kasper K. Berthelsen

Tuesday 4th of January 2005, 9:00-13:00

In the assessment emphasis will be put on both correct methods as wells as correct answer, hence the the method should be clearly stated.

Good luck!

Problem 1. (approx. 20%)
A random variable X has mean 10 and variance 50.

1. Calculate the mean and variance of the random variable $Y = 10 + 5X$.
2. Find the mean of $V = (X - 10)^2$ and $Z = X^2$.

Problem 2. (approx. 10%)
In a survey among house owner, people are asked if they are willing to pay more for snow removal. Among the 84 people who reply the answers are distributed according to age as follow:

Age	No	Yes	No answer
20–25	1	0	0
26–35	0	3	1
36–50	6	10	10
51–60	1	7	1
61–70	2	13	6
> 70	4	13	6

1. Calculate the conditional probability for answering yes, conditionally on the person's age being ≤ 50 years and > 50 years, respectively.
2. Are the events $A = \{\text{Age} \leq 50 \text{ years}\}$ and $B = \{\text{Yes}\}$ independent? Justify your answer.

Problem 3. (approx. 20%)
In an airport, whenever the metal detector goes off, there is a 25% probability that the alarm is caused by coins in the pocket of the passenger walking through the metal detector.

1. During one day the alarm goes off 15 times. What is the probability that at least 3 of these alrams are caused by passengers having coins in their pockets?
2. Question 1 continued: Is it likely that none of these 15 alarms are caused by coins in a pocket? Explain your answer based on the probability of this event.
3. Just before Christmas the airport is unusually busy. On one day the metal detector alarm goes off 50 times. What is the probability that at most $\frac{1}{5}$ of these alarms are caused by coins in a pocket.

Problem 4. (approx. 35%)
As is well-known, the department network is often down. Near the project dead-line some students decide to measure the dayly downtime in minutes. Acordingly they measure how many minutes the network is down each day for 14 days and obtain the following downtimes:

Day	1	2	3	4	5	6	7
downtime (minutes)	229	295	343	337	282	313	262
Day	8	9	10	11	12	13	14
downtime (minutes)	303	201	374	376	343	406	163

The downtimes are assumed to follow a normal distribution with mean μ and variance σ^2.

1. Estimate the mean μ and the standard deviation σ for the dayly downtime.
2. Determine a 95% confidence interval for μ.
3. Determine a 95% confidence interval for σ.
4. The students want the downtime to be as short as possible. Test on the 5% significance level if the expected downtime is significantly less than 4 hours, i.e. 240 minutes.
5. What is the probability that the average down time over a 14 day period is less then 4 hours, i.e. 240 minute? Assume that the 14 downtimes are independent and normal distributed with equal means $\mu = 300$ and unknown and equal variances.

Problem 5. (approx. 15%)
At a Christmas dinner 10 students measure their blood alcohol level by each making two measurements using a breathalyzer. They obtain the following measurements where the differences between measurements are given:

Student	1st measurement	2nd measurement	Difference
1	0.9	0.9	0.0
2	1.0	1.8	-0.8
3	1.8	1.8	0.0
4	1.2	1.6	-0.4
5	0.8	0.8	0.0
6	1.0	0.8	0.2
7	0.9	1.0	-0.1
8	1.2	2.1	-0.9
9	2.2	2.0	0.2
10	1.2	1.5	-0.3
$\bar{x}$	1.22	1.43	-0.21
s^2	0.197	0.260	0.150

It is assumed that the random variables correspoindg to the alchohol level for first and second measurments are independent and noraml distributed with equal mean and variance. Notice that the two measurements for the same student are **not** independent.

1. Find a 90% confidence interval for the difference in the two measurements.
2. Tes at the 5% significance level if the level at the first measurement is different from the secon measurement.

Remember to add student number on all sheets and state how many sheets your solution consists of.

Written exam in Probability & Statistics

PM6 & ET6
Lecturer: Kasper K. Berthelsen

Friday 6th of January 2006, 9:00-13:00

In the assessment emphasis will be put on both correct methods as wells as correct answer, hence the the method should be clearly stated.

Good luck!

Problem 1. (approx. 20%)
A salesman at a used car dealer receives a commission for each car or van he sells. When he sells a car he receives 4200 kr and 4800 kr when he sells a van. He expects to sell a number of cars and vans each day according to the following probabilities:

Number of cars	0	1	2	3
Probability	0.3	0.4	0.2	0.1

Number of vans	0	1	2
Probability	0.4	0.5	0.1

1. Calculate the expected number of cars and vans the salesman is expected to sell each day.
2. Calculate the standard deviation of the number of cars and van the salesman sells in a day.
3. Calculate the expected commission for both cars and vans a salesman will receive in a day.
4. Calculate the standard deviation of the salesman total commission in a day when we assume that the number of sold cars and sold vans are dependent with a correlation coefficient of $\rho = 0.1$.

Problem 2. (approx. 15%)
The length of times it takes to repair a vending machine follows a normal distribution with mean 120 minutes and variance 16 minutes^2. If the vending machine is under repair for more than 125 minutes the machines must be cleaned and emptied which is an unwanted extra expense.

1. What is the probability that the vending machine is under repair for more than 125 minutes?
2. A member of staff wants to find a time interval in which the time it takes to repair the vending machine is with 95% probability. Find such a 95% probability interval which is symmetric around the mean.

Problem 3. (approx. 15%)
Wanting to optimise storage space a seller wants to model the number of orders on a specific product in December. In December the previous year the number of orders was 15.

1. Specify a random variable and its distribution, so that it describes that number of orders in December — explain your choice.
2. What is the probability of 17 or more orders.
3. How large does stock need to be for the seller to have at least a 95% probability of fulfilling all orders? Assume that the seller cannot receive new stock during December.

Problem 4. (approx. 30%)
The walls in a plastic bottle need to have a certain thickness to avoid that the bottle does breaks. An engineer in quality control takes a sample of 25 bottles and measures the wall thickness obtaining a sample average of $\bar{x} = 4.05mm$ and a sample standard deviation of $s = 0.08mm$. He further assumes that the observations are independent and normal distributed.

1. Determine a 95% confidence interval for the mean of the wall thickness.
2. Determine a 95% confidence interval for the standard deviation of the wall thickness.
3. Test at the 5% significance level if the wall thickness is less than $4mm$.
4. Test at the 5% significance level if the standard deviation of the wall thickness equals 0.1

Problem 5. (approx. 20%)
A cement factory wants to buy a new machine for filling bags with 50kg of cement. They have two machines to choose from. From each machine they take a sample of 6 bags and weigh each of them. The measured weight are given in the table below

	Machine I	Machine II
	51.2	29.4
	49.0	50.7
	49.8	49.1
	51.7	48.7
	50.3	48.7
	51.4	50.1
$\bar{x}$	50.57	49.80
s^2	1.0987	0.7520

1. Test at the 10% significance level if the variance of the weights are equal for the two machines.
2. Test at the 10% significance level if the means of the weights are equal for the two machines.

Remember to add student number on all sheets and state how many sheets your solution consists of.

Written exam in Probability Theory and Statistics - K7

Lecturer: P. Svante Eriksen
Thursday 13th of january 2011, 9:00-13:00
In the assessment emphasis will be put on both correct methods as well as correct answers. Hence the method should be clearly stated.

Problem 1. (approx 20%)
The random variable X has a normal distribution with mean 4 and variance 25.

1. Calculate the mean and variance of the variable $4X + 6$.
2. Calculate $P(0 \leq X \leq 4)$.

The random variable Y has mean 5 and variance 10. The correlation coefficient of X and Y is -0.5.

3. Calculate the mean and variance of the variable $4X + 5Y + 1$.

Problem 2. (approx 20%)
The joint probability distribution of X and Y is given by

$$f(x, y) = \frac{2x + y}{27}, \quad x = 0, 1, 2; \; y = 0, 1, 2$$

1. Evaluate the marginal distribution of X.
2. Find $P(Y = 2|X = 1)$ and $P(Y = 2|X = 2)$. Are X and Y statistically independent?
3. Evaluate $E(X^2Y)$.

Problem 3. (approx 10%)
In a certain city the need for money to buy drugs is stated as the reason for 60% of all thefts.
Consider the next 20 theft cases in the city and let X denote the number of cases resulting from the need for money to buy drugs.

1. Calculate the mean and variance of X.
2. Evaluate $P(4 \leq X \leq 12)$.

Problem 4. (approx 30%)
An engineer in quality control takes a sample of 30 bolts and measures their diameter, which yields a sample average of $\bar{x} = 10.023mm$ and a sample standard deviation $s = 0.009mm$. He assumes that the observations are a random sample from the normal distribution.

1. Determine a 95% confidence interval for the mean of the bolt diameter.

2. Determine a 95% confidence interval for the standard deviation of the bolt diameter.

3. Test at the 5% significance level whether the bolts meet a requirement of a mean diameter equal to $10mm$.

4. Test at the 2.5% significance level whether the measurements meet a requirement of a standard deviation below or equal to $0.005mm$.

Problem 5. (approx 20%)
Two methods for measuring the molar heat of fusion of water are being compared. Ten measurements made by method A have a sample mean $\bar{x}_A = 6.025$ kilojoules per mole and sample standard deviation of $s_A = 0.024KJ/mol$. Five measurements made by method B have a sample mean $\bar{x}_B = 6.001KJ/mol$ and sample standard deviation of $s_B = 0.012KJ/mol$.

1. Test at the 5% significance level whether the two methods have the same standard deviation.

2. Test at the 5% significance level whether the mean measurements differ between the two methods.

Remember to add student number on all sheets and state how many sheets your solution consists of

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Written exam in Probability Theory and Statistics

Lecturer: Robert Jacobsen
Friday 18th of January 2013, 9:00-13:00
In the assessment emphasis will be put on both correct methods as well as correct answers. Hence the method should be clearly stated.

Problem 1 (25%) Let X and Y be two stochastic variables with a joint probability distribution partly given in the following table.

	$Y=1$	$Y=2$	$P(X=x)$
$X=1$	?	?	?
$X=2$	0.50	?	0.60
$P(Y=y)$	0.80	?	?

1) Fill in the 6 spots with "?" and explain how you obtain the results.
2) Compute the conditional distribution of X given Y.
3) Compute the mean and variance of X.
4) Are X and Y independent? Justify your answer.
5) Compute the correlation between X and Y.

Problem 2 (15%) On a normal day at Mount Slippery the average number of skiers falling is 2 every 5 minutes.

1) We describe the distribution of the number of falling skiers during 1 minute with a random variable X that follows a Poisson distribution. What is the parameter of this Poisson distribution?
2) What is the expected number of falling skiers during 30 minutes?
3) What is the probability that at least 5 skiers fall during 30 minutes?

Problem 3 (30%) It is claimed that the width of a good ski should be 61 - 63 millimeters wide. The ski company Straight Down claim that their skies are 62 millimeters wide. A sample of seven skies have been selected independently at random and their widths have been measured.

Measurement	1	2	3	4	5	6	7
Width	62.1	62.9	61.7	63.1	60.5	61.3	62.2

Assume that the measurements are normally distributed with the same parameters.

1) Estimate the mean and variance of the width.
2) Calculate a 90% confidence interval for both the mean and the variance.
3) Test on a 10% significance level that the mean is 62 millimeter.
4) If the mean *is* 62 millimeter what should the variance be to ensure that at most 2% of the skies are narrower than 61 millimeters?

Problem 4 (30%) Two skiers want to see who is faster down a mountain side. Their times in seconds have been collected in the following table.

	Skier 1	Skier 2
	21.6	22.2
	22.8	23.4
	26.0	22.4
	23.8	21.6
		21.0
		22.6
$\overline{x}$	23.5	22.2
s^2	3.48	0.688

Assume that they have raced under identical circumstances and that the times are normally distributed and independent.

1) Test at the 5% significance level if the variances are equal.
2) Test at the 5% significance level if the means are equal – assume equal variance.
3) Assume that the estimated means and variances are the true means and variances. What is the probability that Skier 1 will be faster that Skier 2?

On all the pages you turn in, you must write your student ID number as well as the total number of pages.
