



Study number: 2015 1913	Programme: All intro students
Evaluation subject: Dynamic Models of Electrical and Control Systems 21 January at 9:30-13:30	

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Total number of pages, including this page: 22

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Problem 1

$$1) \quad \bar{f}_I = 10 e^{j45} \quad [\text{given}]$$

where \bar{f}_I is defined as current vector.

Let phase currents in phases a, b & c be defined as f_{Ia} , f_{Ib} & f_{Ic}

respectively.

$$f_{Ia} = \frac{\operatorname{Re}(\bar{f}_I)}{e^{j0^\circ}} = \frac{\operatorname{Re}[10 e^{j45}]}{e^{j0}}$$

$$\Rightarrow f_{Ia} = \operatorname{Re}[10 e^{j(45-0)}]$$

$$= 10 \cos 45^\circ$$

$$= 7.071 \text{ A}$$

$$\Rightarrow f_{I_b} = \frac{\operatorname{Re}(\bar{f})}{e^{j2\pi/3}} = \frac{\operatorname{Re}(10 e^{j45^\circ})}{e^{j2\pi/3}}$$

$$= \cancel{10} 10 e^{j(45^\circ - 120^\circ)}$$

$$= 10 \cos(-75^\circ)$$

$$= 2.59 \text{ A}$$

$$\Rightarrow f_{I_c} = \frac{\operatorname{Re}(\bar{f})}{e^{-j2\pi/3}} = \frac{\operatorname{Re}(10 e^{j45^\circ})}{e^{-j2\pi/3}}$$

$$= 10 e^{j(45^\circ + 120^\circ)}$$

$$= 10 \cos(165^\circ)$$

$$= -9.66 \text{ A}$$

~~Q1~~Problem 1.

(2)

(2) Let the dq component of the current vector be defined as i_d & i_q respectively.

As given in question;

d-axis leads a-axis by 90° & we know q-axis leads d-axis by 90° .

$$i_d = \frac{\text{Re}(10 e^{j45^\circ})}{e^{j90^\circ}} = \text{Re}(10 e^{j(45^\circ - 90^\circ)})$$

$$\Rightarrow i_d = 10 \cos(-45^\circ) = 7.071 \text{ A}$$

Now for the q component.

$$f_{I_q} = \frac{\operatorname{Re}(10 e^{j45})}{e^{j180}} = \operatorname{Re}(10 e^{j(45-180)})$$

$$\Rightarrow f_{I_q} = 10 \cos(-135^\circ) \\ = -7.071 \text{ A}$$

Problem 1.

(3) Let the rotating speed of dq reference frame be $\omega_r \text{ rad s}^{-1}$

As given in question;

Angular velocity of current vector
 $\omega_c = 2\pi \cdot 50 = 314.16 \text{ rad}$

Now, the relative speed difference between the dq reference frame & the space current vector will cause the difference in angle.

Problem 1

③

Q:- 3 contd.

So, As per question the d axis leads the space current vector by 90° which means the change in angle i.e. $90^\circ - 45^\circ = 45^\circ = \Delta\theta$, occurred during the time interval of $t = 0.05$ seconds due to the relative speed.

So, we can say that.

$$(\omega_r - \omega_e) \text{ time} = \Delta\theta$$

$$\Rightarrow (\omega_r - 314.16)(0.05) = 45^\circ = \frac{\pi}{4}$$

$$\Rightarrow 0.05 \omega_r - 15.71 = \frac{\pi}{4}$$

$$\Rightarrow 0.05 \omega_r = 15.71 + 0.785 = 16.5$$

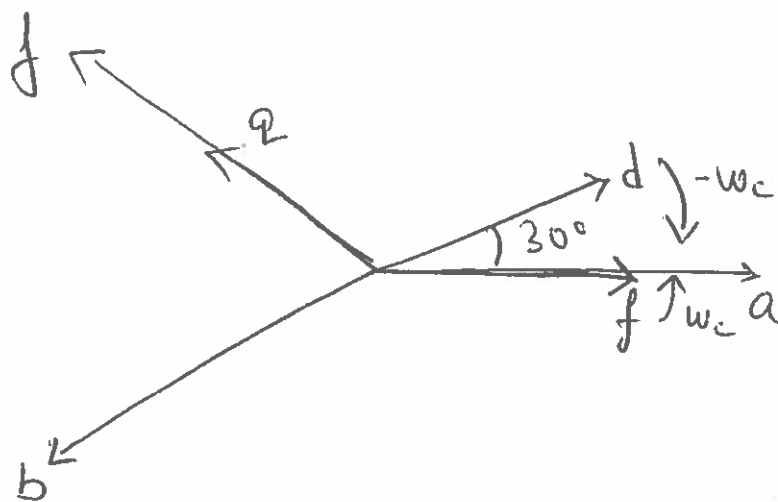
$$\omega_r = \frac{16.5}{0.05}$$

$$= 329.91 \text{ rad s}^{-1}$$

∴ The rotating speed of dq reference frame is $329.91 \text{ rad s}^{-1}$

Problem 1

4) As given in the question:-



Let the corresponding d & q component be f_{Id} & f_{Iq} .

$$f_{Id} = \text{Re} \left(\frac{\bar{f} e^{j\omega_c t}}{e^{-j(\omega_c t - \pi/6)}} \right)$$

Problem 1.

(4)

4) Contd.

$$\Rightarrow f_{Id} = 10 \cos \left[e^{j\omega_e t} \cdot e^{j(\omega_e t - 30)} \right]$$

$$\Rightarrow f_{Id} = 10 \cos \left[e^{j(2\omega_e t - \pi/6)} \right]$$

$$\Rightarrow f_{Id} = 10 \cos (2\omega_e t - \pi/6)$$

Substituting $\omega_e = 2\pi \cdot 50 = 314.16 \text{ rad s}^{-1}$

$$\Rightarrow f_{Id} = 10 \cos (2 \times 314.16 t - \pi/6)$$

Similarly for f_{Iq} .

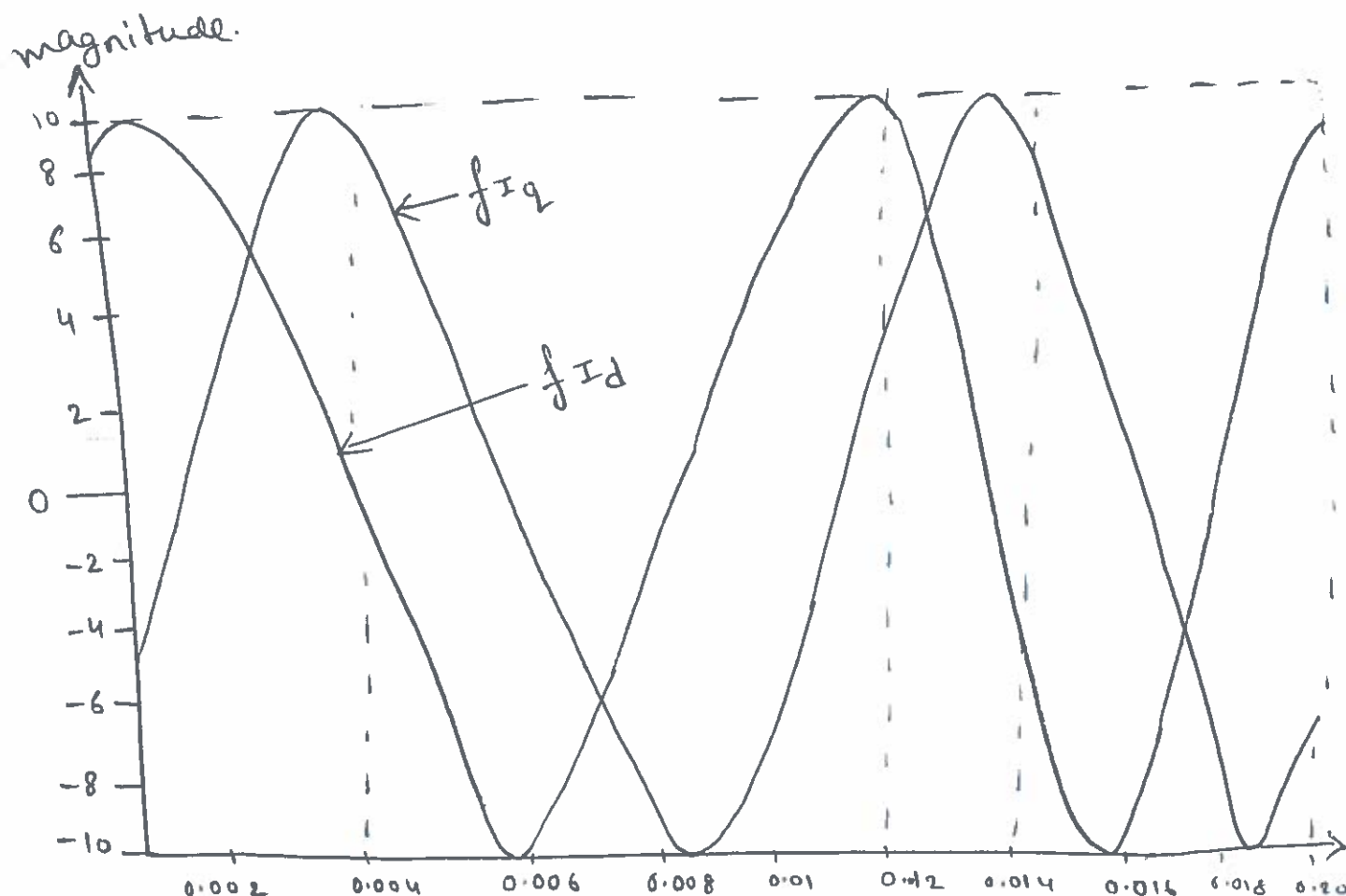
$$f_{Iq} = \text{Re} \left(\frac{\hat{f} e^{j\omega_e t}}{e^{-(j\omega_e t - 2\pi/3)}} \right)$$

$$\Rightarrow f_{Iq} = 10 \cos(2\omega_e t - 2\pi/3)$$

substituting $\omega_e = 314.16 \text{ rad s}^{-1}$

$$\Rightarrow f_{Iq} = 10 \cos(2 \times 314.16 t - 2\pi/3)$$

The sketch of the d-, q- waveforms
for the time period $[0, 0.002]$ second
have been obtained using "Matlab".



Problem 1.

(5)

Question 4 contd

As per the sketch in the previous page, it must be noted that the "blue" waveform ~~refers~~ represents the q axis current component & the "grey" waveform represents the d axis current component.

Problem 1.Question 5

In abc reference frame, we have phase 'a' that always aligns itself with the real axis.

We also know that the space vector used to represent in α, β system is given as.

where $\alpha = \text{alpha}$
 $\beta = \text{Beta}$

$$\vec{f} = f_{\alpha} + j f_{\beta}$$

The above equation clearly states that the ' α ' component is aligned along the Real axis of the Co-ordinate system. Therefore, we can say that the alpha ' α ' component in α, β system is same as the phase 'a' component in a,b,c reference frame.

To corroborate the above explanation. The transformation matrix, that is used to transform α, β components to a,b,c components is used.

Problem 1.

⑥

(5) continued. α, β to a, b, c transformation:-

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix}$$

 \therefore we see that

$$f_a = 1 \cdot f_\alpha + 0 \cdot f_\beta$$

$$\therefore f_a = f_\alpha$$

Proved.

Problem 2

(1) Let the mutual inductance between stator phase - a & stator phase - c be M_{asc}

$$\Rightarrow M_{asc} = L_{aaq} \operatorname{Re} \left(\frac{e^{j\theta}}{e^{j0^\circ}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\theta}}{e^{-j2\pi/3}} \right) + L_{aad} \operatorname{Re} \left(\frac{e^{j(\theta-\pi/2)}}{e^{j0}} \right) \cdot \operatorname{Re} \left(\frac{e^{j(\theta-\pi/2)}}{e^{-j2\pi/3}} \right)$$

$$\Rightarrow L_{aaq} \cos \theta \cdot \cos (\theta + 2\pi/3) + L_{aad} \sin \theta \cdot \sin (\theta + 2\pi/3)$$

[We know that.

$$\left[\begin{array}{l} \cos (A+B) + \cos (A-B) = 2 \cos A \cdot \cos B \\ \cos (A-B) - \cos (A+B) = 2 \sin A \cdot \sin B \end{array} \right]$$

Problem 2

(7)

1) contd.

$$\Rightarrow \frac{L_{aaq}}{2} \left[\cos(\theta + \theta + 2\pi/3) + \cos(\cancel{\theta} - \cancel{\theta} - 2\pi/3) \right]$$

$$\frac{L_{aad}}{2} \left[\cos(\cancel{\theta} - \cancel{\theta} - 2\pi/3) - \cos(\theta + \theta + 2\pi/3) \right]$$

Considering $L_{aad} = L_{aaq} = L_{ms}$

$$\Rightarrow \frac{L_{ms}}{2} \left[\cos(\cancel{2\theta} + 2\pi/3) + \cos(-2\pi/3) + \cos(-2\pi/3) - \cos(\cancel{2\theta} + 2\pi/3) \right]$$

$$\Rightarrow \frac{L_{ms}}{2} \left[2 \cos(-2\pi/3) \right]$$

$$\therefore \boxed{M_{ases} = -\frac{1}{2} L_{ms}}$$

Problem 2.

(2) Let the mutual inductance between motor phase - b & stator phase a be M_{asbr} .

$$M_{asbr} = L_{aaq} \operatorname{Re} \left(\frac{e^{j0}}{e^{j0}} \right) \cdot \operatorname{Re} \left(\frac{e^{j0}}{e^{j(\theta_r + 2\pi/3)}} \right)$$

$$L_{aad} \operatorname{Re} \left(\frac{e^{j(\theta - \pi/2)}}{e^{j0}} \right) \cdot \operatorname{Re} \left(\frac{e^{j(\theta - \pi/2)}}{e^{j(\theta_r + 2\pi/3)}} \right)$$

$$\Rightarrow L_{aaq} \cos(\theta) \cdot \cos(\theta - \theta_r - 2\pi/3) +$$

$$L_{aad} \sin(\theta) \cdot \sin(\theta - \theta_r - 2\pi/3)$$

$$\left[\begin{array}{l} \text{We know that} \\ \cos(A+B) + \cos(A-B) = 2 \cos A \cdot \cos B \\ \cos(A-B) - \cos(A+B) = 2 \sin A \cdot \sin B \end{array} \right]$$

Problem 2

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⑧

(2) continued.

$$\Rightarrow M_{asbr} = \frac{L_{aag}}{2} \left[\cos(\theta + \theta - \theta_r - 2\pi/3) + \cos(\theta - \theta + \theta_r + 2\pi/3) \right] +$$

$$\frac{L_{aad}}{2} \left[\cos(\theta - \theta + \theta_r + 2\pi/3) - \cos(\theta + \theta - \theta_r - 2\pi/3) \right]$$

Considering $L_{aad} = L_{aag} = L_{ms}$

$$\Rightarrow \frac{L_{ms}}{2} \left[\cos(\cancel{2\theta} - \theta_r - 2\pi/3) + \cos(\theta_r + 2\pi/3) + \cos(\theta_r + 2\pi/3) - \cos(\cancel{2\theta} - \theta_r - 2\pi/3) \right]$$

$$\Rightarrow M_{asbr} = \frac{L_{ms}}{2} \left[\cancel{2} \cos(\theta_r + 2\pi/3) \right]$$

∴

$$M_{asbr} = L_{ms} \cdot \cos(\theta_r + 2\pi/3)$$

Problem 2

3) Given:-

rotor speed on shaft = 240 rpm

no. of pole pairs (P_p) = 4

Let rotor angular velocity be ω_r

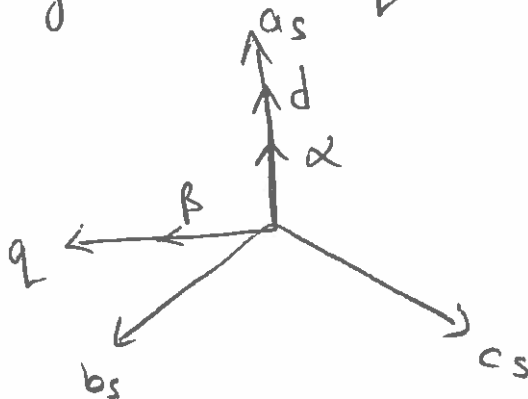
$$\omega_r = \frac{2\pi}{60} \times P_p \times [\text{rotor speed in rpm}]$$

$$\Rightarrow \omega_r = \frac{2\pi}{60} \times 4 \times 240$$

$$= 100.531 \text{ rad s}^{-1}$$

(4) As given in question,

Now.



Problem 2

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(9)

(4) contd.

problem,

Now in this ↑ the space vector representation ~~the vector~~ in q, d reference

is $\bar{f} = f_d + j f_q$ - (i)

the space vector representing in α, β reference is $\bar{f} = f_\alpha + j f_\beta$ - (ii)

Correlating (i) & (ii) we have

$f_d = f_\alpha$ & $f_q = f_\beta$ & since q, d reference frame is now stationary with respect to α, β reference frame of stator, $\boxed{\omega_\theta = 0}$.
Stator voltage expressions in q, d reference frame.

$$V_{qs} = R_s \cdot i_{qs} + p \lambda_{qs} + \cancel{\omega_\theta \lambda_{ds}} + \omega_\theta \lambda_{ds} \quad L \quad (iii)$$

$$V_{ds} = R_s \cdot i_{ds} + p \lambda_{ds} - \omega_\theta \lambda_{qs} \quad (iv)$$

In equation (iii) & (iv) replacing all q & d components with β & α components respectively we get. [$\omega_0 = 0 \text{ rad/s}$]

$$V_{\beta s} = R_s i_{\beta s} + p \lambda_{\beta s} + \cancel{\omega_0} \lambda_{\alpha s}$$

$$V_{\alpha s} = R_s i_{\alpha s} + p \lambda_{\alpha s} - \cancel{\omega_0} \lambda_{\beta s}$$

$$\therefore \begin{cases} V_{\beta s} = R_s i_{\beta s} + p \lambda_{\beta s} \\ V_{\alpha s} = R_s i_{\alpha s} + p \lambda_{\alpha s} \end{cases}$$

Now, writing for rotor side voltage equation.

$$V'_{qr} = R'_r \cdot i'_{qr} + p \lambda'_{qr} + (\omega_0 - \omega_r) \lambda'_{dr} \quad \text{--- (v)}$$

$$V'_{dr} = R'_r \cdot i'_{dr} + p \lambda'_{dr} - (\omega_0 - \omega_r) \lambda'_{qr} \quad \text{--- (vi)}$$

Replacing ~~α, β~~ to q, d components in eqⁿ. (v) & (vi) with β, α components respectively we get ($\omega_0 = 0 \text{ rad/s}$)

$$V'_{\beta r} = R'_r \cdot i'_{\beta r} + p \lambda'_{\beta r} + (\cancel{\omega_0} - \omega_r) \lambda'_{\alpha r}$$

$$V'_{\alpha r} = R'_r \cdot i'_{\alpha r} + p \lambda'_{\alpha r} - (\cancel{\omega_0} - \omega_r) \lambda'_{\beta r}$$

Problem 2

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(10)

(4) contd.

∴

$$V'_{\beta r} = R'_r \cdot i'_{\beta r} + p \lambda'_{\beta r} - \omega_r \lambda'_{\alpha r}$$

$$\Delta V'_{\alpha r} = R'_r \cdot i'_{\alpha r} + p \lambda'_{\alpha r} + \omega_r \lambda'_{\beta r}$$

Problem 3

(1) given:-

Rated shaft power = 0.6 kW

Rated stator voltage (V_L) = 400 V

Rated stator current (I) = 2.1 A

$$\cos \theta = 0.6$$

$$\text{motor's efficiency} = \frac{\text{Output Power}}{\text{Input Power}}$$

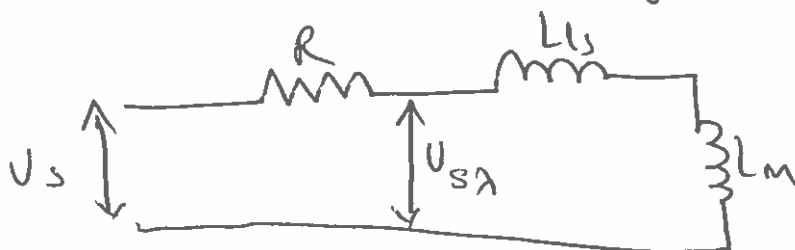
$$\begin{aligned}\text{Output Power} &= 0.6 \text{ kW} \\ &= 600 \text{ W}\end{aligned}$$

$$\begin{aligned}\text{Input Power} &= \sqrt{3} V_L I_L \cos \theta \\ &= \sqrt{3} \times 400 \times 2.1 \times 0.6 \\ &= 872.953 \text{ W}\end{aligned}$$

$$\text{efficiency } (\eta\%) = \frac{\text{Output Power}}{\text{Input Power}} \times 100\%$$

$$= \frac{600}{872.953} \times 100 = 68.73\%$$

(2) The equivalent circuit of an induction motor is given below for stator side only.



(2) Since nothing in question is mentioned about stator resistance voltage compensation, so we have to find the voltage after the stator resistance, i.e. defined in the diagram as U_{s2} .

From the diagram from the previous page, we can clearly state that

$$\cancel{U_{s2pk} = V_{phpk} - I_{phpk} \times \text{stator resistance}}$$

$$\Rightarrow \cancel{V_{s2pk} = \frac{400}{\sqrt{3}} \times \sqrt{2} = 2.1 \times \sqrt{2} \times 12}$$

$$= \cancel{326.599} \quad \cancel{35.64}$$

$$= \cancel{290.961 \text{ V}}_{pk}$$

$$\begin{aligned}
 V_{sA} &= V_s - I_s R_s \\
 &= \frac{400}{\sqrt{3}} - 2.1 \times 12 \\
 &= 230.94 - 25.2 \\
 &= 205.74 \text{ V}
 \end{aligned}$$

As given in question $f_s = 60 \text{ Hz}$.

\therefore the $\frac{V}{f}$ ratio would be $\frac{205.74}{60}$

$$\therefore \frac{V}{f} = 3.429 \text{ V/Hz}$$

3) As the resistance is neglected,

$$\text{So, } V_s = V_{sA}$$

$$V_{s \text{ peak}} = \frac{400}{\sqrt{3}} \times \sqrt{2} = 326.6 \text{ V}$$

Problem 3

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(12)

(3) Contd.

$$\begin{aligned}\omega_s &= 2\pi f_s = 2\pi \times 60 \\ &= 376.99 \text{ rad s}^{-1} \\ &\approx 377 \text{ rad s}^{-1}\end{aligned}$$

Let the stator flux Linkage be ' λ_s '

$$\lambda_s = \frac{V_{s \text{ ph peak}}}{\omega_s} = \frac{326.6}{377} = 0.866 \text{ V/rad s}$$

(4) The motor is required to run at 0.25 Hz.

So, now we have to find f_s .

$$\therefore s_{\text{rated}} = \frac{f_{s \text{ rated}} - f_{\text{rotor speed rated}}}{f_{s \text{ rated}}}$$

Rotor speed in electrical radians/sec

$$\omega_r = 850 \text{ rpm} \times \frac{2\pi}{60} \times P_p \quad \left[P_p = \text{pole pairs} \right]$$

$$\Rightarrow \omega_r = 850 \times \frac{2\pi}{60} \times 4 \quad \left[P_p = 4 \right. \\ \left. \text{given in question} \right]$$

$$\Rightarrow \omega_r = 356.047 \text{ rad s}^{-1}$$

~~S_{rated}~~ $f_r = \frac{\omega_r}{2\pi}$

$$S_{\text{rated}} = \frac{\omega_{s, \text{rated}} - \omega_{r, \text{rated}}}{\omega_{s, \text{rated}}}$$

$$= \frac{377 - 356.047}{377}$$

$$377$$

$$= 0.056$$

4) Since the slip does not change, (13)

Thus,

$$S_{rated} = \frac{f_{s\text{new}} - f_{r\text{new}}}{f_{s\text{new}}}$$

$$\Rightarrow 0.056 = \frac{f_{s\text{new}} - 0.25}{f_{s\text{new}}} \quad \left[\begin{array}{l} f_{r\text{new}} = 0.25 \text{ Hz} \\ \text{given in} \\ \text{question.} \end{array} \right]$$

$$\Rightarrow \cancel{f_{s\text{new}}}$$

$$\Rightarrow f_{s\text{new}} (1 - 0.056) = 0.25$$

$$\therefore f_{s\text{new}} = \frac{0.25}{0.944} = 0.265 \text{ Hz}$$

As per question, the stator flux linkage (λ_s) should remain the same

$$\therefore \lambda_s = 0.866 \text{ V/rad s}^{-1}$$

$$\begin{aligned}\omega_{s\text{new}} &= 2\pi f_{s\text{new}} \\ &= 2\pi \times 0.265 \text{ Hz} \\ &= 1.664 \text{ rad s}^{-1}\end{aligned}$$

We know that

$$\frac{V_{s\lambda}}{\omega_{s\text{new}}} = \lambda_s$$

$$\begin{aligned}\Rightarrow V_{s\lambda} &= \lambda_s \times \omega_{s\text{new}} \\ &= 0.866 \times 1.664 \\ &= 1.44 \text{ V}\end{aligned}$$

$$V_s = V_{s\lambda} + I_s R_s$$

Problem 3

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(14)

$$(4) \quad U_s = 1.44 \angle 0 + (1 \angle -45) \times 12$$

$$U_s = 13.058 \angle -46.53 \text{ V}$$

~~The~~ Magnitude of phase voltage after
phase resistive ^{voltage} drop compensation is

$$\underline{|U_s| = 13.058 \text{ volts.}}$$

$$(5) \quad \text{We know, } S_{\text{rated}} = 0.056$$

$$T_{\text{now}} = \frac{1}{4} T_{\text{rated.}}$$

We know that,

$$\Rightarrow \frac{T_{\text{now}}}{T_{\text{rated}}} = \frac{f_{se}}{S_{\text{rated}} \times f_{\text{rated.}}}$$

$$\Rightarrow \frac{1}{4} \frac{T_{\text{rated}}}{T_{\text{rated}}} = \frac{f_{se}}{0.056 \times 60}$$

Problem 3

(5) contd.

$$\Rightarrow f_{se} = \frac{1}{4} \times 0.056 \times 60$$
$$= 0.84 \text{ Hz}$$

$$f_{se} = f_{s\text{now}} - f_{r\text{now}}$$

~~we know that the shaft speed frequency
now ($f_{r\text{now}}$) = 15 Hz~~

~~So, the stator frequency ($f_{s\text{now}}$) has to be
found.~~

$$~~0.84 = f_{s\text{now}} - 15~~$$

$$~~\therefore f_{s\text{now}} = 15.84 \text{ Hz}~~$$

$$~~\text{New slip } (s_{\text{new}}) = \frac{f_{s\text{now}} - f_{r\text{now}}}{f_{s\text{now}}}~~$$

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(15)

$$\Rightarrow S_{\text{new}} = \frac{15.84 - 15}{15.84} = \frac{0.84}{15.84} = 0.053$$

Problem 3

5) contd.

So, now we know $f_{s \text{ rated}} = 60 \text{ Hz}$

$$\& f_{se} = 0.84 \text{ Hz}$$

∴ The compensated stator frequency

becomes, $f_{s, \text{new}} = f_{s \text{ rated}} + f_{se}$

$$= 60 + 0.84$$

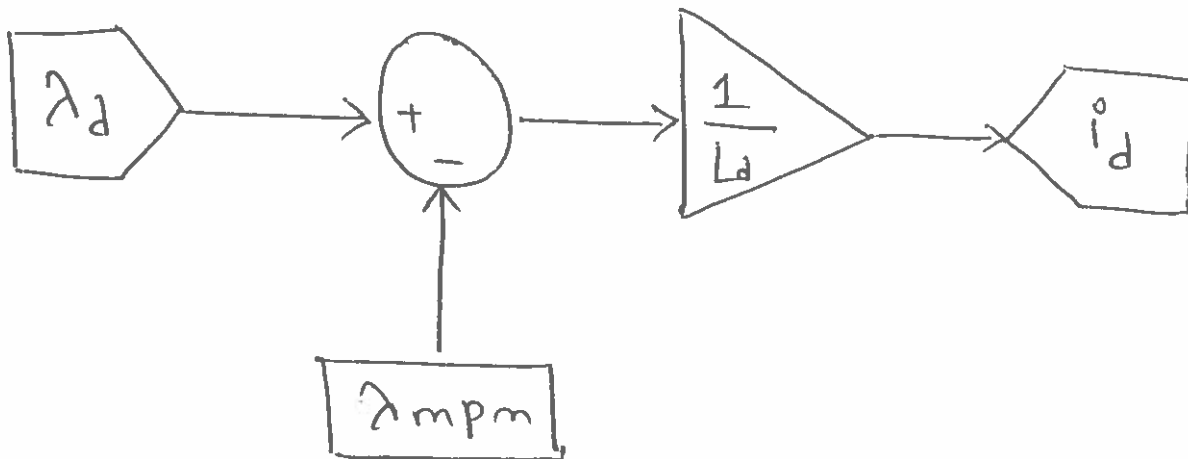
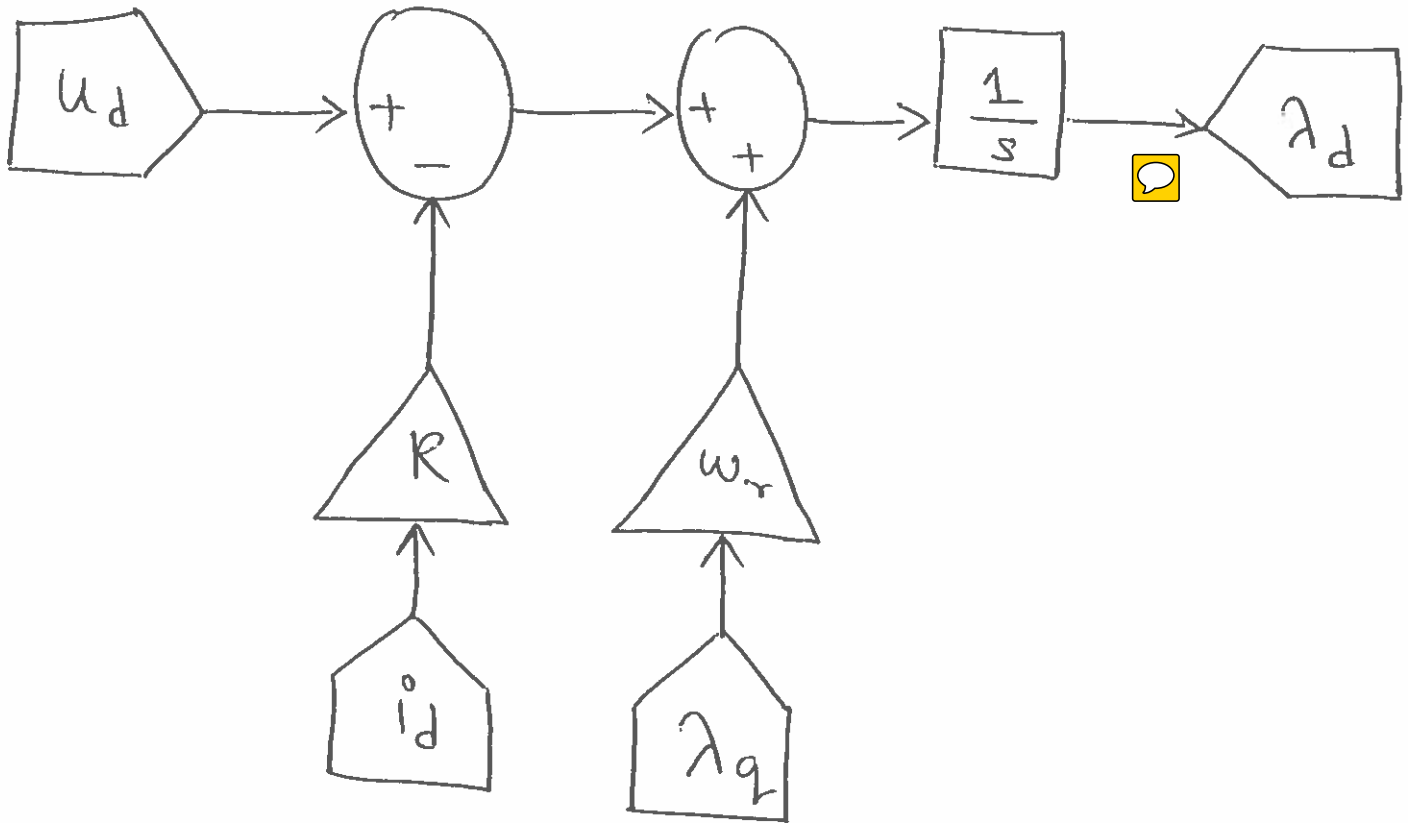
$$= 60.84 \text{ Hz}$$

∴ $f_{se} = 0.84 \text{ Hz}$ is the slip frequency
that needs ^{to be} added to the frequency

command.

Problem 4.

(1) Block diagram in Simulink, to get d-axis current.



Problem 4

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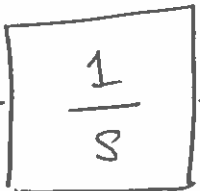
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2.


2. At $t=0$, we need to d axis

flux linkage $\lambda_d = \lambda_{mpm}$. So for

this, in the simulink diagram, in the

integrator block that is 

We put the initial condition as

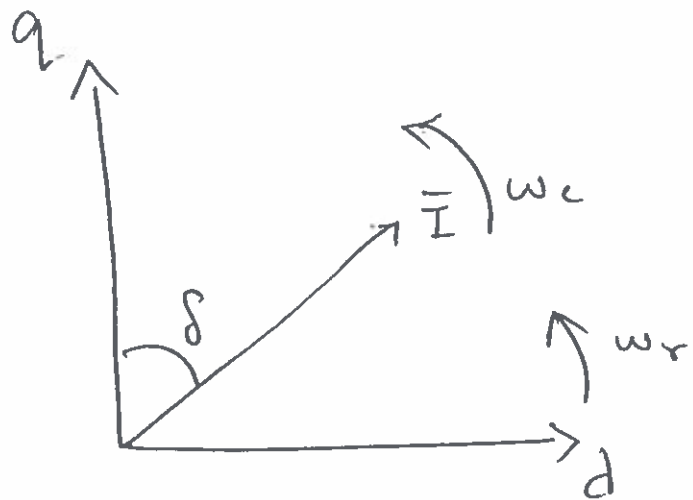
' λ_{mpm} ' in the integrator block 

Thus at $t=0$, we will have

$$\lambda_d = \lambda_{mpm}$$

Problem 4

(3) In I/F control of the PM machine, the current vector should be lagging the q -axis.



Let δ be the angle defined between current vector \bar{I} & q axis.

The electromechanical torque equation:

$$T_e - T_{load} = J \frac{d\omega_r}{dt}$$

We assume that the current vector

\bar{I} rotates with an angular velocity of ' ω_c rad s^{-1} ' & the qd axis rotates with an angular ~~rot~~ velocity of ' ω_r rad s^{-1} '

3) Explanation.

Case I : Required for stable operation.

\bar{I} (current vector) lags the q -axis.

In this case, considering T_{load} increase then the ^{angular velocity} ~~speed~~ of rotation of the current vector (\bar{I}) increases with respect to the angular velocity of rotation of q -axis reference frame. As a result of which ' δ ' decreases, which brings the current vector closer to the q -axis.

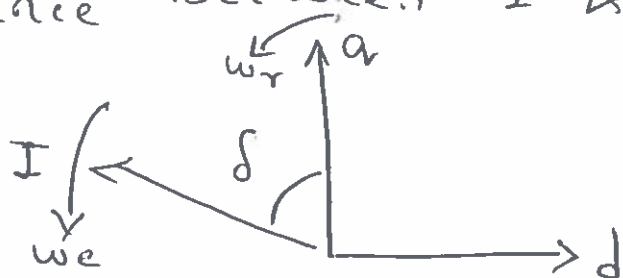
Thereby increasing the q -axis current (i_q). We know that $\underline{T_e} \propto i_q$. So

when i_q increases, electromechanical torque ' T_e ' also increases. Thus the difference in

torque between T_e & T_{load} [$T_e - T_{load}$] is maintained and thus the angular ~~vel~~ velocity of the q d reference frame \uparrow is maintained and stability of the system is observed.

Case: 2 [Unstable operation]
How Not recommended.]

However, if the current vector ' \bar{I} ' leads the q axis. Then, when the load torque ' T_{load} ' increases, the q d reference frame angular velocity decreases. This leads to increase in ' δ ', i.e. the phase difference between \bar{I} & q axis,



3) Contd.

Thus in this condition, we see that as the ' δ ' increases the current vector \vec{I} moves away from the q -axis, thus further reducing the q axis current.

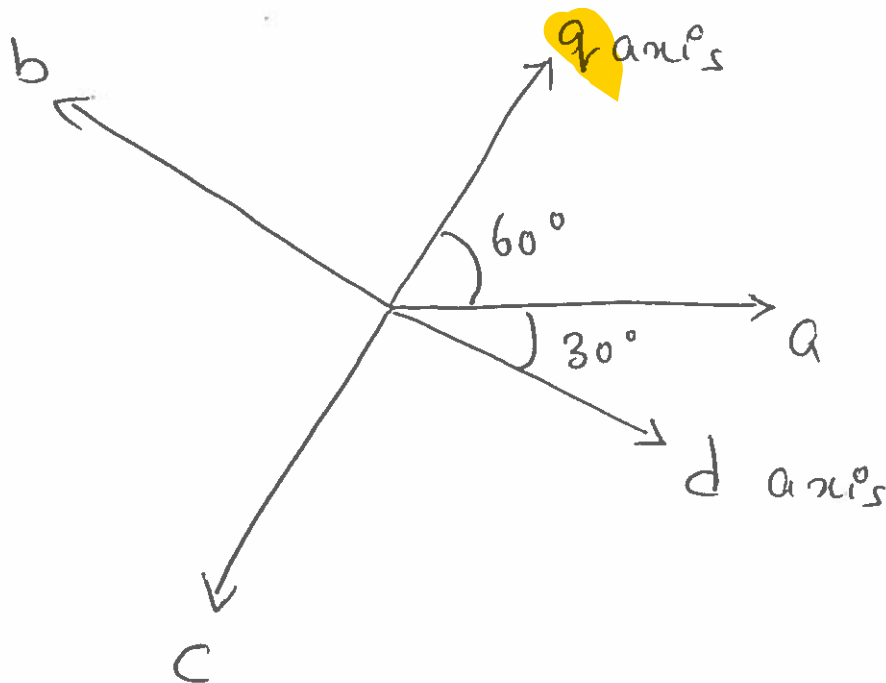
& we know that " $T_e \propto i_q$ ".
So with decrease in i_q , T_e also continuously keeps decreasing. Then the ~~sys~~ difference $T_e - T_{load}$ keeps increasing with time & system becomes unstable.

Thus for stable operation the
current vector ' \vec{I} ' should lag the q -axis.

Problem 4.

4) As given in question, the system is a dq reference frame.

The rotor position (i.e. d axis) is -30 electrical degrees, when phase 'a' voltage crosses 'zero' from negative to positive.



As phase 'a' voltage will be zero, when the voltage space vector will be at $\pm 90^\circ$ to phase 'a' voltage.

~~Va~~

(19)

4) Contd.

$$V_a = \operatorname{Re} \left(\frac{\bar{V}_f e^{j90^\circ}}{e^{j0^\circ}} \right)$$

Let \bar{V}_f be the voltage space vector
 ΔV_a is the phase a voltage.

$$\Rightarrow V_a = \bar{V}_f \cos(90^\circ) = 0$$

$$\text{However, } V_a = \bar{V}_f \cos(-90^\circ) = 0$$

So, ~~\bar{V}_f for V_a~~

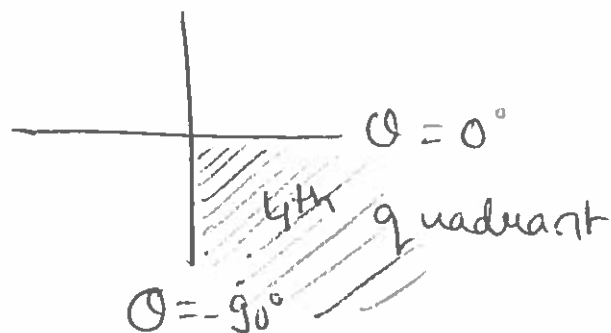
So, for V_a to be '0' volts, \bar{V}_f should
 either be at $+90^\circ$ or at -90° ~~by the~~
~~with respect to a~~
 coordinate [i.e. 90° leading or 90° lagging]
 at the instant when $V_a = 0$.

But from the ^{question} ~~graph~~ we see that,
at ~~zero~~ ~~at~~ the voltage vector \bar{V}_f
has to be represented when phase a
voltage crosses the zero, from
negative to positive, ~~when~~

So, if we consider the voltage space
vector \bar{V}_f to be leading 'a' axis by 90° ,
then after the zero crossing, the phase
'a' voltage should ~~be~~ become more
negative. So, \bar{V}_f cannot lead a-axis by
 90° at the instant when V_a crosses
the zero voltage from negative to positive.

4) contd.

However, if we consider voltage space vector (\bar{V}_f) to be lagging a-phase voltage by 90° at the instant when V_a is zero. Then we observe that with time, \bar{V}_f move towards positive value of V_a phase as now, \bar{V}_f will enter the 4th quadrant, defined between $\theta = -90^\circ$ to $\theta = 0^\circ$.

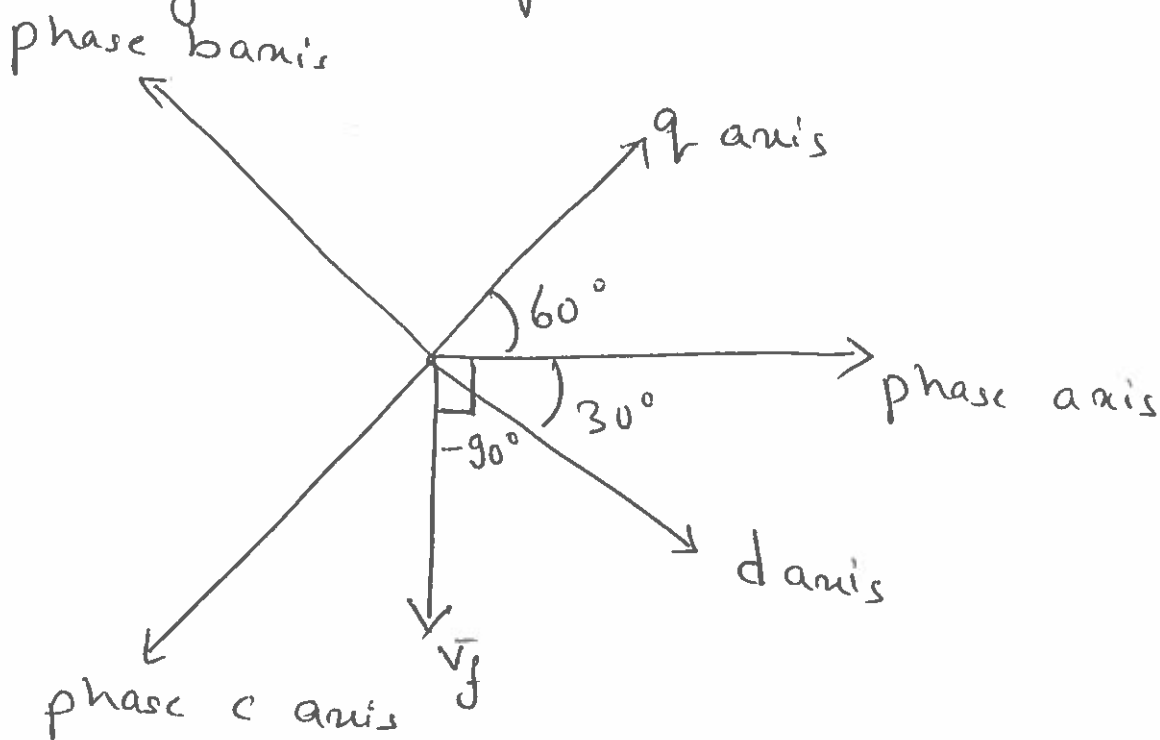


In 4th quadrant $\cos \theta = \text{positive value}$.

$$\therefore V_a = \bar{V}_f \left(\frac{e^{j(0)}}{e^{j0}} \right) \text{ for } -90^\circ < \theta < 0^\circ$$

V_a will have positive value.

Thus \bar{V}_f should be lagging a-axis by 90° at the instant when, the a axis phase voltage crosses **zero**, from negative to positive.



where \bar{V}_f = Voltage space vector.

From the graph we can see that,

$$360 \text{ degrees} = 0.02 \times \frac{\text{degrees}}{\text{division}}$$

$$u) \Rightarrow \frac{\text{degrees}}{\text{division}} = \frac{360^\circ}{0.02} = \frac{18000 \text{ degrees}}{\text{division}}$$

So for $0.00167 \times 18000 = 30 \text{ degrees}$.

As per graph a axis phase current lags a axis phase voltage by 30° .

So, the ~~pha~~ ^{current} ~~voltage~~ space vector (\hat{I}_f) lags the voltage space vector by 30° as well.

