Lecture 9 - contents

- Other topics regarding machine modeling
  - **Chapter 7 – operational impedances and time constants**
  - **Chapter 8 – linearized machine equations**
- Modeling of other types of motors
  - **What to investigate of a ‘new’ type of machine**
  - **Modeling of a SRM**
  - **Modeling of a surface mounted PMTFM**
- Modeling of a 1-phase Induction Machine

Operational impedances and time constants concepts

New names you may find, like (P288):

Transient reactances

$$X'_q = X_{ls} + \frac{X_{mq} X'_{lkq1}}{X'_{lkq1} + X_{mq}}$$

$$X'_d = X_{ls} + \frac{X_{md} X'_{lfd}}{X'_{lfd} + X_{md}}$$

Subtransient reactances

$$X''_q = X_{ls} + \frac{X_{mq} X'_{lkq1} X'_{lkq2}}{X_{mq} X'_{lkq1} + X_{mq} X'_{lkq2} + X'_{lkq1} X'_{lkq2}}$$

$$X''_d = X_{ls} + \frac{X_{md} X'_{lfd} X'_{lkd}}{X_{md} X'_{lfd} + X_{md} X'_{lkd} + X'_{lfd} X'_{lkd}}$$

'normal' reactances

$$X_q = X_{ls} + X_{mq}$$

$$X_d = X_{ls} + X_{md}$$

These impedances may be derived from (P285)

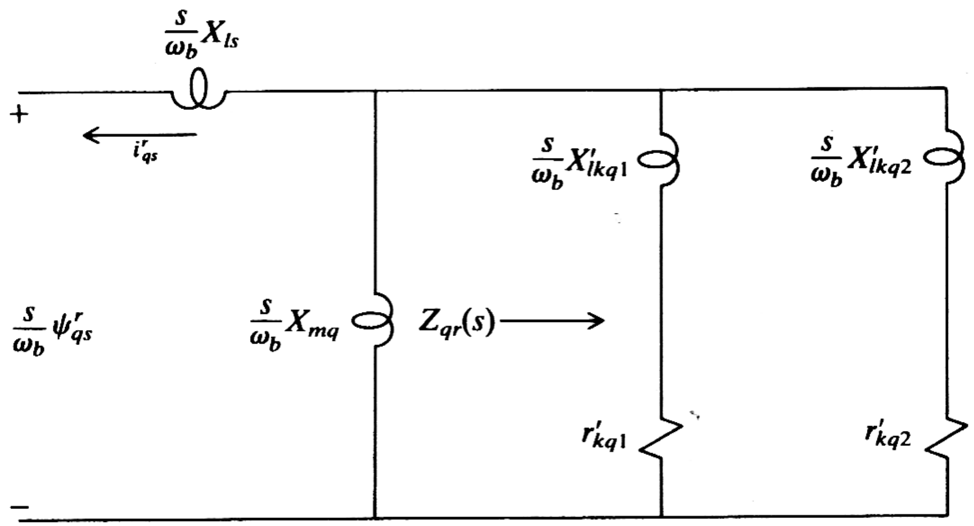


Figure 7.3-1 Equivalent circuit with two damper windings in the quadrature axis.

- This stands for two damping windings on the rotor q-axis

Why we need to do this? – because in the real life, we may have

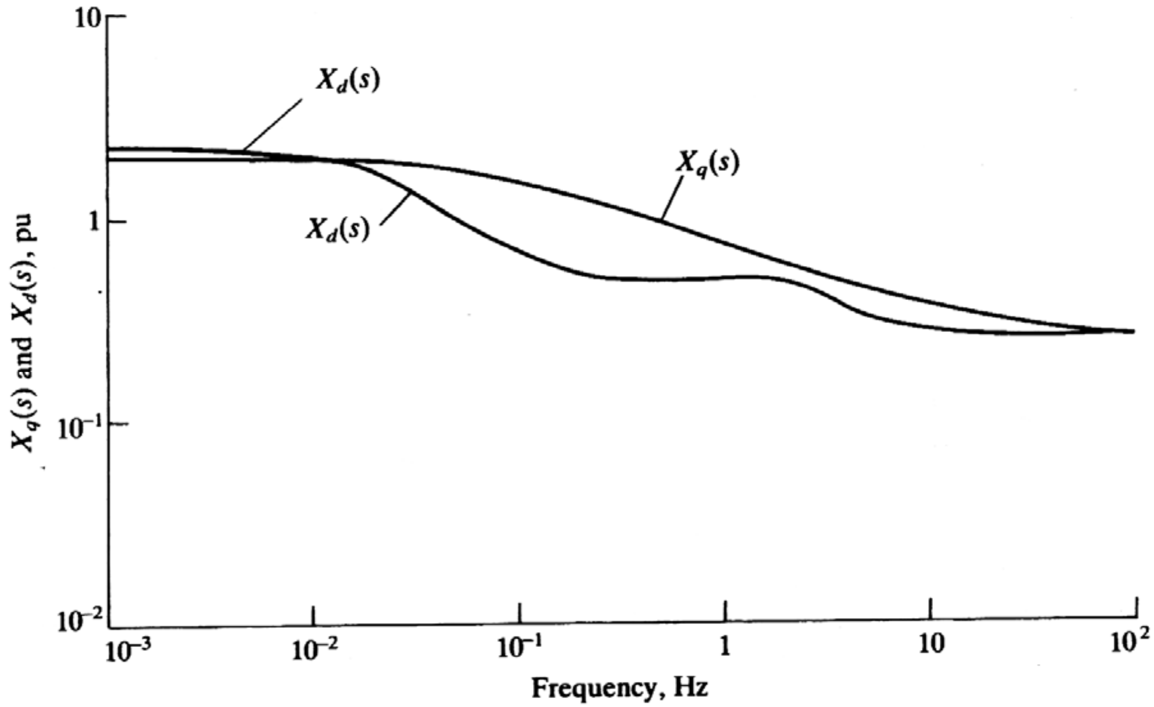


Figure 7.8-1 Plot of $X_q(s)$ and $X_d(s)$ versus frequency for a solid iron synchronous machine.

Typical for a machine with solid iron rotor core!

By using curve fitting, we obtain

PARAMETERS FROM FREQUENCY-RESPONSE CHARACTERISTICS **303**

$$X_q(s) = 2.0\,\frac{(1+0.64s)(1+0.016s)}{(1+1.59s)(1+0.05s)}$$

$$X_q(s) = X_q\,\frac{(1+\tau_q' s)(1+\tau_q'' s)}{(1+\tau_{qo}' s)(1+\tau_{qo}'' s)}$$

Time constant (P290)

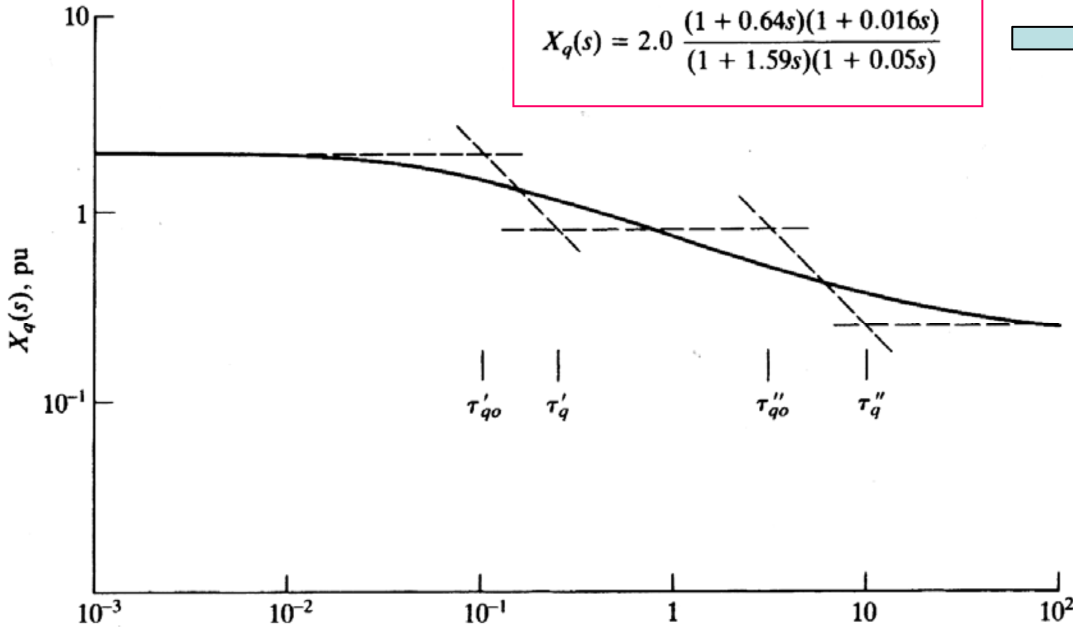
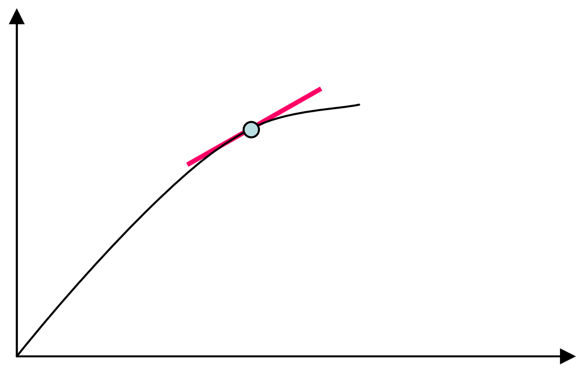


Figure 7.8-2 Two-rotor winding approximation of $X_q(s)$.

Linearized machine equations

Principle – like the 'small signal model' used in analog circuit analysis

Around the working point, any profile may be modelled by a straight line!
For example:



Linearized machine equations

Mathematical approach – Taylor expansion

$$g(f_i) = g(f_{io}) + g'(f_{io})\Delta f_i + \frac{g''(f_{io})}{2!}\Delta f_i^2 + \cdots$$

$$f_i = f_{io} + \Delta f_i$$

The constant offset will disappear on the two sides of the equation

Higher order differentiation may be neglected, like items associated
With co-efficient $\Delta f_i \cdot \Delta f$

Other types of motors

What to investigate of a 'new' type of machine

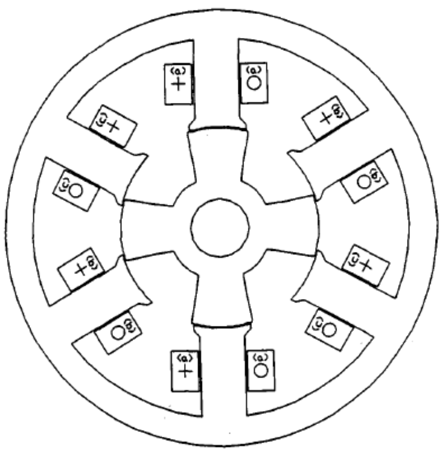
- Rotating air gap flux density waveform – back EMF if there is any
- How the phases are coupled (SS, RS, RR, RS)

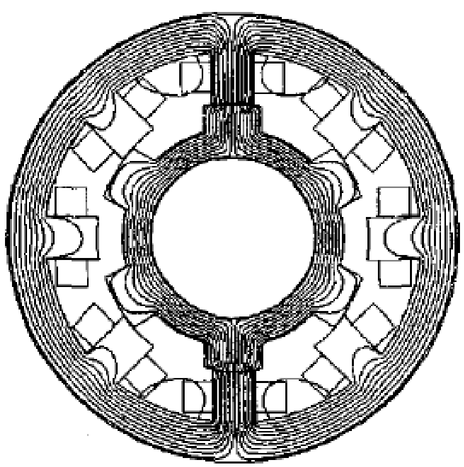
FEM

- Derive the stator and rotor flux linkage equations
- Do reference-frame transformation if necessary (purpose and relationships)
- Set up the voltage equations (easy!)
- Study the change of the stored field energy to derive the torque equation
- Validation – using FEM or measurement

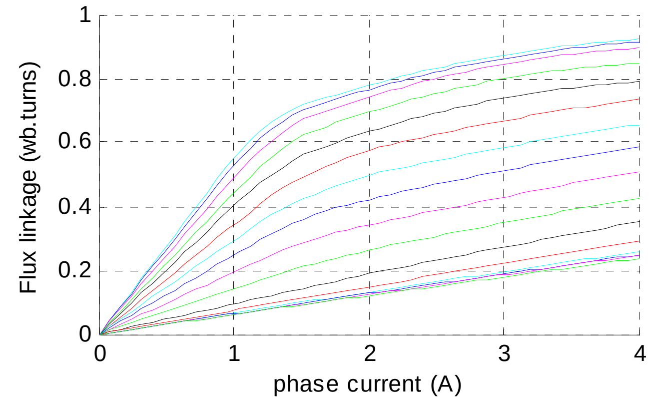
Other types of motors

Modeling of a SRM





- No phase coupling!
- No rotor windings!
- Complicated non-linear flux linkage profiles

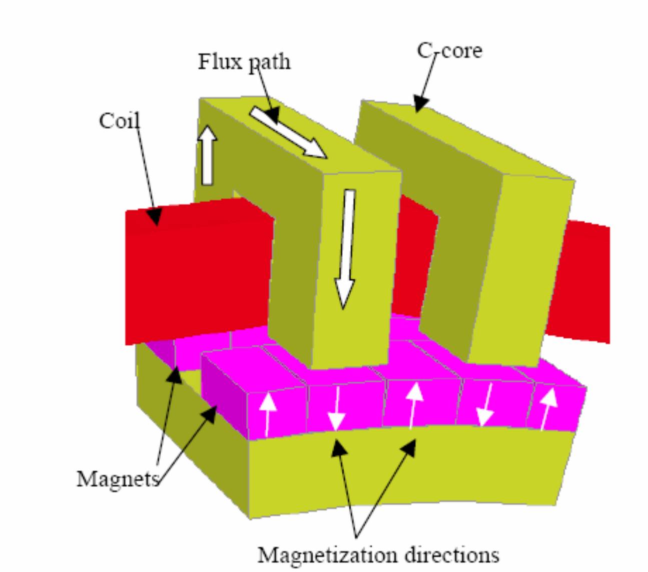


$$u = Ri + \frac{d\lambda}{dt}$$

- Torque calculation using the energy method

Other types of motors

Modeling of a surface mounted PMTFM



- Sinusoidal back EMF – using FEM
- How does this motor work?
- What kind of current should we supply?
- Voltage equations? Torque equation?

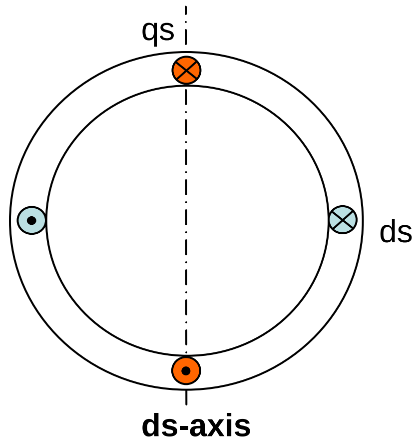
Exactly like the 1-ph surface mounted PM motor that we have discussed!

Modeling of a 1-phase IM

Stator flux linkage equations

$$\begin{bmatrix} \lambda_{das} \\ \lambda_{qas} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_m & 0 \\ 0 & L_{ls} + L_m \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_{dsr} \\ \lambda_{qsr} \end{bmatrix} = \begin{bmatrix} k_n L_m \cos\theta_r & k_n L_m \cos\left(\theta_r + \frac{\pi}{2}\right) \\ k_n L_m \cos\left(\theta_r - \frac{\pi}{2}\right) & k_n L_m \cos\theta_r \end{bmatrix} \cdot \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix}$$

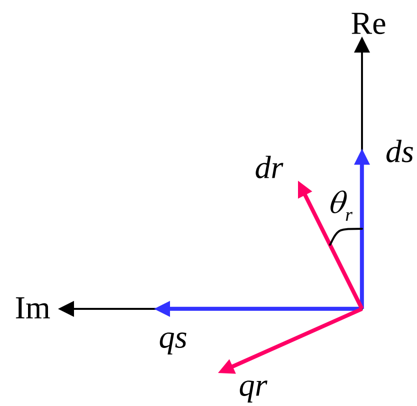


$$\boxed{L_m \operatorname{Re}\left(\frac{e^{j(\theta_r)}}{e^{j\left(\frac{\pi}{2}\right)}}\right) \operatorname{Re}\left(\frac{e^{j(\theta_r)}}{e^{j\,j(\theta_r)}}\right) = L_m \cos\left(\theta_r - \frac{\pi}{2}\right)}$$

$$\boxed{\frac{N_s}{N_r} M_{sr} = L_m}$$

$$\boxed{k_n = \frac{N_r}{N_s}}$$

$$\boxed{\begin{aligned} &N_{sd} = N_{sq} = N_s \qquad N_{rd} = N_{rq} = N_r \\ &L_{md} = L_{mq} = L_m \end{aligned}}$$



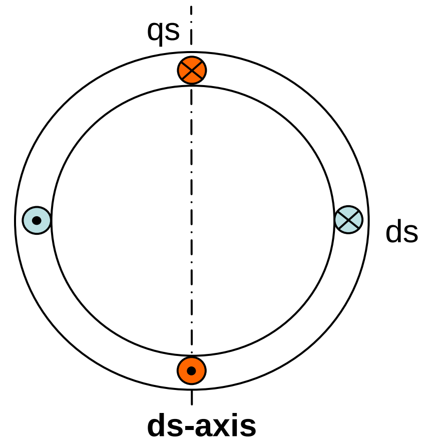
Modeling of a 1-phase IM

Rotor flux linkage equations

$$\begin{bmatrix} \lambda_{dar} \\ \lambda_{qar} \end{bmatrix} = \begin{bmatrix} L_{lr} + k_n^{\,2} L_m & 0 \\ 0 & L_{lr} + k_n^{\,2} L_m \end{bmatrix} \cdot \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix}$$

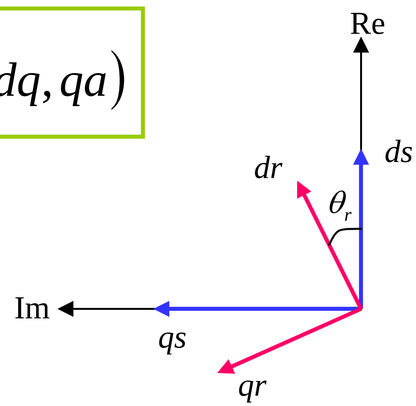
$$\Downarrow$$

$$\begin{bmatrix} \lambda'_{dar} \\ \lambda'_{qar} \end{bmatrix} = \begin{bmatrix} L'_{lr} + L_m & 0 \\ 0 & L'_{lr} + L_m \end{bmatrix} \cdot \begin{bmatrix} i'_{dr} \\ i'_{qr} \end{bmatrix}$$



$$\boxed{\lambda'_{jr} = \frac{N_s}{N_r} \lambda_{jr} = \frac{1}{k_n} \lambda_{jr} \qquad i'_{jr} = k_n i_{jr} \qquad L'_{lr} = \frac{1}{k^2_{\,n}} L_{lr} \qquad (j = dq, qa)}$$

$$\begin{bmatrix} \lambda'_{drs} \\ \lambda'_{qrs} \end{bmatrix} = \begin{bmatrix} L_m \cos\theta_r & L_m \cos\left(\theta_r + \dfrac{\pi}{2}\right) \\ L_m \cos\left(\theta_r - \dfrac{\pi}{2}\right) & L_m \cos\theta_r \end{bmatrix}^T \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$$



Modeling of a 1-phase IM

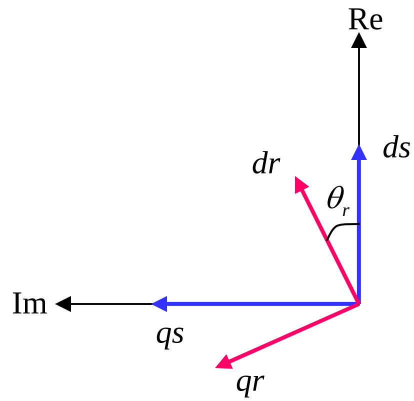
Flux linkage transformation

$$\boxed{i_{ds} + j i_{qs} = \left(i_{dr} + j i_{qr}\right) e^{j\theta_r} \qquad \left(i_{ds} + j i_{qs}\right) e^{-j\theta_r} = i_{dr} + j i_{qr}}$$

$$\begin{bmatrix} i^{s}{}_{dr} \\ i^{s}{}_{qr} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{\pi}{2}\right) \\ \cos\left(\theta_r - \frac{\pi}{2}\right) & \cos\theta_r \end{bmatrix} \cdot \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} \qquad \begin{bmatrix} i^{r}{}_{ds} \\ i^{r}{}_{qs} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{\pi}{2}\right) \\ \cos\left(\theta_r + \frac{\pi}{2}\right) & \cos\theta_r \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_m & 0 \\ 0 & L_{ls} + L_m \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + L_m \cdot \begin{bmatrix} i'^{s}{}_{dr} \\ i'^{s}{}_{qr} \end{bmatrix}$$

$$\begin{bmatrix} \lambda'^{s}{}_{dr} \\ \lambda'^{s}{}_{qr} \end{bmatrix} = \begin{bmatrix} L'_{lr} + L_m & 0 \\ 0 & L'_{lr} + L_m \end{bmatrix} \cdot \begin{bmatrix} i'^{s}{}_{dr} \\ i'^{s}{}_{qr} \end{bmatrix} + L_m \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$$



Modeling of a 1-phase IM

The voltage equations

- Do we need to do anything about the stator voltage equations?

$$\begin{bmatrix} u_{ds} \\ u_{qs} \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix}$$

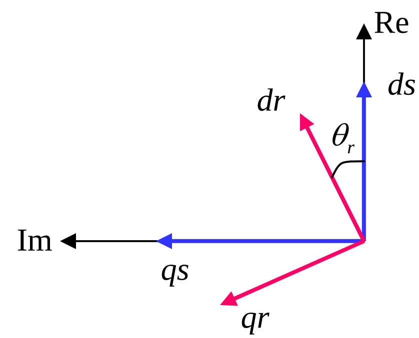
- The rotor voltage equations, according to the slide P12, lecture 8

$$\begin{bmatrix} u'^{s}_{dr} \\ u'^{s}_{qr} \end{bmatrix} = \begin{bmatrix} R'_r & 0 \\ 0 & R'_r \end{bmatrix} \cdot \begin{bmatrix} i'^{s}_{dr} \\ i'^{s}_{qr} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda'^{s}_{dr} \\ \lambda'^{s}_{qr} \end{bmatrix} - \left(p\underline{K}_{sr} \cdot \underline{K}^{-1}_{sr} \right) \begin{bmatrix} \lambda'^{s}_{dr} \\ \lambda'^{s}_{qr} \end{bmatrix}$$

$$\underline{K}_{sr} = \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{\pi}{2}\right) \\ \cos\left(\theta_r - \frac{\pi}{2}\right) & \cos\theta_r \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} u'^{s}_{dr} \\ u'^{s}_{qr} \end{bmatrix} = \begin{bmatrix} R'_r & 0 \\ 0 & R'_r \end{bmatrix} \cdot \begin{bmatrix} i'^{s}_{dr} \\ i'^{s}_{qr} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda'^{s}_{dr} \\ \lambda'^{s}_{qr} \end{bmatrix} + \omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \lambda'^{s}_{dr} \\ \lambda'^{s}_{qr} \end{bmatrix}$$



Modeling of a 1-phase IM – *uneven number of turns for the main and aux. windings*

Stator flux linkage equations

$$\begin{bmatrix}\lambda_{das}\\ \lambda_{qas}\end{bmatrix}=\begin{bmatrix}L_{lsd}+L_{md} & 0\\ 0 & L_{lsq}+L_{mq}\end{bmatrix}\cdot\begin{bmatrix}i_{ds}\\ i_{qs}\end{bmatrix}$$

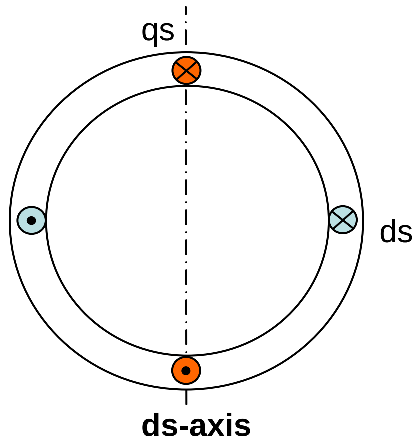
$$\begin{bmatrix}\lambda_{dsr}\\ \lambda_{qsr}\end{bmatrix}=\begin{bmatrix}k_{nd}L_{md}\cos\theta_r & \boxed{k_{nd}L_{md}}\cos\left(\theta_r+\frac{\pi}{2}\right)\\ \boxed{k_{nq}L_{mq}}\cos\left(\theta_r-\frac{\pi}{2}\right) & k_{nq}L_{mq}\cos\theta_r\end{bmatrix}\cdot\begin{bmatrix}i_{dr}\\ i_{qr}\end{bmatrix}$$

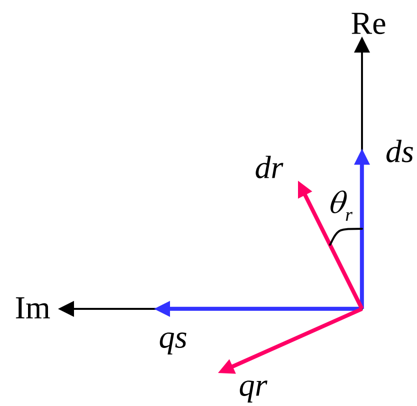
$$k_{nd}=\frac{N_r}{N_{sd}} \qquad k_{nq}=\frac{N_r}{N_{sq}}$$

$$\frac{N_{sq}}{N_r}M_{srq}=L_{mq} \qquad \frac{N_{sd}}{N_r}M_{srd}=L_{md}$$

$$N_{sd}\neq N_{sq} \qquad \boxed{N_{rd}=N_{rq}=N_r}$$

$$L_{md}\neq L_{mq}$$

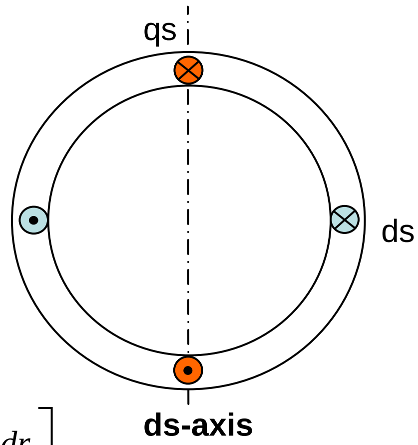




Modeling of a 1-phase IM – *uneven number of turns for the main and aux. windings*

Transformation for the rotor variables

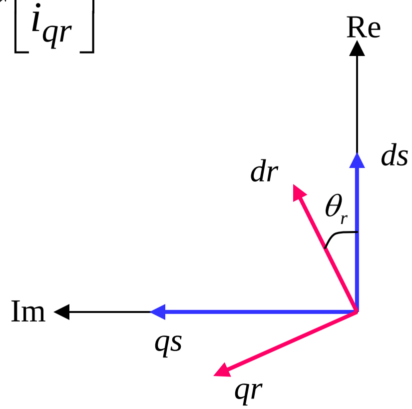
$$K_{sr} = \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{\pi}{2}\right) \\ \cos\left(\theta_r - \frac{\pi}{2}\right) & \cos\theta_r \end{bmatrix} \qquad K_{rs} = K_{sr}^{-1} = \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{\pi}{2}\right) \\ \cos\left(\theta_r + \frac{\pi}{2}\right) & \cos\theta_r \end{bmatrix}$$



$$\begin{bmatrix} \lambda_{dsr} \\ \lambda_{qsr} \end{bmatrix} = \begin{bmatrix} k_{nd} L_{md} \cos\theta_r & k_{nd} L_{md} \cos\left(\theta_r + \frac{\pi}{2}\right) \\ k_{nq} L_{mq} \cos\left(\theta_r - \frac{\pi}{2}\right) & k_{nq} L_{mq} \cos\theta_r \end{bmatrix} \cdot K_{sr}^{-1} K_{sr} \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix}$$

Turns ratio transformation

$$\begin{bmatrix} \lambda_{dsr} \\ \lambda_{qsr} \end{bmatrix} = \begin{bmatrix} k_{nd} L_{md} & 0 \\ 0 & k_{nq} L_{mq} \end{bmatrix} \cdot \begin{bmatrix} i^s{}_{dr} \\ i^s{}_{qr} \end{bmatrix} = \begin{bmatrix} L_{md} & 0 \\ 0 & L_{mq} \end{bmatrix} \cdot \begin{bmatrix} i'^s{}_{dr} \\ i'^s{}_{qr} \end{bmatrix}$$

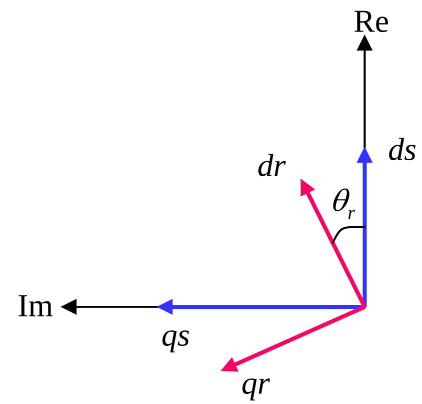


$$\boxed{i'^s{}_{dr} = k_{nd} \cdot i^s{}_{dr} \qquad i'^s{}_{qr} = k_{nq} \cdot i^s{}_{qr}}$$

Modeling of a 1-phase IM – *uneven number of turns for the main and aux. windings*

Rotor flux linkage equations

$$\begin{bmatrix}\lambda_{dar}\\ \lambda_{qar}\end{bmatrix}=\begin{bmatrix}L_{lr}+k_{nd}^{2}L_{md} & 0\\ 0 & L_{lr}+k_{nq}^{2}L_{mq}\end{bmatrix}\cdot\begin{bmatrix}i_{dr}\\ i_{qr}\end{bmatrix}$$



$$\begin{bmatrix}\lambda_{drs}\\ \lambda_{qrs}\end{bmatrix}=\begin{bmatrix}k_{nd}L_{md}\cos\theta_r & \boxed{k_{nq}L_{mq}}\cos\left(\theta_r-\frac{\pi}{2}\right)\\ \boxed{k_{nd}L_{md}}\cos\left(\theta_r+\frac{\pi}{2}\right) & k_{nq}L_{mq}\cos\theta_r\end{bmatrix}\cdot\begin{bmatrix}i_{ds}\\ i_{qs}\end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix}\lambda^{s}{}_{drs}\\ \lambda^{s}{}_{qrs}\end{bmatrix}=K_{sr}\begin{bmatrix}k_{nd}L_{md}\cos\theta_r & k_{nq}L_{mq}\cos\left(\theta_r-\frac{\pi}{2}\right)\\ k_{nd}L_{md}\cos\left(\theta_r+\frac{\pi}{2}\right) & k_{nq}L_{mq}\cos\theta_r\end{bmatrix}\cdot\begin{bmatrix}i_{ds}\\ i_{qs}\end{bmatrix}=\begin{bmatrix}k_{nd}L_{md} & 0\\ 0 & k_{nq}L_{mq}\end{bmatrix}\cdot\begin{bmatrix}i_{ds}\\ i_{qs}\end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix}\lambda'^{s}{}_{drs}\\ \lambda'^{s}{}_{qrs}\end{bmatrix}=\begin{bmatrix}L_{md} & 0\\ 0 & L_{mq}\end{bmatrix}\cdot\begin{bmatrix}i_{ds}\\ i_{qs}\end{bmatrix}$$

$$\boxed{\lambda'^{s}{}_{dr}=\frac{1}{k_{nd}}\cdot\lambda^{s}{}_{dr}\qquad \lambda'^{s}{}_{qr}=\frac{1}{k_{nq}}\cdot\lambda^{s}{}_{qr}}$$

Modeling of a 1-phase IM – *uneven number of turns for the main and aux. windings*

The stator side voltage equations

$$\begin{bmatrix} u_{ds} \\ u_{qs} \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \end{bmatrix} = \begin{bmatrix} \lambda_{das} \\ \lambda_{qas} \end{bmatrix} + \begin{bmatrix} \lambda_{dsr} \\ \lambda_{qsr} \end{bmatrix} = \begin{bmatrix} L_{lsd} + L_{md} & 0 \\ 0 & L_{lsq} + L_{mq} \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \begin{bmatrix} L_{md} & 0 \\ 0 & L_{mq} \end{bmatrix} \cdot \begin{bmatrix} i'^{s}{}_{dr} \\ i'^{s}{}_{qr} \end{bmatrix}$$

The rotor side voltage equations

$$K_{sr} = \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{\pi}{2}\right) \\ \cos\left(\theta_r - \frac{\pi}{2}\right) & \cos\theta_r \end{bmatrix} \quad K_{rs} = K_{sr}{}^{-1} = \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{\pi}{2}\right) \\ \cos\left(\theta_r + \frac{\pi}{2}\right) & \cos\theta_r \end{bmatrix}$$

$$\begin{bmatrix} u^{s}{}_{dr} \\ u^{s}{}_{qr} \end{bmatrix} = K_{sr} \begin{bmatrix} u_{dr} \\ u_{qr} \end{bmatrix} \qquad \begin{bmatrix} u'^{s}{}_{dr} \\ u'^{s}{}_{qr} \end{bmatrix} = \begin{bmatrix} \frac{1}{k_{nd}} & 0 \\ 0 & \frac{1}{k_{nq}} \end{bmatrix} \begin{bmatrix} u^{s}{}_{dr} \\ u^{s}{}_{qr} \end{bmatrix}$$

Modeling of a 1-phase IM – *uneven number of turns for the main and aux. windings*

The rotor side voltage equations

$$K_{srn} = \begin{bmatrix} \frac{1}{k_{nd}} & 0 \\ 0 & \frac{1}{k_{nq}} \end{bmatrix} K_{sr} = \begin{bmatrix} \frac{1}{k_{nd}}\cos\theta_r & \frac{1}{k_{nd}}\cos\left(\theta_r + \frac{\pi}{2}\right) \\ \frac{1}{k_{nq}}\cos\left(\theta_r - \frac{\pi}{2}\right) & \frac{1}{k_{nq}}\cos\theta_r \end{bmatrix} \qquad K^{-1}{}_{srn} = \begin{bmatrix} k_{nd}\cos\theta_r & k_{nq}\cos\left(\theta_r - \frac{\pi}{2}\right) \\ k_{nd}\cos\left(\theta_r + \frac{\pi}{2}\right) & k_{nq}\cos\theta_r \end{bmatrix}$$

$$p\underline{K}_{srn} \cdot \underline{K}^{-1}{}_{srn} = \omega_r \begin{bmatrix} -\frac{1}{k_{nd}}\sin\theta_r & -\frac{1}{k_{nd}}\cos\theta_r \\ \frac{1}{k_{nq}}\cos\theta_r & -\frac{1}{k_{nq}}\sin\theta_r \end{bmatrix} \begin{bmatrix} k_{nd}\cos\theta_r & k_{nq}\cos\left(\theta_r - \frac{\pi}{2}\right) \\ k_{nd}\cos\left(\theta_r + \frac{\pi}{2}\right) & k_{nq}\cos\theta_r \end{bmatrix}$$

$$= \omega_r \begin{bmatrix} 0 & -k_{nqd} \\ \frac{1}{k_{nqd}} & 0 \end{bmatrix}$$

$$\boxed{k_{nqd} = \frac{k_{nq}}{k_{nd}} = \frac{N_{sd}}{N_{sq}}}$$

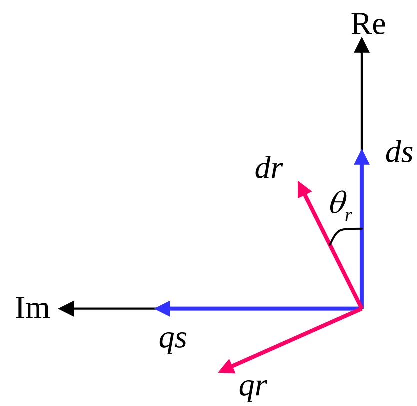
Modeling of a 1-phase IM – *uneven number of turns for the main and aux. windings*

The rotor side voltage equations

$$\begin{bmatrix} u'^s_{dr} \\ u'^s_{qr} \end{bmatrix} = \begin{bmatrix} R'_r & 0 \\ 0 & R'_r \end{bmatrix} \cdot \begin{bmatrix} i'^s_{dr} \\ i'^s_{qr} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \lambda'^s_{dr} \\ \lambda'^s_{qr} \end{bmatrix} - \left(p\underline{K}_{srn} \cdot \underline{K}^{-1}_{srn}\right)\begin{bmatrix} \lambda'^s_{dr} \\ \lambda'^s_{qr} \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} u'^s_{dr} \\ u'^s_{qr} \end{bmatrix} = \begin{bmatrix} R'_r & 0 \\ 0 & R'_r \end{bmatrix} \cdot \begin{bmatrix} i'^s_{dr} \\ i'^s_{qr} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \lambda'^s_{dr} \\ \lambda'^s_{qr} \end{bmatrix} + \omega_r \begin{bmatrix} 0 & k_{nqd} \\ -\dfrac{1}{k_{nqd}} & 0 \end{bmatrix}\begin{bmatrix} \lambda'^s_{dr} \\ \lambda'^s_{qr} \end{bmatrix}$$



$$\begin{bmatrix} \lambda'^s_{dr} \\ \lambda'^s_{qr} \end{bmatrix} = \begin{bmatrix} \lambda'^s_{dar} \\ \lambda'^s_{qar} \end{bmatrix} + \begin{bmatrix} \lambda'^s_{drs} \\ \lambda'^s_{qrs} \end{bmatrix} = \begin{bmatrix} L'_{lrd} + L_{md} & 0 \\ 0 & L'_{lrq} + L_{mq} \end{bmatrix} \cdot \begin{bmatrix} i'^s_{dr} \\ i'^s_{qr} \end{bmatrix} + \begin{bmatrix} L_{md} & 0 \\ 0 & L_{mq} \end{bmatrix} \cdot \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$$

$$\boxed{R'_r = \frac{1}{k^2_{nd}} \cdot R_r} \qquad \boxed{L'_{lrd} = \frac{1}{k^2_{nd}} L_{lr} \quad L'_{lrq} = \frac{1}{k^2_{nq}} L_{lq}} \qquad \boxed{k_{nqd} = \frac{k_{nq}}{k_{nd}} = \frac{N_{sd}}{N_{sq}}}$$