Written examination in

Dynamic Models of Electrical Machines and Control Systems

1st semester M.Sc. (PED / EPSH / WPS / MCE)

Duration: 4 hours

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off
- Internet access is strictly forbidden
- Any kind of communication with other students is not allowed
- Remember to write your study number on all answer sheets
- All intermediate steps and calculations should be included in your answer sheets --- printing the final result is insufficient

---

The set consists of 5 problems

Problem 1 (25%)

For a given space vector $\bar{f} = 10e^{-j\left(\omega_e t + \frac{\pi}{6}\right)}$, where $\omega_e = 2\pi \cdot 50$, please

(1) Find the expressions for its corresponding alfa-, beta-components. Please draw their waveforms as functions of the time.

(2) Find the expressions for its corresponding a-, b-, c-components. Please draw **phase-a** waveform as a function of the time.

(3) Please find the particular moment ($t = ?$) that makes phase-b reach its maximum value. Please draw the location of the space vector at this particular moment with respect to phase-a axis.

(4) Now you are given a dq-reference frame. At time t=0, its d-axis is aligned with phase-a axis. It rotates positively (anti-clockwise direction), at a speed of $\omega_e = 2\pi \cdot 50$. Please find the expressions of the dq-components for the given space vector $\bar{f} = 10e^{-j\left(\omega_e t + \frac{\pi}{6}\right)}$. Please draw the dq-component waveforms as functions of the time.

(5) Find the space vector for the following abc signals (where $\omega_e = 2\pi \cdot 50$ [rad/s])

$$v_a = V_{pk} \sin\left(\omega_e t + \frac{\pi}{6}\right),$$
$$v_b = V_{pk} \sin\left(\omega_e t + \frac{2\pi}{3} + \frac{\pi}{6}\right),$$
$$v_c = V_{pk} \sin\left(\omega_e t - \frac{2\pi}{3} + \frac{\pi}{6}\right)$$

Problem 2 (25%)

A sketch of a synchronous machine is shown below. A **dq-reference frame** is given as well.

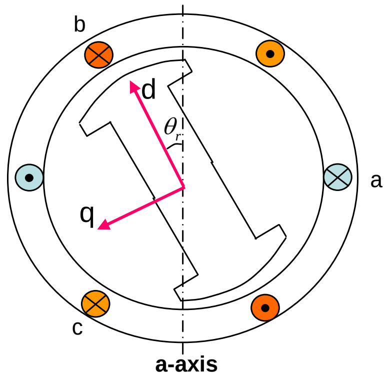

Fig. 1

(1) Please determine the mutual inductance between stator phase-b and stator phase-c.

(2) Please find the maximum value of this mutual inductance and explain at which position, the maximum inductance value is achieved?

A simple single-phase PM machine is shown below.

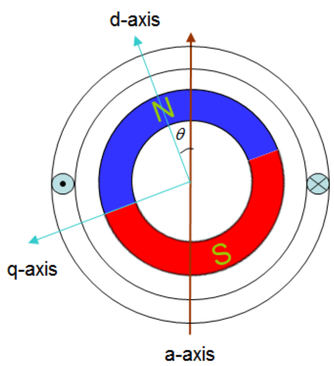

Fig. 2

(3) The PM flux linkage is sinusoidal, as $\lambda_{pm,a} = \lambda_{mpm}\cos(\theta)$. Now phase-a is supplied with 3rd harmonic current component, as $i_a = -I_{m1}\sin(3\theta)$, please give the instantaneous torque expression. Please also sketch this torque waveform (instantaneous torque vs. position θ).

(4) Suppose you supply the machine with a trapezoidal current waveform instead of sinusoidal, will the machine be able to rotate (i.e. with an average torque component that is non-zero)?

(5) At least how many phases are needed in order to make the machine with such a permanent magnet rotor to have constant **instantaneous** torque?

Problem 3 (10%)

A sketch of an induction machine phase axes is given below, where **qd-reference** frame is shown.

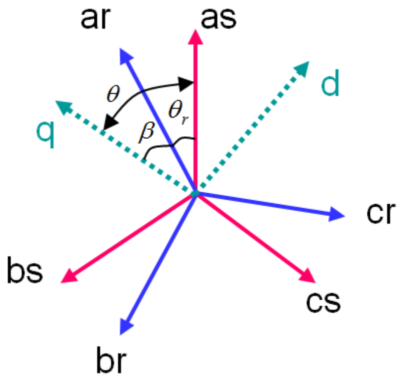

Fig. 3

where notation 's' stands for stator phase axes and notation 'r' stands for rotor phase axis.

Knowing the machine model expressed in an arbitrary qd-reference frame is

Stator side voltage equations:

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{0s} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} - \omega_\theta \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$

(1) Please transform this voltage equation into a vector form, using qd-frame space vector representations, i.e.

$$\overline{f}_{qd} = f_q - jf_d$$

(f is a variable that could stand for the voltage or current.)

(2) Please give the stator voltage equation expressed in the $\alpha\beta$ -reference frame.

Problem 4 (20 %)

A four-pole induction motor has the following data (**the rotor windings are short-circuited**):

Rated shaft power	1.9 kW
Rated speed	1440 rpm
Rated stator frequency	50 Hz
Rated stator voltage	380 V RMS (line-to-line)
Rated stator current	4.1 A RMS
Rated power factor cos φ	0.8 inductive
Stator resistance	2.5 Ohm
Main (magnetization) inductance	0.27 H
Stator leakage inductance	0.01 H

(1) The machine is supplied with the rated voltage and rated frequency. The load torque is zero. What is the rotor speed in rpm? Please calculate the corresponding stator phase current in RMS value and the voltage across the stator reactance.

(2) In V/f control, please explain what must be known in order to compensate the voltage drop on the stator resistance? Why it is needed to compensate the resistance voltage drop? Please explain.

(3) The machine is supplied with the rated voltage and rated frequency. Please calculate the active current value (i_{sd}, *find more information on slide P15 of the lecture discussing about scalar control of induction machine with slip compensation*), which is proportional to the machine torque. Now it is observed that the machine slip is 25% of the rated slip, what is the machine torque now? What is value for this active current (i_{sd}) now? Please also calculate the slip frequency that needs to be compensated.

(4) In V/f control, the stator frequency is now 50% of the rated frequency. The load torque is 25% of the rated torque. What is the slip frequency that needs to be compensated in order to make the electrical shaft speed to be 25Hz? The stator resistance effects may be neglected.

Problem 5 (20 %)

The stator voltage equation of a permanent magnet synchronous machine may be given as (*same notations as used in the lecture slides*):

$$u_q = Ri_q + p\lambda_q + \omega_r\lambda_d \qquad \lambda_q = (L_{ls} + L_{mq})i_q = L_q i_q$$

$$u_d = Ri_d + p\lambda_d - \omega_r\lambda_q \qquad \lambda_d = (L_{ls} + L_{md})i_d + \lambda_{mpm} = L_d i_d + \lambda_{mpm}$$

The machine has **8 poles**.

(1) Are the machine equations the same, when expressed in a dq-reference frame or in a qd-reference frame?

(2) When the machine is running at 1200 rpm, what is the frequency of the stator phase current?

(3) This PM machine is driven by another DC motor and running at a constant speed of 1200 rpm. When the stator windings are open-circuited, measured line-to-line RMS voltage is 120 (Vols). Please determine the value of λ_{mpm} to be used in the above machine equations.

(4) For this machine, the d-, q-axes inductances are equal. What is the torque when the machine q-axis current is 2 (A) and the d-axis current is -1 (A)?

(5) At a particular moment ($t = 0$), it is observed that the machine q-axis current is 4 (A) and its d-axis current is -2 (A). At this moment, the rotor d-axis is leading the stator phase-a by 30 electrical degrees. The speed is constant and is 1200 rpm. Please draw stator phase-b current waveform for one period, starting from $t = 0$ as defined before. (Please indicate clearly *its initial current value at* $t = 0$ *and its peak value.)*
The power factor of this machine at this operation condition is 0.866 (voltage leading current). The phase rms voltage is 100 volts. Please add phase-b voltage waveform for one electrical period to the phase-b current waveform mentioned previously.