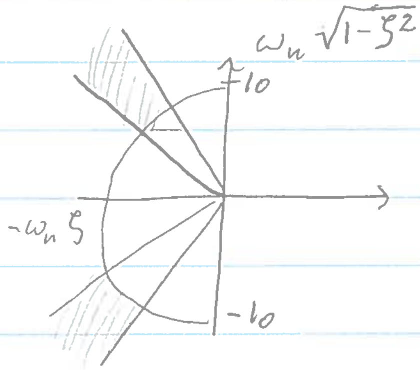
P1

1) a) $\omega_n \geq 0$ and $0.6 \leq \zeta \leq 0.8$

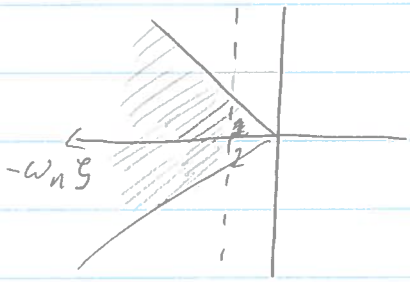


$\theta = \cos^{-1}(\zeta)$

$\theta_1 = \cos^{-1}(0.6) = 53.13^\circ$

$\theta_2 = \cos^{-1}(0.8) = 36.87$

b) $\tau \leq 0.5$ and $\zeta \geq 0.7$



$\omega_n \zeta \geq \frac{1}{\tau} \Rightarrow \omega\zeta \geq \frac{1}{0.5} = 2$

$\theta = \cos^{-1}(0.7) = 45^\circ$

2) Always evaluated for OL! when looking at stability

a) $P_m = 5^\circ$ % Find 0dB crossing $\rightarrow P_m = \text{Phase} - (-180^\circ)$

$G_m = \infty$ % Find -180° phase crossing

b) $\omega_1 = 0.1$ $\omega_n = 10$

$$G(s) = \frac{1}{s} \cdot \frac{(s+\omega_1)}{\omega_1} \cdot \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \cdot K$$

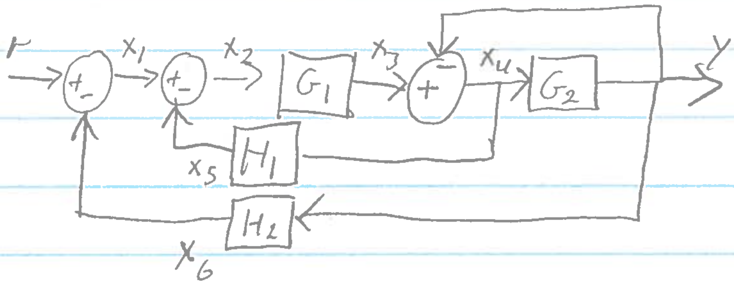
$K = 20\ dB$ $\cancel{Am} = 20\cdot\log(K) \Rightarrow K = 10$

c) "stable" since OL stability margins are positive i.e. $G_m > 0$, $P_m > 0^\circ$

d) $\frac{1}{K_p+1}$ $K_p = \lim_{s\to 0} G(s)_{OL}$

P2

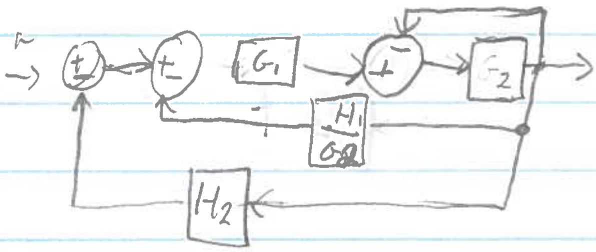
3)

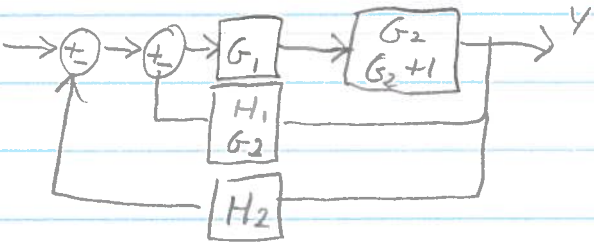


$$\left\{\begin{aligned} x_1 &= r - x_6 \\ x_2 &= x_1 - x_5 \\ x_3 &= G_1 x_2 \\ x_4 &= x_3 - y \\ x_5 &= H_1 x_4 \\ x_6 &= H_2 y \\ y &= G_2 x_4 \end{aligned}\right.$$

By symbolic solver in MathCad

$$\frac{Y}{X} = \frac{G_1 G_2}{G_2 + G_1 H_1 + G_1 G_2 H_2 + 1}$$





P3

4) a) $e_{ss} = 0$ as $\frac{1}{s}$

b) $G_c(s) = K_p \frac{s + K_i}{s}$, $G_p(s) = \frac{1000}{s^2 + 20s + 100}$

use Matlab

$$G_{CL} = \frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) G_p(s)} \overset{\downarrow}{=} \frac{1000 K_i}{s^3 + 20 s^2 + (1000 K_p + 100) s + 1000 K_i K_p}$$

Char. eq.: $s^3 + 20 s^2 + (1000 K_p K_i + 100) s + 1000 K_i K_p$

c)

$$\begin{array}{c|cc} s^3 & 1 & 1000 K_p K_i + 100 \\ s^2 & 20 & 1000 K_p K_i \\ s^1 & b_1 & 0 \\ s^0 & c_1 & 0 \end{array}$$

$$b_1 = -\frac{1}{20} \begin{vmatrix} 1 & 1000 K_p K_i + 100 \\ 20 & 1000 K_p K_i \end{vmatrix} = -50 K_i K_p + 1000 K_p + 100$$

$$c_1 = -\frac{1}{b_1} \begin{vmatrix} 20 & 1000 K_p \\ b_1 & 0 \end{vmatrix} = -\frac{1}{b_1} \left(-1000 K_i K_p b_1 \right)$$

$= 1000 K_i K_p$ ← must be > 0

$K_i > 0$, $K_p > 0$

... see photo

d)

P4

d) $K_i = 40 \Rightarrow$

$$G_{OL}(s) = K_p \cdot \frac{s+40}{s} \cdot \frac{1000}{s^2+20s+100}$$

$$\text{Poles} \begin{cases} 0 \\ -10 \\ -10 \end{cases}, \quad \text{zeros} = -40$$

$$\alpha = \#\text{poles} - \#\text{Zeros} = 3-1=2$$

$$\theta = \frac{r\,180}{\alpha} = 90^\circ, \quad r = \pm 1, \pm 3, \dots$$

$$\sigma = \frac{\sum Re(\text{poles}) - \sum Re(\text{zeros})}{\alpha} = \frac{0-10-10-(-40)}{2} = 20$$

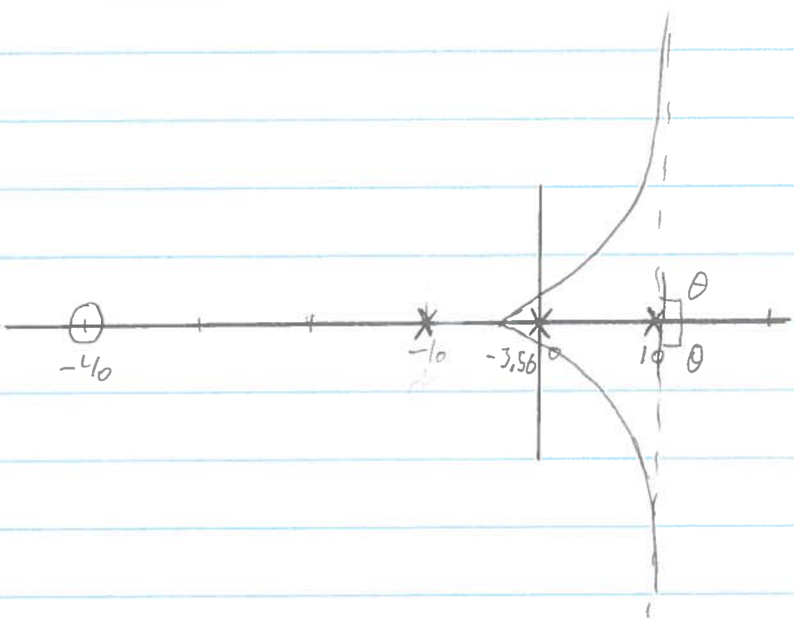
$N(s) =$

$D(s) =$

$$N(s)D'(s) - D(s)N'(s) = 0$$

$$\Rightarrow 0 = 2000(s+10)(s^2+60s+200)$$

$$\begin{cases} -10 \\ -3.56 \\ -56.5 \end{cases}$$

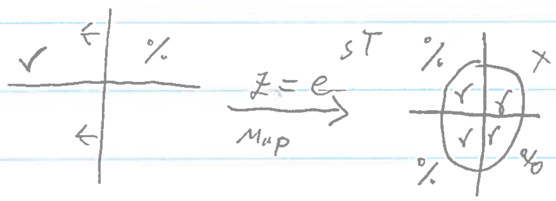


PS

1) For which values of A is it stable

$$D(z) = \frac{M(z)}{E(z)} = \frac{Az+1}{z+A}$$

Bonus info



Proof: $z = e^{\sigma + j\omega T} = e^{\sigma T} \cdot e^{j\omega T} = e^{\sigma T} e^{j\omega T} = |\angle \omega T|$

$E(z) = z + A = 0 \Rightarrow \begin{cases} -1 + A = 0 \rightarrow A = 1 \\ +1 + A = 0 \rightarrow A = -1 \end{cases} \Rightarrow \underline{\underline{-1 < A < 1}}$

i.e. within unit circle

b) $1 + A = 0 \Rightarrow \underline{\underline{A = -1}}$

c) $M(z)(z + A) = E(z)(Az + 1)$

$\Rightarrow M(z)z + M(z)A = E(z)Az + E(z)$

$m(k+1) + m(k)A = e(k+1)A + e(k)$

$\Downarrow$

$\underline{m(k) + A\,m(k-1) = A\,e(k) + e(k-1)}$ when solved for $m(k)$

In difference eq.
$z \rightarrow k+1$
$z^{-1} \rightarrow k-1$
$z^{-2} \rightarrow k-2$

d

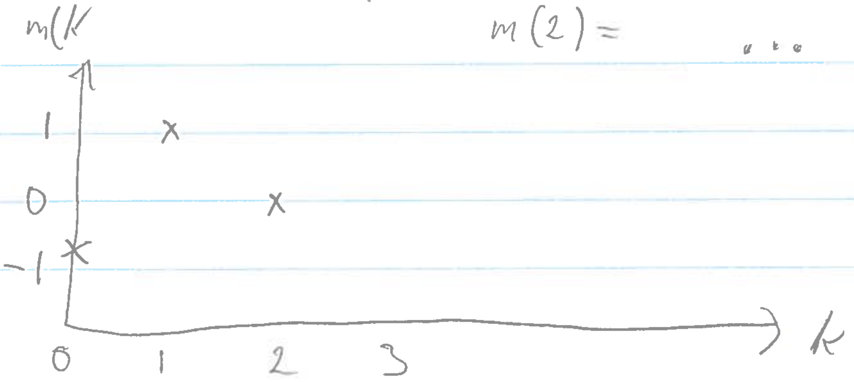
$$e(k) \begin{cases} 1, & k = 0 \\ -1 & k = 1 \\ 0, & k \geq 2 \end{cases}$$

$\Rightarrow m(0) = -A \cdot e(0) = -0.9 \cdot 1 = -0.9$

$m(1) = -A\,m(0) + A \cdot e(1) + e(0)$

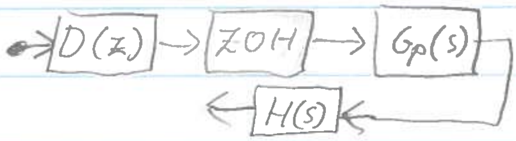
$= 0.9 \cdot (-0.9) + (-0.9) \cdot (-1) + 1 = 1$

$m(2) = \ldots = 0$



P7

b) Sketch root locus



Note: This is always done open loop

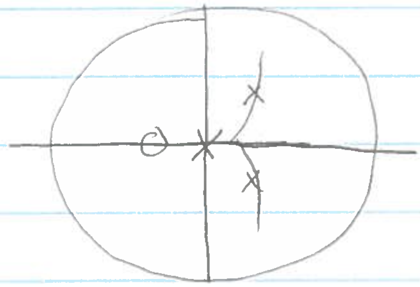
First calculate

$$\mathcal{Z}\left\{ZOH \cdot \left(G_p(s)H(s)\right)\right\}$$

$$G_{OL}(z) = D(z)G(z)H(z) = \frac{1}{z}\,\frac{0.595(z+0.258)}{(z-0.135)\,z} \cdot 0.1$$

complex

$$\text{poles} = \begin{cases} 0 \\ 2 \cdot 0.135 \end{cases}, \quad \text{Zeros} = -0.258$$



c) $\angle ZOH = -30^\circ$

$$\angle G_{ZOH}(j\omega) = \frac{-\pi \cdot \omega}{\omega_s} = \frac{-\pi f}{f_s}$$

$$\Rightarrow -30 \cdot \frac{\pi}{180} = \frac{\pi f}{f_s} \rightarrow \begin{cases} f = 0.833\ Hz \\ \omega = 5.37\ rad/s \end{cases}$$