Written examination in

Dynamic Models of Electrical Machines and Control Systems

INTRO 1st semester M.Sc. (PED/EPSH/WPS/MCE)

Duration: 4 hours

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off
- Internet access is strictly forbidden
- Any kind of communication with other students is not allowed
- Remember to write your study number on all answer sheets
- All intermediate steps and calculations should be included in your answer sheets --- printing the final result is insufficient

The set consists of 6 problems

Problem 1 (25%) – reference frame theory

(1) Suppose now you have a set of 3-phase signals as:

$$v_a = V_{pk}\sin(\omega_e t),\quad v_b = V_{pk}\sin\left(\omega_e t - \frac{2\pi}{3}\right),\quad v_c = V_{pk}\sin\left(\omega_e t + \frac{2\pi}{3}\right)$$

where $\omega_e = 2\pi \cdot 50$ and $V_{pk} = 1$

Please calculate the space vector formed by these three-phase signals at

- $t = 0$ (seconds)
- $t = \frac{1}{300}$ (seconds)

(*Note these signals are 'sin' not 'cos' as we used to use!*)

Please also draw the space vectors obtained at these two special moments in a stationary afa-bet reference frame 

(2) Now phase-c signal is lost, i.e. $v_c = 0$. Please repeat question (1) with $v_c = 0$. What is the zero component now?

(3) Now phase-b and phase-c are exchanged, which yields

$$v_a = V_{pk}\sin(\omega_e t),\quad v_b = V_{pk}\sin\left(\omega_e t + \frac{2\pi}{3}\right),\quad v_c = V_{pk}\sin\left(\omega_e t - \frac{2\pi}{3}\right)$$

Please calculate the space vector formed by these three-phase signals at

- $t = \frac{1}{300}$ (seconds)

Please also draw this space vector in the stationary afa-bet reference frame.

(4) Transform the above a, b, c signals ($v_a = V_{pk}\sin(\omega_e t), v_b = V_{pk}\sin\left(\omega_e t + \frac{2\pi}{3}\right)$, $v_c = V_{pk}\sin\left(\omega_e t - \frac{2\pi}{3}\right)$) to a stationary afa-bet reference frame. Then transform the afa-bet signals to a rotating dq-frame. This dq-frame is rotating positively (anti-clockwise direction) at a frequency of 50 Hz.
(*Remember to give the expressions of the transformed signals.*)

Problem 2 (25%)

A sketch of a synchronous machine is shown below.

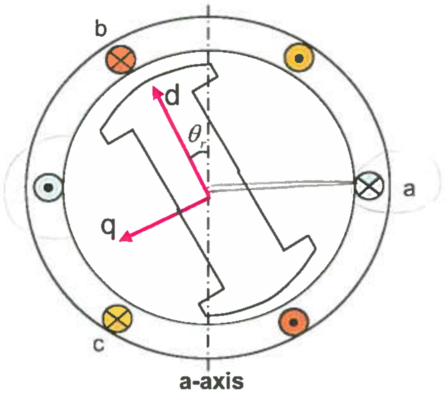
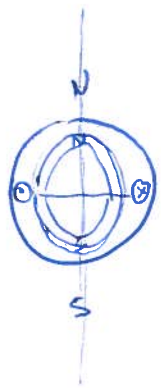


Fig. 1

(1) Please draw the position where phase-a self-inductance reaches its minimum value, and why?
(2) Please draw the position where phase-a and phase-b mutual-inductance reaches its minimum value.

A simple single-phase PM machine is shown below in Fig. 2(a). Now another phase (name it phase-b) is added to this machine, as indicated in Fig. 2(b).



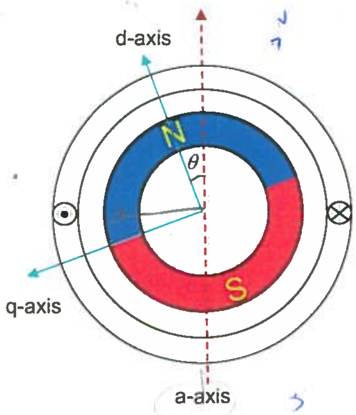


Fig. 2(a)

90 - θ

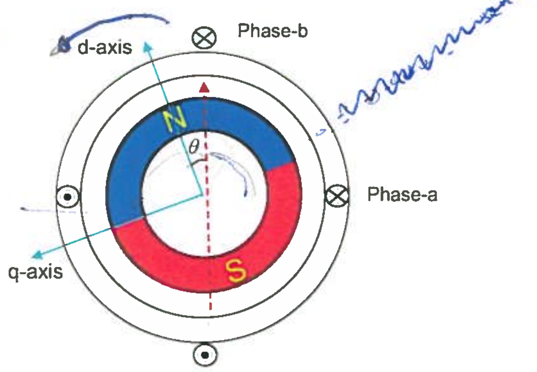


Fig. 2(b)

(3) If the phase-a PM flux linkage waveform is expressed as: $\lambda_{pm,a} = \lambda_{mpm} \cos\theta$. (where λ_{mpm} is its peak value and θ is the rotor position as indicated in Fig. 2), please give the PM flux linkage waveform for phase-b.
(4) If phase-a is now supplied with a current of $i_a = -I_m \sin\theta$ (where I_m is the peak value of the current), please determine the needed current waveform for phase-b.
(5) Please show the instantaneous torque produced by phase-a and phase-b, respectively.
(6) Please give an expression for the total torque produced by phase-a and phase-b together.

Problem 3 (9%)

A three-phase induction motor connected directly to a 400V / 50 Hz power source must be selected for a simple pumping application. The pump must run at approximately 725 rpm and the required torque is ~~100 Nm~~. bơm beo

turke = $\mu \cdot m$

(1) How many pole pairs should the motor have?

$P = \mu \cdot \omega \rightarrow$ rad/s

(2) What is the nominal shaft power requirement for the motor?

$725 \text{ rpm} \cdot \frac{2\pi}{60} =$ rad/sg

(3) Sketch the speed-torque profile for the motor and for the pump
Note: You do not have to calculate numerical values – an overall sketch is sufficient

(4) Does the shaft speed change if the supply voltage is changed to 380 V / 50 Hz?

Problem 4 (8 %)

Consider a three-phase induction motor, where $\boldsymbol{u}_s$ and $\boldsymbol{i}_s$ denote the stator voltage and the stator current space vectors, respectively.

In a synchronous coordinate system rotating with the angular speed $\omega_s = 100\pi$ [rad/s], the voltage and the current space vectors are $\boldsymbol{u}_s = 300 + j50$ [V] and $\boldsymbol{i}_s = 6 + j0$ [A]

(1) Determine the phase angle between $\boldsymbol{u}_s$ and $\boldsymbol{i}_s$

(2) Calculate the active component I_{act} of the stator current

(3) Plot the graph for the real and the imaginary components of $\boldsymbol{u}_s$ in the time interval $0 < t < 20$ [ms].

Problem 5 (8 %)

A six-pole induction motor has these data:

Rated shaft power	25 kW
Rated speed	975 rpm
Rated stator frequency	50 Hz

The machine is supplied from a 50 Hz power source at rated voltage.

(1) Calculate the shaft speed when the load torque is -50 per cent of the rated torque (note that the machine works in generator mode).

By means of a frequency converter the supply frequency is now changed to 25 per cent of the rated value and the supply voltage is changed to 25 per cent of the rated value also. The load torque is still -50 per cent of the rated value.

(2) Calculate the shaft speed for this operating condition

Problem 6 **(25 %)**

Given the differential equation

$$(*) \qquad \frac{d^2x}{dt^2} + f\left(\frac{dx}{dt}\right)h(x) + g(x) = 0$$

where f, g and h are real C^1 functions defined in R. It is assumed that $f(0) = 0$ and $ug(u) > 0$ for $u \neq 0$.

1. Make a state space formulation $(**)$ of $(*)$, using $x_1 = x$ and $x_2 = \frac{dx}{dt}$. Show that $(0,0)$ is the only singular point for $(**)$. We further define

$$V(x_1, x_2) = \frac{1}{2}x_2^2 + \int_0^{x_1} g(u)du \quad ; \quad (x_1, x_2) \in R^2$$

   Determine $\frac{dV(x_1,x_2)}{dt}$ for all $(x_1, x_2) \in R^2$

   {Hint: $\frac{d}{dt}\left(\int_0^{x_1} g(u)du\right) = g(x_1)\frac{dx_1}{dt}$}

   $\int \frac{d}{dt} \int_0^{x_1} g(u)\,du = g(x_1) \cdot \frac{dx_1}{dt}$

2. Now it is assumed that $uf(u) \geq 0$ and $h(u) \geq 0$ for all $u \in R$. Show that $(0,0)$ is a stable singular point for $(**)$. Also show that if f and h only has one zero, then $(0,0)$ is asymptotically stable.

   {Hint: Find the largest invariant set}

   $\int_0^{x_1} g(u) \cdot du = \int \frac{dx_1}{dt} \cdot g(x_1)$

   $= x_1 \cdot g(x_1)$

-----oo0oo-----

Problem 1

Untitled

Q1
```
>>
2/3*(sin(0)+sin(0-2*pi/3)*exp(j*2*pi/3)+sin(0+2*pi/3)*exp(j*4*pi/3
))

ans =

  -0.0000 - 1.0000i


2/3*(sin(pi/3)+sin(pi/3-2*pi/3)*exp(j*2*pi/3)+sin(pi/3+2*pi/3)*exp
(j*4*pi/3))

ans =

   0.8660 - 0.5000i
```

Q3
```
2/3*(sin(pi/3)+sin(pi/3+2*pi/3)*exp(j*2*pi/3)+sin(pi/3-2*pi/3)*exp
(j*4*pi/3))

ans =

   0.8660 + 0.5000i
```

For Q2: Note. for unbalanced system, the zero component is not zero, therefore the original signal should first minus the zero component, then can be used for forming the space vector.

$V_a' = V_a - V_0 \quad U_b' = U_b - V_0 \quad V_c' = V_c - V_0$

Lecture No. 2. P_{11-13} slides.

Qusetion 1 - (4).

Let $V_{pk} = 1$ for simplicity.

We first exam the standard sequence:

$V_a = \cos(W_e t) \quad V_b = \cos(W_e t_i - 120°) \quad V_c = \cos(W_e t_i + 120°)$.

What is the corresponding $\bar{V}_{\alpha\beta}$ space vector ....

We have: $\bar{V}_{\alpha\beta} = \frac{2}{3}\left(V_a + V_b e^{j120°} + V_c e^{-j120°}\right)$

So: $\frac{3}{2}\bar{V}_{\alpha\beta} = \cancel{\cos} V_a + V_b\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + V_c\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$.

$$= V_a - \frac{1}{2}(V_b + V_c) + j\cdot\frac{\sqrt{3}}{2}(V_b - V_c)$$

Note that: $V_b + V_c = -V_a$. and

$$V_b - V_c = \cos(W_e t_i - 120°) - \cos(W_e t_i + 120°)$$

$$= +2\cdot\sin(W_e t_i)\cdot\sin(120°) = \sqrt{3}\sin(W_e t).$$

So: $\bar{V}_{\alpha\beta} = \frac{2}{3}\cdot\left[\frac{3}{2}\cdot V_a + j\cdot\frac{3}{2}\cdot\sin(W_e t)\right]$

$$= \cos W_e t_i + j\sin(W_e t). = e^{jW_e t_i}$$

Now we have:

$V_a = \sin(W_e t) \qquad V_b = \sin(W_e t_i + \frac{2\pi}{3}) \qquad V_c = \sin(W_e t_i - \frac{2\pi}{3})$

So we do a simple transformation.

$$\Rightarrow V_a = \cos(\omega_e t_i - \tfrac{\pi}{2}),\quad V_b = \cos(\omega_e t_i + \tfrac{2\pi}{3} - \tfrac{\pi}{2}),\quad V_c = \cos(\omega_e t_i - \tfrac{2\pi}{3} - \tfrac{\pi}{2})$$

$$\Rightarrow \cancel{V_a = \cos(-(\omega_e t_i - \tfrac{\pi}{2}))},\quad V_b = \cos\left[-(\omega_e t_i - \tfrac{\pi}{2}) - \tfrac{2\pi}{3}\right],$$

$$V_a = \cos\left[-(\omega_e t_i - \tfrac{\pi}{2})\right] \qquad V_c = \cos\left[-(\omega_e t_i - \tfrac{\pi}{2}) + \tfrac{2\pi}{3}\right]$$

Then we define: $\alpha = -(\omega_e t_i - \frac{\pi}{2})$,

the original a.b.c. sequence becomes

$$V_a = \cos\alpha,\quad V_b = \cos(\alpha - \tfrac{2\pi}{3}),\quad V_c = \cos(\alpha + \tfrac{2\pi}{3})$$

balanced ordinary 3-phase,

we know its $\alpha\beta$ space vector is

$$\bar{V}_{\alpha\beta} = e^{j\alpha} = e^{-j\omega_e t_i} \cdot e^{j\frac{\pi}{2}} = j \cdot e^{-j\omega_e t_i}$$

Transform it to dq, the relationship is

$$\bar{V}_{dq} e^{j\omega t_i} = \bar{V}_{\alpha\beta}$$

where, $\omega = 2\pi * 50$, and in this case, $\omega = \omega_e$

rotating speed of the dq frame.

$$\text{So. } \bar{V}_{dq} = \bar{V}_{\alpha\beta} e^{-j\omega t_i} = j e^{-j\omega_e t_i} \cdot e^{-j\omega t_i} = j e^{-j2\omega t_i}$$

Problem 2.

Q – (3): $\lambda_{pm,b} = \lambda_{mpm} \cos(\theta - 90^\circ)$

(because when $\theta = 90^\circ$, $\lambda_{pm,b}$ = max. value)

So if you think let $a = \theta - 90^\circ$ then for phase-b

we have. $\lambda_{pm,b} = \lambda_{mpm} \cos a$.

similar form as phase–a, but with θ replaced by a.

Q – (4)

therefore. it is easy to find that.

current for phase-b. $i_b = -I_m \sin a = -I_m \sin(\theta - 90^\circ)$

Q – (5):

$0 < \cos\theta < 1$

$\tau_a = \frac{1}{2} I_m \lambda_{mpm} \cdot (1 - \cos \underset{90^\circ}{2\theta})$.

$\tau_b = \frac{1}{2} I_m \lambda_{mpm} (1 - \cos 2a) = \frac{1}{2} I_m \lambda_{mpm} (1 - \cos(2\theta - 180^\circ))$

$= \frac{1}{2} I_m \lambda_{mpm} (1 + \cos 2\theta)$

Q – (6)

$\tau_a + \tau_b = I_m \lambda_{mpm}$.

It should be easy to find the waveform if you know the expressions.

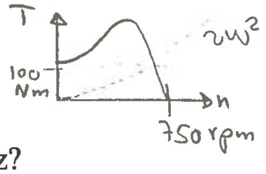
Problem 3 (9%)

A three-phase induction motor connected directly to a 400V / 50 Hz power source must be selected for a simple pumping application. The pump must run at approximately 725 rpm and the required torque is 100 Nm.

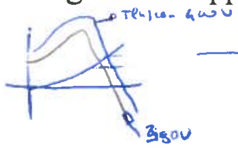
(1) How many pole pairs should the motor have? 4 pole pair ~ 750 rpm @ noload @ 50 Hz

(2) What is the nominal shaft power requirement for the motor? $P = T \cdot \omega = 100 \cdot \frac{725}{60} 2\pi \text{ W} = 7.59 \text{ kW}$

(3) Sketch the speed-torque profile for the motor and for the pump
Note: You do not have to calculate numerical values – an overall sketch is sufficient



(4) Does the shaft speed change if the supply voltage is changed to 380 V / 50 Hz?



Yes, the speed drops a little because U/f - ratio is reduced

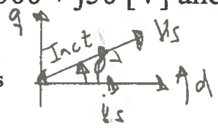
Problem 4 (8 %)

Consider a three-phase induction motor, where $\boldsymbol{u}_s$ and $\boldsymbol{i}_s$ denote the stator voltage and the stator current space vectors, respectively.

In a synchronous coordinate system rotating with the angular speed $\omega_s = 100\pi$ [rad/s], the voltage and the current space vectors are $\boldsymbol{u}_s = 300 + j50$ [V] and $\boldsymbol{i}_s = 6 + j0$ [A]

no importa como sea la tensión y la intensidad, la intensidad está desfasada.

(1) Determine the phase angle between $\boldsymbol{u}_s$ and $\boldsymbol{i}_s$

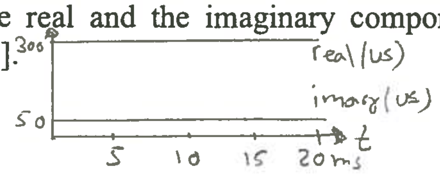


$\varphi = \operatorname{atan}\left(\frac{50}{300}\right) = 0{,}165 \text{ rad} \sim 9{,}5 \text{ deg}$

(2) Calculate the active component I_{act} of the stator current $I_{act} = |i_s| \cos\varphi = 6 \cdot \cos 9{,}5° \text{ A} = 5{,}92 \text{ A}$

Es la proyección de la ~~tensión~~ intensidad en la tensión.

(3) Plot the graph for the real and the imaginary components of $\boldsymbol{u}_s$ in the time interval $0 < t < 20$ [ms].



¡!

Problem 5 (8 %)

A six-pole induction motor has these data:

Rated shaft power	25 kW
Rated speed	975 rpm
Rated stator frequency	50 Hz

rated $n_{slip} = 1000 - 975 = 25$ rpm

@ 50% T_{load} $n_{slip} = 12{,}5$ rpm

$\frac{60 \cdot 50}{3} = 1000 - 975 = 2$

esto es cuando tenemos a tope de torque. a todo

The machine is supplied from a 50 Hz power source at rated voltage.

(1) Calculate the shaft speed when the load torque is -50 per cent of the rated torque (note that the machine works in generator mode). $n = 1000 + 12{,}5 \text{ rpm} = 1012{,}5 \text{ rpm}$

By means of a frequency converter the supply frequency is now changed to 25 per cent of the rated value and the supply voltage is changed to 25 per cent of the rated value also. The load torque is still -50 per cent of the rated value. Rated U/f ratio ⇒ n_{slip} = const

(2) Calculate the shaft speed for this operating condition $n = \frac{1}{4} 1000 + 12{,}5 \text{ rpm} = 262{,}5 \text{ rpm}$

9 January 2013

(*) $\frac{d^2x}{dt^2} + f\left(\frac{dx}{dt}\right) \cdot h(x) + g(x) = 0$

f, g, h are real C_1 functions defined in $\mathbb{R}$.

$f(0) = 0$ and $u \cdot g(u) > 0$ for $u \neq 0$.

(* *) → state vector formulation of (*)

$$\left.\begin{aligned} x_1 &= x \\ x_2 &= \frac{dx}{dt} \end{aligned}\right\} \qquad \begin{aligned} \dot{x}_1 &= x_2 \\ x_2 &= \dot{x} \end{aligned}$$

$$\dot{x}_2 + f(x_2) \cdot h(x_1) + g(x_1)$$

$$\hookrightarrow \dot{x}_2 = -f(x_2) h(x_1) - g(x_1)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -f(x_2) h(x_1) - g(x_1) \end{bmatrix}$$

Singular point

$x_2 = 0$

$-f(x_2) h(x_1) - g(x_1) = 0$

$(f(0) = 0)$ $\quad g(x_1) = 0$

$\boxed{x_1 = 0}$

$$V(x_1, x_2) = \frac{1}{2} x_2^2 + \underbrace{\int_0^{x_1} g(u)\, du}$$

$$\hookrightarrow \frac{d}{dt} \int_0^{x_1} g(u)\, du = g(x_1) \frac{dx_1}{dt}$$

$$\dot{V}(x_1, x_2) = x_2 \cdot \dot{x}_2 + g(x_1) \cdot \dot{x}_1$$

$$\dot{V}(x_1, x_2) = x_2 \left(-f(x_2) h(x_1) - g(x_1)\right) + g(x_1) \cdot x_2$$

$$\dot{V}(x_1, x_2) = -x_2 \cdot f(x_2) \cdot h(x_1) - \cancel{x_2 \cdot g(x_1)} + \cancel{x_2 \cdot g(x_1)}$$

$$\dot{V}(x_1, x_2) = -x_2 \cdot f(x_2) \cdot h(x_1)$$