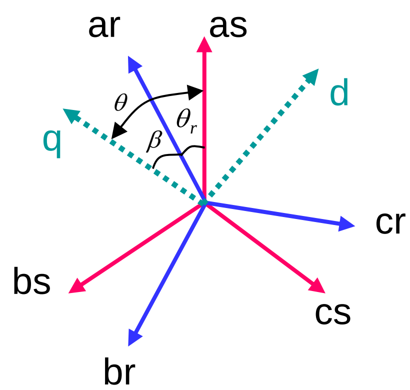
More experience with the IM modeling

Voltage equations

•The stator voltage equations

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{0s} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} - \omega_\theta \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$



•The rotor voltage equations would be

$$\begin{bmatrix} u'_{qr} \\ u'_{dr} \\ u'_{0r} \end{bmatrix} = \begin{bmatrix} R'_r & 0 & 0 \\ 0 & R'_r & 0 \\ 0 & 0 & R'_r \end{bmatrix} \cdot \begin{bmatrix} i'_{qr} \\ i'_{dr} \\ i'_{0r} \end{bmatrix} + p \begin{bmatrix} \lambda'_{qr} \\ \lambda'_{dr} \\ \lambda'_{0r} \end{bmatrix} - (\omega_\theta - \omega_r) \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda'_{qr} \\ \lambda'_{dr} \\ \lambda'_{0r} \end{bmatrix}$$

Flux linkage equations

•The stator side

$$\underline{\lambda}_{qd0s} = \begin{bmatrix} L_{ls}+L_m & 0 & 0 \\ 0 & L_{ls}+L_m & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \cdot \underline{i}_{qd0s} + \begin{bmatrix} L_m & 0 & 0 \\ 0 & L_m & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i'_{qr} \\ i'_{dr} \\ i'_{0r} \end{bmatrix}$$

$$\underline{i}_{qd0s} = \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} \qquad \begin{bmatrix} i'_{qr} \\ i'_{dr} \\ i'_{0r} \end{bmatrix} = \underline{i}'_{qd0r}$$

•The rotor side

$$\underline{\lambda}'_{qd0r} = \begin{bmatrix} L'_{lr}+L_m & 0 & 0 \\ 0 & L'_{lr}+L_m & 0 \\ 0 & 0 & L'_{lr} \end{bmatrix} \cdot \underline{i}'_{qd0r} + \begin{bmatrix} L_m & 0 & 0 \\ 0 & L_m & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \underline{i}_{qd0s}$$

In another form (excluding the zero component)

- The stator side

$$u_{qs} = R_s i_{qs} + p\lambda_{qs} + \omega_\theta \lambda_{ds} \qquad \lambda_{qs} = \left(L_{ls} + L_m\right) i_{qs} + L_m \cdot i'_{qr}$$

$$u_{ds} = R_s i_{ds} + p\lambda_{ds} - \omega_\theta \lambda_{qs} \qquad \lambda_{ds} = \left(L_{ls} + L_m\right) i_{ds} + L_m \cdot i'_{dr}$$

- The rotor side

$$u'_{qr} = R'_r i'_{qr} + p\lambda'_{qr} + \left(\omega_\theta - \omega_r\right)\lambda'_{dr} \qquad \lambda'_{qr} = \left(L'_{lr} + L_m\right) i'_{qr} + L_m i_{qs}$$

$$u'_{dr} = R'_r i'_{dr} + p\lambda'_{dr} - \left(\omega_\theta - \omega_r\right)\lambda'_{qr} \qquad \lambda'_{dr} = \left(L'_{lr} + L_m\right) i'_{dr} + L_m i_{ds}$$

Now in a vector form

- The stator side

$$u_{qs} = R_s i_{qs} + p\lambda_{qs} + \omega_\theta \lambda_{ds}$$

$$u_{ds} = R_s i_{ds} + p\lambda_{ds} - \omega_\theta \lambda_{qs}$$

$$\lambda_{qs} = \left(L_{ls} + L_m\right) i_{qs} + L_m \cdot i'_{qr}$$

$$\lambda_{ds} = \left(L_{ls} + L_m\right) i_{ds} + L_m \cdot i'_{dr}$$

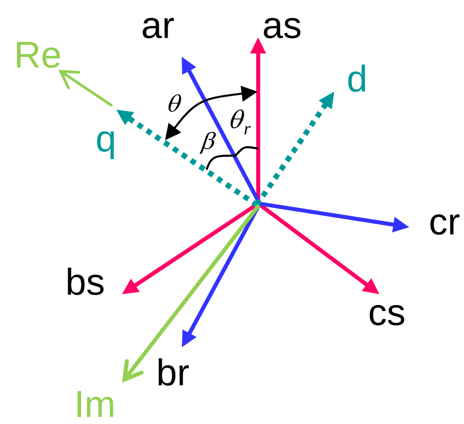
$$u_{qs} = R_s i_{qs} + p\lambda_{qs} + \omega_\theta \lambda_{ds}$$

$$+$$

$$-ju_{ds} = -jR_s i_{ds} - jp\lambda_{ds} + j\omega_\theta \lambda_{qs}$$

$$\boxed{\bar{u}_{qds} = R_s \bar{i}_{qds} + p\bar{\lambda}_{qds} + j\omega_\theta \bar{\lambda}_{qds}}$$

$$\boxed{\bar{\lambda}_{qds} = \left(L_{ls} + L_m\right)\bar{i}_{qds} + L_m \bar{i}'_{qdr}}$$



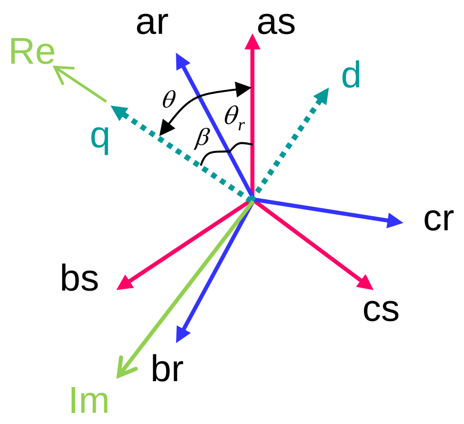
$$\bar{f}_{qd} = f_q - jf_d$$

Similar for the rotor side

- The rotor side

$$\boxed{\bar{u}'_{qdr} = R_s \bar{i}'_{qdr} + p\bar{\lambda}'_{qdr} + j(\omega_\theta - \omega_r)\bar{\lambda}'_{qdr}} \qquad \boxed{\bar{\lambda}'_{qdr} = \left(L'_{lr} + L_m\right)\bar{i}'_{qdr} + L_m \bar{i}_{qds}}$$

When it is in afa-beta



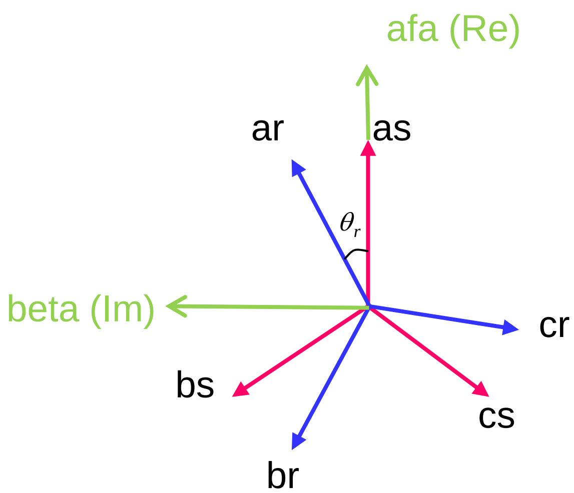
$$\bar{f}_{qd} = f_q - jf_d$$

$$\theta = 0$$

$$\omega_\theta = 0$$

$$f_d = -f_\beta$$

$$f_q = f_\alpha$$



$$\bar{f}_{\alpha\beta} = f_\alpha + jf_\beta$$

For the torque

- Original

$$\tau = \frac{3}{2} p L_m \left(i_{qs} i'_{dr} - i_{ds} i'_{qr} \right)$$

$$f_d = -f_\beta$$
$$f_q = f_\alpha$$

$$\tau = \frac{3}{2} p L_m \left(i_{\beta s} i'_{\alpha r} - i_{\alpha s} i'_{\beta r} \right)$$

This is the number of pole pairs

Practical exercises

Use afa-bet reference frame induction machine equations. Realize them in Simulink. The parameters are:

The data for a 2.2 kW, 4-pole induction motor should be used:

R_s = 3.67, [Om]
R_r = 2.32, [Om]
L_s = 0.2442, [H], note, $L_s = L_{ls}+L_m$, where L_{ls} is the stator leakage inductance
L_r = 0.2473, [H], note, $L_r = L_{lr}+L_m$, where L_{lr} is the rotor leakage inductance
L_M = 0.2350, [H]
J = 0.0069, [kg/m^2]

```
% Induction motor nameplate
Pn = 2200;                % nominal (rated) power [W]
Un = 220;                 % nominal phase voltage RMS[V]
In = 5.1;                 % nominal current RMS[A]
fn = 50;                  % nominal frequency [Hz]
PF = 0.81;                % nominal power factor, current lags voltage
nn = 1430;                % nominal shaft speed, [rpm]
Omegae = 2*pi*fn;         % nominal electrical angular speed [rad/s]
```