Department of Energy Technology

AALBORG UNIVERSITET

Matlab and Control Theory — INTRO 1st semester 2011

February 17, 2012

Written re-exam

09.00-13.00 CET (4 hours)

The set consists of 11 problems

Rules

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions.
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off. Internet access is strictly forbidden.
- Any kind of communication with other students is not allowed.

Remember

1. To write your study number on all sheets handed in.
2. It must be clear from the solutions, which methods you are using, and you must include sufficient intermediate calculations, diagrams, sketches etc. so the line of thought is clear. Printing the final result is insufficient.

Problem 1 (10 %)

In Fig. 1 below is shown the block diagram for a given system.

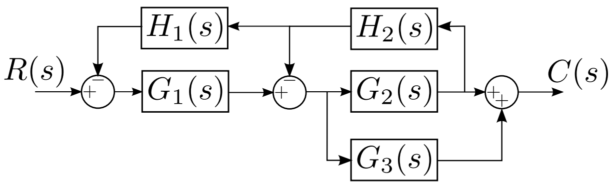

Figure 1: Reduce the block diagram.

(a) Reduce the block diagram and determine the transfer function $\dfrac{C(s)}{R(s)}$

Problem 2 (5 %)

The system shown in Fig. 2 is considered. The transfer function $G(s)$ is given by:

$$G(s) = \frac{5\,(s+5)}{\big(s+6)(s+2\big)}$$

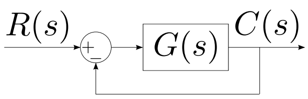

Figure 2: System for which the steady state error should be determined.

(a) Determine the steady state error if the system is applied a step input, i.e. $R(s) = \dfrac{1}{s}$

(b) Determine the steady state error if the system is applied a ramp input, $R(s) = \dfrac{1}{s^2}$

Problem 3 (10 %)

Figure 3 below shows the Bode diagram for a system with the transfer function $G(s)$.

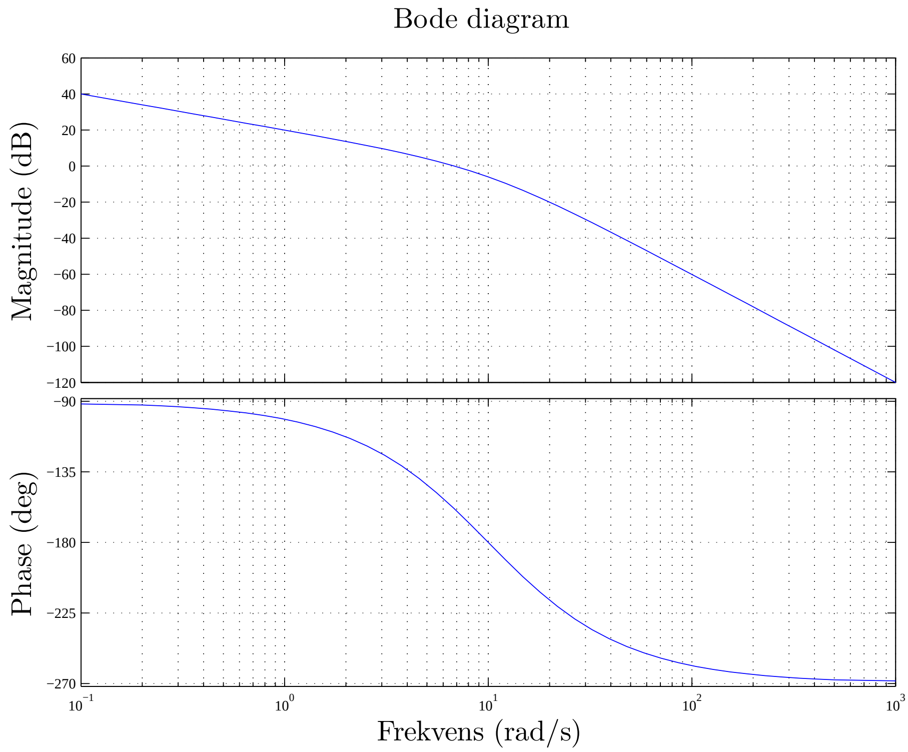

Figure 3: Bode diagram for the system, for which the transfer function should be determined.

(a) Determine the transfer function, $G(s)$, for the system with the Bode diagram shown in Fig. 3.

It is informed that the transfer function $G(s)$ is the open loop transfer function for a closed-loop system with unity gain feedback (similar to the system shown in Fig. 2 in Problem 2).

(b) Determine the gain margin and phase margin for the system - indicate in the solution how these are determined.

(c) Determine whether the closed loop system is stable and justify the answer.

Problem 4 (15 %)

A continuous-time control system is shown in Fig. 4, where $G_c(s)$ represents the controller.

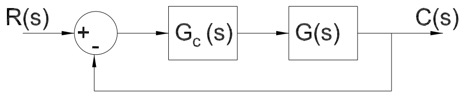

Figure 4: Block diagram for the system, for which the controller $G_c(s)$ should be designed.

The plant transfer function is given by:

$$G(s) = \frac{80}{s\left(s^2 + 8s + 25\right)}$$

(a) Design a lead-controller $G_c(s)$ for the system shown in Fig. 4, which fullfils these requirements:

1. The system should have a phase margin of approximately $45°$
2. The settling time should be $T_s \approx 5\ [s]$.
3. The open-loop dc-gain should not be changed

As a help for the controller design, the system open-loop Bode diagram (without controller) is shown in Fig. 5.

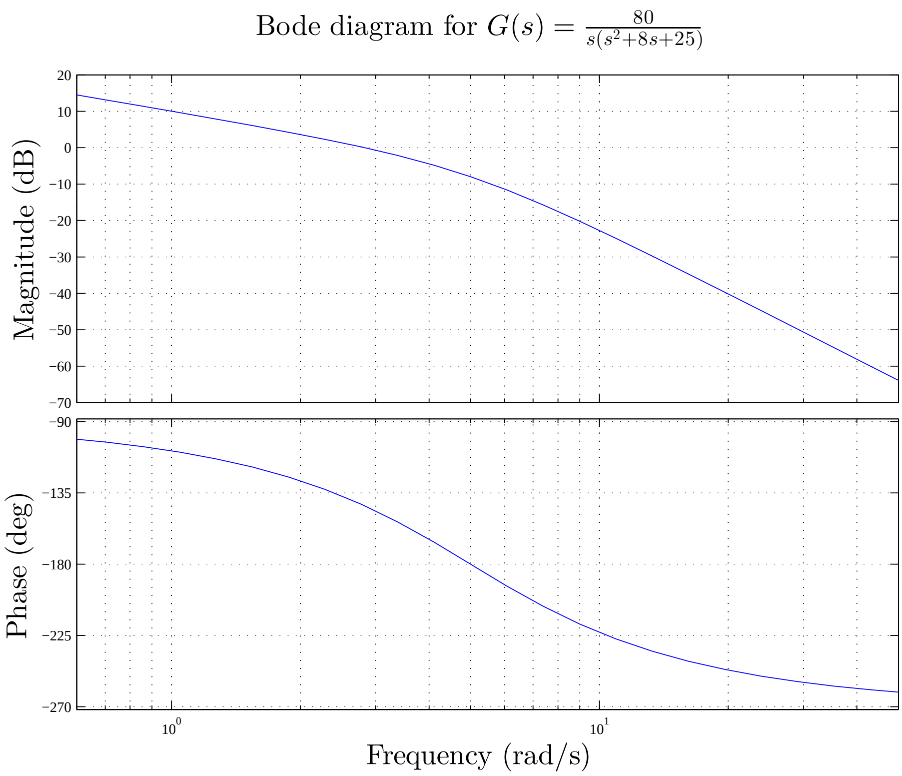

Figure 5: Open-loop Bode diagram for considered system (without controller).

Problem 5 (8 %)

The Z-transform of a number sequence $\{f(k)\}$ is given by

$$F(z) = \frac{2z - 3}{z(z - 0.5)}$$

(a) Determine the partial fraction expansion for $\dfrac{F(z)}{z}$

(b) Use the result of (a) to find the inverse Z-transform, i.e. find an expression for $f(k)$.

Problem 6 (8 %)

A continuous-time plant has the transfer function

$$G_p(s) = \frac{s - 2}{s + 1}$$

and it is preceded by a zero-order hold operating with the sampling time T.

(a) Find an analytic expression for the pulse transfer function $G(z)$

(b) Determine the poles and zeros for $G(z)$ and plot a zero-pole map for $G(z)$ for $T = 0.5$ seconds.

(c) Assume that the input to the plant is a unit step. Will the plant output converge to a finite value for $t \to \infty$?

Problem 7 (12 %)

A discrete controller $K(z)$ is given by

$$K(z) = \frac{M(z)}{E(z)} = \frac{z + 0.2}{z^2 - 0.8z + 0.8}$$

The sampling angular frequency is $\omega_s = 10$ rad/s.

(a) Find the controller's equivalent poles in the Laplace domain and determine the natural frequency (ω_n) and the damping factor (ζ) associated with these poles

(b) Determine the difference equation for $m(k)$ that corresponds to the transfer function $K(z)$

Now, assume that the input sequence to the controller is given by

$$e(k) = \begin{cases} 2, & k = 0 \\ 1, & k = 1 \\ 0, & \text{otherwise} \end{cases}$$

(c) Calculate the output samples $m(k)$ for $0 \le k \le 2$.

Problem 8 (12 %)

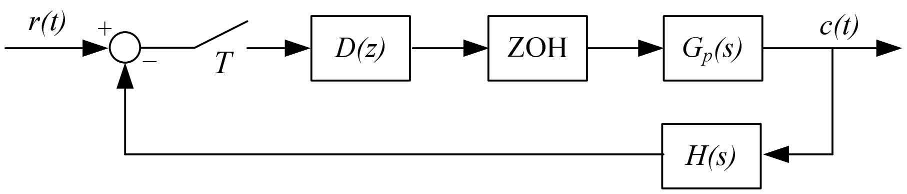

Figure 6:

The continuous plant $G_p(s)$ in Fig. 6 has the transfer function

$$G_p(s) = \frac{10}{s+10}$$

The feedback filter is $H(s) = 2$ and the discrete controller operating with the sampling frequency 20 Hz is represented by the transfer function:

$$D(z) = K\left(1 + \frac{0.6}{z}\right)$$

(a) Determine the total open-loop gain, i.e. find an expression for $D(z)\overline{GH}(z)$

(b) Find the roots of the characteristic equation for $K = 0.5$.

(c) Sketch the root locus (Note: you do *not* need to calculate precise values for breakaway points etc; a simple sketch is sufficient.)

Problem 9 (7 %)

(a) Determine the transfer function $H(z) = \dfrac{Y(z)}{X(z)}$ for the discrete-time system presented in Fig. 7

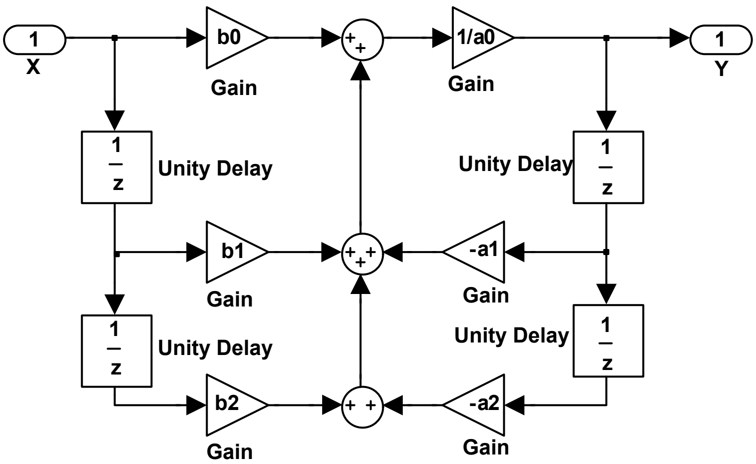

Figure 7: Discrete time system.

Problem 10 (6 %)

Write `Matlab` commands that will

(a) create a workspace vector `avec` with the elements $\texttt{avec}(n) = \frac{(-1)^n}{n}$, $n = 0..9$, without using loops, i.e. `for` and `while` commands must not be used.

Given a 3-by-3 matrix denoted by A. Write `Matlab` commands that

(b) creates a variable S which equals the sum of the diagonal elements of the matrix A

(c) defines a variable `invA`, where each element in `invA` is the inverse of the corresponding element in A. For example, $\texttt{invA}(2,3) = \frac{1}{\mathsf{A}(2,3)}$

Problem 11 (7 %)

A SISO system is given by its transfer function:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0}.$$

$X(s)$ and $Y(s)$ are the input and the output of the system, respectively. Coefficients a_0, a_1, b_0 and b_1 are constant non-zero gains.

(a) Using only Integrator, Gain and Sum blocks shown in Fig. 8, sketch the Simulink implementation of the SISO system. The Sum block may have more than two input ports.

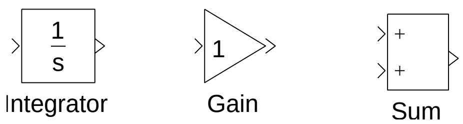

Figure 8: Simulink blocks.