# Opgaver lektion 1 IS



**2.15**

$S = \{C, S, N, P, U, O, Z\}$

$A = \{C, S, Z\}$ $B = \{S, N, P\}$ $C = \{O\}$

a) $A' = \underline{\{N, P, U, O\}}$ ✓

b) $A \cup C = \underline{\{C, S, Z, O\}}$ ✓

c) $(A \cap B') \cup C' = (\{C, S, Z\} \cap \{C, U, O, Z\}) \cup \{C, S, N, P, U, Z\}$

$= \{C, Z\} \cup \{C, S, N, P, U, Z\}$

$= \underline{\{C, S, N, P, U, Z\}}$ ✓

d) $B' \cap C' = \underline{\{C, U, Z\}}$ ✓

e) $A \cap B \cap C = \underline{\emptyset}$ ✓

f) $(A' \cup B') \cap (A' \cap C) = \{N, P, U, O, C, Z\} \cap \{O\} = \underline{\{O\}}$ ✓

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**2.37** 4 drenge og 5 piger skal sidde på **række** stolene

P D P D P D P D P en pige mere end drenge, dvs start med pige

Antal mulige måde: $5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 5!4! = \underline{2880}$ ✓

↑ multiplikationsprincippet (TI-89: 2nd MATH → Probability)

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**2.43** 5 forskellige tæver i cirkel; antal måder: $4! = \underline{24}$ ✓

↑ Theorem 2.5 $\left(\frac{5!}{5} = 4!\right)$ (5 ens cirkler bare rykket)

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**2.45** Antal forskellige permutationer af bogstaverne INFINITY:

i alt 8 bogstaver: 3 I'er, 2 N'er, 1 F, 1 T, 1 Y

# permutationer: $8!$

# forskellige permutationer: $\frac{8!}{3!2!1!1!1!} = \frac{8!}{3!2!} = \underline{3360}$ ✓

↑ $3!2!$ ens permutationer pga. 3 I'er 2 N'er

↑ Theorem 2.6

# Opgaver Lektion 1 IS.



**2.60** Kast med to terninger, antag at vi kan kende forskel på dem. (udtrukket med tilbagelægning)

Antal mulige udfald: $6 \cdot 6 = 36$

a) total på 8, dvs. (2,6) (6,2) (3,5) (5,3) (4,4)

$P(\text{total } 8) = \frac{5}{36}$ $\left(\frac{5 \cdot 1}{6 \cdot 6}\right)$

b) højst 5 i total, dvs (1,1) (1,2) (1,3) (1,4) (2,1) (2,2) (2,3) (3,1) (3,2) (4,1)

(eller total 2, 3, 4, 5: $1 + 2 \cdot 1 + 3 \cdot 1 + 4 \cdot 1 = 10$)

$P(\text{højst 5 total}) = \frac{10}{36} = \frac{5}{18}$

**2.63** Træk 5 kort ud af 52: $\binom{52}{5}$ måder

(udtrukket uden tilbagelægning)

a) $P(3 \text{ es'er}) = \frac{\binom{4}{3} \cdot \binom{48}{2}}{\binom{52}{5}} = \frac{4! \, 48! \, 5! \, 47!}{3! \, 1! \, 2! \, 46! \, 52!} = \frac{4^2 \cdot 47 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{52 \cdot 51 \cdot 50 \cdot 49} = \frac{94}{54145} = 0.0017$ ✓

b) $P(4 \text{ hjerter og 1 ikke}) = \frac{\binom{13}{4} \cdot \binom{13}{1}}{\binom{52}{5}} = \frac{13 \cdot 13! \, 47! \, 5!}{4! \, 9! \, 12! \, 52!} = \frac{13 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 5}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} = \frac{143}{39984} = 0.0036$ ✓

F: female N: nude M: male P: pyjamas T: T-shirt

**2.83** a) $P(F \cap N) = 0.018$ ✓ b) $P(M) = 0.22 + 0.002 + 0.160 + 0.162 + 0.046 + 0.084 = 0.614$ ✓

c) $P(P|M) = \frac{P(P \cap M)}{P(M)} = \frac{0.102}{0.614} = 0.166$ ✓

d) $P(M|PUT) = \frac{P(M \cap (PUT))}{P(PUT)} = \frac{0.102 + 0.046}{0.175 + 0.134} = 0.479$ ✓

# Opgaver lektion 1 I5



2.89 $P(\text{correct diagnose}) = 0.7$ dvs $P((\text{correct diag})') = 0.3$

$P(\text{law suit} \mid (\text{correct diagnose})') = 0.9$

$P((\text{correct diag})' \cap \text{law suit}) = P(\text{law suit} \mid (\text{correct diag})') P((\text{correct diag})')$

$= 0.9 \cdot 0.3$

$= \underline{0.27}$ ✓

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2.93 $P(\#1 \text{ available}) = P(\#2 \text{ available}) = 0.96$

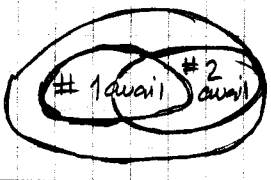
a) $P((\#1 \text{available})' \cap (\#2 \text{ available})') = P(\#1 \text{aval})' \cdot P(\#2 \text{aval})'$ (uafhængige)

$= 0.04 \cdot 0.04$

$= \underline{0.0016}$ ✓

b) $P(\#1 \text{ available} \cup \#2 \text{ availble}) = 1 - P(\#1 \text{avail})' \cap (\#2 \text{avail})')$

$= \underline{0.9984}$ ✓

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2.97 20 mælkekartoner med 5 defekte, udtag 4

a) Theorem 2.15:

$P(4 \text{ gode}) = P(1.\text{god} \cap 2.\text{god} \cap 3.\text{god} \cap 4.\text{god})$

$= P(1.\text{god}) \cdot P(2.\text{god} \mid 1.\text{god}) \cdot P(3.\text{god} \mid 1. \text{ og } 2.\text{god}) \cdot P(4.\text{god} \mid 1., 2. \text{ og } 3.\text{god})$

$= \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{13}{18} \cdot \frac{12}{17} = \underline{\frac{91}{323}}$ ✓

b) Theorem 2.8 + 2.9:

$P(4 \text{ gode}) = \frac{\binom{15}{4}}{\binom{20}{4}} = \frac{15!\,16!\,4!}{4!\,11!\,20!} = \frac{15 \cdot 14 \cdot 13 \cdot 12}{20 \cdot 19 \cdot 18 \cdot 17} = \underline{\frac{91}{323}}$ ✓ ens

# Opgaver lektion 1 IS



**2.101**

$P(\text{cancer}) = 0.05$ o: $P((\text{cancer})') = 0.95$

$P(\text{diag cancer} \mid \text{cancer}) = 0.78$

$P(\text{diag cancer} \mid (\text{cancer})') = 0.06$

$P(\text{diag cancer}) = P(\text{diag cancer} \mid \text{cancer}) \cdot P(\text{cancer}) +$ [eliminationsregel] $P(\text{diag cancer} \mid (\text{cancer})') \cdot P((\text{cancer})')$

$= 0.06 \cdot 0.95 + 0.78 \cdot 0.05 = \underline{\underline{0,096}}$ ✓

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**2.103**

$P(\text{cancer} \mid \text{diag cancer}) = \dfrac{P(\text{diag cancer} \mid \text{cancer})\, P(\text{cancer})}{P(\text{diag cancer})}$ [Bayes' formel]

$= \dfrac{0.78 \cdot 0.05}{0.096} = \underline{\underline{0.40625}}$ ✓

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**2.105**

A : no expiration data

$B_1$ : John $B_2$ : Tom $B_3$ : Jeff $B_4$ : Pat

$P(B_1) = 0.2$ $P(B_2) = 0.6$ $P(B_3) = 0.15$ $P(B_4) = 0.05$

$P(A \mid B_1) = 0.005$ $P(A \mid B_2) = 0.01$ $P(A \mid B_3) = 0.011$ $P(A \mid B_4) = 0.005$

$$P(B_1 \mid A) = \frac{P(A \mid B_1) \cdot P(B_1)}{P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + P(A \mid B_3)P(B_3) + P(A \mid B_4) \cdot P(B_4)}$$ [Bayes' formel udvidet]

$$= \frac{0.2 \cdot 0.005}{0.2 \cdot 0.005 + 0.6 \cdot 0.01 + 0.15 \cdot 0.011 + 0.05 \cdot 0.005}$$

$$= \frac{0\,001}{0.0089} = \underline{\underline{0.1124}}$$ ✓