ESTA EN LO QUE
ME PASO VICTOR.

Written exam in Probability & Statistics

PM6 & ET6

Lecturer: Kasper K. Berthelsen

Tuesday 4th of January 2005. 9:00-13:00

In the assessment emphasis will be put on both correct methods as wells as correct answer, hence the the method should be clearly stated.

Good luck!

Problem 1. (approx. 20%)
A random variable X has mean 10 and variance 50.

1. Calculate the mean and variance of the random variable $Y = 10 + 5X$.
2. Find the mean of $V = (X - 10)^2$ and $Z = X^2$.

$Y \quad E[Y] = E[10+5X] = 10 + 5E[X] = 10 + 5 \cdot 10 = \underline{60}$

2) $Var[X] = E[(X - E[X])^2] = E[(X-10)^2] = \underline{50}$

$Var[X] = E[X^2] - E[X]^2 = 50 \Leftrightarrow E[X^2] = 50 + E[X]^2 = 50 + 10^2 = \underline{150}$

Problem 2. (approx. 10%)
In a survey among house owner, people are asked if they are willing to pay more for snow removal. Among the 84 people who reply the answers are distributed according to age as follow:

Age	No	Yes	No answer
20–25	1	0	0
26–35	0	3	1
36–50	6	10	10
51–60	1	7	1
61–70	2	13	6
> 70	4	13	6

1) $P(\{Yes\} \mid \{Age \leq 50\}) = \dfrac{P(\{Yes\} \cap \{Age \leq 50\})}{P(\{Age \leq 50\})} = \dfrac{13/84}{31/84} = \underline{0{,}42}$

2) $P(\{Yes\} \mid \{Age > 50\}) = \dfrac{P(\{Yes\} \cap \{Age > 50\})}{P(\{Age > 50\})} = \dfrac{33/84}{53/84} = \underline{0{,}62}$

2) Indept. if only if $P(A \cap B) = P(A)P(B)$

$P(A \cap B) = \frac{13}{84} = 0{,}154$

$P(A) = \frac{31}{84}$, $P(B) = \frac{46}{84}$ $\quad P(A)P(B) = 0{,}202$

Conclusion: A and B dependent.

1. Calculate the conditional probability for answering yes, conditionally on the person's age being ≤ 50 years and > 50 years, respectively.
2. Are the events $A = \{\text{Age} \leq 50 \text{ years}\}$ and $B = \{\text{Yes}\}$ independent? Justify your answer.

Problem 3. (approx. 20%)
In an airport, whenever the metal detector goes off, there is a 25% probability that the alarm is caused by coins in the pocket of the passenger walking through the metal detector.

1. During one day the alarm goes off 15 times. What is the probability that at least 3 of these alrams are caused by passengers having coins in their pockets?
2. Question 1 continued: Is it likely that none of these 15 alarms are caused by coins in a pocket? Explain your answer based on the probability of this event.
3. Just before Christmas the airport is unusually busy. On one day the metal detector alarm goes off 50 times. What is the probability that at most $\frac{1}{5}$ of these alarms are caused by coins in a pocket.

1) $X \sim B(15, 0{,}25) \quad P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0{,}2361 = \underline{0{,}7639}$

2) $P(X=0) = 0{,}0134$, fairly unlikely

3) $X \sim B(50, 0{,}25)$, $E[X] = 50 \cdot 0{,}25 = 12{,}5 \quad V[X] = 0{,}25 \cdot (1 - 0{,}25) \cdot 50 = 9{,}375$

$X \sim N(12{,}5,\ 9{,}375) \quad P\left(X \leq \frac{50}{5}\right) = P\left(\frac{Z - 12.5}{\sqrt{9.375}} \leq \frac{10 - 12.5}{\sqrt{9.375}}\right) = P(Z \leq -0{,}82) = \underline{0{,}2061}$

1) $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{4227}{14} = \underline{\underline{301.93}}$ $\quad s^2 = \frac{\sum(x_i-\bar{x})^2}{n-1} = \frac{n\sum x_i^2 - (\sum x_i)^2}{n(n-1)} = \frac{14 \cdot 1339337 - (4227)^2}{14 \cdot 13} = 4832,687$ $\quad s = \sqrt{s^2} = \underline{\underline{69.66}}$

2) $(1-\alpha)100\%$ CI. $\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} = 301.93 \pm 2.160 \cdot \frac{69.66}{\sqrt{14}} = [308.73 , 295.13]$

Problem 4. (approx. 35%)

As is well-known, the department network is often down. Near the project dead-line some students decide to measure the dayly downtime in minutes. Acordingly they measure how many minutes the network is down each day for 14 days and obtain the following downtimes:

Day	1	2	3	4	5	6	7
downtime (minutes)	229	295	343	337	282	313	262

Day	8	9	10	11	12	13	14
downtime (minutes)	303	201	374	376	343	406	163

3) $(1-\alpha)100\%$ CI for σ^2

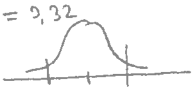
$\left[\frac{(n-1)s^2}{\chi_{\alpha/2,n-1}} , \frac{(n-1)s^2}{\chi_{1-\alpha/2,n-1}}\right] = \left[\frac{13 \cdot 4832,687}{24,74} , \frac{13 \cdot 4832,687}{5,01}\right] = [2539,86 ; 12543,035]$

The downtimes are assumed to follow a normal distribution with mean μ and variance σ^2.

1. Estimate the mean μ and the standard deviation σ for the dayly downtime.
2. Determine a 95% confidence interval for μ.
3. Determine a 95% confidence interval for σ.
4. The students want the downtime to be as short as possible. Test on the 5% significance level if the expected downtime is significantly less than 4 hours, i.e. 240 minutes.
5. What is the probability that the average down time over a 14 day period is less then 4 hours, i.e. 240 minute? Assume that the 14 downtimes are independent and normal distributed with equal means $\mu = 300$ and unknown and equal variances.

4) $H_0: \mu \geq 240 \quad H_1: \mu < 240$

$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{301,93-240}{69,66/\sqrt{14}} = 9,32$



$-t_{\alpha,n-1} = -2,16$

__We cannot reject H_0__

5) $P(\bar{x} < 240) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{240-\mu}{\sigma/\sqrt{n}}\right) = P\left(Z < \frac{-60 \cdot \sqrt{14}}{\sigma}\right)$

Problem 5. (approx. 15%)

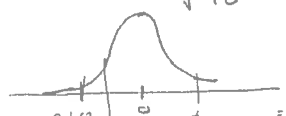
At a Christmas dinner 10 students measure their blood alcohol level by each making two measurements using a breathalyzer. They obtain the following measurements where the differences between measurements are given:

Student	1st measurement	2nd measurement	Difference
1	0.9	0.9	0.0
2	1.0	1.8	-0.8
3	1.8	1.8	0.0
4	1.2	1.6	-0.4
5	0.8	0.8	0.0
6	1.0	0.8	0.2
7	0.9	1.0	-0.1
8	1.2	2.1	-0.9
9	2.2	2.0	0.2
10	1.2	1.5	-0.3
$\bar{x}$	1.22	1.43	-0.21
s^2	0.197	0.260 (forkerte)	0.150

2) $H_0: \mu_1 = \mu_2 \quad \alpha = 0.05$

$H_1: \mu_1 \neq \mu_2$

$t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{-0,21}{\sqrt{\frac{0,150}{10}}} = -1,71$



$-2,62 \quad 0 \quad -1,71$

$t_{\alpha/2,n-1} = t_{0,025,9} = 2,62$

• __Cannot reject H_0__

1) $(1-\alpha)100\%$ CI:

$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$

$-0,21 \pm 1,833 \cdot \sqrt{\frac{0,150}{10}}$

$= [-0,435 ; 0,0145]$

It is assumed that the random variables correspoindg to the alchohol level for first and second measurments are independent and noraml distributed with equal mean and variance. Notice that the two measurments for the same student are **not** independent.

1. Find a 90% confidence interval for the difference in the two measurements.
2. Test at the 5% significance level if the level at the first measurement is different from the second measurement.

Remember to add student number on all sheets and state how many sheets your solution consists of.

4 of January 2005

Problem 1

A rando variabe X has mean 10 $\mu = 10$

an varian ce 50 $\sigma^2 = 50$

1) Calculate the mean and variance of the random variable

$Y = 10 + 5X$

$E(10+5X) = 10 + 5 \cdot E(x) = 10 + 5 \cdot 10 = \underline{\underline{60}}$

$\sigma^2(10+5X) = 5 \cdot \sigma^2 = \underline{\underline{250}}$

2) Find the mean of $V = (x-10)^2$ an $Z = X^2$

$E[(x-10)^2] = \sigma^2_x = 50$

$Z = x^2$ $E(x^2) = \sigma_x^2 + E^2(x) = 50 + 100 = 150$

PROBLEM 2

1) Calculate the conditional probability for answering yes

persona ≤ 50 years and > 50

- $p(\text{yes}/\leq 50) = \frac{13}{31} = \underline{0,42}$

- $p(\text{yes}/> 50)$ $\cancel{84-13}$ $\frac{33}{84-31} = \frac{33}{53} = \underline{0,62}$

se podría hacer este problema de una forma más lógica poniendo

$p(\text{yes}/\leq 50) = \frac{13/84}{31/84}$ $\begin{matrix} \leftarrow p(\text{yes}) \\ \leftarrow p(\leq 50) \end{matrix}$ $\}$ pero se va el 84 así que es lo mismo.

2) $A = \{\text{Age} \leq 50 \text{ years}\}$ and $B = \{\text{yes}\}$ independente!

$p(A \cap B) = p(A) \cdot p(B/A) =$

$\frac{13}{84} = \frac{31}{84} \cdot \frac{13}{31} =$

$0,1547 = 0,1547$

~~yes are independente~~

~~A and B.~~

$p(B) = p(B/A)$

$\frac{46}{84} = \frac{13}{31}$

$0,5476 \neq 4193 \rightarrow$ Dependiente. No are independent

PROBLEMA 3

Metal detector on $\left\langle \begin{array}{l} 25\% \text{ is for coins} \\ 75\% \text{ isn't for coins} \end{array} \right.$

1) $n = 15$ $\quad p(3 \leq x) = 1 - p(x \leq 2)$

$1 - p(x \leq 2) = 1 - (p(1) + p(2)) \to 1 - \left(\frac{15!}{1!\,14!} 0{,}25^1 \cdot 0{,}75^{14} + \frac{15!}{2!\,13!} 0{,}25^2 \cdot 0{,}75^{13} \right)$

$p(x \leq 2)$ $1 - (p(0) + p(1) + p(2))$ $\quad \boxed{\text{Acuerdate de cero}}$

$= \cancel{0{,}7772}$

$1 - \left(\frac{15!}{15!} 0{,}25^0 \cdot 0{,}75^{15} + \frac{15!}{1!\,14!} \cdot 0{,}25^1 \cdot 0{,}75^{14} + \frac{15!}{2!\,13!} \cdot 0{,}25^2 \cdot 0{,}75^{13} \right)$

$= \underline{0{,}763912}$

2) $p(0) = \frac{15!}{15!} \cdot 0{,}25^0 \cdot 0{,}75^{15} = 0{,}0133 \cancel{\%} = 1{,}33\%$

fairly unlikely $\quad$ bastante poco probable.

3) $50 = n$ $\quad E(x) = n \cdot p = 50 \cdot 0{,}25 = \underline{12{,}5 = \bar{x}}$

$p(x \leq 10)$ $\quad \sigma^2 = n \cdot p \cdot q = 50 \cdot 0{,}25 \cdot 0{,}75 = \underline{9{,}375 = \sigma^2}$

$\shortparallel$

$p(z \leq 0{,}8164) =$ $\quad \cancel{z = \frac{x - \bar{x}}{\sigma} = \frac{-12{,}5 + 10}{\sqrt{9{,}375}} = 0{,}8164}$

$1 - p(z \leq 0{,}8164)$ $\quad z = \frac{x - \bar{x}}{\sigma} = \frac{10 - 12{,}5}{\sqrt{9{,}375}} = \underline{-0{,}8164}$

$1 - 0{,}7939 = \underline{0{,}2061}$

Cuando

PROBLEM 4

1) • $E(x) = \sum_x x \, P(x) \Rightarrow$ or $\boxed{E(x) = \sum_{i=1} x_i \frac{1}{n} =}$

$$= \frac{229+295+343+337+282+313+262+303+201+374+376+343+406+163}{14}$$

$$= 301{,}93 = \bar{x}$$

• $\sigma^2(x) = \frac{n \cdot \sum x_i^2 - (\sum x_i)^2}{n \cdot (n-1)} = \frac{14 \cdot 1339337 - 17867529}{14 \cdot 13} = 4852{,}68$

$\sigma^2(x) = 4852{,}68 \qquad \sigma = \sqrt{4852{,}68} = \underline{69{,}661} = S$

2) Determine 95 % confidence interval for mean.

$$\mu \in \left(\bar{x} \pm \underset{13,\,0{,}025}{t_{n-1,\frac{\alpha}{2}}} \cdot \frac{S}{\sqrt{n}}\right) = 301{,}93 \pm 2{,}1604 \cdot \frac{69{,}661}{\sqrt{14}}$$

$$\mu \in (261{,}737 \ , 342{,}122)$$

3) Determine 95% confidence interval for ~~variance~~ standar desviation

$$\sigma^2 \in \left[\frac{(n-1)\, S_x^2}{\chi^2_{n-1,\frac{\alpha}{2}}} , \frac{(n-1)\, S_x^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}\right] =$$

$$= \left[\frac{13 \cdot 4852{,}68}{24{,}7356} \quad \frac{13 \cdot 4852{,}68}{5{,}0087}\right] = \left[2550{,}36 \ , 12595{,}05\right]$$

4) test 5% significance level. Aunque no me digan media simple lo hago con la media.

$H_0 \quad \mu \leq 240$ mm

$H_1 \quad \mu \geqslant 240$ mm

$$\frac{\bar{x} - \mu_0}{S/\sqrt{n}} \leq t_{\alpha, n-1} \rightarrow \frac{301{,}93 - 240}{69{,}661/\sqrt{14}} \leq 1{,}7709$$

$3{,}3264 \leq 1{,}7709$ $\quad H_0$ reject

H_1 Do not reject

5) $p(\bar{x} < 240)$ $\quad z = \frac{x - \bar{x}}{\sigma} = \frac{240 - 300}{300} =$

$p(x < -0{,}2)$ $\quad z = -0{,}2$

$p(z < -0{,}2) = 1 - p(z < 0{,}2) = 1 - 0{,}5793 = \underline{0{,}4207}$

PROBLEMS

1 - Find a 90% confidence interval

$$\mu \in \left[\bar{x} \pm t_{n-1, \frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}} \right]$$

$$\mu \in \left[-0,21 \pm 1,8331 \cdot \frac{\sqrt{0,15}}{\sqrt{10}} \right] = \langle$$

$$\mu \in [-0,4350 \ , \ 0,0145079]$$

2 - test 5% significance level $\mu_1 = \mu_2$

$$\frac{\bar{x}_1 - \bar{x}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \in \left[-t_{\alpha/2, n_1+n_2-2} \ , \ t_{\alpha/2, n_1+n_2-2} \right]$$

$0{,}025 \quad 18 \qquad 0{,}025, \ 18$

$$S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{9 \cdot 0{,}197 + 9 \cdot 0{,}260}{18}$$

$$\frac{1{,}22 - 1{,}43}{0{,}2285 \sqrt{\frac{1}{10} + \frac{1}{10}}} \in [-2{,}1098 \ , \ 2{,}1098]$$

$$-2{,}0550 \ [-2{,}1098 \ , \ 2{,}1098]$$

Ho Don't reject

Written exam in Probability Theory and Statistics - K7

Lecturer: P. Svante Eriksen
Thursday 13th of january 2011, 9:00-13:00
In the assessment emphasis will be put on both correct methods as well as correct answers. Hence the method should be clearly stated.

Problem 1. (approx 20%)
The random variable X has a normal distribution with mean 4 and variance 25. 1. Mean: $4E(X)+6 = 4\cdot4+6 = 22$ Var: $16\cdot\sigma_X^2 = 400$

1. Calculate the mean and variance of the variable $4X + 6$.
2. Calculate $P(0 \leq X \leq 4)$. $= P(X \geq 0) - P(X \geq 4) = 0.5 - 0.212 = 0.288$

The random variable Y has mean 5 and variance 10. The correlation coefficient of X and Y is -0.5. Mean: $4E(X) + 5E(Y) + 1 = 4\cdot4 + 5\cdot5 + 1 = 42$

3. Calculate the mean and variance of the variable $4X + 5Y + 1$. Var: $4^2\sigma_X^2 + 5^2\sigma_Y^2 + 2\cdot4\cdot5\,\sigma_{XY} =$
$16\cdot25 + 25\cdot10 + 40\times(-0.5\cdot\sqrt{25\cdot10})$
≈ 333.8

Dist of X:

x \ y	0	1	2	SUM
0	0	1/27	2/27	3/27
1	2/27	3/27	4/27	9/27
2	4/27	5/27	6/27	15/27
	6/27	9/27	12/27	27/27

Problem 2. (approx 20%)
The joint probability distribution of X and Y is given by

$E(X)$ $E(Y)$
σ_X σ_Y

$$f(x, y) = \frac{2x + y}{27}, \quad x = 0, 1, 2; \; y = 0, 1, 2$$

1. Evaluate the marginal distribution of X. $P(Y=2 \mid X=1) = \frac{f(1,2)}{P(X=1)} = \frac{4}{9}$
2. Find $P(Y = 2|X = 1)$ and $P(Y = 2|X = 2)$. Are X and Y statistically independent? $P(Y=2 \mid X=2) = f(2,2)/P(X=2) = \frac{6}{15}$

No:
$P(Y=2 \mid X=1) \neq P(Y=2 \mid X=2)$

3. Evaluate $E(X^2Y)$. $\sum_{x=0}^{2}\sum_{y=0}^{2} x^2y\, f(x,y) = 1\cdot\frac{3}{27} + 2\cdot\frac{4}{27} + 4\cdot\frac{5}{27} + 8\cdot\frac{6}{27} = \frac{79}{27}$

Problem 3. (approx 10%)
In a certain city the need for money to buy drugs is stated as the reason for 60% of all thefts.
Consider the next 20 theft cases in the city and let X denote the number of cases resulting from the need for money to buy drugs.

1. Calculate the mean and variance of X. $\sim$ binomial $(n=20,\ p=0.6)$
2. Evaluate $P(4 \leq X \leq 12)$. $EX = n\cdot p = 12$ $\sigma_X^2 = n\cdot p(1-p) = 4.8$

$= P(X \leq 12) - P(X \leq 3) \approx 0.548$

Marginal dist X
$X_0 = \frac{3}{27}$
$X_1 = \frac{9}{27}$
$X_2 = \frac{15}{27}$

Problem 4. (approx 30%)
An engineer in quality control takes a sample of 30 bolts and measures their diameter, which yields a sample average of $\bar{x} = 10.023mm$ and a sample standard deviation $s = 0.009mm$. He assumes that the observations are a random sample from the normal distribution.

Walpole p. 279
$v = 29$, $t_{\alpha/2} = 2.045$
$\rightarrow [10.01964, 10.02635]$

1. Determine a 95% confidence interval for the mean of the bolt diameter.

2. Determine a 95% confidence interval for the standard deviation of the bolt diameter.

Walpole p. 307:
$v = 29$,
$\chi^2_{\alpha/2} = 16.05$
$\chi^2_{1-\alpha/2} = 45.72 \rightarrow$
$[0.514, 1.464] \times 10^{-4}$
$\sqrt{\ } \rightarrow [0.717, 1.210] \times 10^{-2}$

3. Test at the 5% significance level whether the bolts meet a requirement of a mean diameter equal to $10mm$.

4. Test at the 2.5% significance level whether the measurements meet a requirement of a standard deviation below or equal to $0.005mm$.

Not in confidence interval $\Rightarrow$ reject.

Problem 5. (approx 20%)
Two methods for measuring the molar heat of fusion of water are being compared. Ten measurements made by method A have a sample mean $\bar{x}_A = 6.025$ kilojoules per mole and sample standard deviation of $s_A = 0.024KJ/mol$. Five measurements made by method B have a sample mean $\bar{x}_B = 6.001KJ/mol$ and sample standard deviation of $s_B = 0.012KJ/mol$.

1. Test at the 5% significance level whether the two methods have the same standard deviation.

Walpole p 368
$F = \left(\frac{24}{12}\right)^2 = 4$
$v_1 = 9 \quad v_2 = 4$
$f_{\alpha/2}(9, 4) = 8.9 > 4$
Do not reject

2. Test at the 5% significance level whether the mean measurements differ between the two methods.

Remember to add student number on all sheets and state how many sheets your solution consists of

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Walpole p. 346: $s_p^2 = 4.43 \times 10^{-4}$
$t = 2.082 < t_{\alpha/2, 13} = 2.16$
Do not reject.

13 January 2011

PROBLEM 1

The random variable X normal distribution with mean → $\mu = 4$

$\sigma^2_{(x)} = 25$

1)- mean → $4X+6 \to E(4x+6) = 6+4\cdot E(x) = 6+4\cdot 4 = 22$

Variance → $6+4x \to \sigma^2(6+4x) = 4^2\cdot \sigma_x^2 = 400$

2) Calculate $P(0 \leq x \leq 4)$

$P(0 \leq x \leq 4) = \cancel{P(0 \leq z} \Rightarrow$ $z = \frac{x-\bar{x}}{\sigma}$ $\frac{0-4}{25} = -0,16$; $\frac{4-4}{25} = 0$

$= P(x \leq 4) - P($

$P(0 \leq z \leq -0,16) \to P(z \leq -0,16) - P(z \leq 0)$

$P(z \leq -0,16) \Rightarrow$ ~~P(z≤~~ $1 - P(z \leq 0,16) = \cancel{1-(1-P(z \leq 0,84))}$

$= 1 - 0,5636 = 0,4364$

$P(z \leq 0) = 0,5$

$P(0 \leq z \leq -0,16) = 0,4364 - 0,5 = -0,636$ X ?!

$\mu_y = 5$ $\sigma_y^2 = 10$ Correlation Coefficient of X and Y = -0,5

correlation coefficien of pirsson. -0,5

$4X+5Y+1 \to E(ax+by+1) = a\cdot E(x) + b(E(y)) + 1 = 4\cdot 4 + 5\cdot 5 + 1 = 42$

$4x+5Y+1 \to \sigma^2(ax+by+1) = \sigma^2(4x+5y) = 4^2\cdot\sigma_x^2 + 5^2\cdot\sigma_y^2 + 2\cdot 4\cdot 5\cdot Cov(xy)$

$\rho = \frac{cov(xy)}{\sigma_x\ \sigma_y} \Rightarrow cov(xy) = \rho\cdot\sigma_x\,\sigma_y = -0,5\cdot\sqrt{10}\cdot\sqrt{25} =$

$\sigma^2(4x+5y) = 16\cdot 25 + 25\cdot 10 + 40\cdot(-7,905) = 333,8$

PROBLEM 2

Joint probability distribution X and Y

$$g(x,y) = \frac{2x+y}{27} \qquad x=0,1,2 \qquad y=0,1,2$$

1)

X \ Y	0	1	2	sum
0	0	1/27	2/27	3/27
1	2/27	3/27	4/27	9/27
2	4/27	5/27	6/27	15/27
sum	6/27	9/27	12/27	1

← Marginal distribution of X

← la suma de la probabilidad de un suceso siempre tiene que ser 1.

2) Find $P(Y=2/X=1)$ $4/27$

$$P(Y=2/X=1) = \frac{P(Y=2 \cap X=1)}{P(X=1)} = \frac{4/27}{9/27} = \boxed{\frac{4}{9}}$$

$$P(Y=2/X=2) = \frac{P(Y=2 \cap X=2)}{P(X=2)} = \frac{6/27}{15/27} = \boxed{\frac{6}{15}}$$

~~3~~) Independent?! $P(Y=2 \cap X=1) = P(X=1) \cdot P(Y=2/_{X=1})$

$$P(X=1 \cap Y=2) = P(X=1) \cdot P(Y=2/_{X=1})$$

$$\overset{\shortparallel}{\frac{4}{27}} \neq \frac{9}{27} \cdot \frac{4}{27}$$

No. Are Dependent.

3) Evaluate $E(X^2 \cdot Y)$

$$\sum_x \sum_y x^2 y \; P(x,y) = \cancel{(0^2 \cdot 0) \cdot 0} + \cancel{(0^2 \cdot 1)} \cdot \frac{1}{27} + \cancel{(0^2 \cdot 2)} \cdot \frac{2}{27} +$$

$$+ \cancel{(1^2 \cdot 0)} \cdot \frac{2}{27} + (1^2 \cdot 1) \cdot \frac{3}{27} + (1^2 \cdot 2) \cdot \frac{4}{27} + (2^2 \cdot 0) \cdot \frac{4}{27} + (2^2 \cdot 1) \cdot \frac{5}{27} +$$

$$+ (2^2 \cdot 2) \cdot \frac{6}{27} =$$

$$= 1 \cdot \frac{3}{27} + 3 \cdot \frac{4}{27} + 4 \cdot \frac{5}{27} + 8 \cdot \frac{6}{27} = \frac{79}{27}$$

$\boxed{\text{PROBLEMA 3}}$

60% Probability of flights $p = 0,6$

20 $n = 20$

1) $\boxed{E(x) = n \cdot p} = 20 \cdot 0,6 = 12$

2) Evaluate $P(4 \leq x \leq 12) = p(x \leq 12) - p(x \leq 4)$

Esto nos daria solo la probabilidad de 12

$$P(12) = \binom{20}{12} \cdot 0,6^{12} \cdot 0,4^{8} = \frac{20!}{12! \cdot 8!} \cdot 0,6^{12} \cdot 0,4^{8} =$$

Esto la probabilidad de $(x \leq 12)$

$$P(x \leq 12) = P(1) + P(2) + P(3) + P(4) + P(5) \ldots$$

$$P(x \leq 12) = 0,5841$$

$$P(x \leq 3) = 0$$

$$P(4 \leq x \leq 12) = \underline{0,5841}$$

Seria una locura calcularlo así que me ayudo de una tabla. En la tabla ~~tom~~ el valor que cojo es como si hiciera la función de distribución.

PROBLEM 4

Sample of 30 bolts $\boxed{n = 30}$

sample average $\boxed{\bar{x} = 10{,}023 \text{ mm}}$

standar desviation $\boxed{s = 0{,}009 \text{ mm}}$

1) 95% confidece interval mean. $\alpha = P-1 = 0{,}5$ $\frac{\alpha}{2} = 0{,}025$ $1-\frac{\alpha}{2} = 0{,}975$

$$\mu \in \left[\bar{x} \pm t_{n-1,\frac{\alpha}{2}} \cdot \frac{S_x}{\sqrt{n}} \right] = \left[10{,}023 \pm 2{,}0542 \cdot \frac{0{,}009}{\sqrt{30}} \right]$$

$$\boxed{\mu \in \left[10{,}019624 \; , \; 10{,}02633 \right]}$$

2) 95% confidende interval standar desviation $\frac{\alpha}{2} = 0{,}025$ $1-\frac{\alpha}{2} = 0{,}975$

$$\sigma^2 \in \left[\frac{(n-1)S^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \; , \; \frac{(n-1)S_x^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}} \right] = \left[\frac{29 \cdot 0{,}009^2}{45{,}7223} \; , \; \frac{29 \cdot 0{,}009^2}{16{,}0471} \right]$$

$$\boxed{\sigma^2 \in \left[5{,}137 \cdot 10^{-5} \; , \; 1{,}4638 \cdot 10^{-4} \right]}$$

3) test 5% mean $= 10$ mm

$H_0 : \mu = \mu_0$ $\quad \mu = 10$

$H_1 \; \mu \neq \mu_0$ $\quad \mu \neq 10$

$$\frac{|\bar{x} - \mu_0|}{s/\sqrt{n}} \leq t_{\alpha/2,\, n-1} \quad {}_{0{,}025,\,29}$$

$\Downarrow$

$$\frac{|10{,}023 - 10|}{0{,}009/\sqrt{30}} \leq 2{,}0452 \rightarrow 13{,}99735 \quad H_0 \text{ is reject}$$

4) test at the 2,5% standed desviation $\sigma^2 \leq 0{,}005$ mm

5

PROBLEM 5

$\bar{x}_A = 6{,}025$ Kilojoules/mol
$S_A = 0{,}024$ KJ/mol
} $n = 10$ Ⓐ method

$\bar{x}_B = 6{,}001$ KJ/mol
$S_B = 0{,}012$ KJ/mol
} $n = 5$ Ⓑ method

1. test 0,05 significance level | $H_0: \sigma_A^2 = \sigma_B^2$

$H_1: \sigma_A^2 \neq \sigma_B^2$

$$\frac{S_A^2}{S_B^2} \in \left[\underset{(0{,}975)\ (9,4)}{F_{1-\frac{\alpha}{2};(n_A-1)(n_B-1)}} ; \underset{(0{,}025)\ (9,4)}{F_{\alpha/2;(n_A-1)(n_B-1)}} \right]$$

$$\frac{0{,}024^2}{0{,}012^2} \in \left[\frac{1}{F_{\alpha/2\ 0{,}025,9,4}} ; F_{\alpha/2\ 0{,}025,9,4} \right]$$

$$4 \in \left[\frac{1}{8{,}9} , 8{,}9 \right] = [0{,}112 , 8{,}9]$$ Do not reject.

2. test at the 0,05 significance level $\mu_A \neq \mu_B$

H_0 $\mu_A \neq \mu_B$ H_1 $\mu_A = \mu_B$

$$\frac{\bar{x}_1 - \bar{x}_2}{S_p\sqrt{1/n_1 + 1/n_2}} \in \left[-t_{\alpha/2, n_1+n_2-2} , + t_{\alpha/2, n_1+n_2-2} \right]$$

$$S_p = \frac{n_1 - 1 \cdot S_1^2 + (n_2-1) S_2^2}{n_1 + n_2 - 2} = \frac{9 \cdot 0{,}024^2 + 4 \cdot 0{,}012^2}{13} = 4{,}4307$$

$$\frac{6{,}025 - 6{,}001}{4{,}4307 \cdot 10^{-4}\sqrt{1/10 + 1/5}} \in \left[-t_{0{,}025,13} , t_{0{,}025,13} \right]$$

$$98{,}89 \in [-2{,}1604 , 2{,}1604]$$

Do not reject.

Written exam in Probability & Statistics

PM6 & ET6

Lecturer: Kasper K. Berthelsen

Friday 6th of January 2006, 9:00-13:00

In the assessment emphasis will be put on both correct methods as wells as correct answer, hence the the method should be clearly stated.

Good luck!

Problem 1. (approx. 20%)
A salesman at a used car dealer receives a commission for each car or van he sells. When he sells a car he receives 4200 kr and 4800 kr when he sells a van. He expects to sell a number of cars and vans each day according to the following probabilities:

X

Number of cars	0	1	2	3
Probability	0.3	0.4	0.2	0.1

Y

Number of vans	0	1	2
Probability	0.4	0.5	0.1

1. Calculate the expected number of cars and vans the salesman is expected to sell each day. $\mu_x = E[X] = \sum_x x\,P(x) = 1{,}1 \quad \mu_y = E[Y] = 0{,}7$
2. Calculate the standard deviation of the number of cars and van the salesman sells in a day. $\sigma_x^2 = \sum_x (x-\mu_x)^2 P(x) = \sum x^2 p(x) - E[x]^2 = 0{,}89 \quad \sigma_x = 0{,}94 \quad \sigma_y^2 = 0{,}41 \quad \sigma_y = 0{,}64$
3. Calculate the expected commission for both cars and vans a salesman will receive in a day. $E[4200X + 4800Y] = 4200\,E[X] + 4800\,E[Y] = 4200 \cdot 1{,}1 + 4800 \cdot 0{,}7 = 7980$
4. Calculate the standard deviation of the salesman total commission in a day when we assume that the number of sold cars and sold vans are dependent with a correlation coefficient of $\rho = 0.1$.

$\rho = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$

$Cov(X,Y) = \rho\,\sigma_x \sigma_y$

$Var(4200X + 4800Y) = 4200^2\,Var(X) + 4800\,Var(Y) + 2 \cdot 4200 \cdot 4800 \cdot Cov(X,Y)$

$= 4200^2 \cdot 0{,}89 + 4800^2 \cdot 0{,}41 + 2 \cdot 4200 \cdot 4800 \cdot 0.1 \cdot 0.94 \cdot 0.64$

$= 27571165\,1$

$\sqrt{} = 5250.87$

Problem 2. (approx. 15%)
The length of times it takes to repair a vending machine follows a normal distribution with mean 120 minutes and variance 16 minutes^2. If the vending machine is under repair for more than 125 minutes the machines must be cleaned and emptied which is an unwanted extra expense.

1. What is the probability that the vending machine is under repair for more than 125 minutes?
2. A member of staff wants to find a time interval in which the time it takes to repair the vending machine is with 95% probability. Find such a 95% probability interval which is symmetric around the mean.

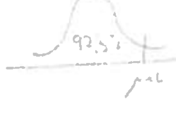
$X \sim N(120, 16)$

1) $P(X > 125) = P\left(\frac{X-120}{4} > \frac{125-120}{4}\right) = P(Z > 1{,}25) = 1 - P(Z < 1{,}25) = 1 - 0{,}89$
$= 0{,}11$

2) $P(\mu - k < X < \mu + k) = 0.95$

$P(X < \mu + k) = 0{,}975$

$P\left(\frac{X-\mu}{\sigma} < \frac{\mu+k-\mu}{\sigma}\right) = P\left(Z < \frac{k}{\sigma}\right) = 0{,}975$



$P(Z < 1{,}96) = 0{,}975$

$\frac{k}{\sigma} = 1{,}96 \quad k = 1{,}96 \cdot \sigma$

$[\mu - k, \mu + k] = [\mu - 1{,}96 \cdot \sigma, \mu + 1{,}96\sigma] = [120 - 1{,}96 \cdot 4, 120 + 1{,}96 \cdot 4] = [112{,}16; 127{,}84]$

Problem 3. (approx. 15%)
Wanting to optimise storage space a seller wants to model the number of orders on a specific product in December. In December the previous year the number of orders was 15.

Begivenhed mangler

1. Specify a random variable and its distribution, so that it describes that number of orders in December — explain your choice. $X \sim \text{Poisson}(15)$ (Tabel A.2)
2. What is the probability of 17 or more orders. $P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.6641 = 0.3359$
3. How large does stock need to be for the seller to have at least a 95% probability of fulfilling all orders? Assume that the seller cannot receive new stock during December.

$P(X \leq k) \geq 0.95$ $\quad P(X \leq 22) = 0.9673$: Needs 22 items in stock.

Problem 4. (approx. 30%)
The walls in a plastic bottle need to have a certain thickness to avoid that the bottle ~~does~~ breaks. An engineer in quality control takes a sample of 25 bottles and measures the wall thickness obtaining a sample average of $\bar{x} = 4.05mm$ and a sample standard deviation of $s = 0.08mm$. He further assumes that the observations are independent and normal distributed.

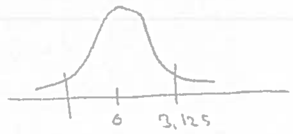
1) $(1-\alpha)100\%$ conf.-int.: $\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} = 4.05 \pm 2.064 \cdot \frac{0.08}{\sqrt{25}} = [4.017; 4.083]$ (Table A.4)

1. Determine a 95% confidence interval for the mean of the wall thickness.
2. Determine a 95% confidence interval for the standard deviation of the wall thickness.
3. Test at the 5% significance level if the wall thickness is less than $4mm$.
4. Test at the 5% significance level if the standard deviation of the wall thickness equals 0.1

2) $\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} ; \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right] = \left[\frac{24 \cdot 0.08^2}{39.364} , \frac{24 \cdot 0.08^2}{12.401}\right] = [0.0039 ; 0.0124]$

Problem 5. (approx. 20%)
A cement factory wants to buy a new machine for filling bags with 50kg of cement. They have two machines to choose from. From each machine they take a sample of 6 bags and weigh each of them. The measured weight are given in the table below

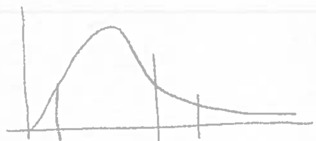
3) $H_0: \mu \leq 4$
$H_1: \mu > 4$
$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{4.05 - 4}{0.08/\sqrt{25}} = 3.125$



$-t_{\alpha/2,n-1} = -1.71$
Do not reject H_1

	Machine I	Machine II
	51.2	29.4
	49.0	50.7
	49.8	49.1
	51.7	48.7
	50.3	48.7
	51.4	50.1
$\bar{x}$	50.57	49.80
s^2	1.0987	0.7520

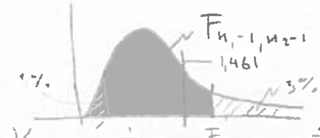
4) $H_0: \sigma = 0.1 \quad H_1: \sigma \neq 0.1$
$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{24 \cdot 0.08^2}{0.1^2} = 15.36$



$\chi^2_{0.975,24} = 12.401$ $\quad$ 15.36 $\quad$ $\chi^2_{0.025,24} = 39.364$
Do not reject H_0

1. Test at the 10% significance level if the variance of the weights are equal for the two machines.
2. Test at the 10% significance level if the means of the weights are equal for the two machines.

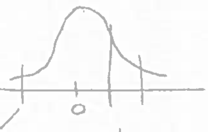
1) $H_0: \sigma_1^2 = \sigma_2^2 \quad H_1: \sigma_1^2 \neq \sigma_2^2$
$f = \frac{s_1^2}{s_2^2} = \frac{1.0987}{0.7520} = 1.461$ 10%
Cannot reject H_0: Assume equal variance!



$F_{n_1-1,n_2-1} = F_{5,5}$
$1/F_{0.05,5,5} = \frac{1}{5.05} = 0.198$ $\quad F_{0.05,5,5} = 5.05$

2) $H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$
$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = 0.92535$
$t = \frac{\bar{x}_1 - \bar{x}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = 1.3864$

Remember to add student number on all sheets and state how many sheets your solution consists of.



-1.812 $\quad t_{\alpha/2,n_1+n_2-1} = t_{0.05,10} = 1.812$
Cannot reject H_0

6 January 2006

Problem 1

each car comision → 4200

each van comision → 4800

1) Number of cars expected to sell each day

$E(x) = \sum_x x \cdot P(x) \to (0 \cdot 0{,}3) + (1 \cdot 0{,}4) + (2 \cdot 0{,}2) + (3 \cdot 0{,}1) = \underline{1.1}$

Number of van expected to sell each day

$E(y) = \sum_y y \cdot P(y) = (0 \cdot 0{,}4) + (1 \cdot 0{,}5) + (2 \cdot 0{,}1) = \underline{0{,}7}$

2) Standar desviation of the cars.

$\sigma_x^2 = \sum_x x^2 \cdot P(x) - E(x)^2 \to (0^2 \cdot 0{,}3) + (1^2 \cdot 0{,}4) + (2^2 \cdot 0{,}2) + (3^2 \cdot 0{,}1) - (1{,}1)^2 = 0{,}89$

$\sigma_x^2 = 0{,}89 \qquad \sigma_x = \sqrt{0{,}89} \Rightarrow \sigma_x = \underline{0{,}94}$

Standar desviation of the vans

$\sigma_y^2 = \sum_y y^2 \cdot P(y) - E(y)^2 \to (0^2 \cdot 0{,}4) + (1^2 \cdot 0{,}5) + (2^2 \cdot 0{,}1) - 0{,}7^2 = 0{,}41$

$\sigma_y^2 = 0{,}41 \qquad \sigma_y = \sqrt{0{,}4} \Rightarrow \sigma_y = \underline{0{,}64}$

3) Comision for car each day

otra forma
$\boxed{E(4200x + 4800y) = E(x) \cdot 4200 + E(y) \cdot 4800}$

$E(x) \cdot \text{comision}_x = 1{,}1 \cdot 4200 = 4620$

$E(y) \cdot \text{comision}_y = 0{,}7 \cdot 4800 = 3360$

total → $(E(x) \cdot c_x) + (E(y) \cdot c_y) = 7980$

4) Stander desviation of salman comision → correlation coefficient of pirson

$p = 0{,}1$

• $\sigma^2(ax + by) = a^2 \cdot \sigma_x^2 + b^2 \cdot \sigma_y^2 + 2 \cdot a \cdot b \cdot Cov(x,y)$

we need to know $Cov(x,y) \to p = \dfrac{cov(x,y)}{\sigma(x) \cdot \sigma(y)}$ so → $cov(x,y) = p \cdot \sigma(x) \cdot \sigma(y)$

• $\sigma^2(ax+by) = a^2 \cdot \sigma_x^2 + b^2 \cdot \sigma_y^2 + 2 \cdot a \cdot b\,(p \cdot \sigma(x)\,\sigma(y)) =$

$= (4200)^2 \cdot 0{,}89 + 4800^2 \cdot 0{,}41 + 2 \cdot 4200 \cdot 4800 \cdot 0{,}1 \cdot 0{,}64 \cdot 0{,}94 = 27571651{,}2$

$\sigma^2(ax+by) = 27571651{,}2 \qquad \sigma(ax+by) = 5250{,}87$

PROBLEM 2

Normal distribution

mean $= 120$ minutes $\rightarrow \mu_x = 120$

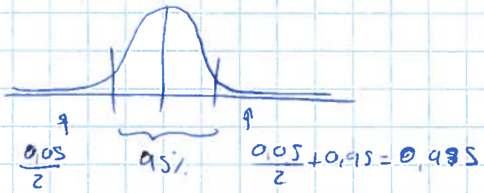
variance $= 16$ $\rightarrow \sigma^2_{(x)} = 16$

1) Probability > 125 minutes. $P(125 > x) = P(1{,}25 > z)$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{125 - 120}{\sqrt{16}} = 1{,}25$$

$$P(1{,}25 > z) = 1 - P(1{,}25 < z) = 1 - 0{,}89 = \underline{0{,}11}$$

2)



$P(1{,}96 > z) = 0{,}975$ so $\dfrac{x - \bar{x}}{\sigma} = 1{,}96$ $x = (1{,}96 \cdot 4) + 120 = \boxed{127{,}84}$

$x = 120 - (1{,}96 \cdot 4) = \boxed{112{,}16}$

$0,003,904 \cdot 10^{-3}$ $0,003904$

$0,1238$

PROBLEM 4

sample 25 bottles

average of samble wall thickness $\bar{x} = 4,05$ mm

sample standar desviation $s = 0,08$ mm

1) 95% interval confiance of the mean

$$\mu \in \left[\bar{x} \pm t_{n-1, \frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}} \right] \qquad \alpha = 1 - p \Rightarrow \alpha = 1 - 0,95 = 0,05$$

$$\frac{\alpha}{2} = 0,025$$

$$\mu \in \left[4,05 \pm 2,0693 \cdot \frac{0,08}{\sqrt{25}} \right] \quad \boxed{\text{Confiance interval } [4,016891, 4,083108]}$$

2) 95% interval confiance of the varance.

$$\sigma^2 \in \left[\frac{(n-1) S^2}{X^2_{n-1, \frac{\alpha}{2}}}, \frac{(n-1) S_x^2}{X^2_{n-1, 1-\frac{\alpha}{2}}} \right] \qquad \frac{\alpha}{2} = 0,025 \quad 1 - \frac{\alpha}{2} = 0,975$$

$$\sigma^2 \in \left[\frac{24 \cdot 0,08^2}{39,341}, \frac{24 \cdot 0,08^2}{12,4011} \right] = \boxed{\text{confiance interval } [0,003904, 0,01238]}$$

3) 5% significate level $H_0 \; \mu \cancel{<} 4$

$H_1 \; \mu \geq 4$

$$\frac{\bar{x} - \mu_0}{S/\sqrt{n}} < t_{\alpha, n-1} \longrightarrow \frac{4,05 - 4}{0,08/\sqrt{25}} = \underbrace{3,125 < 1,719}_{\text{reject}}$$

$\alpha = 0,05$

that $\mu < 4$

$\boxed{\text{Dont reject that } \mu \geq 4}$

PROBLEM 5

1) $n = 6$

$\bar{x}_1 = 50,57$ $\bar{x}_2 = 49,8$

$s_1^2 = 1,0987$ $s_2^2 = 0,7520$

test at 10% significate level $\sigma_1^2 = \sigma_2^2 = H_0$

$$\frac{s_1^2}{s_2^2} \in \left[F_{1-\frac{\alpha}{2}};(n_1-1)(n_1-1), F_{\alpha/2}(n_1-1)(n_2-1)\right]$$

$1-\frac{\alpha}{2} = 1-\frac{0,1}{2} = \underline{0,95, 5 \cdot 5}$ $0,5 \cancel{5}, 5 \cdot 5$

$$\frac{1,0987}{0,7520} \in \left[1/F_{0,05,\cancel{8},5}, F_{0,05,\cancel{8},5}\right] = \left[\frac{1}{5,05}, 5,05\right]$$

$1,4611 \in [0,19801; 5,05]$ Do not reject.

H_0 Do not reject.

2) test at the 10% significance level μ

$H_0 \to \mu_1 = \mu_2$
$H_1 \to \mu_1 \neq \mu_2$

$$\frac{\bar{x}_1 - \bar{x}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \in \left(-t_{\alpha/2, n_1+n_2-2}, +t_{\alpha/2, n_1+n_2-2}\right)$$

$$S_p^2 = \frac{(n_1-1)\cdot S_1^2 + (n_2-1)\cdot S_2^2}{n_1+n_2-2} = \frac{5\cdot 1,0987 + 5\cdot 0,7520}{10} = 0,9253$$

$S_p = 0,96193$

$$\frac{50,57 - 49,8}{0,96195\sqrt{\frac{1}{8}+\frac{1}{6}}} \in \left[-t_{0,05,10}, +t_{0,05,10}\right]$$

$1,3864 \in [-1,8125, 1,8125]$

H_0 Dont reject.

PROBLEM 3

mean = 15 in december of the previous year.

1 - Specify a random variable.

$X \sim$ Poisson (15) → is poisson because is like a Binomial but. n, that is the numbers of the day is big, and p, that is the probability of the exit is small.

2 - What is the probability of 17 or more orders.

$\lambda = 15$

$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0,6641 = \underline{0,3359}$

3 - at least 95%, I take to the table 0,9673 96,73%

and that is 22.

$P(X \leq K) \geq 0,95$ $P(\leq 22) = 0,9673$ Need 22 items in stok.