Digital Control of Switch-Mode Converters

Topic 9.

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Agenda

- Introduction
- Signal processing
- Sampling and ADC
- Pulse Width Modulation
- Controller design
  - Discretization
  - Design

Introduction

- Digital control offers the possibility to implement sophisticated control laws, taking care of system nonlinearities, parameter variations or construction tolerances by means of self-analysis and auto tuning strategies, very difficult or impossible to implement analogically.

- Software based digital controllers are inherently flexible, which allows the designer to modify the control strategy, or even to totally re-program it, without the need for significant hardware modifications. Also very important are the higher tolerance to signal noise and the complete absence of ageing effects or thermal drifts.

- A large variety of electronic devices, from home appliances to industrial instrumentation, require the presence of some form of man to machine interface (MMI). Its implementation is almost impossible without having some kind of embedded microprocessor. The utilization of the computational power, that thus becomes available, also for lower level control tasks is often very convenient.

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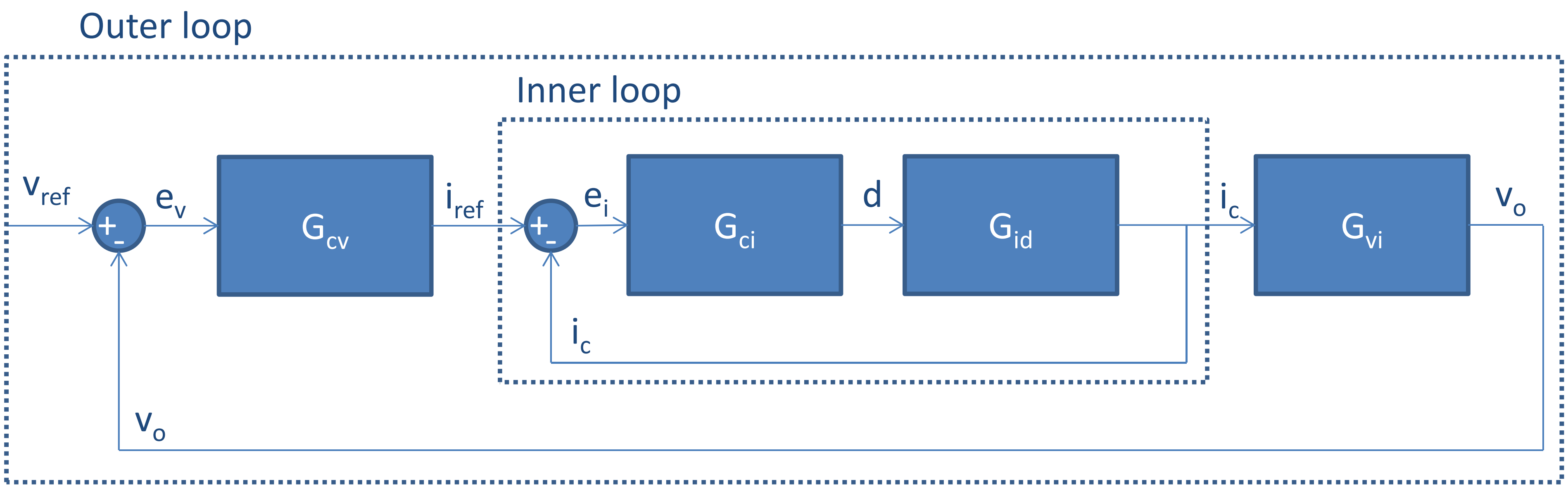
Introduction

- The application of digital controllers has been increasingly spreading and has become the only effective solution for a lot of industrial power supply production areas. Adjustable speed drives (ASDs) and uninterruptible power supplies (UPSs) are nowadays fully controlled by digital means.

- The increasing availability of low cost, high performance microcontrollers and digital signal processors stimulates the diffusion of digital controllers in areas where the cost of the control circuitry is a critical issue, e.g. in power supplies for portable equipment, battery chargers, electronic welders ...

- A significant increase of digital control applications in these very competing markets is not likely to take place until new implementation methods, different from the traditional microcontroller or DSP unit application, prove their viability. From this standpoint, the research efforts need to be focused on the design of custom integrated circuits, more than on algorithm design and implementation. Issues like occupied area minimization, scalability, power consumption minimization, limit cycle containment play a key role in this context.

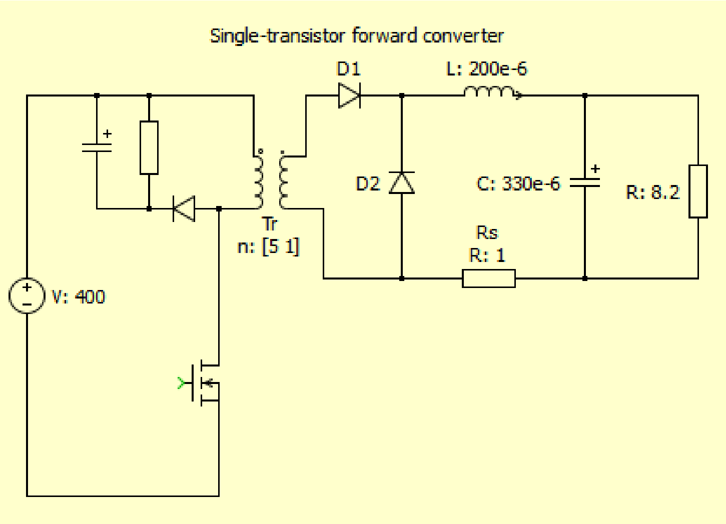
Introduction

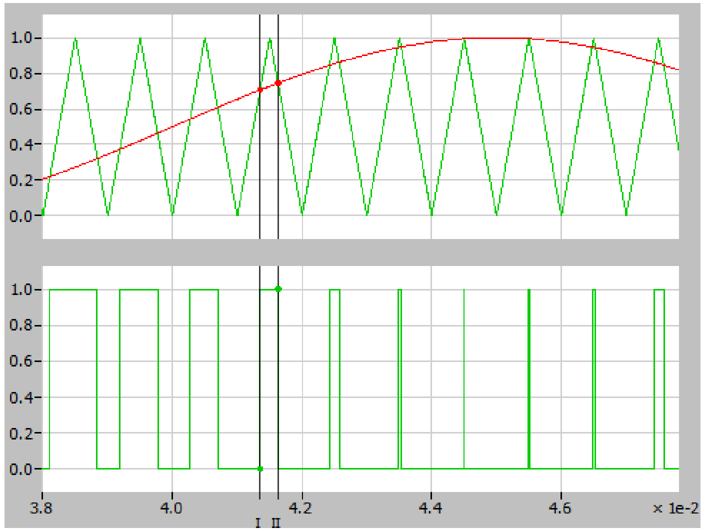


Control structure for a switched mode power converter

Case study

- The single switch single output forward converter – you already know very well ☺
- Signal processing, sampling and ADC issues will be discussed – revision last lecture
- Controller design will be presented in discrete/digital domain
- Pulse Width Modulation (PWM) control strategy will be explained briefly in the continuous time domain, successively in the discrete time domain.





PWM modulation

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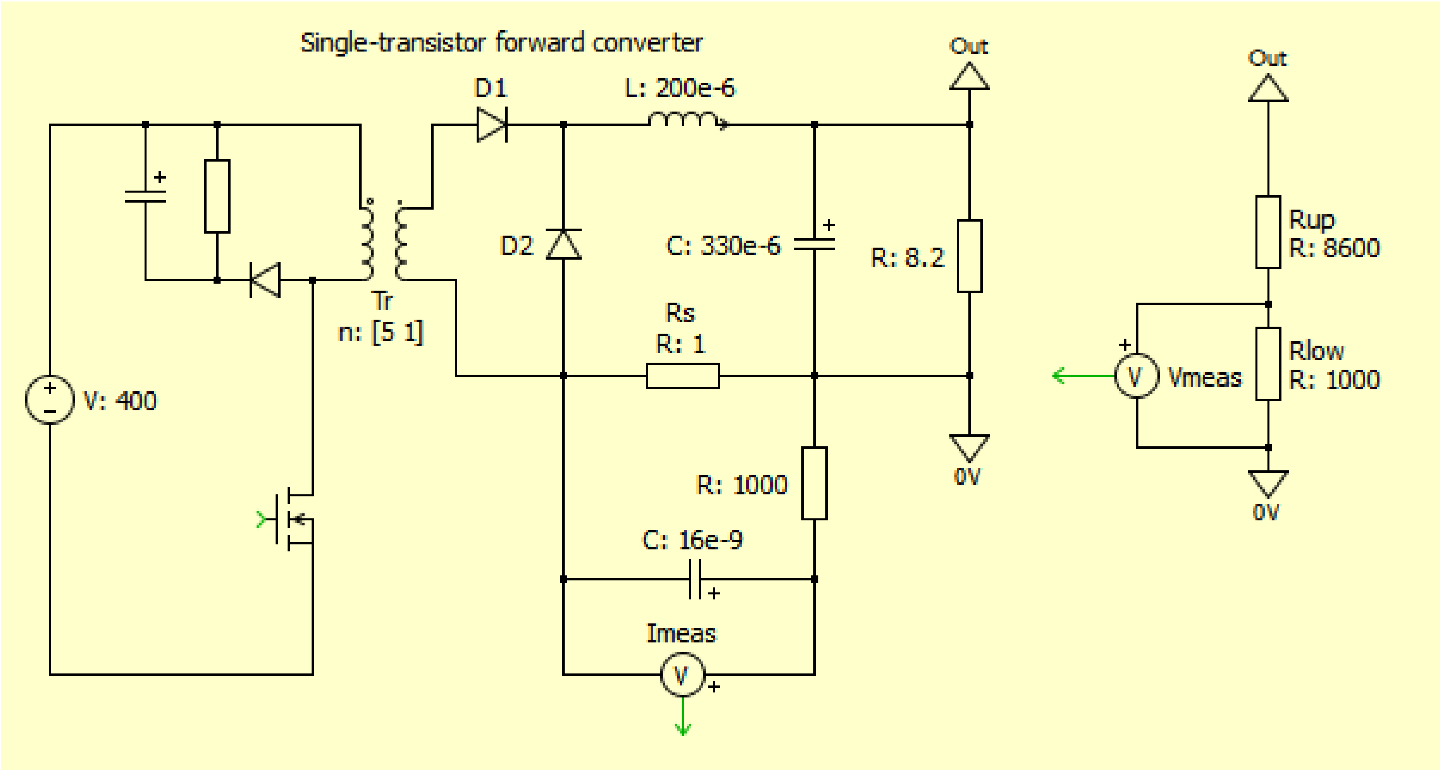
Case study

- **Single switch single output forward converter**



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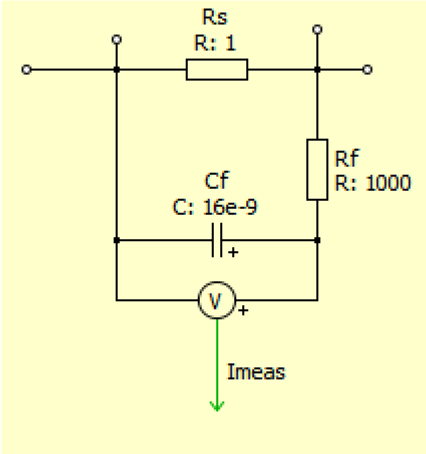
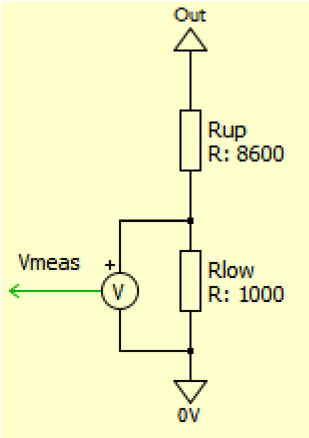
Signal processing – Last lecture

- **Filtering**
  - Low pass
  - High pass
  - Band pass



- **Scaling**
  - 0 - 3,3 V
  - 0 – 5 V



Sampling and ADC – last lecture

- **ADC**
  - Modern microcontrollers with 10 to 16 bit ADC
  - If Vcc = 3.3 / 5 V then:

$$x_{bits} = round\left(x_{volt} \cdot \frac{2^n}{Vcc} \right)$$

$$x_{volt} = 2.5\,V, n = 10 \ and \ Vcc = 3.3V \Rightarrow round\left(2.5 \cdot \frac{1024}{3.3} \right) = 776$$

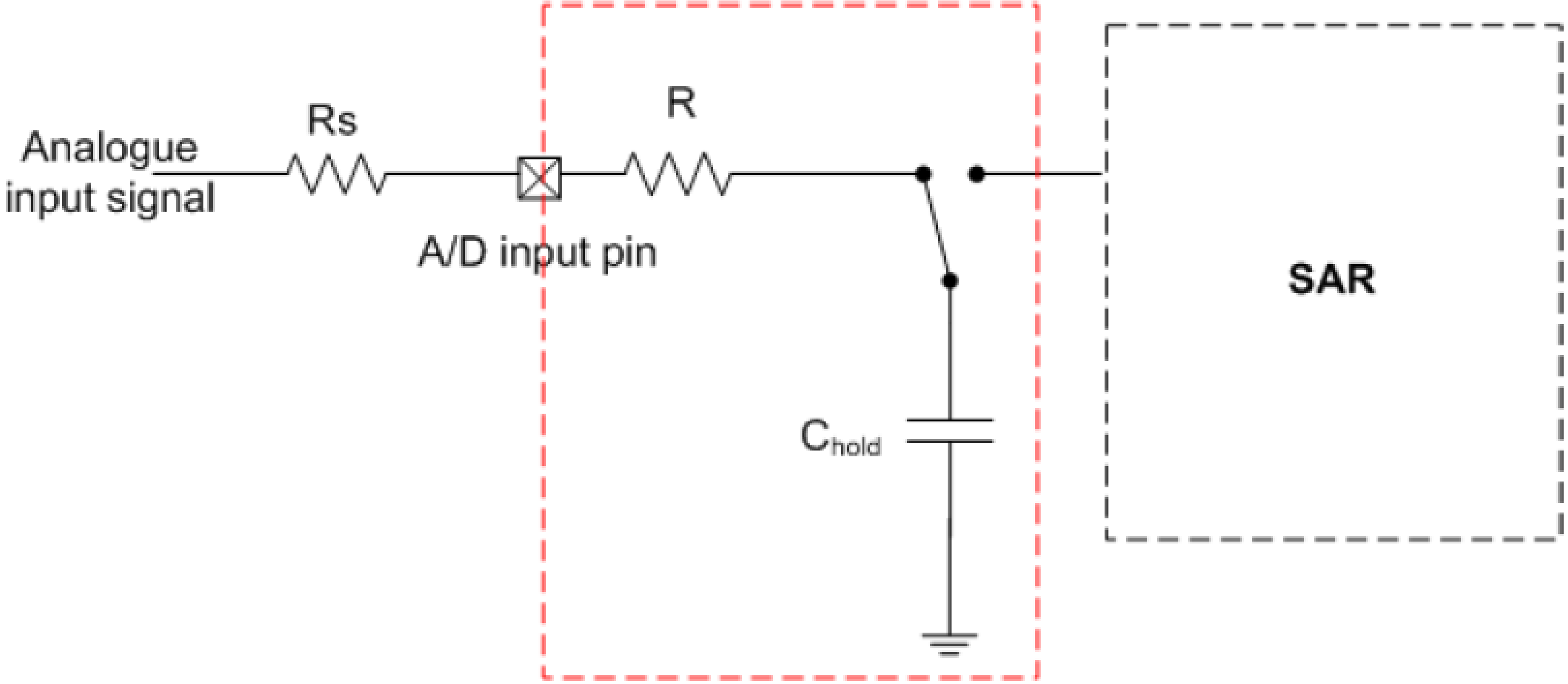
$$x_{bits} = 11\ 0000\ 1000$$

  - ADC resolution: the number of discrete values can be produced over a range of analog values. For 10 bit ADC:

$$\frac{3.3}{1024} = 0.0032V$$

Sampling and ADC – last lecture

- **Sample and hold circuit**



Simplified scheme of a Sample and Hold circuit

http://www.delftek.com/wp-content/uploads/2012/04/National_ABCs_of_ADCs.pdf

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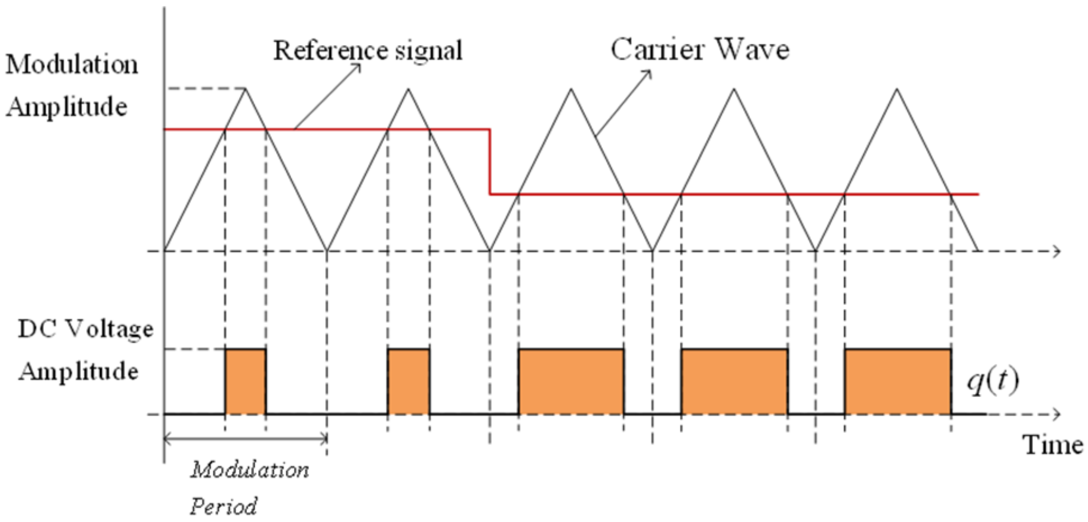
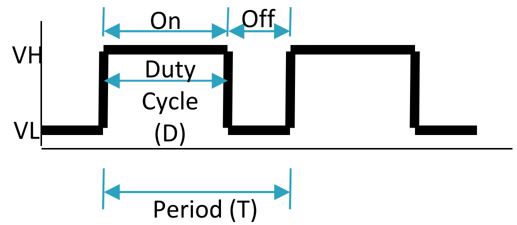
Pulse Width Modulation

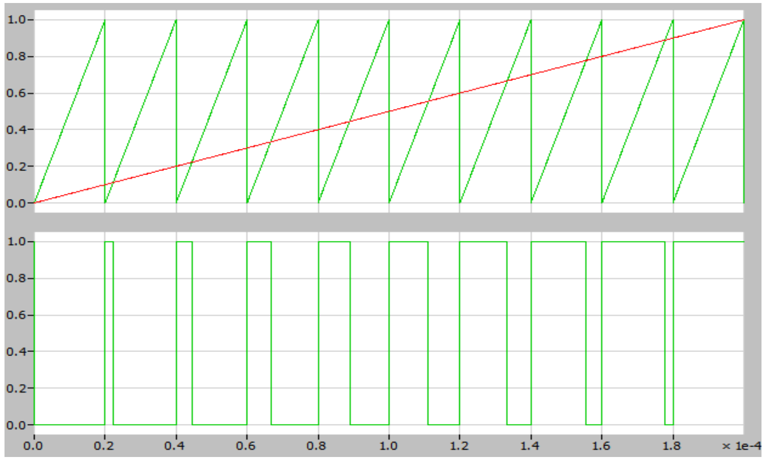




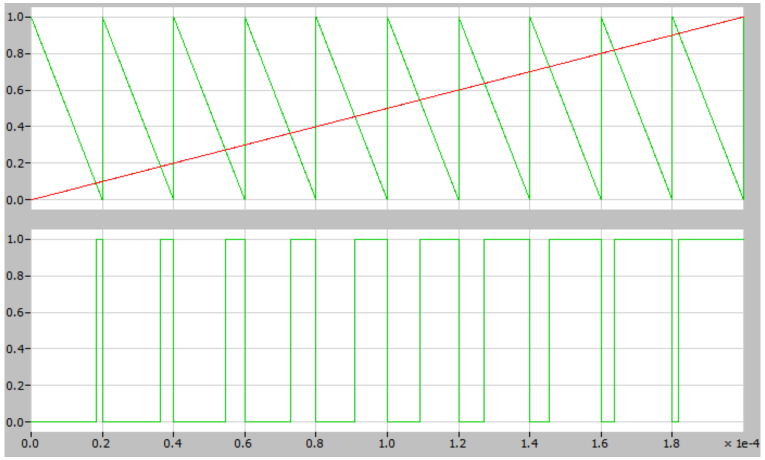
$$Duty\ Cycle = \frac{On\ Time}{Period} \times 100\%$$

- **PWM carrier types**
  - There are three commonly used types of PWM defined by which edge of the analog signal is to be modulated
    - Lead Edge Modulation
    - Trail Edge Modulation
    - Pulse Center Two Edge Modulation/Phase Correct PWM

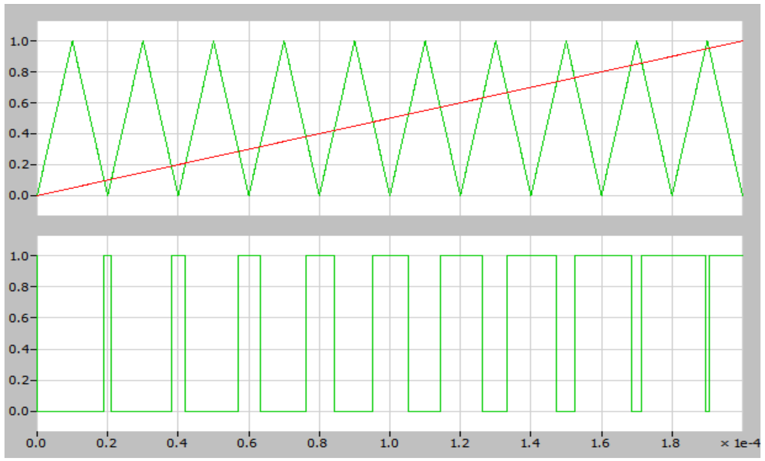
Pulse Width Modulation



Trail Edge Modulation



Lead Edge Modulation



Pulse Center Two Edge Modulation/Phase Correct PWM

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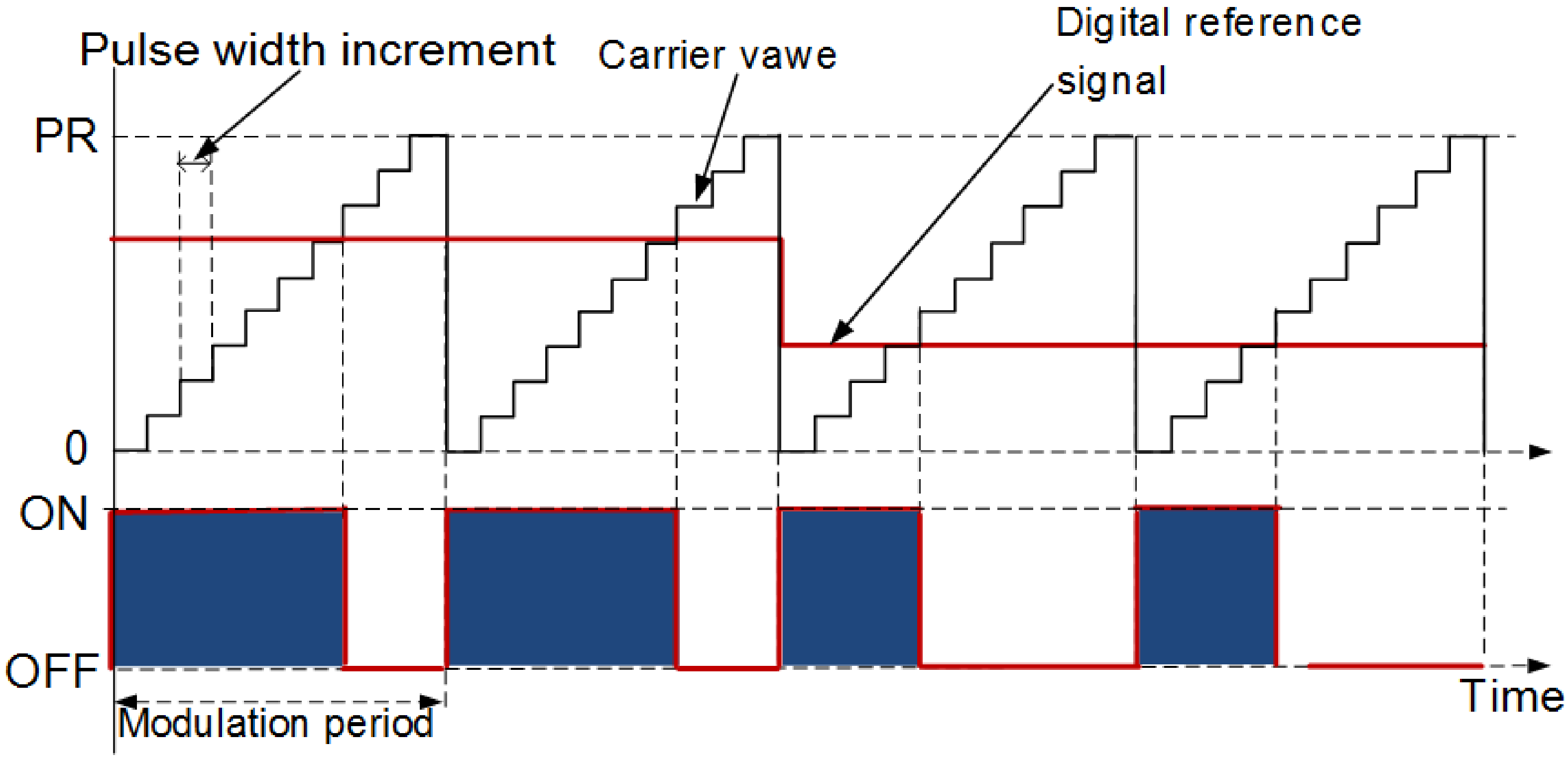
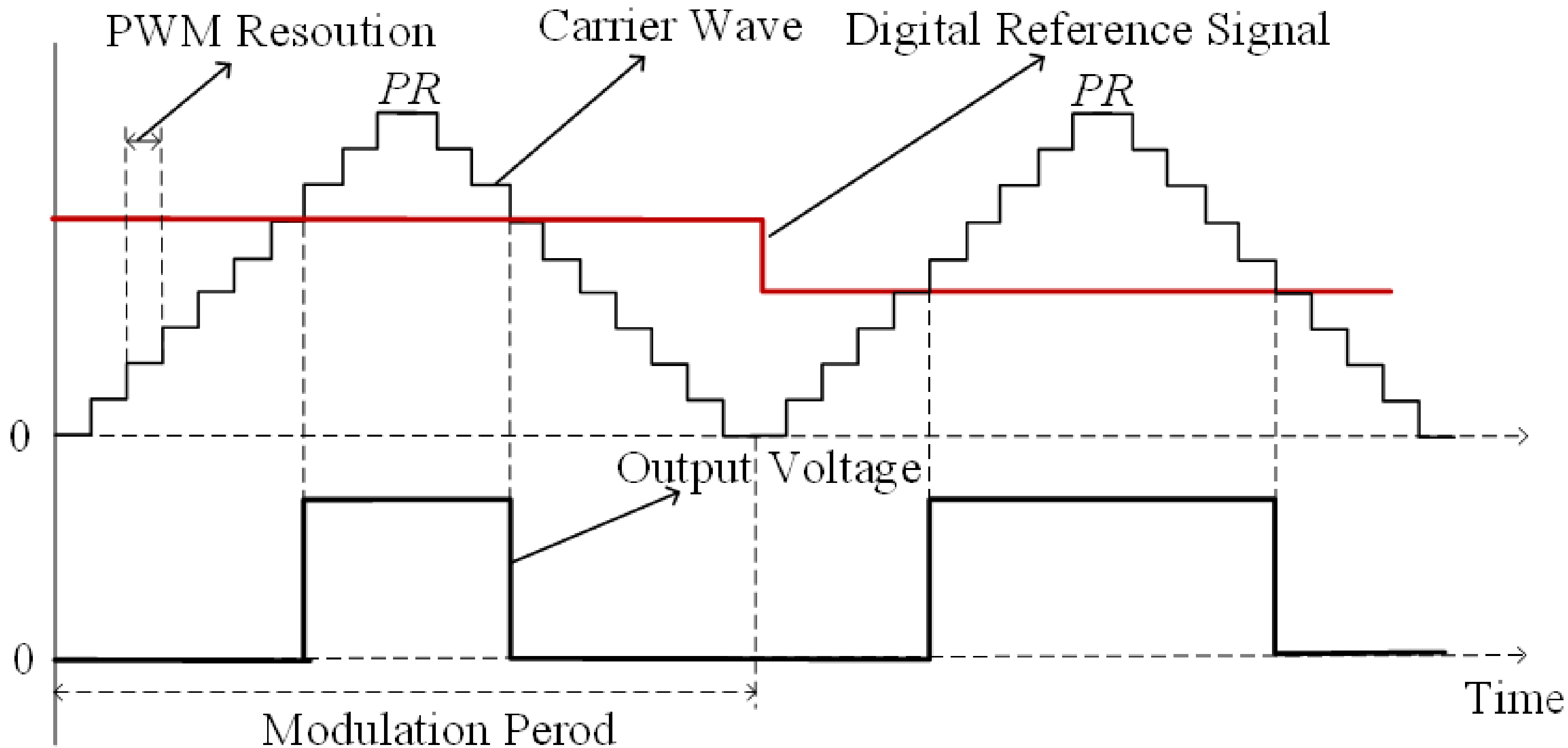
PWM modulation

- **Digital PWM with trailing (rising) carrier**
  - Reference value < Timer count value - OFF
  - Timer count value is zero - ON



Pulse Width Modulation

- **Digital PWM with two edge (triangular) carrier**
  - Reference value > Timer count value - ON
  - Reference value < Timer count value - OFF



Pulse Width Modulation

- **Basic considerations in choosing PWM frequency:**
  - Transitions can only occur on a clock tick
  - Frequency limited by your clock and desired resolution
  - Resolution is defined by clock speed and frequency of the PWM

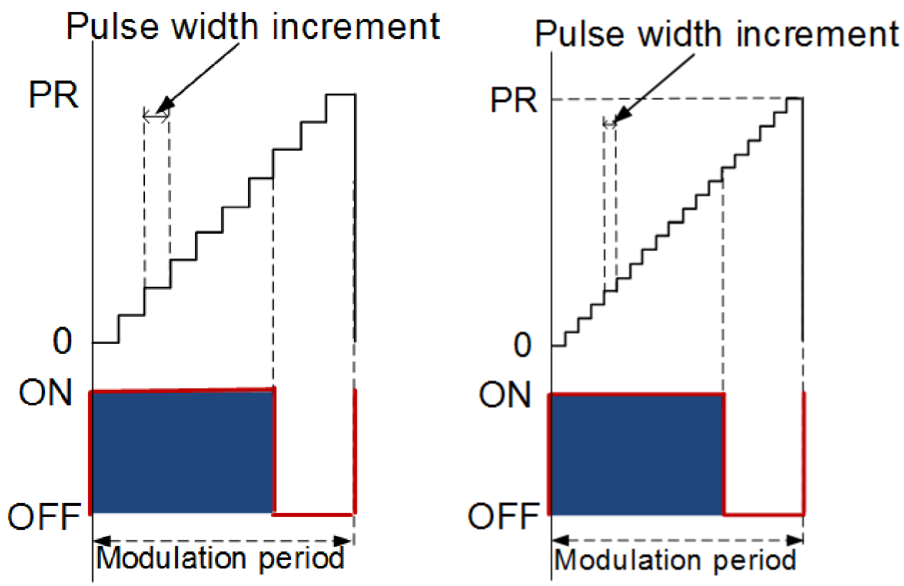
The faster you run the PWM frequency, the fewer clock ticks occur in the period considered so the lower duty cycle resolution

Pulse Width Modulation

- PWM resolution indicates the number of different widths what the output pulse can have

$$PWM_{resolution} = \log_2\left(\frac{F_{CPU}}{F_{PWM}}\right) \quad \text{(in bits)}$$

- **For example**: a 10-bit resolution PWM can have 1024 different widths for the pulses the higher the PWM resolution, the more precise approximation of a given reference is possible



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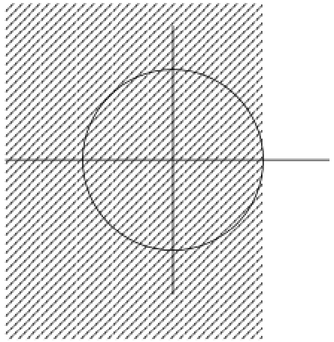
Controller Design

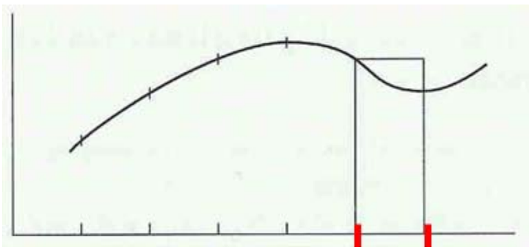
Discretization

- Numerical Integrator

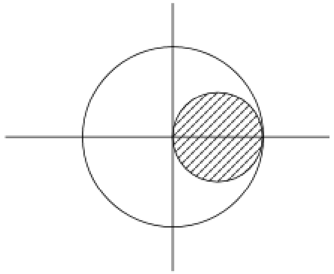
$$s = \frac{z-1}{T_s}$$



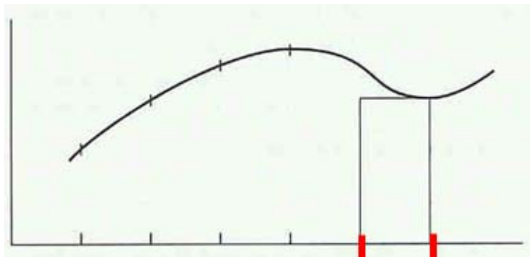
Forward Euler



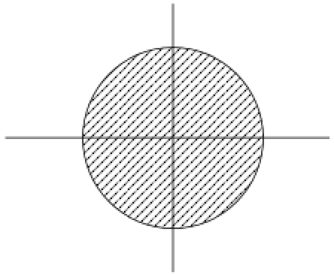
$$s = \frac{z-1}{T_s \cdot z}$$



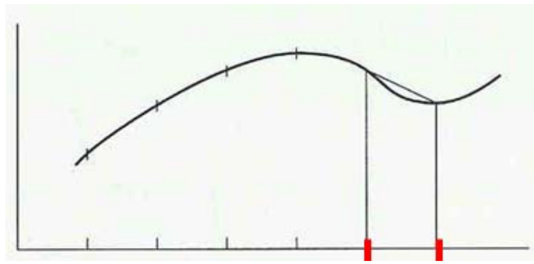
Backward Euler



$$s = \frac{2}{T_s}\frac{z-1}{z+1}$$



Bilinear (Tustin)



Controller Design

Discretization

- Zero-Pole Matching

1. If s = a is the pole: $z = e^{-aT_s}$

2. If s = b is finite zero then: $z = e^{-bT_s}$

3. If the numerator (n) is of lower order than the denominator (m), add powers of (z+1) to the numerator until numerator and denominator are of equal order

4. Identical gain at some critical freq. (typically, s=0)

   H(s) at s=0 => H(z) at z=1

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Controller Design

Discretization

- Zero-Pole Matching example:

$$H(s) = \frac{a}{s+a}$$

Pole (rule 1) $\quad s = -a \quad \rightarrow \quad z = e^{-aT} = \alpha$

Zero (rule 3) $\quad n = 0, m = 1 \rightarrow add\ one\,(z+1)$

Gain (rule 4) $\quad K \left.\frac{z+1}{z-e^{-aT}}\right|_{z=1} = \left.\frac{a}{s+a}\right|_{s=0} \Rightarrow K\frac{2}{1-e^{-aT}} = 1 \quad \Rightarrow \quad K = \frac{1-e^{-aT}}{2}$

$$H(z) = K\frac{z+1}{z-\alpha}$$

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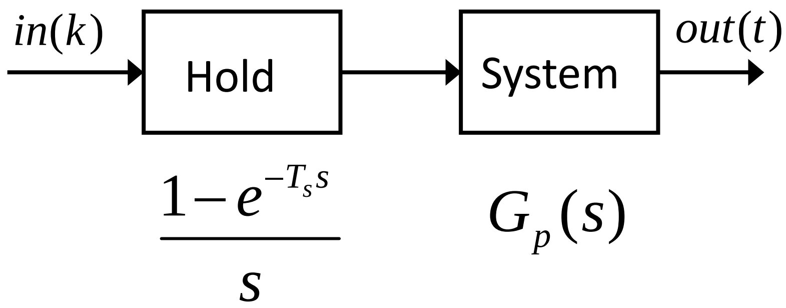
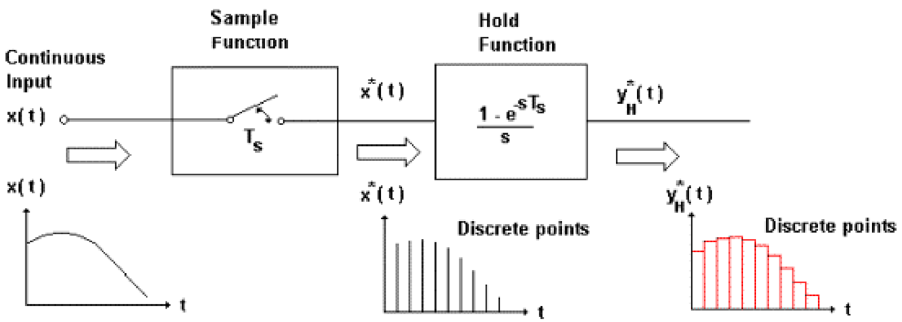
Controller Design

Discretization

- Zero Order Hold





Source: http://www.moultonworld.pwp.blueyonder.co.uk/Lecture16_page_files/image053.gif

$$H(z) = (1 - z^{-1})\, Z\left\{\frac{G_p(s)}{s}\right\}$$

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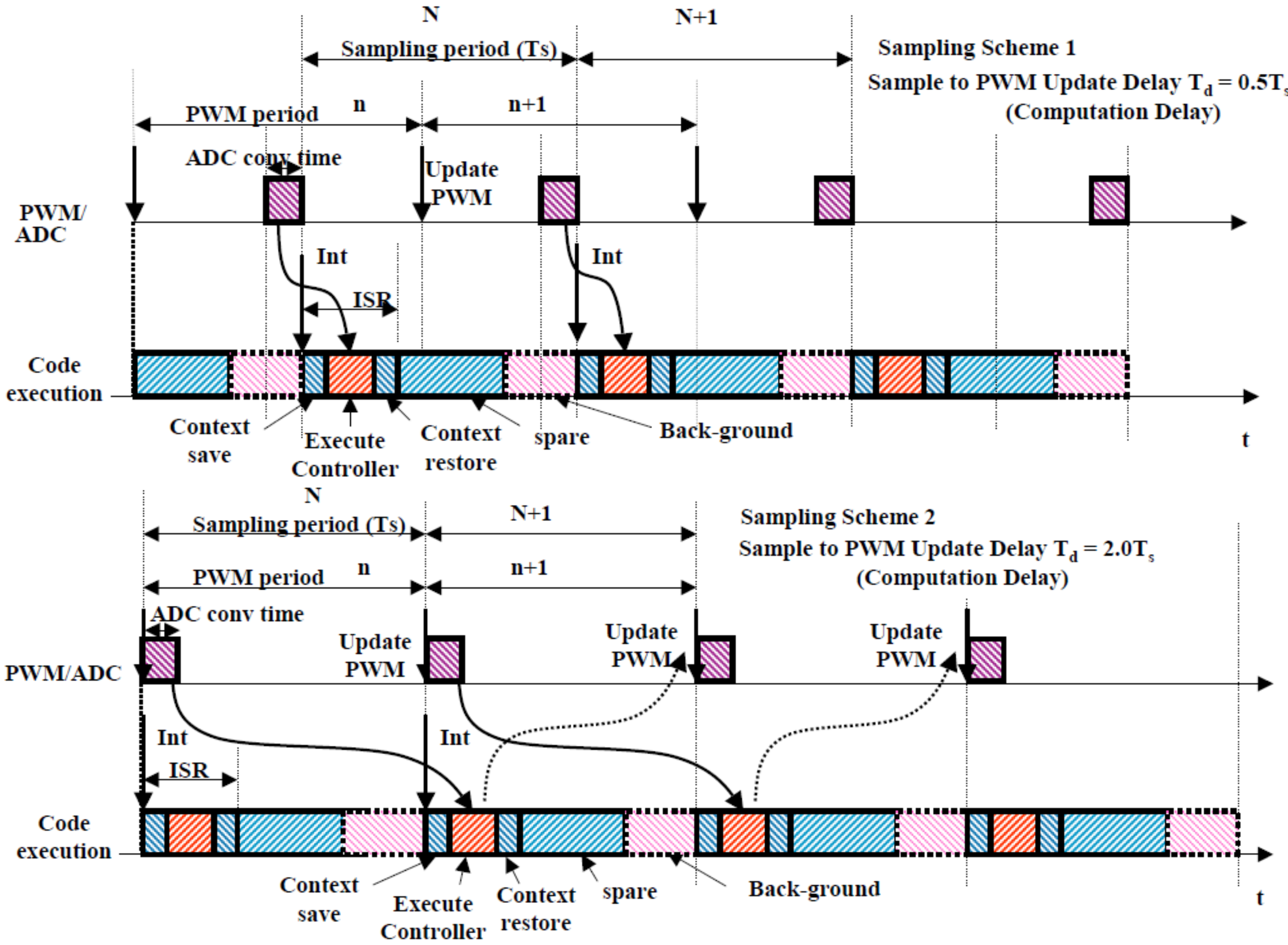
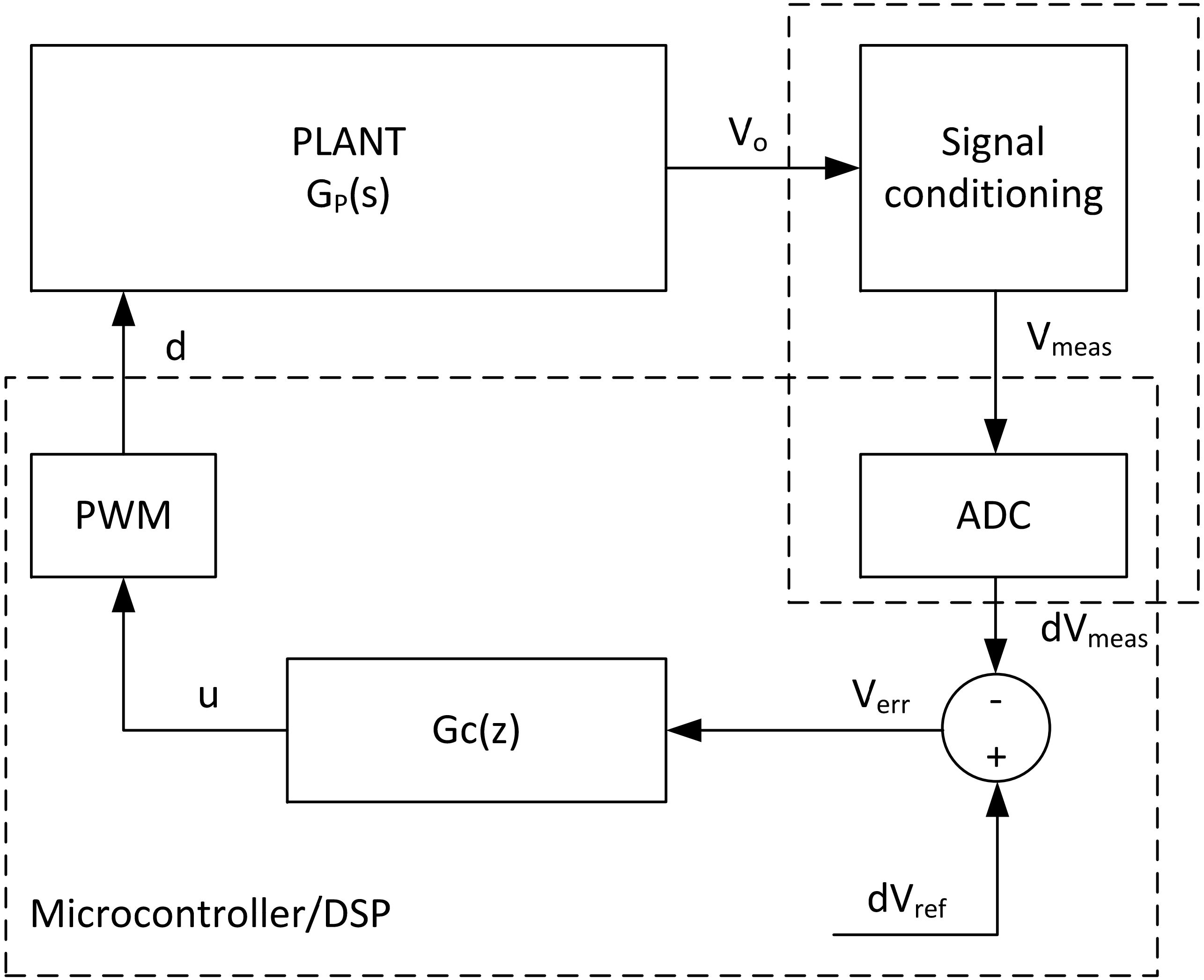
Controller Design



Source: http://www.ti.com/corp/docs/landing/fusion/3_1_Choudhury.pdf

Controller Design



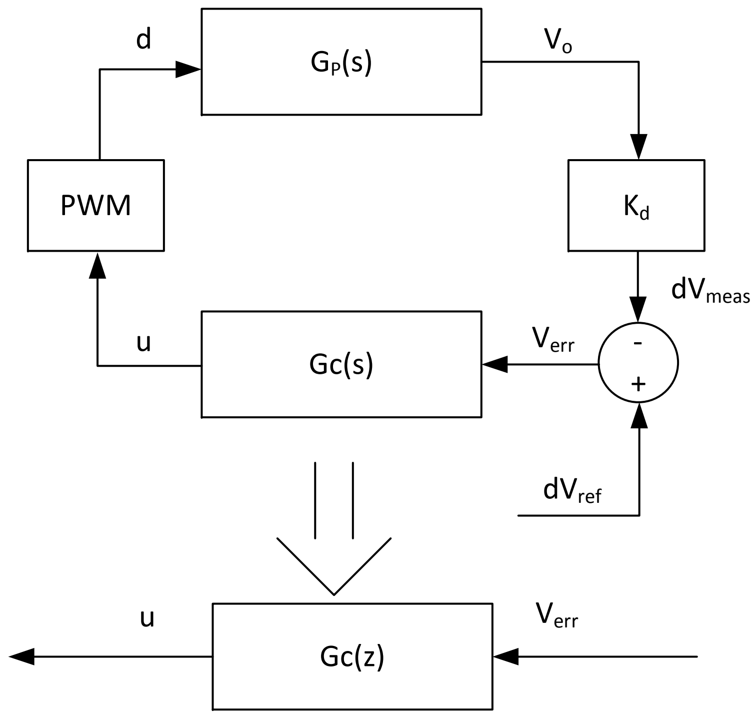
Controller Design

1. Control design by emulation

- An analog controller is firstly designed in the continuous domain as if one were building continuous time control system (ignoring the effects of sampling and hold associated with the AD converter and the digital PWM circuits)

- The analog controller is then converted to a discrete-time compensator by some approximate techniques

- K_d includes all the ADC and signal conditioning gains



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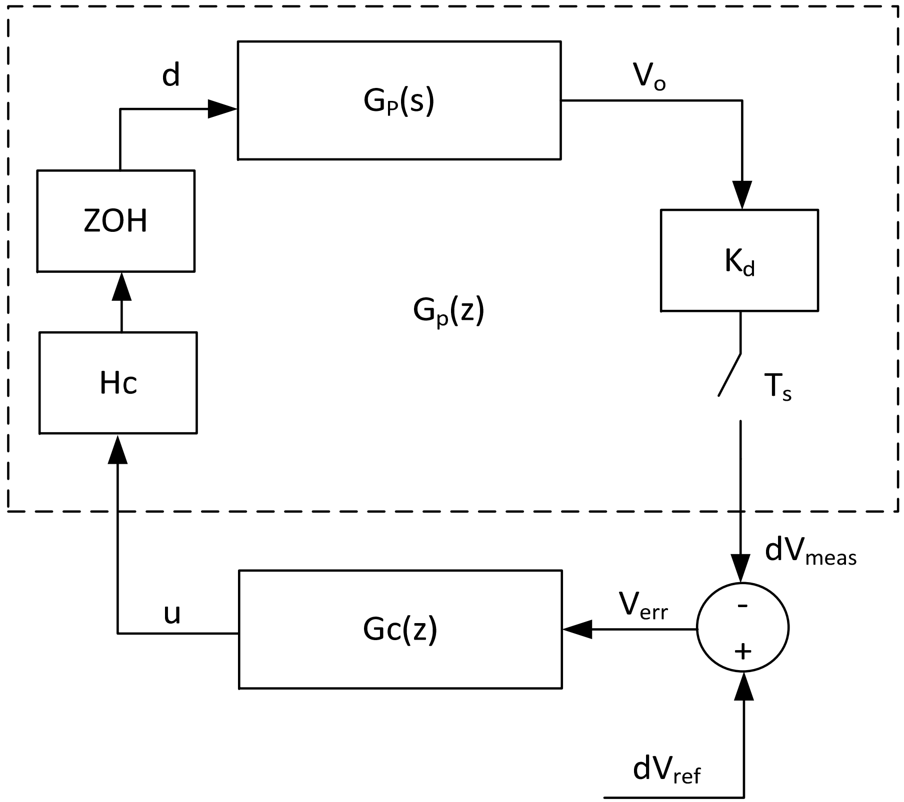
Controller Design

2. Direct controller design

- T_s ideal sampler
- K_d includes signal conditioning and ADC gain
- Hc includes ADC conversion time and computational delay (T_d)
- PWM module acts as a hold device so ZOH represents the sample (ADC) and hold (PWM) device



$$ZOH(s) = \frac{1 - e^{-sT_s}}{s}$$

$$Hc(s) = e^{-sT_d}$$

$$G_P(z) = Z\left\{ZOH(s) \cdot K_d \cdot G_P(s) \cdot Hc(s)\right\}$$

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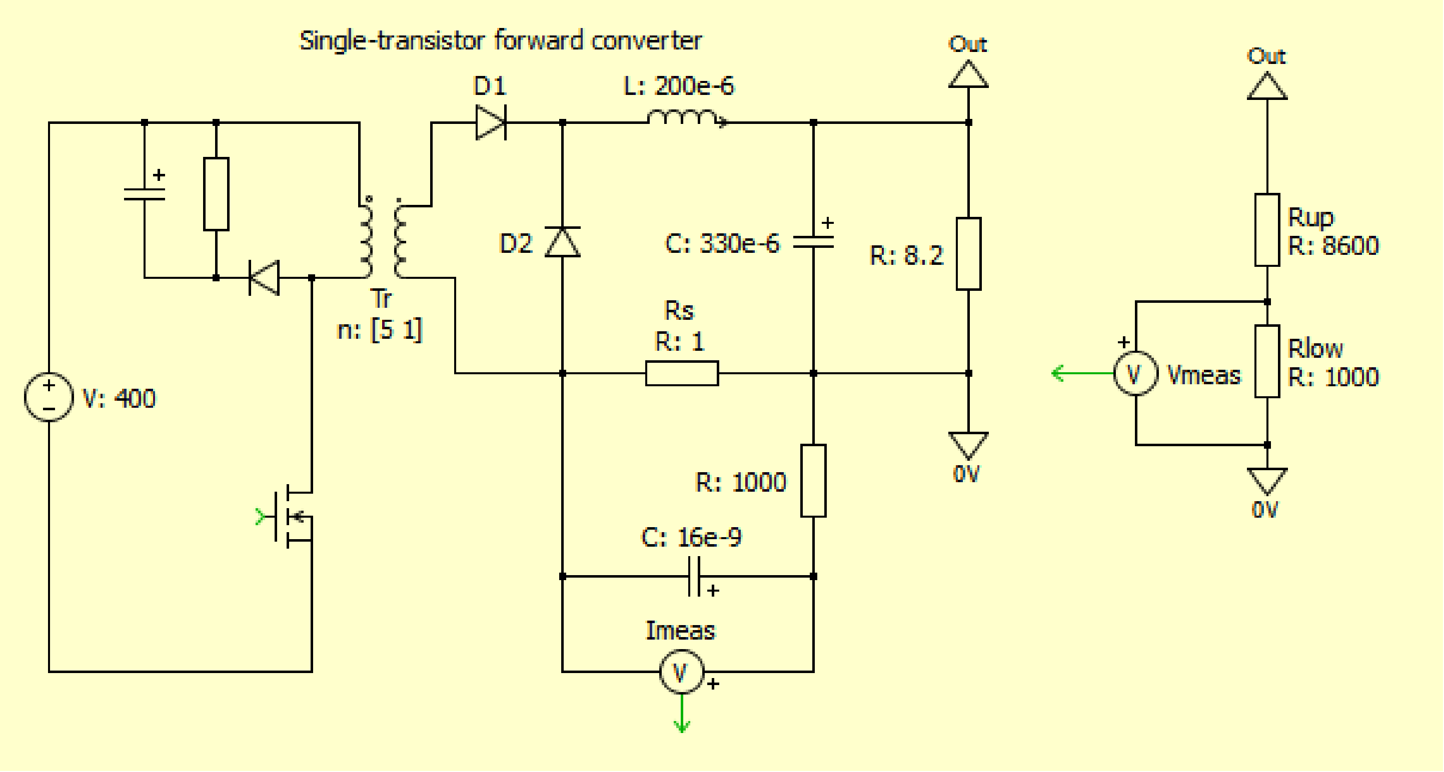
Controller Design

- **Design Example with emulation – iL control of forward converter**



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Controller Design

- **Design Example with emulation – iL control of forward converter**
  - Given is the plant: Vg=400 V, C=330 uF, L=200 uH, R=8.2 Ohm, n=1/5, D=0.3
    Vo=Vin·n·D=24V

$$G_{id}(s)=\frac{i_L(s)}{d(s)}=\frac{nV_g}{DR}\frac{1}{LC}\frac{(RCs+1)}{s^2+\frac{1}{CR}s+\frac{1}{LC}}$$

- Tips:
  - Design usually the controller with the nominal values (currents and voltages)
  - Signal conditioning effects can be eliminated inside the microcontroller by multiplying the scaling factors with the measured values to return to the nominal values

Controller Design

- **Design Example with emulation – iL control of forward converter**

  - Classical PI controller
  - Tips:
    - When designing controller and talk about frequencies: usually we talk about frequencies **f** in **Hz** but when we use it we calculate in **rad/s** so as **ω=2*π*f=1/T**

$$G_C(s) = \frac{V_{err}(s)}{d(s)} = K_P \cdot (1 + \frac{1}{T_I s}) = K_P + K_I \frac{1}{s} \qquad where \;\; K_I = \frac{K_P}{T_I}$$

or

$$G_C(s) = \frac{V_{err}(s)}{d(s)} = \frac{K_P T_I s + K_P}{T_I s}$$

Controller Design

- **Design Example with emulation – iL control of forward converter**
  - Design criteria: design a controller with high bandwidth paying also attention to the sampling issues: Fs=50 kHz so Ts=20us.
  - Tips:
    - Check bode diagram of the plant: usually the bandwidth should be high enough for a current loop.
    - Pay attention to the sampling time/frequency and also the resonant frequency of the plant

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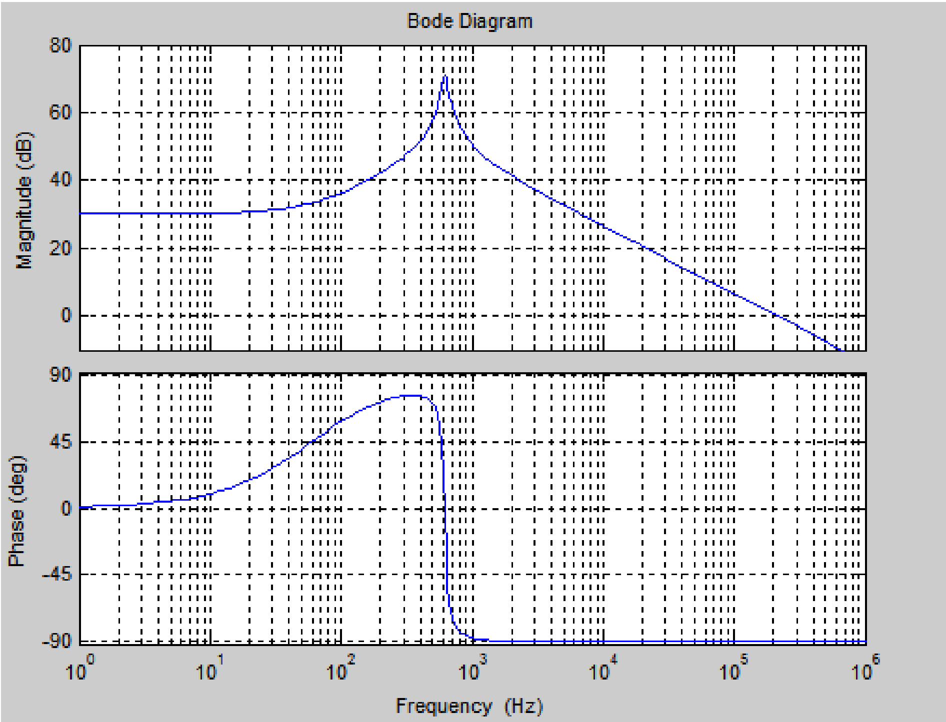
Controller Design

- **Design Example with emulation – iL control of forward converter**

  - Design criteria: design a controller with high bandwidth paying also attention to the sampling issues: Fs=50 kHz so Ts=20us.



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Controller Design

- **Design Example with emulation – iL control of forward converter**

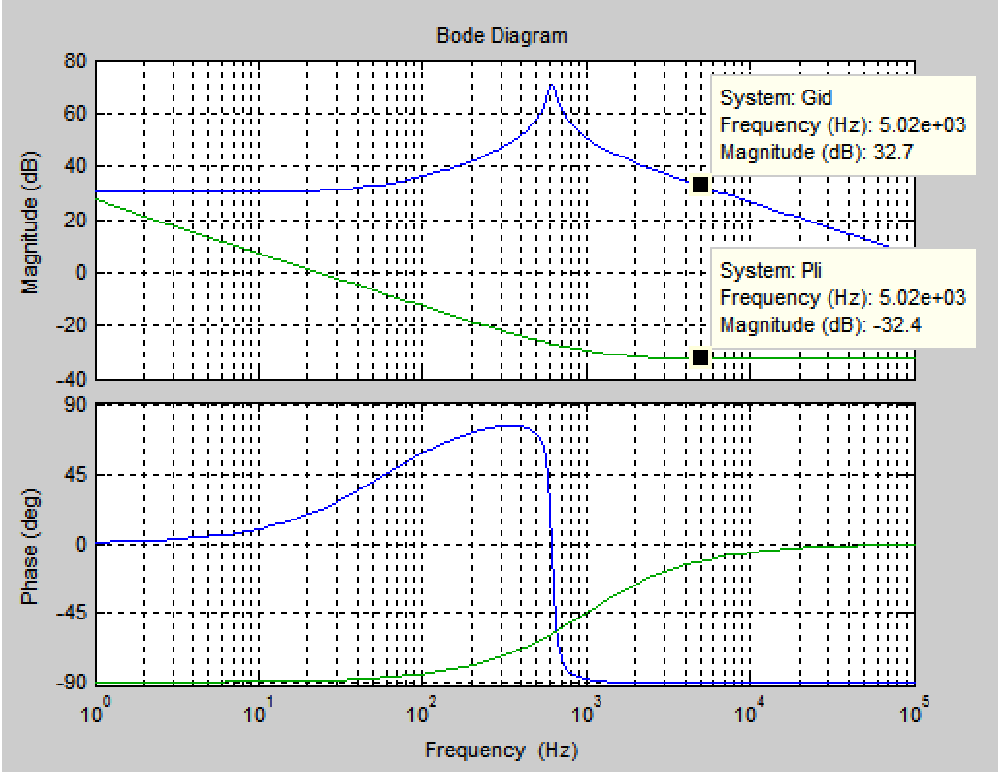
  - Chose the PI parameters based on the plant gain and design requirements

  - At 5000 Hz the Gain=32,7 dB
  - Kp should be -32,7 dB

  - Tips:
    - Bode shows gain in dB
    - We calculate with magnitude

$$mag = 10^{\frac{dB}{20}}$$

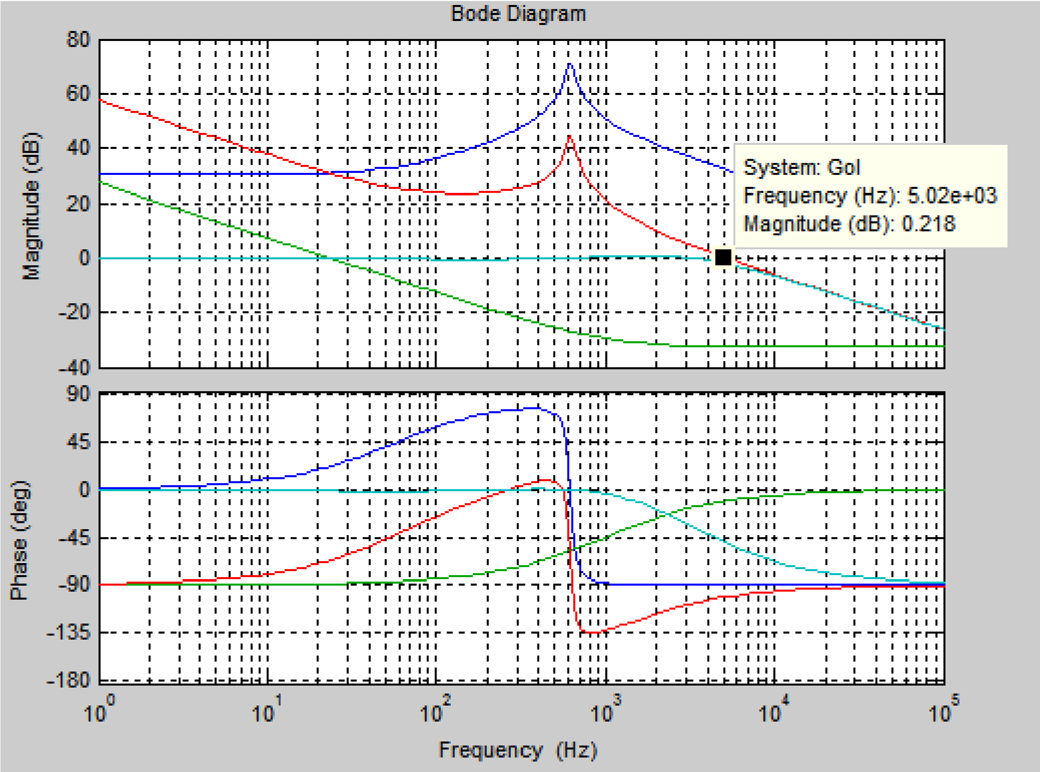
    - Chose Ti valaue at a lower frequency then ωc



Controller Design

- **Design Example with emulation – iL control of forward converter**

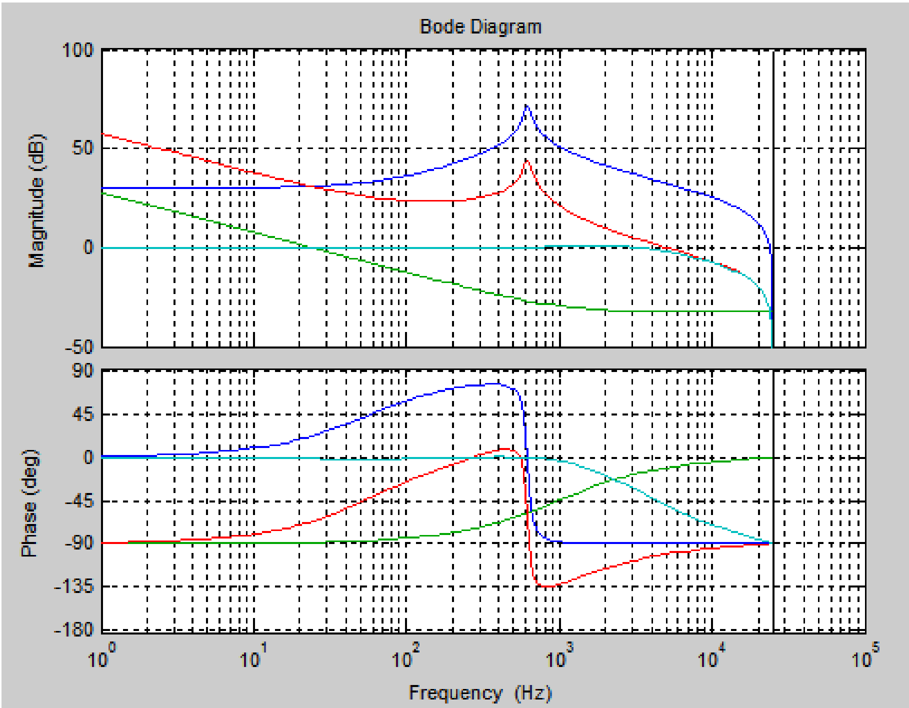
  - Check open and closed loop system



Controller Design

- **Design Example with emulation – iL control of forward converter**

  - Discretize all plant and controller transfer functions and plot discrete open and closed loop bode curves (I have selected Tustin method)



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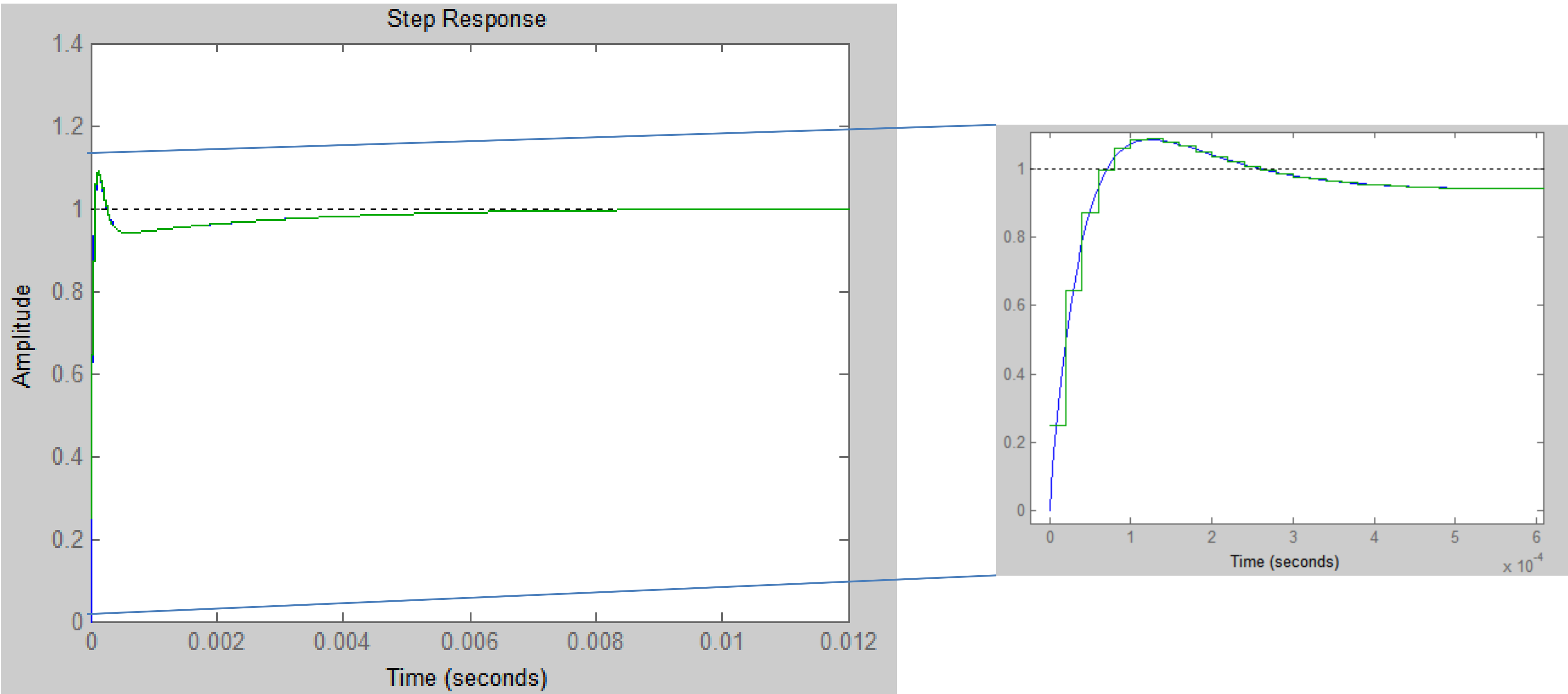
Controller Design

- **Design Example with emulation – iL control of forward converter**

  - Check step response of both continuous and discrete closed current loop:



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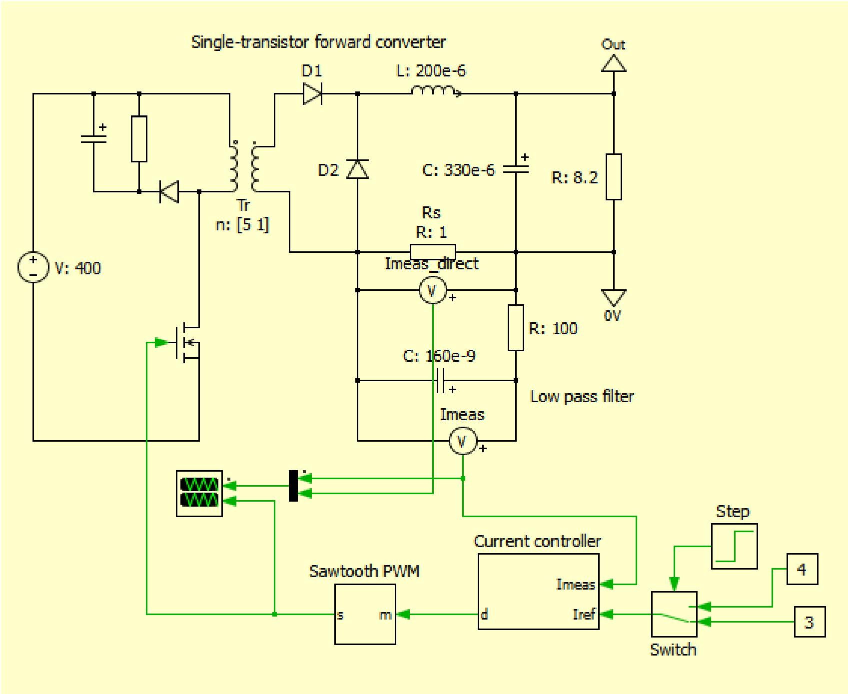
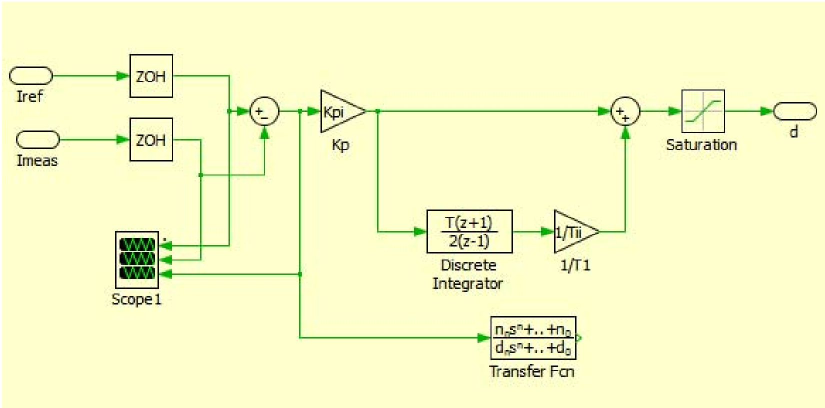
Controller Design

- **Design Example with emulation – iL control of forward converter**

  - Build controller in Plecs





Controller Design

- **Design Example with emulation – iL control of forward converter**

  - Results

Exercise

- Design a discrete current and a voltage PI controller as show below, for your forward converter.
- Design criteria:
  - Current loop: wc = 5000 Hz,
  - Voltage loop: wc = 500Hz
  - Neglect the signal conditioning factors, Rs=1, and measure and control directly the output voltage
  - Inductor current to output voltage transfer function:

$$G_{iv} = \frac{R}{RCs + 1}$$

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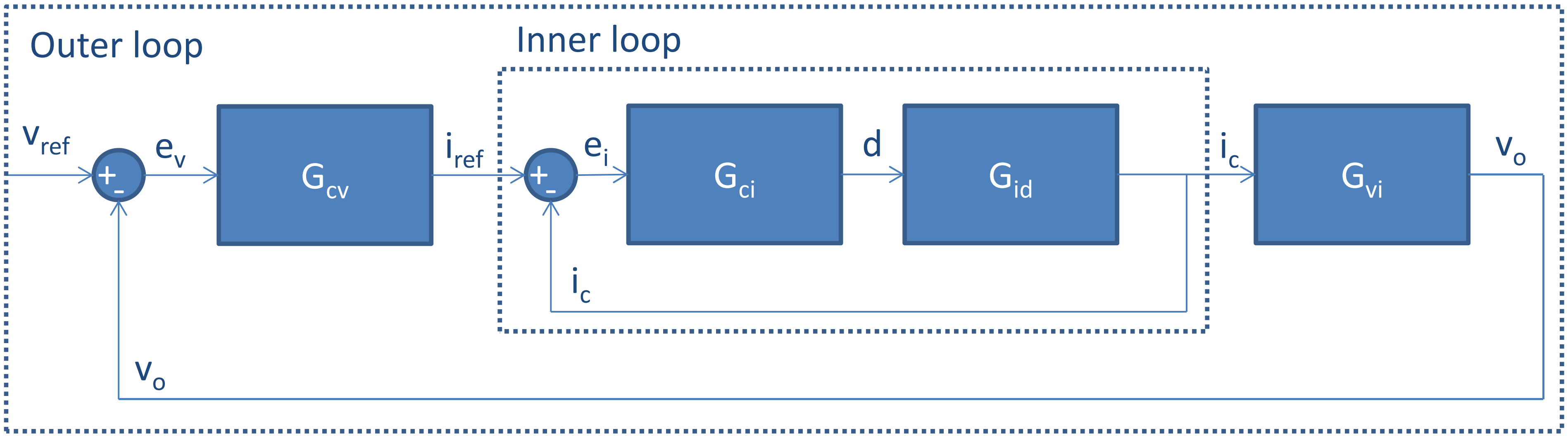
Exercise



Control structure for the forward converter