

**Problem 3 (10 %)**

There is a transformer with two coils sketched as below.

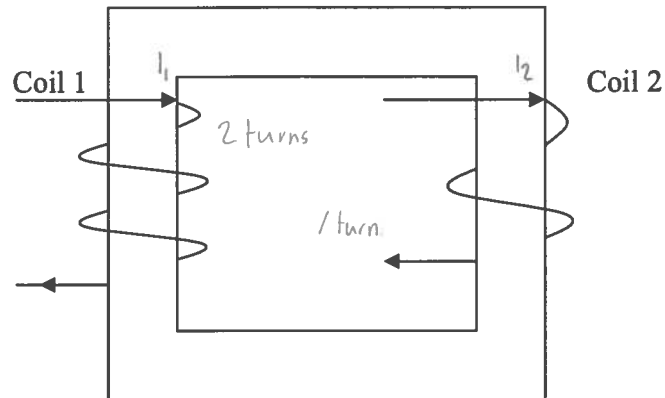


Fig. 3

- (1) Please express the flux linkage for coil 1 using the coils 1 self-inductance ( $L_1$ ) and coil 1 and coil 2 mutual inductance ( $M$ ). The positive current directions are already given in Fig. 3.
- (2) Suppose coil 1 has  $N_1$  turns, and coils 2 has  $N_2$  turns. If it is desired to perform turns ratio transformation, to let coil 2 to have the same number of turns as coil 1. How the flux linkage equation derived before should be modified?
- (3) Which parameters and variables related to coil 2 are affected by the turns ratio transformation?

$$\lambda_1 = L_1 i_1 + M i_2$$

# DMoEM - Jan 14

## Problem 3 - A transformer

1 - The flux linkage of coil 1.

$$\lambda_1 = \underset{\substack{\uparrow \\ \text{Coil 1} \\ \text{self-} \\ \text{inductance}}}{L_1} i_1 + \underset{\substack{\uparrow \\ \text{Mutual} \\ \text{inductance}}}{M} i_2$$

2 - Turns ratio transformation:

- It is desired to let's coil 2 have the same # of turns as coil 1.

- Flux linkage eq.:

$$\lambda_2 = L_2 i_2 + M i_1$$

$$M = \frac{N_1 \phi_2}{i_2}$$

If  $N_1 = N_2$ , then the # of winding for  $N_2$  are doubled, then only half the line current are needed to produce the same flux.

$$M = \frac{N_1 \phi_2}{i_2/2} \Rightarrow 2 \frac{N_1 \phi_2}{i_2} = 2M$$

Thus:

$$\lambda_1 = L_1 i_1 + 2M i_2$$

3 - Parameters/variables affected by the turn ratio transformation:

- If it is desired to produce the same flux

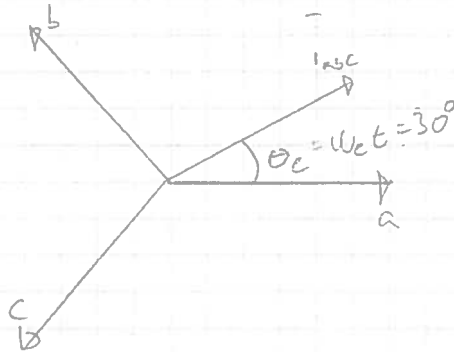
$$N_1 \propto \frac{1}{i_2}$$

# Elektrotechnik Maschinenbau - Jan 2014

## Problem 1

$$\omega_c = 2\pi 50 \text{ rad/s}$$

$$I = 10 \text{ A} \quad \angle 30^\circ$$



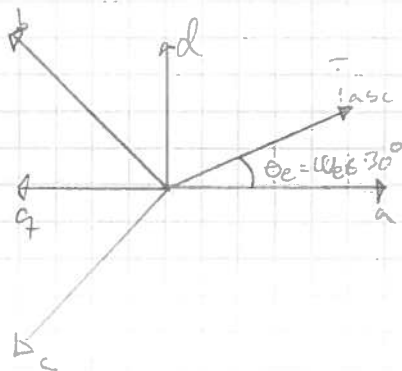
$$\bar{i}_{asc} = 10 \cdot e^{j30^\circ}$$

$$i_a = \operatorname{Re}\left(\frac{10 e^{j30^\circ}}{e^{j0^\circ}}\right) = 10 \cos(30^\circ) = 8,66 \text{ A}$$

$$i_b = \operatorname{Re}\left(\frac{10 e^{j30^\circ}}{e^{j120^\circ}}\right) = 10 \cos(30^\circ - 120^\circ) = 10 \cos(-90^\circ) = 0$$

$$i_c = \operatorname{Re}\left(\frac{10 e^{j30^\circ}}{e^{j240^\circ}}\right) = 10 \cos(30^\circ + 120^\circ) = -8,66 \text{ A}$$

2) Determine dq component values  $\theta = 90^\circ$



$$i_d = \operatorname{Re}\left(\frac{\bar{i}_{asc}}{e^{j\theta}}\right) = 10 \cos(30^\circ - \theta) = 5 \text{ A}$$

$$i_q = \operatorname{Re}\left(\frac{\bar{i}_{asc}}{e^{j\theta+90^\circ}}\right) = 10 \cos(30^\circ - (\theta + 90^\circ)) = -8,66 \text{ A}$$

$$\bar{i}_{dq} = (5 \text{ A} - j 8,66 \text{ A}) e^{j90^\circ}$$

3) At  $t = 0,05 \text{ s}$  d-axis is leading the current  $i_c$  by  $90^\circ$  deg, thus

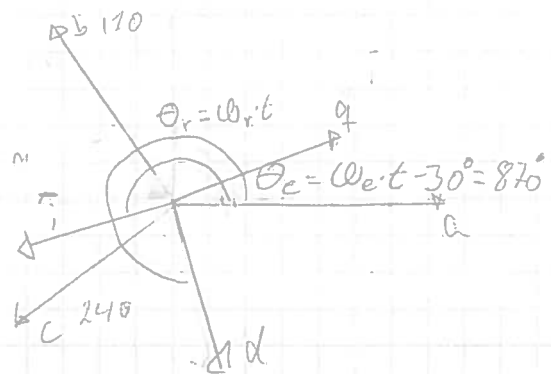
$$\bar{I} = 10 e^{j30^\circ} \cdot e^{j\theta_c}$$

$$= 10 \cos(30^\circ - 150^\circ)$$

$$\theta_r = \omega_c t = \theta_c + 90^\circ (t = 0,05)$$

$$\omega_p = \frac{(8,70 + 90)}{0,05 \text{ s}} = 356,63 \text{ rad/s}$$

$$f_r = 56,67 \text{ Hz}$$

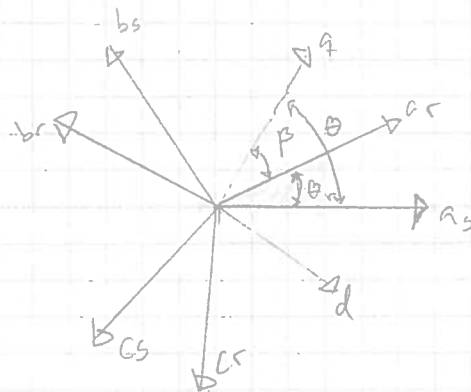


## Problem 2

Induction machine

S stator

r rotor



- 1) What is the mutual inductance (phase b and c) <sup>stator</sup>

$$M_{bscm} = L_{aq} \operatorname{Re} \left( \frac{e^{j\theta}}{e^{j\frac{2}{3}\pi}} \right) \operatorname{Re} \left( \frac{e^{j\theta}}{e^{j\frac{2}{3}\pi}} \right) +$$

$$L_{ad} \operatorname{Re} \left( \frac{e^{j(\theta - \frac{\pi}{2})}}{e^{j\frac{2}{3}\pi}} \right) \operatorname{Re} \left( \frac{e^{j(\theta - \frac{\pi}{2})}}{e^{j\frac{2}{3}\pi}} \right)$$

- 2) Mutual inductance rotor phase c and stator phase b? <sup>a pos</sup>

$$M_{rcsbm} = L_{aq} \operatorname{Re} \left( \frac{e^{j\theta}}{e^{j(\frac{2}{3}\pi - \theta_r)}} \right) \operatorname{Re} \left( \frac{e^{j\theta}}{e^{j(\frac{2}{3}\pi)}} \right) +$$

$$L_{ad} \operatorname{Re} \left( \frac{e^{j(\theta - \frac{\pi}{2})}}{e^{j(-\frac{2}{3}\pi - \theta_r)}} \right) \operatorname{Re} \left( \frac{e^{j(\theta - \frac{\pi}{2})}}{e^{j(\frac{2}{3}\pi)}} \right)$$

- 3) The rotor shaft speed is 120 RPM. No. pole pairs is 2. Please calculate the rotor angular vel.  $\omega_r$ , to be used in this machine model

$$\omega_m = 120 \frac{2\pi}{60} = 4\pi \quad \omega_r = 2 \cdot 4\pi = 8\pi$$

③ Related variables affected?

The current in coil 2 is now  
expressed as  $i_2 = i_1 \frac{N_1}{N_2}$

Problem 4

8 pole = 4 pole pairs induction motor

① Cal efficiency

$$P = 400 V_{RMS} \sqrt{2} \cdot 2,1 A_{RMS} \sqrt{2} \cdot 0,6 = 1,008 \text{ kW}$$

$$\eta = \frac{0,6 \text{ kW}}{1,008 \text{ kW}} = 0,59 \approx 60\%$$

②  $V/f$  control