The inductance as a function of the rotor position θ, may be expressed using Fourier Series expansion, as

$$L(\theta) = a_0 + a_1\cos\theta + b_1\sin\theta + a_2\cos 2\theta + b_2\sin 2\theta$$
$$+ a_3\cos 3\theta + b_3\sin 3\theta + \underbrace{\cdots\cdots\cdots}_{\text{high order harmonics are neglected}} \quad \cdots (*)$$

We notice the inductance satisfies two constrains.

$L(\theta) = L(-\theta)$. $\quad \cdots$ ①

$L(\theta+\pi) = L(\theta)$. $\quad \cdots$ ②

- For the 1st constrain, it eliminates all sin components in (*), because if there ~~exists~~ exits a sin component, e.g. $b_1\sin\theta$.

  then. $L(-\theta) = \cdots + b_1\sin(-\theta)\cdots = \cdots - b_1\sin(\theta)\cdots\cdots$

  where in $L(\theta)$, we have $L(\theta) = \cdots + b_1\sin\theta\cdots\cdots$

  So $L(-\theta) = L(\theta)$ is not possible.

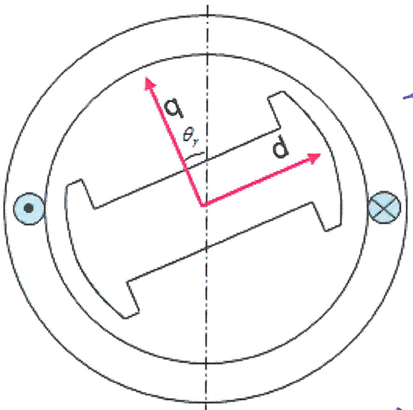
- The 2nd constrain (②) eliminate $\cos\theta$ $\cos 3\theta$ components because $\cos(\theta+\pi) = -\cos\theta$. and $\cos[3(\theta+\pi)] = \cos(3\theta+\pi)$
  $= -\cos 3\theta$.

  Therefore: $L(\theta+\pi) = \cdots - a_1\cos\theta \cdots - a_3\cos 3\theta$.

  $L(\theta) = \cdots + a_1\cos\theta \cdots + a_3\cos 3\theta$.

  Existence of $\cos\theta$ and $\cos 3\theta$ will make $L(\theta+\pi)$, $L(\theta)$ not equal.

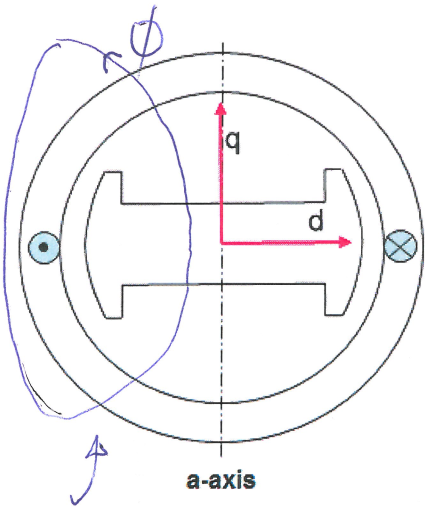
- Therefore the only two components left: → DC component a_0
  → 2nd cos harmonic $a_2\cos 2\theta$

In qdo reference frame
(Defined as )

the inductance is expressed as

$$L(\theta) = L_1 - L_2 \cos 2\theta_r \quad \cdots (**)$$

We check two special positions to validate equation $(**)$.

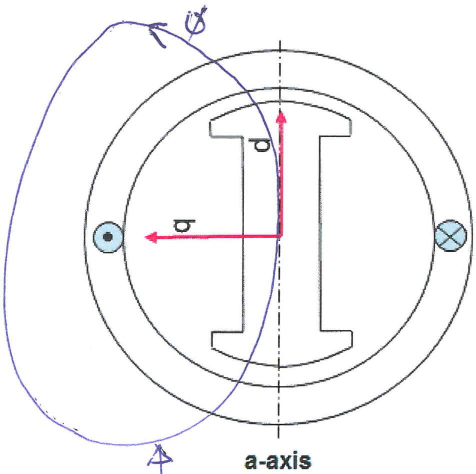


(when $\theta_r = 0$)

when $\theta_r = 0$. $(**)$ gives

$L(\theta) = L_1 - L_2$. , this is the min. inductance value of $L(\theta)$. and this is correct because the phase-a current produces flux traveling through the largest air gap

min. inductance.

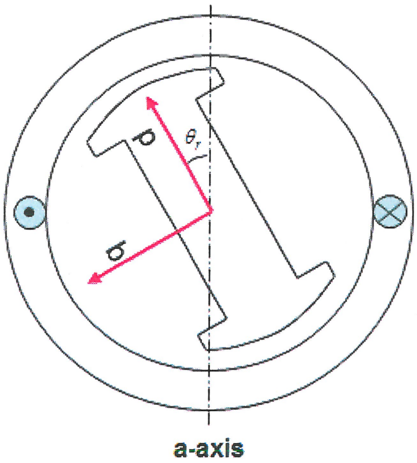


(when $\theta_r = \frac{\pi}{2}$)

when $\theta_r = \frac{\pi}{2}$, $(**)$ gives.

$L(\theta) = L_1 + L_2$. this is the max. inductance value of $L(\theta)$. This is correct because at this position, the flux travels ~~trav~~ through the minimum air gap $\Rightarrow$ largest inductance value.

In dq0 reference frame.
(Defined as)



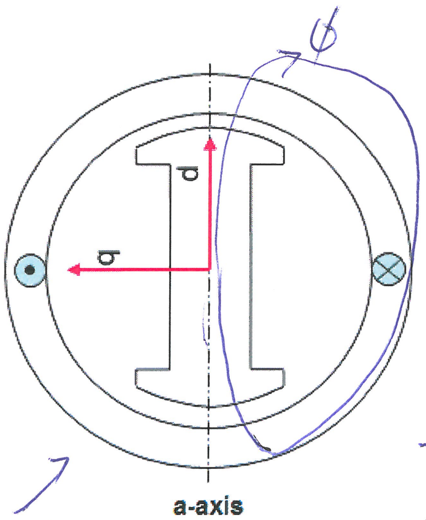
the inductance is expressed:

$$L(\theta) = L_1 + L_2 \cos 2\theta_r \quad \cdots\cdots\cdots (*)$$

L_1: DC component

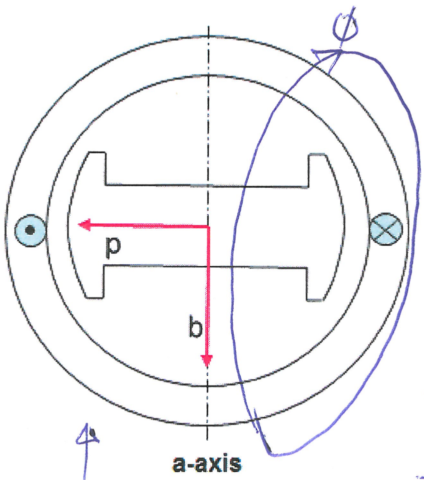
L_2: coefficient for the 2nd order harmonic.

Why this expression is correct?
We can check at two special positions.



(when $\theta_r = 0$)

$\Rightarrow \theta_r = 0$, then $(*)$ gives. $L(\theta) = L_1 + L_2$.
(This is the max. value of $L_1 + L_2 \cos 2\theta_r$)
This is correct, because the flux, produced by phase-a current, travels through the minimum air gap $\rightarrow$ corresponding to max. inductance.



(when $\theta_r = \frac{\pi}{2}$)

$\Rightarrow \theta_r = \frac{\pi}{2}$, then $(*)$ gives $L(\theta) = L_1 - L_2$
(This is the min. value of $L_1 + L_2 \cos 2\theta_r$)
This is correct, because the flux travels through the largest air gap $\rightarrow$ corresponding to the min. inductance value.