Practical Magnetic Design

Topic 6.

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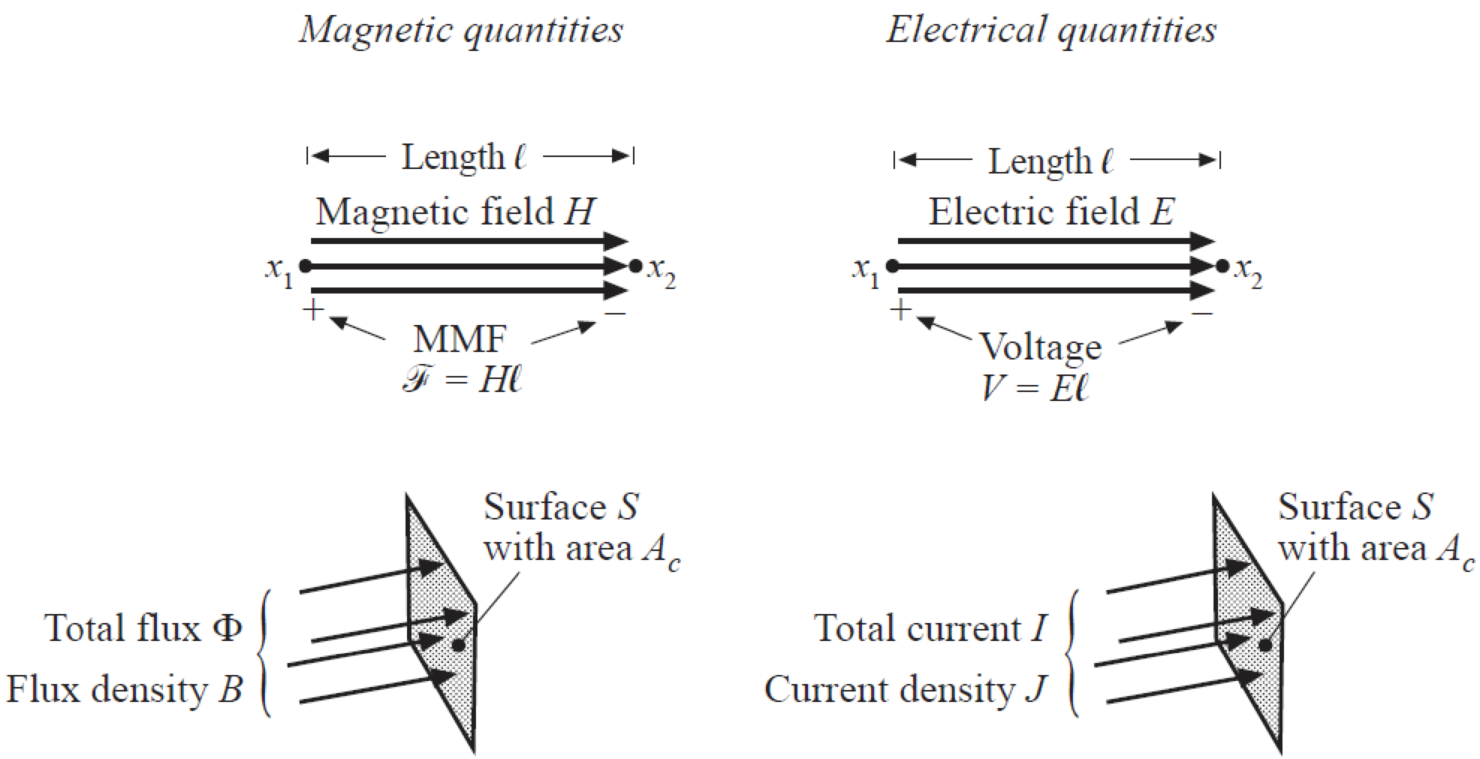
Agenda

- Basic magnetics theory
  - Basic concepts
  - Magnetic circuits
- Inductor design
  - Design constraints
  - A step-by-step procedure
- Transformer design
  - Transformer model
  - Design constraints
  - A step-by-step procedure
- Magnetic materials

Basic magnetics theory

- **Basic concepts**

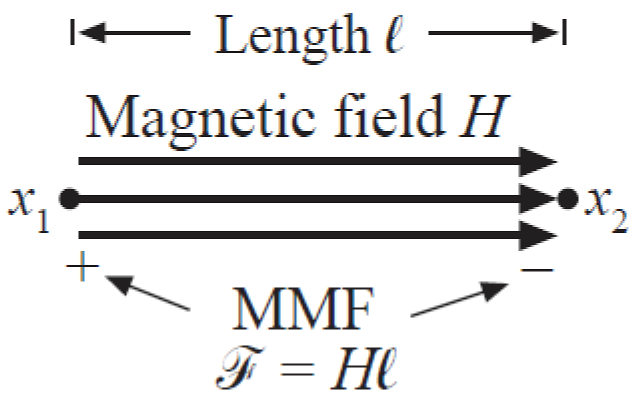


Basic magnetics theory

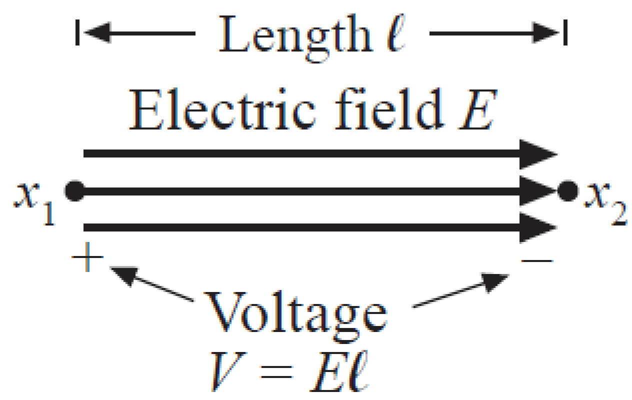
- **Magnetic Field *H* and Magneto Motive Force $\mathscr{F}$**

$$\mathscr{F} = \int_{x_1}^{x_2} \boldsymbol{H} \cdot d\boldsymbol{\ell}$$

Example: uniform magnetic field of magnitude H



Analogous to electric field of strength E, which induces voltage (EMF) V:

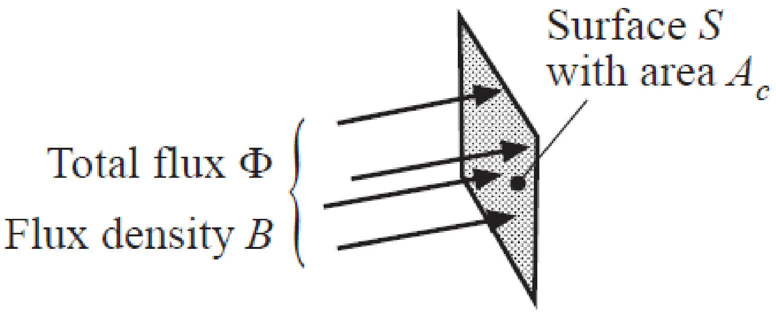


Basic magnetics theory

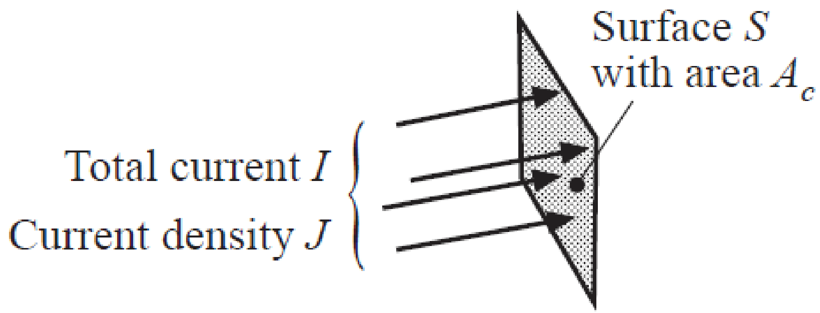
- **Flux Density *B* and Total Flux ϕ**

$$\Phi = \int_{surface\ S} \boldsymbol{B} \cdot d\boldsymbol{A}$$

Example: uniform flux density of magnitude B

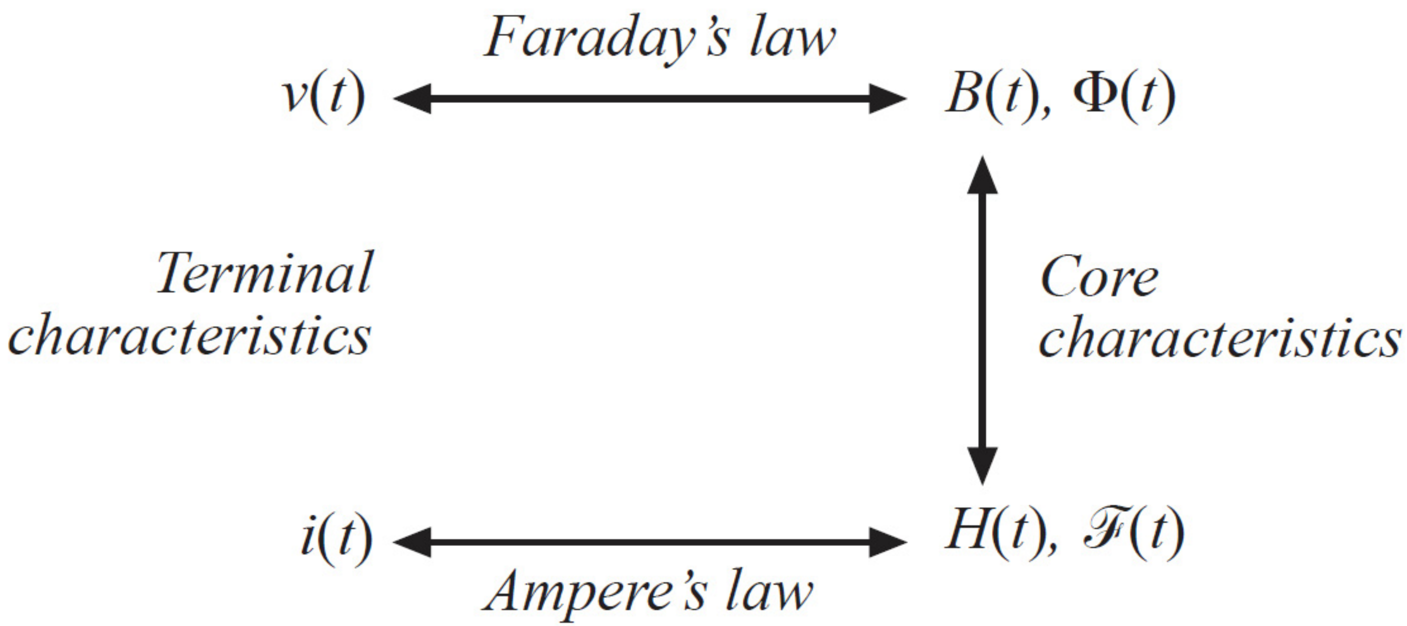


Analogous to electrical conductor current density of magnitude J, which leads to total conductor current I:



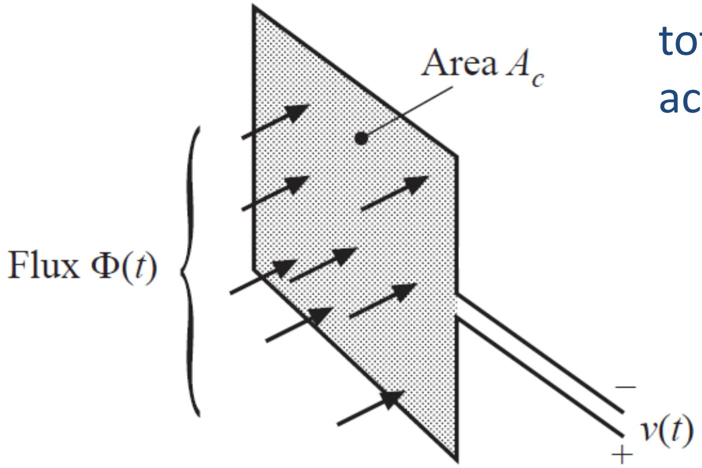
Basic magnetics theory

- **Connection between the electrical and magnetic quantities**



Basic magnetics theory

- **Faraday's law**

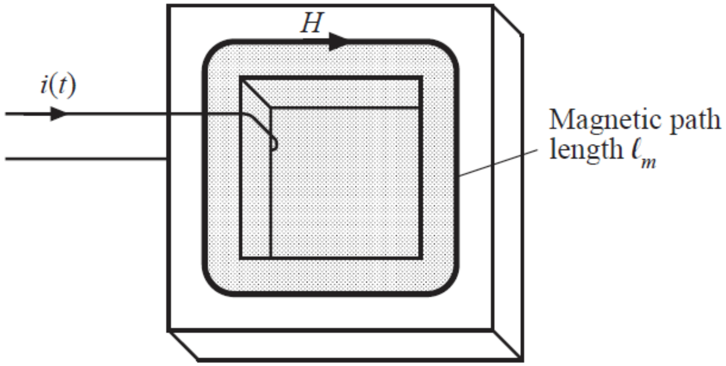


Voltage $v(t)$ is induced in a loop of wire by change in the total flux $\phi(t)$ passing through the interior of the loop, according to:

$$v(\mathrm{t}) = \frac{d\Phi(\mathrm{t})}{dt} = A_c \frac{dB(\mathrm{t})}{dt}$$

Basic magnetics theory

- **Ampere's law**



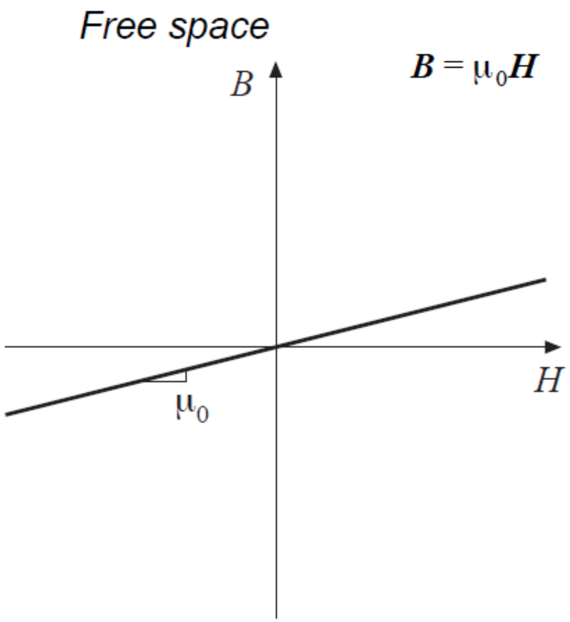
The net MMF around a closed path is equal to the total current passing through the interior of the path:

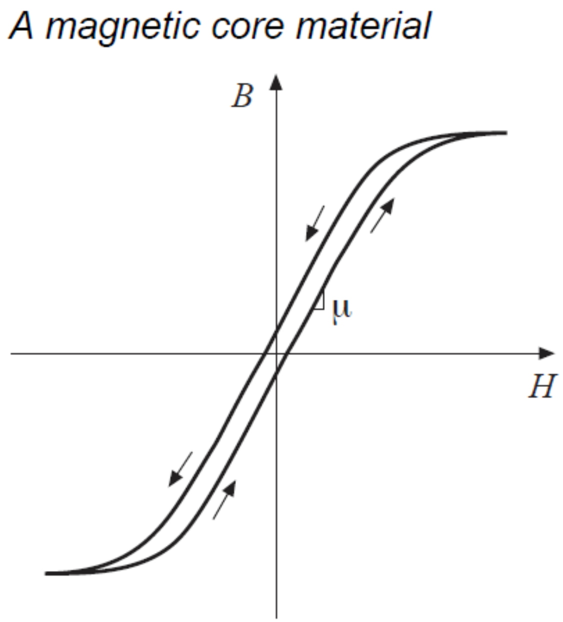
$$\int_{closed\ path} H \cdot dl = \text{total current passing through interior of path}$$

$$H(t)\,l_m = i(t)$$

Basic magnetics theory

- **Core material characteristics: the relation between *B* and *H***



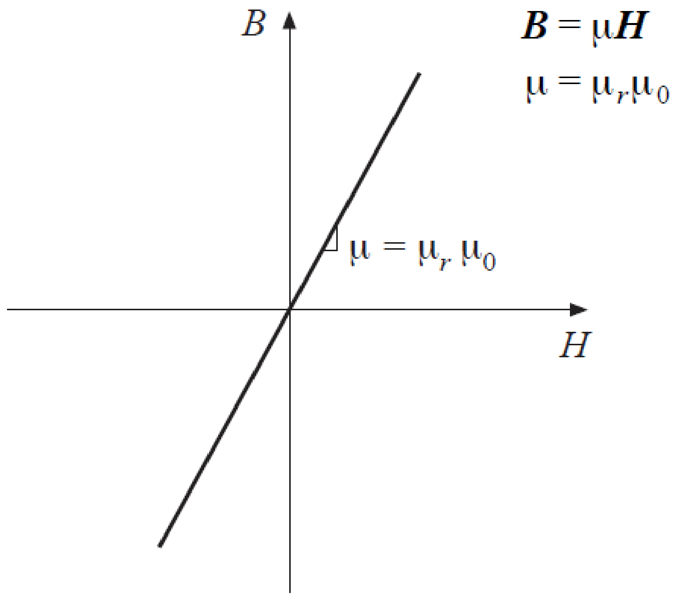


$\mu_0 = 4 \cdot \pi \cdot 10^{-7}$ permeability of the free space [H/m]

Basic magnetics theory

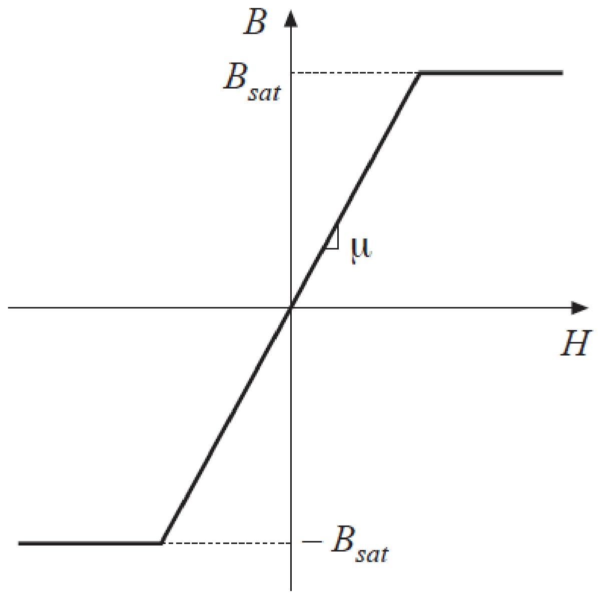
- **Piecewise-linear modelling of core material characteristics:**

No hysteresis or saturation



Typical relative permeability
$\mu_r = 10^3$ to 10^5

Saturation, no hysteresis



Typical values for saturation flux densities:
B_{sat} = 0,3T to 0,5T ferrite
B_{sat} = 0,5T to 1T powder iron
B_{sat} = 1T to 2T iron lamination

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Basic magnetics theory

- **Units**

quantity	*MKS*	*unrationalized cgs*	*conversions*
core material equation	$B = \mu_0\, \mu_r\, H$	$B = \mu_r\, H$	
B	Tesla	Gauss	$1\mathrm{T} = 10^4\mathrm{G}$
H	Ampere / meter	Oersted	$1\mathrm{A/m} = 4\pi \cdot 10^{-3}\ \mathrm{Oe}$
Φ	Weber	Maxwell	$1\mathrm{Wb} = 10^8\ \mathrm{Mx}$ $1\mathrm{T} = 1\mathrm{Wb}\ /\ \mathrm{m}^2$

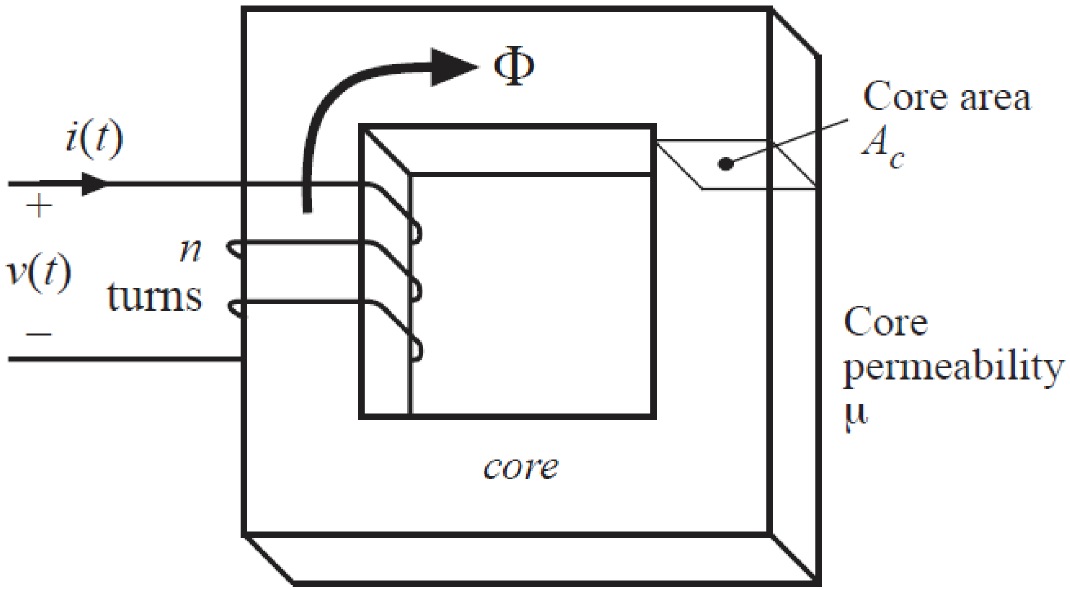
Basic magnetics theory

- **Example – simple inductor – Faraday's law:**

Faraday's law:

For each turn of wire, we can write

$$v_{turn}(t) = \frac{d\Phi(t)}{dt}$$



Total winding voltage is

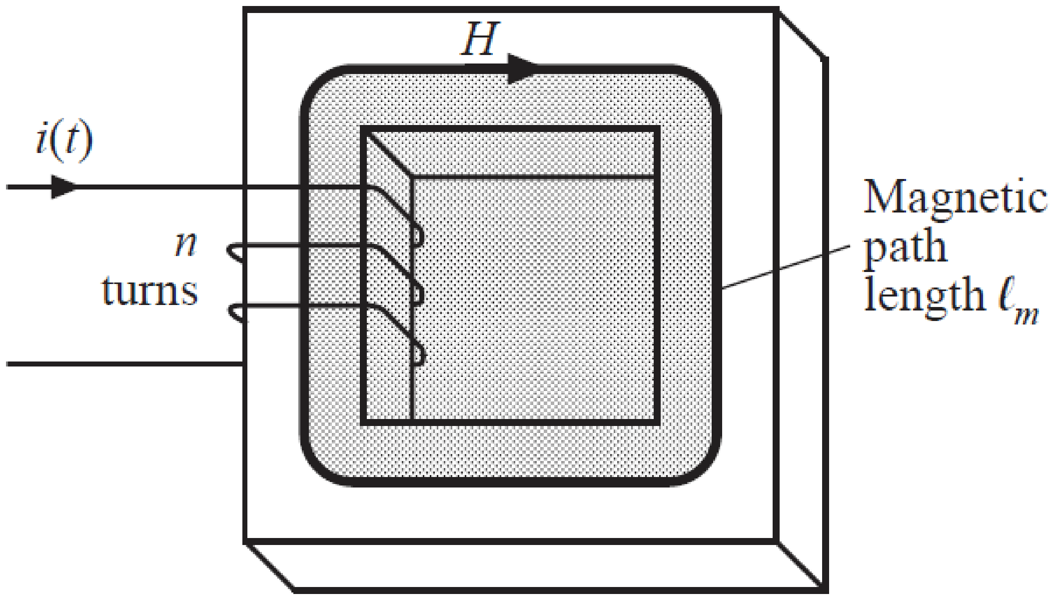
$$v(t) = n v_{turn}(t) = n \frac{d\Phi(t)}{dt}$$

Express in terms of the average flux density $B(t) = \mathscr{F}(t)/A_c$

$$v(t) = n A_c \frac{dB(t)}{dt}$$

Basic magnetics theory

- **Example – simple inductor – Ampere's law:**



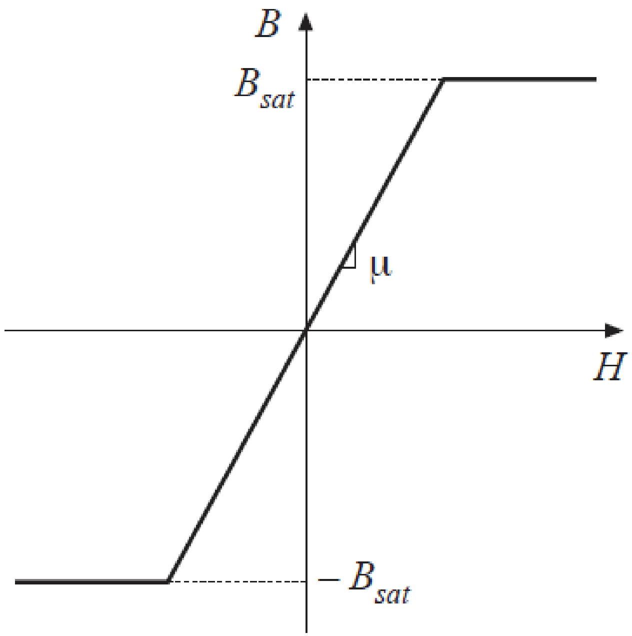
$$H(t)\cdot l_m = n\cdot i(t)$$

Basic magnetics theory

- **Example – simple inductor – core material model:**

$$B = \begin{cases} B_{sat} & \text{for } H \geq B_{sat}/\mu \\ \mu H & \text{for } |H| < B_{sat}/\mu \\ -B_{sat} & \text{for } H \leq -B_{sat}/\mu \end{cases}$$

$$I_{sat} = \frac{B_{sat} l_m}{\mu n}$$



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Basic magnetics theory

- **Example – simple inductor – electrical terminal characteristics:**

$$v(t) = nA_c \frac{dB(t)}{dt} \qquad H(t)\,\ell_m = n\,i(t) \qquad B = \begin{cases} B_{sat} & \text{for } H \geq B_{sat}/\mu \\ \mu H & \text{for } |H| < B_{sat}/\mu \\ -B_{sat} & \text{for } H \leq -B_{sat}/\mu \end{cases}$$

$$|i| < I_{sat}: \quad v(t) = \mu n A_c \frac{dH(t)}{dt} \longrightarrow v(t) = \frac{\mu n^2 A_c}{\ell_m}\frac{di(t)}{dt}$$

$$v(t) = L\frac{di(t)}{dt}$$ where $$\boxed{L = \frac{\mu n^2 A_c}{l_m}}$$ **Inductance**

$$|i| \geq I_{sat}: \quad v(t) = nA_C \frac{dB_{sat}(t)}{dt} = 0$$

Flux density is constant and equal to B_{sat}

Short circuit

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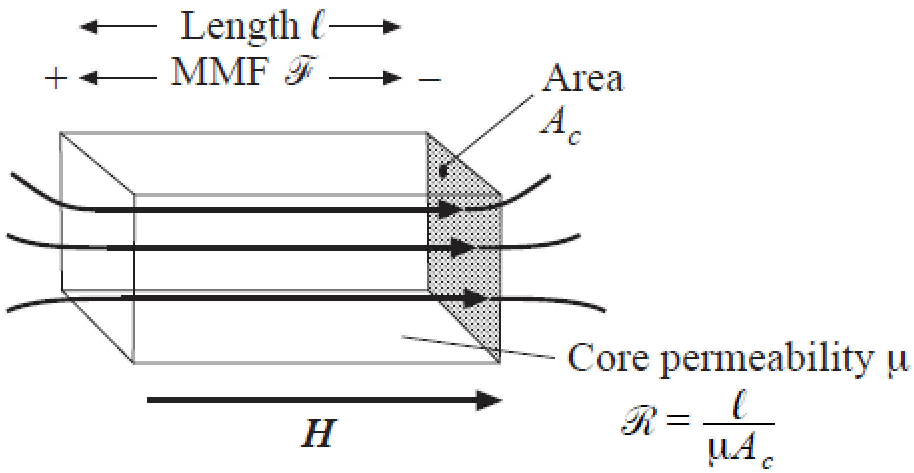
Basic magnetics theory

- **Magnetic circuits**

Uniform flux and magnetic field inside a rectangular element:



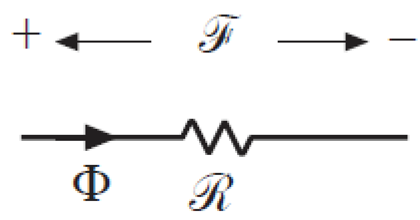
MMF between ends of element is

$$\mathscr{F} = H\ell$$

Since $H = B / \mu$ and $B = \Phi / A_c$, we can express $\mathscr{F}$ as

$$\mathscr{F} = \Phi\mathscr{R} \quad \text{with} \quad \mathscr{R} = \frac{\ell}{\mu A_c}$$

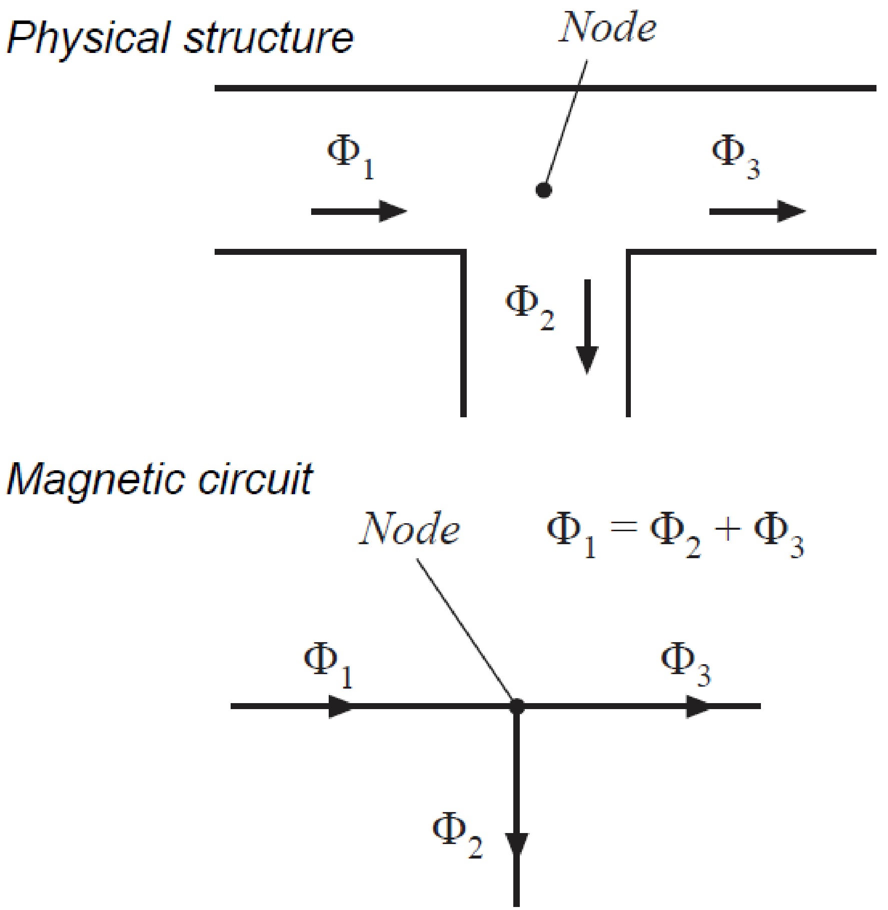
A corresponding model:



$\mathscr{R}$ = reluctance of element

Basic magnetics theory

- **Magnetic analog of Kirchhoff's current law:**



Basic magnetics theory

- **Magnetic analog of Kirchhoff's voltage law:**

$$\oint_{closed\ path} H \cdot dl = \text{total current passing through interior of path}$$

Left-hand side: sum of MMF's across the reluctances around the closed path

Right-hand side: currents in windings are sources of MMF's. An ***n***-turn winding carrying current ***i*(*t*)** is modelled as an MMF (voltage) source, of value ***n·i*(*t*)**.

Total MMF's around the closed path add up to zero.

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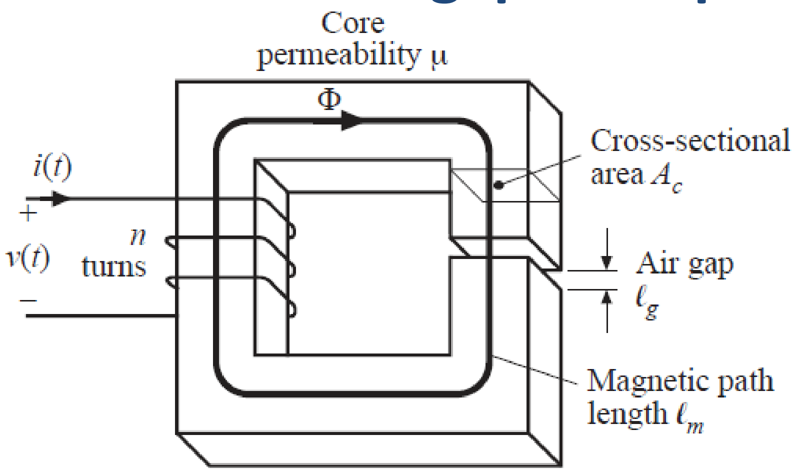
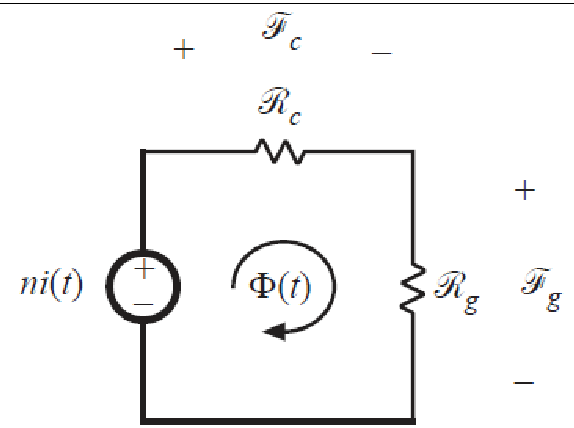
Basic magnetics theory

- **Inductor with air-gap example:**





$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

$$\mathcal{R}_c = \frac{\ell_c}{\mu A_c}$$

$$\mathcal{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

Faraday's law: $$v(t) = n\frac{d\Phi(t)}{dt}$$

Substitute for Φ: $$v(t) = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}\frac{di(t)}{dt}$$

Hence inductance is

$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g}$$

Basic magnetics theory

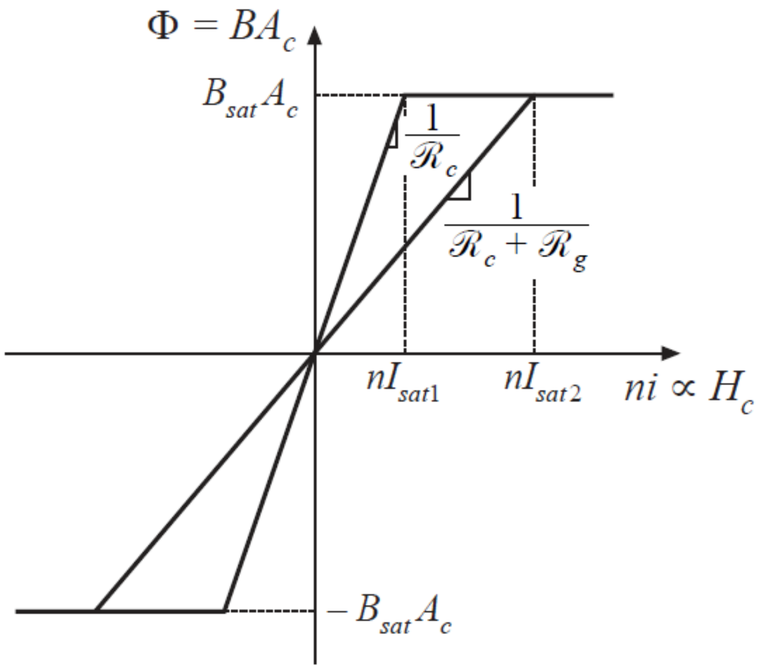
- **Effect of the air-gap:**

$$ni = \Phi\left(\mathscr{R}_c + \mathscr{R}_g\right)$$

$$L = \frac{n^2}{\mathscr{R}_c + \mathscr{R}_g}$$

$$\Phi_{sat} = B_{sat}A_c$$

$$I_{sat} = \frac{B_{sat}A_c}{n}\left(\mathscr{R}_c + \mathscr{R}_g\right)$$

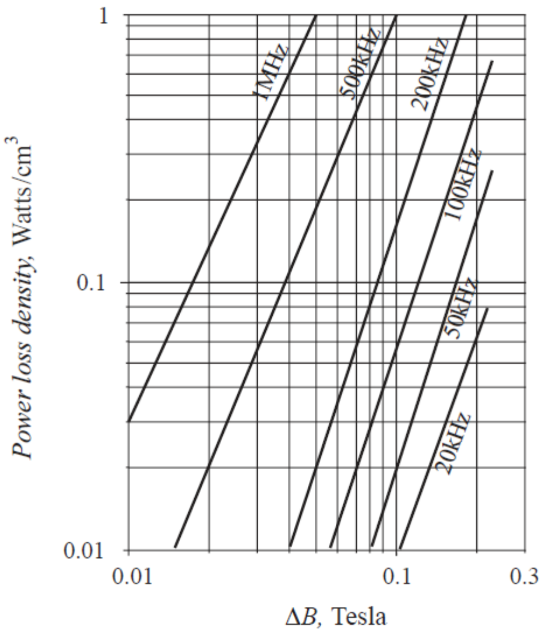


- Decrease inductance
- Increase saturation current
- Inductance is less dependent on core permeability

Basic magnetics theory

- **Loss mechanisms in magnetic devices**

Total core losses – usually check manufacturer data
- Hysteresis losses
- Eddy current losses



Empirical equation, at a fixed frequency:

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m$$

Basic magnetics theory

- **Core losses**

In literature other formulas can also be found to the calculate P_{fe}

- Some depend on the volume of the core multiplied with some coefficient
- Some depend on the mass of the core multiplied with some coefficient

Those coefficients just as $\mathbf{K_{fe}}$ and **β** should be found from datasheet of the core material (ferrite ex.)

Example (see datasheets below):

$V_{core} = 78{,}65\ cm^3$ **

$P_{fe\_unity} = 400\ kW/m^3$ (at ΔB=200mT, 100°C, f_{ws}=100kHz – see next page) *

So: $\mathbf{P_{fe\_tot} = P_{fe\_unity} \cdot 2 \cdot V_{core}}$ (usually the magnetics are built of even nr. of identical cores)

$$P_{fe\_tot} = 400 \cdot 10^3\ W / 10^6\ cm^3 \cdot 2 \cdot 78{,}65\ cm^3 = 62{,}92\ W$$

Pay attention to units!

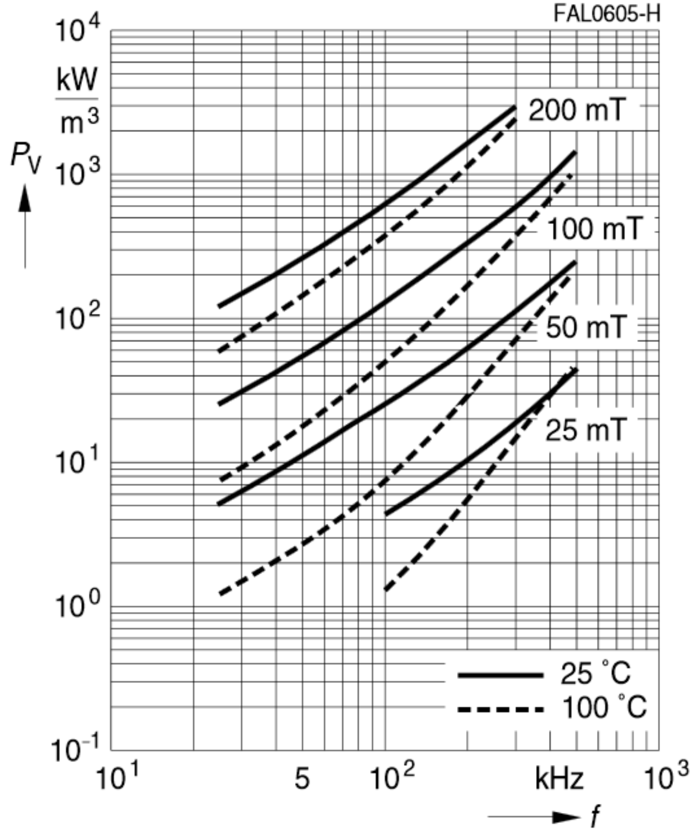
*Material N87: http://en.tdk.eu/blob/528882/download/4/pdf-n87.pdf

**Core E65: http://docs-europe.electrocomponents.com/webdocs/13c0/0900766b813c0f15.pdf

Basic magnetics theory

- **Core losses**

Relative core losses
versus frequency
(measured on R34 toroids)



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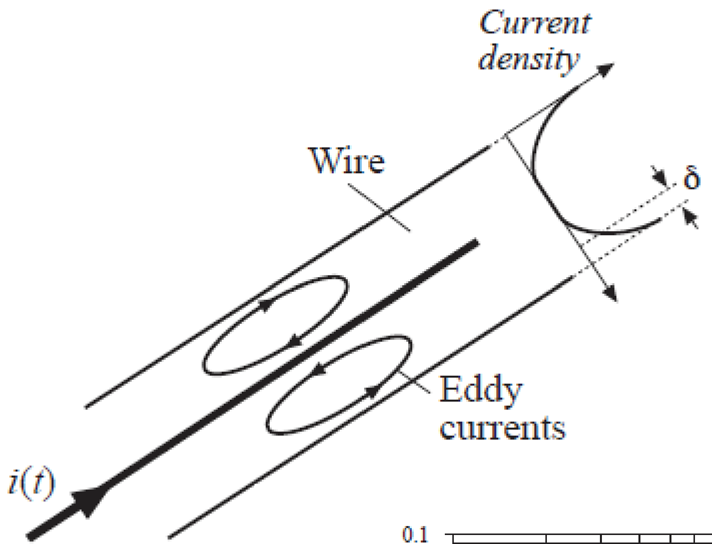
Basic magnetics theory

- **Copper losses**

Total copper losses

- Eddy current losses
  - Skin effect
  - Proximity effect

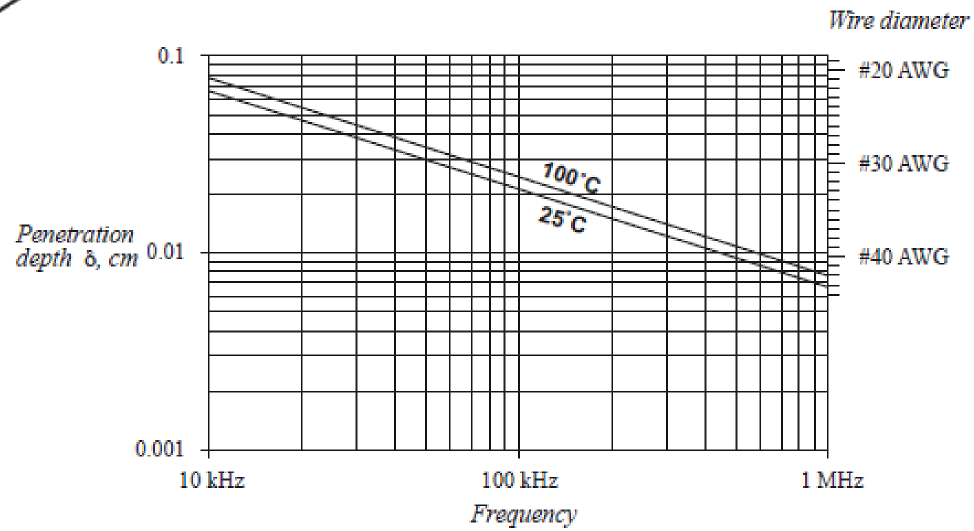


Penetration depth δ, cm

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}$$

For copper at room temperature:

$$\delta = \frac{7.5}{\sqrt{f}}\ \text{cm}$$



- DC losses – wire resistance

$$P_{Cu} = I_{RMS}{}^{2} R$$

Basic magnetics theory

- **Wire diameter (#AWG and mm) and A_w**

Conversion table - American Wire Gauge - mm. - mm^2					
AWG N°	Diam. mm.	Area mm^2	AWG N°	Diam. mm.	Area mm^2
1	7,350	42,400	16	1,290	1,3100
2	6,540	33,600	17	1,150	1,0400
3	5,830	26,700	18	1,024	0,8230
4	5,190	21,200	19	0,912	0,6530
5	4,620	16,800	20	0,812	0,5190
6	4,110	13,300	21	0,723	0,4120
7	3,670	10,600	22	0,644	0,3250
8	3,260	8,350	23	0,573	0,2590
9	2,910	6,620	24	0,511	0,2050
10	2,590	5,270	25	0,455	0,1630
11	2,300	4,150	26	0,405	0,1280
12	2,050	3,310	27	0,361	0,1020
13	1,830	2,630	28	0,321	0,0804
14	1,630	2,080	29	0,286	0,0646
15	1,450	1,650	30	0,255	0,0503

Tnt-Audio Internet HiFi Review http://www.tnt-audio.com

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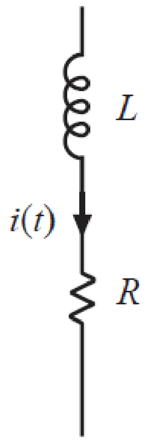
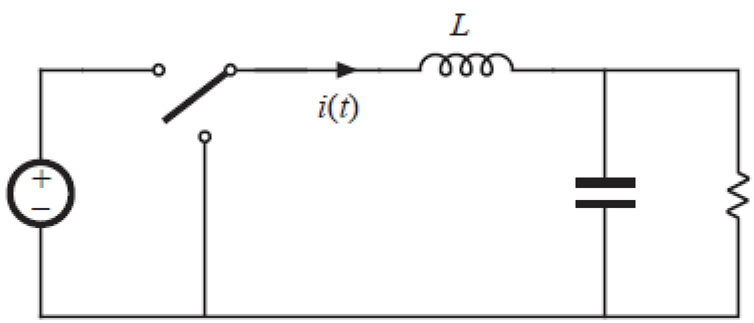
Inductor design

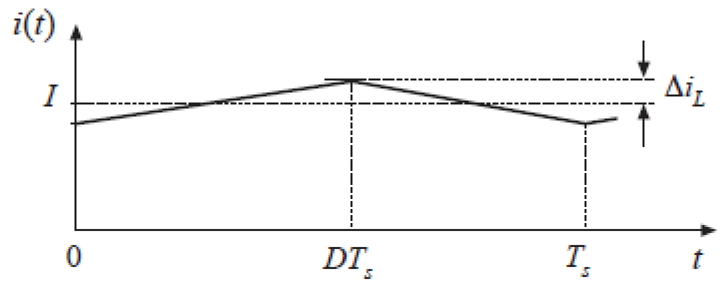
- **Objectives:**



Design inductor having a given inductance ***L***, which carries worst-case current ***I_{max}*** without saturating, and which has a given winding resistance ***R***, or, equivalently, exhibits a worst-case copper loss of:

$$P_{Cu} = I_{RMS}{}^{2}R$$





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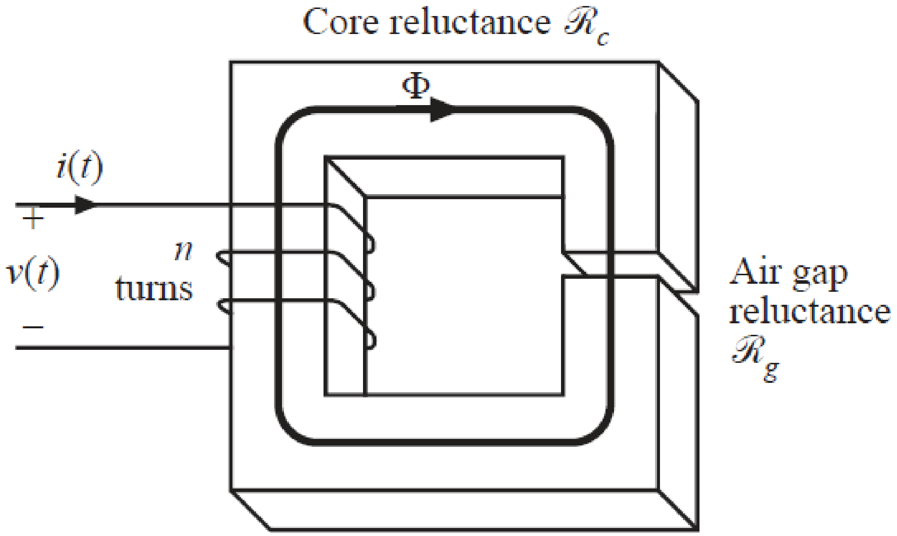
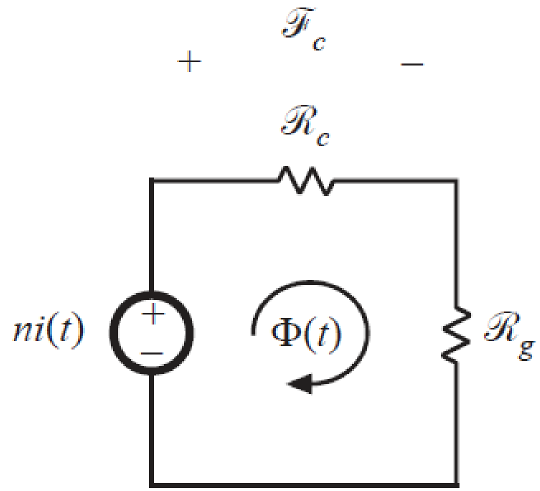
Inductor design

- **Assumed geometry**



$$\mathcal{R}_c = \frac{\ell_c}{\mu_c A_c}$$

$$\mathcal{R}_g = \frac{\ell_g}{\mu_0 A_c}$$



Solve magnetic circuit:

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

Usually $\mathcal{R}_c \ll \mathcal{R}_g$ and hence

$$ni \approx \Phi \mathcal{R}_g$$

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Inductor design

- **Constraint 1.**

  Maximum flux density: Given a peak winding current I_{max}, it is desired to operate the core flux density at a peak value B_{max}. The value of B_{max} is chosen to be less than the worst-case saturation flux density B_{sat} of the core material.

$$ni = BA_c\mathcal{R}_g$$

$$B = B_{max}:$$

$$nI_{max} = B_{max}A_c\mathcal{R}_g = B_{max}\frac{\ell_g}{\mu_0}$$

The turns ratio n and air gap length l_g are unknown.

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Inductor design

- **Constraint 2.**

  Specified inductance:

$$L = \frac{n^2}{\mathcal{R}_g} = \frac{\mu_0 A_c n^2}{\ell_g}$$

The turns ratio n, core area A_c, and air gap length l_g are unknown.

Inductor design

- **Constraint 3.**

Winding area: wires must fit into the core window

- Total area of the copper in the window

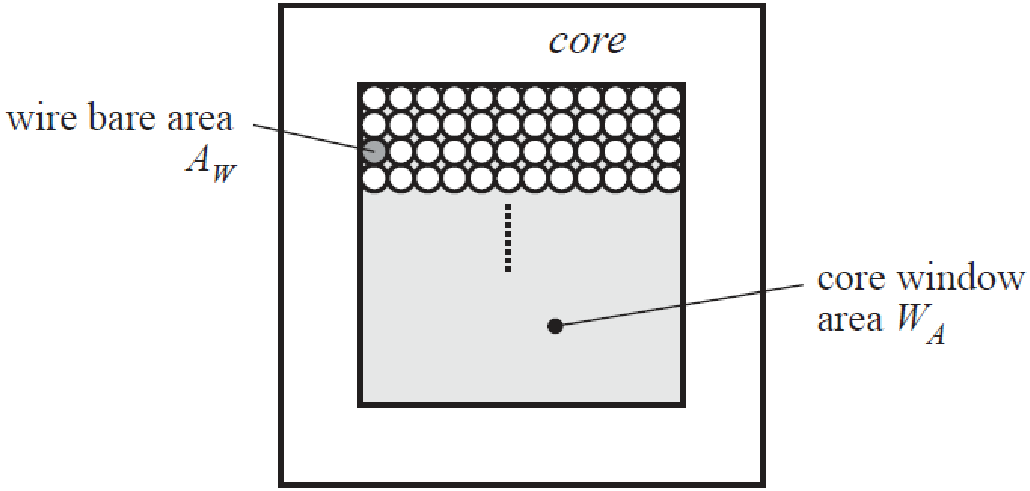
$$nA_w$$

- Area available for windings (*K_u is the fill factor*)

$$K_u W_A$$

- Constraint

$$\boxed{K_u W_A \geq nA_w}$$



Inductor design

- **Constraint 4: Winding resistance**

$$R = \rho \frac{l_b}{A_W}$$

where ρ is the resistivity of the conductor material, l_b is the length of the wire, and A_W is the wire bare area. The resistivity of copper at room temperature is $1.724 \cdot 10^{-6}$ Ωcm. The length of the wire comprising an n-turn winding can be expressed as:

$$l_b = n(MLT)$$

where (MLT) is the mean-length-per-turn of the winding. The mean length-per-turn is a function of the core geometry. The above equations can be combined to obtain the fourth constraint:

$$R = \rho \frac{n(MLT)}{A_W}$$

Inductor design

- **Core geometrical constant K_g**

$$nI_{max} = B_{max} A_c \mathcal{R}_g = B_{max} \frac{\ell_g}{\mu_0}$$

$$L = \frac{n^2}{\mathcal{R}_g} = \frac{\mu_0 A_c n^2}{\ell_g}$$

$$K_u W_A \geq n A_W$$

$$R = \rho \frac{n\,(MLT)}{A_W}$$

- A_c, W_A, and MLT are functions of the core geometry
- I_{max}, B_{max}, μ_0, L, K_u, R, and ρ are given specifications or other known quantities
- n, l_g, and A_W *are unknowns*

$$\frac{A_c^2 W_A}{(MLT)} \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u}$$

$$K_g = \frac{A_c^2 W_A}{(MLT)}$$

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Inductor design

- **Core geometrical constant K_g**

K_g is a figure-of-merit that describes the effective electrical size of magnetic cores, in applications where the following quantities are specified:

- Copper loss
- Maximum flux density

How specifications affect the core size:

A smaller core can be used by increasing

B_{max} -> use core material having higher B_{sat}

R -> allow more copper loss

How the core geometry affects electrical capabilities:

A larger K_g can be obtained by increase of

A_c -> more iron core material, or

W_A -> larger window and more copper

Inductor design

- **Step-by-step procedure: *see attached StepByStepInductorDesign.docx file***

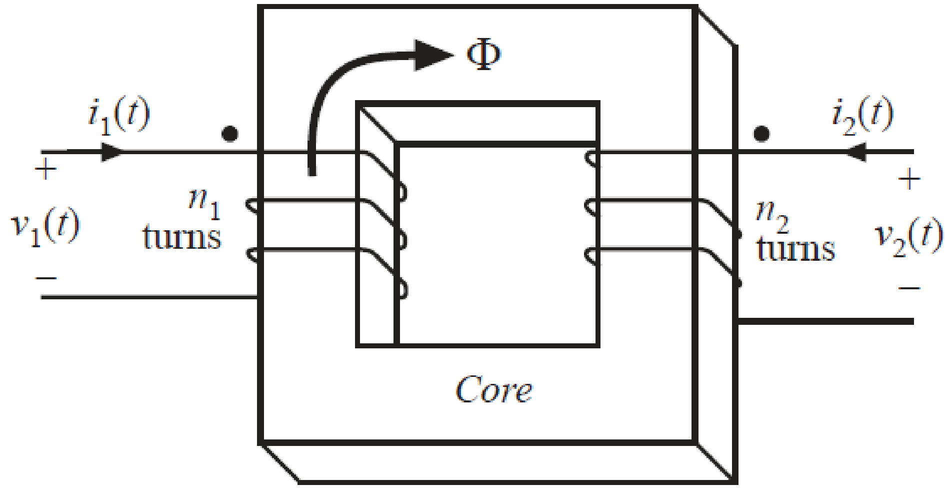
Transformer design

Two windings, no air gap:

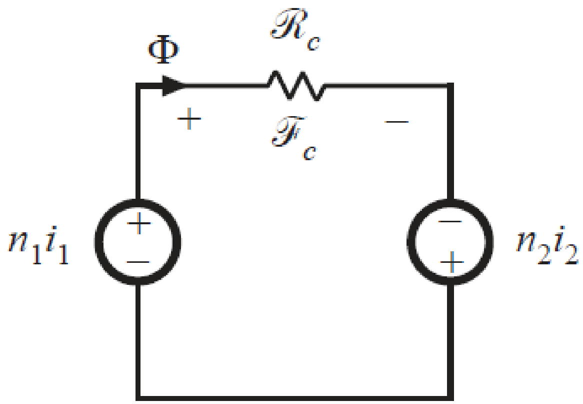
$$\mathscr{R} = \frac{\ell_m}{\mu A_c}$$

$$\mathscr{F}_c = n_1 i_1 + n_2 i_2$$

$$\Phi \mathscr{R} = n_1 i_1 + n_2 i_2$$



Magnetic circuit model:



Transformer design

- **Ideal transformer**

In the ideal transformer, the core reluctance $\mathscr{R}$ approaches zero.

MMF $\mathscr{F}_c = \Phi\,\mathscr{R}$ also approaches zero. We then obtain

$$0 = n_1 i_1 + n_2 i_2$$

Also, by Faraday's law,

$$v_1 = n_1 \frac{d\Phi}{dt}$$

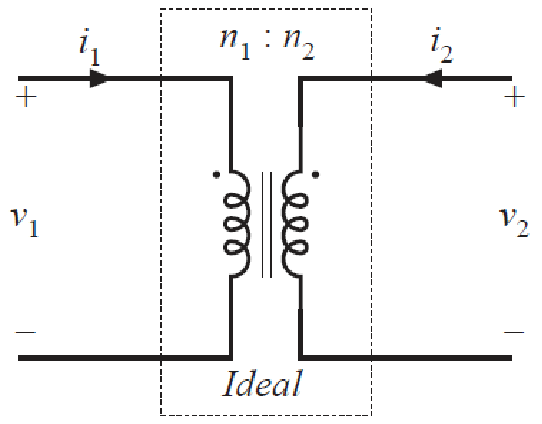
$$v_2 = n_2 \frac{d\Phi}{dt}$$

Eliminate Φ :

$$\frac{d\Phi}{dt} = \frac{v_1}{n_1} = \frac{v_2}{n_2}$$

Ideal transformer equations:

$$\frac{v_1}{n_1} = \frac{v_2}{n_2} \quad \text{and} \quad n_1 i_1 + n_2 i_2 = 0$$



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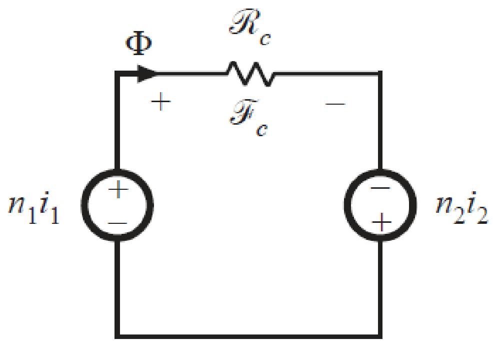
Transformer design

- **Magnetizing inductance**

For nonzero core reluctance, we obtain

$$\Phi\mathcal{R} = n_1 i_1 + n_2 i_2 \quad \text{with} \quad v_1 = n_1 \frac{d\Phi}{dt}$$



Eliminate Φ:

$$v_1 = \frac{n_1^2}{\mathcal{R}} \frac{d}{dt}\left[i_1 + \frac{n_2}{n_1} i_2 \right]$$

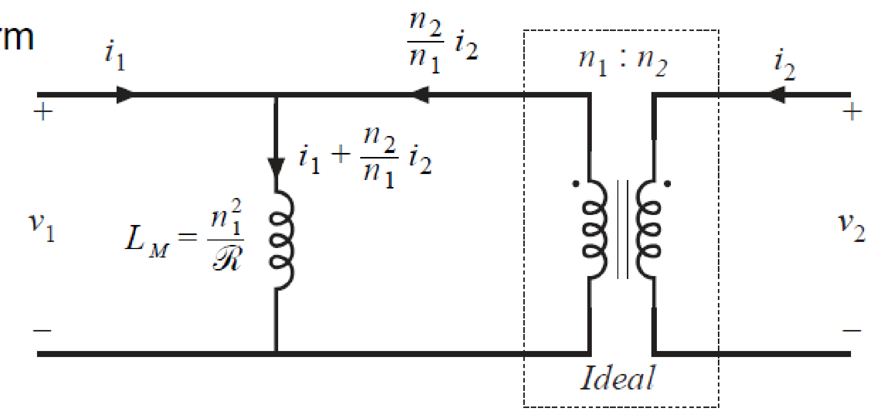
This equation is of the form

$$v_1 = L_M \frac{di_M}{dt}$$

with

$$L_M = \frac{n_1^2}{\mathcal{R}}$$

$$i_M = i_1 + \frac{n_2}{n_1} i_2$$



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Transformer design

- **Constraint 1:**

Core losses:

Typical value of β for ferrite materials: 2.6 or 2.7

ΔB is the peak value of the ac component of *B*(*t*), i.e., the peak ac flux density

So increasing Δ*B* causes core loss to increase rapidly

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m$$

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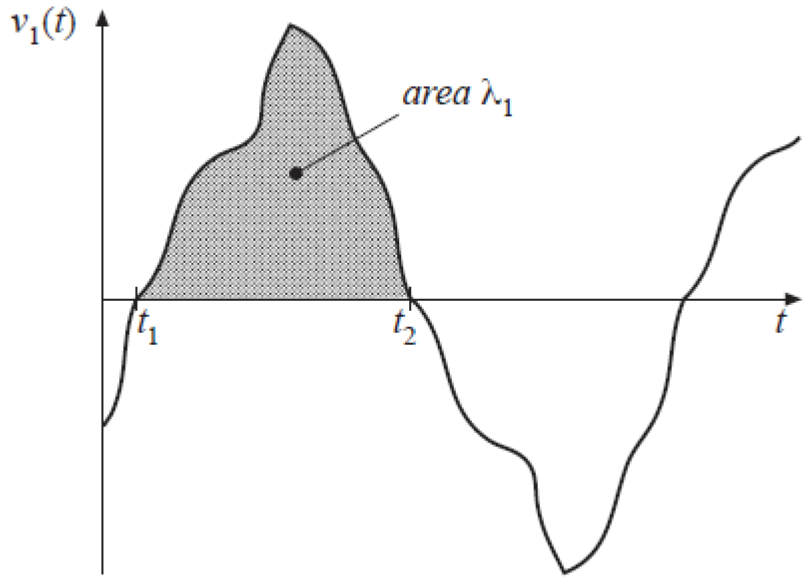
Transformer design

- **Constraint 2:**

Flux density:
Flux density $B(t)$ is related to the applied winding voltage according to Faraday's Law. Denote the voltseconds applied to the primary winding during the positive portion of $v_1(t)$ as λ_1.

This causes the flux to change from its negative peak to its positive peak. From Faraday's law, the peak value of the ac component of flux density is ΔB.

To attain a given flux density, the primary turns should be chosen as n_1.



$$\lambda_1 = \int_{t_1}^{t_2} v_1(t)\,dt$$

$$\Delta B = \frac{\lambda_1}{2 n_1 A_c} \qquad n_1 = \frac{\lambda_1}{2 \Delta B A_c}$$

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Transformer design

- **Constraint 3:**

Copper losses:
Total copper loss is then equal to:

$$P_{cu} = I_{tot}{}^2 R \qquad R = \rho \frac{n(MLT)}{A_W}$$

$$P_{cu} = \frac{\rho (MLT) n_1^2 I_{tot}^2}{W_A K_u} \qquad K_u W_A \geq n A_W$$

with:

$$I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_1} I_j$$

Eliminate n_1 (previous slide):

$$P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{4 K_u} \right) \left(\frac{(MLT)}{W_A A_c^2} \right) \left(\frac{1}{\Delta B} \right)^2$$

Thus the total loss will be:

$$P_{tot} = P_{fe} + P_{cu}$$

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Transformer design

- **The core geometrical constant K_{gfe}**

Core and material properties

Define

$$K_{gfe} = \frac{W_A (A_c)^{(2(\beta-1)/\beta)}}{(MLT)\ell_m^{(2/\beta)}} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)} \right]^{-\left(\frac{\beta+2}{\beta}\right)}$$

Design procedure: select a core that satisfies

$$K_{gfe} \geq \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4 K_u (P_{tot})^{((\beta+2)/\beta)}}$$

Design properties

This equation and inequality are the result of deriving the P_{tot} in function of the ΔB to find the optimum ac flux density

Transformer design

- **Step-by-step procedure:** ***see attached StepByStepTransformerDesign.docx file***

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Magnetic materials

Metal Alloy Tape-Wound Cores

- low frequency cores – 50, 60, 400 Hz
- high saturation flux density B_{sat} = 0,9 T
- extremely high permeability – 60 000
- build of tape wound laminations

Powdered Metal Cores

- used mostly for inductor design
- internal distributed air-gap
- low permeability – 15-200, not suitable for transformer application
- relative high B_{sat} up to 1 T
- tend to have increased core loss – depends on application
- pricy

Ferrite Cores

- most popular for SMPS
- fair permeability 1500-3000
- high frequency range up ti 1-2 MHZ
- low B_{sat} up to 0,5 T
- low cost and losses

Magnetic materials manufacturers

https://product.tdk.com/info/en/products/ferrite/catalog.html

http://www.micrometalsarnoldpowdercores.com/

http://www.mag-inc.com/

	Kool Mu	MPP	High Flux	XFLUX	AmoFlux
Perm	14-125	14-550	14-160	26-60	60
Core Loss	Low	Very Low	Moderate	High	Low
DC Bias	Good	Better	Best	Best	Better
Saturation Flux Density (Tesla)	1.0	0.75	1.5	1.6	1.5
Curie Temperature (°C)	500	460	500	700	400
Operation Temp. Range (°C)	-55~200	-55~200	-55~200	-55~200	-55~155
60u, u flat to...	900 kHz	2 MHz	1 MHz	500 kHz	2 MHz

*Source: http://www.mag-inc.com/

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Magnetic materials datasheets

Ferrite

http://docs-europe.electrocomponents.com/webdocs/13c0/0900766b813c0f15.pdf

CoolMμ

http://www.mag-inc.com/File%20Library/Product%20Datasheets/Powder%20Core/New%20Powder%20Cores/Toroids/620%20Size/0077617A7.pdf