AALBORG UNIVERSITET
Energy

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Evaluation subject: Dynamic Models of Electrical Machines and Control Systems 29 January 2018 at 9:30-13:30	

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Total number of pages, including this page: 10

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NB! Your paper must be easy to read. If this is not the case, your paper may be evaluated as "not passed".

All usual aids are allowed (notes, books, tables, calculator and PC). You are not allowed to communicate amongst each other or with the outside world which means that the use of mobile phone, Wi-Fi, internet, email is not allowed.

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Problem 1

1

- The current space vector rotates at anti-clockwise direction because

- The current space vector with respect to phase-a axis at time $t = 0{,}0075$ sec. is given by

$i_a = I_m \cdot \sin(\omega t)$, sin becaus it starts at zero at t=0

I_m is equal to 2 A and period = 0,02 sec

$\omega = \frac{2\pi}{0{,}02} = 314{,}15$ rad/s

$i_a = 2A \sin(314.15 \cdot 0{,}0075) = \underline{1{,}414\ A}$

-

Problem 1

(2) The space vector of the following a, b and c signals is found.

$$V_a = V_{ph}\cos(\omega_e t)\ ,\ V_b = V_{ph}\cos(\omega_e t + 2\pi/3)\ ,\ V_c = V_{ph}\cos(\omega_e t - \tfrac{2\pi}{3})$$

We call the space vector for $\bar{f}_{abc}$

$$\bar{f}_{abc} = \frac{2}{3}\left(V_a + V_b e^{j2\pi/3} + V_c e^{-j2\pi/3}\right)$$

$$\bar{f}_{abc} = \frac{2}{3}V_{ph}\left(\underbrace{\cos(\omega_e t) + \cos(\omega_e t + 2\pi/3)\cdot e^{j2\pi/3} + \cos(\omega_e t - 2\pi/3)\cdot e^{-j2\pi/3}}_{(*)}\right)$$

Then we find the corresponding α and β component.
The α component is given by Re(*) and β is given by Im(*)

Real part of (*)

$$= \cos(\omega_e t) + \cos(\omega_e t + 2\pi/3)\cdot\cos(2\pi/3) + \cos(\omega_e t - 2\pi/3)\cdot\cos(2\pi/3)$$

$$= \cos(\omega_e t) - \frac{1}{2}\left(\cos(\omega_e t + 2\pi/3) + \cos(\omega_e t - 2\pi/3)\right)$$

$$= \cos(\omega_e t) - \frac{1}{2}\left(\cos(\omega_e t)\cdot\cos(2\pi/3) - \cancel{\sin(\omega_e t)\cdot\sin(2\pi/3)} + \cos(\omega_e t)\cos(\tfrac{2\pi}{3}) + \cancel{\sin(\omega_e t)\sin(2\pi/3)}\right)$$

$$= \cos(\omega_e t) - \frac{1}{2}\left(2\cos(\omega_e t)\cdot\cos(2\pi/3)\right)$$

$$= \cos(\omega_e t) - \cos(\omega_e t)\cdot\underbrace{\cos(2\pi/3)}_{-\frac{1}{2}} = \frac{3}{2}\cos(\omega_e t)$$

The imaginary part of (*):

$$= \cos(\omega_e t + 2\pi/3)\cdot\sin(2\pi/3) - \cos(\omega_e t - 2\pi/3)\sin(2\pi/3)$$

$$= \frac{\sqrt{3}}{2}\left(\cos(\omega_e t + 2\pi/3) - \cos(\omega_e t - 2\pi/3)\right) = \frac{\sqrt{3}}{2}\Big(\cos(\omega_e t)\cos(\tfrac{2\pi}{3}) - \sin(\omega_e t)\sin(\tfrac{2\pi}{3})$$

$$\ldots - \cos(\omega_e t)\cos(\tfrac{2\pi}{3}) + \sin(\omega_e t)\sin(\tfrac{2\pi}{3})\Big)$$

$$= \frac{\sqrt{3}}{2}\cdot 0 = 0$$

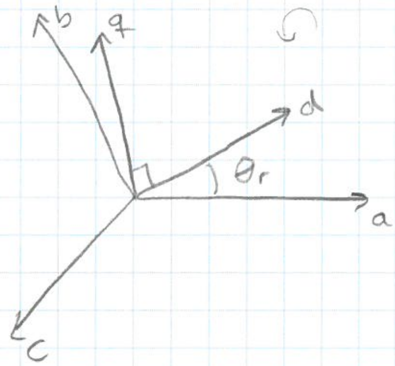
Therefore $\bar{f}_{\alpha\beta}$ is

$$\bar{f}_{\alpha\beta} = V_{ph}\cdot\frac{2}{3}\cdot\left(\frac{3}{2}\cos(\omega_e t) + j\cdot 0\right) = V_{ph}\left(\cos(\omega_e t) + j0\right)$$

Problem 2

(1) The mutual inductance between phase-a and phase-c

$$M_{ascs} = L_{aaq} \, Re\left(\frac{e^{j\theta_r + \pi/2}}{e^{j0°}}\right) Re\left(\frac{e^{j\theta_r + \pi/2}}{e^{j2\pi/3}}\right) + L_{aad} \, Re\left(\frac{e^{j\theta_r}}{e^{j0°}}\right) Re\left(\frac{e^{j\theta_r}}{e^{-j2\pi/3}}\right)$$



$$M_{ascs} = L_{aaq} \cdot \cos(\theta_r + \pi/2) \cdot \cos(\theta_r + \pi/2 + 2\pi/3) + L_{aad} \cos(\theta_r) \cdot \cos(\theta + 2\pi/3)$$

$$M_{ascs} = L_{aaq} \sin(\theta_r) \cdot \sin(\theta_r + 2\pi/3) + L_{aad} \cos(\theta_r) \cdot \cos(\theta + 2\pi/3)$$

(2) The inductance vs. rotor position wave form is sketched below

Where the values of L_{aaq} and L_{aad} is assumed to be

$L_{aaq} = 1,2$

$L_{aad} = 1,5$

Problem 2

3) If phase-a has $\lambda_{pm,a} = \lambda_{mpm} \cos\theta$

then $\lambda_{pm,b} = \lambda_{mpm} \cdot \cos(\theta + 90°)$ as there are 90° betwen the two phases.

$\lambda_{pm,b} = \lambda_{mpm} \cdot \sin(\theta)$

4) If phase-a supplied with current of $i_a = -I_m \sin\theta$

then the current phase-b should be

$$i_b = -I_m \sin(\theta + 90°) = -I_m \cos(\theta)$$

5) The instantaneous torques produced by phase-a and phase-b, respectively is shown in the following

The fundamental equation for the torque is

$$\tau = p \cdot i \cdot \frac{d(\lambda_{mpm} \cos\theta)}{d\theta}$$

pole pair

so

phase-a: $\tau_a = 1 \cdot (-I_m \cdot \sin\theta) \cdot \lambda_{mpm} \cdot (-\sin\theta)$

$\tau_a = \lambda_{mpm} \cdot I_m \cdot \sin\theta^2$

phase-b: $\tau_b = 1 \cdot (-I_m \cos\theta) \cdot \lambda_{mpm} \cdot \cos\theta$

$\tau_b = -\lambda_{mpm} \cdot I_m \cos\theta^2$

Problem 2

(6) The total torque produced by phase-a and phase-b will be given by

$$\tau_e = \lambda_{mpm} I_m (\sin\theta^2 - \cos\theta^2)$$

Problem 3

(1) We give the vector form of the stator side eq.

We have

$$u_{qs} = R_s \cdot i_{qs} + \overset{\frac{d}{dt}}{p} \cdot \lambda_{qs} + \omega_\theta \lambda_{ds} \quad , \quad \lambda_{qs} = (L_{ls} + L_m) i_{qs} + L_m i'_{qr}$$

$$u_{ds} = R_s \cdot i_{ds} + p \lambda_{ds} - \omega_\theta \lambda_{qs} \quad , \quad \lambda_{ds} = (L_{ls} + L_m) i_{ds} + L_m i'_{dr}$$

$\downarrow$

$$u_{qs} = R_s i_{qs} + p \lambda_{qs} + \omega_\theta \lambda_{ds} \qquad \text{fx} \quad \boxed{\bar{i}_{qds} = i_{qs} - j i_{ds}}$$

$+$

$$-j u_{ds} = -j R_s i_{ds} - j p \lambda_{ds} + j \omega_\theta \lambda_{qs}$$

$$\bar{u}_{qds} = R_s (i_{qs} - j i_{ds}) + p(\lambda_{qs} - j \lambda_{ds}) + \omega_\theta (\lambda_{ds} + j \lambda_{qs})$$

$$\bar{u}_{qds} = R_s \bar{i}_{qds} + p \bar{\lambda}_{qds} + j\omega_\theta \underbrace{(\lambda_{qs} - j \lambda_{ds})}_{\bar{\lambda}_{qds}}$$

and

$$\bar{\lambda}_{qds} = (L_{ls} + L_m) \bar{i}_{qds} + L_m \bar{i}'_{qdr}$$

(2) Now we express the stator voltage equation in vector form in the $\alpha\beta$-reference frame

When we go from qd to $\alpha\beta$ we do following

$$\bar{f}_{qd} = f_q - j f_d \rightarrow \begin{matrix} f_d = -f_\beta \\ f_q = f_\alpha \end{matrix} \rightarrow \bar{f}_{\alpha\beta} = f_\alpha + j f_\beta$$

$$\text{fx} \quad \bar{i}_{\alpha\beta} = i_\alpha + j i_\beta$$

$$\bar{u}_{\alpha\beta s} = R_s (i_{\alpha s} - j \cdot (-i_{\beta s})) + p(\lambda_{\alpha s} - j \cdot (-\lambda_{\beta s})) + j\omega_\theta (\lambda_{\alpha s} - j \cdot (-\lambda_{\beta s}))$$

$$\bar{u}_{\alpha\beta s} = R_s \bar{i}_{\alpha\beta s} + p \bar{\lambda}_{\alpha\beta s} + j\omega_\theta \bar{\lambda}_{\alpha\beta s}$$

We calculate the magnitude of the stator flux linkage $|\bar{\lambda}_{\alpha\beta s}|$ at rated steady state operation condition

Problem 3 contionuous.

(2) contn.

The voltage equation in steady state becomes.

$$\bar{V}_{\alpha\beta s} = j\omega_\theta \bar{\lambda}_{\alpha\beta s}$$

$$|\bar{\lambda}_{\alpha\beta s}| = \frac{|\bar{V}_{\alpha\beta s}|^{rated}}{\omega_{\theta,rated}} = \frac{380 \cdot \frac{1}{\sqrt{3}} \cdot \sqrt{2}\,V}{60 \cdot 2\pi \text{ rad/s}} = 0,823 \text{ V/rad}\cdot s^{-1}$$

The magnitude of stator flux linkage in rated steady state is 0,823

(3) Now the magnitude of the phase voltage after phase resistive voltage dropcompensation.
This is do by the following equation

$$|\bar{u}_s| = u_s = R_s \cdot I_s \cdot \cos\varphi + \sqrt{U_{s\lambda} - (r_s I_s \sin\varphi)^2}$$

where $I_s = 1A$, $R_s = 0,28\,\Omega$, $\varphi = 30°$ and

$$U_{s\lambda} = |\bar{\lambda}_{\alpha\beta s}| \cdot \omega_{snew} = 0,823 \text{ V/rad}s^{-1} \cdot 2\pi \cdot 0,1 \text{ rad/s} = 0,5171\,V$$

$$u_s = 0,28\,\Omega \cdot 1A \cdot \cos(30°) + \sqrt{(0,5171V)^2 - (0,28\,\Omega \cdot 1A \cdot \sin(30°))^2}$$

$$u_s = 0,7402\,V$$

(4) The slip frequency is found. want the mechanical rotor speed to be 10 Hz

$$f_s = 0,5 \cdot 60\,Hz = 30\,Hz \rightarrow \omega_s = 2\pi \cdot 30 = 60\pi$$

$$\tau_{now} = \frac{1}{4}\tau_{rated}$$

We use

$$\frac{\tau_{now}}{\tau_{rated}} = \frac{f_{se}}{S_{rated} \cdot f_{srated}}$$

And

$$\omega_{se} = \omega_s - \omega_m$$

ω_{se} - slip frequency

ω_s - stator frequency

ω_m - machine frequency

Problem 3 contn.

(4) contn.

need to find: s_{rated} by following

$$s_{rated} = \frac{\omega_{s\,rated} - \text{pole pair} \cdot \omega_{r,mec,rated}}{\omega_{s\,rated}}$$

$$s_{rated} = \frac{60 \cdot 2\pi - 3 \cdot 1160 \cdot \frac{2\pi}{60}}{60\, 2\pi} = 0{,}0333 = 3{,}33\%$$

Then f_{se} can be found

$$f_{se} = \frac{\tau_{now}}{\tau_{rated}} \cdot s_{rated} \cdot f_{s\,rated} = \frac{1}{4} \cdot 0{,}0333 \cdot 60\,Hz = 0{,}499\ Hz$$

The slip frequency needs to be $0{,}499\ Hz$

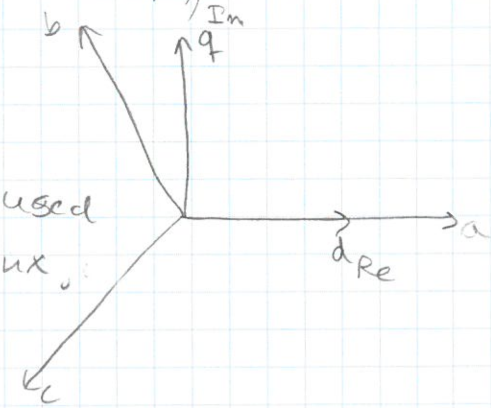
Problem 4

(1) λ_{mpm} is determined in the following

$$\bar{u}_{qd} = R(i_q - j i_d) + \underbrace{p(\lambda_q - j\lambda_d)}_{\to 0\ s.s} + j\omega_r(\lambda_q - j\lambda_d)$$

$$\bar{u}_{qd} = R(i_q - j i_d) + j\omega_r\left(L_q i_q - j(L_d i_d + \lambda_{mpm})\right)$$

$$\bar{u}_{dq} = R(i_d + j i_q) + \omega_r(\lambda_d - j\lambda_q)$$



λ_{mpm} is a peak value and is used to describe the permanent magnet flux linkag: $\lambda_{pm} = \lambda_{mpm} \cos\theta$

use

$$\lambda_{mpm} = \frac{V_{peak}}{\omega_r} = \frac{120 \frac{1}{\sqrt{3}} \sqrt{2}}{1200 \cdot \frac{2\pi}{60}} = 0{,}7796\ V/rad\,s^{-1}$$

Problem 4 contr.

(2) The block diagram on how the d-axis would be implemental to solve for i_d:

First the equations is rewriten into the following

$\frac{d}{dt}$

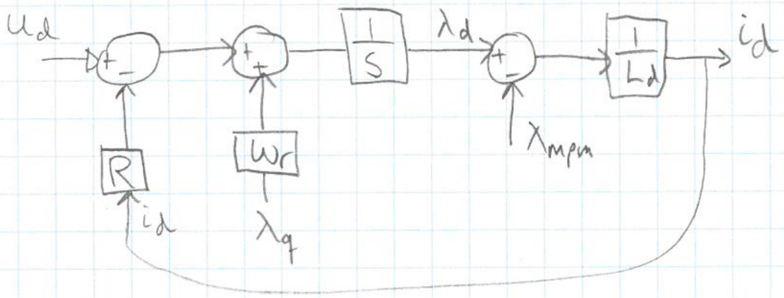
$$p\lambda_d = u_d + \omega_r \lambda_q - R i_d$$

$$\lambda_d = \frac{1}{s}(u_d + \omega_r \lambda_q - R i_d)$$

and $\lambda_d = L_d i_d + \lambda_{mpm}$

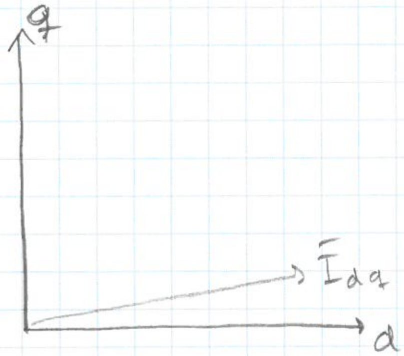
$$i_d = \frac{1}{L_d}(\lambda_d - \lambda_{mpm})$$

Block diagram:



The initial value for the integrator should be set equal to λ_{mpm} so that for $t=0$ the current $i_d = 0$.

(3) It is found that for this PM machine that $L_d > L_q$



It should be placed close to the d-axis such that I_q is small so L_d is multiplyed with a bigger numer and λ_d becomes even bigger