Department of Energy Technology

AALBORG UNIVERSITET

Mechatronics and discrete control — MCE5 semester 2015

January 6, 2016

Written exam

09.30 - 13.30 CET (4 hours)

Rules

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions.
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off. Internet access is strictly forbidden.
- Any kind of communication with other students is not allowed.

Remember

1. To write your study number on all sheets handed in.
2. It must be clear from the solutions, which methods you are using, and you must include sufficient intermediate calculations, diagrams, sketches etc. so the line of thought is clear. Printing the final result is insufficient.

Problem 1 (12.5 %)

A discrete-time controller $D(z)$ is given by the transfer function

$$D(z) = \frac{M(z)}{E(z)} = \frac{Az+1}{z+A}$$

where $M(z)$ og $E(z)$ are the output and input, respectively. A is a constant. The sampling frequency is 10 Hz.

(a) For which values of A is the controller stable?

(b) Which value must A have if the controller should have integral action?

(c) Find the difference equation that corresponds to $D(z)$

Suppose now that $A = -0.9$, and the input to the controller $e(k)$ is

$$e(k) = \begin{cases} 1, & k = 0 \\ -1, & k = 1 \\ 0, & k \geq 2 \end{cases}$$

(d) Calculate the output $m(k)$ for $0 \leq k \leq 2$ and draw diagrams showing $e(k)$ and $m(k)$

(e) Write a `Matlab` script that can plot the frequency response for $D(z)$
(it is not necessary to show the results)

(f) Draw a block diagram that represents $D(z)$ using only gain, summation, and delay (that is z^{-1}) blocks.

Problem 2 (12.5 %)

A continuous-time controller $C(s)$ is defined by

$$C(s) = \frac{M(s)}{E(s)} = 100 + \frac{3s}{0.3s+1}$$

$C(s)$ must be converted to an equivalent discrete-time controller $C(z)$ using the Forward Euler method (*forward rectangular rule*). The sampling time is denoted T.

(a) What is the gain at 0 Hz for the continuous-time controller $C(s)$?

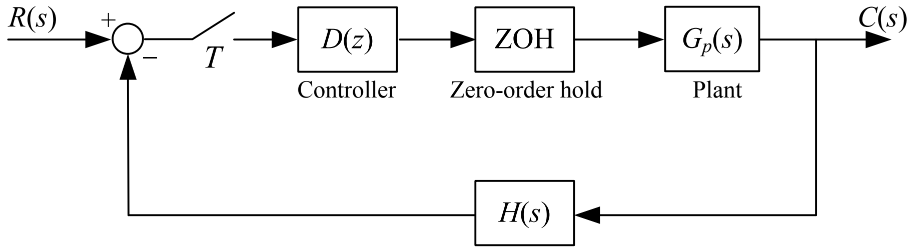
(b) Find an expression for $C(z)$ and list all zeros and poles for $C(z)$

(c) For which values of the sampling time T is $C(z)$ stable?

(d) What is the gain at 0 Hz for the discrete-time controller $C(z)$?

(e) Which sampling time T will you recommend to use?

Problem 3 (12.5 %)



The figure shows a closed-loop control system having the plant transfer function

$$G_p(s) = \frac{100}{(s+10)^2}$$

Also, $H(s) = 0.1$.

The discrete controller is $D(z) = K/z = Kz^{-1}$, where K is a positive constant. The sampling angular frequency is $\omega_s = 10\pi$ rad/s.

(a) Find an analytical expression for the pulse transfer function $G(z)$

(b) Sketch the root locus for the system in the z-domain and clarify whether or not the closed-loop response is stable when $K \to \infty$

(c) At which frequency (measured in Hz) does the ZOH give a phase shift equal to -30 degrees?

Problem 4 (12.5 %)

Consider the continuous-time system given by the differential equation

$$\frac{d^3y(t)}{dt^3} + 5\frac{d^2y(t)}{dt^2} + 9\frac{dy(t)}{dt} + 5y(t) = 2u(t)$$

where the input is $u(t)$ and the output is $y(t)$.

(a) Obtain a state-variable (state-space) model of this system

(b) Find the transfer function from input $U(s)$ to output $Y(s)$

(c) Calculate the eigenvalues of the system matrix $\mathbf{A}$

A controller is now to be designed using the pole-placement method and the design specifications are met if the closed-loop poles are placed at $s_1 = -3$, $s_2 = -4$ and $s_3 = -5$.

(d) Find the desired closed-loop characteristic equation (that is write up the expression for $\alpha_c(s) = 0$)

(e) Write a `Matlab` script that can calculate the closed loop gain vector $\mathbf{K}$
(it is not necessary to show the results of the calculation)
