Written examination in

Dynamic Models of Electrical Machines and Control Systems

1st semester M.Sc. (PED/EPSH/WPS/MCE)

Duration: 4 hours

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off
- Internet access is strictly forbidden
- Any kind of communication with other students is not allowed
- Remember to write your study number on all answer sheets
- All intermediate steps and calculations should be included in your answer sheets --- printing the final result is insufficient

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The set consists of 4 problems

Problem 1 (25%)

(1) A space current vector rotates at an angular velocity of $\omega_e = 2\pi \cdot 50$ [rad/s]. Please tell what are the phase a, b , c current instantaneous values at a particular moment where the vector is found to be leading phase-a axis by 45 degrees, (as shown in Fig. 1-(a)). The amplitude of this current vector is 10 (A).

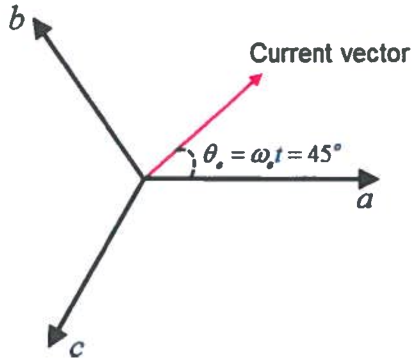

Fig. 1-(a)

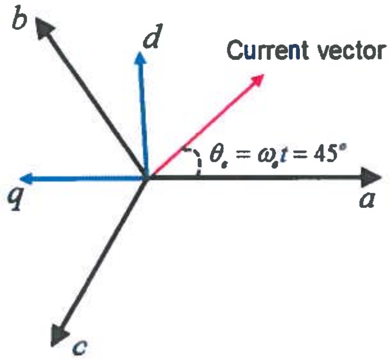

Fig. 1-(b)

(2) A dq-rotating reference frame is added to Fig. 1. At a particular moment, it is found that the d-axis is leading phase-a axis by 90 degrees while the current vector is leading phase-a axis by 45 degrees (as shown in Fig. 1-(b)). Please determine its instantaneous dq current component values.

(3) Starting from the moment described in case (2) (Fig. 1-(b)), after 0.05 seconds, it is observed that now the d-axis is leading the current vector by 90 degrees (in case (2), this angle is 45 degrees as may be observed from Fig. 1-(b)). Please determine the rotating speed of this dq-reference frame. (Note, both the current vector and the dq-frame are rotating but at different speeds.)

(4) Defining that at time t=0, the current vector is aligned with the phase-a axis and the d-axis is leading phase-a axis by 30 degrees. The current vector will start to rotate at an angular velocity of $\omega_e = 2\pi \cdot 50$ [rad/s] and the dq-frame will start to rotate at an angular velocity of $\omega_e = -2\pi \cdot 50$ [rad/s] (negative rotating!). Please sketch the d-, q-component waveforms for time period [0, 0.02] seconds.

(5) Please tell the relationship between the alfa-component in the $\alpha\beta$ reference frame and the phase-a component in the abc-reference frame. Give your proofs.

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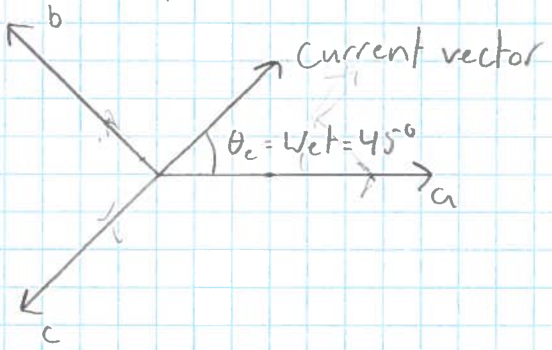
Problem 1 - A space current vector



$I = 10\,A$

$\hat{I}_{abc} = 10\, e^{j45°}$

1 - Instantanious current at 45°

- Phase a:

$$i_a = Re\left(\frac{10\, e^{j45}}{e^{j0}}\right) = 10 \cdot \cos(45°) = \underline{7{,}0711\ A}$$

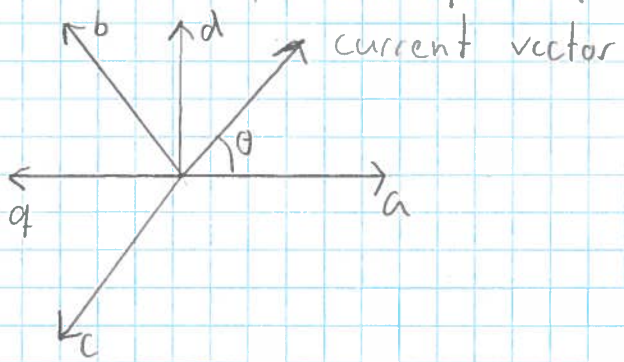
- Phase b:

$$i_b = Re\left(\frac{10\, e^{j45}}{e^{j120}}\right) = 10 \cdot \cos(45-120) = \underline{2{,}5882\ A}$$

- Phase c:

$$i_c = Re\left(\frac{10\, e^{j45}}{e^{j-120}}\right) = 10 \cdot \cos(45+120) = \underline{-9{,}659\ A}$$

2 - Instantanious dq components



- At a particular moment the d-axis is leading the a-axis by 90°.
- Current vector leads the a-axis by 45°.

- d-component:

$$\bar{f}_d = Re\frac{\bar{f}}{e^{j\theta}} = \frac{10\, e^{j45°}}{e^{j90}} = 10\cos(45-90) = 7{,}0711\ A$$

- q-component

$$\bar{f}_q = Re\left(\frac{\bar{f}}{e^{j\theta+90}}\right) = \frac{10\, e^{j45}}{e^{j90+90}} = 10\left(\cos(45-180)\right) = -7{,}0711\ A$$

$\bar{f}_{d} \quad (7{,}07 + 7{,}07)\, e^{j90°}$

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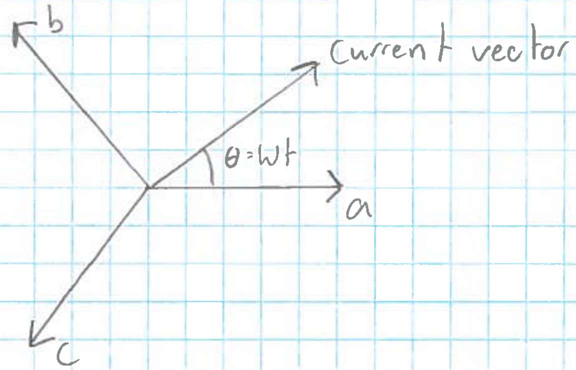
3 - Rotating speed in dq-reference frame:



- At $t=0{,}05$ s, the d-axis is leading the current vector by 90°.

- Determine the rotating speed of this dq-frame.

$w_e = 2\pi 50$ rad/s - Ang. vel. of current vector.

Thus at $t = 0{,}05$ s, it is at

$\theta_e = w_e t \Rightarrow 2\pi 50 \frac{rad}{s} \cdot 0{,}05\,s = 900°$

- Since the dq-frame leads by 90° at $t=0{,}05$ s it is at: 990°.

- Thus the speed of the dq-frame is:

$$w = \frac{(900° + 90° - 45°)\,\pi/180}{0{,}05\,s} = 329{,}9 \text{ rad/s}$$

$$= 52{,}5 \text{ Hz.}$$

Subtract 45° due to the fact that it already lead 45° initially (t=0).

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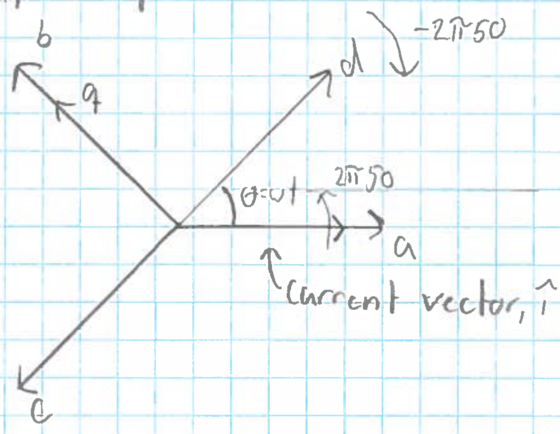
4 - dq-components



· At $t=0$ the current vector is aligned with the a-axis and d-axis leads the a-axis by 30°

- At $t=0$ the current vector starts rotating by $\omega_e = 2\pi \cdot 50$ rad/s, and the dq frame with $\omega_e = -2\pi \cdot 50$ rad/s

- Sketch the d and q-components waveform for time period [0, 0,02].

$\hat{i} = 10 e^{j 2\pi 50 t}$

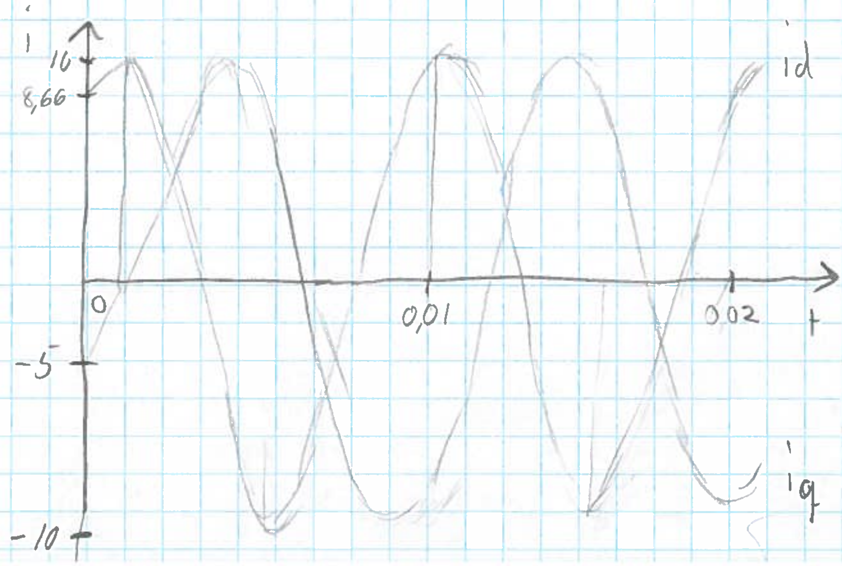
$$\cdot\ i_d = Re\left(\frac{10 e^{j2\pi 50 t}}{e^{j(-2\pi 50 t + \pi/6)}}\right)$$

$$= Re\left(e^{j(2\pi 50 t + 2\pi 50 t - \pi/6)}\right)$$

$$= 10\cos(4\pi 50 t - \pi/6)$$

i_q - Since $\omega < 0$ it lags i_d by 90°

$$i_q = 10\cos(4\pi 50 t - \pi/6 - \pi/2)$$



$i_d(t=0) = 8{,}66$

$i_d(t=0{,}02) = 8{,}66$

- Peak values:

$\cos(4\pi 50 t - \pi/6) = 1$

$t = \frac{1}{1200} + \frac{2\pi}{4\pi 50}$

$i_q(t=0) = -5$

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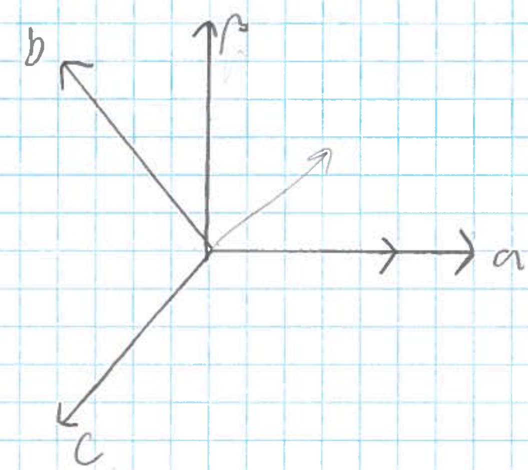
5 - $\alpha\beta$ - reference frame

- The relation between the alfa-component in $\alpha\beta$-reference frame and the phase a-component in the abc-reference frame:



- abc-reference frame and $\alpha\beta$-reference frame are both stationary, and since phase a and α component are aligned, there are no difference.

- Prove

  - Considering the space vector

$$\bar{f}_{\alpha\beta} = A e^{j\omega t} = A(\cos\omega t + j\sin\omega t)$$

$$\Downarrow \; \bar{f}_{\alpha} = Re(\bar{f}_{\alpha\beta}) = A\cos\omega t$$

$$f_{\beta} = Im(\bar{f}_{\alpha\beta}) = A\sin\omega t$$

  - For phase a

$$f_a = Re\left(\frac{f}{e^{j0}}\right) = A\cos\omega t$$

Problem 2 (25%)

A sketch of an induction machine phase axes is given below (**same to the course slides**).

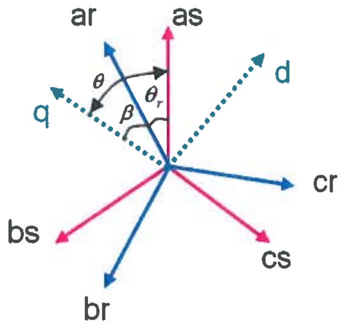


Fig. 2

where notation 's' stands for stator phase axes and notation 'r' stands for rotor phase axis.

(1) Please describe how the mutual inductance between stator phase-a and stator phase-c is obtained?

(2) How the mutual inductance between the rotor phase-b and stator phase-a is obtained?

Knowing the machine model expressed in an arbitrary **qd-reference** frame is

Stator side voltage equations:

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{0s} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} - \omega_\theta \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$

Rotor side voltage equations:

$$\begin{bmatrix} u'_{qr} \\ u'_{dr} \\ u'_{0r} \end{bmatrix} = \begin{bmatrix} R'_r & 0 & 0 \\ 0 & R'_r & 0 \\ 0 & 0 & R'_r \end{bmatrix} \cdot \begin{bmatrix} i'_{qr} \\ i'_{dr} \\ i'_{0r} \end{bmatrix} + p \begin{bmatrix} \lambda'_{qr} \\ \lambda'_{dr} \\ \lambda'_{0r} \end{bmatrix} - (\omega_\theta - \omega_r) \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda'_{qr} \\ \lambda'_{dr} \\ \lambda'_{0r} \end{bmatrix}$$

(3) Suppose the rotor speed measured on the shaft is 240 rpm. The number of **pole pairs** of this induction machine is 4. Please calculate the value of rotor angular velocity ω_r, to be used in this machine model.

(4) Observed from Fig. 2 that when $\theta = 90$ degrees, the d-axis is aligned with stator phase-a axis and the q-axis is leading phase-a axis by 90 degrees. Let's fix this qd-frame at this position (let it become stationary). This makes the d-axis now become the alfa-axis and the q-axis is now the beta-axis. Please give the machine stator and rotor voltage equations expressed in this alfa, beta-reference frame. (*Please leave the zero component equations for simplicity.*)

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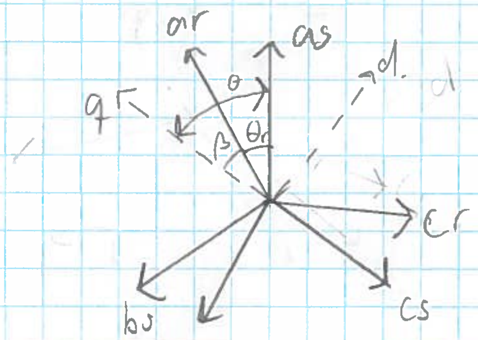
Problem 2 - Inductor machine



S - Stator

r - Rotor

1 - Describe the mutual inductance between stator phase a and stator phase c is obtained:

$$M_{csasm} = L_{aad}\, Re\left(\frac{e^{j\theta}}{e^{j0}}\right) Re\left(\frac{e^{j\theta}}{e^{j-120}}\right) + L_{aaq}\, Re\left(\frac{e^{j\theta+90}}{e^{j0}}\right) Re\left(\frac{e^{j\theta+90}}{e^{j-120}}\right)$$

$$= L_{aad}\, Re\left(e^{j\theta}\right) Re\left(e^{j\theta-120}\right) + L_{aaq}\, Re\left(e^{j\theta+90}\right) Re\left(e^{j\theta+90-120}\right)$$

$$= L_{aad} \cos(\theta) \cos(\theta-120) + L_{aaq} \cos(\theta+90) \cos(\theta-30)$$

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2 - The mutual inductance between rotor phase b and stator phase a:

$$M_{rbas} = L_{aad}\, Re\left(\frac{e^{j\theta}}{e^{j0}}\right) Re\left(\frac{e^{j\theta}}{e^{j-120-\theta_r}}\right) + L_{aaq}\, Re\left(\frac{e^{j\theta+90}}{e^{j0}}\right) Re\left(\frac{e^{j\theta+90}}{e^{j-120-\theta_r}}\right)$$

Angle between stator and rotor.

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3 - The machine model in an arbitrary qd-reference

- Rotor shaft speed is 240 rpm, $w_{r,mec}$
- # of pole pairs of the induction machine is 4

- Calculate the value of rotor angular velocity, w_r

$w_r = p\, w_{r,mec}$

$w_r = 4 \cdot 240 \cdot \frac{2\pi}{60} = 32\pi$ rad/s

$= 16$ Hz

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4 - $\alpha\beta$ - reference frame

- At $\theta = 90°$ → d-axis aligned with phase a axis
  ↳ This dq-frame is now fixed (stationary), which makes the d, q s-axis become the α, β -axis respectively.

- Express the machine stator and rotor voltage eq. in the $\alpha\beta$ - reference frame:

$f_d = f_\alpha$, $f_q = f_\beta$, $w_\theta = 0$, since it is fixed

- Stator

$$\begin{cases} u_{qs} = R_s i_{qs} + p\lambda_{qs} + \cancel{w_\theta \lambda_{ds}}^{\,0} \\ u_{ds} = R_s i_{ds} + p\lambda_{ds} - \cancel{w_\theta \lambda_{qs}}^{\,0} \end{cases}$$

⇓

$u_\alpha = R_s i_\alpha + p\lambda_\alpha$

$u_\beta = R_s i_\beta + p\lambda_\beta$

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- Rotor

$$\begin{cases} u_{qr} = R_r i_{qr} + p\lambda_{qr} + (\cancelto{0}{\omega_\theta} - \omega_r)\lambda_{dr} \\ u_{dr} = R_r i_{dr} + p\lambda_{dr} - (\cancelto{0}{\omega_\theta} - \omega_r)\lambda_{qr} \end{cases}$$

$$\Downarrow$$

$$u_\alpha = R_r i_\alpha + p\lambda_\alpha + \omega_r \lambda_\beta$$

$$u_\beta = R_r i_\beta + p\lambda_\beta - \omega_r \lambda_\alpha$$

Problem 3 (25 %)

An eight-pole induction motor has the following data (**the rotor windings are short-circuited**):

Rated shaft power	0.6 kW
Rated speed	850 rpm Rated shaft speed, $\omega_{r,mec}$
Rated stator frequency	60 Hz $\omega_s = 2\pi 60$
Rated stator voltage	400 V RMS (line-to-line)
Rated stator current	2.1 A RMS
Rated power factor cos φ	0.6 inductive
Stator resistance	12 Ohm
Main (magnetization) inductance	1.4 H
Stator leakage inductance	0.2 H

(1) Calculate the motor's efficiency η at the rated operating condition and the rated shaft torque, rated slip.

(2) You will apply V/f control to this machine. The output of your V/f controller is the peak phase voltage command. What is the value of the constant V/f ratio you will use in your controller?

(3) At the rated operation condition, please calculate the stator flux linkage magnitude. The resistance may be neglected. (*Remember to use the steady state induction machine equations we used for discussing scalar control of induction machine.*)

(4) Now it is asked to control the motor running at 0.25 Hz (mechanical shaft frequency). In order to maintain the same stator flux level as experienced at the rated condition, compensation of the voltage drop on the stator resistance needs to be introduced to the V/f control. In steady state, the phase-a current is found to be 1.0 (A) and phase-a current is lagging phase-a voltage by 45 degrees. Please determine the magnitude of the phase voltage after phase resistive voltage drop compensation.

(5) The stator frequency command is now 60 Hz and the machine is supplied with the rated voltage. When the machine is loaded by 1/4 of its rated load, please calculate the slip that needs to be added to the frequency command in order to make the shaft speed to be 60/4 = 15 Hz (mechanical frequency).

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Problem 3 - An 8-pole induction motor:

1 - Motor effeciency, η, at rated operating condition

$$\eta = \frac{\text{Rated shaft Power}}{m \, U_{rms} \cdot I_{rms} \cdot \phi}$$

m ↑ # of phases

$$= \frac{0{,}6 \text{ kW}}{3 \cdot 400V \cdot 2{,}1A \cdot \frac{1}{\sqrt{3}} \cdot 0{,}6} = 68\,\%$$

- Rated shaft torque:

$$\tau_{rated} = \frac{P_{s,rated}}{\omega_{r,mec,rated}} =$$

$$= \frac{0{,}6 \text{ kW}}{850 \cdot \frac{2\pi}{60} \text{ rad/s}} = 6{,}74 \text{ Nm}$$

- Rated slip:

$$S_{rated} = \frac{\omega_{s,rated} - \overbrace{P\,\omega_{r,mec,rated}}^{\omega_{r,el}}}{\omega_{s,rated}}$$

$$= \frac{60 \cdot 2\pi \text{ rad/s} - 4 \cdot 850 \cdot \frac{2\pi}{60} \text{ rad/s}}{60 \cdot 2\pi \text{ rad/s}} = 5{,}5\,\%$$

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2 - V/F - control

- Calculate the value of the constant V/f - ratio
- in the controller
  - Controller Output Peak Phase Voltage

$$\underbrace{U_{s,new}}_{RMS/Peak} = f_{s,new} \frac{U_{s,rated}}{f_{s,rated}}$$

- Thus:

$$\frac{U_{s,rated}}{f_{s,rated}} \Rightarrow \frac{400 \cdot \overset{\text{Line to Neutral}}{\frac{1}{\sqrt{3}}} \overset{\text{Peak}}{\sqrt{2}} V}{60 \cdot 2\pi \; rad/s} = 0{,}866 \; Vs$$

3 - Stator flux linkage at rated operation conditions

- Steady state induction machine eq.

$$U_s = \cancel{r_s i_s} + j w_s \lambda_s , \quad r_s = 0$$

$$U_s = j w_s \lambda_s$$

$$\lambda_s = \frac{U_s}{j w_s}$$

$$\lambda_s = \frac{U_s}{|j w_s|} \Rightarrow \lambda_s = \frac{400 \cdot \frac{1}{\sqrt{3}} V}{60 \cdot 2\pi \cdot rad/s} = 0{,}6126 \; wb$$

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4 - Stator resistance compensation; slip = 0.

- Control the motor running at 0,25 Hz
  ↳ Desired to maintain cst flux
- The voltage is given as:

$$U_s = r_s I_s \cos(\phi) + \sqrt{U_{s\lambda}^2 - r_s^2 (I_s \cos(\phi))^2} \quad (*)$$

$$U_{s\lambda} = \frac{U_{s,rated}}{f_{s,rated}} \underbrace{f_{s,now}}_{\text{Reference speed}}, \quad \cancel{f_{s,now} = \frac{f_r}{1-s} \Rightarrow f_{s,now} = \frac{0{,}25 p \cdot 2\pi}{1-0{,}055} = 6{,}649 \frac{rad}{s}}$$

$$U_{s\lambda} = 0{,}866 \frac{V}{s} \cdot \underset{\text{Pole pair}}{4} \cdot 0{,}25 \cdot 2\pi \frac{rad}{s} = 5{,}44 \ V$$

- Inserting in (*) to find phase a - Voltage:

Using:

$$I_s = \hat{I}_a \cdot \sqrt{2}$$

$$U_s = 12\Omega (1 \cdot \sqrt{2} A \cos(45) + \sqrt{0{,}866 V^2 - 12\Omega^2 (1\sqrt{2} \cdot \cos(45))^2})$$

$$= 12 + 10{,}696 j \ V$$

$$|U_s| = 16{,}07 V$$

Thus the magnitude of the phase-a Voltage is:

$|U_s| = 16{,}07V$.

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5 - Slip compensation

- The stator frequency command is 60 Hz.
- The machine is supplied with the rated voltage
- The machine is loaded by 1/4 of its rated load

- Calculate the slip that needs to be added to the frequency command in order to make the shaft speed to be 60/4 = 15 Hz (mechanical frequency).

$W_{se} = W_s - W_m$,

W_{se} - Slip frequency

W_s - Stator speed

W_m - Machine frequency

$$\frac{T_e}{T_{rated}} = \frac{f_{se}}{S_{rated}\, f_{s,rated}}$$

$\Downarrow$

$$f_{sc} = \frac{T_e\, S_{rated}\, f_{s,rated}}{T_{rated}}$$

$$f_{sc} = \frac{\frac{1}{4}\, 6{,}74\,Nm\; 5{,}5\% \cdot 60\, 2\pi\ rad/s}{6{,}74\,Nm} = 0{,}76910\ rad/s$$

$= 0{,}1224\ Hz$

Thus;

$W_s = W_m + W_{se}$

Needs to be added to machine frequency to maintain the desired frequency.

Problem 4 (25 %)

The stator voltage equation of a permanent magnet synchronous machine may be given as (*same notations as used in the lecture slides*):

$$u_q = Ri_q + p\lambda_q + \omega_r\lambda_d \qquad \lambda_q = (L_{ls} + L_{mq})i_q = L_q i_q$$
$$u_d = Ri_d + p\lambda_d - \omega_r\lambda_q \qquad \lambda_d = (L_{ls} + L_{md})i_d + \lambda_{mpm} = L_d i_d + \lambda_{mpm}$$

(1) Please show the block diagram as you may implement in Simulink by using the d-axis voltage equation to solve for the d-axis current.

(2) If, at time t=0 in Simulink, you want the d-axis flux linkage λ_d to be equal to the rotor peak permanent magnet flux linkage λ_{mpm}, How can you achieve this in your Simulink model?

(3) In I/f control of the PM machine, should the current vector to be placed lagging the q-axis or should it be leading the q-axis? Please give your explanations.

(4) In steady state, you observe the phase-a voltage and current waveforms are as shown in Fig.4 (10 V for voltage and 2 A for current peak values). In addition, at the moment when phase-a voltage crosses the zero from negative to positive,* the corresponding rotor position found is -30 electrical degrees. Please add the voltage space vector, the current space vector and the rotor dq-axes to Fig. 5, which should represent the instantaneous waveforms shown at t=0 in Fig. 4.

* t=0

The vector, V is placed there, such that at t=0, $V_a = 0$, due to the projection of V to V_a at t=0 should be 0.

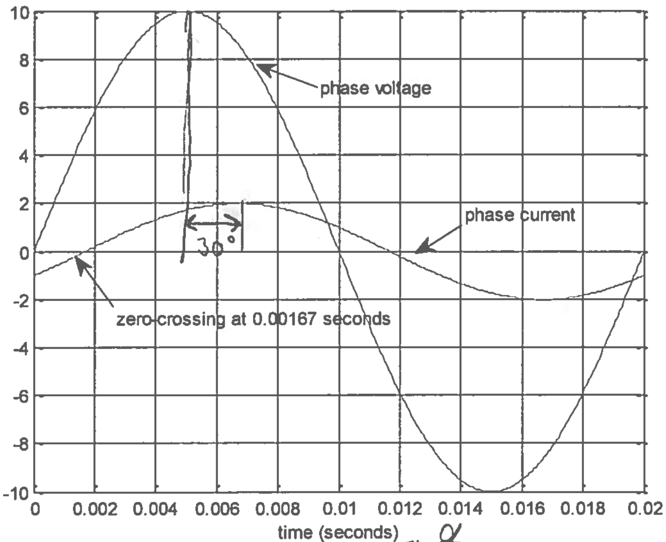

Fig. 4

$\omega = \frac{1}{0,02} 2\pi$ rad/s $= 2\pi 50$ rad/s

$V = 10\cos(\omega t)$

$I = 2\cos(\omega t - 30)$

$\theta = \omega t = 2\pi 50 \cdot 0,00167 = 30°$

↑Angle between V and I.

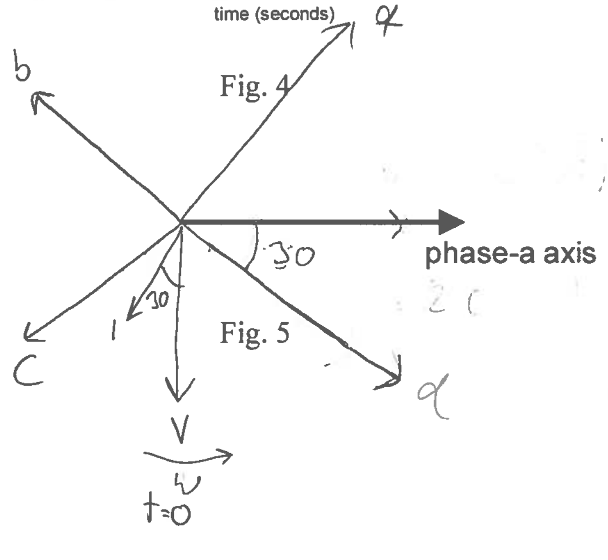

Fig. 5

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Problem 4 - Permanent magnet synchronous machine.

$$u_q = R i_q + p\lambda_q + W_r \lambda_d \qquad \lambda_q = (L_{ls} + L_{mq}) i_q = L_q i_q$$

$$u_d = R i_d + p\lambda_d - W_r \lambda_q \qquad \lambda_d = (L_{ls} + L_{md}) i_d + \lambda_{mpm}$$

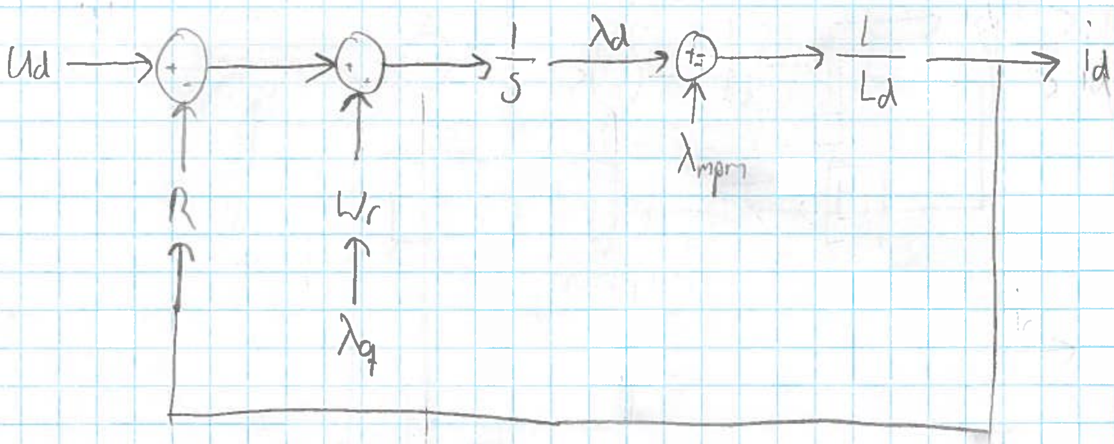
$$= L_d i_d + \lambda_{mpm}$$

1 - Block diagram (SimuLink implementation)

u_d - Input, i_d - Output

$$\lambda_q = \frac{1}{s}\left(u_q - R i_q - W_r \lambda_d\right), \qquad i_q = \lambda_q \frac{1}{L_q}$$

$$\lambda_d = \frac{1}{s}\left(u_d - R i_d + W_r \lambda_q\right), \qquad i_d = (\lambda_d - \lambda_{mpm}) \frac{1}{L_d}$$



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2 - Initial Conditions.

- At $t=0$, it is desired to have the d-axis flux linkage, λ_d, to be equal to the rotor peak permanent magnet flux linkage, λ_{mpm}, so:

$$\lambda_d = \lambda_{mpm} \quad \text{at} \quad t=0.$$

- Since

$$\lambda_d = L_d i_d + \lambda_{mpm}, \qquad i_d = 0$$

This can be achieved by setting the initial condition of the integrator block in simulink to λ_{mpm}, so that

$$\lambda_d = \lambda_{mpm}$$

$$\Downarrow$$

$$\lambda_{mpm} = \cancel{L_d i_d}^{\,0} + \lambda_{mpm}, \qquad \xrightarrow{} \frac{1}{s} \xrightarrow{\lambda_d = \lambda_{mpm}} \quad ic = \lambda_{mpm}$$

Thus $i_d = 0$.

3 - V/f-control of the PM-machine

- The current vector should be placed lagging the q-axis