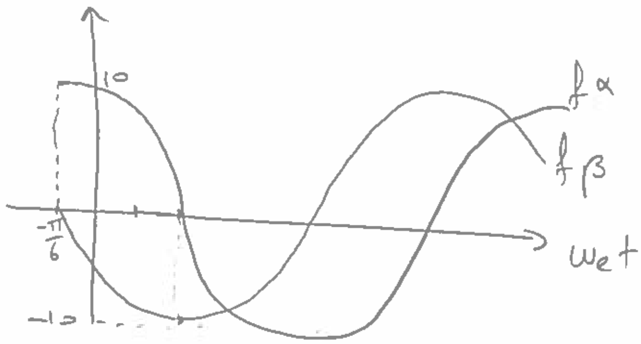
Problem 1)

(1) knowing $\overline{f_{\alpha\beta}} = f_\alpha + j f_\beta$ (the definition)

now $\overline{f} = 10 e^{-j(\omega_e t + \frac{\pi}{6})} = \overline{f_{\alpha\beta}}$

therefore $\overline{f_{\alpha\beta}} = 10^{-j(\omega_e t + \frac{\pi}{6})} = 10 \left[\cos\left(\omega_e t + \frac{\pi}{6}\right) - j \sin\left(\omega_e t + \frac{\pi}{6}\right) \right]$



$= \frac{\pi}{2} - \frac{\pi}{6}$

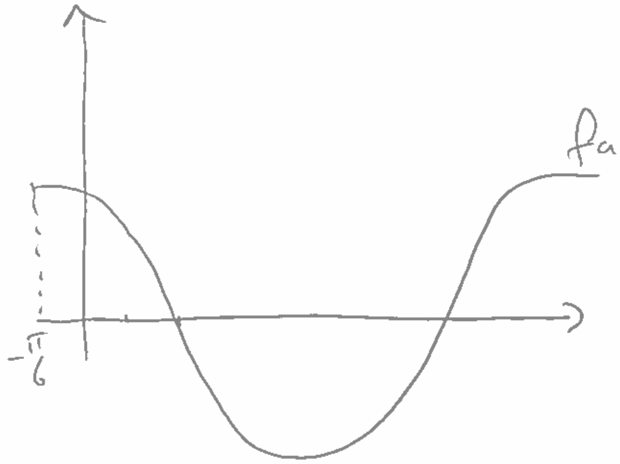
$= \frac{\pi}{6}$

(2) Using the "vector projection method"

$f_a = Re\left(\frac{\overline{f_{\alpha\beta}}}{e^{j0^\circ}}\right) = 10 \cos\left(\omega_e t + \frac{\pi}{6}\right)$ $f_b = Re\left(\frac{\overline{f_{\alpha\beta}}}{e^{j120^\circ}}\right) = 10 \cos\left(\omega_e t + \frac{\pi}{6} + 120^\circ\right)$

Real part taken ↑ ↑ location of phase a axis

$f_c = Re\left(\frac{\overline{f_{\alpha\beta}}}{e^{-j120}}\right) = 10 \cos\left(\omega_e t + \frac{\pi}{6} - 120^\circ\right)$

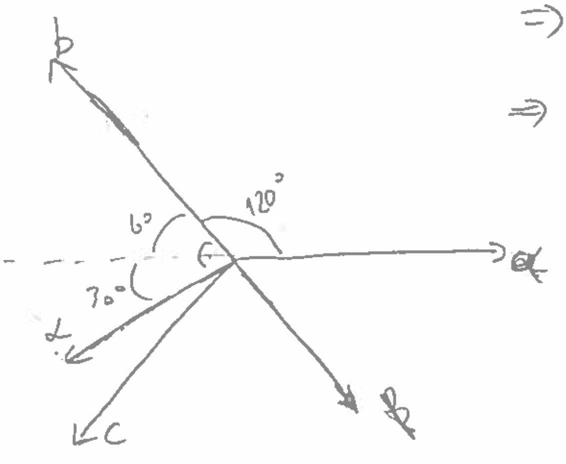


(3) $f_b = 10 \cos\left(\omega_e t + \frac{\pi}{6} + 120^\circ\right) \Rightarrow \cos\left(\omega_e t + \frac{\pi}{6} + 120\right) = 1$ $T = \frac{1}{f} = \frac{1}{50} = 0{,}02\ s$

$\Rightarrow \omega_e t + \frac{\pi}{6} + 120 = 0$

$\Rightarrow 2\pi \cdot 50 t + \frac{\pi}{6} + \frac{2\pi}{3} = 0$

$t \cdot 2\cancel{\pi}\, \cancel{50}^{\,10} = -\frac{5\cancel{\pi}}{6} \Rightarrow t = -\frac{1}{120}\ s \Rightarrow$

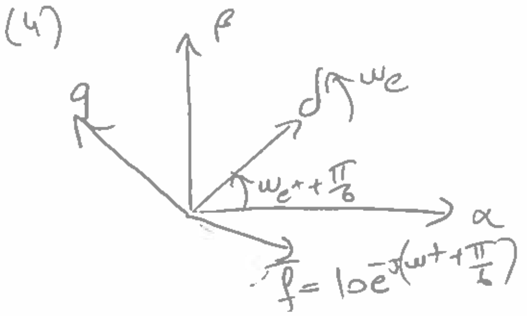


$t = -\frac{1}{120} + 0{,}02 = \underline{0{,}01166\ s}$

$\theta = 360 \cdot \frac{t}{T} = 210^\circ$

EXAM JAN 2018

(4)



This is how the reference frames may be defined. It can be seen that at $t=0$, $w_e t + \frac{\pi}{6} = \frac{\pi}{6}$ and d-axis has $+\frac{\pi}{6}$ degree difference between phase-a axis (and α-axis).

Same vector protection method used and if "t" increases $\bar{f} = 10 e^{-j(wt + \frac{\pi}{6})}$ will rotate CCW respectivly.

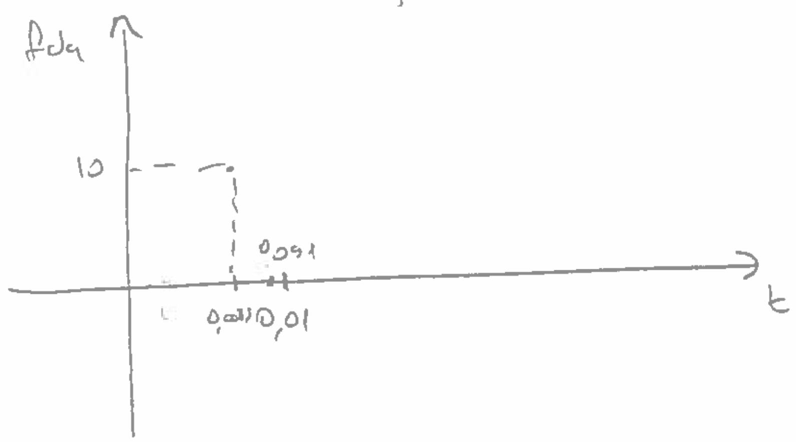
According to vector protection method

$$f_d = Re\left(\frac{\overline{f_{\alpha\beta}}}{e^{j(wt + \frac{\pi}{6})}}\right) = 10 \cos\left(2 w_e t + \frac{\pi}{3}\right) \quad \left(\text{Note } \overline{f_{\alpha\beta}} = \bar{f} = 10 e^{-j(w_e t + \frac{\pi}{6})}\right)$$

location of the d-axis

$$f_q = Re\left(\frac{\overline{f_{\alpha\beta}}}{e^{j(wt + \frac{\pi}{6} + \frac{\pi}{2})}}\right) = -10 \sin\left(2 w_e t + \frac{\pi}{3}\right)$$

It can be drawn for $f = 2 w_e t \Rightarrow 100\,Hz$



for f_d max

$2 w_e t + \frac{\pi}{3} = 0$

$2.2\pi 50.t = -\frac{\pi}{3}$

$t = -\frac{1}{600} + 0,01 = 0,0083$

min

$2 w_e t + \frac{\pi}{3} = \frac{\pi}{2}$

$2.2\pi.50.t = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q1) (3) $t = 0{,}05$

$w_e t = 2\pi \cdot x\, 0{,}05 = 5\pi = 180°$

$\theta_d = \underbrace{180}_{\text{time}} + \underbrace{30}_{\text{beginning}} + \underbrace{90}_{\text{difference}} = 300°$

I for current

$\vec{I} = 10\,(e^{jwt})$

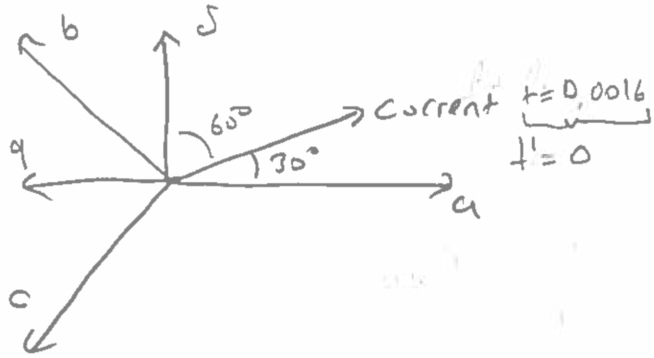
$\vec{f_d} = Re\left(\frac{\vec{I}}{e^{j300}}\right) = Re\left(10\, e^{jwt} \cdot e^{-j300°}\right)$

$= 10\cos(w_e t - 300)$

$\vec{f_q} = Re\left(\frac{\vec{I}}{e^{j390}}\right) = Re\left(10 e^{jwt} \cdot e^{-j30°}\right)$

$= 10\cos(w_e t - 30)$

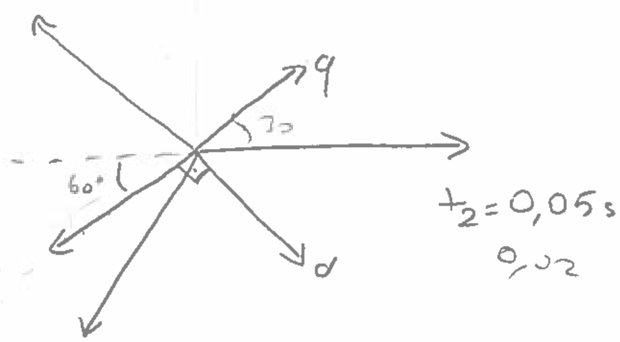
Case 1



$\theta_{d1} = 90°$

$\theta_{c1} = 30°$

case 2



$\theta_{d2} = 300°$

$\theta_{c2} = 210°$

d frame is always leading current frame

$0{,}05\,s \Rightarrow \frac{0{,}05}{0{,}02} = 2{,}5$ rotation

$\theta_{lead} = \Delta d - \Delta c = (300 - 90) - (210 - 30) = 30°$ in 2,5 rotation

$12° \Leftarrow 1$ rotation

$\Delta W_e \cdot t = \theta_{lead}$

$2\pi\,(f_{dq} - f_c) \cdot t = \theta_{lead}$

$2\pi\,(f_{dq} - 50) \cdot t = 12$

$f_{dq} - 50 = 1{,}66$

$f_{dq} = 51{,}66$

$W_e = 2\pi \cdot 51{,}66$ rad/s

JAN EXAM 2016

Q2)

(1) Determine mutual inductance between stator b and stator c

$$M_{bscsm} = L_{aad}\,Re\left(\frac{e^{j\theta}}{e^{j120}}\right) Re\left(\frac{e^{j\theta}}{e^{-j120}}\right) + L_{aaq}\,Re\left(\frac{e^{j\theta+90}}{e^{j120}}\right) Re\left(\frac{e^{j\theta+90}}{e^{-j120}}\right)$$

$$M_{bscsm} = L_{aad}\cos(\theta-120)\cos(\theta+120) + L_{aaq}\cos(\theta+90-120)\cos(\theta+90+120)$$

$$= L_{aad}\cos(\theta-120)\cos(\theta+120) + L_{aaq}\sin(\theta-120)\sin(\theta+120)$$

(2) When d-axis on phase a it gives out the maximum inductances and they are together when $\theta = 0$

$$M_{bscsm} = L_{aad}\cos(0-120)\cos(0+120) + L_{aaq}\sin(0-120)\sin(0+120)$$

$$= 0{,}25L_{aad} - 0{,}75\,L_{aaq}$$

from Lecture 4 notes $\Rightarrow = -\frac{1}{2}L_1 - L_2\,\cancel{\cos}(2\theta)$

$$= -\frac{1}{2}L_1 - L_2$$

$\boxed{L_{aad} = L_1 - L_2 \quad L_{aaq} = L_1 + L_2}$

Minimum when $\theta = \frac{\pi}{2}$

$$M_{bscsm} = L_{aad}\cos(90-120)\cos(90+120) + L_{aaq}\sin(90-120)\sin(90+120)$$

$$= 0{,}75L_{aad} + 0{,}25L_{aaq}$$

(3) $\lambda_{pm,a} = \lambda_{mpm}\cos(\theta)$ $\qquad \overline{f_{abc}} = \overline{f_{da}} = \overline{f_{qd}} = \overline{f_{\alpha\beta}}$

Phase-a: $I_a = -I_m\sin(3\theta)$

Instantaneous Torque — electrical rotor angle

$$\tau = p \cdot i_a \cdot \lambda_{mpm}(-\sin\theta)$$

$$= +p\,I_{m_1}\sin(3\theta)\sin\theta \cdot \lambda_{mpm}\cdot\cos(\theta)$$

$$= 2\underbrace{I_{m_1}\lambda_{mpm_1}}_{\text{constant} = x}\sin(3\theta)\sin(\theta)\cos(\theta)$$

At $\theta = 0$ & $90° \Rightarrow I_{m_1}\lambda_{mpm} = x$

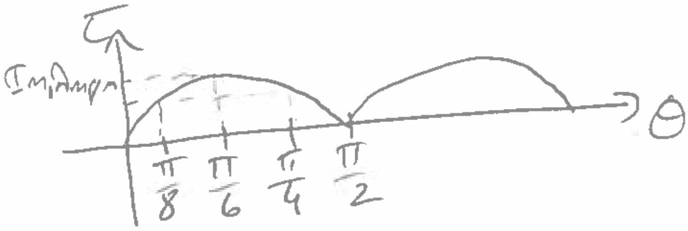
$\tau = 2x\sin(0)\cdot\sin(0)\cdot\cos(0) \qquad \tau = 0$

$\tau = 2x\sin(90)\cdot\sin(90)\cdot\cos(90) \qquad \tau = 0$

$\theta = \frac{\pi}{2} \qquad \tau = 2x\,0{,}353$

$\theta = \frac{\pi}{6} \qquad \tau = 2x\,0{,}433$

$\theta = \frac{\pi}{8} \qquad \tau = 2x\,0{,}31$



(4) Yes

(5) 3ϕ

Q3) (1) The second row "$\times j$" and then plus the first row.

We have

$$U_{qs} - jU_{ds} = R_s(i_{qs} - jI_{ds}) + p(\lambda_{qs} - j\lambda_{qs}) + \omega_0 \lambda_{ds} + j\omega_0\lambda_{qs}$$

(2) $f_{\alpha\beta} = f_{dq} e^{j\theta}$

$$\vec{U}_\alpha = Re\left(\frac{\vec{U}_{qds}}{e^{j0}}\right) = R_s i_{qds} + p\lambda_{qds} + j\omega\theta\lambda_{qds}$$

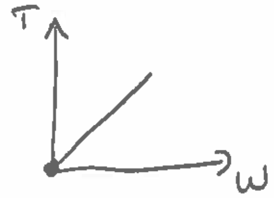
$$\vec{U}_\beta = Re\left(\frac{\vec{U}_{qds}}{e^{j90}}\right) = R_s i_{qds} \cancel{\cos 90}^{\,0} + p\lambda_{qds}\cancel{\cos 90}^{\,0} + j\omega\theta\lambda_{qds}\sin 90$$

$$\vec{U}_{\alpha\beta} = U_\alpha + j U_\beta = R_s i_{qds} + p\lambda_{qds} + \cancel{j\omega\theta\lambda_{qds}} - \cancel{j\omega\theta\lambda_{qds}}$$

$$= R_s i_{qds} + p\lambda_{qds}$$

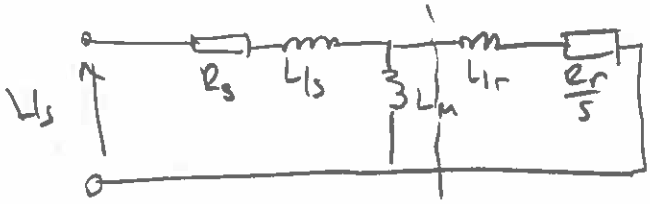
EXAM 2015

Q4) (1)



If $T=0$ $\omega_{slip}=0$ that means $\frac{\omega_s}{p}=\omega_m$

$n_s = \omega_s \cdot \frac{60}{\pi} \cdot \frac{1}{2p} = 2\pi \cdot f \cdot \frac{60}{\pi} \cdot \frac{1}{2 \cdot 2} = 1500 \text{ rpm} = n_m$



s=0 rotor side is neglected

$U_s = I_s (R_s + (2\pi f (L_{ls} + L_M)))$

$\frac{880}{\sqrt{3}} = I_s (2{,}5 + (2\pi f (0{,}281)))$

$I_s = 2{,}43$ A

$\Rightarrow U_x = I_s \cdot X_{TOTAL} = 2{,}43 \times 8 = 213{,}75$ V

(2) Voltage drop compensation

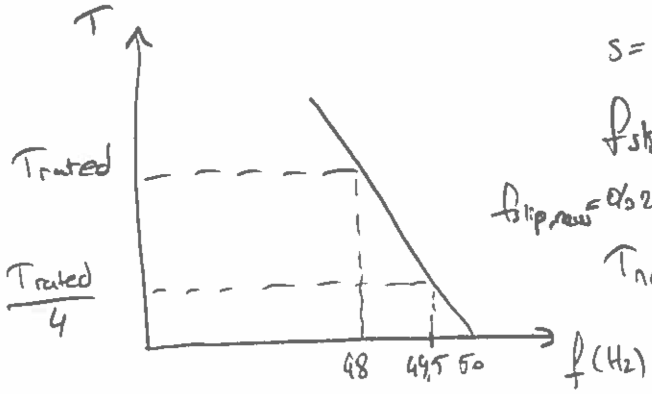
$U_s = R_s \cdot I_s + j\omega_s \lambda_s$

$\underbrace{R_s \cdot I_s}$ → R_s and I_s must be known

To obtain a constant flux linkage $R_s \cdot I_s$ needs to be taking into account, and added to the input voltage

(3) $V_{rated} = \frac{V_{l-l}}{\sqrt{3}} = 220$ $I_{rated} = \frac{1{,}9 \text{ kW}}{3 \times 220} \cong 4{,}1$ A

$I_{act,rated} = I_{rated} \times \sqrt{2} \times \cos\varphi = 4{,}1 \times \sqrt{2} \times 0{,}8 = 4{,}638$



$s = \frac{n_s - n_{rated}}{n_s} = 0{,}04$

$f_{slip} = f_s \cdot s = 50 \cdot 0{,}04 = 2$ Hz

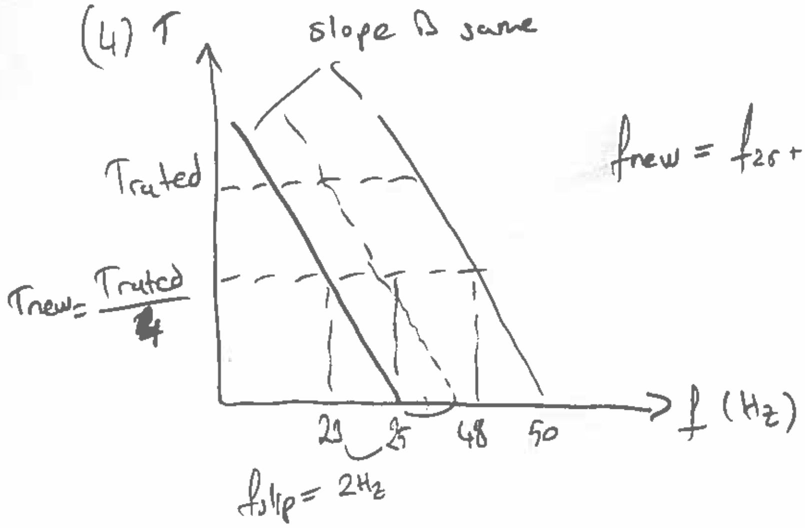
$f_{slip,new} = 0{,}25 \times f_{slip} = 0{,}5$ Hz

$T_{new} = \%0{,}25$ of T_{rated}

$\frac{i_{sd,new}}{i_{sd}} = \frac{T_{new}}{T_{rated}} = \frac{i_{sd,new}}{4{,}63} = \frac{1}{4}$

$i_{sd,new} = 1{,}157$ A

(4)



$f_{new} = f_{25} + f_{slip} = 25 + 2 = 27$ Hz

Q(5) EXAM 2015

(1) Yes, Lecture 5 exercise answer p 1-2-3

(2) $f = \frac{n \cdot p}{60} = \frac{1200 \cdot 4}{60} = 80\ Hz$

(3) $\omega_r = 2\pi \cdot p \left(\frac{n}{60}\right) = 502{,}65\ rad/s$

$V_{ph,peak} = \frac{120}{\sqrt{3}} \times \sqrt{2} = 98\ V$

from equations it canbe seen that $\lambda_{mpm} = \frac{V_{ph,peak}}{s\ \omega_r} = \frac{98}{502{,}65} = 0{,}195$

(4) $L_{mq} = L_{md}$

q-axis current $= 2A$

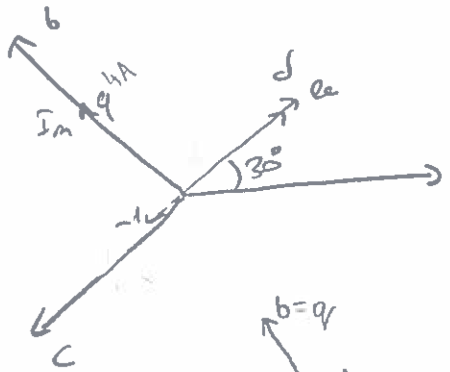
d-axis current $= -1A$

$T = ?$

$T_e = \frac{3}{2} p \left(\lambda_{mpm} \cdot i_q + \cancel{(L_d - L_q)}\, i_d \cdot i_q\right)$

$T_e = \frac{3}{2} \cdot 4\ (0{,}195 \times 2 + 0) = 2{,}34\ Nm$

(5)

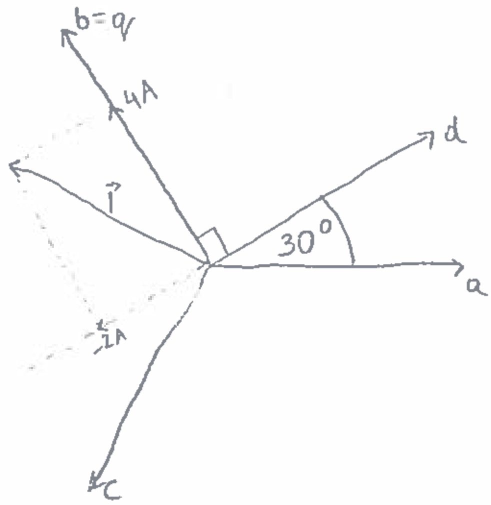


As b-axis is aligned with q-axis, the initial value is $i_q = 4A$.

We can calculate current peak value as the length of current vector:

$|\vec{i}| = \sqrt{4^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20}$

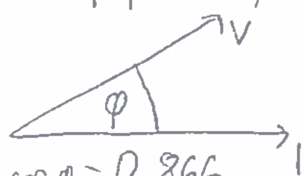
$|\vec{i}| = 4.47\ A$



$f_{me} = 1200\ rpm = \frac{1200}{60}\ Hz = 20\ Hz$

This is mechanical frequency. In order to obtain electrical frequency, we need to multiply it by number of pole pairs:

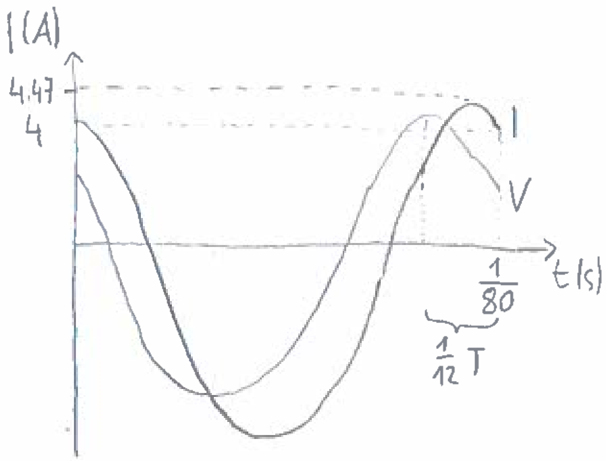
$f_{el} = p \cdot f_{me} \quad f_{el} = 4 \cdot 20 Hz = 80 Hz$



$\cos\varphi = 0.866$

$\varphi = 30°$

$30° = \frac{360°}{12}$



Peak voltage:

$V_{peak} = V_{rms} \cdot \sqrt{2} = 100\sqrt{2}\ V$