AALBORG UNIVERSITET
Energy

Study number: 20151913	**Programme:** All intro students
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Please write your study no. on all pages. Do not write your name as your evaluation is anonymous!

Total number of pages, including this page: 22

NB! Your paper must be easy to read. If this is not the case, your paper may be evaluated as "not passed".

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Problem 1

1) $\bar{f}_I = 10\, e^{j45}$ [given]

where $\bar{f}_I$ is defined as current vector.

Let phase currents in phases a, b & c be defined as f_{Ia}, f_{Ib} & f_{Ic} respectively.

$$f_{Ia} = \frac{Re(\bar{f}_I)}{e^{j0^\circ}} = \frac{Re\left[10\, e^{j45}\right]}{e^{j0}}$$

$$\Rightarrow f_{Ia} = Re\left[10\, e^{j(45-0)}\right]$$

$$= 10 \cos 45^\circ$$

$$= 7.071 \text{ A}$$

$$\Rightarrow f_{I_b} = \frac{Re(\bar{f})}{e^{j2\pi/3}} = \frac{Re(10e^{j45})}{e^{j2\pi/3}}$$

$$= 10\, e^{j(45^\circ - 120^\circ)}$$

$$= 10 \cos(-75^\circ)$$

$$= 2.59 \text{ A}$$

$$\Rightarrow f_{I_c} = \frac{Re(\bar{f})}{e^{-j2\pi/3}} = \frac{Re(10e^{j45})}{e^{-j2\pi/3}}$$

$$= 10\, e^{j(45^\circ + 120^\circ)}$$

$$= 10 \cos(165^\circ)$$

$$= -9.66 \text{ A}$$

~~Q.1~~

Problem 1.

(2) Let the dq component of the current vector be defined as f_{I_d} & f_{I_q} respectively.

As given in question;

d-axis leads a-axis by 90° & we know q axis leads d axis by 90°.

$$f_{I_d} = \frac{Re\left(10e^{j45}\right)}{e^{j90^\circ}} = Re\left(10e^{j(45^\circ - 90^\circ)}\right)$$

$$\Rightarrow f_{I_d} = 10\cos(-45^\circ)$$

$$= 7.071 \text{ A}$$

Now for the q component.

$$f_{I_q} = \frac{Re\left(10\, e^{j45}\right)}{e^{j180^\circ}} = Re\left(10\, e^{j(45-180^\circ)}\right)$$

$$\Rightarrow f_{I_q} = 10\cos(-135^\circ)$$

$$= -7.071 \text{ A}$$

Problem 1.

(3) Let the rotating speed of dq reference frame be ω_r rad s^{-1}

As given in question,

angular velocity of current vector

$$\omega_c = 2\pi \cdot 50 = 314.16 \text{ rad}$$

Now, the relative speed difference between the dq reference frame & the space current vector will cause the difference in angle.

Problem 1

Q:- 3 contd.

So, As per question the d axis leads the space current vector by 90° which means the change in angle i.e. $90° - 45° = 45° = \Delta\theta$, occured during the time interval of $t = 0.05$ seconds due to the relative speed.

So, we can say that.

$$(\omega_r - \omega_e)\, time = \Delta\theta$$

$$\Rightarrow (\omega_r - 314.16)(0.05) = 45° = \frac{\pi}{4}$$

$$\Rightarrow 0.05\,\omega_r - 15.71 = \frac{\pi}{4}$$

$$\Rightarrow 0.05\,\omega_r = 15.71 + 0.785 = 16.5$$

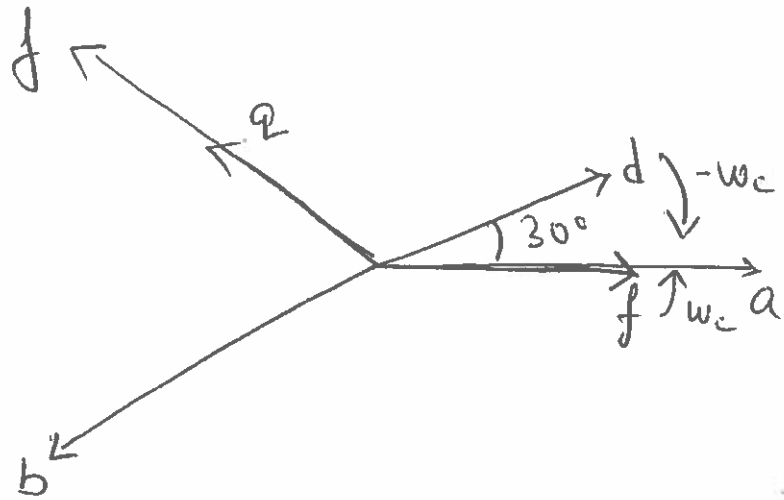
$$\omega_r = \frac{16.5}{0.05}$$

$$= 329.91 \text{ rad s}^{-1}$$

∴ The rotating speed of dq reference frame is $329.91 \text{ rad s}^{-1}$

Problem 1

4) As given in the question:-



Let the corresponding d & q component be f_{Id} & f_{Iq}.

$$f_{Id} = Re\left(\frac{\bar{f}\, e^{j\omega_e t}}{e^{-j(\omega_e t - \pi/6)}}\right)$$

Problem 1.

4) Contd.

$$\Rightarrow f_{rd} = 10 \cos \text{Re}\left[e^{j\omega t} \cdot e^{j(\omega_e t - 30)} \right]$$

$$\Rightarrow f_{rd} = 10 \cos \text{Re}\left[e^{j(2\omega_e t - \pi/6)} \right]$$

$$\Rightarrow f_{Id} = 10 \cos (2\omega_e t - \pi/6)$$

Substituting $\omega_e = 2\pi \cdot 50 = 314.16 \text{ rads}^{-1}$

$$\Rightarrow f_{Id} = 10 \cos (2 \times 314.16\, t - \pi/6)$$

Similarly for f_{Iq}.

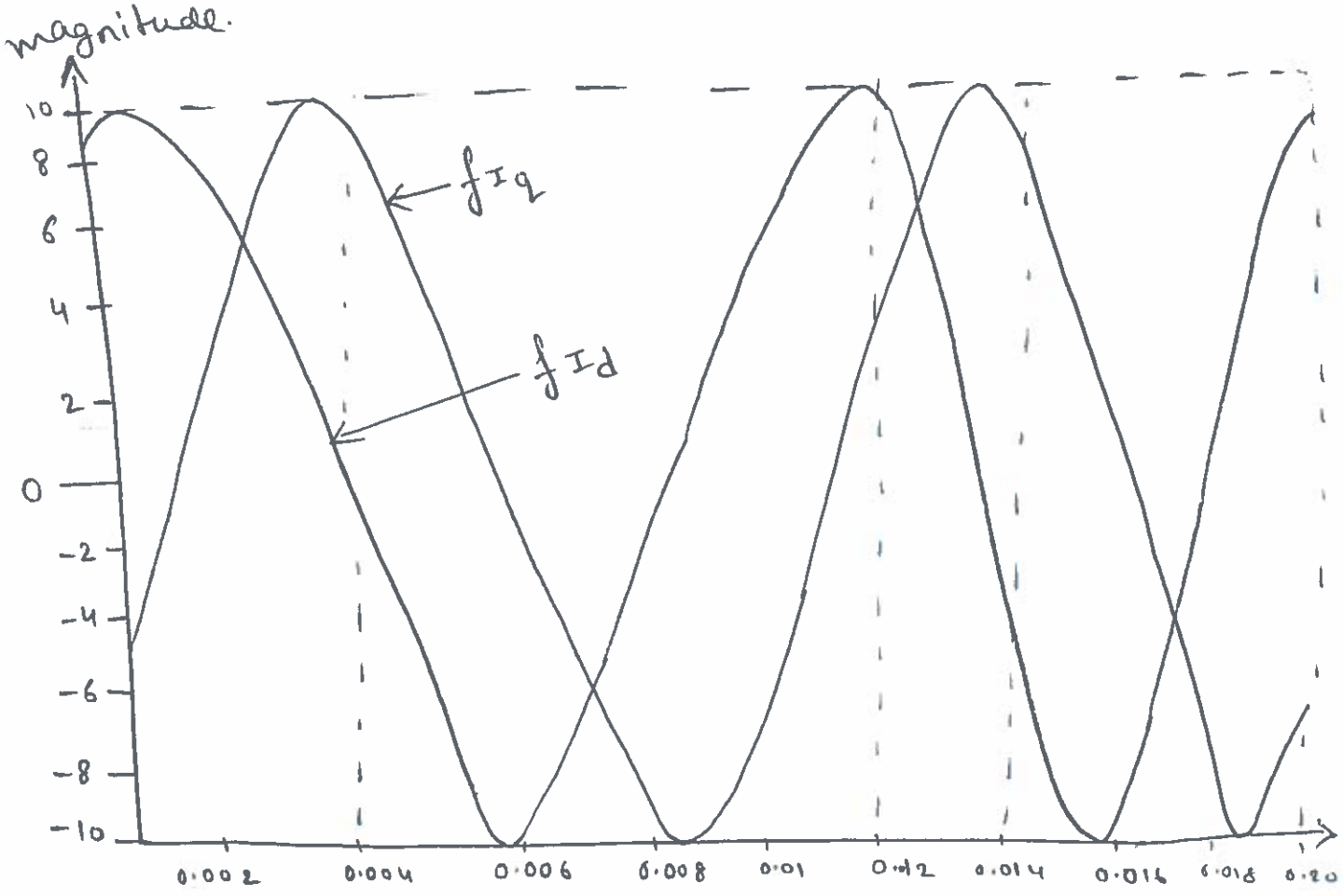
$$f_{Iq} = \text{Re}\left(\frac{j e^{j\omega_e t}}{e^{-(j\omega_e t - 2\pi/3)}} \right)$$

$$\Rightarrow f_{Iq} = 10 \cos\left(2\omega_e t - 2\pi/3\right)$$

Substituting $\omega_e = 314.16 \text{ rad s}^{-1}$

$$\Rightarrow f_{Iq} = 10 \cos\left(2 \times 314.16 - 2\pi/3\right)$$

The sketch of the d-, q- waveforms for the time period [0, 0.002] second have been obtained using "Matlab".



Problem 1.

Question 4 contd

As per the sketch in the previous page, it must be noted that the "blue" waveform ~~reference~~ represents the q axis current component & the "grey" waveform represents the d axis current component.

Problem 1.

Question 5

In abc reference frame, we have phase 'a' that always aligns itself with the real axis.

We also know that the space vector used to represent in α, β system is given as.

where α = alpha
& β = Beta

$$\bar{f} = f\alpha + j f\beta$$

The above equation clearly states that the 'α' component is aligned along the Real axis of the Co-ordinate system. Therefore, we can say that the alfa 'α' component in α, β system is same as the phase 'a' component in a,b,c reference frame.

To corroborate the above explanation. The transformation matrix, that is used to transform α, β components to a,b,c components it is is used.

Problem 1.

(5) continued.

α, β to a, b, c transformation :-

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/2 & \frac{\sqrt{3}}{2} \\ -1/2 & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix}$$

$\therefore$ we see that

$$f_a = 1 \cdot f_\alpha + 0 \cdot f_\beta$$

$$\therefore f_a = f_\alpha$$

Proved.

Problem 2

(1) Let the mutual inductance between stator phase-a & & stator phase-c be Masc

$$\Rightarrow M_{asc} = L_{aag} Re\left(\frac{e^{j\theta}}{e^{j0^\circ}}\right) \cdot Re\left(\frac{e^{j\theta}}{e^{-j2\pi/3}}\right) +$$

$$L_{aad}\, Re\left(\frac{e^{j(\theta-\pi/2)}}{e^{j0}}\right) \cdot Re\left(\frac{e^{j(\theta-\pi/2)}}{e^{-j2\pi/3}}\right)$$

$$\Rightarrow L_{aag} \cos\theta \cdot \cos(\theta + 2\pi/3) +$$

$$L_{aad} \sin\theta \cdot \sin(\theta + 2\pi/3)$$

[We know that.
$\cos(A+B) + \cos(A-B) = 2\cos A \cdot \cos B$
$\cos(A-B) - \cos(A+B) = 2\sin A \cdot \sin B$]

Problem 2

1) Contd.

$$\Rightarrow \frac{L_{aaq}}{2}\left[\cos\left(\theta+\theta+2\pi/3\right)+\cos\left(\theta-\theta-2\pi/3\right)\right]$$

$$\frac{L_{aad}}{2}\left[\cos\left(\theta-\theta-2\pi/3\right)-\cos\left(\theta+\theta+2\pi/3\right)\right]$$

Considering $L_{aad} = L_{aaq} = L_{ms}$

$$\Rightarrow \frac{L_{ms}}{2}\left[\cos\left(2\theta+2\pi/3\right)+\cos\left(-2\pi/3\right)+\cos\left(-2\pi/3\right)-\cos\left(2\theta+2\pi/3\right)\right]$$

$$\Rightarrow \frac{L_{ms}}{2}\left[2\cos\left(-2\pi/3\right)\right]$$

$$\therefore \boxed{M_{asc} = -\frac{1}{2}L_{ms}}$$

Problem 2.

(2) Let the mutual inductance between motor phase-b & stator phase a be Masbr.

$$Masbr = L_{aag}\, Re\left(\frac{e^{j\theta}}{e^{j0}}\right) \cdot Re\left(\frac{e^{j\theta}}{e^{j(\theta_r + 2\pi/3)}}\right)$$

$$L_{aad}\, Re\left(\frac{e^{j(\theta - \pi/2)}}{e^{j0}}\right) \cdot Re\left(\frac{e^{j(\theta - \pi/2)}}{e^{j(\theta_r + 2\pi/3)}}\right)$$

$$\Rightarrow L_{aag} \cos(\theta) \cdot \cos(\theta - \theta_r - 2\pi/3) +$$

$$L_{aad} \sin(\theta) \cdot \sin(\theta - \theta_r - 2\pi/3)$$

[We know that

$\cos(A+B) + \cos(A-B) = 2\cos A \cdot \cos B$

$\cos(A-B) - \cos(A+B) = 2\sin A \cdot \sin B$]

Problem 2

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(8)

(2) continued.

$$\Rightarrow M_{asbr} = \frac{L_{aaq}}{2}\left[\cos(\theta+\theta-\theta_r-2\pi/3)+\cos(\cancel{\theta}-\cancel{\theta}+\theta_r+2\pi/3)\right]+$$

$$\frac{L_{aad}}{2}\left[\cos(\cancel{\theta}-\cancel{\theta}+\theta_r+2\pi/3)-\cos(\theta+\theta-\theta_r-2\pi/3)\right]$$

Considering $L_{aad} = L_{aaq} = L_{ms}$

$$\Rightarrow \frac{L_{ms}}{2}\left[\cancel{\cos(2\theta-\theta_r-2\pi/3)}+\cos(\theta_r+2\pi/3)+\cos(\theta_r+2\pi/3)-\cancel{\cos(2\theta-\theta_r-2\pi/3)}\right]$$

$$\Rightarrow M_{asbr} = \frac{L_{ms}}{\cancel{2}}\left[\cancel{2}\cos(\theta_r+2\pi/3)\right]$$

$\therefore$ $$\boxed{M_{asbr} = L_{ms}\cdot\cos(\theta+2\pi/3)}$$

Problem 2

3) Given :-

rotor speed on shaft = 240 rpm

no. of pole pairs (P_p) = 4

Let rotor angular velocity be ω_r

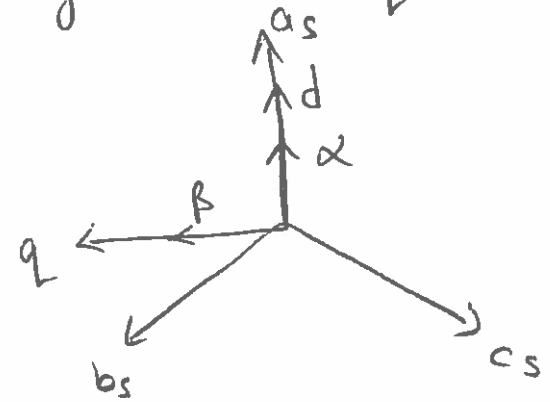
$$\omega_r = \frac{2\pi}{60} \times P_p \times [\text{rotor speed in rpm}]$$

$$\Rightarrow \omega_r = \frac{2\pi}{60} \times 4 \times 240$$

$$= 100.531 \text{ rad } s^{-1}$$

(4) As given in question,

Now.



(4) contd.

Now in this ↑ problem, the space vector representation ~~the vector~~ in qd reference ~~is~~ is $\bar{f} = f_d + j f_q$ -(i) ∴

the space vector representing in α, β reference is $\bar{f} = f_\alpha + j f_\beta$. -(ii)

Correlating (i) & (ii) we have

$f_d = f_\alpha$ & $f_q = f_\beta$ ∴ since qd reference frame is now stationary with respect to a, bc reference frame of stator, so $\omega_\theta = 0$.

Stator voltage expressions in qd reference frame.

$$V_{qs} = R_s \cdot i_{qs} + p\lambda_{qs} + \omega_\theta \lambda_{ds} \quad \text{(iii)}$$

$$V_{ds} = R_s \cdot i_{ds} + p\lambda_{ds} - \omega_\theta \lambda_{qs} \quad \text{(iv)}$$

In equation (iii) & (iv) replacing all q & d components with β & α components respectively we get. [$\omega_\theta = 0$ rads^{-1}]

$$V_{\beta s} = R_s i_{\beta s} + p\lambda_{\beta s} + \omega_\theta^{\,0} \lambda_{\alpha s}$$

$$V_{\alpha s} = R_s i_{\alpha s} + p\lambda_{\alpha s} - \omega_\theta^{\,0} \lambda_{\beta s}$$

$$\therefore V_{\beta s} = R_s . i_{\beta s} + p\lambda_{\beta s}$$

$$\& \; V_{\alpha s} = R_s . i_{\alpha s} + p\lambda_{\alpha s}$$

Now, writing for rotor side voltage equation.

$$V'_{qr} = R'_r \cdot i'_{qr} + p\lambda'_{qr} + (\omega_\theta - \omega_r)\lambda'_{dr} \quad \text{(v)}$$

$$V'_{dr} = R'_r \cdot i'_{dr} + p\lambda'_{dr} \mp (\omega_\theta - \omega_r)\lambda'_{qr} \quad \text{(vi)}$$

Replacing ~~α, β~~ q, d components in eq^n. (v) & (vi) with β, α components respectively we get ($\omega_\theta = 0$ rads^{-1})

$$V'_{\beta r} = R'_r \cdot i'_{\beta r} + p\lambda'_{\beta r} + (\omega_\theta^{\,0} - \omega_r)\lambda'_{\alpha r}$$

$$V'_{\alpha r} = R'_r \cdot i'_{\alpha r} + p\lambda'_{\alpha r} \mp (\omega_\theta^{\,0} - \omega_r)\lambda'_{\beta r}$$

Problem 2

(4) contd.

$$\therefore \quad v'_{\beta r} = R'_r \cdot i'_{\beta r} + p\lambda'_{\beta r} - \omega_r \lambda'_{\alpha r}$$

$$\& \quad v'_{\alpha r} = R'_r \cdot i'_{\alpha r} + p\lambda'_{\alpha r} + \omega_r \lambda'_{\beta r}$$

Problem 3

(1) given:-

Rated shaft power = 0.6kw

Rated stator voltage (V_L) = 400 V

Rated stator current (I) = 2.1 A

$\cos\theta = 0.6$

$$\text{motor's efficiency} = \frac{\text{Output Power}}{\text{Input Power}}$$

Output Power = 0.6 kw

= 600 w

Input Power = $\sqrt{3} V_L I_L \cos\theta$

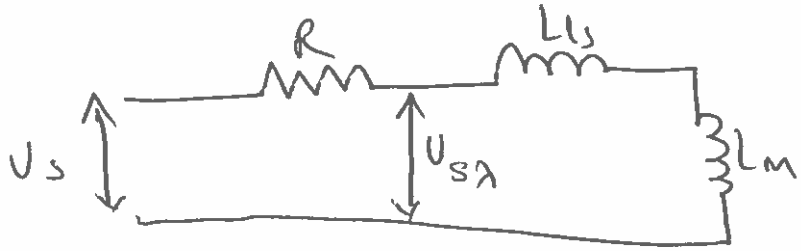
= $\sqrt{3} \times 400 \times 2.1 \times 0.6$

= 872.953 w

efficiency ($\eta\%$) = $\frac{\text{Output Power}}{\text{Input Power}} \times 100\%$

= $\frac{600}{872.953} \times 100 = 68.73\%$

(2) The equivalent circuit of an induction motor is given below for stator side only.



Problem 3

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(2) Since nothing in question is mentioned about stator resistance voltage compensation, so we have to find the voltage after the stator resistance, i.e. defined in the diagram as V_{s2}.

From the diagram from the previous page, we can clearly state that

$$V_{s2pk} = V_{ph\,pk} - I_{ph\,pk} \times \text{stator resistance}$$

$$\Rightarrow V_{s2pk} = \frac{400}{\sqrt{3}} \times \sqrt{2} - 2.1 \times \sqrt{2} \times 12$$

$$= 326.599 - 35.64$$

$$= 290.961\ V_{pk}$$

$$V_{sA} = V_s - I_s R_s$$

$$= \frac{400}{\sqrt{3}} - 2.1 \times 12$$

$$= 230.94 - 25.2$$

$$= 205.74 \text{ V}$$

As given in question $f_s = 60 \text{ Hz}$.

∴ the $\frac{V}{f}$ ratio would be $\frac{205.74}{60}$

∴ $\frac{V}{f} = 3.429 \text{ V/Hz}$

3) As the resistance is neglected,

So, $V_s = V_{sA}$

$$V_{s_{ph}peak} = \frac{400}{\sqrt{3}} \times \sqrt{2} = 326.6 \text{ V}$$

Problem 3

(3) Contd.

$$\omega_s = 2\pi f_s = 2\pi \times 60$$
$$= 376.99 \text{ rad s}^{-1}$$
$$\simeq 377 \text{ rad s}^{-1}$$

Let the stator flux Linkage be 'λ_s'

$$\lambda_s = \frac{U_{s\,ph\,peak}}{\omega_s} = \frac{326.6}{377} = 0.866 \text{ V/rad s}$$

(4) The motor is required to run at 0.25 Hz.

So, now we have to find f_s.

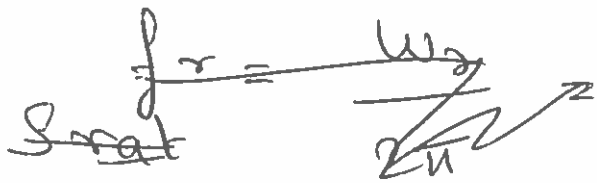
$$\therefore \text{Srated} = \frac{f_{s\,rated} - f_{rotor\,speed\,rated}}{f_{s\,rated}}$$

Rotor speed in electrical radians/sec

$$\omega_r = 850 \text{ rpm} \times \frac{2\pi}{60} \times P_p \quad \left[P_p = \text{pole pairs}\right]$$

$$\Rightarrow \omega_r = 850 \times \frac{2\pi}{60} \times 4 \quad \left[P_p = 4 \text{ given in question}\right]$$

$$\Rightarrow \omega_r = 356.047 \text{ rad s}^{-1}$$



$$S_{rated} = \frac{\omega_{s\,rated} - \omega_{r\,rated}}{\omega_{s\,rated}}$$

$$= \frac{377 - 356.047}{377}$$

$$= 0.056$$

Problem #3

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4) Since the slip doesnot change, (13)

Thus.

$$S_{rated} = \frac{f_{snew} - f_{rnew}}{f_{snew}}$$

$$\Rightarrow 0.056 = \frac{f_{snew} - 0.25}{f_{snew}} \quad \left[f_{rnew} = 0.25 Hz \text{ given in question.} \right]$$

$\Rightarrow$ ~~$f_{s(new)}$~~

$$\Rightarrow f_{snew}(1 - 0.056) = 0.25$$

$$\therefore f_{snew} = \frac{0.25}{0.944} = 0.265 Hz$$

As per question, the stator flux linkage (λ_s) should remain the same

$\therefore \lambda_s = 0.866 \text{ V/rad s}^{-1}$

$\omega_{snew} = 2\pi f_{snew}$

$= 2\pi \times 0.265 \text{ Hz}$

$= 1.664 \text{ rad s}^{-1}$

We know that

$$\frac{V_{s\lambda}}{\omega_{snew}} = \lambda_s$$

$\Rightarrow V_{s\lambda} = \lambda_s \times \omega_{snew}$

$= 0.866 \times 1.664$

$= 1.44 \text{ V}$

$V_s = V_{s\lambda} + I_s R_s$

Problem 3

(4) $V_s = 1.44 \angle 0 + (1 \angle -45) \times 12$

$$V_s = 13.058 \angle -40.53 \text{ V}$$

∴ Magnitude of phase voltage after phase resistive voltage drop compensation is

$$|V_s| = 13.058 \text{ volts.}$$

(5) We know, $S_{rated} = 0.056$

$$T_{e\,now} = \frac{1}{4} T_{e\,rated.}$$

We know that,

$$\Rightarrow \frac{T_{e\,now}}{T_{e\,rated}} = \frac{f_{se}}{S_{rated} \times f_{s\,rated.}}$$

$$\Rightarrow \frac{1}{4} \frac{T_{e\,rated}}{T_{e\,rated}} = \frac{f_{se}}{0.056 \times 60}$$

Problem 3

(5) contd.

$$\Rightarrow f_{se} = \frac{1}{4} \times 0.056 \times 60$$

$$= 0.84 \text{ Hz}$$

$$f_{se} = f_{snow} - f_{rnow}$$

we know that the shaft speed frequency now (f_{rnow}) = 15 Hz

So, the stator frequency (f_{snow}) has to be found.

$$0.84 = f_{snow} - 15$$

$$\therefore f_{snow} = 15.84 \text{ Hz}$$

$$\text{New slip } (s_{new}) = \frac{f_{snow} - f_{rnow}}{f_{snow}}$$

$\Rightarrow S_{new} = \frac{15.84 - 15}{15.84} = \frac{0.84}{15.84} = 0.053$

Problem 3

5) Contd.

So, now we know $f_{s\,rated} = 60\,Hz$

& $f_{se} = 0.84\,Hz$

∴ The compensated stator frequency becomes, $f_{s,new} = f_{s\,rated} + f_{se}$

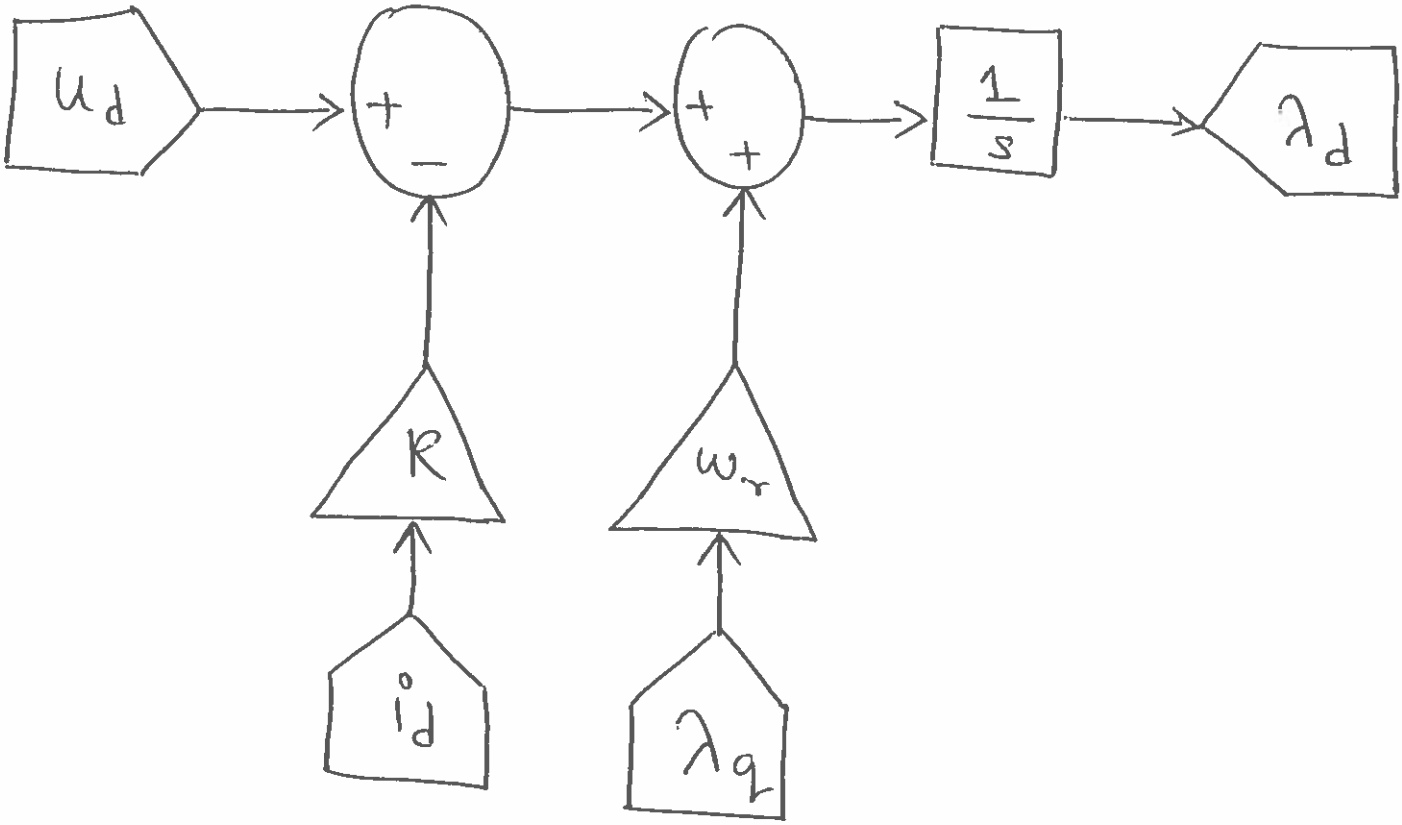
$= 60 + 0.84$

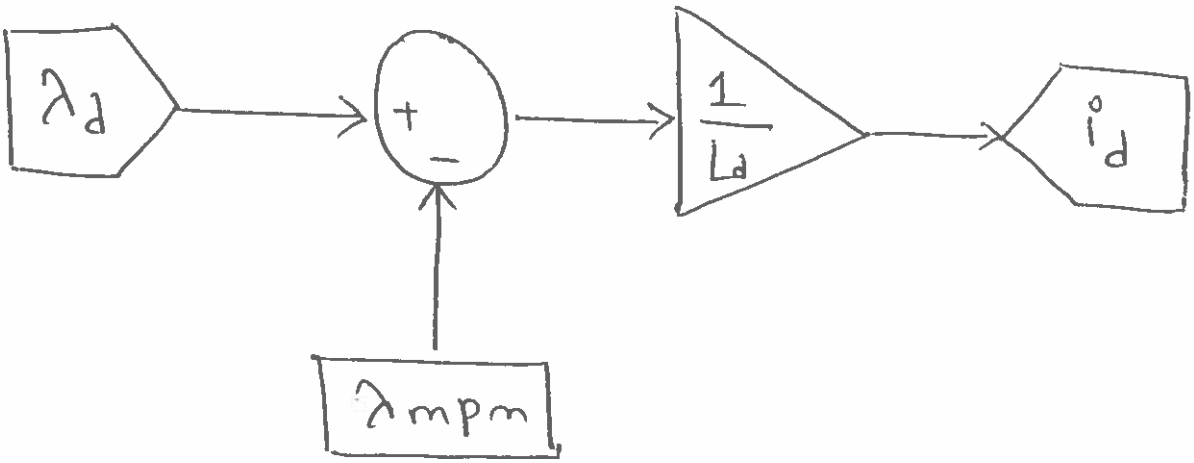
$= 60.84\,Hz$

∴ $f_{se} = 0.84\,Hz$ is the slip frequency that needs to be added to the frequency command.

Problem 4.

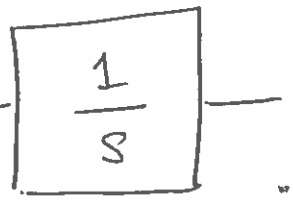
(1) Block diagram in Simulink. to get d-axis current.





Problem 4

2.

2. At $t=0$, we need d axis flux linkage $\lambda_d = \lambda_{mpm}$. So for this, in the simulink diagram, in the integrator block that is 

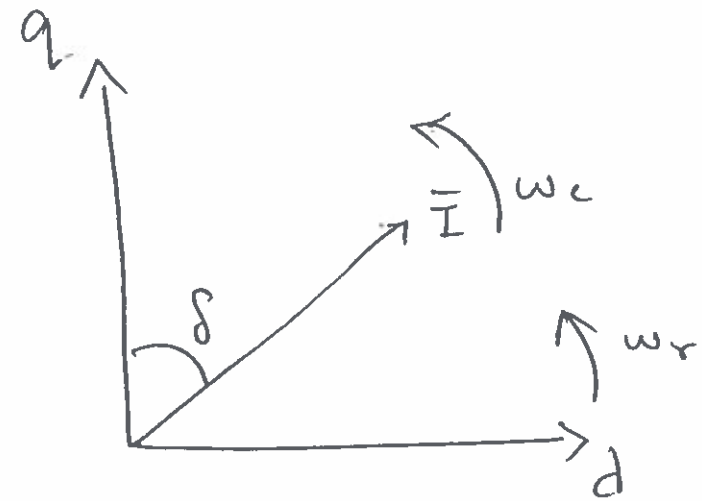
We put the initial condition as 'λ_{mpm}' in the integrator block 

Thus at $t=0$, we will have

$$\lambda_d = \lambda_{mpm}$$

Problem 4

(3) In I/F control of the PM machine, the current vector should be lagging the q-axis.



Let δ be the angle defined between current vector $\bar{I}$ & q axis.

The electromechanical torque equation:

$$T_e - T_{load} = J\frac{d\omega_r}{dt}$$

We assume that the current vector $\bar{I}$ rotates with an angular velocity of 'ω_e' rad s^{-1} & the qd axis rotates with an angular ~~rad~~ velocity of 'ω_r' rad s^{-1}

3) explaination.

Case I : Required for stable operation.

$\bar{I}$ (current vector) lags the q-axis.

In this case, considering T_{Load} increase then the ~~speed~~ angular velocity of rotation of the current vector ($\bar{I}$) increases with respect to the angular velocity of rotation of qd reference frame. As a result of which 'δ' decreases, which brings the current vector closer to the q axis.

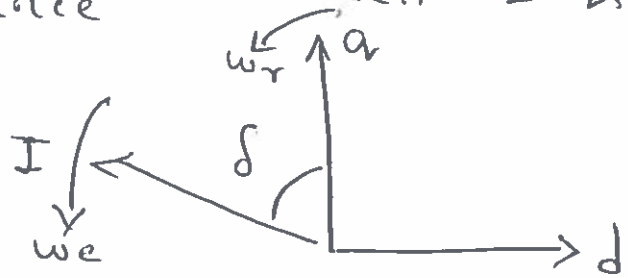
Thereby increasing the q axis current(i_q). We know that "$T_e \propto i_q$". So when i_q increases, electromechanical torque 'T_e' also increases. Thus the difference in

torque between Te & T_{Load} [$T_e - T_{load}$] is maintained and thus the angular ~~vel~~ velocity of the qd reference frame ↑ increases and is maintained and stability of the system is observed.

Case: 2 [Unstable operation Not recommended.]

~~How~~

However, if the current vector 'I' leads the q axis. Then, when the load torque 'T_{Load}' increases, the qd reference frame angular velocity decreases. This leads to increase in 'δ', i.e the phase difference between $\bar{I}$ & q axis.



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3) δ contd.

Thus in this condition, we see that as the 'δ' increases the current vector $\bar{I}$ moves away from the q-axis, thus further reducing the q axis current.

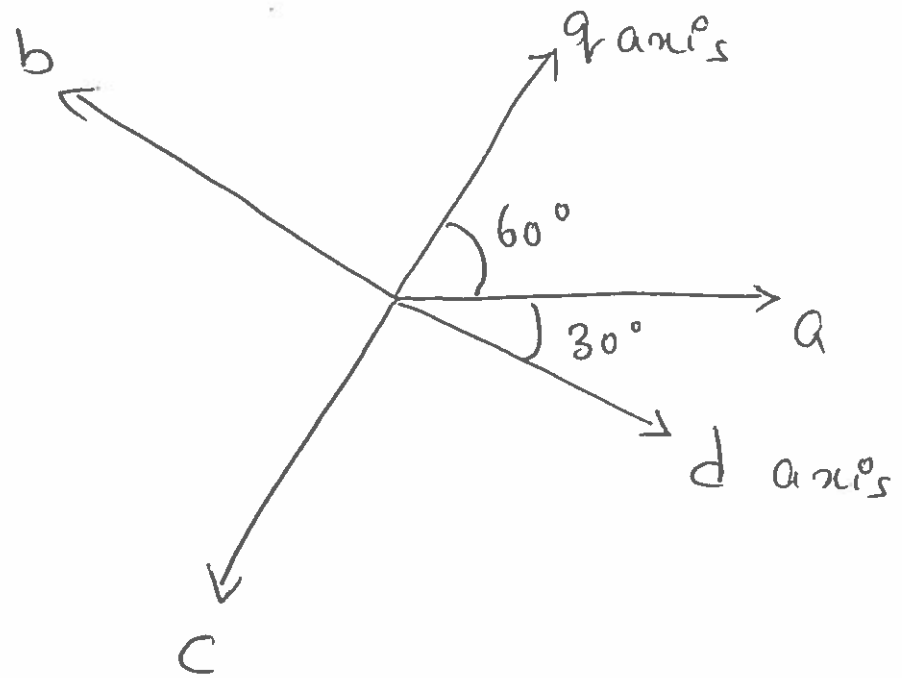
& we know that " $T_e \propto i_q$ ".

So with decrease in i_q, T_e also continuously keeps decreasing. Then the ~~sys~~ difference $T_e - T_{load}$ keeps increasing with time & system becomes unstable.

Thus for stable operation the current vector 'I' should lag the q-axis.

Problem 4.

4) As given in question, the system is a dq reference frame. The rotor position (i.e. d axis) is -30 electrical degrees, when phase 'a' voltage crosses 'zero' from negative to positive.



~~b~~ Phase 'a' voltage will be zero, when the voltage space vector will be at $\pm 90°$ to phase 'a' voltage.

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~~V_a~~

4) Contd.

$$V_a = Re\left(\frac{\bar{V}_f\, e^{j90^\circ}}{e^{j0^\circ}}\right)$$

Let $\bar{V}_f$ be the voltage space vector & V_a is the phase a voltage.

$\Rightarrow V_a = \bar{V}_f \cos(90^\circ) = 0$

However, $V_a = \bar{V}_f \cos(-90) = 0$

~~So, $\bar{V}_f$ for V_a~~

So, for V_a to be '0' volts. $\bar{V}_f$ should either be at $+90^\circ$ or at -90° ~~w.r.t the with respect to a~~ coordi [i.e. 90° leading or 90° lagging] at the instant when $V_a = 0$.

But from the question we see that, at ~~zero cr~~ the voltage vector $\bar{V}_f$ has to be represented when phase a voltage crosses the zero, from negative to positive, ~~what~~

So, if we consider the voltage space vector $\bar{V}_f$ to be leading 'a' axis by 90°, then after the zero crossing, the phase 'a' voltage should ~~be to~~ become more negative. So, $\bar{V}_f$ cannot lead a-axis by 90° at the instant when Va crosses the zero voltage from negative to positive.

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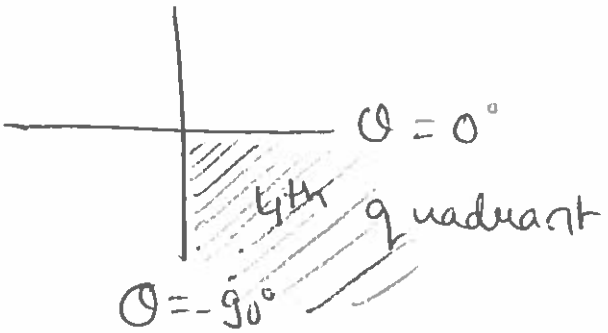
4) contd.

However, if we consider voltage space vector ($\bar{V}_f$) to be lagging a-phase voltage by 90° at the instant when V_a is zero. Then we observe that with time, $\bar{V}_f$ move towards positive value of V_a phase as now, $\bar{V}_f$ will enter the 4th quadrant, defined between $\theta = -90°$ to ~~$\theta = -90$~~ $\theta = 0°$.

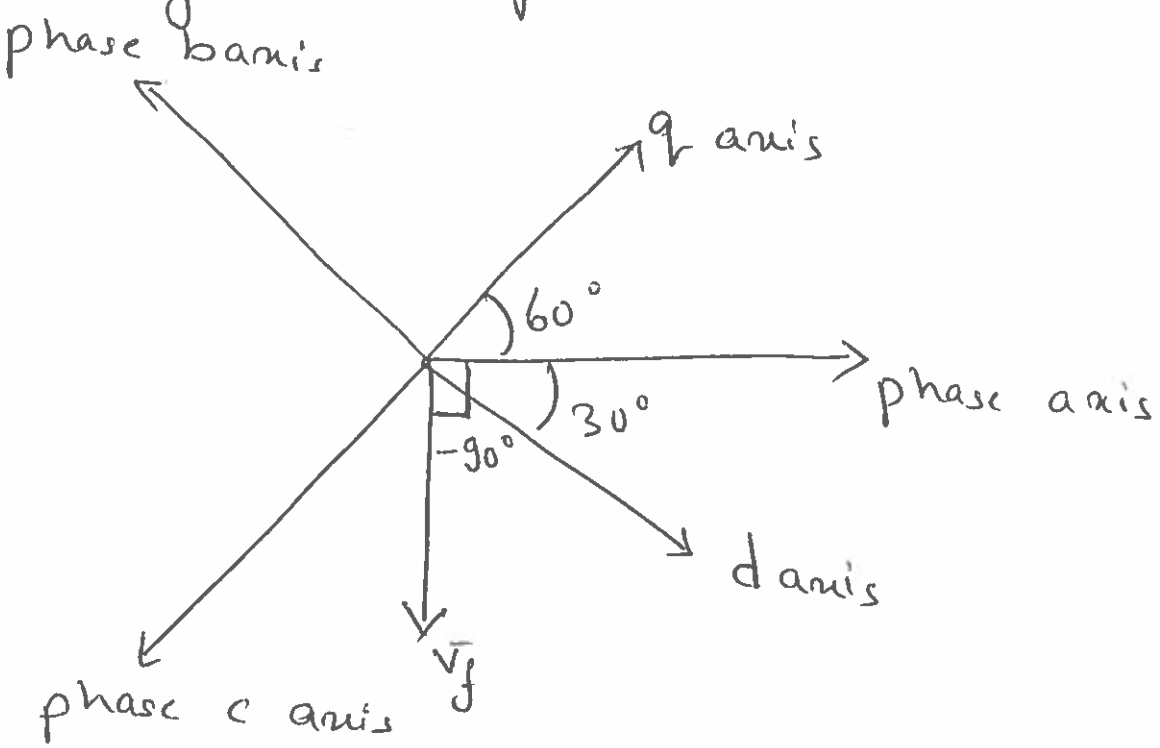


In 4th quadrant @ cos θ = positive value.

$\therefore V_a = \bar{V}_f \left(\frac{Re\, e^{j(\theta)}}{e^{j0}} \right)$ for ~~θ but~~ $-90 < \theta < 0°$

V_a will have positive value.

Thus $\bar{V}_f$ should be lagging a-axis by 90° at the instant when, the a axis's phase voltage crosses zero, from negative to positive.



where $\bar{V}_f$ = Voltage space vector.

From the graph we can see that,

$$360 \text{ degrees} = 0.02 \times \frac{\text{degrees}}{\text{division}}$$

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4)

$$\Rightarrow \frac{\text{degrees}}{\text{division}} = \frac{360^\circ}{0.02} = \frac{18000 \text{ degrees}}{\text{division}}$$

So for $0.00167 \times 18000 = 30$ degrees.

As per graph a axis phase current lags a axis phase voltage by 30°.

So, the ~~pha~~ current ~~voltage~~ space vector ($\bar{I}_f$) lags the voltage space vector by 30° as well.