Written examination in

Dynamic Models of Electrical Machines and Control Systems

1st semester M.Sc. (PED/EPSH/WPS/MCE)

Duration: 4 hours

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off
- Internet access is strictly forbidden
- Any kind of communication with other students is not allowed
- Remember to write your study number on all answer sheets
- All intermediate steps and calculations should be included in your answer sheets --- printing the final result is insufficient

The set consists of 5 problems

Problem 1 (25%)

For a given space vector $\bar{f} = 10e^{-j\left(\omega_e t + \frac{\pi}{6}\right)}$, where $\omega_e = 2\pi \cdot 50$, please

(1) Find the expressions for its corresponding alfa-, beta-components. Please draw their waveforms as functions of the time.

(2) Find the expressions for its corresponding a-, b-, c-components. Please draw **phase-a** waveform as a function of the time.

(3) Please find the particular moment ($t = ?$) that makes phase-b reach its maximum value. Please draw the location of the space vector at this particular moment with respect to phase-a axis.

(4) Now you are given a dq-reference frame. At time t=0, its d-axis is aligned with phase-a axis. It rotates positively (anti-clockwise direction), at a speed of $\omega_e = 2\pi \cdot 50$. Please find the expressions of the dq-components for the given space vector $\bar{f} = 10e^{-j\left(\omega_e t + \frac{\pi}{6}\right)}$. Please draw the dq-component waveforms as functions of the time.

(5) Find the space vector for the following abc signals (where $\omega_e = 2\pi \cdot 50$ [rad/s])

$$v_a = V_{pk} \sin\left(\omega_e t + \frac{\pi}{6}\right),$$
$$v_b = V_{pk} \sin\left(\omega_e t + \frac{2\pi}{3} + \frac{\pi}{6}\right),$$
$$v_c = V_{pk} \sin\left(\omega_e t - \frac{2\pi}{3} + \frac{\pi}{6}\right)$$

Problem 2 (25%)

A sketch of a synchronous machine is shown below. A dq-reference frame is given as well.

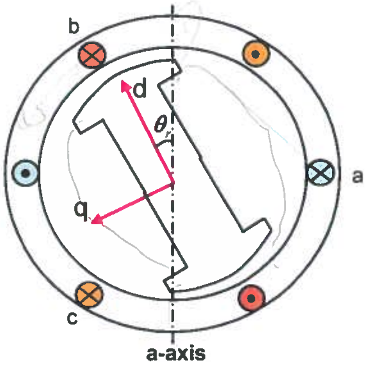
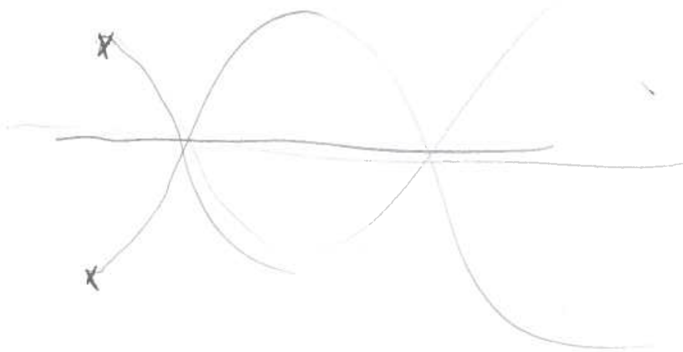


Fig. 1



(1) Please determine the mutual inductance between stator phase-b and stator phase-c.

(2) Please find the maximum value of this mutual inductance and explain at which position, the maximum inductance value is achieved?

A simple single-phase PM machine is shown below.

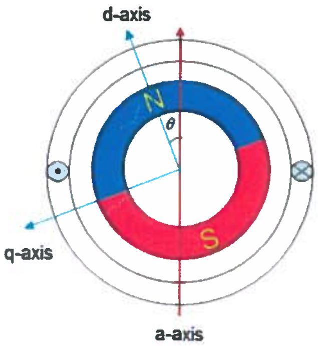


Fig. 2

(3) The PM flux linkage is sinusoidal, as $\lambda_{pm,a} = \lambda_{mpm}\cos(\theta)$. Now phase-a is supplied with 3[rd] harmonic current component, as $i_a = -I_{m1}\sin(3\theta)$, please give the instantaneous torque expression. Please also sketch this torque waveform (instantaneous torque vs. position θ).

(4) Suppose you supply the machine with a trapezoidal current waveform instead of sinusoidal, will the machine be able to rotate (i.e. with an average torque component that is non-zero)? ?

(5) At least how many phases are needed in order to make the machine with such a permanent magnet rotor to have constant **instantaneous** torque? ?

Problem 5 (20 %)

The stator voltage equation of a permanent magnet synchronous machine may be given as (*same notations as used in the lecture slides*):

$$u_q = Ri_q + p\lambda_q + \omega_r\lambda_d \qquad \lambda_q = (L_{ls} + L_{mq})i_q = L_q i_q$$
$$u_d = Ri_d + p\lambda_d - \omega_r\lambda_q \qquad \lambda_d = (L_{ls} + L_{md})i_d + \lambda_{mpm} = L_d i_d + \lambda_{mpm}$$

The machine has 8 poles.

(1) Are the machine equations the same, when expressed in a dq-reference frame or in a qd-reference frame?

(2) When the machine is running at 1200 rpm, what is the frequency of the stator phase current?

(3) This PM machine is driven by another DC motor and running at a constant speed of 1200 rpm. When the stator windings are open-circuited, measured line-to-line RMS voltage is 120 (Vols). Please determine the value of λ_{mpm} to be used in the above machine equations.

(4) For this machine, the d-, q-axes inductances are equal. What is the torque when the machine q-axis current is 2 (A) and the d-axis current is -1 (A)?

(5) At a particular moment ($t = 0$), it is observed that the machine q-axis current is 4 (A) and its d-axis current is -2 (A). At this moment, the rotor d-axis is leading the stator phase-a by 30 electrical degrees. The speed is constant and is 1200 rpm. Please draw stator phase-b current waveform for one period, starting from $t = 0$ as defined before. (Please indicate clearly *its initial current value at $t = 0$ and its peak value.)*
The power factor of this machine at this operation condition is 0.866 (voltage leading current). The phase rms voltage is 100 volts. Please add phase-b voltage waveform for one electrical period to the phase-b current waveform mentioned previously.

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1 - A given space vector:

$$f = 10e^{-j(w_e t + \frac{\pi}{6})}, \quad \text{where} \quad w_e = 2\pi 50$$

- The corresponding alfa - beta - component:

By definition:

$$\bar{f}_{\alpha\beta 0} = \bar{f}_\alpha + j\bar{f}_\beta, \quad \text{thus} \quad \bar{f} = \bar{f}_{\alpha\beta}$$

$$\bar{f} = \bar{f}_{\alpha\beta}$$

By using $e^{ix} = \cos(x) + j\sin(x)$, it yields:

$$\bar{f}_{\alpha\beta} = 10^{-j(w_e t + \frac{\pi}{6})} = \bar{f}_{\alpha\beta} = 10 \cdot \cos(-w_e t + \frac{\pi}{6}) + j\, 10 \cdot \sin(-w_e t + \frac{\pi}{6})$$

omitted $\cos(-\theta) = \cos(\theta)$

$$\Downarrow$$

$$f_\alpha = Re(f_{\alpha\beta}) = 10 \cdot \cos(w_e t + \frac{\pi}{6})$$

$$f_\beta = Im(f_{\alpha\beta}) = -10 \cdot \sin(w_e t + \frac{\pi}{6})$$

- Sketch of waveform

$f_\alpha(0) = 8{,}66$

$f_\beta(0) = -5$

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2 - Find the corresponding abc-components:
By the "vector project method":

- Phase

$$\begin{cases} f_a = Re\left(\dfrac{\bar{f}_{\alpha\beta}}{e^{j0}}\right), & 0° \text{ - Location of phase a} \\ f_b = Re\left(\dfrac{\bar{f}_{\alpha\beta}}{e^{j120}}\right) \\ f_c = Re\left(\dfrac{\bar{f}_{\alpha\beta}}{e^{-j120}}\right) \end{cases}$$

$\Downarrow$

Amplitude — Phase shift

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} |f_a| \cos(w_e t - \delta) \\ |f_b| \cos(w_e t - \delta - 120°) \\ |f_c| \cos(w_e t - \delta + 120°) \end{bmatrix}$$

Thus:

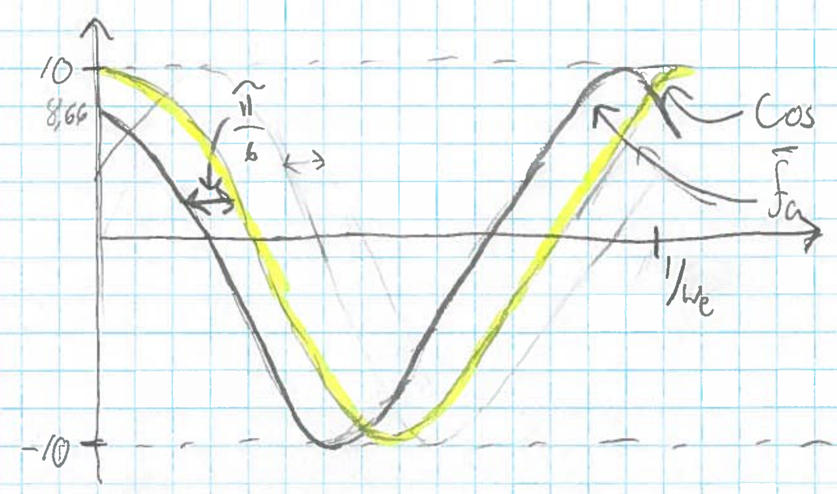
$$f_a = 10 \cos(-w_e t - \tfrac{\pi}{6})$$

$$f_b = 10 \cos(-w_e t - \tfrac{\pi}{6} - \tfrac{2\pi}{3}) = 10 \cos(-w_e t - \tfrac{5\pi}{6})$$

$$f_c = 10 \cos(-w_e t - \tfrac{\pi}{6} + \tfrac{2\pi}{3}) = 10 \cos(-w_e t + \tfrac{2\pi}{3})$$

Sketch phase a:

$f_a(0) = 8{,}66$



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3 - The instance of time where b reaches its maximum value:

$$\bar{f}_b = 10 \cdot \cos\left(\cdot \omega_e t + \frac{5\pi}{6}\right)$$

Its maximum value, $|\bar{f}_b| = 10$, so it is at max, whenever $\cos\left(\cdot \omega_e t \cdot \frac{5\pi}{6}\right) = 1$, so:

$$\cos\left(\cdot 2\pi \cdot 50\, t + \frac{5\pi}{6}\right) = 1$$

$$\Downarrow$$

$$\cdot 2\pi 50 t + \frac{5\pi}{6} = 0$$

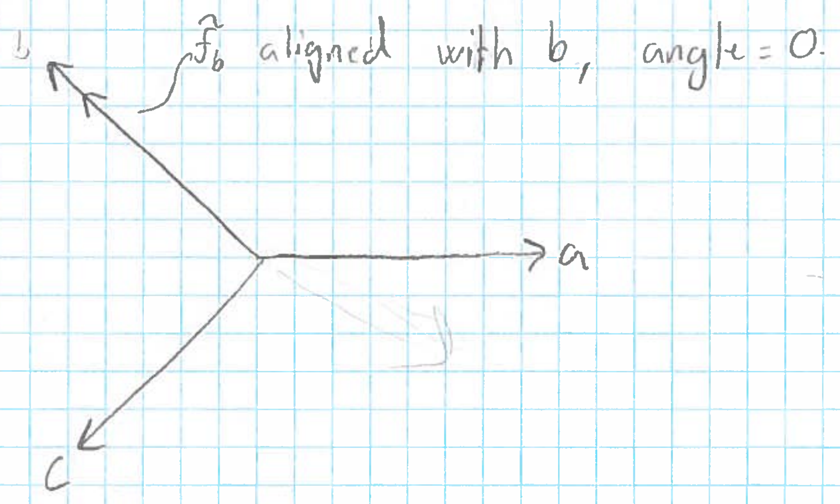
$$t = \frac{-5\pi}{2\pi 50 \cdot 6}$$

$$t = -\frac{1}{120}$$ - Negative since it leads, only for 1. iter. — cos by $30°$

So $\bar{f}_b$ is at its max for:

$$t = -\frac{1}{120} + \frac{2n\pi}{50 \cdot 2\pi} \qquad n = 1, 2, 3 \ldots n$$

- Sketch



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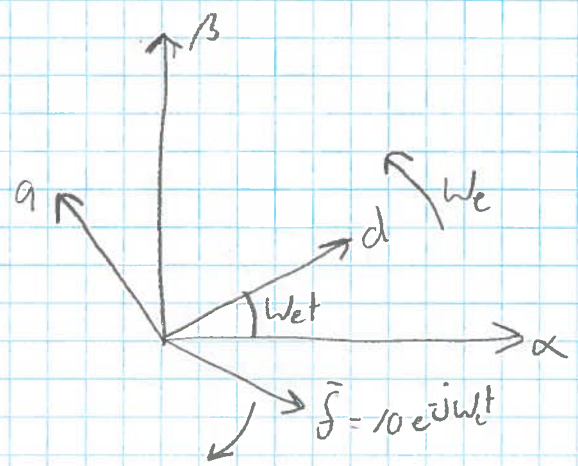
4 - dq-reference frame.

At $t=0$, the d-axis is aligned with the phase a-axis.

$$\bar{f} = 10 e^{-j(\omega_e t + \frac{\pi}{6})}$$

→ Rotates in negative direction (CW)



By "vector projection method"

$$f_d = Re\left(\frac{\bar{f}_{\alpha\beta}}{e^{j\omega_e t}}\right)$$ ← Location of d-axis.

$$= Re\left(\frac{10 e^{-j(\omega_e t + \frac{\pi}{6})}}{e^{j\omega_e t}}\right)$$

$$= \cancel{Re}\left(10 e^{-j(\omega_e t + \frac{\pi}{6})} e^{-j(\omega_e t)}\right) = 10 e^{-j(\omega_e t + \frac{\pi}{6} + \omega_e t)} = 10 e^{-j(2\omega_e t + \frac{\pi}{6})}$$

$$= 10 \cos\left(-2\omega_e t - \frac{\pi}{6}\right) = 10 \cos\left(2\omega_e t + \frac{\pi}{6}\right)$$

$$f_q = Re\left(\frac{10 e^{-j(\omega_e t + \frac{\pi}{6})}}{e^{j\omega_e t + 90}}\right)$$

$$= Re\left(10 e^{-j(\omega_e t + \frac{\pi}{6})} e^{-j(\omega_e + \frac{\pi}{2})}\right) = 10 e^{-j(\omega_e t + \frac{\pi}{6} + \omega_e + \frac{\pi}{2})}$$

$$= 10 \cos\left(2\omega_e + \frac{\pi}{6} + \frac{\pi}{2}\right)$$

$$= 10 \sin\left(2\omega_e + \frac{\pi}{6}\right)$$

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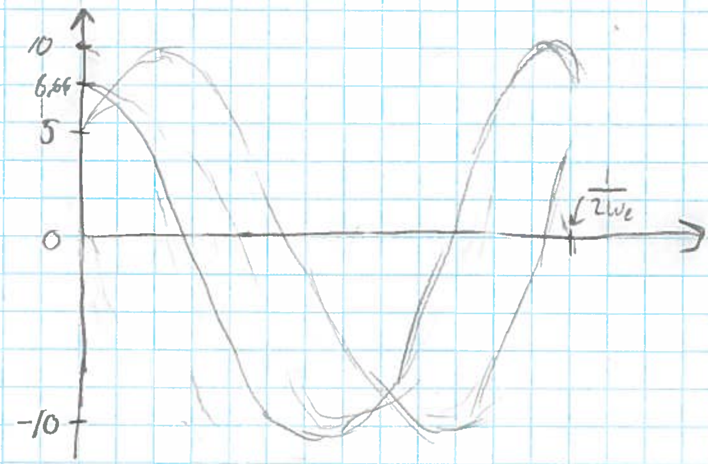
- Sketch i_d and i_q:

$\delta_d(0) = 8{,}66$

$\delta_q(0) = 5$



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5 - ABC - components

$$-V_a = V_{pk} \sin\left(\omega_e t + \frac{\pi}{6}\right)$$
$$V_b = V_{pk} \sin\left(\omega_e t + \frac{2\pi}{3} + \frac{\pi}{6}\right)$$
$$V_c = V_{pk} \sin\left(\omega_e t - \frac{2\pi}{3} + \frac{\pi}{6}\right) \quad , \quad \omega_e = 2\pi \cdot 50 \ [rad/s]$$

- Find the space vector

$$\bar{f}_0 = \frac{1}{3}\left(\bar{f}_a + f_b + f_c\right)$$
$$\bar{f} = \frac{2}{3}\left(f_{a0} e^{j0} + f_{b0} e^{j120} + f_{c0} e^{-j120}\right), \quad f_{a0} = f_a - f_0, \quad f_{b0} = f_b - f_0, \quad f_{c0} = f_c - f_0$$

For symmetrical abc variables $\Rightarrow f_0 = 0$.

$$f = \frac{2}{3} V_{pk}\left(\sin\left(\omega_e t + \frac{\pi}{6}\right)e^{j0} + \sin\left(\omega_e t + \frac{\pi}{6} + \frac{2\pi}{3}\right)e^{j120} + \sin\left(\omega_e t - \frac{2\pi}{3} + \frac{\pi}{6}\right)e^{-j120}\right)$$

$\Downarrow$

$$\bar{f} = \frac{2}{3} V_{pk} \underbrace{\left(\sin\left(\omega_e t + \frac{\pi}{6}\right)e^{j0} + \sin\left(\omega_e t + \frac{5\pi}{6}\right)e^{j120} + \sin\left(\omega_e t - \frac{3\pi}{6}\right)e^{-j120}\right)}_{*}$$

$\Downarrow$

Real part of (*):

$$\sin\left(\omega_e t + \frac{\pi}{6}\right) + \sin\left(\omega_e t + \frac{5\pi}{6}\right)\cos(120) + \sin\left(\omega_e t - \frac{3\pi}{6}\right)\cos(-120)$$
$$= \sin\left(\omega_e t + \frac{\pi}{6}\right) - \frac{1}{2}\left(\sin\left(\omega_e t + \frac{5\pi}{6}\right) + \sin\left(\omega_e t - \frac{3\pi}{6}\right)\right)$$
$$\Downarrow$$
$$= \sin\left(\omega_e t + \frac{\pi}{6}\right) - \sin\left(\omega_e + \frac{\pi}{6}\right) \cdot \underbrace{\cos(120)}_{-1/2}$$
$$= \sin\left(\omega_e t + \frac{\pi}{6}\right) + \frac{1}{2}\sin\left(\omega_e + \frac{\pi}{6}\right)$$
$$= \frac{3}{2}\sin\left(\omega_e t + \frac{\pi}{6}\right)$$

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Imaginary part of (*):

$$\sin(\omega_e t + \tfrac{\pi}{6}) \cdot \sin(0) + \sin(\omega_e t + \tfrac{5\pi}{6}) \sin(120) + \sin(\omega_e t - \tfrac{3\pi}{6}) \underset{\sin(-120)}{}$$

$$+ \frac{\sqrt{3}}{2}\left(\sin(\omega_e t + \tfrac{5\pi}{6}) - \sin(\omega_e t - \tfrac{3\pi}{6})\right)$$

$$= \sqrt{3}\cos(\omega_e t + \tfrac{\pi}{6}) \cdot \sin(\tfrac{2\pi}{3})$$

$$= \frac{3}{\sqrt{2}}\cos(\omega_e t + \tfrac{\pi}{6})$$

Thus:

$$\bar{f} = V_{pk}\frac{2}{3}\frac{3}{2}\left(\sin(\omega_e t + \tfrac{\pi}{6}) + j\cos(\omega_e t + \tfrac{\pi}{6})\right)$$

$$= V_{pk}\left(\sin(\omega_e t + \tfrac{\pi}{6}) + j\cos(\omega_e t + \tfrac{\pi}{6})\right)$$

$$= V_{pk}\left(\cos(\tfrac{\pi}{2} - (\omega_e t + \tfrac{\pi}{6})) + j\sin(\tfrac{\pi}{2} - (\omega_e t + \tfrac{\pi}{6}))\right)$$

$$= V_{pk}\left(\cos(\tfrac{1}{3}\pi - \omega_e t) + j\sin(\tfrac{1\pi}{3} - \omega_e t)\right)$$

$$\Downarrow \quad e^{ix} = \cos x + j\sin x$$

$$\bar{f}_{ap} = V_{pk}\, e^{-j(\omega_e t + \frac{1}{3}\pi)}$$

- Alternatively:

$$V_a = V_{pk}\sin(\omega_e t + \tfrac{\pi}{6})$$

$$= V_{pk}\cos(-\omega_e t - \tfrac{\pi}{6} + \tfrac{\pi}{2})$$

$$= V_{pk}\cos(-\omega_e t + \tfrac{1\pi}{3})$$

Thus

$$f_{abc} = V_{pk}\, e^{-j(\omega_e t + \frac{1}{3}\pi)}$$

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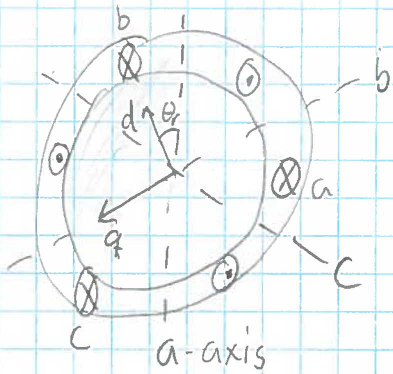
Problem 2 - A synchronous machine:



Non - Salient

$L_{aaq} \rightarrow$ flux through max. air gap length $\rightarrow$ Min Inductance

$L_{aad} \rightarrow$ Flux through min. length $\rightarrow$ Max Inductance.

1 - Determine the mutual inductance between stator phase b and stator phase c.

$$M_{bscom} = L_{aaq} \cos\left(\theta_r - \frac{2\pi}{-3}\right) \cos\left(\theta_r + \frac{2\pi}{3}\right) + L_{aad} \sin\left(\theta_r - \frac{2\pi}{3}\right) \sin\left(\theta_r + \frac{2\pi}{3}\right)$$

$$= L_{aad} \, Re\left(\frac{e^{j\theta}}{e^{j\frac{2\pi}{3}}}\right) \cdot Re\left(\frac{e^{-j\theta}}{e^{-j\frac{2\pi}{3}}}\right) + L_{aaq} \, Re\left(\frac{e^{j\theta + \pi/2}}{e^{j\frac{2\pi}{3}}}\right) Re\left(\frac{e^{j\theta_r + \pi/2}}{e^{-j2\pi/3}}\right)$$

Position of d-axis

Position of b-axis

Position of c-axis

Position of q-axis

$e^{j(\theta - \frac{2\pi}{3})}$ $\quad e^{j(\theta + \frac{2\pi}{3})}$ $\quad j\theta_r + \frac{\pi}{2} + \frac{2\pi}{3}$

$$= L_{aad} \, Re\left(e^{j(\theta + 120)}\right) Re\left(e^{-j(\theta + 120)}\right) + L_{aaq} \, Re\left(e^{j(\theta + 90 - 120)}\right) Re\left(e^{-j(\theta + \frac{7\pi}{6})}\right)$$

$$= L_{aad} \cos(\theta - 120) \cos(\theta + 120) + L_{aaq} \cos\left(\theta - \frac{\pi}{6}\right) \cos(\theta + 210)$$

90° phase shift

$$= L_{aad} \underbrace{\cos(\theta - 120)}_{b} \underbrace{\cos(\theta + 120)}_{c} + L_{aaq} \sin(\theta - 120) \sin(\theta + 120)$$

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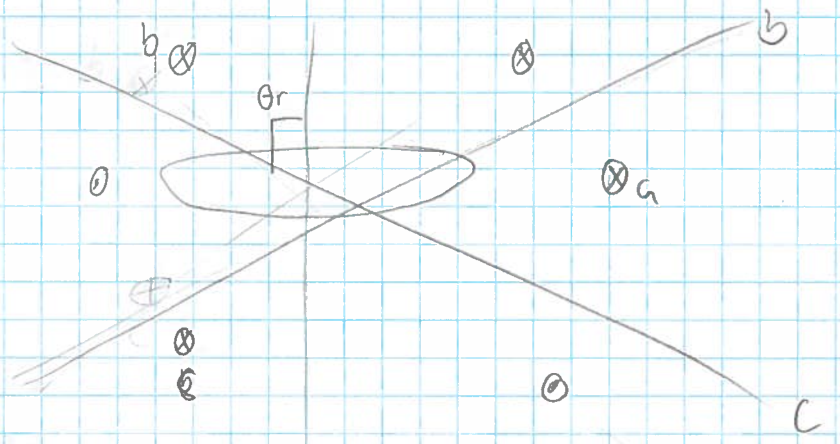
2 - Maximum Value of this mutual inductance:

The mutual inductance between b and c is greatest, when the inductance from a is lowest, or when the flux through b and c is greatest, which is in the middle of b and c.



Thus at $\theta_r = 90^0$

The mutual inductance is:

$$M_{bscsm} = \cos(90-120)\cos(90+120)L_{aad} + \sin(\theta-120)\sin(\theta+120)L_{aaq}$$

The angle can be obtain by:

$\text{diff}(\cos(\theta-120)\cos(\theta+120)) = 0$, since L_{aad} is desired to be greater.

$$= \begin{cases} -\frac{1}{2}\pi \rightarrow \text{Max} \\ 0 \rightarrow \text{min} \end{cases}$$

Thus:

$$M_{bscsm}\Big|_{\theta=-\frac{1}{2}\pi} = -\frac{3}{4}L_{aad} + \frac{1}{4}L_{aaq}$$

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- Alternatively:

$$M_{bscsm} = L_{aaq} \cos(\theta_r - 120)\cos(\theta_r + 120) + L_{aad} \sin(\theta_r - 120)\sin(\theta_r + 120)$$

- $L_{aaq} = L_1 - L_2$
- $L_{aad} = L_1 + L_2$

- Inserting

$$= (L_1 - L_2)\cos(\theta_r - 120)\cos(\theta_r + 120) + (L_1 + L_2)\sin(\theta_r - 120)\sin(\theta_r + 120)$$

$$= L_1(\cos(\theta_r - 120)\cos(\theta_r + 120) + \sin(\theta_r - 120)\sin(\theta_r + 120))$$
$$+ L_2(-\cos(\theta_r - 120)\cos(\theta_r + 120) + \sin(\theta_r - 120)\sin(\theta_r + 120))$$

$\Downarrow$ By trigonometric relation

$$= L_1 \cos(\theta_r - 120 - (\theta_r + 120)) + L_2 \cos(\theta_r - 120 + \theta_r + 120)$$

$$= L_1 \cos(240) + L_2 \cos(2\theta_r)$$

$$= -\frac{1}{2} L_1 + L_2 \cos(2\theta_r)$$

Thus:

$$M_{bscsm} = -\frac{1}{2} L_1 + L_2 \cos(2\theta_r)$$

- Maximum value of the mutual inductance:
  - Max value when $\cos(2\theta_r) = 1$

$$\cos(2\theta_r) = 1$$

$$2\theta_r = \begin{cases} 0 \\ \pi \end{cases}$$

- So the max. value occur when $\theta_r = 0 \vee \theta_r = \pi$

$$-M_{bscsm}\big|_{max} = -\frac{1}{2} L_1 + L_2 \quad , \quad L_1 = \frac{L_{aad} + L_{aaq}}{2} \quad , \quad L_2 = \frac{L_{aad} - L_{aaq}}{2}$$

$$= -\frac{1}{2} \frac{L_{aad} + L_{aaq}}{2} + \frac{L_{aad} - L_{aaq}}{2}$$

$$= -\frac{1}{4} L_d - \frac{3}{4} L_q$$

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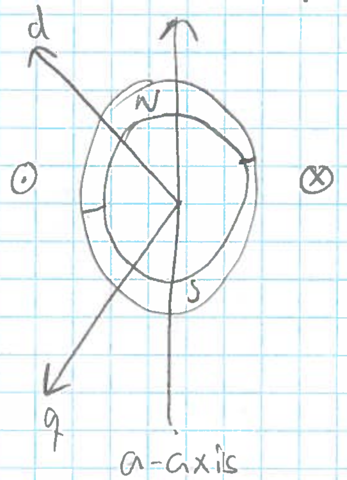
3 - A single-phase PM machine:



- PM - flux linkage is sinusoidal,

$\lambda_{pm,a} = \lambda_{mpm} \cos(\theta)$

- Phase a is supplied with a current component

$I_a = -I_{m} \sin(3\theta)$ ← 3. harmonic

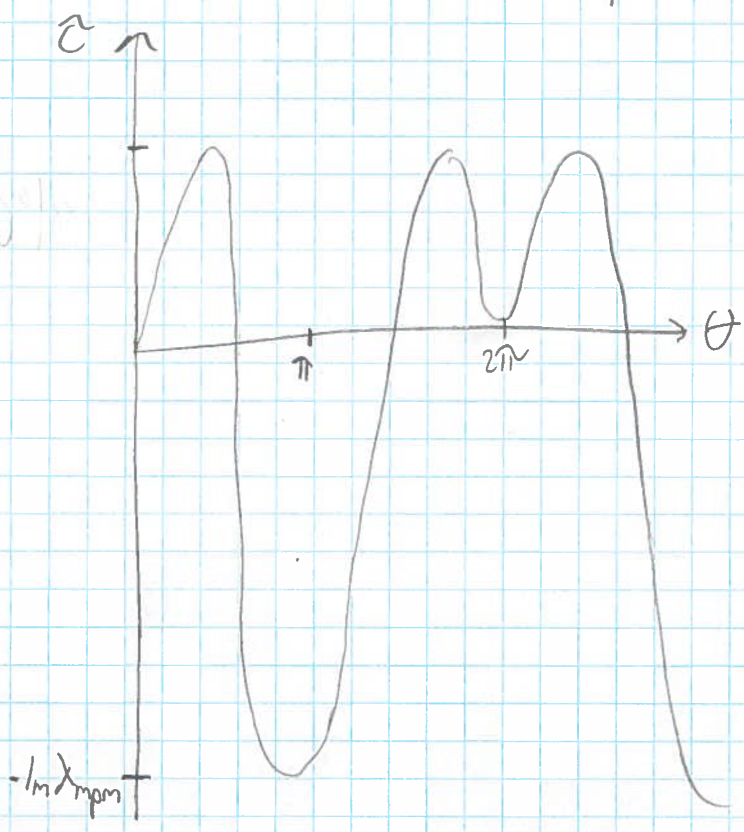
- Derive the instantaneous torque expression:

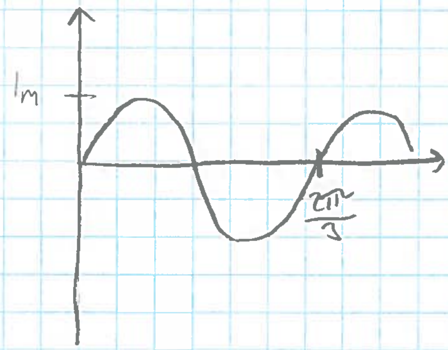
$\tau = p\, i\, \lambda_{mpm}(-\sin(\theta))$

$\Rightarrow \tau = -I_m \sin(3\theta)\lambda_{mpm}(-\sin(\theta))$

$= I_m \sin(3\theta)\sin(\theta)\,\lambda_{mpm}$

- Sketch instantanious torque vs. position:





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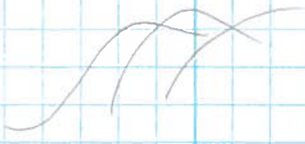
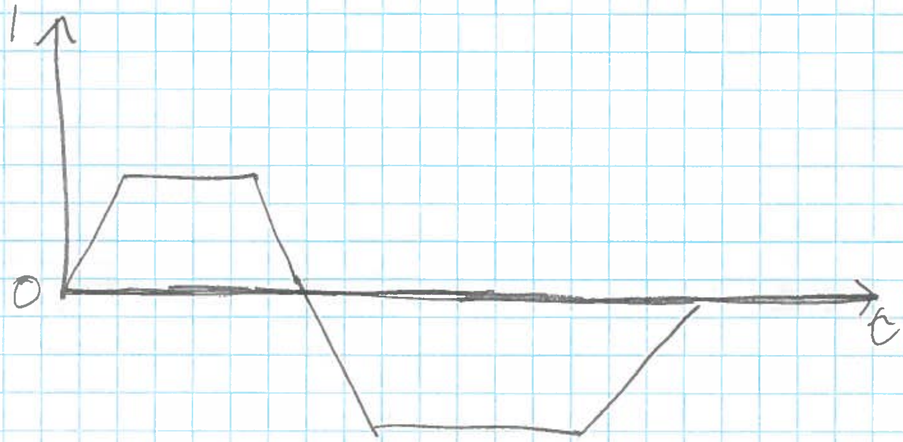


4 - The machine is supplied with a trapezoidal input current, is the machine still able to rotate.



Yes,

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5 - # of phase

- Determine the # of phases of the PM-machine to have constant instantaneous torque.

- Considering the torque eq.:

$\tau = p I_m \lambda_m \sin^2(\theta)$, the current and λ are in phase.

- Considering 2 phases:

$$\tau = p I_m \lambda_m \sin^2(\theta) + p I_m \lambda_m \sin^2(\theta + 90)$$

$$= 2 p I_m \lambda_m \underbrace{(\sin^2(\theta) + \cos^2(\theta))}_{=1}$$

$$\tilde{\tau} = 2 p I_m \lambda_m$$

Since the is a constant, so at least 2 phases are needed in order to have constant instantaneous torque.

11,5

Problem 3 (10%)

A sketch of an induction machine phase axes is given below, where qd-reference frame is shown.

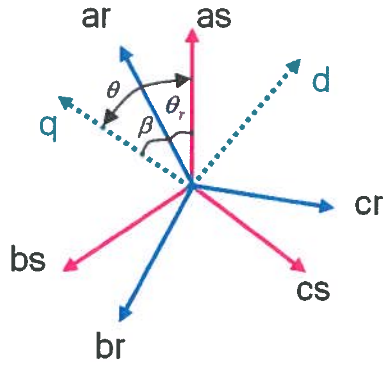

Fig. 3

where notation 's' stands for stator phase axes and notation 'r' stands for rotor phase axis.

Knowing the machine model expressed in an arbitrary qd-reference frame is

Stator side voltage equations:

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{0s} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} - \omega_\theta \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$

(1) Please transform this voltage equation into a vector form, using qd-frame space vector representations, i.e.

$$\overline{f}_{qd} = f_q - jf_d$$

(f is a variable that could stand for the voltage or current.)

(2) Please give the stator voltage equation expressed in the $\alpha\beta$ -reference frame. ?

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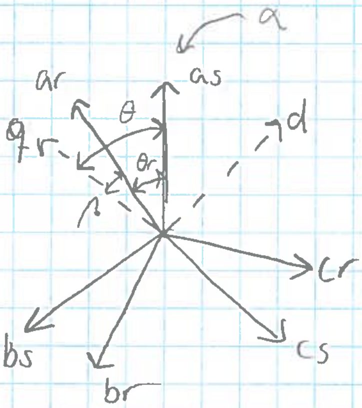
Problem 3 - An induction machine



S: Stator

R: Rotor

- The machine model in an arbitrary qd-frame
  - Stator side Voltage eq:

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{0s} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + \underset{\frac{d}{dt}}{p} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} - \omega_\theta \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$

- Transform the voltage eq. into a vector form,

$$\bar{f}_{qd} = f_q - j f_d$$

Inserting:

$$u_{qs} - j u_{ds} = R(i_{qs} - j i_{ds}) + p(\lambda_{qs} - j \lambda_{ds}) + \omega_\theta (\lambda_{ds} - j \lambda_{qs})$$

$$\bar{f}_{qd} = R_s \bar{i}_{qds} + p \bar{\lambda}_{qds} + j \omega_\theta \lambda_{qds}$$

Note: $\omega_\theta \lambda_{ds} + j \omega_\theta \lambda_{qs}$

$= j\omega_\theta (\lambda_{qs} - j \lambda_{ds})$

$= j \omega_\theta \lambda_{qds}$

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- Express the stator voltage eq. in $\alpha\beta$-reference frame:

$\bar{f} = f_\alpha + j f_\beta$

$\Downarrow$

$f_\alpha + j f_\beta = (f_d + j f_q)\, e^{j\theta}$

- $f_\alpha = Re(\bar{f})$

- $f_\beta = Im(\bar{f})$

$\bar{f} = f_d \cos(\theta) + j f_d \sin(\theta) - j f_q \cos(\theta) + f_q \sin(\theta)$

$f_\alpha = f_d \cos(\theta)\, f_q \sin(\theta)$

$f_\beta = f_d \sin(\theta) - j f_q \cos(\theta)$

Inserting

$\bar{f} = f_\alpha + j f_\beta$

$= u_{qs} \cos(\theta)\, u_{ds} \sin(\theta) + j\left(u_{ds} \sin(\theta) - u_{qs} \cos(\theta)\right)$

Problem 4 (20 %)

A four-pole induction motor has the following data (**the rotor windings are short-circuited**):

$P = VI$

Rated shaft power	1.9 ~~k~~W
Rated speed	1440 rpm . W_{mec}
Rated stator frequency	50 Hz , $50 \cdot 2\pi = W_s$
Rated stator voltage	380 V RMS (line-to-line)
Rated stator current	4.1 A RMS
Rated power factor cos φ	0.8 inductive
Stator resistance	2.5 Ohm
Main (magnetization) inductance	0.27 H
Stator leakage inductance	0.01 H

- Max. speed to work properly

(1) The machine is supplied with the rated voltage and rated frequency. The load torque is zero. What is the rotor speed in rpm? Please calculate the corresponding stator phase current in RMS value and the voltage across the stator reactance.

(2) In V/f control, please explain what must be known in order to compensate the voltage drop on the stator resistance? Why it is needed to compensate the resistance voltage drop? Please explain.

(3) The machine is supplied with the rated voltage and rated frequency. Please calculate the active current value (i_{sd} , *find more information on slide P15 of the lecture discussing about scalar control of induction machine with slip compensation*), which is proportional to the machine torque. Now it is observed that the machine slip is 25% of the rated slip, what is the machine torque now? What is value for this active current (i_{sd}) now? Please also calculate the slip frequency that needs to be compensated.

(4) In V/f control, the stator frequency is now 50% of the rated frequency. The load torque is 25% of the rated torque. What is the slip frequency that needs to be compensated in order to make the electrical shaft speed to be 25Hz? The stator resistance effects may be neglected.

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Problem 4 - Four-pole induction motor

1. The machine is supplied with the rated voltage and frequency.
   - The rotor speed
     - Loaded torque is zero. → Slip = 0

$$W_m = p V_{r,mec} \Rightarrow V_{r,mec} = \frac{W_m}{p},$$

W_m = Rated mec. shaft speed

$$n_s = n_r = \frac{50\,\text{Hz} \cdot \frac{60}{2\pi}}{p} = 1500 \text{ rpm}$$

p: pole pair

W_r : Rotor speed

- The corresponding stator phase current in RMS and voltage across the stator reactance.
  - Voltage:

$$\hat{V}_s = \hat{i}_s (r_s + j(\underbrace{X_{ls}}_{\text{Stator reactance}} + \underbrace{X_m}_{\text{Main reactance}})), \quad X_{ls} = 2\pi f \underbrace{L_{ls}}_{\text{Leakage inductance}}$$

$$X_m = 2\pi f \underbrace{L_m}_{\text{Main Inductance}}$$

  - Current

$$i_s = \frac{\hat{V}_s}{r_s + j(X_{ls} + X_m)} \Rightarrow \frac{\overbrace{380}^{\text{Line-to-line}}/\sqrt{3} \ V_{rms}}{2{,}5\,\Omega + j 2\pi \cdot 50 (0{,}01 + 0{,}27) H}$$

$$= 0{,}07083 - 2{,}4922 j$$

$$I_{rms} = 2{,}49 \text{ A}$$

  - Voltage across the stator reactance:

$$\hat{V}_s = i_s (r_s + j(X_{ls})) \Rightarrow (0{,}07083 - 2{,}4922j)(2{,}5 + j\, 2\pi 50 (0{,}01 + 0{,}27) H)$$

$$= 219{,}4 - 0{,}0000609j \text{ V}$$

$$V_{rms} = 219{,}4 \text{ V} \Rightarrow V_{rated} \cdot \frac{1}{\sqrt{3}}$$

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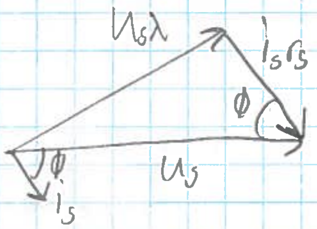
2 - V/f control

- What is required to compensate the voltage drop across the stator resistance.

- The voltage is given as:

$$U_s = r_s(I_s \cos\phi) + \sqrt{U_{s\lambda}^2 - r_s^2(I_s \sin\phi)^2}$$
$$= r_s(I_s \cos\phi) + \sqrt{U_{s\lambda}^2 - (r_s I_s)^2 + r_s^2(I_s \cos\phi)^2}$$



- r_s - Stator resistance
  I_s - Stator phase current (Measured)
  ϕ - Power factor angle
  U_s - Stator Voltage
  $U_{s\lambda}$ - Stator reactance Voltage

- Reason why the resistance Voltage drop is compensated for:

  - The voltage across the resistance dependens on the current, which is load dependent. This will give a steady state error, and will vary according to the load, thus varying the flux. A constant flux is desired.

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3 - The machine is supplied with rated Voltage and frequency:

- Active current value:

  - Slip is assumed 0, $s=0$

  d-axis aligned with $V_s\lambda$, thus

  $i_{sd} = Re(I_s) = 0{,}07083\,A$

  $$T_m = \frac{P_{mech}}{W_{r,mec}} = \frac{1{,}9\,kW}{1440 \cdot \frac{2\pi}{60}\,rad/s} = 12{,}6\,Nm, \text{ machine } \tau$$

$\frac{n_s - n_r}{n_s}$

$$\frac{T_e}{T_{m,rated}} = \frac{W_{se}}{W_{se,rated}}$$

$$s = \frac{W_s - W_r}{W_s}, \quad W_{se} = W_s - W_r \Rightarrow W_{se} = s \cdot W_s$$

W_{se} = Slip frequency

W_r = Rotor speed

W_s = Synchronous speed

s = Slip

$$s_{rated} = \frac{W_{s,rated} - W_{r,rated}}{W_{s,rated}}$$

$W_{r,rated} = W_{r,mec} \cdot p$

$W_m = W_r$

$\Downarrow$

$$s_{\text{E}} = \frac{50 \cdot 60\,rpm - 1440 \cdot 2\,rpm}{50 \cdot 60\,rpm} = 0{,}040 \quad \text{- Rated slip}$$

$$\underbrace{W_{se,rated}} = s_{rated} \cdot W_{s,rated} = 0{,}04 \cdot (3000 \cdot 2\pi/60) = 4\pi\ rad/s$$

Rated slip frequency.

Thus

$T_{e,rated} = T_{m,rated}$

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The slip is 25% of the rated slip,

- The machine torque

$$\tau = \frac{\tau_{rated} \cdot 25\% \, \omega_{se,rated}}{\omega_{se,rated}}$$

$T_m = 3{,}15 \, Nm \Rightarrow T_{el} = \frac{T_m}{2}$? $\Rightarrow T_{el} = 1{,}67 \, Nm$

- The active current, i_{sd}

$$\omega_{se} = \frac{i_{sd}}{i_{sd,rated}} \omega_{se,rated}$$

$\Downarrow$

$$i_{sd} = \frac{\omega_{se} \, i_{sd,rated}}{\omega_{se,rated}}$$

$i_{sd,rated} = I_s \cdot \cos(\theta) \Rightarrow 4{,}1 \, A \cdot 0{,}8 = 3{,}28 \, A$

$$i_{sd} = \frac{25\% \, \cancel{\omega_{se,rated}} \cdot 3{,}28 A}{\cancel{\omega_{se,rated}}} = 0{,}82 \, A \quad \text{- RMS}$$

- The slip frequency that needs to be compensated:

$\omega_{se} = s \, \omega_s$

$\omega_{se} = 0{,}25 \cdot s_{rated} \cdot 50 \cdot 2\pi \, rad/s = \pi \, rad/s = \frac{1}{2} \, Hz.$

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4 - V/F-control

- $\omega_s = 50\% \, \omega_{s,rated}$
- $\tau_1 = 25\% \, \tau_{rated}$

- Determine the slip frequency that needs to be compensated in order to make the electrical shaft speed

$$\frac{T_e}{T_{e,rated}} = \frac{\omega_{se}}{\omega_{se,rated}} \quad , \quad f_{se} = f_s \cdot s$$

$$\Downarrow$$

$$\omega_{se} = \frac{T_e}{T_{e,rated}} \omega_{se,rated} \Rightarrow \frac{1/4 \cancel{T_{e,rated}}}{\cancel{T_{e,rated}}} 4\pi \, rad/s = \pi \, rad/s$$

$$f_{se} = \pi \cdot \frac{1}{2\pi} = \frac{1}{2} \, Hz.$$

17,5

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Problem 5 - The stator voltage eq. of a PM synchronous machine may be given as:

$$u_q = R i_q + p\lambda_q + \omega_r \lambda_d, \qquad \lambda_q = (L_{ls} + L_{mq}) i_q = L_q i_q$$

$$u_d = R i_d + p\lambda_d - \omega_r \lambda_q, \qquad \lambda_d = (L_{ls} + L_{md}) i_d + \lambda_{mpm}$$

$$= L_d i_d + \lambda_{mpm}$$

Permanent magnet flux

The machine has 8 poles.

1 - Are the machine eq. the same, when expressed in a dq-reference or in qd-reference frame:

- dq0 - frame

$$\bar{f} = (f_d + j f_q) e^{j\theta}$$

- qd0 - frame

$$\bar{f} = (f_q - j f_d) e^{j\theta}$$

Thus:

$$(f_d + j f_q) e^{j\theta} = (f_q - j f_d) e^{j\theta} \Rightarrow f_d = f_q, \quad f_q = -f_d$$

$$d \to q \qquad q \to -d$$

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2 - When the machine is running at 1200 rpm, what is the frequency of the stator phase current:

- It is a synchronous machine

$W_{r,el} = p W_{r,mec} = 4 \cdot 1200 = 4800$ rpm

Since it is a synchronous machine, the slip is 0, thus:

$W_{r,el} \cdot W_m$

$\cancel{W_{sc}}^{0} = W_s - W_m = W_s = W_m$

$\uparrow$
$s=0$

$W_s = \frac{4800}{60} = 80\,Hz = 160\pi\ rad/s$

3 . The stator windings are OC, measured line-to-line RMS Voltage is 120 V - PM driven by a DC-motor.

- Determine λ_{mpm}
  - Since it is OC $\rightarrow$ $i_q = 0$, $i_d = 0$

  Thus the voltage eq. becomes:

$$u_q = W_r \lambda_d \quad , \quad \lambda_q = 0$$
$$u_d = -W_r \lambda_q \quad , \quad \lambda_d = \lambda_{mpm}$$

$\Downarrow$

$$u_q = W_r \lambda_{mpm} = \bar{U}_{dq} = \underbrace{\sqrt{2} V_s}_{Peak} e^{j\theta}$$

$\Downarrow$

$$\lambda_{mpm} = \frac{u_q}{W_r} = \frac{\sqrt{2}\,120/\sqrt{3}\ V}{80 \cdot 2\pi \frac{rad}{s}} = 0,1949\ Vs = 0,1949\ Wb$$

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4 - The inductances of the machine are equal.

- The torque when $i_q = 2A$ & $i_d = -1A$

$$T_e = \frac{3}{2} p (\lambda_d i_d - \lambda_q i_q)$$

$\Downarrow$ Inserting the expression of λ

$$T_e = \frac{3}{2} p \left(L_q i_q i_d - (L_d i_d + \lambda_{mpm}) i_q \right)$$

$\Downarrow$ $\rightarrow 0$ - $L_q = L_d$

$$T_e = \frac{3}{2} p \left(\lambda_{mpm} i_q + (\cancel{L_q - L_d}) i_d i_q \right)$$

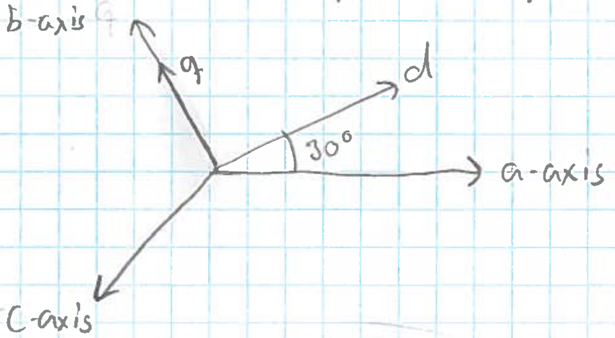
$\Downarrow$ Since this inductances are equal

$$T_e = \frac{3}{2} p \left(\lambda_{mpm} i_q \right)$$

$$= \frac{3}{2} \cdot 4 \cdot 0{,}1949\ Wb \cdot 2A = 2{,}3388\ VsA = 2{,}3388\ Nm$$

5 - Stator phase b - current

At $t = 0 \rightarrow i_q = 4A$, $i_d = -2A$, d lead a-axis by 30°.



$$\hat{i}_{dq} = (-2 + 4j) e^{j(\theta + 30°)}, \quad f_{dq} = (f_d + j f_q) e^{j\theta}$$

$$\hat{i}_b = Re\left(\frac{\hat{i}_{dq}}{e^{j120}} \right)$$

$$= Re(-2 + 4j) e^{j\theta 30 - 120}$$

$$= -2(\cos(\theta - 90) + j\sin(\theta - 90)) + 4j(\cos(\theta - 90) + j\sin(\theta - 90))$$

Re: $= -2\cos(\theta - 90) - 4\sin(\theta - 90)$

Thus

$$\hat{i}_b = -2\cos(\theta - 90) - 4\sin(\theta - 90)$$

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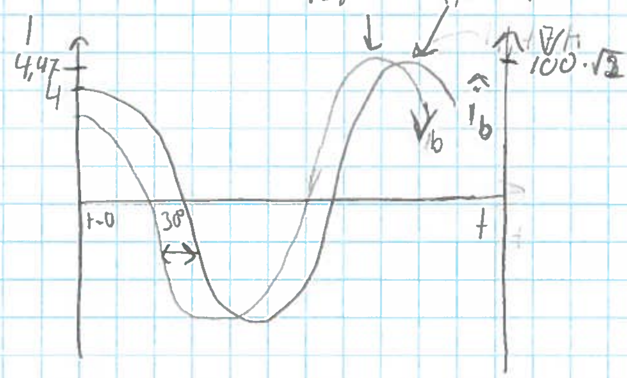
- i_b at $t=0$: (initial value)

  $i_b(t=0) = 4A$ - Since at this instance it is aligned with the q-axis.

- Peak value :

  $$|I_{dq}| = |-2+4j| = 4{,}47A$$

- Sketch $\hat{i}_b$:



- Power factor angle at this operating point is 0,866

- Phase RMS Voltage is 100V

  Since PF = 0,866 the power factor angle is:

  $$\phi = \cos^{-1}(PF) = \cos^{-1}(0{,}866) = 30°$$

  Thus the phase b voltage, V_b lead i_b with 30°.