Department of Energy Technology

AALBORG UNIVERSITET

Matlab and Control Theory — INTRO 1st semester 2011

January 13, 2012

Written exam

09.00-13.00 CET (4 hours)

The set consists of 12 problems

Rules

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions.
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off. Internet access is strictly forbidden.
- Any kind of communication with other students is not allowed.

Remember

1. To write your study number on all sheets handed in.
2. It must be clear from the solutions, which methods you are using, and you must include sufficient intermediate calculations, diagrams, sketches etc. so the line of thought is clear. Printing the final result is insufficient.

Problem 1 (5 %)

In Fig. 1 below is shown an electric filter.

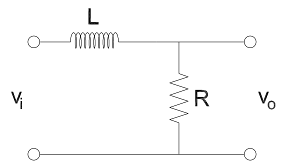

Figure 1: Filter, for which the transfer function $G(s) = \frac{v_o(s)}{v_i(s)}$ should be determined.

Determine the transfer function $G(s) = \frac{v_o(s)}{v_i(s)}$ relating the output voltage to the input voltage, when it is given that no current is drawn at the output and all initial conditions are zero.

Problem 2 (5 %)

The force in a solenoid is a function of the plunger position (x) and the current (i) in the solenoid and may be described by:

$$F(x,i) = c\frac{i^2}{x^2}$$

Where c is a solenoid specific constant (related to number of coil windings, airgap area and the permability of vacuum).

Set up a linearised expression for the solenoid force as a function of position and current, when linearising at the operating point (x_o, i_0), and determine the linearisation coefficients.

Problem 3 (15 %)

For a given system the transfer function is given by:

$$G(s) = \frac{2000}{s\left(s^2 + 20s + 100\right)}$$

1) Sketch asymptotically the Bode diagram (straight-line approximation) for the system. Notice that you may find a logarithmic coordinate system in the back of the exam set.

2) Write a piece of Matlab code, which plots the Bode diagram for the system (you do not need to execute the code or plot the results).

3) Determine analytically the gain of the system when the frequency is $\omega = 10$ [rad/s].

4) Determine the steady state output $c_{ss}(t)$ of the system (in the time domain) shown in the Fig. 2 below, when $r(t) = 4\cos(10t)$.

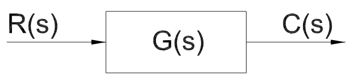

Figure 2:

Problem 4 (10 %)

A PI-controller should be designed (*using the frequency-response design method*) for the system shown in Fig. 3, where $G_c(s)$ represents the controller.

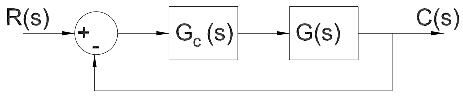

Figure 3: Block diagram for the system, for which the controller $G_c(s)$ should be designed.

The plant transfer function is given by:

$$G(s) = \frac{50}{(0.1s+1)\left(s^2+8s+25\right)}$$

It is required that the system obtain a phase margin of approximately $45°$. As a help the system open-loop Bode diagram (without) controller is shown below in Fig. 4.

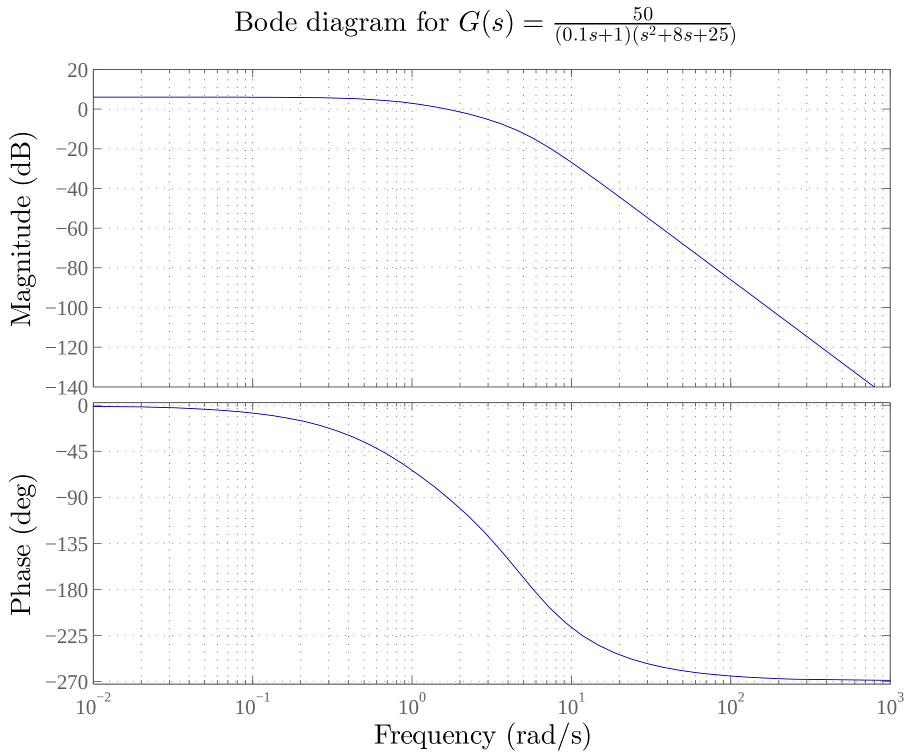

Figure 4: Open-loop Bode diagram for considered system (without controller).

Problem 5 (5 %)

A given system has the following open loop transfer function, where K is the gain:

$$KG(s) = K\frac{10(s+10)}{(s+1)(s^2+14s+100)}$$

Determine for what values of K the closed loop system is stable. As a help the root locus for the system is shown in Fig. 5.

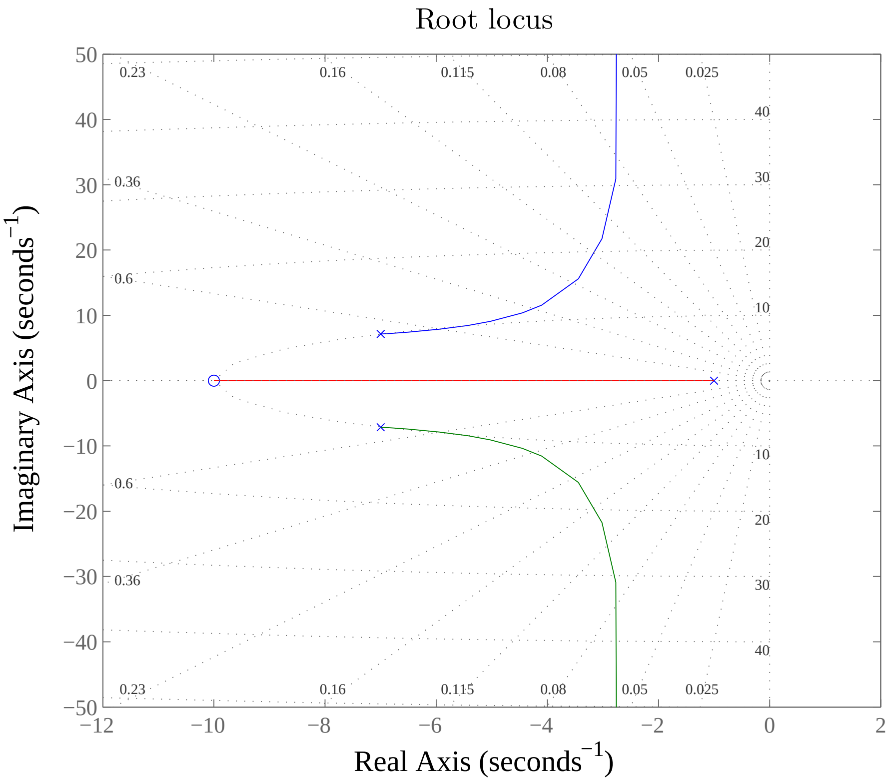

Figure 5: Root locus for the system.

Problem 6 (5 %)

The Z transform of a number sequence $\{e(k)\}$ is given by

$$E(z) = \frac{z^2+0.5}{z^2-0.5}$$

(a) Determine and sketch $e(k)$ for $0 \leq k \leq 4$.

(b) Calculate $e(\infty)$

Problem 7 (10 %)

The discrete transfer function for a digital controller is

$$H(z) = \frac{Y(z)}{U(z)} = \frac{K(z-0.3)z}{(z-0.7)(z-0.9)}$$

where K is a positive constant. The sampling angular frequency is $\omega_s = 10$ rad/s.

(a) Determine the controller gain K so that the DC gain for the controller $H(z)$ is equal to 10 dB

(b) Using the value for K determined in (a), find the gain for $H(z)$ at the Nyquist frequency

(c) Using $K = 1$, write a `Matlab` script with the commands needed to plot the frequency response. (You do not need to execute the commands or plot the result.)

Problem 8 (10 %)

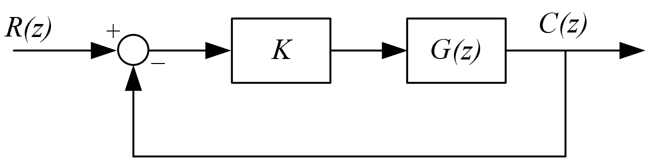

Figure 6:

The plant in Fig. 6 with unity feedback has the discrete transfer function $G(z)$

$$G(z) = \frac{z}{(z-0.8)^2}$$

The controller is a simple proportional gain K.

(a) Sketch (not necessarily to scale) the root locus for the system

(b) Determine the closed-loop transfer function $T(z) = C(z)/R(z)$

(c) Find the closed-loop poles for $K = 10$ and comment on the stability

Problem 9 (15 %)

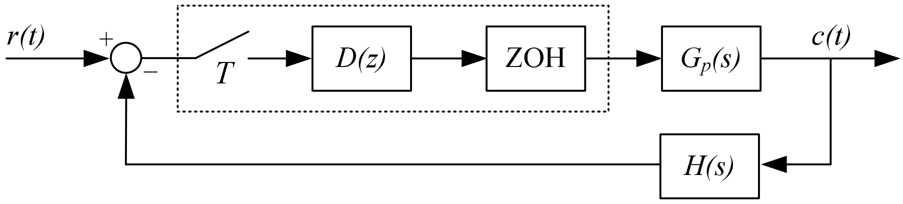

Figure 7:

A closed loop digital control system operating with the sampling frequency $F_s = 1/T = 20$ Hz is shown in Fig. 7. $G_p(s) = 6/(s+3)$ is the continuous plant transfer function. The feedback filter is given by $H(s) = 1$. The digital controller has the transfer function $D(z) = K/(z-1)$, where K is a constant.

(a) Determine an analytic expression for the open-loop gain given by $D(z)G(z)$, where $G(z)$ is the pulse transfer function related to the plant

(b) Is the closed-loop system stable for any positive gain values K ?

(c) Write the difference equation, which represents the controller $D(z)$

(d) How large is the phase lag caused by the zero-order hold (ZOH) at 5 Hz?

Problem 10 (4 %)

Write Matlab statements that will generate the following variables in the workspace:

$$
\begin{aligned}
a &= [4\ \ 5\ \ 6\ \cdots\ 99\ \ 100] \\
b &= [45\ \ 40\ \ 35\ \cdots\ -30\ \ -35\ \ -40] \\
c &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}
\end{aligned}
$$

Problem 11 (8 %)

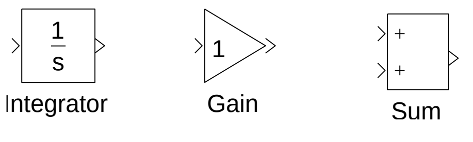

Figure 8: Simulink blocks.

Sketch the Simulink implementation of a SISO system using the basic blocks given in Fig. 8. The sum block may have more than two input ports. The system is described by the following differential equation:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + G\frac{\mathrm{d}y}{\mathrm{d}t} + Hy + Ux + V\frac{\mathrm{d}x}{\mathrm{d}t} = 0,$$

where x is input, y is output and G, H, U and V are constant non-zero gains.

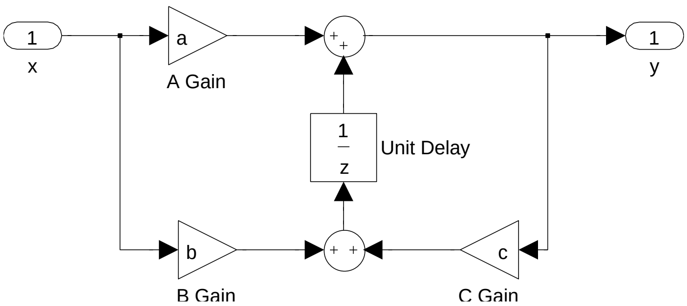

Figure 9: Discrete time system.

Problem 12 (8 %)

Find an expression for the transfer function $H(z) = \frac{Y(z)}{X(z)}$ of the discrete time system shown in Fig. 9, and determine where possible poles and zeros are located.

Logarithmic coordinate system for Problem 3