Opgaver lektion 2

3.1 $\underline{X}$: diskret P : diskret

Y : kontinuert Q : kontinuert

M : kontinuert

N : diskret ✓

3.5 a) $f(x) = c(x^2+4) \quad x = 0, 1, 2, 3$

$\sum_x f(x) = 1$ dvs. $c(0^2+4)+c(1^2+4)+c(2^2+4)+c(3^2+4) = 1$

$\Updownarrow$

$4c + 5c + 8c + 13c = 1$

$\Updownarrow$

$30c = 1$

$\Updownarrow$

$\underline{\underline{c = \frac{1}{30}}}$ ✓

b) $f(x) = c\binom{2}{x}\binom{3}{3-x} \quad x = 0, 1, 2$

$\sum_x f(x) = 1$ dvs. $c\binom{2}{0}\cdot\binom{3}{3} + c\binom{2}{1}\binom{3}{2} + c\binom{2}{2}\binom{3}{1} = 1$

$\Updownarrow$

$c \cdot 1 + c \cdot 2 \cdot 3 + c \cdot 1 \cdot 3 = 1$

$\Updownarrow$

$10c = 1$

$\Updownarrow$

$\underline{\underline{c = \frac{1}{10}}}$ ✓

3.7 $\underline{X}$: antal 100 timers brug af støvsuger (kontinuert)

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \le x < 2 \\ 0 & \text{ellers} \end{cases}$$ tæthedsfunktion

a) $P(\underline{X} \le 1{,}20) = \int_0^{1,2} f(t)\,dt = \int_0^1 t\,dt + \int_1^{1,2}(2-t)\,dt = \left[\frac{1}{2}t^2\right]_0^1 + \left[2t - \frac{1}{2}t^2\right]_1^{1,2}$

def 3.7

$= \frac{1}{2} + 2{,}4 - 0{,}72 - 2 + \frac{1}{2} = \underline{\underline{0.68}}$ ✓

b) $P(0.5 < \underline{X} \le 1) = \int_{0.5}^1 t\,dt = \left[\frac{1}{2}t^2\right]_{0.5}^1 = \frac{1}{2} - \frac{1}{8} = \frac{3}{8} = \underline{\underline{0{,}375}}$ ✓

2/5

Opgaver lektion 2

3.9 X : andel mennesker der reagerer på postordre reklame (kontinuert)

$$f(x) = \begin{cases} \frac{2(x+2)}{5} & 0<x<1 \\ 0 & \text{ellers} \end{cases}$$ tæthedsfunktion

a) VIS $P(0<X<1) = 1$

$$P(0<X<1) = \int_0^1 \frac{2(t+2)}{5}\,dt = \frac{2}{5}\left[\frac{1}{2}t^2+2t\right]_0^1 = \frac{2}{5}\left(\frac{1}{2}+2\right) = \underline{\underline{1}}$$ ✓

b) $$P\left(\frac{1}{4}<X<\frac{1}{2}\right) = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{2}{5}(t+2)\,dt = \frac{2}{5}\left[\frac{1}{2}t^2+2t\right]_{\frac{1}{4}}^{\frac{1}{2}} = \frac{2}{5}\left(\frac{1}{8}+1-\frac{1}{32}-\frac{1}{2}\right) = \underline{\underline{\frac{19}{80}}}$$ ✓

3.13 X : antal fejl pr. 10 m stof (diskret)

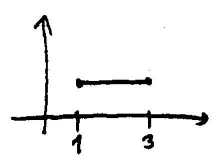
sandsynligheds funktion $\longrightarrow$ $f(x)$

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

Bestem fordelingsfunktionen:

$$F(x) = \begin{cases} 0 & x<0 \\ 0.41 & 0\le x<1 \\ 0.78 & 1\le x<2 \\ 0.94 & 2\le x<3 \\ 0.99 & 3\le x<4 \\ 1 & x\ge 4 \end{cases}$$ $(F(x) = P(X\le x))$ ✓

3.17 X kan antage værdier i $[1;3]$ kontinuert med tæthed $f(x) = \frac{1}{2}$



a) $$\int_1^3 \frac{1}{2}\,dt = \left[\frac{1}{2}t\right]_1^3 = \frac{3}{2}-\frac{1}{2} = \underline{\underline{1}}$$ ✓

b) $$P(2<X<2.5) = \int_2^{2.5} \frac{1}{2}\,dt = \frac{1}{2}[t]_2^{2.5} = \frac{1}{2}\cdot\frac{1}{2} = \underline{\underline{\frac{1}{4}}}$$ ✓

c) $$P(X<1.6) = \int_1^{1.6} \frac{1}{2}\,dt = \frac{1}{2}[t]_1^{1.6} = \frac{1}{2}\cdot 0.6 = \underline{\underline{0.3}}$$ ✓

Opgaver lektion 2

3.37 a)

$f(x,y) = c \cdot x \cdot y \quad x = 1,2,3 \quad y = 1,2,3$

$\sum_x \sum_y f(x,y) = 1$ dvs. $c\cdot1\cdot1 + c\cdot1\cdot2 + c\cdot1\cdot3 + c\cdot2\cdot1 + c\cdot2\cdot2 + c\cdot2\cdot3$

$+ c\cdot3\cdot1 + c\cdot3\cdot2 + c\cdot3\cdot3 = 1$

$\Updownarrow$

$6c + 12c + 18c = 1$

$\Updownarrow$

$\underline{\underline{c = \frac{1}{36}}}$ ✓

b)

$f(x,y) = c|x-y| \quad x = -2,0,2 \quad y = -2,3$

$\sum_x \sum_y f(x,y) = 1$ dvs $c(|-2+2| + |-2-3| + |0+2| + |0-3| + |2+2| + |2-3|) = 1$

$\Updownarrow$

$c(0+5+2+3+4+1) = 1$

$\Updownarrow$

$\underline{\underline{c = \frac{1}{15}}}$ ✓

3.38 $f(x,y) = \frac{(x+y)}{30} \quad x = 0,1,2,3 \quad y = 0,1,2$

a) $P(X \leq 2, Y = 1) = \sum_{x=0}^{2} \sum_{y=1}^{1} \frac{(x+y)}{30} = \sum_{x=0}^{2} \frac{x+1}{30} = \frac{1}{30} + \frac{2}{30} + \frac{3}{30} = \underline{\underline{\frac{1}{5}}}$

b) $P(X > 2, Y \leq 1) = \sum_{x=3}^{3} \sum_{y=0}^{1} \frac{(x+y)}{30} = \sum_{y=0}^{1} \frac{3+y}{30} = \frac{3}{30} + \frac{4}{30} = \underline{\underline{\frac{7}{30}}}$

c) $P(X > Y) = f(1,0) + f(2,1) + f(2,0) + f(3,1) + f(3,2) + f(3,0)$

$= \frac{1}{30} + \frac{3}{30} + \frac{2}{30} + \frac{4}{30} + \frac{5}{30} + \frac{3}{30} = \frac{18}{30} = \underline{\underline{\frac{3}{5}}}$

d) $P(X+Y=4) = f(2,2) + f(3,1) = \frac{4}{30} + \frac{4}{30} = \underline{\underline{\frac{4}{15}}}$

Opgaver lektion 2

3.42 $\underline{X}$: levetid af komponent 1

$\underline{Y}$: levetid af komponent 2

Hint: se Ex 3.14

$$f(x,y) = \begin{cases} e^{-(x+y)} & x>0,\ y>0 \\ 0 & \text{ellers} \end{cases}$$ simultan tæthed

$$P(0<X<1 \mid Y=2) = \int_0^1 f(x|2)\,dx$$

$$f(x|y) = \frac{f(x,y)}{h(y)} \quad \text{hvor} \quad \boxed{h(y)} = \int_0^\infty f(x,y)\,dx$$

$$= \int_0^\infty e^{-(x+y)}\,dx$$

$$= \left[-e^{-(x+y)}\right]_0^\infty$$

$$= 0 + e^{-y} = \boxed{e^{-y}}$$

dvs $$\boxed{f(x|y) = \begin{cases} e^{-x} & x>0,\ y>0 \\ 0 & \text{ellers} \end{cases}}$$

hermed: $$P(0<X<1 \mid Y=2) = \int_0^1 f(x|2)\,dx = \int_0^1 e^{-x}\,dx = \left[-e^{-x}\right]_0^1$$

$$= -e^{-1} + 1 = \underline{\underline{0{,}6321}}$$

Opgaver lektion 2

3.43 X: reaktionstid i sek.

Y: reaktionstemp i °F

$$f(x,y) = \begin{cases} 4xy & 0<x<1;\ 0<y<1 \\ 0 \end{cases}$$ simultan tæthed

a) $$P(0 \le X \le \tfrac{1}{2}, \tfrac{1}{4} \le Y \le \tfrac{1}{2}) = \int_{\frac{1}{4}}^{\frac{1}{2}}\int_{0}^{\frac{1}{2}} 4xy \, dx\,dy = \int_{\frac{1}{4}}^{\frac{1}{2}} 4y \int_{0}^{\frac{1}{2}} x\,dx\,dy$$

$$= \int_{\frac{1}{4}}^{\frac{1}{2}} 4y \left[\tfrac{1}{2}x^2\right]_0^{\frac{1}{2}} dy = \int_{\frac{1}{4}}^{\frac{1}{2}} \tfrac{1}{2}y\,dy = \left[\tfrac{1}{4}y^2\right]_{\frac{1}{4}}^{\frac{1}{2}} = \tfrac{1}{16} - \tfrac{1}{64} = \underline{\underline{\tfrac{3}{64}}}$$ ✓

b) $$P(X<Y) = \int_0^1\int_0^y 4xy\,dx\,dy = \int_0^1 4y \int_0^y x\,dx\,dy$$

$$= \int_0^1 4y\left[\tfrac{1}{2}x^2\right]_0^y dy = \int_0^1 2y\cdot(y^2-0)\,dy = \int_0^1 2y^3 = \left[\tfrac{1}{2}y^4\right]_0^1 = \underline{\underline{\tfrac{1}{2}}}$$ ✓

3.50

$f(x,y)$		$x=2$	$x=4$
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

$$g(x) = \sum_y f(x,y) = \begin{cases} 0.40 & x=2 \\ 0.60 & x=4 \end{cases}$$

$$h(y) = \sum_x f(x,y) = \begin{cases} 0.25 & y=1 \\ 0.50 & y=3 \\ 0.25 & y=5 \end{cases}$$

3.58

$$f(x,y) = \begin{cases} 6x & 0<x<1,\ 0<y<1-x \\ 0 & \text{ellers} \end{cases}$$ eller $0<x<1-y$

$$\boxed{\begin{aligned} h(y) &= \int_0^{1-y} 6x\,dx = \left[3x^2\right]_0^{1-y} \\ &= 3\cdot(1-y)^2 \\ f(x|y) &= \frac{f(x,y)}{h(y)} = \begin{cases} \frac{2x}{(1-y)^2} & 0<x<1-y \\ 0 & \text{ellers} \end{cases} \end{aligned}}$$

Da $f(x|y)$ mænhænger af y er X og Y ikke uafhængige

dvs

$$P(X>0.3 \mid Y=0.5) = \int_{0.3}^{1-0.5} \frac{2x}{(1-0.5)^2}dx = 8\int_{0.3}^{0.5} x\,dx = 8\left[\tfrac{1}{2}x^2\right]_{0.3}^{0.5} = 4\cdot 0.16 = \underline{0.64}$$

Opgaver lektion 3

4.12 X: bilforhandlers profit (enhed \$5.000)

tæthed
$$f(x) = \begin{cases} 2(1-x) & 0<x<1 \\ 0 & \text{ellers} \end{cases}$$

Forventet værdi X: $E(X) = \int_0^1 x\, 2(1-x)\, dx = \int_0^1 2x - 2x^2\, dx$

$$= \left[x^2 - \tfrac{2}{3}x^3\right]_0^1 = 1 - \tfrac{2}{3} = \tfrac{1}{3}$$

Forventet profit: $\frac{1}{3} \cdot 5.000\,\$ = \underline{\underline{1667\,\$}}$

4.22 $Y = X+4$ hvor X har tæthed $f(x) = \begin{cases} \frac{32}{(x+4)^3} & x>0 \\ 0 & \text{ellers} \end{cases}$

$E(Y) = E(X+4) = \int_0^\infty (x+4)\frac{32}{(x+4)^3}\, dx$

$$= 32\int_0^\infty \frac{1}{(x+4)^2}\, dx = 32\left[\frac{-1}{x+4}\right]_0^\infty = 0 - \left(-32\cdot\tfrac{1}{4}\right) = 8$$

dvs. gennemsnitlig hospitalsindlæggelse er $\underline{\underline{8 \text{ dage}}}$

4.23 X og Y har simultan sandsynlighedsfkt.

y \ x	2	4
1	0.1	0.15
3	0.2	0.3
5	0.1	0.15

$\rightarrow g(x) = \begin{cases} 0.4 & x= \\ 0.6 & x= \end{cases}$

$h(y) = \begin{cases} 0.25 & x=1 \\ 0.5 & x= \\ 0.25 & x= \end{cases}$

a) $E(XY^2) = \sum_x \sum_y x y^2 f(x,y) = 2(1^2 0,1 + 3^2 0,2 + 5^2 0,1)$
$+ 4(1^2 0,15 + 3^2 0,3 + 5^2 0,15)$
$= 8.8 + 26.4 = \underline{\underline{35.2}}$ ✓

b) $\mu_X = \sum x f(x) = 2\cdot 0.4 + 4\cdot 0.6 = \underline{3.2}$ ✓ $\quad \mu_Y = 0.25 + 3\cdot 0.5 + 5\cdot 0.25 = \underline{3}$ ✓ ($= \sum y f(y)$)