

Trigonometric Relations

$$\cos \theta = \cos(-\theta) = \sin(\pi/2 - \theta)$$

$$\sin \theta = -\sin(-\theta) = \cos(\pi/2 - \theta)$$

$$\tan \theta = -\tan(-\theta) = \cot(\pi/2 - \theta)$$

$$\sin 2\theta = 2 \cos \theta \sin \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = 2 \tan \theta / (1 - \tan^2 \theta)$$

$$\sin \theta/2 = \pm \sqrt{[(1 - \cos \theta)/2]}$$

$$\cos \theta/2 = \pm \sqrt{[(1 + \cos \theta)/2]}$$

$$\tan \theta/2 = \sin \theta / (1 + \cos \theta)$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\tan^2 \theta = (1 - \cos 2\theta) / (1 + \cos 2\theta)$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = (\tan A + \tan B) / (1 - \tan A \tan B)$$

$$\tan(A - B) = (\tan A - \tan B) / (1 + \tan A \tan B)$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\tan A + \tan B = \sin(A + B) / (\cos A \cos B)$$

$$\tan A - \tan B = \sin(A - B) / (\cos A \cos B)$$

$$\sin^2 A + \sin^2 B = 1 - \cos(A + B)\cos(A - B)$$

$$\sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)$$

$$\cos^2 A + \sin^2 B = 1 - \sin(A + B)\sin(A - B)$$

$$\cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B)$$

$$\cos^2 A + \cos^2 B = 1 + \cos(A + B)\cos(A - B)$$

$$\cos^2 A - \cos^2 B = -\sin(A + B)\sin(A - B)$$

For a triangle with sides a, b, c , and angles A, B, C opposite sides a, b and c respectively, the following relations hold.

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

$$a/\sin A = b/\sin B = c/\sin C.$$

$$(a - b)/(a + b) = \tan \frac{1}{2}(A - B) / \tan \frac{1}{2}(A + B)$$

$$\cos^2 x + \cos^2\left(x - \frac{2\pi}{3}\right) + \cos^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin^2 x + \sin^2\left(x - \frac{2\pi}{3}\right) + \sin^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin x \cos x + \sin\left(x - \frac{2\pi}{3}\right) \cos\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) \cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x + \sin\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) = 0$$

$$\cos x + \cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x \cos y + \sin\left(x - \frac{2\pi}{3}\right) \cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) \cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2} \sin(x - y)$$

$$\sin x \sin y + \sin\left(x - \frac{2\pi}{3}\right) \sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) \sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2} \cos(x - y)$$

$$\cos x \cos y + \cos\left(x - \frac{2\pi}{3}\right) \cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right) \cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2} \cos(x - y)$$

$$\sin x \cos y + \sin\left(x + \frac{2\pi}{3}\right) \cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right) \cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2} \sin(x + y)$$

$$\cos x \sin y + \cos\left(x + \frac{2\pi}{3}\right) \sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right) \sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2} \sin(x + y)$$

$$\sin x \sin y + \sin\left(x + \frac{2\pi}{3}\right) \sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right) \sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2} \cos(x + y)$$

$$\cos x \cos y + \cos\left(x + \frac{2\pi}{3}\right) \cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right) \cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2} \cos(x + y)$$