## Written examination in

# Dynamic Models of Electrical Machines

**Duration: 2 hours**

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- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off
- Internet access is strictly forbidden
- Any kind of communication with other students is not allowed
- Remember to write your study number on all answer sheets
- All intermediate steps and calculations should be included in your answer sheets --- printing the final result is insufficient

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The set consists of 3 problems

## Problem 1 (20%)

For a given space vector $\bar{f} = 10e^{-j\omega_e t}$, where $\omega_e = 2\pi \cdot 50$, please

(1) Find the expressions for its corresponding afa-, beta-components. Please draw their waveforms as functions of the time.

(2) Find the expressions for its corresponding a-, b-, c-components. Please draw phase-a waveform as a function of the time.

(3) Now you are given a dq-reference frame. At time t=0, its d-axis is aligned with phase-a axis. It rotates positively (anti-clockwise direction), at a speed of $\omega_e = 2\pi \cdot 50$. Please find the expressions for the dq-components when the original space vector $\bar{f} = 10e^{-j\omega_e t}$ is transformed to this dq-reference frame. Please draw the dq-component waveforms as functions of the time.

(4) Transform the following three-phase signals (where $\omega_e = 2\pi \cdot 50$ [rad/s])

$$v_a = V_{pk}\sin(\omega_e t), \qquad v_b = V_{pk}\sin\left(\omega_e t + \frac{2\pi}{3}\right), \qquad v_c = V_{pk}\sin\left(\omega_e t - \frac{2\pi}{3}\right)$$

to a stationary afa-bet reference frame. Then transform the afa-bet signals to a rotating dq-frame. This dq-frame is rotating positively (anti-clockwise direction) at a frequency of 50 Hz.
(*Remember to give the expressions of the transformed signals.*)

## Problem 2 (20%)

A sketch of a synchronous machine is shown below.

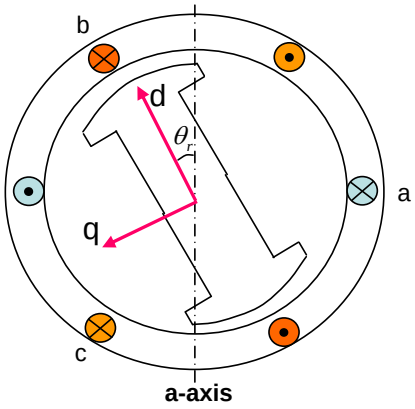


Fig. 1

(1) Please determine the mutual inductance between stator phase-a and stator phase-c.

(2) Please find the minimum and maximum values of this mutual inductance and explain at which position, the minimum and maximum values are achieved respectively?

A simple single-phase PM machine is shown below.

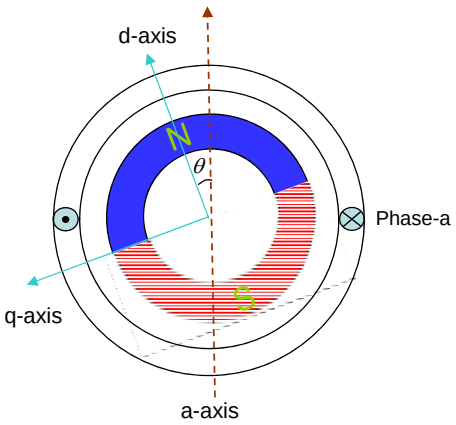


Fig. 2

(3) It is known that the PM flux linkage for phase-a is $\lambda_{pm,a} = \lambda_{m1}\cos(\theta)$. When phase-a is supplied with current $i_a = -I_{m1}\sin\theta$, what is the instantaneous torque? Please sketch this torque waveform.

Please answer the following

(4) In an unbalanced system, i.e. there exists zero-component of the voltage and current. Will this zero-component current produce any torque? and why?

**Problem 3 (10%)**

A sketch of an induction machine phase axes is given below.

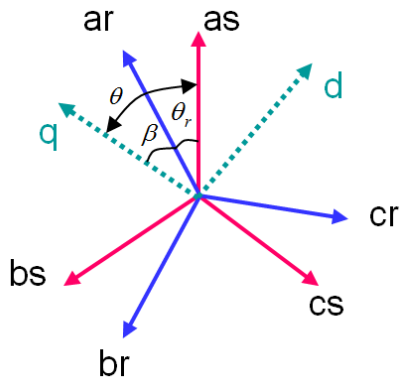


Fig. 3

*where notation 's' stands for stator phase axes and notation 'r' stands for rotor phase axis.*

Knowing the machine model expressed in an arbitrary qd-reference frame is

*Stator side voltage equations:*

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{0s} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} - \omega_\theta \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$

(1) Please transform this voltage equation into a vector form, using qd-frame space vector representations, i.e.

$\overline{f}_{qd} = f_q - jf_d$

( $f$ is a variable that could stand for the voltage or current.)

(2) Please give the stator voltage equation when used for steady state performance analysis.