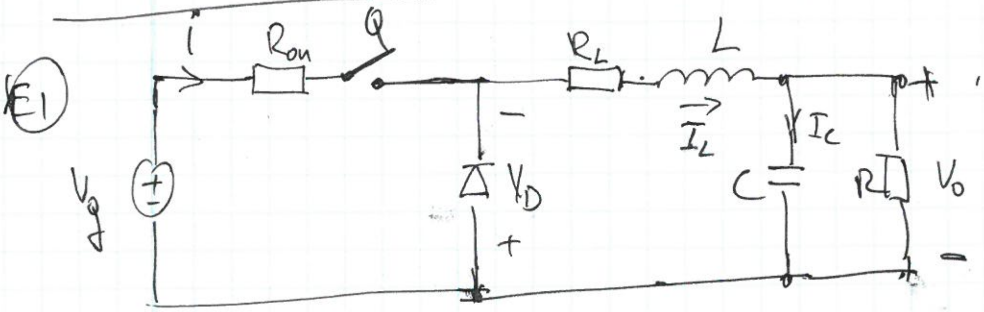
exercice solution

E1



$Q_A = 1$ $\quad (0 < t < DT_s)$

$$\begin{cases} V_L(t) = V_g(t) - I_L(t)(R_{on} + R_L) - V_o(t) \\ I_C(t) = I_L(t) - \frac{V_o(t)}{R} \\ I_g(t) = I_L(t) \end{cases}$$

$Q = 0$ $\quad (DT_s < t < T_s)$

$$\begin{cases} V_L(t) = -V_o(t) - V_D \mp I_L(t) R_L \\ I_C(t) = I_L(t) - \frac{V_o(t)}{R} \\ I_g(t) = 0 \end{cases}$$

CORRECTION: last term in inductor voltage expression should also be with **minus!** Accordingly, the inductor volt-second balance equations below need to be updated.

Averaging over one switching period

$$\langle V_L(t) \rangle_{T_s} = \left[\langle V_g(t) \rangle - \langle I_L(t) \rangle (R_{on} + R_L) - \langle V_o(t) \rangle \right] \cdot D$$

$$- \left[\langle V_o(t) \rangle + V_D - \langle I_L(t) \rangle \cdot R_L \right] \cdot D'$$

$$\langle I_C(t) \rangle_{T_s} = \langle I_L(t) \rangle - \frac{\langle V_o(t) \rangle}{R}$$

$$\langle I_g(t) \rangle_{T_s} = \langle I_L(t) \rangle \cdot D$$

E2

~~$\langle v_L(t) \rangle$~~

Inductor voltage perturbation & linearization

$$V_L + \hat{v}_L = \left[V_g + \hat{v}_g - (I_L + \hat{i}_L)(R_{on}+R_L) - V_0 - \hat{v}_0\right]\cdot(D+\hat{d}) -$$

$$- \left[V_0 + \hat{v}_0 + V_D - (I_L + \hat{i}_L)R_L\right]\cdot(D' - \hat{d}) =$$

$$= V_g \cdot D + \hat{v}_g \cdot D - I_L\cdot(R_{on}+R_L)\cdot D - \hat{i}_L(R_{on}+R_L)\cdot D - V_0\cdot D - \hat{v}_0\cdot D +$$

$$+ V_g\cdot\hat{d} + \cancel{\hat{v}_g\cdot\hat{d}} - I_L\cdot(R_L+R_{on})\cdot\hat{d} - \cancel{\hat{i}_L(R_{on}+R_L)\cdot\hat{d}} - V_0\cdot\hat{d} - \cancel{\hat{v}_0\cdot\hat{d}} -$$

$$- V_0\cdot D' - \hat{v}_0\cdot D' - V_D\cdot D' + I_L\cdot R_L\cdot D' + \hat{i}_L\cdot R_L\cdot D' +$$

$$+ V_0\cdot\hat{d} + \cancel{\hat{v}_0\cdot\hat{d}} + V_D\cdot\hat{d} - I_L\cdot R_L\cdot\hat{d} - \cancel{\hat{i}_L\cdot R_L\cdot\hat{d}}$$

DC:

$$0 = V_g\cdot D - I_L\cdot(R_{on}+R_L)\cdot D - \underbrace{V_0\cdot D - V_0\cdot D'}_{V_0} - V_D\cdot D' + I_L\cdot R_L\cdot D'$$

$$0 = V_g\cdot D - I_L(R_{on}+R_L)D - V_0 - V_D\cdot D' + I_L\cdot R_L\cdot D'$$

$$\boxed{V_0 = V_g\cdot D - I_L\cdot(R_{on}+R_L)\cdot D - V_D\cdot D' + I_L\cdot R_L\cdot D'}$$

$\Downarrow$

D (in steady state $I_L = I_R$)

E3

$$m_1 = \frac{\cancel{V_g}\ V_L \cdot \cancel{DT_s}}{L} = \frac{V_g - I_L(R_{on} + R_L) - V_0}{L}$$

$$-m_2 = \frac{-V_0 - V_D + I_L \cdot R_L}{L}$$

b)

inductor voltage ac component

$$\hat{v}_L = \hat{v}_g \cdot D - \hat{i}_L \cdot (R_{on} + R_L) D - \hat{v}_0 \cdot D + V_g \cdot \hat{d} - I_L (R_L + R_{on}) \hat{d} -$$

$$- V_0 \cdot \hat{d} - \hat{v}_0 \cdot D' + \hat{i}_L \cdot R_L \cdot D' + V_0 \cdot \hat{d} + V_D \cdot \hat{d} -$$

$$- I_L \cdot R_L \cdot \hat{d}$$

Capacitor current perturbation and linearization

$$I_C + \hat{i}_C = I_L + \hat{i}_L - \frac{V_0 + \hat{v}_0}{R}$$

$$DC: \quad I_C = I_L - \frac{V_0}{R} = 0 \Rightarrow I_L = \frac{V_0}{R}$$

$$AC: \quad \boxed{\hat{i}_C = \hat{i}_L - \frac{\hat{v}_0}{R}}$$

E4 Input current

$$\cancel{I_g}\; I_g + \hat{i}_g = (I_L + \hat{i}_L)(D + \hat{d})$$

$$\cancel{DC:}\; I_g + \hat{i}_g = \underline{I_L \cdot D} + \hat{i}_L \cdot D + I_L \cdot \hat{d} + \cancel{\hat{i}_L \cdot \hat{d}}$$

$$DC: \boxed{I_g = I_L \cdot D}$$

$$AC: \boxed{\hat{i}_g = \hat{i}_L \cdot D + I_L \cdot \hat{d}}$$

~~b)~~ **c)** $M_a = 0.5 \cdot M_2$

$$\alpha = -\frac{1 - \frac{m_a}{m_2}}{\frac{D'}{D} + \frac{m_a}{m_2}}$$ [12.27 from Erickson]

stability margin: $\alpha = 1$. for stability should be $\underline{\alpha < 1}$

~~should $\alpha < 1$~~

(E5) d.) From capacitor current equation

$$\boxed{\hat{i}_C = \hat{i}_L - \frac{\hat{V}_0}{R}}$$

in Laplace domain:

$$\hat{i}_C(s) = \hat{i}_L(s) - \frac{\hat{V}_0(s)}{R} \quad ; \quad \hat{i}_C(s) = s \cdot C \cdot \hat{V}_0(s)$$

$$s \cdot C \cdot \hat{V}_0(s) = \hat{i}_L(s) - \frac{\hat{V}_0(s)}{R}$$

$$s \cdot C \cdot \hat{V}_0(s) + \frac{\hat{V}_0(s)}{R} = \hat{i}_L(s) \Rightarrow \left(s \cdot C + \frac{1}{R}\right)\hat{V}_0(s) = \hat{i}_L(s)$$

simple approximation: $\hat{i}_L(s) \simeq i_{ctrl}(s)$

$$\boxed{G_{vc}(s) = \frac{\hat{V}_0(s)}{\hat{I}_{ctrl}(s)} = \frac{1}{sC + \frac{1}{R}}}$$

Control to output transfer function doesn't depend on loss elements.