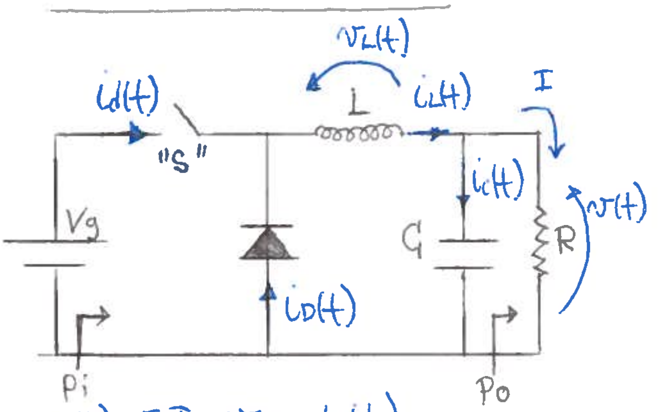
Buck Converter

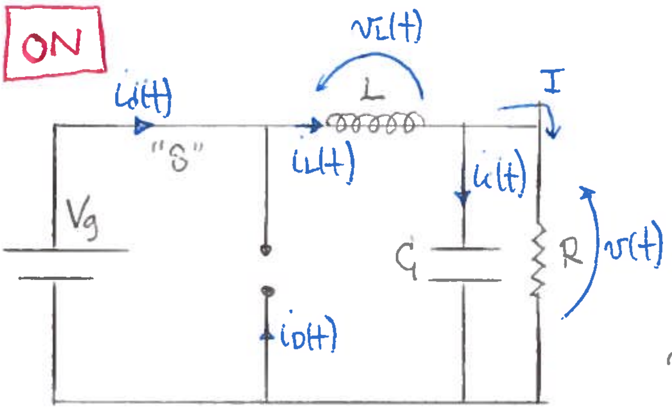


$v(t) = V + v_{ripple}(t)$

$|V| \gg |v_{ripple}| \Rightarrow \boxed{v(t) \approx V}$

$\boxed{I = \frac{V}{R}}$ $\boxed{P_i = P_o}$

ON

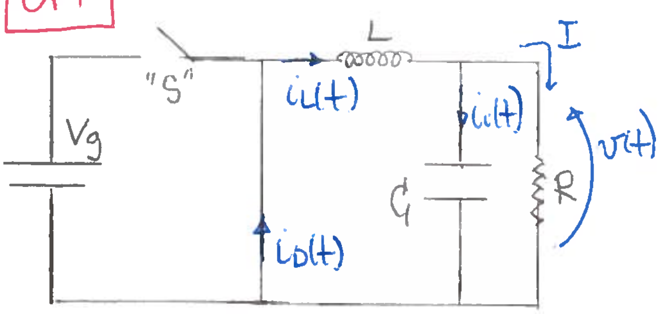


$\boxed{V_L = V_g - V}$

$$v_L(t) = L\frac{di_L(t)}{dt} \Rightarrow \boxed{\frac{di_L(t)}{dt} = \frac{V_g - V}{L}}$$

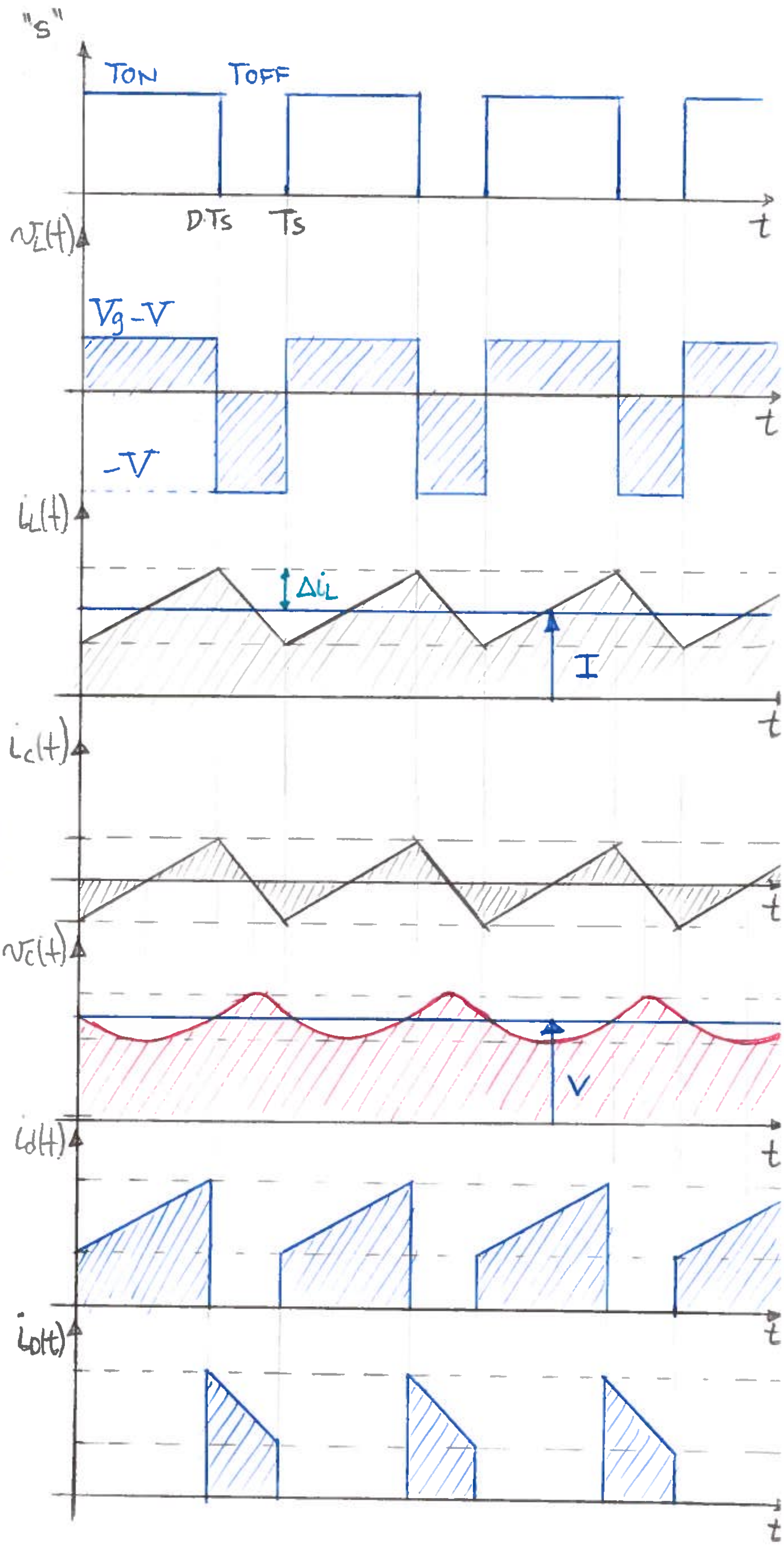
slope

OFF



$\boxed{V_L = -V}$

$$v_L(t) = L\frac{di_L(t)}{dt} \Rightarrow \boxed{\frac{di_L(t)}{dt} = \frac{-V}{L}}$$

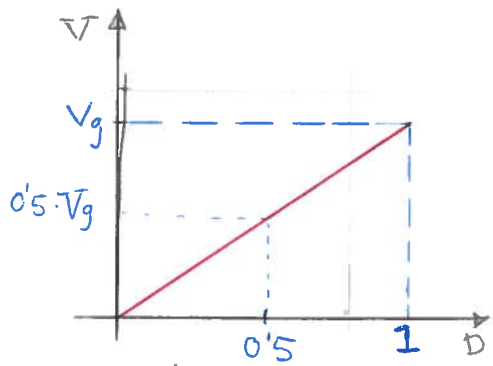


- Inductor volt-second balance:

$$V_{L,avg} = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = \emptyset \Rightarrow V_{L,avg} = \frac{1}{T_s}\left[\int_0^{D\cdot T_s}(V_g - V)\,dt + \int_{D\cdot T_s}^{T_s}(-V)\,dt\right] =$$

$$= \frac{1}{\cancel{T_s}}\left[(V_g - V)\,D\cdot\cancel{T_s} - V(\cancel{T_s} - D\cdot\cancel{T_s})\right] = V_g\cdot D - \cancel{V\cdot D} + \cancel{V\cdot D} - V = \emptyset$$

$$\Rightarrow \boxed{V = V_g\cdot D} \quad \text{where} \quad V = I\cdot R \qquad \frac{V}{V_g} = D$$



- By analizing the inductor current:

$$i_L(T_s) - i_L(0) = \frac{1}{L}\int_0^{T_s} v_L(t)\,dt = \emptyset \Rightarrow |\Delta i_{LON}| = |\Delta i_{LOFF}|$$

$$\left.\begin{aligned} |\Delta i_{LON}| &= \frac{V_g - V}{L}\cdot D\cdot T_s \\ |\Delta i_{LOFF}| &= \frac{V}{L}(1-D)T_s \end{aligned}\right\} \frac{V_g - V}{\cancel{L}}\cdot D\cdot\cancel{T_s} = \frac{V}{\cancel{L}}(1-D)\cdot\cancel{T_s} \Rightarrow V_g\cdot D - \cancel{V\cdot D} = V - \cancel{V\cdot D}$$

$$\Rightarrow \boxed{V = V_g\cdot D}$$

- Inductor current ripple:

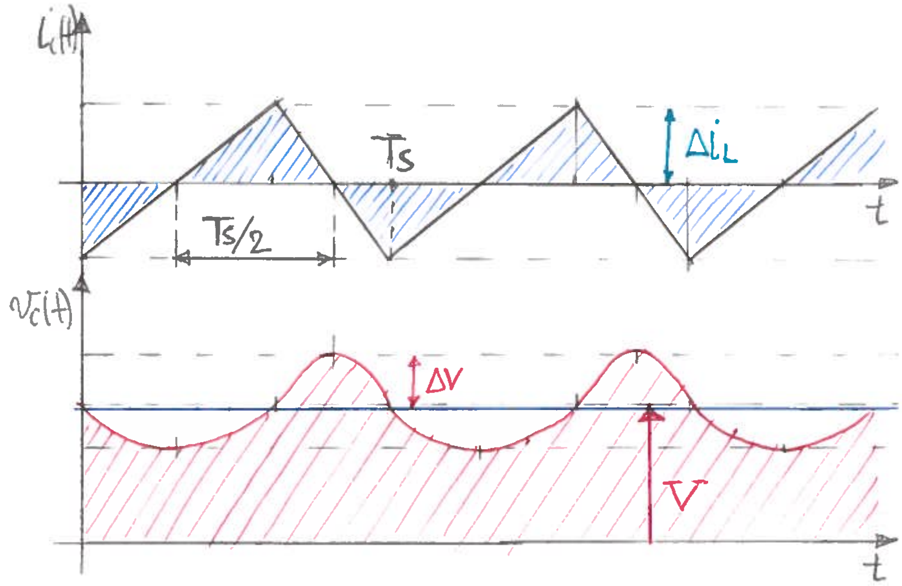
$$2\Delta i_L = \left(\frac{V_g - V}{L}\right)D\cdot T_s \Rightarrow \Delta i_L = \left(\frac{V_g - V}{2\cdot L}\right)D\cdot T_s = \frac{V_g(1-D)\cdot D\cdot T_s}{2\cdot L}$$

The inductor value can be choosen such that a desired current ripple Δi_L is obtained:

$$\boxed{L = \left(\frac{V_g - V}{2\cdot\Delta i_L}\right)D\cdot T_s = \frac{V_g(1-D)\cdot D\cdot T_s}{2\cdot\Delta i_L}}$$

- Output voltage ripple:

$$i_L(t) = I + i_{L\,ripple}(t)$$



$$Z_C\big|_{f=f_s} \lll Z_R\big|_{f=f_s}$$

$$\Downarrow$$

$$\boxed{i_{L\,ripple}(t) = i_C(t)}$$

$$Q = C \cdot V \Rightarrow \Delta Q = C \cdot 2 \cdot \Delta V$$

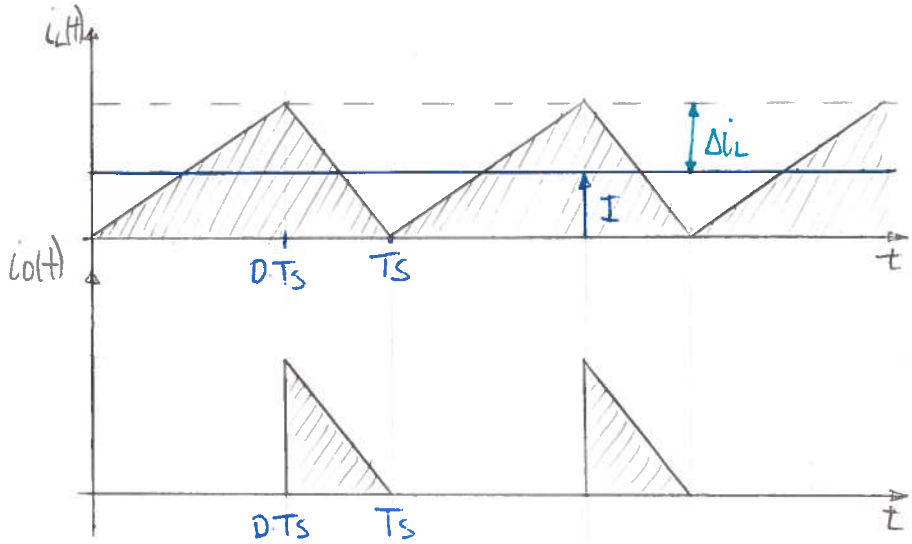
$$i(t) = \frac{dq(t)}{dt} \Rightarrow q(0 - T_S/2) = \int_0^{\frac{T_S}{2}} i(t)\,dt = \frac{1}{2} \cdot \Delta i_L \cdot \frac{T_S}{2} = \frac{\Delta i_L \cdot T_S}{4}$$

$$\Delta V = \frac{\Delta Q}{2 \cdot C} = \frac{\Delta i_L \cdot T_S}{8C}$$

The value for the capacitance C can be chooven such that a given voltage ripple ΔV is obtained:

$$\boxed{C = \frac{\Delta i_L \cdot T_S}{8 \cdot \Delta V}}$$

- Boundary mode between the discontinuous conduction mode and continuous conduction mode.



$I = \frac{V}{R} \Rightarrow$ if $R\uparrow\uparrow$ then $I\downarrow\downarrow$ but Δi_L remain the same. Eventually the boundary situation is reached:

$$\boxed{I_B = \Delta i_L = \left(\frac{V_g - V}{2L}\right) \cdot D \cdot T_S}$$

Hence:

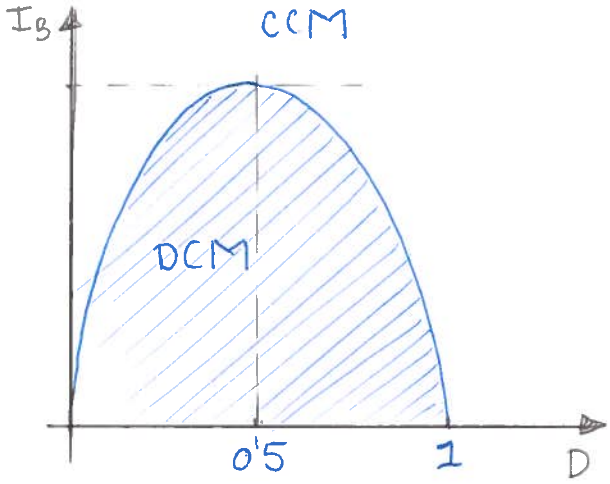
if $I > \Delta i_L$ the converter works in CCM

if $I < \Delta i_L$ the converter work in DCM

• If V_g is constant and V is variable

$$I_B = \left(\frac{V_g - V}{2 \cdot L}\right) \cdot D \cdot T_s \Rightarrow I_B = \left(\frac{V_g - D \cdot V_g}{2 \cdot L}\right) \cdot D \cdot T_s = \frac{V_g(1-D) \cdot D \cdot T_s}{2 \cdot L}$$

$$I_B = \frac{V_g \cdot T_s}{2 \cdot L}(D - D^2) \Rightarrow \frac{dI_B}{dD} = \frac{V_d \cdot T_s}{2 \cdot L}(1 - 2D) = \emptyset$$



$\Rightarrow$ There is a maximum $1 - 2D = \emptyset \Rightarrow D = 1/2$

$$\boxed{I_{Bmax} = \frac{V_g \cdot T_s}{8 \cdot L_s}}$$

The boundary current is a function of the duty cicle

For a given duty cicle the load R can increase or decrease

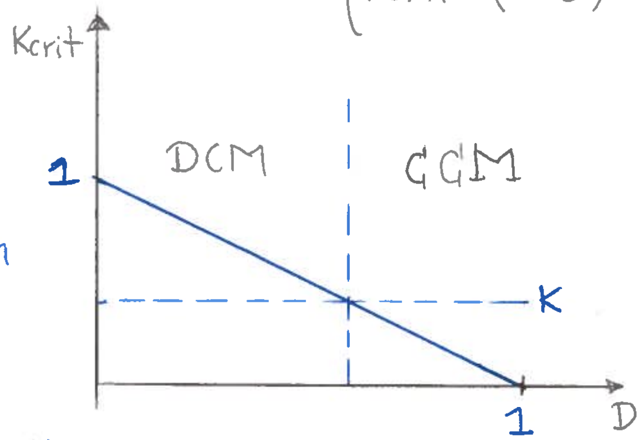
if $I < \Delta i_L \Rightarrow$ the converter is working in DCM

$$\frac{V}{R} < \left(\frac{V_g - V}{2L}\right) \cdot D \cdot T_s \Rightarrow \frac{V_g \cdot \cancel{D}}{R} < \left(\frac{\cancel{V_g} - \cancel{V_g} \cdot D}{2 \cdot L}\right) \cancel{D} \cdot T_s \Rightarrow \frac{1}{R} < \frac{(1-D)}{2 \cdot L} \cdot T_s$$

$$\Rightarrow \boxed{\frac{2 \cdot L}{T_s \cdot R} < (1 - D)} \Rightarrow K < K_{crit} \Rightarrow \begin{cases} K = \dfrac{2 \cdot L}{R \cdot T_s} \\ K_{crit} = (1 - D) \end{cases}$$

$$\boxed{\emptyset \leq K_{crit} \leq 1}$$

if $K > 1$ the converter is working allways in CCM



It is natural to express the boundary mode in terms of the load resistance R rather than the dimensionless parameter K:

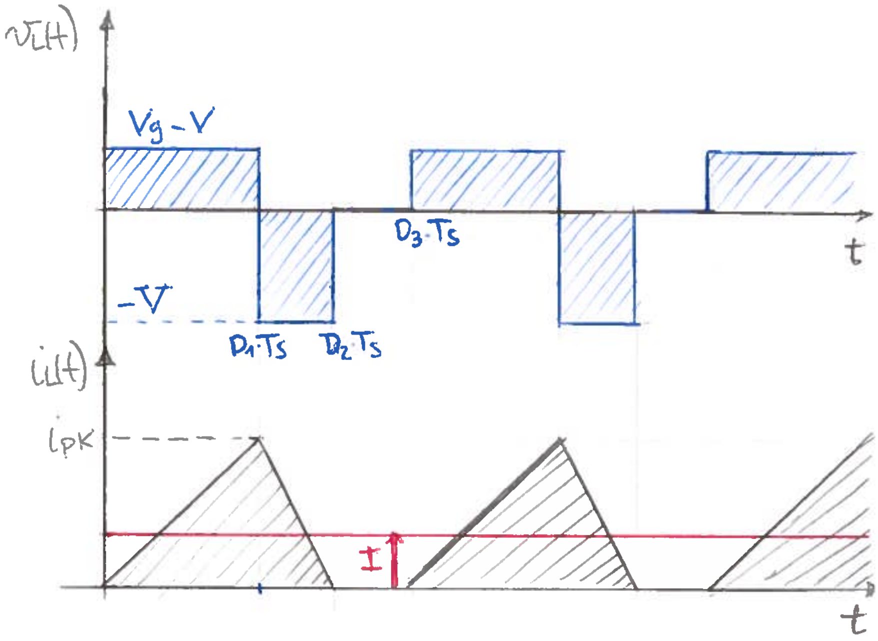
$$R > \frac{2 \cdot L}{(1-D) \cdot T_s} \Rightarrow \begin{cases} R > R_{crit}(D) \Rightarrow \text{the converter is working in DCM} \\ R < R_{crit}(D) \Rightarrow \text{the converter is working in CCM} \end{cases}$$

$$\boxed{\frac{2 \cdot L}{T_s} \leq R_{crit} \leq \infty}$$

• Conversion ratio when working in DCM.

inductor volt-second balance:

$$V_{L,avg} = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = \emptyset$$



$$V_{L,avg} = \frac{1}{T_s}\left[\int_0^{D_1 \cdot T_s}(V_g - V)\,dt + \int_0^{D_1 \cdot T_s}(-V)\,dt\right] =$$

$$= \frac{1}{\cancel{T_s}}\left[(V_g - V)\cdot D_1 \cdot \cancel{T_s} + (-V)\,D_2 \cdot \cancel{T_s}\right] = \emptyset$$

$$\Rightarrow V_g \cdot D_1 - V \cdot D_1 - V \cdot D_2 = V_g \cdot D_1 - V(D_1 + D_2) = \emptyset$$

$$\Rightarrow \boxed{V = V_g\left(\frac{D_1}{D_1 + D_2}\right)} \quad (1)$$

The average current through the inductor:

$$I = I_{L,avg} = \frac{1}{T_s}\int_0^{T_s} i_L(t)\,dt = \frac{1}{\cancel{T_s}}\left[\frac{1}{2}\cdot i_{pK}(D_1 + D_2)\cdot \cancel{T_s}\right] = \frac{(V_g - V)}{2 \cdot L}\cdot D_1 \cdot T_s (D_1 + D_2)$$

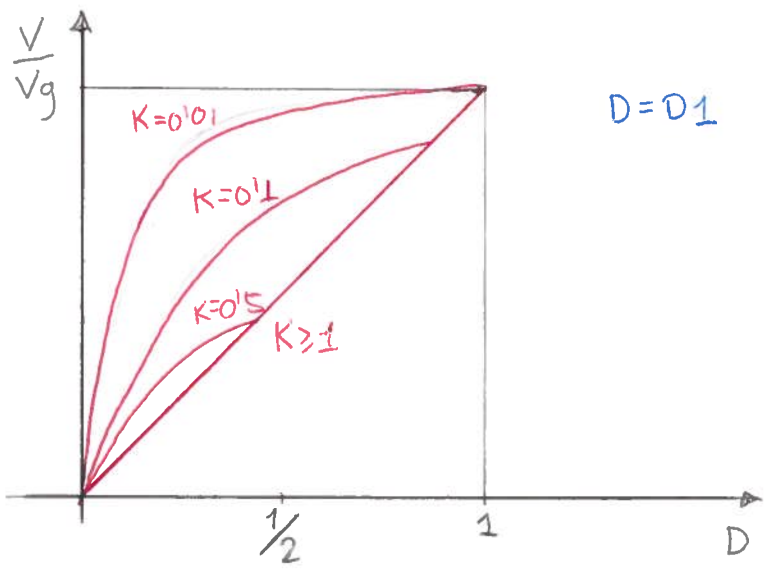
$$\boxed{\frac{V}{R} = \frac{D_1 \cdot T_s}{2 \cdot L}(D_1 + D_2)(V_g - V)} \quad (2)$$

By combining the equations (1) and (2) the following expression is obtained:

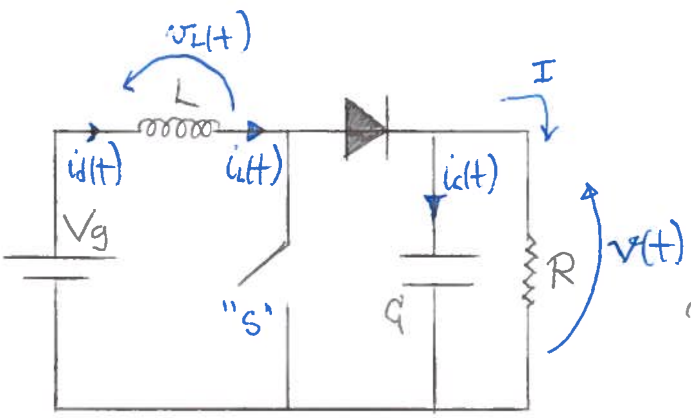
$$\boxed{\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + \frac{4K}{D_1^2}}}}$$ for DCM $K < K_{crit}$

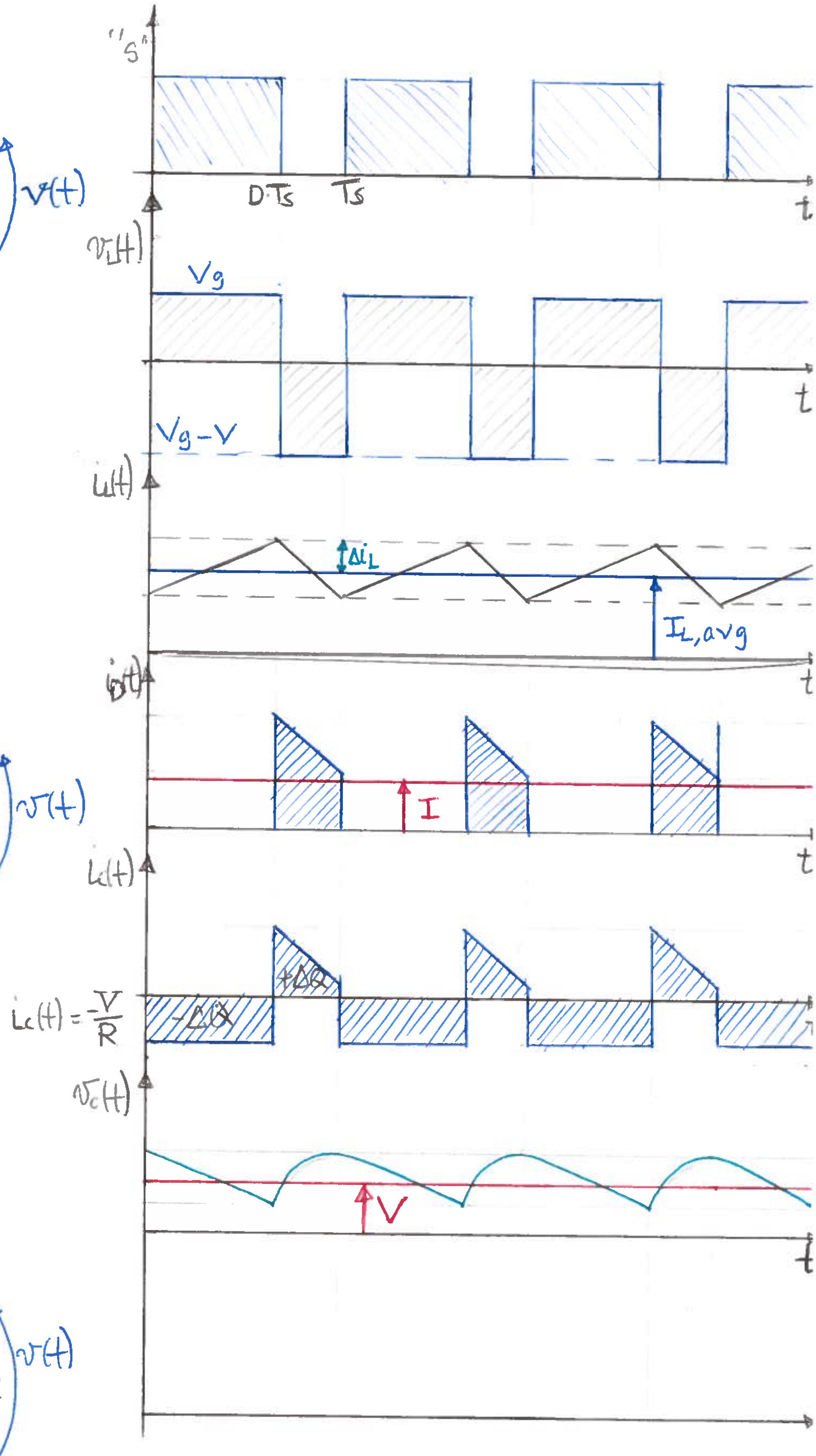
where $K = \dfrac{2 \cdot L}{R \cdot T_s}$

$$\boxed{\frac{V}{V_g} = D}$$ for CCM $K > K_{crit}$



Boost Converter



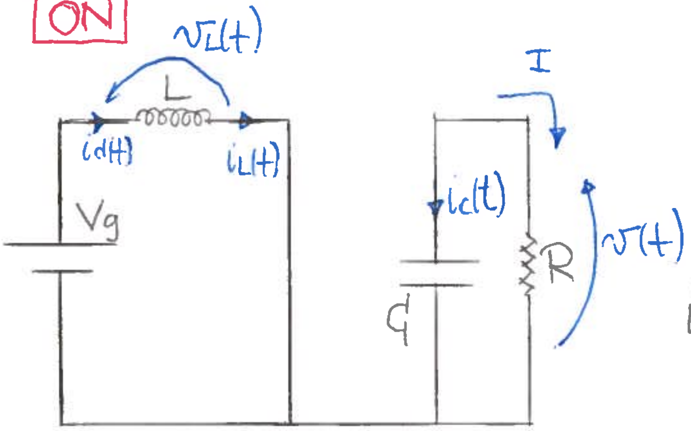


$v(t) = V + v_{ripple}(t)$

$|V| \gg |v_{ripple}| \Rightarrow v(t) = V$

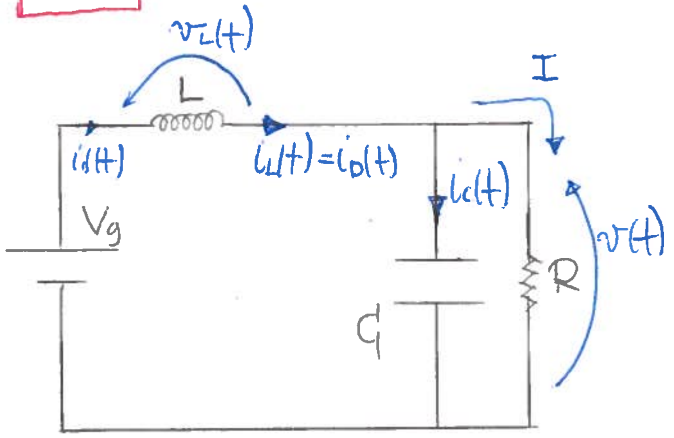
$I = \frac{V}{R}$

ON



$\boxed{V_L = V_g}$ $\boxed{\frac{di_L(t)}{dt} = \frac{V_g}{L}}$

OFF



$\boxed{V_L = V_g - V}$ $\boxed{\frac{di_L(t)}{dt} = \frac{V_g - V}{L}}$

$i_c(t) = i_L(t) - \frac{V}{R}$

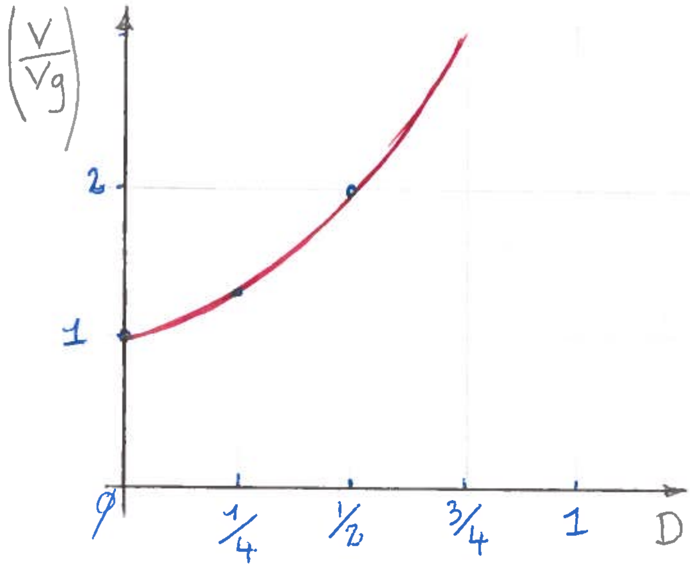
• Inductor recon-recond balance:

$$V_{L,avg} = \frac{1}{T_s}\int_{\emptyset}^{T_s} v_L(t)dt = \emptyset \Rightarrow V_{L,avg} = \frac{1}{T_s}\left[\int_{\emptyset}^{D\cdot T_s} V_g dt + \int_{D\cdot T_s}^{T_s} (V_g - V)\, dt = \right.$$

$$= \frac{1}{\cancel{T_s}}\left[V_g \cdot D \cdot \cancel{T_s} + (V_g - V)(1-D)\cancel{T_s}\right] = \cancel{V_g \cdot D} + V_g - \cancel{V_g \cdot D} - V + V \cdot D =$$

$$= V_g - V + V \cdot D = V_g + V(D-1) = \emptyset \Rightarrow V(D-1) = -V_g$$

$$\Rightarrow \boxed{\frac{V}{V_g} = \frac{1}{1-D}}$$



• By analizing the current:

$$|\Delta i_{LON}| = |\Delta i_{LOFF}|$$

$$|\Delta i_{LON}| = \frac{V_g}{L} \cdot D \cdot T_s$$

$$|\Delta i_{LOFF}| = \frac{V - V_g}{L}(1-D)\, T_s$$

$$\frac{V_g}{\cancel{L}} \cdot D \cdot \cancel{T_s} = \frac{V - V_g}{\cancel{L}}(1-D)\cancel{T_s} \Rightarrow V_g \cdot D = V - V_g(1-D)$$

$$\Rightarrow \cancel{V_g \cdot D} = V - V \cdot D - V_g + \cancel{V_g \cdot D} \Rightarrow V(1-D) = V_g \Rightarrow \boxed{\frac{V}{V_g} = \frac{1}{1-D}}$$

• Inductor current ripple:

$$2 \cdot \Delta i_L = \frac{V_g}{L} \cdot D \cdot T_s \Rightarrow \boxed{\Delta i_L = \frac{V_g}{2 \cdot L} \cdot D \cdot T_s}$$

• The inductor value:

$$\boxed{L = \frac{V_g}{2 \cdot \Delta i_L} \cdot D \cdot T_s}$$

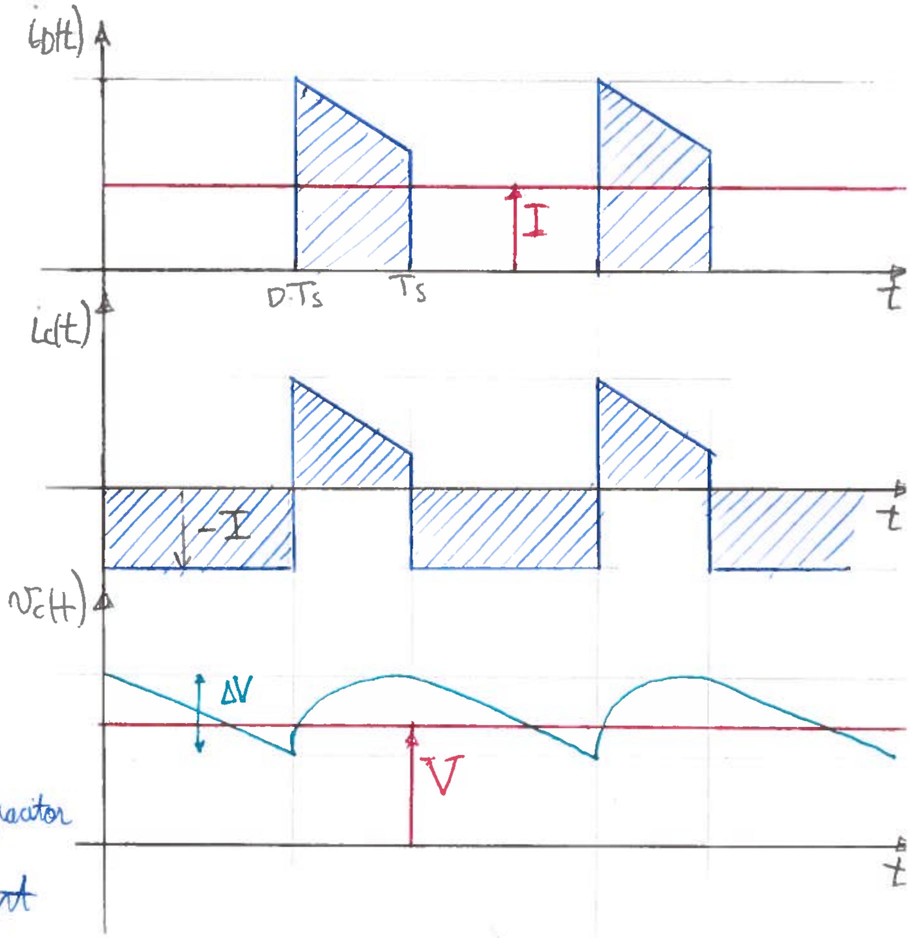
Output voltage ripple:

$$i_D(t) = I + i_C(t)$$

$$Q = C \cdot V \Rightarrow \Delta Q = C \cdot \Delta V$$

$$q(t) = \int_0^{D \cdot T_s} i(t)\,dt = \int_{\emptyset}^{D \cdot T_s} I \cdot dt =$$

$$= I \cdot D \cdot T_s$$



$$\Delta V = \frac{\Delta Q}{C} = \frac{I \cdot D \cdot T_s}{C} = \frac{V}{R} \cdot \frac{D \cdot T_s}{C}$$

$$\boxed{\Delta V = \frac{V}{RC} \cdot D \cdot T_s}$$ ⇒ can be used to select the capacitor C

The dc component of the inductor current is derived by using the principle of capacitor charge balance.

$$I_{c,avg} = \frac{1}{T_s}\int_0^{T_s} i_c(t)\,dt = \frac{1}{\cancel{T_s}}\left[-\frac{V}{R} \cdot D \cdot \cancel{T_s} + \left(I_L - \frac{V}{R}\right)(1-D)\cancel{T_s}\right] = \emptyset$$

$$-\frac{V}{R} \cdot D + \left(I_L - \frac{V}{R}\right)(1-D) = \emptyset \Rightarrow -\cancel{\frac{V}{R} D} + I_L - I_L D - \frac{V}{R} + \cancel{\frac{V}{R} D} = \emptyset$$

$$\Rightarrow I_L(1-D) = \frac{V}{R} \Rightarrow \boxed{I_L = \frac{V}{(1-D) \cdot R}}$$

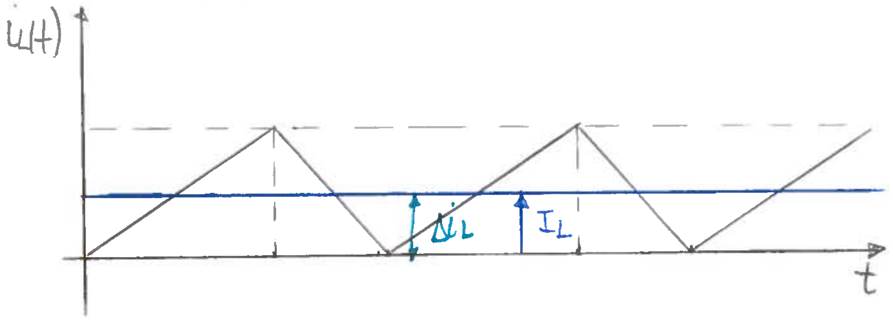
this is obtained using the following approximation

$$v(t) = V$$
$$i_L(t) = I_L$$

$$\boxed{I_L = \frac{V_g}{(1-D)^2 \cdot R}}$$

• Boundary mode between the DCM and CCM

$$\boxed{I_B = \Delta i_L = \frac{V_g}{2 \cdot L} D \cdot T_s}$$



Hence:

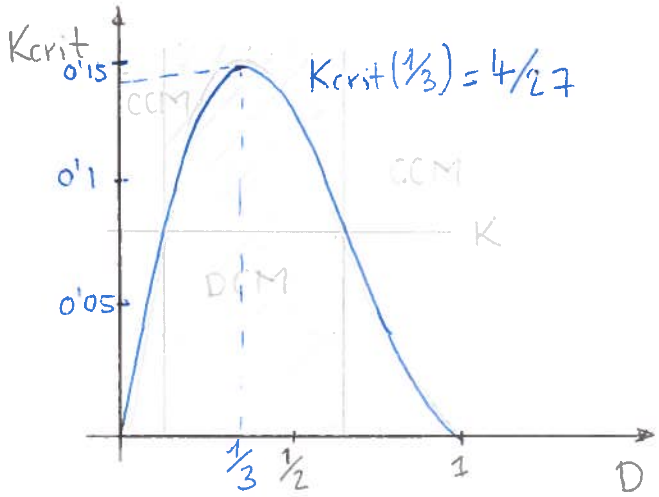
if $I_L > \Delta i_L$ the converter works in CCM

if $I_L < \Delta i_L$ the converter works in DCM

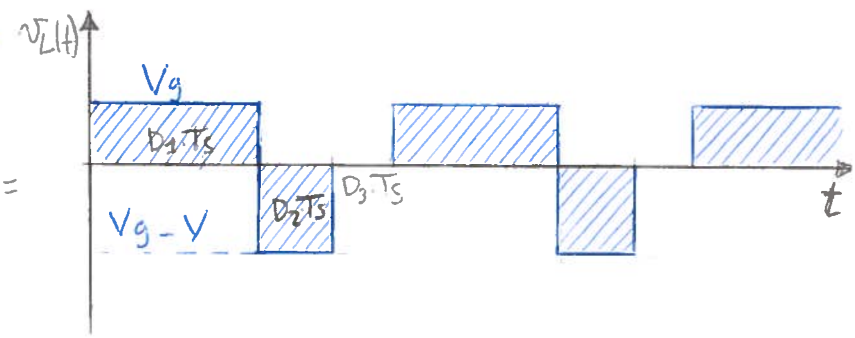
$$\frac{\cancel{V_g}}{(1-D)^2 \cdot R} > \frac{\cancel{V_g}}{2 \cdot L} \cdot D \cdot T_s \Rightarrow \frac{2 \cdot L}{R \cdot T_s} > D(1-D)^2 \Rightarrow K > K_{crit} \Rightarrow \begin{cases} K = \dfrac{2 \cdot L}{R \cdot T_s} \\ K_{crit} = D(1-D)^2 \end{cases}$$

$$K_{crit} = D(1 - 2D + D^2) = D^3 - 2D^2 + D$$

$$\frac{d K_{crit}}{d D} = 3D^2 - 4D + 1 = \emptyset \Rightarrow D = \frac{4 \pm \sqrt{16-12}}{2 \cdot 3} = \frac{4 \pm 2}{6} \rightarrow D_1 = 1; \quad D_2 = 1/3$$



Conversion ratio when working in DCM



inductor volt-second balance:

$$V_{L,avg} = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = \frac{1}{T_s}\left[\int_0^{D_1 \cdot T_s} V_g\,dt \int_0^{D_2 \cdot T_s} (V_g - V)\,dt\right] =$$

$$= \frac{1}{\cancel{T_s}}\left[V_g \cdot D_1 \cdot \cancel{T_s} + (V_g - V) D_2 \cdot \cancel{T_s}\right] =$$

$$= V_g \cdot D_1 + V_g \cdot D_2 - V \cdot D_2 = V_g(D_1 + D_2) - V D_2 = \emptyset \Rightarrow \boxed{V = V_g\left(\frac{D_1 + D_2}{D_2}\right)} \quad (1)$$

The diode current will be:

$$i_D(t) = i_C(t) + \frac{v(t)}{R}$$

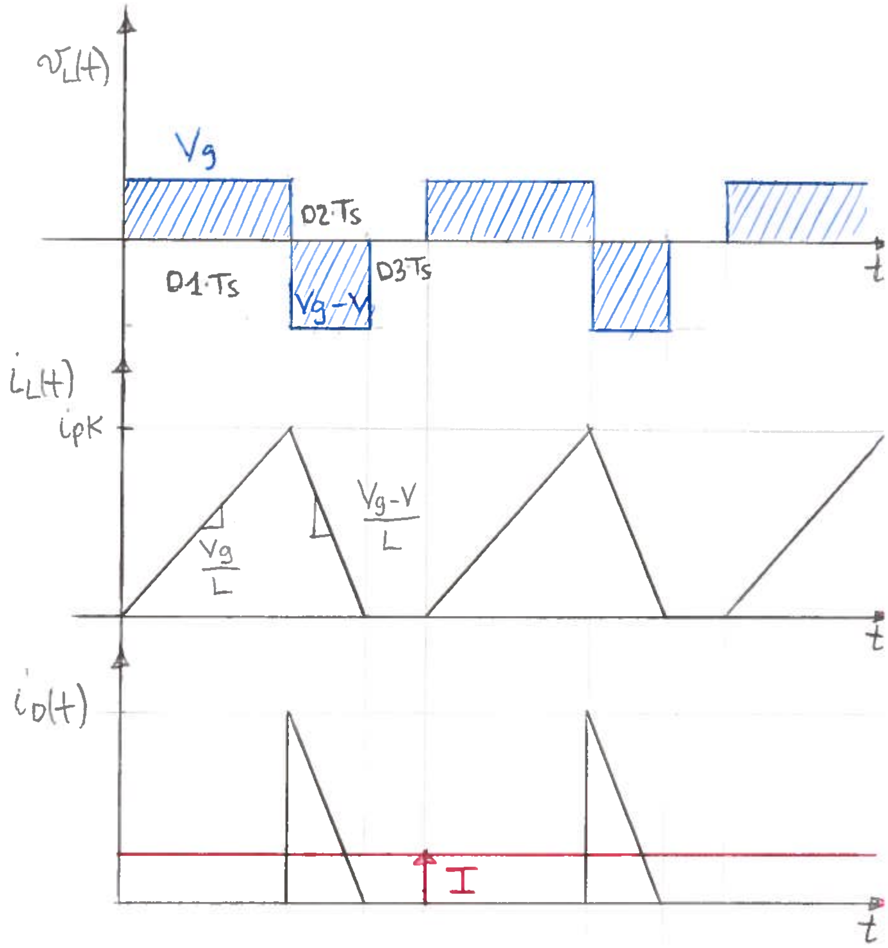
$$I = \frac{V}{R} = i_{D,avg}$$

$$I = \frac{1}{T_s}\int_0^{T_s} i_D(t)\,dt =$$

$$= \frac{1}{T_s}\left[\frac{1}{2}\, i_{pK}\cdot D_2 T_s\right] =$$

$$= \frac{1}{2}\, i_{pK}\cdot D_2 = \frac{1}{2}\left(\frac{V_g\cdot D_1 T_s}{L}\right) D_2$$

$$\boxed{I = \frac{V}{R} = \frac{V_g\cdot D_1\cdot D_2\cdot T_s}{2\cdot L}} \quad (2)$$

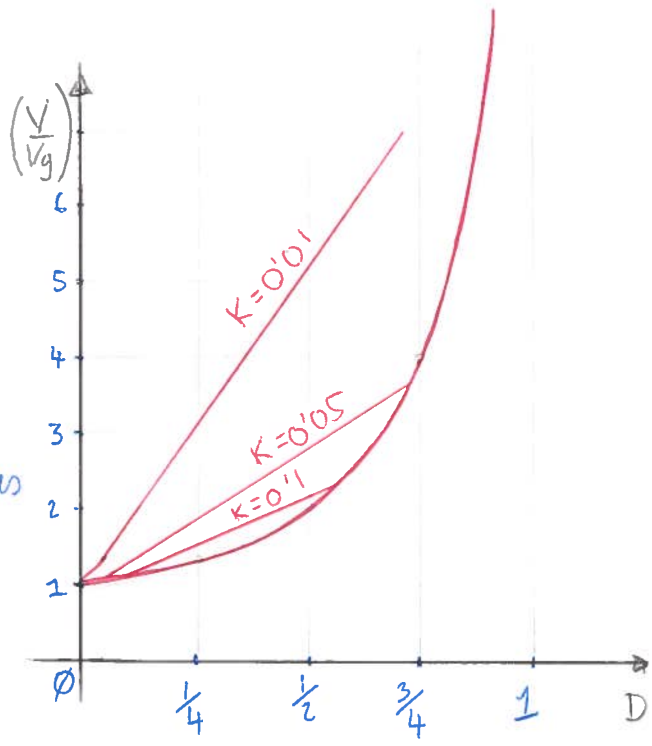


By combining (1) and (2) the following expression is obtained:

$$\boxed{\frac{V}{V_g} = \frac{1+\sqrt{1+\frac{4D_1^2}{K}}}{2}}$$

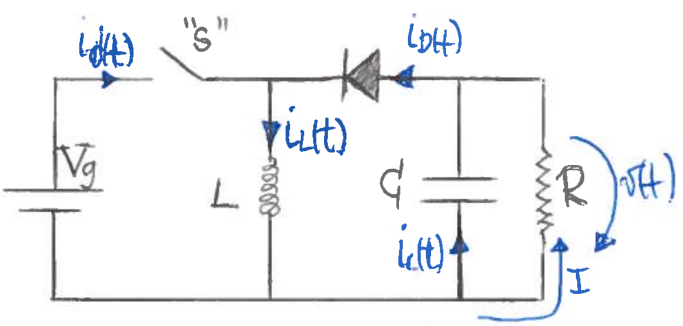
where $K = \frac{2L}{R\,T_s}$

$$M = \frac{V}{V_g}\begin{cases} \dfrac{1}{1-D} & \text{for } K > K_{crit} \\[2ex] \dfrac{1+\sqrt{1+\frac{4D_1^2}{K}}}{2} & \text{for } K < K_{crit} \end{cases}$$



As in the buck converter, the effect of the discontinuous mode is to cause the output voltage to increase.

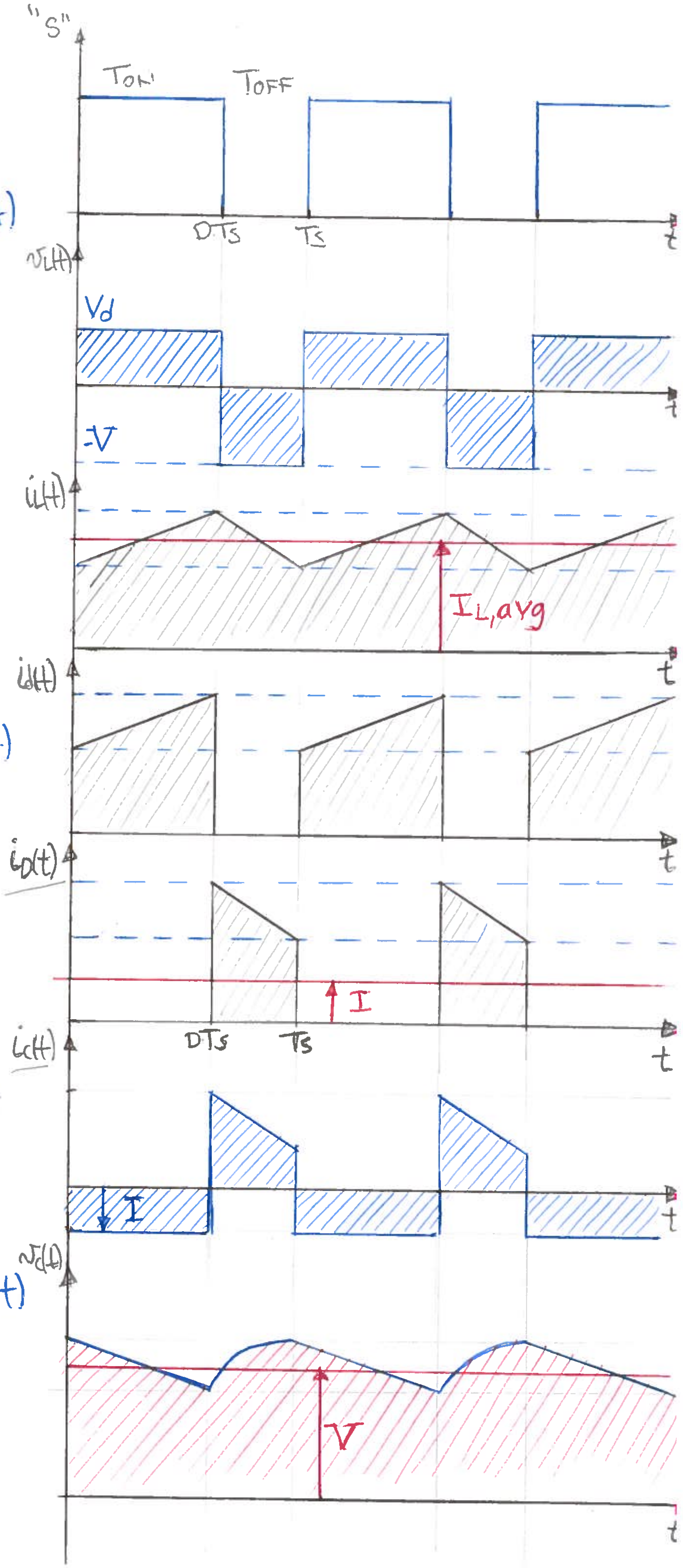
Buck-Boost Converter



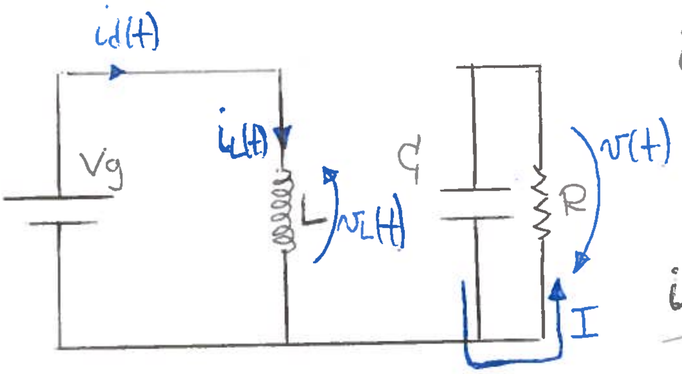
$$v(t) = V + v_{ripple}(t)$$

$$|V| \gg |v_{ripple}| \Rightarrow v(t) = V$$

$$I = \frac{V}{R} = (1-D)\cdot I_{L,avg}$$



ON

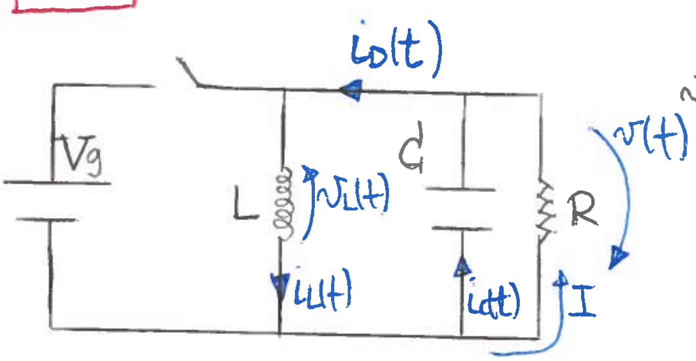


$$V_L = V_g$$

$$i_d(t) = i_L(t)$$

$$v_L(t) = L \cdot \frac{di_L(t)}{dt} \Rightarrow \frac{di_L(t)}{dt} = \frac{V_g}{L}$$

OFF



$$V_L = -V$$

$$i_D(t) = i_L(t)$$

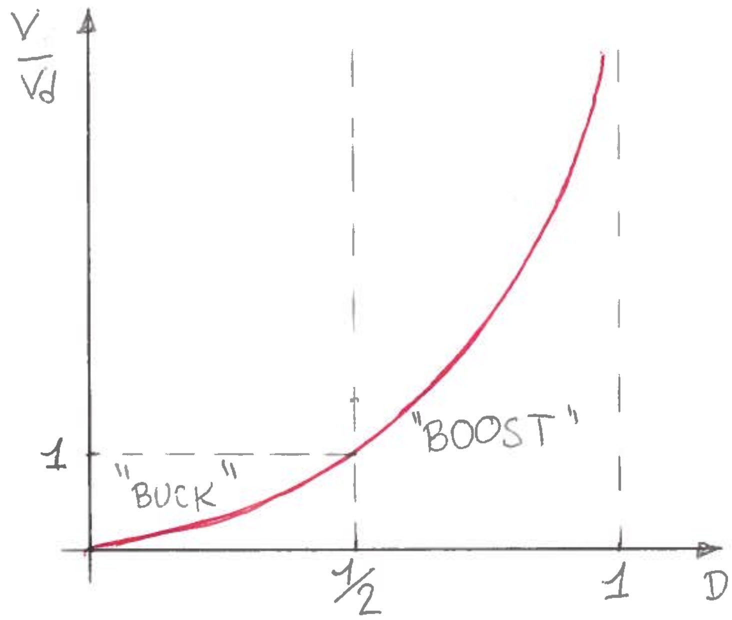
$$\frac{di_L(t)}{dt} = \frac{-V}{L}$$

- Inductor volt-second balance

$$V_{L,avg} = \frac{1}{T_s}\int_0^{T_s} v_L(t) = \emptyset \Rightarrow V_{L,avg} = \frac{1}{T_s}\left[\int_0^{D\cdot T_s} V_g \cdot dt + \int_{D\cdot T_s}^{T_s} -V \cdot dt\right] =$$

$$= \frac{1}{\cancel{T_s}}\left[V_g\,(D\cdot \cancel{T_s}) - V(1-D)\cancel{T_s}\right] = V_g \cdot D - V + VD = \emptyset$$

$$\Rightarrow \boxed{V = -V_g\left(\frac{D}{1-D}\right)}$$



- By analyzing the inductor current:

$$|\Delta i_{LON}| = |\Delta i_{LOFF}|$$

$$\left.\begin{aligned} |\Delta i_{LON}| &= \frac{V_g}{L}\cdot D\cdot T_s \\ |\Delta i_{LOFF}| &= \frac{V}{L}(1-D)\cdot T_s \end{aligned}\right\}$$

$$\frac{V_g}{\cancel{L}}\cdot D\cdot\cancel{T_s} = \frac{V}{\cancel{L}}(1-D)\cancel{T_s} \Rightarrow V_g\cdot D = V - V\cdot D \Rightarrow \boxed{V = V_g\left(\frac{D}{1-D}\right)}$$

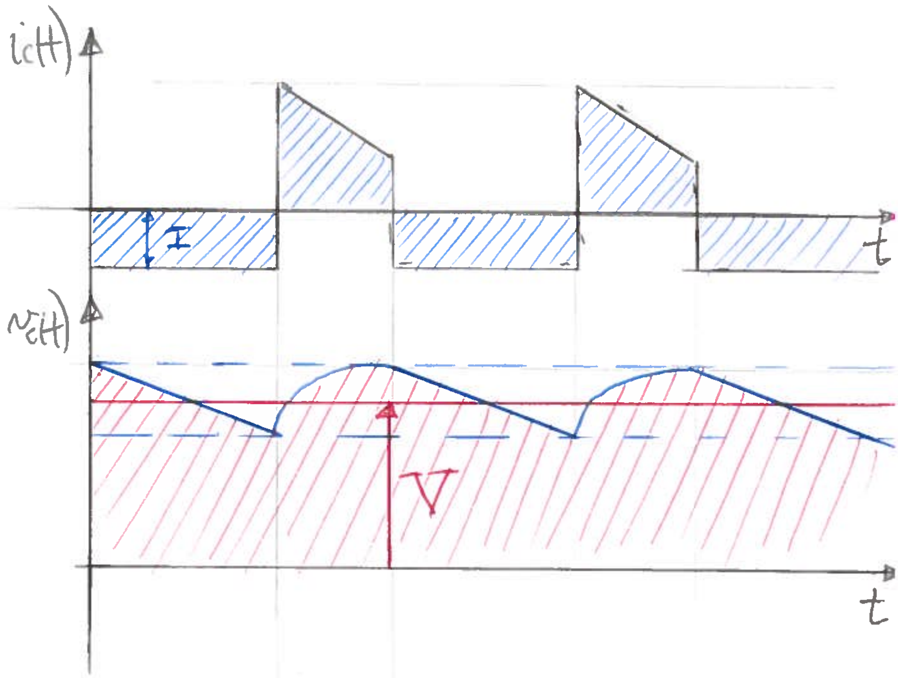
- Inductor current ripple:

$$2\cdot\Delta i_L = \frac{V_g}{L}\cdot D\cdot T_s \Rightarrow \Delta i_L = \frac{V_g}{2\cdot L}\cdot D\cdot T_s$$

The inductor value:

$$\boxed{L = \frac{V}{2\cdot\Delta i_L}\cdot D\cdot T_s}$$

· Output voltage ripple:



$$i_D(t) = i_c(t) + I$$

$$Q = C_1 \cdot V \Rightarrow \Delta Q = C_1 \cdot \Delta V$$

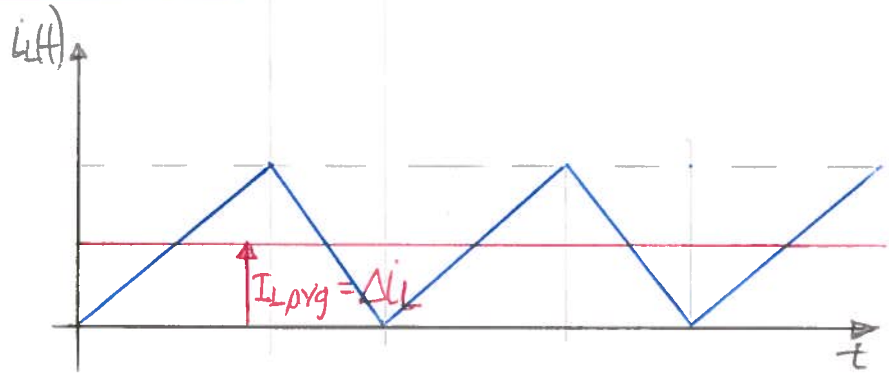
$$Q = I \cdot D \cdot T_s$$

$$\Delta V = \frac{\Delta Q}{C_1} = \frac{I \cdot D \cdot T_s}{C_1} = \frac{V}{R} \cdot \frac{D \cdot T_s}{C_1}$$

$$\boxed{\Delta V = \frac{V}{R \cdot C_1} \cdot D \cdot T_s}$$

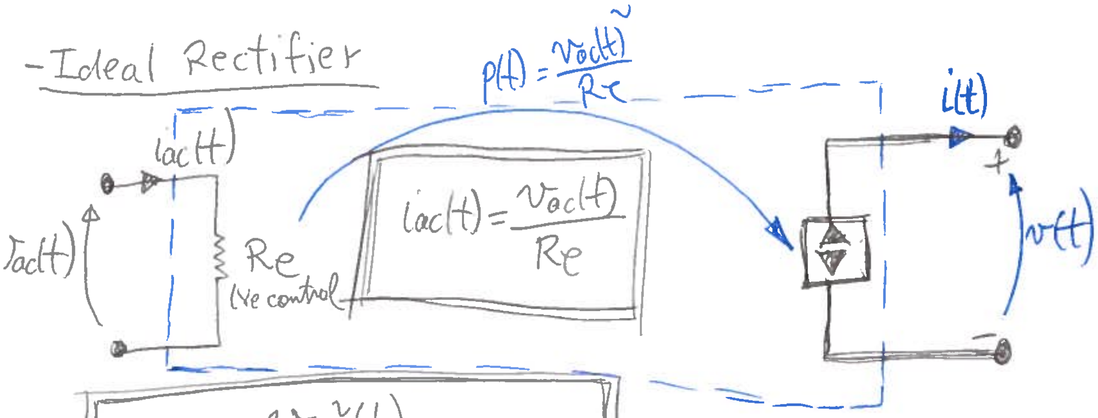
· Boundary mode between the DCM and CCM.

$$I_B = \Delta i_L = \frac{V_g}{2L} \cdot D \cdot T_s$$



LECTURE 2

- Ideal Rectifier



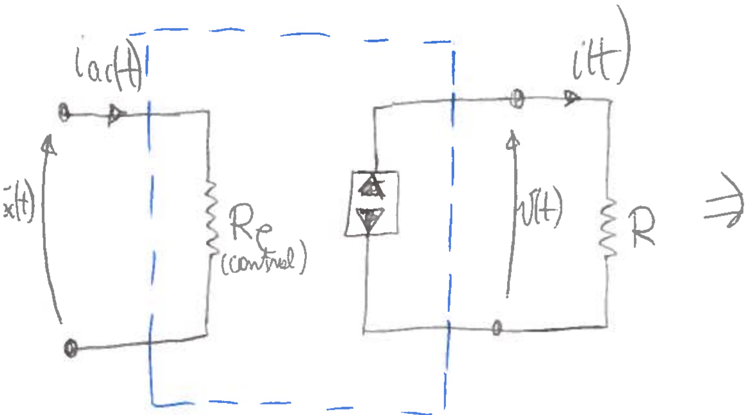
$$\boxed{P_{Re}(t) = \frac{v_{ac}^2(t)}{Re(v_{control}(t))}}$$

LFR ⇒ loss free resistor

$p(t) = v(t) \cdot i(t)$

$i_{ac}(t) = \frac{v_{ac}(t)}{Re}$

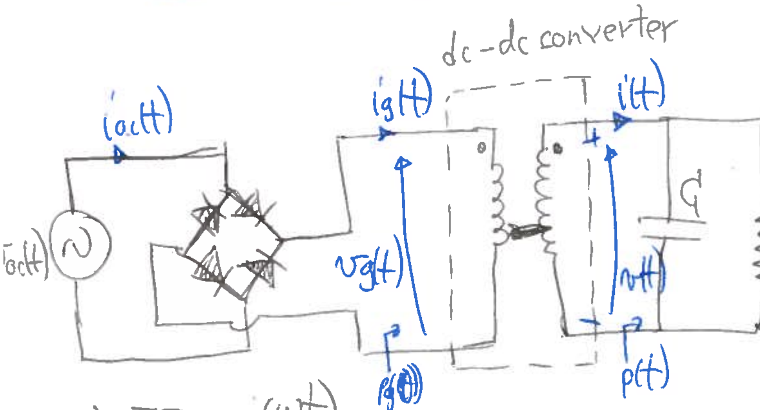
$$\boxed{p(t) = \frac{v_{ac}^2(t)}{Re[v_{control}(t)]}}$$

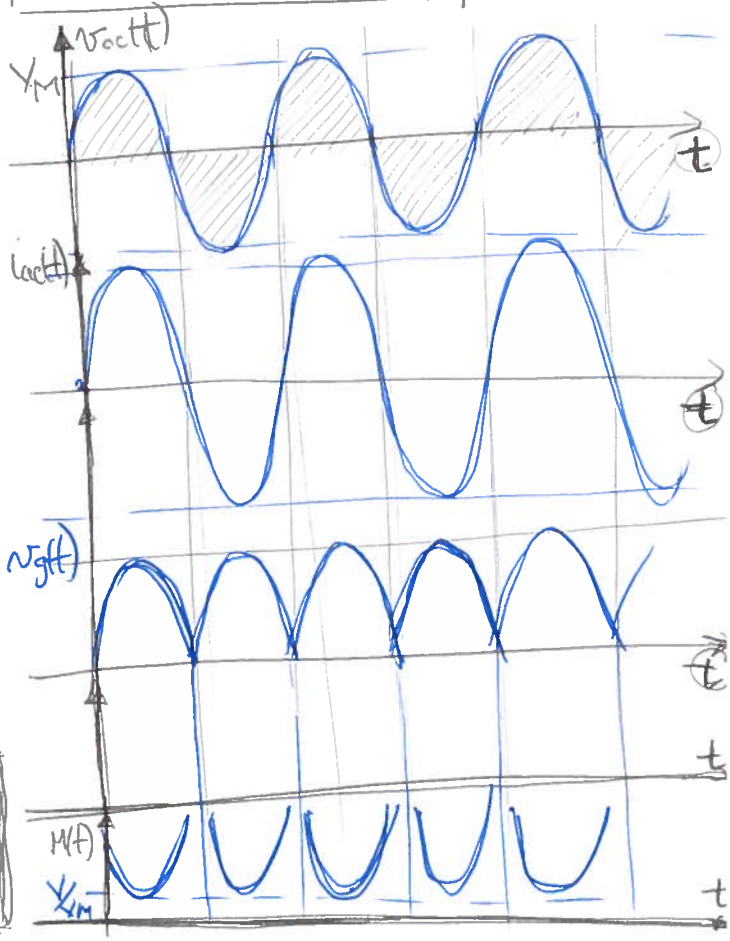


⇒

$$\boxed{\frac{V_{rms}}{V_{ac,rms}} = \sqrt{\frac{R}{Re}}}$$

$$\boxed{\frac{I_{ac,rms}}{I_{rms}} = \sqrt{\frac{R}{Re}}}$$





$v_{ac}(t) = V_M \cdot \sin(\omega t)$

$v_g(t) = V_M |\sin(\omega t)|$

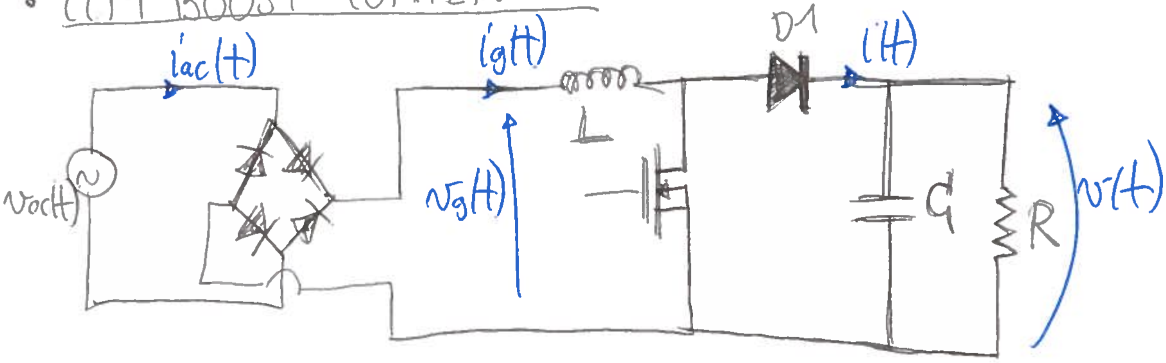
$M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M(\sin(\omega t))} \Rightarrow \boxed{\frac{V}{V_M} \quad M < \infty}$

$P_g(t) = p(t) \Rightarrow v_g(t) \cdot i_g(t) = i(t) \cdot v(t) \Rightarrow i(t) = \frac{v_g(t) \cdot i_g(t)}{v(t)}$

$\boxed{i(t) = \frac{v_g^2(t)}{V \cdot Re}} \Rightarrow \boxed{i(t) = \frac{V_M^2 \cdot \sin^2(\omega t)}{V \cdot Re} = \frac{V_M^2}{2 \cdot V \cdot Re}(1 - \cos(2\omega t))}$

$$i(t) = \frac{V_M^2}{2\,V\cdot R_e}\left[1-\cos(2\omega t)\right] \Rightarrow \boxed{I_{avg} = \frac{V_M^2}{2\,V\cdot R_e}} \Rightarrow \boxed{P = \frac{V_M^2}{2\cdot R_e}}$$

- CCM BOOST CONVERTERS



$$M(d(t)) = 1/1-d(t) \Rightarrow \frac{V}{v_g(t)} = \frac{1}{(1-d(t))} \Rightarrow 1-d(t) = \frac{v_g(t)}{V} \Rightarrow \boxed{d(t) = 1-\frac{v_g(t)}{V}}$$

If CCM $\Rightarrow \Delta i_L = \frac{v_g(t)}{2\cdot L}\cdot d(t)\cdot T_S$

The converter operates in CCM:

$$I_{g,avg} > \Delta i_{Lg}(t) \Rightarrow \frac{\cancel{v_g(t)}}{R_e} > \frac{\cancel{v_g(t)}}{2\cdot L}\cdot d(t)\cdot T_S \Rightarrow \frac{2\cdot L}{R_e\cdot T_S} > d(t)$$

$$\Rightarrow \boxed{R_e < \frac{2L}{T_S\left(1-\frac{v_g(t)}{V}\right)}} \text{ for CCM}$$

since $\Rightarrow$ $\emptyset < v_g(t) < V_M$ the converter always operates in CCM if:

$$\boxed{R_e < \frac{2L}{T_S}}$$

And always operates in DCM if:

$$\boxed{R_e > \frac{2L}{T_S\left(1-\frac{V_M}{V}\right)}}$$