AALBORG UNIVERSITET
Energy

Study number: 20126645	Programme: EPSH/PED/WPS/MCE
Evaluation subject: Dynamic Models of Electrical Machines and Control Systems Monday 23 January at 9:30-13:30	

Please write your study no. on all pages. Do not write your name as your evaluation is anonymous!

Total number of pages, including this page: 9

Please, only write on one side of the papers that you hand in.

NB! Your paper must be easy to read. If this is not the case, your paper may be evaluated as "not passed".

All usual aids are allowed (notes, books, tables, calculator and PC). You are not allowed to communicate amongst each other or with the outside world which means that the use of mobile phone, Wi-Fi, internet, email is not allowed.

You are allowed to take the examination questions with you. But you are NOT allowed to take them with you if you leave the room before the examination has ended.

(1)

$$I_a = Re\left(\frac{e^{j30°}}{e^{j0°}}\right) = \cos(-30°) = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

$$I_b = Re\left(\frac{e^{j30°}}{e^{j120°}}\right) = \cos(-30° - 120°) = \underline{\underline{-\frac{\sqrt{3}}{2}}}$$

$$I_c = Re\left(\frac{e^{-j30°}}{e^{-120°}}\right) = \cos(-30° + 120°) = \underline{\underline{0}}$$

$$I_\alpha = Re\left(\frac{e^{j-30°}}{e^{j0}}\right) = \cos(-30°) = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

$$I_\beta = Re\left(\frac{e^{j-30}}{e^{j\frac{\pi}{2}}}\right) = \sin(-30°) = \underline{\underline{-\frac{1}{2}}}$$

$$I_d = Re\left(\frac{e^{-30°}}{e^{+30°}}\right) = \cos(-60°) = \frac{1}{2}$$

$$I_q = Re\left(\frac{e^{-30°}}{e^{+30°+90°}}\right) = \cos(-30-30-90) = \underline{\underline{-\frac{\sqrt{3}}{2}}}$$

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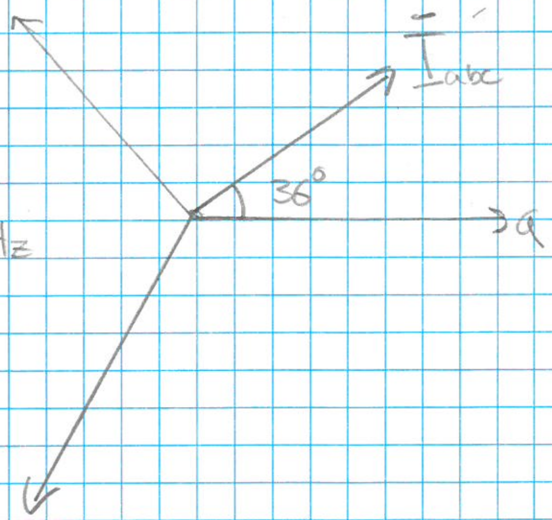
ex 1

(2)

$$\frac{0{,}0025}{0{,}025} \cdot 360° = 36°$$

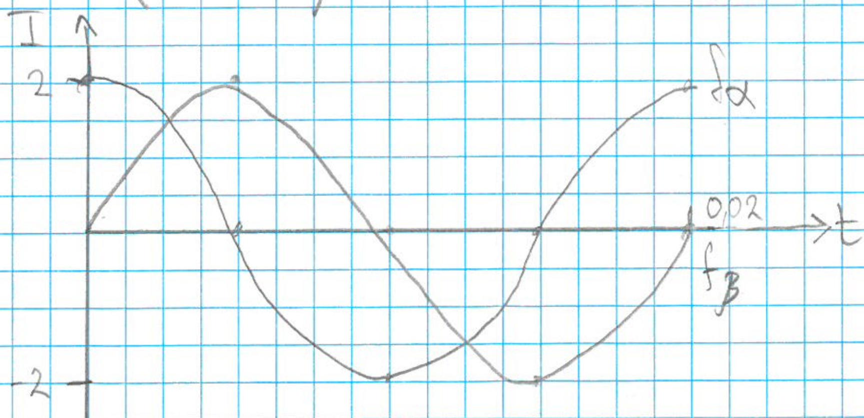
$$f = \frac{1}{T_p} = \frac{1}{0{,}025} = 50Hz$$



$$\bar{I}_{abc} = 2e^{j(w_e t)} \quad , \; w_e = 2\pi 50Hz$$

$$f_\alpha = Re\left(\frac{2e^{j(w_e t)}}{e^{j0}}\right) = 2\cos(w_e t)$$

$$f_\beta = Re\left(\frac{2e^{j(w_e t)}}{e^{j90°}}\right) = 2\sin(w_e t)$$



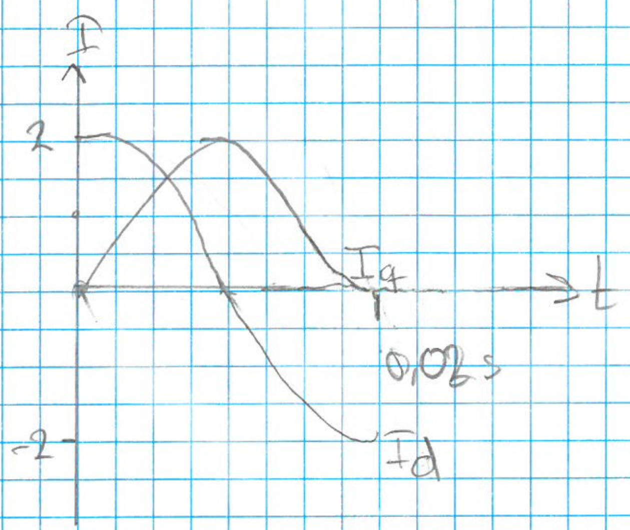
Rotating Ref frame d-q at 25 Hz

$$I_d = Re\left(\frac{2e^{j\omega_{e1}t}}{e^{j\omega_{e2}t}}\right) = 2\cos((\omega_{e1}-\omega_{e2})\cdot t)$$

$\omega_{e1} = 2\pi\, 50\, Hz$

$\omega_{e2} = 2\pi\, 25\, Hz$

$$I_q = Re\left(\frac{2e^{j\omega_{e1}t}}{e^{j\omega_{e2}t}}\right) = 2\sin((\omega_{e1}-\omega_{e2})t)$$



① Mutual inductance ph. c ph. a

$$M_{ascs} = L_{aaq} Re\left(\frac{e^{j(\theta_r + \frac{\pi}{2})}}{e^{j0}}\right) \cdot Re\left(\frac{e^{j(\theta_r + \frac{\pi}{2})}}{e^{j\frac{2\pi}{3}}}\right)$$

$$+ L_{aad} Re\left(\frac{e^{j\theta_r}}{e^{j0}}\right) Re\left(\frac{e^{j\theta_r}}{e^{-j\frac{2\pi}{3}}}\right)$$

$$\underline{\underline{M_{ascs} = L_{aaq} \cos\left(\theta_r + \frac{\pi}{2}\right) \cos\left(\theta_r + \frac{7\pi}{6}\right) + L_{aad} \sin\left(\theta_r + \frac{\pi}{2}\right) \sin\left(\theta_r + \frac{7\pi}{6}\right)}}$$

Find minimum.

$L_{aaq} = L_1 - L_2 \qquad L_{aad} = L_1 + L_2$

Collect terms of L_1 and L_2

$$L_1 \Big[\underbrace{\cos\left(\theta_r + \frac{\pi}{2}\right) \cos\left(\theta_r + \frac{7\pi}{6}\right) + \sin\left(\theta_r + \frac{\pi}{2}\right) \sin\left(\theta_r + \frac{7\pi}{6}\right)}_{\to \text{ euler rule}} \Big]$$

for L_1

$$\Rightarrow L_1 \cos\left(\frac{\pi}{2} - \frac{7\pi}{6}\right) = L_1 \cos\left(-\frac{2\pi}{3}\right) = -L_1 \cdot \frac{1}{2}$$

$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A) \cdot \sin(B)$

for L_2

$$L_2 \cos\left(2\theta_r + \frac{\pi}{2} + \frac{7\pi}{6}\right) = L_2 \cos\left(2\theta_r + \frac{5\pi}{3}\right) = L_2 \sin\left(2\theta_r + \frac{\pi}{6}\right)$$

$$\underline{M_{ascs} = -\frac{1}{2} L_1 + L_2 \sin\left(2\theta_r + \frac{\pi}{6}\right)}$$

So to get the mimimum of M_{ascs} then $\sin\left(2\theta_r + \frac{\pi}{6}\right) = -1$ which happens at $-\frac{\pi}{2}$

So

$$2\theta_r + \frac{\pi}{6} = -\frac{\pi}{2} \Rightarrow \theta_r = \frac{1}{2}\left(-\frac{\pi}{6} - \frac{\pi}{2}\right) = -\frac{\pi}{3}$$

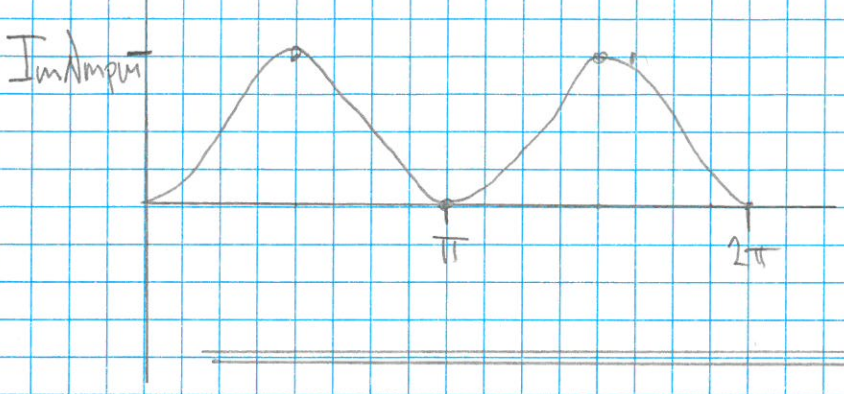
So the position is $\underline{\underline{-\frac{\pi}{3}}}$ or $\underline{\underline{-60^\circ}}$

(3) determine the Torque waveform

$$\tau = p \cdot i \frac{d\lambda_{pm}}{d\theta} \quad \text{in general}$$

So $$\tau = p \cdot i_a \cdot \frac{d\lambda_{pm,a}}{d\theta} = p \cdot (-I_m \sin\theta) \cdot \frac{d \cdot \lambda_{mpm} \cos\theta}{d\theta}$$

$$\tau = 1 \cdot I_m \cdot \sin\theta \cdot \lambda_{mpm} \sin\theta = \underline{\underline{I_m \lambda_{mpm} \sin^2\theta}}$$



(4) flux linkage for ph. c and current phase c.
Since we always want to produce max torque. the flux linkage after differentiation should match the current waveform.
the rotor is the same so $\lambda_{mpm,c} = \lambda_{mpm,a}$

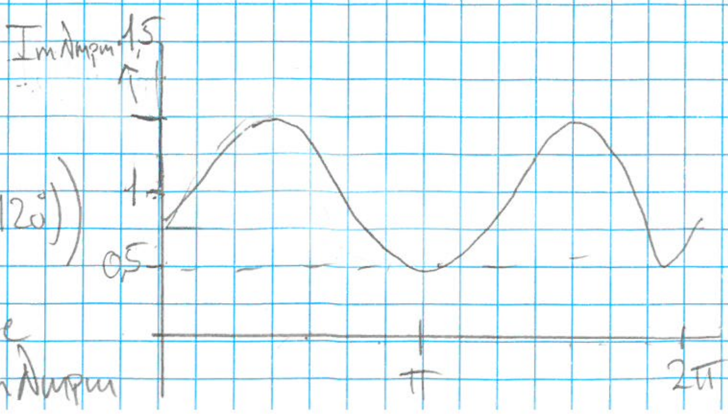
So $$\frac{d\lambda_{pm,c}}{d\theta} = \lambda_{mpm} - (\sin(\theta - \overset{\text{location of the winding}}{120^\circ}))$$

So $$\underline{\underline{\lambda_{pm,c} = \lambda_{mpm} \cos(\theta - 120^\circ)}}$$

and the current for phace c shoulde be

$$\underline{\underline{i_c = -\sin(\theta - 120^\circ)}}$$

(5) $$\tau_{a+c} = I_m \lambda_{mpm} \left(\sin\theta^2 + \sin^2(\theta - 120^\circ)\right)$$



The values on y-axis should be multiplied by $I_m \lambda_{mpm}$

1 Induction motor

For $\alpha\beta$ frame $\omega_\theta = 0,\ \theta = 0$, $f_d = -f_\beta$ $f_q = f_\alpha$

in qd: and $U_{qs} = R_s i_{qs} + \omega_\theta \lambda_{ds} + p\lambda_{qs}$

and $U_{ds} = R_s i_{ds} + p\lambda_{ds} - \omega_\theta \lambda_{qs}$

$\lambda_{qs} = (L_{ls} + L_m) i_{qs} + L_m i'_{qr}$

$\lambda_{ds} = (L_{ls} + L_m) i_{ds} + L_m i'_{dr}$

So in $\alpha\beta$

$$U_{\alpha s} = R_s i_{\alpha s} + p\lambda_{\alpha s}$$
$$U_{\beta s} = R_s i_{\beta s} + p\lambda_{\beta s}$$

So
$$\bar{U}_{\alpha\beta} = (R_s i_{\alpha s} + p\lambda_{\alpha s}) + j(R_s i_{\beta s} + p\lambda_{\beta s})$$

$$\underline{\underline{\bar{U}_{\alpha\beta} = R_s i_{\alpha\beta s} + p \cdot \lambda_{\alpha\beta s}}}$$

, $i_{\alpha\beta s} = i_{\alpha s} + j i_{\beta s}$

$\lambda_{\alpha\beta s} = \lambda_{\alpha s} + j\lambda_{\beta s}$

2 V/f - Control

$$\frac{V}{f} = \frac{V_{rated} \cdot \sqrt{2}}{\sqrt{3} \cdot f_{rated}} = \frac{380\,V \cdot \sqrt{2}}{\sqrt{3} \cdot 60\,Hz} = \underline{\underline{5{,}17\ V/Hz}}$$

for Peak

for Y-Connection

3 Motor at 1 Hz

$$U_s = I_s \cdot R_s \cdot \cos(\varphi) + \sqrt{\left[f_{in} \cdot \left(\frac{\sqrt{2} \cdot V_{in}}{\sqrt{3} f_{rated}}\right)\right]^2 - (I_s R_s \sin\varphi)^2}$$

P = Numper pole pairs $= \frac{\text{Poles}}{2} = \frac{6}{2} = 3$, $f_{in} = 1\,Hz$, $I_s = \sqrt{2} \cdot 0{,}5A$

$$U_s = 0{,}5 \cdot \sqrt{2} \cdot 0{,}28\Omega \cdot \cos(30^\circ) + \sqrt{(1\,Hz \cdot 5{,}17\,V/Hz)^2 - (0{,}5 \cdot \sqrt{2} \cdot 0{,}28\Omega \cdot \sin(30^\circ))^2} = \underline{\underline{5{,}34V}}$$

$$\underline{\underline{U_s = 5{,}34V}}$$

4 Slip compensation

At full load

The slip is: $S = \frac{n_r - n}{n_r}$, $n_r = \frac{60Hz \cdot 60\, s/m}{3} = 1200$

$$S = \frac{1200 - 1160}{1200} = \frac{1}{30} = 0{,}03$$

$$\frac{T_e}{T_{rated}} = \frac{f_{se}}{f_{se\,rated}} \Rightarrow f_{se} = \frac{T_e}{T_{rated}} \cdot f_{se\,rated}, \quad f_{se\,rated} = S \cdot f_s = 0{,}03 \cdot 60Hz = 2\,Hz$$

$$T_e = \frac{1}{2} T_{rated} \Rightarrow$$

$$f_{se} = \frac{\frac{1}{2} T_{rated}}{T_{rated}} \cdot 2\,Hz = \frac{1}{2} \cdot 2\,Hz = \underline{\underline{1\,Hz}}$$

① Since it is open circuited and steady state the voltage at the terminals is equal.

pole pairs $= \frac{\text{Poles}}{2}$

$\hat{U}_{s\,peak} = \omega_r \lambda_{pm\,peak}$, $\omega_r = \frac{1200\ \text{rpm} \cdot 2\pi}{60\ ^{s}/_{m}} \cdot 4 = 160\pi$

and $\hat{U}_{s\,peak} = \frac{\sqrt{2} \cdot 100\text{V}}{\sqrt{3}} = 81{,}6\ \text{V}$

So

$\lambda_{pm\,peak} = \frac{\hat{U}_{s\,peak}}{\omega_r} = \frac{81{,}6\text{V}}{160\pi} = \underline{\underline{0{,}162}}$

② Since $I_d = 0$, $I_q = \hat{I}_a = 2A$

$\tau = \frac{3}{2} P \cdot \lambda_{pm} \cdot \hat{i}_q = \frac{3}{2} \cdot 4 \cdot 0{,}162 \cdot 2A = \underline{\underline{1{,}949\ \text{Nm}}}$

③ it should be placed lagging.

When additional load is added the rotor speed decreases and since this is the d-q speed the current will get closer to q-axis, if it is lagging. thereby will the torque produced go up by $(\frac{3}{2} P \cdot \lambda_{pm} \cdot \hat{i}_q)$, since $\hat{i}_q$ increases, and rotor will overcome the new load and be selvballancing

(4) The ph. b Current is lagging the Voltage by $\cos^{-1}(0{,}707) = 45^\circ$

Since the period is 0,02 the frequency is $\frac{1}{T_p} = \frac{1}{0{,}02} = 50\,Hz$

The time for reaching max is when it is alligned with the b-axis which happens 45° after the Voltage so:

$$\frac{45^\circ}{360^\circ} \cdot T_p = \frac{1}{8} \cdot 0{,}02\,s = \underline{\underline{0{,}0025\ s}}$$