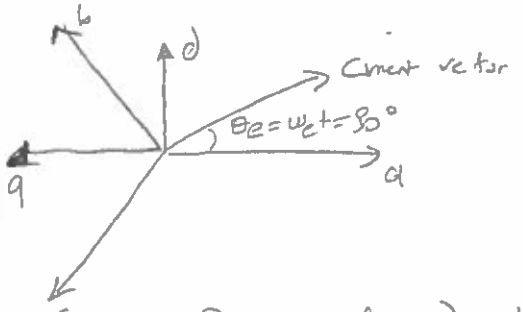
Q1) (1) $\omega_e = 2\pi 50$



$\omega_e t = 30 \quad T = \frac{\pi/6}{2\pi 50} = \frac{1}{600} = 1,66 \times 10^{-3} = 0,0016\,s$

$T = \frac{1}{50} = 0,02\,s$

And $I_{pk} = 10\,A \quad \omega_e t = 30°$

$I_a = I_{pk}\cos(\omega_e t) = 10\cos(30^\circ) = 8,66\,A$

$I_b = I_{pk}\cos(\omega_e t - 120°) = 10\cos(30-120) = 0\,A$

$I_c = I_{pk}\cos(\omega_e t + 120°) = 10\cos(30+120) = -8,66\,A$

(2) for current;

$$\bar{f} = \frac{2}{3}\left(I_a e^{j0} + I_b e^{j\frac{2\pi}{3}} + I_c e^{-j\frac{2\pi}{3}}\right)$$

$$\bar{f} = \frac{2}{3} I_{pk}\left[\cos(\omega_e t) + \cos(\omega_e t - 120)\, e^{j120} + \cos(\omega_e t + 120)\, e^{-j120}\right]$$

$$\bar{f} = \frac{2}{3} I_{pk}\left[\cos(\omega_e t) + \cos(\omega_e t - 120)\cos(120) + \cos(\omega_e t + 120)\cos(-120)\right]$$

$$\bar{f} = \frac{2}{3} I_{pk}\left[\cos(\omega_e t) - \frac{1}{2}\left(\cos(\omega_e t - 120) + \cos(\omega_e t + 120)\right)\right]$$

from $\cos\alpha \cdot \cos\beta = \frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2}$

$$\bar{f} = \frac{2}{3} I_{pk}\left[\cos(\omega_e t) - \cos(\omega_e t)\cdot\cos(120^\circ)\right]$$

$$\bar{f} = \frac{2}{3}\cdot 10\cdot\frac{3}{2}\cos(\omega_e t) = 10\cos(\omega_e t) \quad \text{(Real part)}$$

$$\bar{f} = \frac{2}{3} I_{pk}\left[\cos(\omega_e t - \tfrac{2\pi}{3})\, e^{j\frac{2\pi}{3}} + \cos(\omega_e t + \tfrac{2\pi}{3})\, e^{-j\frac{2\pi}{3}}\right] \quad \text{(Imaginary Part)}$$

$$= \frac{2}{3} I_{pk}\left[\cos(\omega_e t - \tfrac{2\pi}{3})\, j\sin\tfrac{2\pi}{3} + \cos(\omega_e t + \tfrac{2\pi}{3})(-j\sin\tfrac{2\pi}{3})\right]$$

$$= \frac{2}{3} I_{pk}\left[\cos(\omega_e t - \tfrac{2\pi}{3})\, j\frac{\sqrt{3}}{2} + \cos(\omega_e t + \tfrac{2\pi}{3})\, -j\frac{\sqrt{3}}{2}\right]$$

$$= \frac{2}{3}\cdot\frac{\sqrt{3}}{2}\, 10\, j\left[\cos \ldots\right.$$

long version

(2)

$f_a = f_m \cos(\omega t + 30^\circ)$

$f_b = f_m \cos(\omega t - 120^\circ + 30^\circ)$ $\quad (\omega t + 30^\circ = \omega t')$

$f_c = f_m \cos(\omega t + 120^\circ + 30^\circ)$

$\underline{f}_{abc} = \frac{2}{3} f_m \left(\cos(\omega t)' \cdot e^{j0} + \cos((\omega t)' - 120^\circ) e^{j120} + \cos((\omega t)' + 120^\circ) e^{-j120} \right)$

$\underline{I}_{abc} = \frac{2}{3} I_{pk} \left(\frac{1}{2} \left(e^{j\omega t'} + e^{-j\omega t'} \right) e^{j0} + \frac{1}{2} \left(e^{j\omega t' - 120^\circ} + e^{-j(\omega t' + 120^\circ)} \right) e^{j120} + \dots \right.$

$\left. \dots + \frac{1}{2} \left(e^{j\omega t' + 120^\circ} + e^{-j\omega t' + 120^\circ} \right) e^{-j120} \right)$

$\hookrightarrow = \frac{2}{3} I_{pk} \frac{1}{2} \left[\left(e^{j(\omega t)' + 0} + e^{-j\omega t' + 0} \right) + \left(e^{j\omega t' + 120 + 120} + e^{-j\omega t' + 120 + 120} \right) + \dots \right.$

$\left. \dots + \left(e^{j\omega t' + 120 - 120} + e^{-j\omega t' + 120 - 120} \right) \right)$

$= \frac{2}{3} I_{pk} \frac{1}{2} \cdot 3 \left(e^{j\omega t'} + e^{-j\omega t'} \left(1 + e^{-j120} \cdot 0 + e^{j120} \right) \right)$

$= \frac{2}{3} I_{pk} \frac{3}{2} e^{j\omega t'}$

$\underline{I}_{abc} = I_{pk} e^{j\omega t'} = \underline{I_{pk} e^{j\omega t + 30}}$

$I_d = Re\left(\frac{\underline{I}_{abc}}{e^{j\omega t + 90}} \right) = Re\left(I_{pk} e^{j(\omega t + 30) - j(\omega t - 90)} \right) = Re\left(I_{pk} e^{-j60} \right) = I_{pk} \cdot \cos(-60)$

$= 10 \cdot 0{,}5 = \boxed{5\,A}$

$I_q = Re\left(\frac{\underline{I}_{abc}}{e^{j\omega t + 180}} \right) = Re\left(I_{pk} e^{j(\omega t + 30)} \cdot e^{-j(\omega t + 180)} \right) = Re\left(I_{pk} e^{-j150} \right) = I_{pk} \cos(-150)$

$= 10 \cdot -8{,}66 = \boxed{-8{,}66\,I}$

shorter version of (2)

91) (3) $t = 0,0T$

$\omega_e t = 2\pi \times 0,05 = 5\pi = 180°$

$\theta_d = \underbrace{180}_{\text{time}} + \underbrace{30}_{\text{beginning}} + \underbrace{90}_{\text{difference}} = 300°$

I. for current

$\vec{I} = 10\,(e^{j\omega t})$

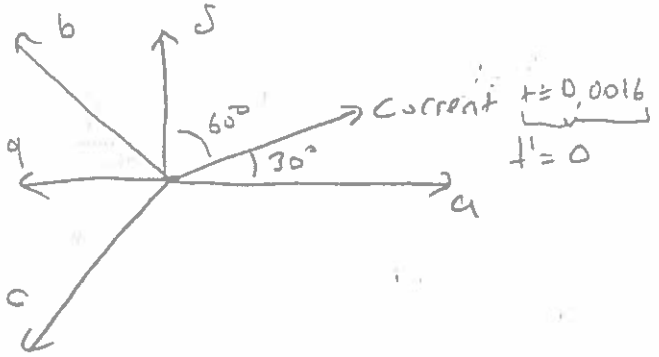
$\vec{f_d} = Re\left(\frac{\vec{I}}{e^{j300}}\right) = Re\left(10\,e^{j\omega t}.e^{-j300°}\right)$

$= 10\cos(\omega_e t - 300)$

$\vec{f_q} = Re\left(\frac{\vec{I}}{e^{j390}}\right) = Re\left(10\,e^{j\omega t}.e^{-j30°}\right)$

$= 10\cos(\omega_e t - 30)$

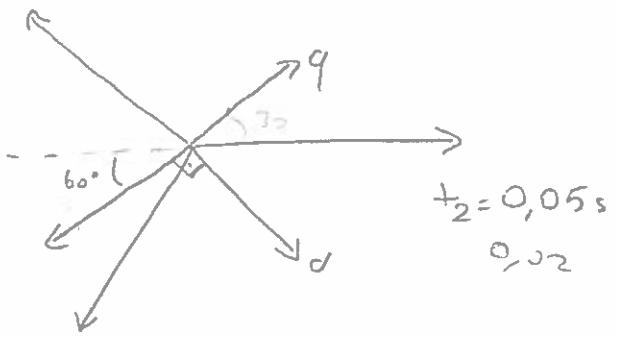
Case 1



$\theta_{d1} = 90°$

$\theta_{c1} = 30°$

case 2



$\theta_{d2} = 300°$

$\theta_{c2} = 210°$

d frame is always leading current frame

$0,05\,s \Rightarrow \frac{0,05}{0,02} = 2,5$ rotation

$\theta_{lead} = \Delta_d - \Delta_c = (300-90) - (210-30) = 30°$ in 2,5 rotation

$12°$ ⇐ 1 rotation

$\Delta\omega_e . t = \theta_{lead}$

$2\pi (f_{dq} - f_c) . t = \theta_{lead}$

$2\pi (f_{dq} - 50) . t = 12$

$f_{dq} - 50 = 1,66$

$f_{dq} = 51,66$ → $\omega_e = 2\pi . 51,66$ rad/s

P2 2016

i) $M_{bscs} = L_{aaq} R_c\left(\frac{e^{j\theta}}{e^{j120}}\right) R_c\left(\frac{e^{j\theta}}{e^{-j120}}\right) + L_{aad} R_c\left(\frac{e^{j(\theta-90)}}{e^{j120}}\right) R_c\left(\frac{e^{j(\theta-90)}}{e^{-j120}}\right)$

$= L_{aaq} \cos(\theta-120)\cos(\theta+120) + L_{aad} \sin(\theta-120)\sin(\theta+120)$

$= L_{aaq} \frac{(\cos(2\theta)+\cos(-240))}{2} + L_{aad} \frac{(\cos(-240)-\cos(2\theta))}{2}$

$= \frac{1}{2}\left(L_1\cos(2\theta) + L_1\cos(-240) - L_2\cos(2\theta) - L_2\cos(-240) + L_1\cos(-240) - L_1\cos(2\theta) + L_2\cos(-240) - L_2\cos 2\theta\right)$

$L_{aaq} = L_1 - L_2$

$L_{aad} = L_1 + L_2$

$\frac{1}{2}(-L_1 - 2L_2\cos(2\theta)) = -\frac{1}{2}L_1 - L_2\cos(2\theta)$

M_{max} is at $\theta = \frac{\pi}{2} \Rightarrow -\frac{1}{2}L_1 + L_2$

M_{min} is at $\theta = 0 \Rightarrow -\frac{1}{2}L_1 - L_2$

2) $m_{crsb} = ?$

or

$m_{bscr} = ? \Rightarrow L_{aaq} R_c\left(\frac{e^{j\theta_m}}{e^{j120^\circ}}\right) R_c\left(\frac{e^{j\theta}}{e^{j(\theta+240)}}\right) + L_{aad} R_c\left(\frac{e^{j(\theta-90)}}{e^{j120}}\right) R_c\left(\frac{e^{j(\theta-90)}}{e^{j(\theta+240)}}\right)$

$\theta_m = \theta - 120$

$\Rightarrow L_{aaq}\cos(\theta-120)\cos(-240) + L_{aad}\sin(\theta-120)\sin(-240)$

$\Rightarrow -0.5 L_{aaq}\cos(\theta-120) + \frac{\sqrt{3}}{2} L_{aad}\sin(\theta-120)$

$-0.5\left(L_1\cos(\theta-120) - L_2\cos(\theta-120)\right) + \frac{\sqrt{3}}{2}\left(L_1\sin(\theta-120) + L_2\sin(\theta-120)\right)$

$L_{aaq}\left(\frac{1}{2}(\cos(\theta) + \cos \ldots\right.$

3)

P2 2016

$\frac{2\pi\omega}{60} = 120$ rpm $\qquad \frac{60}{2\pi}\omega = 120$ rpm

$\omega = 12.57$ rad/s

$p = 2$

$\omega_r = ?$

$f_{\alpha} + j f_{\beta} = (f_q - j f_d)\, e^{j\theta}$

$\Rightarrow \bar{V}_{qds} = \left(R_s(i_{qs} - j i_{ds}) + p(\lambda_{qs} - j\lambda_{ds}) + \omega\lambda_{ds} + j\omega\lambda_{qs}\right) e^{j\theta}$

$V_{qs} - jV_{ds}$

$\Rightarrow R_s(i_{qs} - j i_{ds})(\cos\theta + j\sin\theta)$

$+ p(\lambda_{qs} - j\lambda_{ds})(\cos\theta + j\sin\theta)$

$(\omega\lambda_{ds} + j\omega\lambda_{qs})(\cos\theta + j\sin\theta)$

$F_{\alpha\beta} = R_s(j i_{qs} + i_{ds}) + p(j\lambda_{qs} + \lambda_{ds}) + \omega(j\lambda_{qs} + \lambda_{ds})$

$F_{\alpha} = R_e\left(\frac{\cdot}{e^{j\theta}}\right)$

Q4) (1) $P_{in} = \sqrt{3} \times U_{n1-1} \times I_n \times \cos\varphi = \sqrt{3} \cdot 380 \times 2.1 \times 0,6 \simeq 873$ W

$P_{out} = 600$ W

$\eta = \% \frac{600}{873} = \% \frac{P_{out}}{P_{in}} = \% 68,7$

(2) $\frac{V_{n,rated}}{f_{rated}} = \frac{400/\sqrt{3}}{60} = 3,83$

(3) $|U_s| = |R_s \cdot I_s + j\omega_s \Psi_s|$ ($R_s \cdot I_s \to 0$)

$|U_s| = |j\omega_s \Psi_s|$

$U_s = \omega_s \Psi_s \Rightarrow 230 = 2\pi 60 \cdot \Psi_s \Rightarrow \Psi_s = 0,61$

(4) Lecture 11 p. 12 Munoz-Garcia

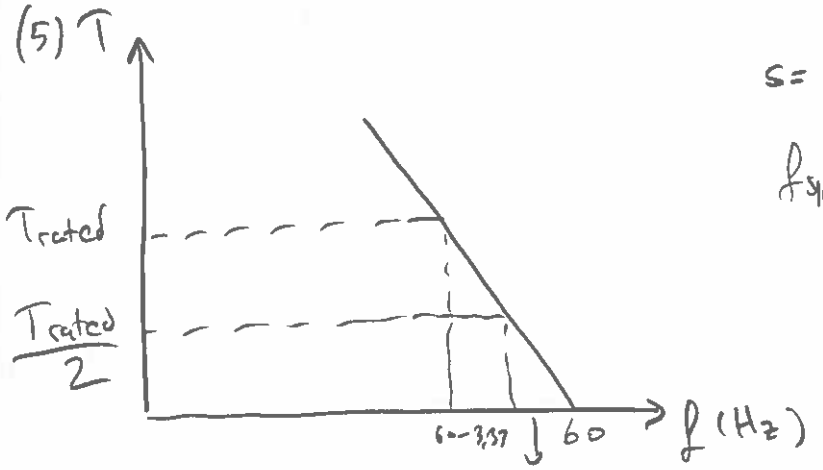
$U_s = r_s (I_s \cos\phi) + \sqrt{U_{s\Psi}^2 - (r_s I_s)^2 + r_s^2 (I_s \cos\phi)^2}$

Magnitude of $U_s \Rightarrow \sqrt{r_s^2 I_s^2 \cos\phi^2 + U_{s\Psi}^2 - r_s I_s^2 + r_s^2 I_s^2 \cos\phi^2}$

$|U_s| = \sqrt{12^2 \cdot 0,5^2 \, 0,866^2 + 0,96^2 - 12 \cdot 0,5^2 + 12^2 \, 0,5^2 \, 0,866^2} = 6,57$ V

$U_{s\Psi} = f_{s\Psi} \cdot \frac{U_{s\,rated}}{f_{s\,rated}} = 0,25 \frac{400/\sqrt{3}}{60} = 0,96$ V

(5)



$s = \frac{n_s - n_M}{n_s} = \frac{900 - 850}{900} = 0,055$

$f_{slip} = f_s \cdot s = 3,33$ Hz

$\frac{3,33}{2} = 1,66$ Hz need to be added stator frequency

$f_{new} = f_s + f_{slip,comp} = 60 + 1,66 = 61,66$ Hz