Space Vector Modulation Analysis

Ref. Control of Induction Motor, by A. Trzynadlowski, Academic Press, 2001

1. P.71, The definition of switching variables

Each leg (two switchers) is assigned a switching variable (a, b, c). Value 1 means the upper switcher is on. Value 0 means the lower switcher is on.

Then it is very interesting to know that

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = V_D \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{1}$$

V_D is the instantaneous DC-link voltage. This equation is valid for any instance and it is only for the line-to-line voltage (V_D could only directly appear in the line-to-line voltage expression, not for the line-to-neutral expression!).

According to the definition of the line-to-line voltage, it could be found (unconditionally) that:-

$$V_{AB} = V_{AN} - V_{BN} \tag{2}$$

$$V_{BC} = V_{BN} - V_{CN} \tag{3}$$

and from these two equations, the definition for V_{CA} should be:-

$$V_{CA} = V_{CN} - V_{AN} \tag{4}$$

If the three-phase is balanced, then we have:-

$$V_{AN} + V_{BN} + V_{CN} = 0 \tag{5}$$

Please note that this equation is applied to the line-to-neutral voltage only.

Thus use (2), (3) and (5), the transformation between the line-to-line voltage and line-to-neutral voltage could be obtained:-

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} \tag{6}$$

The relationship between the line-to-neutral voltage and the switching variables could be found as:-

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \frac{V_D}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{7}$$

2. P73, The switching interval is defined as the switching period as

$$T_{sw} = \frac{1}{f_{sw}} \tag{8}$$

Thus the number of N, the switching intervals per cycle of the output voltage, is given by:-

$$N = \frac{f_{sw}}{f} \tag{9}$$

3. P74, Understanding the space vector

$$\vec{v} = \frac{2}{3}\left(v_a + e^{j120} v_b + e^{-j120} v_c\right)$$

This space vector is $\frac{2}{3}$ of the vector sum of the three instantaneous voltage vectors ($v_a e^{j0}, v_b e^{j120}, v_c e^{-j120}$). It has the important characteristic that its projection on each phase would be the actual instantaneous value for that phase minus the instantaneous zero component. For example, the projection to phase B:-

$$\mathrm{Re}\left(\frac{\vec{v}}{e^{j120}}\right) = \frac{2}{3}\mathrm{Re}\left(v_a e^{-j120} + v_b + e^{j120} v_c\right) = \frac{2}{3}\left(-\frac{1}{2}v_a + v_b - \frac{1}{2}v_c\right) \tag{10}$$

Defining the zero component as $v_0 = \frac{1}{3}\left(v_a + v_b + v_c\right)$, (10) could be further transformed to:-

$$\mathrm{Re}\left(\frac{\vec{v}}{e^{j120}}\right) = \frac{2}{3}\left(v_b - \frac{3}{2}v_0 + \frac{1}{2}v_b\right) = \frac{2}{3}\left(\frac{3}{2}v_b - \frac{3}{2}v_0\right) = v_b - v_0 \tag{11}$$

If the three phases are balanced, the instantaneous zero component becomes zero and the projection of this space vector $\vec{v}$ results in the actual instantaneous phase voltage.

If the three phases are balanced, the space vector could be further expressed as:-

$$\vec{v} = v_d + jv_q$$

and

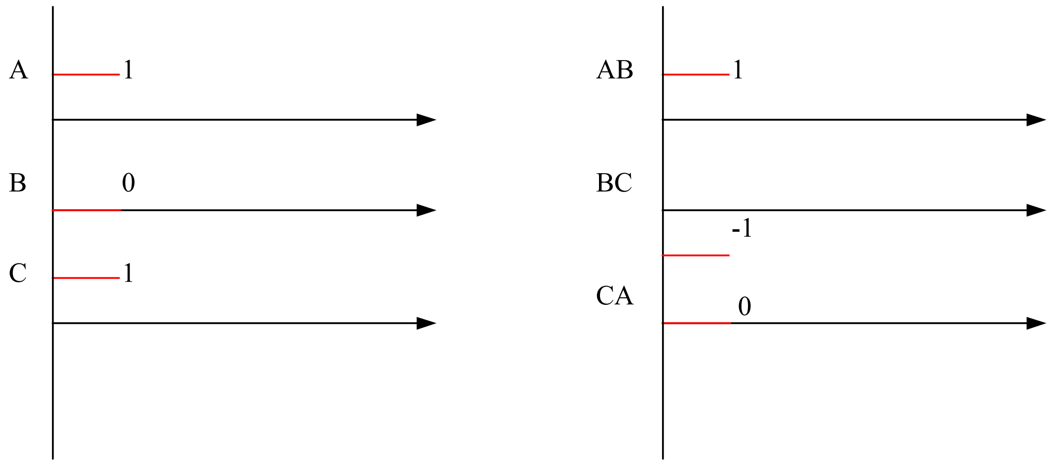
$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{12}$$

Compared to equation (4.11) on P.74, the d, q, values given by (12) are $\frac{2}{3}$ times smaller.

Similarly, the inverse transformation could be:-

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix} \tag{13}$$

Compared to equation (4.12) on P.75, the a, b, c values given by (13) are $\frac{3}{2}$ times bigger.
For example, for switching state 5 (corresponding to 101),



Note, the voltage A, B, C shown in the left figure do not mean the line-to-neutral voltage! From the output of the three-phase inverter, only the line-to-line voltage could be obtained!

Thus

$$v_{AN} = \frac{1}{3}\left(v_{AB} - v_{CA}\right) = \frac{v_D}{3}$$

$$v_{BN} = \frac{1}{3}\left(-v_{AB} + v_{BC}\right) = -\frac{2v_D}{3}$$

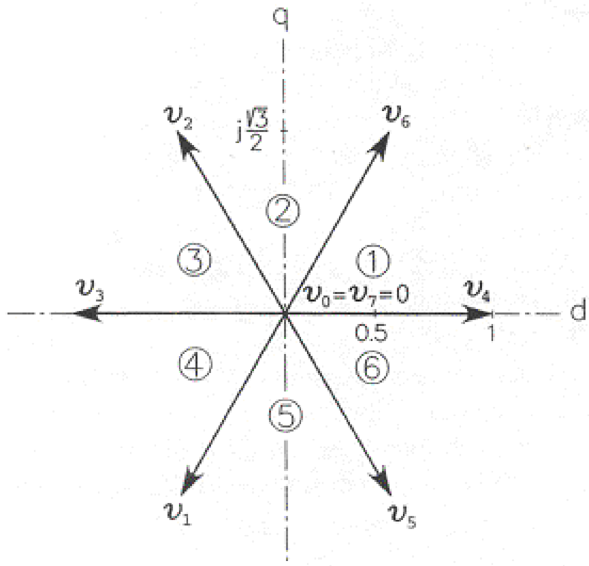
$$v_{CN} = \frac{1}{3}\left(-v_{BC} + v_{CA}\right) = \frac{v_D}{3}$$

therefore

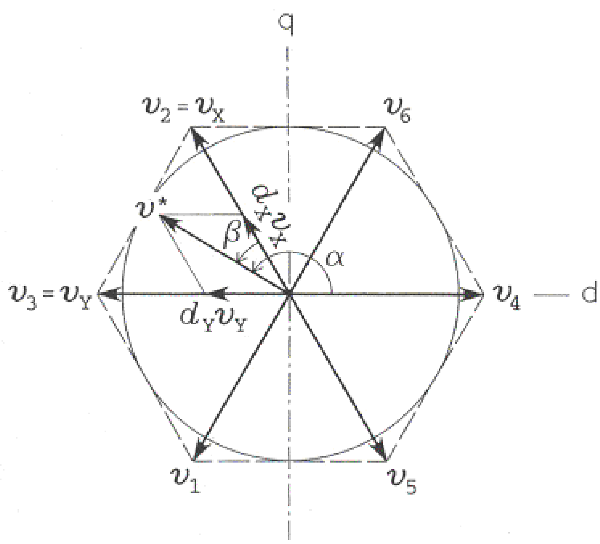
$$\vec{v} = \frac{2}{3}v_D\left(\frac{1}{3} - \frac{2}{3}e^{j120} + e^{-j120}\frac{1}{3}\right) = \frac{2}{3}v_D\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \tag{14}$$

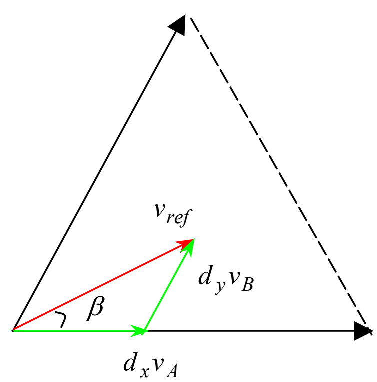
This is the vector that has the correct magnitude.

All the available space vectors could be plotted as:-



4. P.77, The SVM algorithm





For any local triangle, the vector $\vec{v_A}$ could always set to be aligned with the real-axis. Therefore it could be found that:-

$$\vec{v_{ref}} = d_x v_A + d_y \vec{v_B} = d_x v_A + d_y v_B e^{j60} \tag{15}$$

$$\vec{v_{ref}} = v_{ref} \cos\beta + j v_{ref} \sin\beta = d_x v_A + d_y \left(\frac{1}{2} v_B + j\frac{\sqrt{3}}{2} v_B \right) \tag{16}$$

Equation (16) forms two equations. Solving them for d_x and d_y, we have

$$d_x = \frac{\sqrt{3} v_{ref}}{v_A} \frac{2}{3} \sin(60 - \beta) \tag{17.1}$$

$$d_y = \frac{\sqrt{3}v_{ref}}{v_B}\frac{2}{3}\sin\beta \tag{17.2}$$

From (14), it could be observed that the magnitude of the six non-zero vectors is $\frac{2}{3}v_D$. Thus

$$v_A = v_B = \frac{2}{3}v_D \tag{18}$$

Thus (17) could be transformed to:-

$$d_x = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\beta) \tag{19.1}$$

$$d_y = \frac{\sqrt{3}v_{ref}}{v_D}\sin\beta \tag{19.2}$$

5. Application in the dSPACE system

Normally, it is in the balanced condition and v_{ref} is given by v_d and v_q. It is valid that:-

$$\vec{v}_{ref} = v_d + jv_q = \frac{\cos\alpha}{v_{ref}} + j\frac{\sin\alpha}{v_{ref}} \tag{20}$$

$$v_{ref} = \sqrt{v_d^{\,2} + v_q^{\,2}} \tag{21}$$

You should notice that angle β is defined in the local system. It belongs to $[0,60^o]$. Angle α is defined in the global syste,.

From (19.1) and (19.2), due to the fact that $|d_x|,|d_y| \le 1$, we could observe that the maximum value of v_{ref} should be:-

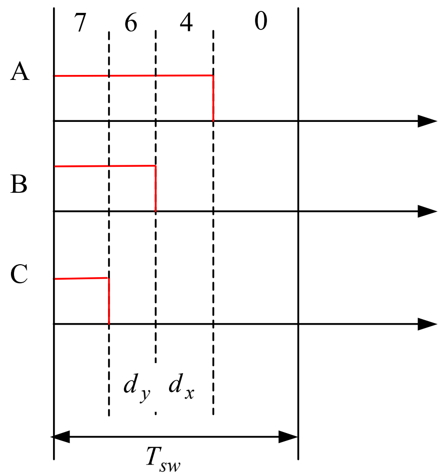
$$|v_{ref}| \le \frac{v_D}{\sqrt{3}} \tag{22}$$

Then we have:-

$$v_d = \frac{v_d}{\sqrt{v_d^{\,2}+v_q^{\,2}}}v_{ref} \le \frac{v_d}{\sqrt{v_d^{\,2}+v_q^{\,2}}}\frac{v_D}{\sqrt{3}} \tag{23.1}$$

$$v_q = \frac{v_q}{\sqrt{v_d^{\,2}+v_q^{\,2}}}v_{ref} \le \frac{v_q}{\sqrt{v_d^{\,2}+v_q^{\,2}}}\frac{v_D}{\sqrt{3}} \tag{23.2}$$

For sector 1, $v_d \ge 0$, $v_q \ge 0$, and $\frac{v_q}{v_d} < \tan(60^o) \Rightarrow v_q < \sqrt{3}v_d$, the two non-zero vectors are 4 (100, corresponding to d_x) and 6(110, corresponding to d_y). Thus according to the relationship between the duty cycle and the generated pulses, the only possible combination of three-phase pulses are:-



It should be guaranteed that
$D_a \geq D_b \geq D_c$

Thus we have:-

$$\begin{bmatrix} d_0 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

where $d_0 = 1 - d_x - d_y$ and D_a, D_b, D_c are the duty cycles for phase A, B and C. Note that the maximum value for a duty cycle is 1.

We could find that the 3×3 matrix is singular and cannot be inversed. Actually, for the same value of d_x and d_y, the duty cycle for phase C could be arbitrary. We need to add another constrain, such as let the length for state 7 and 0 be equal. Then we have:-

$$D_c = 1 - D_a \tag{25}$$

$$\begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} \tag{26}$$

The expressions for the duty cycles could be obtained as:-

$$\begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix} \tag{27}$$

The d_x and d_y could be calculated as:-

$$d_x = \frac{\sqrt{3} v_{ref}}{v_D} \sin(60 - \beta) = \frac{\sqrt{3} v_{ref}}{v_D} \left(\frac{\sqrt{3}}{2} \cos\beta - \frac{1}{2} \sin\beta \right) = \frac{3}{2 v_D} \left(\frac{2}{3} \sqrt{3} \frac{\sqrt{3}}{2} v_d - \frac{2}{3} \sqrt{3} \frac{1}{2} v_q \right)$$

$$= \frac{3}{2 v_D} \left(v_d - \frac{1}{\sqrt{3}} v_q \right)$$

$$d_y = \frac{\sqrt{3}v_{ref}}{v_D}\sin\beta = \frac{3}{2v_D}\frac{2}{3}\sqrt{3}v_q = \frac{3}{2v_D}\frac{2}{\sqrt{3}}v_q$$

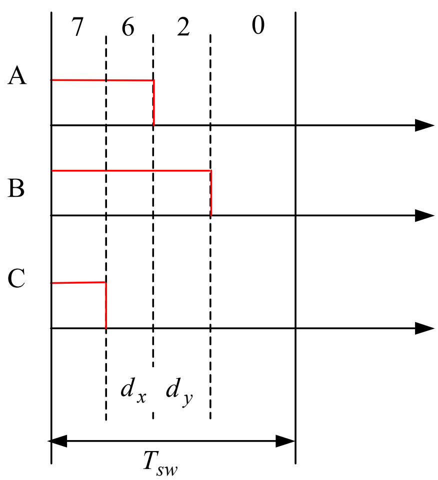
Use voltage $\frac{2}{3}v_D$ to normalized the d, q voltages. Denote the normalized voltages as v_{dn}, v_{qn}. The final expressions for the duty cycles could be found as:-

$$D_a = \frac{1}{2}+\frac{1}{2}d_x+\frac{1}{2}d_y = \frac{1}{2}+\frac{1}{2}\left(v_{dn}-\frac{1}{3}v_{qn}\right)+\frac{1}{\sqrt{3}}v_{qn} \qquad (28.1)$$

$$D_b = \frac{1}{2}-\frac{1}{2}d_x+\frac{1}{2}d_y = D_a - d_x \qquad (28.2)$$

$$D_c = \frac{1}{2}-\frac{1}{2}d_x-\frac{1}{2}d_y \qquad (28.3)$$

For sector 2, $v_d \geq 0$, $v_q \geq 0$, and $\frac{v_q}{v_d} \geq \tan(60^o) \Rightarrow v_q \geq \sqrt{3}v_d$, the two non-zero vectors are 2 (010, corresponding to d_y) and 6(110, corresponding to d_x). The only possible combination of three-phase pulses are:-



It should be guaranteed that
$D_b \geq D_a \geq D_c$

Correspondingly, we have (please pay attention to the sequence of d_x and d_y! d_y should correspond to the vector that always anti-clockwise leading the vector corresponding to d_x):-

$$\begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} \Rightarrow \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix}$$

$$d_x = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\beta) = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\alpha+60) = \frac{\sqrt{3}v_{ref}}{v_D}\left(\frac{\sqrt{3}}{2}\cos\alpha + \frac{1}{2}\sin\alpha\right),$$

$$= \frac{3}{2v_D}\left(\frac{2}{3}\sqrt{3}\frac{\sqrt{3}}{2}v_d + \frac{2}{3}\sqrt{3}\frac{1}{2}v_q\right) = \frac{3}{2v_D}\left(v_d + \frac{1}{\sqrt{3}}v_q\right)$$

$$d_y = \frac{\sqrt{3}v_{ref}}{v_D}\sin\beta = \frac{\sqrt{3}v_{ref}}{v_D}\sin(\alpha-60) = \frac{3}{2v_D}\left(\frac{1}{\sqrt{3}}v_q - v_d\right)$$

Please note that for calculating d_x and d_y, angle β is needed. This is the angle defined referred to the vector corresponding to dx in that sector. Angle α is defined in the global system. $\alpha = a\tan\frac{v_q}{v_d}$, or refer (20).

You should also notice that v_d, v_q could be minus or plus, but the calculated d_x, d_y are always positive.

Thus

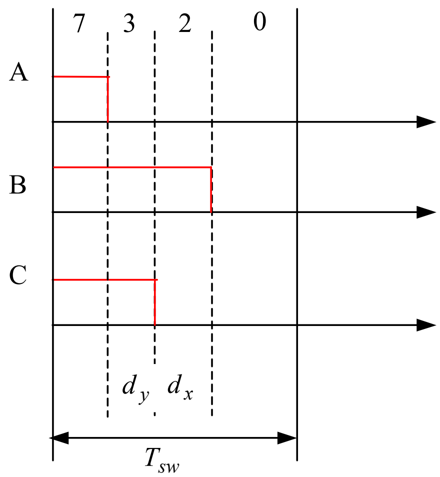
$$D_a = \frac{1}{2} + \frac{1}{2}d_x - \frac{1}{2}d_y = \frac{1}{2} + \frac{1}{2}\left(\frac{v_{qn}}{\sqrt{3}} + v_{dn}\right) - \frac{1}{2}\left(\frac{v_{qn}}{\sqrt{3}} - v_{dn}\right) = \frac{1}{2} + v_{dn} \quad (29.1)$$

$$D_b = \frac{1}{2} + \frac{1}{2}d_x + \frac{1}{2}d_y = \frac{1}{2} + \frac{v_{qn}}{\sqrt{3}} \quad (29.2)$$

$$D_c = \frac{1}{2} - \frac{1}{2}d_x - \frac{1}{2}d_y = \frac{1}{2} - \frac{v_{qn}}{\sqrt{3}} \quad (29.3)$$

For sector 2, $v_d < 0$, $v_q > 0$, and $\frac{v_q}{v_d} \le \tan(120^o) \Rightarrow v_q \ge \sqrt{3}|v_d|$, the two non-zero vectors are 2 (010, corresponding to d_y) and 6(110, corresponding to d_x), the calculation of the duty cycles are the same as (29).

For sector 3, $v_d < 0$, $v_q > 0$, and $\frac{v_q}{v_d} > \tan(120^o) \Rightarrow v_q < \sqrt{3}|v_d|$, the two non-zero vectors are 2 (010, corresponding to d_x) and 3 (011, corresponding to d_y). The only possible combination of three-phase pulses are:-



It should be guaranteed that
$D_b \ge D_c \ge D_a$

Then we have:-

$$\begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} \Rightarrow \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix}$$

$$d_x = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\beta) = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\alpha+120) = \frac{\sqrt{3}v_{ref}}{v_D}\sin\alpha$$

$$= \frac{3}{2v_D}\frac{2}{3}\sqrt{3}v_q = \frac{3}{2v_D}\frac{2}{\sqrt{3}}v_q$$

$$d_y = \frac{\sqrt{3}v_{ref}}{v_D}\sin\beta = \frac{\sqrt{3}v_{ref}}{v_D}\sin(\alpha-120) = \frac{3}{2v_D}\frac{2}{3}\sqrt{3}\left(-\frac{1}{2}v_q - \frac{\sqrt{3}}{2}v_d\right)$$

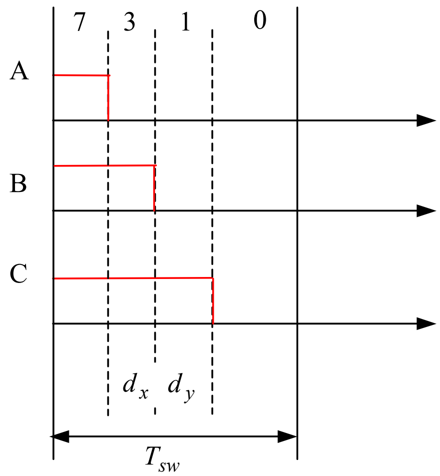
$$= -\frac{3}{2v_D}\left(\frac{v_q}{\sqrt{3}} + v_d\right)$$

$$D_a = \frac{1}{2} - \frac{1}{2}d_x - \frac{1}{2}d_y = \frac{1}{2} - \frac{v_{qn}}{\sqrt{3}} + \frac{1}{2}\left(\frac{v_{qn}}{\sqrt{3}} + v_{dn}\right) = \frac{1}{2} + \frac{1}{2}\left(v_{dn} - \frac{v_{qn}}{\sqrt{3}}\right)$$

$$D_b = \frac{1}{2} + \frac{1}{2}d_x + \frac{1}{2}d_y = \frac{1}{2} - \frac{1}{2}\left(v_{dn} - \frac{v_{qn}}{\sqrt{3}}\right)$$

$$D_c = \frac{1}{2} - \frac{1}{2}d_x + \frac{1}{2}d_y = \frac{1}{2} - \frac{v_{qn}}{\sqrt{3}} - \frac{1}{2}\left(\frac{v_{qn}}{\sqrt{3}} + v_{dn}\right)$$

For sector 4, $v_d < 0$, $v_q \le 0$, and $\frac{v_q}{v_d} \le \tan(240^o) \Rightarrow |v_q| \le \sqrt{3}|v_d|$, the two non-zero vectors are 3 (110, corresponding to d_x) and 1 (001, corresponding to d_y). The only possible combination of three-phase pulses are:-



It should be guaranteed that
$D_c \ge D_b \ge D_a$

$$\begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} \Rightarrow \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix}$$

$$d_x = \frac{\sqrt{3} v_{ref}}{v_D} \sin(60 - \beta) = \frac{\sqrt{3} v_{ref}}{v_D} \sin(60 - \alpha + 180) = -\frac{\sqrt{3} v_{ref}}{v_D} \sin(60 - \alpha)$$

$$= \frac{3}{2 v_D} \left(\frac{v_q}{\sqrt{3}} - v_d \right)$$

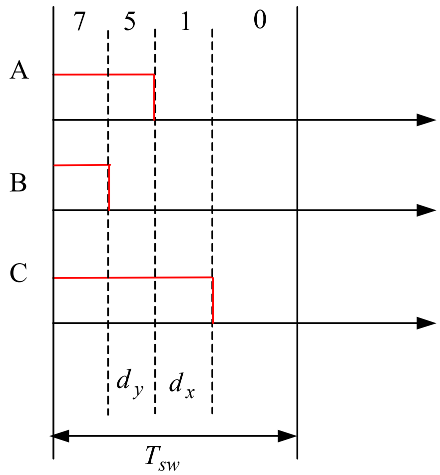
$$d_y = \frac{\sqrt{3} v_{ref}}{v_D} \sin \beta = \frac{\sqrt{3} v_{ref}}{v_D} \sin(\alpha - 180) = -\frac{\sqrt{3} v_{ref}}{v_D} \sin(\alpha) = -\frac{3}{2 v_D} \frac{2}{\sqrt{3}} v_q$$

$$D_a = \frac{1}{2} - \frac{1}{2} d_x - \frac{1}{2} d_y = \frac{1}{2} + \frac{1}{2} \left(v_{dn} + \frac{v_q}{\sqrt{3}} \right)$$

$$D_b = \frac{1}{2} + \frac{1}{2} d_x - \frac{1}{2} d_y = \frac{1}{2} + \frac{1}{2} \left(\frac{v_{qn}}{\sqrt{3}} - v_d \right) + \frac{v_{qn}}{\sqrt{3}}$$

$$D_c = \frac{1}{2} + \frac{1}{2} d_x + \frac{1}{2} d_y = \frac{1}{2} - \frac{1}{2} \left(\frac{v_{qn}}{\sqrt{3}} + v_{dn} \right)$$

For sector 5, $v_d < 0$, $v_q \le 0$, and $\frac{v_q}{v_d} > \tan(240^o) \Rightarrow |v_q| > \sqrt{3} |v_d|$, the two non-zero vectors are 1 (001, corresponding to d_x) and 5 (101, corresponding to d_y). The only possible combination of three-phase pulses are:-



It should be guaranteed that $D_c \geq D_a \geq D_b$

$$\begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} \Rightarrow \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix}$$

$$\begin{aligned} d_x &= \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\beta) = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\alpha+240) = -\frac{\sqrt{3}v_{ref}}{v_D}\sin(120-\alpha) \\ &= -\frac{3}{2v_D}\left(v_d + \frac{1}{\sqrt{3}}v_q\right) \end{aligned},$$

$$d_y = \frac{\sqrt{3}v_{ref}}{v_D}\sin\beta = \frac{\sqrt{3}v_{ref}}{v_D}\sin(\alpha-240) = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\alpha) = \frac{3}{2v_D}\left(v_d - \frac{1}{\sqrt{3}}v_q\right)$$

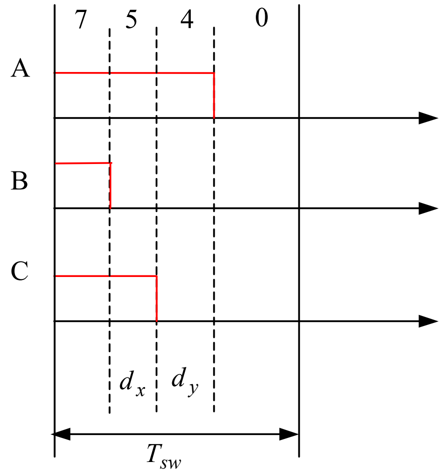
$$D_a = \frac{1}{2} - \frac{1}{2}d_x + \frac{1}{2}d_y = \frac{1}{2} + \frac{1}{2}\left(v_d + \frac{1}{\sqrt{3}}v_q\right) + \frac{1}{2}\left(v_d - \frac{1}{\sqrt{3}}v_q\right)$$

$$D_b = \frac{1}{2} - \frac{1}{2}d_x - \frac{1}{2}d_y = \frac{1}{2} + \frac{1}{2}\left(v_d + \frac{1}{\sqrt{3}}v_q\right) - \frac{1}{2}\left(v_d - \frac{1}{\sqrt{3}}v_q\right)$$

$$D_c = \frac{1}{2} + \frac{1}{2}d_x + \frac{1}{2}d_y$$

For sector 5, $v_d \geq 0$, $v_q < 0$, and $\frac{v_q}{v_d} < \tan(300^o) \Rightarrow |v_q| > \sqrt{3}|v_d|$, the two non-zero vectors are 1 (001, corresponding to d_x) and 5 (101, corresponding to d_y). The duty cycles are the same as for the previous sector 5.

For sector 6, $v_d \geq 0$, $v_q < 0$, and $\frac{v_q}{v_d} \geq \tan(300^o) \Rightarrow |v_q| \leq \sqrt{3}|v_d|$, the two non-zero vectors are 5 (101, corresponding to d_x) and 4 (100, corresponding to d_y). The only possible combination of three-phase pulses are:-



It should be guaranteed that
$D_a \ge D_c \ge D_b$

$$\begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} \Rightarrow \begin{bmatrix} D_a \\ D_b \\ D_c \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix}$$

$$d_x = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\beta) = \frac{\sqrt{3}v_{ref}}{v_D}\sin(60-\alpha+300) = -\frac{\sqrt{3}v_{ref}}{v_D}\sin\alpha$$

$$= -\frac{3}{2v_D}\frac{2}{\sqrt{3}}v_q$$

$$d_y = \frac{\sqrt{3}v_{ref}}{v_D}\sin\beta = \frac{\sqrt{3}v_{ref}}{v_D}\sin(\alpha-300) = \frac{\sqrt{3}v_{ref}}{v_D}\sin(120-\alpha) = \frac{3}{2v_D}\left(v_d + \frac{1}{\sqrt{3}}v_q\right)$$

$$D_a = \frac{1}{2} + \frac{1}{2}d_x + \frac{1}{2}d_y = \frac{1}{2} - \frac{v_{qn}}{\sqrt{3}} + \frac{1}{2}\left(v_{dn} + \frac{v_{qn}}{\sqrt{3}}\right) = \frac{1}{2} + \frac{1}{2}\left(v_{dn} - \frac{v_{qn}}{\sqrt{3}}\right)$$

$$D_b = \frac{1}{2} - \frac{1}{2}d_x - \frac{1}{2}d_y = \frac{1}{2} + \frac{v_{qn}}{\sqrt{3}} - \frac{1}{2}\left(v_{dn} + \frac{v_{qn}}{\sqrt{3}}\right) = \frac{1}{2} - \frac{1}{2}\left(v_{dn} - \frac{v_{qn}}{\sqrt{3}}\right)$$

$$D_c = \frac{1}{2} + \frac{1}{2}d_x - \frac{1}{2}d_y = \frac{1}{2} - \frac{v_{qn}}{\sqrt{3}} - \frac{1}{2}\left(v_{dn} + \frac{v_{qn}}{\sqrt{3}}\right)$$