Opgaver lektion 3 $\quad$ 2/3

4.37 Opgave 4.12 fortsat $\mu = \frac{1}{3}$

$$Var(X) = E(X^2) - \mu^2 = \int_0^1 x^2\, 2(1-x)\,dx - \frac{1}{3}^2$$

$$= \left[\frac{2}{3}x^3 - \frac{1}{2}x^4\right]_0^1 - \frac{1}{3}^2 = \frac{2}{3} - \frac{1}{2} - \frac{1}{9} = \underline{\underline{\frac{1}{18}}} \checkmark$$

4.43 $Y = 3X - 2$ $\qquad f(x) = \begin{cases} \frac{1}{4}e^{-x/4} & x>0 \\ 0 & \text{ellers} \end{cases}$

$$E(Y) = E(3X-2) = \int_0^\infty (3x-2)\cdot\frac{1}{4}e^{-x/4}\,dx = \frac{1}{4}\int_0^\infty (3x-2)e^{-\frac{x}{4}}\,dx$$

$$= \frac{1}{4}\left[(-12x-40)e^{-\frac{x}{4}}\right]_0^\infty = 0 - \frac{1}{4}\cdot(-40) = \underline{\underline{10}} \checkmark$$

$$Var(Y) = E[(3X-2-10)^2] = \int_0^\infty (3x-12)^2\cdot\frac{1}{4}e^{-\frac{x}{4}}\,dx$$

$$= \frac{9}{4}\int_0^\infty (x-4)^2\cdot e^{-\frac{x}{4}}dx = \frac{9}{4}\left[(-4x^2-64)e^{-\frac{x}{4}}\right]_0^\infty = -\frac{9}{4}(-64) = \underline{\underline{144}} \checkmark$$

4.47

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & 0\le x\le 1,\ 0\le y\le 1 \\ 0 & \text{ellers} \end{cases}$$

$$g(x) \overset{\text{int. y ud.}}{=} \int_0^1 \frac{2}{3}(x+2y)\,dy = \frac{2}{3}\left[xy+y^2\right]_0^1 = \frac{2}{3}(x+1) \checkmark \quad 0\le x\le 1$$

$$\mu_X = \int_0^1 \frac{2}{3}x(x+1)\,dx = \frac{2}{3}\left[\frac{1}{3}x^3+\frac{1}{2}x^2\right]_0^1 = \frac{2}{3}\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{5}{9} \checkmark$$

$$h(y) \overset{\text{int. x ud}}{=} \int_0^1 \frac{2}{3}(x+2y)\,dx = \frac{2}{3}\left[\frac{1}{2}x^2+2yx\right]_0^1 = \frac{2}{3}\left(\frac{1}{2}+2y\right) \checkmark$$

$$\mu_Y = \int_0^1 \frac{2}{3}y\left(\frac{1}{2}+2y\right)dy = \frac{2}{3}\left[\frac{1}{4}y^2+\frac{2}{3}y^3\right]_0^1 = \frac{2}{3}\left(\frac{1}{4}+\frac{2}{3}\right) = \frac{11}{18} \checkmark$$

$$E(X\cdot Y) = \int_0^1\int_0^1 x\cdot y\,\frac{2}{3}(x+2y)\,dx\,dy = \frac{2}{3}\int_0^1 y\int_0^1 x^2+2xy\,dx\,dy$$

$$= \frac{2}{3}\int_0^1 y\left[\frac{1}{3}x^3+\cancel{2}y\,x^2\right]_0^1 dy = \frac{2}{3}\int_0^1 y\left(\frac{1}{3}+y\right)dy = \frac{2}{3}\left[\frac{1}{6}y^2+\frac{1}{3}y^3\right]_0^1$$

$$= \frac{2}{3}\cdot\left(\frac{1}{6}+\frac{1}{3}\right) = \frac{1}{3} \checkmark$$

$$Cov(X,Y) = E(X\cdot Y) - \mu_X\mu_Y = \frac{1}{3} - \frac{5}{9}\cdot\frac{11}{18} = -\frac{1}{162} = \underline{\underline{-0.0062}} \checkmark$$

dvs. negativ korrelation

4.53 5 kartoner skummet mælk til \$1.2 pr karton en gros
sælger detail til \$1.65 pr. karton
Over salgsdatoen sendes tilbage og får $\frac{3}{4} \cdot \$1.2$ pr. karton

X : antal kartoner solgt, har flg. sandsynlighedfunktion

x	0	1	2	3	4	5
$f(x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{3}{15}$

Y: profit $Y = \underbrace{1.65X + 0.9 \cdot (5-X)}_{\text{indtægt}} - \underbrace{5 \cdot 1.2}_{\text{udgift}} = 0.75X - 1.5$

$E(Y) = E(0.75X - 1.5) = 0.75E(X) - 1.5 = 0.75 \sum_x x f(x) - 1.5$

$= 0.75 \cdot \left(\frac{2}{15} + \frac{4}{15} + \frac{9}{15} + \frac{16}{15} + \frac{15}{15}\right) - 1.5 = 0.8$

dvs. forventet profit på 5 kartoner mælk er $\underline{\underline{\$0.8}}$ ✓

4.64 X og Y uafh. med $Var(X) = 5$ $Var(Y) = 3$

$Z = -2X + 4Y - 3$ $Var(Z) = (-2)^2 Var(X) + 4^2 Var(Y) = 20 + 48 = \underline{\underline{68}}$

4.65 X og Y afh. med $Var(X) = 5$ $Var(Y) = 3$ $Cov(X,Y) = 1$

$Z = -2X + 4Y - 3$ $Var(Z) = (-2)^2 Var(X) + 4^2 Var(Y) - 16 Cov(X,Y) = \underline{\underline{52}}$ ✓

Opgaver lektion 4 1/2

5.5 30 % af alle fiaskoer er pga. operatorfejl

$n = 20 \quad P(\text{fiasko}) = 0.3 \quad X: \text{antal fiaskoer} \sim b(20, 0.3)$

a) $P(X \geq 10) = 1 - P(X < 10) = 1 - 0.9520 = \underline{\underline{0.0480}}$

b) $P(X \leq 4) = \underline{\underline{0.2375}}$

c) $P(X = 5) = \binom{20}{5} \cdot 0.3^5 \cdot 0.7^{15} = \frac{20!}{5!\,15!}\, 0.3^5 \cdot 0.7^{15} = 0.1789$

dvs. rimelig ss. at $X=5$ for $b(20, 0.3)$ dvs. 30% fiaskoer passer rimelig godt til den her fabrik.

Tabelopslag: eller $P(X=5) = P(X\leq 5) - P(X \leq 4) = 0.4164 - 0.2375 = 0.1789$

5.35 Udtag $n=5$ enheder ud af $N=50$ ens produkter, hvoraf 20% er defekte, dvs $k = \frac{50 \cdot 20}{100} = 10$.

Inspektionsprocedure: udtag 5 enheder, hvis mindre end eller netop 2 defekte elementer findes godtages partiet på 50.

Hvad er sandsynlighed for at partier der indeholder 20 % defekte enheder accepteres?

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$ (hyper-geometrisk)

$$= \frac{\binom{10}{0}\binom{40}{5}}{\binom{50}{5}} + \frac{\binom{10}{1}\binom{40}{4}}{\binom{50}{5}} + \frac{\binom{10}{2}\binom{40}{3}}{\binom{50}{5}}$$

$$= \frac{40!\,45!\,5!}{5!\,35!\,50!} + \frac{10!\,40!\,5!\,45!}{9!\,4!\,36!\,50!} + \frac{10!\,40!\,5!\,45!}{2!\,8!\,3!\,37!\,50!}$$

$$= \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36}{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46} + \frac{10 \cdot 5 \cdot 40 \cdot 39 \cdot 38 \cdot 37}{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46} + \frac{10 \cdot 9 \cdot 5 \cdot 4 \cdot 40 \cdot 39 \cdot 38}{2 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46}$$

$= 0,3106 + 0,4313 + 0.2099$

$= \underline{\underline{0,9518}}$

Opgaver lektion 4

5.71 X : antal kunder i bilservice pr. time $X \sim po$ med mv. 7

dvs. $X \sim pois(7 \cdot 1)$

a) $Y \sim Po(7 \cdot 2)$ Y : antal kunder pr. 2 timer

$P(Y > 10) = 1 - P(Y \leq 10) = 1 - 0.1757 = 0.8243$ ✓

b) $E(Y) = \underline{\underline{14}}$ dvs. gennemsnitslig 14 kunder pr 2. timer

5.58 I et givet vejkryds er der gennemsnitlig 3 uheld pr. måned

X : antal uheld $\sim pois(3 \cdot 1)$

a) $P(X = 5) = \dfrac{e^{-3} \cdot 3^5}{5!} = 0.1008$ eller tabelopslag

$P(X = 5) = P(X \leq 5) - P(X \leq 4) = 0.9161 - 0.8153 = \underline{\underline{0.1008}}$

b) $P(X < 3) = P(X \leq 2) = \underline{\underline{0.4232}}$

c) $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1) = 1 - 0.1991 = \underline{\underline{0.8009}}$