

4.37

Oppgave 4.12 fortsat $\mu = \frac{1}{3}$

$$\text{Var}(X) = E(X^2) - \mu^2 = \int_0^1 x^2 \cdot 2(1-x) dx - \frac{1}{3}^2$$

$$= \left[\frac{2}{3}x^3 - \frac{1}{2}x^4 \right]_0^1 - \frac{1}{9} = \frac{2}{3} - \frac{1}{2} - \frac{1}{9} = \underline{\underline{\frac{1}{18}}}$$

4.43

$$Y = 3X - 2$$

$$f(x) = \begin{cases} \frac{1}{4} e^{-x/4} & x > 0 \\ 0 & \text{ellers} \end{cases}$$

$$E(Y) = E(3X - 2) = \int_0^{\infty} (3x - 2) \cdot \frac{1}{4} e^{-x/4} dx = \frac{1}{4} \int_0^{\infty} (3x - 2) e^{-x/4} dx$$

$$= \frac{1}{4} \left[(-12x - 40) e^{-x/4} \right]_0^{\infty} = 0 - \frac{1}{4} \cdot (-40) = \underline{\underline{10}}$$

$$\text{Var}(Y) = E[(3X - 2 - 10)^2] = \int_0^{\infty} (3x - 12)^2 \cdot \frac{1}{4} e^{-x/4} dx$$

$$= \frac{9}{4} \int_0^{\infty} (x - 4)^2 \cdot e^{-x/4} dx = \frac{9}{4} \left[(-4x^2 - 64) e^{-x/4} \right]_0^{\infty} = -\frac{9}{4} (-64) = \underline{\underline{144}}$$

4.47

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{ellers} \end{cases}$$

$$g(x) \stackrel{\text{int. y ud.}}{=} \int_0^1 \frac{2}{3}(x + 2y) dy = \frac{2}{3} [xy + y^2]_0^1 = \frac{2}{3}(x + 1) \quad 0 \leq x \leq 1$$

$$\mu_X = \int_0^1 \frac{2}{3}x(x + 1) dx = \frac{2}{3} \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 \right]_0^1 = \frac{2}{3} \left(\frac{1}{3} + \frac{1}{2} \right) = \frac{5}{9}$$

$$h(y) \stackrel{\text{int. x ud.}}{=} \int_0^1 \frac{2}{3}(x + 2y) dx = \frac{2}{3} \left[\frac{1}{2}x^2 + 2yx \right]_0^1 = \frac{2}{3} \left(\frac{1}{2} + 2y \right) \quad 0 \leq y \leq 1$$

$$\mu_Y = \int_0^1 \frac{2}{3}y \left(\frac{1}{2} + 2y \right) dy = \frac{2}{3} \left[\frac{1}{4}y^2 + \frac{2}{3}y^3 \right]_0^1 = \frac{2}{3} \left(\frac{1}{4} + \frac{2}{3} \right) = \frac{11}{18}$$

$$E(X \cdot Y) = \int_0^1 \int_0^1 x \cdot y \cdot \frac{2}{3}(x + 2y) dx dy = \frac{2}{3} \int_0^1 y \int_0^1 (x^2 + 2xy) dx dy$$

$$= \frac{2}{3} \int_0^1 y \left[\frac{1}{3}x^3 + \frac{2}{2}x^2 y \right]_0^1 dy = \frac{2}{3} \int_0^1 y \left(\frac{1}{3} + y \right) dy = \frac{2}{3} \left[\frac{1}{6}y^2 + \frac{1}{3}y^3 \right]_0^1$$

$$= \frac{2}{3} \cdot \left(\frac{1}{6} + \frac{1}{3} \right) = \frac{1}{3}$$

$$\text{Cov}(X, Y) = E(X \cdot Y) - \mu_X \mu_Y = \frac{1}{3} - \frac{5}{9} \cdot \frac{11}{18} = -\frac{1}{162} = -0.0062 \quad \checkmark$$

alts negativ korrelasjon

Opgaver lektion 3

3/3

4.53 5 kartoner skummet mælk til \$1.2 pr. karteon en gras sælger detail til \$1.65 pr. karteon
Over salgsdatoen sendes tilbage og får $\frac{3}{4} \cdot \$1.2$ pr. karteon

X : antal kartoner solgt, har flg. sandsynlighedsfunktion

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|
| $f(x)$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{4}{15}$ | $\frac{3}{15}$ |

Y : profit $Y = \underbrace{1.65X + 0.9 \cdot (5-X)}_{\text{indtægt}} - \underbrace{5 \cdot 1.2}_{\text{udgift}} = 0.75X - 1.5$

$$E(Y) = E(0.75X - 1.5) = 0.75E(X) - 1.5 = 0.75 \sum_x x f(x) - 1.5$$

$$= 0.75 \cdot \left(\frac{2}{15} + \frac{4}{15} + \frac{9}{15} + \frac{16}{15} + \frac{15}{15} \right) - 1.5 = 0.8$$

dis. forventet profit på 5 kartoner mælk er \$0.8 ✓

4.64 X og Y uafh. med $\text{Var}(X) = 5$ $\text{Var}(Y) = 3$

$$Z = -2X + 4Y - 3 \quad \text{Var}(Z) = (-2)^2 \text{Var}(X) + 4^2 \text{Var}(Y) = 20 + 48 = \underline{\underline{68}}$$

4.65 X og Y afh. med $\text{Var}(X) = 5$ $\text{Var}(Y) = 3$ $\text{Cov}(X, Y) = 1$

$$Z = -2X + 4Y - 3 \quad \text{Var}(Z) = (-2)^2 \text{Var}(X) + 4^2 \text{Var}(Y) - 16 \text{Cov}(X, Y) = \underline{\underline{52}} \quad \checkmark$$

Opgaver lektion 4

1/2

5.5 30 % af alle flasker er pga. operatørfek
 $n = 20$ $P(\text{flaske}) = 0.3$ X : antal flasker $\sim \text{bi}(20, 0.3)$

a) $P(X \geq 10) = 1 - P(X < 10) = 1 - 0.9520 = \underline{\underline{0.0480}}$

b) $P(X \leq 4) = \underline{\underline{0.2375}}$

c) $P(X = 5) = \binom{20}{5} \cdot 0.3^5 \cdot 0.7^{15} = \frac{20!}{5!15!} \cdot 0.3^5 \cdot 0.7^{15} = 0.1789$

abelopsky:

eller $P(X=5)$

$P(X \leq 5) - P(X \leq 4)$

$= 0.4164 - 0.2375 = 0.1789$

des. rimelig ss. at $X=5$ for $\text{bi}(20, 0.3)$ des. 30%
 flasker passer rimelig godt til den her fabrik.

5.35 Udtag $n=5$ enheder ud af $N=50$ ens produkter, hvoraf 20%
 er defekte, des $K = \frac{50 \cdot 20}{100} = 10$.

Inspektionsprocedure: Udtag 5 enheder, hvis mindre end
 eller netop 2 defekte elementer findes
 godkendes partiet på 50.

Hvad er sandsynlighed for at partiet der indeholder 20%
 defekte enheder accepteres?

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$ (hyper-geometrisk)

$= \frac{\binom{10}{0} \binom{40}{5}}{\binom{50}{5}} + \frac{\binom{10}{1} \binom{40}{4}}{\binom{50}{5}} + \frac{\binom{10}{2} \binom{40}{3}}{\binom{50}{5}}$

$= \frac{10 \cdot 71 \cdot 38 \cdot 37 \cdot 36}{50! \cdot 45! \cdot 5!} + \frac{10 \cdot 9 \cdot 40 \cdot 39 \cdot 38 \cdot 37}{10! \cdot 40! \cdot 5! \cdot 45!} + \frac{10 \cdot 9 \cdot 5 \cdot 4 \cdot 40 \cdot 39 \cdot 38}{10! \cdot 40! \cdot 5! \cdot 45!}$

$= 0.3106 + 0.4313 + 0.2099$

$= \underline{\underline{0.9518}}$

Opgaver lektion 4

2/2

5.71 \bar{X} : antal kunder i bilservice pr. time $\bar{X} \sim \text{po}$ med mv. 7
dvs. $\bar{X} \sim \text{pois}(7.1)$

a) $\bar{Y} \sim \text{po}(7.2)$ \bar{Y} : antal kunder pr. 2 timer

$$P(\bar{Y} > 10) = 1 - P(\bar{Y} \leq 10) = 1 - 0.1757 = 0.8243 \checkmark$$

b) $E(\bar{Y}) = \underline{14}$ dvs. gennemsnitlig 14 kunder pr 2. timer

5.58 I et givet vejkræs er der gennemsnitlig 3 uheld pr måned
 \bar{X} : antal uheld $\sim \text{pois}(3.1)$

a) $P(\bar{X} = 5) = \frac{e^{-3} \cdot 3^5}{5!} = 0.1008$ eller tabelgæng

$$P(\bar{X} = 5) = P(\bar{X} \leq 5) - P(\bar{X} \leq 4) = 0.9161 - 0.8153 = \underline{\underline{0.1008}}$$

b) $P(\bar{X} < 3) = P(\bar{X} \leq 2) = \underline{\underline{0.4232}}$

c) $P(\bar{X} \geq 2) = 1 - P(\bar{X} < 2) = 1 - P(\bar{X} \leq 1) = 1 - 0.1991 = \underline{\underline{0.8009}}$