The Use of Harmonic Distortion to Increase the Output Voltage of a Three-Phase PWM Inverter

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Abstract—**By adding a measure of third harmonic to the output of each phase of a three-phase inverter, it is possible to obtain a line-to-line output voltage that is 15 percent greater than that obtainable when pure sinusoidal modulation is employed. The line-to-line voltage is undistorted. The method permits the inverter to deliver an output voltage approximately equal to the voltage of the ac supply to the inverter. Thus an induction motor of standard rating with respect to the ac supply to the inverter can deliver very nearly full power at rated speed when supplied from the inverter. This is achieved without pulse dropping or any other form of mode-changing.**

I. INTRODUCTION

A PULSEWIDTH modulated (PWM) inverter, employing pure sinusoidal modulation, cannot supply sufficient voltage to enable a standard motor to operate at rated power and rated speed. Sufficient voltage can be obtained from the inverter by overmodulating, but this produces distortion of the output waveform. This paper demonstrates that the necessary increase in output voltage can be obtained without recourse to overmodulation and without distortion of the line-to-line output waveform.

The problem is illustrated in Fig. 1. The dc supply for an inverter is usually obtained by rectifying a three-phase mains supply. Let the line-to-line voltage of this supply be V_{in}. The dc link voltage will then be approximately equal to the peak voltage of the three-phase supply. Therefore,

$$V_{dc} = \sqrt{2}\ V_{in}\ . \tag{1}$$

In practice, the V_{dc} will be slightly less than this due to the forward voltage drop of the rectifier and to the ripple on the dc link.

As Fig. 2 shows, the peak-to-peak output voltage obtainable from each phase of the PWM inverter is equal to the dc link voltage. Thus the rms output voltage of each phase is given by

$$V_{out(phase)} = \frac{V_{dc}}{2} \cdot \frac{1}{\sqrt{2}}\ .$$

Substituting from (1), we obtain

$$V_{out(phase)} = \frac{\sqrt{2}V_{in}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{V_{in}}{2}\ . \tag{2}$$

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The line-to-line output voltage is therefore given by

$$V_{out} = \sqrt{3}\ \frac{V_{in}}{2} = 0.866\ V_{in}. \tag{3}$$

From the foregoing, it can be seen that an inverter, operating from a supply of standard voltage and driving a standard voltage motor, can maintain the correct relationship between output voltage and output frequency only up to 0.866 of the supply frequency (e.g., 52 Hz for 60-Hz mains, and 43 Hz for 50-Hz mains).

It is, of course, possible to increase the output voltage of an inverter beyond that which is obtainable with sinusoidal PWM. The traditional method of achieving this is by dropping pulses from the PWM waveform. This method is successfully employed in present state-of-the-art PWM waveform generators [1], [2]. The disadvantages of this method, however, are that the abrupt dropping of pulses may result in a step change in output voltage, which can cause motor current instability [3], and the harmonic content of the output increases as pulses are removed from the waveform, thereby increasing motor losses. The problem of step changes in the output voltage may be negligible [1], [2] or may be overcome by various techniques [4], [5], but the problem of increased output distortion remains. The aim of this paper is to show that the line-to-line output voltage of a PWM inverter can be increased by up to 15 percent without the need for pulse dropping or any other form of overmodulation, and without any material effect on the harmonic content of the line-to-line waveform.

II. DESCRIPTION OF METHOD

A. General

The method involves the addition of triplen harmonics to the phase voltage waveforms. It is shown in the Appendix that all triplen harmonics in a three-phase supply are cophasal and are therefore eliminated from the line-to-line waveforms. Although the triplen harmonics are eliminated from the line waveforms, their effect on the phase waveform is to decrease the peak voltage. King has used harmonic distortion of this type to produce flat-topped phase waveforms which improve the efficiency of a Class B transistor inverter [6]. This is achieved by the addition of various amounts of third, ninth, fifteenth, etc., harmonics. The method clearly has potential for extending the rating of all PWM inverters. However, King does not identify the optimum amount of each triplen harmonic to be used. Hodkinson and Mills have similarly employed harmonic distortion to increase the output voltage of an inverter, but do not analyze the process [7]. The object of this present paper is to determine the manner of distorting the

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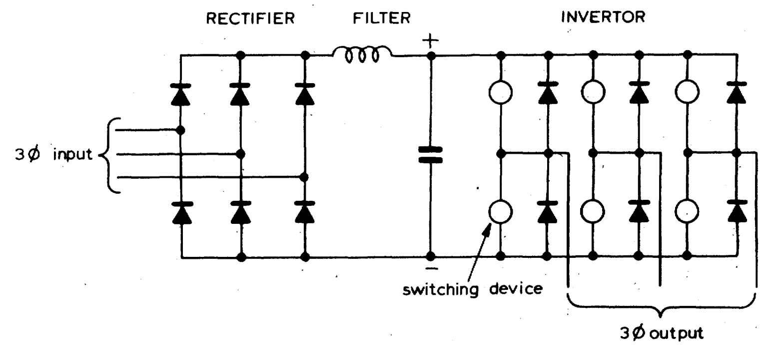


Fig. 1. Inverter circuit.

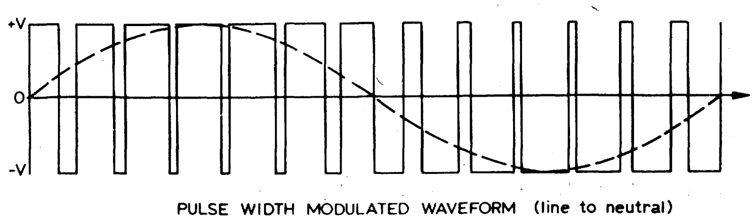


Fig. 2. PWM waveform with maximum output using sinusoidal modulation.

phase waveform so as to obtain optimum performance from the inverter.

B. Calculation of Optimum Distortion

The best modification that can be made to the inverter phase output waveform is assumed *a priori* to be the addition of a measure of third harmonic. It will be subsequently shown that no further improvement can be derived from the addition of other triplen harmonics.

Assume a phase waveform of the type

$$y = \sin\theta + a\sin 3\theta \tag{4}$$

where $\Theta = \omega t$ and a is the parameter to be determined. First we locate the turning points of this function by differentiating y with respect to Θ and equating the result to zero. Thus

$$\frac{dy}{d\theta} = \cos\theta + 3a\cos 3\theta = 0. \tag{5}$$

The maxima and minima of the waveform therefore occur at

$$\cos\theta = 0 \tag{6}$$

and

$$\cos\theta = \left(\frac{9a-1}{12a}\right)^{1/2} \tag{7}$$

From (6) we have

$$\sin\theta = 1 \tag{8}$$

and from (7) we have

$$\sin\theta = \left(\frac{1+3a}{12a}\right)^{1/2} \tag{9}$$

(using the identify that $\sin\theta = (1-\cos^2\theta)^{1/2}$). The peak value of y can be found by substituting the values obtained for $\sin\theta$ in (8) and (9) into (4). This is facilitated by first manipulating (4) using the identity

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta. \tag{10}$$

Thus (4) becomes

$$y = (1+3a)\sin\theta - 4a\sin^3\theta. \tag{11}$$

Substituting for $\sin\theta$, the values obtained in (8) and (9), we have:

$$\hat{y} = 1 - a \tag{12}$$

$$\hat{y} = 8a\left(\frac{1+3a}{12a}\right)^{3/2} \tag{13}$$

where $\hat{y}$ is the peak value of y.

The optimum value for a is that value which minimises $\hat{y}$. The optimum value of a can therefore be found by differentiating the expression for $\hat{y}$ and equating the result to zero. Thus, from (13),

$$\frac{d\hat{y}}{da} = \left(\frac{1+3a}{12a}\right)^{1/2}\left(2 - \frac{1}{3a}\right) = 0 \tag{14}$$

from which we obtain

$$a = -\frac{1}{3} \tag{15}$$

$$a = \frac{1}{6}.$$

From (12), we can see that negative values of a give values of $\hat{y}$ greater than unity and can therefore be disregarded. The required value of a is therefore 1/6, and the required waveform is

$$y = \sin\theta + \frac{1}{6}\sin 3\theta. \tag{17}$$

To demonstrate that no further reduction in $\hat{y}$ is possible by the addition of other triplen harmonics, the values of θ at which the peaks of y occur are found by substituting for a in (6) and (7). As may be expected, (6) gives $\theta = \pi/2$, independent of a, but (7) gives

$$\cos\ \theta = \tfrac{1}{2},$$

i.e.,

$$\theta = \pi/3,\ 2\pi/3,\ \text{etc.} \tag{18}$$

All triplen harmonics pass through zero at these values of θ. Thus no further reduction in $\hat{y}$ can be effected by the addition of other triplen harmonics, and the original assumption is justified.

If we now substitute these values of $\theta(=n\pi/3)$ in (17), we obtain the peak values of y. Hence

$$y = \pm\sqrt{3}/2$$

and

$$\hat{y} = \pm 0.866. \tag{19}$$

C. Increasing the Output Voltage

As has been shown, the addition of one-sixth of third harmonic to the modulating waveform has the effect of reducing the peak value of the output waveform by a factor of 0.866 without changing the amplitude of the fundamental. This process is illustrated in Fig. 3. It is then possible to increase the amplitude of the modulating wave by a factor K so that the full output voltage range of the inverter is again utilized. Thus the modulating waveform becomes

$$y = K\ \left(\sin\ \theta + \frac{1}{6}\ \sin\ 3\theta\right). \tag{20}$$

Assuming no minimum pulsewidth limitations, $\hat{y}$ can equal unity. From (19) we know that the previous peak value of y was 0.866. Therefore, we have

$$1 = K \times 0.866$$

$$K = 1/0.866 = 1.155. \tag{21}$$

Thus as Fig. 3 shows, the addition of one-sixth of third harmonic produces a 15.5- percent increase in the amplitude of the fundamental of the phase voltage waveform and, therefore, in the line voltage waveform. Even when minimum pulsewidth limitations are taken into consideration, a similar increase in output voltage is possible. The line-to-line waveform is undistorted since the third harmonic components in the phase waveforms cancel. Note that it is necessary for the inverter to have a bandwidth equal to three times the wanted output frequency in order to satisfactorily handle the third harmonic.

III. TEST CIRCUIT

The principle has been proved on a 1.9-kVA three-phase inverter employing a high carrier frequency and filtering of the phase waveforms. The output waveforms are shown in Fig. 4.

The power circuit of the inverter is shown in Fig. 5. The inverter uses power MOSFET's as the switching devices. The

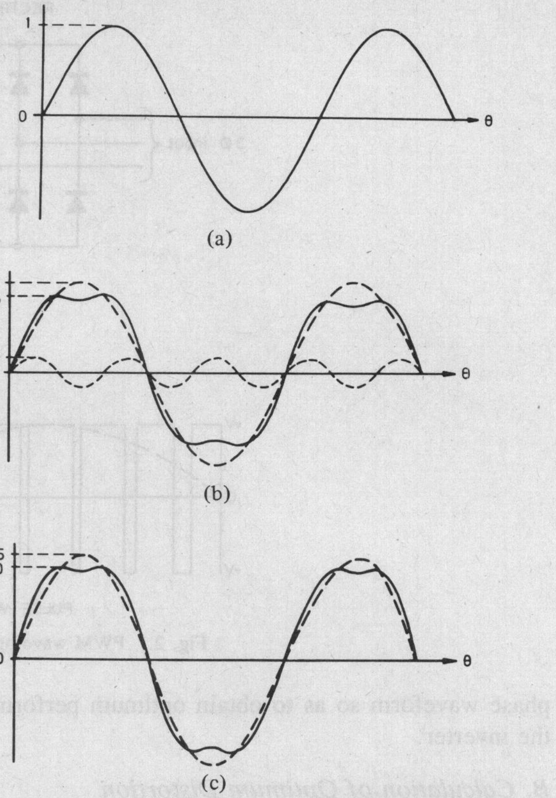


Fig. 3. Increasing fundamental output voltage by addition of third harmonic. (a) No third harmonic. Peak value = 1. Amplitude of fundamental = 1. (b) With one-sixth of third harmonic added. Peak value = 0.866. Amplitude of fundamental = 1. (c) With one-sixth of third harmonic added and peak value restored to one. Peak value = 1. Amplitude of fundamental = 1.155.

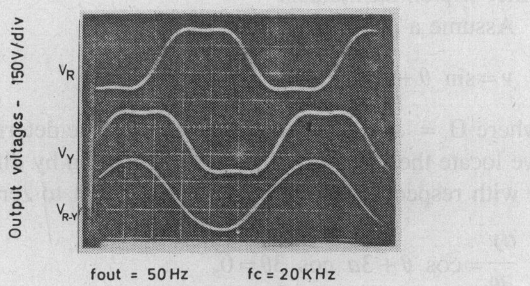
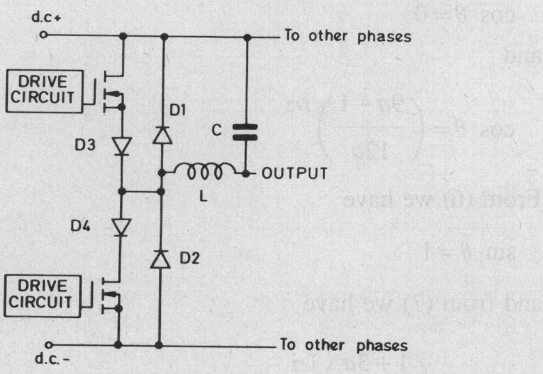


Fig. 4. Output voltage waveforms.



Transistors - BUZ45
Diodes D1, D2, BYV29 - 400
Diodes D3, D4, BYV29 - 150
L = 6 mH
C = 2·2 μF

Fig. 5. Inverter circuit.

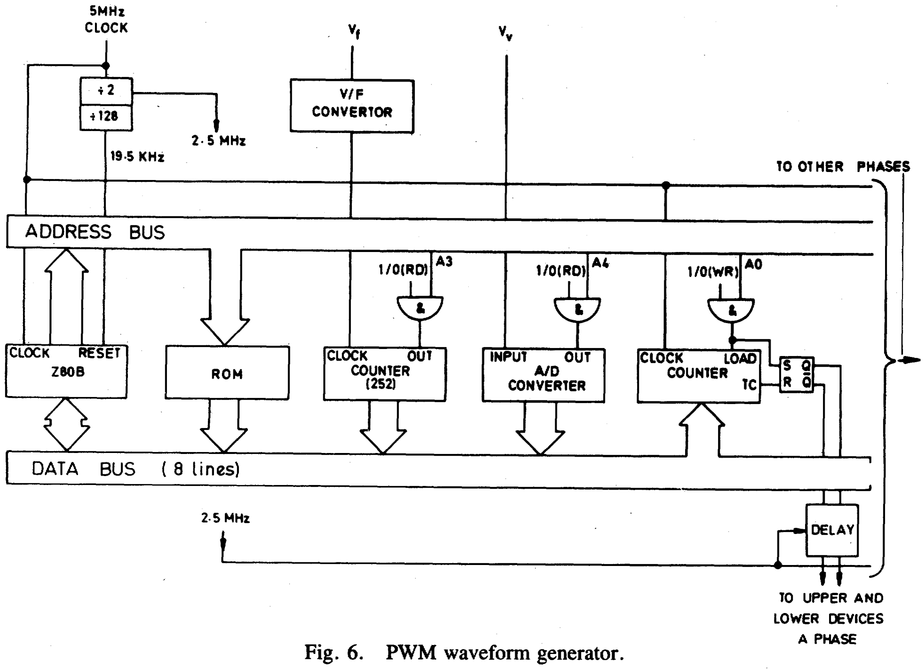


Fig. 6. PWM waveform generator.

high switching speed of the MOSFET's permits the use of an ultrasonic carrier. This gives the inverter a bandwidth which encompasses both the fundamental and its third harmonic for output frequencies far beyond normal mains frequency.

The PWM waveform was generated digitally using microprocessor-based waveform generator. Fig. 6 shows a schematic outline of the waveform generator. Using a carrier frequency the carrier period is only 51 μs. To minimize the harmonic content of the PWM waveform, the positions of the edges of the waveform must be calculated every cycle for all three phases. To make maximum use of the time available for computation, sine wave values are stored in a lookup table in th read-only memory (ROM). The one-sixth of third harmonic is incorporated in this table. The output of a counter external to the microprocessor is used to address the lookup table. To obtain the waveform values for each of the three phases, an algorithm in the microprocessor programme modifies these values to implement voltage control. The modified values are then supplied to counter timers to determine the instants of switching of the power devices.

IV. CONCLUSION

It is possible to increase the output voltage of a PWM inverter beyond that obtainable using pure sinusoidal modulation by adding a measure of third harmonic to the modulating waveform. The maximum increase in output voltage is obtained when the amplitude of the third harmonic is one-sixth that of the fundamental. The method permits a 15-percent increase in the output voltage of PWM inverters without causing distortion of the line-to-line waveforms.

APPENDIX

CANCELLATION OF TRIPLEN HARMONICS

It can be shown that triplen harmonics contained in the phase waveforms of the output of a three-phase inverter are eliminated from the line-to-line voltage waveforms. Consider a three-phase supply. Let the output waveforms be composed of the fundamentals plus an arbitrary amount of any triplen harmonic. Therefore,

$$V_A = \sin\,\omega t + a\,\sin\,3n\omega t \tag{A1}$$

$$V_B = \sin\,(\omega t - 2\pi/3) + a\,\sin\,3n(\omega t - 2\pi/3)$$

$$= \sin\,(\omega t - 2\pi/3) + a\,\sin\,(3n\omega t - 2n\pi) \tag{A2}$$

$$V_C = \sin\,(\omega t - 4\pi/3) + a\,\sin\,3n(\omega t - 4\pi/3)$$

$$= \sin\,(\omega t - 4\pi/3) + a\,\sin\,(3n\omega t - 4n\pi). \tag{A3}$$

The fundamental components of angular frequency ω form a three-phase set, whereas the triplen components are cophasal. At triplen frequency, therefore, no voltage difference exists between phases and all triplen harmonics are eliminated from the line-to-line voltage waveform.

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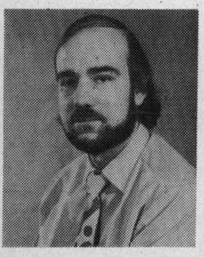

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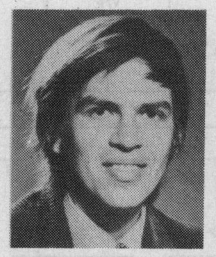

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