Dynamic modeling of electric machines

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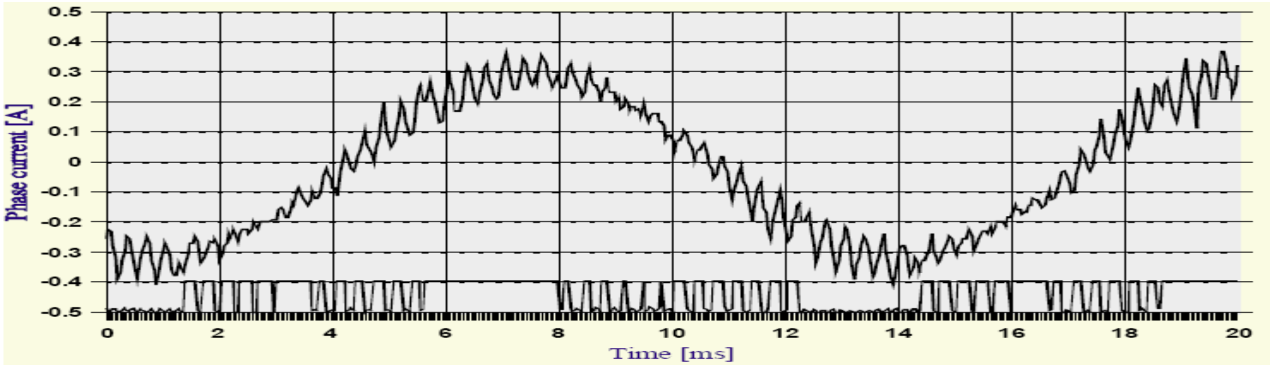
Lecture 1 - content

Introduction & dynamic modeling of transformer

- Introduction – why dynamic modeling? How?
- Case study - transformer
    - How to model?
    - How to solve?

Introduction – why dynamic modeling?



PWM voltage and current

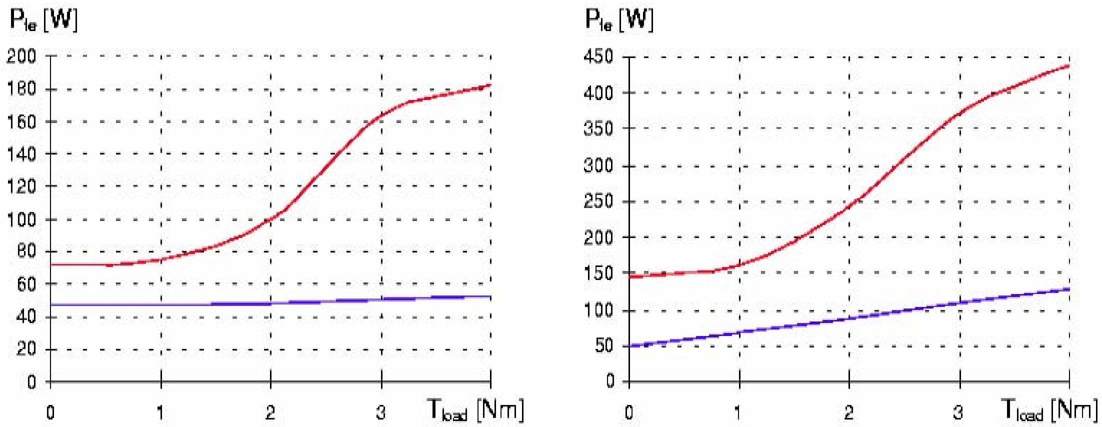


Figure 7.11 Comparison of calculated iron losses as function of the load for sinusoidal voltage, blue, and PAM-voltage, red. To the left for f = 30 Hz. To the right for f = 50 Hz.

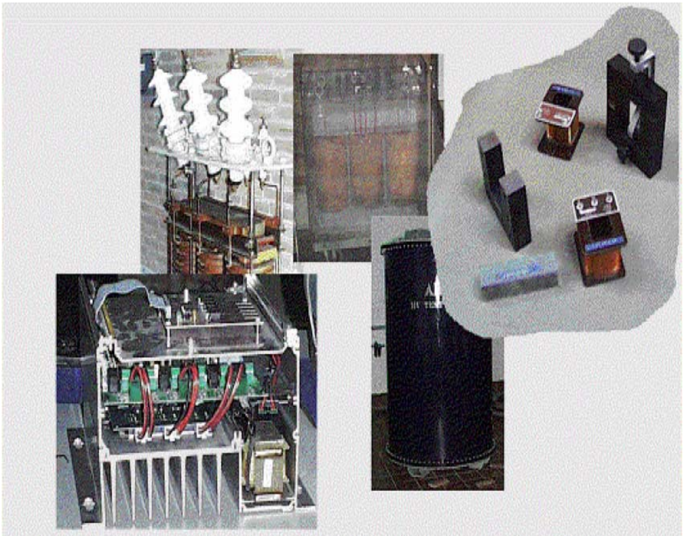
Introduction – why dynamic modeling?

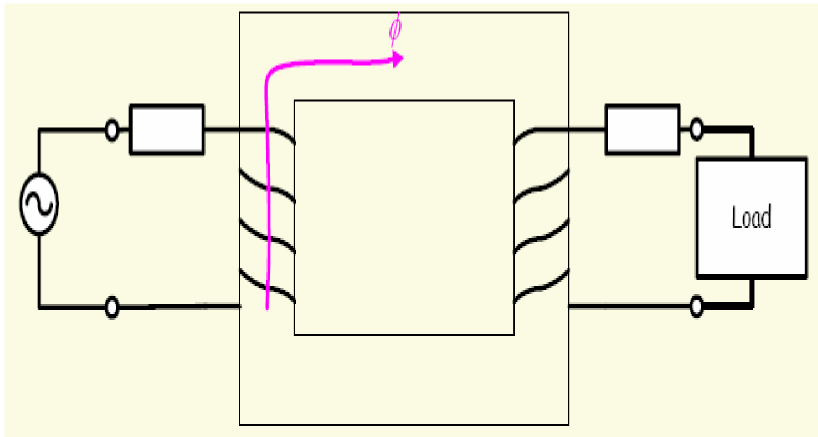
- Torque transient response, interested for dynamic performance of a motor drive system
- Transient current analysis, - detection of the dangerous peak current
- Dangerous situations, e.g. winding short-circuited of a generator
- ……

A dynamic model is a more 'general' model than the steady-state model

Understanding dynamic modeling case study – transformer

- Transformer is an analog to the Induction Motor
- Cover some of the important issues regarding motor modeling





‘modeling’ – what we are looking for?

Equations linking terminal voltage and line current

For ANY electromagnetic devices, a general equation can always be written as

$$v = Ri + \frac{d\lambda}{dt}$$

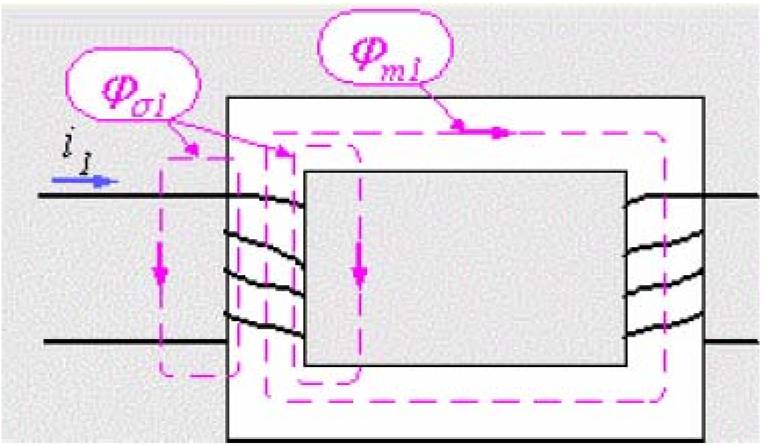
Out task is to find how the flux linkage is obtained.

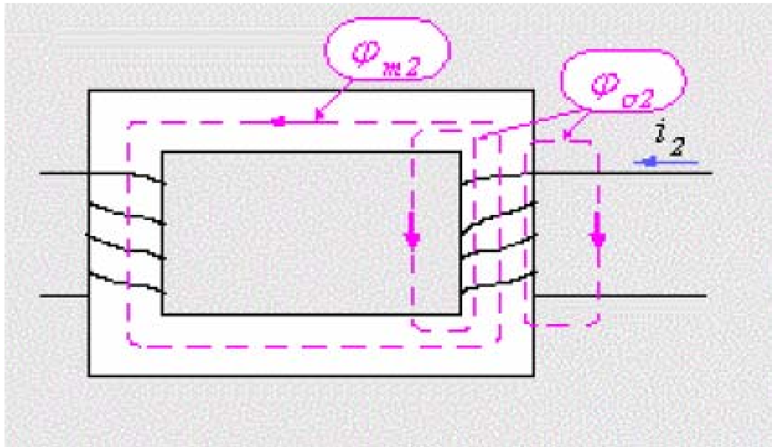
Remember, flux linkage is often linked to the inductance

$$\lambda = L_{11} i_1 + M_{12} i_2 + \ldots$$

Find parameters based on the flux pattern

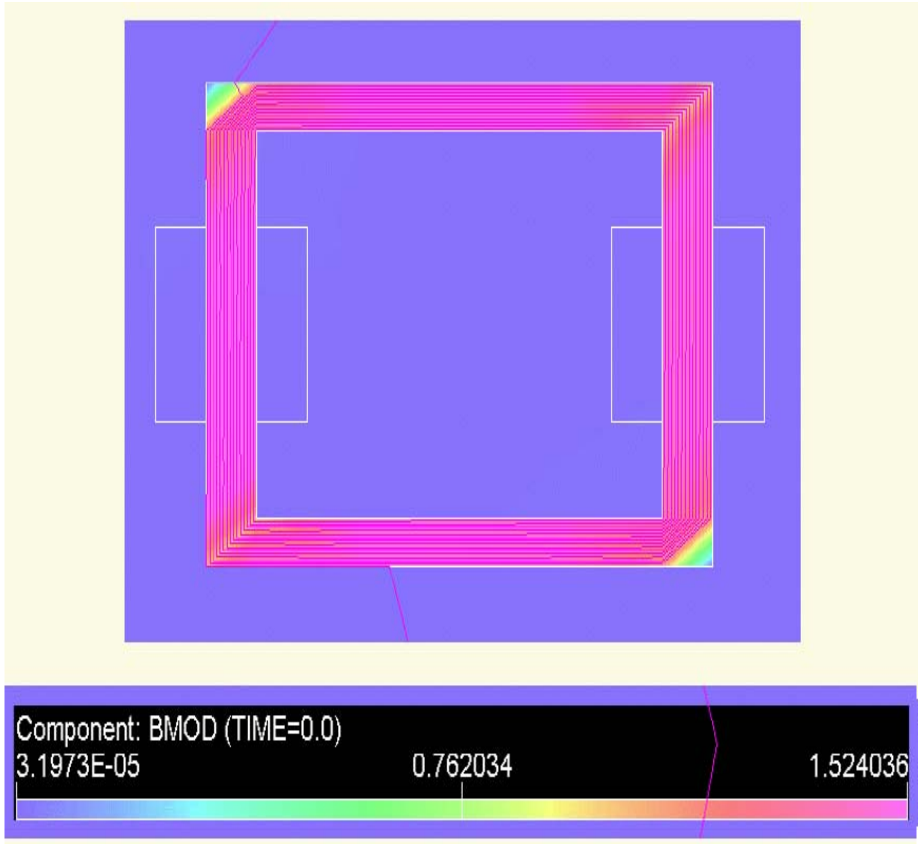
- Mutual inductance (M) <<< flux linking both windings
- magnetization inductance (L_m) <<< flux linking both windings
- Leakage inductance (L_σ) <<< flux traveling in the air
- Winding inductance (self-inductance) $L = L_\sigma + L_m$
- $L_m = M$ only valid when both windings have 1-turn only



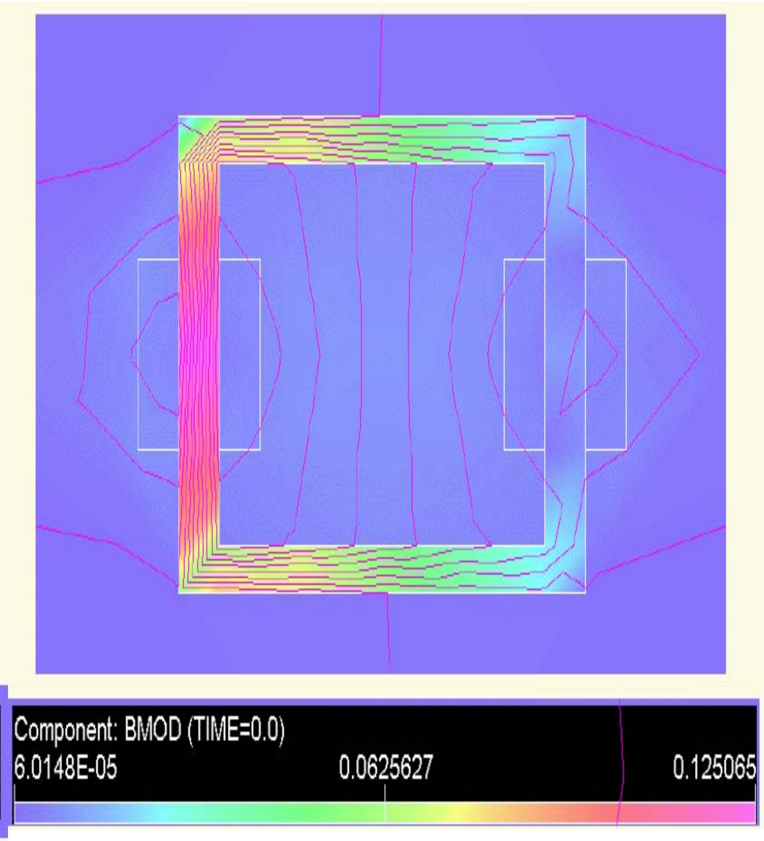


FE modeling of a transformer – observe the leakage flux

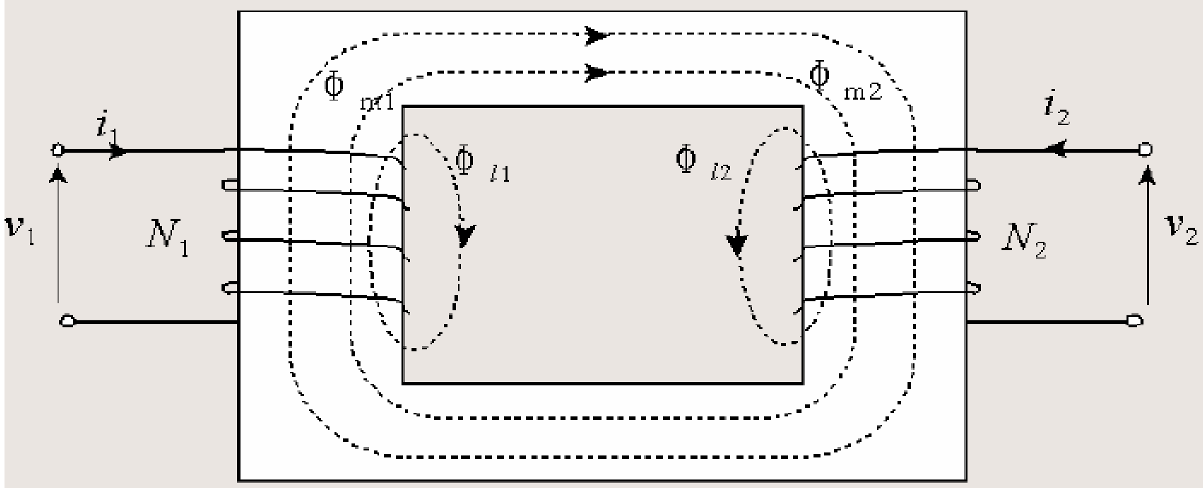
At no-load (Primary 25 A)



Short-circuited (Primary 25 A)



Transformer – how the windings are coupled



$$\Phi_1 = \Phi_{l1} + \Phi_{m1} + \Phi_{m2}$$

$$\Phi_2 = \Phi_{l2} + \Phi_{m2} + \Phi_{m1}$$

Don't forget how the positive directions are defined!

Very important – flux linkage expression is based on the positive directions defined!

Step. 1 define all the voltage, current, and flux positive directions

Step. 2 Write down the flux linkage expression for each winding

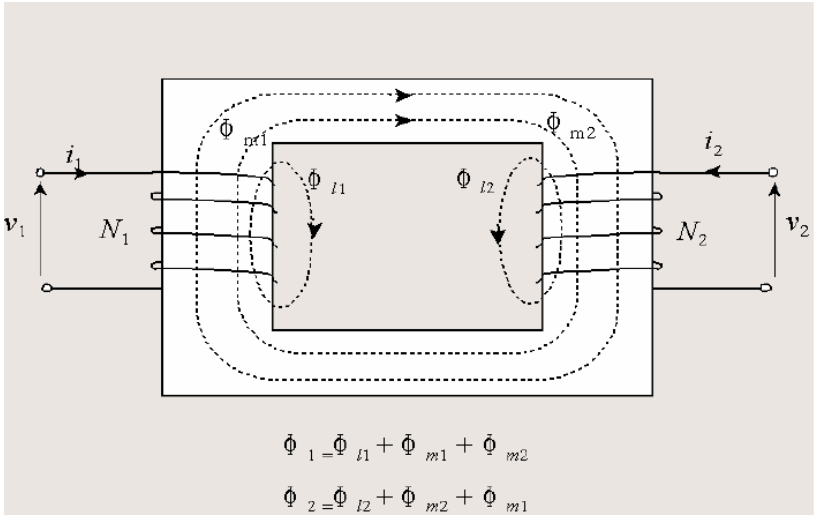
E.g. winding 1

When no current in winding 2.
winding 1 carries positive current

$$\phi_1 = \phi_{l1} + \phi_{m1}$$

Flux linkage becomes

$$\lambda_1 = N_1 \phi_1$$



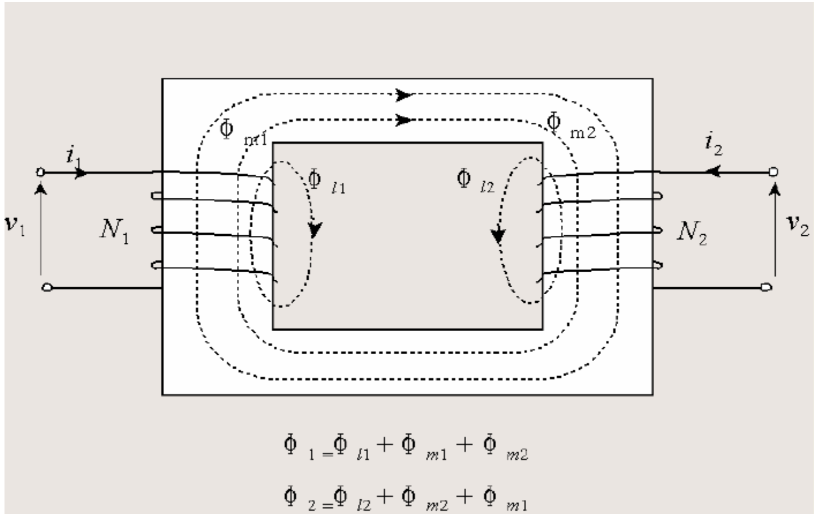
Very important – flux linkage expression is based on the positive directions defined!

Each flux linkage corresponds to an inductance

$$\lambda_1 = N_1\phi_1 = N_1\phi_{l1} + N_1\phi_{m1} = L_{1\sigma}i_1 + L_{m1}i_1$$

$$\boxed{L_{1\sigma} = \frac{N_1\phi_{l1}}{i_1} \qquad L_{m1} = \frac{N_1\phi_{m1}}{i_1}}$$

Defined!



Then no winding 1 current, but winding 2 carries current

$$\phi_1 = \phi_{m2}$$

Flux linkage becomes

$$\lambda_1 = N_1\phi_1 = N_1\phi_{m2}$$

The corresponding inductance

$$\lambda_1 = N_1\phi_1 = N_1\phi_{m2} = Mi_2$$

$$\boxed{M = \frac{N_1\phi_{m2}}{i_2}}$$

Very important – flux linkage expression is based on the positive directions defined!

When both windings (1 and 2) carry current, we have

$$\lambda_1 = L_{1\sigma} i_1 + L_{m1} i_1 + M i_2$$

$$\boxed{L_{11} = L_{1\sigma} + L_{m1}}$$

Defined!

Winding 1 self inductance

$$\lambda_1 = L_{11} i_1 + M i_2$$

Similarly, we can write down the flux linkage expression for winding 2

$$\lambda_2 = L_{22} i_2 + M i_1$$

What if we change the winding configuration?

Number of turns vs. inductance

We have

$$L_{m1} = \frac{N_1 \phi_{m1}}{i_1}$$

Double the number of turns

$$L_{m1,new} = \frac{2N_1 \phi_{m1}}{\boxed{i_1/2}} \Rightarrow L_{m1,new} = 2^2 \frac{N_1 \phi_{m1}}{i_1} = 2^2 L_{m1}$$

Half of the line current is needed in order to produce the same flux ϕ_{m1}

Inductance is proportional to square of the number of turns

Number of turns vs. inductance

Mutual inductance

$$M = \frac{N_1 \phi_{m2}}{i_2}$$

Double the number of turns of winding 2

$$M_{new} = \frac{N_1 \phi_{m2}}{\boxed{i_2/2}} \quad \Rightarrow \quad M_{new} = 2\frac{N_1 \phi_{m2}}{i_2} = 2M$$

Half of the line current is needed in order to produce the same flux ϕ_{m1}

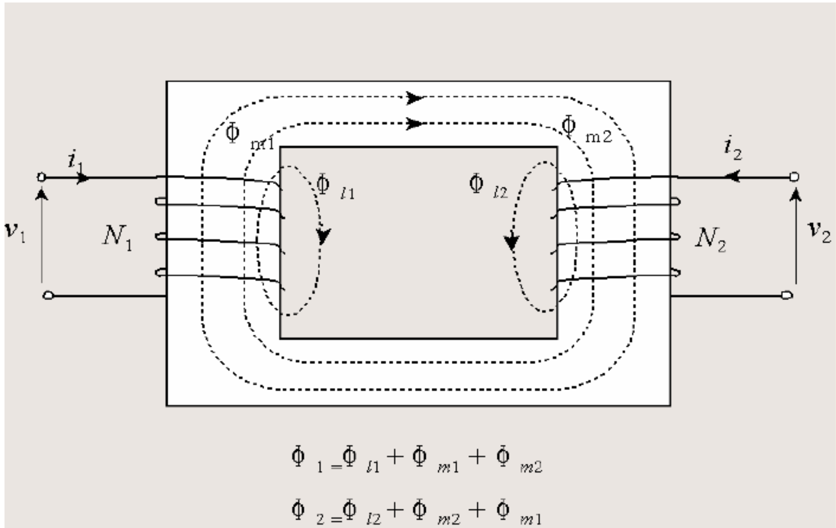
Double the number of turns of winding 1

$$M_{new} = \frac{2N_1 \phi_{m2}}{i_2} = 2M$$

Therefore $M \propto N_1 N_2$

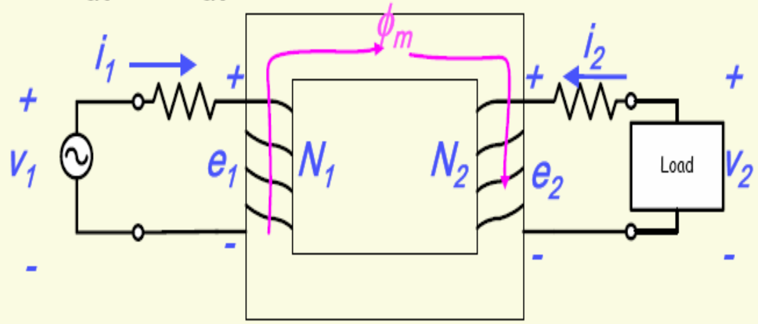
Ideal transformer

- No leakage flux
- Winding resistance is small and may be neglected
- Core loss is small and may be neglected
- Permeability of the core is very high - MMF to establish the field is neglected



Ideal transformer

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi_m}{dt} \qquad e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi_m}{dt}$$



$$v_1 = i_1 R_1 + e_1 = i_1 R_1 + \frac{d\lambda_1}{dt} = i_1 R_1 + N_1 \frac{d\Phi_m}{dt}$$

$$v_2 = i_2 R_2 + e_2 = i_2 R_2 + \frac{d\lambda_2}{dt} = i_2 R_2 + N_2 \frac{d\Phi_m}{dt}$$

$$\frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{N_1}{N_2} \qquad v_1 = \frac{N_1}{N_2} v_2$$

$$F = N_1 i_1 + N_2 i_2 = 0$$

$$-i_1 = \frac{N_2}{N_1} i_2$$

$$v_1 i_1 = v_2 i_2$$

Transformer – differential equations

- Important but different concepts – flux vs. flux linkage
- The voltage differential equations

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt} \qquad v_2 = R_2 i_2 + \frac{d\lambda_2}{dt}$$

$$\lambda_1 = L_{11} i_1 + M i_2 \qquad \lambda_2 = L_{22} i_2 + M i_1$$

$$L_{11} = L_{m1} + L_{1\sigma} = N_1{}^2 L_{sgl} + L_{1\sigma}$$

$$L_{22} = L_{m2} + L_{2\sigma} = N_2{}^2 L_{sgl} + L_{2\sigma}$$

$$M = N_1 N_2 L_{sgl}$$

$$L_{sgl} = \frac{\Phi}{I_{tol}}$$

Modeling techniques using Matlab/Simulink - basics

How to solve differential equations?

- A Matlab M-file example (can be found in the help documents)

$$
\begin{aligned}
y'_1 &= y_2\, y_3 & y_1(0) &= 0 \\
y'_2 &= -y_1\, y_3 & y_2(0) &= 1 \\
y'_3 &= -0.51\, y_1\, y_2 & y_3(0) &= 1
\end{aligned}
$$

```
options = odeset('RelTol',1e-4,'AbsTol',[1e-4 1e-4 1e-5]);
[T,Y] = ode45(@rigid,[0 12],[0 1 1],options);
```

```
function dy = rigid(t,y)
dy = zeros(3,1);    % a column vector
dy(1) = y(2) * y(3);
dy(2) = -y(1) * y(3);
dy(3) = -0.51 * y(1) * y(2);
```

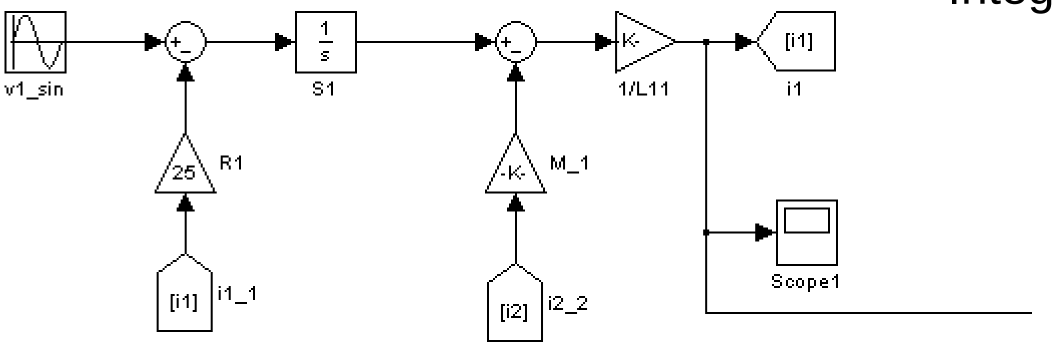
Modeling techniques using Matlab/Simulink - basics

It might be easier to build a Simulink model

- Always try to use integrator block instead of derivative block!
- Define the input and output variables
- Change the differential equation as follow:

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt} \quad \Rightarrow \quad (v_1 - R_1 i_1)\cdot \boxed{\frac{1}{S}} = \lambda_1 = L_{11} i_1 + M i_2$$

Integral operator



Understanding the turns ratio transformation

- Why?
- Defining a 'main inductance'

$$L_m = N_1^{\,2} L_{sgl}$$

$$L_{11} = L_m + L_{1\sigma} \qquad L_{22} = \left(\frac{N_2}{N_1}\right)^2 L_m + L_{2\sigma} \qquad M = L_m \frac{N_2}{N_1}$$

$$\lambda_2 = L_{2\sigma} i_2 + \left(\frac{N_2}{N_1}\right)^2 L_m i_2 + \frac{N_2}{N_1} L_m i_1$$

$$\Downarrow$$

$$\lambda'_2 = \frac{N_1}{N_2}\lambda_2 = \frac{N_1}{N_2} L_{2\sigma} i_2 + L_m\left(\frac{N_2}{N_1} i_2\right) + L_m i_1$$

$$\Downarrow$$

$$\lambda'_2 = L'_{2\sigma} i'_2 + L_m i'_2 + L_m i_1$$

$$L'_{2\sigma} = \left(\frac{N_1}{N_2}\right)^2 L_{2\sigma}$$

$$i'_2 = \frac{N_2}{N_1} i_1$$

Understanding the turns ratio transformation

$$\lambda_1 = L_{1\sigma} i_1 + L_m i_1 + L_m i'_2$$

$$\lambda'_2 = L'_{2\sigma} i'_2 + L_m i'_2 + L_m i_1$$

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt}$$

$$v'_2 = R'_2 i'_2 + \frac{d\lambda'_2}{dt}$$

$$v'_2 = \frac{N_1}{N_2} v_2 = \frac{N_1}{N_2} R_2 \frac{N_1}{N_2}\left(\frac{N_2}{N_1} i_2\right) + \frac{d\lambda'_2}{dt}$$

$$L'_{2\sigma} = \left(\frac{N_1}{N_2}\right)^2 L_{2\sigma}$$

$$R'_2 = \left(\frac{N_1}{N_2}\right)^2 R_2$$

$$i'_2 = \frac{N_2}{N_1} i_1$$

$$v'_2 = \frac{N_1}{N_2} v_2$$

$$\lambda'_2 = \frac{N_1}{N_2} \lambda_2$$

Excises

- Build a Simulink model to solve the differential equation governing a R-L circuit with sinusoidal excitation. Assume: R = 0.9; L=0.0021; Input voltage: Vpk=100 (V), f=50 (Hz).

- Build a Simulink model to model a single phase transformer, described by the following differential equations:

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt} \qquad v_2 = -R_2 i_2 + \frac{d\lambda_2}{dt}$$

$$\lambda_1 = L_{11} i_1 - M i_2 \qquad \lambda_2 = -L_{22} i_2 + M i_1$$

- How the positive directions are defined for the voltage and current on the secondary side?

1. Assuming the secondary is short-circuited
2. Assuming the secondary output is connected to a R-L load. How this load may be connected to the transformer model?

Parameters for the transformer are given in the next page.

Excises

```
Omega = 2*pi*50;           %primary side angular frequency

Nratio  = 10;              % Turns ratio, primary vs. secondary

R1     = 0.72;             % Primary resistance
L1lkg  = 0.92/Omega;       % Primary leakage inductance
L1m    = 4370/Omega;       % Primary magnetizing inductance
L11    = L1m+L1lkg;        % Primary total inductance

R2     = 0.007;            % Secondary resistance
L2lkg  = 0.009/Omega;      % Secondary leakage inductance
L2m    = 43.7/Omega;       % Secondary magnetizing inductance
L22    = L2m+L2lkg;        % Secondary total inductance

M      = L1m/Nratio;       % Calculation of the mutual inductance

RL     = 0.8962;           % Secondary load resistance
LL    = 0.6721/Omega*0;    % Secondary load inductance
```