Written examination in

Dynamic Models of Electrical Machines

Duration: 2 hours

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off
- Internet access is strictly forbidden
- Any kind of communication with other students is not allowed
- Remember to write your study number on all answer sheets
- All intermediate steps and calculations should be included in your answer sheets --- printing the final result is insufficient

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The set consists of 2 problems

Problem 1 (25%)

(1) Please draw the reference frame axes for abc reference frame, dq rotating reference frame, and αβ stationary reference frame.

(2) Suppose now you have a set of 3-phase signals as:

$$v_a = V_{pk}\cos(\omega_e t),\quad v_b = V_{pk}\cos\left(\omega_e t - \frac{2\pi}{3}\right),\quad v_c = V_{pk}\cos\left(\omega_e t + \frac{2\pi}{3}\right)$$

where $\omega_e = 2\pi \cdot 50$ and $V_{pk} = 1$

Please draw the signal waveforms viewed in dq-frame for

- when the dq-frame is rotating at 50 Hz (in the anti-clockwise direction which is the positive rotational direction)
- when the dq-frame is rotating at –50Hz (which means it rotates in the negative (clockwise) direction).

(3) Transform the v_a, v_b, v_c signals in (2) to αβ-reference frame.

(4) For the following 3-phase signals

$$V_a = V_{pk}\cos\left(\omega_e t + \frac{\pi}{6}\right),\quad V_b = V_{pk}\cos\left(\omega_e t - \frac{2\pi}{3} + \frac{\pi}{6}\right),\quad V_c = V_{pk}\cos\left(\omega_e t + \frac{2\pi}{3} + \frac{\pi}{6}\right)$$

Please draw its space vector at time $t = \frac{1}{50}$ (assuming $\omega_e = 2\pi \cdot 50$). Assuming $V_{pk} = 1$, what is the amplitude of this space vector?

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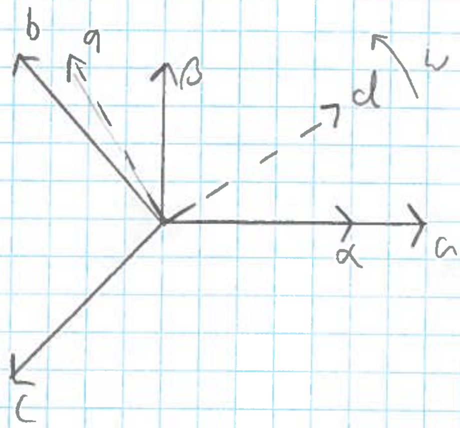
Problem 1 - Reference frame

1 - Different reference frames:



2 - 3 phase signal:

$V_a = V_{pk} \cos(\omega t)$, $\omega = 2\pi 50$ rad/s

$V_b = V_{pk} \cos(\omega t - 120)$ $V_{ph} = 1$

$V_c = V_{pk} \cos(\omega t + 120)$

- Sketch the signal viewed in dq-frame:
  - When the frame rotates with 50 Hz.

$\bar{f}_{abc} = V_p e^{j\omega t + \theta} \Rightarrow \bar{f}_{abc} = 1 e^{j\omega t}$

- Transform to dq

$f_d = Re\left(\frac{\bar{f}}{e^{j\theta}}\right)$

$= Re\left(\frac{1 e^{j\omega t}}{e^{j\theta}}\right)$

$= Re\left(\frac{1 e^{j2\pi 50 t}}{e^{j2\pi 50 t}}\right)$

$= Re\left(e^{j(2\pi 50 t - 2\pi 50 t)}\right)$

$= \cos(2\pi 50 t - 2\pi 50 t)$

$= 1$

$f_q = Re\left(\frac{\bar{f}}{e^{j\theta + 90}}\right)$

$= Re\left(\frac{1 e^{j\omega t}}{e^{j\theta + 90}}\right)$

$= Re\left(\frac{1 e^{j2\pi 50 t}}{e^{j2\pi 50 t + \pi/2}}\right)$

$= Re\left(e^{j(2\pi 50 t - (2\pi 50 t + \pi/2))}\right)$

$= \cos(\pi/2)$

$= 0$

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- When the frame rotates with -50 Hz

$f_d = 1\cos(2\pi 50t + 2\pi 50t)$

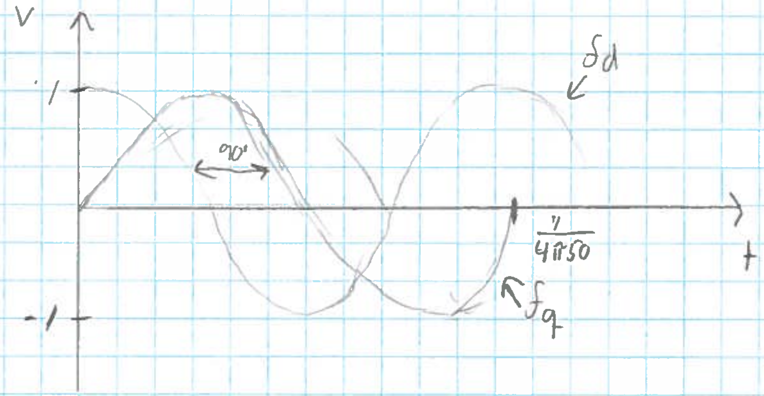
$f_d = \cos(4\pi 50t)$

$f_q = \cos(2\pi 50t + 2\pi 50t + \pi/2)$

$f_q = \cos(4\pi 50t + \pi/2)$

$= \sin(4\pi 50t)$

- Sketch



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3 - $\alpha\beta$ - reference frame:

$\bar{f} = f_\alpha + j f_\beta$

$\bar{f} = 1 \cdot e^{j\omega t}$

$= \cos(\omega t) + j\sin(\omega t)$

$f_\alpha = Re(\bar{f}) \Rightarrow \cos(\omega t) = \cos(2\pi 50t)$

$f_\beta = Im(\bar{f}) \Rightarrow \sin(\omega t) = \sin(2\pi 50t)$

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4 - 3-phase signal

$V_a = V_{ph} \cos\left(\omega_e t + \frac{\pi}{6}\right)$, $V_{ph} = 1\ V$

$V_b = V_{ph} \cos\left(\omega_e t - \frac{2\pi}{3} + \frac{\pi}{6}\right)$ $\omega_e = 2\pi 50\ rad/s$

$V_c = V_{ph} \cos\left(\omega_e t + \frac{2\pi}{3} + \frac{\pi}{6}\right)$

- Space Vector:

$\bar{f} = 1 e^{j2\pi 50 t + \pi/6}$

- At $t = 1/50\ s$

$\bar{f} = e^{j2\pi 50 \cdot 1/50 + \pi/6}$

$= e^{j2\pi + \pi/6}$

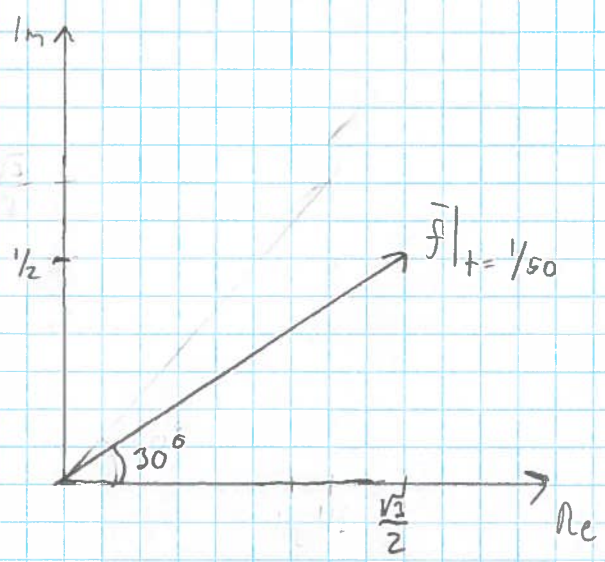
$= \cos(2\pi + \pi/6) + j\sin(2\pi + \pi/6)$

$\bar{f} = \frac{\sqrt{3}}{2} + \frac{1}{2} j$

- Magnitude of $\bar{f}$:

$|\bar{f}| = \sqrt{\frac{\sqrt{3}}{2}^2 + \frac{1}{2}^2} = 1$

- Sketch of $\bar{f}$ at $t = 1/50\ s$



Problem 2 (25%)

A sketch of an induction machine phase axes is given below.

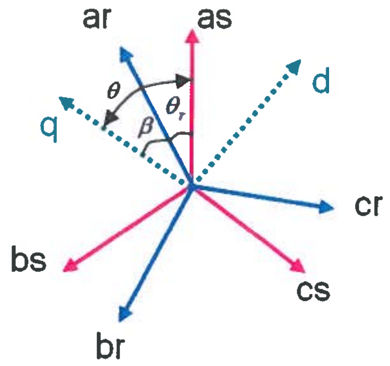


Fig. 1

where notation 's' stands for stator phase axes and notation 'r' stands for rotor phase axes.

Knowing the machine model expressed in an arbitrary qd-reference frame is

Stator side voltage equations:

$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u_{0s} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} - \omega_\theta \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$

(1) Please re-express the above stator side voltage equations using αβ-reference frame. The α-axis is aligned with stator phase-as axis and the β-axis is leading phase-as axis by 90 electrical degrees (as usual).

(2) Please write down the stator side α, β flux linkage expressions.

(3) Please sketch how you may implement the stator side voltage and flux linkage equations in Simulink in order to solve these equations. The input signals to your model are stator α, β voltage components and you want to solve the model to find the stator side α, β current components. Assuming all the rotor currents and rotor flux linkage components are known – you may use these rotor side variables directly in sketching your Simulink model. (*Like what we did in our last workshop day*).

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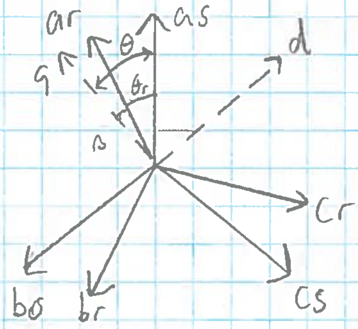
Problem 2 - Induction machine



- Stator Voltage eq.

$$u_{qs} = R_s i_q + p\lambda_q + \omega_\theta \lambda_d$$
$$u_{ds} = R_s i_d + p\lambda_d - \omega_\theta \lambda_q$$

1 - $\alpha\beta$ - reference frame

$$\bar{f} = (f_q - f_d)\, e^{j\theta} = f_\alpha + j f_\beta$$

$\Downarrow$ when in $\alpha\beta$, $\theta = 0$, $\omega_\theta = 0$

$$f_q = f_\alpha \quad , \quad f_d = -f_\beta$$

$$u_\alpha = R_s i_\alpha + p\lambda_\alpha$$
$$u_\beta = R_s i_\beta + p\lambda_\beta$$

2 - Stator side $\alpha\beta$ - flux linkage:

- In qd - reference frame:

$$\begin{cases} \lambda_{qs} = (L_{ls} + L_m)\, i_{qs} + L_m \cdot i_{qr} \\ \lambda_{ds} = (L_{ls} + L_m)\, i_{ds} + L_m \cdot i_{dr} \end{cases}$$

$\Downarrow$

$$\lambda_{\alpha s} = (L_{ls} + L_m)\, i_{\alpha s} + L_m\, i_{\alpha r}$$
$$\lambda_\beta = (L_{ls} + L_m)\, i_{\beta s} + L_m\, i_{\beta r}$$

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B - Simulink Model

- Input: $\bar{U}_{\alpha\beta}$, output $\bar{I}_{\alpha\beta}$

$$U_\alpha = R_s i_\alpha + p\lambda_\alpha \Rightarrow \lambda_\alpha = \frac{1}{s}\left(U_\alpha - R_s i_\alpha\right)$$

$$\lambda_\alpha = (L_{ls} + L_m) i_{\alpha s} + L_m i_{\alpha r} \Rightarrow i_{\alpha s} = \frac{\lambda_\alpha - L_m i_{\alpha r}}{L_{ls} + L_m}$$

$$U_\beta = R_s i_\beta + p\lambda_\beta \Rightarrow \lambda_\beta = \frac{1}{s}\left(U_\beta - R_s i_\beta\right)$$

$$\lambda_\beta = (L_{ls} + L_m) i_{\beta s} + L_m i_{\beta r} \Rightarrow i_{\beta s} = \frac{\lambda_\beta - L_m i_{\beta r}}{L_{ls} + L_m}$$