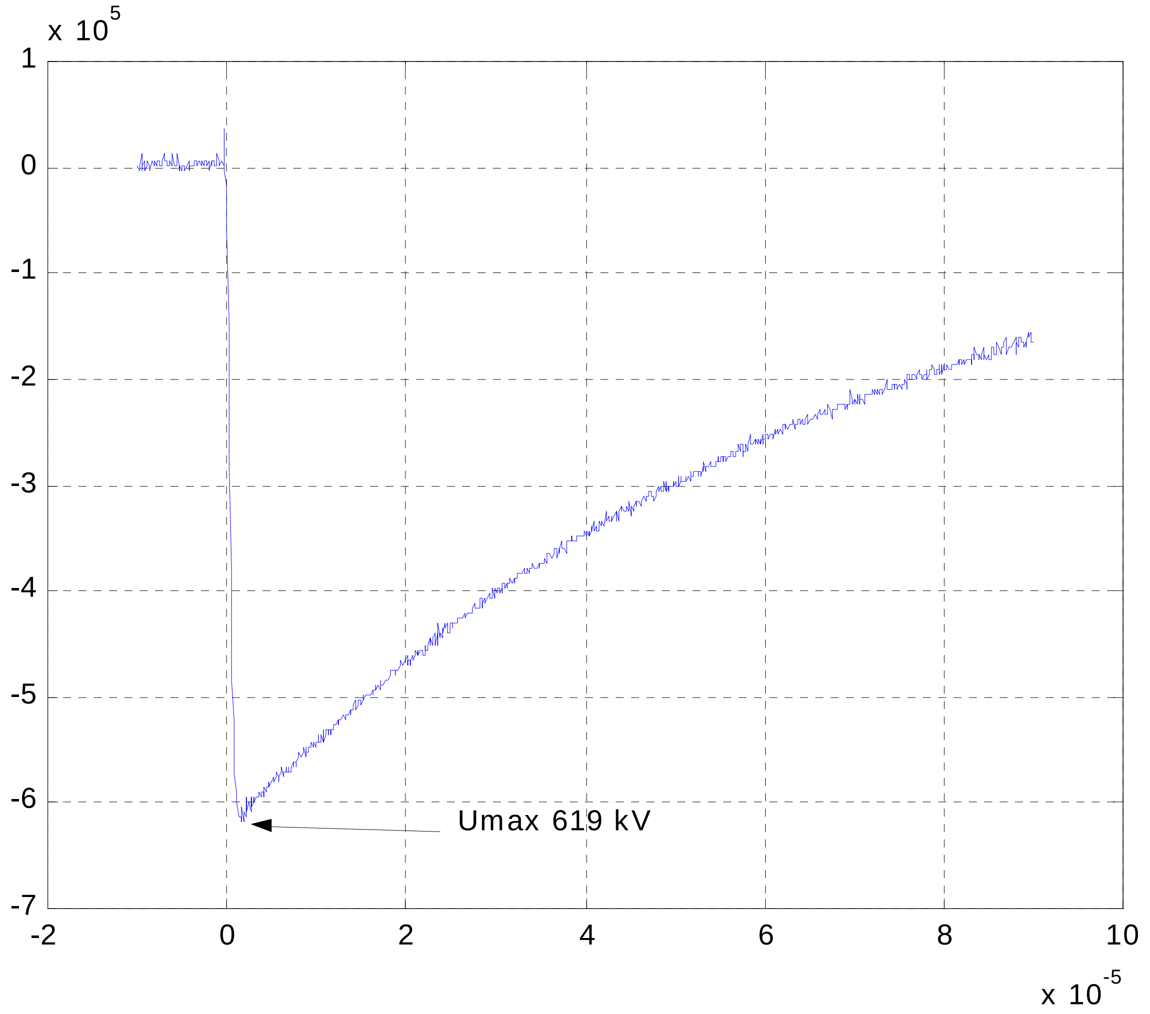
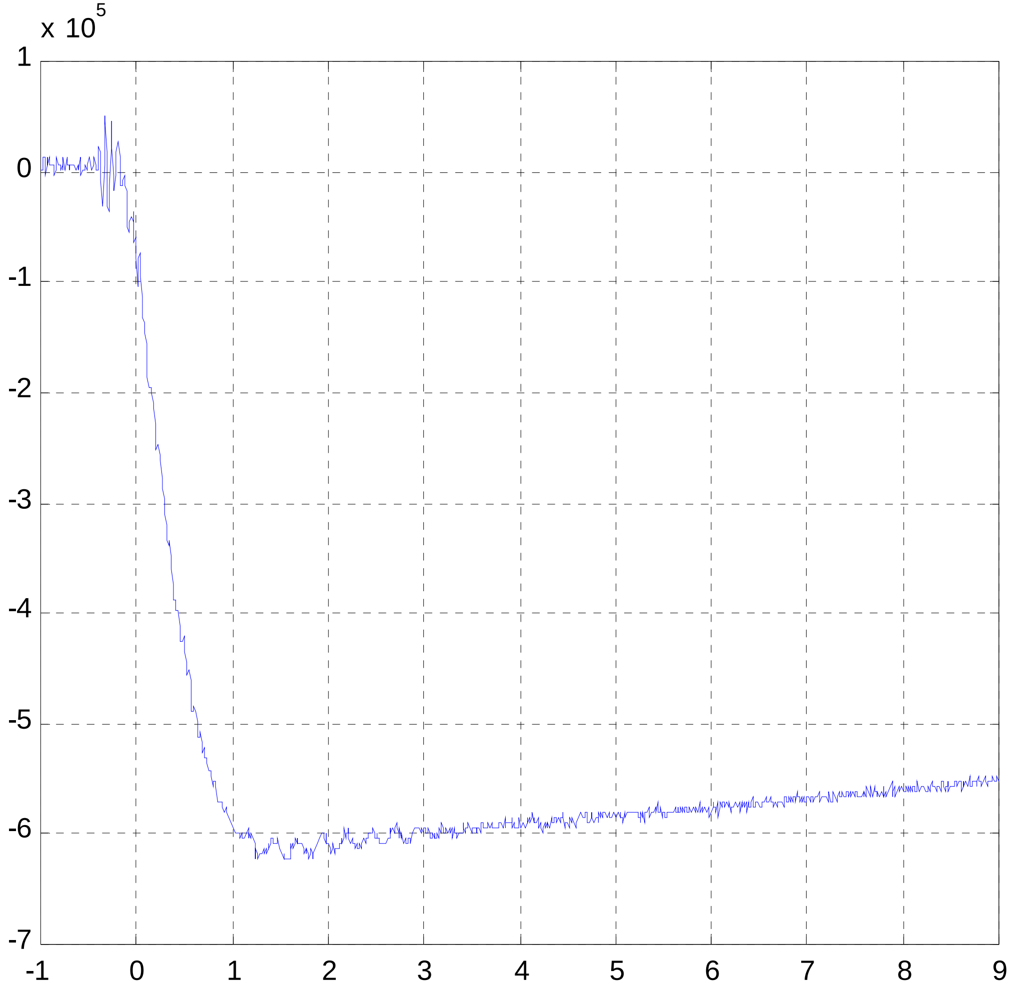
Real –life negative lightning impulse 1,2/50 µs



Real –life negative lightning impulse 1,2/50 μs with higher horisontal resolution



Is this the ACTUAL voltage impulse ?

Or is it just some-thing distorted/ invented by the measuring system ?

We must analyze !

Impulse measuring system

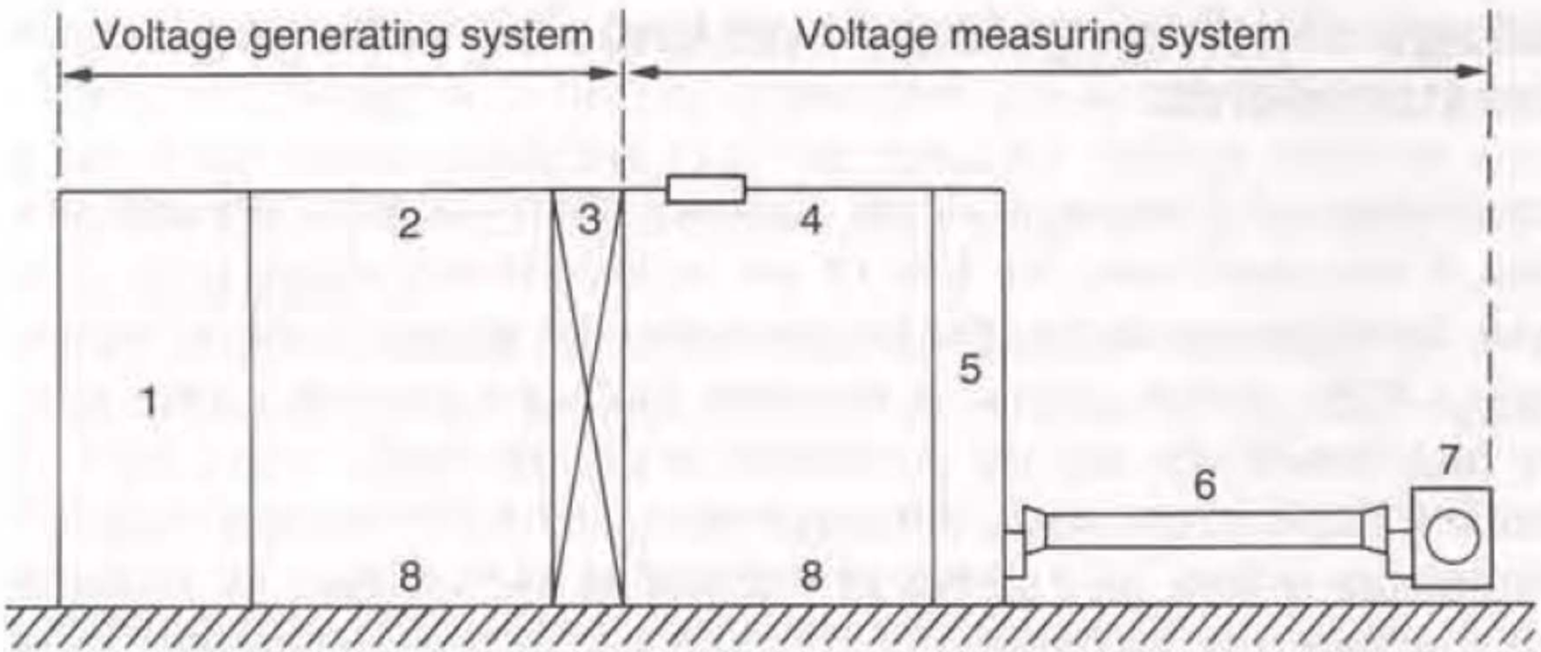


Figure 3.23 *Basic voltage testing system. 1. Voltage supply. 2. Lead to test object. 3. Test object. 4. Lead to voltage divider. 5. Voltage divider. 6. Signal or measuring cable. 7. Recording instrument. 8. Ground return*

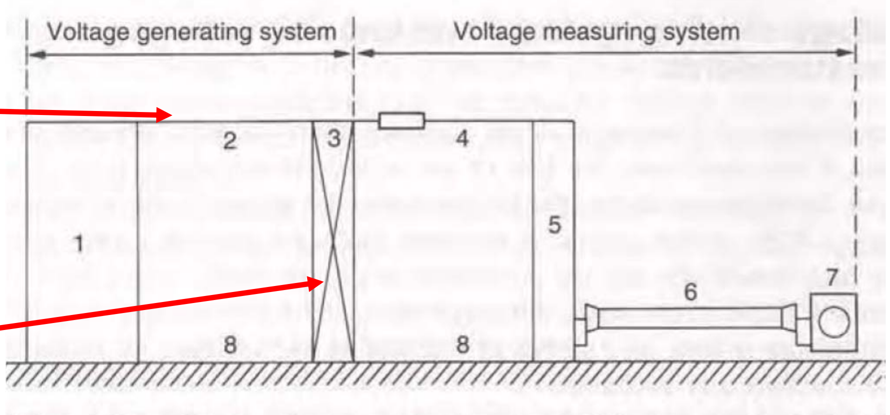
DC: Often include voltage divider in generator
AC: Often likewise – check inductive voltage drop of lead to test object
IMPULSE: No good idea !

part of the impulse generator. The simple reasons can easily be understood from the impulse generator circuits (see Chapter 2, Fig. 2.25). There, the wave shaping load capacitance C_2 is often combined with an l.v. capacitor connected in series, thus forming an adequate voltage divider. An undamped connection to the object under test then leads to the erroneous assumption that negligible voltage drop can occur across the lead. This assumption may be correct for slowly rising impulse voltages and quite short leads. Connecting leads with lengths of many metres, however, are often used and thus this assumption may become unacceptable. It must be remembered that the test object is a capacitor become unacceptable. It must be remembered that the test object is a capacitor and thus the circuit formed by the lead and test object is a series resonant circuit. These oscillations are likely to be excited by firing the generator, but will only partly be detected by the voltage divider. Completely wrong is the assumption that such a voltage divider being a part of the generator

is measuring the correct voltage across the test object following a voltage collapse or disruptive discharge. The whole generator including voltage divider will be discharged by this short-circuit at the test object and thus the voltage divider is loaded by the voltage drop across lead 2. This lead forms to first

Lead=inductor

EUT=capacitor



Resonant circuit !

Voltage divider should be placed away from any energized objects

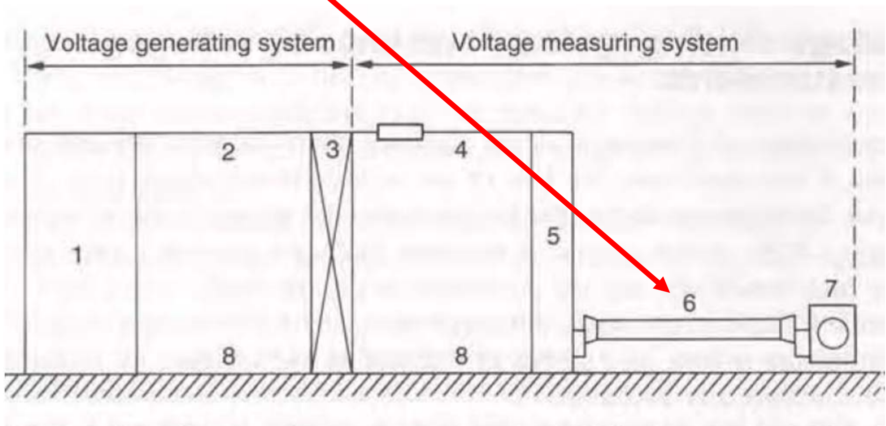
There is a further reason for placing the voltage dividers away from any energized objects. High-voltage dividers consist of 'open' networks and cannot be shielded against external fields. All objects in the vicinity of the divider which may acquire transient potentials during a test will disturb the field distribution and thus the divider performance. The lead from the voltage divider to the test object 4 is therefore an integral part of the measuring system. The

Which means high-impedant, as this is necessary to handle the very HV

influence of this lead will theoretically be treated in section 3.6.3. There it will be established that a damping resistor at the input end of this lead contributes to improved transfer characteristics of the system.

Shielded cables for divider-oscilloscope

In order to avoid heavy electromagnetic interactions between the recording instrument and the h.v. test area as well as safety hazards, the length of the signal cable 6 must be adequately chosen. For any type of voltage to be measured, the signal cable should be of a coaxial and shielded type. The shield or outer conductor picks up the transient *electrostatic* fields and thus prevents the penetration of this field to the inner conductor. Although even transient *magnetic* fields can penetrate into the cable, no appreciable voltage (noise) is induced due to the symmetrical arrangement. Ordinary coaxial cables

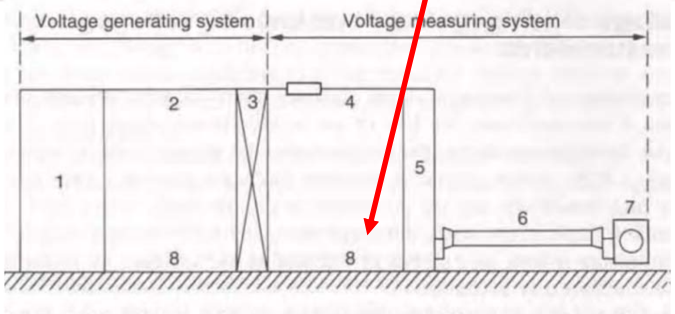


Use double-screened cables with rigid screen!

the d.c. resistance of the shield. If the frequency of these currents increases, the coupling impedance will continuously decrease if the shield is of rigid cross-section; then the eddy currents will attenuate the current density at the inner surface of the cylindrical shield. Hence rigid or corrugated shields, i.e. flexwell cables, are best suited for noise reduction. For braided shields, the coupling impedance is in general not a stable quantity, as the current distribution within the shield is likely to be influenced by resistive contacts within the braid. Double-shielded cables with predominantly two insulated braided shields will improve the behaviour. Best conditions are gained by placing the coaxial cable into an additional, non-braided metal tube, which is connected to ground potential at least at the input end of the measuring cable and also at its end.

Ground return

In Fig. 3.23 there is finally the ground return 8. For h.v. test circuits disruptive discharge must always be taken into account. Large and heavily oscillating short-circuit currents are developed and hence every ground return with simple leads only cannot keep the voltage drops small. The impedance, therefore, must be reduced. Large metal sheets of highly conducting material such as copper or aluminium are best. Many h.v. laboratories provide such ground returns in combination with a Faraday cage for a complete shielding of the laboratory. Expanded metal sheets give similar performance. At least metal tapes of large width should be used to reduce the impedance. A parallel connection of tapes within flat areas will further decrease the inductance and thus approximate the efficiency of huge metal sheets.

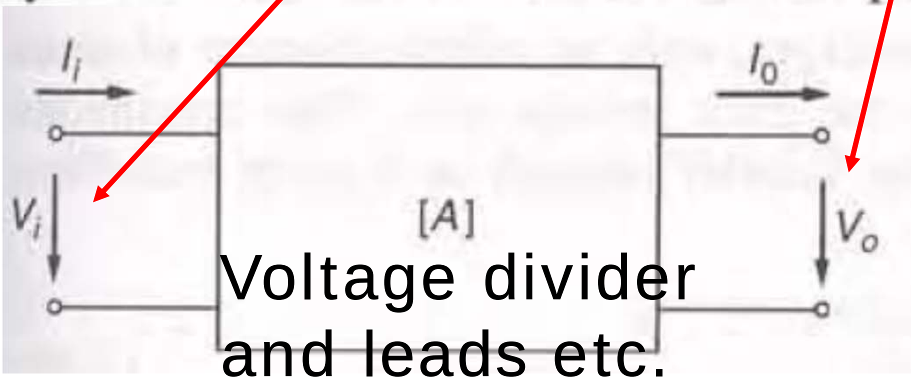


Demands upon transfer characteristics of the measuring system

The voltage measuring system defined in Fig. 3.23 is a four-terminal network and can thus be represented as shown in Fig. 3.24. V_i indicates the voltage across the test object (3 in Fig. 3.23), and the output voltage V_0 appears at the recording instrument, i.e. at the screen of a CRO or transient recorder.

The input voltages V_i are either continuous steady state voltages for d.c. and a.c. generating systems, or single events for impulse voltages. In both cases, the instantaneous amplitudes will change with time, even for d.c. voltages with a periodic ripple.

For a sinusoidal input voltage $v_i(t) = V_{mi} \sin(\omega t + \phi_i)$ the magnitude V_{m0} and phase angle ϕ_0 of the output voltage $v_0(t) = V_{m0} \sin(\omega t + \phi_0)$ can be determined either by calculation with known network parameters or by

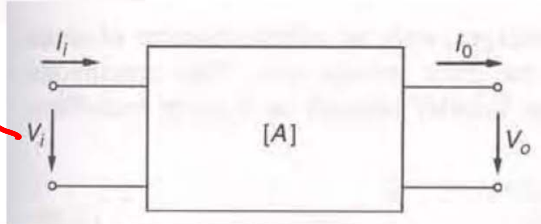


At oscilloscope

Voltage divider and leads etc.

Transfer function for HV measuring system

$$\mathbf{H}(j\omega) = \frac{V_0}{V_i} = |\mathbf{H}(j\omega)| \exp\{j[\phi_0(\omega) - \phi_i(\omega)]\}$$



Neither d.c. voltages with ripple nor a.c. testing voltages are pure sinusoidal, but periodic in nature. The input voltages may then be described by a – in general – limited number of complex amplitudes $\mathbf{V}_{ik}$ obtained by the application of Fourier series,

$$\mathbf{V}_{ik} = \frac{1}{T}\int_{-T/2}^{T/2} v_i(t)\exp(-jk\omega t)\,dt \qquad (3.37)$$

$$= |\mathbf{V}_{ik}|\exp(j\phi_{ik}),$$

Output voltage $v_0(t)$ i.e. at oscilloscope

$$v_0(t) = \sum_{k=-\infty}^{\infty} \mathbf{V}_{ik}\mathbf{H}(j\omega_k)\exp(jk\omega t). \qquad (3.38)$$

A direct comparison between $v_0(t)$ and $v_i(t)$ can thus be made and the errors evaluated.

This was for AC - sinusoidals

Impulse voltages – lightning 1,2/50 μs

For the single events of impulse voltages, only an infinite number of sinusoidal voltages are able to represent the input voltage $v_i(t)$. This continuous frequency spectrum is defined by the Fourier integral or Fourier transform of $v_i(t)$

$$\mathbf{V}_i(j\omega) = \int_{t=-\infty}^{\infty} v_i(t)\exp(-j\omega\tau)\,\mathrm{d}\tau \tag{3.39}$$

and contains amplitude and phase spectra. The linearity and homogeneity of the time invariant systems assumed enable us again to calculate the time response of the system by a convolution of the continuous frequency spectrum with the network response function and the transition from frequency to time domain by means of the inverse Fourier transform:

$$v_0(t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \mathbf{V}_i(j\omega)\mathbf{H}(j\omega)\exp(j\omega t)\,\mathrm{d}\omega. \tag{3.40}$$

No series – transform and convolution

Of course we don't know what's really at EUT ☺

In practice, the real input quantity $v_i(t)$ is not known, as only $v_0(t)$ can be measured. This output voltage, however, has suffered from the loss of information contained in $\mathbf{H}(j\omega)$. No appreciable transmission errors could occur, if at least the amplitude frequency response $H(\omega) = |\mathbf{H}(j\omega)|$ would be constant within a frequency range, in which the line or continuous frequency spectra, $\mathbf{V}_{ik}$ or $\mathbf{V}_i(j\omega)$, cannot be neglected. Thus the computation of the spectra of an estimated input quantity is a very efficient tool to judge the necessary frequency range or bandwidth of our measuring system and its individual components.

So by knowing the transfer function H(jω) gives us access to the dynamics of the measuring system !

Input voltage double-exponential with/without chopping at T_c

$$v_i(t) = \begin{cases} 0 & \text{for } < 0; t > T_c \\ A[\exp(-t/\tau_1) - \exp(-t/\tau_2)] & \text{for } 0 \leq t \leq T_c. \end{cases}$$

Performing the Fourier transform acc. to (3.39)

For the special case of a non-chopped voltage ($T_c \rightarrow \infty$), the Fourier transform of the input voltage is merely

$$\mathbf{V}_i(j\omega) = A\left[\left(\frac{\tau_1}{1+(\omega\tau_1)^2} - \frac{\tau_2}{1+(\omega\tau_2)^2}\right) - j\left(\frac{\omega\tau_1^2}{1+(\omega\tau_1)^2} - \frac{\omega\tau_2^2}{1+(\omega\tau_2)^2}\right)\right]. \quad (3.43)$$

The relative amplitudes for a full lightning impulse ($T_c \to \infty$) become already very small in a frequency range of about 0.5–1 MHz; hence an

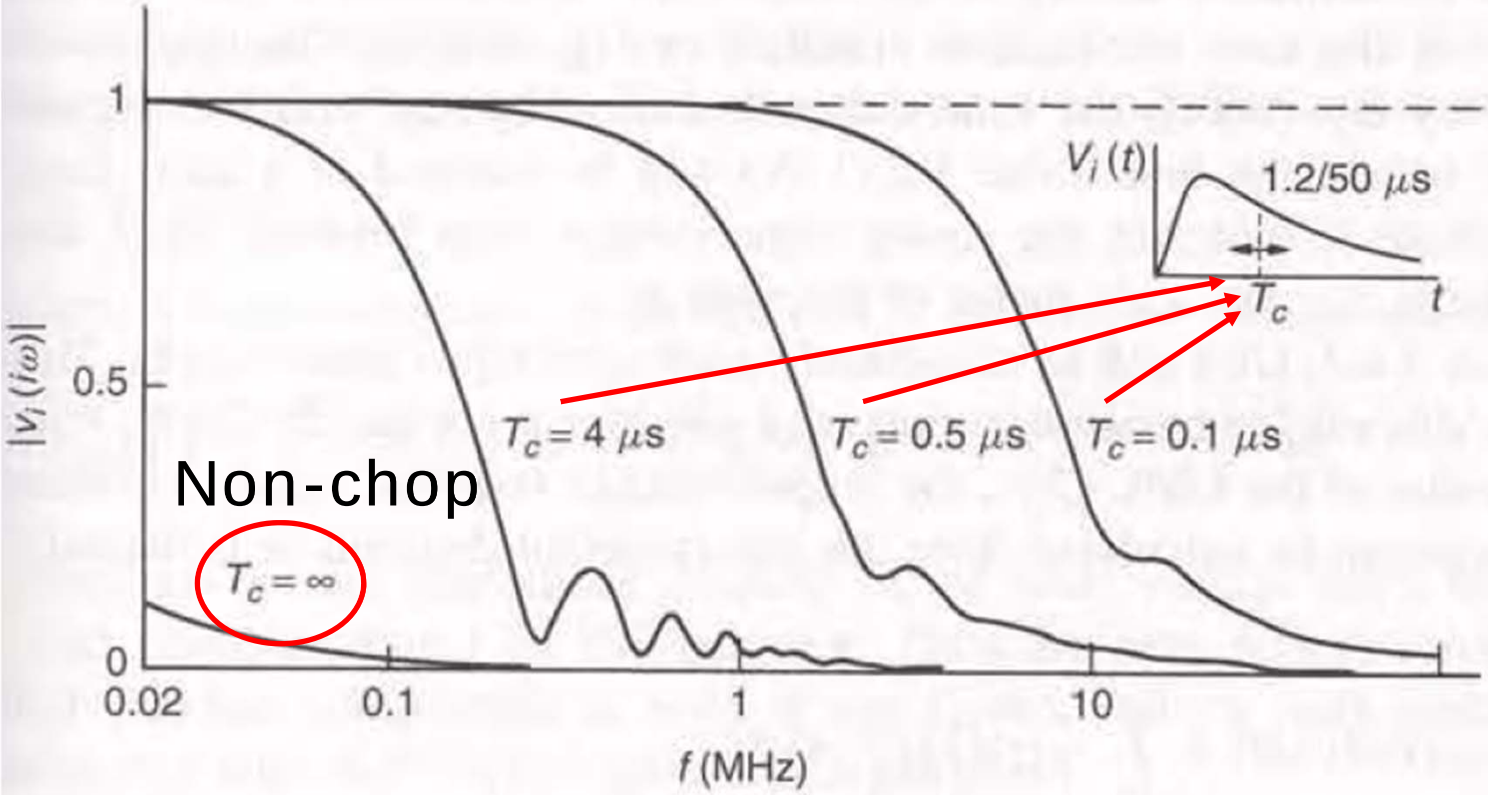


Figure 3.25 *Normalized amplitude frequency spectra (Fourier transform) of a lightning impulse voltage of 1.2/50 µsec, wave full and chopped*

Bandwidth for non-chopped impulse !

Depending upon the decay of the amplitude frequency response, the bandwidth (−3 dB point) has to be much higher, i.e. about 5–10 MHz.

Bandwidth for chopped impulse !!!

The chopping of the voltage introduces a heavy increase of the harmonics content. For $T_c = 4\,\mu\text{sec}$, i.e. a chopping at the impulse tail, an accurate measurement of the crest voltage may still be provided by the above-mentioned amplitude response, although appreciable errors might appear during the instant of chopping. The voltages chopped within the front ($T_c = 0.5$–$0.1\,\mu\text{sec}$), however, will require a very wide bandwidth which must obviously increase with decreasing chopping time. Desirable values of f_B for $T_c = 0.5\,\mu\text{sec}$ only shall obviously reach magnitudes of about 100 MHz, but such large values cannot be achieved with measuring systems for very high voltages.

● The response time of the measuring system

The reponse time of a measuring system is <u>defined</u> as the area:

$$T = \int_0^\infty (1 - g(t))\, dt$$

So – this is the sum of the difference between a unit step and the answer !

● **The response time of the measuring system**

The reponse time of a measuring system is defined as the area:

Simplified measuring system !

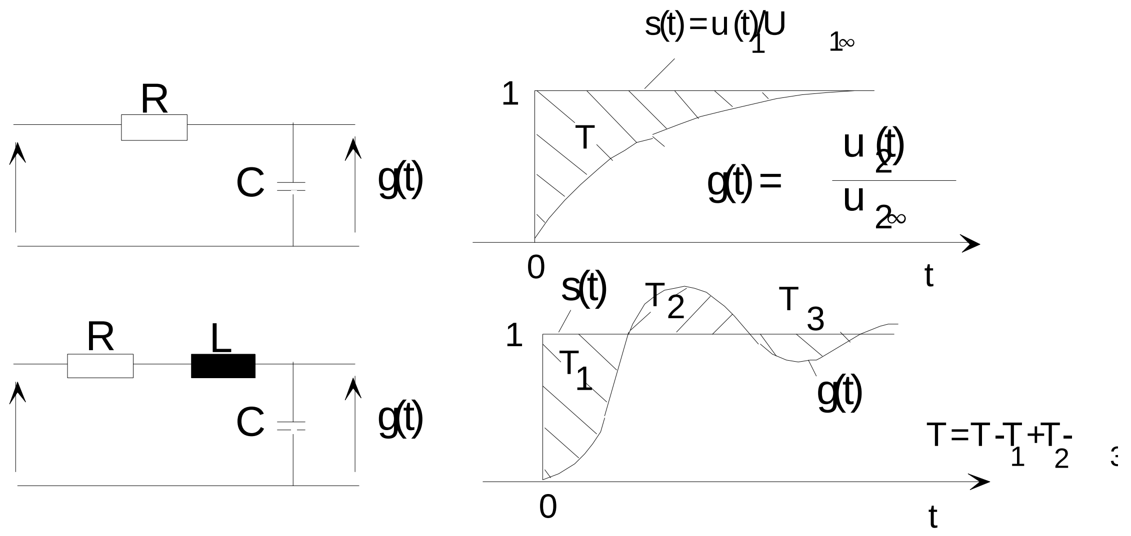


Fig. 7.8 Equivalent scheme and unit step response for voltage dividers a) RC- circuit, b) RLC- circuit

High bandwidth and fairly damping demands that T_1 is small and the ratio $T/T_1 \rightarrow 1$.

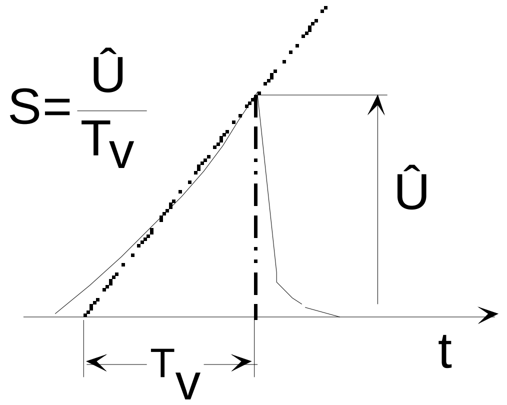
- **Rules of thumb for the response time to keep the error within 5 %**

  - Non-chopped and tail chopped lightning impulse voltages with rated value 1,2/50 µs:

$$\boxed{T \le 200\ ns}$$

  - Wedge shaped (lightning impulse chopped at the front) impulse voltages with an approximative rise from 0 ➜ maximum value within time T_v:

$$\boxed{T \le 0{,}05 \bullet T_v}$$



Unit Step Response USR method instead of frequency domain method

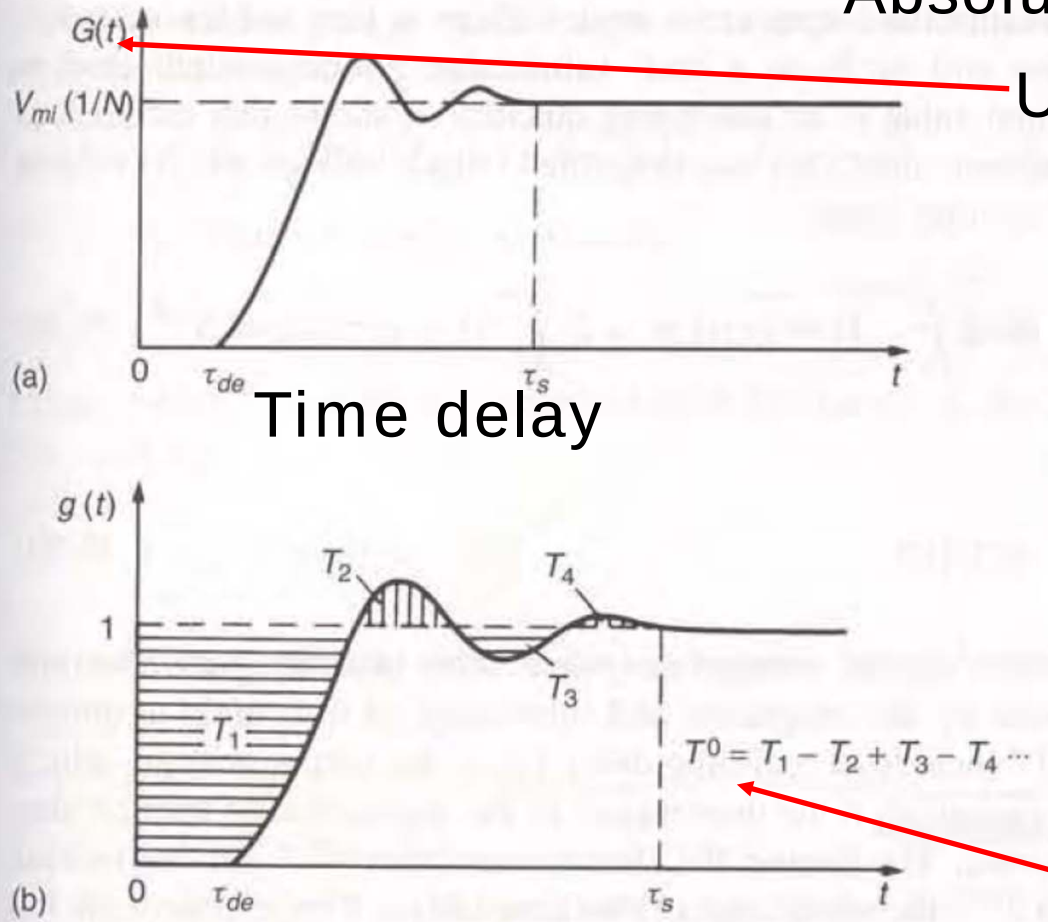
This frequency domain method described so far for determining a transfer characteristic quantity to estimate measuring errors is difficult to use, as the two quantities, $H(\omega)$ and $\phi(\omega)$, are difficult to measure due to the large 'scale factors' of the measuring systems. For h.v. measuring systems, the transfer characteristic is therefore evaluated by means of a measured (experimental unit) 'step response'.(57,53) This time-domain method is based upon the fact that the Fourier transform (eqn (3.39)) of a single-step function is proportional to $1/j\omega$ and thus all frequencies are contained. Let us, therefore, represent the input voltage of our measuring system by such a step function:

$$v_i(t) = \begin{cases} 0 & \text{for } t < 0 \\ V_{mi} & \text{for } t > 0. \end{cases} \tag{3.44}$$

Unit step at t=0

Absolute values

Unit step response G(t)



Time delay

$$v_0(t) = v_i(t)G(+0) + \int_0^t v_i(\tau)G'(t-\tau)\,d\tau$$
$$= G(t)v_i(+0) + \int_0^t v_i'(t-\tau)G(\tau)\,d\tau.$$

Normalized values

Response time

Figure 3.26 *Unit step response and definition of the response time T'.*
(a) Unit step response as output voltage. (b) Normalized unit step response

Chopped pulses – Linearly rising with S

The chopping of a lightning impulse voltage at the front ($T_c \leq 1\,\mu$sec in Fig. 3.25) is sometimes used for h.v. testing and the demands upon the measuring circuits become severe. The chopping on front provides a nearly linearly rising voltage up to T_c. Let us assume an ideally linearly rising voltage,

$$\boxed{v_i(t) = St,} \tag{3.46}$$

Duhamels integral (3.45) gives

$$v_0(t) = S\int_0^t G(\tau)\,\mathrm{d}\tau = \frac{S}{N}\int_0^t g(\tau)\cdot \mathrm{d}\tau$$

Expansion gives

$$Nv_0(t) = S\left[t - \int_0^t [1 - g(\tau)]\,\mathrm{d}\tau\right]$$

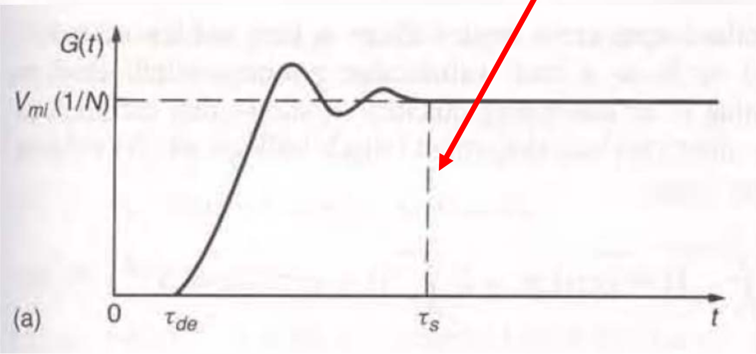
This expression relates the output to the input voltage as long as (St) increases. The integral term will settle to a final value after a time τ_s indicated in Fig. 3.26. This final value is an interesting quantity, it shows that differences in amplitudes between input (St) and magnified output voltage $Nv_0(t)$ remain constant. Hence we may write

$$v_i(t) - Nv_0(t) = S\int_0^{t>\tau_s}[1-g(\tau)]\,d\tau = S\int_0^{\infty}[1-g(\tau)]\,d\tau = ST^0 \quad (3.49)$$

where

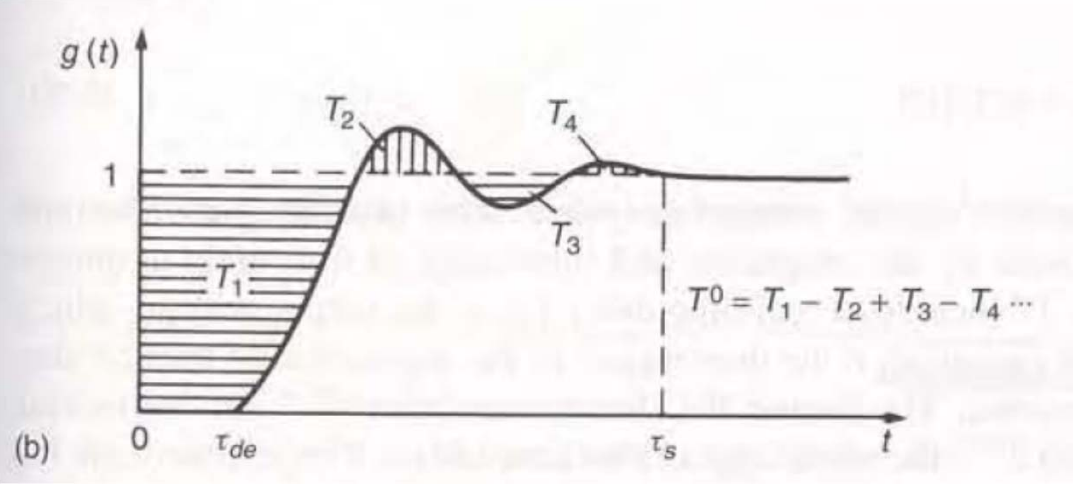
$$T^0 = \int_0^{\infty}[1-g(\tau)]\,d\tau \quad (3.50)$$

RESPONSE TIME

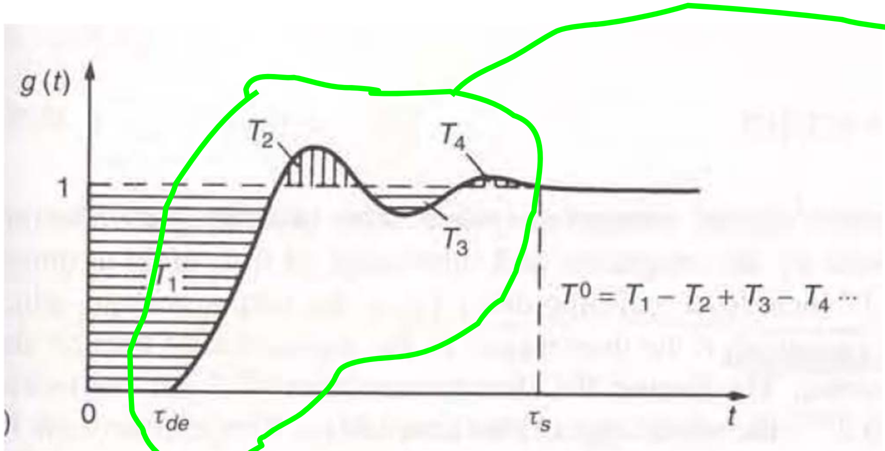


$$T^0 = \int_0^\infty [1 - g(\tau)]\,\mathrm{d}\tau \tag{3.50}$$

is the 'response time' of the measuring system. This quantity gives the time which can be found by the integration and summation of time areas as shown in Fig. 3.26(b). T^0 includes a real time delay τ_{de} of the output voltage, which is in general not measured, if the time instant of the application of the unit step input is not recorded. The former IEC Recommendations[57] and the newest IEC Standard 60-2[53] therefore neglect this time delay. The justifications for



neglecting this delay are shown in Fig. 3.27. There, the linearly rising input voltage is suddenly chopped, and the output voltage multiplied by N is approximately sketched for the USR of Fig. 3.26. Equation (3.48) can be applied up to the instant of chopping, T_c; for later times, eqn (3.45) must be rearranged, and it can easily be seen that a superposition of three terms (response to St, negative USR with amplitude ST_c, and negative response to St for $t > T_c$) will govern this output voltage.



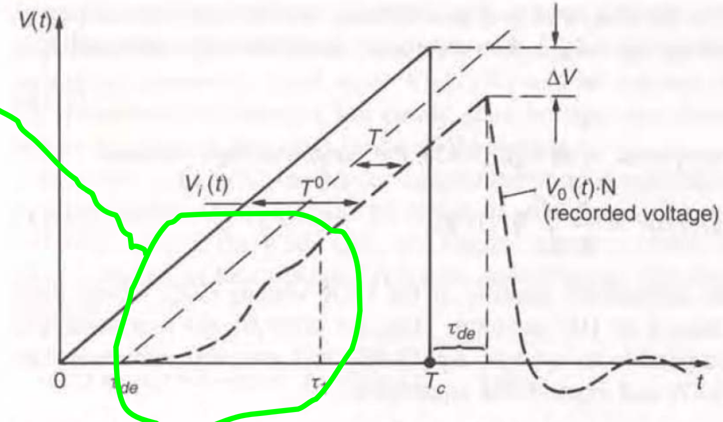


Figure 3.27 *Measuring error ΔV for linearly rising voltages chopped at T_c. Definition of response time T^0 and T*

Unit step G(t)

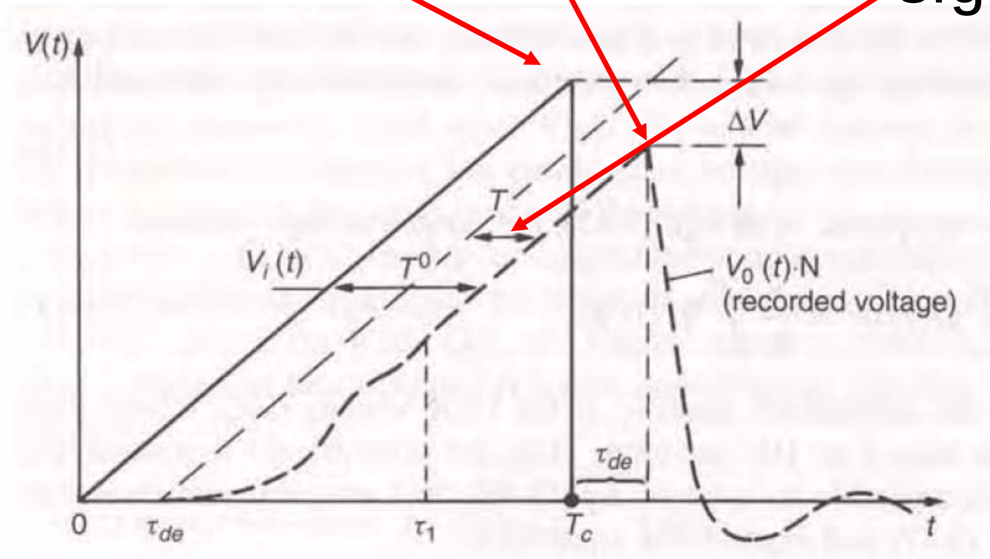
Response to S*t

As the sudden change in the output voltage is also delayed, the amplitude error ΔV is obviously given by

Remember: $v_i(T_C)=S*T_C$

$$\Delta V = v_i(T_c) - N v_0(T_c + \tau_{de}) = S(T^0 - \tau_{de}) = ST \qquad v_i(t) = St, \qquad (3.46)$$

* sign here

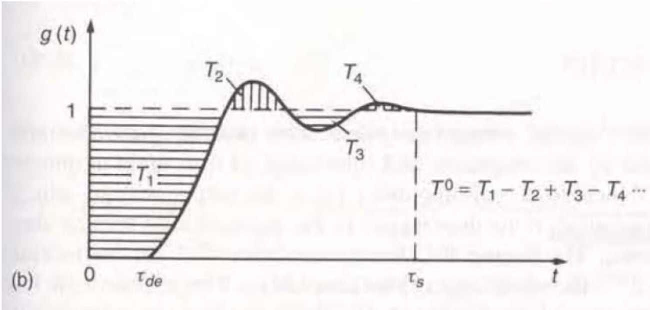


if $T_c > \tau_s$. Thus the simple relationship

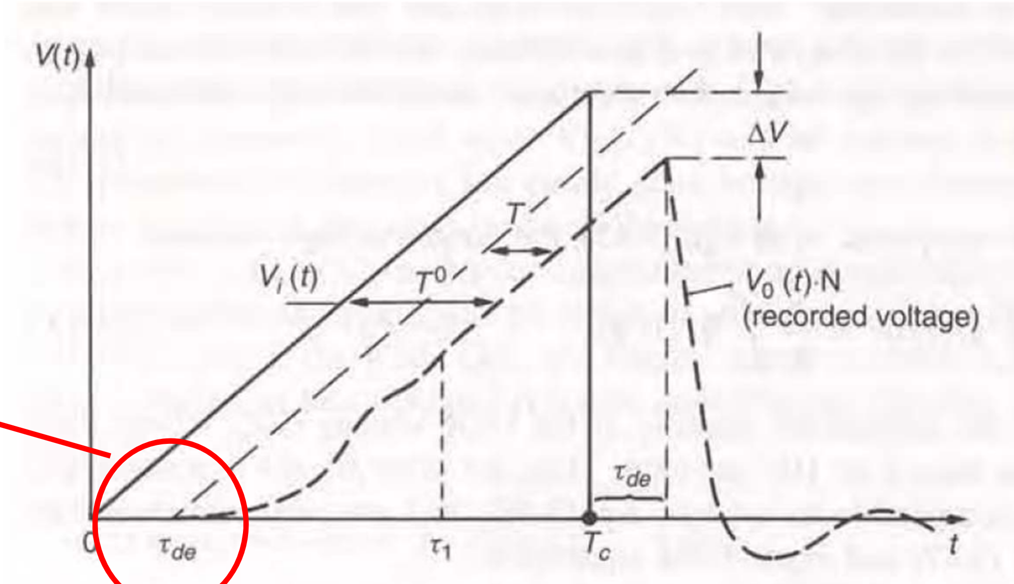
$$T = T^0 - \tau_{de}$$

Oscillations have settled!

exists, where T is equal to a response time similar to T_0, but integrated from Fig. 3.26 by



$$T = \int_{\tau_{de}}^{\infty} [1 - g(\tau)]\,d\tau$$



So response time is defined to be calculated from USR of measuring system and starting integration at τ_{de}

Voltage error – amplitude error δ for linearly rising voltages – front chopped voltages

The relative amplitude error δ for a chopped linearly rising voltage thus becomes

$$\delta = \frac{\Delta V}{ST_c} = \frac{T}{T_c}.$$

Relative error!

Realistic response time can be e.g. 50 ns and the chopping at the middle of the front of a lightning impulse 1,2/50 μs gives T_C app. equal to 0,5 μs so the relative error is 50 ns/500 ns which is 10 %

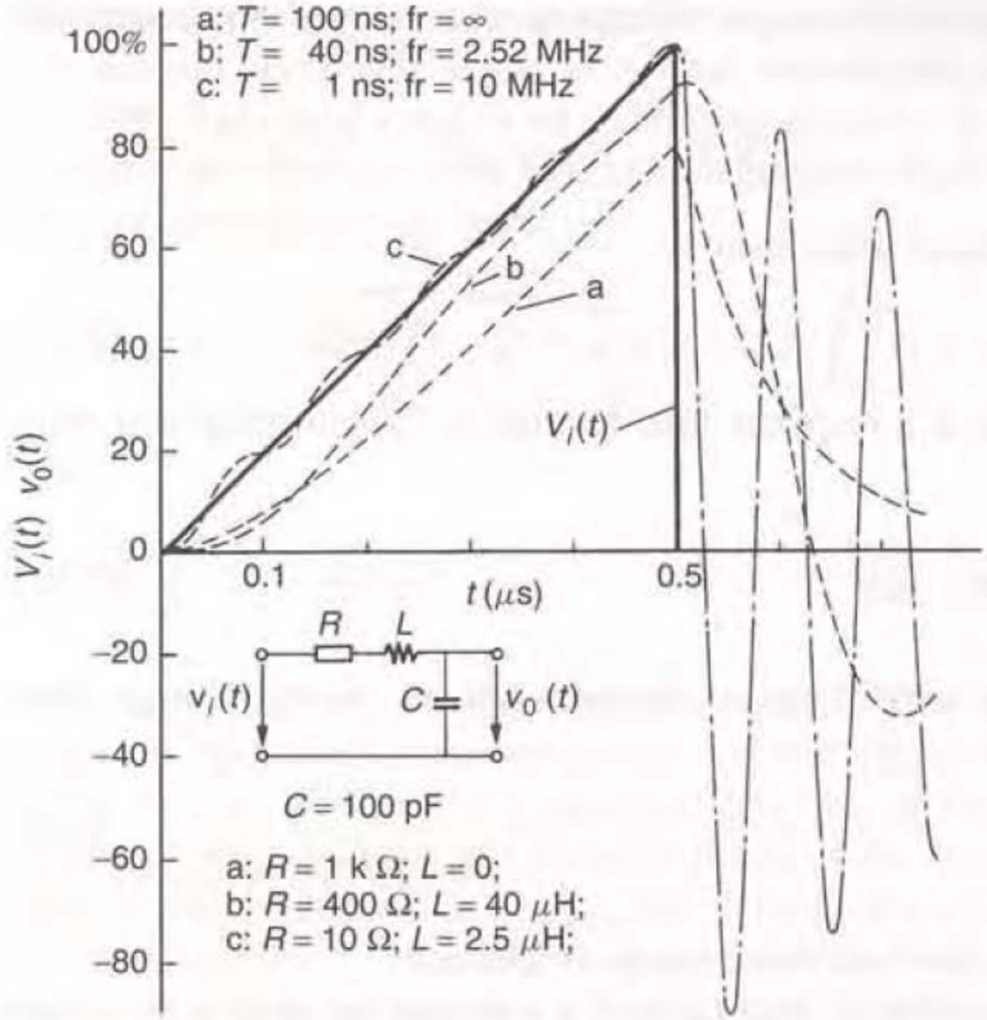


Figure 3.28 *Computed response $V_0(t)$ of an R–L–C circuit with given parameters to a linearly rising input voltage $V_i(t)$ chopped at $T_c = 0.5$ μsec*

Measurement dynamics – damping in the lead between EUT and voltage divider

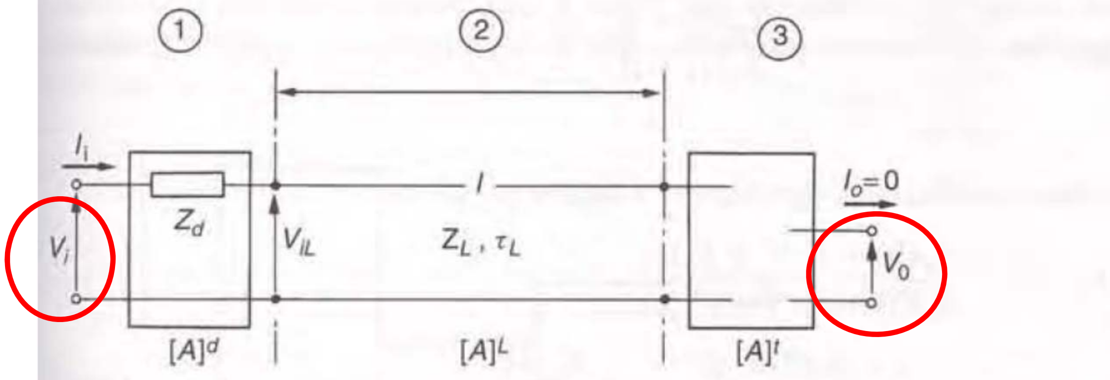
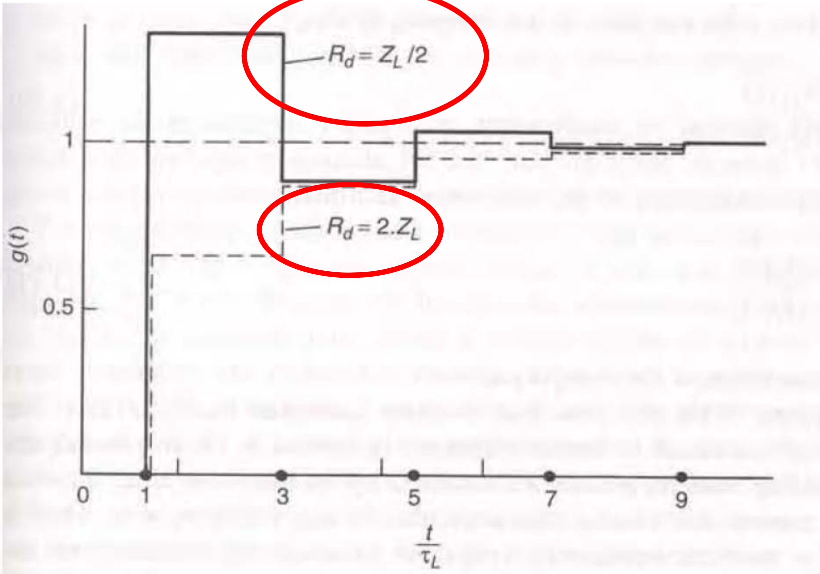


Figure 3.29 *The 'three-component system' comprised of a (1) damping, (2) transmission and (3) terminating system*



$$g(t) = L^{-1}\left\{\frac{\exp(-\tau_L s)}{s}(1 - K_t K_d)[1 + (K_t K_d)e^{-2\tau_L s} + \ldots\right.$$

$$\left.\ldots + (K_t K_d)^2 e^{-4\tau_L s} + (K_t K_d)^3 e^{-6\tau_L s} + \ldots]\right\}$$

$R_d = Z_L$ makes all reflections disappear ☺

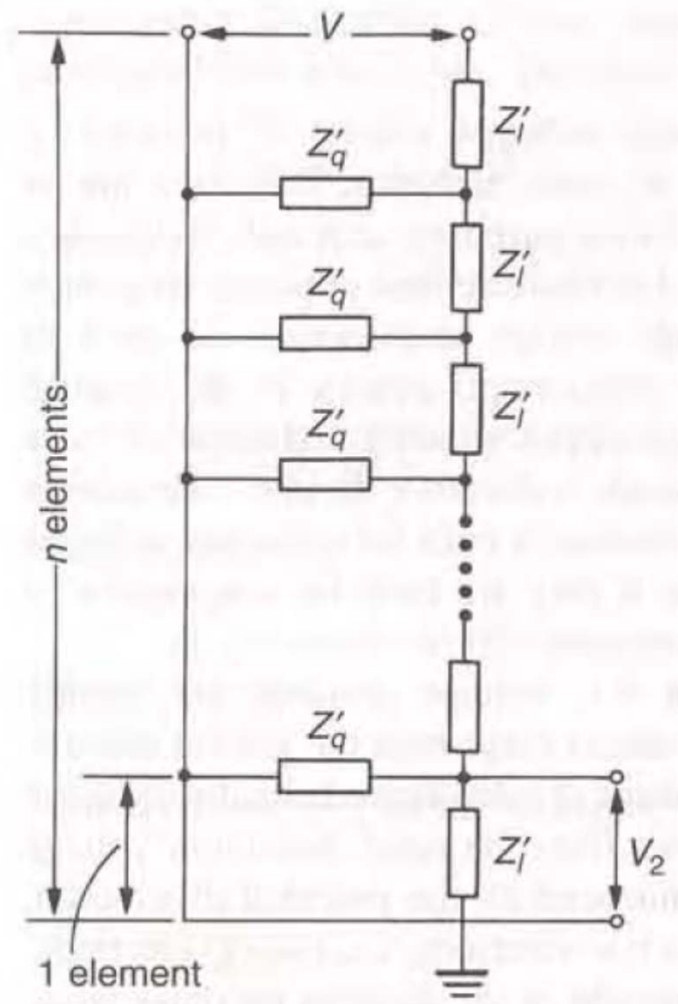
Voltage dividers

Voltage dividers for d.c., a.c. or impulse voltages consist of resistors or capacitors or convenient combinations of these elements. Inductors are in general not used for voltage dividers for testing purposes, although 'inductance voltage dividers' do exist and are used for the measurement of power frequency voltages,[135] independent from inductive voltage transformers as used in power transmission. Inductance voltage transformers consist in the simplest case of a high-quality toroidal core with a tapped winding and some of these elements can be cascaded to form a 'cascade inductance divider'. Measuring uncertainties down to a few ppm can be reached if built for quite low voltages (1 kV or less), but lots of problems arise if they are built for magnitudes of 100 kV or more. Therefore, no further treatment follows here.

2.5 to 3 m/MV for d.c. voltages;
2 to 2.5 m/MV for lightning impulse voltages;
up to or more than 5 m/MV (r.m.s.) for a.c. voltages;
up to and more than 4 m/MV for switching impulse voltages.

Large devices makes parasitic fields important!

Equally distributed parameter equivalent circuit for voltage divider



$n = N = V/V_2$

$$Z_l = \sum Z_l' = nZ_l';$$

$$Z_q = \left(\sum \frac{1}{Z_q'}\right)^{-1} = \frac{Z_q'}{n}$$

$$h_t(s) = \frac{nV_2}{V} = \frac{n \sinh \frac{1}{n}\sqrt{Z_l(s)/Z_q(s)}}{\sinh \sqrt{Z_l(s)/Z_q(s)}}.$$

$$g_t(t) = L^{-1}\left[\frac{1}{s}h_t(s)\right].$$

Normalized USR

Resistive voltage dividers

Normalized transfer function based on (3.73)

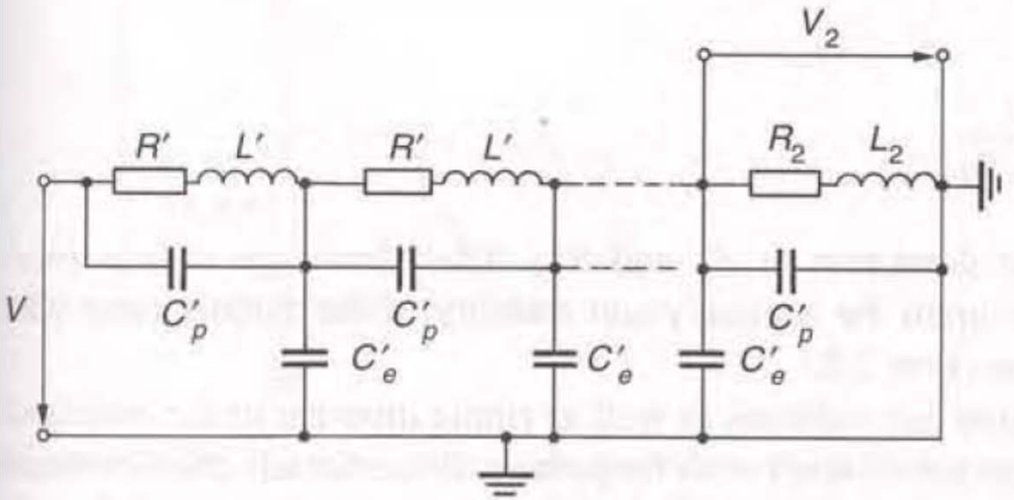


Figure 3.32 *Equivalent circuit for resistor voltage dividers.* $R = nR'$; $L = nL'$; $C_e = nC'_e$; $C_p = C'_p/n$; $R_2 = R'$; $L_2 = L'$; $R_1 = (n-1)R'$

$$h_t(s) = n\frac{\sinh \frac{1}{n}\sqrt{\frac{(R+sL)sC_e}{1+(R+sL)sC_p}}}{\sinh\sqrt{\frac{(R+sL)sC_e}{1+(R+sL)sC_p}}}.$$

$$g_t(t) = 1 + 2e^{-at}\sum_{k=1}^{\infty}(-1)^k \frac{\cosh(b_k t) + \dfrac{a}{b_k}\sinh(b_k t)}{1 + \dfrac{C_p}{C_e}k^2\pi^2};$$

where

$$a = R/2L;$$

$$b_k = \sqrt{a^2 - \frac{k^2\pi^2}{LC_e[1 + (C_p/C_e)k^2\pi^2]}};$$

$$k = 1, 2, 3, \ldots, \infty.$$

First, it is clear that resistor dividers are ideal for d.c. voltage measurements. The transfer function $h_t(s)$ for high R values and accordingly small values of L/R increase steadily with a decrease of the frequency. For $s \to 0$, $h_t(s) \hat{=} 1$ and therefore

$$\boxed{V_2 = \frac{V}{n} = V\frac{R_2}{R_1 + R_2}}$$

Simplified bandwidth and response time R-divider

The ability to measure a.c. voltages as well as ripple inherent in d.c. voltages depends upon the decrease of $h_t(s)$ with frequency. Since for all constructions of high ohmic resistor dividers the L/R values are lower than about 0.1 μsec, and also $C_p \ll C_e$, the controlling factor of the transfer function is given by the product RC_e. We can thus neglect L and C_p in eqn (3.74) as well as in

$$h_t(s) \approx n \frac{\sinh \frac{1}{n}\sqrt{sRC_e}}{\sinh \sqrt{sRC_e}}$$

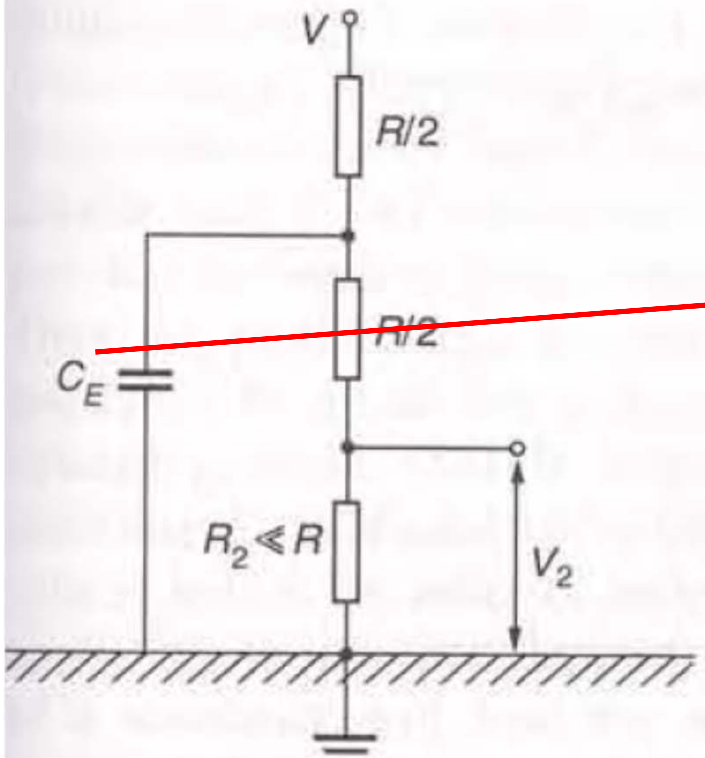
$$g_t(t) = 1 + 2\sum_{k=1}^{\infty}(-1)^k \exp\left(-\frac{k^2\pi^2}{RC_e}t\right)$$

$$|g_t(s)| = 1/\sqrt{2}$$ Which means -3 dB

$$f_B = \frac{1.46}{RC_e}$$

$$T^0 = \frac{RC_e}{6} \approx T$$

Even simpler equivalent circuit for R-divider



$$g_t(t) = 1 - \exp(-t/\tau); \qquad \tau = RC_E/4$$

$$T^0 = \frac{RC_e}{6} = \frac{RC_E}{4}; \quad \rightarrow C_E = \frac{2}{3}C_e,$$

NOT distributed capacitance to G

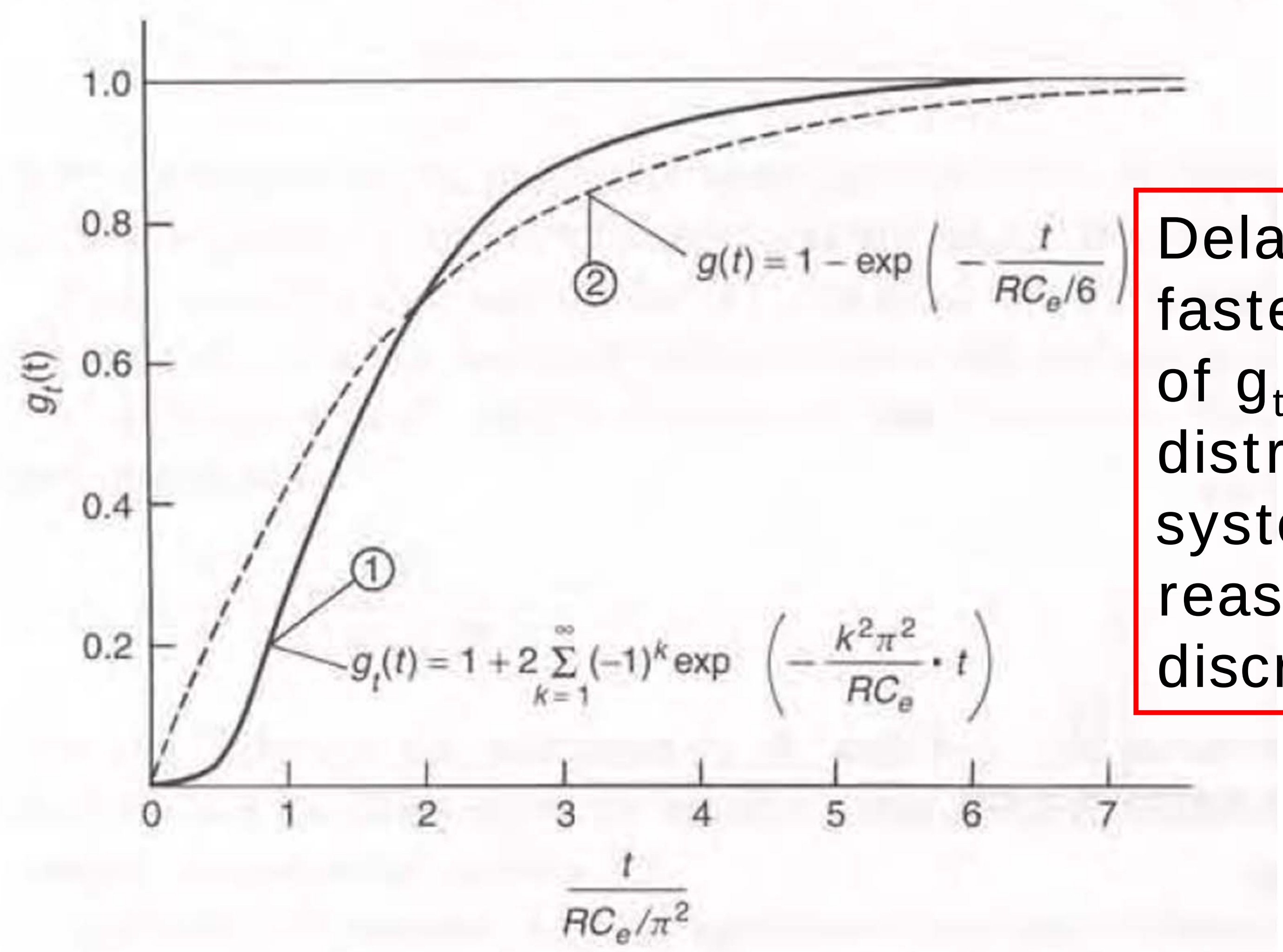
$$f_B = 1/2\pi\tau$$

And T=τ for a first order system

$$\frac{4}{2\pi RC_E} = \frac{1.46}{RC_e}; \quad \rightarrow C_E = 0.44C_e$$

Figure 3.33 *Common equivalent circuit representing approximately the distributed parameter circuit, Fig. 3.32, with $L = C_p = 0$. $C_E = \left(\frac{2}{3}\right) C_e$ for equal response times (eqn (3.80)). $C_E = 0.44\, C_e$ for equal bandwidth (eqn (3.81))*

Reason for discrepancies between simple RC and distributed model



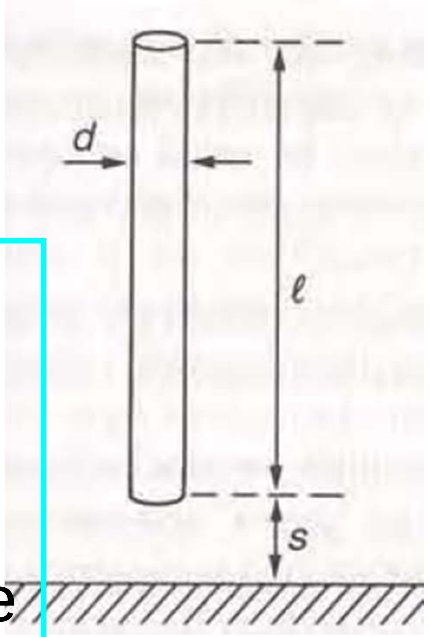
Delayed, but faster increase of $g_t(t)$ for the distributed system is the main reason for the discrepancies!

Rules of thumb for R-divider bandwidth

$$\frac{C_e}{[\mathrm{pF}]} \approx (10-15)\frac{H}{[\mathrm{m}]}; \qquad \frac{R}{[\mathrm{G\Omega}]} \approx (1-2)\frac{V}{[\mathrm{MV}]};$$

$$C_e = \frac{2\pi\varepsilon l}{\ln\left[\dfrac{2l}{d}\sqrt{\dfrac{4s+l}{4s+3l}}\right]}$$

$$f_B \approx \frac{50\ldots150}{HV} \quad \text{with} \begin{cases} f_B \text{ in Hz} \\ H \text{ in m.} \\ V \text{ in MV} \end{cases}$$



Resistor dividers cannot be constructed for very HV lightning impulse because of shortage in sufficient bandwidth. Shielding (lower C_e) plus a reduction in R is necessary to lower τ and thereby increase bandwidth. SIZE MATTERS ☺

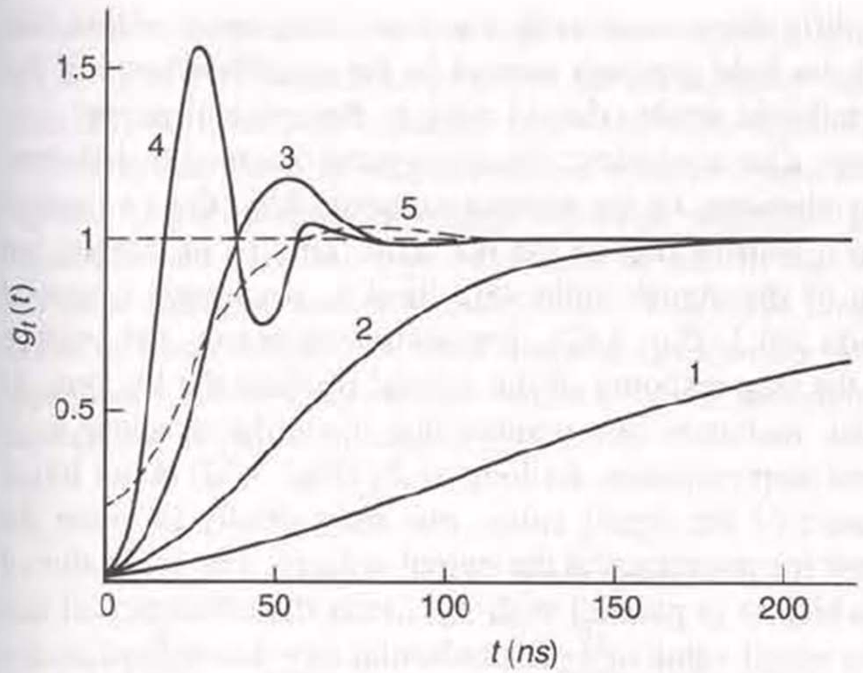


Figure 3.35 *Calculated unit step response for resistor dividers. Equivalent circuit according to Fig. 3.32*

$L/R = 10$ nsec;	$C_e = 40$ pF;	$C_p = 1$ pF;	R_{crit}
(1) $R = 30\,k\Omega$			$15.5\,k\Omega$
(2) $R = 10\,k\Omega$			$8.9\,k\Omega$
(3) $R = 3\,k\Omega$			$4.85\,k\Omega$
(4) $R = 1\,k\Omega$			$2.8\,k\Omega$
$L/R = 10$ nsec;	$C_e = 12$ pF;	$C_p = 1$ pF;	
(5) $R = 10\,k\Omega$			$13.4\,k\Omega$

Critical resistance in order to have aperiodically damping

$$R_{crit} \approx R \leq 2\pi\sqrt{\frac{L}{C_e}\frac{1}{1+\pi^2 C_p/C_e}}$$

Parallel-mixed resistor-capacitor dividers

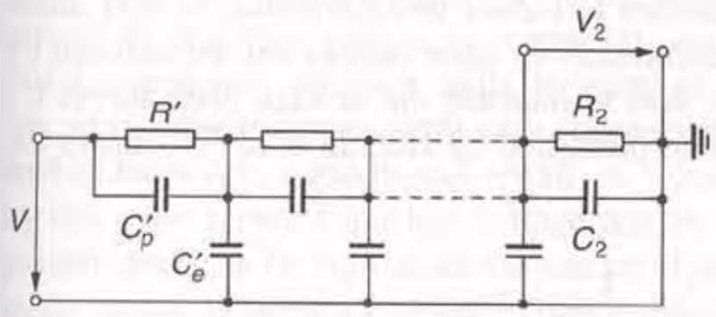


Figure 3.36 *Simplified equivalent circuit for parallel-mixed resistor–capacitor dividers.* $R = nR'$; $C_p = C'_p/n$; $C_e = nC'_e$; $R_2 = R'$; $C_2 = C'_p$

$$h_t(s) = n\frac{\sinh \frac{1}{n}\sqrt{\frac{sRC_e}{1+sRC_p}}}{\sinh \sqrt{\frac{sRC_e}{1+sRC_p}}}$$

$$g_t(t) = 1 + 2\sum_{k=1}^{\infty}(-1)^k \frac{\exp(-a_k t)}{1+k^2\pi^2 C_p/C_e}$$

$$a_k = \frac{k^2\pi^2}{RC_e(1+k^2\pi^2 C_p/C_e)}$$

$$k = 1, 2, 3, \ldots$$

The idea is to reduce the influence of the stray capacitances to earth thus improving bandwidth

The peculiar effect of this circuit is detected by the calculation of the limiting values for very high and very low frequencies, or very short and very long times:

$$\lim_{s\to 0}[h_t(s)] = 1; \quad \lim_{t\to\infty}[g_t(t)] = 1.$$

But

$$\lim_{s\to\infty}[h_t(s)] \cong 1 - \frac{C_e}{6C_p}; \quad \lim_{t\to 0}[g_t(t)] \cong 1 - \frac{C_e}{6C_p}.$$

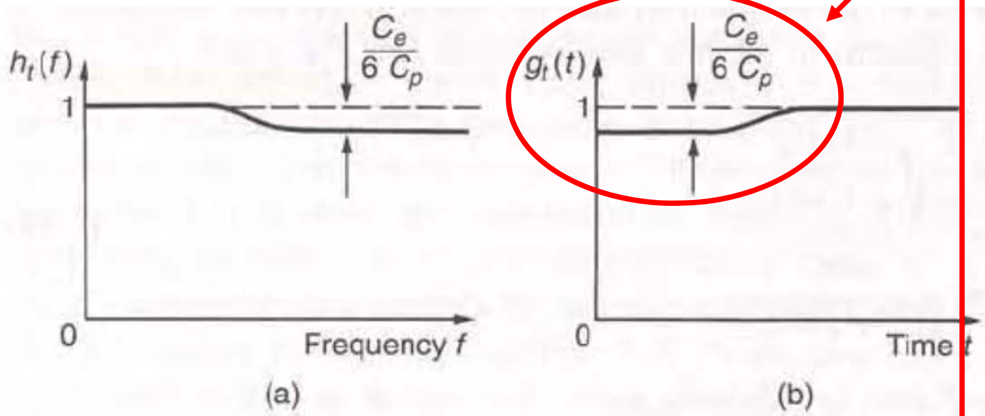


Figure 3.37 *Schematic diagrams for the normalized amplitude frequency response: (a) and unit step response, (b) for voltage dividers according to Fig. 3.36*

Mixed resistor/capacitor dividers NOT recommended for high impulse voltages !!

In other words; the compensation of resistor dividers with parallel capacitors should be avoided!!

Capacitive impulse voltage dividers

Capacitor voltage dividers

It was shown in section 3.5.4 that pure capacitor voltage dividers could be made either by using single h.v. capacitance units, i.e. a compressed gas capacitor, in series with a l.v. capacitor, or by applying many stacked and series connected capacitor units to form an h.v. capacitor. The absence of any stray capacitance to earth with compressed gas capacitors provides a very well-defined h.v. capacitance, small in value and small in dimensions, and by this even a pure capacitor voltage divider with quite good high-frequency performance can be built if the l.v. arm or capacitor is constructively integrated in the layout of such a capacitor. This means that this capacitor must be very close to the h.v. capacitance, and this can be provided for instance by inserting

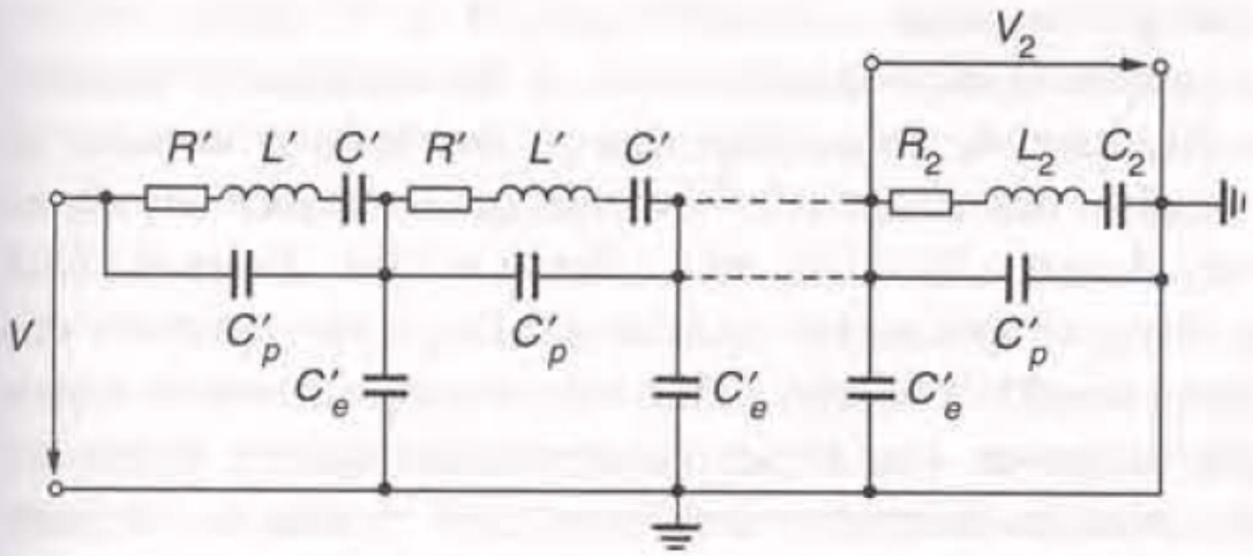


Figure 3.38 *Equivalent circuit for capacitor voltage dividers.* $R = nR'$; $L = nL'$; $C_e = nC'_e$; $C = C'/n$; $C_p = C'_p/n$; $R_2 = R'$; $L_2 = L'$; $C_2 = C'$

The absence of any stray capacitance to earth provides a very-well defined HV capacitance

Unit step response for capacitive divider

$$g_t(t) = 1 - \frac{C_e}{6(C + C_p)} + 2\exp(-at)\sum_{k=1}^{\infty}(-1)^k \frac{\cosh(b_k t) + \dfrac{a}{b_k}\sinh(b_k t)}{AB},$$

where

$$A = \left(1 + \frac{C_p}{C} + \frac{C_e}{Ck^2\pi^2}\right), \quad a = \frac{R}{2L},$$

$$B = \left(1 + \frac{C_p k^2\pi^2}{C_e}\right), \quad b_k = \sqrt{\frac{k^2\pi^2 \cdot A}{LC_eB}}, \qquad (3.87)$$

Becomes 1 when R=0

Evaluation of (3.87) almost undamped !

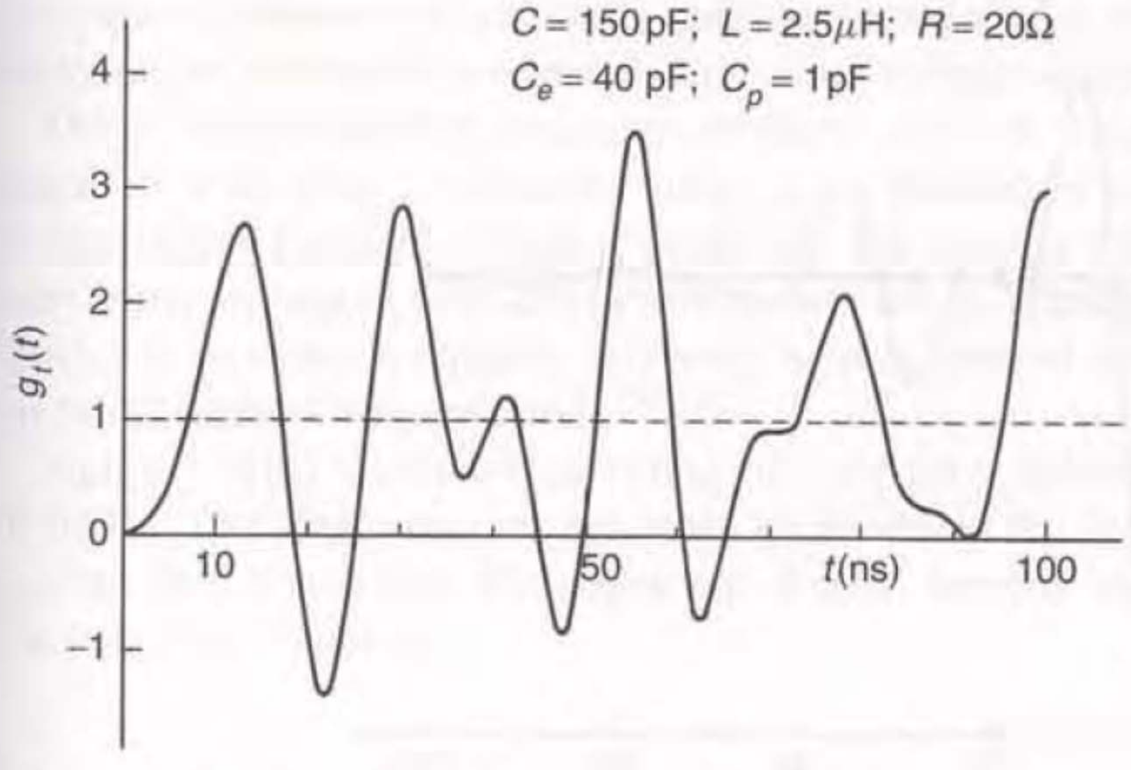


Figure 3.39 *Calculated unit step response for a capacitor voltage divider; the equivalent circuit is Fig. 3.38.* $R = 20\ \Omega$; $L = 2.5\ \mu H$; $C = 150\ pF$; $C_e = 40\ pF$; $C_p = 1\ pF$

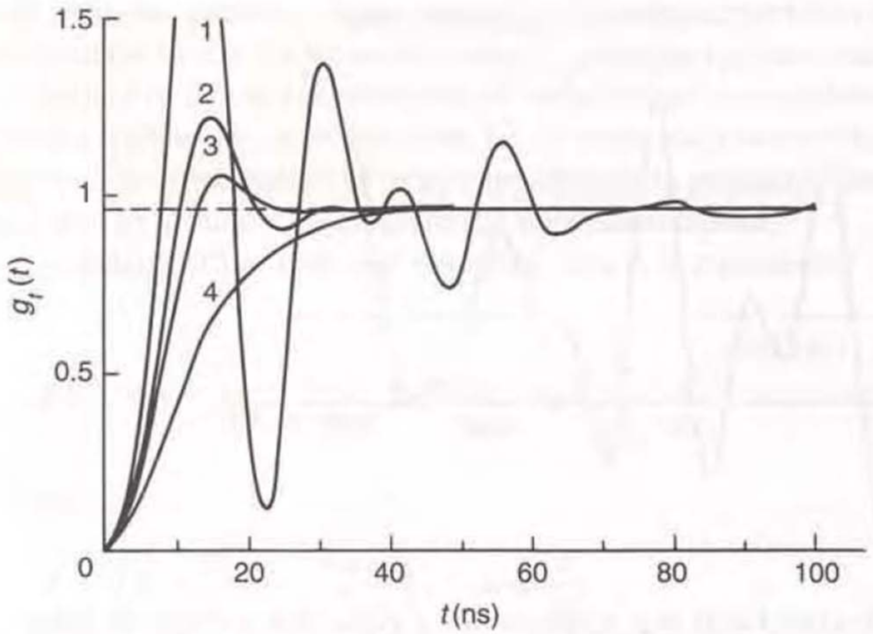


Figure 3.40 *Computed unit step response $G_t(t)$ for damped capacitor dividers according to equivalent circuit, Fig. 3.38*

$C = 150\ pF;\ L = 2.5\ \mu H;\ C_e = 40\ pF;\ C_p = 1\ pF$

(1) $R = 250\ \Omega$

(2) $R = 750\ \Omega$ $\qquad 4\sqrt{\frac{L}{C_e}} = 1000\ \Omega$

(3) $R = 1000\ \Omega$

(4) $R = 2000\ \Omega$

A very well damped response is reached with resistance R as:

$$R \approx 4\sqrt{\frac{L}{C_e}}$$

Interaction between voltage divider and its lead

The analytical treatment of our measuring system presented so far is not yet complete. Whereas the USR of the voltage dividers could readily be calculated, similar results are missing for the entire circuit. Now it can be shown that the generalized expression for the response time T and its interaction with the circuit elements, eqn (3.66), can effectively be applied in practice.

As already mentioned in section 3.6.3, it is too difficult to apply an analytical solution to the USR of the whole measuring system, which was represented by the 'three-component system' of Fig. 3.29. Numerical solutions by

is assumed to be an extended plane. Many experiments[50] demonstrated that the travel time τ_L is controlled by the velocity of light c_0. As $Z_L = \sqrt{L_L/C_L}$ and $\tau_L = \sqrt{L_L C_L} = l/c_0$, with L_L being the total inductance and C_L the total capacitance of this lead, $Z_L = l/c_0 C_L$, with l being the length of the lead. The capacitance of the lead can be computed assuming that a cylindrical lead of diameter d is at height H above a plane, which is earthed. The well-known

Capacitance of cylindrical lead of diameter d and heigth H above grounded plane

WARNING !
Erroneous square root line extension !

capacitance formula

$$C_L = \frac{2\pi\varepsilon_0 l}{A};$$

where

$$A = \ln\left[\frac{2l}{d}\sqrt{\frac{\sqrt{1+(2H/l)^2}-1}{\sqrt{1+(2H/l)^2}+1}}\right]$$

$$= \ln\left(\frac{4H}{d}\right) - \ln\frac{1}{2}\left(1+\sqrt{1+2(H/l)^2}\right)$$

may well be used, although this lead is placed between the test object and the voltage divider. As $c_0 = (\varepsilon_0 \mu_0)^{-0.5}$, where ε_0 = permittivity and μ_0 = permeability of free space, the surge impedance becomes

$$(Z_L)_{\text{hor}} = A\frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}} = 60 \times A(l, d, H) \quad [\Omega] \tag{3.89}$$

for this *horizontal* lead. Sometimes, the horizontal lead is lengthened by a vertical lead to measure the experimental USR of the system. Thus we need Z_L for a vertical lead also. According to Fig. 3.22 and eqn (3.33), this capacitance is known. With the same assumptions as made above, we obtain

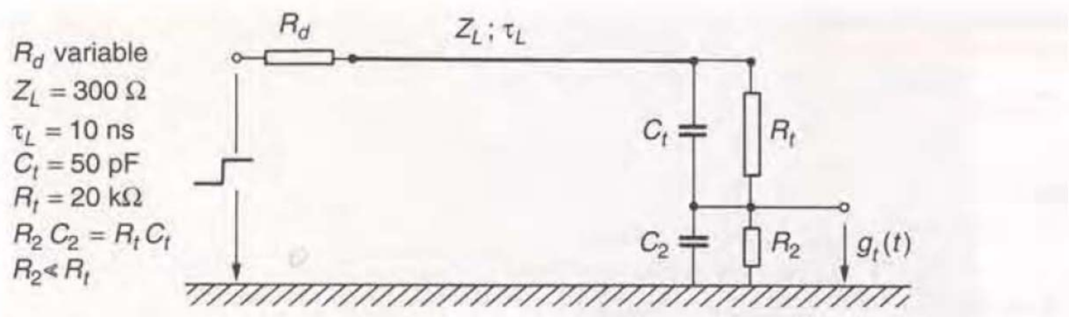
$$(Z_L)_{\text{vert}} = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}} \ln\left[\frac{2l}{d}\sqrt{\frac{4s+l}{4s+3l}}\right]$$

$$\approx 60 \ln\left(\frac{1.15l}{d}\right) \quad [\Omega] \quad \text{for } \cancel{s \gg l}. \quad s \ll l \tag{3.90}$$

The differences in the surge impedances are not large if the usual dimensions are taken into account.

Resistive divider – USR with lead

In Fig. 3.42(a), a very simplified equivalent circuit represents a 20-kΩ resistor divider with a lead length of 3 m ($\tau_L = 10$ ns). The divider is idealized by the omission of any stray capacitances or inductances, but a parallel capacitance of $C_t = 50$ pF across the whole divider represents a top electrode which may shield the divider. A pure resistor R_d provides ideal damping conditions for travelling waves. Figure 3.42(b) shows some computed results of the USR. For $R_d = 0$, no noticeable damping effect is observed within the exposed time scale. Although the oscillations are non-sinusoidal, the fundamental frequency can clearly be seen. This frequency is obviously close to the resonance frequency f_r, generated by the lead inductance L_L and the divider's capacitance C_t. As $L_L = Z_L \tau_L$, this inductance is 3 μH, giving $f_r = 13$ MHz.



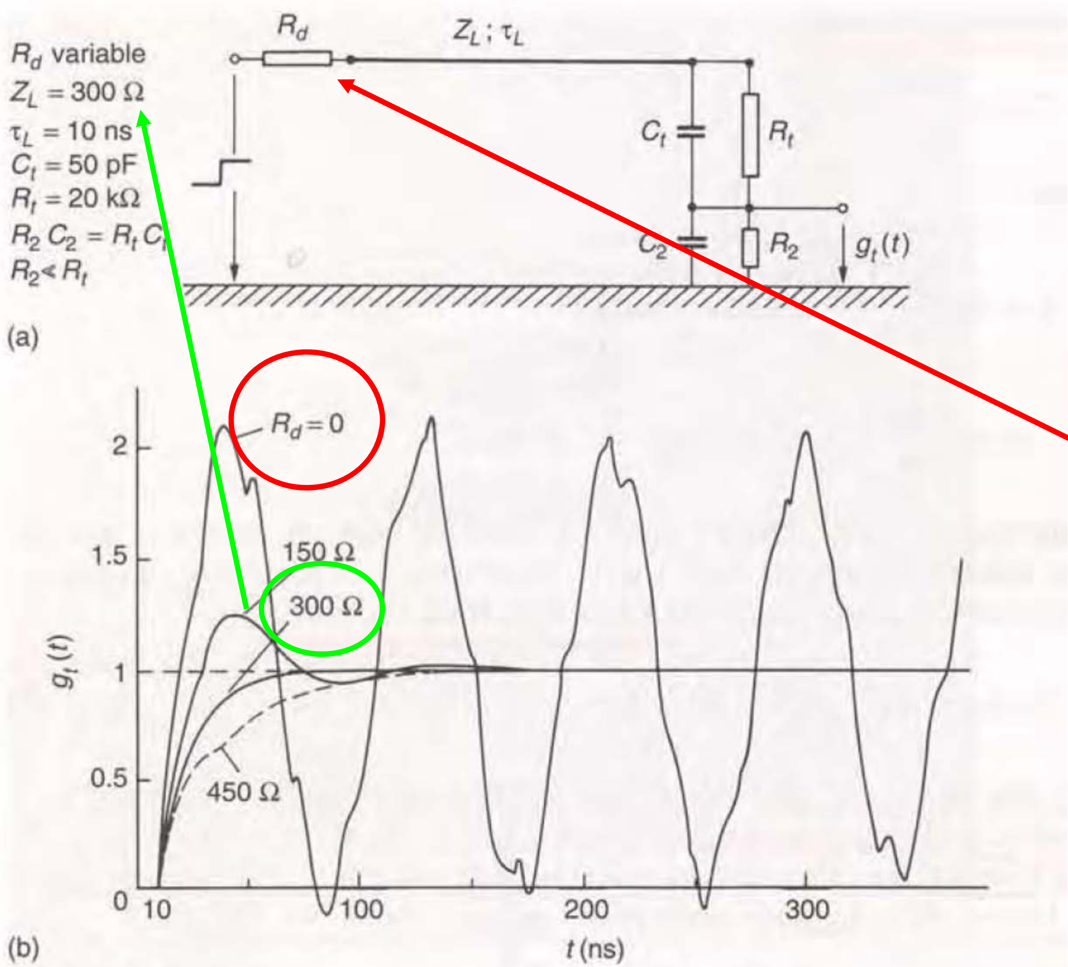


Figure 3.42 *Computed unit step response for idealized resistor or parallel-mixed resistor–capacitor divider with lead. (a) Equivalent circuit. (b) Computed USR*

Matching resistor R_d = Z_L makes no reflections.........!

Response time for divider and its lead

$$T = T^0 - \tau_L = \frac{1}{(1 + R_d/R_t)}\left[R_d C_t - \tau_L\left(1 - \frac{Z_L}{R_t}\right)\left(1 - \frac{R_d}{Z_L}\right)\right]. \quad (3.91)$$

Some remarkable findings can be observed.

For $R_d = Z_L$, the length of the lead has *no* influence upon the response time. This case corresponds to the 'infinite line response', as the same result would be achieved if a step voltage supplied from an extremely long lead would be applied to the dividing system.

With no damping resistance, or $R_d < Z_L$, the response time taken from the actual beginning of the USR will always decrease proportionally with the lead length $l = \tau_L c_0$. This decrease of T is clearly produced by an overshoot of the USR. As is seen from the computed USR, the determining factor is $R_d C_t$ providing a positive contribution to T. For capacitor dividers, $R_t \to \infty$ and the same equation can be applied.

!

Distributed resistance

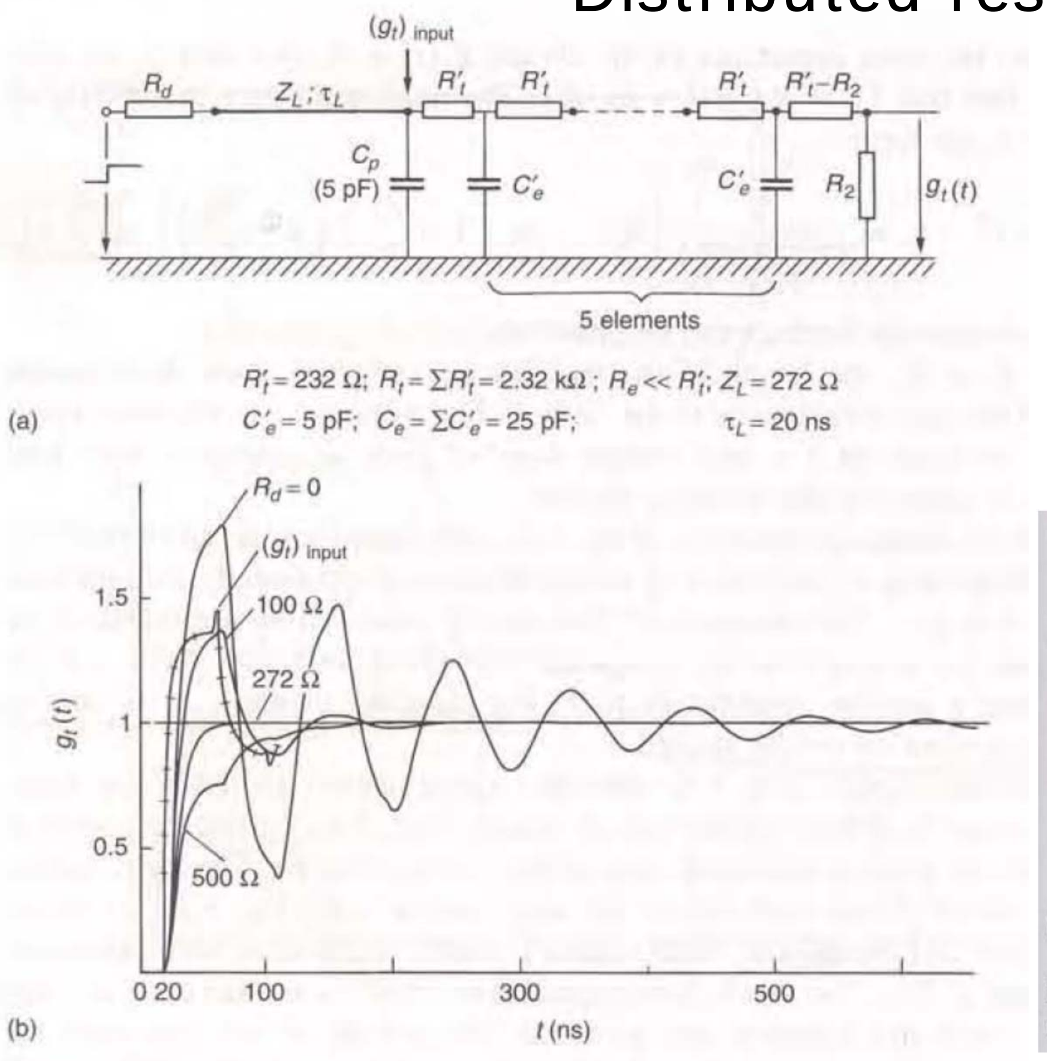


Figure 3.43 *Computed USR for low-value resistor voltage divider. (a) Equivalent circuit. (b) Computed USR (for divider input)*

Low resistance divider with negligible inductance, but stray capacitance

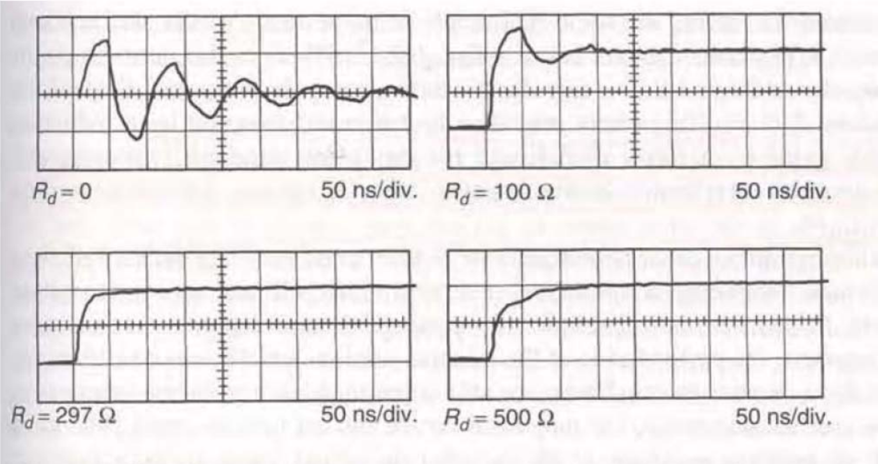


Figure 3.44 *Measured unit step response for the resistor voltage divider. $R = 2320\ \Omega$, with 6-m lead, according to Fig. 3.43*

Resistive voltage divider with parasitic inductance

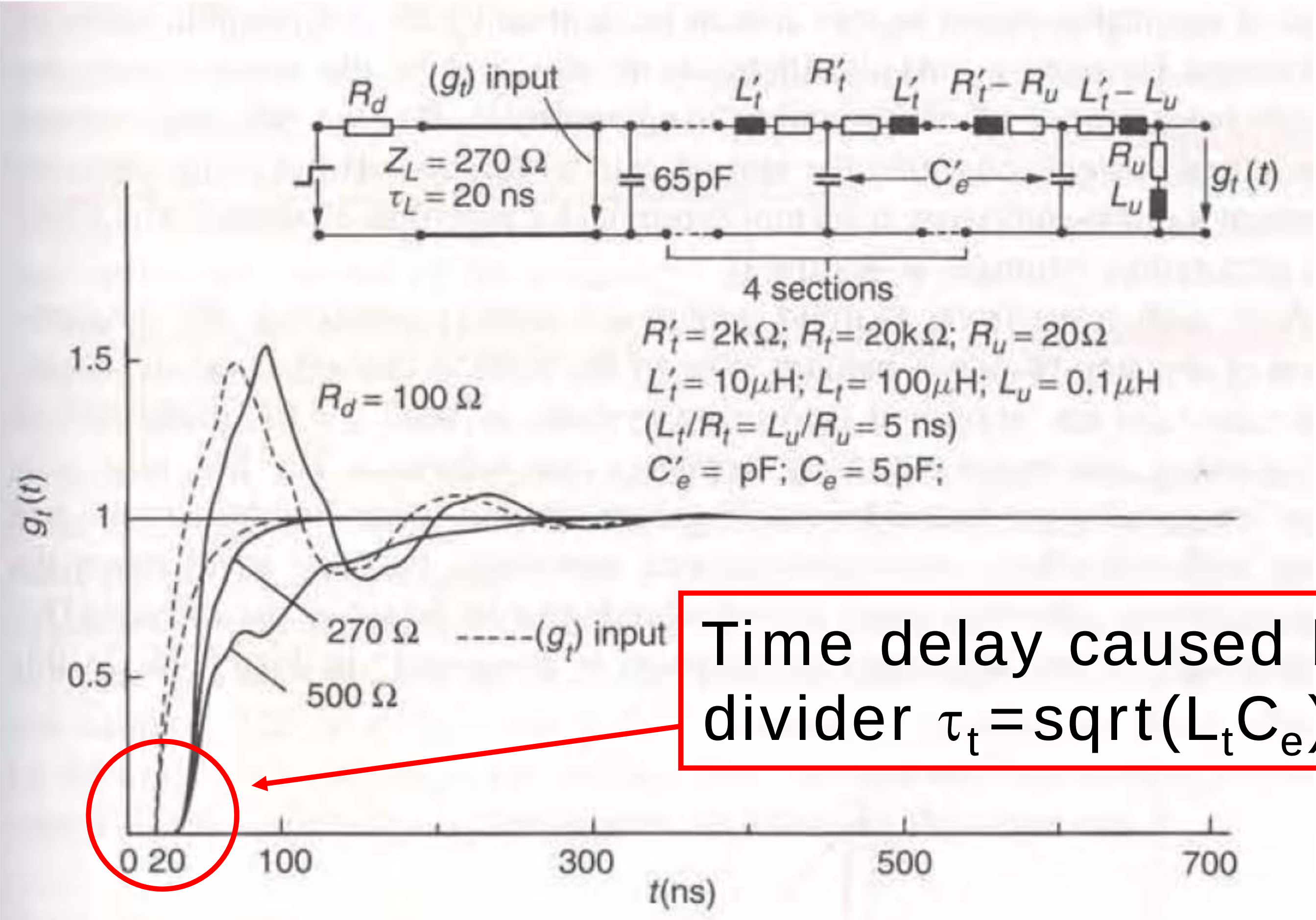
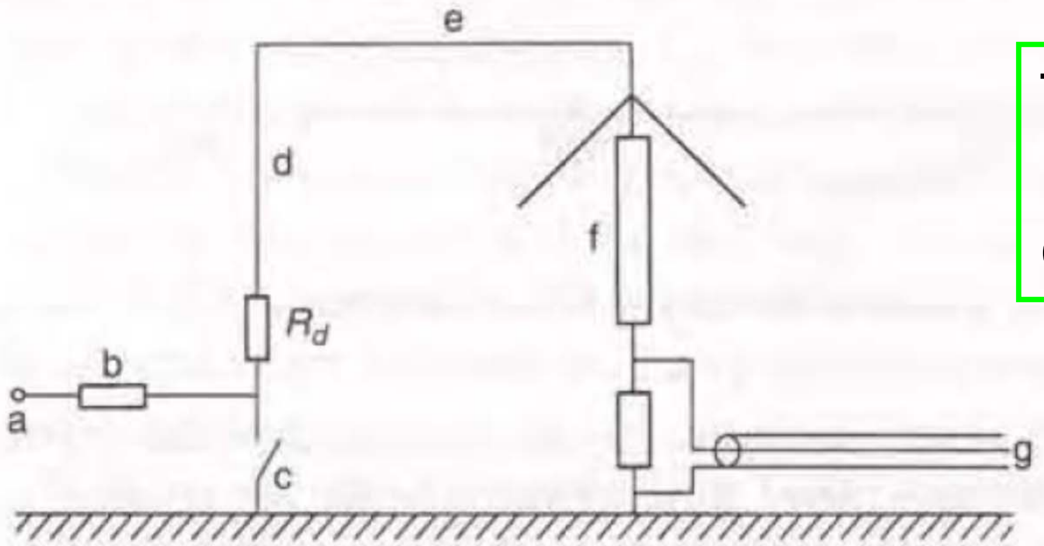


Figure 3.45 *Computed USR of resistor divider with inductance*

Time delay caused by travel time of divider $\tau_t = \mathrm{sqrt}(L_t C_e)$

Setup for measuring USR of voltage divider systems



This is what we do in the laundry wire experiment ☺

Figure 3.46 *The unit step method. (a) To d.c. supply. (b) Charging resistor. (c) Fast switch. (d) (Added) vertical lead. (e) High-voltage lead. (f) Voltage divider. (g) To recording instrument* the earlier standards.(6.57) The step generator a to c must have approximately zero impedance while generating the voltage step and during the subsequent response. Any fast switching device, c, which short-circuits a constant d.c. voltage as used to charge the measuring system before the short-circuit occurs is applicable. Very suitable switches are mercury-wetted relays but also a uniform field gap of about 1 mm spacing at atmospheric air or a uniform

shall serve as the earth return between divider and the step generator. The length of the lead, represented by d and e, shall be equal to the length as used during actual impulse voltage measurement, if the response time is an essential parameter during the measurements. If only part e is used during

Use metallic wall/plate at step generator

Starting point of USR must be approximated

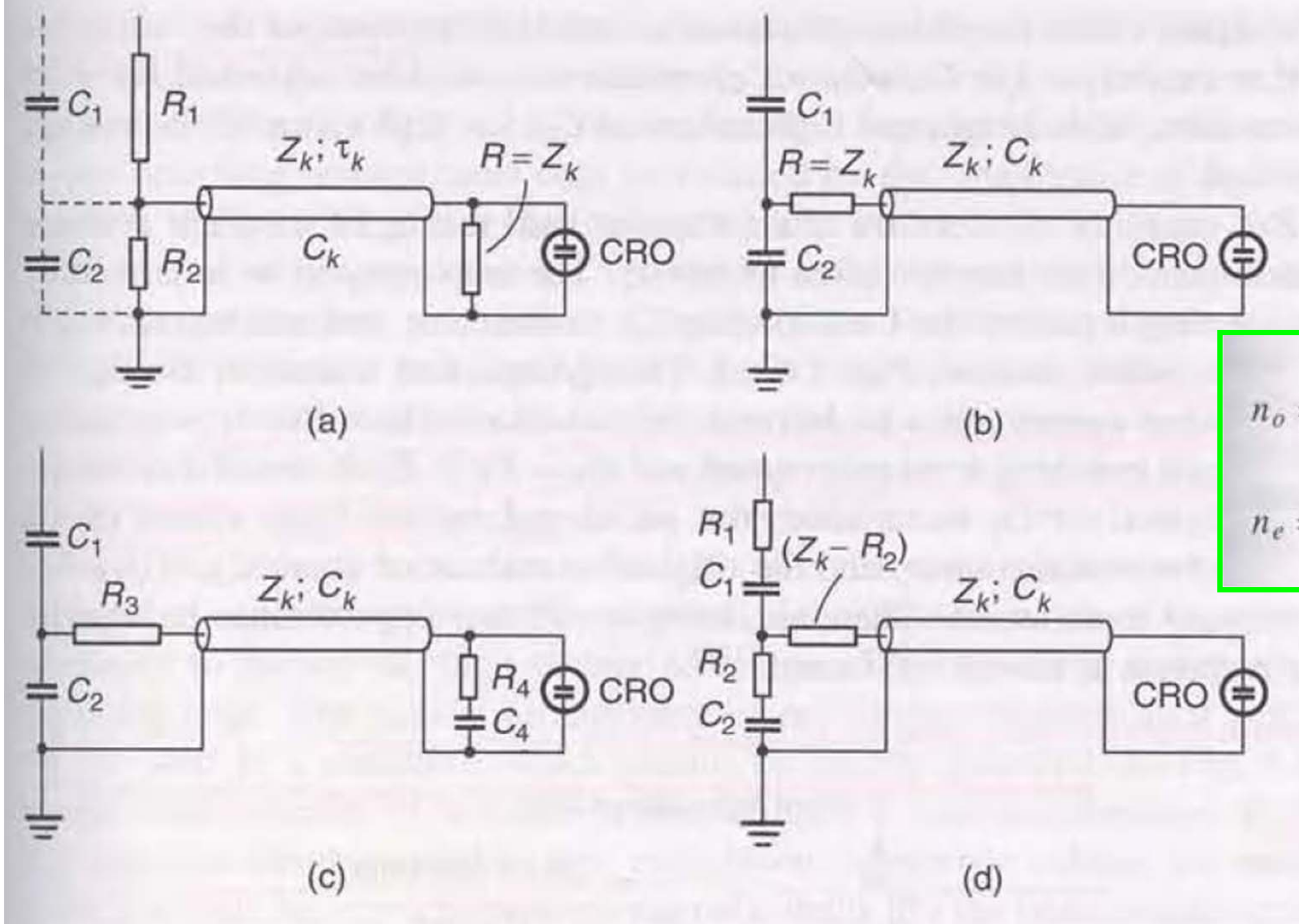


Figure 3.47 *Circuits for signal cable matching. (a) Resistor or parallel-mixed capacitor-dividers. (b) Capacitor dividers, simple matching. (c) Capacitor dividers, compensated matching. (d) Damped capacitor divider, simple matching*

$$n_o = \frac{C_1 + C_2}{C_1} \quad \text{for } t = 0;$$

$$n_e = \frac{C_1 + C_2 + C_k}{C_1} \quad \text{for } t \geq 2\tau_k.$$