Department of Energy Technology

AALBORG UNIVERSITET

Matlab and Control Theory — INTRO 1st semester 2014

January 13, 2015

Written exam

09.00-13.00 CET (4 hours)

The set consists of 11 problems

Rules

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions.
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off. Internet access is strictly forbidden.
- Any kind of communication with other students is not allowed.

Remember

1. To write your study number on all sheets handed in.
2. It must be clear from the solutions, which methods you are using, and you must include sufficient intermediate calculations, diagrams, sketches etc. so the line of thought is clear. Printing the final result is insufficient.
3. In problems 9, 10 and 11 points are awarded not only for correctly solving the problem but also for the coding style. Your code should be scalable (can work with different sizes of input vectors). It should be self explanatory. It should be well commented where it needs it. Use vector operations wherever possible.

Problem 1 (5 %)

In Figure 1 below is shown an electric filter, where R_L represent the load resistance and $v_o(s)$ is the voltage across the load.

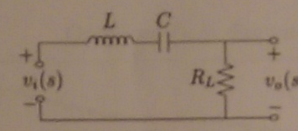


Figure 1: Circuit for which the transfer function $G(s) = \frac{v_o(s)}{v_i(s)}$ should be determined.

1. Determine the transfer function $G(s) = \frac{v_o(s)}{v_i(s)}$ when it is given that no current is drawn at the output and all initial conditions are zero.

Problem 2 (15 %)

The system shown in Figure 2 is considered. K is the gain and $G(s)$ is given by:

$$G(s) = \frac{1}{(s+2)(s+4)(s+6)}$$

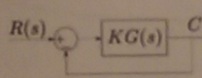


Figure 2: System considered in exercise 2.

1. Sketch the root locus for the system, when varying the gain K. Notice that it is ***not*** sufficient to use Matlab!
2. Determine, based on the root locus, whether it is possible that the dominant closed loop poles obtain a damping of $\zeta = 0.7$ using only a proportional controller (i.e. by varying the gain K).
3. Determine for which values of K the system is stable.

Problem 3 (12 %)

A system has the open loop frequency response shown in Figure 3 and the closed loop frequency response shown in Figure 4.

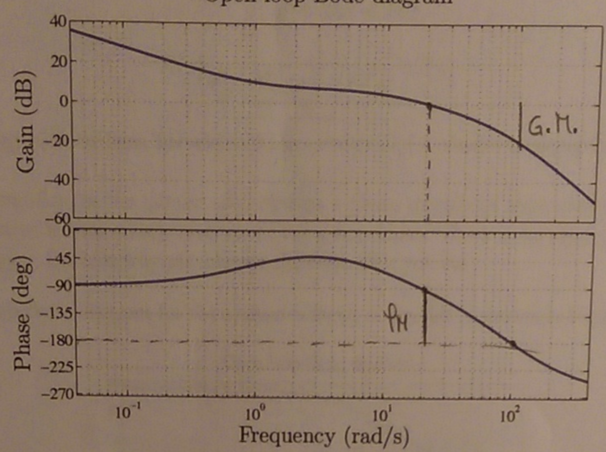


Figure 3: Open loop Bode diagram for the considered system.

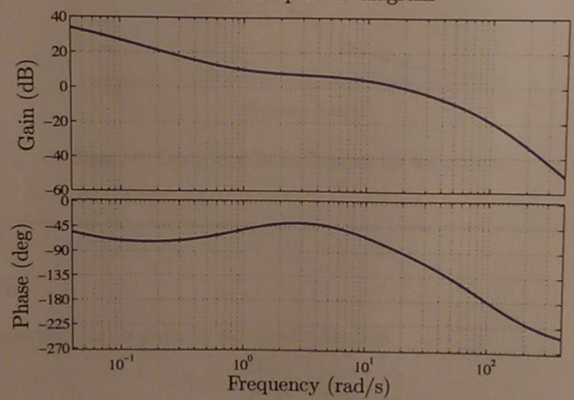


Figure 4: Closed loop Bode diagram for the considered system..

1. Determine the gain and phase margin for the system. Describe how these are determine (this page may be handed in with the solution).
2. Determine whether the closed loop system is stable or not. Justify the answer.
3. Determine the steady state error for the system, if this is given a unit step input. Justify the answer.

Problem 4 (8 %)

The system shown in Figure 5 is considered, where $G_c(s)$ represents the controller and the system transfer function is given by:

$$G(s) = \frac{800}{s^2 + 14s + 100}$$

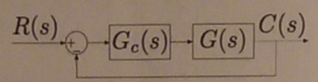


Figure 5: System, for which the controller $G_c(s)$ should be designed.

1. Design a suited controller for the system so it obtains a phase margin of approximately $\phi_m = 40°$ and so the closed loop system has ***no steady state error*** for a step input. There is *no need to improve the systems transient performance*. Explain why you choose the controller you do.

As a help the open loop Bode diagram for the system without controller is shown in Figure 6.

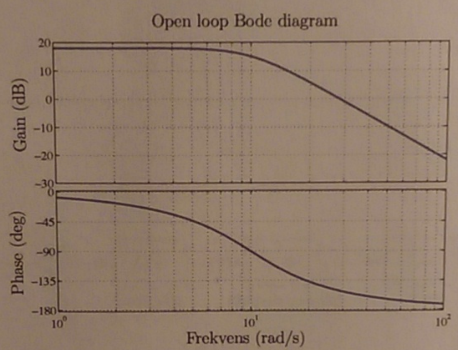


Figure 6: Open loop Bode diagram for the system.

Problem 5 (10 %)

A continuous-time controller $C(s)$ is given by the transfer function

$$C(s) = \frac{M(s)}{E(s)} = K\frac{s + \omega_0}{s + \omega_1}$$

where K, ω_0 and ω_1 are positive constants.

$C(s)$ must be discretized to an equivalent discrete-time controller $C(z)$ using the Backward Euler method (*backward rectangular rule*). The sampling time is denoted T.

1. Determine an expression for $C(z)$ and list all zeroes and poles for this controller
2. For which values of T does $C(z)$ have a zero in the left half-plane of the z-domain?
3. If $K = 0.01$, $\omega_0 = 10$ rad/s and $\omega_1 = 100$ rad/s, sketch the phase characteristics for $C(s)$
4. Which value will you select (approximately) for the sampling time, if the constants have the values listed above?

Problem 6 (8 %)

A dicrete-time controller is given by the transfer function

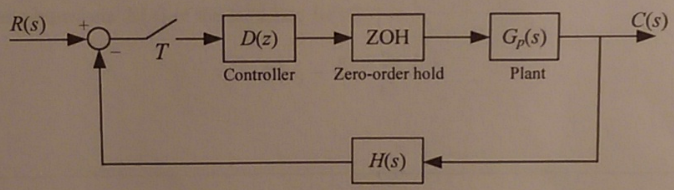
$$D(z) = \frac{U(z)}{E(z)} = \frac{z - 0.5}{z(z-1)}$$

where $U(z)$ and $E(z)$ are the output and the input, respectively. The controller is updated with a sampling frequency equal to 1 kHz.

1. Determine the gain for the discrete controller at $f = 0$ Hz and at $f = F_n/5$, where F_n denotes the Nyquist-frequency.
2. Find the difference equation, which corresponds to the transfer function $D(z)$, i.e. find a formula for $u(k)$
3. Determine and plot the output $u(k)$ when the input $e(k)$ is

$$e(k) = \begin{cases} 1, & k = 0 \\ 0, & k \geq 1 \end{cases}$$

Problem 7 (12 %)



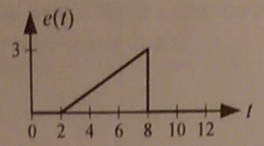
The figure shows a closed-loop control system, where the plant is given by

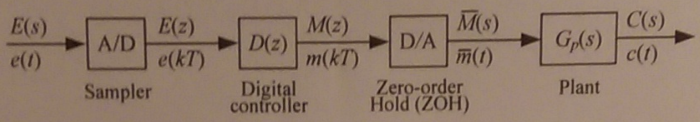
$$G_p(s) = \frac{10}{0.1s + 1}$$

Also, the filter in the feedback path is $H(s) = 0.1$. The discrete-time controller is $D(z) = K/z$, where K is a positive constant. The sampling time is $T = 20$ ms.

1. Determine an analytical expression for $G(z)$ — the plant's pulse transfer function
2. Sketch the root locus in the z-domain
3. Determine an analytical expression for the z-domain closed loop transfer function $T(z) = C(z)/R(z)$
4. Find the closed-loop poles when $K = 4$ and discuss the stability properties for $K = 4$
5. Write a `Matlab` script that can plot the closed-loop step response for the system (you do not need to show any results - just print the needed commands)

Problem 8 (10 %)





The continuous-time signal $e(t)$ shown in the top of the figure is input to the shown system. The sampling time for both the A/D and D/A is $T = 2$ seconds. Also, the discrete controller $D(z)$ and the plant $G_p(s)$ are given by

$$D(z) = \frac{z}{z-1} \quad \text{and} \quad G_p(s) = \frac{1}{s},$$

respectively.

1. Explain in your own words which functionality the A/D and the D/A converters have
2. Sketch to scale (i.e. accurately) the signals $e(kT)$, $m(kT)$, $\overline{m}(t)$ og $c(t)$ in the interval $0 \leq t \leq 10$ seconds.
3. What is the steady-state value (as $t \to \infty$) for the output $c(t)$?
4. Prove that the z-transform $E(z)$ of the sequence $\{e(kT)\}$ is

$$E(z) = \frac{z^2 + 2z + 3}{z^4}$$

Problem 9 (5 %)

Given a code below:

```
a = rand(100000000,1);

s = 0;
for i = 1:length(a)
  if a(i) >= 0.5
    s = s+1;
  end
end
```

1. Vectorize this code
2. Time this code and your code and find the speed increase.

Problem 10 (5 %)

You are given a vector of 46 random exam scores. Write a function *topnaverage* to calculate the mean of only n highest grades. Assume you always get more scores than n.

```
scores = randi(100,1,46)
n = 8
topnaverage(scores,n)
```

Problem 11 (10 %)

Write a function *weightedmean* that, given two vectors, will determine the weighted arithmetic mean.

Example, for

```
a = [10 15 20];
weights = [0.1 0.2 0.5]
weightedmean(a,weights)
```

your function should return

$$\frac{0.1 \cdot 10 + 0.2 \cdot 15 + 0.5 \cdot 20}{0.1 + 0.2 + 0.5} = 17.5$$

Remember to make the function work with arbitrarily long input vectors.