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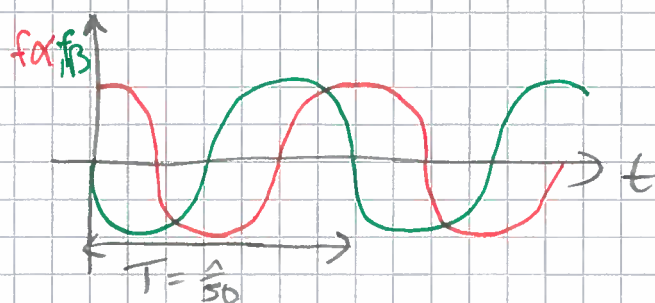
Re-exam 22nd Feb 13

Problem 1:

$$1) \quad \bar{f} = e^{-j\omega t} = \cos(\omega t) - j \sin(\omega t)$$

$$f_a = \operatorname{Re}(\bar{f}) = \cos(\omega t)$$

$$f_b = \operatorname{Im}(\bar{f}) = -\sin(\omega t)$$



$$2) \quad f_{a0} = \operatorname{Re}\left(\frac{\bar{f}}{e^{j0}}\right) = \operatorname{Re}(e^{-j\omega t}) = \cos(\omega t)$$

$$f_{b0} = \operatorname{Re}\left(\frac{\bar{f}}{e^{j120^\circ}}\right) = \operatorname{Re}(e^{-j(\omega t + 120^\circ)}) = \cos(\omega t + 120^\circ)$$

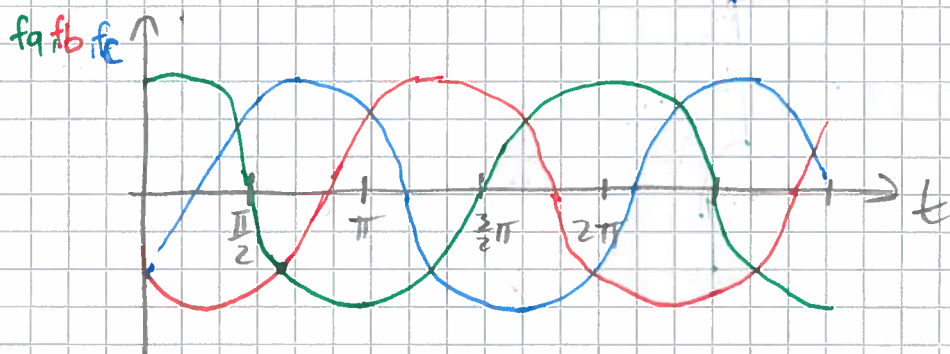
$$f_{c0} = \operatorname{Re}\left(\frac{\bar{f}}{e^{j240^\circ}}\right) = \operatorname{Re}(e^{-j(\omega t - 120^\circ)}) = \cos(\omega t - 120^\circ)$$

In balanced systems:

$$f_a = f_{a0}$$

$$f_b = f_{b0}$$

$$f_c = f_{c0}$$



(2)

3.)

$$\bar{V}_{dqs} = \bar{V}_{dqs} e^{j\theta}$$

$$\bar{I}_{dq} = \frac{\bar{V}_{dq}}{e^{j\theta}} =$$

$$\ominus = 2\pi 50$$

$$\bar{f}_d = \operatorname{Re}\left(\frac{\bar{f}}{e^{j\theta}}\right) = \operatorname{Re}(\bar{f} e^{-j\theta})$$

$$f_d = \operatorname{Re}\left(\frac{\bar{f}}{e^{j\theta}}\right) = \operatorname{Re}\left(\frac{e^{-j\theta}}{e^{j\theta}}\right) = \operatorname{Re}(e^{-j2\theta})$$

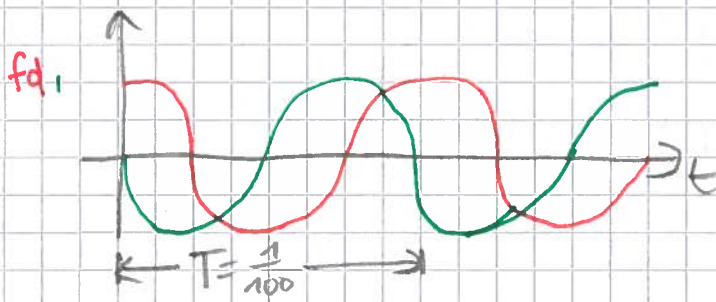
$$= \cos(2\theta) = \cos(2\pi 100 t)$$

$$f_q = \operatorname{Re}\left(\frac{\bar{f}}{e^{j(\theta+90^\circ)}}\right) = \operatorname{Re}\left(\frac{e^{-j\theta}}{e^{j(\theta+90^\circ)}}\right) = \operatorname{Re}(e^{-j(2\theta+90^\circ)})$$

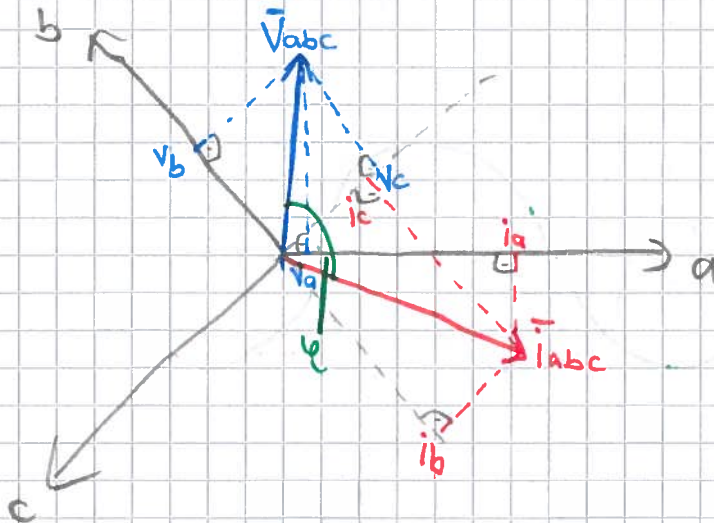
$$= \cos(2\theta + 90^\circ)$$

$$= -\sin(2\theta)$$

$$= -\sin(2\pi 100 t)$$



4.)



If current $i_a = I_m \cos(\omega t - \phi)$ and the parameters of the RL-circuit are identical, then:

$$i_b = I_m \cos(\omega t - \frac{2\pi}{3} - \phi)$$

$$i_c = I_m \cos(\omega t + \frac{2\pi}{3} - \phi)$$

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Power: $\bar{f}_{ap} = \bar{f}_{dq} \cdot e^{j\theta}$ and $\bar{f}_{ap}^* = \bar{f}_{dq}^* \cdot e^{-j\theta}$

$$\begin{aligned}
 P &= V_a i_a + V_b i_b + V_c i_c \\
 &= \frac{3}{2} (V_d i_d + V_q i_q + \cancel{V_0 i_0}) \quad \xrightarrow{\theta=0} \\
 &= \frac{3}{2} \operatorname{Re}[(V_d + jV_q)(i_d + j i_q)^*] \\
 &= \frac{3}{2} \operatorname{Re}[(V_d + jV_q)(i_d - j i_q)] \\
 &= \frac{3}{2} \operatorname{Re}[(V_d + jV_q) \cdot e^{j\theta} \cdot (i_d - j i_q) e^{-j\theta}] \\
 &= \frac{3}{2} \operatorname{Re} \left[\underbrace{V_{dq} \cdot e^{j\theta}}_{\bar{V}_{\alpha\beta}} \cdot \underbrace{I_{dq}^* \cdot e^{-j\theta}}_{\bar{I}_{\alpha\beta}^*} \right] \\
 &= \frac{3}{2} \operatorname{Re}[(V_\alpha + jV_\beta) \cdot (i_\alpha - j i_\beta)] \\
 &= \frac{3}{2} \operatorname{Re}[V_\alpha i_\alpha + V_\beta i_\beta + j(V_\beta i_\alpha - V_\alpha i_\beta)] \\
 &= \frac{3}{2} (V_\alpha i_\alpha + V_\beta i_\beta)
 \end{aligned}$$

Balanced system, since same RL parameters
 \Rightarrow hence $V_0 = 0$
 $i_0 = 0$

Problem 2

1) By vector projection of the L_q and L_d values on the ~~each~~ respective stator axes
 \Rightarrow ~~equation~~

$$\begin{aligned}
 2) \quad M_{bscs} &= L_{ad} \cdot \operatorname{Re}\left(\frac{e^{j\theta}}{e^{j120^\circ}}\right) \operatorname{Re}\left(\frac{e^{j\theta}}{e^{j-120^\circ}}\right) \\
 &\quad + L_{aq} \cdot \operatorname{Re}\left(\frac{e^{j(\theta+90^\circ)}}{e^{j120^\circ}}\right) \operatorname{Re}\left(\frac{e^{j(\theta+90^\circ)}}{e^{j-120^\circ}}\right)
 \end{aligned}$$

$$\begin{aligned}
 &= L_{ad} \cdot \cos(\theta - 120^\circ) \cos(\theta + 120^\circ) \\
 &\quad + L_{aq} \sin(\theta - 120^\circ) \sin(\theta + 120^\circ)
 \end{aligned}$$

with $L_{ad} = L_1 + L_2$
 $L_{aq} = L_1 - L_2$

$$\begin{aligned}
 &= L_1 (\cos(\theta - 120^\circ) \cos(\theta + 120^\circ) + \\
 &\quad \sin(\theta - 120^\circ) \sin(\theta + 120^\circ))
 \end{aligned}$$

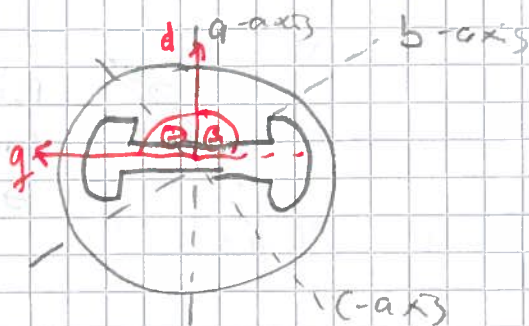
$$\begin{aligned}
 &\quad + L_2 (\cos(\theta - 120^\circ) \cos(\theta + 120^\circ) \\
 &\quad - \sin(\theta - 120^\circ) \sin(\theta + 120^\circ))
 \end{aligned}$$

$$\begin{aligned}
 &= L_1 \cdot \cos(\theta - 120^\circ - (\theta + 120^\circ)) + L_2 \cos(\theta - 120^\circ + \theta + 120^\circ) \\
 &= L_1 \cdot \cos(-240^\circ) + L_2 \cos(2\theta)
 \end{aligned}$$

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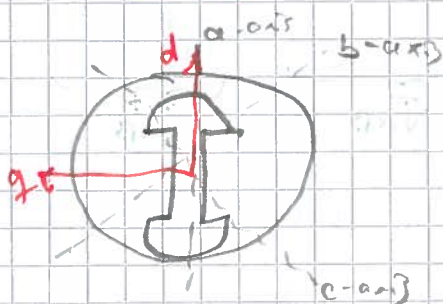
Max. value if $M_{bscs} = -\frac{1}{2}L_1 - L_2$

\Rightarrow obtained if $\Theta = -90^\circ$ and $\Theta = 90^\circ$



Min. value if $M_{bscs} = -\frac{1}{2}L_1 + L_2$

\Rightarrow obtained if $\Theta = 0^\circ$ and $\Theta = 180^\circ$



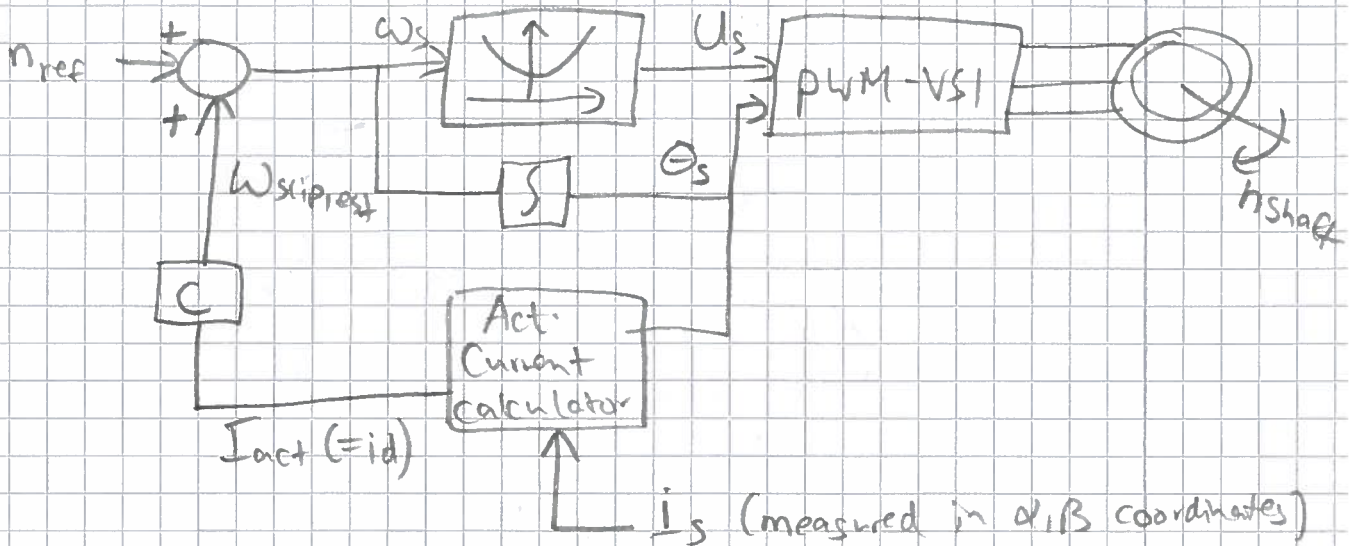
3) see "20 Jan 12" problem 2, ex. 6 !

4.)

No, because the zero component of the current is only related to the leakage flux which will not affect the torque !

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Problem 3:



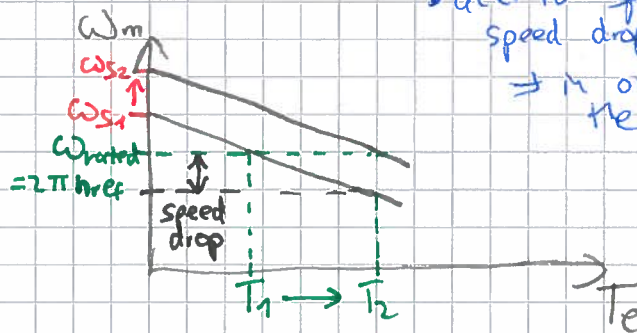
- Targets:
 - constant shaft speed for any load torque!
 - constant stator flux for high torque production capability

→ Target 2 obtained by adapt the stator voltage U_s for any frequency in order to:

$$\cancel{\Phi = \frac{U_s}{\omega_s}} \quad |\vec{\lambda}_s| = \left| \frac{\vec{U}_s}{\omega_s} \right| = \Phi \quad (\text{assuming } r_s = 0)$$

→ Target 1 obtained by adapting the stator frequency at load changes in order to keep $n_{mech} = \Phi$

- acc. to speed-torque-characteristic the speed drops at increasing torque
 ⇒ in order to compensate this, increase the stator frequency and hence the synchronous speed



- obtained by following constraints:
 - $T_e \sim \omega_{slip}$
 - $T_e \sim i_{sd}$

$$\Rightarrow \omega_{slip} \sim i_{sd}$$

- measure stator current
- calculate active part of it
- use $\omega_{slip} - i_{sd}$ relationship with rated values to calculate an estimated slip that is added to the reference speed in order to calculate the supplied frequency

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Steady state:

- at no load there is no estimated slip because $i_{sd}=0$
 - motor runs at syn. speed which is then the rated speed
- at load the slip is estimated and added to the reference speed
 - syn. speed acc. to the required ref. speed

Dynamics

- sudden load step will force the speed to drop
 - but $i_{sd} \sim T_e$ increases which will increase the estimated slip
 - hence, the syn. speed is adapted after a while (depends on PI controller of the system) in order to have the reference speed again which is normally not exactly the ref. speed but with an error $< \pm 0\%$ (due to non-exact rated values)

2.)

- Rated speed as reference speed
- Rated voltage and rated frequency for calculating the supplied voltage depending on the stator frequency
- Rated active current (hence rated current \pm power factor) for calculating the estimated slip
- (- Rated power in order to calculate rated torque for checking the model)