Opgaver lektion 6

8.4 10 patienters ventid i minutter: 5, 11, 9, 5, 10, 15, 6, 10, 5, 10

a) stikprøve gennemsnit: $\bar{X} = \frac{1}{10}\sum_{i=1}^{10} x_i = 8.6$ ✓ TI-89: 2nd MATH → Statistics → mean

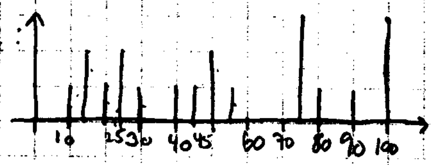
b) median: 5, 5, 5, 6, 9, 10, 10, 10, 11, 15, dvs. $\tilde{X} = \frac{9+10}{2} = 9.5$ ✓ TI-89 → ... → median

c) modes: modes er 5 og 10 ✓

8.7 donation i $ til United Fund: 100, 40, 75, 15, 20, 100, 75, 50, 30, 10, 55, 75, 25, 50, 90, 80, 15, 25, 45, 100

a) mean: $\bar{X} = \frac{1}{20}\sum_{i=1}^{20} x_i = 53.75$ $ ✓

b) mode:



dvs. modes 75 $ og 100 $ ✓

8.13 20 point:

3.2	1.9	2.7	2.4
2.8	2.9	3.8	3.0
2.5	3.3	1.8	2.5
3.7	2.8	2.0	3.2
2.3	2.1	2.5	1.9

$$\bar{x} = \frac{1}{20}\sum_{i=1}^{20} x_i = 2.665$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}(x_i^2 + \bar{x}^2 - 2\bar{x}x_i)$$

$$= \frac{1}{n-1}\left[\left(\sum_{i=1}^{n} x_i^2\right) + n\bar{x}^2 - 2\bar{x}\underbrace{\sum_{i=1}^{n} x_i}_{\bar{x}\cdot n}\right]$$

$$= \frac{1}{n-1}\left[\left(\sum_{i=1}^{n} x_i^2\right) - n\bar{x}^2\right] \qquad n\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n^2}$$

$$= \frac{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$$

$$= \frac{20\left(\sum_{i=1}^{20} x_i^2\right) - (53.3)^2}{20\cdot 19}$$

$$= \frac{20\cdot 148.55 - 2840.89}{20\cdot 19}$$

$$= \underline{\underline{0.342}}$$ ✓

$$\Rightarrow \underline{\underline{s = 0.585}}$$ ✓

Opgaver lektion 6

8.19 Styrke af tråd: middelværdi 78.3 kg $= \mu$
standard afvigelse 5.6 kg $= \sigma$

Varians af stikprøve middelværdi: $\boxed{\frac{\sigma^2}{n}}$ (slide 4)

a) $n: 64 \to 196$, dvs. varians af stikprøvemiddelværdi ændres fra $\frac{5.6^2}{64} = 0.49$ til $\frac{5.6^2}{196} = 0.16$
dvs. variansen på stikprøvemiddelværdi bliver mindre og mindre for større og større n. dvs. man bliver mere og mere sikker på sit gæt på middelværdien.
Virker meget logisk.

b) $n: 784 \to 49$, dvs. variansen af stikprøvemiddelværdien ændres fra $\frac{5.6^2}{784} = 0.04$ til $\frac{5.6^2}{49} = 0.64$
se i øvrigt kommentar under a)

8.21 $\mu = 240$ og $\sigma = 15$ for X: antal ml. sodavand fra sodavandsmaskine
$n = 40$
$\bar{X} \sim N(\underset{\mu_{\bar{x}}}{240}, \underset{\sigma^2_{\bar{x}}}{\frac{15^2}{40}})$
Hvis $\bar{x}$ ligger i $\mu_{\bar{x}} \pm 2\sigma_{\bar{x}} = [235{,}257 ; 244{,}743]$, så fungerer maskinen ok, dvs for $\bar{x} = 236$ ml er maskinen ok

8.26 X tid ved automat med $\mu = 3.2$ min og $\sigma = 1.6$
$n = 64$ kunder $\bar{X} \sim N(3.2, \frac{1.6^2}{64})$

a) $P(\bar{X} < 2.7) = P(Z < \frac{2.7-3.2}{1.6/8}) = P(Z \leq -2.5) \overset{\text{tabel A.3}}{=} \underline{\underline{0.0062}}$

b) $P(\bar{X} > 3.5) = P(Z > \frac{3.5-3.2}{1.6/8}) = 1 - P(Z \leq 1.5) = 1 - 0.9332 = \underline{\underline{0.0668}}$

c) $P(3.2 < \bar{X} < 3.4) = P(\bar{X} \leq 3.4) - P(\bar{X} \leq 3.2) = P(Z < \frac{3.4-3.2}{1.6/8}) - P(Z \leq \frac{3.2-3.2}{1.6/8})$
$= P(Z \leq 1) - P(Z \leq 0) = 0.8413 - 0.5 = \underline{\underline{0.3413}}$

Opgaver lektion 6

8.29 X : højde af terrier hund $\mu_X = 72$ cm $\sigma_X = 10$ cm $n_X = 64$

Y : –"– af puddel hund $\mu_Y = 28$ cm $\sigma_Y = 5$ cm $n_Y = 100$

$\bar{X} \sim \mathcal{N}(72, \frac{10^2}{64})$ $\bar{Y} \sim \mathcal{N}(28, \frac{5^2}{100})$

Theorem 7.11 ⇓ NB!+

$\bar{X} - \bar{Y} \sim \mathcal{N}(72-28, \frac{10^2}{64} + \frac{5^2}{100}) = \mathcal{N}(44, \frac{29}{16})$ ✓

$P(\bar{X} - \bar{Y} \leq 44.2) = P(Z \leq \frac{44.2-44}{\sqrt{29/16}}) = P(Z \leq 0.15) = \underline{\underline{0.5596}}$ ✓

8.39 $\chi^2_{0.025}(15) = \underline{\underline{27.488}}$ ✓, $\chi^2_{0.01}(7) = \underline{\underline{18.475}}$ ✓, $\chi^2_{0.05}(24) = \underline{\underline{36.415}}$ ✓

8.43 $n = 25$ normalfordeling med $\sigma^2 = 6 \rightarrow \chi^2 = \frac{S^2 \cdot 24}{6} \sim \chi^2(24)$ (slide 8)

a) $P(S^2 > 9.1) = P(\underbrace{\frac{S^2 \cdot 24}{6}}_{\chi^2} > \frac{9.1 \cdot 24}{6}) = P(\chi^2 > 36.4) = \underline{\underline{0.05}}$ ✓

$\chi^2_{0.05}(24) = 36.415$

b) $P(3.462 < S^2 < 10.745) = P(13.848 < \chi^2 < 42.98) = 0.95 - 0.01 = \underline{\underline{0.94}}$ ✓

8.52 fedtindhold $\mu = 0.5$ g 8 stikprøver: 0.6, 0.7, 0.7, 0.3, 0.4, 0.5, 0.4, 0.2

$\bar{X} = 0.475$ $S^2 = \frac{8 \cdot \sum_{i=1}^{8} x_i^2 - 3.8^2}{8 \cdot 7}$

$= \frac{8 \cdot 2.04 - 3.8^2}{8 \cdot 7} = 0.0336$

$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim T(n-1)$ (slide 10) ⇓ $S = 0.1832$

hvis $\mu = 0.5$ så $\frac{\bar{X} - 0.5}{0.1832/\sqrt{8}} \sim T(7)$ 0.95% interval $[-2,365 ; 2,365]$

$\bar{X} = 0.475$ ↪ $= -0.3859$ ligger i → dvs. rimelig at antage at $\mu = 0.5$ udfra stikprøve

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8.53 $f_{0.05}(7,15) = \underline{2.71}$ ✓; $f_{0.05}(15,7) = \underline{3.51}$ ✓; $f_{0.01}(24,19) = \underline{2.92}$

$f_{0.95}(19,24) = \frac{1}{f_{0.05}(24,19)} = \frac{1}{2.11} = \underline{\underline{0.47}}$ ✓, $f_{0.99}(28,12) = \frac{1}{f_{0.01}(12,28)} = \frac{1}{2.90} = \underline{\underline{0.34}}$ ✓

8.55 $S_1^2 = 15750$ $S_2^2 = 10.920$ (lommeregner)

Antag ens varians dvs $F = \frac{\cancel{\sigma_2^2} S_1^2}{\cancel{\sigma_1^2} S_2^2} = 1.44$ ✓ $\sim F(4,5)$ $f_{0.05}(4,5) = 5.19$ ✓

altså $F = 1.44$ er rimelig i $F(4,5)$ altså ens varians.