Department of Energy Technology

AALBORG UNIVERSITET

Matlab and Control Theory — INTRO 1st semester 2016

February 13, 2017

Written re-exam

09.30-13.30 CET (4 hours)

The set consists of 10 problems

Rules

- All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions.
- Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off. Internet access is strictly forbidden.
- Any kind of communication with other students is not allowed.

Remember

1. To write your study number on all sheets handed in.
2. It must be clear from the solutions, which methods you are using, and you must include sufficient intermediate calculations, diagrams, sketches etc. so the line of thought is clear. Printing the final result is insufficient.

Problem 1 (9 %)

The block diagram shown in Fig. 1 is considered.

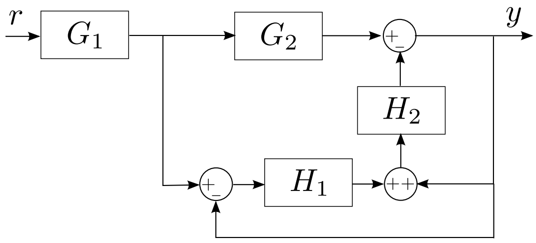


Figure 1: Reduce the block diagram, to determine the transfer function.

1. Determine the system transfer function $G(s) = \frac{y(s)}{r(s)}$ using block diagram reductions.

Problem 2 (9 %)

A second order system is given by the transfer function:

$$G(s) = \frac{360}{20s^2 + 48s + 180}$$

1. Determine the systems natural eigenfrequency, damping ratio and dc-gain.
2. The system is given a step input. Determine the percentage overshoot of the step response.

Problem 3 (14 %)

The system shown in Fig. 2 is considered. $G(s)$ is given by:

$$G(s) = \frac{s+8}{(s+2)(s+4)}$$

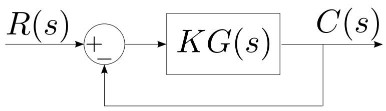


Figure 2: System considered.

1. Determine the minimum gain K, required for the system to have a steady state error of $e_{ss,s} \leq 0.1$, when applied a unit step input, i.e. $R(s) = \frac{1}{s}$.
2. Determine the characteristic equation for the system.
3. Sketch the root locus for the system, when it is the gain K that is varied along the root locus.

   Notice, it is not sufficient just to use Matlab!
4. On the root locus diagram, indicate the pole locations that will yield the smallest settling time and explain (in words) why it is so.

Problem 4 (8 %)

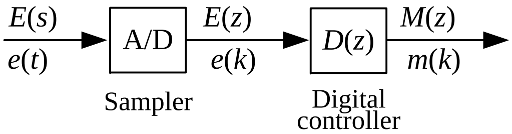
A closed loop system with the transfer function $G(s)$ has two dominating complex poles.

Sketch the area in the complex plane, where the roots are to be located to fulfill the specifications:

1. $\omega_n \geq 4$ and $\zeta > 0.8$.
2. $T_s \leq 4$ (settling time according to 2% criterion) and $\zeta = 1/\sqrt{(2)}$.

Problem 5 (13 %)

The figure below shows a continuous signal $e(t)$ fed to a digital controller through a sampler.



The sampling time is $T = 5$ milli-seconds. The controller is represented by the transfer function

$$D(z) = \frac{M(z)}{E(z)} = \frac{3z^2 - 0.3z}{z^2 - 1.2z + 0.27}$$

where $M(z)$ and $E(z)$ are the output and input, respectively.

1. Find all zeros and poles for the controller
2. Determine the controller's gain at low frequency ($f = 0$ Hz)
3. Draw a block diagram that represents $D(z)$ using only summation, gain, and unity delay (z^{-1}) blocks
4. Does the controller have problems with integrator wind-up?
5. Find the difference equation that corresponds to $D(z)$, i.e. find an expression for $m(k)$

Suppose now that the input error is defined by

$$e(t) = \begin{cases} 2e^{-200t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

6. Sketch $e(t)$ in the interval $0 \leq t \leq 5T$
7. Calculate $e(k)$ for $k = 0$ and $k = 1$
8. Find an expression for $E(z)$

Problem 6 (13 %)

A continuous-time filter $C(s)$ is defined by

$$C(s) = \frac{M(s)}{E(s)} = \frac{\omega_0^2}{s^2 + 0.3\omega_0 s + \omega_0^2}$$

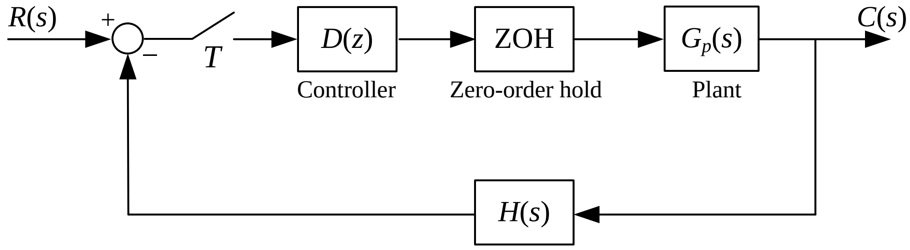
where ω_0 is a positive parameter. $C(s)$ must be converted to an equivalent discrete-time filter $C(z)$ using the Tustin method (*trapezoidal or bilinear rule*). The sampling time is denoted T.

1. Find an expression for $C(z)$

Now, suppose $\omega_0 = 100$ rad/s and $T = 0.005$ s.

2. Calculate the zeros and poles of $C(z)$
3. Calculate that damping factor for the poles of $C(z)$.
   Compare this to the damping factor for the corresponding poles of $C(s)$
4. Is it a good idea to use $T = 0.005$ s when $\omega_0 = 100$ rad/s, if $C(z)$ should mimic the continuous filter $C(s)$?

Problem 7 (14 %)



The figure shows a closed-loop control system having the plant transfer function

$$G_p(s) = \frac{10}{s}$$

Also, the feedback filter is $H(s) = 1/(10s + 1)$

The discrete controller is a simple proportional gain $D(z) = K$, where K is a positive constant. The sampling time is $T = 1$ second.

1. Sketch the frequency response for $G_p(s)$ and for $H(s)$

   Note: It is sufficient to show the amplitude response

2. Find an analytical expression for the pulse transfer function $G(z)$ for the plant $G_p(s)$

3. Determine the characteristic equation

4. Plot all open loop poles and zeros in a diagram and sketch the root locus for the system

   Note: You do not need to calculate precise values for breakaway points, asymptotes, etc. — just a rough sketch

5. An engineer has figured out that $K = 0.02$ fulfills the closed-loop performance requirements. Estimate the settling time for the closed-loop step response

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Problem 8 (6 %)

Write a **sumofcubes(n)** MATLAB function that will calculate the sum of cubes from 1^3 to n^3 where *n* is the function input.

$$y = 1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3$$

```
>> sumofcubes(100)

ans =

25502500
```

Problem 9 (6 %)

Write a **sumofevencubes(n)** MATLAB function that will calculate the sum of **even** cubes from 1^3 to n^3 where *n* is the function input.

$$y_{all} = 1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = 1 + 8 + 27 + 64 + \ldots + n^3$$

$$y_{even} = 8 + 64 + \ldots + n^3$$

```
>> sumofevencubes(100)

ans =

13005000
```

Problem 10 (8 %)

Implement a Heron's method for finding a square root of a number as a MATLAB function **heronsqrt(n,accuracy)**.
Example: to calculate $\sqrt{S}$, where S = 100, follow the algorithm:

$$\begin{aligned}
x_0 &= \frac{S}{2} = \frac{100}{2} = 50\\
x_1 &= \frac{1}{2}(x_0 + \frac{S}{x_0} = \frac{1}{2}(50.000 + \frac{100}{50.000}) = 26.000\\
x_2 &= \frac{1}{2}(x_1 + \frac{S}{x_1} = \frac{1}{2}(26.000 + \frac{100}{26.000}) = 14.923\\
x_3 &= \frac{1}{2}(x_2 + \frac{S}{x_2} = \frac{1}{2}(14.923 + \frac{100}{14.923}) = 10.812\\
x_4 &= \frac{1}{2}(x_3 + \frac{S}{x_3} = \frac{1}{2}(10.812 + \frac{100}{10.812}) = 10.030\\
x_5 &= \frac{1}{2}(x_4 + \frac{S}{x_4} = \frac{1}{2}(10.030 + \frac{100}{10.030}) = 10.000
\end{aligned}$$

First guess (x_0) is just the input number *S* divided by two. Consecutive steps are done by following the algorithm. This algorithm needs a stop condition. In this case check if your x_n^2 is close to the *n* number.

```
>> heronsqrt(100,0.001)

ans =

10.000
```
