Hints

The back-EMF is equal to the differentiation of the PM flux link with respect to time, as

$$BackEMF = \frac{d\lambda_{pm}}{dt}$$

The instantaneous torque equation is still the same as the one given on P6 of the slides, as

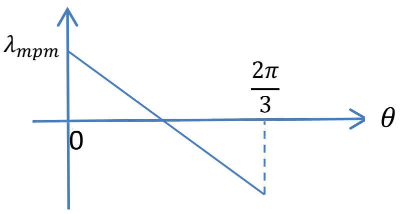
$$\tau = \frac{1}{\Omega} i \frac{d\lambda_{pm}}{dt}$$

where Ω is the mechanical shaft speed (rad/s), and i is the instantaneous current. Its waveform is to be determined.

From the above equation, we understand that, when back-EMF is zero, the torque is zero, which is independent on the current value given. Therefore, the most economical solution will be to let the current to be zero in the period when the back-EMF is zero, which results in minimized copper loss.

Observed from the original PM flux linkage waveform, we can see that the back-EMF will not be zero in the period of 30 ~ 150 degrees, and 210 ~330 degrees.

We will focus on the period of 30 ~ 150 degrees. For simplicity, we use a local axis, which starts from 30 degrees in the original axis. The PM flux linkage waveform in the range of 30 ~ 150 degrees will be a straight line, as



(represented in the local axis, 0 degrees here corresponds to 30 degrees in the original axis.)

Therefore, the corresponding function for this straight line could be found as:

$$\lambda_{pm} = \lambda_{mpm}(1 - \frac{\theta}{\pi/3})$$

(You can check at two special positions, $\theta = 0$, and $\theta = \frac{\pi}{3}$, to validate easily the above equation.)

Therefore, we find that the back-EMF (the differentiation of the PM flux linkage function) becomes:

$$\frac{d\lambda_{pm}}{dt} = -\lambda_{mpm}\frac{3}{\pi} \cdot \frac{d\theta}{dt} = -\lambda_{mpm}\frac{3}{\pi} \cdot \omega_\theta$$

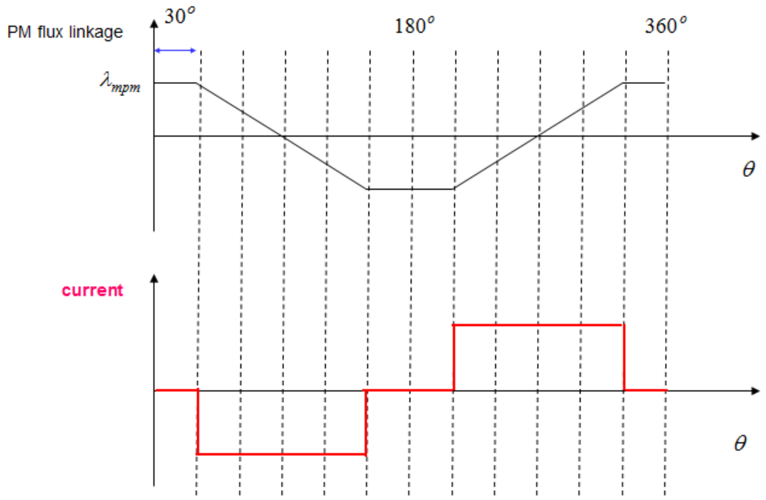
$\omega_\theta$is the electrical rotor speed. It equals to $\omega_\theta = p\Omega$, where p is the number of pole pairs. The torque equation given above can then be extended to:

$$\tau = \frac{1}{\Omega} i \frac{d\lambda_{pm}}{dt} = \frac{1}{\Omega} i(-\lambda_{mpm}) \frac{3}{\pi} \cdot \omega_\theta = pi(-\lambda_{mpm}) \frac{3}{\pi}$$

In this torque equation, $p(-\lambda_{mpm})\frac{3}{\pi}$ is a negative constant. Therefore, an obvious choice, for obtaining a target positive average torque and having minimum torque ripple will be to let the current to be $i = -I_m$, where I_m is a constant.

The same procedure can be repeated for the period 30 ~ 150 degrees.

Therefore the current waveform becomes:



Note, you will find the needed current waveform is exactly the same shape as the back-EMF waveform. This is generally valid for any kinds of PM machines!

(What about its instantaneous torque waveform?)