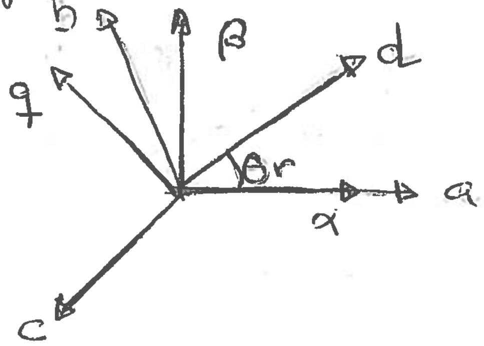
DYNAMIC MODELS EXERCISES

QUESTION 1

- Please draw the ref. frame axes for abc ref, dq rotati ref frame and α-β stationary rf.



- Suppose you have the fallowing 3-phase voltages:

$Va = V_{pk} \cdot \cos\left(w_e t + \frac{\pi}{6}\right)$

$Vb = V_{pk} \cdot \cos\left(w_e t + \frac{\pi}{6} - \frac{2\pi}{3}\right)$

$Vc = V_{pk} \cdot \cos\left(w_e t + \frac{\pi}{6} + \frac{2\pi}{3}\right)$

where $w_e = 2\pi \cdot 50$

When $t = 0$ please draw the space vector for the above abc system

Please draw the space vector again at $t = \frac{1}{300}$

$t = 0$; $V_{pk} = 1$

$Va = \cos(\pi/6)$

$Vb = \cos(\pi/6 - 2\pi/3)$

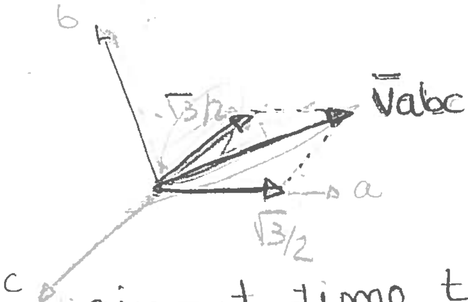
$Vc = \cos(\pi/6 + 2\pi/3)$

$\bar{V}abc = \frac{2}{3}(Va \cdot e^{j0} + Vb \cdot e^{j120} + Vc \cdot e^{-j120})$

$= \frac{2}{3}(\cos(\pi/6) \cdot 1 + \cos(\pi/6 - 2\pi/3) \cdot 1\angle 120$

$+ \cos(\pi/6 + 2\pi/3) \cdot 1\angle{-120})$ $\quad \frac{2\pi}{3}$

$\boxed{\bar{V}abc = \frac{\sqrt{3}}{2} + \frac{1}{2} j}$ ⟶



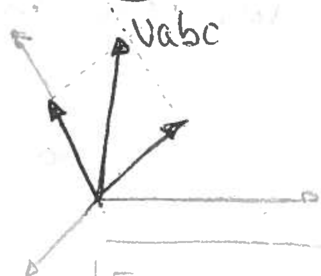
- Please draw the vector again at time $t = 1/300$

$w_e \cdot t = 2\pi \cdot 50 \cdot \frac{1}{300} = \frac{\pi}{3}$

$Va = \cos(\pi/3 + \pi/6) = 0$

$Vb = \cos(\pi/3 + \pi/6 + 2\pi/3) = \frac{\sqrt{3}}{2}$

$Vc = \cos(\pi/3 + \pi/6 + 2\pi/3) = -\frac{\sqrt{3}}{2}$



$\boxed{\bar{V}abc = 1j}$

· Please mention how the transformation ...
abc-dq and dq-abc are obtained.

abc-dq0

$$\begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix} = \begin{bmatrix} \cos\theta & \cos(\theta-120) & \cos(\theta+120) \\ -\sin\theta & -\sin(\theta-120) & -\sin(\theta+120) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

Igual está mejor desarrollado en los ejercicios de preparación del examen. (anteriores)

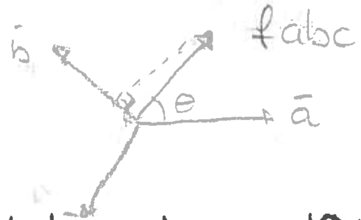
dq0-abc

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin(\theta) & 1 \\ \cos(\theta-120) & -\sin(\theta-120) & 1 \\ \cos(\theta+120) & -\sin(\theta+120) & 1 \end{bmatrix} \begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix}$$

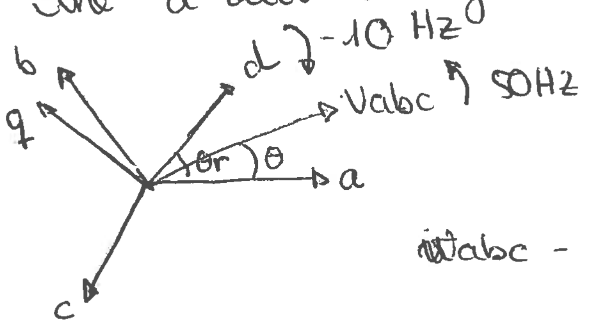
· When the space vector expressed in abc is known how the instantaneous phase b variable may be obtained?

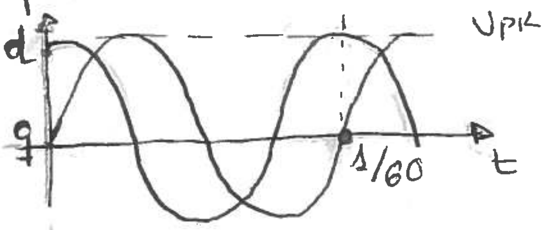
$$\bar{f}_{abc} = \frac{2}{3}\left(f_a \cdot e^{j0} + f_b \cdot e^{j120} + f_c \cdot e^{-j120} \right)$$

$$f_b = \bar{f}_b \cdot \cos(\theta - 120)$$



· If the dq-frame is rotating and it rotates at $-2\pi \cdot 10$ rad/s. For a balanced set of 3-phase voltages at 50 Hz. What are the voltages waveforms in the dq frame? Suppose at t=0, the voltage space vector is aligned with the phase-a axis, and the d-axis is aligned with phase-a axis





$$\omega_{abc} - \omega_{dq} = 50 - (-10) = 60\,\text{Hz}$$

• How the mutual inductance between the rotor winding and the stator phase-b is obtained.

We just have to prayect the rotor inductance over the phas b ofe the stator.

d-axis

$$M_{bsfdm} = L_{sfd} \cdot \cos\left(\theta r - \frac{\pi}{2}\right) = L_{bb} . Re\left(\frac{e^{j(\theta r - \pi/2)}}{e^{j\frac{2\pi}{3}}}\right)$$

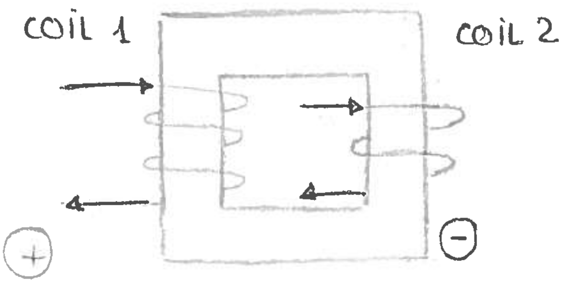
b-axis

$$= L_{sfd} \cdot \sin\left(\theta r - \frac{2\pi}{3}\right)$$

• Please explain from the physical point of view, for such a machine, why the equivalent d q axes inductances will be position independent?

From the rotor point of view, we see that the windings rotate at the same speed that the rotor, and therefore will see no change in the air gap and hence the inductances will not change neither.

| QUESTION 4 |



• Please express the flux linkage for coil 1 using the coils 1 sel. induct. (L_1) and coil 2 mutual inductance (M) $\Rightarrow \lambda_1 = L_{11} \cdot i_1 - M \cdot i_2$

Suppose coil 1 has N_1 turns, and coil 2 has N_2 turns. If it is desired to perform turns ratio transformation, to let the coil 2 to have the same number of turns as coil 1, which parameters and variables related to coil 2 are affected by the turns ratio transformation.

¿? all the ones that define the charact. of the coil?

· Is the leakage inductance a CTE? For the main winding inductance, which one is more important and predominant?

L_{ls} → The LEAKAGE inductance is a CTE. It afects the air around the phase a. When you design a machine you want it to be as small as possible.

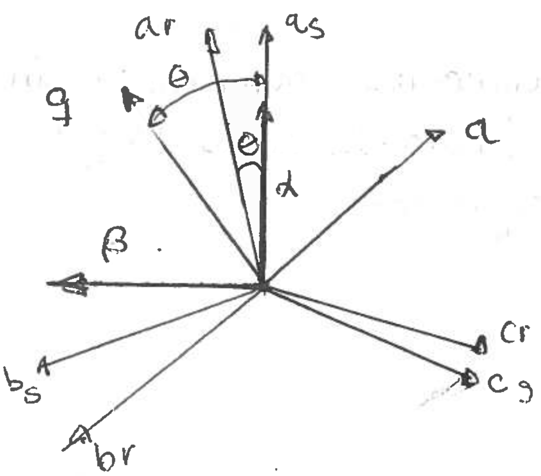
¿¿ Is it also CTE in the SM >>>

LM is more important an predominant in the machine as when you design it you want it to be as big as possible in order to have more flux in the machine

· Why the speed related coefficient in the rotor side voltage is $(\omega_\theta - \omega_r)$ → ¿ because we didnt choose

ω_θ is the speed of the choosen ref frame: SYNCHRONOUS SPEED. As you need a difference of speed to operate the machine you choose to rotate the rotor at ω_r. >>>

· give the IM model in SS and represent the α-β.



· Suppose you supply the machine with a trapezoidal current waveform instead of sinusoidal, what is the consequence in the output torque? Can you still rotate the machine

QUESTION 8

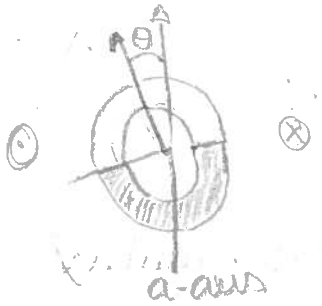
· In the model of a PM machine, how the rotor PM flux linkage is used in the model? Is it RMS or peak value? Please give explanations.

MODEL of PM MACHINE:

$$u = R \cdot i + \frac{d\lambda}{d\theta} \cdot \omega$$

$$\lambda = \lambda_{pm} + \lambda_a$$

$$\lambda_{p.m} = \lambda_{mpm} \cdot \cos\theta$$



PEAK VALUES are used, since we are working in steady state

· Please use the voltage / flux linkage eq. of the machines to drive the following: When the PM machine is driven by a DC motor, and is rotating at the rated speed with stator windings open-circuited, what will be the winding terminal voltages measured on the dq-frame?

If it is driven by a DC MOTOR, we can affirm that it is working as a GENERATOR with no load to supply. Therefore, the voltage measured on the dq-frame:

P.9

$$u_{qs} = R_s \cdot i_{qs} + \frac{d}{dt} \cdot \lambda_{qs} + \omega_\theta \cdot \lambda_{ds}$$

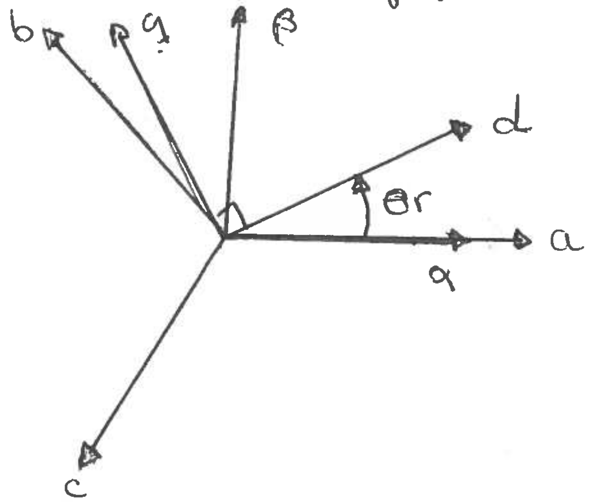
$$u_{ds} = R_s \cdot i_{ds} + \frac{d}{dt} \cdot \lambda_{ds} - \omega_\theta \cdot \lambda_{qs}$$

EXAMS - DYNAMIC MODELS + NON-LINEAR + SCALAR CONTROL THEORY.

EXAM JANUARY 2012

PROBLEM 1.

(1) Please draw the ref. frame axes for abc, dq and αβ.



(2) Suppose now you have a set of 3-phase signals as:

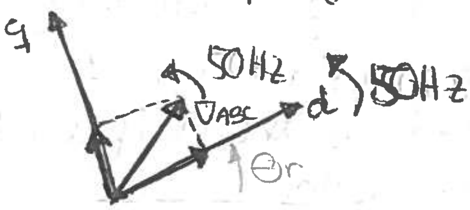
$V_a = V_{pk} \cdot \cos(\omega_e t)$ $\qquad \omega_e = 2\pi \cdot 50$

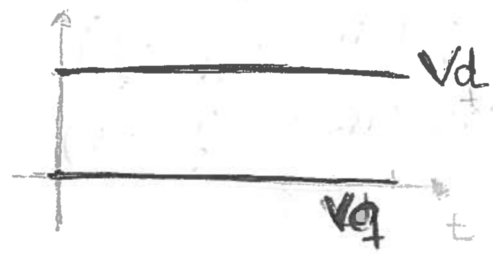
$V_b = V_{pk} \cdot \cos(\omega_e t - \frac{2\pi}{3})$ $\qquad V_{pk} = 1.$

$V_c = V_{pk} \cdot \cos(\omega_e t + \frac{2\pi}{3})$

Please draw the signal waveforms viewed in dq frame

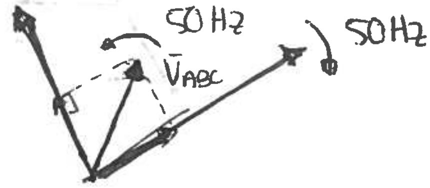
· When dq-frame is rotating at 50 Hz ↺

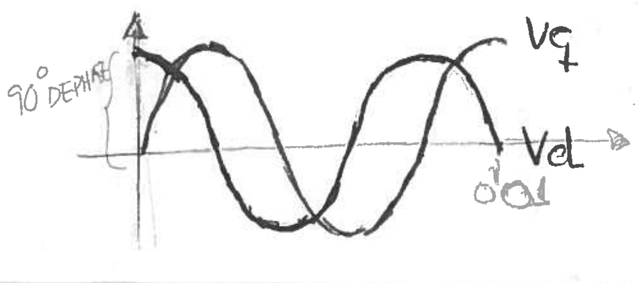




If we suppose the at t=0 is aligned with α axis

· When dq-frame is rotating at 50Hz ↻





$f = 50-(-50)$

$= 100 Hz$

$T = \frac{1}{f} = \frac{1}{100}$

$T = 0'01\, s.$

(c) Transform the following power expression, representation ... to dq frame.

$$P = \bar{V}_{\alpha\beta} \cdot (\bar{I}_{\alpha\beta})^* \qquad (\bar{I}_{\alpha\beta})^* = (I_\alpha + jI_\beta)^* = I_\alpha - jI_\beta$$

The relationship between the $\alpha\beta$ frame and dq-frame may be expressed as: $\bar{f}_{\alpha\beta} = \bar{f}_{dq}\, e^{j\theta}$

We are trying to demonstrate that the POWER is POSITION INDEPENDENT.

$$P = (V_{dq}\, e^{j\theta})(I_{dq}\, e^{j\theta})^* = V_{dq}\, e^{j\theta} \cdot I_{dq}^* \cdot e^{j\theta *}$$

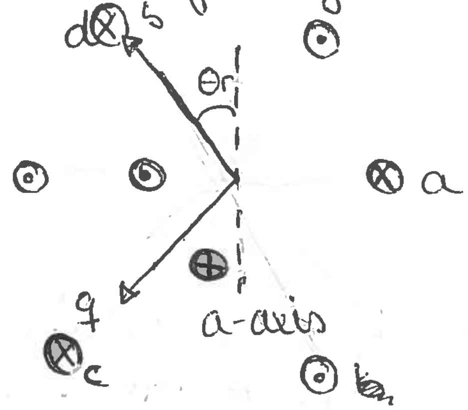
$$= V_{dq} \cdot e^{j\theta} \cdot I_{dq}^* \cdot (\cos\theta + j\sin\theta)^*$$

$$= V_{dq} \cdot e^{j\theta}\, I_{dq}^* \cdot (\cos\theta - j\sin\theta) = V_{dq} \cdot e^{j\theta}\, I_{dq} \cdot e^{-j\theta}$$

$$\boxed{P = V_{dq} \cdot I_{dq}}$$

PROBLEM 2

A sketch of a synchronous machine is shown below



(1) Please describe how the mutual inductance between phase-a and phase-b is obtained?

L3 MUTUAL INDUCTANCES OF THE STATOR PHASES.

To find the mutual inductance between the phases of the stator, we have to understand that it will contain a d-inductance component and a q-inductance component. In this way the d and q axis will have a constant inductance term (L_{aad} and L_{aaq}) and the position dependent terms of the projections of the MMF of phase-a and the flux linkage of phase-b

$$M_{asbsm} = L_{aad} \cos\theta_r \cdot \cos(\theta_r - 120) + L_{aaq} \cdot \sin(\theta_r) \sin(\theta_r - 120)$$

$$L_1 = \frac{L_{aad} + L_{aaq}}{2} \qquad L_2 = \frac{L_{aad} - L_{aaq}}{2}$$

$$M_{asbsm} = -\frac{1}{2} L_1 - L_2 \cdot \cos(2\theta_r + 2\pi/3)$$

(5) Does the ... ... have any influence on the output torque? Please explain.

No, as we can see the flux that intervens in the torque eq. (λ_{mpm}) is just produced by the permanent magnet of the rotor.

(6) If the armature current contains a 3rd harmonic. Will this 3rd harmonic current component produce any torque? What is the instantaneous and average torque corresponding to this harmonic current?

Adding a 3rd harmonic:

$$\tau_{IN} = p \cdot \lambda_{mpm} \cdot I_m \cdot [-\sin(\theta + \theta_L) + \overbrace{\sin 3\theta}^{3^{rd} \text{ harmonic}}] \cdot [-\sin\theta]$$

$$= p \lambda_{mpm} \cdot I_m \cdot [\underbrace{\sin(\theta + \theta_L) \cdot \sin\theta}_{\text{The same that the originall}} + \underbrace{\sin 3\theta \cdot \sin\theta}_{\text{The added part!}}]$$

- $-\sin\theta \cdot \sin 3\theta = \dfrac{\cos 4\theta - \cos 2\theta}{2}$

- $\tau_{AVERAGE} = \dfrac{1}{2} \underbrace{p \cdot I_m \cdot \lambda_{mpm}}_{CTE} \cdot \cos\theta_L = \dfrac{1}{2} \displaystyle\int_0^{2\pi} \dfrac{\cos 4\theta - \cos 2\theta}{2} \, d\theta$

$$= \frac{1}{2} \cdot p \cdot I_m \cdot \lambda_{mpm} \cdot [0] = \underline{\underline{0}}$$ NO CHANGE

stator voltage frequency while the shaft load torque is zero. Determine the

a) Stator voltage amplitud

$$U_s = \frac{U_0}{W_{e0}} \cdot \sqrt{we^2 + \left(\frac{r_s}{L_s}\right)^2} = \frac{400}{2\pi \cdot 50} \sqrt{(2\pi \cdot 2)^2 + \frac{10'1}{(1'5+0'2)^2}}$$

$U_s = 17'7 \ V_{L-L}$

$|U_s| = \boxed{\frac{17'7}{\sqrt{3}}} = 10'22 \ A$

b) Stator current vector I_s

$$I_s = \frac{U_s}{R_s + jwL_s} = \frac{10'22}{10'1 + j(2\pi 2) \cdot 1'7} = 0'43 \angle -64^0$$

c) Voltage drop across the stator resistance

$V_{Rs} = |R_s \cdot I_s| = 10'1 \cdot 0'43 \angle -64 \ = 4'36 \ V$

d) shaft speed in rpm

$$\left.\begin{array}{l} 50 \ Hz \longrightarrow 1000 \ rpm \\ 2 \ Hz \longrightarrow n_e \end{array}\right\} \ n_e = 40 \ rpm$$

(4) assume the load torque is 50% of the rated torque and that the motor shaft speed must 500 rpm. Calculate the required stator freq.

- $n_{slip} = n_s - n_m = \left(\frac{60 \cdot 50}{3}\right) - 925 = 1000 - 925 = 75 \ rpm$
- $0'5 \ Z \rightarrow 0'5 n_{slip} \Rightarrow 75 \cdot 0'5 = 37'5 \ rpm$
- $$\left.\begin{array}{l} 1000 \longrightarrow 50 \\ 37'5 \longrightarrow f_{slip} \end{array}\right\} \ f_{slip} = 1'875 \ Hz$$
- $f_{STATOR} = f_{slip} + f_m = 1'875 + 25 = 26'875 \ Hz$

$f_{mec} \rightarrow 0'5 \ Z \rightarrow 0'5 \ f = 25$

The function will be a ... following two conditions are accomplished.

1) $V(x_1, x_2)$ PD

2) $\dot{V}(x_1, x_2)$ SND

CONDITION #1: POSITIVE DEFINITE (V)

a function is PD if it accomplished

1) $V(\underline{0}) = 0$

$$V(x_1, x_2) = \frac{1}{2} x_2^2 + \alpha . F(-x_1 - \beta x_2)$$

$$V(0,0) = 0 + \alpha . \underbrace{\int_0^u f(\xi) . d\xi}_{u=0 \rightarrow f(u)=0} = 0$$

2) $V(x_1, x_2) > 0$ $x \neq 0$

$$V(x_1, x_2) = \underbrace{\frac{1}{2} x_2^2}_{+} + \alpha . \int_0^u f(\xi) . d\xi \quad \begin{cases} u>0 & f(u)>0 \\ u<0 & f(u)<0 \end{cases}$$

We try with the only case that can breake the rule

$(u<0, f<0) \rightarrow$ $\left.\begin{matrix} y = -u \\ g(u) = -f(u) \end{matrix}\right]$

$$F(u) = \int_0^{-y} -g(u) . du = \int_{-y}^{0} g(u) . du > 0 \quad \boxed{PD}$$

CONDITION #2: NEGATIVE SEMI DEFINITE ($\dot{V}$)

a function is NSD if it accomplished

1) $\dot{V}(\underline{0}) = 0$

$$\dot{V}(x_1, x_2) = x_2 . \dot{x}_2 + \alpha . \underbrace{\dot{F}(-x_1 - \beta x_2)}_{= f(-x_1 - \beta x_2)} (-\dot{x}_1 - \beta \dot{x}_2)$$

with the STATE SPACE FORMULATION $\rightarrow$

$$= x_2 . \alpha . f(-x_1 - \beta x_2) + \alpha . f(-x_1 - \beta x_2)(-x_2 - \beta(\alpha . f(-x_1 - \beta x_2$$

$$= \cancel{x_2 . \alpha . f(-x_1 - \beta x_2)} - \cancel{x_2 . \alpha . f(-x_1 - \beta x_2)} - \alpha^2 . \beta . f^2(-x_1 - \beta x_2)$$

$$\dot{V}(x_1, x_2) = -\alpha^2 . \beta . f^2(-x_1 - \beta x_2)$$

Knowing that α and β are positive numbers and $\rightarrow$

PROBLEM 1

(1) Suppose now you have a set of 3-phase signals as:

$V_a = V_{pk} \cdot \sin(w_e t)$

$V_b = V_{pk} \cdot \sin(w_e t - \frac{2\pi}{3})$

$V_c = V_{pk} \cdot \sin(w_e t + \frac{2\pi}{3})$

where $w_e = 2\pi \cdot 50$ and $V_{pk} = 1$

Please calculate the space vector formed by these three-phase signals at $t = 0$, $t = \frac{1}{300}$.

$t = 0$

$V_a = \sin(0) = 0$

$V_b = \sin(-2\pi/3) = -\sqrt{3}/2$

$V_c = \sin(2\pi/3) = \sqrt{3}/2$

$\bar{V}_{abc}(t=0) = \frac{2}{3}(\cancel{V_a \cdot e^{j0}} + V_b \cdot e^{j120} + V_c \cdot e^{j120})$

$$\bar{V}_{abc} = \frac{2}{3}\left(-\frac{\sqrt{3}}{2}(\cos(120) + j\sin(120)) + \frac{\sqrt{3}}{2} \cdot (\cos(120) - j\sin(120))\right)$$

$$= \frac{2}{3}\left(-\frac{\sqrt{3}}{2}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right)$$

$$= \frac{2}{3}\left[\left(\cancel{\frac{\sqrt{3}}{4}} - \frac{3}{4}j\right) + \left(\cancel{-\frac{\sqrt{3}}{4}} - \frac{3}{4}j\right)\right] = \frac{\cancel{2}}{\cancel{3}} \cdot \frac{-3}{2}j = \boxed{-j}$$

$\boxed{\bar{V}_{abc} = 1 \angle -90}$

$t = 1/300 \rightarrow w_e t = 2\pi \cdot 50 \cdot \frac{1}{300} = \frac{\pi}{3}$

$V_a = \sin\left(\frac{\pi}{3}\right) = \sqrt{3}/2$

$\bar{V}_{abc}(t = \frac{1}{300}) = (V_a \cdot e^{j0} + V_b \cdot e^{j120} + \cancel{V_c \cdot e^{j-120}}$

$V_b = \sin\left(\frac{\pi}{3} - \frac{2\pi}{3}\right) = -\sqrt{3}/2$

$V_c = \sin\left(\frac{\pi}{3} + \frac{2\pi}{3}\right) = 0$

$$\bar{V}_{abc} = \frac{2}{3}\left(\frac{\sqrt{3}}{2} \cdot e^{j0} - \frac{\sqrt{3}}{2} \cdot e^{j120}\right)$$

$$= \frac{2}{3}\left(\frac{\sqrt{3}}{2} \cdot (\cos(0) + j\sin(0)) - \frac{\sqrt{3}}{2}(\cos(120) + j\sin(120))\right)$$

$$= \frac{2}{3}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right) = \frac{2}{3}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} - \frac{3}{4}j\right) = \frac{\sqrt{3}}{2} - \frac{1}{2}j$$

$\boxed{\bar{V}_{abc} = 1 \angle -30^\circ}$

(3) Now phase b and phase c are exchanged which yields

$V_a = V_{pk} \sin(w_e t)$

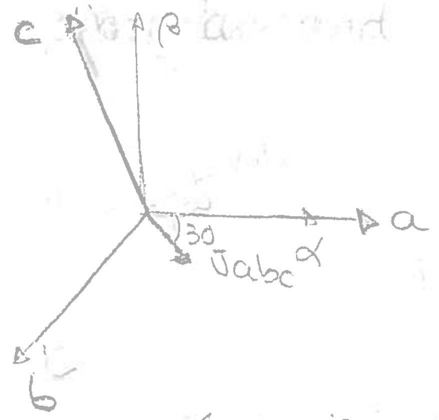
$V_b = V_{pk} \sin\left(w_e t + \frac{2\pi}{3}\right)$

$V_c = V_{pk} \sin\left(w_e t - \frac{2\pi}{3}\right)$

Please calculate the space vectors formed by these 3-phase signals at $t = 1/300$

Please also draw this space vector in the stationary $\alpha\beta$ ref. f

Ans. The phase b and c are exchanged:



$V_a = \sin\left(\frac{\pi}{3}\right) = \sqrt{3}/2$

$V_b = \sin\left(\frac{\pi}{3} + \frac{2\pi}{3}\right) = 0$

$V_c = \sin\left(\frac{\pi}{3} - \frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$$\vec{V}_{abc} = \frac{2}{3}\left(V_a\, e^{j0} + V_b\, e^{-j120} + V_c\, e^{j120}\right)$$

$$= \frac{2}{3}\left(\frac{\sqrt{3}}{2}\left(\cos(0) + j\sin(0)\right) - \frac{\sqrt{3}}{2}\left(\cos(120) + j\sin(120)\right)\right)$$

$$= \frac{2}{3}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\left(-1/2 - \sqrt{3}/2\, j\right)\right) = 1\angle{-30}$$

(4) Transform the above a, b, c signals to a stationary α-β ref frame. Then transform the $\alpha\beta$ ref frame to a rotating dq frame. This dq frame is rotating positively at 50Hz.

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$V_\alpha = \left(V_a - \frac{V_c}{2} - \frac{V_b}{2}\right)\frac{2}{3}$

$V_\beta = \left(\frac{\sqrt{3}}{2} V_b - \frac{\sqrt{3}}{2} V_c\right)\frac{2}{3}$

$V_0 = \left(\frac{V_a}{2} + \frac{V_b}{2} + \frac{V_c}{2}\right)\frac{2}{3}$

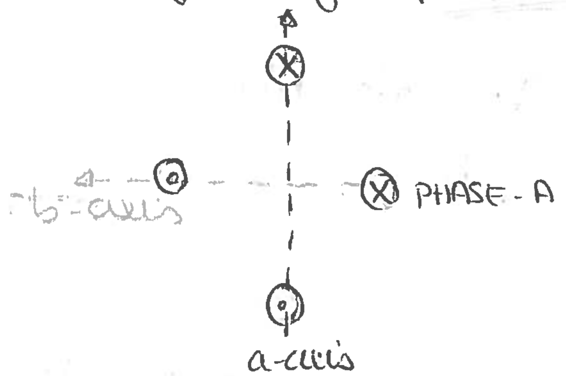
$$V_d = \frac{V_\alpha .\, e^{j0} + V_\beta .\, e^{j(0+90)}}{e^{j\theta}}$$

$$V_q = \frac{V_\alpha .\, e^{j0} + V_\beta .\, e^{j(0+90)}}{e^{j(\theta+90)}}$$

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} f_\alpha \\ f_\beta \\ f_0 \end{bmatrix}$$

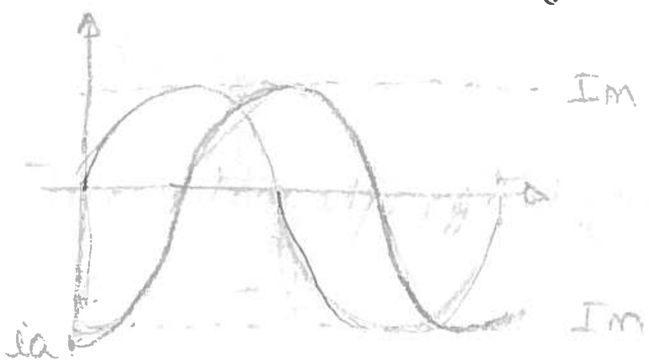
1 a: simple single-phase PM machine is shown below in Fig 2a
Now another phase (phase b) is added to this machine Fig 2b

(3) If the phase-a PM flux linkage waveform is expressed as $\lambda_{pm,a} = \lambda_{mpm} \cos\theta$. Please give the PM flux linkage waveform for phase b.



$\lambda_{pm,b} = \lambda_{mpm} \sin\theta$

(4) If phase-a is now with a current of $i_a = -I_m \sin\theta$ (where Im is the peak value of the current), please determine the needed current for phase-b.



$i_b = I_m \cos\theta$

(5) Please show the instantaneous torque produced by phase-a and phase-b respectively.

$\theta_t = 0$

(6) Please give an expression for the total torque produced by phase a and phase b.

PROBLEM 4

Consider a 3-phase induction motor, where u_s and i_s denote the stator voltage and the stator current space vectors.

In a synchronous coordinate system rotating with the angular velocity $\omega_s = 100\pi$ rad/s the voltage and the current vectors are $u_s = 300 + j50$ [V] $\quad i_s = 6 + j0$ [A]

(1) Determine the phase angle between u_s and i_s

$u_s = 300 + j50 = 304'14 \angle 9'46$

$i_s = 6 + j0$

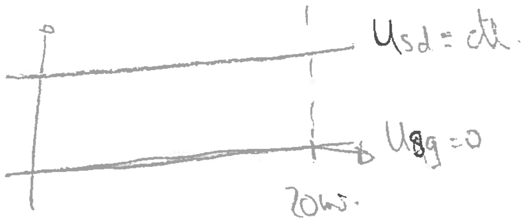
(2) Calculate the active component I_{act} of the stator current

$I_{act} = 6$ [A]

$u_s = 300 + j\,2\pi\,50\,(t) \rightarrow 1/2\pi$

$u_s = 300 + j\,50$

(3) Plot the graph for the real and the imaginary components of u_s in the time interval $0 < t < 20$ [ms]



$U_s = U_d + jU_q$

PROBLEM 5

6 pole induction motor

Rated shaft power 25 KW
Rated speed 975 rpm
Rated stator f 50 Hz

The machine is suplied from a 50 Hz power source at rated voltage

(1) Calculate the shaft speed when the load is −50% of the rated Torque.

$T_L = 0'5T \longrightarrow n'_{slip} = 0'5\, n_{slip}$

$n_s = \frac{60 \cdot 50}{3} = 1000$ rpm

$n_{slip} = \cancel{n_{RATED} - n_{SINC}} = 975 - 1000 = -25$ rpm

$n'_{slip} = -12'5$ rpm

$n_{SHAFT} = n_m + n_{slip}$

$n'_{SHAFT} = 1000 + 12'5$

$\boxed{n'_{SHAFT} = 1012'5 \text{ rpm}}$

- STATE SPACE FORMULATION

$x_1 = x \longrightarrow \dot{x}_1 = \dot{x} = x_2$

$x_2 = \dot{x} \longrightarrow \dot{x}_2 = -f(\dot{x}_2).h(x_1) - g(x_1)$

- SHOW (0, 0) is the only singular point

SINGULAR POINT is an equilibrium point in the phase plane.

$\dot{x}_1 = 0 = x_2$ $\quad f(0)=0$

$\dot{x}_2 = 0 = -f(\cancel{x_2}) \cdot h(x_1) - g(\cancel{x_1})$

$u>0 \quad g(u)>0$

$u<0 \quad g(u)<0$

$u=0 \quad g(u)=0$

Therefore, knowing that h, f, g are C^1 real functions, it means that (0,0) will be the only SINGULAR POINT

- DETERMINE $\dot{V}(x_1, x_2)$

$\dot{V}(x_1,x_2) = x_2 \cdot \dot{x}_2 + \frac{d}{dt}\left(\int_0^{x_1} g(u)\,du\right)$

$\dot{V}(x_1,x_2) = x_2 \cdot \dot{x}_2 + g(x_1) \cdot \dot{x}_1$

2. Now it is assumed that $u.f(u) \geq 0$ and $h(u) \geq 0$ for all $u \in \mathbb{R}$. Show that (0,0) is a stable ~~point~~ singular point for (* *). Also show that if f and h only have one zero, then (0,0) is asymptotically stable.

- GLOBALLY ASYMPTOTICALLY STABLE in (0,0)
  - $V(\underline{x}) \to \infty$ as $\|\underline{x}\| \to \infty$
  - $\dot{V}(\underline{x}) \leq 0$ over the whole state space. NSD

- $V(x_1,x_2) = \frac{1}{2}x_2^2 + \int_0^{x_1} g(u)\,du \qquad (x_1,x_2) \in R^2$

  $V(x_1,x_2) \to \infty \quad , \|x_1, x_2\| \to \infty$

- $\dot{V}(x_1,x_2) = x_2 \cdot (-f(x_2).h(x_1) - g(x_1)) + g(x_1) \cdot x_2$

  $= -x_2 f(x_2) \cdot h(x_1) - \cancel{x_2 \cdot g(x_1)} + \cancel{g(x_1) \cdot x_2}$