Ugly Notes of Power Electronics Converters

Benoît Bidoggia
bbi@et.aau.dk

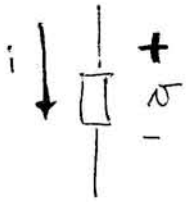
Date: 7 September 2012

CAVEAT

This is only to be intended as a collection of notes.
There might be (and there are!) mistakes, imprecisions and simplifications.
Some parte are also incomplete.
Please, send me an email when you find something wrong.

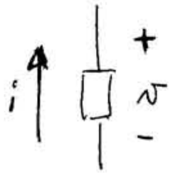
Turning the page and using these notes is at your own risk.
I decline all responsabilities for that :-)

"LOAD" convention



$p = v \cdot i$

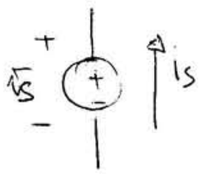
"SOURCE" convention



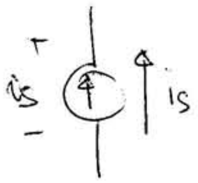
$p = v \cdot i$

$$\begin{cases} P > 0 \Rightarrow \text{consumed power} \\ P < 0 \Rightarrow \text{generated power} \end{cases} \qquad \begin{cases} P > 0 \Rightarrow \text{generated power} \\ P < 0 \Rightarrow \text{consumed power} \end{cases}$$

- Components. "Source" convention

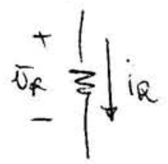


"Voltage source" $v_S = V_S \quad \forall i_S$

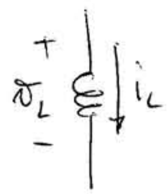


"Current source" $i_S = I_S \quad \forall v_S$

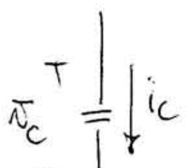
"LOAD" convention



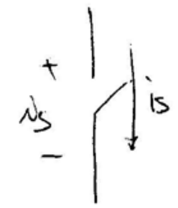
"Resistor" $v_R = R\, i_R$



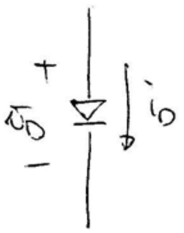
"Inductor" $v_L = L \dfrac{di_L}{dt} \qquad i_L = \dfrac{1}{L}\displaystyle\int v_L\, dt + \underline{\underline{I_1}}$



"Capacitor" $i_C = C \dfrac{dv_C}{dt} \qquad v_C = \dfrac{1}{C}\displaystyle\int i_C\, dt + \underline{\underline{V_1}}$

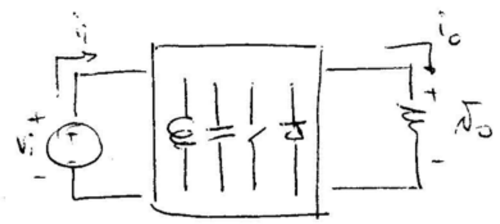


"Switch" $\begin{cases} = \text{ } & i_S = 0 \quad v_S \neq 0 \\ = \text{ } & i_S \neq 0 \quad v_S = 0 \end{cases}$

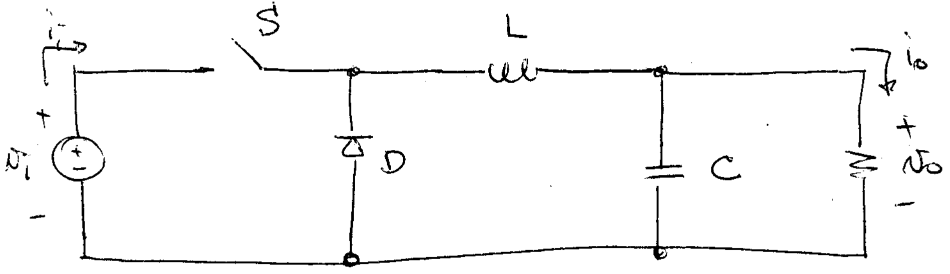


"Diode" $\begin{cases} = \text{ } & i_D = 0 \quad \underline{\underline{v_D < 0}} \\ = \text{ } & v_D = 0 \quad \underline{\underline{i_D > 0}} \end{cases}$

DC/DC CONVERTERS
THE SIMPLEST TOPOLOGIES

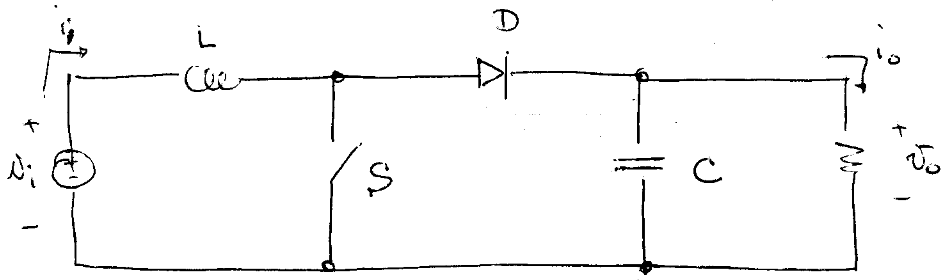


$v_o \leqslant v_i$	$v_o \geqslant v_i$	$v_o \lesseqgtr v_i$
"BUCK"	"BOOST"	"BUCK-BOOST"



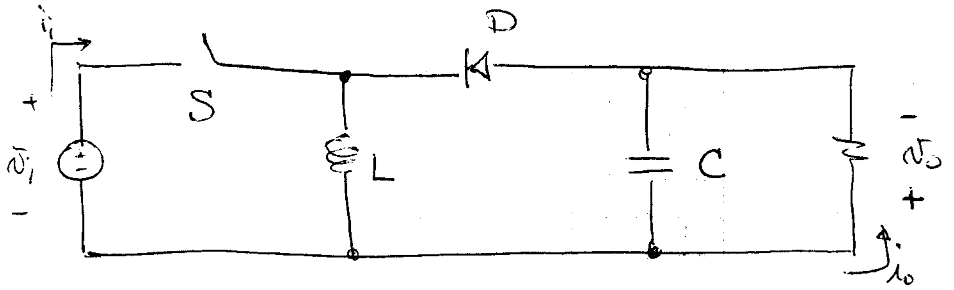
$\boxed{v_o \leq v_i}$

"BUCK"



$\boxed{v_o \geq v_i}$

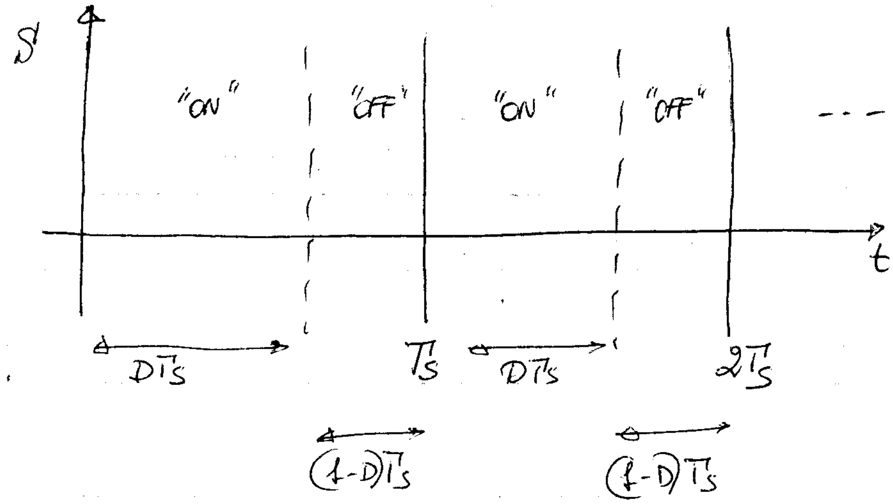
"BOOST"



$\boxed{v_o \lessgtr v_i}$

"BUCK-BOOST"

In all of them S is "on" and "off" regularly:



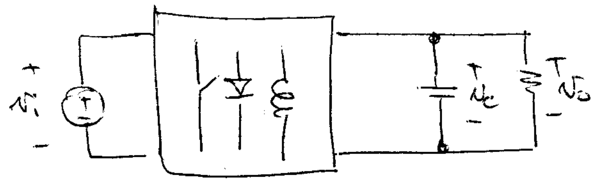
D: duty cycle $\quad 0 \leq D \leq 1$

T_s: switching period

$f_s = \frac{1}{T_s}$: switching frequency $\quad [1\,kHz;\ 1\,MHz]$

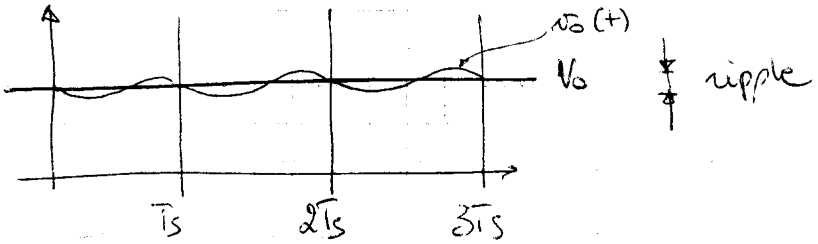
HOW TO STUDY THESE CIRCUITS

Fundamental hypothesis



"C" is "big enough" to assume that $v_c = v_o \triangleq V_o$ (constant voltage).

Actually, $v_c = v_o = \underbrace{V_o}_{\text{constant}} + \tilde{v}_c(t)$ ← ripple

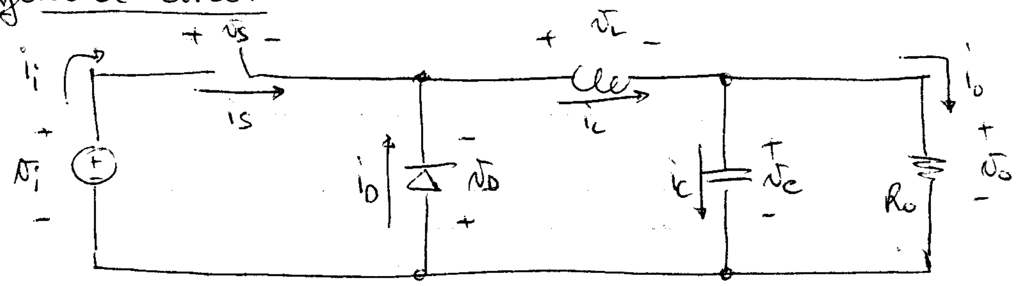


If ripple is small → it can be neglected

"small ripple approximation" or "linear ripple approx."

BUCK

1) General circuit



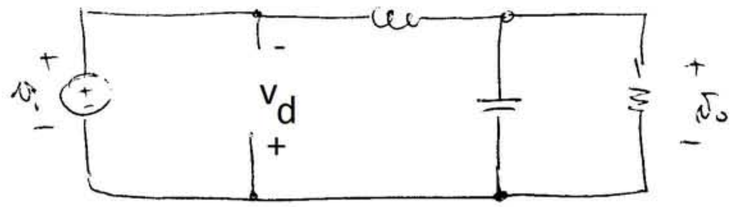
non linear circuit because of S and D

S can be	"ON" or	"OFF"
D can be	"ON" or	"OFF"

⇒ We try all the possibilities, excluding the ones that cannot work.

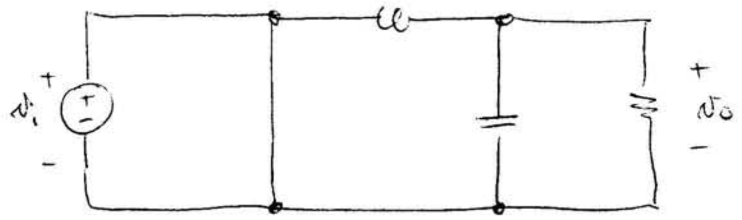
Switch \ DIODE	"on"	"off"
"on"	B	A
"off"	C	D

(A)



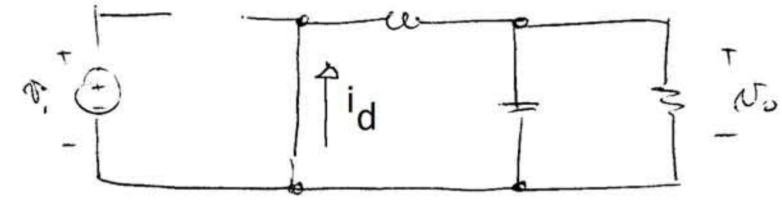
$v_d < 0$

(B)



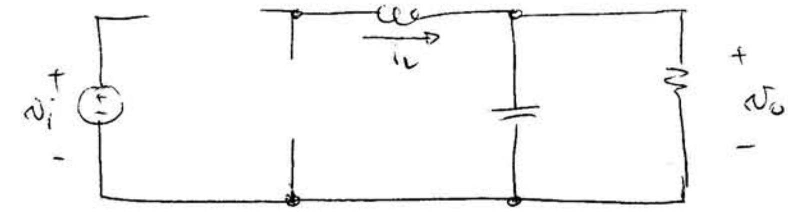
cannot work: v_i is shortcircuited

(C)



$i_d > 0$

(D)

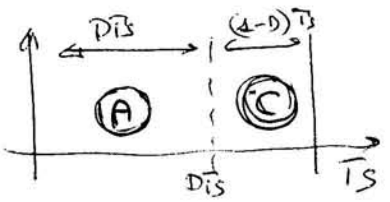


works only if $i_L = 0$: special case we'll consider in the future

⇒ Only (A) and (C) are possible:

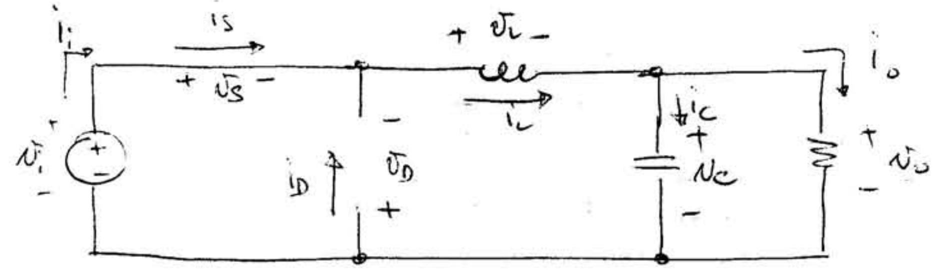
(A) when the switch is "on" (between 0 and DT_s)

(C) when the switch is "OFF" (between DT_s and T_s)



(cont.) Studying the two possible circuits.

(A)



Voltage (cfr v/i diagrams)

$v_S = 0 \qquad v_D = -V_i \qquad v_i = V_i$

$v_C = v_o \simeq V_o$

$v_L = v_i - v_o = V_i - V_o$

$$V_i > V_o \Rightarrow V_i - V_o > 0 \Rightarrow \boxed{v_L > 0}$$

Current

$i_D = 0$

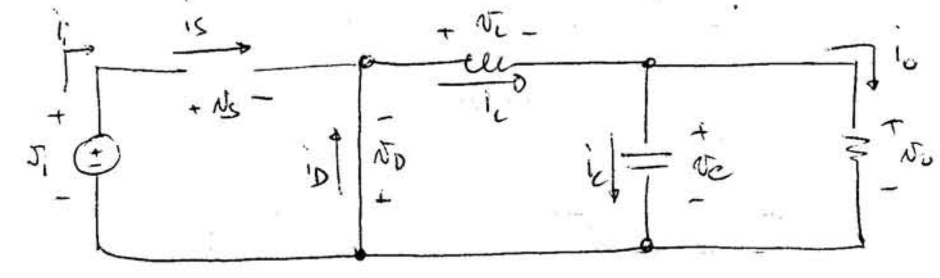
$$i_i = i_S = i_L = \frac{1}{L}\int_0^t v_L(t)\,dt + I_1 = \frac{1}{L}\int_0^t (V_i - V_o)\,dt + I_1$$

$$= \frac{V_i - V_o}{L}\,t + I_1$$

$$i_o \simeq \frac{V_o}{R} = I_o$$

$$i_C = i_L - i_o \simeq i_L - I_o$$

(C)



Voltage (cfr v/i diagrams)

$v_S = v_i = V_i \qquad v_D = 0$

$v_C = v_o \simeq V_o$

$v_L = -v_o \simeq -V_o \qquad \boxed{v_L < 0}$

Current

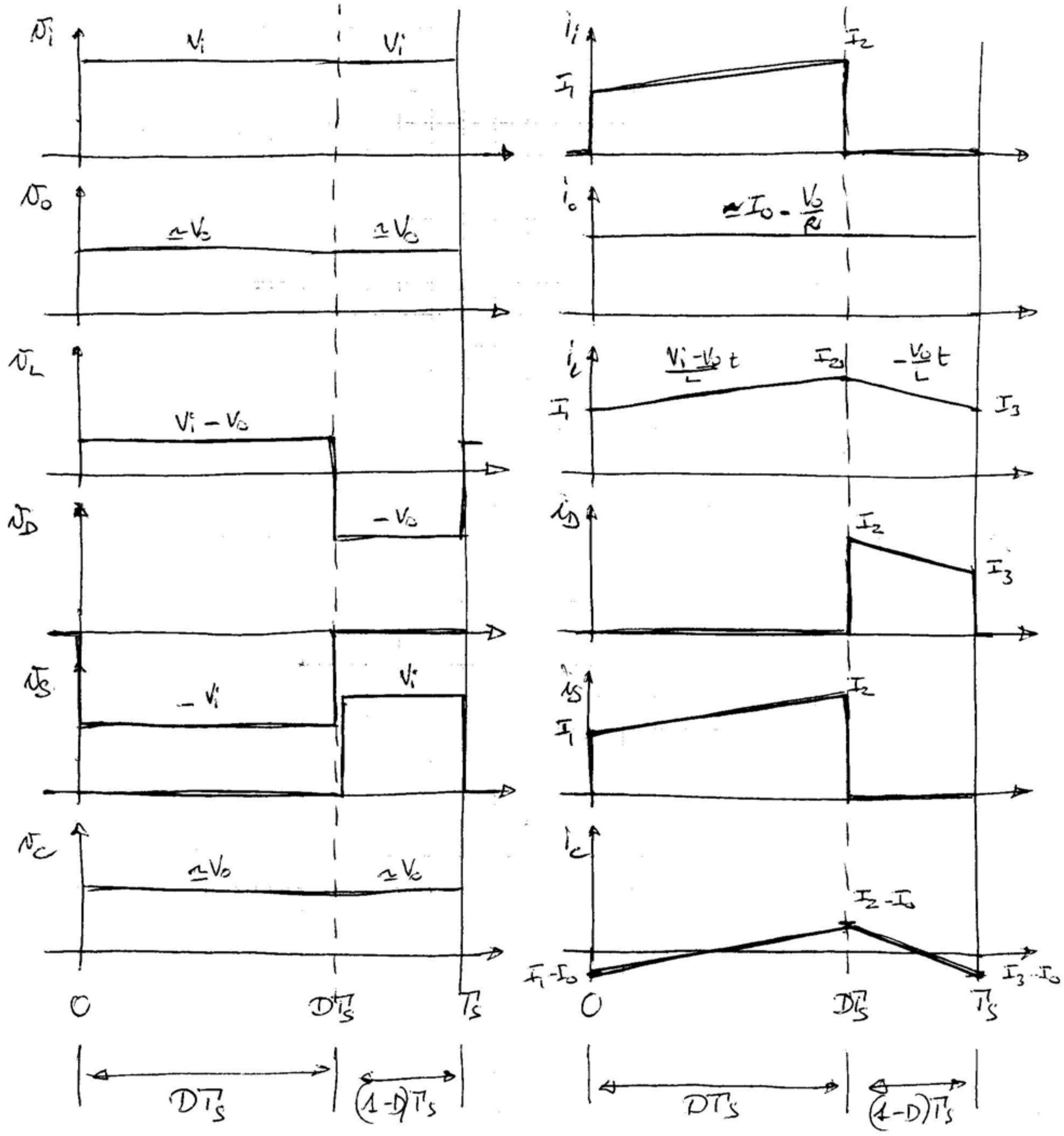
$$i_i = i_S = 0$$

$$i_D = i_L$$

$$i_L = \frac{1}{L}\int_0^t v_L \, dt + I_2 = \frac{1}{L}\int_0^t (-V_o)\, dt + I_2 = -\frac{V_o}{L} t + I_2$$

$$i_C = i_L - i_o \simeq i_L - I_o$$

3) Voltage and current diagrams



4) **Steady-state conditions**

Inductor

$$v_L = L \frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L} \int_0^t v_L(t)\,dt + i_L(0)$$

$$i_L(T_s) = \frac{1}{L} \int_0^{T_s} v_L(t)\,dt + i_L(0)$$

Steady state condition: $i_L(T_s) = i_L(0)$

$$\cancel{i_L(T_s)} = \frac{1}{L} \int_0^{T_s} v_L(t)\,dt + \cancel{i_L(0)} \Longrightarrow \frac{1}{L} \int_0^{T_s} v_L(t)\,dt = 0 \Longrightarrow \boxed{\int_0^{T_s} v_L(t)\,dt = 0}$$

"Inductor volt-second balance"

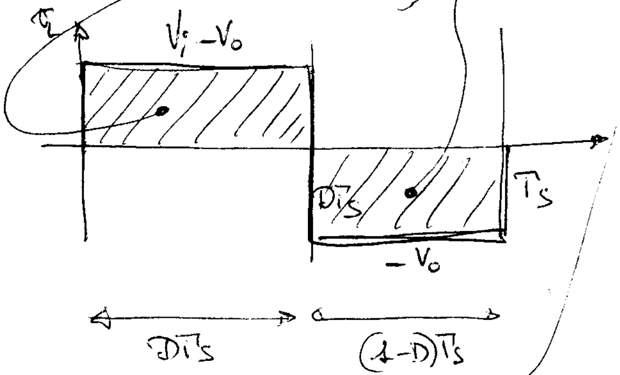
$$\int_0^{DT_s} v_L(t)\,dt + \int_{DT_s}^{T_s} v_L(t)\,dt = 0$$

$$\int_0^{DT_s} v_L(t)\,dt = -\int_{DT_s}^{T_s} v_L(t)\,dt$$

$$\int_0^{DT_s} (V_i - V_o)\,dt = -\int_{DT_s}^{T_s} (-V_o)\,dt$$

$$(V_i - V_o) DT_s = (T_s - DT_s) V_o$$

$$(V_i - V_o) DT_s = V_o (1 - D) T_s$$



$$(V_i - V_o) D = V_o (1 - D)$$

$$\boxed{V_o = D V_i}$$

Implication:

$$\int_0^{T_s} v_L(t) = 0 \Rightarrow \langle v_L \rangle = \frac{1}{T_s} \int_0^{T_s} v_L(t) = 0 \qquad \langle v_L \rangle = 0$$

Capacitor

$$i_c = C \frac{dv_c}{dt}$$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(t)dt + v_c(0)$$

$$v_c(T_s) = \frac{1}{C}\int_0^{T_s} i_c(t)\,dt + v_c(0)$$

Steady State Condition : $v_c(0) = v_c(T_s)$

$$\cancel{v_c(T_s)} = \frac{1}{C}\int_0^{T_s} i_c(t)dt + \cancel{v_c(0)} \Rightarrow \frac{1}{C}\int_0^{T_s} i_c(t)dt = 0$$

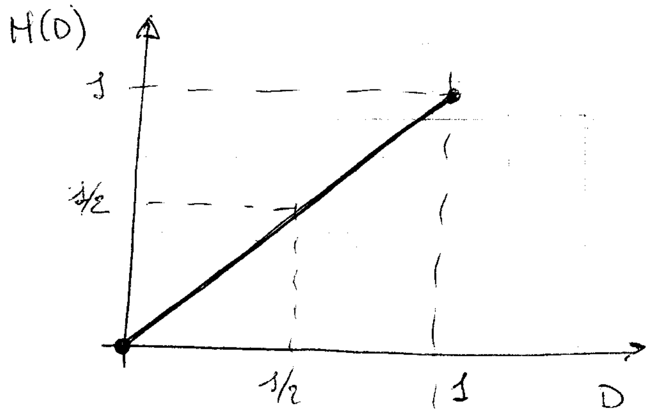
$$\Rightarrow \boxed{\int_0^{T_s} i_c(t)\,dt = 0}$$ "Capacitor charge balance"

Implication:

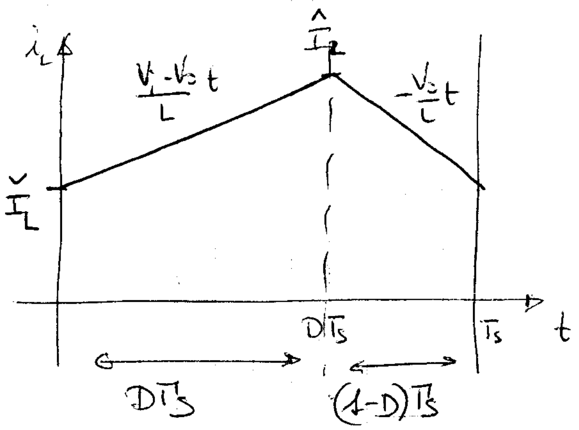
$$\langle i_c \rangle = \frac{1}{T_s}\int_0^{T_s} i_c(t)dt = 0$$ $$\boxed{\langle i_c \rangle = 0}$$

5) Conversion ratio

$$H(D) = \frac{V_o(D)}{V_i} = D$$



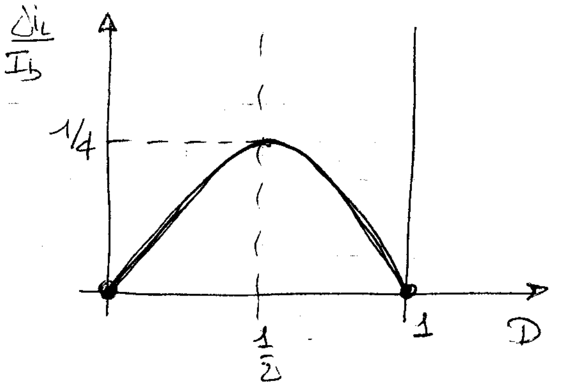
6) Current ripple (inductor)



$$\Delta i_L = \hat{I}_L - \check{I}_L$$

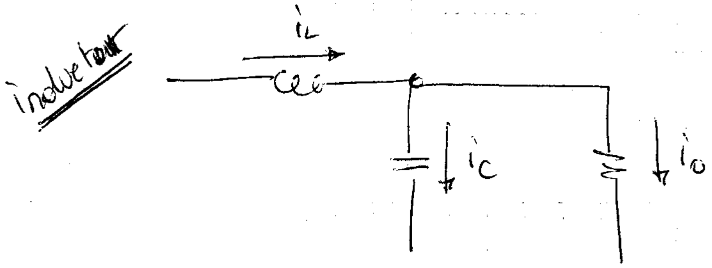
$$\Delta i_L = \frac{V_i - V_o}{L} \cdot DT_s = \frac{V_i - DV_i}{L} DT_s = \underbrace{\frac{V_i T_s}{L}}_{I_b} (1-D)D$$

$$= I_b (1-D)D$$



$$\boxed{\hat{\Delta i}_L = \frac{V_i T_s}{4L}}$$

7) Average and maximum current



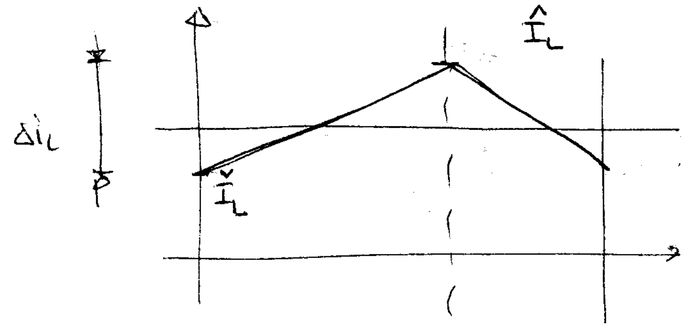
$$i_L = i_C + i_o$$

$$\frac{1}{T_s}\int i_L \, dt = \frac{1}{T_s}\int i_C \, dt + \frac{1}{T_s}\int i_o \, dt$$

$$\langle i_L \rangle = \cancel{\langle i_C \rangle} + \langle i_o \rangle$$

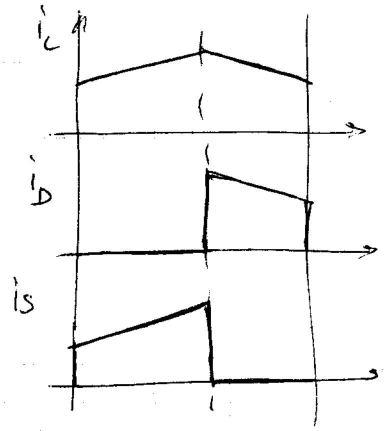
$$\langle i_L \rangle = \langle i_o \rangle = I_o = \frac{V_o}{R}$$

max



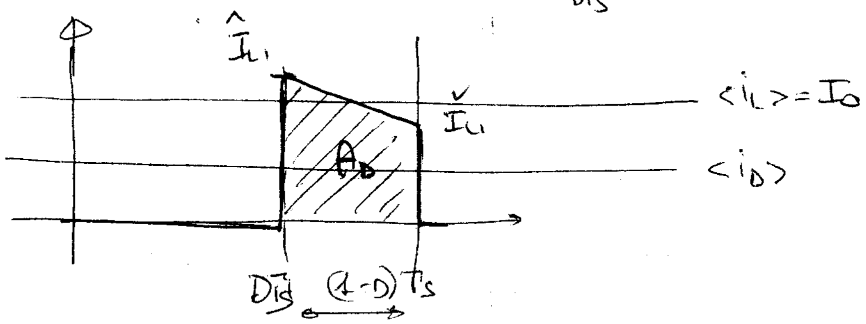
$$\hat{I}_L = \langle i_L \rangle + \frac{\Delta i_L}{2} = I_o + \frac{V_i T_s}{2L}(1-D)D$$

$$\hat{I}_L = \hat{I}_D = \hat{I}_S$$



Diode

$$\langle i_D \rangle = \frac{1}{T_s}\int_0^{T_s} i_D(t)\,dt = \cancel{\frac{1}{T_s}\int_0^{DT_s} i_D(t)\,dt} + \frac{1}{T_s}\int_{DT_s}^{T_s} i_D(t)\,dt$$

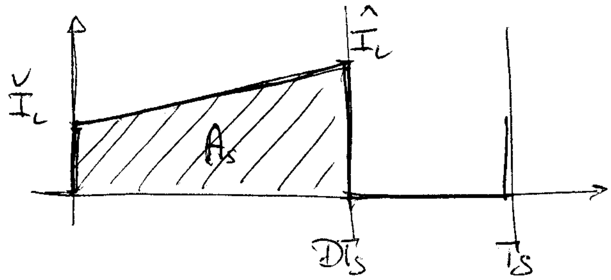


$$= \frac{1}{T_s} A_D = \frac{1}{\cancel{T_s}}\left[\frac{\hat{I}_L + \check{I}_L}{2}(1-D)\cancel{T_s}\right] = I_o(1-D)$$

$$\boxed{\langle i_D \rangle = I_o(1-D)}$$

Switch

$$\langle i_S \rangle = \frac{1}{T_s}\int_0^{T_s} i_S(t)\,dt = \cancel{\frac{1}{T_s}\int_0^{DT_s} i_S(t)\,dt} + \frac{1}{T_s}\int_{DT_s}^{T_s} i_S(t)\,dt =$$



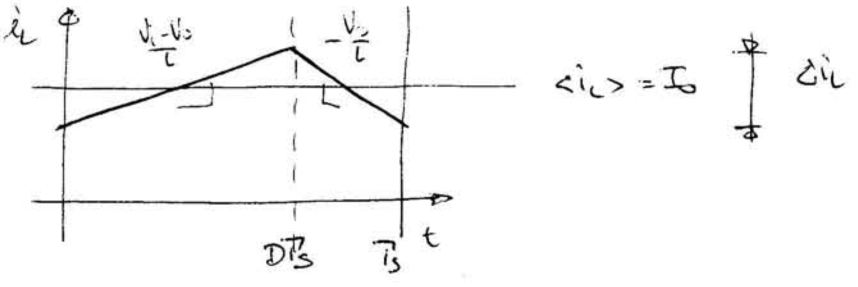
$$= \frac{1}{T_s} A_s = \frac{1}{\cancel{T_s}} \left[\frac{\check{I}_L + \hat{I}_L}{2} \cdot D \cancel{T_s} \right] = I_o D$$

$$\boxed{\langle i_s \rangle = I_o D} \qquad \boxed{\langle i_i \rangle = \langle i_s \rangle = I_o D}$$

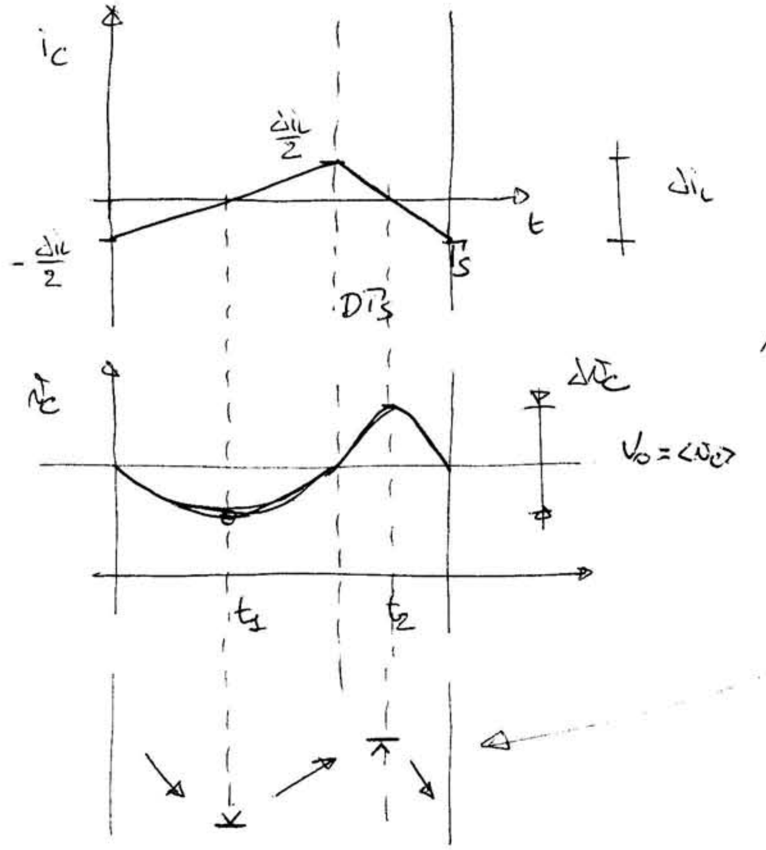
8) Maximum voltage

$$|\hat{v}_S| = V_i \qquad |\hat{v}_D| = V_i$$

a) Output voltage ripple



$$i_C = i_L - I_o$$



$$v_C = \frac{1}{C}\int_0^t i_C\,dt + V_o$$

$$i_C = \frac{dv_C}{dt}$$

$$v_C(t_2) = \frac{1}{C}\int_0^{t_2} i_C\,dt + V_o$$

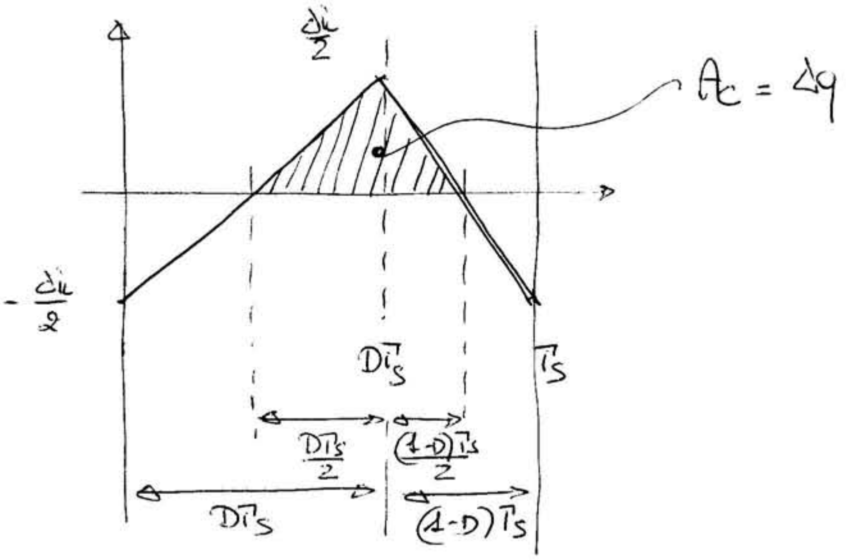
$$v_C(t_1) = \frac{1}{C}\int_0^{t_1} i_C\,dt + V_o$$

$$\Delta v_C = v_C(t_2) - v_C(t_1) = \frac{1}{C}\int_0^{t_2} i_C\,dt + \cancel{V_o} - \frac{1}{C}\int_0^{t_1} i_C\,dt - \cancel{V_o}$$

$$= \frac{1}{C}\int_0^{t_2} i_C\,dt + \frac{1}{C}\int_{t_1}^{0} i_C\,dt$$

$$= \frac{1}{C}\int_{t_1}^{0} i_C\,dt + \frac{1}{C}\int_0^{t_2} i_C\,dt = \frac{1}{C}\int_{t_1}^{t_2} i_C\,dt$$

$$\Delta v_C = \frac{1}{C}\int_{t_1}^{t_2} i_C\,dt = \frac{1}{C} A_C = \frac{\Delta q}{C}$$



$$A_C = \frac{1}{2}\left[\frac{DT_S}{2} + \frac{(1-D)T_S}{2}\right]\cdot\frac{\Delta i_L}{2}$$

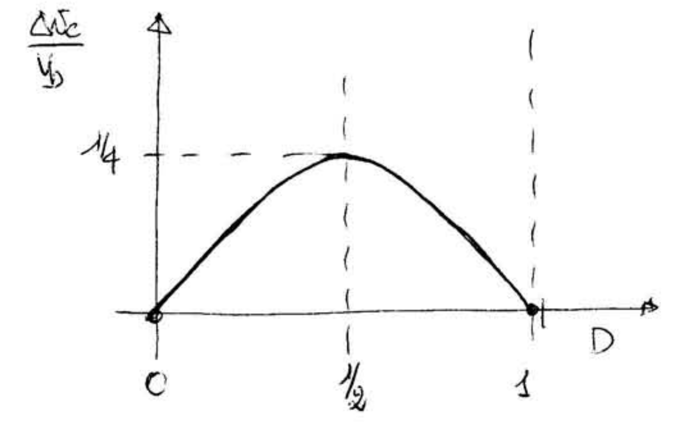
$$= \frac{1}{2}\cdot\frac{1}{2}\left[\cancel{D} + (1-\cancel{D})\right]T_S\cdot\frac{\Delta i_L}{2} = \frac{1}{8}T_S\cdot\Delta i_L$$

$$\Delta i_L = \frac{V_i T_S}{L}(1-D)D$$

$$\Delta v_C = \frac{1}{C}\,\frac{1}{8}\cdot T_S\cdot\frac{V_i T_S}{L}(1-D)D$$

$$\boxed{\Delta v_C = \frac{V_i T_S^2}{8LC}(1-D)D}$$

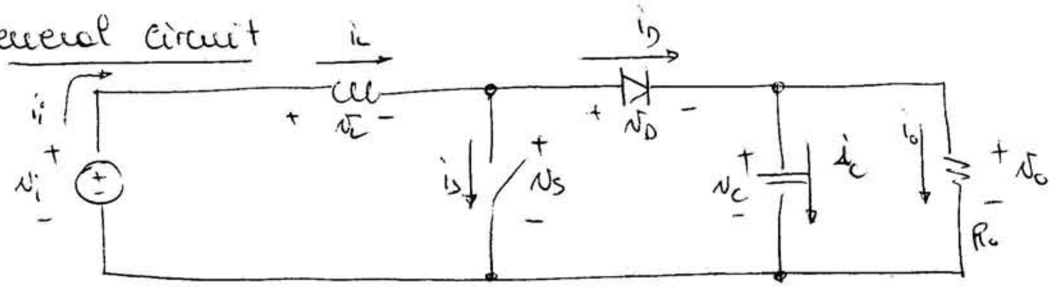
$$\frac{V_i T_S^2}{8LC} = V_b$$



$$\Delta \hat{v}_C = \frac{V_i T_S^2}{8LC}\,\frac{1}{4} = \frac{V_i T_S^2}{32LC}$$

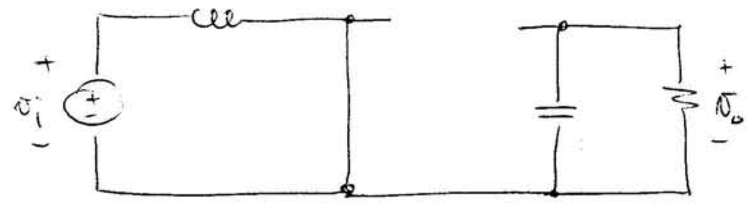
BOOST

1) General circuit

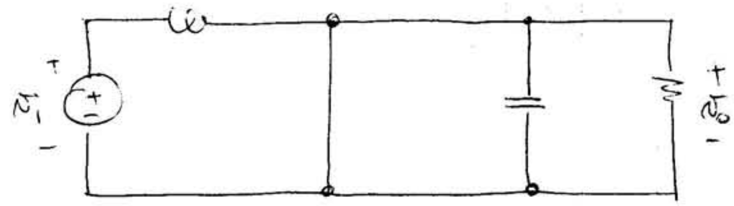


		DIODE "on"	DIODE "off"
switch	"on"	B	A
	"off"	C	D

A

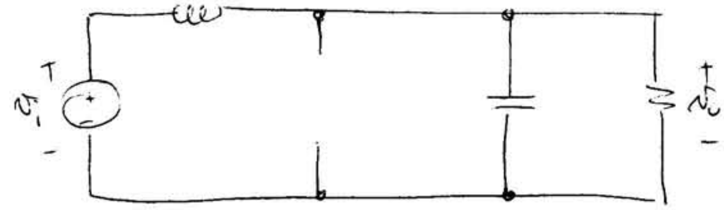


B

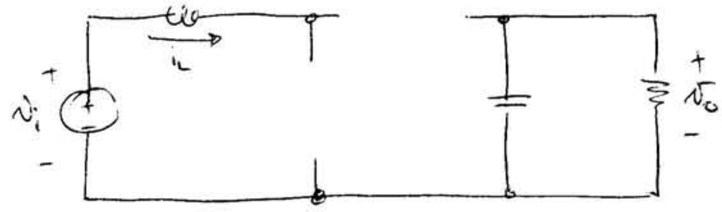


cannot work: "v_o" shortcircuited

C

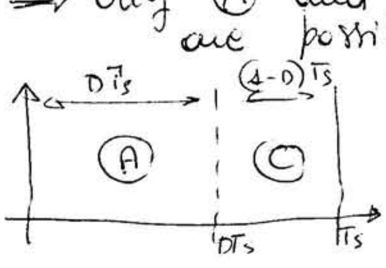


D



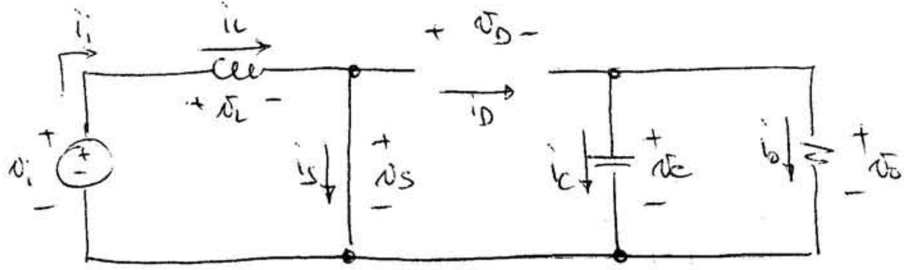
works only if $i_L = 0$ special case we'll consider in the future

→ Only A and C are possible:



2) Studying the two possible circuits

(A)



Voltage (cfr v/i diagrams)

$v_S = 0 \qquad v_C = v_o \simeq V_o \qquad v_D = -v_o \simeq -V_o$

$v_L = v_i = V_i \qquad V_i > 0 \quad \Rightarrow \quad \boxed{v_L > 0}$

Current

$i_D = 0$

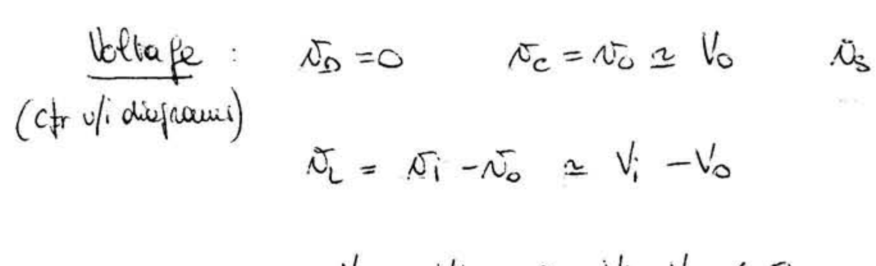
$i_i = i_S = i_L = \frac{1}{L}\int_0^t v_L(t)dt + I_1 = \frac{1}{L}\int_0^t V_i\,dt + I_1$

$= \frac{V_i}{L} t + I_1$

$i_o \simeq \frac{V_o}{R} = I_o$

$i_C = -i_o \simeq -I_o = -\frac{V_o}{R}$

(C)



Voltage : (cfr v/i diagrams)

$v_D = 0 \qquad v_C = v_o \simeq V_o \qquad v_S = v_o \simeq V_o$

$v_L = v_i - v_o \simeq V_i - V_o$

$V_o > V_i \Rightarrow V_i - V_o < 0 \Rightarrow \boxed{v_L < 0}$

Current

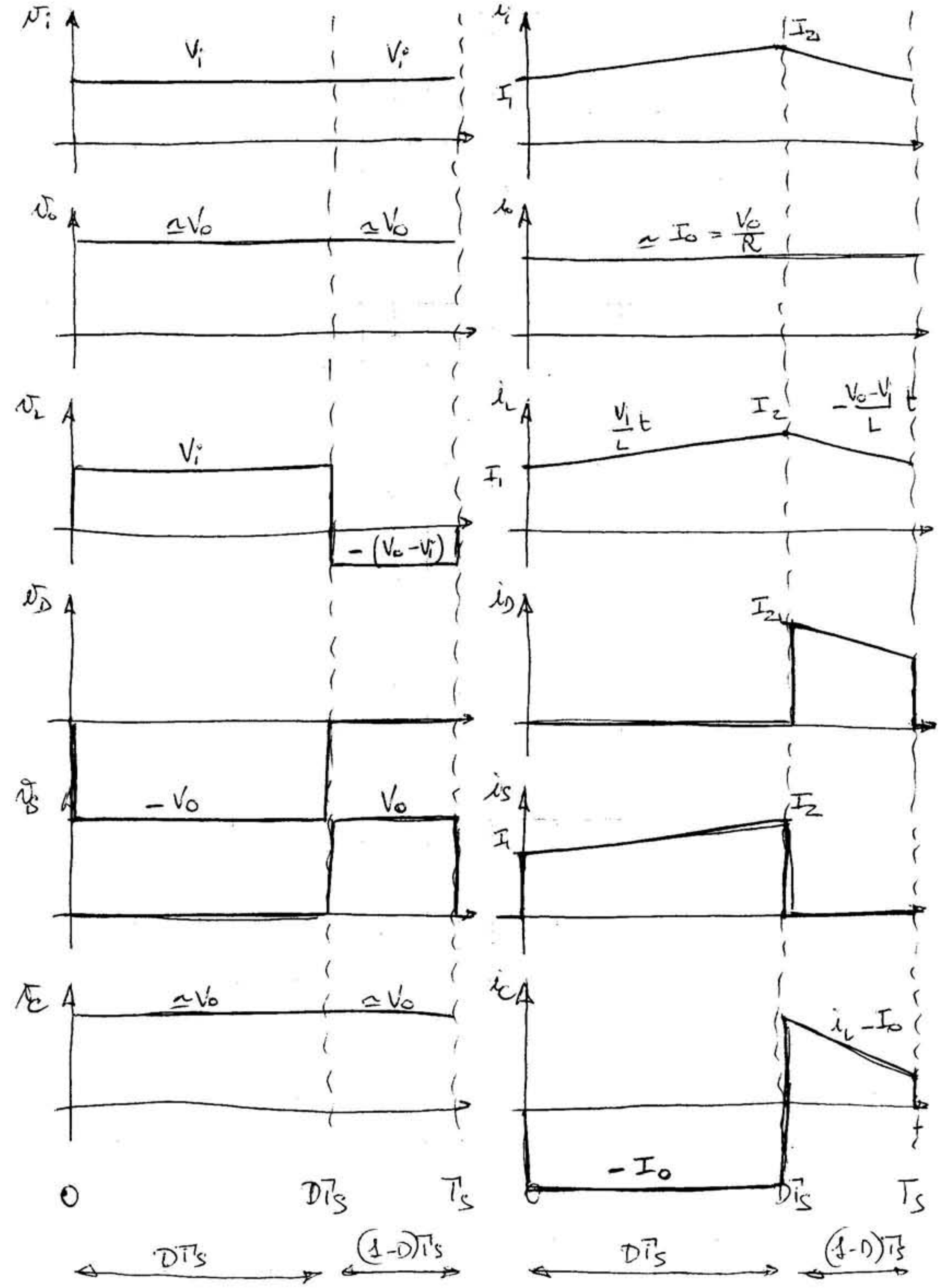
$i_S = 0$

$i_i = i_D = i_L = \frac{1}{L}\int_0^t v_L(t)dt + I_2$

$$\frac{1}{L}\int_0^t (V_i - V_o)\,dt + I_2 = -\frac{V_o - V_i}{L}\,t + I_2$$

$$i_C = i_D - i_o = i_L - i_o \simeq i_L - I_o$$

3) Voltage and current diagrams



4) Steady-state conditions

Inductor $\quad v_L = L \dfrac{di_L}{dt}$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0)$$

$$i_L(T_s) = \frac{1}{L}\int_0^{T_s} v_L(t)dt + i_L(0)$$

Steady state condition: $\quad i_L(T_s) = i_L(0)$

$$\cancel{i_L(T_s)} = \frac{1}{L}\int_0^{T_s} v_L(t)dt + \cancel{i_L(0)} \Rightarrow \frac{1}{L}\int_0^{T_s} v_L(t)dt = 0 \rightarrow \boxed{\int_0^{T_s} v_L(t) = 0}$$

$$\int_0^{DT_s} v_L(t)dt + \int_{DT_s}^{T_s} v_L(t)dt = 0$$

"Inductor volt-second balance"

$$\int_0^{DT_s} v_L(t)dt = -\int_{DT_s}^{T_s} v_L(t)dt$$

$$\int_0^{DT_s} V_i \, dt = -\int_{DT_s}^{T_s} -(V_o - V_i)\, dt$$

$$V_i D\cancel{T_s} = (V_o - V_i)(1-D)\cancel{T_s}$$

$$V_i(\cancel{D} + 1 - \cancel{D}) = V_o(1-D)$$

$$\boxed{V_o = \frac{1}{1-D} V_i}$$

Implication

$$\int_0^{T_s} v_L(t)dt \Rightarrow \langle v_L \rangle = \frac{1}{T_s}\int_0^{T_s} v_L(t)dt = 0 \qquad \langle v_L \rangle = 0$$

Capacitor

$$i_c = C\frac{dv_c}{dt}$$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(t)dt + v_c(0)$$

$$v_c(T_S) = \frac{1}{C}\int_0^{T_S} i_c(t)dt + v_c(0)$$

Steady-state condition. $v_c(0) = v_c(T_S)$

$$\cancel{v_c(T_S)} = \frac{1}{C}\int_0^{T_S} i_c(t)dt + \cancel{v_c(0)} \Rightarrow \frac{1}{C}\int_0^{T_S} i_c(t)\,dt = 0$$

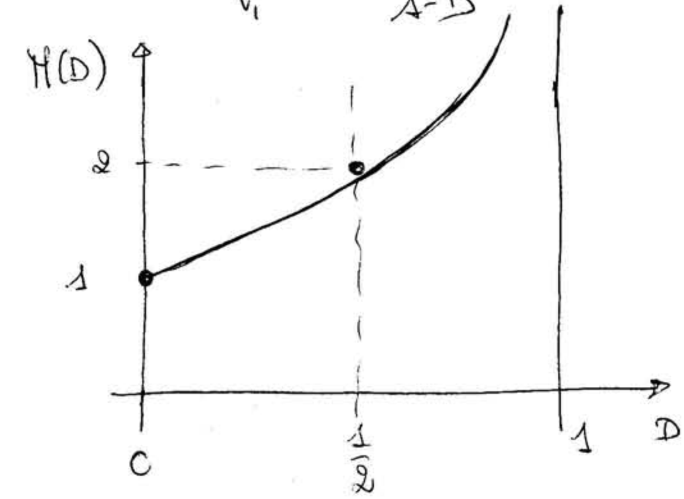
$$\Rightarrow \boxed{\int_0^{T_S} i_c(t)dt = 0}$$ "Capacitor charge balance"

Implication $$\langle i_c\rangle = \frac{1}{T_S}\int_0^{T_S} i_c(t)dt = 0 \qquad \boxed{\langle i_c\rangle = 0}$$

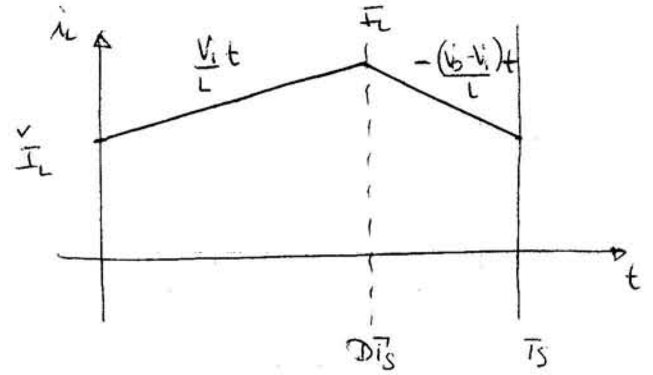
---

g) Conversion ratio

$$M(D) = \frac{V_o(D)}{V_i} = \frac{1}{1-D}$$

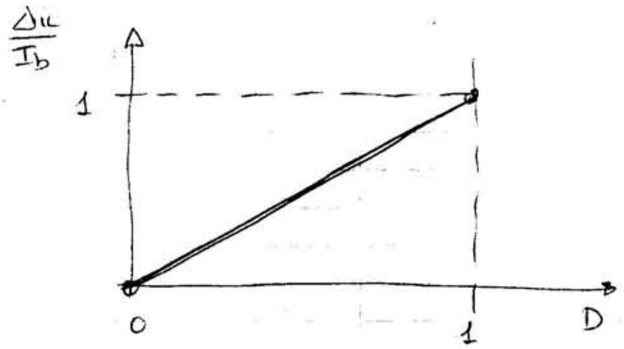


6) Current ripple (inductor)



$$\Delta i_L = \hat{I}_L - \check{I}_L$$

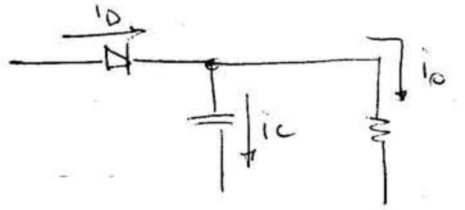
$$\Delta i_L = \frac{V_i}{L} D T_s = \underbrace{\frac{V_i T_s}{L}}_{I_b} D = I_b D$$



$$\boxed{\hat{\Delta i_L} = \frac{V_i T_s}{L}}$$

7) Average and maximum current

diode

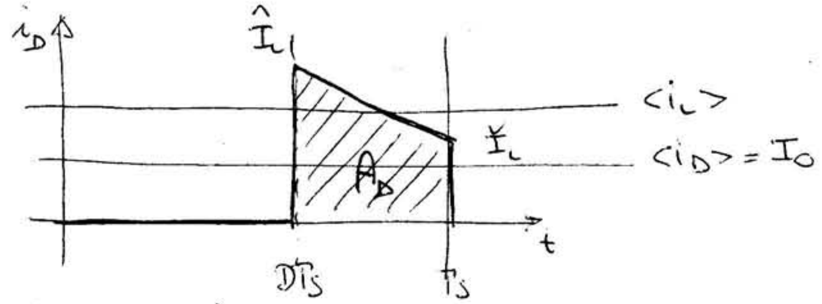


$$i_D = i_C + i_o$$

$$\rightarrow \quad \langle i_D \rangle = \cancel{\langle i_C \rangle} + \langle i_o \rangle$$

$$\langle i_D \rangle = \langle i_o \rangle = I_o = \frac{V_o}{R}$$

inductor



$$\langle i_D \rangle = \frac{1}{T_s}\int_0^{T_s} i_D(t)\,dt = \cancel{\frac{1}{T}\int_0^{DT_s} i_D(t)\,dt} + \frac{1}{T_s}\int_{DT_s}^{T_s} i_D(t)\,dt = \frac{1}{T_s} A_D$$

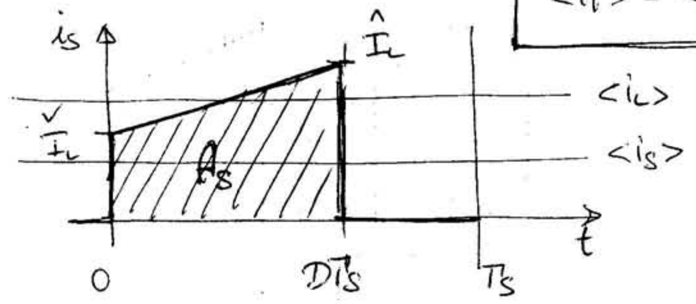
$$\frac{1}{\cancel{T_S}}\left[\frac{\hat{I}_L+\check{I}_L}{2}(1-D)\cancel{T_S}\right]=\langle i_L\rangle(1-D)$$

$$\langle i_D\rangle = I_O$$

$$\langle i_L\rangle(1-D) = I_O \implies \boxed{\langle i_L\rangle = \frac{1}{1-D} I_O}$$

$$\boxed{\langle i_i\rangle = \langle i_L\rangle = \frac{1}{1-D} I_O}$$

Switch

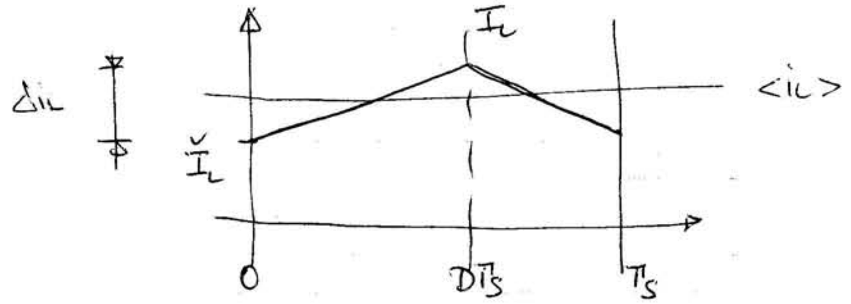


$$\langle i_S\rangle = \frac{1}{T_S}\int_0^{T_S} i_S(t)\,dt = \frac{1}{T_S}\int_0^{DT_S} i_S(t)\,dt + \cancel{\frac{1}{T_S}\int_{DT_S}^{T_S} i_S(t)\,dt} = \frac{1}{T_S} A_S$$

$$\frac{1}{\cancel{T_S}}\left[\frac{\check{I}_L+\hat{I}_L}{2} D\cancel{T_S}\right] = \langle i_L\rangle D = \frac{D}{1-D} I_O$$

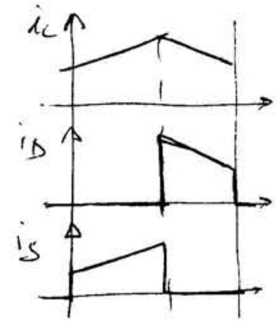
$$\boxed{\langle i_S\rangle = \frac{D}{1-D} I_O}$$

max



$$\hat{I}_L = \langle i_L\rangle + \frac{\Delta i_L}{2} = \frac{1}{1-D} I_O + \frac{V_i T_S}{L} \cdot D$$

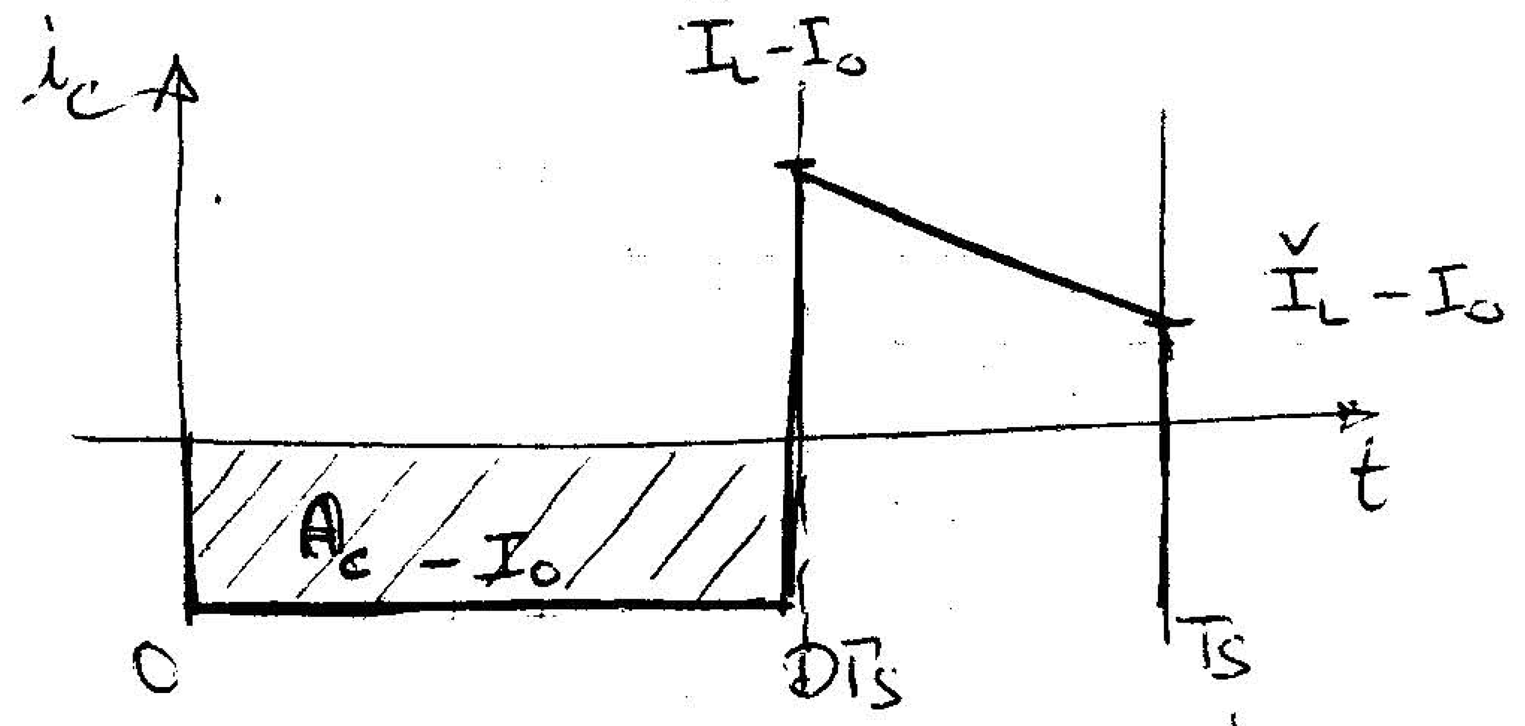
$$\hat{I}_L = \hat{I}_S = \hat{I}_D$$



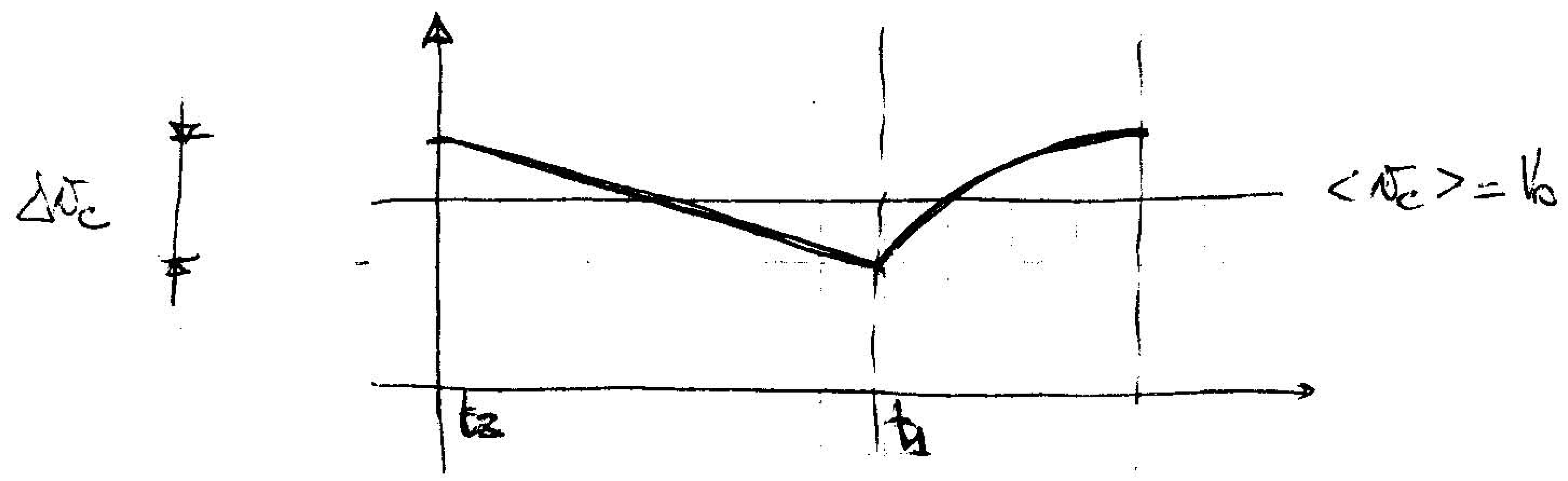
8) Maximum voltage

$$|\hat{v}_S| = V_o \qquad |\hat{v}_D| = V_o$$

9) Output voltage ripple



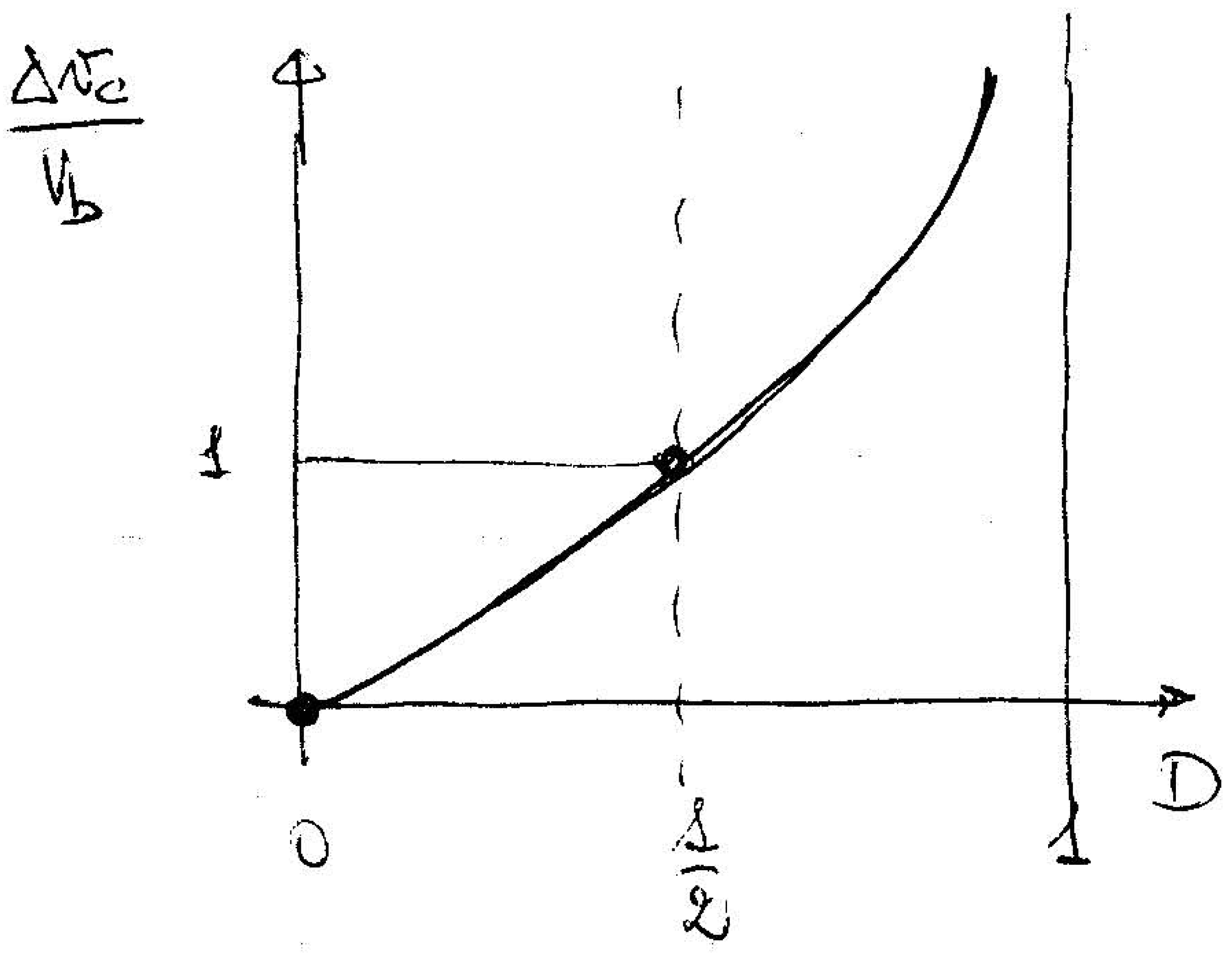
$$i_C = C\frac{dv_C}{dt} \Rightarrow v_C = \frac{1}{C}\int_0^t i_C\,dt + V_1$$



$$\Delta v_C = \frac{1}{C}\int_{t_1}^{t_2} i_C\,dt = \frac{1}{C}\int_{DT_S}^{0} i_C\,dt = -\frac{1}{C}\int_0^{DT_S} i_C\,dt = -\frac{1}{C}\int_0^{DT_S}(-I_o)\,dt = \frac{I_o DT_S}{C} = \frac{V_o DT_S}{RC}$$

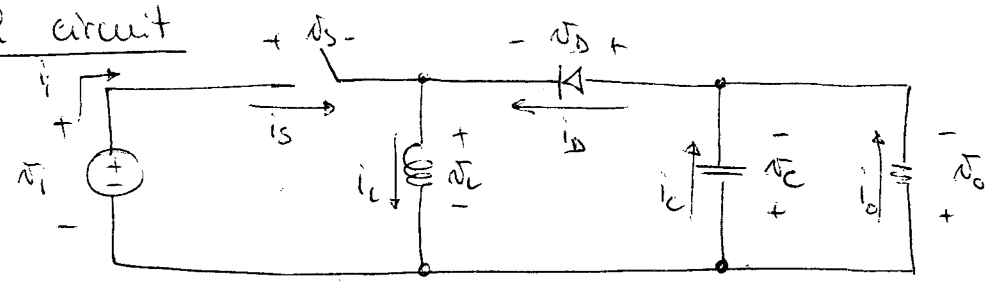
$$= V_i \frac{1}{1-D}\frac{DT_S}{RC} = \underbrace{\frac{V_i T_S}{RC}}_{V_b}\frac{D}{1-D}$$

$$\boxed{\Delta v_C = \frac{V_i T_S}{RC}\frac{D}{1-D}}$$



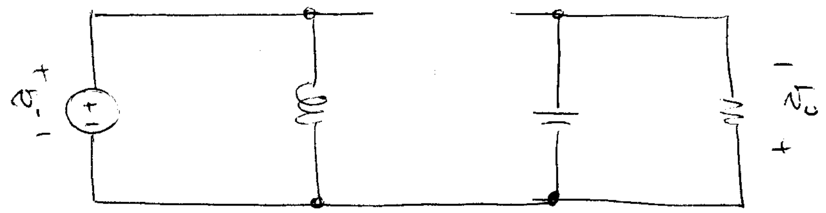
BUCK-BOOST

1) General circuit

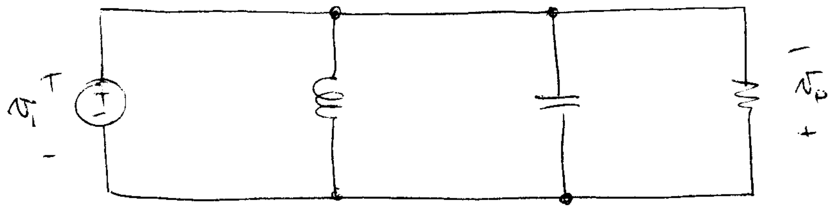


	Diode "on"	Diode "off"
Switch "on"	B	A
Switch "off"	C	D

A

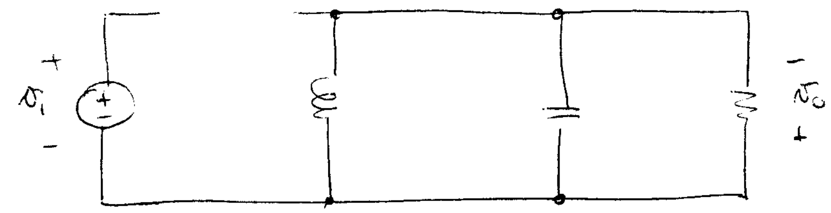


B

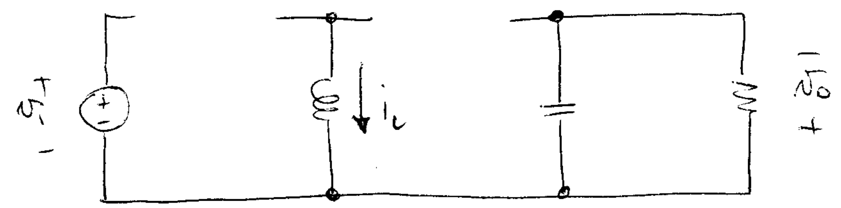


cannot work: $v_i = -v_o$

C

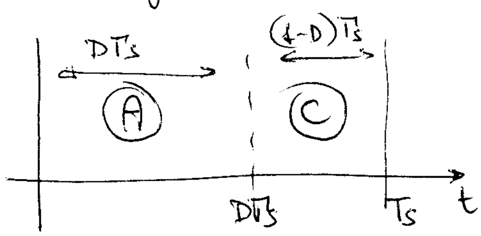


D



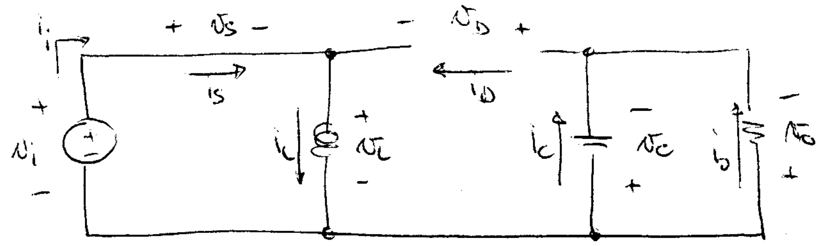
works only if $i_L = 0$
special case we'll consider in the future.

→ only A and C are possible.



2) Studying the two possible circuits

Ⓐ



Voltage (cfr v/i diagrams) $v_S = 0$ $\quad v_C = v_o \simeq V_o \quad v_D = -v_i - v_o \simeq -V_i - V_o$

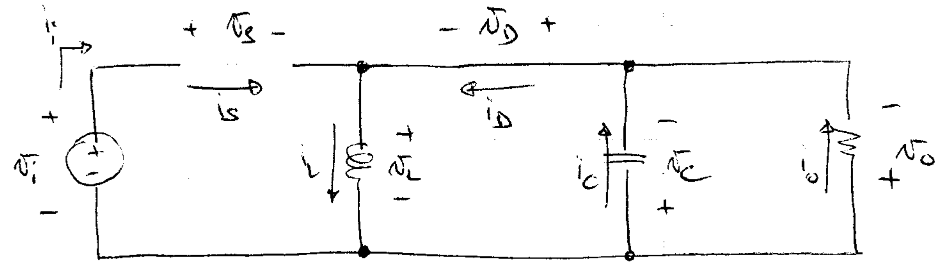
$v_L = V_i$

Current $i_D = 0 \quad i_i = i_S = i_L = \frac{1}{L}\int_0^t v_L(t)\,dt + I_1$

$$= \frac{1}{L}\int_0^t V_i\,dt + I_1 = \frac{V_i}{L}t + I_1$$

$$i_C = -i_o \simeq -I_o = -\frac{V_o}{R}$$

Ⓒ



Voltage (cfr v/i diagrams) $v_D = 0 \quad v_C = v_o \simeq V_o$

$v_S = v_i + v_o = V_i + V_o$

$v_L = -v_o \simeq -V_o$

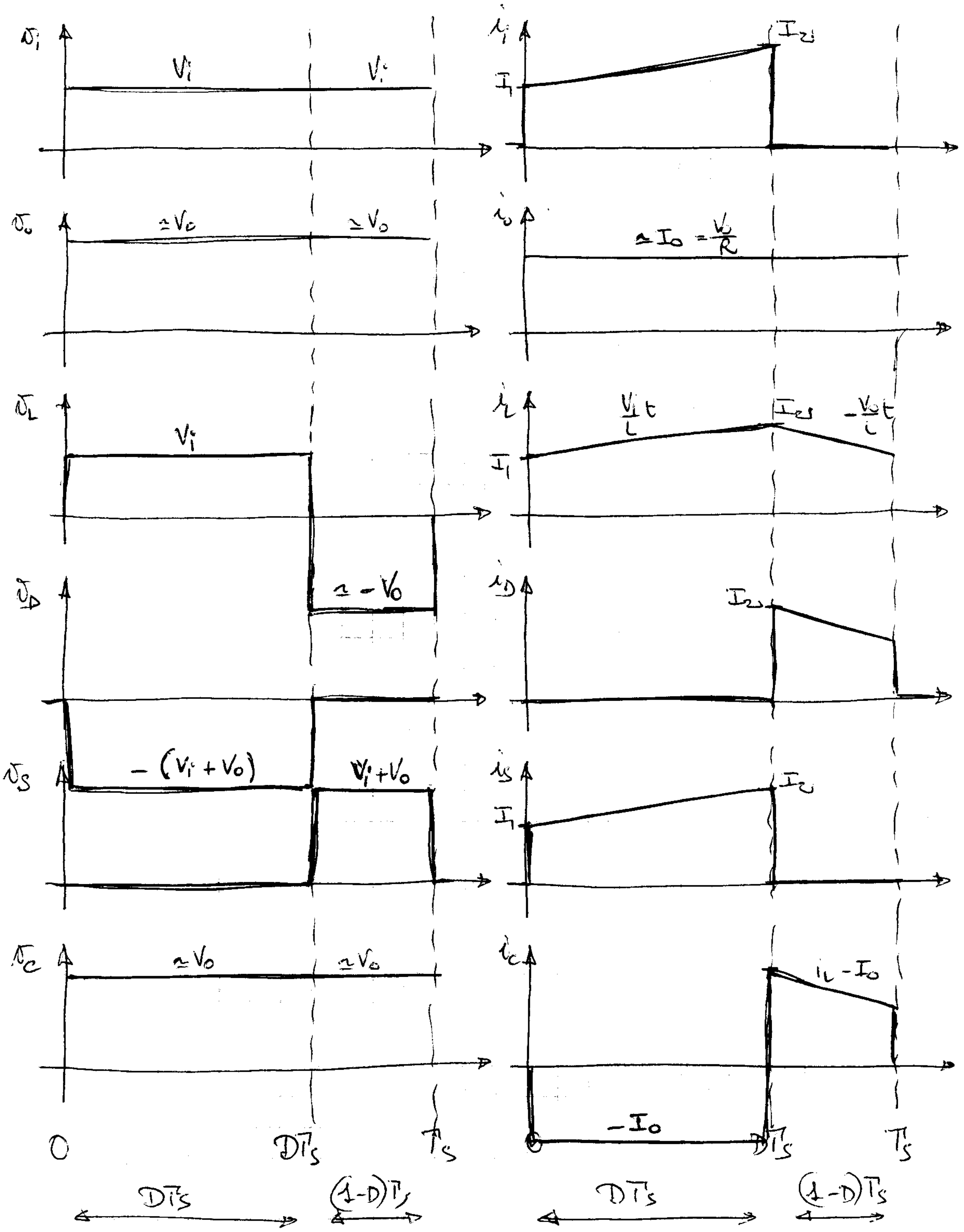
Current $i_S = 0$

$$i_D = i_L = \frac{1}{L}\int_0^t v_L(t)\,dt + I_2 = \frac{1}{L}\int_0^t (-V_o)\,dt + I_2$$

$$= -\frac{V_o}{L}t + I_2$$

$$i_C = i_D - i_o \simeq i_D - I_o$$

3) Voltage and current diagrams



4) Steady-state condition

Inductor

$$v_L = L\frac{di}{dt}$$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)\,dt + i_L(0)$$

$$i_L(T_S) = \frac{1}{L}\int_0^{T_S} v_L(t)dt + i_L(0)$$

Steady-state condition : $i_L(T_S) = i_L(0)$

$$\cancel{i_L(T_S)} = \frac{1}{L}\int_0^{T_S} v_L(t)dt + \cancel{i_L(0)} \Rightarrow \frac{1}{L}\int_0^{T_S} v_L(t)dt = 0 \Rightarrow \boxed{\int_0^{T_S} v_L(t)dt = 0}$$

"Inductor volt second balance"

$$\int_0^{DT_S} v_L(t)dt + \int_{DT_S}^{T_S} v_L(t)dt = 0$$

$$\int_0^{DT_S} v_L(t)dt = -\int_{DT_S}^{T_S} v_L(t)dt$$

$$\int_0^{DT_S} V_i\, dt = -\int_{DT_S}^{T_S} (-V_o)dt$$

$$V_i D \cancel{T_S} = V_o (1-D)\cancel{T_S}$$

$$\boxed{V_o = \frac{D}{1-D} V_i}$$

Implication

$$\int_0^{T_S} v_L(t)dt = 0 \Rightarrow \langle v_L \rangle = \frac{1}{T_S}\int_0^{T_S} v_L(t)dt = 0 \qquad \langle v_L \rangle = 0$$

Capacitor

$$i_C = C\frac{dv_C}{dt}$$

$$v_C(t) = \frac{1}{C}\int_0^{t} i_C(t)dt + v_C(0)$$

$$v_C(T_S) = \frac{1}{C}\int_0^{T_S} i_C(t)dt + v_C(0)$$

Steady-state condition. $v_c(0) = v_c(T_s)$

$$v_c(T_s) = \frac{1}{C}\int_0^{T_s} i_c(t)\,dt + v_c(0) \Rightarrow \frac{1}{C}\int_0^{T_s} i_c(t)\,dt = 0$$

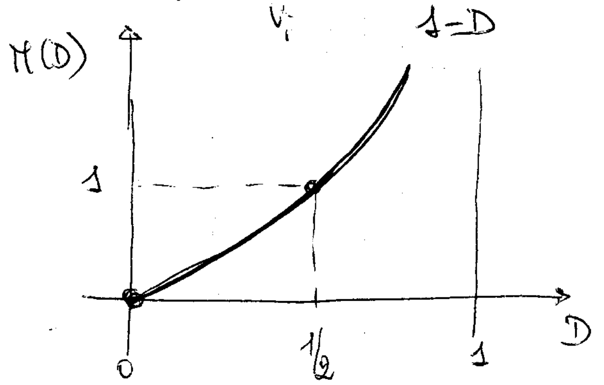
$$\Rightarrow \boxed{\int_0^{T_s} i_c(t)\,dt = 0}$$ "Capac charge balance"

Implication $\langle i_c \rangle = \frac{1}{T_s}\int_0^{T_s} i_c(t)\,dt = 0 \Rightarrow \boxed{\langle i_c \rangle = 0}$

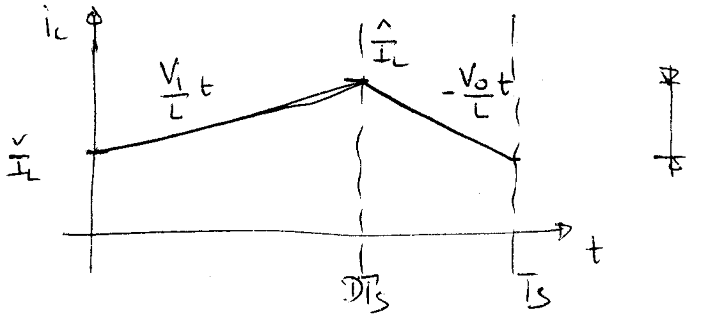
---

5) Conversion ratio

$$M(D) = \frac{V_o(D)}{V_i} = \frac{D}{1-D}$$

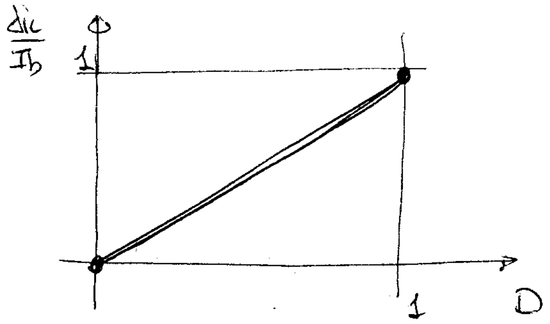


6) Current ripple (inductor)



$\Delta i_L = \hat{I}_L - \check{I}_L$

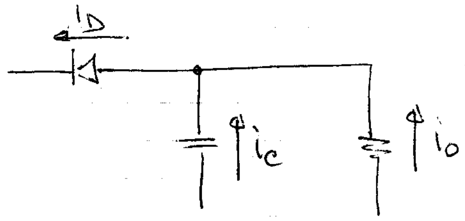
$$\Delta i_L = \frac{V_i}{L} DT_s = \underbrace{\frac{V_i T_s}{L}}_{I_b} D = I_b D$$



$$\boxed{\Delta \hat{i}_L = \frac{V_i T_S}{L}}$$

7) Average and maximum current

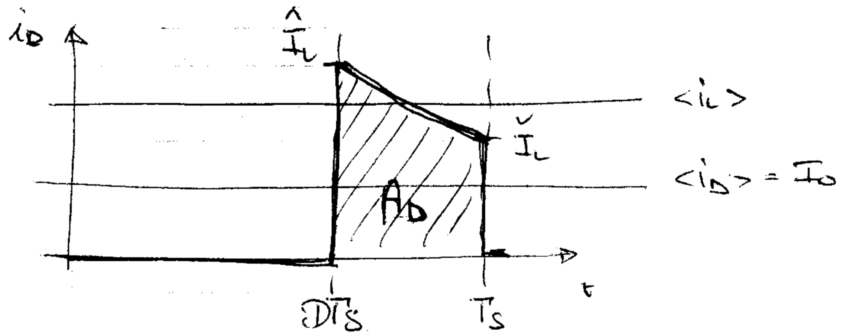
diode



$$i_D = i_C + i_o \implies \langle i_D \rangle = \cancel{\langle i_C \rangle} + \langle i_o \rangle$$

$$\langle i_D \rangle = \langle i_o \rangle = I_o = \frac{V_o}{R} \qquad \boxed{\langle i_D \rangle = I_o}$$

inductor



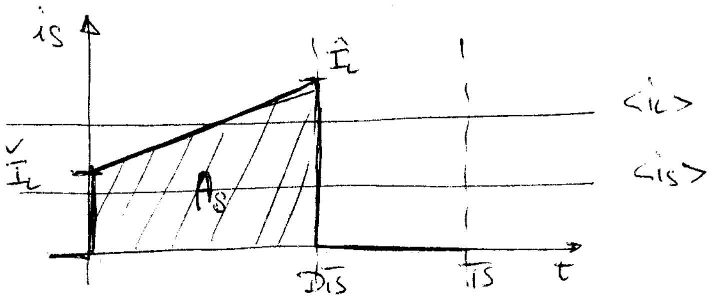
$$\langle i_D \rangle = \frac{1}{T_S}\int_0^{T_S} i_D(t)dt = \frac{1}{T}\int_0^{DT_S} \cancel{i_D(t)\,dt} + \frac{1}{T_S}\int_{DT_S}^{T_S} i_D(t)dt = \frac{1}{T_S} A_D$$

$$= \frac{1}{\cancel{T_S}}\left[\frac{\hat{I}_L + \check{I}_L}{2}(1-D)\cancel{T_S}\right] = \langle i_L \rangle (1-D)$$

$$\langle i_D \rangle = I_o$$

$$\langle i_L \rangle (1-D) = I_o \implies \boxed{\langle i_L \rangle = \frac{1}{1-D} I_o}$$

switch

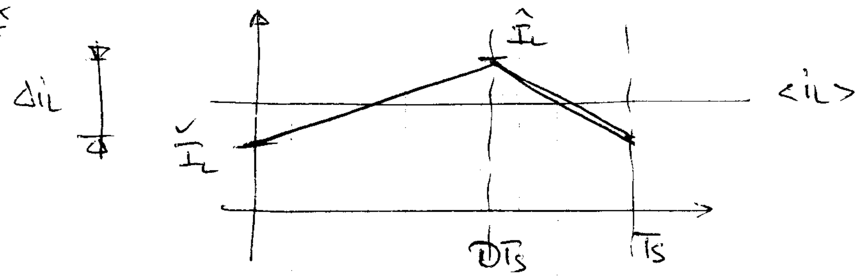


$$\langle i_S \rangle = \frac{1}{T_S}\int_0^{T_S} i_S(t)\,dt = \frac{1}{T_S}\int_0^{DT_S} i_S(t)\,dt + \cancel{\frac{1}{T_S}\int_{DT_S}^{T_S} i_S(t)\,dt} = \frac{1}{T_S} A_S$$

$$= \frac{1}{\cancel{T_S}}\left[\frac{\check{I}_L + \hat{I}_L}{2} D \cancel{T_S}\right] = \langle i_L \rangle D = \frac{D}{1-D} I_o$$

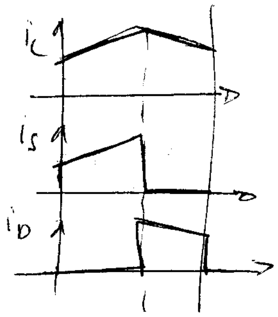
$$\boxed{\langle i_S \rangle = \frac{D}{1-D} I_o} \qquad \boxed{\langle i_i \rangle = \langle i_S \rangle = \frac{D}{1-D} I_o}$$

max



$$\hat{I}_L = \langle i_L \rangle + \frac{\Delta i_L}{2} = \frac{1}{1-D} I_o + \frac{V_i T_S}{L} D$$

$$\hat{I}_L = \hat{I}_S = \hat{I}_D$$



8) Maximum voltage

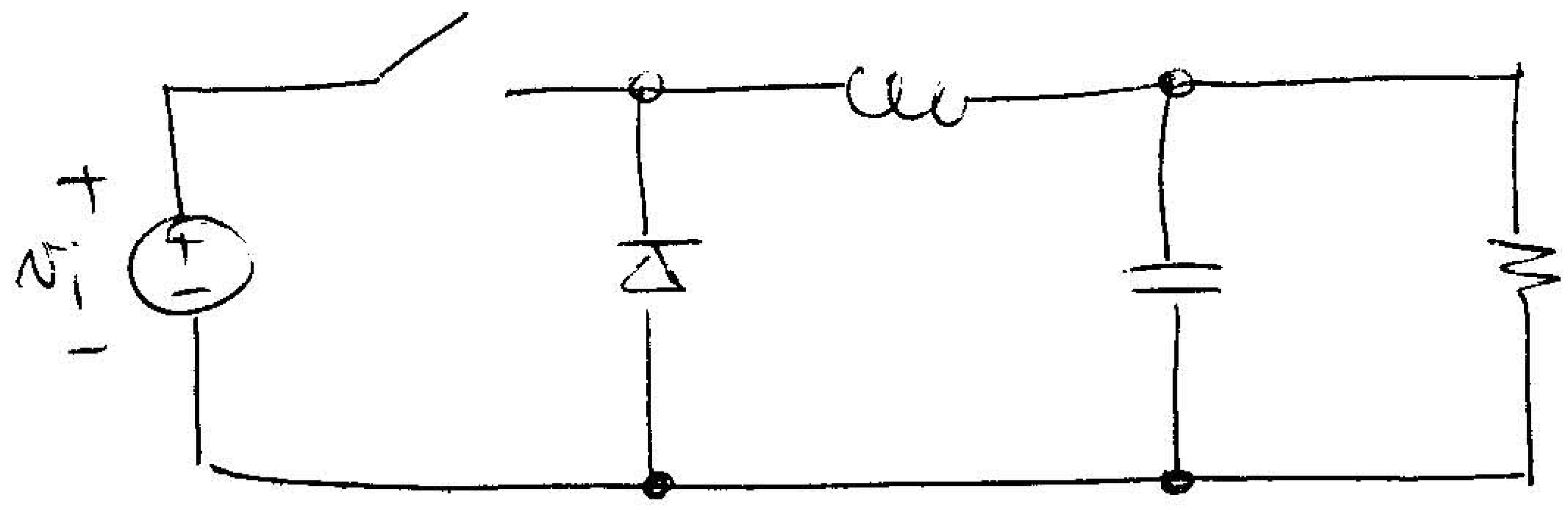
$$|\hat{v}_S| = V_i + V_o \qquad |\hat{v}_D| = V_i + V_o$$

9) Output voltage ripple

same as boost

$$\boxed{\Delta v_C = \frac{V_i T_S}{R_o C} \frac{D}{1-D}}$$

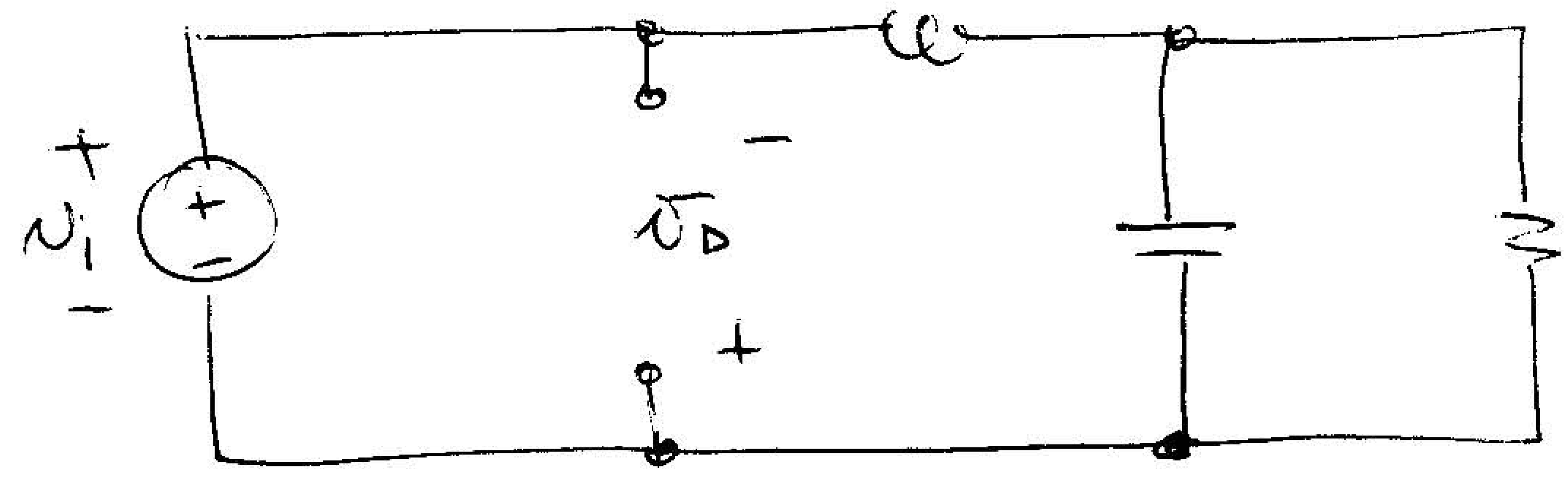
DISCONTINUOUS CONDUCTION MODE



Son / Doff $\quad$ $v_D < 0$?

Soff/Don $\quad$ $i_D > 0$?

Son/Doff

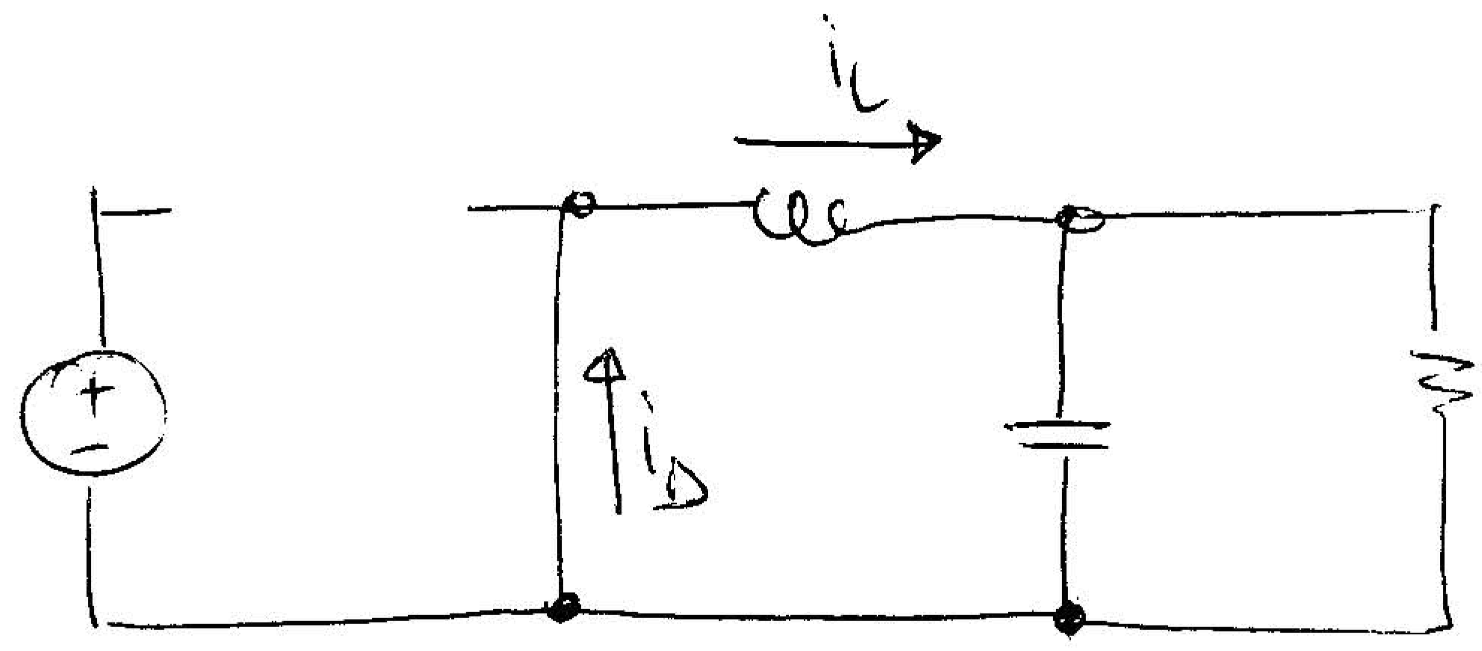


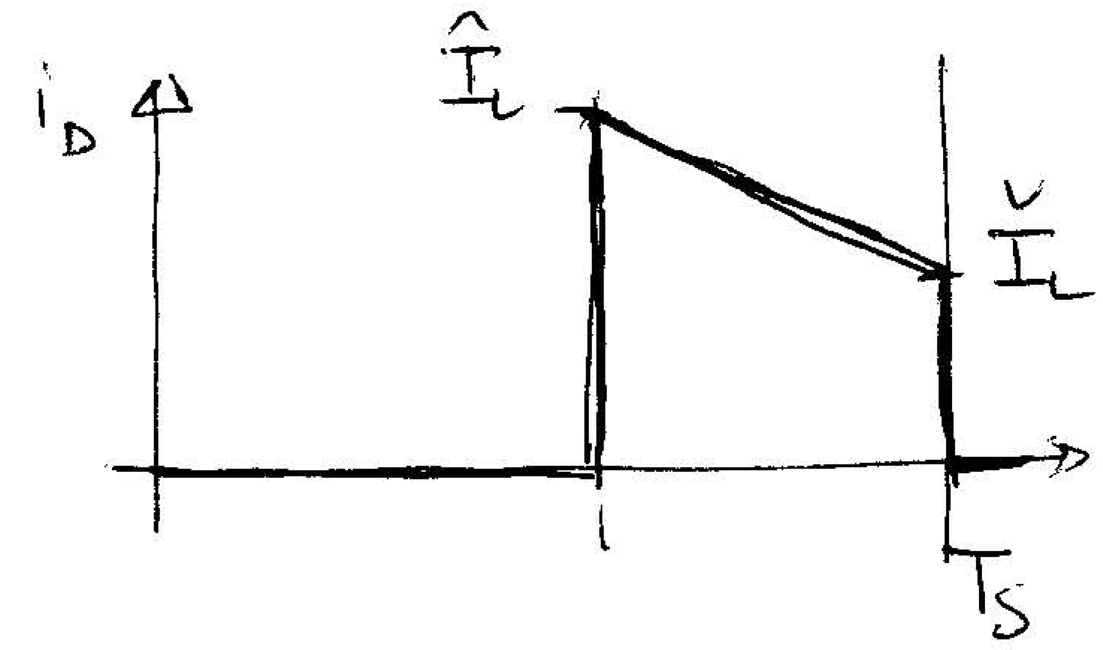
$v_i = V_i$

$v_D = -V_i$

$v_D < 0 \rightarrow$ OK

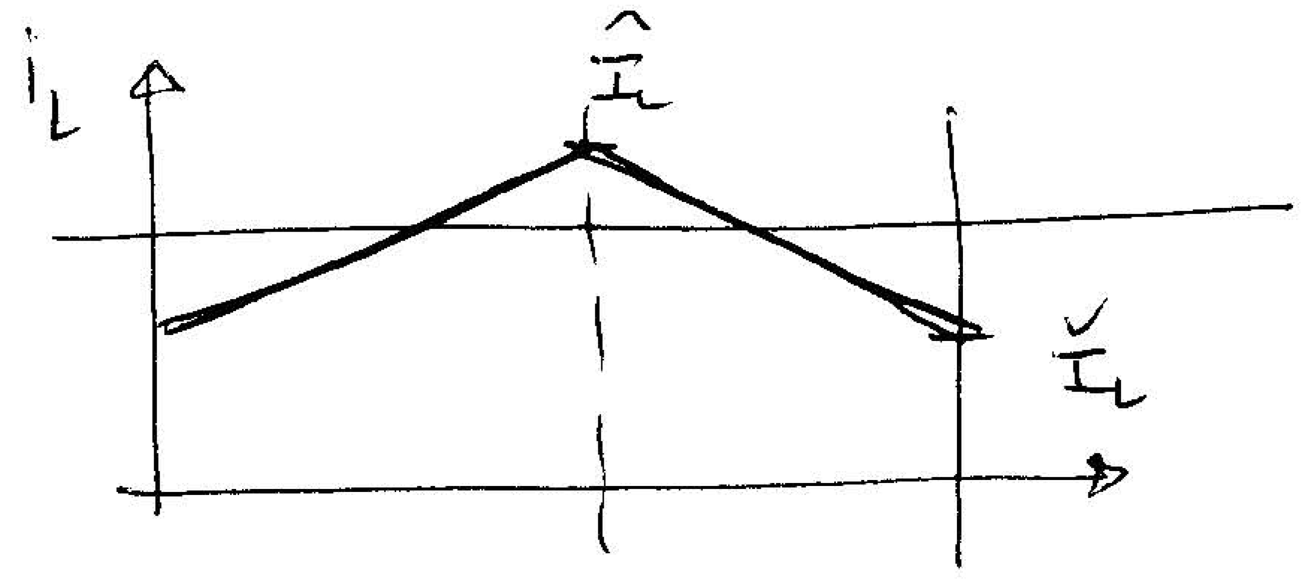
Soff/Don





$i_D > 0 \Leftrightarrow \check{I}_L > 0$

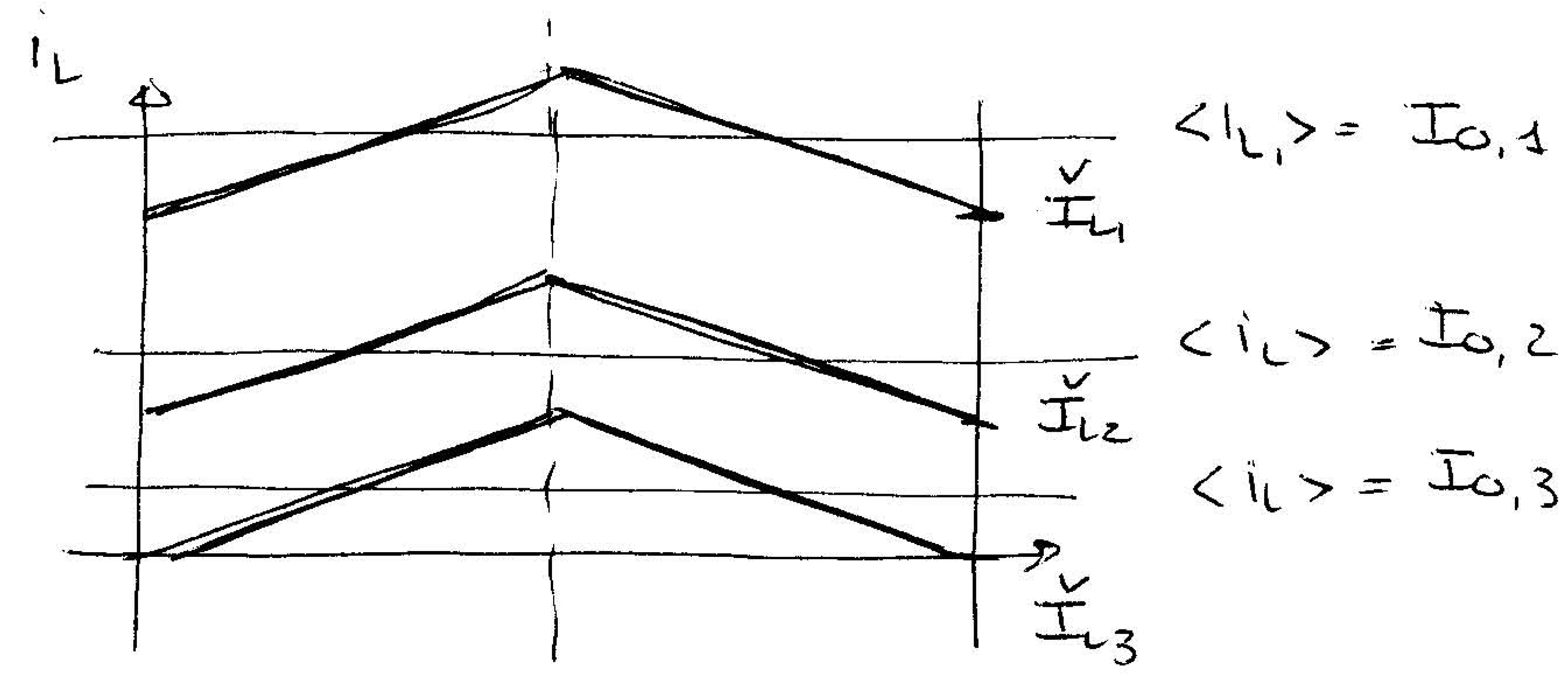
$\check{I}_L > 0$ not always



$\langle i_L \rangle = I_o$

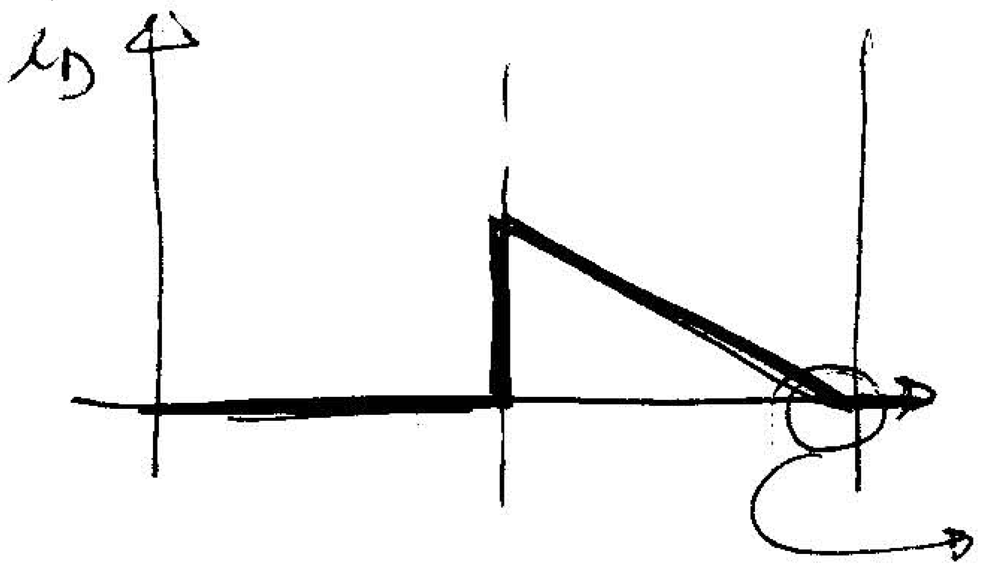
$$\check{I}_L = I_o - \frac{\Delta i_L}{2}$$ (I_o : output current)

$$I_o \downarrow \longrightarrow \check{I}_L \downarrow$$



$I_{o3} < I_{o2} < I_{o1}$

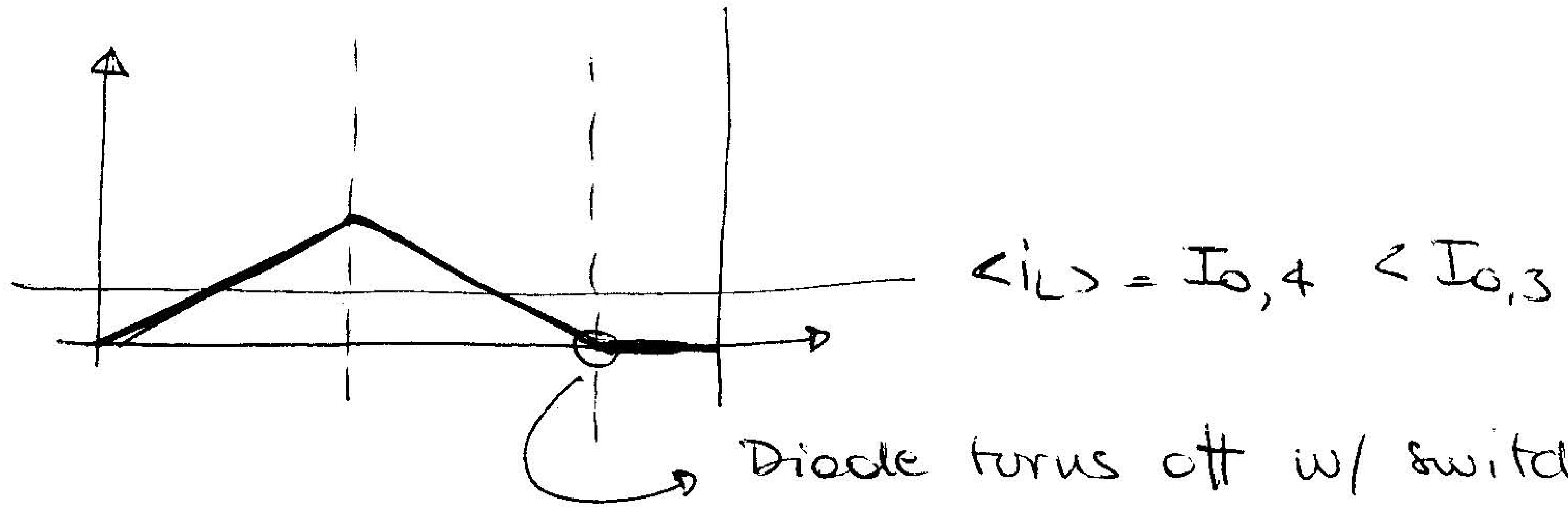
$I_{o3} \Rightarrow \check{I}_{L3} = 0$



$\boxed{i_D = 0}$ Diode turns off (in the period when Soff)

BUT in this case limit, switch turns on immediately after

BUT if $I_o \downarrow$ even more :



Diode turns off w/ switch off

Doff	Don	Doff
Son	Soff	Soff

case D

$i_D = 0 \quad i_S = 0 \quad i_L = i_S + i_D = 0$

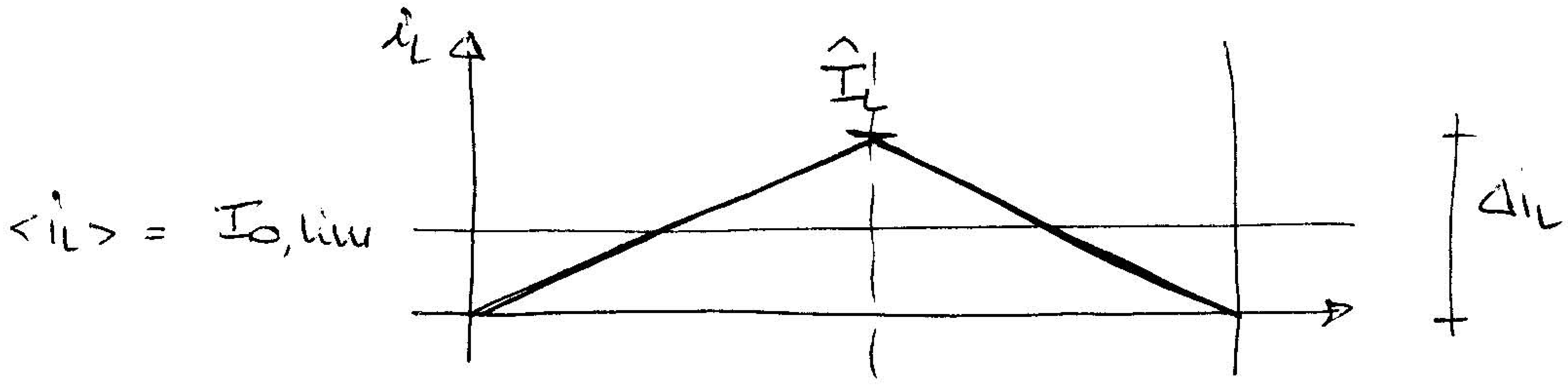
the current in L goes (and stays) zero

⟶ "Discontinuous conduction mode" (DCM)

[versus the previous on "Continuous cond. mode" (CCM)]

The case $I_o = I_{o,3}$ is the "border" between CCM and DCM.

I_{o3} is called $I_{o,lim}$ (limit current).

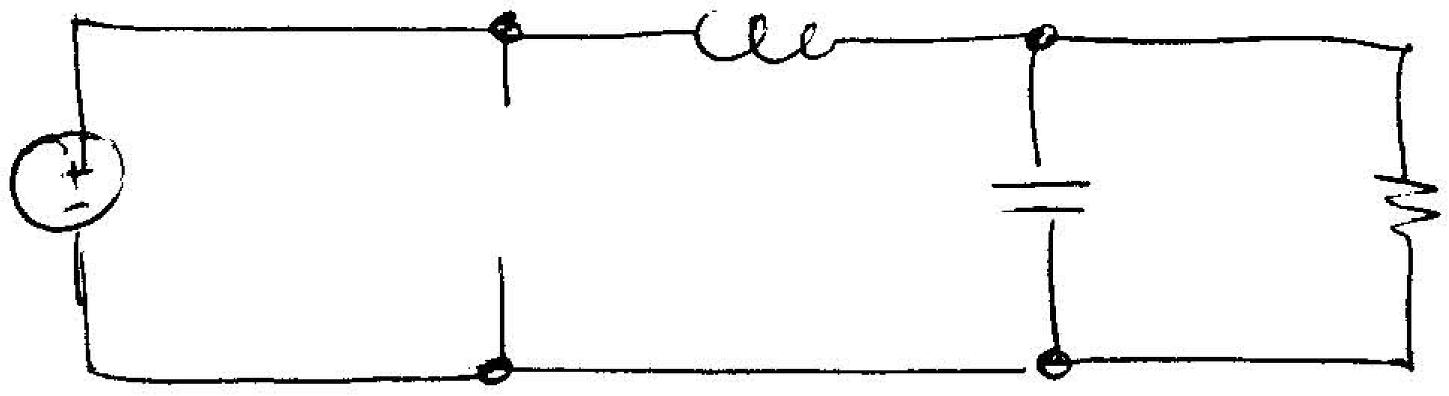


$I_o < I_{o,lim} \longrightarrow$ DCM (to study)

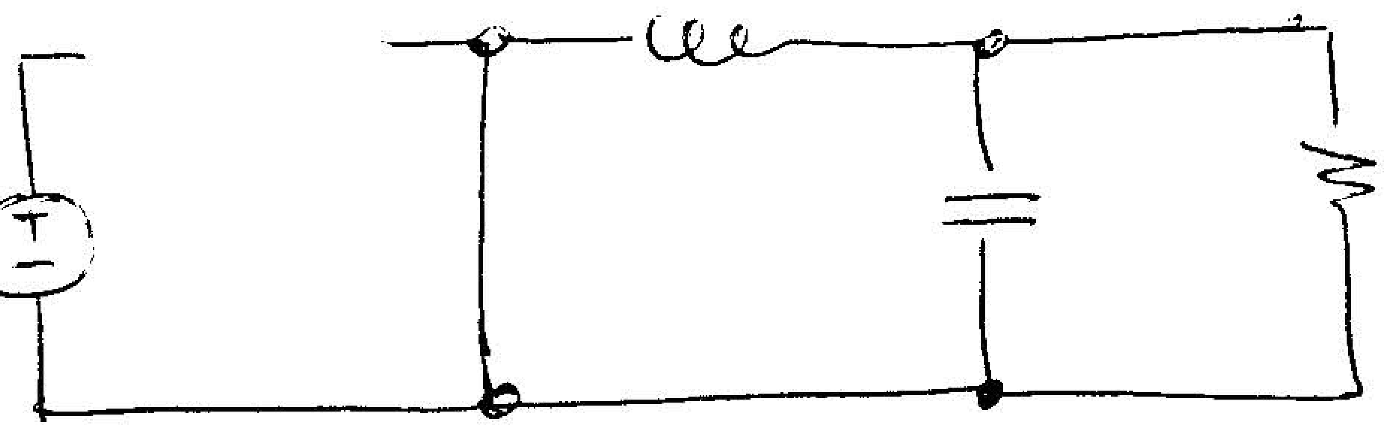
$I_o > I_{o,lim} \longrightarrow$ CCM (prev. studied)

2) Studying the three possible circuits

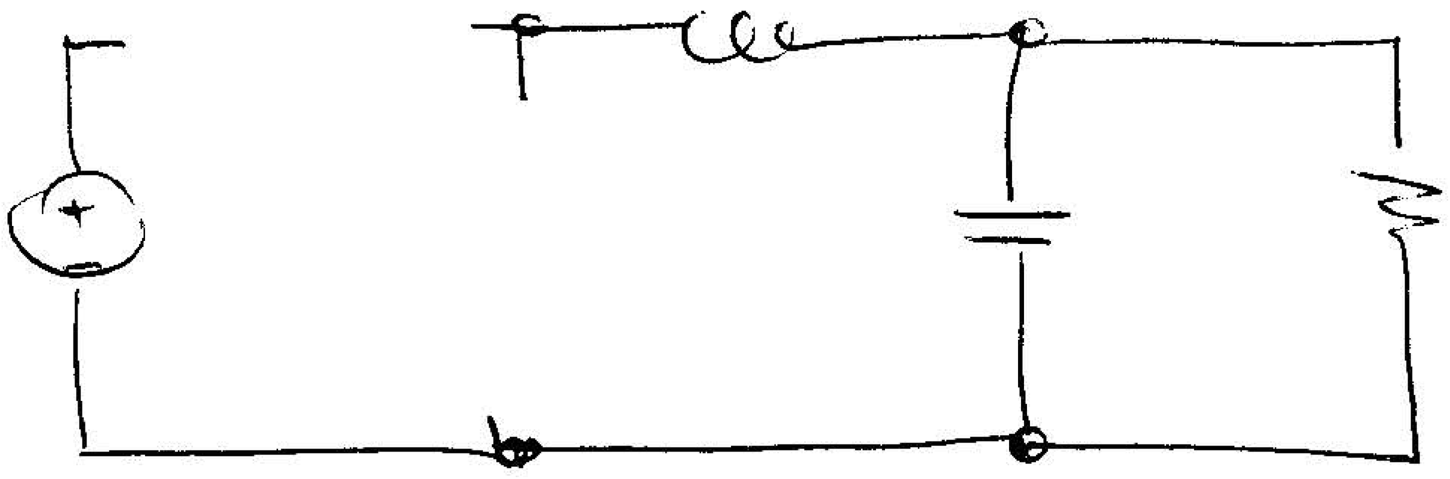
(A)

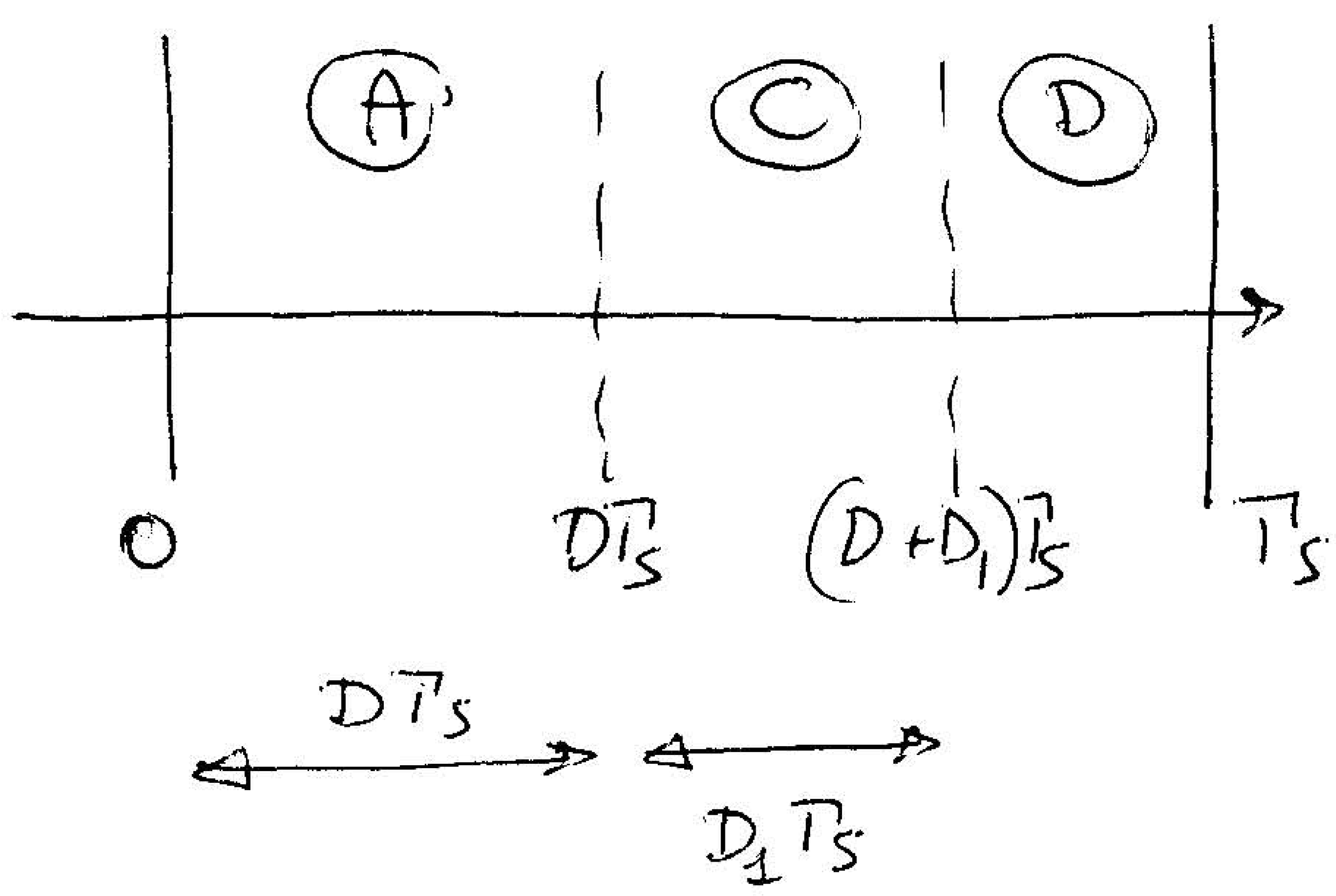


(C)

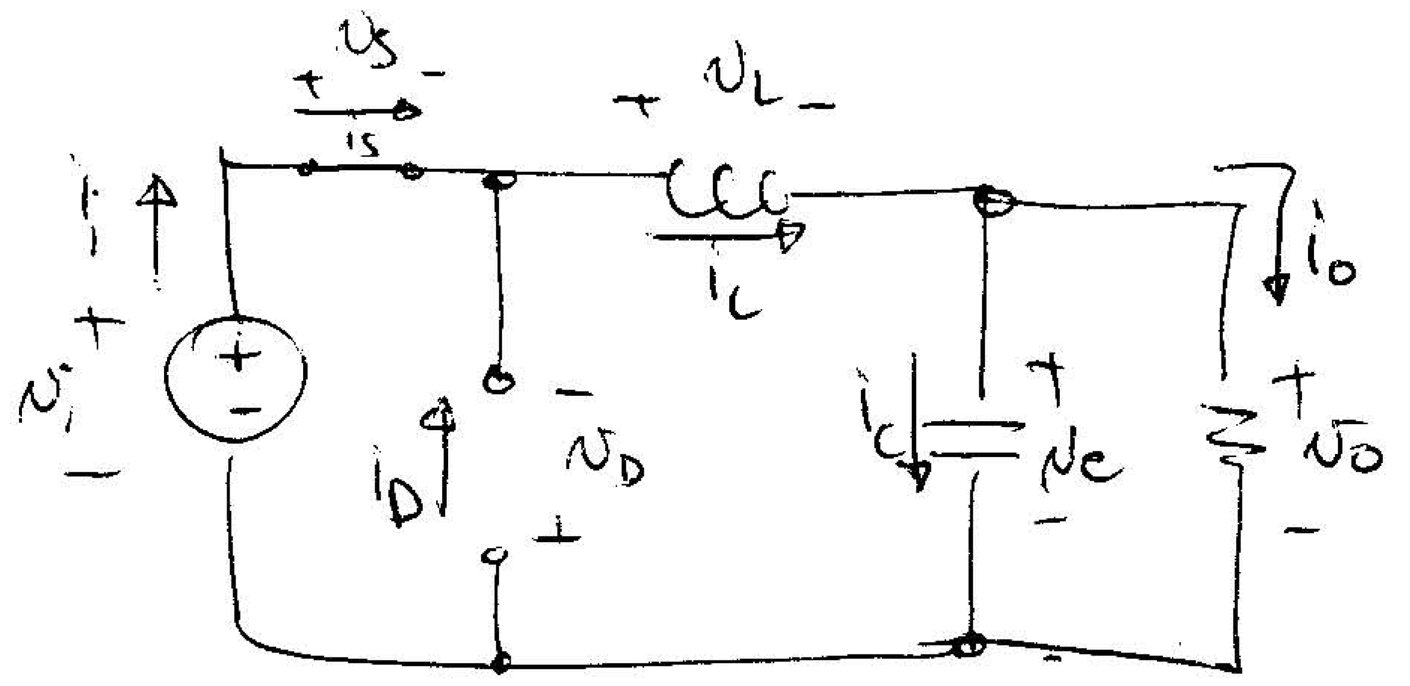


(D)





A



Voltages

$v_i = V_i$

$v_o \simeq V_o$

$v_L = V_i - V_o$

$v_D = -V_i$

$v_S = 0$

$v_C \simeq V_o$

Currents

$\check{I}_L = 0$

$i_L = \frac{V_i - V_o}{L} t + \check{I}_L$

$= \frac{V_i - V_o}{L} t$

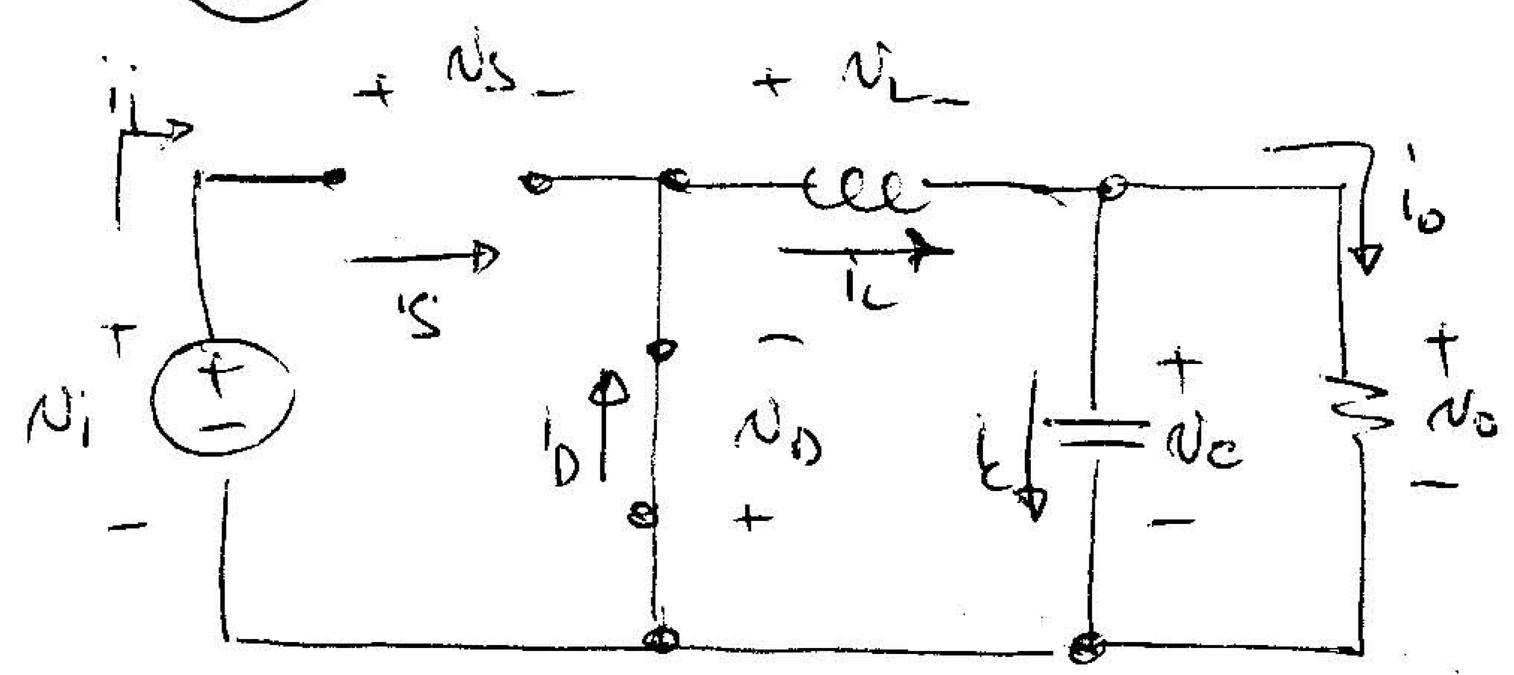
$i_i = i_S = i_L$

$i_D = 0$

$i_o = I_o = \frac{V_o}{R}$

$i_C = i_L - I_o$

C



$v_i = V_i$

$v_o \simeq V_o$

$v_L = -V_o$

$v_D = 0$

$v_S = V_i$

$v_C \simeq V_o$

$i_L = -\frac{V_o}{L} t + \hat{I}_L$

$i_i = 0$

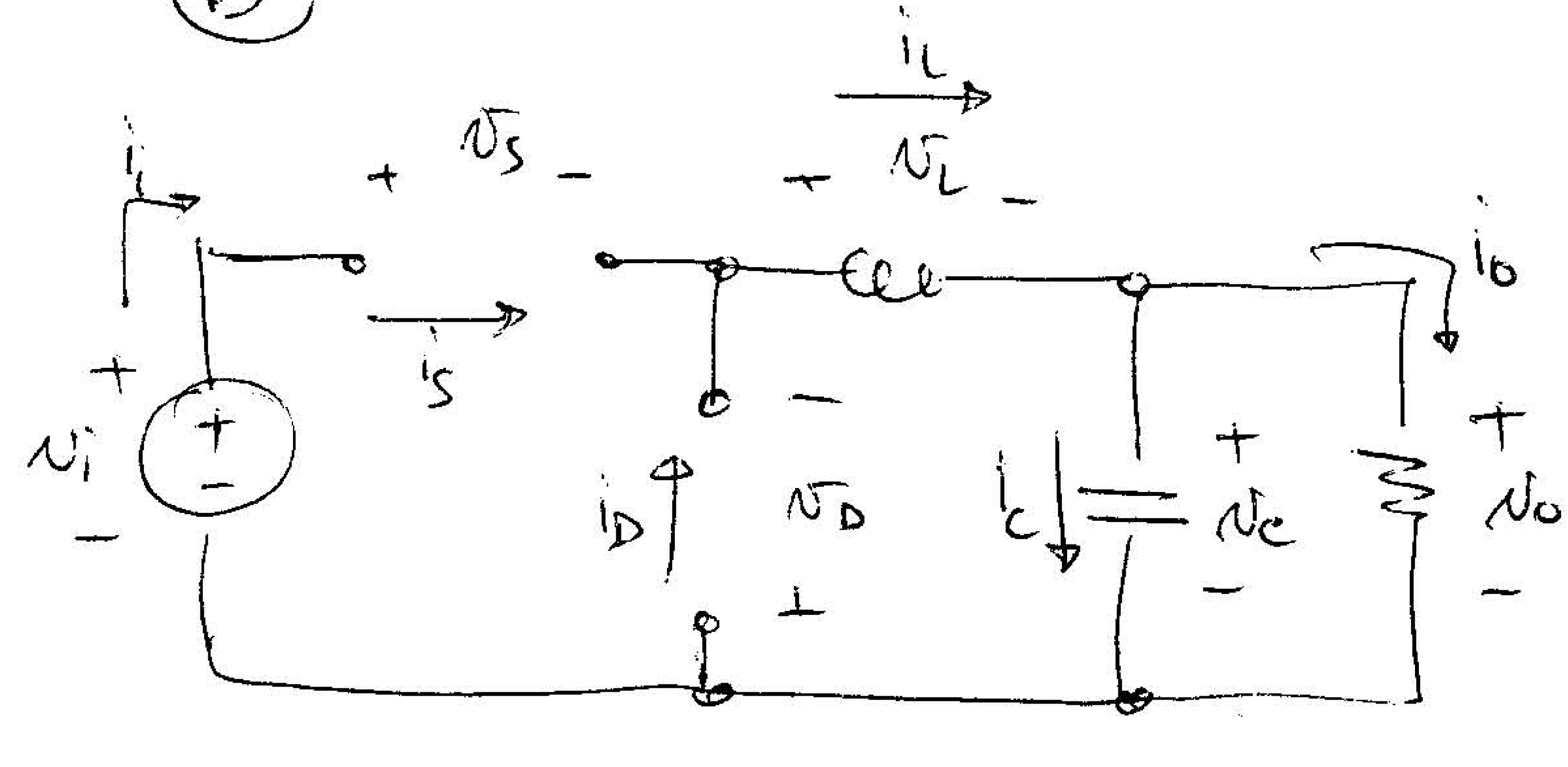
$i_S = 0$

$i_D = i_L$

$i_o = I_o = \frac{V_o}{R}$

$i_C = i_L - I_o$

D



$v_i = V_i$

$v_o \simeq V_o$

$i_L = 0$

$v_L = L\frac{di_L}{dt} = 0$

$v_D = -V_o$

$v_S = V_i - V_o$

$v_C = V_o$

$i_i = 0$

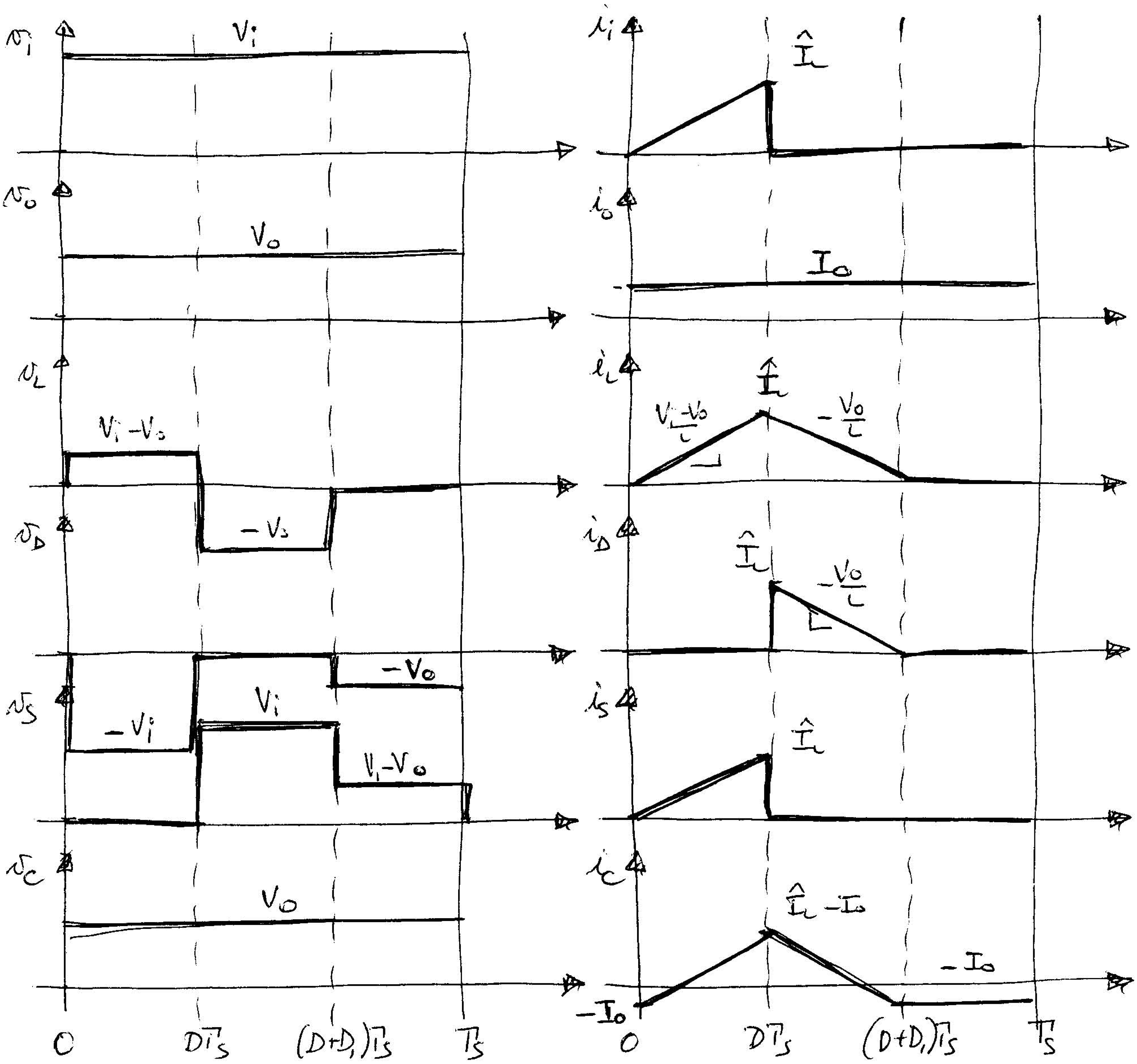
$i_S = 0$

$i_D = 0$

$i_o = I_o = \frac{V_o}{R}$

$i_C = -I_o$

3) Voltage and current diagrams



4) Steady state conditions

<u>inductor</u> $\quad \cancel{i_L(T_S)} = \frac{1}{L}\int_0^{T_S} v_L(t)\,dt + \cancel{i_L(0)}$

$$\Rightarrow \boxed{\int_0^{T_S} v_L\,dt = 0}$$ Still valid

$$\Rightarrow \langle v_L \rangle = \frac{1}{T_S}\int_0^{T_S} v_L\,dt = 0 \qquad \langle v_L \rangle = 0$$ still valid

$$\int_0^{T_S} v_L\,dt = 0$$

$$\int_0^{DT_S} v_L\,dt + \int_{DT_S}^{(D+D_1)T_S} v_L\,dt + \cancel{\int_{(D+D_1)T_S}^{T_S} v_L\,dt} = 0$$

$$\int_0^{DT_S} (V_i - V_o)\,dt = + \int_{DT_S}^{(D+D_1)T_S} (\cancel{+} V_o)\,dt$$

$$(V_i - V_o)\,D\cancel{T_S} = V_o\,(\cancel{D} + D_1 - \cancel{D})\,\cancel{T_S}$$

$$\boxed{(V_i - V_o)\,D = V_o\,D_1}$$

2 unknowns: D and D_1

<u>capacitor</u>

$$\cancel{v_C(T_S)} = \frac{1}{C}\int_0^{T_S} i_C\,dt + \cancel{v_C(0)}$$

$$\boxed{\int_0^{T_S} i_C\,dt = 0}$$

still valid

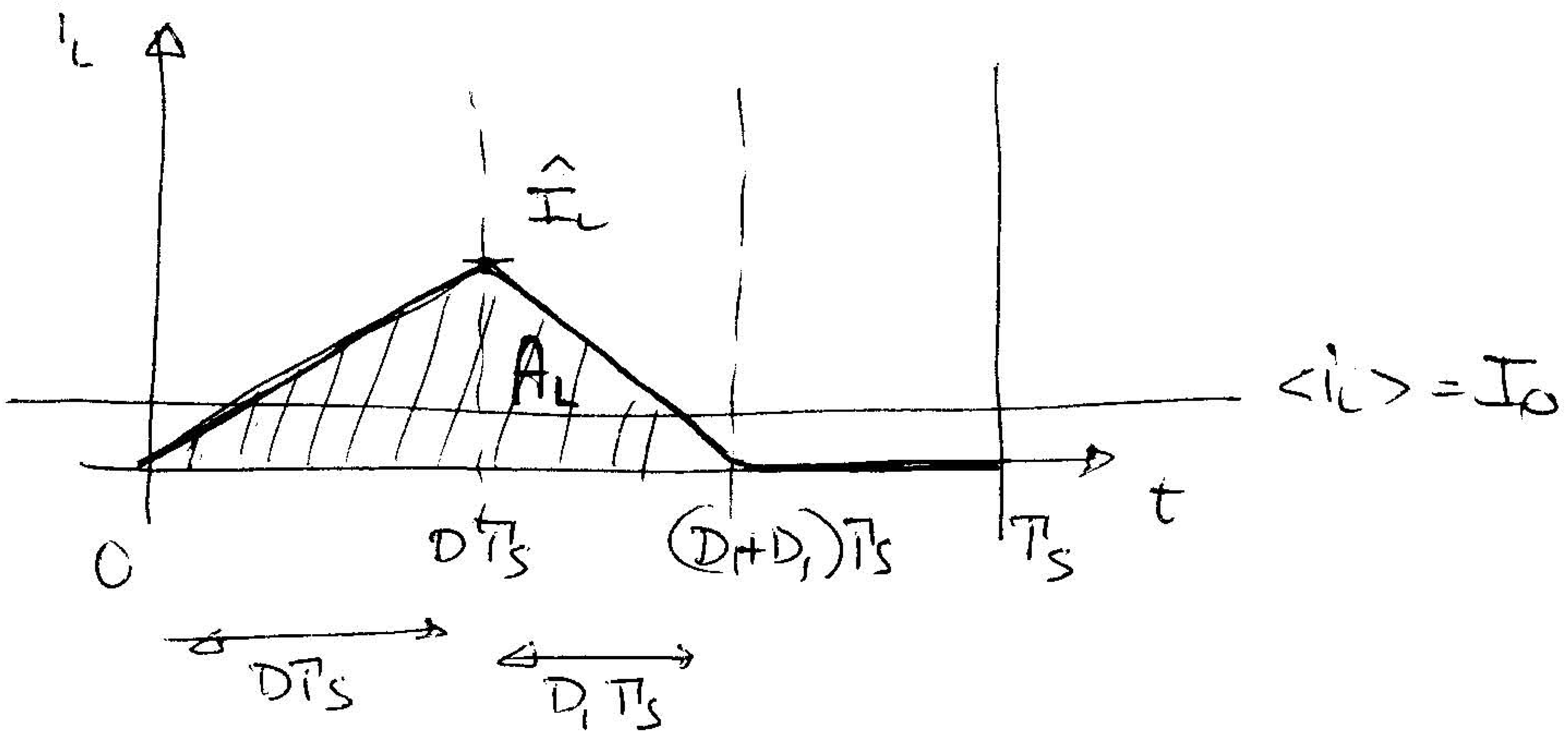
$$\Longrightarrow \quad \langle i_C \rangle = \frac{1}{T_S}\int_0^{T_S} i_C\,dt = 0 \qquad \boxed{\langle i_C \rangle = 0}$$

still valid

$$i_C = i_L - I_o$$

$$\cancel{\langle i_C \rangle} = \langle i_L \rangle - I_o \qquad \boxed{\langle i_L \rangle = I_o}$$

still valid



$$\langle i_L \rangle = \frac{1}{T_S} \int_0^{T_S} i_L \, dt = \frac{1}{T_S} A_L$$

$$A_L = \frac{1}{2} (D + D_1) T_S \cdot \hat{I}_L$$

$$\langle i_L \rangle = \frac{1}{\cancel{T_S}} \frac{1}{2} (D + D_1) \cancel{T_S} \cdot \hat{I}_L = \frac{D + D_1}{2} \hat{I}_L$$

$$\langle i_L \rangle = I_o \implies \frac{D + D_1}{2} \hat{I}_L = I_o$$

$$\hat{I}_L = \frac{V_i - V_o}{L} D T_S$$

$$\boxed{\frac{D + D_1}{2} \frac{V_i - V_o}{L} \cdot D T_S = I_o}$$ 2 unknowns: D & D_1

---

5) <u>Conversion ratio</u> $M = \frac{V_o}{V_i}$

$$\begin{cases} (V_i - V_o) D = V_o D_1 \\ \frac{D + D_1}{2} \frac{V_1 - V_o}{L} \cdot D T_S = I_o \end{cases}$$

$$(V_i - V_o) D = V_o D_1 \qquad \frac{V_i - V_o}{V_o} \cdot D = D_1 \qquad D_1 = D\left(\frac{V_i}{V_o} - 1\right)$$

$$\frac{D + D\left(\frac{V_i}{V_o} - 1\right)}{2} \cdot \frac{V_i - V_o}{L} \cdot D T_S = I_o$$

$$\frac{D\left(\cancel{1} + \frac{V_i}{V_o} - \cancel{1}\right)}{2} \frac{V_i - V_o}{L} \cdot D T_S = I_o$$

$$\frac{D}{2} \cdot \frac{V_i}{V_o} \cdot (V_i - V_o) \cdot \frac{D T_S}{L} = I_o$$

$$\frac{V_i - V_o}{V_o} \cdot \frac{V_i T_S}{2L} \cdot D^2 = I_o$$

$$\left(\frac{V_i}{V_o} - 1\right) \underbrace{\frac{V_i T_S}{2L}}_{I_b} D^2 = I_o$$

$$\left(\frac{V_i}{V_o} - 1\right) I_b\, D^2 = I_o$$

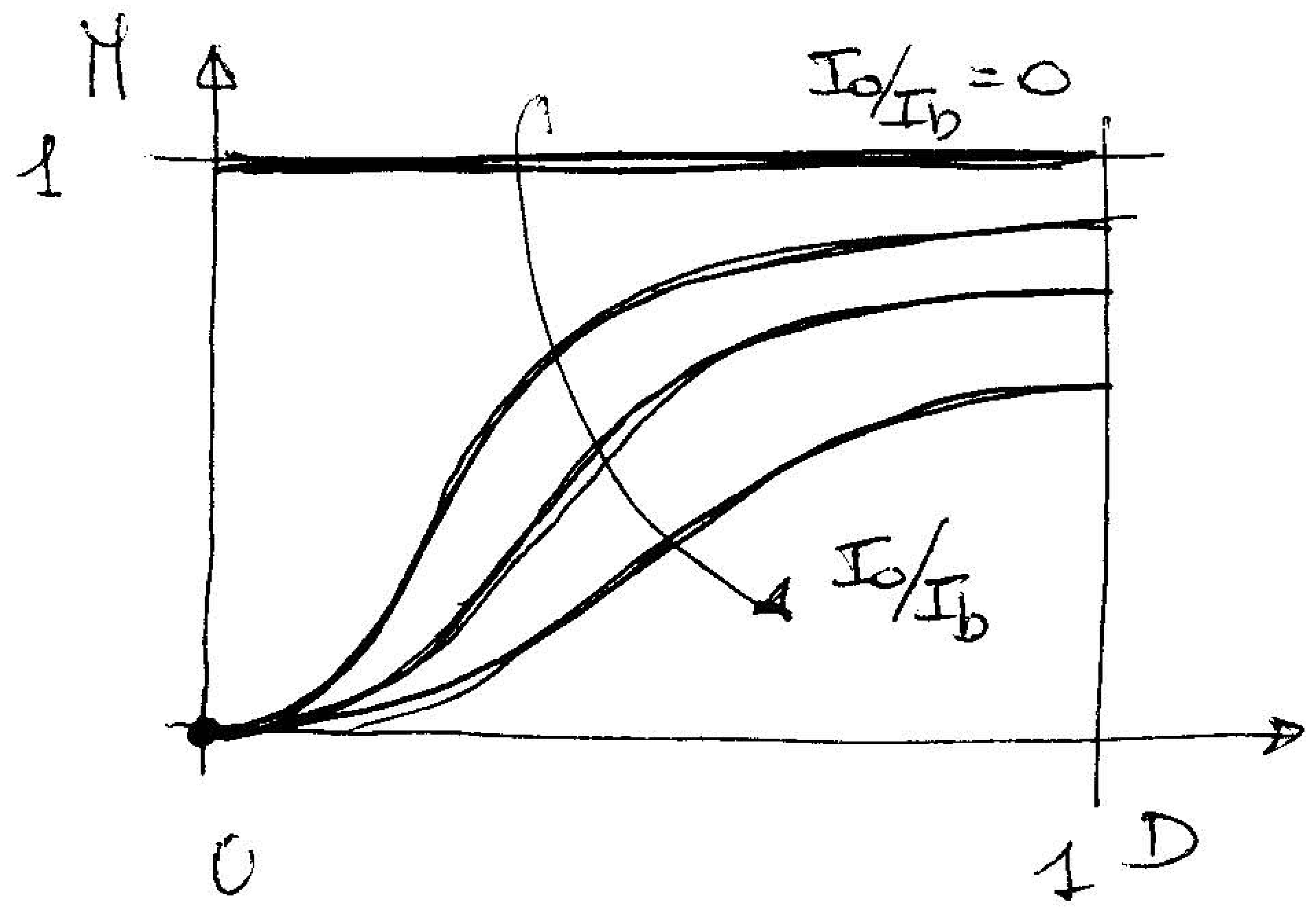
$$\left(\frac{1}{M} - 1\right) I_b \cdot D^2 = I_o$$

$$\frac{1}{M} - 1 = \frac{I_o}{I_b} \frac{1}{D^2}$$

$$\frac{1}{M} = \frac{I_o}{I_b D^2} + 1 = \frac{I_o + I_b D^2}{I_b D^2}$$

$$M = \frac{I_b D^2}{I_o + I_b D^2} = \frac{D^2}{D^2 + \frac{I_o}{I_b}}$$

$$\boxed{M = \frac{D^2}{D^2 + \frac{I_o}{I_b}}}$$



$$(V_1 - V_o) D = V_o D_1$$

$$D_1 = \frac{V_1 - V_o}{V_o} D = \left(\frac{V_1}{V_o} - 1\right) D = \left(\frac{1}{M} - 1\right) D$$

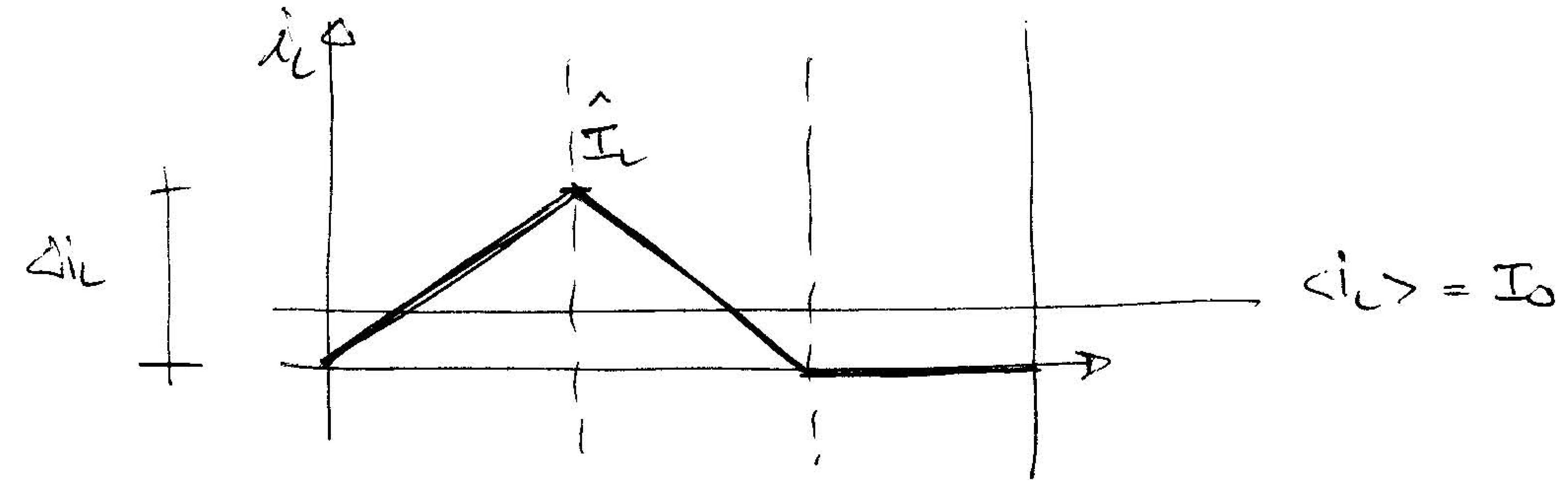
$$D_1 = \left(\frac{1}{M}-1\right)D = \left(\frac{1-M}{M}\right)D$$

$$= \left(\frac{1-\dfrac{D^2}{D^2+I_o/I_b}}{\dfrac{D^2}{D^2+I_o/I_b}}\right)\cdot D = \left(\frac{\cancel{D^2}+I_o/I_b-\cancel{D^2}}{D^{\cancel{2}}}\right)\cdot \cancel{D}$$

$$= \frac{I_o/I_b}{D}$$

$$\boxed{D_1 = \frac{1}{D}\,\frac{I_o}{I_b}}$$

6) Inductor current ripple and maximum current

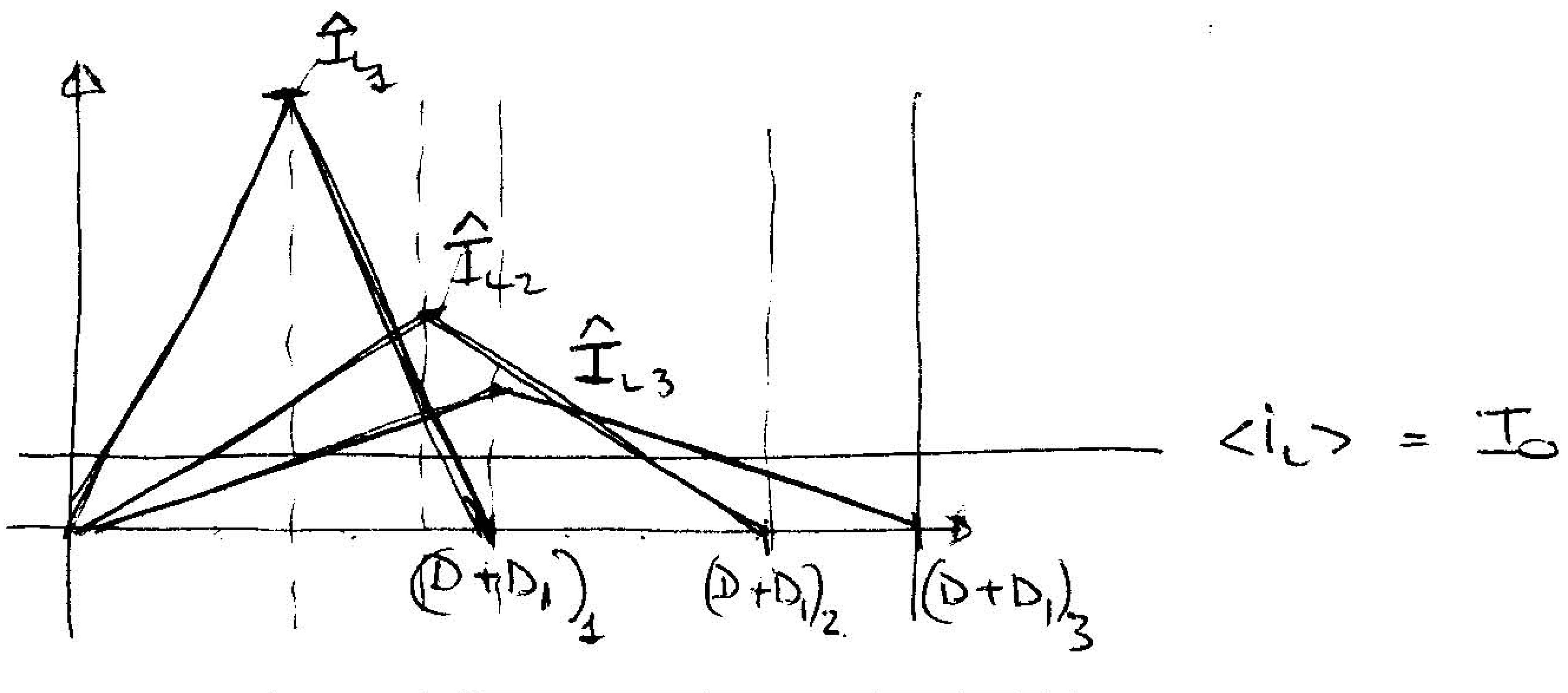


$$\langle i_L \rangle = \frac{1}{2}\,\hat{I}_L\,(D+D_1)\cancel{T_s}\,\frac{1}{\cancel{T_s}}$$

$$= \frac{\hat{I}_L}{2}(D+D_1)$$

$$\langle i_L \rangle = I_o \implies \frac{\hat{I}_L}{2}(D+D_1) = I_o$$

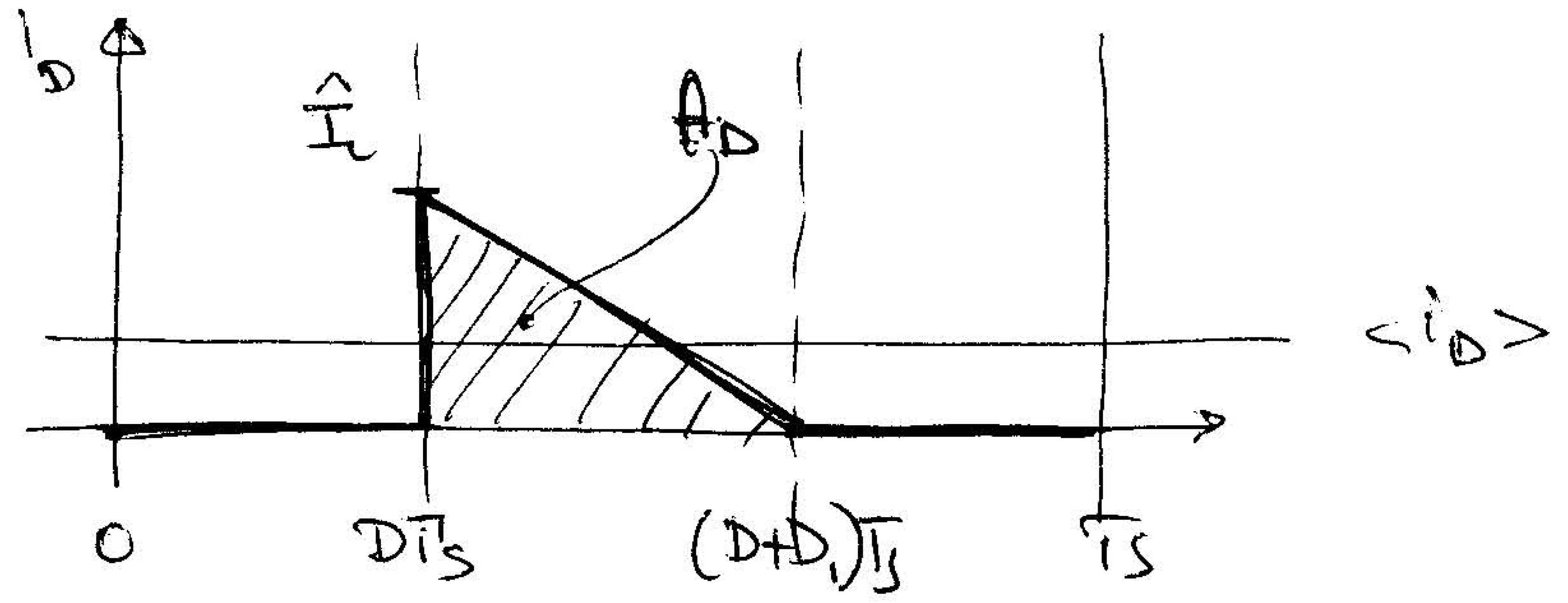
$$\boxed{\Delta i_L = \hat{I}_L = 2\,I_o\,\frac{1}{D+D_1}}$$



$$(D+D_1) \rightarrow 1 \qquad \hat{I}_L = \Delta i_L \downarrow$$

$$(D+D_1) \rightarrow 0 \qquad \hat{I}_L = \Delta i_L \uparrow$$

Average current in the diode

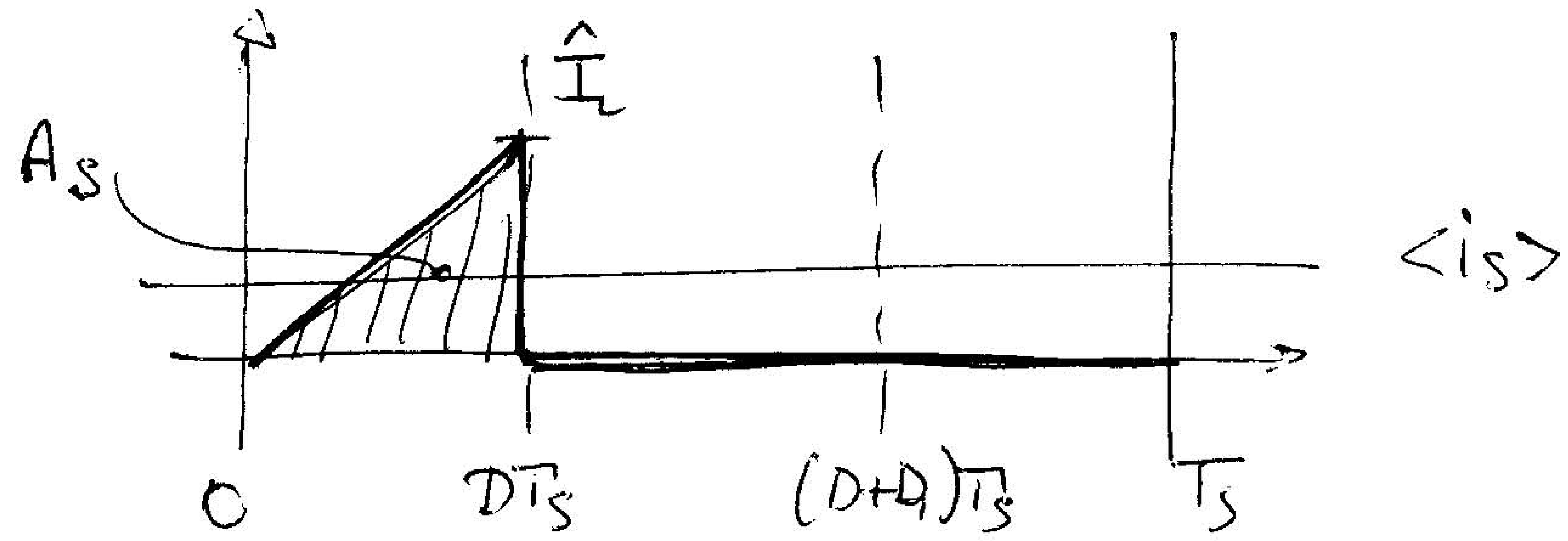


$$<i_D> = \frac{1}{T_S}\int_0^{T_S} i_D\,dt = \frac{1}{T_S} A_D = \frac{1}{\cancel{T_S}} \frac{1}{2} \cdot \hat{I}_L \cdot D_1 \cancel{T_S} = \frac{\hat{I}_L}{2} \cdot D_1$$

$$= 2 I_o \frac{1}{D+D_1} \frac{1}{2} \cdot D_1 = \frac{D_1}{D+D_1} I_o$$

$$\boxed{<i_D> = \frac{D_1}{D+D_1} I_o}$$

Average current in the switch



$$<i_S> = \frac{1}{T_S}\int_0^{T_S} i_S\,dt = \frac{1}{T_S} A_S = \frac{1}{\cancel{T_S}} \cdot \frac{1}{2} \hat{I}_L D \cancel{T_S} = \frac{\hat{I}_L}{2} \cdot D$$

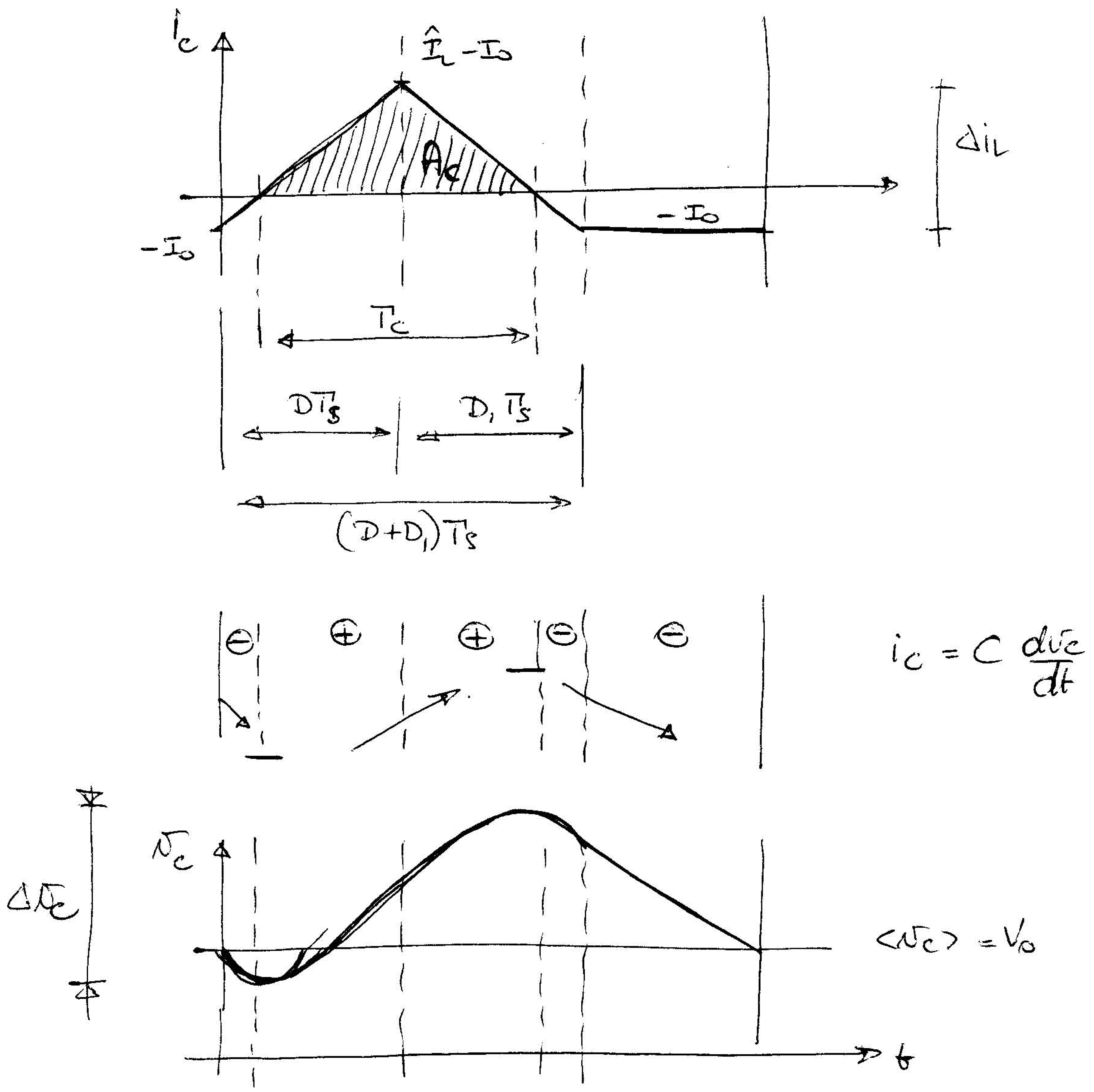
$$= 2 I_o \frac{1}{D+D_1} \frac{1}{2} \cdot D = \frac{D}{D+D_1} I_o$$

$$\boxed{<i_S> = <i_i> = \frac{D}{D+D_1} I_o}$$

8) Maximum voltage

$$|\hat{v}_S| = V_i \qquad |\hat{v}_D| = V_i$$

9) Output voltage ripple



$$i_c = C \frac{dv_c}{dt}$$

$$\Delta v_c = \frac{\Delta q_c}{C} = \frac{A_c}{C}$$

$$A_c = \frac{1}{2} T_c (\Delta i_L - I_o)$$

$$\frac{T_c}{(D+D_1)T_s} = \frac{\Delta i_L - I_o}{\Delta i_L} \longrightarrow \boxed{T_c = \frac{\Delta i_L - I_o}{\Delta i_L}(D+D_1)T_s}$$

$$(V_i - V_o) D = V_o D_1$$

$$D_1 = \frac{V_i - V_o}{V_o} D = \left(\frac{V_i}{V_o} - 1\right) D$$

$$D + D_1 = D + \left(\frac{V_i}{V_o} - 1\right) D = \frac{V_i}{V_o} D = \frac{1}{M} D$$

$$\boxed{D + D_1 = \frac{1}{M} D}$$

$$A_C = \frac{1}{2}\left[\frac{\Delta i_L - I_o}{\Delta i_L}(D + D_1) T_S\right](\Delta i_L - I_o)$$

$$= \frac{1}{2} \frac{(\Delta i_L - I_o)^2}{\Delta i_L} \cdot \frac{1}{M} D \cdot T_S$$

$$\Delta v_C = \frac{A_C}{C} = \frac{1}{2C} \frac{(\Delta i_L - I_o)^2}{\Delta i_L} \cdot \frac{1}{M} \cdot D \cdot T_S$$

$$\Delta i_L = \hat{I}_L = 2 I_o \frac{1}{D + D_1} = 2 I_o \frac{M}{D}$$

$$\Delta v_C = \frac{1}{2C} \frac{\left(2\frac{M}{D} I_o - I_o\right)^2}{2\frac{M}{D} I_o} \cdot \frac{1}{M} D \, T_S$$

$$= \frac{1}{2C} \frac{\left(2\frac{M}{D} - 1\right)^2 I_o^{\cancel{2}}}{2\left(\frac{M}{D}\right)^2 \cancel{I_o}} T_S$$

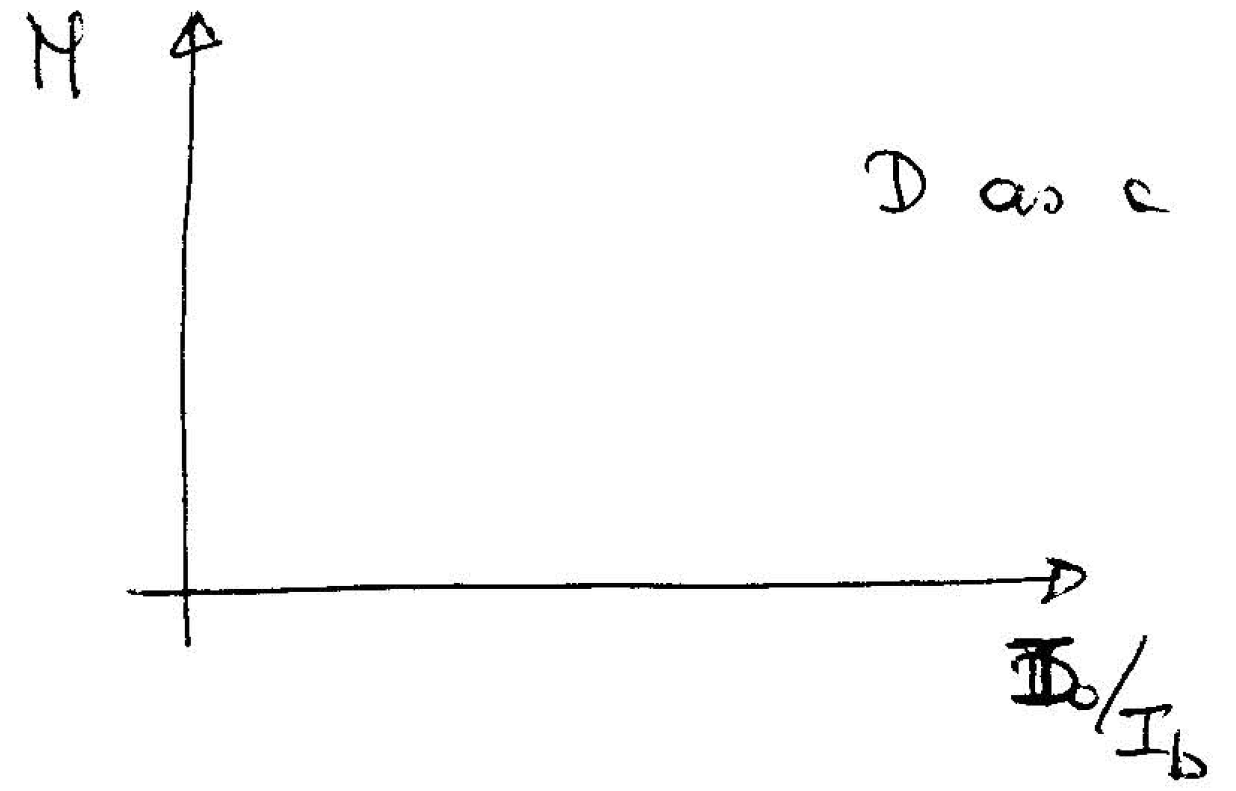
$$= \frac{1}{C} \frac{\left(2\frac{M}{D} - 1\right)^2 I_o}{\left(2\frac{M}{D}\right)^2} T_S = \frac{1}{C}\left(\frac{2\frac{M}{D} - 1}{2\frac{M}{D}}\right)^2 I_o T_S$$

$$= \frac{1}{C}\left(1 - \frac{D}{2M}\right)^2 I_o T_S$$

$$\boxed{\Delta v_C = \frac{T_S \cdot I_o}{C}\left(1 - \frac{D}{2M}\right)^2}$$

DCM AND CCM TOGETHER

1) Output characteristic



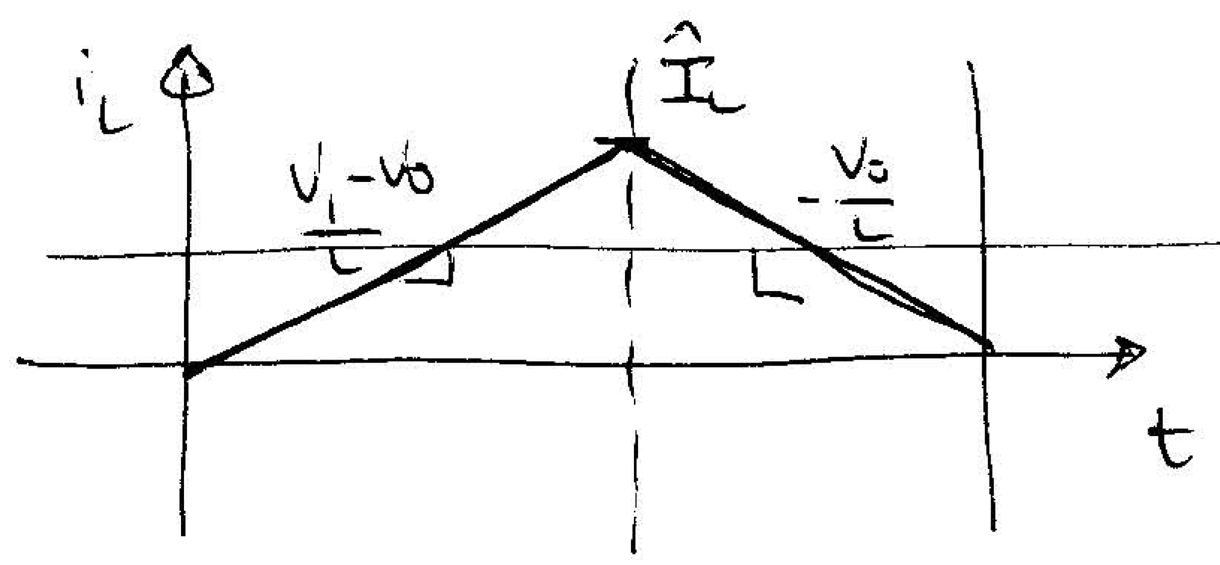
D as a parameter

1. CCM | if $I_o > I_{olim} \Rightarrow$ CCM $\qquad M = D \quad \forall \, I_o/I_b$

2. DCM | if $I_o < I_{olim} \Rightarrow$ DCM $\qquad M = \dfrac{D^2}{D^2 + I_o/I_b}$

Border between CCM and DCM |

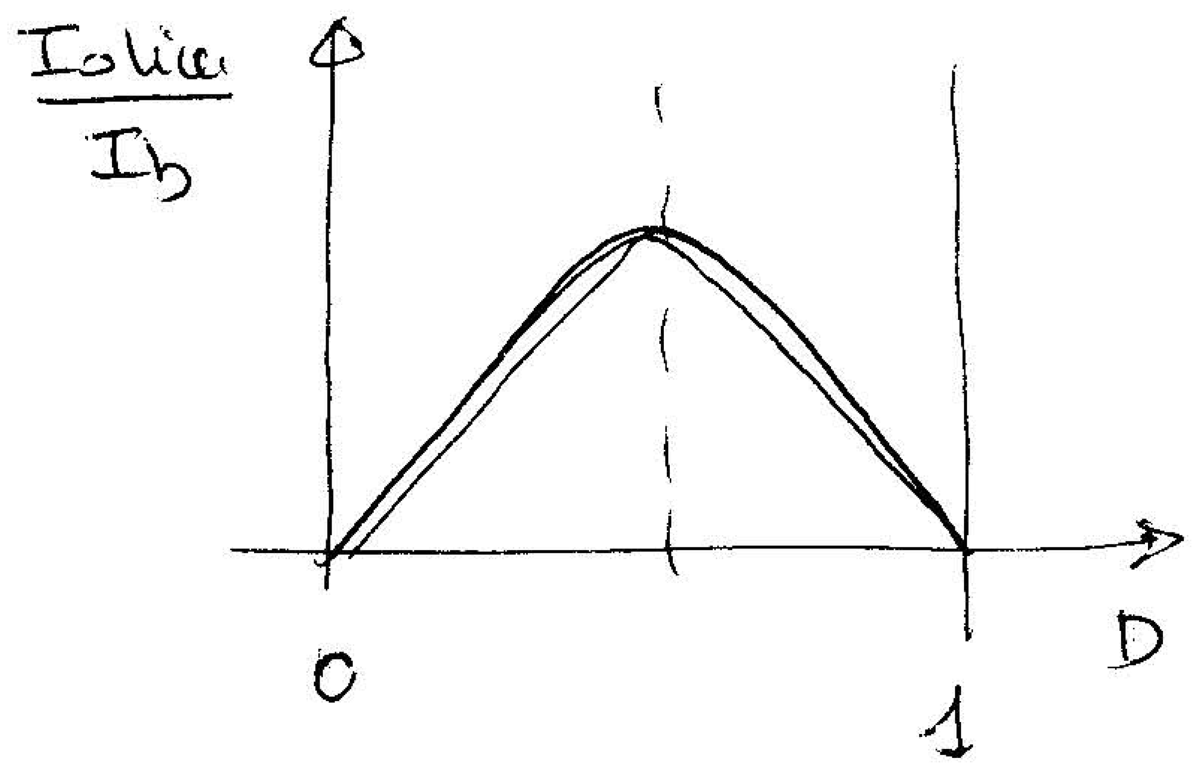
$$I_o = I_{olim}$$



$\langle i_L \rangle = I_{olim}$

$$I_{olim} = \frac{\Delta i_L}{2} = \frac{\hat{I}_L}{2} = \frac{V_i - V_o}{2L} \cdot DT_s = \frac{V_i(1-D)}{2L} \cdot DT_s$$

$$= \underbrace{\frac{V_i T_s}{2L}}_{I_b}(1-D)D = I_b(1-D)D$$



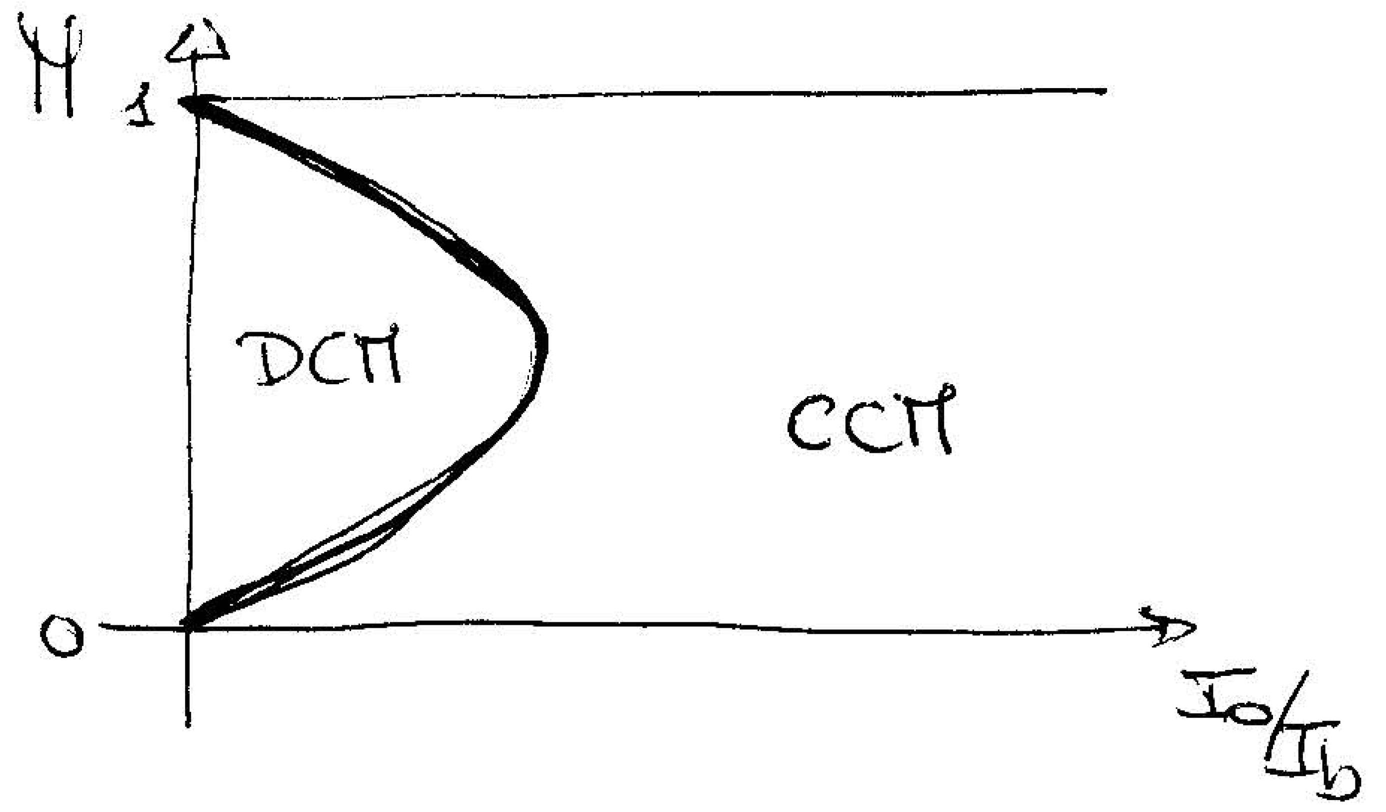
$$I_o = I_{o\,lim}$$

$$I_o = I_b (1-D) D$$

$$\frac{I_o}{I_b} = (1-D) D$$

At the border $M = D$ is still valid:

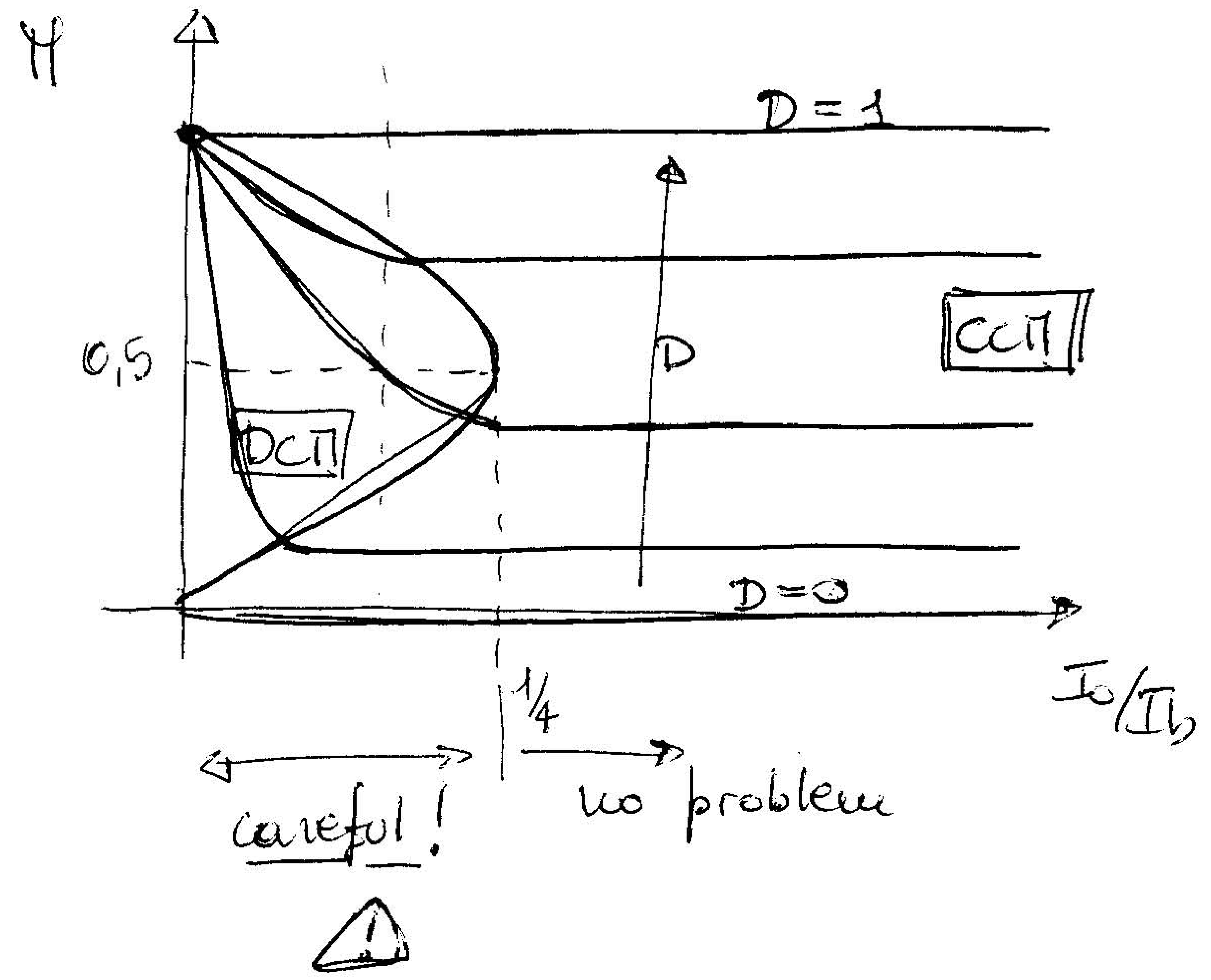
$$\hookrightarrow \quad \frac{I_o}{I_b} = (1-M) M$$



$$CCM \iff I_o > I_{o\,lim} \iff I_o > I_b (1-D) D$$

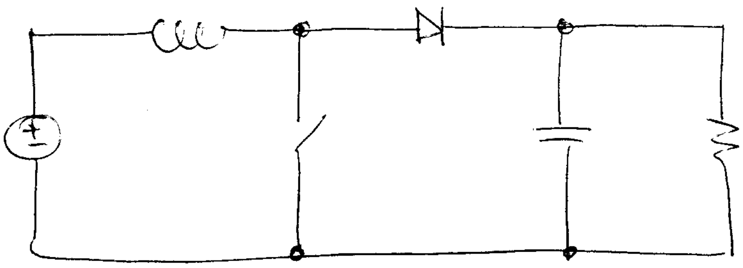
$$I_o > I_b (1-M) M$$

$$\frac{I_o}{I_b} > (1-M) M$$



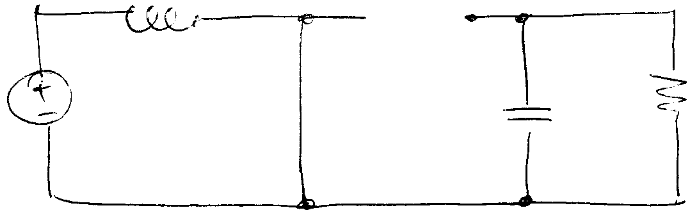
Boost DCM

1) General circuit

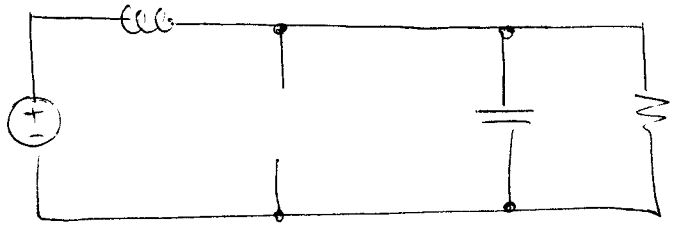


2) Study of the three possible circuits

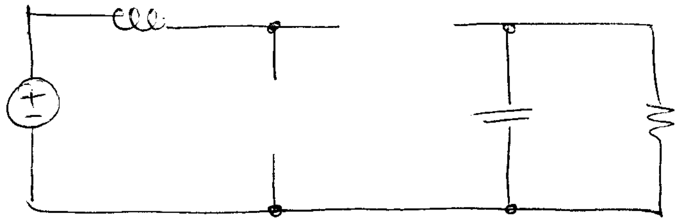
A

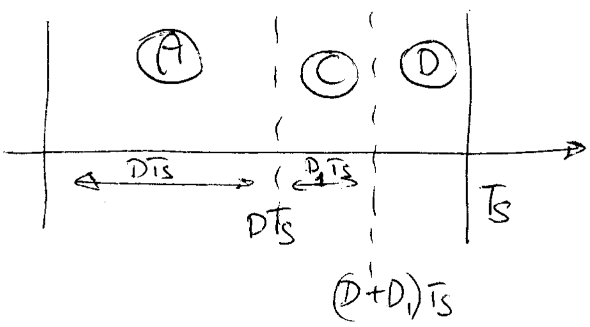


C

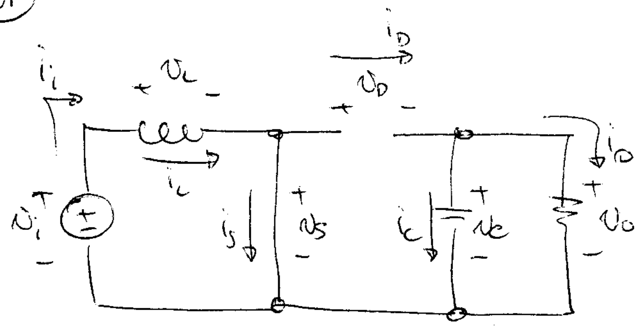


D



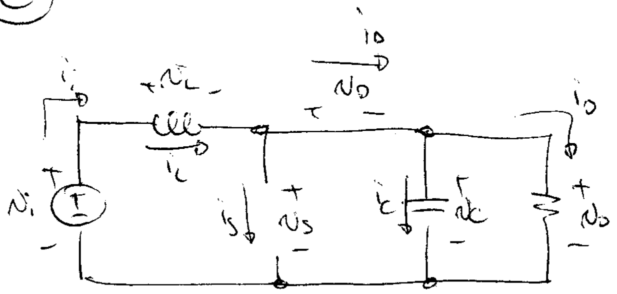


A



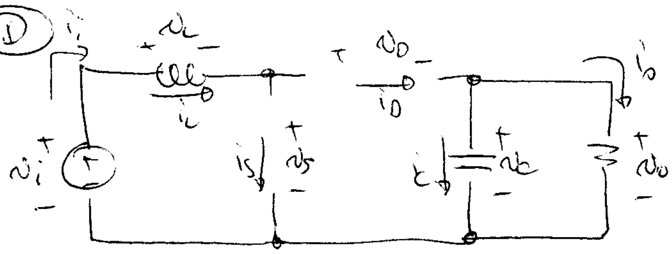
Voltages	Currents
$v_i = V_i$	$\check{I}_L = 0$
$v_o \simeq V_o$	$i_L = \frac{V_i}{L} t + \check{I}_L = \frac{V_i}{L} t$
$v_L = V_i$	
$v_D = -V_o$	$i_i = i_L = i_S$
$v_S = 0$	$i_D = 0$
$v_C = V_o$	$i_o = I_o = \frac{V_o}{R}$
	$i_C = -I_o$

C



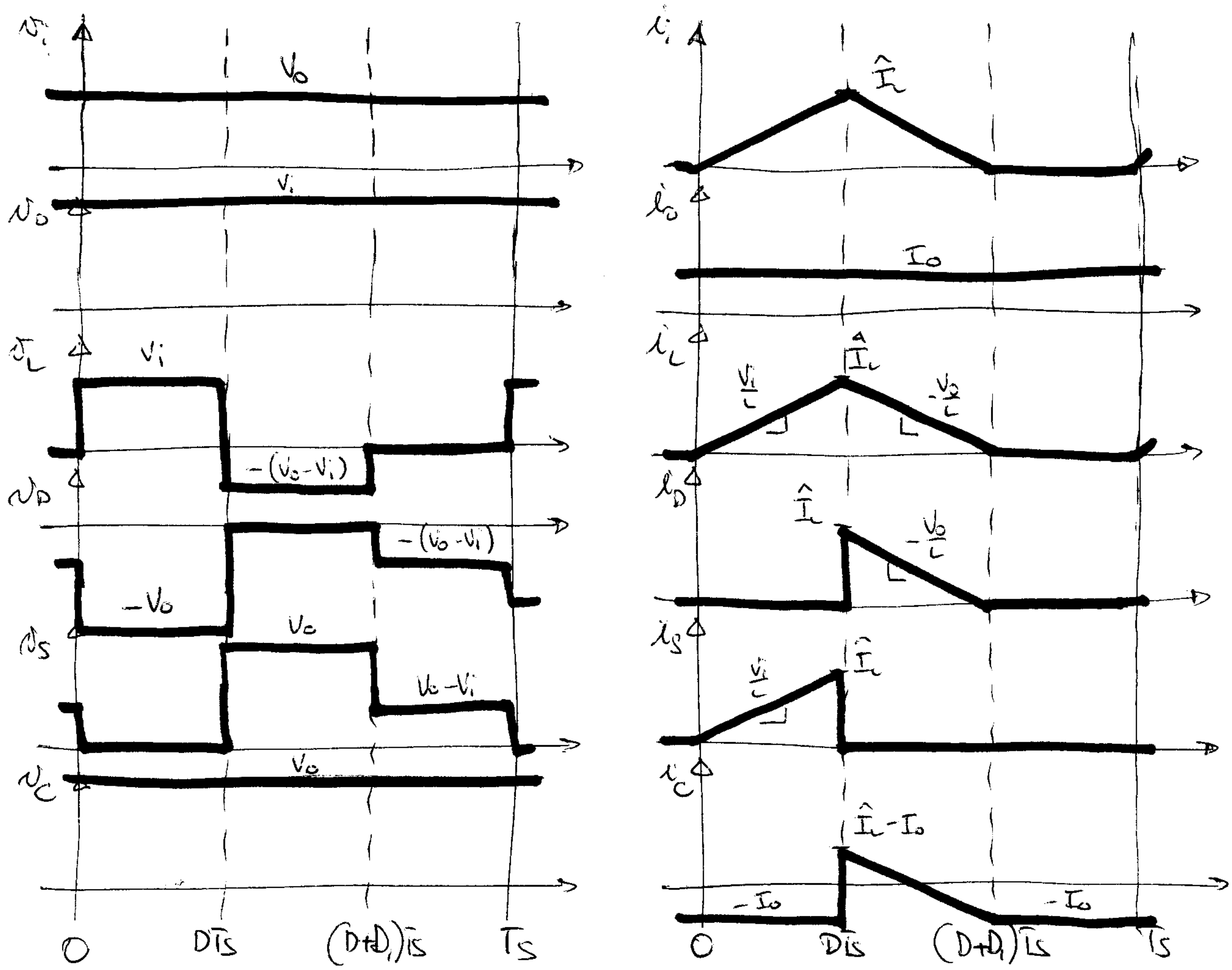
$v_i = V_i$	$i_L = -\frac{V_o - V_i}{L} t + \hat{I}_L$
$v_o \simeq V_o$	$i_i = i_L$
$v_L = -(V_o - V_i)$	$i_S = 0$
$v_D = 0$	$i_D = i_L$
$v_S = V_o$	$i_o = I_o = \frac{V_o}{R}$
$v_C = V_o$	$i_C = i_L - I_o$

D



$v_i = V_i$	
$v_o \simeq V_o$	$i_L = 0$
$v_L = L\frac{di}{dt} = 0$	$i_i = 0$
$v_D = -(V_o - V_i)$	$i_S = 0$
$v_S = V_i$	$i_D = 0$
$v_C = V_o$	$i_o = I_o = \frac{V_o}{R}$
	$i_C = -I_o$

3) Voltage and current diagrams



4) Steady state conditions

inductor
$$i_L(\cancel{T_S}) = \frac{1}{L}\int_0^{T_S} v_L(t)\,dt + \cancel{i_L(0)}$$

$$\Rightarrow \boxed{\int_0^{T_S} v_L\,dt = 0} \quad \text{still valid}$$

$$\Rightarrow \langle v_L\rangle = \frac{1}{T_S}\int_0^{T_S} v_L\,dt = 0 \qquad \langle v_L\rangle = 0 \quad \text{still valid}$$

$$\int_0^T v_L\,dt = 0$$

$$\int_0^{DT_S} v_L\,dt + \int_{DT_S}^{(D+D_1)T_S} v_L\,dt + \cancel{\int_{(D+D_1)T_S}^{T_S} v_L\,dt} = 0$$

$$\int_0^{DT_S} V_i\,dt = + \int_{DT_S}^{(D+D_1)T_S} + (V_o - V_i)\,dt$$

$$V_i\,D\,T_{\cancel{S}} = (V_o - V_i)(\cancel{D} + D_1 - \cancel{D})\,\cancel{T_S}$$

$$\boxed{V_i D = (V_o - V_i)\,D_1}$$ 2 unknowns. D and D_1

---

capacitor

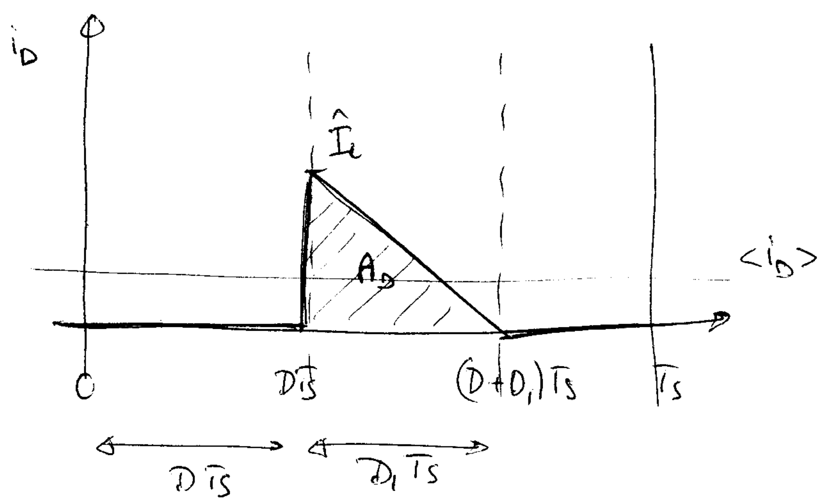
$$\cancel{v_C(T_S)} = \frac{1}{C}\int_0^{T_S} i_C\,dt + \cancel{v_C(0)}$$

$$\Rightarrow \boxed{\int_0^{T_S} i_C\,dt = 0}$$ still valid

$$\Rightarrow \langle i_C\rangle = \frac{1}{T_S}\int_0^{T_S} i_C\,dt = 0$$ $\boxed{\langle i_C\rangle = 0}$ still valid

$$i_D = i_C + I_o$$

$$\langle i_D\rangle = \cancel{\langle i_C\rangle} + I_o$$ $\boxed{\langle i_D\rangle = I_o}$ still valid



$$\langle i_D \rangle = \frac{1}{T_S}\int_0^{T_S} i_D\,dt = \frac{1}{T_S} A_D$$

$$A_D = \frac{1}{2} D_1 T_S \hat{I}_L$$

$$\langle i_D \rangle = \frac{1}{\cancel{T_S}} \frac{1}{2} D_1 \cancel{T_S} \hat{I}_L = \frac{D_1}{2} \hat{I}_L$$

$$\langle i_D \rangle = I_o \implies \frac{D_1}{2} \hat{I}_L = I_o$$

$$\hat{I}_L = \frac{V_i}{L} \cdot D T_S$$

$$\boxed{\frac{D_1}{2} \frac{V_i}{L} D T_S = I_o}$$

2 unknowns: D & D_1

5) <u>Conversion ratio</u> $\quad \Pi = \frac{V_o}{V_i}$

$$\begin{cases} V_i D = (V_o - V_i) D_1 \\ \frac{D_1}{2} \frac{V_i}{L} D T_S = I_o \end{cases}$$

$$V_i D = (V_o - V_i) D_1$$

$$D_1 = \frac{V_i}{V_o - V_i} D$$

$$\frac{V_i}{V_o - V_i} \frac{D}{2} \frac{V_i}{L} D T_S = I_o$$

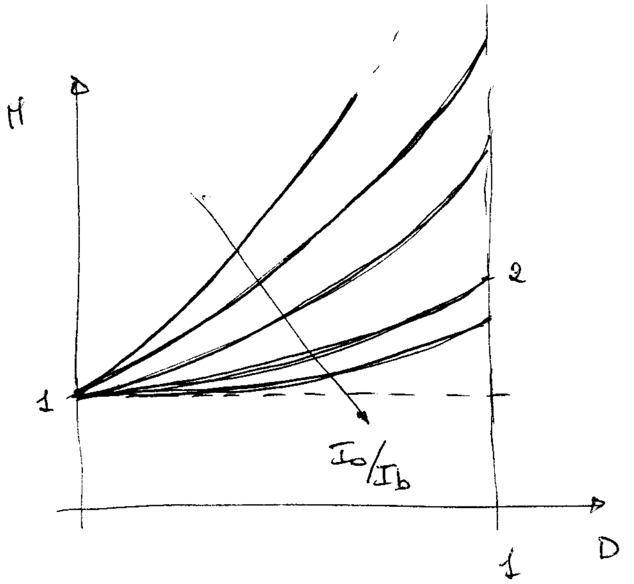
$$\frac{V_i}{V_o - V_i} D^2 \underbrace{\frac{V_i T_S}{2L}}_{I_b} = I_o$$

$$\frac{V_i}{V_o - V_i} D^2 = \frac{I_o}{I_b}$$

$$\frac{D^2}{\frac{I_o}{I_b}} = \frac{V_o}{V_i} - 1$$

$$M = \frac{V_o}{V_i} = 1 + \frac{D^2}{\frac{I_o}{I_b}}$$

$$\boxed{M = 1 + \frac{D^2}{\frac{I_o}{I_b}}}$$



$$V_i D = (V_o - V_i) D_1$$

$$D_1 = \frac{V_i}{V_o - V_i} D = \frac{D}{\frac{V_o - V_i}{V_i}} = \frac{D}{M - 1}$$

$$= \frac{\cancel{D}}{\cancel{1} + \frac{D^{\cancel{2}}}{\frac{I_o}{I_b}} - \cancel{1}} = \frac{1}{D} \frac{I_o}{I_b}$$

$$\boxed{D_1 = \frac{1}{D} \frac{I_o}{I_b}}$$

6) Inductor current ripple and maximum current

(omitted)

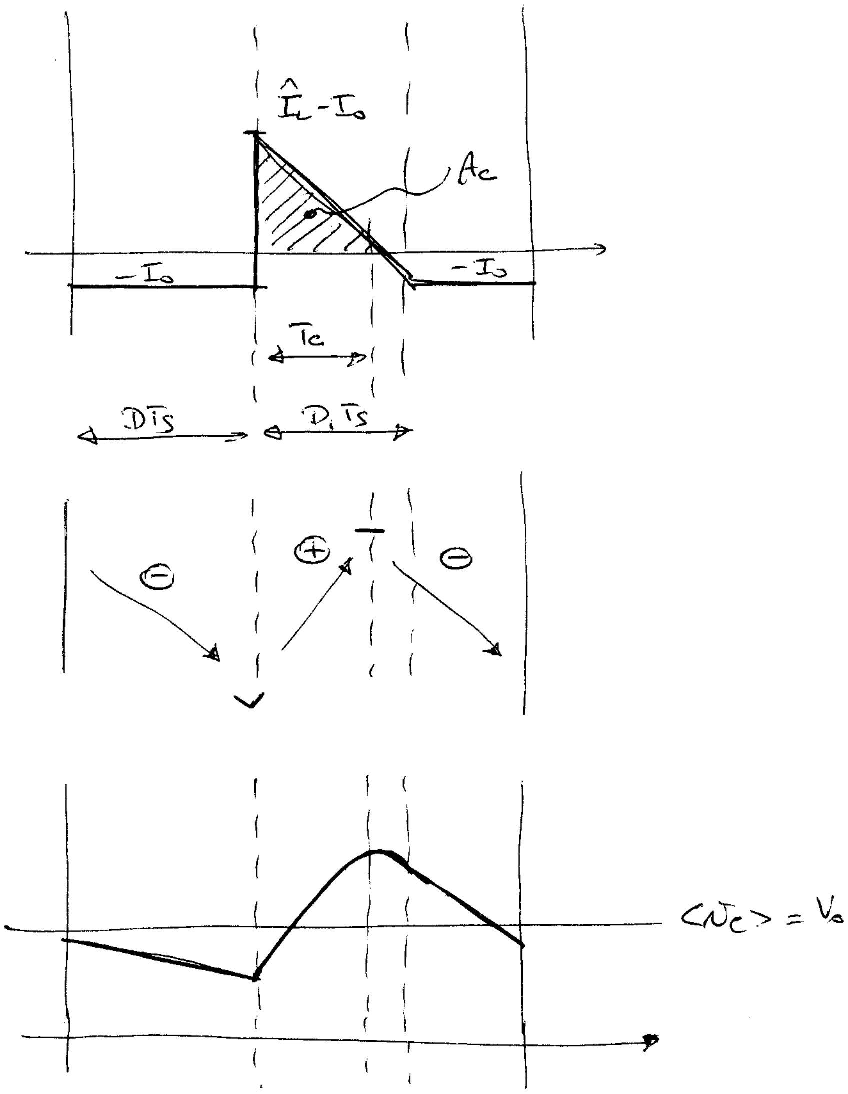
7) Average and maximum currents

(omitted)

8) Maximum voltage

$$|\hat{v}_S| = V_o \qquad |\hat{v}_o| = V_o \qquad |\hat{v}_c| = V_o$$

9) Output voltage ripple



$$\Delta v_c = \frac{\Delta q_c}{C} = \frac{A_c}{C}$$

$$A_c = \frac{1}{2} T_c \left(\hat{I}_L - I_o \right)$$

$$\frac{T_c}{D_1 T_S} = \frac{\hat{I}_L - I_o}{\hat{I}_L}$$

$$\boxed{T_c = \frac{\hat{I}_L - I_o}{\hat{I}_L} \cdot D_1 \cdot T_S}$$

$$\hat{I}_L = 2\frac{I_o}{D_1}$$

$$T_c = \frac{2\frac{I_o}{D_1} - I_o}{2\frac{I_o}{D_1}} \cdot D_1 T_S = \frac{\frac{2-D_1}{D_1}}{\frac{2}{D_1}} \cdot D_1 T_S$$

$$\boxed{T_C = \frac{1}{2}(2-D_1) D_1 \cdot T_S}$$

$$A_C = \frac{1}{2} T_C (\hat{I}_L - I_o) = \frac{1}{2}\left[\frac{1}{2}(2-D_1) D_1 T_S\right]\left[\frac{2 I_o}{D_1} - I_o\right]$$

$$= \frac{1}{4}(2-D_1)\cancel{D_1} T_S \left(\frac{2-D_1}{\cancel{D_1}}\right) I_o$$

$$= \frac{1}{4}(2-D_1)^2 \cdot T_S \cdot I_o$$

$$\Delta v_C = \frac{A_C}{C} = \frac{1}{4C}(2-D_1)^2 \cdot T_S \cdot I_o$$

$$= \frac{T_S \cdot I_o}{C}\left(\frac{2-D_1}{2}\right)^2 = \frac{T_S \cdot I_o}{C}\left(1 - \frac{D_1}{2}\right)^2$$

— · —

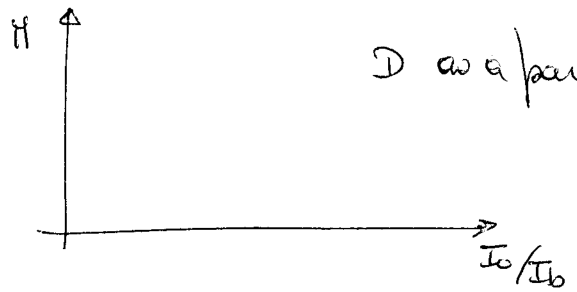
$$\begin{cases} D_1 = \dfrac{1}{D}\dfrac{I_o}{I_b} \\ M = 1 + \dfrac{D^2}{\frac{I_o}{I_b}} \longrightarrow \dfrac{I_o}{I_b} = \dfrac{D^2}{M-1} \end{cases}$$

$$D_1 = \frac{1}{\cancel{D}}\frac{D^{\cancel{2}}}{M-1} = \frac{D}{M-1}$$

$$\boxed{\Delta v_C = \frac{T_S I_o}{C}\left(1 - \frac{D}{M-1}\right)^2}$$

DCM AND CCM TOGETHER

1) Output characteristic



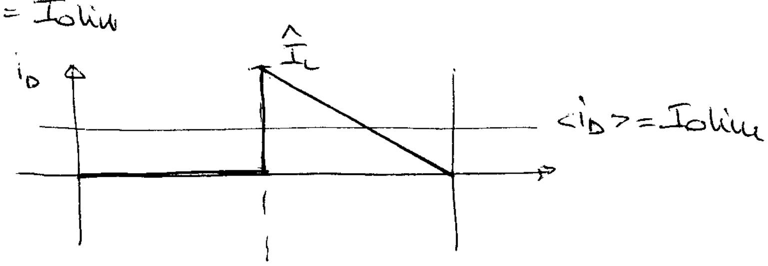
D as a parameter

1. CCM | if $I_o > I_{olim}$ $\Rightarrow$ CCM $\quad M = \frac{1}{1-D} \quad \forall \frac{I_o}{I_b}$

2. DCM | if $I_o < I_{olim}$ $\Rightarrow$ DCM $\quad M = 1 + \frac{D^2}{I_o/I_b}$

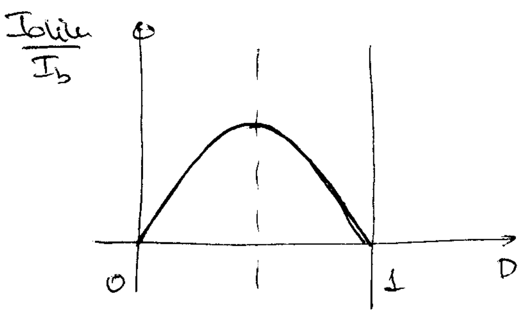
Border between CCM and DCM |

$I_o = I_{olim}$



$$I_{olim} = \frac{\hat{I}_L}{2}(1-D) = \frac{V_i}{2L} \cdot D \cdot T_S(1-D) = \frac{V_i T_S}{2L}(1-D)D$$

$$\stackrel{!}{=} I_b (1-D)D$$



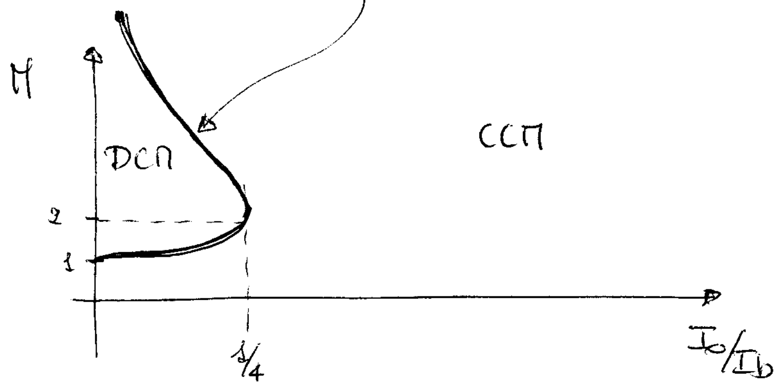
$$I_o = I_{o\,lim} = I_b(1-D)D$$

At the border $M = \dfrac{1}{1-D}$ is still valid:

$$\hookrightarrow D = \frac{M-1}{M} \quad \rightarrow \quad \frac{I_o}{I_b} = (1-D)D = \left(1 - \frac{M-1}{M}\right)\frac{M-1}{M}$$

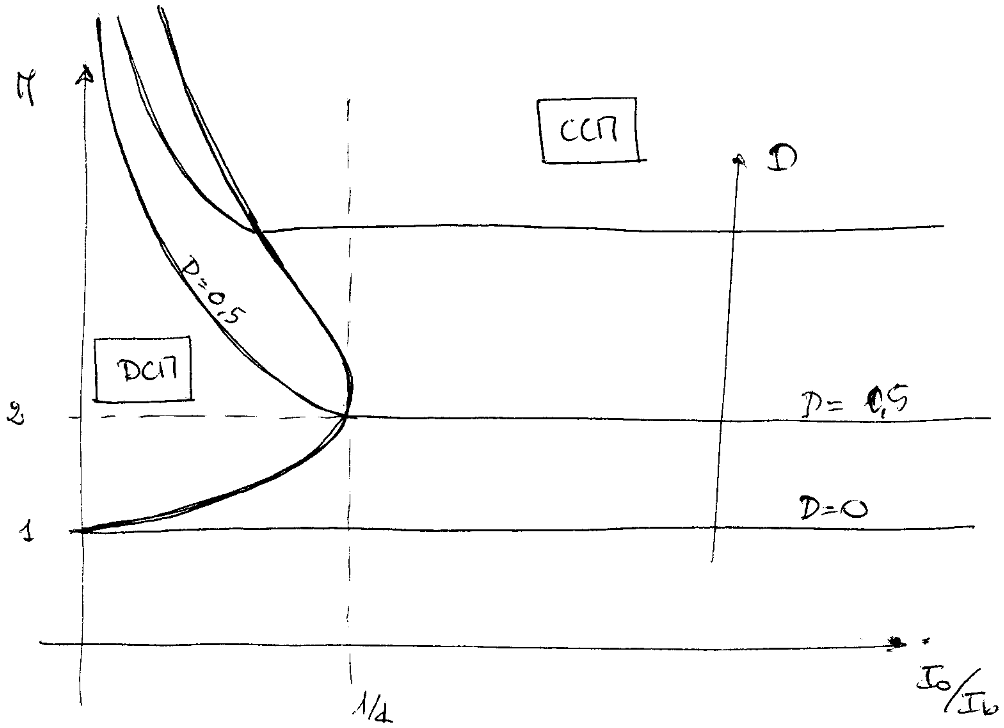
$$\frac{I_o}{I_b} = \left(\frac{\cancel{M}-\cancel{M}+1}{M}\right)\frac{M-1}{M} = \frac{M-1}{M^2}$$

$$\boxed{\frac{I_o}{I_b} = \frac{M-1}{M^2}}$$



$$CCM \iff I_o > I_{o\,lim} \iff I_o > I_b(1-D)D$$

$$I_o > I_b\,\frac{M-1}{M^2}$$



BUCK-BOOST DCM

(omitted)

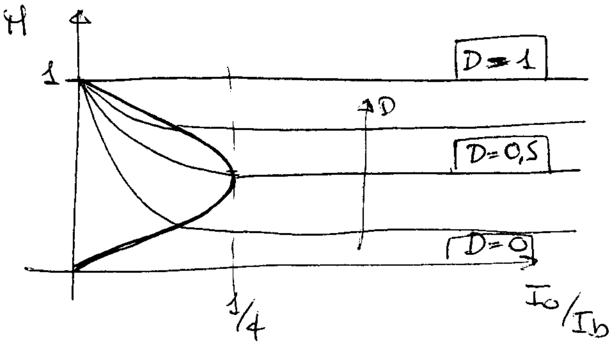
SUMMARY

Buck

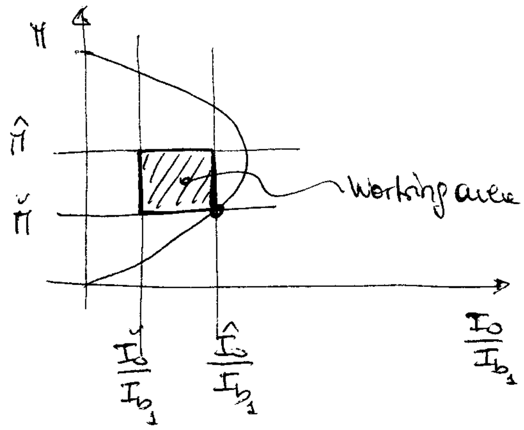
in CCM: $M = D$

in DCM: $M = \dfrac{D^2}{D^2 + \frac{I_o}{I_b}}$

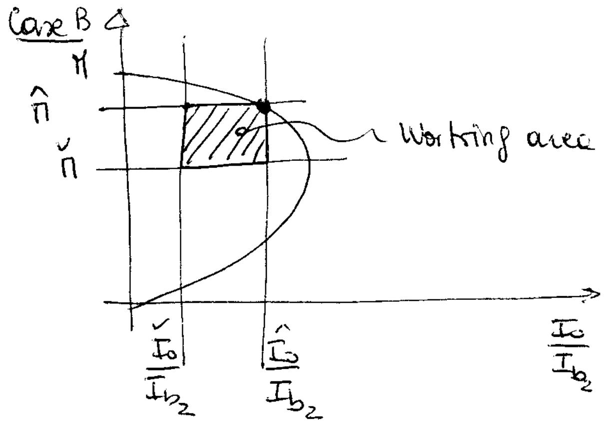
limit: $\dfrac{I_o}{I_b} = M(1-M)$



Case A



Case B



$$I_{b_1} = \frac{\hat{I_o}}{\check{M}(1-\check{M})}$$

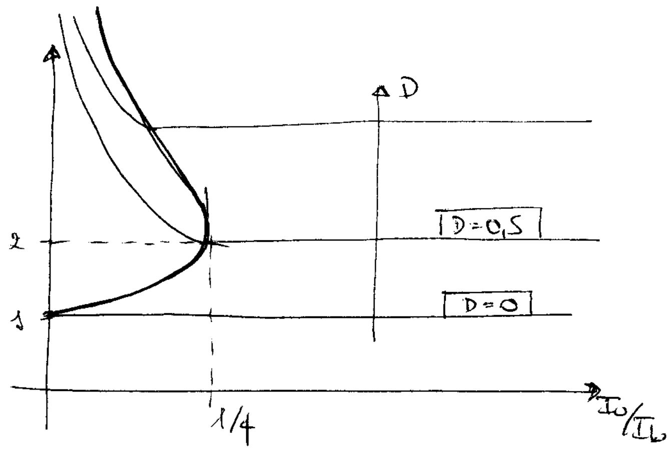
$$I_{b_2} = \frac{\hat{I_o}}{\hat{M}(1-\hat{M})}$$

Boost

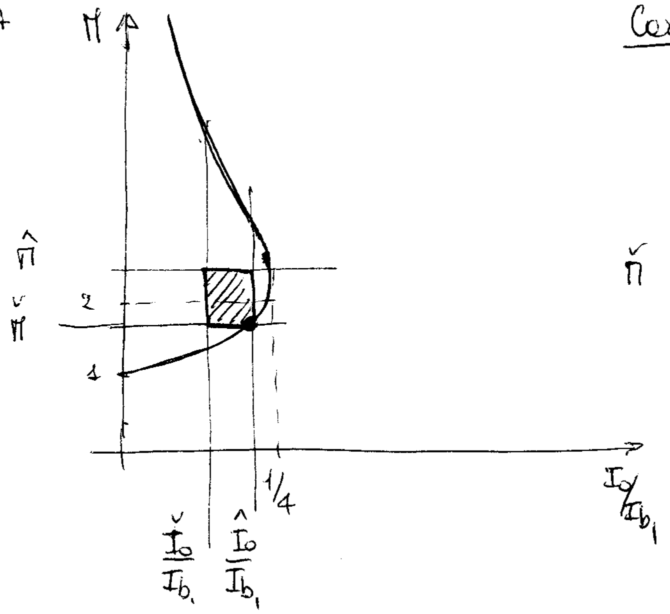
in CCM = $M = \dfrac{1}{1-D}$

in DCM = $M = 1 + \dfrac{D^2}{I_o/I_b}$

limit: $\dfrac{I_o}{I_b} = \dfrac{M-1}{M^2}$

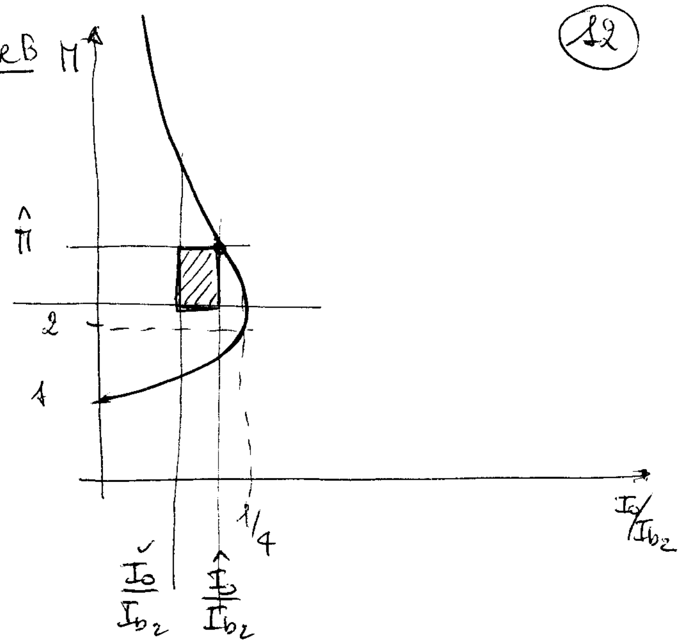


Case A



$$I_{b_1} = \hat{I}_o \frac{\check{M}^2}{\check{M}-1}$$

Case B



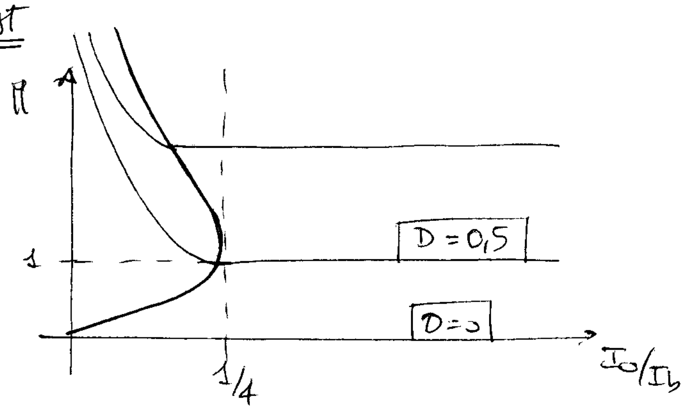
$$I_{b_2} = \hat{I}_o \frac{\hat{M}^2}{\hat{M}-1}$$

Buckboost

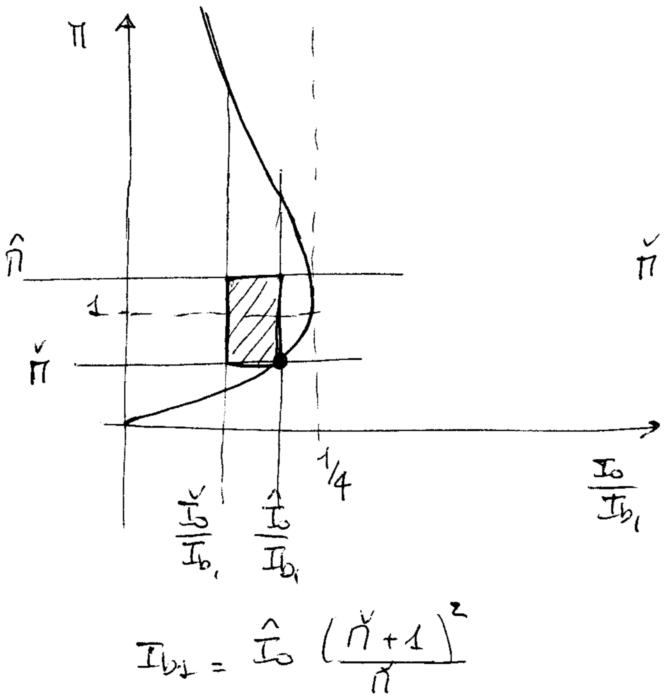
in CCM : $M = \frac{D}{1-D}$

in DCM : $M = \frac{D^2}{I_o/I_b}$

limit : $\frac{I_o}{I_b} = \frac{M}{(M+1)^2}$

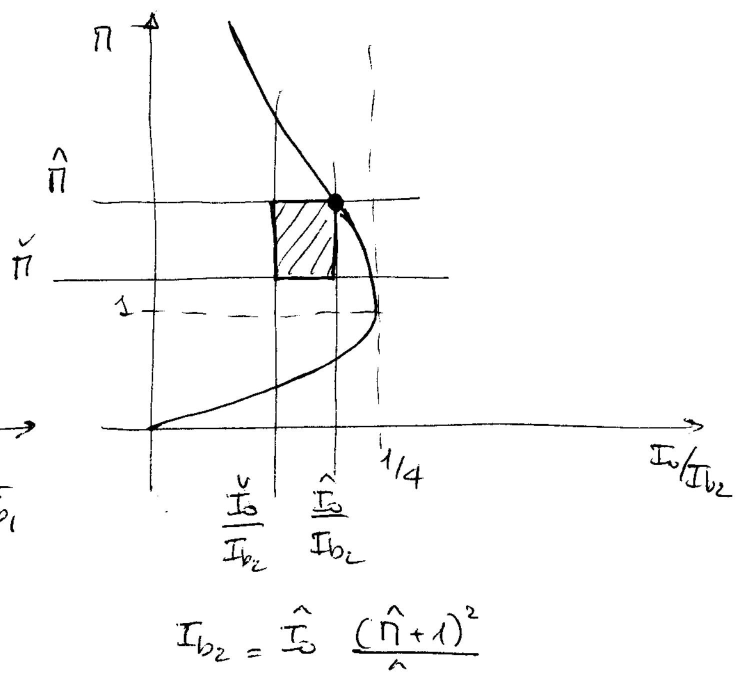


Case A



$$I_{b_1} = \hat{I}_o \frac{(\check{M}+1)^2}{\check{M}}$$

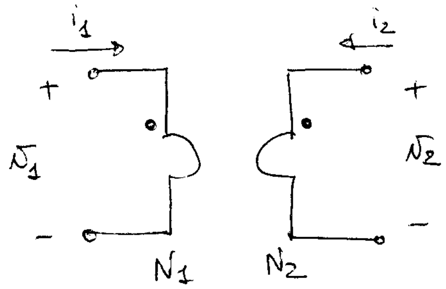
Case B



$$I_{b_2} = \hat{I}_o \frac{(\hat{M}+1)^2}{\hat{M}}$$

TRANSFORMERS

Ideal transformer w/ 2 windings



⚠ Always "load" convention!

Def:

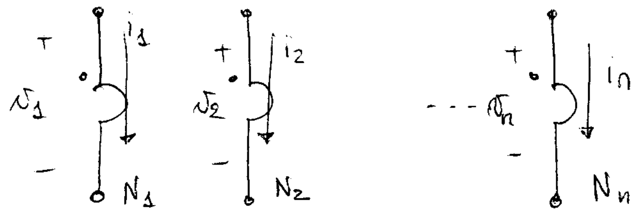
$$\boxed{\frac{v_1}{N_1} = \frac{v_2}{N_2}} \iff v_1 = \frac{N_1}{N_2} v_2$$

$P_1 + P_2 = 0 \qquad v_1 i_1 + v_2 i_2 = 0 \qquad v_1 = \frac{N_1}{N_2} v_2$

$$\frac{N_1}{N_2} \cancel{v_2} i_1 + \cancel{v_2} i_2 = 0$$

$$\boxed{N_1 i_1 + N_2 i_2 = 0} \iff i_1 = -\frac{N_2}{N_1} i_2$$

Ideal transformer w/ "n" windings



Def:

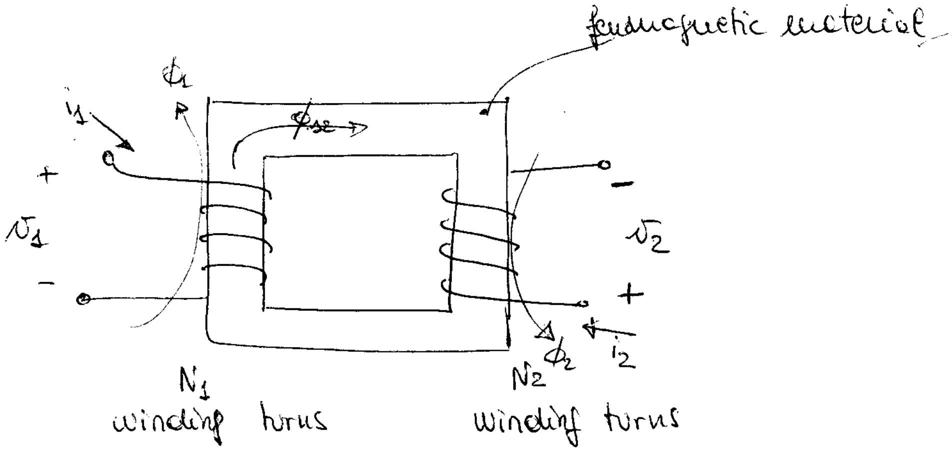
$$\boxed{\frac{v_1}{N_1} = \frac{v_2}{N_2} = \dots = \frac{v_n}{N_n}} \Rightarrow \begin{array}{l} v_1 = \frac{N_1}{N_2} v_2 \\ v_n = \frac{N_n}{N_2} v_2 \end{array}$$

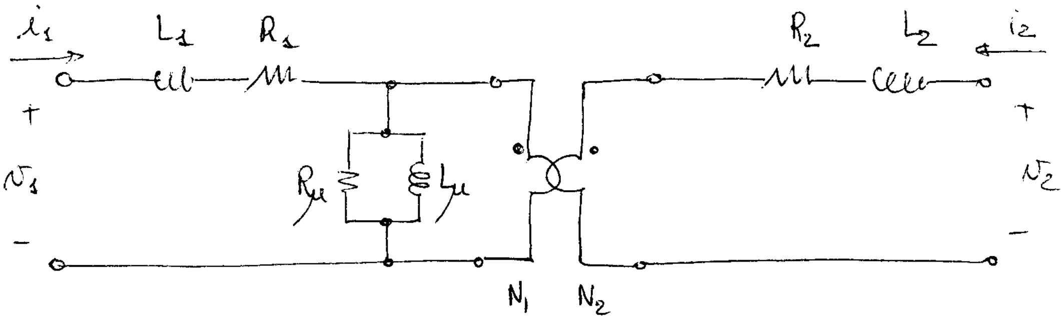
$P_1 + P_2 + \dots + P_n = 0 \qquad v_1 i_1 + v_2 i_2 + \dots + v_n i_n = 0$

$$\frac{N_1}{N_2} \cancel{v_2} i_1 + \cancel{v_2} i_2 + \dots + \frac{N_n}{N_2} \cancel{v_2} i_n = 0$$

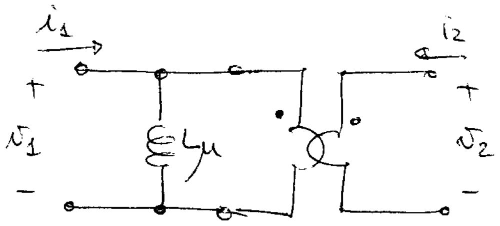
$$\boxed{N_1 i_1 + N_2 i_2 + \dots + N_n i_n = 0}$$

Real transformer with 2 windings

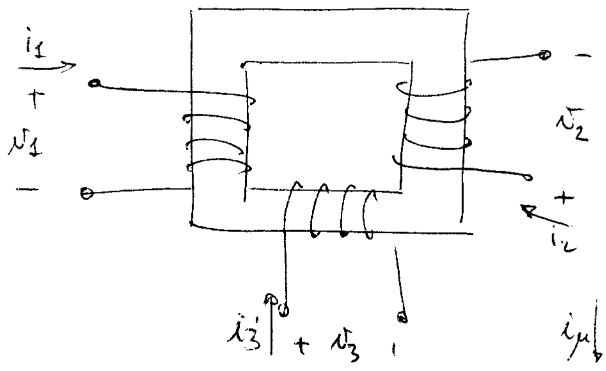


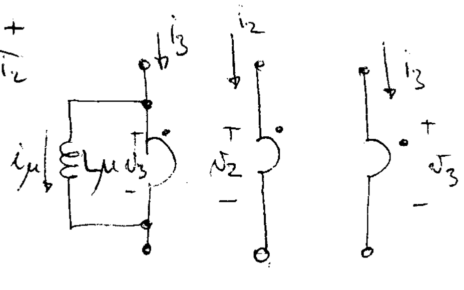


It can be simplified in order to study power converters w/ transf.:



Real transformer with 3 windings

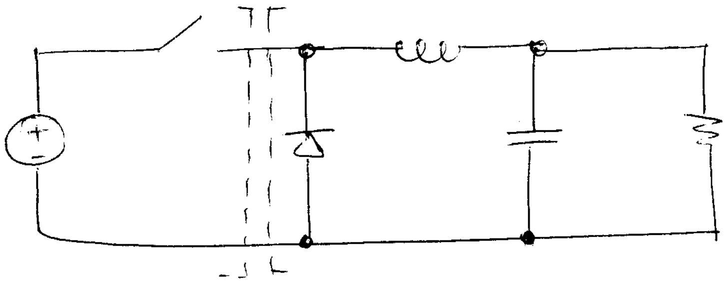


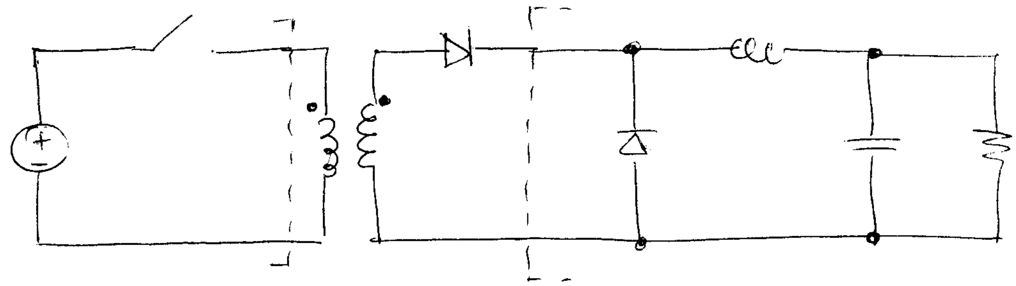


FORWARD

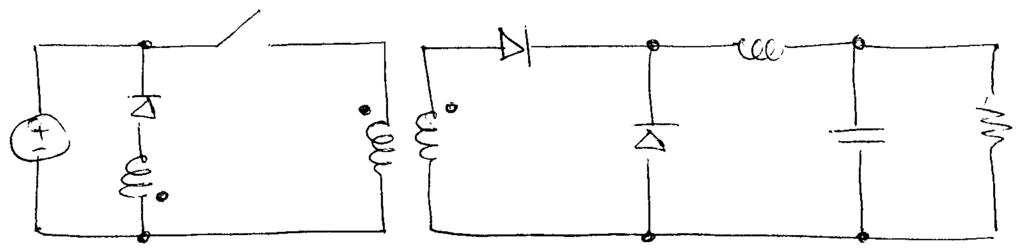
- Topology derived from the buck
- Transformer
- Different variations on the theme, we present the one with "third demagnetizing winding".

0) <u>Derivation from buck</u>



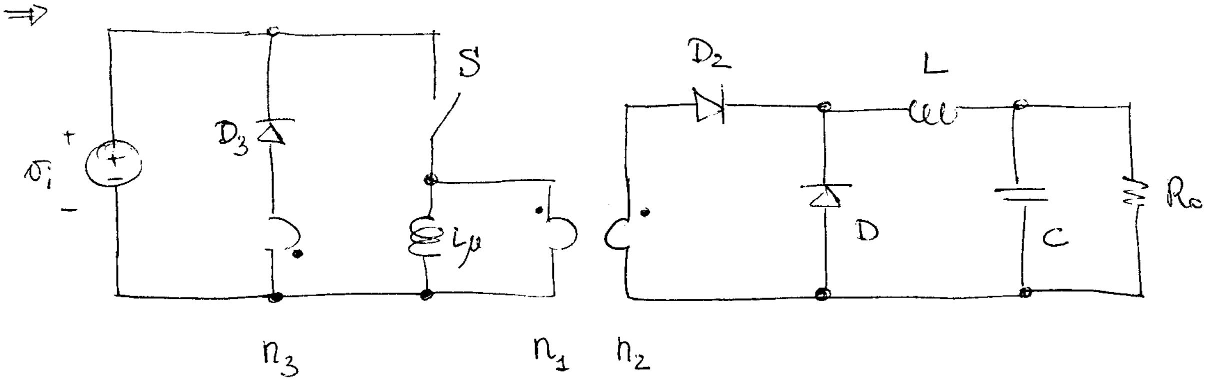


it still cannot work: the core cannot be demagnetized
→ we need circuit to demagnetize the transformer.

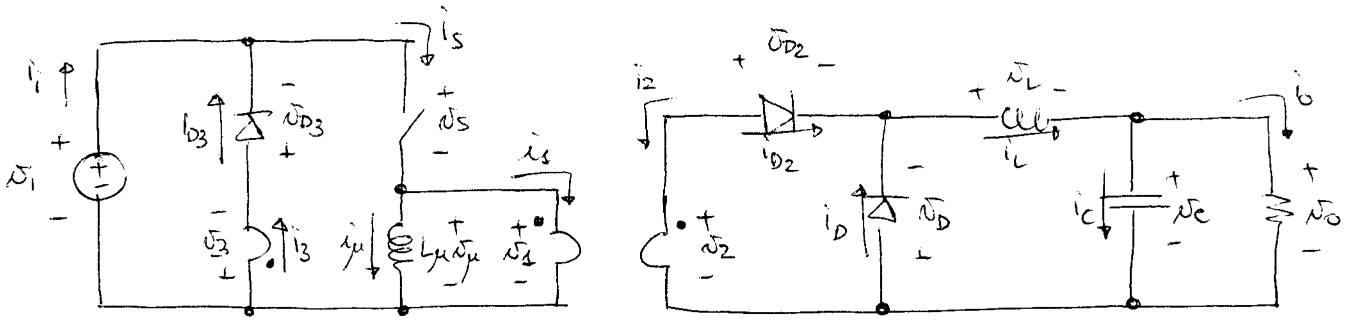


- Now with model (the simplest) of the real transformer

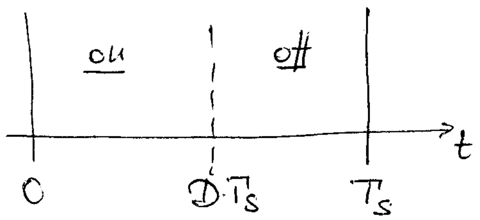
⇒



1) General circuit



Switch :



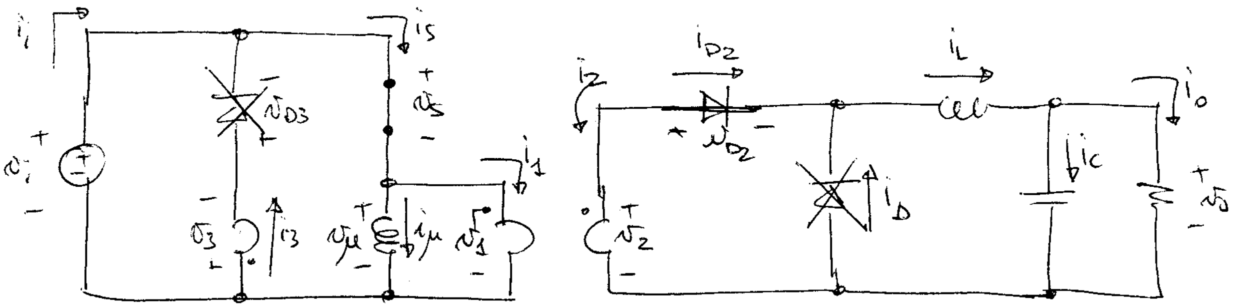
Diodes :

3 diodes + 1 switch = 16 cases.

↳ let's try a more intuitive way

2) Studying the possible circuits

(A) S_{on}, D_3 ?, D_2 ? D ?



$v_S = 0$ $\quad v_\mu = V_i$ $\quad v_1 = V_i$ $\quad v_2 = \frac{N_2}{N_1} v_1 = \frac{N_2}{N_1} V_i$ $\quad v_3 = \frac{N_3}{N_1} v_1 = \frac{N_3}{N_1} V_i$

$v_{D3} = -V_i - v_3 = -V_i - \frac{N_3}{N_1} V_i = -\left(1 + \frac{N_3}{N_1}\right) V_i$ $\quad v_{D3} < 0 \rightarrow D_3$ off

$\longrightarrow i_{D3} = 0 \quad i_3 = 0$

$$i_L > 0 \Rightarrow \begin{cases} i_{D2} > 0 & \Leftrightarrow D_2 \text{ on} \\ i_D > 0 & \Leftrightarrow D \text{ on} \\ i_D > 0 \wedge i_{D2} > 0 & \Leftrightarrow D_2 \wedge D \text{ on} \end{cases}$$

$\boxed{D_2 \wedge D \text{ on}}$ $\Leftrightarrow$ $v_2 = 0 \Rightarrow \frac{N_2}{N_1} V_i = 0 \Rightarrow V_i = 0 \Rightarrow \underline{\text{NO}}$

$\boxed{D \text{ on}}$ $\Rightarrow$ $v_{D_2} = v_2 = \frac{N_2}{N_1} V_i > 0 \Rightarrow \underline{\text{NO}}$

<u>Condition:</u> <u>D_2 on</u>

$$v_{D_2} = 0 \Rightarrow v_D = -v_2 = -\frac{N_2}{N_1} V_i < 0 \Rightarrow \text{OK} \quad D \text{ off}$$

$$V_i' = \frac{N_2}{N_1} V_i \qquad v_D = V_i'$$

$$v_L = v_2 - v_o = \frac{N_2}{N_1} V_i - V_o = V_i' - V_o \qquad \text{it must be } V_i' - V_o > 0$$

$$i_L = \frac{1}{L}\int_0^t v_L \, dt = \frac{1}{L}\int (V_i' - V_o)\,dt = \frac{V_i' - V_o}{L} t + \check{I}_L$$

$$i_{D_2} = i_L$$

$$i_o = \frac{V_o}{R} = I_o$$

$$i_C = i_L - i_o = i_L - I_o$$

$$i_2 = -i_{D2} = -i_L$$

$$N_1 i_1 + N_2 i_2 + \cancel{N_3 i_3} = 0 \rightarrow i_1 = -\frac{N_2}{N_1} i_2 = \frac{N_2}{N_1} i_L$$

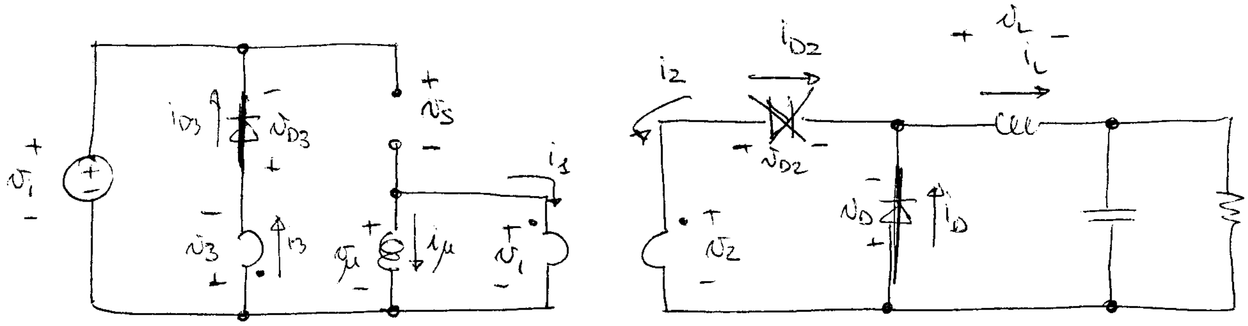
$$v_\mu = V_i \rightarrow i_\mu = \frac{1}{L_\mu}\int_0^t v_\mu \, dt = \frac{V_i}{L_\mu} t$$

$$i_S = i_1 + i_\mu = \frac{N_2}{N_1} i_L + \frac{V_i}{L_\mu} t$$

$$i_i = i_S$$

Conclusion : (A) S on , D_3 off, D_2 on , D off

(C_1) S off , D_3 ? , D_2 ? , D ?



D_3 off $\iff i_3 = 0 \qquad i_1 = -i_\mu \qquad i_2 = -\frac{N_1}{N_2} i_1 = \frac{N_1}{N_2} i_\mu$

$i_{D2} = -i_2 = -\frac{N_1}{N_2} i_\mu \Rightarrow D_2$ off $\Rightarrow i_2 = 0 \Rightarrow i_\mu = 0$

→ not possible

→ D_3 on

$v_3 = -V_i \qquad v_1 = \frac{N_1}{N_3} v_3 = -\frac{N_1}{N_3} V_i \qquad v_2 = \frac{N_2}{N_3} V_i = -\frac{N_2}{N_3} V_i$

$v_\mu = v_1 = -\frac{N_1}{N_3} V_i < 0$

$i_{L\mu} = \frac{1}{L_\mu} \int_0^t v_\mu + \hat{I}_\mu = -\frac{N_1}{N_3} \frac{V_i}{L_\mu} t + \hat{I}_\mu$

$i_L > 0$
- $i_{D2} > 0 \iff D_2$ on
- $i_D > 0 \iff D$ on
- $i_D > 0 \wedge i_{D2} > 0 \iff D \wedge D_2$ on

$\boxed{D \wedge D_2 \text{ on}} \iff v_2 = 0 \Rightarrow \frac{N_2}{N_3} V_i = 0 \Rightarrow V_i = 0 \Rightarrow$ No

$\boxed{D_2 \text{ on}} \qquad v_D = -v_2 = -\left(-\frac{N_2}{N_3}\right) V_i = \frac{N_2}{N_3} V_i > 0$ no

$\boxed{D \text{ on } (D_2 \text{ off})}$

$i_{D_2} = 0 \qquad i_2 = -i_{D2} = 0 \qquad i_1 N_1 + \cancel{i_2 N_2} + i_3 N_3 = 0$

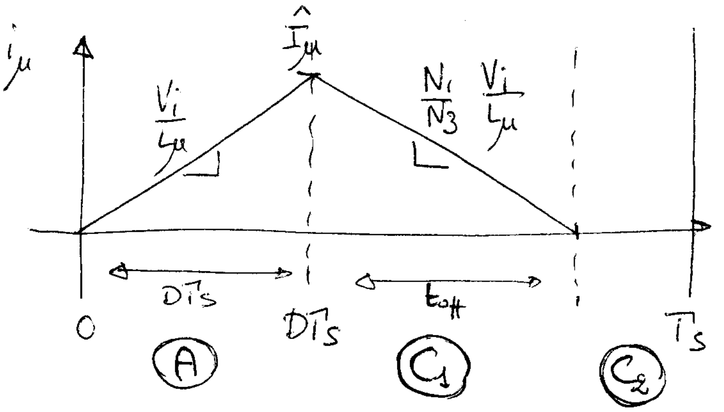
$v_{D2} = v_2 = -\frac{N_2}{N_3} V_i < 0 \rightarrow$ OK

$v_L = -V_0 \qquad i_L = \frac{1}{L}\int_0^t v_L\,dt + \hat{I}_L = -\frac{V_0}{L}t + \hat{I}_L$

$i_1 = -i_\mu$

$i_3 = -i_1 \frac{N_1}{N_3} = i_\mu \frac{N_1}{N_3} = \left[-\frac{N_1}{N_3}\frac{V_i}{L_\mu}t + \hat{I}_\mu\right]\frac{N_1}{N_3}$

$\frac{N_1}{N_3}\hat{I}_\mu - \left(\frac{N_1}{N_3}\right)^2 \frac{V_i}{L_\mu}t \qquad i_{D3} = i_3$



t_{off} : $\quad \hat{I}_\mu - \frac{N_1}{N_3}\frac{V_i}{L_\mu}t_{off} = 0$

$\hat{I}_\mu = \frac{N_1}{N_3}\frac{V_i}{L_\mu}t_{off} \qquad \hat{I}_\mu = \frac{V_i}{L_\mu}DT_s$

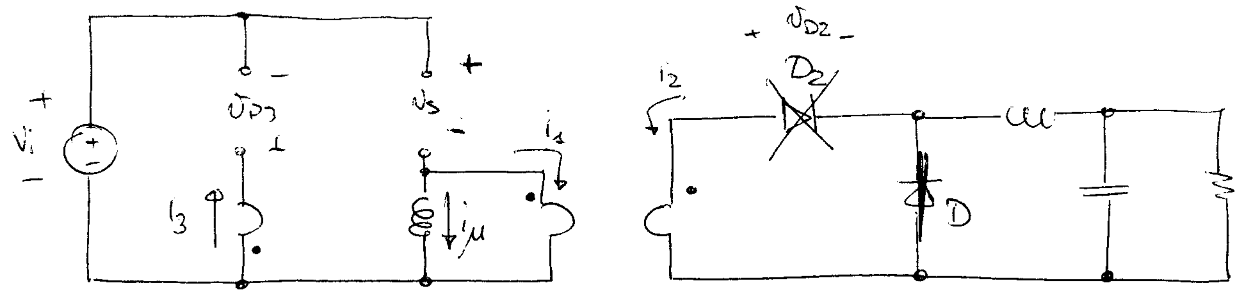
$\cancel{\frac{V_i}{L_\mu}}DT_s = \frac{N_1}{N_3}\cancel{\frac{V_i}{L_\mu}}t_{off}$

$$\boxed{t_{off} = \frac{N_3}{N_1}DT_s}$$

<u>Conclusion</u>. C1 $\quad S$ off, D_3 on, D_2 off, D on

$v_S = v_i - v_1 = V_i - \left(-\frac{N_1}{N_3}\right)V_i = \left(1 + \frac{N_1}{N_3}\right)V_i$

C_2 S off, D_3 off, D_2 ?, D ?



$i_3 = 0$ $\quad$ $i_\mu = 0$ $\quad$ $i_1 = -i_\mu = 0$ $\quad$ $\cancel{i_1 N_1} + i_2 N_2 + \cancel{i_3 N_3} = 0$ $\quad$ $i_2 = 0$

$\longrightarrow D_2$ off

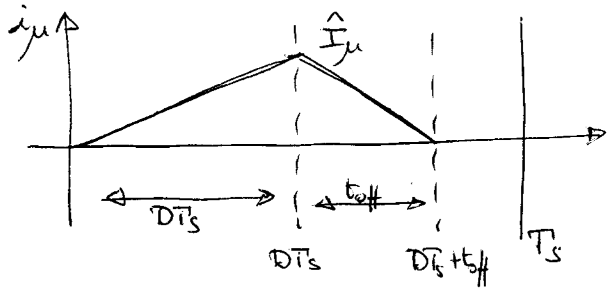
$i_\mu \equiv 0 \Rightarrow v_\mu = 0 \longrightarrow v_1 = 0 \quad v_2 = 0 \quad v_3 = 0$

$i_L > 0$ $\quad$ D on

$v_{D3} = -V_i$ $\quad$ $v_S = V_i$ $\quad$ $v_{D2} = 0$

Conclusion: C_2 S off, D_3 off, D_2 off, D on

IMPORTANT CONDITION

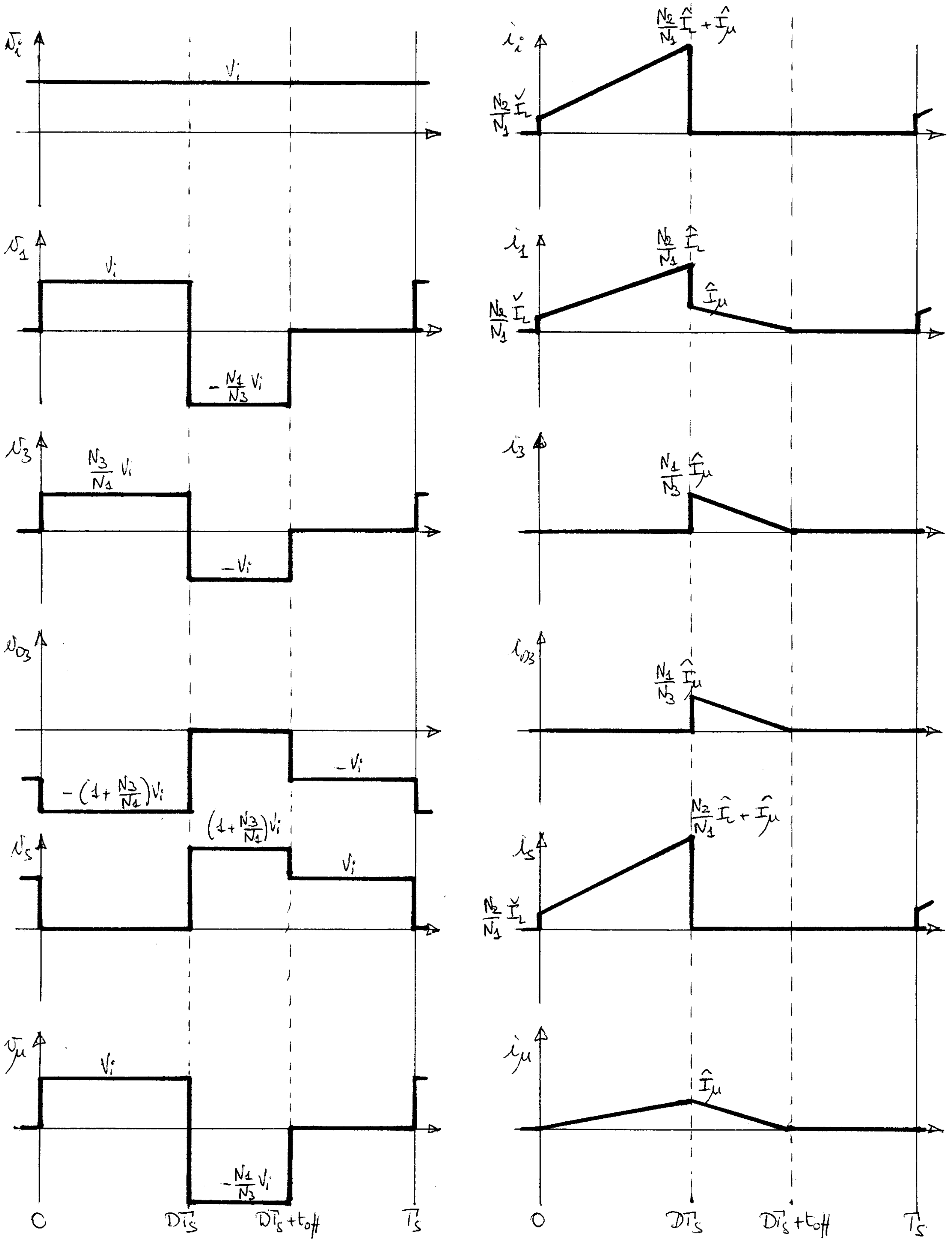


$DT_s + t_{off} < T_s$ $\quad\quad$ $t_{off} = \frac{N_3}{N_1} DT_s$

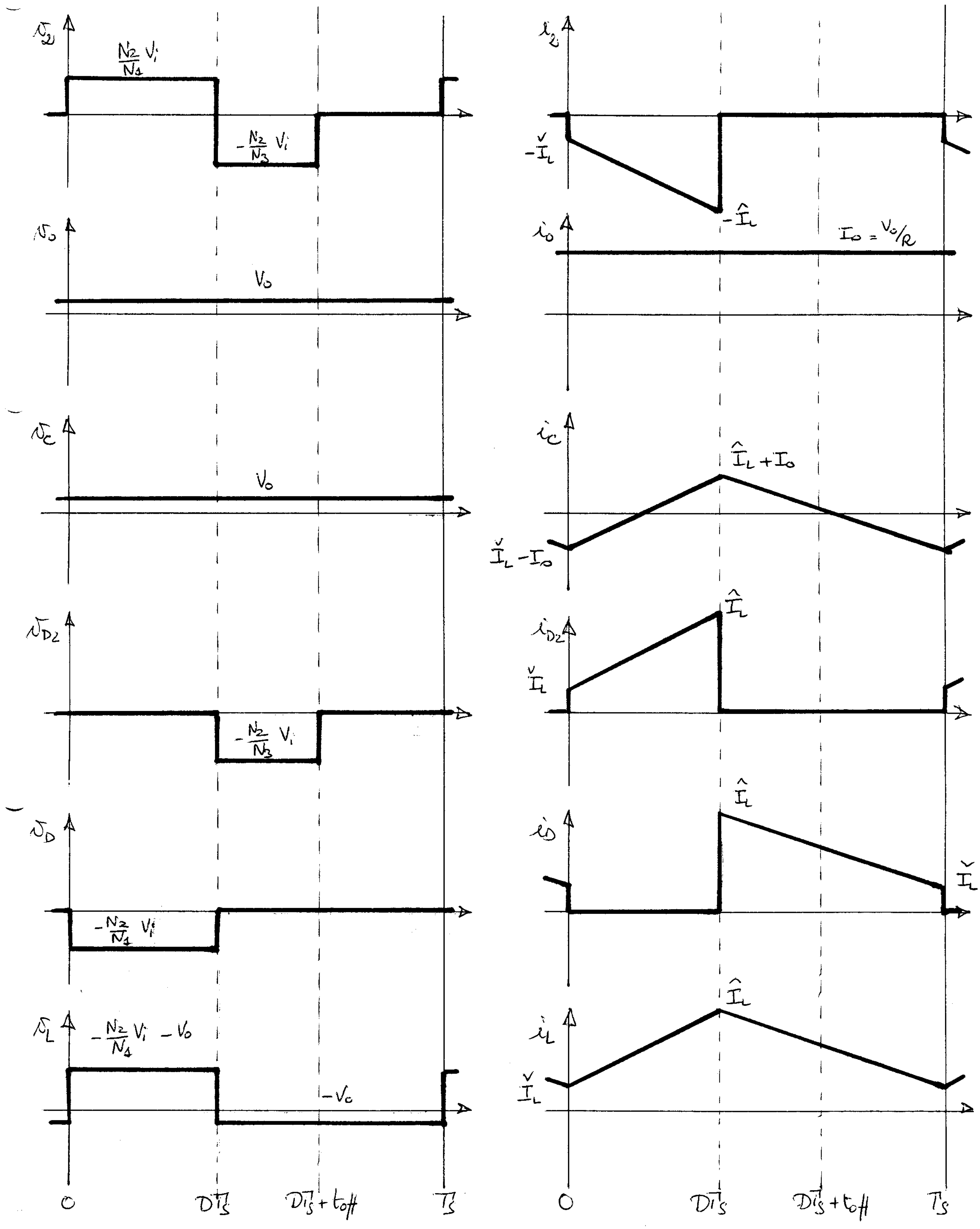
$D\cancel{T_s} + \frac{N_3}{N_1} D\cancel{T_s} < \cancel{T_s}$

$D\left(1 + \frac{N_3}{N_1}\right) < 1 \quad\quad 1 + \frac{N_3}{N_1} < \frac{1}{D} \quad\quad \frac{N_3}{N_1} < \frac{1}{D} - 1$

$$\boxed{\frac{N_3}{N_1} < \frac{1-D}{D}}$$



v_i
V_i
v_1
V_i
$-\frac{N_1}{N_3} V_i$
v_3
$\frac{N_3}{N_1} V_i$
$-V_i$
v_{D3}
$-(1+\frac{N_3}{N_1})V_i$
$-V_i$
v_S
$(1+\frac{N_3}{N_1})V_i$
V_i
v_μ
V_i
$-\frac{N_1}{N_3} V_i$
0
DT_S
DT_S+t_{off}
T_S
i_i
$\frac{N_2}{N_1}\hat{I}_L+\hat{I}_\mu$
$\frac{N_2}{N_1}\check{I}_L$
i_1
$\frac{N_2}{N_1}\hat{I}_L$
$\hat{I}_\mu$
$\frac{N_2}{N_1}\check{I}_L$
i_3
$\frac{N_1}{N_3}\hat{I}_\mu$
i_{D3}
$\frac{N_1}{N_3}\hat{I}_\mu$
i_S
$\frac{N_2}{N_1}\hat{I}_L+\hat{I}_\mu$
$\frac{N_2}{N_1}\check{I}_L$
i_μ
$\hat{I}_\mu$
0
DT_S
DT_S+t_{off}
T_S


v_2
$\frac{N_2}{N_1} V_i$
$-\frac{N_2}{N_3} V_i$
v_o
V_o
v_C
V_o
v_{D2}
$-\frac{N_2}{N_3} V_i$
v_D
$-\frac{N_2}{N_1} V_i$
v_L
$-\frac{N_2}{N_1} V_i - V_o$
$-V_o$
0
DT_S
$DT_S + t_{off}$
T_S
i_2
$-\check{I}_L$
$-\hat{I}_L$
i_o
$I_o = V_o/R$
i_C
$\hat{I}_L + I_o$
$\check{I}_L - I_o$
i_{D2}
$\hat{I}_L$
$\check{I}_L$
i_D
$\hat{I}_L$
$\check{I}_L$
i_L
$\hat{I}_L$
$\check{I}_L$
0
DT_S
$DT_S + t_{off}$
T_S

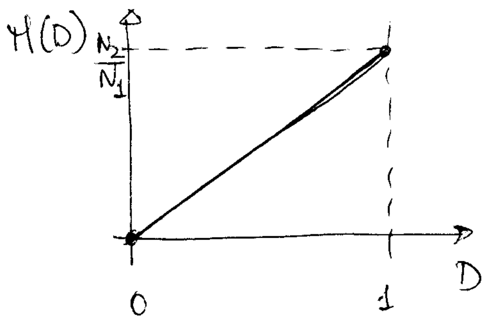
4) Steady-state conditions

Defined $V_i' = \frac{N_2}{N_1} V_i$, the second part of the converter behaves like a buck with an input voltage V_i'. All the equations are then the same.

$$\boxed{V_o = D V_i' = D \frac{N_2}{N_1} V_i}$$

5) Conversion ratio

$$H(D) = \frac{V_o}{V_i} = D \frac{N_2}{N_1}$$



6) Current ripple (inductor)

$$\Delta i_L = \frac{V_i' - V_o}{L} D T_s = \frac{V_i' - D V_i'}{L} D T_s = \frac{V_i' T_s}{L}(1-D)D$$

$$= \frac{N_2}{N_1} \frac{V_i T_s}{L} (1-D)D$$

7) Average and maximum current

$$\langle i_L \rangle = I_o = \frac{V_o}{R}$$

$$\hat{I}_L = I_o + \frac{\Delta i_L}{2} = I_o + \frac{V_i' T_s}{L}(1-D)D$$

$$= I_o + \frac{N_2}{N_1} \frac{V_i T_s}{L}(1-D)D$$

$$\hat{I}_D = \hat{I}_{D2} = \hat{I}_L$$

$$\hat{I}_1 = \hat{I}_S = \frac{N_2}{N_1}\hat{I}_L + \hat{I}_\mu$$

$$\hat{I}_{D3} = \frac{N_1}{N_3}\hat{I}_\mu$$

$$|\hat{I}_2| = \hat{I}_L \qquad |\hat{I}_1| = \frac{N_2}{N_1}\hat{I}_L \qquad |\hat{I}_3| = \frac{N_1}{N_3}\hat{I}_\mu$$

$$\langle i_D \rangle = I_o(1-D)$$

$$\langle i_{D2} \rangle = I_o D$$

$$\langle i_2 \rangle = -\langle i_{D2} \rangle = -I_o D$$

$$\langle i_1 \rangle = \langle i_S \rangle = \frac{1}{T_S}\int_0^t i_S(t)\,dt = \frac{1}{T_S}A_S =$$

$$= \frac{1}{\cancel{T_S}}\left[\frac{N_2}{N_1}\check{I}_L + \frac{N_2}{N_1}\hat{I}_L + \hat{I}_\mu\right]\frac{1}{2}\cdot D\cancel{T_S}$$

$$= \frac{N_2}{N_1}\langle i_L \rangle + \frac{\hat{I}_\mu}{2} = \frac{N_2}{N_1}I_o + \frac{\hat{I}_\mu}{2}$$

8) Maximum voltage

$$|\hat{v}_S| = \left(1 + \frac{N_3}{N_1}\right)V_i$$

$$|\hat{v}_D| = \frac{N_2}{N_1}V_i$$

$$|\hat{v}_{D2}| = \frac{N_2}{N_3}V_i$$

$$|\hat{v}_{D3}| = \left(1 + \frac{N_3}{N_1}\right)V_i$$

9) Output voltage ripple

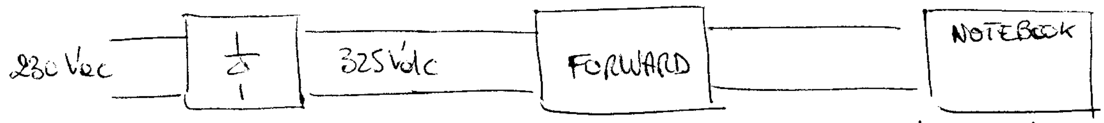
$$\Delta v_C = \frac{V_i' T_S^2}{8LC}(1-D)D = \frac{N_2}{N_1}\frac{V_i T_S^2}{8LC}(1-D)D$$

Exercise

1) Complete study of forward

2) Study of forward in DCM

3) Design of a forward.



$V_o = 20\,V$

$P = (40 \div 90)\,W$

$\Delta V_C = 0{,}2\,V$

$N_1 = 325$

$L_\mu = 60\,mH$

$f_s = 40\,kHz$

CCM operation

D: free choice

$N_2 = ?$	$N_3 = ?$	$I_o = ?$	$I_{o\,lim} = ?$
$D = ?$	$L = ?$	$C = ?$	$\Delta i_L = ?$
$\langle i_L \rangle = ?$	$\langle i_S \rangle = ?$	$\langle i_D \rangle = ?$	$\langle i_{D_3} \rangle = ?$
$\langle i_{D_2} \rangle = ?$	$\hat{I}_L = ?$	$\hat{I}_S = ?$	$\hat{I}_D = ?$
$\hat{I}_{D_3} = ?$	$\hat{I}_{D_2} = ?$	$\lvert \hat{v}_S \rvert = ?$	$\lvert \hat{v}_D \rvert = ?$
$\lvert \hat{v}_{D_3} \rvert = ?$	$\lvert \hat{v}_{D_2} \rvert = ?$		

Compare your choice and your results with your colleagues and decide on the best choice.

$\boxed{\text{LECTURE 4}}$

RMS (Root Mean Square) Values

For all the topologies we have found

$\hat{I}_D$ $\quad \langle i_D \rangle \quad$ $|\hat{V}_D|$

$\hat{I}_S$ $\quad \langle i_S \rangle \quad$ $|\hat{V}_S|$

& $\quad |\hat{V}_C|$

This is not enough for choosing a comp. & evaluating losses. → We also need the $\boxed{I_{RMS}}$

↳ Generally related with ohmic losses.

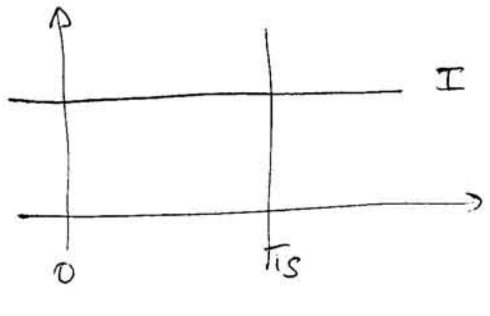
Calculation of RMS ROOT MEAN SQUARE

$i(t)$

$$I_{RMS} = \sqrt{\frac{1}{T_S}\int_0^{T_S} i^2(t)\,dt}$$

Examples

1) $i(t) = I$

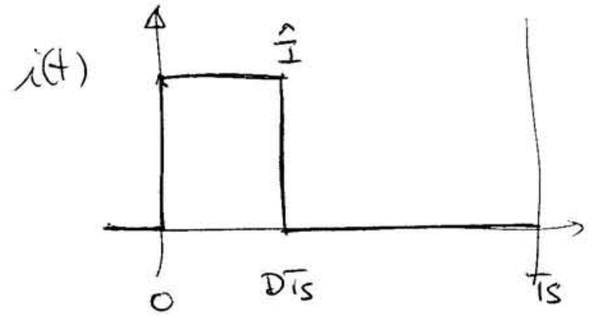


$$I_{RMS} = \sqrt{\frac{1}{T_S}\int_0^{T_S} I^2\,dt} = \sqrt{\frac{1}{T_S} I^2 T_S} = I$$

$$I_{RMS} = |i(t)|$$

2) $\langle i \rangle = I$

2)

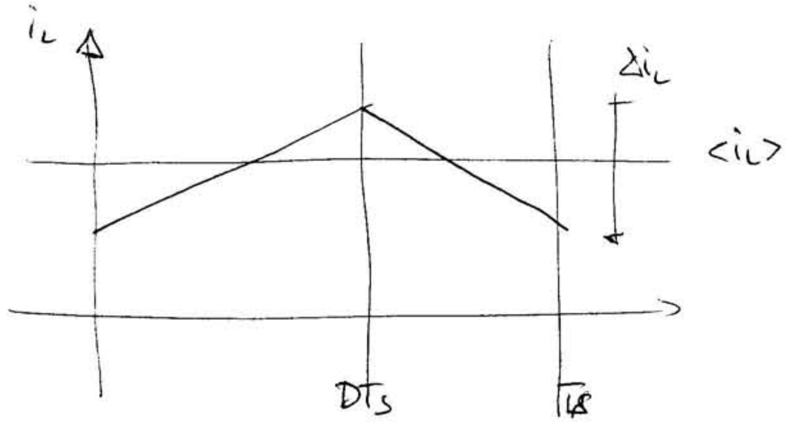


$$I_{RMS} = \sqrt{\frac{1}{T_S}\int_0^{T_S} i^2(t)dt} = \sqrt{\frac{1}{T_S}\int_0^{DT_S} i^2 dt + \cancel{\frac{1}{T_S}\int_{DT_S}^{T_S} i^2 dt}} =$$

$$= \sqrt{\frac{1}{T_S}\int_0^{DT_S} \hat{I}^2 dt} = \sqrt{\frac{1}{\cancel{T_S}}\hat{I}^2 D \cancel{T_S}} = |\hat{I}|\sqrt{D}$$

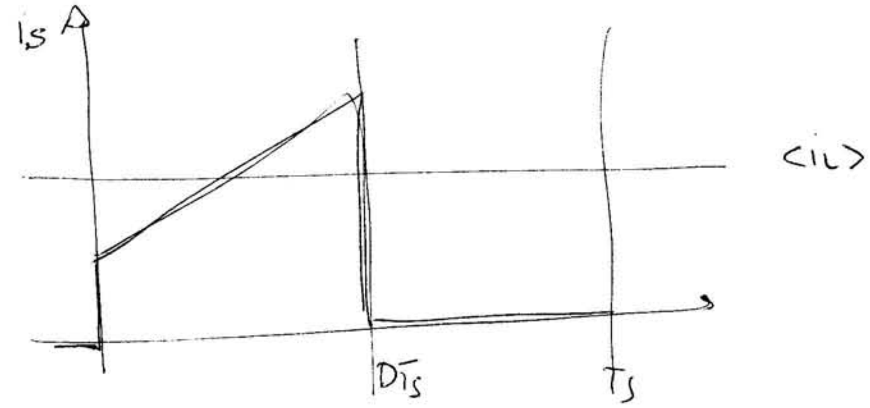
$$\langle i \rangle = \frac{1}{T_S}\int_0^{T_S} i(t)\,dt = \frac{1}{T_S}\int_0^{DT_S}\hat{I}\,dt = D\hat{I}$$

3)



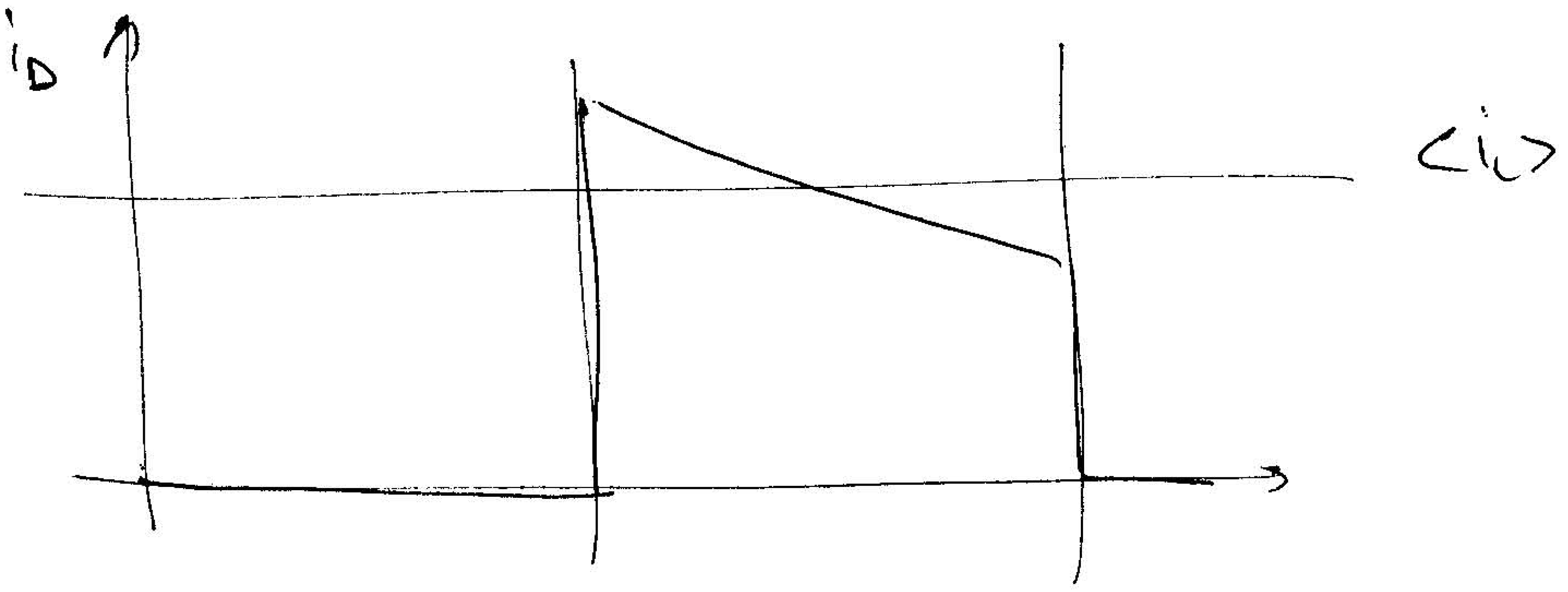
$$I_{RMS} = \langle i_L \rangle \sqrt{1 + \frac{1}{12}\left(\frac{\Delta i_L}{\langle i_L \rangle}\right)^2}$$

4)



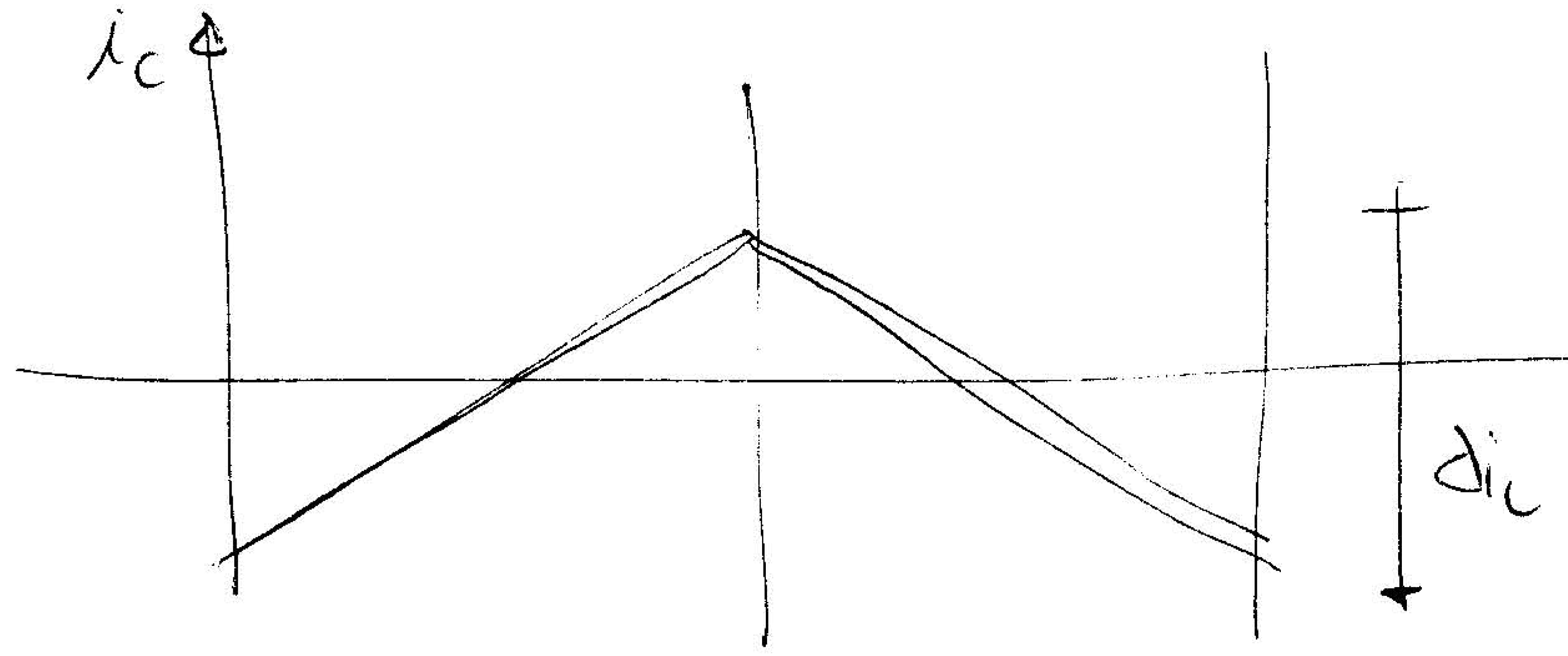
$$I_{S,RMS} = \langle i_L \rangle \sqrt{D} \sqrt{1 + \frac{1}{12}\left(\frac{\Delta i_L}{\langle i_L \rangle}\right)^2}$$

5)



$$I_{D,RMS} = \langle i_L \rangle \sqrt{1-D} \sqrt{1 + \frac{1}{12}\left(\frac{\Delta i_L}{\langle i_L \rangle}\right)^2}$$

6)



$$I_{C,RMS} = \frac{1}{\sqrt{3}}\left(\frac{\Delta i_L}{2}\right) = \frac{\Delta i_L}{2\sqrt{3}}$$

7) GENERAL PIECEWISE WAVEFORM

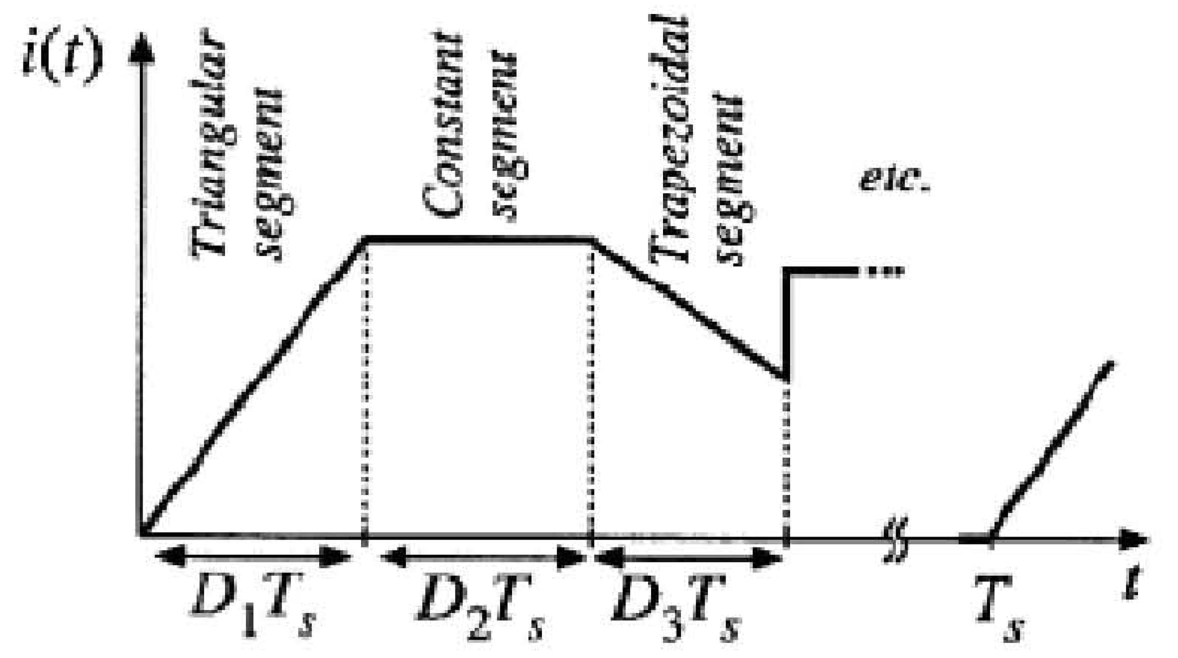


Fig. A.12

For a periodic waveform composed of n piecewise segments as in Fig. A.12, the rms value is

$$rms = \sqrt{\sum_{k=1}^{n} D_k u_k} \qquad \text{(A.12)}$$

where D_k is the duty cycle of segment k, and u_k is the contribution of segment k. The u_ks depend on the shape of the segments—several common segment shapes are listed **in Erickson, Maksimovic, Fundamentals of Power Electronics, Appendix A.2 (available at www.aub.aau.dk).**

Components

- (Power) Diodes
- (Power) Switches
- Switch drivers
- Capacitors

Power diodes

- Slightly different from normal diodes
  - They have one more layer to block higher voltages
- Manufacturers
  - ST Microelectronics www.st.com
  - Microsemi www.microsemi.com
  - International rectifiers www.irf.com
  - Vishay www.vishay.com
  - many others...

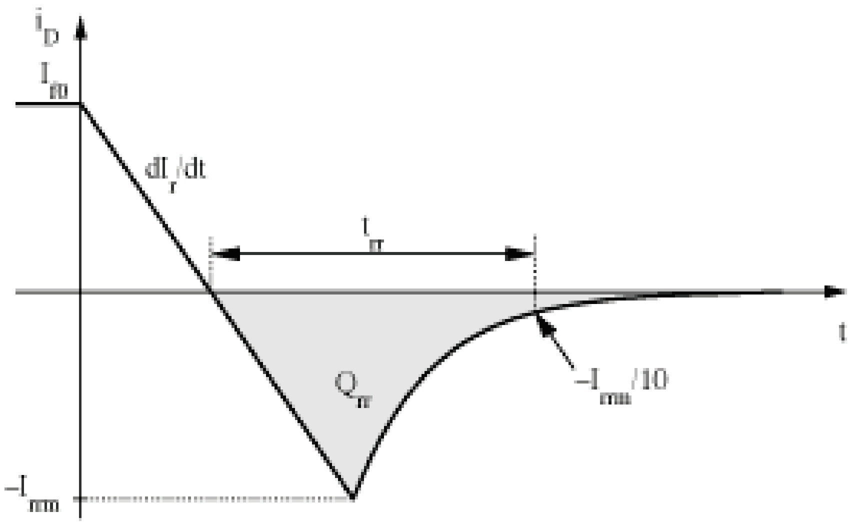
Criteria

- Diode's maximum reverse voltage $V_r > |V_{D,max}|$ (with margin)
- Diode's nominal current $I_D > I_{D,max}$ (with margin)
  - Can be $I_D < \langle i_{D,max} \rangle$ (but other limitations also on $I_{D,max}$)
- Diode's forward voltage V_F "low enough"
  - Related to power losses
- Recovery time t_{rr} compatible with f_s
- Temperature can be higher than 25 °C
- Package/Modules
- Cost

Schottky diodes

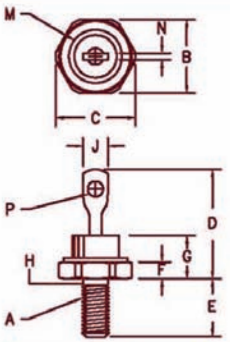
- Special diodes (metal-semiconductor junction)
- Advantages
  - Extremely fast
- Disadvantages
  - Low reverse voltage
  - High reverse leakage current

Recovery time (diode's turn-off)



Example of a power diode datasheet
(www.microsemi.com)

Ultra Fast Recovery Rectifiers
UFR30, 31 & 32



Notes:
1. 10–32 UNF3A threads
2. Full threads within 2 1/2 threads Standard Polarity: Stud is Cathode Reverse Polarity: Stud is Anode

Dim.	Inches Minimum	Inches Maximum	Millimeter Minimum	Millimeter Maximum	Notes
A	----	----	----	----	1
B	.424	.437	10.77	11.10	
C	----	.505	----	12.82	
D	.600	.800	15.24	20.32	
E	.422	.453	10.72	11.50	
F	.075	.175	1.91	4.44	
G	----	.405	----	10.29	
H	.163	.189	4.15	4.80	2
J	.100	.310	2.54	7.87	
M	----	.350	----	8.89	Dia.
N	.020	.065	.510	1.65	
P	.070	.100	1.78	2.54	Dia.

DO203AA (DO4)

Microsemi Catalog Number		Working Peak Reverse Voltage	Peak Reverse Voltage
UFR3010*		100V	100V
UFR3015*		150V	150V
UFR3020*	UFR3120*	200V	200V
	UFR3130*	300V	300V
	UFR3140*	400V	400V
	UFR3150*	500V	500V
UFR3260*		600V	600V
UFR3270*		700V	700V
UFR3280*		800V	800V

*Add Suffix R For Reverse Polarity

- Ultra Fast Recovery Rectifier
- 175°C Junction Temperature
- V_{RRM} 100 to 800V
- High Reliability
- 30 Amps current rating
- t_{RR} 35 to 60 nsec maximum

Electrical Characteristics

		UFR30	UFR31	UFR32	
Average forward current	$I_{F(AV)}$	30A	30A	30A	Square wave, $R_{\theta JC}$ = 1.8°C/W
Case Temperature	T_C	127°C	110°C	107°C	
Maximum surge current	I_{FSM}	500A	400A	300A	8.3 ms, half sine, T_J = 175°C
Max peak forward voltage	V_{FM}	.975V	1.25V	1.35V	I_{FM} = 30A: T_J = 25°C*
Max reverse recovery time	t_{RR}	35 ns	50 ns	60 ns	1/2A, 1A, 1/4A, T_J = 25°C
Max peak reverse current	I_{RM}	———	1.0 mA	———	V_{RRM}, T_J = 125°C
Max peak reverse current	I_{RM}	———	15 μA	———	V_{RRM}, T_J = 25°C
Typical Junction Capacitance	C_J	140 pF	115 pF	100 pF	V_R = 10V, f = 1Mhz, T_J = 25°C

*Pulse test: Pulse width 300 μsec, Duty cycle 2%

Thermal and Mechanical Characteristics

Storage temp range	T_{STG}	−65°C to 175°C	
Operating junction temp range	T_J	−65°C to 175°C	
Max thermal resistance	$R_{\theta JC}$	1.8°C/W	Junction to Case
Typical thermal resistance	$R_{\theta JC}$	1.3°C/W	Junction to Case
Typical thermal resistance (greased)	$R_{\theta CS}$	0.4°C/W	Case to sink
Mounting torque		12–15 inch pounds	
Weight		0.2 ounces (6.0 grams) typical	

LAWRENCE Microsemi
6 Lake Street
Lawrence, MA 01841
PH: (978) 620–2600
FAX: (978) 689–0803
www.microsemi.com

05–07–07 Rev. 2

Figure 1
Typical Forward Characteristics

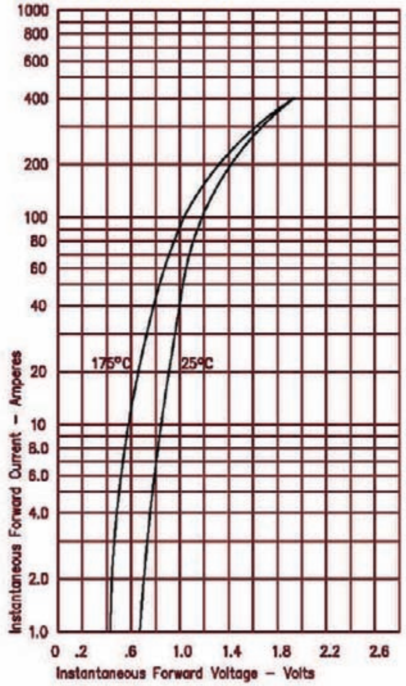


Figure 3
Typical Junction Capacitance

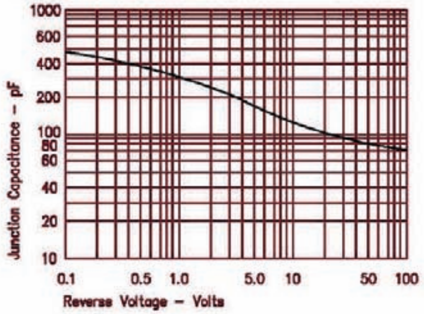


Figure 4
Forward Current Derating

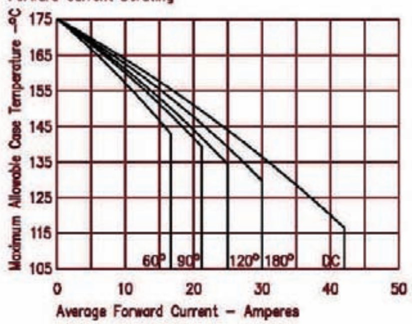


Figure 2
Typical Reverse Characteristics

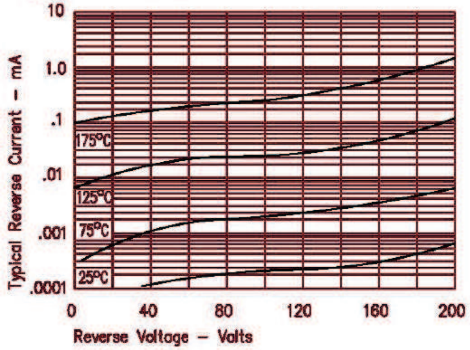
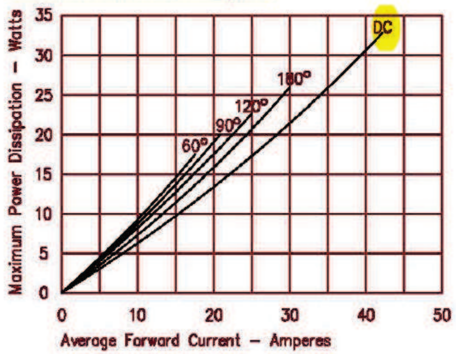
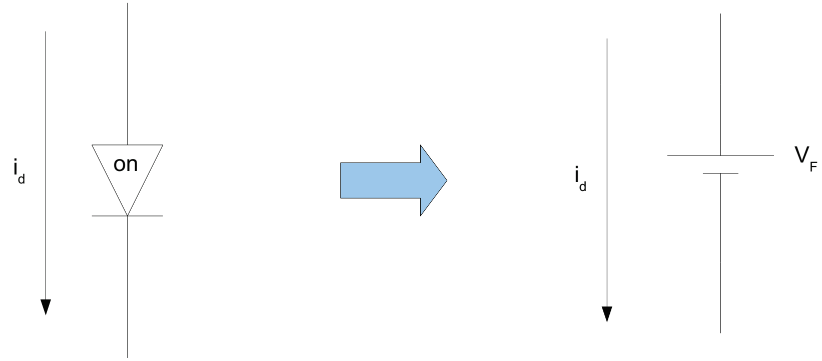


Figure 5
Maximum Forward Power Dissipation



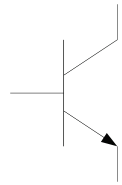
Conduction losses



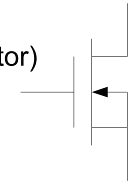
$$Pd = V_F * \langle i_d \rangle$$

Power switches

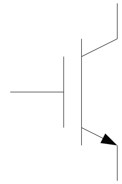
- **BJT** (Bipolar Junction Transistor)
  - (Almost) not used anymore



- **MOSFET** (Metal Oxide Semiconductor Field Effect Transistor)
  - Currently used (lower voltage, higher frequency)
  -



- **IGBT** (Insulated Gate Bipolar Transistor)
  - Currently used (higher voltage, lower frequency)



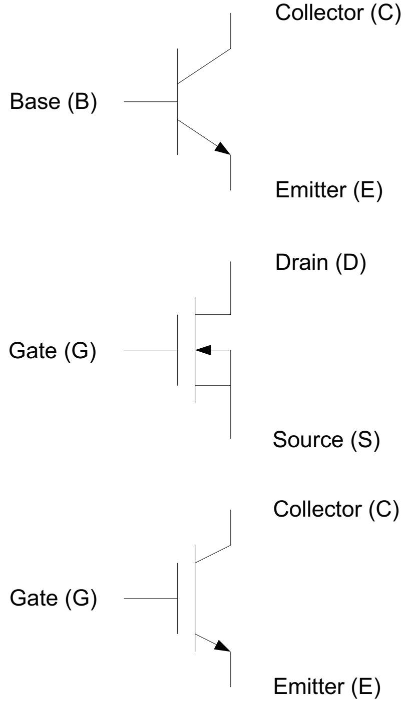
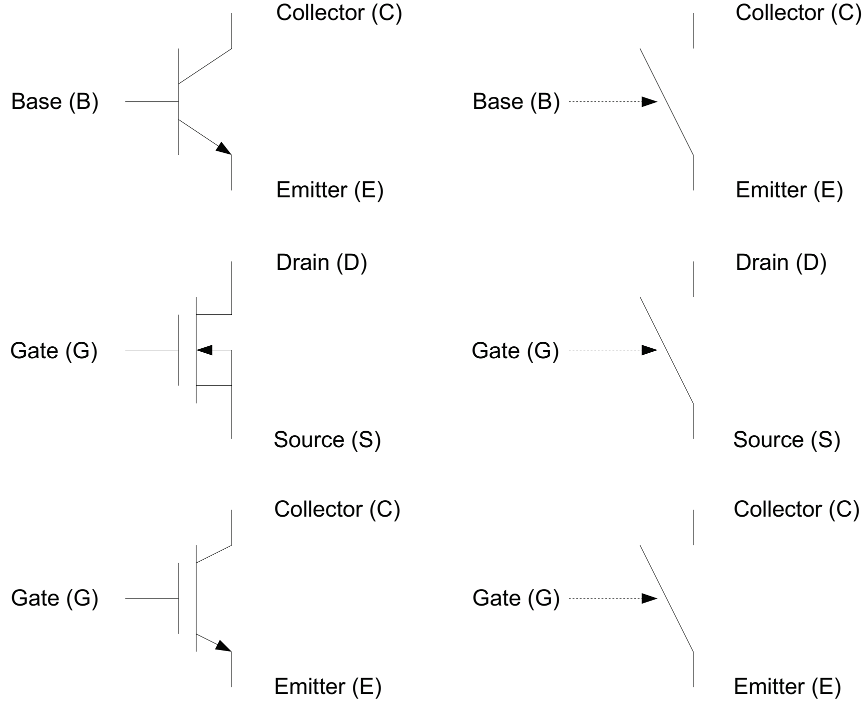
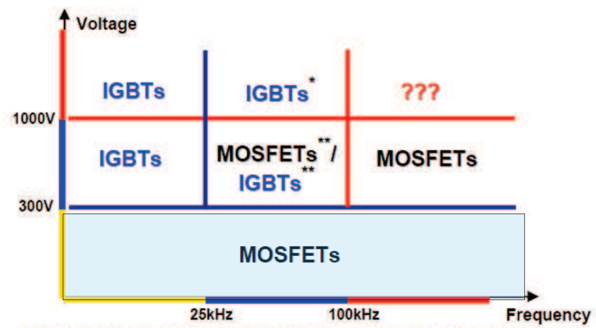


Table 1: Charge Technology Comparison

COMPARISON CRITERIA	BJTs	MOSFETs	IGBTs
Drive Method	Current	Voltage	Voltage
Drive Circuit Complexity	High	Low	Low
Switching Speeds	Slow (µs)	Very Fast (ns)	Medium
Switching Frequencies	few kHz	Up to 1 MHz	< 50 kHz
Forward Voltage Drop	Low	Medium	Low
Current Carrying Capability	High	Medium	High
Breakdown Voltage	High	Medium (<1500V)	Very High (~5000V)



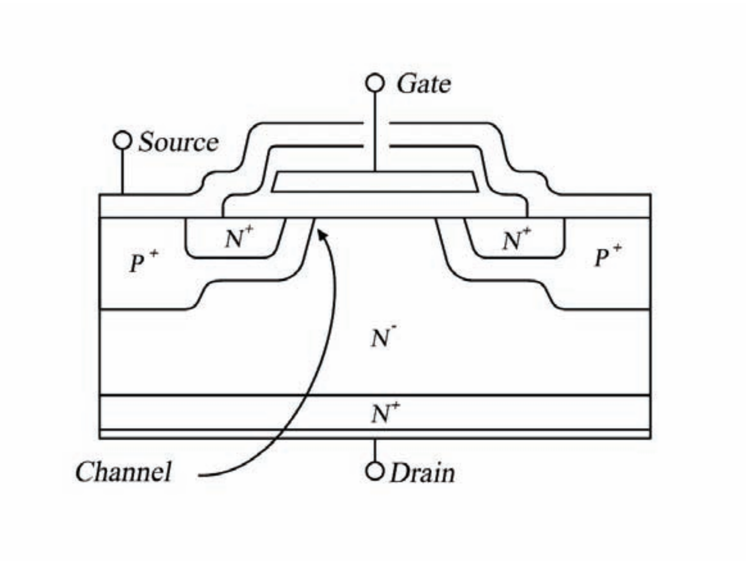


* Switching frequencies above 25kHz only achievable with soft switching techniques
** The choice of MOSFETs vs IGBTs depends mainly on the application requirements

Fig. 6: MOSFETs and IGBTs Application Range

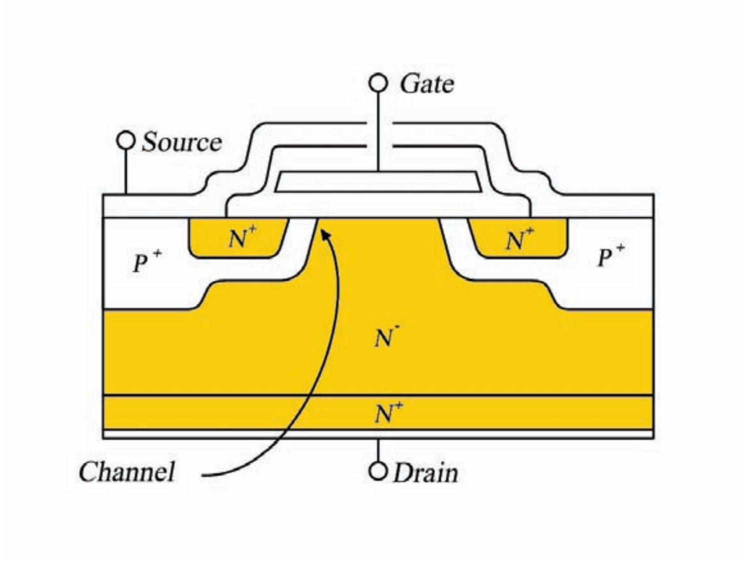
Technical Note, High Efficiency Power Switches, Nasser Kutkut, PowerDesigners, LLC – Madison, WI

Structure of a MOSFET



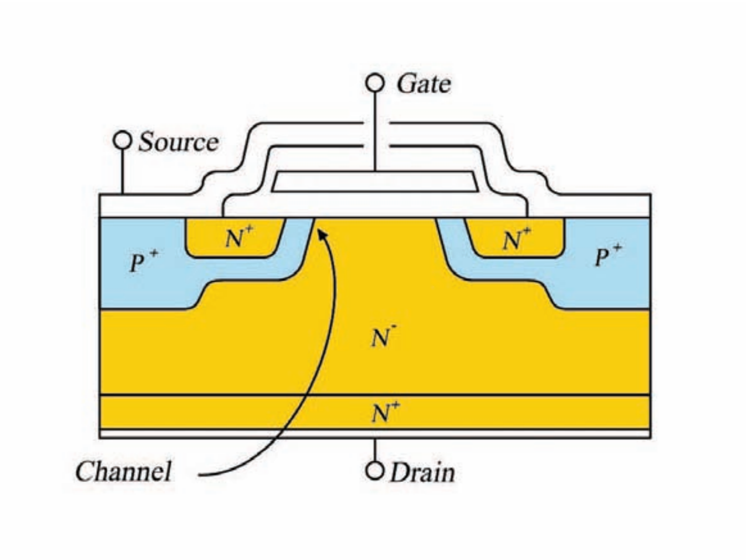
http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

Structure of a MOSFET



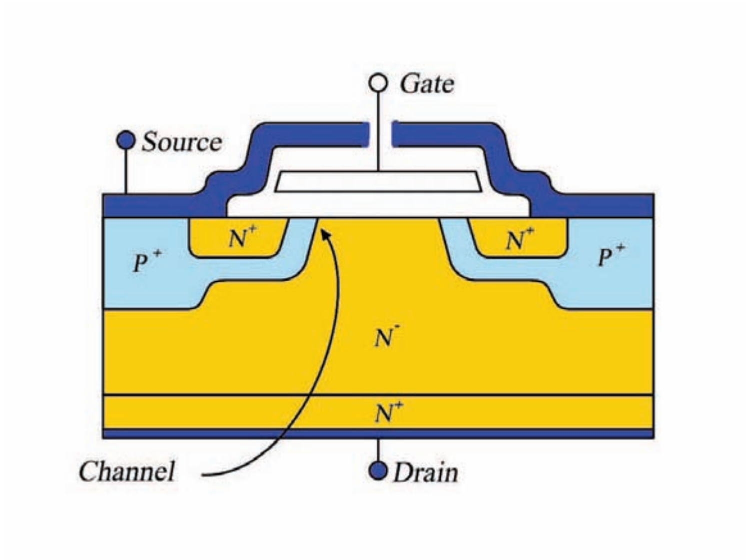
http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

Structure of a MOSFET



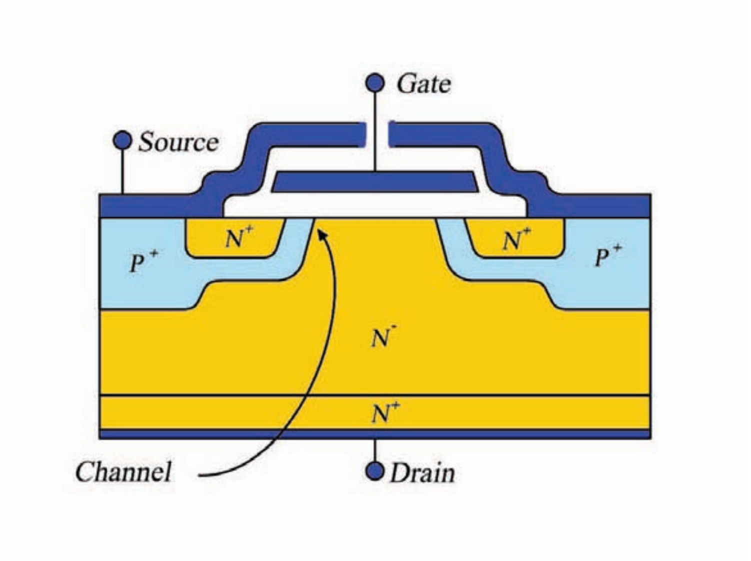
http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

Structure of a MOSFET



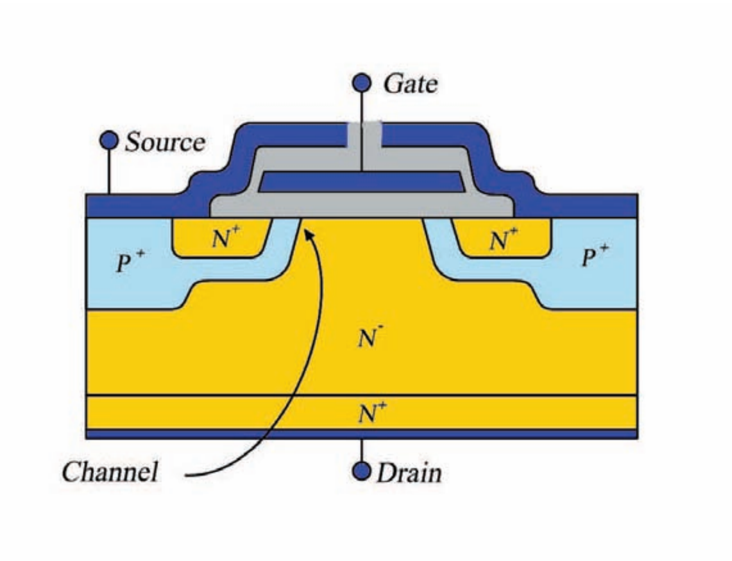
http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

Structure of a MOSFET



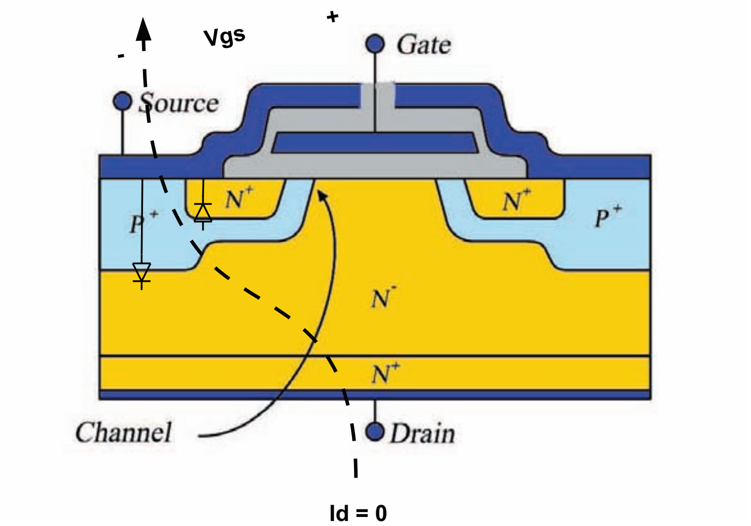
http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

Structure of a MOSFET



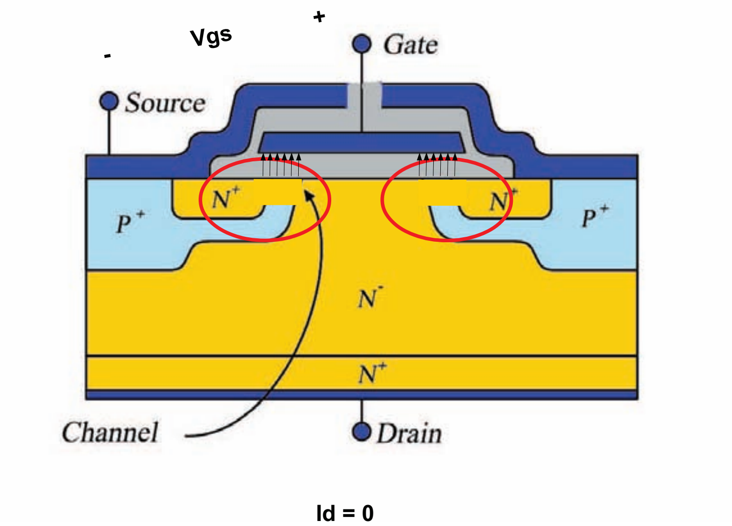
http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

Operation of a MOSFET *Vgs < Vth => OFF*



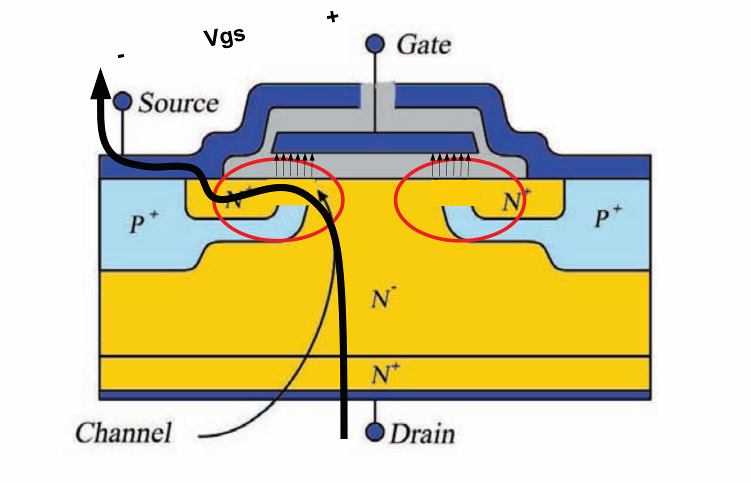
http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

Operation of a MOSFET *Vgs > Vth => On*



http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

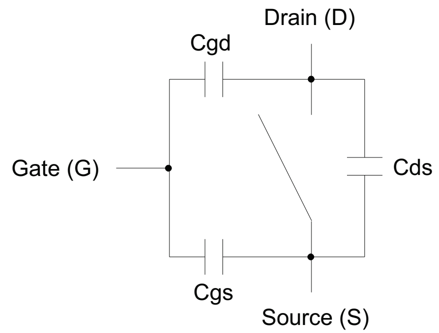
Operation of a MOSFET
Vgs > Vth => On



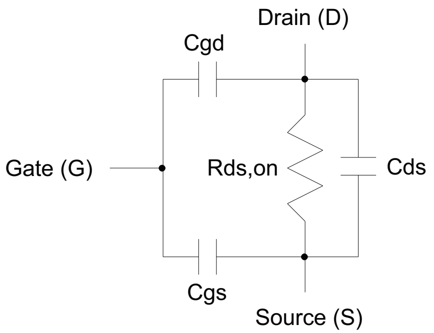
Id = 0

http://upload.wikimedia.org/wikipedia/commons/1/1e/Vdmos_cross_section_en.svg, Modified by Benoit Bidoggia

Off behavior (Vgs < Vth)

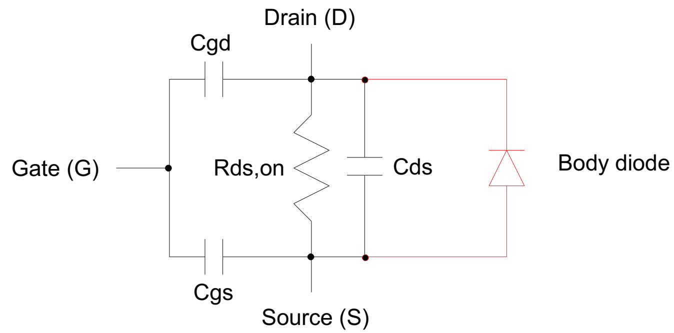


On behavior (Vgs > Vth)

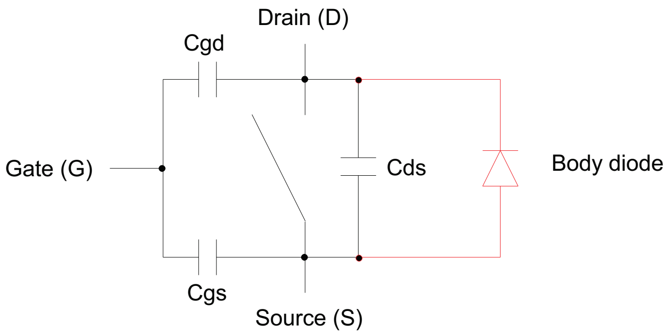


Rds,on = f(Vgs)

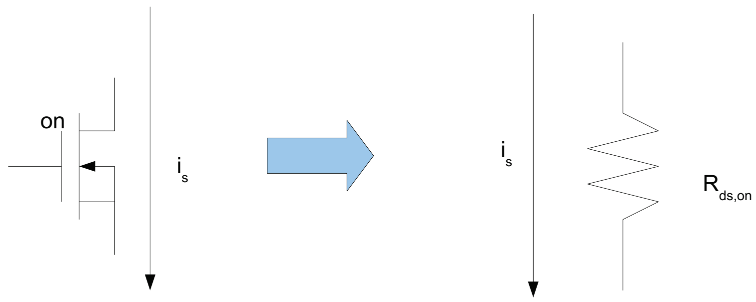
On behavior (Vgs > Vth)



Off behavior (Vgs < Vth)



Conduction losses



$$\boxed{P_s = R_{ds,on} * I_{s,rms}^2}$$

MOSFETs

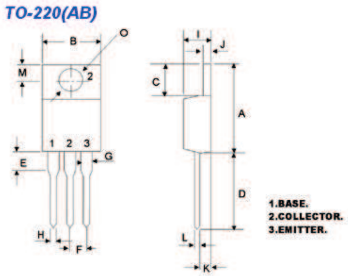
- Two types:
  - N-channel (most commonly used)
  - P-channel (less commonly used)
- Manufacturers:
  - International Rectifiers www.irf.com
  - Microsemi www.microsemi.com
  - many others...

Criteria

- Mosfet's breakdown voltage $V_{DSS} > |V_{S,max}|$ (with margin)
- Mosfet's max drain current $I_D < I_{S,max}$ (with margin)
- $R_{DS,on}$ "low enough"
  - Related to power losses
- Temperature can be higher than 25 °C
- Package/Modules
- Cost

TO220 Package

- Through hole component

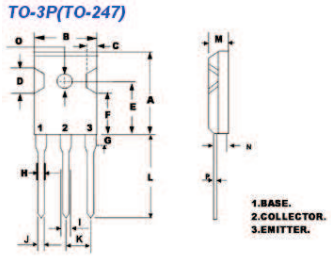


DIM	MILLIMETERS	
	MIN	MAX
A	14.68	15.32
B	9.78	10.42
C	5.02	6.52
D	13.06	14.62
E	3.57	4.07
F	2.42	2.66
G	1.12	1.36
H	0.72	0.96
I	4.22	4.98
J	1.14	1.36
K	2.20	2.97
L	0.33	0.55
M	2.48	2.98
O	3.70	3.90

http://www.mospec.com.tw/package.htm

TO247 Package

- Through hole component

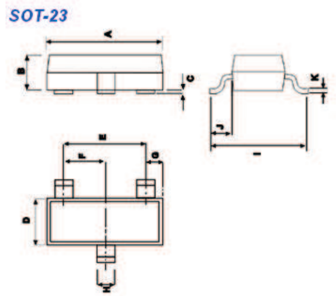


DIM	MILLIMETERS	
	MIN	MAX
A	20.63	22.38
B	15.38	16.20
C	1.90	2.70
D	5.10	6.10
E	14.81	15.22
F	11.72	12.84
G	4.20	4.50
H	1.82	2.46
I	2.92	3.23
J	0.89	1.53
K	5.26	5.66
L	18.50	21.50
M	4.68	5.36
N	2.40	2.80
O	3.25	3.65
P	0.55	0.70

http://www.mospec.com.tw/package.htm

SOT23 Package

- Surface Mounted Technology (SMT)

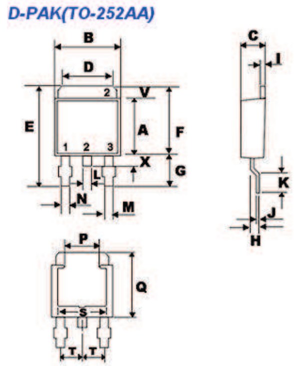


DIM	MILLIMETERS	
	MIN	MAX
A	2.67	3.05
B	0.64	1.30
C	---	0.10
D	1.20	1.60
E	1.70	2.10
F	0.89	1.05
G	0.35	0.65
H	0.30	0.54
I	2.10	2.75
J	0.55	0.64
K	0.085	0.18

http://www.mospec.com.tw/package.htm

D-PAK Package

- Surface Mounted Technology (SMT)

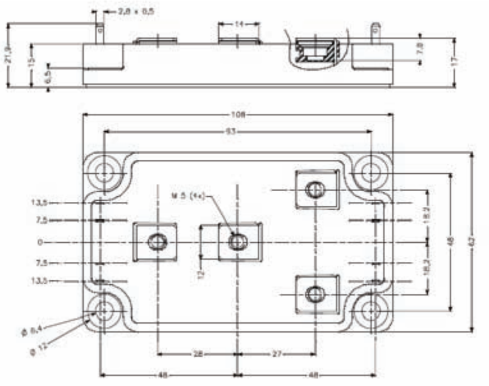


DIM	MILLIMETERS	
	MIN	MAX
A	5.40	5.60
B	6.30	6.70
C	2.20	2.40
D	5.20	5.50
E	9.00	10.00
F	6.60	7.00
G	2.40	3.00
H	0.90	1.50
I	0.45	0.50
J	0.45	0.60
K	0.90	1.50
L	0.70	0.90
M	0.50	0.70
N	0.60	0.90
P	2.70	3.10
Q	5.00	5.40
S	4.80	5.20
T	---	2.30
V	1.20	1.40
X	0.80	1.20

http://www.mospec.com.tw/package.htm

Specific modules

- Connection with screws (very high currents)



http://www.microsemi.com/PackagePDFs/SP6Mechanical.pdf

Example of a MOSFET datasheet
(www.irf.com)

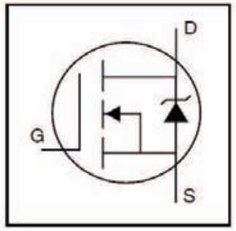
International
IOR Rectifier

PD - 95456

IRLIZ44NPbF

HEXFET® Power MOSFET

- Logic-Level Gate Drive
- Advanced Process Technology
- Isolated Package
- High Voltage Isolation = 2.5KVRMS ⑤
- Sink to Lead Creepage Dist. = 4.8mm
- Fully Avalanche Rated
- Lead-Free



V_{DSS} = 55V

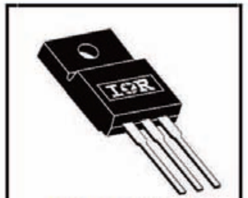
$R_{DS(on)}$ = 0.022Ω

I_D = 30A

Description

Fifth Generation HEXFETs from International Rectifier utilize advanced processing techniques to achieve extremely low on-resistance per silicon area. This benefit, combined with the fast switching speed and ruggedized device design that HEXFET Power MOSFETs are well known for, provides the designer with an extremely efficient and reliable device for use in a wide variety of applications.

The TO-220 Fullpak eliminates the need for additional insulating hardware in commercial-industrial applications. The moulding compound used provides a high isolation capability and a low thermal resistance between the tab and external heatsink. This isolation is equivalent to using a 100 micron mica barrier with standard TO-220 product. The Fullpak is mounted to a heatsink using a single clip or by a single screw fixing.



TO-220 FULLPAK

Absolute Maximum Ratings

	Parameter	Max.	Units
I_D @ T_C = 25°C	Continuous Drain Current, V_{GS} @ 10V	30	A
I_D @ T_C = 100°C	Continuous Drain Current, V_{GS} @ 10V	22	
I_{DM}	Pulsed Drain Current ①⑥	160	
P_D @ T_C = 25°C	Power Dissipation	45	W
	Linear Derating Factor	0.3	W/°C
V_{GS}	Gate-to-Source Voltage	± 16	V
E_{AS}	Single Pulse Avalanche Energy②⑥	210	mJ
I_{AR}	Avalanche Current①⑥	25	A
E_{AR}	Repetitive Avalanche Energy①	4.5	mJ
dv/dt	Peak Diode Recovery dv/dt ③⑥	5.0	V/ns
T_J T_{STG}	Operating Junction and Storage Temperature Range	-55 to + 175	°C
	Soldering Temperature, for 10 seconds	300 (1.6mm from case)	
	Mounting torque, 6-32 or M3 screw	10 lbf•in (1.1N•m)	

Thermal Resistance

	Parameter	Typ.	Max.	Units
$R_{\theta JC}$	Junction-to-Case	——	3.3	°C/W
$R_{\theta JA}$	Junction-to-Ambient	——	65	

Electrical Characteristics @ T_J = 25°C (unless otherwise specified)

	Parameter	Min.	Typ.	Max.	Units	Conditions
$V_{(BR)DSS}$	Drain-to-Source Breakdown Voltage	55	–––	–––	V	V_{GS} = 0V, I_D = 250μA
$\Delta V_{(BR)DSS}/\Delta T_J$	Breakdown Voltage Temp. Coefficient	–––	0.070	–––	V/°C	Reference to 25°C, I_D = 1mA ⑤
$R_{DS(on)}$	Static Drain-to-Source On-Resistance	–––	–––	0.022	Ω	V_{GS} = 10V, I_D = 17A ④
		–––	–––	0.025		V_{GS} = 5.0V, I_D = 17A ④
		–––	–––	0.035		V_{GS} = 4.0V, I_D = 14A ④
$V_{GS(th)}$	Gate Threshold Voltage	1.0	–––	2.0	V	V_{DS} = V_{GS}, I_D = 250μA
g_{fs}	Forward Transconductance	21	–––	–––	S	V_{DS} = 25V, I_D = 25A ⑤
I_{DSS}	Drain-to-Source Leakage Current	–––	–––	25	μA	V_{DS} = 55V, V_{GS} = 0V
		–––	–––	250		V_{DS} = 44V, V_{GS} = 0V, T_J = 150°C
I_{GSS}	Gate-to-Source Forward Leakage	–––	–––	100	nA	V_{GS} = 16V
	Gate-to-Source Reverse Leakage	–––	–––	-100		V_{GS} = -16V
Q_g	Total Gate Charge	–––	–––	48	nC	I_D = 25A
Q_{gs}	Gate-to-Source Charge	–––	–––	8.6		V_{DS} = 44V
Q_{gd}	Gate-to-Drain ("Miller") Charge	–––	–––	25		V_{GS} = 5.0V, See Fig. 6 and 13 ④⑤
$t_{d(on)}$	Turn-On Delay Time	–––	11	–––	ns	V_{DD} = 28V
t_r	Rise Time	–––	84	–––		I_D = 25A
$t_{d(off)}$	Turn-Off Delay Time	–––	26	–––		R_G = 3.4Ω, V_{GS} = 5.0V
t_f	Fall Time		15			R_D = 1.1Ω, See Fig. 10 ④⑤
L_D	Internal Drain Inductance	—	4.5	—	nH	Between lead, 6mm (0.25in.)
L_S	Internal Source Inductance	—	7.5	—		from package and center of die contact
C_{iss}	Input Capacitance	–––	1700	–––	pF	V_{GS} = 0V
C_{oss}	Output Capacitance	–––	400	–––		V_{DS} = 25V
C_{rss}	Reverse Transfer Capacitance	–––	150	–––		f = 1.0MHz, See Fig. 5 ⑤
C	Drain to Sink Capacitance	–––	12	–––		f = 1.0MHz

Source-Drain Ratings and Characteristics

	Parameter	Min.	Typ.	Max.	Units	Conditions
I_S	Continuous Source Current (Body Diode)	–––	–––	30	A	MOSFET symbol showing the
I_{SM}	Pulsed Source Current (Body Diode) ①⑤	–––	–––	160		integral reverse p-n junction diode.
V_{SD}	Diode Forward Voltage	–––	–––	1.3	V	T_J = 25°C, I_S = 17A, V_{GS} = 0V ④
t_{rr}	Reverse Recovery Time	–––	80	120	ns	T_J = 25°C, I_F = 25A
Q_{rr}	Reverse RecoveryCharge	–––	210	320	μC	di/dt = 100A/μs ④⑤
t_{on}	Forward Turn-On Time	Intrinsic turn-on time is negligible (turn-on is dominated by L_S+L_D)				

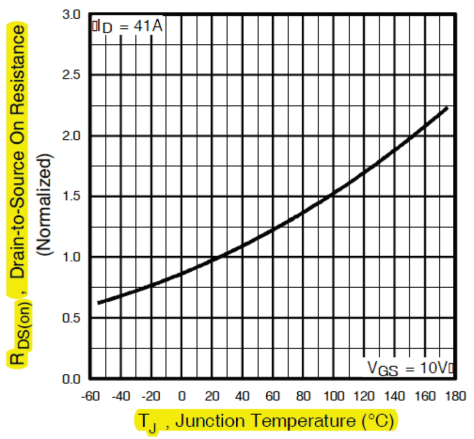


Fig 4. Normalized On-Resistance Vs. Temperature

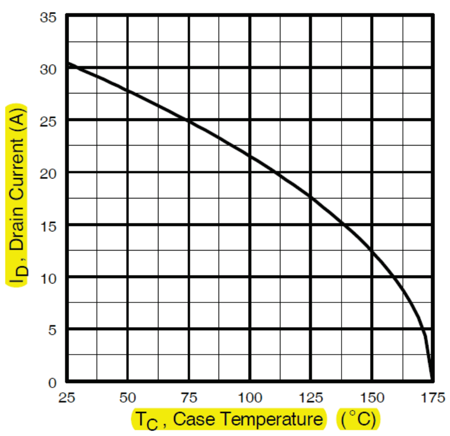


Fig 9. Maximum Drain Current Vs. Case Temperature

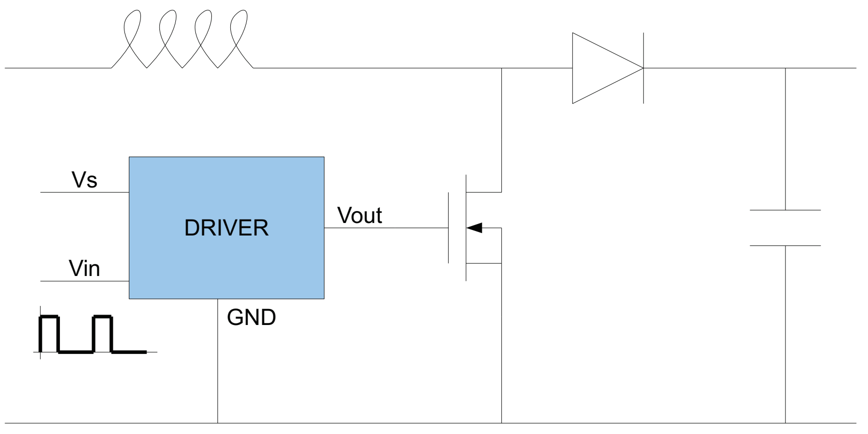
MOSFET Drivers

- Mosfet off if $V_{gs} < V_{th}$
- Mosfet turns on if $V_{gs} > V_{th}$
- Then, $R_{ds,on} = f(V_{gs})$ $\quad V_{gs}\uparrow \leftrightarrow R_{ds,on}\downarrow$

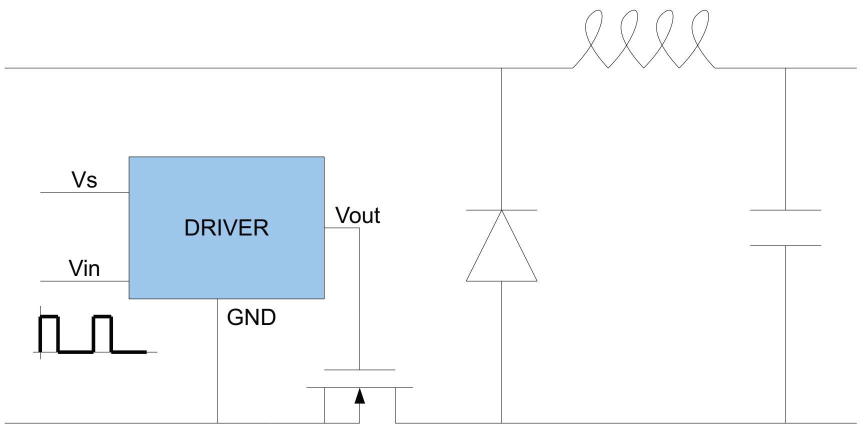
- Turning on: V_{gs} = 0 → 10 V (charging eq. capacitor)
- Turning off: V_{gs} = 10 → 0 V (discharging eq. cap.)

- MOSFET drivers provides energy to turn on/off

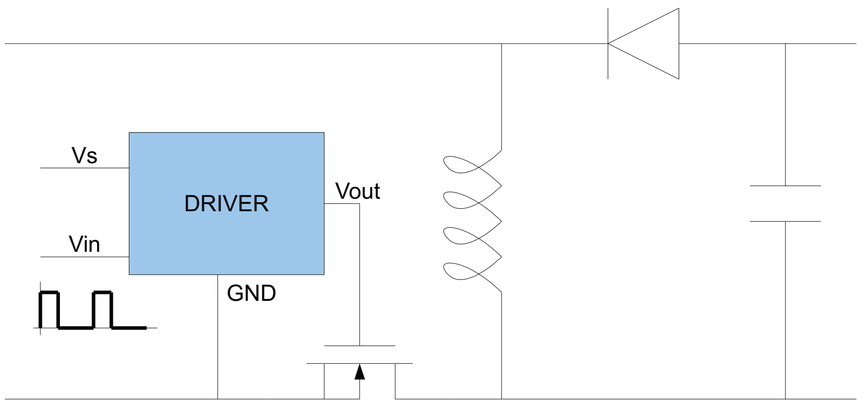
Example of connection of a driver (Boost)



Example of connection of a driver (Buck)



Example of connection of a driver (Buck-boost)



Drivers

- Manufacturers:
  - Microchip www.microchip.com
  - International rectifier www.irf.com
  - Maxim www.maxim-ic.com
  - Micrel www.micrel.com
  - many others...
- Peak current: $I_{pk} = \frac{dQ}{dt}$

Criteria

- High-side/Low-side
- Peak output current
- Max supply voltage
- Temperature can be higher than 25 °C
- Package/Modules
- Cost

Example of a MOSFET driver datasheet
(www.micrel.com)

MICREL®

MIC4126/27/28

Dual 1.5A-Peak Low-Side MOSFET Drivers in Advanced Packaging

General Description

The MIC4126, MIC4127, and MIC4128 family are highly-reliable dual 1.5A low-side MOSFET drivers fabricated on Micrel's BiCMOS/DMOS process. The devices feature low power consumption and high efficiency. The MIC4126/27/28 translate TTL or CMOS input logic levels to output voltage levels that swing within 25mV of the positive supply or ground whereas comparable bipolar devices are capable of swinging only to within 1V of the supply. The MIC4126/7/8 is available in three configurations: dual inverting, dual non-inverting, and complimentary output.

The MIC4126/27/28 offer pin-compatible as well as smaller footprint replacements for the MIC4426/27/28 with improved packaging and electrical performance. The MIC4126/27/28 are available in exposed pad, EPAD, SOIC-8L and MSOP-8L options as well as a small-size 3mm x 3mm MLF™-8L option. The devices have an input operating range of 4.5V to 20V.

Primarily intended for driving power MOSFETs, MIC4426/7/8 drivers are suitable for driving other loads (capacitive, resistive, or inductive) which require low-impedance, high peak current, and fast switching time. The devices can withstand up to 500mA of reverse current (either polarity) without latching and up to 5V noise spikes (either polarity) on ground pins.

Data sheets and support documentation can be found on Micrel's web site at www.micrel.com.

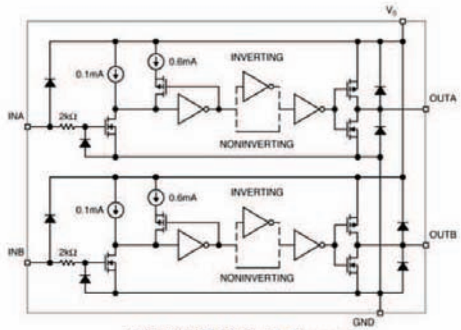
Features

- Dual 1.5A-peak drivers
- 4.5V to 20V operating range
- Exposed backside pad packaging reduces heat
  - ePAD SOIC-8L (θ_{JA} = 58°C/W)
  - ePAD MSOP-8L (θ_{JA} = 60°C/W)
  - 3mm x 3mm MLF™-8L (θ_{JA} = 60°C/W)
- Bipolar/CMOS/DMOS construction
  - 25mV maximum output offset from supply or ground
- Latch-up protection to >200mA reverse current
- Switches 1000pF in 25ns
- Logic-input threshold independent of supply voltage
- Logic-input protection to –5V
- 6pF typical equivalent input capacitance
- Dual inverting, dual non-inverting, and complementary configurations
- -40°C to +125°C operating junction temperature range

Applications

- DC/DC converters
- Motor drivers
- Clock line driver

Functional Diagram



MIC4126/27/28 Block Diagram

Absolute Maximum Ratings (1)

Supply Voltage (V_S)	+24V
Input Voltage (V_{IN})	V_S + 0.3V to GND – 5V
Junction Temperature (T_J)	150°C
Storage Temperature	–65°C to +150°C
Lead Temperature (10 sec.)	300°C
ESD Rating, **Note 3**	

Operating Ratings (2)

Supply Voltage (V_S)	+4.5V to +20V
Temperature Range (T_J)	–40°C to +125°C
Package Thermal Resistance	
3X3 MLF™ θ_{JA}	60°C/W
EPAD MSOP-8L θ_{JA}	60°C/W
EPAD SOIC-8L θ_{JA}	58°C/W

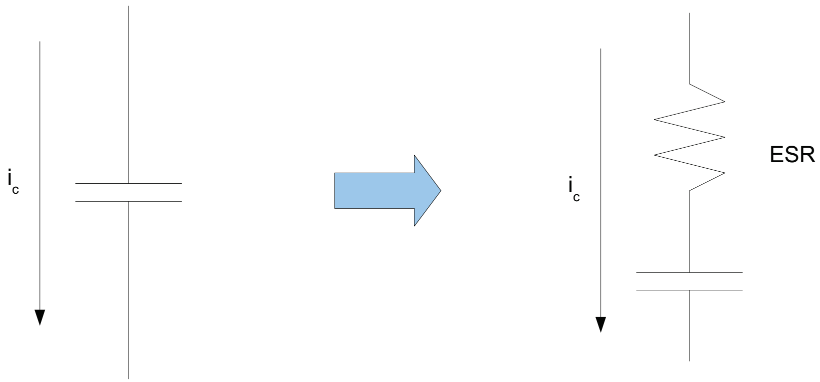
Capacitors

- One of the most unreliable components
- Types:
  - Aluminium electrolytic (cheap but unreliable)
  - Film (more expensive but more reliable)
  - Ceramic
  - Tantalum
- Manufacturers:
  - Kemet, AVX...

Criteria

- Rated voltage > $V_{c,max}$
- Maximum rms current > $I_{cRMS,max}$
- Peak current > $I_{c,max}$
- ESR (Eq. Series Resistance) “low enough”
  - Related to power losses
- Cost

(Conduction) Losses



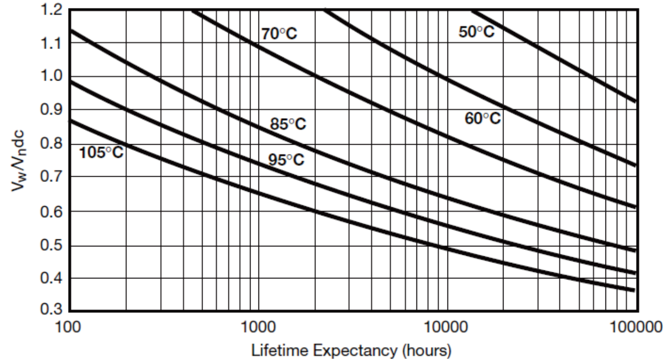
$$\boxed{P_c = ESR*I_{c,rms}^2}$$

Example of a Capacitor datasheet
(www.avx.com)

RATINGS AND PART NUMBER REFERENCE – POLYESTER DIELECTRIC

Part Number	Capacitance (µF)	Case Style	I_{rms} max. (A)	R_s (mΩ)	R_{th} (°C/W)	Typical Weight (g)
$V_n$dc 75V Vrms max.: 45 volts Voltage Code: D						
FFB14D0336K--	33	PO	3	3	40.7	15
FFB24D0476K--	47	18	4.3	2	33.3	20
FFB34D0686K--	68	19	6.2	1.7	29.9	25
FFB44D0826K--	82	26	7.4	1.6	26.7	32
FFB54D0117K--	110	R68 (2 terminals)	10	1.4	22.9	40
FFB54D0117KJC	110	R68 (4 terminals)	10	1.4	22.9	40
$V_n$dc 100V Vrms max.: 60 volts Voltage Code: E						
FFB14E0206K--	20	PO	2.6	3	40.5	15
FFB24E0276K--	27	18	3.5	2.5	33.3	20
FFB34E0396K--	39	19	5	2	29.8	25
FFB44E0476K--	47	26	6	1.7	26.6	32
FFB54E0686K--	68	R68 (2 terminals)	9	1.4	22.8	40
FFB54E0686KJC	68	R68 (4 terminals)	9	1.4	22.8	40
$V_n$dc 300V Vrms max.: 90 volts Voltage Code: H						
FFB14H0755K--	7.5	PO	2.4	16	40.7	15
FFB24H0116K--	11	18	3.6	11	33.5	20
FFB34H0166K--	16	19	5.2	8	29.9	25
FFB44H0186K--	18	26	6	7	27.1	32
FFB54H0276K--	27	R68 (2 terminals)	9	5	22.9	40
FFB54H0276KJC	27	R68 (4 terminals)	9	5	22.9	40

LIFETIME EXPECTANCY vs VOLTAGE AND HOT SPOT TEMPERATURE – POLYESTER DIELECTRIC

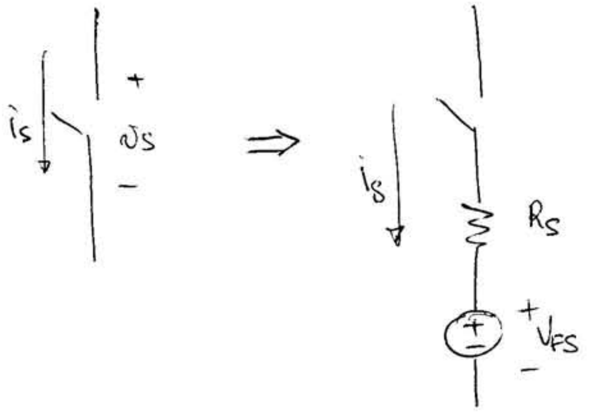


Vw = Permanent working or operating DC voltage.

LECTURE 5

Conduction losses

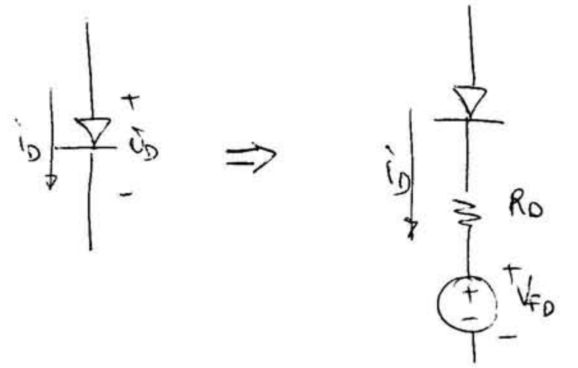
Switch



$$P_S = R_S I_{S,RMS}^2 + V_{FS} \langle i_S \rangle$$

[for a mosfet: $V_{FS} = 0$, $R_S = R_{DS,on}$]

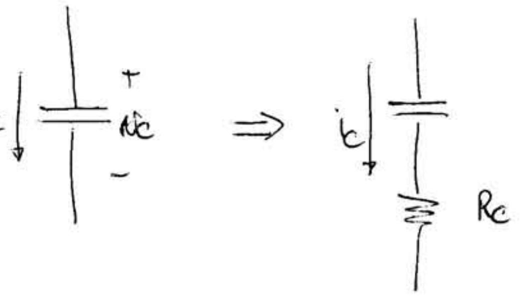
Diode



$$P_D = R_D I_{D,RMS}^2 + V_{FD} \langle i_D \rangle$$

[generally $R_D \simeq 0$]

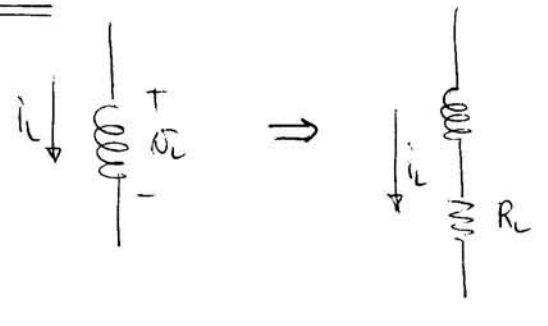
Capacitor



$$P_C = R_C \cdot I_{C,RMS}^2$$

[R_C is generally called ESR]

Inductor

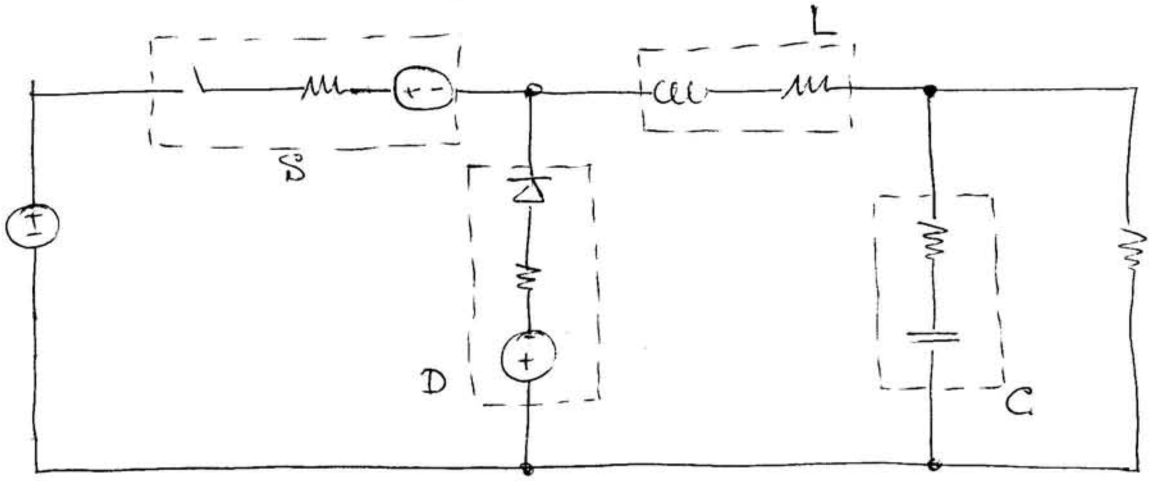


$$P_L = R_L \cdot I_{L,RMS}^2$$

$$\boxed{P_T = P_S + P_D + P_C + P_L}$$

Modified circuits

Buck

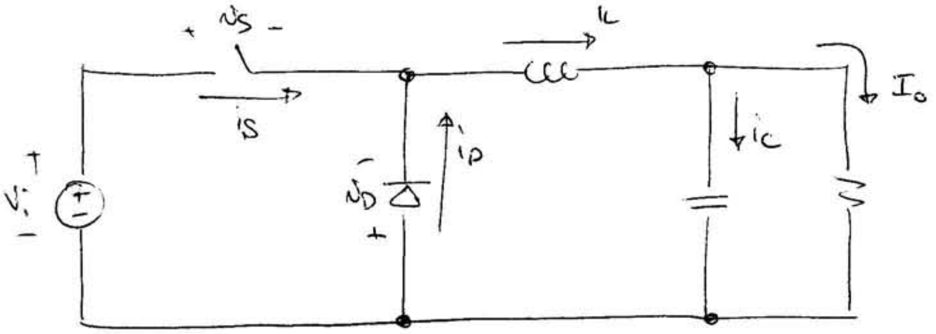


Hp: V_{FS}, V_{FD}, R_S, R_D, R_L, R_C don't significantly influence the behavior of the circuit.

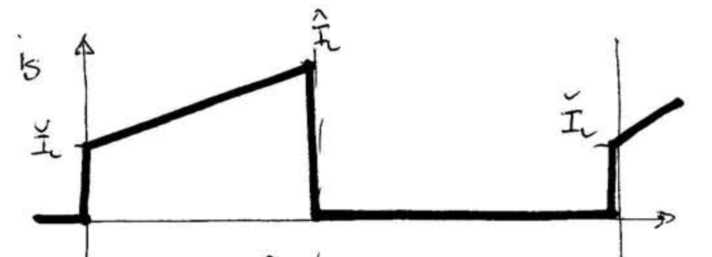
Switching losses

- Origins. – it takes time to turn off and on switches and diodes
  - power diodes need an extra negative current to be turned off

- Simplified analysis
- Switching losses depend on the circuit: ex w/ buck.



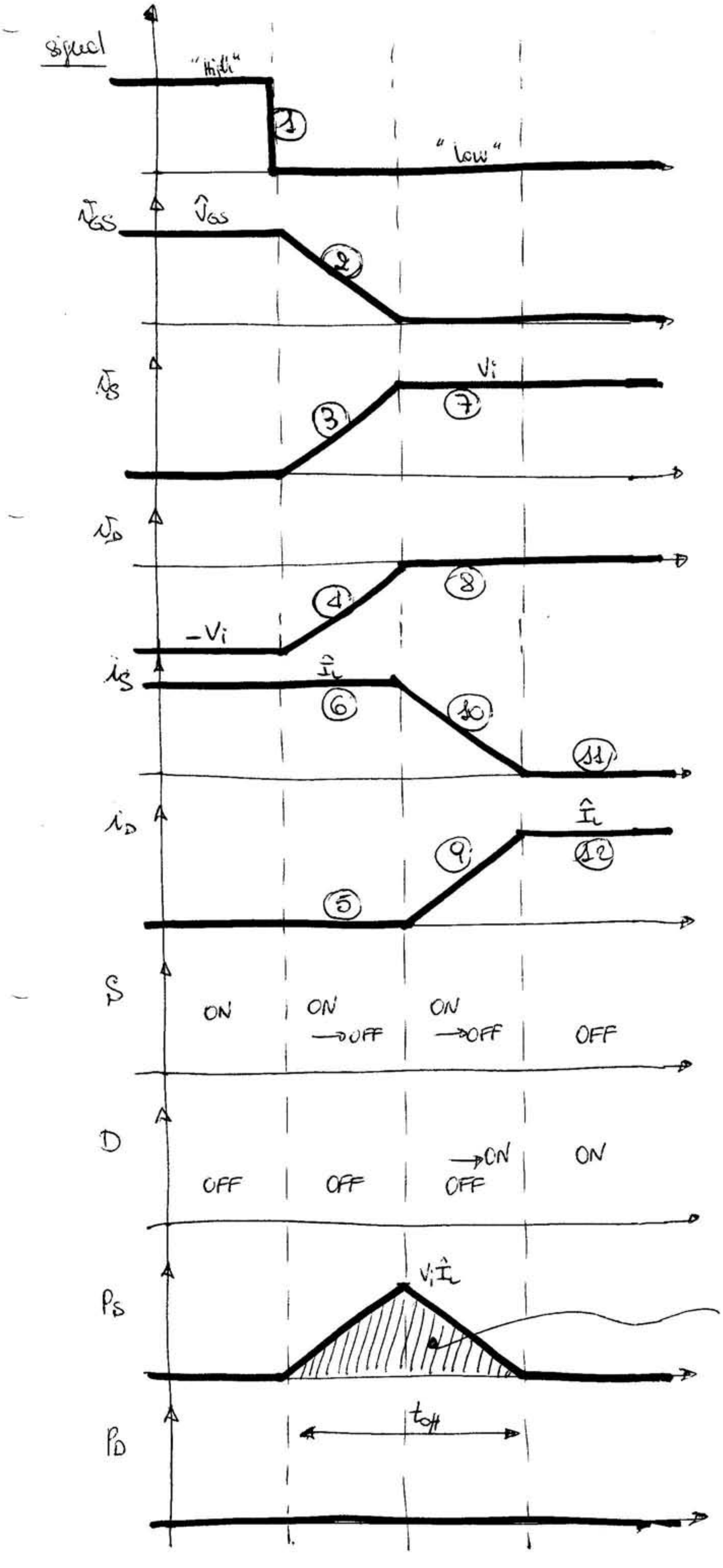
$v_D = -V_i + v_S$



S TURN-OFF } transient
D TURN-ON

S TURN-ON } transient
D TURN-OFF

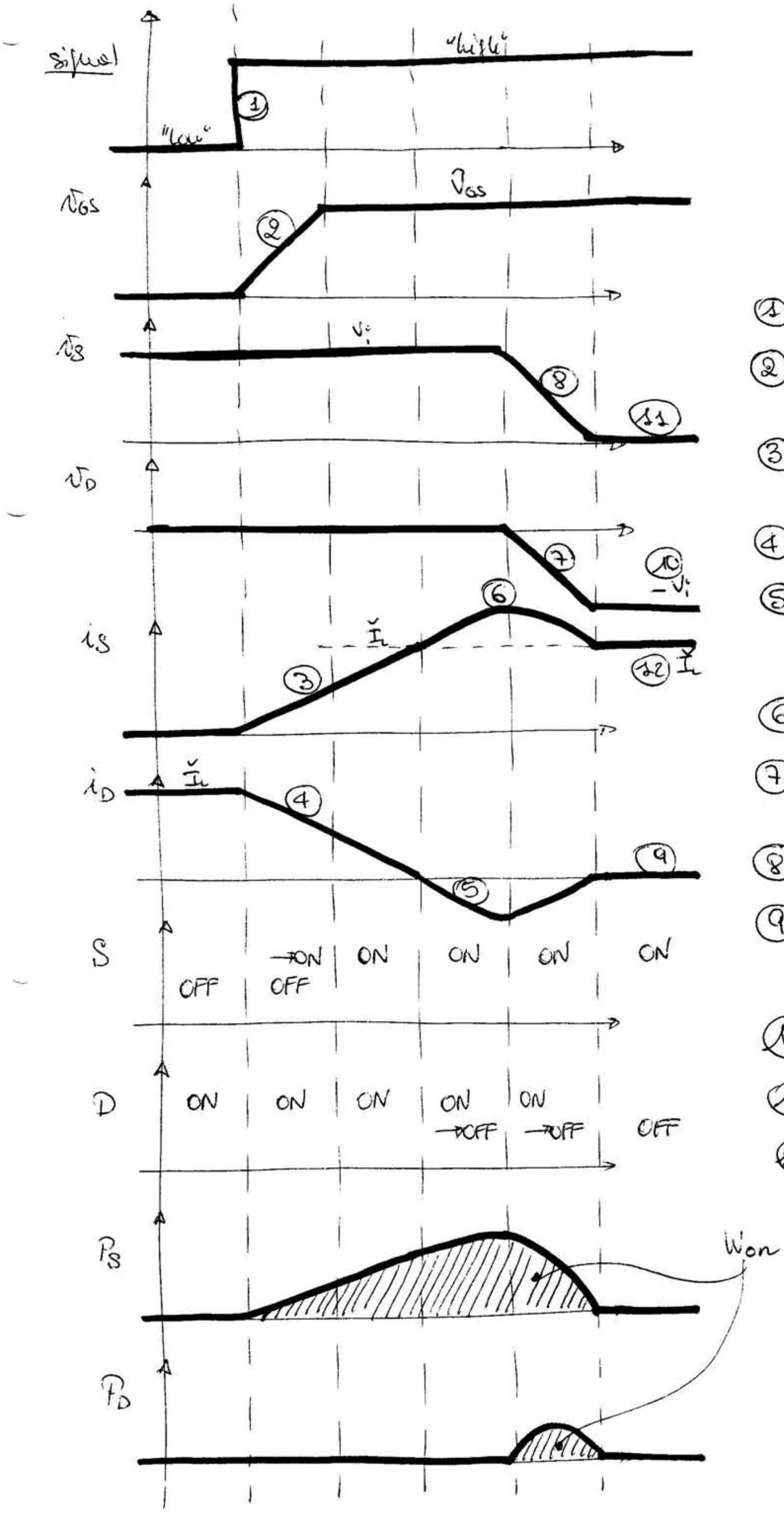
Switch turn-off / Diode turn-on transient



$$W_{off} = \frac{1}{2} V_i \hat{I}_L \cdot t_{off}$$

1. signal "High" → "Low"
2. Driver extracts charges → v_{GS} decreases linearly
3. R_{ds} increases, $v_S = R_{ds} \cdot \hat{I}_L$ increases
4. $v_D = -(V_i - v_S) = -V_i + v_S$
5. if $v_D < 0 \Rightarrow D_{off} \Rightarrow i_D = 0$
6. $i_S + i_D = i_L \Rightarrow i_S = i_L = \hat{I}_L$
7. $v_S = V_i$
8. $v_D = 0$: diode can turn-on
9. diode turns on
10. $i_S = i_L - i_D$
11. $i_S = 0 \rightarrow S_{off}$
12. $i_D = \hat{I}_L \rightarrow$ D fully on

Switch turn-on / Diode turn-off transient



1. Signal "low" → "high"
2. Driver injects charges → v_{gs} increases linearly
3. Switch starts conducting → i_s increases
4. $i_D = \check{I}_L - i_S$
5. i_D becomes negative to remove all the charges
6. $i_S > \check{I}_L$
7. the diode starts opening → $|v_D|$ increases
8. $v_S = v_D + V_i$ → v_S decreases
9. $i_D = 0$ recovery finished ↳ D is fully off
10. $v_D = -V_i$
11. $v_S = 0$
12. $i_S = \check{I}_L$ → S is fully on

$$W_T = W_{on} + W_{off} + W_{anc}$$

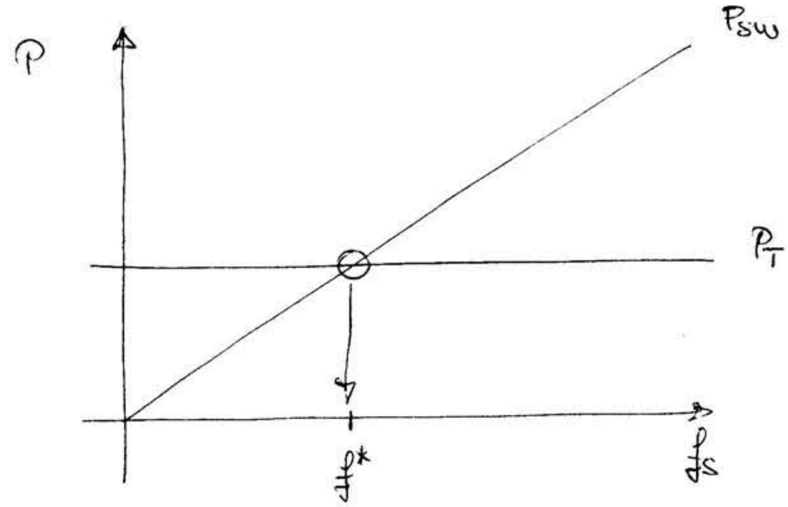
W_{anc}: ancillary losses (mosfet drivers)

$$\boxed{P_{SW} = \frac{W_T}{T_S} = W_T \cdot f_S}$$

Total losses

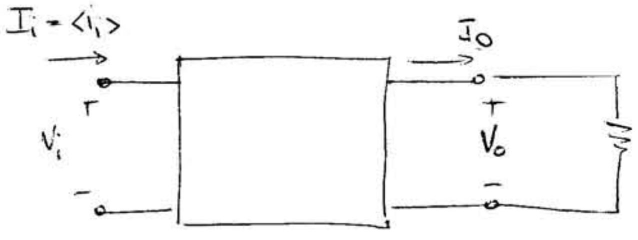
$$P_{TL} = P_T + P_{SW}$$

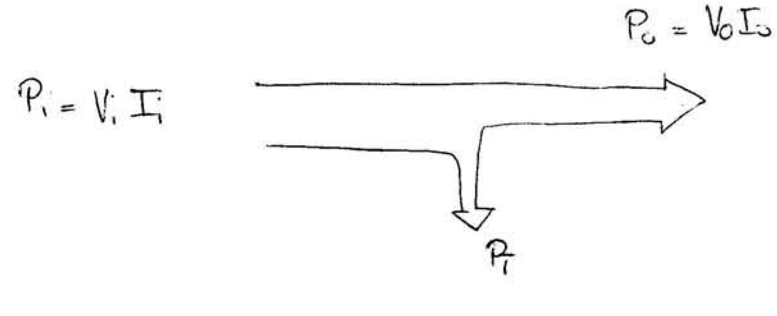
Critical frequency



"Critical frequency"

Efficiency





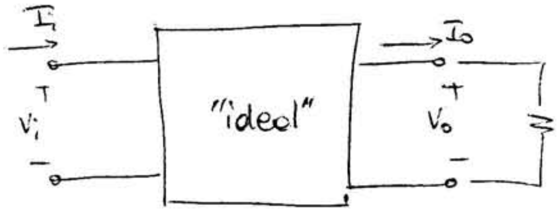
$$P_i = P_o + P_T$$

$$\eta = \frac{P_o}{P_i} = \frac{P_o}{P_o + P_T}$$

— . —

Average input voltage

Theoretically : $I_i = \langle i_i \rangle = D I_o$



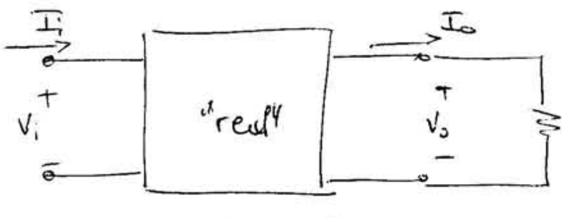
$$V_o = D V_i$$

$$P_i = P_o + \cancel{P_{TL}} = P_o$$

$$\cancel{V_i} I_i = V_o I_o = D \cancel{V_i} I_o$$

$$I_i = D I_o$$

Actually : ("trick")



$$V_o = D V_i$$

$$P_i = P_o + P_{TL}$$

$$V_i I_i = V_o I_o + P_{TL}$$

$$I_i = \frac{V_o}{V_i} I_o + \frac{P_{TL}}{V_i} = D I_o + \boxed{\frac{P_{TL}}{V_i}}$$

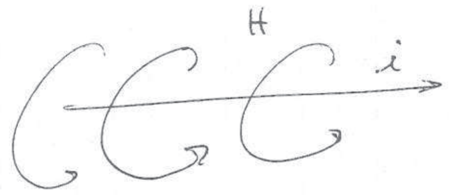
LECTURE 8

Magnetics

$\boxed{H}$ A current "i" generates a magnetic field "$\vec{H}$"

$$H \propto i \qquad i\,[A] \qquad H\left[\frac{A}{m}\right]$$

Magnetic field of a conductor with current



Right hand rule

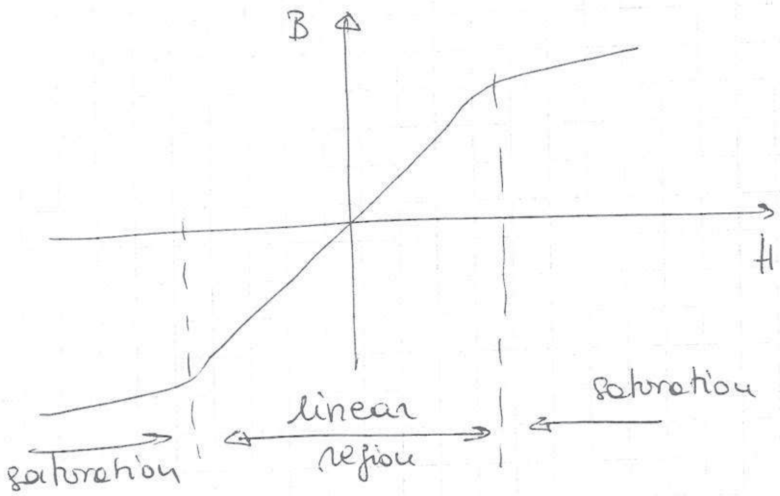
$\boxed{B}$ Magnetic field in different materials has different effects: $\longrightarrow$ "Flux density" or "B-field"

$$B = \mu H = \mu_0 \mu_r H$$

μ_0 : permeability of vacuum $4\pi \cdot 10^{-7} \frac{H}{m}$

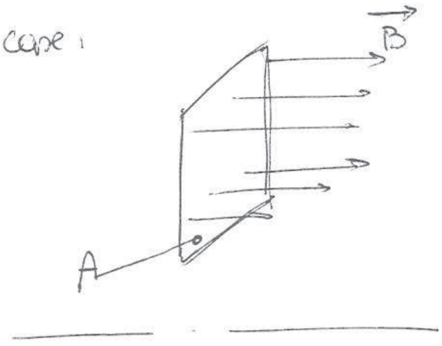
μ : absolute permeability
μ_r = relative permeability

} property of the medium



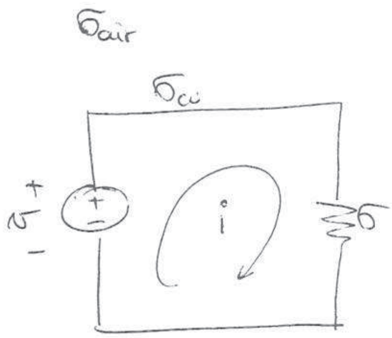
$\boxed{\phi}$ Flux: always referred to a surface

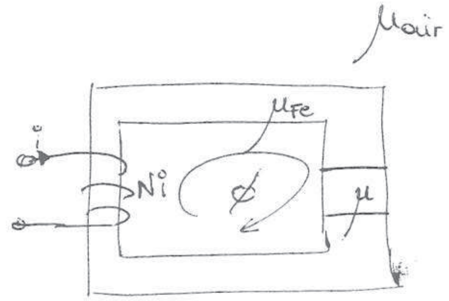
particular case:



$$\phi = A \cdot B = A \cdot \mu H = A \cdot \mu_0 \mu_r H \propto i$$

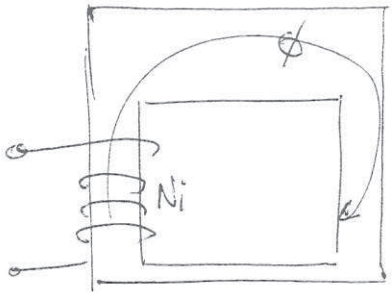
Similarities between electric and magnetic circuits

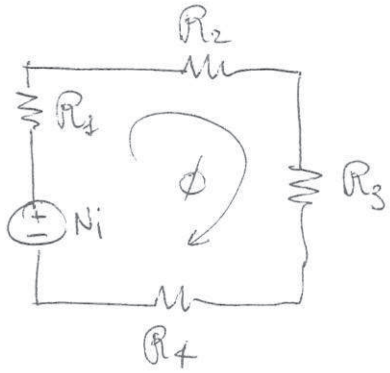




$\sigma_{cu} \ggg \sigma_{air}$ $\sigma \ll \sigma_{cu}$	Conductivity	$\mu_{Fe} \gg \mu_{air}$ $\mu < \mu_{Fe}$	Permeability
i	Current	ϕ	Flux
v	Voltage	Ni	Mmf
$R = \frac{v}{i}$	Resistance	$\mathcal{R} = \frac{Ni}{\phi}$	Reluctance
$G = \frac{1}{R}$	Conductance	$\mathcal{P} = \frac{1}{\mathcal{R}}$	Permeance
$\Sigma i = 0$ @nodes	KVL	$\Sigma \phi = 0$	
$\Sigma v = 0$ on loops	KCL	$\Sigma Ni = 0$	
$R = \rho \frac{\ell}{S} = \frac{1}{\sigma} \frac{\ell}{S}$		$\mathcal{R} = \frac{1}{\mu} \frac{\ell}{S}$	

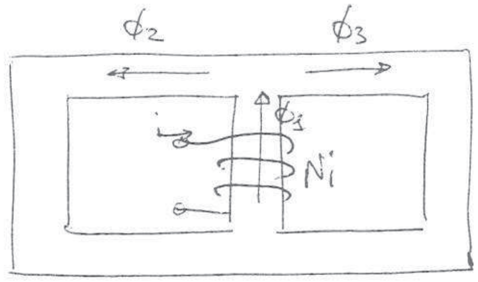
Ex

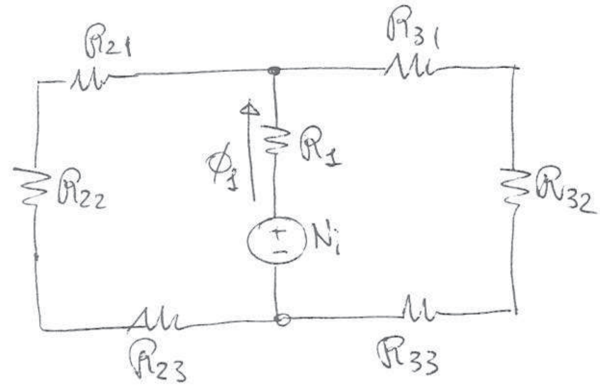




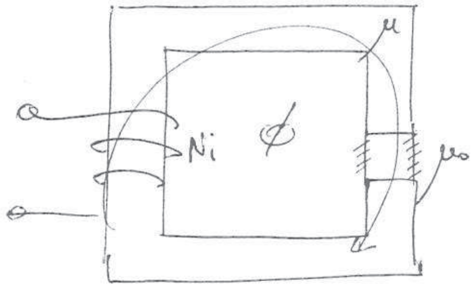
$$\phi = \frac{Ni}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

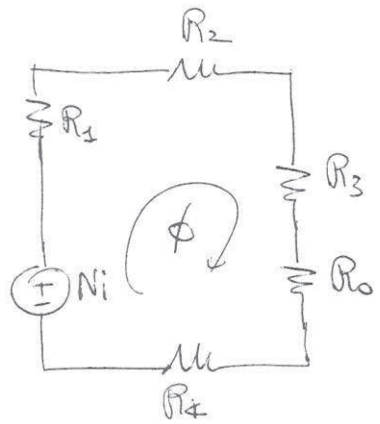
Ex





Ex (important)





$$\mathcal{R} = \frac{1}{\mu}\frac{\ell}{S} \qquad \mathcal{R}_0 = \frac{1}{\mu_0}\frac{\ell}{S}$$

$$\mu \gg \mu_0$$

$$\mathcal{R} \ll \mathcal{R}_0$$

$$\phi = \frac{Ni}{\mathcal{R}_0 + \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \simeq \frac{Ni}{\mathcal{R}_0}$$

Inductance

Def: $L = \frac{N\phi}{i}$

$$\mathcal{R} = \frac{Ni}{\phi} \longrightarrow \phi = \frac{Ni}{\mathcal{R}}$$

$$L = \frac{N}{\cancel{i}}\left[\frac{N\cancel{i}}{\mathcal{R}}\right] = \frac{N^2}{\mathcal{R}} \qquad \boxed{L = \frac{N^2}{\mathcal{R}}}$$

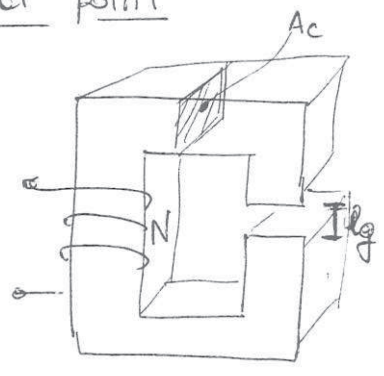
Inductance is a property of:
- the magnetic circuit ($\mathcal{R}$)
- the number of turns

INDUCTOR DESIGN

Starting point

- Desired inductance. L
- Maximum current: $\hat{I}_L$
- RMS current: $I_{L,RMS}$
- Maximum copper losses: $P_{L,w}$
- Winding ~~material~~ resistivity ρ
- Maximum B-field $\hat{B}$
- Fill factor (winding technique) k_u $[< 1]$

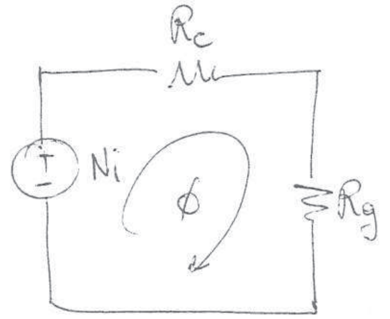
Arrival point



- Core geometry
- Number of turns N
- Air gap length l_g
- Conductor section A_W

Equation reminder

1)



$$\phi = \frac{Ni}{R_c + R_g} \simeq \frac{Ni}{R_g}$$

$$R_g = \frac{1}{\mu_0} \frac{l_g}{A_c}$$

2) $$\phi = B \cdot A_c$$

The four Conditions

① Maximum flux density

$$Ni = \phi R_g = B A_c R_g$$

$$N\hat{I} = \hat{B} A_c R_g = \hat{B} \cancel{A_c} \frac{1}{\mu_0} \frac{l_g}{\cancel{A_c}} = \frac{\hat{B} l_g}{\mu_0}$$

$$\boxed{N\hat{I} = \hat{B} \frac{l_g}{\mu_0}}$$

---

② Inductance

$$L = \frac{N^2}{R_g} = \frac{\mu_0 A_c}{l_g} N^2$$

$$\boxed{L = \frac{\mu_0 A_c N^2}{l_g}}$$

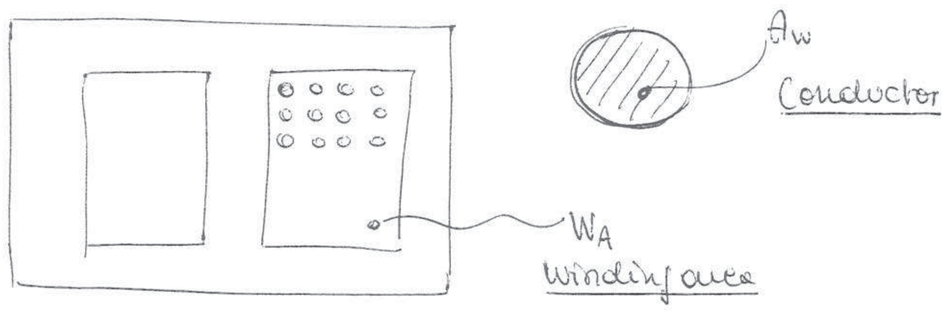
③ Winding resistance

$$R = \rho \frac{l_w}{A_w} = \rho \frac{N l_T}{A_w}$$

l_T : mean length per turn (function of core geometry)

$$\boxed{R = \frac{\rho N l_T}{A_w}}$$

④ Winding area



$$k_U W_A \geq N A_W$$

k_U · fill factor

[$k_U \simeq 0{,}5$ for low voltage inductors]

Derivation of the expressions

$$\begin{cases} N\hat{I} = \hat{B}\dfrac{l_g}{\mu_0} & \longrightarrow N = \dfrac{\hat{B}}{\hat{I}}\dfrac{l_g}{\mu_0} \\ L = \dfrac{\mu_0 A_c N^2}{l_g} & \longrightarrow L = \dfrac{\cancel{\mu_0} A_c}{\cancel{l_g}}\dfrac{\hat{B}^2}{\hat{I}^2}\dfrac{l_g^{\cancel{2}}}{\mu_0^{\cancel{2}}} \end{cases}$$

$$\boxed{l_g = \frac{L\hat{I}^2\mu_0}{A_c\hat{B}^2}}$$

$$N = \frac{\hat{B}}{\hat{I}}\frac{l_g}{\mu_0} = \frac{\cancel{\hat{B}}}{\cancel{\hat{I}}}\frac{1}{\cancel{\mu_0}}\frac{L\hat{I}^{\cancel{2}}\cancel{\mu_0}}{A_c\hat{B}^{\cancel{2}}} = \frac{L\hat{I}}{A_c\hat{B}}$$

$$\boxed{N = \frac{L\hat{I}}{A_c\hat{B}}}$$

$$R = \frac{\rho N l_T}{A_w} \longrightarrow \boxed{A_w = \frac{\rho N l_T}{R}}$$

$$k_U W_A \geq N A_W$$

$$\geq N\frac{\rho N l_T}{R} = \rho\frac{N^2 l_T}{R} = \frac{\rho l_T}{R}\left[\frac{L\hat{I}}{A_c\hat{B}}\right]^2 = \frac{\rho l_T}{R}\frac{L^2\hat{I}^2}{A_c^2\hat{B}^2}$$

$$k_U W_A \geq \frac{\rho l_T}{R}\frac{L^2\hat{I}^2}{A_c^2\hat{B}^2}$$

$$\frac{W_A\, A_c^2}{\ell_T} \geq \frac{L^2\, \hat{I}^2\, \rho}{\hat{B}^2\, R\, K_U}$$

Depends on the geometry

Depends on the problem.

$$K_g = \frac{W_A\, A_c^2}{\ell_T} \qquad \check{K_g} = \frac{L^2\, \hat{I}^2\, \rho}{\hat{B}^2\, R\, K_U}$$

- Finding a geometry whose $K_g \geq \dfrac{L^2\, \hat{I}^2\, \rho}{\hat{B}^2 R\, K_U}$.

then we can find the rest.

Precision: ~~Mx~~ Magnetic core with distributed gap.

The manufacturer gives the permeance of the core, often indicated with A_L.

$$A_L = P = \frac{1}{\mathcal{R}} = \frac{\mu_0 A_c}{l_g} \qquad \cancel{l_g = \frac{\mu_0 A_c}{A_L}}$$

$$A_L = \cancel{\mu_0} A_c \left[\frac{A_c\, \hat{B}^2}{L\, \hat{I}^2\, \cancel{\mu_0}}\right] = \frac{A_c^2\, \hat{B}^2}{L\, \hat{I}^2}$$

$$\boxed{A_L = \frac{A_c^2\, \hat{B}^2}{L\, \hat{I}^2}}$$

(instead of using l_g)

SIZING PROCEDURE
(with example)

$\hat{I}_L = 76\ A$

$I_{L_{RMS}} = 43\ A$

$L = 2{,}4\ \mu H$

$T = 100^\circ C \longrightarrow \rho_{100C} = 23 \cdot 10^{-9}\ \Omega \cdot m$ ⚠

$f_s = 60\ kHz$

$R = 0{,}14\ m\Omega$

$k_U = 0{,}5$ DCM operation

---

① Switching frequency → material [site: Ferroxcube]

Ex: 3C90 → $B_{sat} = 380\ mT$ → $\hat{B} = 250\ mT$

---

② Finding k_g minimum

$$\check{k}_g = \frac{\rho L^2 \hat{I}^2}{\hat{B}^2 R k_U} = \frac{23 \cdot 10^{-9}\ (2{,}4 \cdot 10^{-6})^2 \cdot 76^2}{(250 \cdot 10^{-3})^2 \cdot 0{,}14 \cdot 10^{-3} \cdot 0{,}5} = 1{,}75 \cdot 10^{-10}$$

---

③ Finding the core whose $k_g \geq \check{k}_g$

$$k_g = \frac{A_c^2 W_A}{\ell_T}$$

A_c : cross-sectional area

W_A : core window area

ℓ_T : mean length per turn

From Ferroxcube site, we try with

$\boxed{E47/20/16}$ $\quad W_A = 130 \cdot 10^{-6}\, m^2$

$l_T = 93{,}3 \cdot 10^{-3}\, m$

$A_C = 234 \cdot 10^{-6}\, m^2$

$$k_g = \frac{A_C^2 W_A}{l_T} = 7{,}63 \cdot 10^{-11} \longrightarrow \text{too small (but close)}$$

→ we try with the next

$\boxed{E55/28/21}$ $\quad W_A = 250 \cdot 10^{-6}\, m^2$

$l_T = 116 \cdot 10^{-3}\, m$

$A_C = 353 \cdot 10^{-6}\, m^2$

$$k_g = \frac{A_C^2 W_A}{l_T} = 2{,}69 \cdot 10^{-10} \rightarrow \underline{\text{ok}}$$

$$\boxed{E55/28/21}$$

(the core has been chosen)

④ Finding A_L (or l_g)

$$l_g = \frac{L \hat{I}^2 \mu_0}{A_C \hat{B}^2}$$

$$l_g = \frac{24 \cdot 10^{-6} \cdot 76^2 \cdot 4\pi \cdot 10^{-7}}{353 \cdot 10^{-6} \cdot 250^2 \cdot 10^{-6}} = 0{,}79\,mm$$

$l_{g1} = 0{,}7\, mm$

$l_{g2} = 0{,}8\, mm$

$$L_1 = \frac{l_{g1} \cdot A_C \cdot \hat{B}^2}{\hat{I}^2 \cdot \mu_0} = \frac{0{,}7 \cdot 10^{-3} \cdot 353 \cdot 10^{-6} \cdot 250^2 \cdot 10^{-6}}{76^2 \cdot 4\pi \cdot 10^{-7}} \doteq 2{,}13\, \mu H$$

$$L_2 = \frac{l_{g2} \cdot A_C \cdot \hat{B}^2}{\hat{I}^2 \cdot \mu_0} = \frac{0{,}8 \cdot 10^{-3} \cdot 353 \cdot 10^{-6} \cdot 250^2 \cdot 10^{-6}}{76^2 \cdot 4\pi \cdot 10^{-7}} \doteq 2{,}43\, \mu H$$

if CCM $\rightarrow$ l_{g_2} & $L_2 = 2{,}4\ \mu H$

if DCM $\rightarrow$ l_{g_1} & $L_1 = 2{,}1\ \mu H$

$$\boxed{L_1 = 2{,}1\ \mu H \quad \& \quad l_{g_1} = 0{,}7\,mm}$$

---

(5) <u>Number of turns</u>

$$N = \frac{L\,\hat{I}}{A_c\,\hat{B}} = \frac{2{,}1\cdot\cancel{10^{-6}}\cdot 76}{353\cdot\cancel{10^{-6}}\cdot 250\cdot 10^{-3}} = 1{,}81$$

$N_1 = 1$

$N_2 = 2$

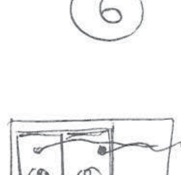
$$\hat{B}_1 = \frac{L\,\hat{I}}{N_1\,A_c} = 452\ mT \qquad \text{NO : SATURATION}$$

$$B_2 = \frac{L\,\hat{I}}{N_2\,A_c} = 226\ mT \qquad \underline{\text{OK}}$$

$$\boxed{N = 2}$$

---

(6) <u>Copper cross-sectional area</u>



$$A_W = \frac{k_u W_A}{N} = \frac{0{,}5\cdot 250\cdot 10^{-6}}{2} = 62{,}5\cdot 10^{-6}\,m^2$$

$$\boxed{A_W = 62{,}5\ mm^2}$$

(7) <u>Current density</u>

$$J_{L,RMS} = \frac{I_{L,RMS}}{A_W} = \frac{43\,A}{62{,}5\,mm^2} = 0{,}688\ A/mm^2$$

$$\boxed{J_{L,RMS} = 0{,}688\ A/mm^2}$$ (quite low)

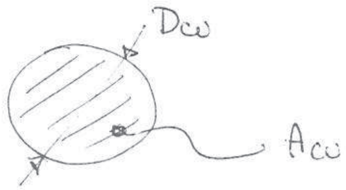
8 Number of parallel wires N_W

Eddy currents in conductor $\Rightarrow$ $D_{cu} \leq 2 \cdot D_{skin}$

$$D_{skin} = \sqrt{\frac{\rho}{\pi \cdot f_s \cdot \mu_0}} = \sqrt{\frac{23 \cdot 10^{-9}}{\pi \cdot 60 \cdot 10^{3} \cdot 4\pi \cdot 10^{-7}}} = 0{,}3 \cdot 10^{-3}\,m \doteq 0{,}3\,mm$$

$D_{cu} = 2 \cdot D_{skin}$

$D_{cu} = 0{,}6\,mm$

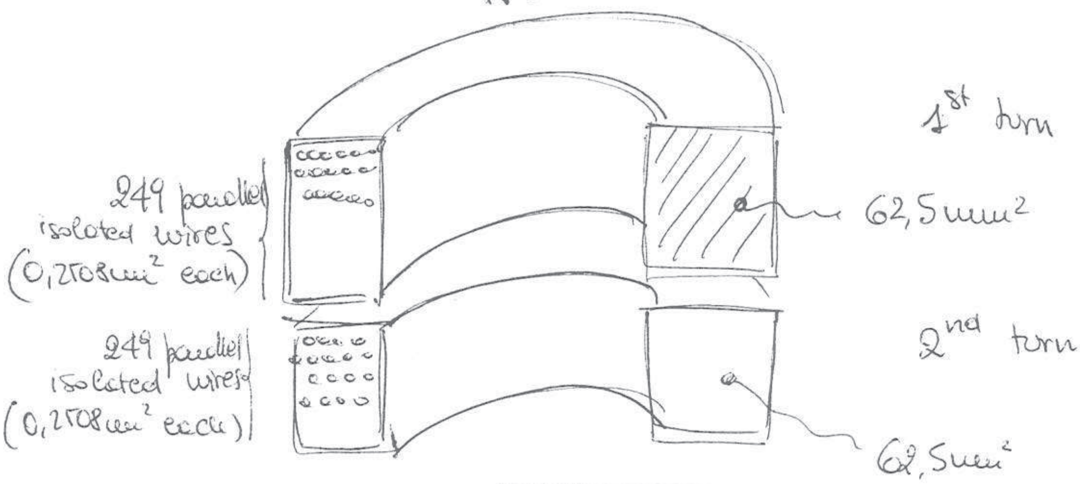


$$A_{cu} = \pi \frac{D_{cu}^2}{4} = \pi \cdot \frac{0{,}6^2 \cdot 10^{-6}}{4} = 0{,}2826 \cdot 10^{-6}\,m^2 \doteq 0{,}2826\,mm^2$$

Closer commercial (available) conductor

$A^*_{cu} = 0{,}2508\,mm^2$ (AWG 23)

$$N_W = \frac{A_W}{A^*_{cu}} = \frac{62{,}5\,mm^2}{0{,}2508\,mm^2} = 249$$



$$\boxed{\begin{aligned} A^*_{cu} &= 0{,}2508 \\ N_W &= 249 \end{aligned}}$$

(9) DC Resistance

$$R_L = \frac{\rho N \ell_T}{A_W} = \frac{\rho N \ell_T}{N_W \cdot A_{cu}^{*}} = \frac{23\cdot 10^{-9}\cdot 2 \cdot 116\cdot 10^{-3}}{249 \cdot 0{,}2508\cdot 10^{-6}} = 85\cdot 10^{-6}$$

$$= 0{,}085\,m\Omega$$

(quite small!)

(10) Copper losses

$$P_{cu} = R_L \cdot I^2_{L,RMS} = 85\cdot 10^{-6} \cdot 43^2 = 0{,}157\,mW$$

(11) Core losses

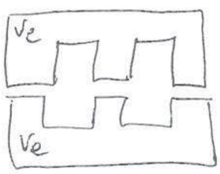
From datasheet core: $2\cdot V_e = 88\cdot 10^{-6}\,m^3$

From datasheet material: $P_V \cong 350\,kW/m^3$

@ $\hat{B} = 250\,mT$ $\quad f_s = 60\,kHz$

$$P_{co} = 2\cdot V_e \cdot P_V = 2\cdot 44\cdot 10^{-6} \cdot 350\cdot 10^3 = 30{,}8\,W$$

The core is made of two "E", and V_e is the volume of only one "E"



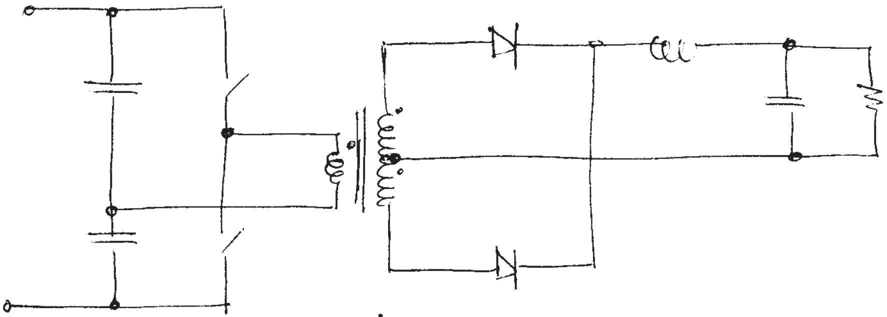
$$\boxed{P_{cu} + P_{co} = 31\,W}$$

(Mostly in the core!)

HALF BRIDGE

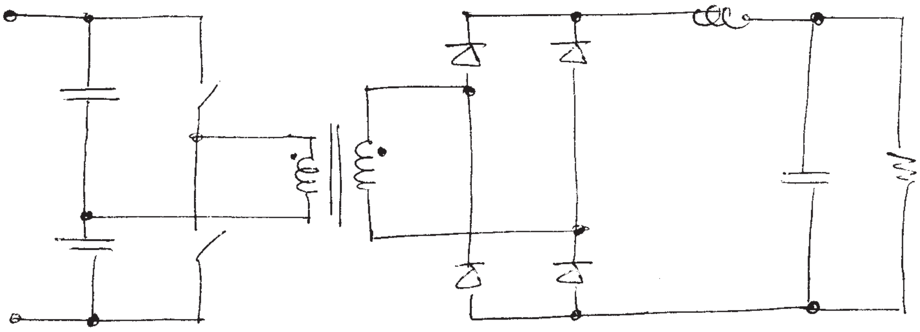
- Topology derived from the buck
- Transformer
- Different variations on the secondary side:

(1)



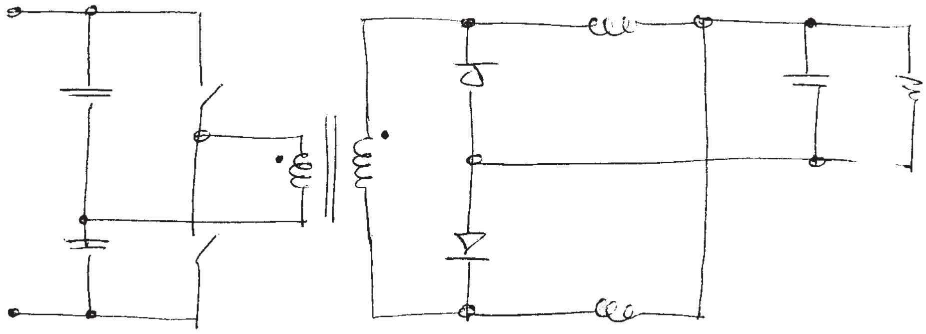
"FULL WAVE RECTIFIER"

(2)



"BRIDGE RECTIFIER"

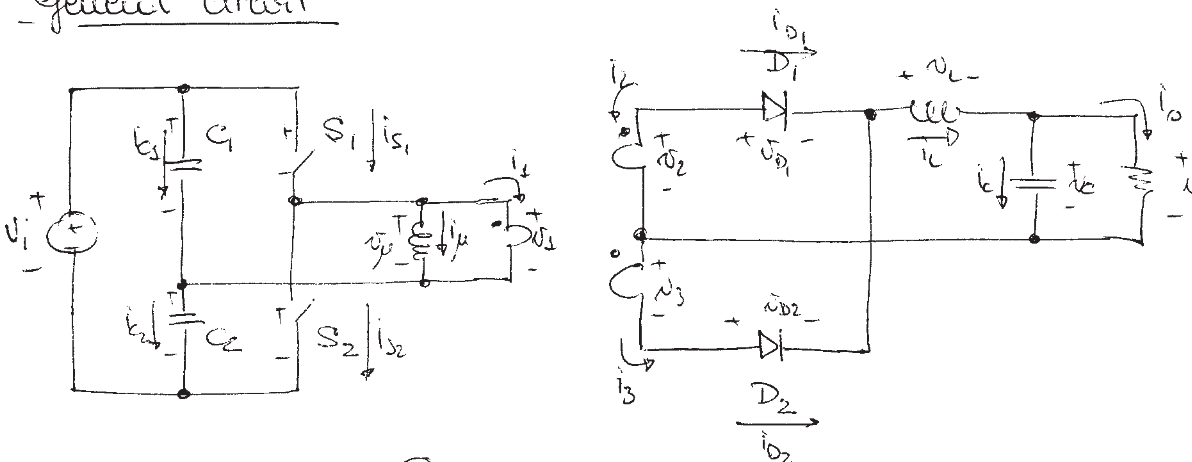
(3)



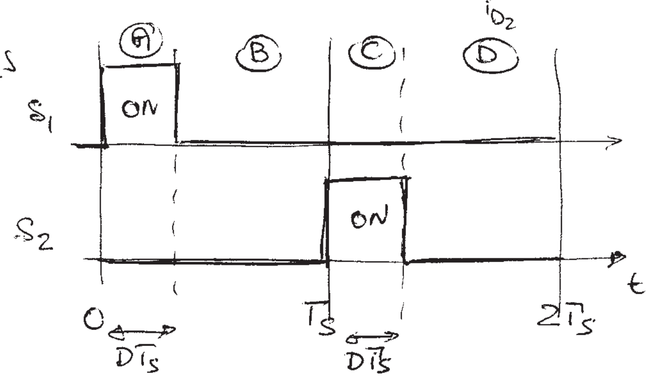
"CURRENT DOUBLER"

Half bridge with full-wave rectifier
CCM

1) General circuit



Switches



$\boxed{N_2 = N_3}$

2) Studying the possible circuits

(A) S_1 on, S_2 off, D_1? D_2?

$v_{C_1} = v_{C_2} = \frac{V_i}{2}$ $\quad v_1 = \frac{V_i}{2}$ $\quad v_\mu = \frac{V_i}{2}$ $\quad v_{S_1} = 0$ $\quad v_{S_2} = V_i$

$\frac{v_1}{N_1} = \frac{v_2}{N_2} = \frac{v_3}{N_2}$ $\quad v_2 = \frac{N_2}{N_1}\frac{V_i}{2}$ $\quad v_3 = \frac{N_2}{N_1}\frac{V_i}{2}$

$i_L > 0 \implies \left[\begin{array}{l} D_1 \text{ on } \wedge D_2 \text{ on} \\ D_1 \text{ on} \\ D_2 \text{ on} \end{array}\right.$

$\boxed{D_1 \text{ on } \wedge D_2 \text{ on ?}}$ $\iff v_3 + v_2 = 0$ $\quad v_2 = -v_3$ $\quad \frac{N_2}{N_1}\frac{V_i}{2} = -\frac{N_2}{N_1}\frac{V_i}{2}$

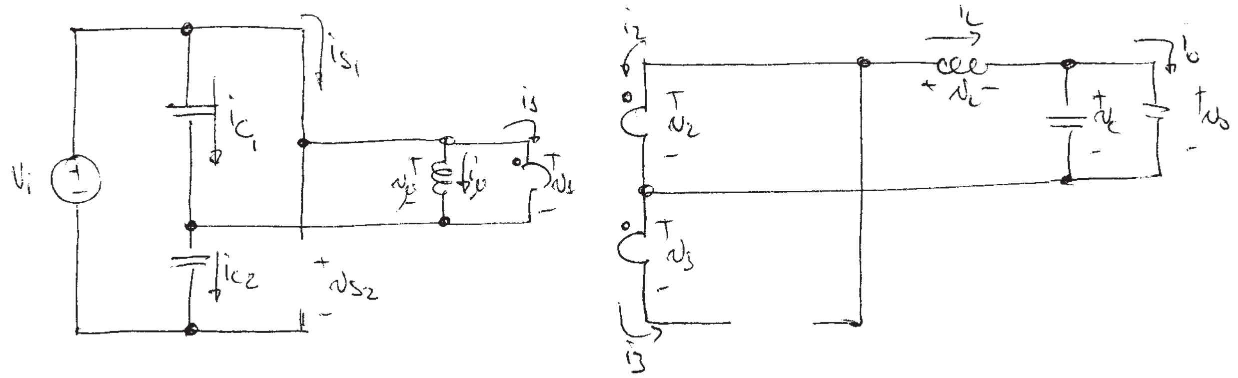
$\implies V_i = -V_i \implies V_i = 0$ $\quad$ No

$\boxed{D_2 \text{ on } \wedge D_1 \text{ off ?}}$ $\implies v_3 + v_2 - v_{D_1} = 0$

$v_{D_1} = v_2 + v_3 = \frac{N_2}{N_1}\left(\frac{V_i}{2} + \frac{V_i}{2}\right) = \frac{N_2}{N_1} V_i > 0$ $\quad$ No

$\boxed{D_2 \text{ off} \wedge D_1 \text{ on}} \Rightarrow \quad v_3 + v_2 + v_{D2} = 0$

$$v_{D2} = - v_3 - v_2 = - \frac{N_2}{N_1} V_i < 0 \quad \rightarrow \underline{\text{ok}}$$



$$v_L = v_2 - V_0 = \frac{N_2}{N_1}\frac{V_i}{2} - V_0 = V_i' - V_0$$

$$i_L = \frac{V_i' - V_0}{L} t + \check{I}_L$$

$i_0 = I_0 = \frac{V_0}{R}$ $\quad i_C = i_L - I_0$ $\quad i_{D2} = 0$ $\quad i_{D1} = i_L$ $\quad i_2 = - i_L$

$i_3 = 0$

$N_1 i_1 + N_2 i_2 = 0 \qquad i_1 = - \frac{N_2}{N_1} i_2 = - \frac{N_2}{N_1}(-i_L) = \frac{N_2}{N_1} i_L$

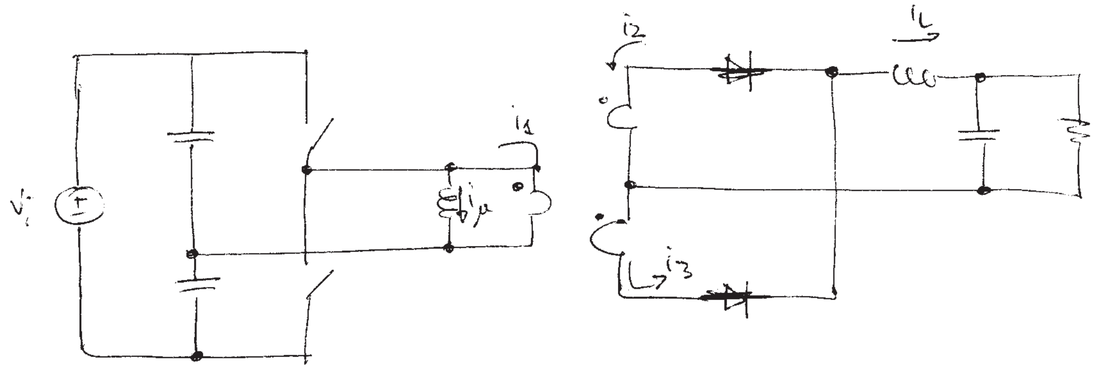
$i_\mu = \frac{V_i}{L_\mu} t + \check{I}_\mu \qquad i_{S1} = i_1 + i_\mu$

$i_{C2} = i_{C_1} + i_{S1} \qquad |i_{C_1}| = |i_{C2}|$ $\left| \begin{array}{l} i_{C1} = i_{C2} \Rightarrow i_{S1} = 0 \quad \underline{\text{No}} \\ i_{C1} = - i_{C2} \Rightarrow i_{C2} = \frac{i_{S_1}}{2} \quad \underline{\text{ok}} \end{array} \right.$

$$i_{C1} = - \frac{i_{S1}}{2}$$

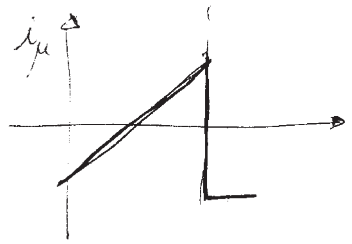
$$i_i = i_{C1} + i_{S1} = \frac{i_{S1}}{2}$$

(B) S_1 off, S_2 off, D_1 ? , D_2 ?



$$i_L > 0 \Rightarrow \left[\begin{array}{l} D_1 \text{ on} \wedge D_2 \text{ on} \\ D_1 \text{ on} \\ D_2 \text{ on} \end{array}\right.$$

D_1 on ? $\quad i_\mu = -i_1 \quad i_1 = -\frac{N_2}{N_1} i_2 = \frac{N_2}{N_1} i_L \quad i_\mu = -\frac{N_2}{N_1} i_L$



$v_\mu = L\frac{di_\mu}{dt} \quad \frac{di_\mu}{dt} = \pm\infty \Rightarrow v_\mu = \pm\infty$

No

("i_μ" shouldn't have jumps")

D_2 on? $\quad i_\mu = \frac{N_2}{N_1} i_L \quad$ similar $\to$ No

D_1 on $\wedge$ D_2 on

$$i_1 = -i_\mu$$
$$i_3 = i_2 + i_L$$

$$N_1 i_1 + N_2 i_2 + N_2 i_3 = 0$$
$$N_1(-i_\mu) + N_2 i_2 + N_2(i_2 + i_L) = 0$$
$$-N_1 i_\mu + 2N_2 i_2 + N_2 i_L = 0$$

$$i_2 = \frac{-N_2 i_L + N_1 i_\mu}{2N_2} = -\frac{i_L}{2} + \frac{N_1}{N_2}\frac{i_\mu}{2}$$

$$i_{D2} = -i_2 = \frac{i_L}{2} - \frac{N_1}{N_2}\frac{i_\mu}{2}$$

$$i_{D2} > 0 \iff \frac{i_L}{2} - \frac{N_1}{N_2}\frac{i_\mu}{2} > 0 \iff \boxed{i_L > \frac{N_1}{N_2} i_\mu}$$

(Heavy load)

$$i_3 = i_2 + i_L = -\frac{i_L}{2} + \frac{N_1}{N_2}\frac{i_\mu}{2} + i_L = \frac{i_L}{2} + \frac{N_1}{N_2}\frac{i_\mu}{2}$$

$i_{D3} = i_3 > 0$ <u>always</u>

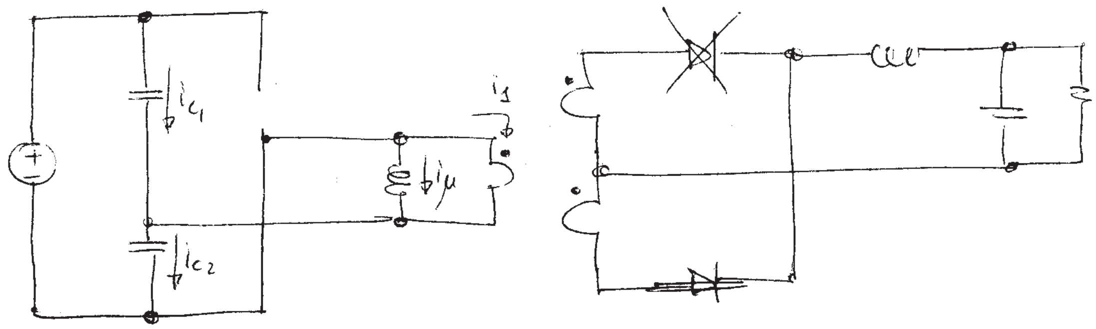
$$v_3 = -v_2 \Rightarrow \frac{N_2}{N_1} v_1 = -\frac{N_2}{N_1} v_1 \Rightarrow v_1 = -v_1 \Rightarrow v_1 = 0$$

$$\Rightarrow \quad v_2 = 0 \qquad v_3 = 0 \qquad v_\mu = 0$$

$$i_\mu \equiv \text{cost} \equiv \hat{I}_\mu \qquad v_L = -V_0 \qquad i_L = -\frac{V_0}{L} t + \hat{I}_L$$

$$v_{S1} = v_{S2} = \frac{V_i}{2}$$

(c) S_1 off, S_2 on, D_1 ?, D_2 ?



$$v_\mu = -\frac{V_i}{2} \qquad v_1 = -\frac{V_i}{2} \qquad v_2 = \frac{N_2}{N_1} v_1 = -\frac{N_2}{N_1}\frac{V_i}{2} \qquad v_3 = v_2$$

D_1 off, D_2 on ?

$$v_{D_1} = v_2 = -\frac{N_2}{N_1}\frac{V_i}{2} < 0 \quad \rightarrow \underline{\text{ok}}$$

$$v_L = -v_3 - V_0 = \frac{N_2}{N_1}\frac{V_i}{2} - V_0 = V_i' - V_0 \qquad i_L = \frac{V_i' - V_0}{L}t + \check{I}_L$$

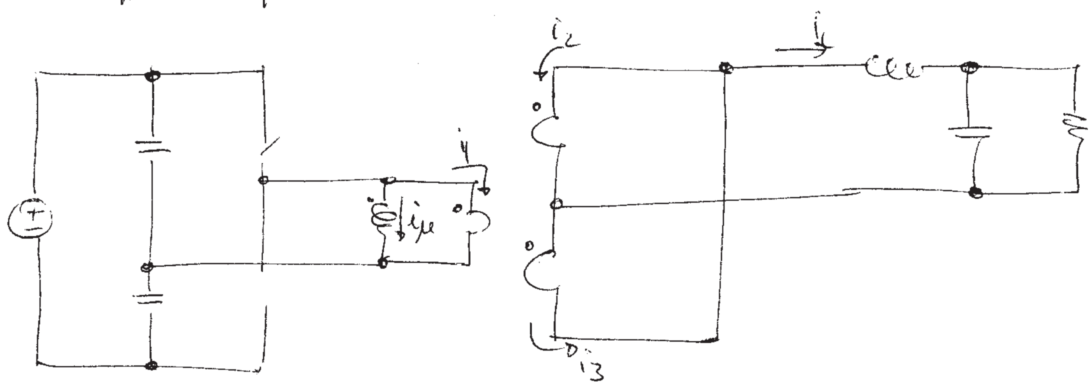
$$i_2 = 0 \qquad i_3 = i_L \qquad i_1 = -\frac{N_2}{N_1}\, i_L$$

$$i_\mu = \hat{I}_\mu - \frac{V_i}{2L_\mu}t$$

$$i_{S2} = -i_\mu - i_1 = -i_\mu + \frac{N_2}{N_1}\, i_L$$

$$i_{C_1} = i_{S2}/2 \qquad i_{C2} = -i_{S2}/2 \qquad i_i = i_{C1} = i_{S2}/2$$

D S_1 off S_2 off D_1? D_2?



$$i_1 = -i_\mu$$

$$N_1 i_1 + N_2 i_2 + N_2 i_3 = 0$$

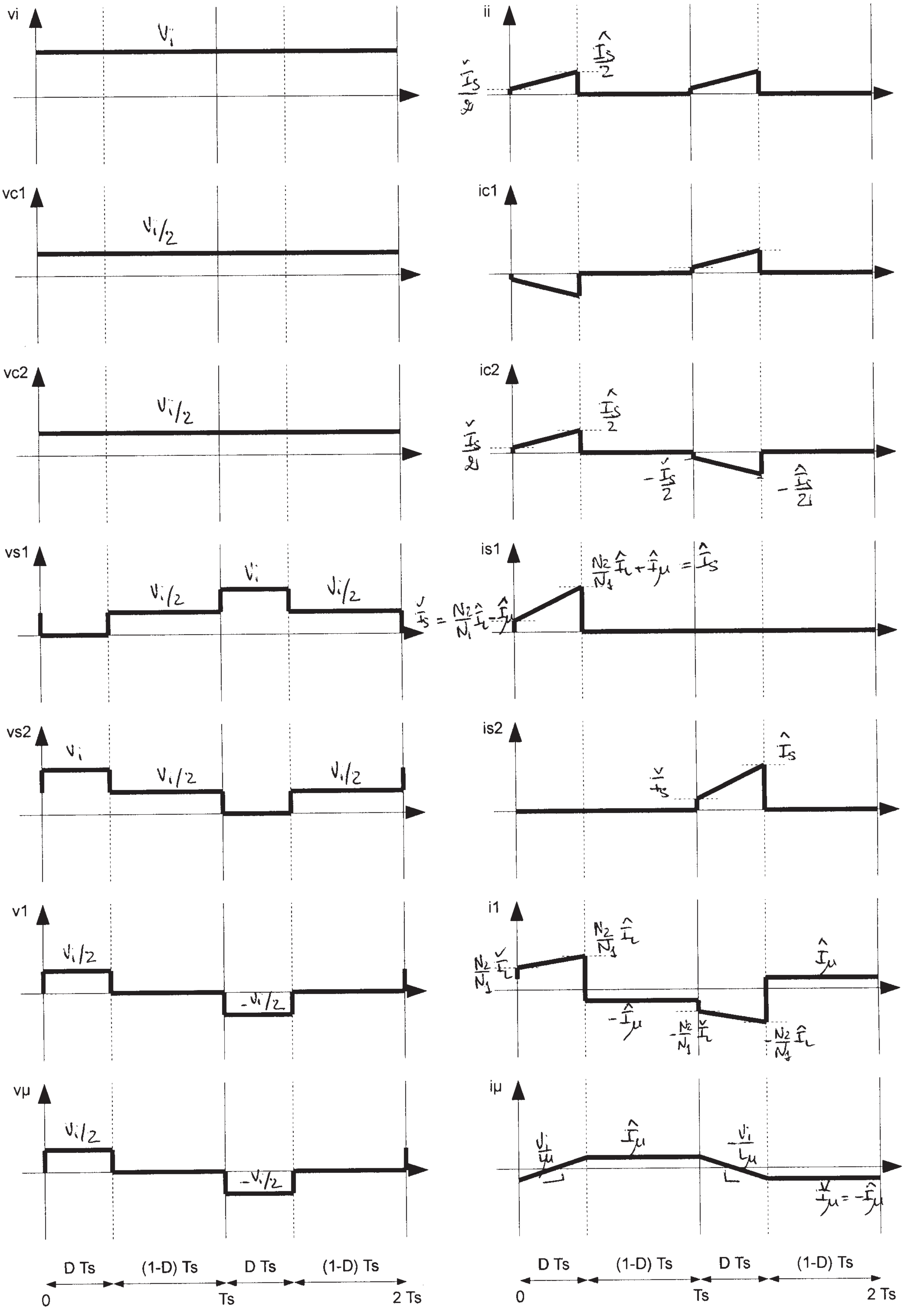
$$N_1(-i_\mu) + N_2 i_2 + N_2(i_2 + i_L) = 0$$

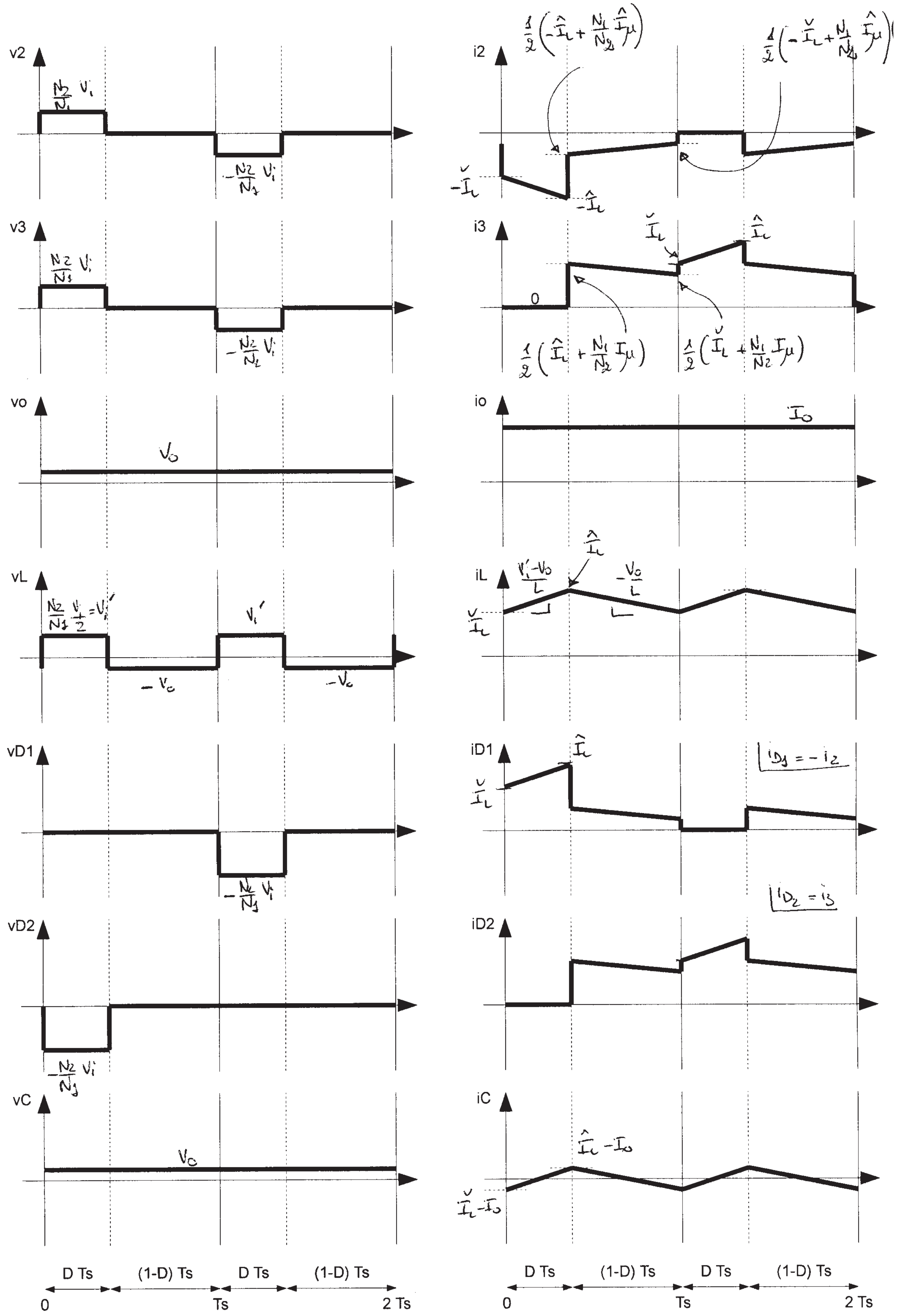
$$-N_1 i_\mu + 2N_2 i_2 + N_2 i_L = 0$$

$$i_2 = -\frac{i_L}{2} + \frac{N_1}{N_2}\frac{i_\mu}{2}$$

$$i_3 = i_2 + i_L = \frac{i_L}{2} + \frac{N_1}{N_2}\frac{i_\mu}{2}$$

$$v_1 = v_2 = v_3 = 0 \;\rightarrow\; v_\mu = 0 \;\rightarrow\; i_\mu \equiv cost \equiv \check{I}_\mu \equiv -\hat{I}_\mu$$


vi
vc1
vc2
vs1
vs2
v1
vμ
ii
ic1
ic2
is1
is2
i1
iμ
D Ts
(1-D) Ts
D Ts
(1-D) Ts
0
Ts
2 Ts



v2
v3
vo
vL
vD1
vD2
vC
i2
i3
io
iL
iD1
iD2
iC
Vo
Io
-Vo
Vi'
0
iD1 = -i2
iD2 = i3
D Ts
(1-D) Ts
D Ts
(1-D) Ts
0
Ts
2 Ts

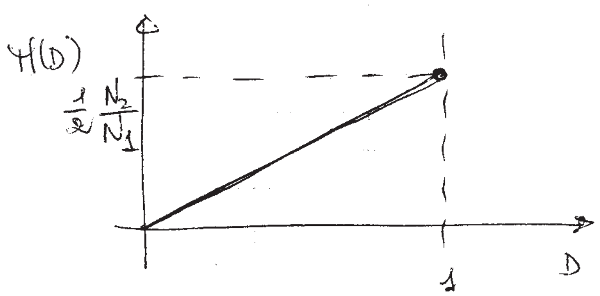
4) Steady-state conditions

Defined $V_i' = \frac{N_2}{N_1}\frac{V_i}{2}$, the second part of the converter behaves like a buck with an input voltage V_i'. All the equations are then similar.

$$\boxed{V_o = D V_i' = D \frac{N_2}{N_1}\frac{V_i}{2}}$$

5) Conversion ratio

$$H(D) = \frac{V_o}{V_i} = D\frac{N_2}{N_1}\cdot\frac{1}{2}$$



6) Current ripple (inductor)

$$\Delta i_L = \frac{V_i' T_s}{L}(1-D)D = \frac{N_2}{N_1}\frac{V_i T_s}{2L}(1-D)D$$

7) Average and maximum current

$$\langle i_L \rangle = I_o = \frac{V_o}{R}$$

$$\hat{I}_L = I_o + \frac{\Delta i_L}{2}$$

$$\hat{I}_{D_1} = \hat{I}_{D_2} = \hat{I}_L$$

$$|\hat{I}_2| = \hat{I}_L \qquad |\hat{I}_1| = \frac{N_2}{N_1}\hat{I}_L \qquad |\hat{I}_3| = \hat{I}_L$$

$$\hat{I}_{S_1} = \hat{I}_{S_2} = \frac{N_2}{N_1} \hat{I}_L + \hat{I}_\mu$$

$$<i_{D1}> = - <i_2> =$$

$$<i_{D2}> = <i_3> =$$

$$<i_1> = 0$$

$$<i_{S1}> =$$

$$<i_{S2}> =$$

$$<i_i> =$$

8) Maximum voltage

$$|\hat{v}_{S1}| = |\hat{v}_{S2}| = V_i$$

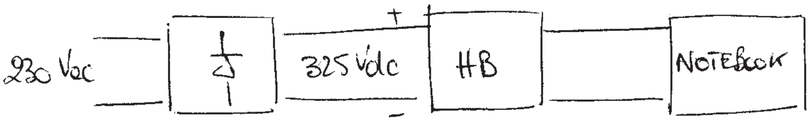
$$|\hat{v}_{D1}| = |\hat{v}_{D2}| = \frac{N_2}{N_1} V_i$$

9) Output voltage ripple

$$\Delta v_C = \frac{V_i' T_S^2}{8LC} (1-D) D = \frac{N_2}{N_1} \frac{V_i T_S^2}{16 LC} (1-D) D$$

Exercise

1) Design the following converter using a HB



$N_1 = 325$

$L_\mu = 60\,mH$

$f_s = 40\,kHz$

Heavy load operation

$V_o = 90\,V$

$P_o = (40 \div 90)\,W$

$\Delta v_C = 0{,}2\,V$

$N_2 = ?$ $\quad I_o = ?$ $\quad D = ?$ $\quad L = ?$ $\quad C = ?$

$\Delta i_L = ?$ $\quad \langle i_L \rangle = ?$ $\quad \langle i_{S_1} \rangle = ?$ $\quad \langle i_{S_2} \rangle = ?$ $\quad \langle i_{D_1} \rangle = ?$

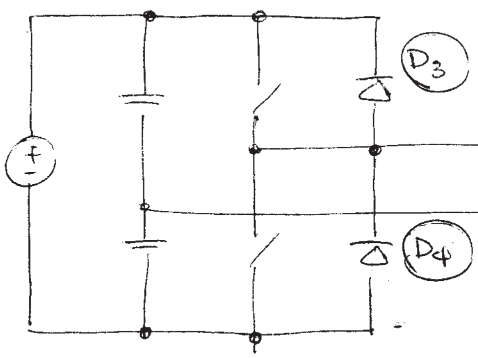
$\langle i_{D_2} \rangle = ?$ $\quad \hat{I}_L = ?$ $\quad \hat{I}_{S_1} = ?$ $\quad \hat{I}_{S_2} = ?$ $\quad \hat{I}_{D_1} = ?$

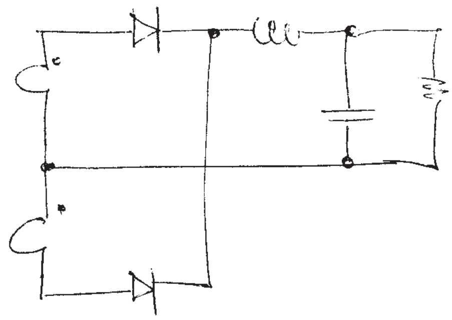
$\hat{I}_{D_2} = ?$ $\quad |\hat{v}_{S_1}| = ?$ $\quad |\hat{v}_{S_2}| = ?$ $\quad |\hat{v}_{D_1}| = ?$ $\quad |\hat{v}_{D_2}| = ?$

Light load

$$\boxed{i_L \leq \frac{N_1}{N_2'} i_\mu}$$

- Need of more diodes (to let i_μ circulate)





↳ the behavior of the circuits changes

- There is also a discontinuous conduction mode, like for the buck.

	CCM	DCM
Heavy load	I	
Light load	II	III

The case I has been presented.

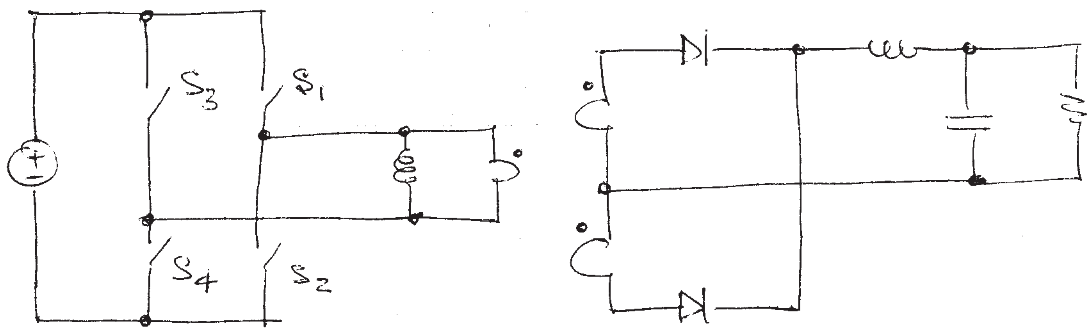
<u>Exercises</u>. - study the other cases.

FULL BRIDGE

- Topology derived from the buck
- Transformer
- Similar variations on the secondary side as for the HB

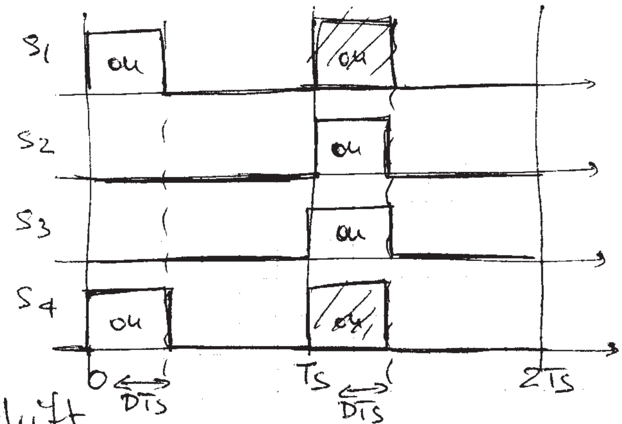
Full bridge with full-wave rectifier

1) General circuit



Two control strategies

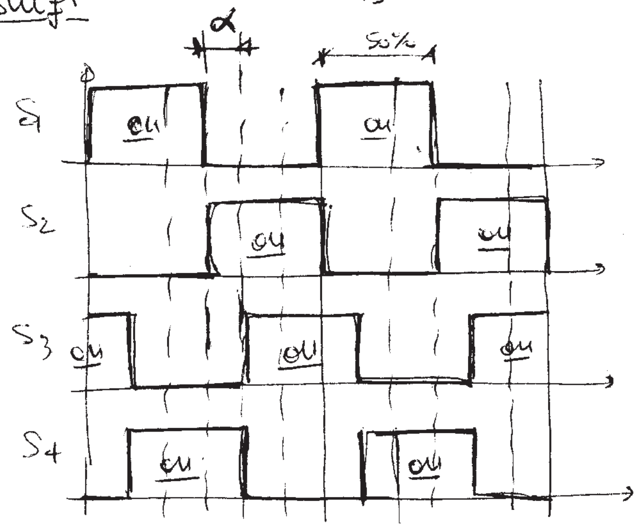
A) Diagonal switches



$S_1 = S_4$

$S_2 = S_3$

B) Phase shift



$S_1 = \overline{S_2}$

$S_3 = \overline{S_4}$