

Lecture 3 - contents

A first step to motor modeling

- Determine the position dependent inductance matrix for a salient pole motor
- Obtain a qd0 model – a first look

Overview of dynamic modeling of el. machines

We are looking for??? (in abc reference frame)

1. Voltage equation

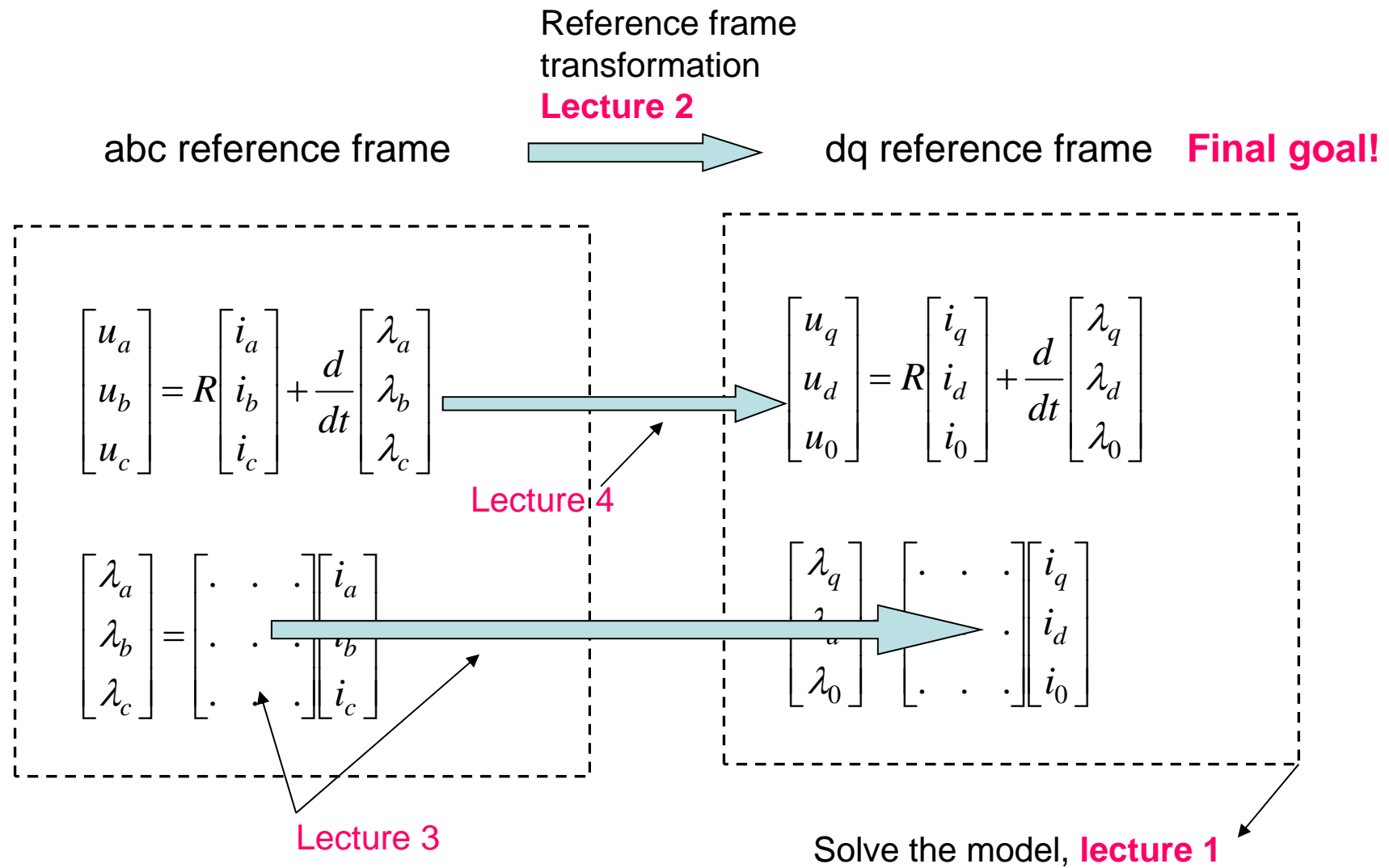


$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix}$$

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

2. Torque equation

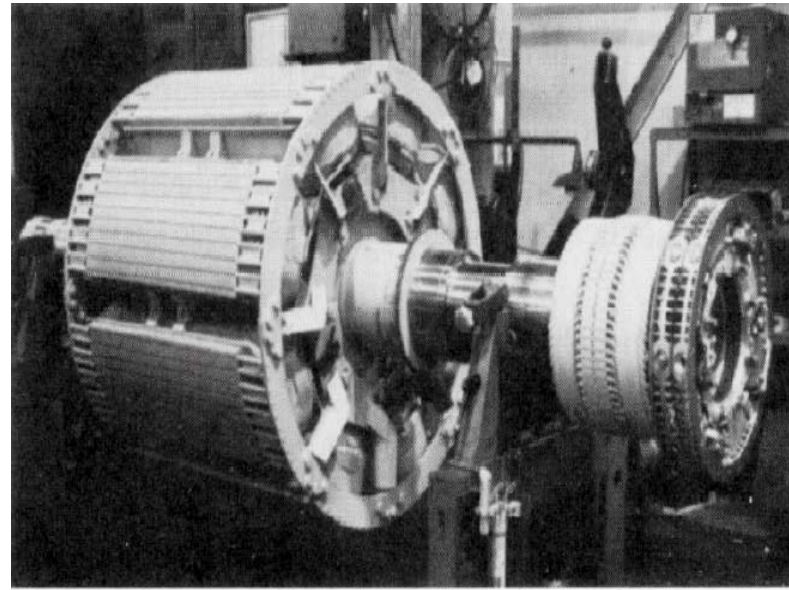
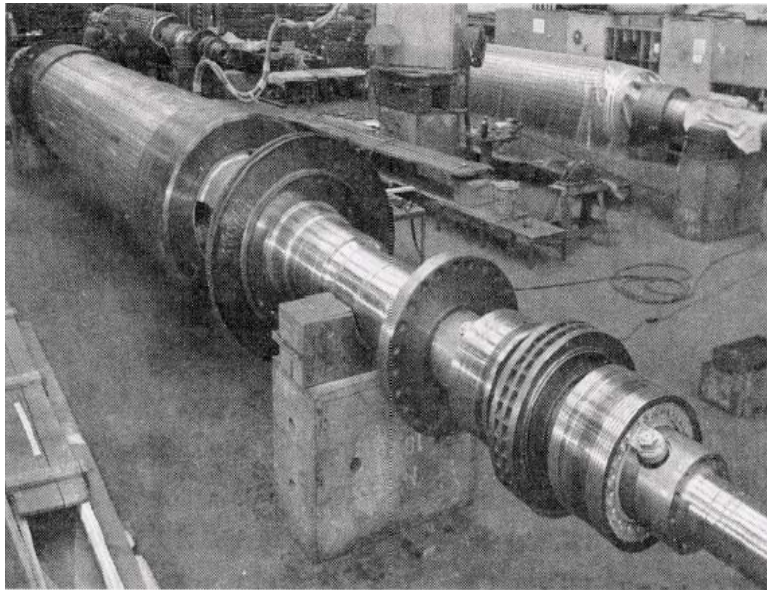
.....?



Complete system modelling and analysis of the salient rotor, synchronous Machine, - **lecture 5**

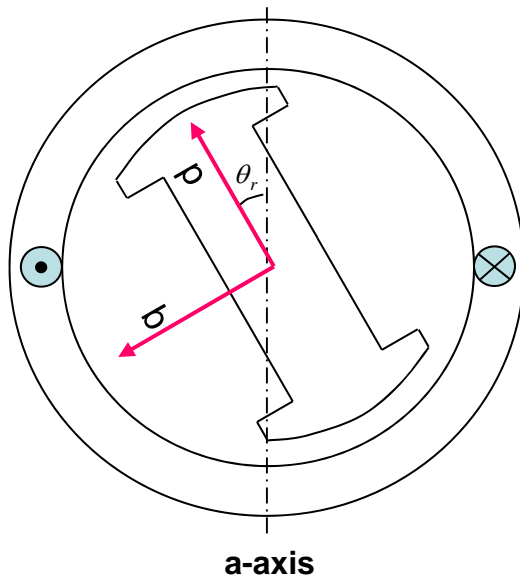
Determine the position dependent inductance matrix

Photos of the non-salient and salient rotors



Determine the position dependent inductance matrix

Basic guidelines - continued



$$L(\theta) = L_1 + L_2 \cos(2\theta)$$

↓ In qd0 system

$$L(\theta) = L_1 - L_2 \cos(2\theta_r)$$

$$\theta_r = 0 \quad \rightarrow \quad L(0) = L_{aaq} = L_1 - L_2$$

$$\theta_r = \frac{\pi}{2} \quad \rightarrow \quad L\left(\frac{\pi}{2}\right) = L_{aad} = L_1 + L_2$$

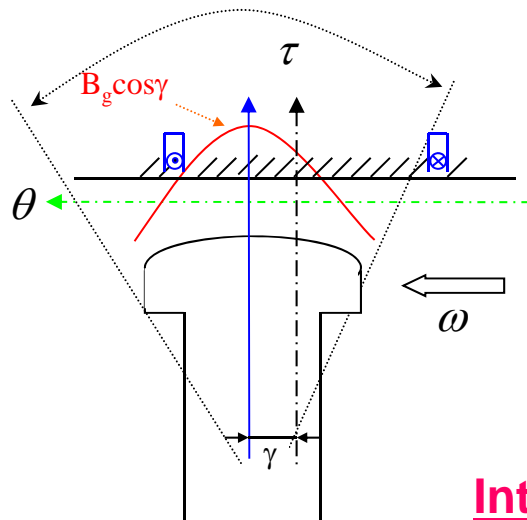
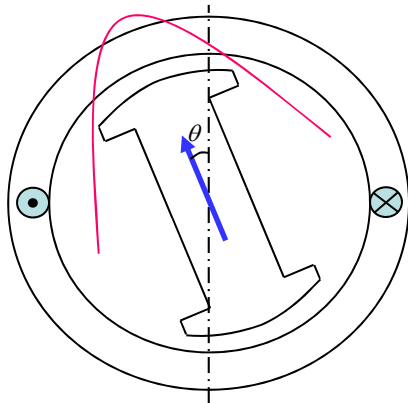


$$L_1 = \frac{L_{aad} + L_{aaq}}{2} \quad L_2 = \frac{L_{aad} - L_{aaq}}{2}$$

$$L(\theta) = L_{aaq} \cos^2 \theta_r + L_{aad} \sin^2 \theta_r$$

Determine the position dependent inductance matrix

Basic guidelines



$$\lambda_a = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B_g \cos(\gamma - \theta) d\gamma \frac{1}{p} r l_{axis} = \frac{2}{\pi} B_g l_{axis} \cos(\theta) = \lambda_m \cos(\theta)$$

$$\lambda_b = \int_{-\frac{\pi}{2} + \frac{2\pi}{3}}^{\frac{\pi}{2} + \frac{2\pi}{3}} B_g \cos(\gamma - \theta) d\gamma \frac{1}{p} r l_{axis} = \lambda_m \cos\left(\theta - \frac{2\pi}{3}\right)$$

$$\lambda_m \cos(\theta) \Rightarrow \operatorname{Re} \left(\frac{e^{j\theta}}{e^{j0}} \right)$$

Position of the flux axis

$$\lambda_m \cos\left(\theta - \frac{2\pi}{3}\right) \Rightarrow \operatorname{Re} \left(\frac{e^{j\theta}}{e^{j\frac{2\pi}{3}}} \right)$$

Position of the stator phase axis

Integration could be replaced by vector projection!

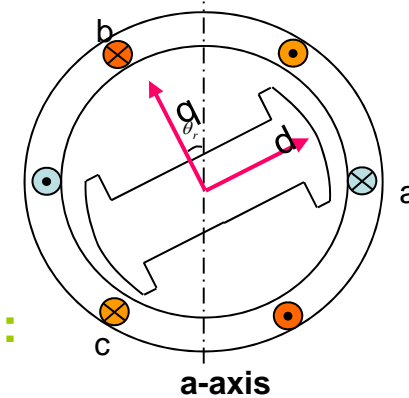
Determine the position dependent inductance matrix

Self-inductances of stator phases

From the book, you may see the following expressions:



A preferred form:



$$L_{asasm} = L_1 - L_2 \cos(2\theta_r)$$

$$L_{bsbsm} = L_1 - L_2 \cos 2\left(\theta_r - \frac{2}{3}\pi\right)$$

$$L_{cscsm} = L_1 - L_2 \cos 2\left(\theta_r + \frac{2}{3}\pi\right)$$

$$= L_{aaq} \cos^2 \theta_r + L_{aad} \sin^2 \theta_r$$

$$= L_{aaq} \cos^2 \left(\theta_r - \frac{2\pi}{3}\right) + L_{aad} \sin^2 \left(\theta_r - \frac{2\pi}{3}\right)$$

$$= L_{aaq} \cos^2 \left(\theta_r + \frac{2\pi}{3}\right) + L_{aad} \sin^2 \left(\theta_r + \frac{2\pi}{3}\right)$$

Determine the position dependent inductance matrix

How can we easily obtain the inductance expressions?

Observations:

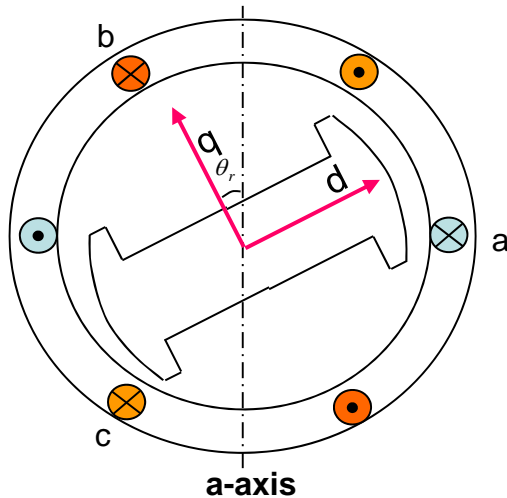
- Each inductance expression has L_{aad} and L_{aaq} components.
- The rest coefficients are sin or cos functions of the rotor position.
- More precisely, there are **two** sin or cos coefficients before L_{aad} and L_{aaq}

Question left:

How can we determine the exact expression for sin or cos coefficients?

Determine the position dependent inductance matrix

Self-inductances of stator phases



Example:

$$L_{cscsm} = L_1 - L_2 \cos 2 \left(\theta_r + \frac{2}{3} \pi \right)$$

$$= L_{aaq} \cos^2 \left(\theta_r + \frac{2\pi}{3} \right) + L_{aad} \sin^2 \left(\theta_r + \frac{2\pi}{3} \right)$$

$$L_{aaq} \cos^2 \left(\theta_r + \frac{2\pi}{3} \right) \longleftrightarrow L_{aaq} \operatorname{Re} \left(\frac{e^{j\theta_r}}{e^{-j\frac{2\pi}{3}}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\theta_r}}{e^{-j\frac{2\pi}{3}}} \right)$$

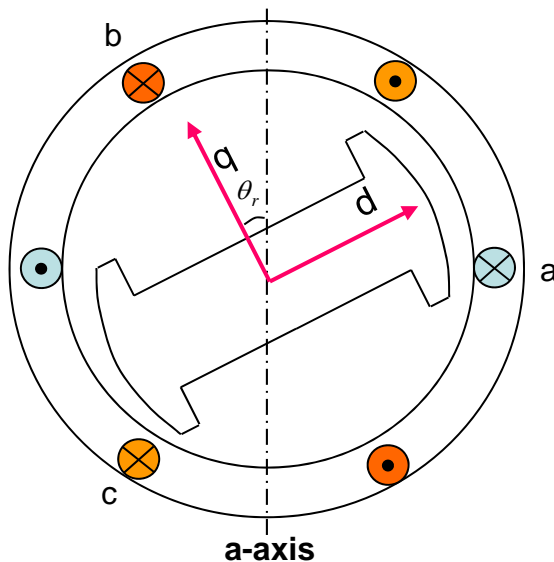
← Position of the q-axis
← Position of the c-axis

$$L_{aad} \sin^2 \left(\theta_r + \frac{2\pi}{3} \right) \longleftrightarrow L_{aad} \operatorname{Re} \left(\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{-j\frac{2\pi}{3}}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{-j\frac{2\pi}{3}}} \right)$$

← Position of the d-axis
← Position of the c-axis

Determine the position dependent inductance matrix

What about mutual inductance? – replace the location of the phase axis!



$$M_{asbsm} = L_{aaq} \cos \theta_r \cos \left(\theta_r - \frac{2\pi}{3} \right) + L_{aad} \sin \theta_r \sin \left(\theta_r - \frac{2\pi}{3} \right)$$

$$L_{aaq} \operatorname{Re} \left(\frac{e^{j\theta_r}}{e^{j0}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\theta_r}}{e^{j\frac{2\pi}{3}}} \right)$$

Position of the q-axis

Position of the a-axis

Position of the b-axis

$$L_{aad} \operatorname{Re} \left(\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{j0}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{j\frac{2\pi}{3}}} \right)$$

Position of the d-axis

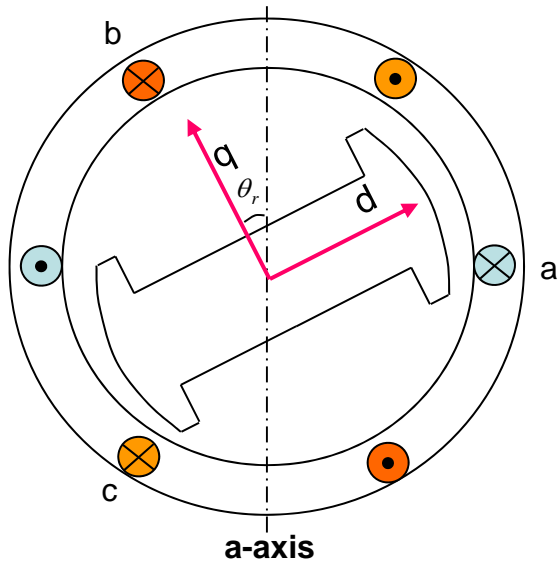
Position of the a-axis

Position of the b-axis

$$M_{asbsm} = -\frac{1}{2} L_1 - l_2 \cos \left(2\theta_r - \frac{2\pi}{3} \right)$$

Determine the position dependent inductance matrix

Examples:



$$M_{csasm} = L_{aaq} \cos \theta_r \cos \left(\theta_r + \frac{2\pi}{3} \right) + L_{aad} \sin \theta_r \sin \left(\theta_r + \frac{2\pi}{3} \right)$$

$$L_{aaq} \operatorname{Re} \left(\frac{e^{j\theta_r}}{e^{j0}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\theta_r}}{e^{-j\frac{2\pi}{3}}} \right)$$

Position of the q-axis

Position of the a-axis

Position of the c-axis

$$L_{aad} \operatorname{Re} \left(\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{j0}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{-j\frac{2\pi}{3}}} \right)$$

Position of the d-axis

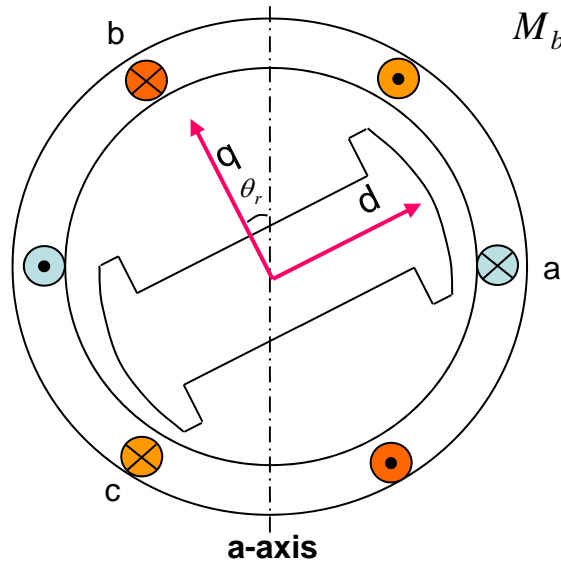
Position of the a-axis

Position of the c-axis

$$M_{asbsm} = -\frac{1}{2} L_1 - l_2 \cos \left(2\theta_r + \frac{2\pi}{3} \right)$$

Determine the position dependent inductance matrix

Examples:



$$M_{bscsm} = L_{aaq} \cos\left(\theta_r - \frac{2\pi}{3}\right) \cos\left(\theta_r + \frac{2\pi}{3}\right) + L_{aad} \sin\left(\theta_r - \frac{2\pi}{3}\right) \sin\left(\theta_r + \frac{2\pi}{3}\right)$$

$$L_{aaq} \operatorname{Re} \left(\frac{e^{j\theta_r}}{e^{j\frac{2\pi}{3}}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\theta_r}}{e^{-j\frac{2\pi}{3}}} \right)$$

Position of the q-axis

Position of the b-axis

Position of the c-axis

$$L_{aad} \operatorname{Re} \left(\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{j\frac{2\pi}{3}}} \right) \cdot \operatorname{Re} \left(\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{-j\frac{2\pi}{3}}} \right)$$

Position of the d-axis

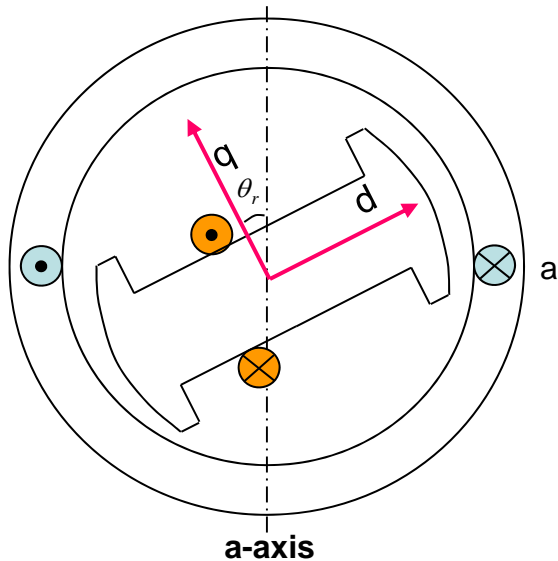
Position of the b-axis

Position of the c-axis

$$M_{asbsm} = -\frac{1}{2} L_1 - l_2 \cos(2\theta_r)$$

Determine the position dependent inductance matrix

Mutual-inductances between rotor and stator phases



$$M_{asfdm} = L_{sfd} \cos\left(\theta_r - \frac{\pi}{2}\right) = L_{sfd} \sin \theta_r$$

$$L_{sfd} \operatorname{Re} \left[\frac{e^{j\left(\theta_r - \frac{\pi}{2}\right)}}{e^{j0}} \right]$$

Position of the d-axis

Position of the a-axis

$$M_{bsfdm} = L_{sfd} \cos\left(\theta_r - \frac{\pi}{2} - \frac{2\pi}{3}\right) = L_{sfd} \sin\left(\theta_r - \frac{2\pi}{3}\right)$$

$$M_{csfdm} = L_{sfd} \cos\left(\theta_r - \frac{\pi}{2} + \frac{2\pi}{3}\right) = L_{sfd} \sin\left(\theta_r + \frac{2\pi}{3}\right)$$

Compare all these results with Paul C Krause's book on P53
Definition of the rotor position and qd-axis is given on P49


Obtain a qd0 model – a first look

Stator voltage equations


- Neglect all the all the rotor windings, and rotor permanent magnets (if any)

- General voltage equation
$$u = Ri + \frac{d\lambda}{dt}$$

$$\underline{\lambda} = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \begin{bmatrix} L_{aal} & -M_{ml} & -M_{ml} \\ -M_{ml} & L_{bbll} & -M_{ml} \\ -M_{ml} & -M_{ml} & L_{ccl} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_{asasm} & M_{asbsm} & M_{ascsm} \\ M_{bsasm} & L_{bsbsm} & M_{bscsm} \\ M_{csasm} & M_{csbsm} & L_{cscsm} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$


 \underline{L}_l

$$\underline{\lambda}_l = \underline{L}_l \cdot \underline{i}$$


 \underline{L}_m

$$\underline{\lambda}_m = \underline{L}_m \cdot \underline{i}$$


Leakage flux linkage

Main flux linkage

Obtain a qd0 model – a first look

Leakage mutual inductance is zero!


$$\underline{\lambda} = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \begin{bmatrix} L_{aal} & 0 & 0 \\ 0 & L_{bbl} & 0 \\ 0 & 0 & L_{ccl} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_{asasm} & M_{asbsm} & M_{ascsm} \\ M_{bsasm} & L_{bsbsm} & M_{bscsm} \\ M_{csasm} & M_{csbsm} & L_{cscsm} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$



\underline{L}_l

$\underline{\lambda}_l = \underline{L}_l \cdot \underline{i}$

Leakage flux linkage



\underline{L}_m

$\underline{\lambda}_m = \underline{L}_m \cdot \underline{i}$

Main flux linkage

Take care of this part first!

Obtain a qd0 model – a first look

Transformation of the main inductance matrix

$$\underline{\lambda}_m = \underline{L}_m \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} L_{asasm} & M_{asbsm} & M_{ascsm} \\ M_{bsasm} & L_{bsbsm} & M_{bscsm} \\ M_{csasm} & M_{csbsm} & L_{cscsm} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$qd0 \sim \frac{2}{3} \begin{bmatrix} \cos \theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin \theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \sim abc$$

$$\underline{\lambda}_m = L_{aaq} \begin{bmatrix} \cos \theta_r \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} \cos \theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} +$$

$$L_{aad} \begin{bmatrix} \sin \theta_r \\ \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} \sin \theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Obtain a qd0 model – a first look

Transformation of the main inductance matrix

$$\underline{\lambda}_m = \underline{L}_m \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} L_{asasm} & M_{asbsm} & M_{ascsm} \\ M_{bsasm} & L_{bsbsm} & M_{bscsm} \\ M_{csasm} & M_{csbsm} & L_{cscsm} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$qd0 \sim \frac{2}{3} \begin{bmatrix} \cos \theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin \theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \sim abc$$

$$\underline{\lambda}_m = L_{aaq} \begin{bmatrix} \cos \theta_r \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} \cos \theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} +$$

$$L_{aad} \begin{bmatrix} \sin \theta_r \\ \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} \sin \theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$\frac{3}{2} i_q$
 $\frac{3}{2} i_d$

Obtain a qd0 model – a first look

Transformation of the main inductance matrix - continued

$$\underline{\lambda}_m = \begin{bmatrix} \cos \theta_r \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \left(\frac{3}{2} L_{aaq} i_q \right) + \begin{bmatrix} \sin \theta_r \\ \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \left(\frac{3}{2} L_{aad} i_d \right)$$

$$\begin{bmatrix} \lambda_{am} \\ \lambda_{bm} \\ \lambda_{cm} \end{bmatrix} = \begin{bmatrix} \cos \theta_r \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \lambda_{qm} + \begin{bmatrix} \sin \theta_r \\ \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \lambda_{dm}$$

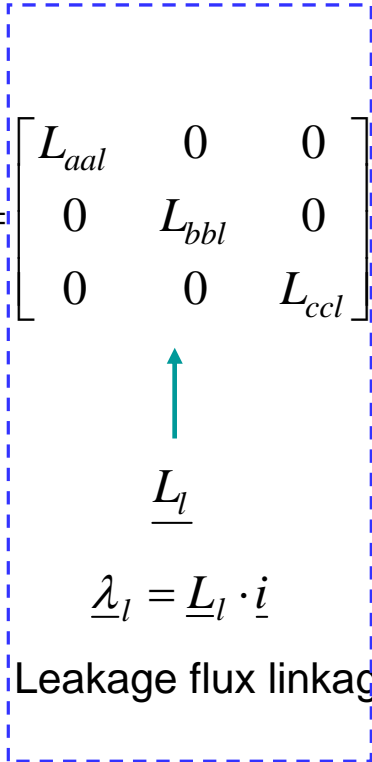
$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \begin{bmatrix} \lambda_q \\ \lambda_d \\ \lambda_0 \end{bmatrix}$$

$$\begin{aligned} \lambda_{qm} &= L_{mq} i_q & L_{mq} &= \frac{3}{2} L_{aaq} \\ \lambda_{dm} &= L_{md} i_d & L_{md} &= \frac{3}{2} L_{aad} \end{aligned}$$

Obtain a qd0 model – a first look

Leakage mutual inductance is zero!

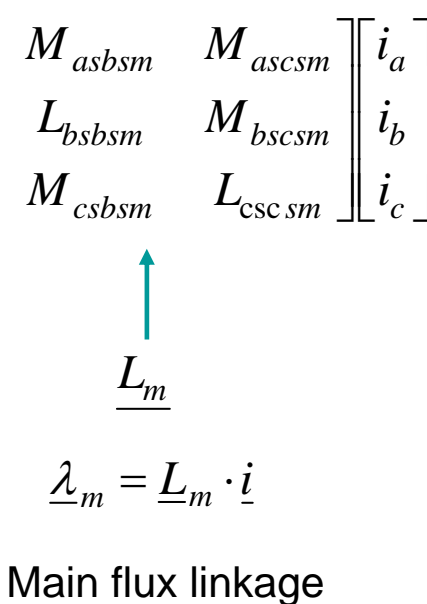
$$\underline{\lambda} = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \begin{bmatrix} L_{aal} & 0 & 0 \\ 0 & L_{bbl} & 0 \\ 0 & 0 & L_{ccl} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_{asasm} & M_{asbsm} & M_{ascsm} \\ M_{bsasm} & L_{bsbsm} & M_{bscsm} \\ M_{csasm} & M_{csbsm} & L_{cscsm} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$



\underline{L}_l

$\underline{\lambda}_l = \underline{L}_l \cdot \underline{i}$

Leakage flux linkage



\underline{L}_m

$\underline{\lambda}_m = \underline{L}_m \cdot \underline{i}$

Main flux linkage

Now take care of this part first!

Obtain a qd0 model – a first look

Transformation of the leakage inductance matrix

$$\underline{\lambda}_l = \underline{L}_l \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} L_{aal} & 0 & 0 \\ 0 & L_{bbl} & 0 \\ 0 & 0 & L_{ccl} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$qd0 \sim \frac{2}{3} \begin{bmatrix} \cos \theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin \theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \sim abc$$

Multiply both sides by the transformation matrix

$$\underline{\lambda}_{lq} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} L_{ls} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\lambda_{lq} = L_{aal} i_q = L_{ls} i_q$$

$$\lambda_{ld} = L_{aal} i_d = L_{ls} i_d$$

$$L_{aal} = L_{bbl} = L_{ccl} = L_{ls}$$

Obtain a qd0 model – a first look

Obtain the 0-sequence equation

$$\lambda_0 = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} L_{aal} & 0 & 0 \\ 0 & L_{bbl} & 0 \\ 0 & 0 & L_{ccl} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + 0$$

$$\lambda_0 = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} L_{aal} & L_{bbl} & L_{ccl} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\lambda_0 = L_0 i_0 \qquad i_0 = \frac{1}{3} (i_a + i_b + i_c)$$

$$u_0 = R i_0 + \frac{d\lambda_0}{dt} \qquad L_0 = L_{aal} = L_{bbl} = L_{ccl}$$

Obtain a qd0 model – a first look

Now we should be able to answer the following questions

- Why there is a coefficient $3/2$ before the q- or d-axis inductance?
- The essential part of the qd0 model is that the inductance becomes position independent – how this is achieved?
- Does the qd0 model have a decoupled q-axis and d-axis flux linkage?
- Does the qd0 model have a decoupled q-axis and d-axis voltage?

Excises

- Derive the self and mutual stator inductances, and mutual inductance between stator phase a and rotor phase b for an induction motor using the 'projection' method presented in the previous slides.
- If the resistance of stator phase a, b and c is R, what is the resistance in the 0-sequence equation? Please prove it.

Induction machine stator and rotor phase axes may be represented as

