Power electronics for control of AC machines

1. Introduction
2. Basic understanding of a power converter
3. Analysis of a 3-phase power converter
4. Space Vector modulation technique

Introduction

- Recall from the 1st mini module:
  - we want to control e.g. ? $_1$, U_1 by means of a static power electronic converter

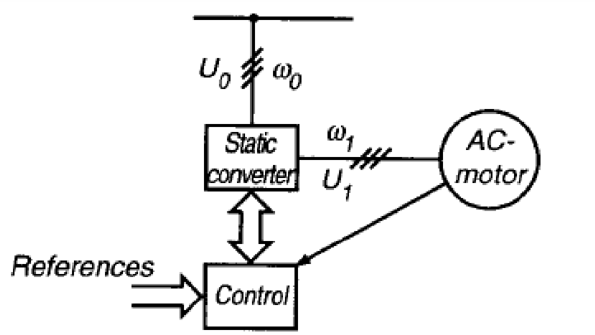


Fig. 11.1. General scheme of AC motor control

- In this lecture we mainly study the machine-side converter

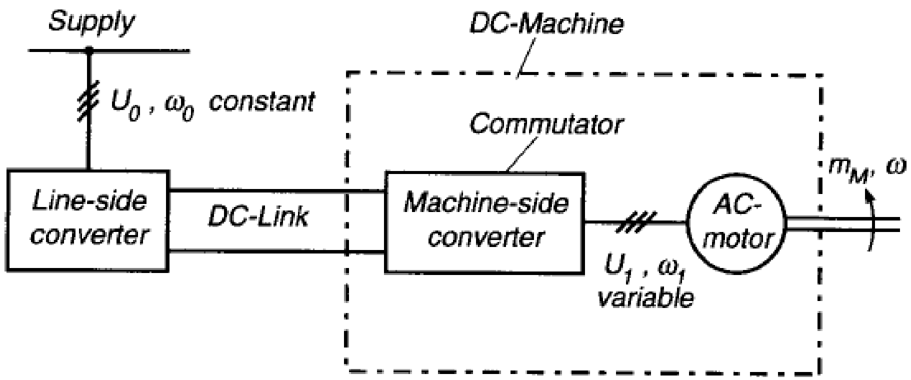


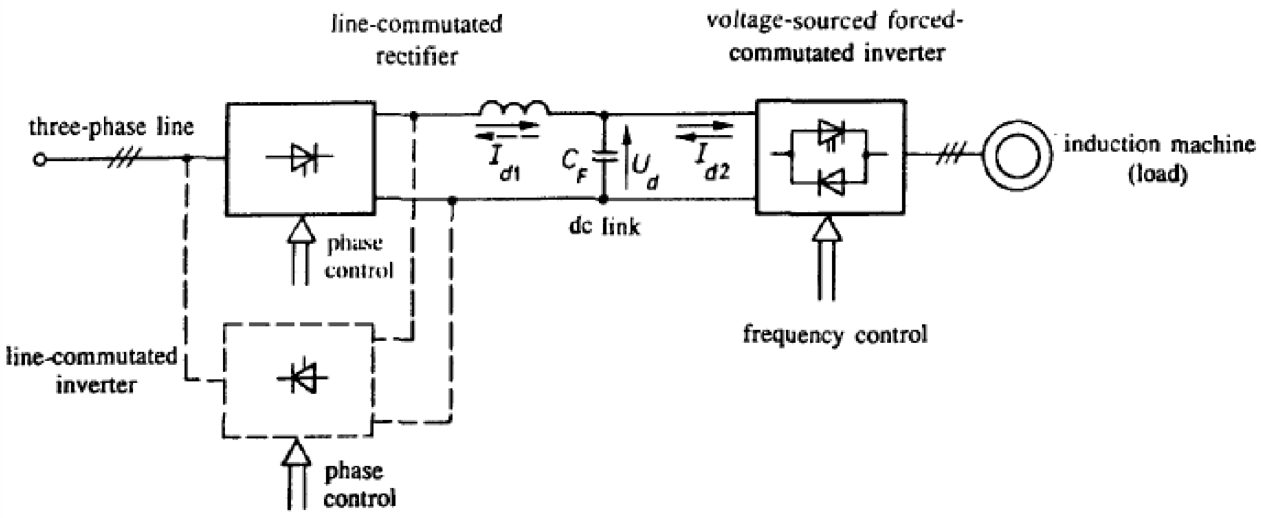
Fig. 11.2. Controlled AC-drive with DC-link converter

Converters / Machines	DC link converters				Cycloconverters
	Voltage-source converters		Current-source converters		
	Transistor inverters (IGBT)	Thyristor inverters (GTO)	Force-commutated thyristor inverters (GTO)	Naturally-commutated thyristor inverters	(with line commutation)
Synchronous motor with permanent magnet excitation	Low power (10 kW) very good dynamic performance (servo drives)	Medium power (1 MW), high power density			
Reluctance motor		Low to medium power (100 kW)			
Squirrel-cage induction motor	Low to medium power (500 kW), high speed, very good dynamic performance (spindle and servo drives)	Medium to high power (2 MW) good dynamic performance (Traction drives)	Medium to high power (4 MW), high speed		High power (7,5 MW) low speed, very good dynamic performance
Doubly fed slip-ring induction motor		Shaft generators on ships (2 MW)		High power (20 MW), subsynchronous operation	High power (100 MW) limited speed control range
Synchronous motor with field and damper windings				High power (40 MW), high speed	High power (5 MW), low speed, good dynamic performance

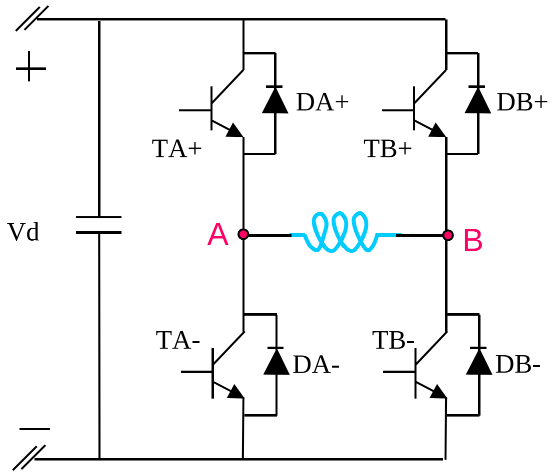
Matrix converters — low-power appl.
(force commutated cyclo converter)

Voltage-source inverters

- Widely used in drive applications
- The input is a "stiff" dc link voltage
- Typically, the front-end is a diode-bridge rectifier (uni-directional power flow)
- Some applications, however, may require bi-directional power flow (thyristor/transistorised rectifier)



Basic understanding of a power converter



- Any time, only one switch on each bridge can be switched on

⇓

- We may introduce a variable D

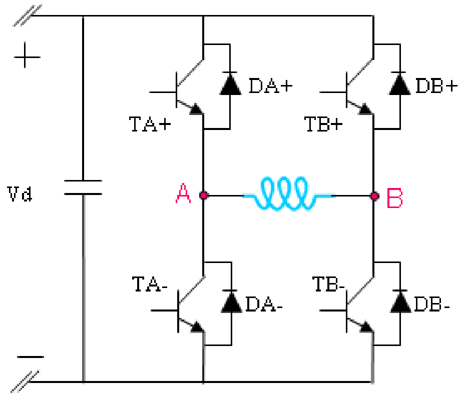
$V_A = V_{dc} D_A$ $\boxed{D_A = 0 \text{ or } 1 \quad \textit{Switching status}}$

Voltage is always a relative value! Where we are referring to when we use the above equation?

If referring to the negative DC bus, $V_{AN} = V_{dc} D_A$

> We may set the rule (for our understanding): for a converter, we always start from considerations on the line-to-line output voltage!

Basic understanding of a power converter



Example 1. bipolar voltage switching

D_A=1, is always coupled with D_B=0

D_A=0, is always coupled with D_B=1

$$\boxed{V_{AB} = V_{AN} - V_{BN} = D_A V_d - D_B V_d = D_A V_d - (1 - D_A) V_d = (2D_A - 1) V_d}$$

Instantaneous meaning!

Average DC voltage during one switching period

$$V_{AB,Ts} = V_{AN,Ts} - V_{BN,Ts} = \frac{D_1 T_s V_d + (1 - D_1) T_s 0}{T_s} - \frac{D_2 T_s V_d}{T_s} = (D_1 - D_2) V_d$$

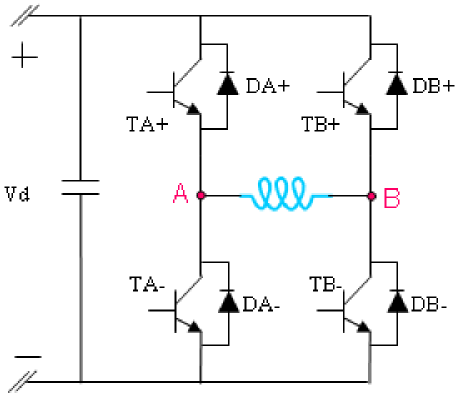
$$D_2 = 1 - D_1$$

$$= (2D_1 - 1) V_d$$

Duty ratio of bridge A *Duty ratio of bridge B*

$$\boxed{0 \le D_{1,2} \le 1}$$

Basic understanding of a power converter



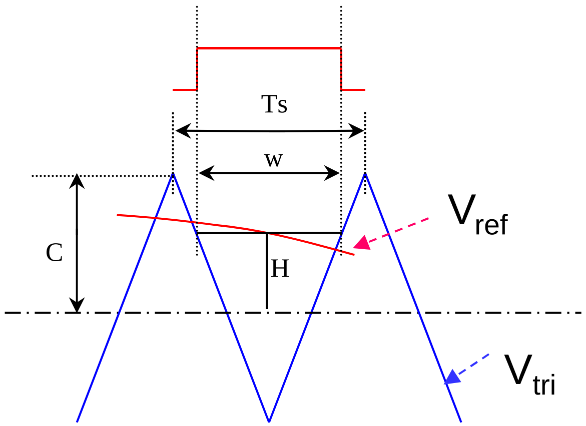
Example 1. bipolar voltage switching

D_A=1, is always coupled with D_B=0

D_A=0, is always coupled with D_B=1

$V_{ref}>V_{tri}$, TA+ on and TB- on, $V_{AB}=V_d$

$V_{ref}<V_{tri}$, TA- on and TB+ on, $V_{AB}=-V_d$



$$\frac{H+C}{2C}=\frac{w}{T_s}=D_1 \Longrightarrow m_a \overset{def}{=} \frac{V_{ref}}{V_{tri,pk}}=\frac{H}{C}=2D_1-1$$

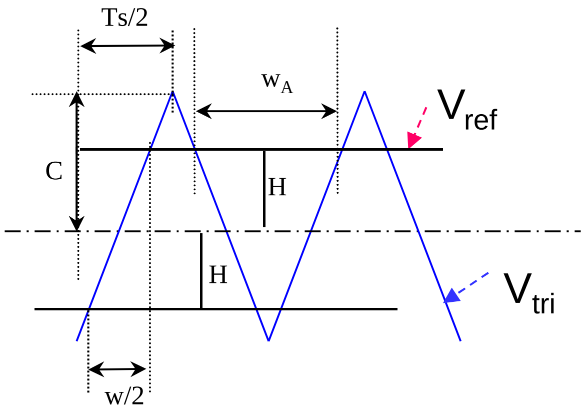
$$\boxed{V_o=V_{AB,Ts}=\frac{V_d}{V_{tri,pk}}V_{ref}=kV_{ref}}$$

$$\frac{V_{AB,Ts}}{V_d}=2D_1-1$$

<u>This is why a sinusoidal reference voltage will result in a sinusoidal output voltage. (not continuous - constituted from a lot of different DC values!)</u>

Basic understanding of a power converter

Example 2. unipolar voltage switching

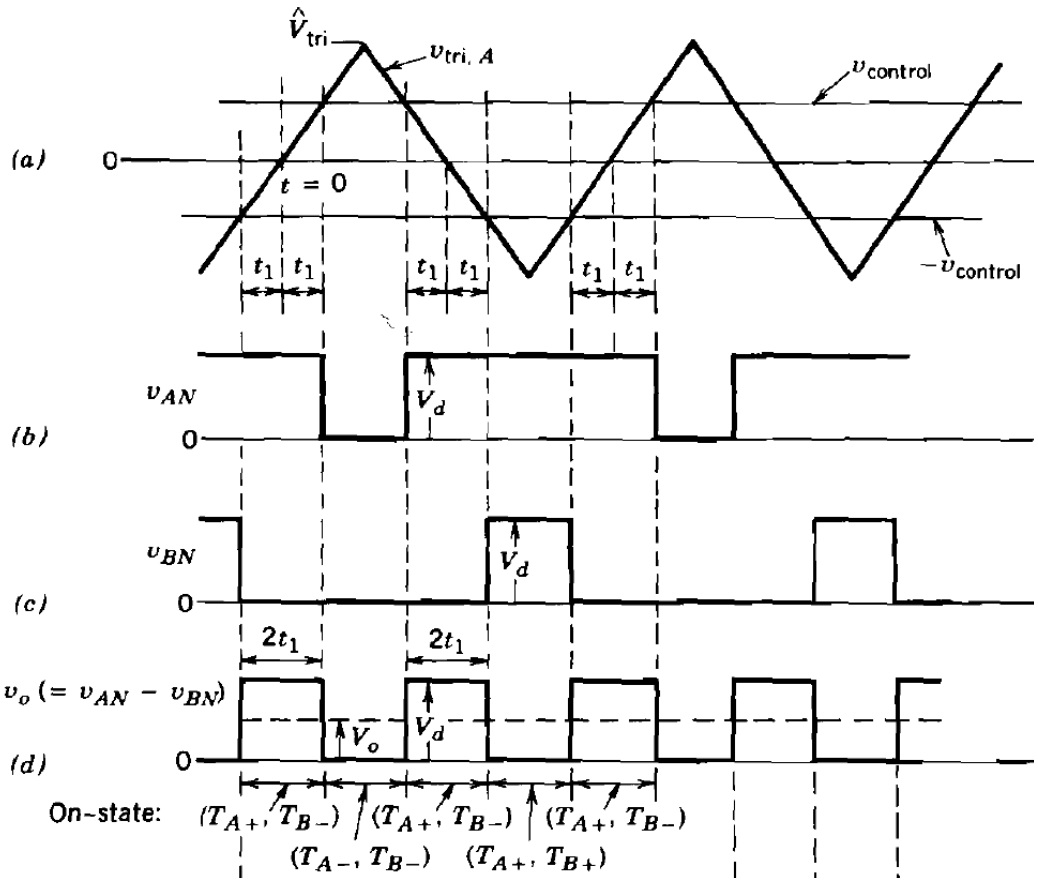


$$\frac{2H+(C-H)}{2C}=\frac{w/2}{T_s/2}=D_1=1-D_2$$

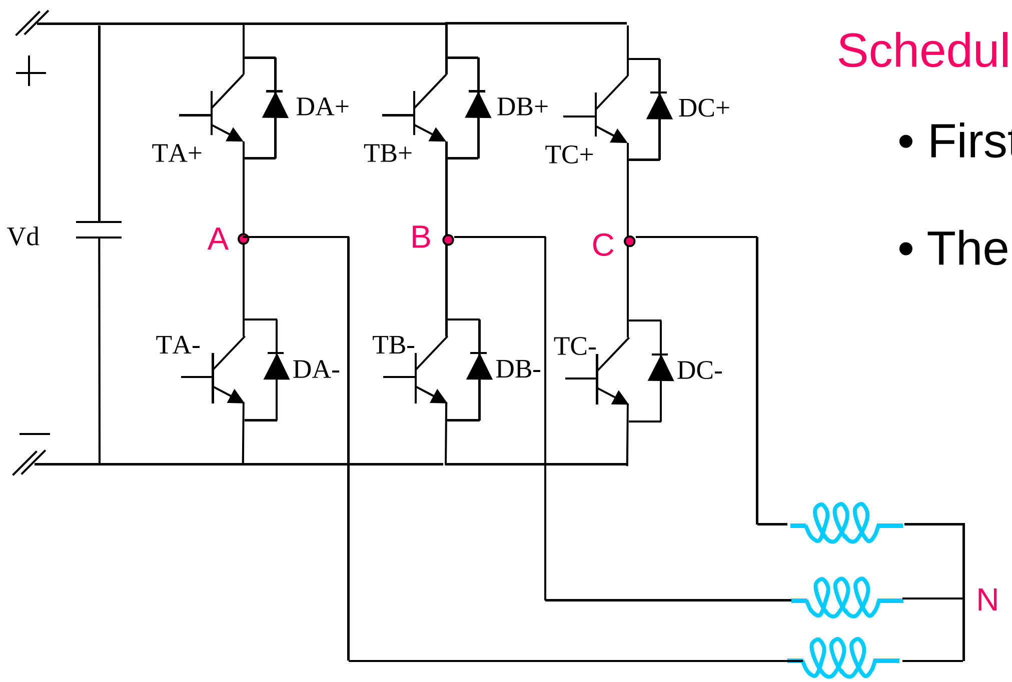
$$\boxed{V_o=V_{AB,Ts}=\frac{V_d}{V_{tri,pk}}V_{ref}=kV_{ref}}$$

The same average DC value per switching period!

RMS value? Harmonics?



Analysis of a 3-phase power converter



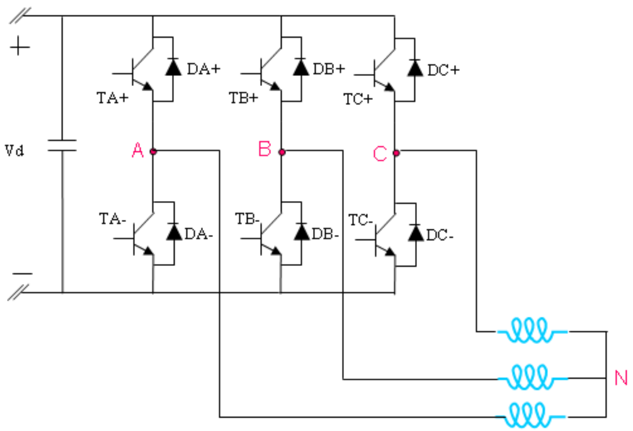
Schedule our analysis as

- First the line-to-line voltage
- Then the line-to-neutral voltage

$$V_{AB} = V_{AN} - V_{BN}$$
$$V_{BC} = V_{BN} - V_{CN}$$
$$V_{CA} = V_{CN} - V_{AN}$$
$$V_{AN} + V_{BN} + V_{CN} = 0$$

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix}$$

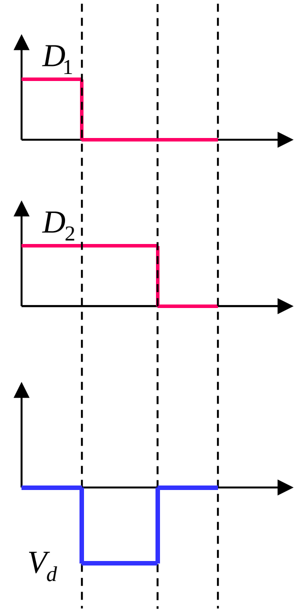
Analysis of a 3-phase power converter



$$V_{AB} = (D_1 - D_2)V_d$$

$$V_{BC} = (D_2 - D_3)V_d$$

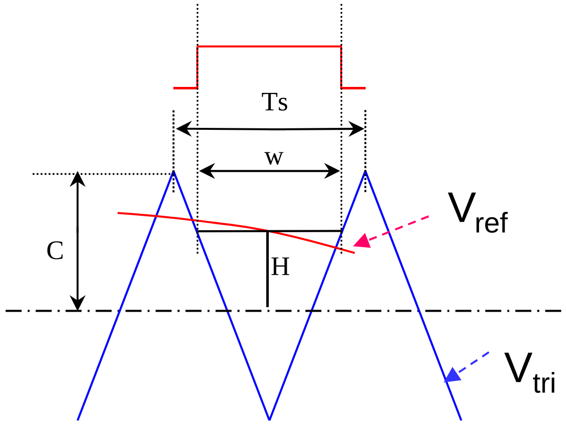
$$V_{CA} = (D_3 - D_1)V_d$$



$$V_{AN} = \frac{1}{3}(2D_1 - D_2 - D_3)V_d \overset{def}{=} V_1 \cos \omega t = V_1 \cos \theta$$

$$V_{BN} \overset{def}{=} V_1 \cos\left(\theta - \frac{2\pi}{3}\right)$$

$$V_{CN} \overset{def}{=} V_1 \cos\left(\theta + \frac{2\pi}{3}\right)$$



$$\frac{H + C}{2C} = \frac{w}{T_s} = D_1$$

Valid, independent of the load!

$$\frac{V_{ref}}{V_{tri,pk}} = \frac{H}{C} = 2D_1 - 1$$

Analysis of a 3-phase power converter

$$V_{AN} = \frac{1}{3}(2D_1 - D_2 - D_3)V_d = \frac{V_d}{3V_{tri,pk}}\left[V_{ref1} + V_{tri,pk} - \left(\frac{V_{ref2}}{2} + \frac{V_{tri,pk}}{2}\right) - \left(\frac{V_{ref3}}{2} + \frac{V_{tri,pk}}{2}\right)\right]$$

$$\boxed{V_{ref1,2,3} = V_{ref,pk}\cos(\theta + \phi_{1,2,3})}$$

$$V_{AN} = \frac{V_d}{3V_{tri,pk}}\left[V_{ref1} - \frac{V_{ref2}}{2} - \frac{V_{ref3}}{2}\right] = \frac{V_d V_{ref,pk}}{3V_{tri,pk}}\left[\cos(\theta+\phi_1) - \frac{1}{2}\cos(\theta+\phi_2) - \frac{1}{2}\cos(\theta+\phi_3)\right]$$

$$= \frac{V_d V_{ref,pk}}{3V_{tri,pk}}\left[\cos(\theta+\phi_1) - \cos\left(\theta + \frac{\phi_2+\phi_3}{2}\right)\cos\left(\frac{\phi_2-\phi_3}{2}\right)\right] \overset{def}{=} V_1\cos\theta$$

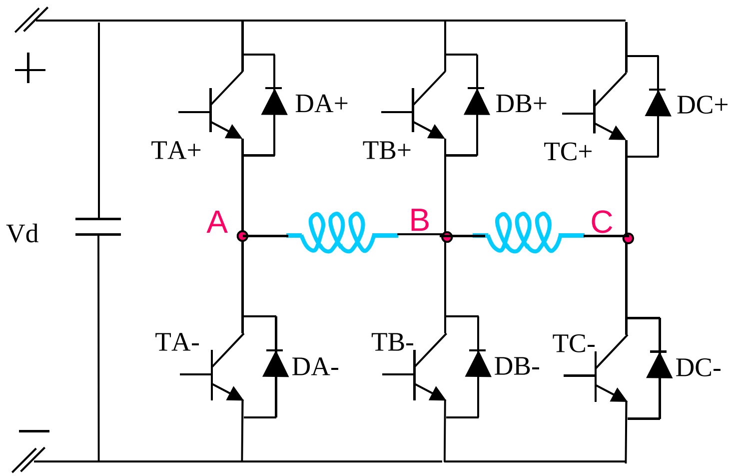
$$\boxed{\text{Let} \quad \phi_1 = 0,\ \phi_2 = -\frac{2\pi}{3},\ \phi_3 = +\frac{2\pi}{3}}$$

$$\boxed{\text{If } m_a \le 1, \quad V_{AN} \le \frac{V_d}{2},}$$

$$V_{AN} = \frac{V_d V_{ref,pk}}{3V_{tri,pk}}\left[\cos(\theta) + \frac{1}{2}\cos(\theta)\right] = \left(\frac{V_{ref,pk}}{V_{tri,pk}}\right)\cdot\frac{V_d}{2}\cos(\theta) = m_a\frac{V_d}{2}\cos(\theta)$$

Analysis of a 3-phase power converter

Example: A 3-phase inverter controlling a 2-phase motor



Requirement: V_{BC} lagging V_{AB} By 90 degrees

$$V_{AB} = V_1 \cos(\theta + \phi_0) \qquad V_{BC} = V_1 \cos\left(\theta - \frac{\pi}{2} + \phi_0\right)$$

$$V_{ref\,1,2} = V_{ref,pk} \cos(\theta + \phi_{1,2})$$

$$V_{AB} = (D_1 - D_2)V_d = \frac{V_d}{2}\frac{V_{ref,pk}}{V_{tri,pk}}[\cos(\theta+\phi_1) - \cos(\theta+\phi_2)] = V_d \frac{V_{ref,pk}}{V_{tri,pk}} \sin\left(\theta + \frac{\phi_1+\phi_2}{2}\right)\sin\left(\frac{\phi_2-\phi_1}{2}\right)$$

$$V_{BC} = (D_2 - D_3)V_d = \frac{V_d}{2}\frac{V_{ref,pk}}{V_{tri,pk}}[\cos(\theta+\phi_2) - \cos(\theta+\phi_3)] = V_d \frac{V_{ref,pk}}{V_{tri,pk}} \sin\left(\theta + \frac{\phi_2+\phi_3}{2}\right)\sin\left(\frac{\phi_3-\phi_2}{2}\right)$$

$$\phi_3 - \phi_2 = \phi_2 - \phi_1$$

$$\phi_1 + \phi_2 = \phi_2 + \phi_3 - \pi$$

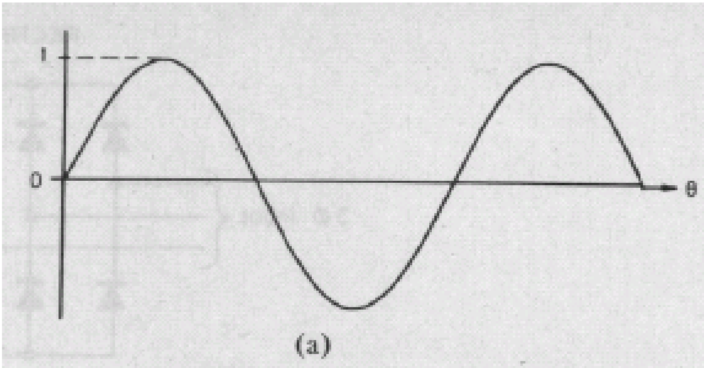
A possible solution is $\phi_1 = 0, \quad \phi_2 = \frac{\pi}{2}, \quad \phi_3 = \pi$

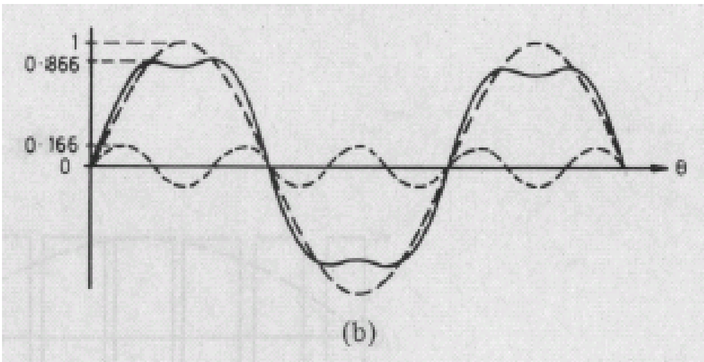
$$V_1 = m_a \frac{\sqrt{2}V_d}{2}$$

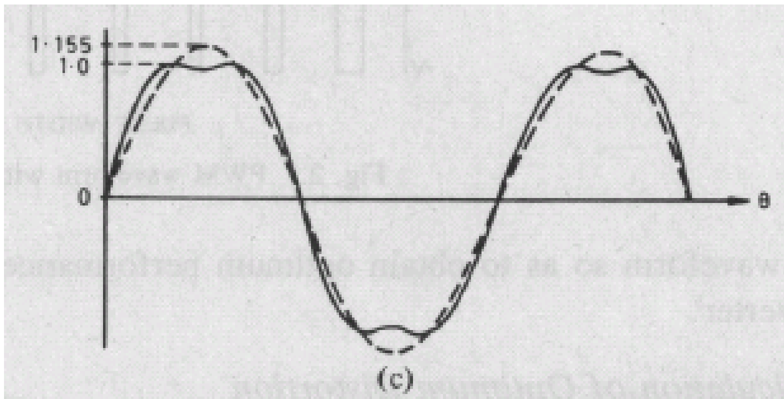
Injection with 3rd harmonic [*Ref-1*]

Better utilization of the DC linkage voltage

- The output peak voltage is limited by the DC-link voltage
- Can we reduce the peak voltage while maintain the required fundamental voltage component?







Injection with 3rd harmonic [*Ref-1*]

- In normal condition:

$$\frac{V_{ref}}{V_{tri,pk}} = \frac{H}{C} = 2D_1 - 1 \quad \Rightarrow \quad \mathrm{D}_1 = \frac{1}{2} + \frac{1}{2} m \sin\theta$$

$$\boxed{\mathrm{m} = \frac{V_{ref,pk}}{V_{tri,pk}} \subset [-1,1]}$$

with harmonic injection

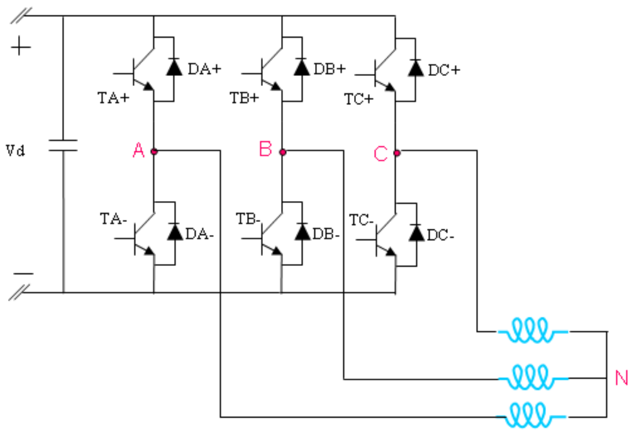
$$\mathrm{D}_1 = \frac{1}{2} + \frac{1}{2}\left(m\frac{2}{\sqrt{3}}\left(\sin\theta + \frac{1}{6}\sin 3\theta\right)\right)$$

- For other phases (bridges)

$$\theta \rightarrow \theta \pm \frac{2\pi}{3}$$

Space Vector Modulation

- **It may be found that**



$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = V_D \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_A \\ D_B \\ D_c \end{bmatrix}$$

Balanced 3-phase load

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = \frac{V_D}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} D_A \\ D_B \\ D_c \end{bmatrix}$$

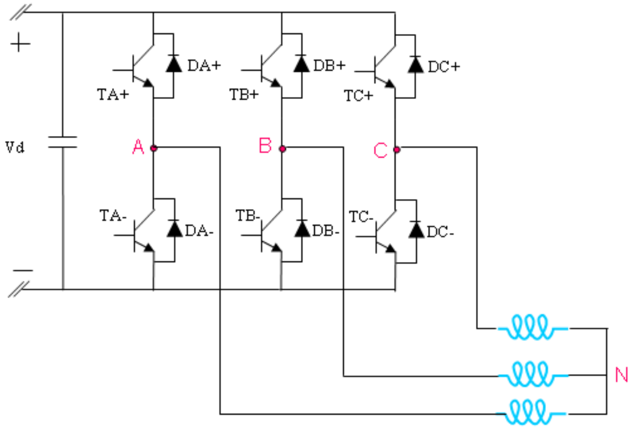
where D_A, D_B, D_c are the switch status (value 0 or 1)

- **There are only 8 possible different output voltage status**

Space Vector Modulation

Use space vector to describe the balanced 3-ph voltages

$$\vec{v} = \frac{2}{3}\left(v_a + e^{j\frac{2\pi}{3}} v_b + e^{-j\frac{2\pi}{3}} v_c\right)$$



To the α, β system

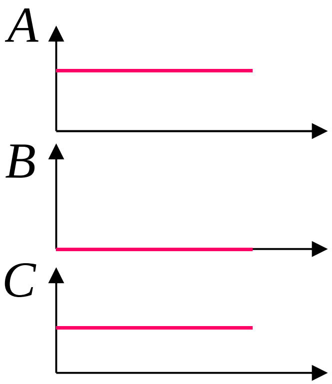
$$v_\alpha + jv_\beta = \frac{2}{3}\left(v_a + e^{j\frac{2\pi}{3}} v_b + e^{-j\frac{2\pi}{3}} v_c\right)$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}$$

Space Vector Modulation

Each voltage output status may correspond to a space vector

For example: Vector 5 (output switch status 101)

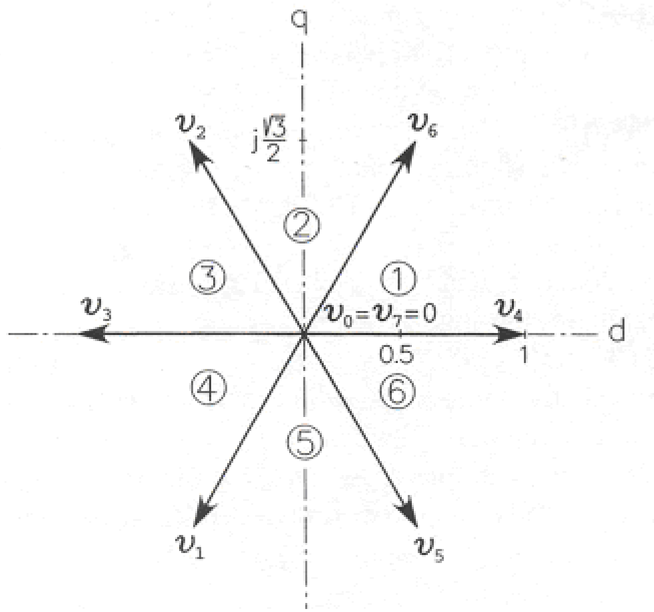


$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \frac{V_D}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{V_D}{3}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{V_D}{3}\begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$$

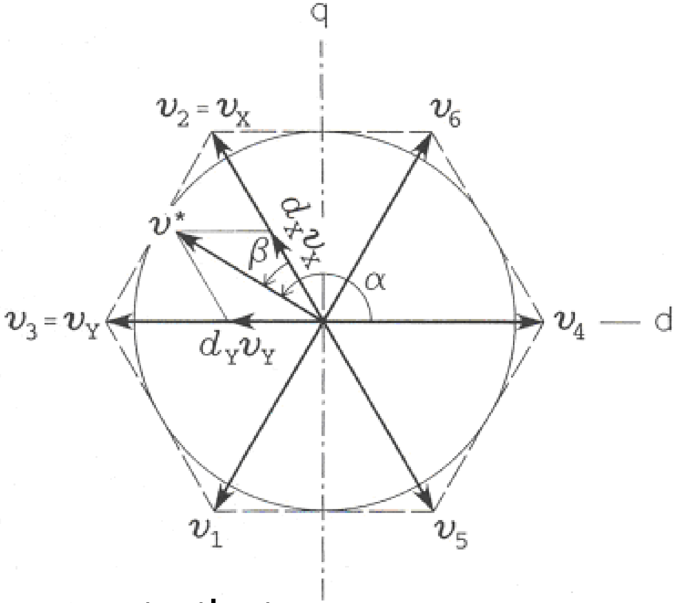
$$\vec{v}_5 = v_\alpha + jv_\beta = \frac{2V_D}{3}\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

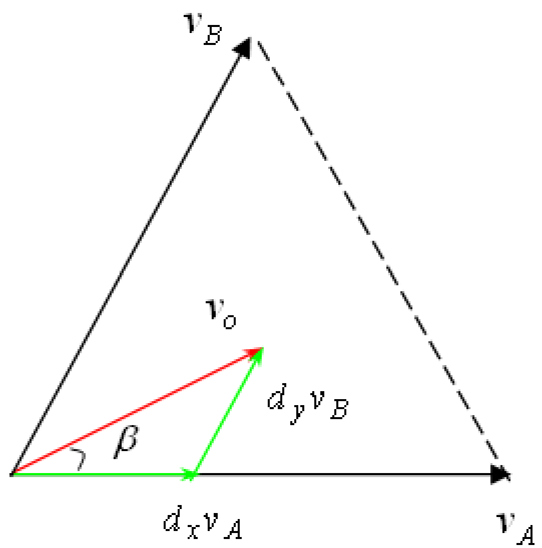
$$= \frac{2V_D}{3}\left(\cos\left(-\frac{\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)\right)$$



Space Vector Modulation

Let an arbitrary voltage vector to be constitued from its two neighbouring vectors





Please note that

$$\left|\vec{v}_i\right| \le \frac{2V_D}{3}, \quad i = 1,\ldots,6 \qquad\qquad |d_x| \le 1, \quad |d_y| \le 1$$

Space Vector Modulation

Suppose the Re-axis is rotated to be aligned with the starting vector of any sector along the positive rotational angle

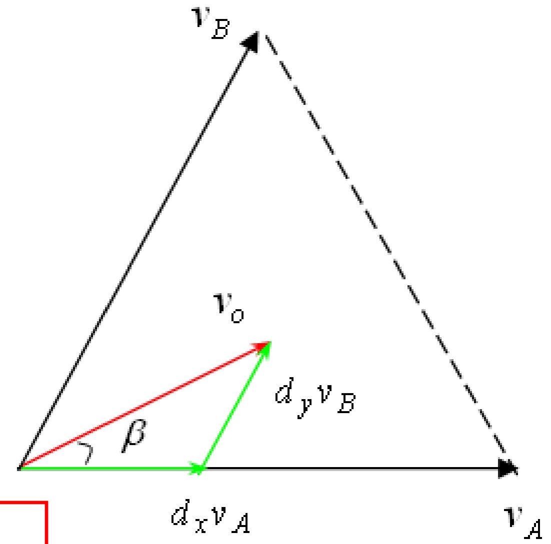
$$\vec{v}_o = d_x v_A + d_y \vec{v}_B = d_x v_A + d_y v_B e^{j60}$$

$$\Downarrow$$

$$\vec{v}_o = v_o \cos\beta + j v_o \sin\beta = d_x v_A + d_y \left(\frac{1}{2} v_B + j\frac{\sqrt{3}}{2} v_B \right)$$

$$\Downarrow$$

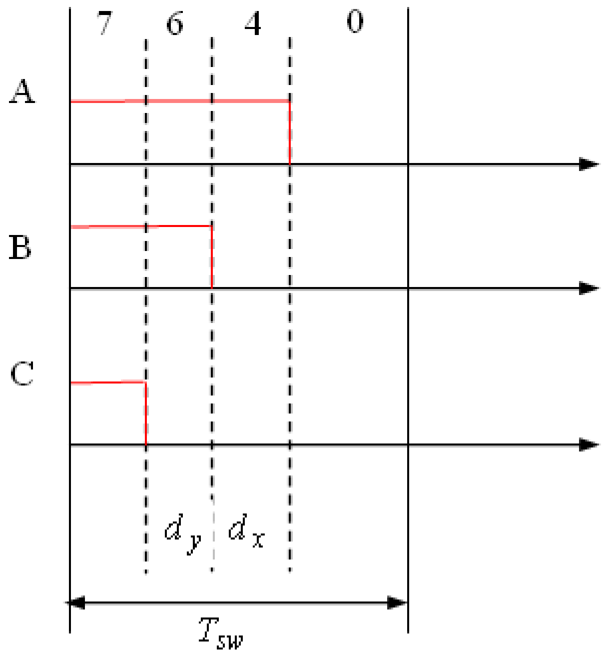
$$\boxed{\begin{aligned} d_x &= \frac{\sqrt{3} v_o}{v_A} \frac{2}{3} \sin(60-\beta) \\ d_y &= \frac{\sqrt{3} v_o}{v_B} \frac{2}{3} \sin\beta \end{aligned}} \Rightarrow \boxed{v_o \le \frac{V_D}{\sqrt{3}}}$$



knowning v_a, v_b, v_c ⇒ knowning v_o, β ⇒ Calculte d_x, d_y ⇒ determine the duty cycles for each leg

Space Vector Modulation

Example: for sector 1, vector 4, 6 and 0 or 7 can be used. If the output PWM duty cycle starts from the begining of a switching period:



$$\boxed{d_0 = 1 - d_x - d_y}$$

$$\begin{bmatrix} d_0 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Duty cycles for leg A, B and C

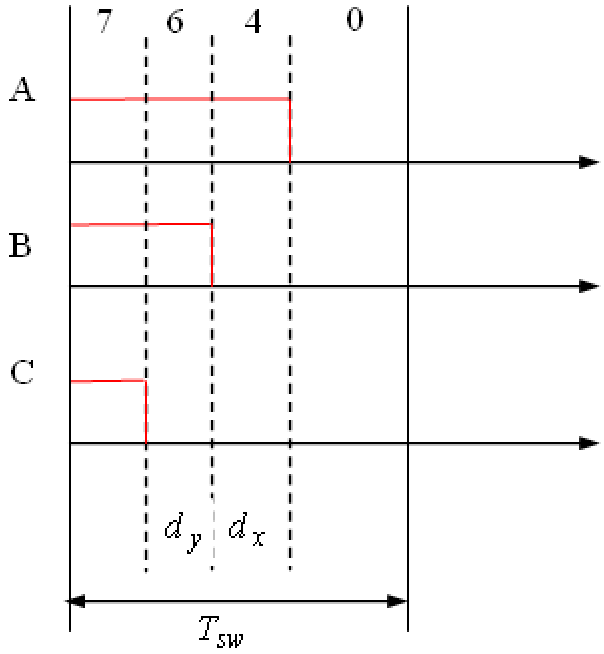
D_1, D_2, D_3 cannot be calculated from d_x, d_y, d_0 **why?**

One more constrain to be added

$$D_{v7} = 1 - D_{v0}$$

Space Vector Modulation

Example - continoued



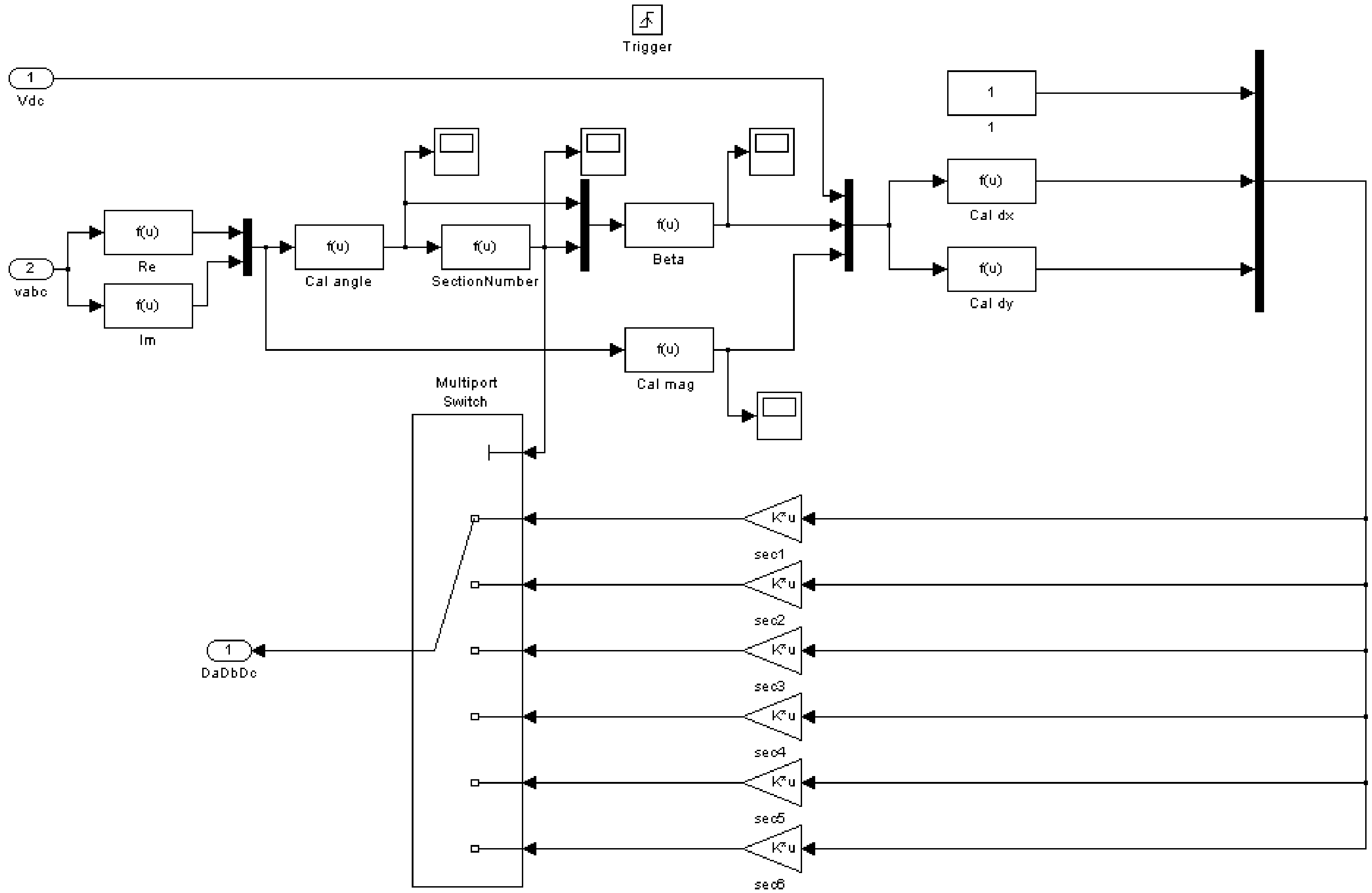
$$\begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{2} \boxed{\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}} \begin{bmatrix} 1 \\ d_x \\ d_y \end{bmatrix}$$

Exercise - please find this matrix for the rest of the sectors!

Space Vector Modulation

A Simulink model



Exercises

1. Finish the IM model model.
2. Find the transformation matrix from calculated dx, dy to D1, D2, D3 and finish the provided Simulink model.