Department of Energy Technology

AALBORG UNIVERSITET

Matlab and Control Theory — INTRO 1st semester 2013

January 28, 2014

Written exam

09.00-13.00 CET (4 hours)

The set consists of 11 problems

Rules

1. All usual helping aids are allowed, including text books, slides, personal notes, and exercise solutions.
2. Calculators and laptop computers are allowed, provided all wireless and wired communication equipment is turned off. Internet access is strictly forbidden.
3. Any kind of communication with other students is not allowed.

Remember

1. To write your study number on all sheets handed in.
2. It must be clear from the solutions, which methods you are using, and you must include sufficient intermediate calculations, diagrams, sketches etc. so the line of thought is clear. Printing the final result is insufficient.

Problem 1 (8 %)

A system has the unit step response shown in Figure 1.

1. Determine the transfer function for the system.

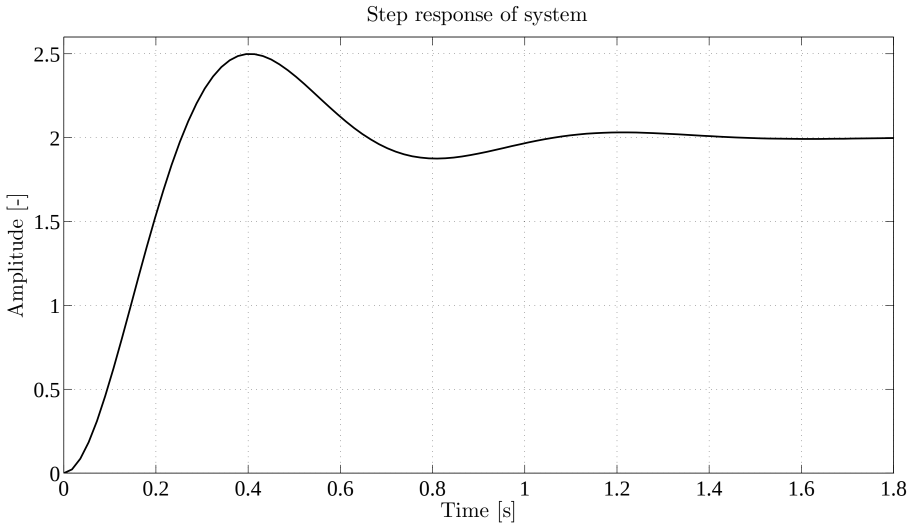

Figure 1: Unit step response for system, for which the transfer function should be determined.

Problem 2 (14 %)

The system shown in Figure 2 is considered, for which the transfer function for the plant is given by:

$$G(s) = \frac{10000}{s\left(s^2 + 14s + 100\right)}$$

For the first two questions the controller is simply a proportional controller with the gain 1, i.e. $G_c(s) = 1$

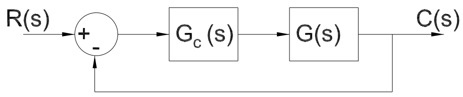

Figure 2: Block diagram for the considered system.

1. Sketch asymptotically the Bode diagram for the **open loop system**. A logarithmic graph paper may be found at the end of the exam set.

2. Determine the steady state error for the system, when given (a) a step input, $R(s) = \frac{1}{s}$, and (b) a ramp input, $R(s) = \frac{1}{s^2}$.

3. The controller is now changed to a PD-controller, so $G_c(s) = K_D s + K_P$, where the differential gain is set to $K_D = \frac{199}{100}$. Determine, using Routh-Hurwitz's stability criterion, for which values of K_P the closed loop system is stable.

Problem 3 (6 %)

The friction, F, in a system may be described as a function of velocity, v, by the expression:

$$F = F_c + (F_s - F_c)\, e^{-\alpha \cdot v} + B \cdot v$$

where F_c is the Coulomb friction, F_s is the Stribeck frictionen, α is a coefficient describing how fast the Stribeck friction decays and B is the viscous friction coefficient. All these parameters are constants.

1. Linearise the above expression and determine the linearisation coefficient.

Problem 4 (12 %)

The system shown in Figure 3 is considered, where $G_c(s)$ represents the controller and the system transfer function is given by:

$$G(s) = \frac{0.8}{s\left(s^2 + 1.2s + 1\right)}$$

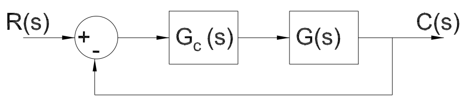

Figure 3: Block diagram for the system, for which the controller $G_c(s)$ should be designed.

1. Determine the gain margin and phase margin for the uncompensated system, i.e. for $G_c(s) = 1$, and explain how these are found.
2. Design a suited controller for the system so it obtains a phase margin of approximately $45°$and so the closed loop system has no steady state error for a step input. There is no need to improve the systems transient performance. Explain why you choose the controller you do.

As a help the open loop Bode diagram for the system without controller is shown in Figure 4.

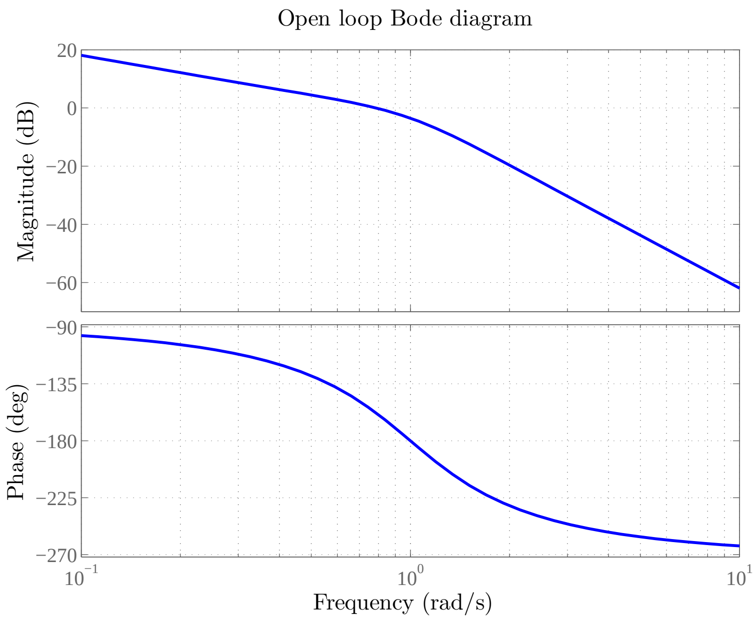

Figure 4: Open loop Bode diagram for considered system (without controller).

Problem 5 (6 %)

Consider a general discrete-time closed-loop control system. In this context, explain in your own words:

1. What a zero-order hold is and what it is used for
2. The role of the sampler
3. The factors that affect the proper selection of the sampling rate

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Problem 6 (8 %)

A discrete-time system is represented by the difference equation

$$y(k) - 0.6y(k-1) + 0.7y(k-2) - 0.8y(k-3) = x(k)$$

where the input is $x(k)$ and the output is denoted by $y(k)$.

1. Find the discrete transfer function $Y(z)/X(z)$
2. Is the system stable?
3. List the `Matlab` commands that will plot the frequency response for system.

   Note: You do <u>not</u> need to execute the commands or show the results

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Problem 7 (14 %)

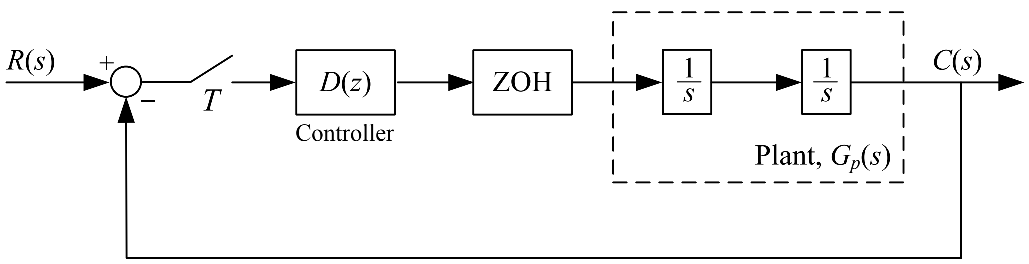

Figure 5: Closed-loop discrete control system.

Figure 5 shows a closed-loop control system, where the controller is a proportional controller, i.e. $D(z) = K$. The sampling time is $T = 3$ sec.

1. Determine the pulse transfer function $G(z)$ for the system in Figure 5.
2. Sketch the root locus
3. Assuming $K = 1$, find the characteristic equation
4. Calculate all closed-loop poles when $K = 1$

Problem 8 (12 %)

A continuous-time filter is given by the following differential equation:

$$\ddot{u} + b\dot{u} = \dot{e} + ae$$

where the continuous-time input and output are e and u, respectively. a and b are positive constants.

This filter is discretized using the forward rectangular rule (forward Euler method) using the sampling time T.

1. Find the discrete transfer function for the equivalent discrete filter
2. Determine the difference equation for $u(k)$
3. For the special case where $a = 2$ and $b = 20$, what sampling time T would you recommend to use?

Problem 9 (4 %)

Given the following vectors and matrix:

$$V_1 = \begin{bmatrix} 57 & 46 & 35 & \cdots & -20 \end{bmatrix}$$
$$V_2 = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$
$$V_3 = \begin{bmatrix} 10 & 12 & 14 & 16 \end{bmatrix}$$
$$V_4 = \begin{bmatrix} 25 & 24 & 23 & 22 \end{bmatrix}$$
$$M_1 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 10 & 12 & 14 & 16 \\ 25 & 24 & 23 & 22 \end{bmatrix}$$

Write `Matlab` commands that:

1. Generate a 4-by-3 matrix where all elements are zero
2. Make the vectors V_1, V_2, V_3, and V_4
3. Make the matrix M_1 from the vectors
4. Get the size of a vector (Use V_1 as an example)
5. Get the size of a matrix (Use M_1 as an example)

Problem 10 (8 %)

Assume you need to simulate the drag effect on a vehicle, which is described by the following equations:

$$F_{\text{drag}} = \frac{1}{2}\rho\, C_{\text{drag}}\, A_{\text{front}}\, v^2$$
$$F_{\text{res}} = ma = m\frac{dv}{dt} = F_{\text{motor}} - F_{\text{drag}}$$

where the motor force F_{motor} is the input and the velocity v is considered the output. ρ, C_{drag}, A_{front} and m are vehicle constants. a is the acceleration.

1. Sketch the `Simulink` implementation of the described system using only the Constant, Sum, Gain, Product and Integrator blocks shown in Figure 6 below.

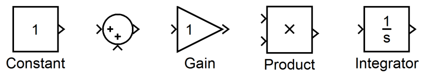

Figure 6: Simulink blocks.

Problem 11 (8 %)

Determine the transfer function $H(z) = \frac{Y(z)}{X(z)}$ for the system in Figure 7.

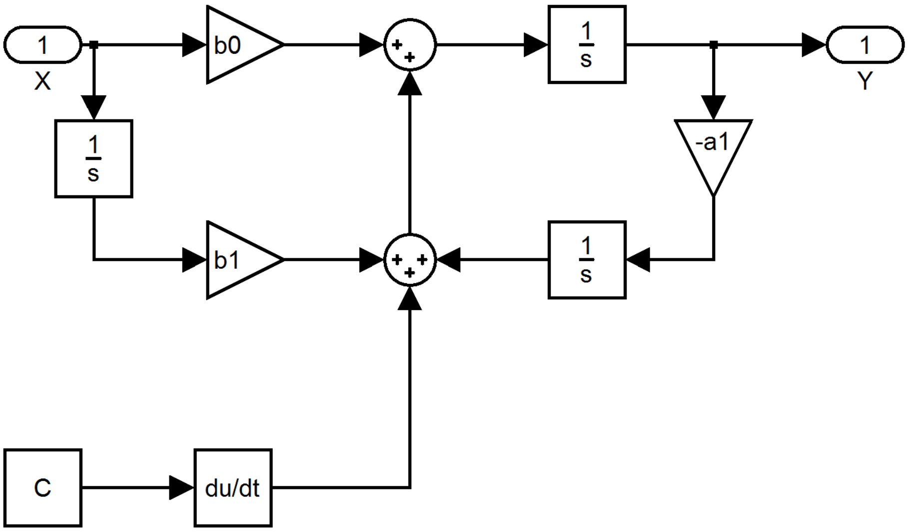

Figure 7: Problem 11.

Logarithmic graph paper for Problem 2

This page may be handed in with the solution.