

Course: _____

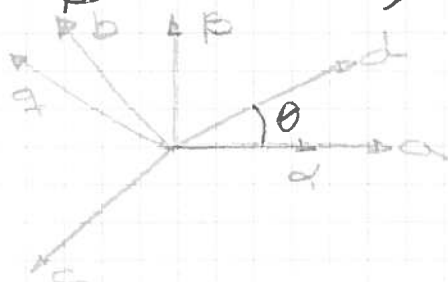
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EXAM 2012

2 problems de Dynamical
1 de Non control

Problem 1

- Please draw the reference frame axes for abc reference frame, dq rotating reference frame and α - β stationary reference frame.



- Suppose now you have a set of 3-phase signals as:

$$V_a = V_{pk} \cdot \cos(\omega t)$$

$$\omega = 2\pi 50$$

$$V_b = V_{pk} \cdot \cos(\omega t - \frac{2\pi}{3})$$

$$V_{pk} = 1$$

$$V_c = V_{pk} \cdot \cos(\omega t + \frac{2\pi}{3})$$

Please draw the signal waveforms viewed in dq reference frame for

- dq reference frame is rotating at 50 Hz (↺)

abc2dq $i = (I_d + jI_q) \cdot e^{j\theta}$

$V_a =$
 $V_b =$
 $V_c =$ we define this

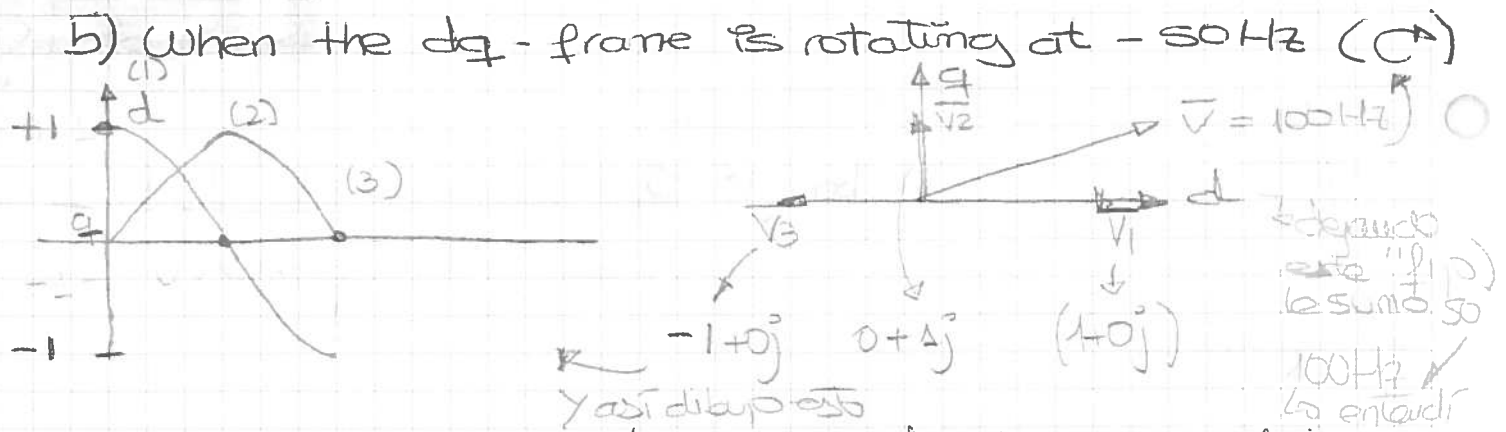
$$V_{dq} = \text{abc2dq}(I_a, I_b, I_c)$$

$$I_d = \frac{2}{3}(I_a \cos(\theta) + I_b \cos(\theta - 120) + I_c \cos(\theta + 120))$$

$$I_q = \frac{2}{3}(-I_a \sin(\theta) - I_b \sin(\theta - 120) - I_c \sin(\theta + 120))$$

$$V_{dq} = I_d + jI_q$$





Transform the V_a, V_b, V_c signals in 2 to alpha-beta reference frame.

(Demonstration) I'm not going to do it

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} V_a - \frac{V_b}{2} - \frac{V_c}{2} \\ V_b \cdot \frac{\sqrt{3}}{2} - V_c \cdot \frac{\sqrt{3}}{2} \end{bmatrix}$$

For the following 3-phase signals

$$V_a = V_{pk} \cdot \cos(\omega_e t + \frac{\pi}{6})$$

$$V_b = V_{pk} \cdot \cos(\omega_e t - \frac{2\pi}{3} + \frac{\pi}{6})$$

$$V_c = V_{pk} \cdot \cos(\omega_e t + \frac{2\pi}{3} + \frac{\pi}{6})$$

$$\omega_e = 2\pi 50$$

$$V_{pk} = 1$$

Please draw its space vector at time $t = \frac{1}{50}$, what is the amplitude of this space vector?

$$V_a = 1 \cdot \cos(2\pi 50 \cdot \frac{1}{50} + \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$$

1 RAD

$$V_b = 1 \cdot \cos(2\pi - \frac{2\pi}{3} + \frac{\pi}{6}) = 0$$

$$V_c = 1 \cdot \cos(2\pi + \frac{2\pi}{3} + \frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$$

DEG

$$\vec{f}_{t=0} = \frac{2}{3} \left(\frac{\sqrt{3}}{2} \cdot e^{j0} - \frac{\sqrt{3}}{2} \cdot e^{-j120} \right) = \frac{2}{3} \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} (\cos 120 - j \sin 120) \right)$$

$$= \frac{2}{3} \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cos 120 + \frac{\sqrt{3}}{2} \sin 120 j \right) = 1 \angle 30^\circ$$

es igual que $\cos(-120) + j \sin(-120)$

c Transform the following power eq. in α - β frame to dq frame.

$$P = V_{\alpha\beta} (I_{\alpha\beta})^*$$

$$\text{where } (I_{\alpha\beta})^* = (I_\alpha + j I_\beta)^* = I_\alpha - j I_\beta$$

The relationship between the α - β frame and the dq frame is $I_{\alpha\beta} = I_{dq} \cdot e^{j\theta}$

$$P = V_{\alpha\beta} (I_{\alpha\beta})^*$$

$$I_{\alpha\beta} = I_{dq} \cdot e^{j\theta}$$

$$P = (V_{dq} \cdot e^{j\theta}) (I_{dq} \cdot e^{j\theta})^* = V_{dq} \cdot e^{j\theta} \cdot (I_{dq})^* (e^{j\theta})^* = V_{dq} I_{dq}^*$$

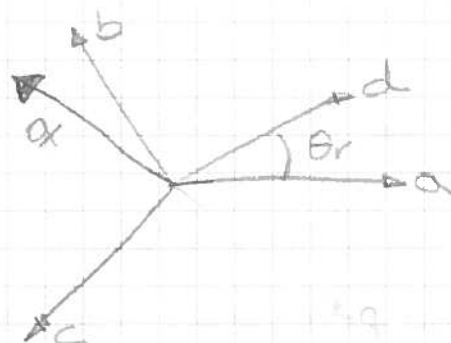
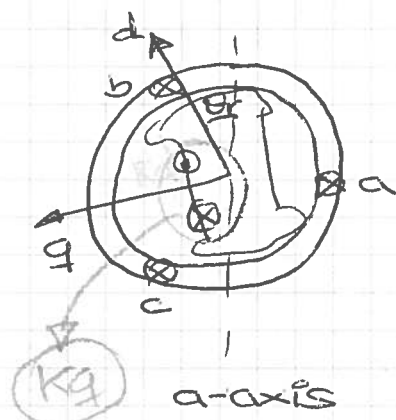
↳ Think the power doesn't depend on position

$$\begin{aligned} I_\alpha + j I_\beta \\ (I_d + j I_q) \cdot e^{j\theta} \end{aligned}$$

$$\boxed{\begin{aligned} I_\alpha &= I_d \\ I_\beta &= I_q \end{aligned}}$$

Problem 2

A sketch of a synchronous machine is shown below



$$(f_d + jf_q) \cdot e^{j\theta}$$

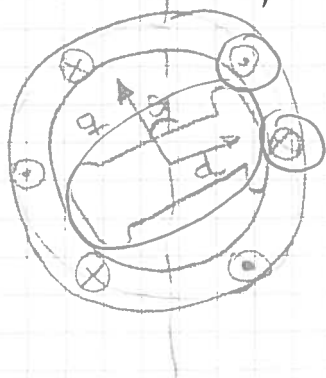
- Please describe how the mutual inductance between phase a and phase b is obtained

$$\begin{aligned}
 M_{ab} &= L_{aa} \cdot \text{Re} \left(\frac{e^{j\theta}}{e^{j0}} \right) \left(\frac{e^{j\theta}}{e^{j120}} \right) + L_{aq} \cdot \text{Re} \left(\frac{e^{j(\theta-90)}}{e^{j0}} \right) \left(\frac{e^{j(\theta-90)}}{e^{j120}} \right) \\
 &= L_{aa} \cdot \cos \theta \cdot \cos(\theta - 120) + L_{aq} \cdot \cos(\theta - 90) \cdot \cos(\theta - 90 - 120) = \\
 &= L_{aa} \cdot \cos \theta \cdot \cos(\theta - 120) + L_{aq} \cdot \sin \theta \cdot \sin(\theta - 120)
 \end{aligned}$$

- How the mutual inductance between the rotor winding and the stator phase-a winding is obtained?

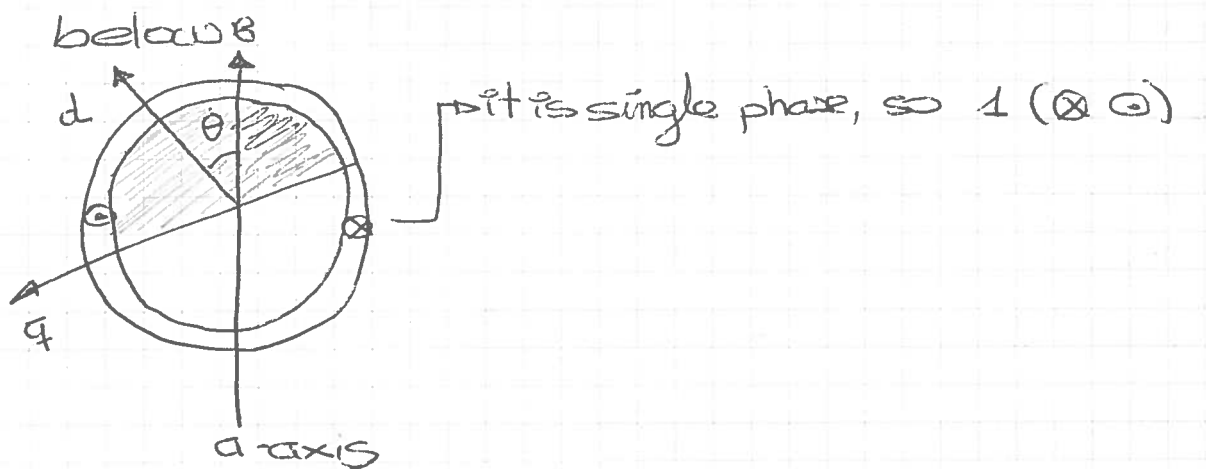
$$M_{akq} = M_{kq} \cdot \text{Re} \left(\frac{e^{j(\theta_r+90)}}{e^{j0}} \right) \text{Re} \left(\frac{e^{j(\theta_r+90)}}{e^{j(\theta_r+90)}} \right) = M_{kq} \cdot \cos(\theta_r+90) = M_{kq} \cdot \sin(\theta_r)$$

- Please explain from the physical point of view, for such a machine, why the eg. dq axis inductances will be position independent?



They rotate together, at the same speed, so airgap doesn't change and hence inductance doesn't change

A simple single-phase PM machine is shown



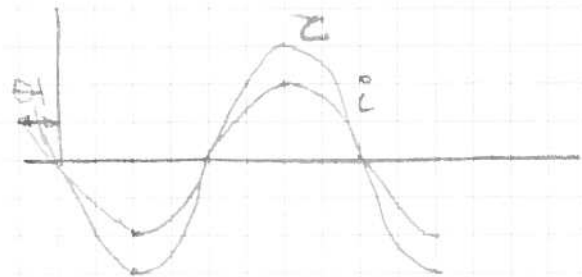
- Please show its instantaneous torque waveform when the machine delivers max torque for a given sinusoidal armature current. Please also show the current waveform in relation of the rotor pos.

$$i = -I_m \cdot \sin(\theta + \theta_t) \rightarrow 270^\circ$$

$$\tau = p \cdot i \cdot \lambda_{mpm} (-\sin \theta) \Rightarrow \text{max torque } \sin(270^\circ) = -1$$

$$\theta = \omega t; 270^\circ = 3\pi/2$$

$$\tau = i \cdot p \cdot \lambda_{mpm} \quad \text{OK}$$



- Does the inductance value have any influence on the output torque? Please explain

$$\tau_{ave} = \frac{1}{2} p I_m \lambda_{mpm} \cos \theta_t$$

$$\tau = p i \cdot \lambda_{mpm} (-\sin \theta)$$

→ No influence

"The power is influence by inductance"

◦ If the armature current contains a 3th harmonic.
 Will this 3th harmonic current component produce
any torque? What is the inst. and average torque
 corresponding to this harmonic current?

$$\tau = p \cdot i \lambda_{mpm} (-\sin \theta)$$

i = has a 3th harmonic.

$$\begin{aligned} \tau &= p \lambda_{mpm} \cdot \text{Im} [-\sin(\theta + \theta_1) + \sin 3\theta] [-\sin \theta] \\ &= p \lambda_{mpm} \text{Im} [\sin(\theta + \theta_1) \cdot \sin \theta - \sin \theta - \sin 3\theta] \end{aligned}$$

The 3th harmonic contributes

$$-\sin \theta \cdot \sin 3\theta = \frac{\cos 4\theta - \cos 2\theta}{2} \quad \text{will interact}$$

with the rotor $\left(\frac{d}{d\theta} (\lambda_{mpm} \cos \theta) \right) \xrightarrow{2} 4^{\text{th}} \text{ and } 2^{\text{th}} \text{ harmonic}$

$$\begin{aligned} \tau_{\text{ave}} &= \frac{1}{2\pi} \int_0^{2\pi} \tau d\theta = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos 4\theta - \cos 2\theta}{2} d\theta \\ &= 0 \end{aligned}$$

I have no average torque, it is a waste, we try to avoid this.

Ripe and no gain

Question 1 (– Lecture 2)

- Please draw the reference frame axes for abc-reference frame, dq rotating reference frame, and afa-bet stationary reference frame.
- Suppose you have the following three phase voltages:

$$V'_a = I'_m \cos\left(\omega_s t + \frac{\pi}{6}\right)$$

$$V'_b = I'_m \cos\left(\omega_s t - \frac{2\pi}{3} + \frac{\pi}{6}\right)$$

$$V'_c = I'_m \cos\left(\omega_s t + \frac{2\pi}{3} + \frac{\pi}{6}\right)$$

Where $\omega_s = 2\pi \cdot 50$

At time $t = 0$, please draw the space vector (also called the general vector) for the above abc-system. Please draw the space vector again at time $t = \frac{1}{300}$.

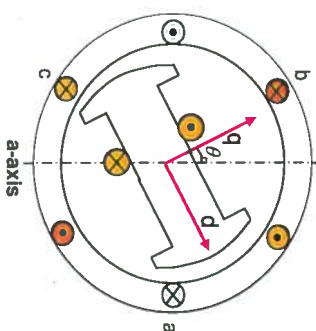
- Please roughly indicate what is the waveform in afa-bet reference frame and in a dq-reference frame that rotates at 50 (Hz).

Question 2 (– Lecture 2)

- Using the space vector (also called the general vector) method to describe how different reference frame transformation matrices are obtained
- Please write the expressions for the space vectors expressed in the abc-reference frame, and in a rotating dq reference frame.
- Please tell how these two vectors are related to each other.
- Please then indicate how the transformation matrices between abc-2-dq, and dq-2-abc may be obtained.
- When the space vector expressed in the abc-reference frame is known, how the instantaneous phase b variable may be obtained from the known space vector?
- If the dq-frame is a rotating reference frame and it rotates at a speed of $-2\pi \cdot 10$ radian per second. For a balanced set of 3-phase voltages at a frequency of 50 Hz, what are the voltage waveforms in this rotating dq reference frame? Suppose at $t=0$, the voltage space vector is aligned with the phase-a axis, and the d-axis is also aligned with the phase-a axis.

Question 3 (– Lecture 3 & 4)

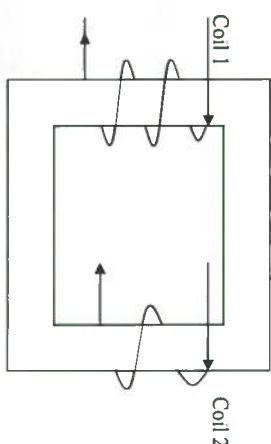
Given such a motor drawing as below



- Please describe how the mutual inductance between phase-b and phase-c is obtained.
- How the mutual inductance between the rotor winding and the stator phase-b is obtained.
- Please explain from the physical point of view, for such a machine, why the equivalent d, q-axes inductances will be position independent?

Question 4 (– Lecture 1, 4 & 5)

There is a transformer with two coils sketched as below.



- Please express the flux linkage for coil 1 using the coils 1 self-inductance (L_1) and coil 1 and coil 2 mutual inductance (M)
- Suppose coil 1 has N_1 turns, and coil 2 has N_2 turns. If it is desired to perform turns ratio transformation, to let coil 2 to have the same number of turns as coil 1, which parameters and variables related to coil 2 are affected by the turns ratio transformation?

- Please give the voltage and flux linkage equations for coil No.2 after turns ratio transformation.

Question 5 (– Lecture 5)

Regarding the Simulink example model discussed in lecture 5 (P20-25)

- Please write down the voltage and flux linkage equations on the d-axis of this machine.
- Please explain how the equations could be modeled in Simulink.
- Please tell us there any initial conditions to be set in the Simulink model?
- Please tell what is the mechanical equation that should be used?
- Please give the machine equation in steady state.
- What is the rated voltage for the rotor field winding based on the data given on P20?

Question 6 (– Lecture 6)

- For an induction machine, its stator self-inductance matrix is (slide P11)

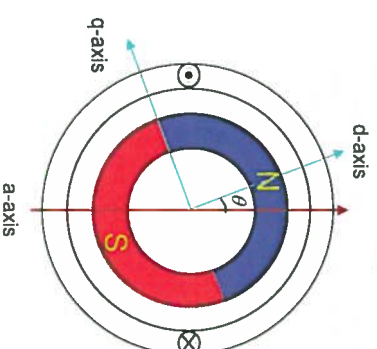
$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = L_s \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & L_{ls} + L_{ms} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} & L_{ls} + L_{ms} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

Please explain from the physical point of view, why there is a co-efficient -1/2 before the mutual inductance.

- Is the leakage inductance a constant? For the main inductance and the leakage inductance, which one is more important and is more predominant?
- Why the speed related coefficient in the rotor side voltage equation is $\omega_b(1 - \alpha)$? (Slide P17)
- Please given the induction machine model in steady state and is represented in the α - β reference frame.

Question 7 (– Lecture 7)

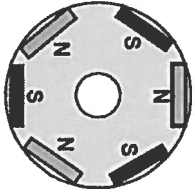
For a single-phase PM machine, as sketched below



- Please describe how its instantaneous torque equation is obtained.
- Does the inductance value have any influence on the output torque for this machine?
- Please sketch how its torque waveform looks like?
- Suppose you have a 3rd harmonic component in the winding current, please demonstrate its instantaneous torque waveform and give its average torque equation.
- Suppose you supply the machine with a trapezoidal current waveform instead of sinusoidal, what is the consequence in the output torque? Can you still be able to rotate the machine?

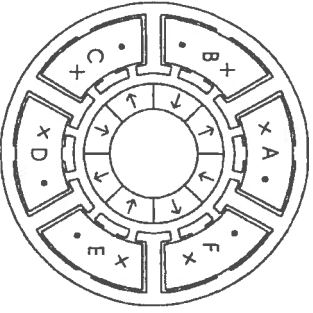
Question 8 (– Lecture 8)

- In the model of a synchronous PM machine, how the rotor PM flux linkage is used in the model? Is its RMS value or its peak value used? Please give explanations.
- Please use the voltage / flux linkage equations of the machine to drive the following when the PM machine is driven by a DC motor, and is rotating at the rated speed with stator windings open-circuited, what will be the winding terminal voltages measured on the d,q-frame?
- For the following PM machine rotor, please indicate its flux lines and determine if $L_d < L_q$ or $L_d > L_q$?



Question 9 (– Lecture 9)

Examine the following machine

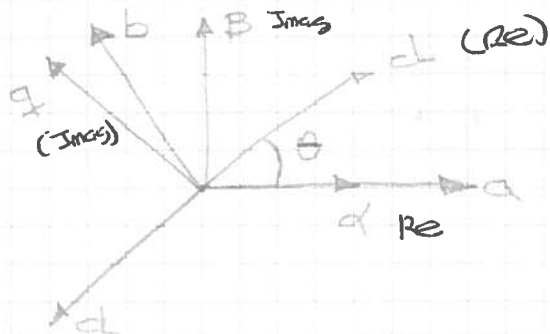


- How the rotor magnet flux will link the stator poles? Please draw a sketch.
- How the flux produced by a stator winding will link its neighboring poles? (Remember that for examining the flux produced by stator winding current, the rotor magnets are replaced by air.)
- What is the mutual inductance between different windings?
- What will be the voltage equation for this machine?
- What will be the torque equation for this machine?

Exam - All Questions

1

Please draw the reference frame axes for abc reference frame, dg rotating reference frame and α - β stationary reference frame.



Suppose you have the following 3-phase voltages:

$$V_a = V_{peak} \left(\omega t + \frac{\pi}{6} \right) \quad \omega = 2\pi 50$$

$$V_b = V_{pk} \left(\omega t - \frac{2\pi}{3} + \frac{\pi}{6} \right)$$

$$V_c = V_{pk} \left(\omega t + \frac{2\pi}{3} + \frac{\pi}{6} \right)$$

At time $t=0$, please draw the space vector (also called the general vector) for the above abc system. Please draw the space vector again at time $t = 1/300$

$$\vec{f} = \frac{2}{3} (f_a \cdot e^{j0} + f_b \cdot e^{j+2\pi/3} + f_c \cdot e^{-j+2\pi/3})$$

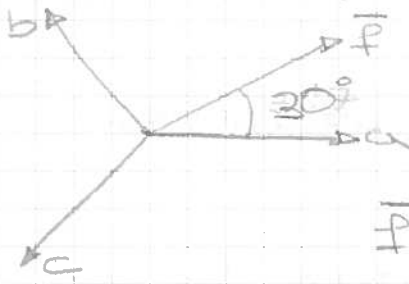
$$f_a = \cos(\omega t + \frac{\pi}{6}) \xrightarrow{t=0} f_a = \sqrt{3}/2$$

$$f_b = \cos(\omega t - \frac{2\pi}{3} + \frac{\pi}{6}) \xrightarrow{t=0} f_b = 0$$

$$f_c = \cos(\omega t + \frac{2\pi}{3} + \frac{\pi}{6}) \xrightarrow{t=0} f_c = -\sqrt{3}/2$$

$$\begin{aligned} \vec{f}_{t=0} &= \frac{2}{3} \left(\sqrt{3}/2 \cdot e^{j0} - \sqrt{3}/2 \cdot e^{-j+2\pi/3} \right) = \frac{2}{3} \left(\sqrt{3}/2 - \sqrt{3}/2 (\cos 2\pi/3 - j \sin 2\pi/3) \right) \\ &= \frac{2}{3} \left(\frac{3\sqrt{3}}{2} + j \frac{3}{4} \right) = 1 \angle 30^\circ \end{aligned}$$

The vector is rotated 30° with the phase-a-axis, which is the reference.



$$\bar{F} = \frac{2}{3} \left(\frac{\sqrt{3}}{2} \cdot (\cos 120^\circ + j \sin 120^\circ) - \frac{\sqrt{3}}{2} (\cos 240^\circ - j \sin 240^\circ) \right)$$

$$t = 1/200$$

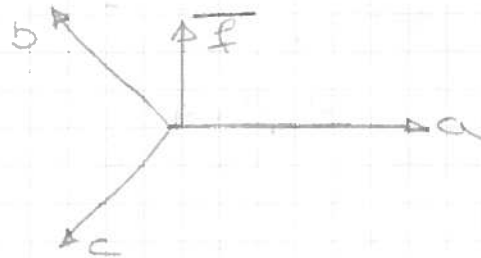
$$f_a = 0$$

$$f_b = \frac{\sqrt{3}}{2}$$

$$f_c = -\frac{\sqrt{3}}{2}$$

$$\bar{F} = \frac{2}{3} \left(-\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} + j \left(\frac{\sqrt{3}}{2} \sin 120^\circ + \frac{\sqrt{3}}{2} \sin 240^\circ \right) \right) =$$

$$= 1 \angle 90^\circ$$



Please roughly indicate what is the waveform in d-q reference frame and in d-g reference frame that rotates at 50 Hz.

$$f = 50 \text{ Hz}$$

$$T = 1/50 = 0,02$$

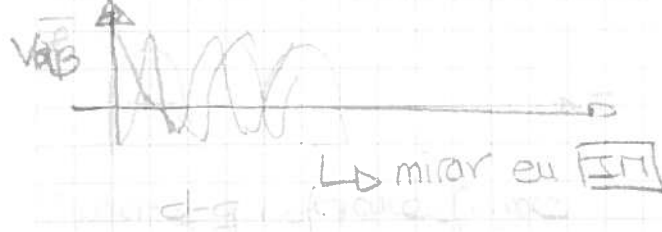


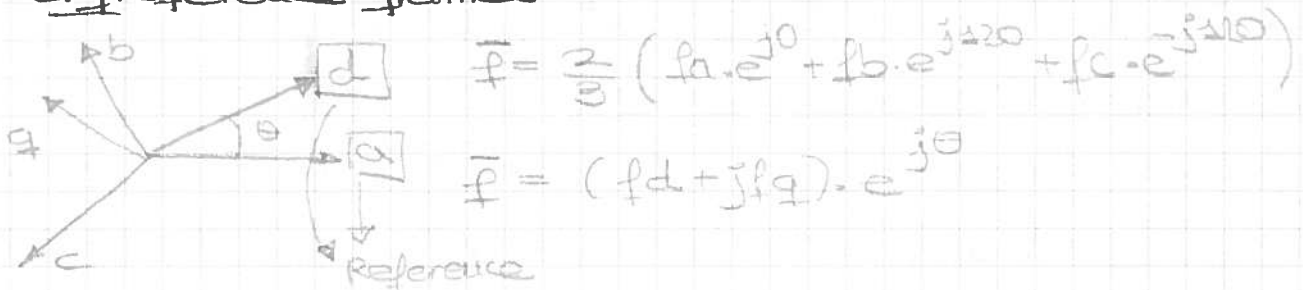
Figura 8.11

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2 Question 2

Using the space vector (also called the general vector) method to describe how different reference frame transformation matrices are obtained.

• Please write the expressions for the space vector expressed in the abc-reference frame and in a rotating dq reference frame.



• Please tell how these two vectors are related to each other.

θ is the angle rotated between the reference of c both vectors.

• Please then indicate how the transformation matrices between abc \leftrightarrow dq and dq \leftrightarrow abc may be obtained.

Easy

• When the space vector expressed in the abc-reference frame is known, how the instantaneous phase b variable may be obtained from the known space vector?

$$\vec{f} = \frac{2}{3} (f_a \cdot e^{j0} + f_b \cdot e^{j120} + f_c \cdot e^{-j120})$$

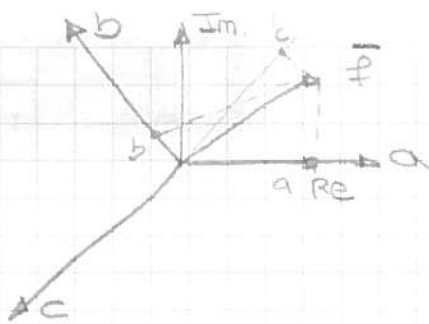
↑



The vector and phase α -axis is rotated α which is known. so phase

Kike

Ang b
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$$\bar{f} = \frac{2}{3} (f_{a0} \cdot e^{j0} + f_{b0} e^{j120} + f_{c0} \cdot e^{-j120})$$

↓

$$\bar{f} + \bar{f}^* \cdot e^{j240} = 2f_{b0} \cdot e^{j120}$$

⊕

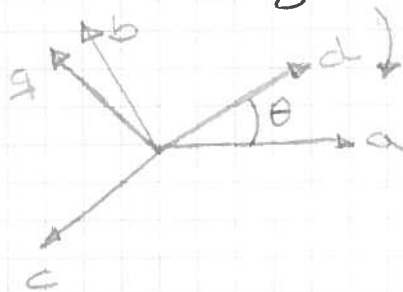
$$\bar{f} \cdot e^{-j120} + \bar{f}^* \cdot e^{j120} = 2f_{b0} \rightarrow \text{we know } f^*$$

$$f_{b0} = f_b - f_0$$

so we know f_{b0}

$$f_0 = \frac{1}{3} (f_a + f_b + f_c) = 0 \quad (\text{in symmetrical systems})$$

• If the dq frame is a rotating reference frame and it rotates at a speed of $-2\pi 150 \frac{\text{rad}}{\text{s}}$. For a balanced set of 3-phase voltages at a $f = 50 \text{ Hz}$, what are the voltages wave forms in this dq reference frame? Suppose at $t=0$, the voltage space vector is aligned with the phase a-axis, and the d-axis is also aligned with the phase-a axis.



$$\omega = -2\pi 150 = -20\pi$$

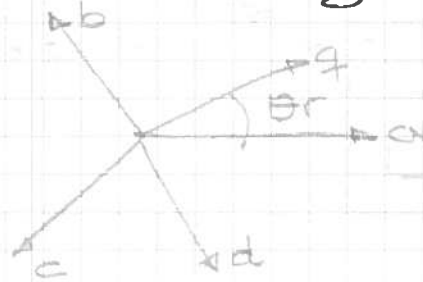
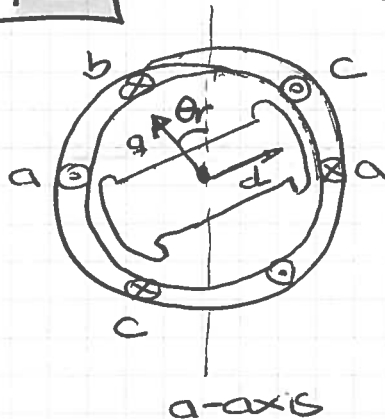
$$\rightarrow \omega = 2\pi 50 = 100\pi$$

Reference frame, so I fix a $\rightarrow \omega = 0 \frac{\text{rad}}{\text{s}}$

$$-20\pi - 100\pi = -120\pi; \omega = -120\pi \frac{\text{rad}}{\text{s}}$$

$$\bar{f} = (f_d + jf_q) \cdot e^{j\theta}$$

3 Given such a motor drawing as belows



o Please describe how the mutual inductance between phase b and phase c is obtained.

$$\begin{aligned}
 M_{bscs} &= L_{aaq} \cdot \text{Re} \left(\frac{e^{j\theta_r}}{e^{j120}} \right) \cdot \text{Re} \left(\frac{e^{j\theta_r}}{e^{-j120}} \right) + L_{aad} \cdot \text{Re} \left(\frac{e^{j(\theta_r-40)}}{e^{j120}} \right) \cdot \text{Re} \left(\frac{e^{j(\theta_r-40)}}{e^{-j120}} \right) = \\
 &= L_{aaq} \cdot \cos(\theta_r - 120) \cdot \cos(\theta_r + 120) + L_{aad} \cdot (\cos\theta \cdot \cos 120 + \sin 90 \cdot \sin 120) \cdot (\cos\theta \cdot \cos 120 - \sin 90 \cdot \sin 120)
 \end{aligned}$$

$$M_{bscs} = -\frac{1}{2} L_A - L_2 \cdot \cos(2\theta_r)$$

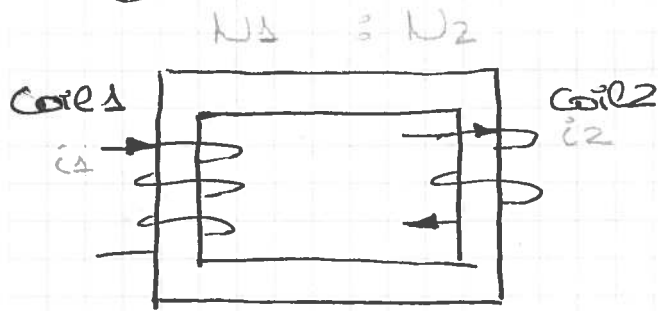
o How the mutual inductance between the rotor winding and the stator phase b is obtained

$$\begin{aligned}
 M_{bsfdm} &= L_{sfd} \cdot \text{Re} \left(\frac{e^{j(\theta_r-120)}}{e^{j120}} \right) = L_{sfd} \cdot (\cos(\theta_r-120) - j \sin(\theta_r-120)) \cdot (\cos 120 + j \sin 120) = \\
 &= L_{sfd} \cdot (\cos(\theta_r-120) \cdot \cos 120 + \sin(\theta_r-120) \cdot \sin 120) = \\
 &= L_{sfd} \cdot \sin(\theta_r - 120) \quad \cos(\theta_r - 90 - 120) = \sin(\theta_r - 120)
 \end{aligned}$$

o why the equivalent dq axes inductances are position independent?

$$\lambda = \lambda_l + \lambda_m = L_l \cdot i + L_m \cdot i = \begin{cases} \lambda_{gm} = L_{mq} \cdot i_q \rightarrow L_{mq} = \frac{3}{2} L_{aaq} \\ \text{position independent} \end{cases}$$

4 There is a transformer with two coils sketched as below.



The current goes into the transformer the direction is OK, positive

o Please express the flux linkage for coil 1 using the coils 1- self inductance (L_1) and coil 1 and coil 2 mutual inductance

$$\lambda_1 = L_{11} \cdot \vec{i}_1 + L_m \cdot \vec{i}_2 \pm M \vec{i}_2$$

\pm depending of the direction

$$\boxed{\lambda_1 = L_{11} \cdot \vec{i}_1 + M \cdot \vec{i}_2}$$

o Suppose coil 1 has N_1 turns and coil 2 N_2 turns, if it is desired to perform turns ratio transformation, to let coil 2 to have the same number of turns as coil 1, which parameters and variables related to coil 2 are affected by the turns ratio transformation?

$$L_{11} = L_m + L_{1\sigma} ; L_{22} = \left(\frac{N_2}{N_1} \right)^2 L_m + L_{2\sigma} ; M = L_m \cdot \frac{N_2}{N_1}$$

$$\lambda_1 = L_{1\sigma} \vec{i}_1 + L_m \cdot \vec{i}_1 - L_m \cdot \vec{i}_2'$$

$$\lambda_2' = -L_{2\sigma} \cdot \vec{i}_2' - L_m \vec{i}_2' + L_m \cdot \vec{i}_1$$

Parameters affected: $L_{2\sigma}$, R_2' , \vec{i}_2' , v_2' , λ_2'

↑ Te lo pae en las dispositivas

o Please give the voltage and flux linkage eq. for coil 2 after turns ratio transformation.

$$\lambda_2' = -L_{2\sigma}' \cdot \dot{i}_2' - L_m \cdot \dot{i}_2' + L_m \cdot \dot{i}_1$$

$$v_2' = -R_2' \dot{i}_2' + \frac{d\lambda_2'}{dt}$$

check if the sign is ok.

5 Skip it, do it later (simulink model)

6 For an induction machine, its stator self-inductance matrix is

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = L_s \times \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -L_{ms}/2 & -L_{ms}/2 \\ -L_{ms}/2 & L_{ls} + L_{ms} & -L_{ms}/2 \\ -L_{ms}/2 & -L_{ms}/2 & L_{ls} + L_{ms} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

Please explain from the physical point of view, why there is a coefficient $-\frac{1}{2}$ before the mutual inductance.

\rightarrow ? what do I mean?

$$M_{ab} = L_{mag} \cdot \text{Re} \left(\frac{e^{j\theta}}{e^{j0}} \right) \text{Re} \left(\frac{e^{j\theta}}{e^{j\pi/2}} \right) + L_{rad} \cdot \text{Re} \left(\frac{e^{j(\theta-\pi)}}{e^{j0}} \right) \text{Re} \left(\frac{e^{j(\theta-\pi/2)}}{e^{j\pi/2}} \right)$$

$L_{mag} = L_{rad}$ (I think) because the airgap is constant so the inductance has the same value in every point = $-L_{ms}/2$

o Is the leakage inductance a constant? For the main inductance and the leakage inductance which one is more important and is more predominant?

L_{ls} is a constant

The most predominant L_{ms} (mutual inductance) more important I think is the same answer.

o Why the speed related coefficient in the rotor side voltage equation is $(\omega_s - \omega_r)$?

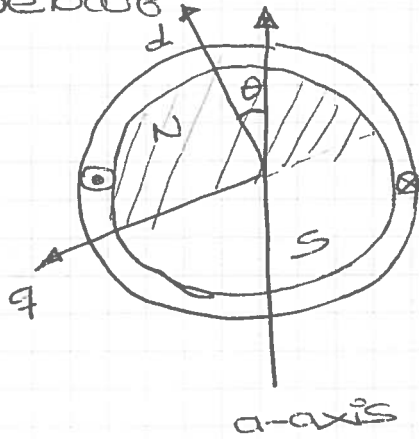
$\omega_B = \omega_s - \omega_r \Rightarrow$ It is explained.

o Please give the induction machine model in steady state and represented in the α - β reference frame.

Same question as in the exercises

7

For a single-phase PM machine, as sketched below:



o Please describe how its instantaneous torque eq. is obtained.

$$\tau = \frac{1}{\omega} P_{mec} = \frac{1}{\omega} \cdot i \frac{d\lambda_{pm}}{dt} = p \cdot i \lambda_{pm} (-\sin \theta)$$

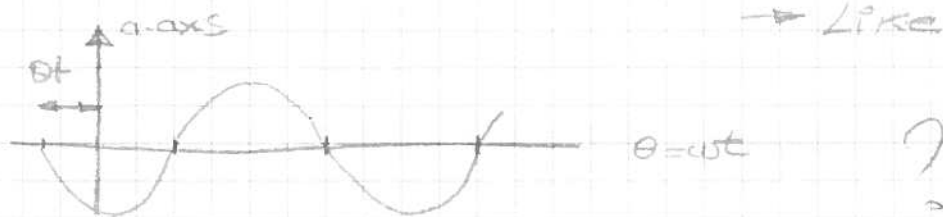
$$\downarrow i = -I_m \sin(\theta + \theta_e)$$

$$\tau = p I_m \lambda_{pm} \sin \theta \sin(\theta + \theta_e) = \frac{1}{2} p I_m \lambda_{pm} \times [\cos \theta_e - \cos(2\theta + \theta_e)]$$

o Does the inductance value have any influence on the output torque for this machine?

NO

o Please sketch how its torque waveform looks like.



→ Like the current

o Suppose you have a 3th harmonic component in the winding current, please demonstrate its instantaneous torque waveform and give its average torque eq.

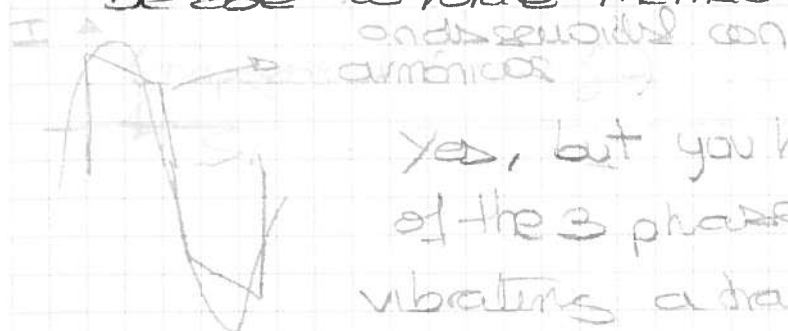
$\omega = 2\pi f \rightarrow$ fundamental

$3\omega = 3 \times 2\pi f \rightarrow 3^{\text{th}}$ harmonic

Average torque: $\frac{1}{2} p \sin \Delta \text{mpm} \cdot \cos 3\theta t$
 ↳ due to 3th harmonic

Ya se ha hecho

o Suppose you supply the machine with a trapezoidal current waveform instead of sinusoidal, what is the consequence in the output torque? Can you still be able to rotate the machine? → I think so



Yes, but you have to control the seq. of the 3 phases. The torque will be vibrating a trapez signal contains a sinusoidal with harmonics.

Therefore the torque will not be cte. It will be vibrating.