

# Written examination in the course Optimisation Theory and Modern Reliability from a Practical Approach Wednesday June 11th 2014

kl. 9 - 13 (4 hours)

The examination set consist of two parts, one for »Optimisation Theory« and one for »Modern Reliability from a Practical Approach«.

For the part in Modern Reliability from a Practical Approach: No computers or other helping aids as slides, textbooks etc. may be used. This means that you should hand in your solution for this part, before starting up your computer or bringing out your notes if you wish to use these for the Optimisation Theory part. The pages in the reliability part are meant to be handed in.

For the Optimisation Theory part: All usual helping aids are allowed, i.e. books, notes, calculator, computer etc. All communication equipment and computer communication protocols must be turned off.

The questions should be answered in English.

**REMEMBER** to write your study number and page number on all sheets handed in.

The set consists of four exercises in the optimisation part (totalling 50% of the examination set) and 25 questions in the reliability part. The weighting for each exercise is stated in percentage. You need in total 50 % of the entire set (optimisation and reliability part) correct in order to pass the exam.

It should be clear from the solution, which methods are used, and there should be a sufficient number of intermediate calculations, so the line of thought is clear.

# Exercise 1: (25 %)

A company produces two products: 1) a MP3 player and 2) a Watch-TV (a wristwatch TV).

The production process for each product is similar in that both require a certain number of hours of electronic work and a certain number of labor-hours in the assembly department. Each MP3 player takes 4 hours of electronic work and 2 hours in the assembly shop. Each Watch-TV requires 3 hours in electronics and 1 hour in assembly.

During the current production period, 240 hours of electronic time are available, and 100 hours of assembly department time are available.

Each MP3 player sold yields a profit of 7 Euro; each Watch-TV produced may be sold for a 5 Euro profit.

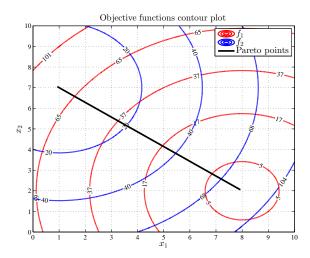
- a) Formulate the problem of determining the best possible combination of MP3 players and Watch-TVs to manufacture to reach the maximum profit.
- b) Solve the problem formulated in a) using graphical methods.
- c) Set up the Lagrangian function for the problem formulated in a) and find point(s) satisfying the KKT necessary conditions. You may make use of results from b) when checking/solving the KKT necessary conditions.

### Exercise 2: (7 %)

The following multi-objective optimisation problem is considered:

minimise 
$$f_1(\mathbf{x}) = (x_1 - 8)^2 + (x_2 - 2)^2 + 3$$
  
 $f_2(\mathbf{x}) = (x_1 - 1)^2 + (x_2 - 7)^2 + 10$ 

Figure 1 shows the Pareto optimal points in the design space and figure 2 shows the Pareto optimal set in the criterion space.



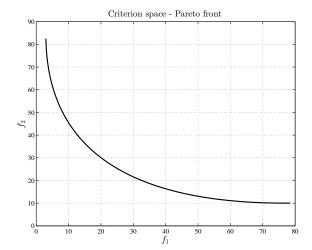


Figure 1: Contour curves and Pareto optimal points for the problem of exercise 3.

Figure 2: Pareto set in criterion space for the problem of exercise 3.

- a) Determine the objective function values of the utopia point.
- b) Assume that the multi-objective problem is solved as single objective problem using the weighting method (i.e. minimise  $U(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x})$ ), with  $w_1 = w_2 = 1$ . Determine the minimum objective function value  $U(\mathbf{x}^*)$ , and the optimum set of design variables  $\mathbf{x}^*$ .

# Exercise 3: (10 %)

An optimisation problem is formulated as:

minimise 
$$f(\mathbf{x}) = 4x_1^2 + 2x_2^2 - 2x_1x_2 + 2x_1 - x_2$$

Subject to:

$$g_1(\mathbf{x}) = x_1 - 12 \le 0$$
  
 $g_2(\mathbf{x}) = x_1^2 - x_1 - 2x_2 - 8 \le 0$   
 $g_3(\mathbf{x}) = x_1 + x_2 - 20 \le 0$ 

The initial starting point for an optimisation is  $\mathbf{x}^{(0)} = (-1, -2)$ , for which the search direction is determined to  $\mathbf{d}^{(0)} = \begin{bmatrix} 2 & 7 \end{bmatrix}^T$  and the vector of Lagrange multipliers for the constraints is  $\mathbf{u} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ . Let  $R_0 = 1$  and  $\gamma = 0.5$ . Choose the trial step according to the sequence  $t_0 = 1$ ,  $t_1 = \frac{1}{2}$ ,  $t_2 = \frac{1}{4}$ ,  $t_3 = \frac{1}{8}$ ...

Calculate the step size using the inexact line search procedure (approximate step size procedure) and determine the new design variables,  $\mathbf{x}^{(1)}$ , for the next iteration.

## Exercise 4: (8 %)

Solve the following problem by setting up the solution tree and using the Branch & Bound Method with Local Minimization:

minimise 
$$f(\mathbf{x}) = -3x_1 - 8x_2$$

Subject to:

$$g_1(\mathbf{x}) = -0.5x_1 + x_2 - 20 \le 0$$
  
 $g_2(\mathbf{x}) = 2x_1 + x_2 - 20 \le 0$   
 $g_3(\mathbf{x}) = 4x_1 - x_2 - 20 \le 0$   
Both  $x_1$  and  $x_2$  should be integer values

As a help for solving the problem the objective function contours and constraints are plotted in figure 3.

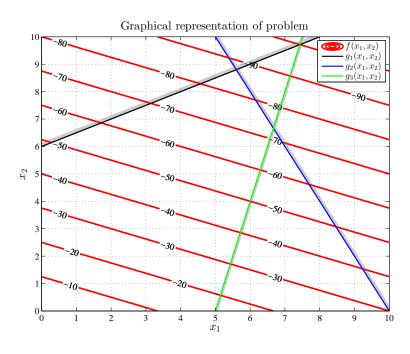


Figure 3: Graphical representation of problem in exercise 4.

### 1) What describe best the difference between Quality and Reliability?

- a) Brand
- b) Level
- c) Time
- d) Capability
- e) Customer satisfaction

### 2) In a Weibull analysis:

### **INFORMATION:**

If you want you can use the Weibull plot and medium rank table on the next page.

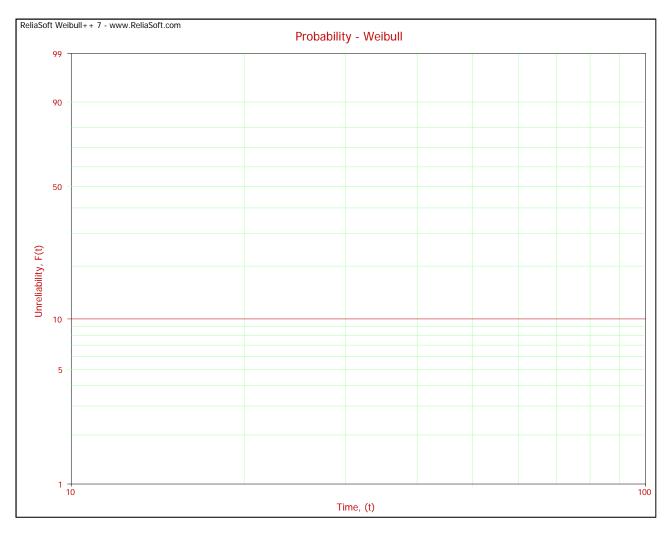
But it is not mandatory to use.

An accelerated test has been done on 10 Printed Circuit Board for testing solder joints. The overall first analysis gave Beta = 3.1 and Eta = 68 days. The test was stopped after 65 days. A deeper analysis of the data and the PCB's showed the following days to fail:

Component location	Pcb1	Pcb2	Pcb3	Pcb4	Pcb5	Pcb6	Pcb7	Pcb8	Pcb9	Pcb10	Failures on solder joint
C14	40		30			20	41				4
L7		36		46	49			55	63	28	6
connector	64	50				58	58		64		5
27 other did not fail											0

Which answer will you choose from the median rank table, for doing correct Weibull analyses?

- a) 4,5 and 6
- b) 10
- c) 15
- d) 30
- e) 300



Rank order	4	5	6	10	30	300
1	15,9%	13,0%	10,9%	6,7%	2,3%	0,2%
2	38,6%	31,5%	26,6%	16,3%	5,6%	0,6%
3	61,4%	50,0%	42,2%	26,0%	8,9%	0,9%
4	84,1%	68,5%	57,8%	35,6%	12,2%	1,2%
5		87,0%	73,4%	45,2%	15,5%	1,6%
6			89,1%	54,8%	18,8%	1,9%
7				64,4%	22,0%	2,2%
8				74,0%	25,3%	2,6%
9				83,7%	28,6%	2,9%
10				93,3%	31,9%	3,2%
11					35,2%	3,6%
12					38,5%	3,9%
13					41,8%	4,2%
14					45,1%	4,6%
15					48,4%	4,9%
16					51,6%	5,2%
17					54,9%	5,6%
18					58,2%	5,9%
19					61,5%	6,2%
20					64,8%	6,6%
21					68,1%	6,9%
22					71,4%	7,2%
23					74,7%	7,6%
24					78,0%	7,9%
25					81,3%	8,2%
26					84,5%	8,6%
27	ĺ				87,8%	8,9%
28					91,1%	9,2%
29					94,4%	9,6%
30					97,7%	Etc.

3) From the test (questions no 2) above. The test equipment is able to perform between 65 and 72 cycles per 24 hours inclusive the time used for analysis. Delta t in the test is 85 Kelvin.

### What kind of data below in such a test are less informative or are they all equal informative?

- a) Delta t
- b) Number of cycles to fail
- c) Number of cycles to fail  $^*\Delta$   $t^2$
- d) They are all equal informative
- e) Number of days to fail
- 4) From the test (questions no 2) above. When you are looking at the numbers or at the plot.

### Can you see indications on bimodal Weibull distribution?

- a) There are no indication for a bimodal distributions
- b) The C14 seems to be a bimodal distribution
- c) The L7 seems to be a bimodal distribution
- d) The Connector seems to be a bimodal distribution
- e) All the components failed in the test seems to be a bimodal distribution
- 5) Failure In Time (FIT) value is still used in the reliability prediction toolbox.

### Which statement below is wrong information in modern reliable works:

- a) The FIT value describe the constant risk (failure level) using the component.
- b) The FIT value is information from the vendor and it is based on no observed failures in qualitative test.
- c) The FIT value is easy accessed information from handbooks and the predictions methods is easy to execute.
- d) The FIT values are covering fatigue and they are related to the single component quality.
- e) The designers are not following the product in the field outside the warranty period, and due to that they are not informed about FIT and fatigue/lifetime problems.
- 6) Weibull function:

When plotting a constant failure rate with:  $\beta$ =1 and  $\eta$ =10 in a plot where both the Y- and x-scale are linear. See eventual the table:

### Why is the unreliability line as a function of time un-linear?

- a) The MTBF is not a constant value?
- b) The Weibull operate with repaired items?
- c) The Weibull operate with no replacement for a defect
- d) The Weibull function was invented on bearings and they cannot function after wear out.
- e) The Weibull function is only used for fatigue or wear outs.

	Unreliability			
time	for beta =	and Eta=		
	1	10		
0	0,00			
1	0,10			
2	0,18			
3	0,26			
4	0,33			
5	0,39			
6	0,45			
7	0,50			
8	0,55			
9	0,59			
10	0,63			
11	0,67			
12	0,70			
13	0,73			
14	0,75			
15	0,78			

7) The proactive approach for design for reliability is based on 4 of these statements (parameters).

Which of the statement is not used directly in the proactive design work?

- a) Product-Performance Specification
- b) Existing Knowledge / Robust demands
- c) Quality Target
- d) Reliability Demonstration Test
- e) Mission Profile / Load Descriptions
- 8) A new product under development. You have to consider a robustness margin.

### Which of the statement below will need no robustness margin:

- a) The component chosen will not degrade in the specified life time.
- b) There are no variations in the technology strength.
- c) The robustness margin is not specified.
- d) End of life functionalities are not specified.
- e) None of the statements can eliminate the robustness margin.
- 9) In the Danish statistic you can find the following information:

### Age-related mortality rates.

The age-related Death rate (M) is calculated as the number of deaths (D) among the people who were alive in that age group at beginning of the period divided by the number of person-year.

The age-specific mortality rates are normally set at 100,000 person-years in order to avoid many decimal places in the presentation.

The calculation of the age-specific mortality rates for men who had a job in the period 1996-2000.

See the numbers in the table below:

		Amount of	Amount of	Amount of	Deaths
age		persons	personyear	deaths	coefficient
		(1)	(2)	(3)	(4)=(3)/(2)
20-24	year	148.919	722.640	494	0,00068
25-29	year	175.011	851.863	579	0,00068
30-34	year	188.707	924.488	815	0,00088
35-39	year	172.119	847.508	1.164	0,00137
40-44	year	167.751	827.324	1.841	0,00223
45-49	year	180.499	889.470	3.035	0,00341
50-54	year	163.097	800.603	4.497	0,00562
55-59	year	115.441	563.338	5.140	0,00912
60-64	year	61.650	297.754	4.133	0,01388

What is the MTBF (Mean Time Between Failures) value for persons in the group age 25-29 years. The correct answer is:

- a) 851.862 years
- b) 173 years
- c) 1.470 years
- d) 302 years
- e) The MTBF cannot be calculated.

### 10) The term "Severe User" (from General Motors) has been defined as:

a) The worst case situations the development team can imagine.

- b) The customer who always are complaining.
- c) A theoretical value based on statistics from the stress measured randomly in the field.
- d) The combinations of stresses during the whole specified lifetime.
- e) The specified stresses at the Product Design Requirement Specifications.

### 11) In the model (Rimmen model, "the pipe") the Robustness Margin is defined in the following way:

- a) Between the functional performance and the end of life performance.
- b) Between the "Lower Operational Level" and the "Lower Destruction Level"
- c) Between the tail of the distribution of "Lower Operational Level" and tail of the distribution of the "Lower Destruction Level"
- d) Between the tail of the distribution of "Lower- or Upper- Operational Level" and "Product Design Requirement Specifications"
- e) Between the "Performance Customer Requirement Specifications" and "Product Design Requirement Specifications"

# 12) The model (Rimmen model, "the pipe") shall give an overview for the test possibilities. Which of the following statement is correct?

- a) All weakness found inside the model are probably design weakness.
- b) All weakness found inside the model are probably component weakness.
- c) Inside the model it is easy to prove the reliability.
- d) You can measure the degradation by physical test inside the model.
- e) Lifetime can be measured inside the model by using accelerated stress test.
- 13) You are doing a new design and need reliability information for a component.

The Mission profile for this component has been send to the vendor, so the vendor can reply with his knowledge about the unreliability for the component as a function of time.

### Which of the following parameters are less informative if the other 4 information's are fulfilled?

- a) Beta from a Weibull distributions
- b) Eta from a Weibull distributions
- c) The Confidence
- d) The Failure in Time (FIT-value)
- e) The performance level for the functional parameters
- **14)** The Mean Cumulative Function (MCF) model for your Reliability budget is built up of some Weibull distributions.

### Which of the following distributions does not belong to the sum called Lack of Customer Satisfaction?

- a) Lack of Installations- and Transport Robustness
- b) Lack of Field information
- c) Lack of Production Capabilities
- d) Lack of Design Robustness

- e) Lack of Lifetime
- **15)** The sudden death area.

My washing machine has stopped working properly.

### Which one of the following statements does not belong to the sudden death description?

- a) The lightning stroke yesterday at the neighbor has also destroyed something in his electrical installation and now my washing machine will not start anymore.
- b) The gasket at the door is leaking after a piece of cloth was trapped in the in the gasket 2 weeks ago.
- c) The washing machine has started making noise when the drum is spinning, even when there is no laundry in the machine.
- d) After the washing machine was crowded, it has not washed as clean as before.
- e) The glass in the door is cracked after we washed the boots.
- 16) When designing a function which will go into mass production, it is important to make sure, that all produced items will work to the end of life.

# Which of the following methods cannot be used to improve the robustness against component variation?

- a) Monte Carlo simulations on schematic diagram
- b) Temperature test on functional level
- c) Power de-rating on the components
- d) Worst case simulations on interfaces
- e) Worst case component testing on functional level
- 17) During the design of a new product there have been made a Weibull-wear-out-budget (lack of lifetime budget). Calculation with the given Mission Profile showing the most dominating components for the unreliability at the time 10 years are:

One relay with unreliability at 30 %

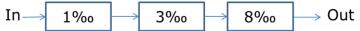
One opto-coupler with unreliability at 40 %

One solder-joint at the a coil 20 %

### What is the accumulated unreliability at the time at 10 years for these 3 components?

- a) 90 % unreliability
- b) 66 % unreliability
- c) 45 % unreliability
- d) 40 % unreliability
- e) 34 % unreliability
- 18) One customer complains over the unreliability he has experienced with a control-product in his production line. Even he has got 5 years of warranty and has got all failed modules replaced for free; the breakdowns in the production flow have been unacceptable. The analysis of the system showed the

failed modules over the installations period have giving the following failure result:



### What is the accumulated failure level per system this customer has expired?

- a) 12 ‰ per system
- b) 10,6 % per system
- c) 4,5 % per system
- d) 4 ‰ per system
- e) 2,4 % per system
- 19) The Vendor of the system agreed that they have to make immediate improvements. The fastest way will be to make a redundancy. This is possible to do together with an updated SW and having the modules powered up continuously. Due to the interfaces it will also be necessary to change some component in the middle of the modules, which unfortunately increased the failure intensity for this module with a factor of 10.

The future schematic could look like this:



### With such a modification the production breakdown can be expected to be around:

- a) 240 ‰
- b) 120 ‰
- c) 24 ‰
- d) 2,4 ‰
- e) 1,2 ‰
- **20)** With the modification (from questions no 18 to question no 19) implemented it might have an impact for both the vendor and the customer.

### The amount of repairs will increase with a factor around:

- a) 30-40 times
- b) 15-20 times
- c) 10-12 times
- d) 6 to 7 times
- e) 2-3 times
- 21) What are the parameters which can take life out of the solder joints? We will here assume that the capability of the soldering-process is high.

### Please mark the parameter which expect to have the lowest impact on the solder-joint-life:

a) Component operating temperature between 80-90°C. The product runs 24/7/365.

- b) Continuously vibration due to the environment for the application. The product runs 16 hours on working days in 25°C and component operating temperature is not higher than 60°C
- c) Continuously vibration due to the environment for the application. The product runs 24/7/365 days in 25°C and component operating temperature is not higher than 60°C
- d) Component operating temperature is changing from -20 to 60°C. The product runs 2 h on working days.
- e) The product is switched on and off several times a day. The temperature change form off to On is less than 30 Kelvin for the solder joints.
- 22) The project group doing a new development has decided to do a HALT test as a part of the test plan.

### What is the purpose for doing the HALT test?

- a) To be able to find the "overdesign" as part of a cost down of the product.
- b) To be able to find the weakest point and then decide if this is strong enough for the product and then stop the expensive test.
- c) To find the weakest point, eliminate the weakness and then increase the stress to find the second weakness etc. continuously doing this until the most of the components are breaking down.
- d) To be able to advertise in the product description, that they have done the HALT test, indicating that the failure level will be low.
- e) To be able to estimate the expected Lifetime for the product.
- 23) When using Multi-Stress Degradation Analysis, you are finding the test time or cycles to failure.

The task is to estimate the expected life for the product in severe user operation.

### Which of the inputs for the analysis cannot be used in the lifetime calculation?

- a) High Stress, the component failed at once.
- b) High stress, the component failed during a week
- c) Lower stress, the component failed over 5 weeks
- d) Specification level in normal operation is not specified in the Mission profile
- e) The fatigue failure mechanism has been the same.

### 24) Which of the following statements about Cpk is correct:

- a) Cpk is always greater than or equal to Cp
- b) Cpk is always less than or equal to Cp
- c) Cpk is always greater than Cp
- d) Cpk is always less than Cp
- e) Cpk is always equal to Cp
- 25) Cost of poor Quality/reliability.

Company "Transform" has produced a converter system in many years for the power-industry.

The field failure level has been steady around 5% in the warranty period.

The price for this product is 60K€.

In the contract with the customer the company "Transform" has to cover for production losses, if the

converter system is not able to perform up to Performance-specification. This cost has varied from 10K€ to 90K€ with an average at 40K€.

It has been decided that a redesign of the converter shall start up. The established project group has been allowed to use up to 25% of "Transform's" saved warranty expenses for this product, if the redesign can reduce the warranty failure level from 5% to 1%.

How many Euros can the project group use to increase the production price if necessary?

- a) 100€
- b) 250 €
- c) 500 €
- d) 600€
- e) 1000€