A New Matlab Simulation of Induction Motor

Mohamad H. Moradi [1] Pouria G. Khorasani [1,2]

1. University of Bu Ali Sina, Department of Electrical Engineering, Hamedan, Iran
2. West Regional Electric Co, Kermanshah, Iran

Abstract—**This paper is introducing a very fast simulation approach to analysis the transient behavior of three-phase squirrel-cage type induction motor. The well known model with four differential equations of voltage and current in rotor reference frame along with the torque equation is used to model the motor. Also, in the proposed model, the saturation of both magnetizing and leakage inductances is considered as function of magnetizing current. The commercial software package, MATLAB, is used to implement the transient behavior of the model. The validation and speed of obtained model is tested and compared with Matlab/Simulink conventional model and published papers. The results show that an excellent agreement is obtained demonstrating the accuracy of the proposed model. Also, the simulation speed of the proposed approach is shown an excellent result compared with conventional approaches.**

I. Introduction

Three-phase squirrel-cage type induction motors (IM) are commonly utilized in the industries from the capacity of several kilowatts to thousands of kilowatt as the driving units for fans, pumps, and compressors. This motor has been favored because of its good self-starting capability, simple and rugged structure, low cost and reliability, etc.

Usually, the motors are maintained periodically. However, when the ground fault occurs at the motor terminal, a serious damage may be brought to the motor. In the worst case, the motor is unable to start after the restoration of the power supply. Also it has been known that re-switching the supply onto a squirrel cage induction motor can result in the production of large negative torque transients (Wood, Flynn [1], Landy [2]). Therefore, it is significant to understand the transient phenomena under abnormal conditions for the optimal design of induction motors. The traditional method of modeling induction machines has been applied by several authors [3]-[5]. In this method of analysis, it is assumed that the effect of saturation due to either the magnetizing inductance or the leakage inductances is negligible. Using this assumption, the values of the magnetizing inductance, stator leakage, and rotor leakage inductances are constant and thus do not vary with the magnetizing current. Also several methods have been developed for the modeling of saturation effects in induction machines [6]-[9]. For example in [6]-[7] induction motor with saturable leakage reactance is modeled and simulated by IGSPICE and an analog computer, respectively. In Levi [9] the effect of the main flux saturation is investigated.

This paper presents a very fast approach to simulate transient behavior of the induction motor using state-space model along with considering saturation of both the magnetizing and leakage inductances. Matlab/Simulink is used to simulate the transient performances of an induction motor by assuming that the main flux inductance, stator, and rotor leakage inductances vary with the magnetizing current.

The paper has been organized as follows; Section II describes the mathematical model of induction motor. Section III shows the effects of inductances saturation in the model. The implemented model in Matlab has been shown in section IV.

From the comparisons between the simulation results and the reported ones, the accuracy of the proposed scheme has been clarified in section V. Namely, the simulation results have been compared with the reported results of 36KW motor in [10] and 2250 hp motor in [11].

In Section VI, the simulation speed of the proposed model is compared with Matlab/Simulink conventional model. Finally, conclusions close the paper.

II. Mathematical Model of IM

The mathematical description of an ideal symmetrical squirrel-cage type induction motor in an arbitrary reference frame is given by the following matrix differential equation shown in (1).

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (R_s + L_s p) & \omega L_s & L_m p & \omega L_m \\ -\omega L_s & (R_s + L_s p) & -\omega L_m & L_m p \\ L_m p & -(\omega-\omega_r)L_m & (R_r + L_r p) & (\omega-\omega_r)L_r \\ -(\omega-\omega_r)L_m & L_m p & -(\omega-\omega_r)L_r & (R_r + L_r p) \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \quad (1)$$

Where,

$$L_s = L_{ls} + L_m \quad (2)$$

$$L_r = L_{lr} + L_m \quad (3)$$

$$p = \frac{d}{dt} \quad (4)$$

V_{qs} The stator voltages refer to the q axis of reference frame
V_{ds} The stator voltages refer to the d axis of reference frame
i_{qs} The stator currents refer to the q axis of reference frame
i_{ds} The stator currents refer to the d axis of reference frame
i_{qr} The rotor currents refer to the q axis of reference frame
i_{dr} The rotor currents refer to the d axis of reference frame
R_s Resistance of stator winding
R_r Resistance of rotor conductor
L_s Stator self inductance

L_r Rotor self inductance
L_{ls} Stator leakage inductance
L_{lr} Rotor leakage inductance
L_m Magnetizing inductance
ω Angle velocity of arbitrary reference frame
ωr Electrical angle velocity of rotor

An arbitrary reference frame may rotate at any angular speed or remain fixed to the stator. However it is important to note that the choice of a reference frame will affect the waveforms of all d-q variables and also the simulation speed as well as the accuracy of the results. The following guidelines are suggested in [11],

- Use the stationary reference frame if the stator voltages are either unbalanced or discontinuous and the rotor voltages are balanced (or zero).
- Apply the rotor reference frame if the rotor voltages are either unbalanced or discontinuous and the stator voltages are balanced.
- Apply either the synchronous or stationary reference frames if all voltages are balanced and continuous.

Also, for analysis involving saturation and deep bar effect, a reference frame fixed to the rotor is recommended [12].

In this paper the arbitrary reference frame fixed to the rotor, in other hand the arbitrary reference frame rotate with electrical angle velocity of rotor (ω_r), therefore, the electrical equation (1) of the squirrel-cage induction motor becomes,

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (R_s + L_s p) & \omega L_s & L_m p & \omega L_m \\ -\omega L_s & (R_s + L_s p) & -\omega L_m & L_m p \\ L_m p & 0 & (R_r + L_r p) & 0 \\ 0 & L_m p & 0 & (R_r + L_r p) \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \quad (5)$$

For fast simulation purpose, (5) should be represented in state space form with currents as state variables. Therefore, (5) is expressed as,

$$p[i] = -[L]^{-1}([R] + \omega_r [G])[i] + [L]^{-1}[V] \quad (6)$$

Where,

$$[i] = [i_{qs} \quad i_{ds} \quad i_{qr} \quad i_{dr}]^T \quad (7)$$

$$[V] = [V_{qs} \quad V_{ds} \quad 0 \quad 0]^T \quad (8)$$

$$[R] = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix} \quad (9) \quad [L] = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix} \quad (10)$$

$$[G] = \begin{bmatrix} 0 & L_s & 0 & L_m \\ -L_s & 0 & -L_m & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

The electromagnetic torque and mechanical speed of motor are given, respectively,

$$T_e = 1.5 P L_m (i_{qs} i_{dr} - i_{ds} i_{qr}) \quad (12)$$

$$\dot{\omega}_m = -F \cdot \omega_m + (T_e - T_L)/J_m \quad (13)$$

Where,
P Number of pole pairs
ω_m Mechanical angle velocity of rotor
F Friction factor of induction motor
J_m Combined rotor and load inertia torque
T_L Applied load torque

The electrical angular velocity and electrical angular position of rotor are calculated by,

$$\omega_r = P\omega_m \quad (14)$$

$$\theta_r(t) = \int_0^t \omega_r dt + \theta_r(0) \quad (15)$$

Where, θ_r represent electrical rotor angular position.

III. SATURATION EFFECT

The values of the inductances used in the development of the mathematical equations for the conventional induction motor model were assumed to be constant. By so doing, the models fail to take into consideration the saturation effects of both the magnetizing inductance and the leakage inductances. In this paper, it is assumed that the magnetizing inductance, stator, and rotor leakage inductances saturate. This implies that the magnetizing inductance and the leakage inductances vary with the magnetizing current. Therefore, the saturation curves of the test motor have to be determined. The magnetizing current, im is defined as (16).

$$i_m = \sqrt{i_{qt}^2 + i_{dt}^2} \quad (16)$$

Where,

$$i_{qt} = i_{qs} + i_{qr} \quad (17)$$

$$i_{dt} = i_{ds} + i_{dr} \quad (18).$$

A. Saturation of Stator Leakage Inductance

When the current flowing into the stator windings increases than the certain value, the wedge part in the stator slots is magnetically saturated, this phenomenon is called as stator leakage inductance saturation, and can be defined as a curve that shown dependency of stator leakage inductance with magnetizing current. This curve can be obtained from measurement of a typical test motor [13]. A sample curve for a 36KW test motor is reported in [10], the measurement points for this curve are extracted in Table I.

B. Saturation of Rotor Leakage Inductance

When the current in the rotor bar increases beyond the certain limitation, the leakage reactance of the tip part of rotor slot teeth becomes small by increasing of the magnetizing current, this phenomenon is called as rotor leakage inductance saturation, and can be defined as a curve that shown dependency of rotor leakage inductance with magnetizing current. This curve can be obtained from measurement of a typical test motor [13]. A sample curve for a 36KW test motor is reported in [10], the measurement points for this curve are extracted in Table II.

C. Saturation of Magnetizing Inductance

When the magnetizing current of the induction motor increases beyond the certain limitation, the specific permeability of iron core decreases by larger currents, this means that the magnetizing inductance decreases with increase magnetizing current. This phenomenon is called as magnetizing inductance saturation, and can be defined as a curve that shown dependency of magnetizing inductance with magnetizing current. This curve can be obtained from measurement of a typical test motor [13]. A sample curve for a 36KW test motor is reported in [10], the measurement points for this curve are extracted in Table III.

D. *Affect of Saturation in Mathematical Model of IM*

By storing the value expressions of tables (I-III) in the computer and use interpolation methods, the values of the magnetizing inductance and leakage inductances in the mathematical model of induction motor related by value of magnetizing current can be updated at each integration step.

In other hand the values of L_{ls}, L_{lr}, L_m in (6)-(11) are not constant and must be changing at each integration step. For this problem several authors in [6]-[10], have different suggestion, in this paper a simple method are proposed to solve this problem, as simulation speed can be rise up in compare with other methods.

By storing the value expressions of tables (I-III) in the three look up tables namely "L_{ls}", "L_{lr}", "L_m", and use interpolation method, inductances value related by a specified value of magnetizing current can be obtained from this look up tables and set in (10)-(11). In this process, the most important factor that would be affected the saturation in mathematical model of induction motor is integration step.

TABLE I
SEVERAL POINTS OF SATURATION CURVE OF STATOR LEAKAGE INDUCTANCE

Magnetizing current (A)	0	20	40	60	80	100	120	140	160	180	200
Stator leakage inductance (mH)	0.3750	0.3745	0.3730	0.3717	0.3708	0.3666	0.3630	0.3583	0.3530	0.3460	0.3377

TABLE II
SEVERAL POINTS OF SATURATION CURVE OF ROTOR LEAKAGE INDUCTANCE

Magnetizing current (A)	0	20	40	60	80	100	120	140	160	180	200
rotor leakage inductance (mH)	0.1200	0.1199	0.1198	0.1196	0.1190	0.1185	0.1177	0.1166	0.1160	0.1150	0.1133

TABLE III
SEVERAL POINTS OF SATURATION CURVE OF MAGNETIZING INDUCTANCE

Magnetizing current (A)	0	20	40	60	80	100	120	140	160	180	200
Magnetizing inductance (mH)	8.400	8.350	8.100	7.700	6.950	5.950	5.000	4.350	4.100	4.000	3.900

IV. SIMULATIONS

The main goal of this paper is present a fast Matlab simulation of induction motor with saturation effect; therefore in this section of paper for a complete expression, all block diagrams of simulated model are explained.

For more simplify electrical part simulation and mechanical part simulations of induction motor are separated.

A. *Electrical Part Simulation of IM*

In order to simulate the electrical part of IM, (6) and (12) must be solved, but for solving (6), it is assumed that three-phase voltages of motor supply are symmetrical, then with determine electrical rotor angular position by (14) and (15), voltages term (8) can be obtained by applying d-q transformation on stator voltages. Because of neglected skin effect in this paper it is assumed that Resistance matrix (9) is constant and determine for all time, also [L] and [G] matrices, (10) and (11), are obtained from look up tables and lastly (14) can be obtained from mechanical part of IM.

For implement this part of simulation in Matlab/Simulink the block diagram which shown in Fig. 1. is suggestion for a fast simulation of (6).

As shown in Fig. 1. L and G must be determined; Fig. 2 is illustrated relevancy between (6) by terms of (10) and (11).

Saturation block Where shown in Fig. 2 contain three look up table and several matrix gains.

In Fig.1 ω_r and θ_r or term (14) and (15) of this paper, also must be determined, determination of this term will explain later in simulate of mechanical part.

For completion simulate of electrical part of IM, it is necessary to solving the (12) which the block diagram has been shown in Fig. 3. As shown in Fig. 3, for obtaining of electromagnetic torque, matrix [i] and L_m must be determined, where matrix [i] is obtained by (6) or Fig. 1, and L_m can be obtained directly by Fig. 2.

B. *Mechanical Part Simulation of IM*

In order to simulate the mechanical part of IM, (13), (14) and (15) must be solved, for solving (13), values of T_L, Te, J_m and F must be determined, the values of T_L, J_m and F are determined as inputs data of motor and driven load and value of T_e is determined by (12) or Fig. 3. at each integration step. The block diagram related to (13) is shown in Fig. 4.

For solving (14) and determination of electrical angular velocity of rotor (ω_r), the output of (13) or ω_m must multiply by P, where P is number of pole pairs of induction motor. The block diagram related to (14) is shown in Fig. 5.

Lastly for determination of θ_r, (15) should be solved by taking integral of ω_r, Fig. 6.

Thus all of the electrical and mechanical equations of IM were simulated in matlab/simulink, and now this simulation can be run on the any capable digital computer.

V. SIMULATED MODELS VERIFICATION

The transient selected for verification of the proposed saturation modeling procedure is acceleration of unloaded three-phase 36KW squirrel-cage type induction motor, where motor data are reported in [10].

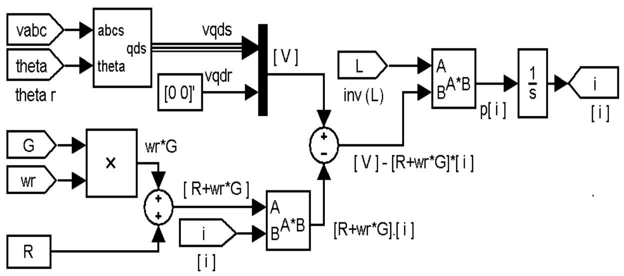

Fig. 1. Block diagram of (3) to determination current vector

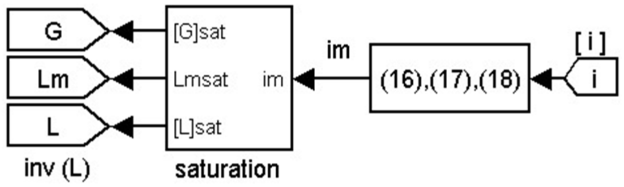

Fig. 2. Block diagram of relevancy between (6) by terms (10), (11)

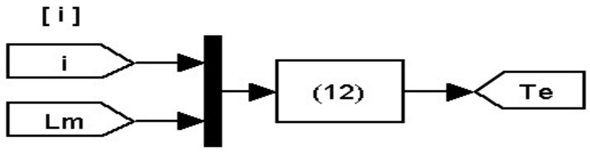

Fig. 3. Block diagram of (12) to determination electromagnetic torque

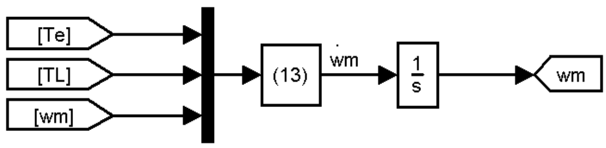

Fig. 4. Block diagram of (13) to determination of ω_m

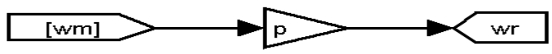

Fig. 5. Block diagram of (14) to determination of ω_r

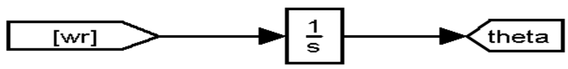

Fig. 6. Block diagram of (15) to determination of θ_r

By assuming that the magnetizing and leakage inductances are constant the conventional machine model is simulated and results of this simulation are compared by those results where obtained by simulation of proposed model (applying effects of the magnetizing and leakage inductances). Final results of this comparison are shown in Fig. 7 and 8, for the electromagnetic torque and mechanical speed as a function of time. Also, the stator current has been shown in Fig. 9.

By comparison of these results with reported ones in [10] an excellent agreement was obtained demonstrating the accuracy of the proposed simulation.

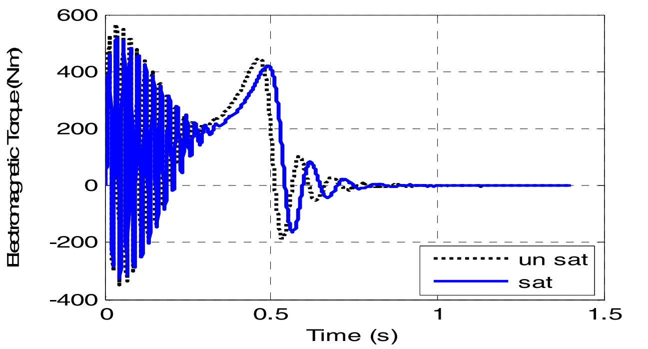

Fig. 7. Electromagnetic Torque with (sat) and with out (un sat) saturation

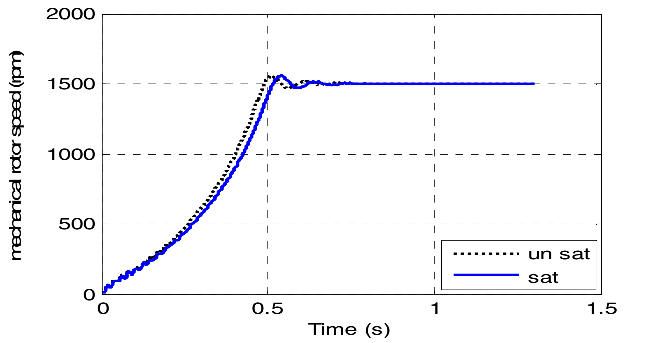

Fig. 8. Mechanical Rotor Speed with (sat) and with out (un sat) saturation

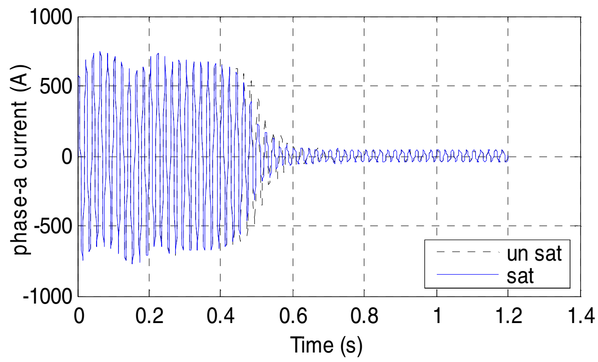

Fig. 9. Stator current (phase-a) with (sat) and with out (un sat) saturation

VI. SIMULATION SPEED VERIFICATION

To verify the speed of proposed model, the transient behavior of a 2250 hp IM, free acceleration test, was compared with reported one, where motor data are reported in [11].

The simulation results for proposed model were presented in Fig. 10, 11 and 12. By comparison of these results with reported ones in [11] and obtained ones by running Matlab/Simulink conventional model an excellent agreement was obtained re-demonstrating the accuracy of the proposed simulation. Also, the running time of the proposed model was compared with Matlab/Simulink conventional model. The results were extracted in Table IV for two different value of supply inductance.

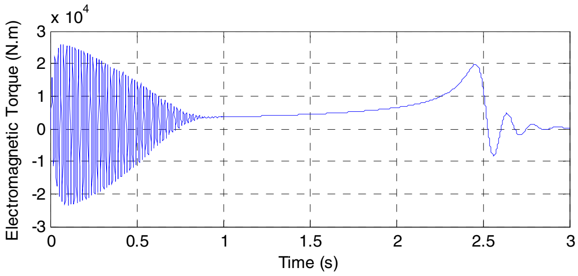

Fig. 10. Electromagnetic torque of 2250 hp IM

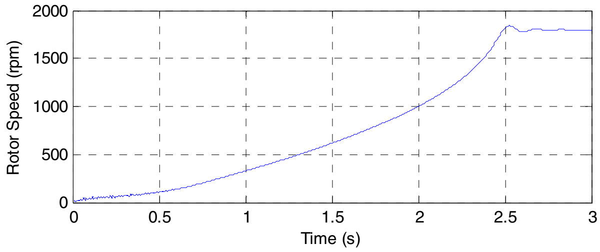

Fig. 11. Rotor speed of 2250 hp IM

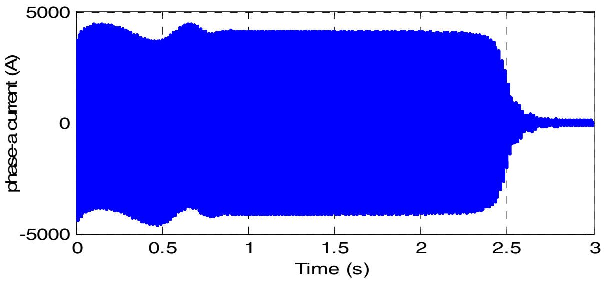

Fig. 12. stator current (phase_a) of 2250 hp IM

All of simulations are running under the same condition as below,

Motor type: 2250 hp squirrel cage IM
Transient phenomena: free acceleration test
Simulation time: 3 second
Maximum step size: 0.001
Solver: ode15s(stiff/NDF)
Method of measure elapsed time: stopwatch timer
Number of scope: one three axes scope
Computer ram and processor: 512 MB and 3 GHz

By comparing of simulation speed of both proposed and conventional model As shown in Table IV, can be observe that the proposed model is faster than Matlab/Simulink conventional model, and this priority will be more noticeable by increasing of AC source inductance than zero. Several reasons are interfere in this phenomenon that beyond the space limitations of a single paper.

TABLE IV
COMPARING OF SIMULATION SPEED OF TWO 2250 HP IM MODELS

	Matlab conventional model		Proposed model	
Supply inductance (mH)	0	0.0001	0	0.0001
Simulation time (sec)	3	3	3	3
Total elapsed time (sec)	1.296	97.44	0.142	0.3885

VII. CONCLUSIONS

In this paper, a fast simulation approach was proposed in consideration of both magnetizing and leakage inductances saturation to analyze the transient behavior of three-phase squirrel-cage type induction motors. for realization of this approach the mathematical model of electrical and mechanical part of induction motor were presented by state-space equations, and then these equations were implemented in Matlab/Simulink with applying both magnetizing and leakage inductances saturation by three look up tables.

To validate the model, a 36 KW saturable induction motor was simulated and the results were compared with reported results.

Finally, running time of proposed simulated model was compared with matlab/simulink conventional model by simulation of an experimental 2250 hp IM. The results were shown that the proposed model is faster. The priority of proposed simulated model will be more noticeable by increasing of supply inductance than zero.

Therefore, the proposed model could be employed to simulate the saturable squirrel-cage IM faster than other conventional model such as Matlab/Simulink model.

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