AALBORG UNIVERSITET
Energy

| Study number: | Programme: MSc2 |
| --- | --- |
| **Evaluation subject:**<br>Optimisation Theory and Modern Reliability from a Practical Approach<br>Friday 9 June 2017 at 9:00 – 13:00 | |

Please write your study no. on all pages. Do not write your name as your evaluation is anonymous!

Total number of pages, including this page: 20

Please, only write on one side of the papers that you hand in.

**NB! Your paper must be easy to read. If this is not the case, your paper may be evaluated as "not passed".**

***All usual aids are allowed (notes, books, tables, calculator and PC). You are not allowed to communicate amongst each other or with the outside world which means that the use of mobile phone, Wi-Fi, internet, email is not allowed.***

**You are allowed to take the examination questions with you. But you are NOT allowed to take them with you if you leave the room before the examination has ended.**

9. juni Exam optimisation and reliability



Exercise 1

Maximize profit for a company based on the specifications

* Two types A and B
* 5000 hours available per day
  ↳ 20 hours per A type & 24 hours per B-type
* Cost for each type
  ↳ A: start expence 500€ and 300€ pr unit
  ↳ B: start expence 900€ and 400€ pr unit
* There need to be made a certain amount to maintain market shares.
  ↳ A: at least 50 stoves pr day
  ↳ B: at least 30 stoves pr day
* Sell price pr type:
  ↳ A: 500 € pr stove
  ↳ B: 600 € pr stove

With these info we need to make an profit Objective function that takes the selling price minus the expenses Since this gives the profit: for this we set $x_1$ = # type A units $x_2$ = # type B units

$$f(x) = \underbrace{500x_1 + 600x_2}_{\text{sell price}} - \underbrace{(300x_1 + 500) - (400x_2 + 900)}_{\text{expences}}$$

Now we need to set up the constraints which is the minimum number and the man hours available exercise 1 cont

constraints for the man hours, we know how long it takes per unit and we know the available amount so we get:

$$g_1(x) = 20x_1 + 24x_2 \leq 5000$$

further they need to produce a minimum number of each type:

$$g_2(x) = x_1 \geq 50$$

$$g_3(x) = x_2 \geq 30$$

So in total we get the following:

maximize: (objective is simplified)

$$f(x) = 200x_1 + 200x_2 - 1400$$

Subject to:

$$g_1(x) = 20x_1 + 24x_2 \leq 5000$$

$$g_2(x) = x_1 \geq 50$$

$$g_3(x) = x_2 \geq 30$$

Exercise 2: unconstrained function to minimi

$$f(x) = (x_1 - 1)^2 + 2x_2^2$$

with a starting point $x^{(0)} = [3 \ \ 1]^T$

a) complete first iteration of steepest descent

1: first we need to chose a convergence criteria at $\varepsilon$ is used. we start with $k=0$ (iteration counter)

2: calculate the gradient of $f(x)$ and evaluate at the point and set $c^{(k)} = \nabla f(x^{(k)})$

$$\nabla f(x) = \begin{bmatrix} 2x_1 - 2 \\ 4x_2 \end{bmatrix} \Rightarrow c^{(0)} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

3: calculate the length of $c^{(0)}$, $\|c^{(0)}\|$ if this is less than $\varepsilon$ then stop. Otherwise continue

$$\|c^{(0)}\| = \sqrt{4^2 + 4^2} \Rightarrow 5{,}6568 \quad > \varepsilon \quad \text{so continue}$$

4: Set search direction at point to be $d^{(k)} = -c^{(k)}$

$$d^{(0)} = \begin{bmatrix} -4 \\ -4 \end{bmatrix}$$

5: calculate stepsize $\alpha_k$ that minimize $f(\alpha) = f(x^{(k)} + \alpha_k d$
→ we find minimum by setting the derivative of $f(\alpha)$ to 0

$$f(\alpha) = ((3 + \alpha \cdot (-4)) - 1)^2 + 2(1 + \alpha \cdot (-4))^2 \rightarrow \text{simplifies to}$$

$$= 48\alpha^2 - 32\alpha + 6$$

$$f'(\alpha) \quad 96\alpha - 32 = 0 \quad \Rightarrow \quad \alpha_0 = \frac{1}{3}$$

6: Update design as $x^{(k+1)} = x^{(k)} + \alpha_k d^{(k)}$

So: $x^{(1)} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} -4 \\ -4 \end{bmatrix} = \begin{bmatrix} 5/3 \\ -1/3 \end{bmatrix}$

then we would start over from 2

Exercice 2b complete first iteration with the modified Newton's method

Step 1| Same as in a) $k=0$, $\varepsilon$ convergence criteria

Step 2| Same as in a)

$$C^{(0)} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \qquad \|c^{(0)}\| = 5{,}6568 > \varepsilon$$

Step 3: Calculate the hessian $H^{(k)}$ at the point $x^{(k)}$

$$H^{(k)} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

we can see that it is constant, we calculate the eigenvalue to check properties!

$$\lambda = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

we see that it is positive definite which means that the descent condition is always satisfied

descent condition $c^{(k)} \cdot d^{(k)} < 0$

Step 4: calculate search direction $d^{(k)}$ by:

$d^{(k)} = -\left(H^{(k)}\right)^{-1} c^{(k)}$ can be solved by:

$H^{(k)} \cdot d^{(k)} = -c^{(k)}$ (solved in Matlab gives)

$$d^{(0)} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

Step 5: Calculate step size, (using same method as step 5 in a))

$f'(\alpha_0) = 12\alpha_0 - 12 = 0 \Rightarrow \alpha_0 = 1$

Step 6: Update design: $x^{(k+1)} = x^{(k)} + \alpha_k d^{(k)}$

$$x^{(1)} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Start again from step 2 to complete

Exercise 3: Using simplex solve the following

minimize: $f(x) = -5x_1 - 2x_2$

Subject to: $g_1(x) = -x_1 + x_2 \leq 10$

$g_2(x) = 2x_1 - x_2 \leq 20$

$x_i \geq 0 \quad \forall \; x_i = \{1, 2\}$

the problem is already linear so we start by introducing slack variables $x_3$ and $x_4$ to make the inequality constraints, equality constrai

minimize: $f(x) = -5x_1 - 2x_2 + 0x_3 + 0x_4$

S.t. $g_1(x) = -x_1 + x_2 + x_3 = 10$

$g_2(x) = 2x_1 - x_2 + x_4 = 20$

$x_i \geq 0 \quad \forall \; x_i = \{1, 2\}$

Now we can set up the tableau:

| basic | $x_1$ | $x_2$ | $x_3$ | $x_4$ | b | $b/a_i$ |
|---|---|---|---|---|---|---|
| $x_3$ | -1 | 1 | 1 | 0 | 10 | $10/_{-1} = -10$ |
| $x_4$ | 2 | -1 | 0 | 1 | 20 | $20/2 = 10$ |
| C | -5 | -2 | 0 | 0 | 0 | |

pick the pivot column as the one with the most negative value, calculate ratio $b/a_i$ and select the smallest none negative row

then we need to make the pivot element 1 and the remaining entries in that column 0

$$\xrightarrow{\frac{1}{2}R_2 \to R_2} \left[\begin{array}{cccc|c} -1 & 1 & 1 & 0 & 10 \\ 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 10 \\ -5 & -2 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_2 + R_1 \to R_1]{\text{obtain zero in other entries}} \left[\begin{array}{cccc|c} 0 & \frac{1}{2} & 1 & \frac{1}{2} & 20 \\ 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 10 \\ -5 & -2 & 0 & 0 & 0 \end{array}\right]$$

Exercise 3:

$$\xrightarrow{5R_2+R_3 \to R_3} \begin{array}{c} \left[\begin{array}{cccc|c} 0 & \frac{1}{2} & 1 & \frac{1}{2} & 20 \\ 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 10 \\ 0 & -\frac{9}{2} & 0 & \frac{5}{2} & 50 \end{array}\right] \begin{array}{c} 40 \\ -20 \\ \phantom{0} \end{array} \end{array}$$

we see that we still have a negative number so we continue

$$\xrightarrow{2R_1 \to R_1} \left[\begin{array}{cccc|c} 0 & 1 & 2 & 1 & 40 \\ 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 10 \\ 0 & -\frac{9}{2} & 0 & \frac{5}{2} & 50 \end{array}\right] \xrightarrow{\frac{1}{2}R_1+R_2 \to R_2}$$

$$\left[\begin{array}{cccc|c} 0 & 1 & 2 & 1 & 40 \\ 1 & 0 & 1 & 1 & 30 \\ 0 & -\frac{9}{2} & 0 & \frac{5}{2} & 50 \end{array}\right] \xrightarrow{\frac{9}{2}R_1+R_3 \to R_3} \left[\begin{array}{cccc|c} 0 & 1 & 2 & 1 & 40 \\ 1 & 0 & 1 & 1 & 30 \\ 0 & 0 & 9 & 7 & 230 \end{array}\right]$$

our final tableau with switched variables
we need to switch the basic variables when doing the operations,

Final tableau:

| basic | $x_1$ | $x_2$ | $x_3$ | $x_4$ | b |
|---|---|---|---|---|---|
| $x_2$ | 0 | 1 | 2 | 1 | 40 |
| $x_1$ | 1 | 0 | 1 | 1 | 30 |
| C | 0 | 0 | 9 | 7 | f+230 |

so we get:

$$x = \begin{bmatrix} 30 \\ 40 \end{bmatrix}$$ with a function value of

$$f(x) = -230$$

Exercise 4 Multiobjective

minimize: $f_1(x) = (x_1 - 5)^2 + (x_2 - 15)^2$

$f_2(x) = (x_1 - 15)^2 + (x_2 - 5)^2$

Subject to: $g_1(x) = x_1 \leq 10$

a) Draw the constraints

See pink line on attached plots Fig 1

b) Draw the pareto optimum points.

- See black line on attached plots Fig 1

c) Sketch the pareto front

See fig 2 in attached plots

d) determine utopia point function values

Utopia is defined as the minimum of each function when the other is not taken into account

Utopia $f_i = \min\{f_i(x) \mid \text{for all } x \text{ in the set } S\}$

$f_1^\circ = 1$ $f_2^\circ = 49$ because of constraint

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## NOTICE THIS PAGE MAY BE HANDED IN WITH THE SOLUTION

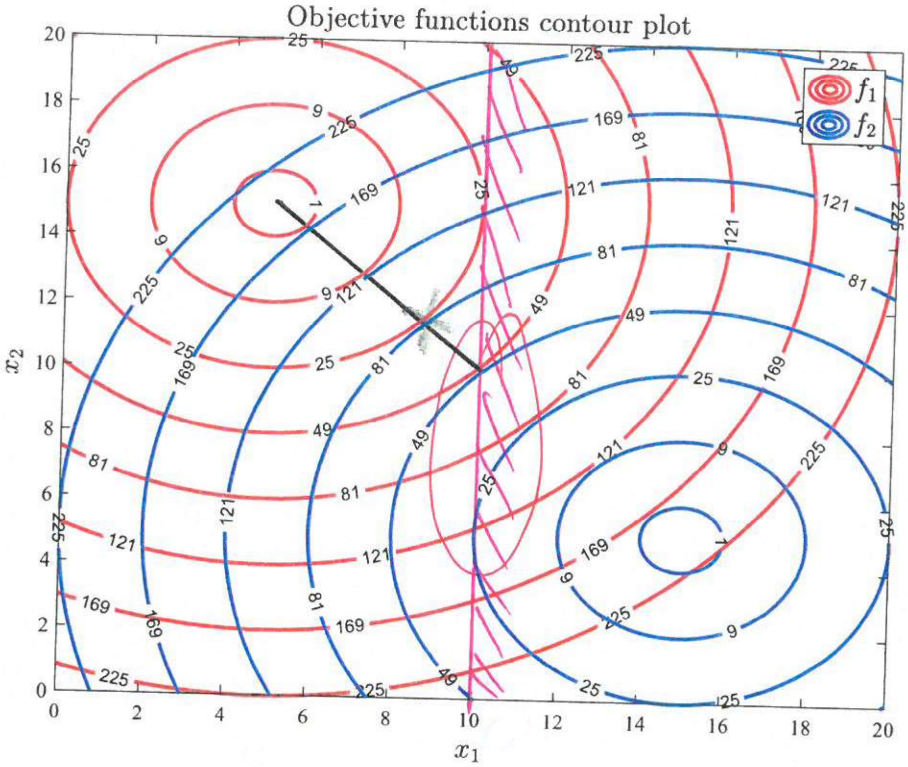



Figure 1: Contour curves and Pareto optimal points.

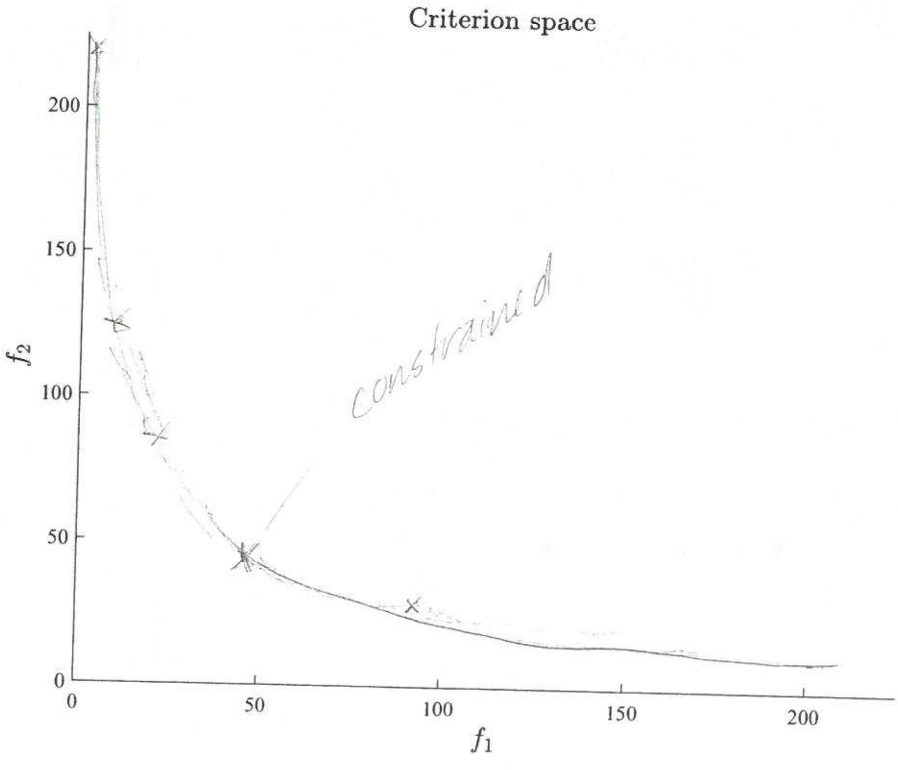



Figure 2: Coordinate system where the Pareto set in criterion space may be plotted.

Exercise 4

e) Assume the multiobjective problem is solved as a single objective problem $U(x)$, using weigthed sum method. $w_1 = 2$ $w_2 = 1$ the weigthed sum function is

$$U(x) = \sum_{i=1}^{2} w_i f_i(x)$$

So our new function will be:

$$U(x) = 2 \cdot \left((x_1 - 5)^2 + (x_2 - 15)^2\right) + \left((x_1 - 15)^2 + (x_2 - 5)^2\right)$$

which simplifies to:

minimize: $U(x) = 3x_1^2 + 3x_2^2 - 50x_1 - 70x_2 + 750$

s.t. $g_1(x) = x_1 \leq 10$

This can be solved by various methods, but i choose KKT:

Lagrangian function: $L(x, u, s) = f(x) \sum_{i=1}^{m} u_i g_i(x) - s_i^2$

$L(x, u, s) = 3x_1^2 + 3x_2^2 - 50x_1 - 70x_2 + 750 + u_1(x_1 - 10 + s_1^2)$

then the conditions are

$$\frac{\partial L}{\partial x_1} = 6x_1 - 50 + u_1 = 0$$

$$\frac{\partial L}{\partial x_2} = 6x_2 - 70 = 0$$

$$\frac{\partial L}{\partial u_1} = s_1^2\ x_1 - 10 = 0$$

$$\frac{\partial L}{\partial s_1} = 2u_1 s_1 = 0$$

which gives 4 eq w. 4 unknown, solved in maple

the first solution gives $u < 0$ so not feasible

the second gives:

$u = 0$, $s = 1{,}2910$

$x_1 = \frac{25}{3} = 8{,}333$

$x_2 = \frac{35}{3} = 11{,}667$

Exercise 4

f) Determine if it is a global optimum

When looking at the function $U(x)$ and plotting it we see that it is convex, and by that, we know that it is the global minimum

Exercise 5 8 paper clips are tested to figure out the wear out

* test stops at 60 times of bending
* Results : 1 - 6 , 15 , 50, 40, 10, 30, and 60

the 7. and 8. still survived at 60 bends

a) Relationship between failure and time

$$F(t) = 1 - \exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right)$$

where $t$ is the time , $F(t)$ is accumulated failure

$\eta$ is the charakteristic life, i.e. the time $t$, where 63,2 % have failed

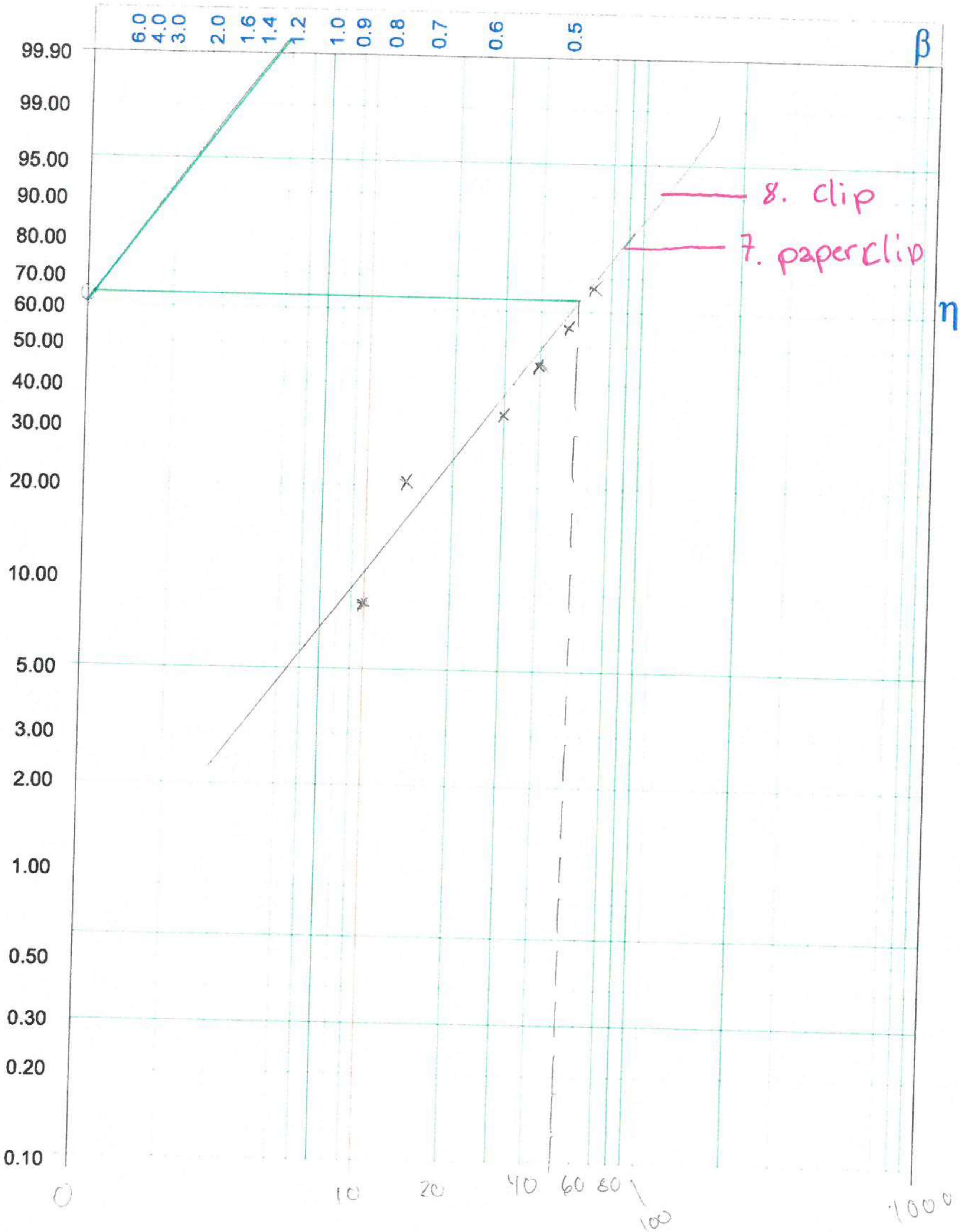
$\beta$ is the shaping factor, i.e. the slope when plotted in the double log diagram

b) Arrange by median rank

Since we have a sample size of 8, we know that, we in the table need to pick that column

| | | |
|---|---|---|
| 1 | 8,30 | 10 |
| 2 | 20,21 | 15 |
| 3 | 32,13 | 30 |
| 4 | 44,04 | 40 |
| 5 | 55,96 | 50 |
| 6 | 67,87 | 60 |
| 7 | 79,79 | 60+ |
| 8 | 91,70 | 60+ |

7. and 8 paper clip are not plotted since they are not failed


6.0
4.0
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2.0
1.6
1.4
1.2
1.0
0.9
0.8
0.7
0.6
0.5
β
99.90
99.00
95.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
5.00
3.00
2.00
1.00
0.50
0.30
0.20
0.10
8. clip
7. paperclip
η
0
10
20
40
60
80
100
1000

Exercise 5

c) See weibull plot

$\beta \approx 1{,}3$ and $\eta = 52$

d)

* $\beta$, is the shapefactor i.e. the Slope when plotted in the double log weibull plot.

Therefore $\beta$ can be found by parallel translocate the plotted line to the $\beta$ plot in the top left corner

* $\eta$ is the charcteristic line and is found as the value, where the accumulated failure reaches 63,2 %

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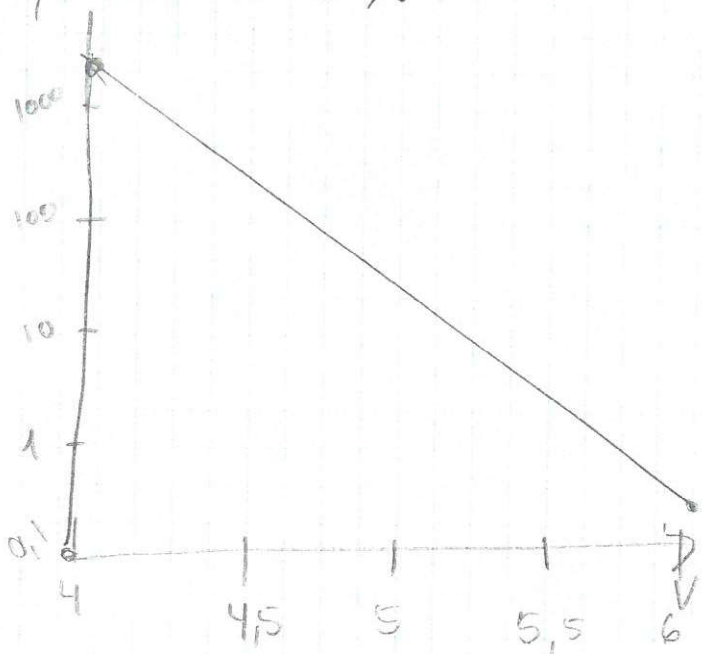
Exercise 6

a) the difference in the slope for the 6V test, shows that the failure mechanism for this is different than for the other 3 test

Which means that some of the tested devices in this 6V test lasted longer, compared to when the first device in same test failed.

b) B5 lifetime is the time where realiability is 0,95 so when looking in the Weibull plot, when failure is 5%

| test | B5 |
|---|---|
| 4V | 2200 h |
| 5V | 35 h |
| 5,5V | 4,7 h |
| 6V | 0,25 h |




When plotted we get the plot to the right and the relationship can be determined based on the following eq.

$$T = A \cdot \exp(-\alpha V)$$

Exercise 6

b) cont.

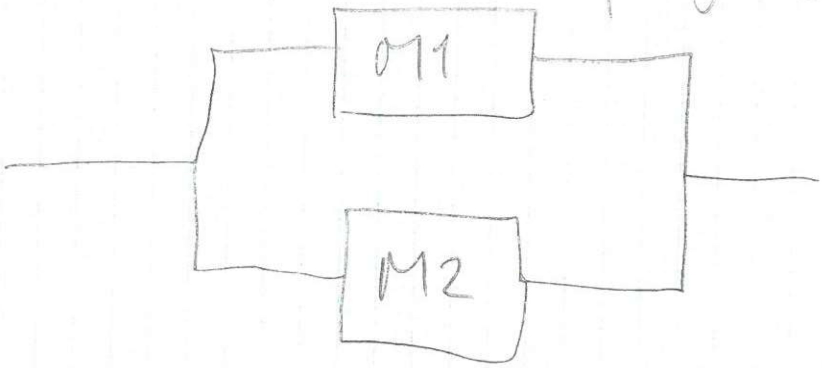
Where $T$ is the time
$A$ is a material constant
$V$ is the stress parameter
$\alpha$ is Eyring parameter

The resulting relationship from regression is

$$T = 1,3955 \cdot 10^{11} \cdot \exp(-4,45044 \cdot V)$$

Exercise 7

a) Sketch up RBD of system.




Since active redundancy we know it should be in parallel

b) Reliability of each component after 2000 hours

$M_1$ is a weibull with $\beta = 0{,}5$ and $\eta = 3000$ so we should use, $R(t) = \exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right)$
i assume $\gamma$ to be zero

$$R_{M1}(2000) = \exp\left(-\left(\frac{2000}{3000}\right)^{0{,}5}\right) = \underline{\underline{0{,}442}}$$

Component M2 is a eksponential with MTBF = 2500, MTBF = $1/\lambda \Rightarrow \lambda = \frac{1}{2500}$

Since eksponential we use:
$R(t) = \exp(-\lambda t)$

$$R_{M2}(2000) = \exp\left(-\frac{1}{2500} \cdot 2000\right) = \underline{\underline{0{,}449}}$$

Exercise 7

c) Reliability of the overall system.

The system is two components in parallel therefore we can use

$$R_{sys} = 1 - \prod_{i=1}^{n} (1 - R_i) \quad , \quad \Rightarrow ,$$

$$R_{sys} = 1 - (1 - R_{M1})(1 - R_{M2}) = \underline{\underline{0{,}693}}$$

Exercise 8

a)

| Stress lvl | Tavg | ΔT | cycles | $N_f$ |
|---|---|---|---|---|
| 1 | 80 | 20 | 5 | $6 \cdot 10^3$ |
| 2 | 50 | 20 | 15 | $5 \cdot 10^4$ |
| 3 | 65 | 30 | 5 | $1 \cdot 10^3$ |

b)

See table ↑

c) accumulated damage in 1h

Using Miners rule, and looking in the graph

Miners rule: $AD = \sum_{i=1}^{n} \frac{n_i}{N_i}$ $n_i$ # cycles at that stress, $N_i$ # cycles to failure

1 blok 5 cycles $T_{avg} = 80°C$ $\Delta T = 20$

2 blok 15 cycles $T_{avg} = 50°C$ $\Delta T = 20$

3 blok 5 cycles $T_{avg} = 65°C$ $\Delta T = 30$

4 blok 10 cycles $T_{avg} = 20°C$ $\Delta T = 0$?

c)

$$AB|_{1h} = \frac{5}{6\cdot 10^5} + \frac{15}{5\cdot 10^4} + \frac{5}{1\cdot 10^3}$$

$$AD|_{1h} = 0{,}53\%$$

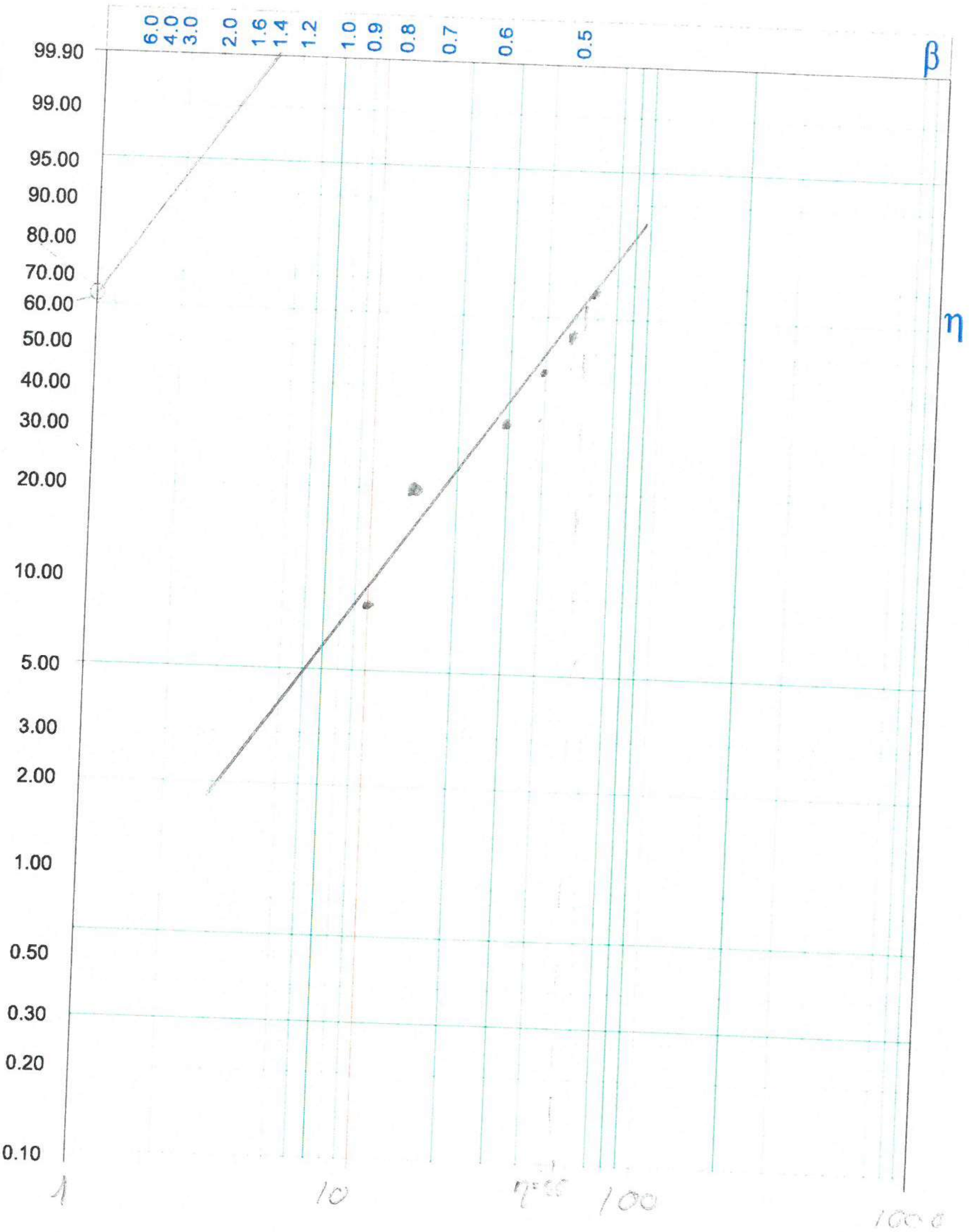
d) expected life time

Total time to failure $= \frac{100\%}{\;} = 188{,}679$ hours

Which with 20 hours pr day, 7 days a week gives

$$\frac{188{,}679 \text{ hours}}{20 \text{ hours/day}} = 9{,}4339 \text{ days}$$

Which is really low but it could be due to misreadings of the plot


6.0
4.0
3.0
2.0
1.6
1.4
1.2
1.0
0.9
0.8
0.7
0.6
0.5
β
η
99.90
99.00
95.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
5.00
3.00
2.00
1.00
0.50
0.30
0.20
0.10
1
10
100
1000

$\beta = 1{,}4$