AALBORG UNIVERSITY
DENMARK

Notice this version only includes the optimisation part.
No solution will be provided.

Written examination in the course

Optimisation Theory and

Modern Reliability from a Practical Approach

Friday June 9th 2017

kl. 9 - 13 (4 hours)

All usual helping aids are allowed, i.e. books, notes, calculator, computer etc. All communication equipment and computer communication protocols must be turned off.

The questions should be answered in English.

REMEMBER to write your study number and page number on all sheets handed in.

The set consists of eight exercises. The total weighting for each of the exercises is stated in percentage. You need 50 % in order to pass the exam.

It should be clear from the solution, which methods are used, and there should be a sufficient number of intermediate calculations, so the line of thought is clear.

Exercise 1: (10 %)

A company wants to maximize its day-to-day profit from manufacturing woodburning stoves. Formulate the profit maximization problem as a standard optimization problem based on the following information:

- The company produces two stoves: 1) Type A and 2) Type B.
- The company has a total of 5000 man hours available per day. It takes 20 man hours to produce a stove of Type A and 24 man hours to produce a stove of Type B.
- The daily cost of producing Type A is a fixed cost of 500 Euro and a further 300 Euro per stove. The daily cost of producing Type B is a fixed cost of 900 Euro and a further 400 Euro per stove.
- To maintain market shares the company needs to produce at least 50 stoves of Type A every day and at least 30 stoves of Type B every day.
- The company sells the stoves at the following prices: 500 Euro per stove of type A and 600 Euro per stove of type B.

Note: you should only formulate the problem, not solve it.

Exercise 2: (15 %)

We will consider gradient-based minimisation of the following unconstrained function:

$$f(\mathbf{x}) \;=\; (x_1 - 1)^2 + 2x_2^2 \tag{1}$$

The starting point is: $\mathbf{x}^{(0)} = [\ 3 \quad 1\ ]^T$.

a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.

b) Complete the first iteration of the modified Newton's method for the function. The 1D line search problem should be solved analytically.

Exercise 3: (10 %)

Solve the following linear optimisation problem using the basic steps of the Simplex method and tableau's:

$$\text{minimise} \qquad f(\mathbf{x}) = -5x_1 - 2x_2$$

Subject to the constraints:

$$\begin{aligned} g_1(\mathbf{x}) &= -x_1 + x_2 \leq 10 \\ g_2(\mathbf{x}) &= 2x_1 - x_2 \leq 20 \\ & x_i \geq 0 \quad \forall \quad x_i = \{1, 2\} \end{aligned}$$

Exercise 4: (15 %)

The following multi-objective optimisation problem is considered:

$$\text{minimise} \qquad \begin{aligned} f_1(\mathbf{x}) &= (x_1 - 5)^2 + (x_2 - 15)^2 \\ f_2(\mathbf{x}) &= (x_1 - 15)^2 + (x_2 - 5)^2 \end{aligned}$$

Subject to the constraint:

$$g_1(\mathbf{x}) = x_1 \leq 10$$

Figure 1 (next page) shows the contour curves for the objective functions.

a) Draw the constraint in the design space (i.e. objective functions contour plot).

b) Draw the Pareto optimal points in the design space.

c) Sketch the Pareto front in the Criterion space (Figure 2 may be used).

d) Determine the objective function values of the utopia point.

e) Assume that the multi-objective problem is solved as single objective problem, $U(\boldsymbol{x})$, using the weighting method with $w_1 = 2$ and $w_2 = 1$. Determine analytically the minimum objective function value $U(\mathbf{x}^*)$, and the optimum set of design variables $\mathbf{x}^*$.

f) Determine whether the found optimum is a global optimum or not - justify the answer.

NOTICE THIS PAGE MAY BE HANDED IN WITH THE SOLUTION

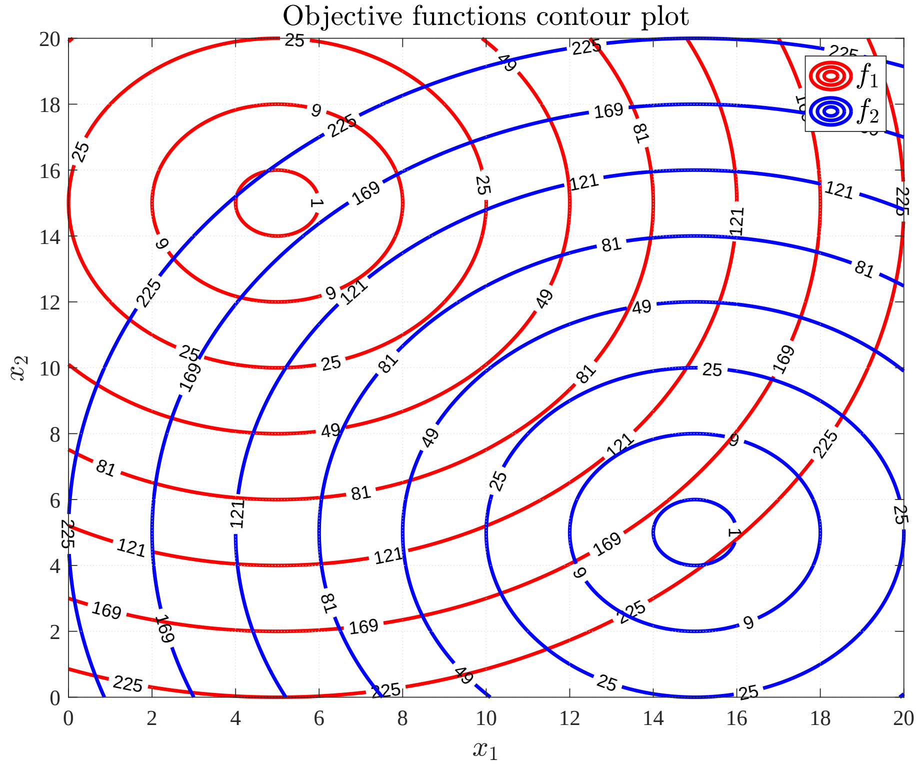

Figure 1: Contour curves and Pareto optimal points.

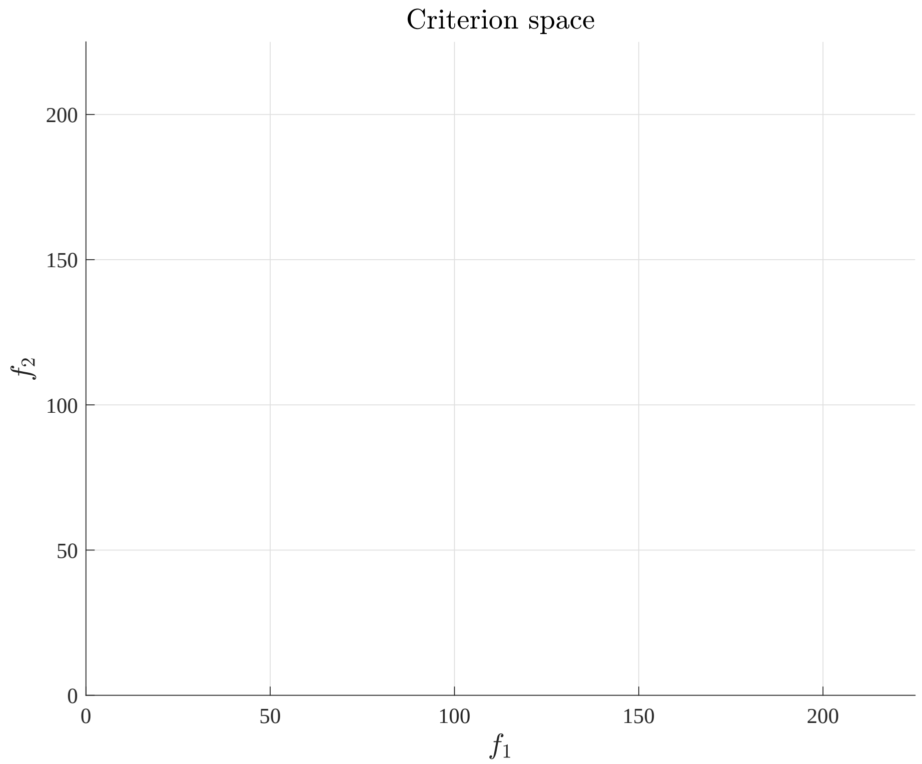

Figure 2: Coordinate system where the Pareto set in criterion space may be plotted.