Exercise 1: (10 %)

The following optimization problem is considered:

$$\begin{aligned} \text{Minimize} \quad f(\mathbf{x}) &= (x_1-1)^2 + (x_2-1)^2 \\ \text{Subject to} \quad h(\mathbf{x}) &= x_1 + x_2 - 4 = 0 \end{aligned} \tag{1}$$

a) Set up the Lagrangian function and find point(s) satisfying the KKT necessary conditions.

b) Check if the point(s) is an optimum point using the graphical method (make a simple sketch).

Exercise 2: (15 %)

We will consider gradient-based minimization of the following unconstrained function:

$$f(\mathbf{x}) = (1-x_1)^2 + (x_2-2)^2 + 2 \cdot x_1 \tag{2}$$

The starting point is: $\mathbf{x}^{(0)} = [\ 3 \quad 1\ ]^T$.

a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.

b) Determine the search direction for the first iteration of Newton's method for the function.

Exercise 3: (13 %)

An optimisation problem is given as:

$$\text{minimise} \qquad f(\mathbf{x}) = -x_1^2 + 3x_2^2 + x_1x_2 - 3 \tag{3}$$

Subject to the constraints:

$$\begin{aligned} g_1(\mathbf{x}) &= \tfrac{1}{x_1} - 2x_2 \leq 0 \\ g_2(\mathbf{x}) &= x_1 - 2x_2^3 \leq 0 \\ & x_i \geq 0 \quad \forall \quad x_i = \{1, 2\} \end{aligned} \tag{4}$$

a) Linearise the problem at the point $(x_1, x_2) = (1, 1)$, and write up the linearised subproblem. Note that there is no need to do a normalisation of the problem!

b) Solve the linearised sub-problem using tableaus and the basic steps of the Simplex method

Page to be handed in with the solution!

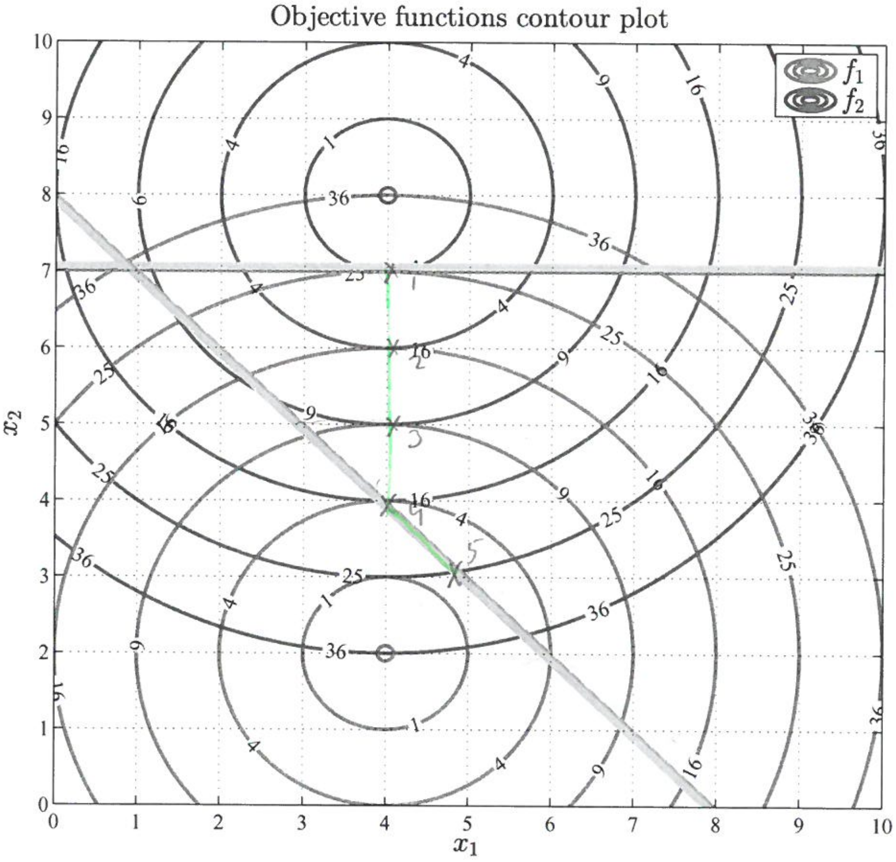

Figure 1: Contour curves for the problem of exercise 4.

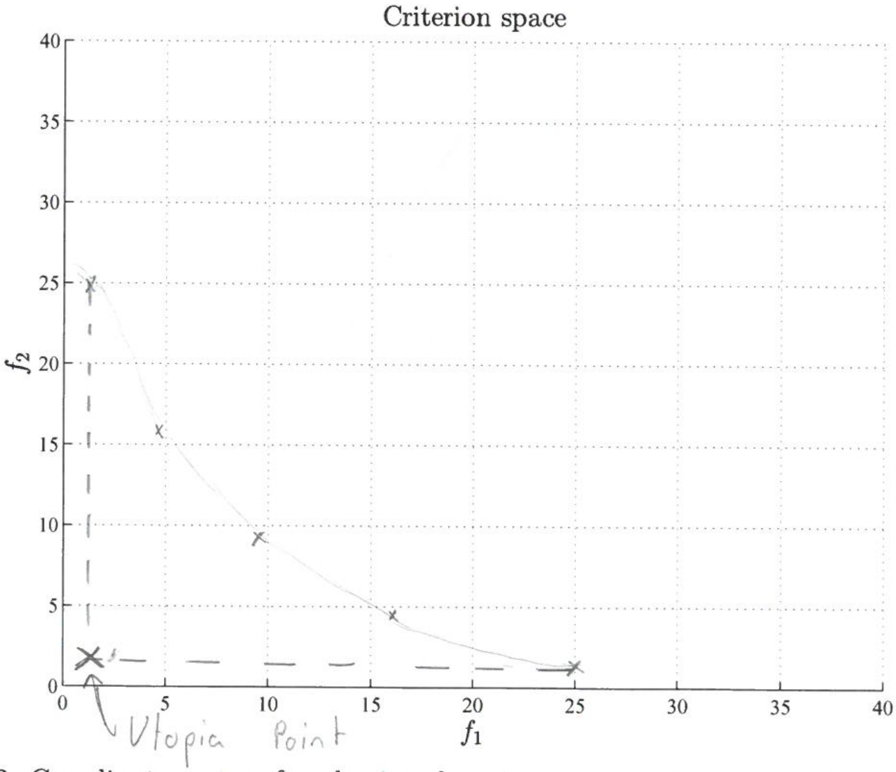

Figure 2: Coordinate system for plotting the criterion space Pareto front in exercise 4.