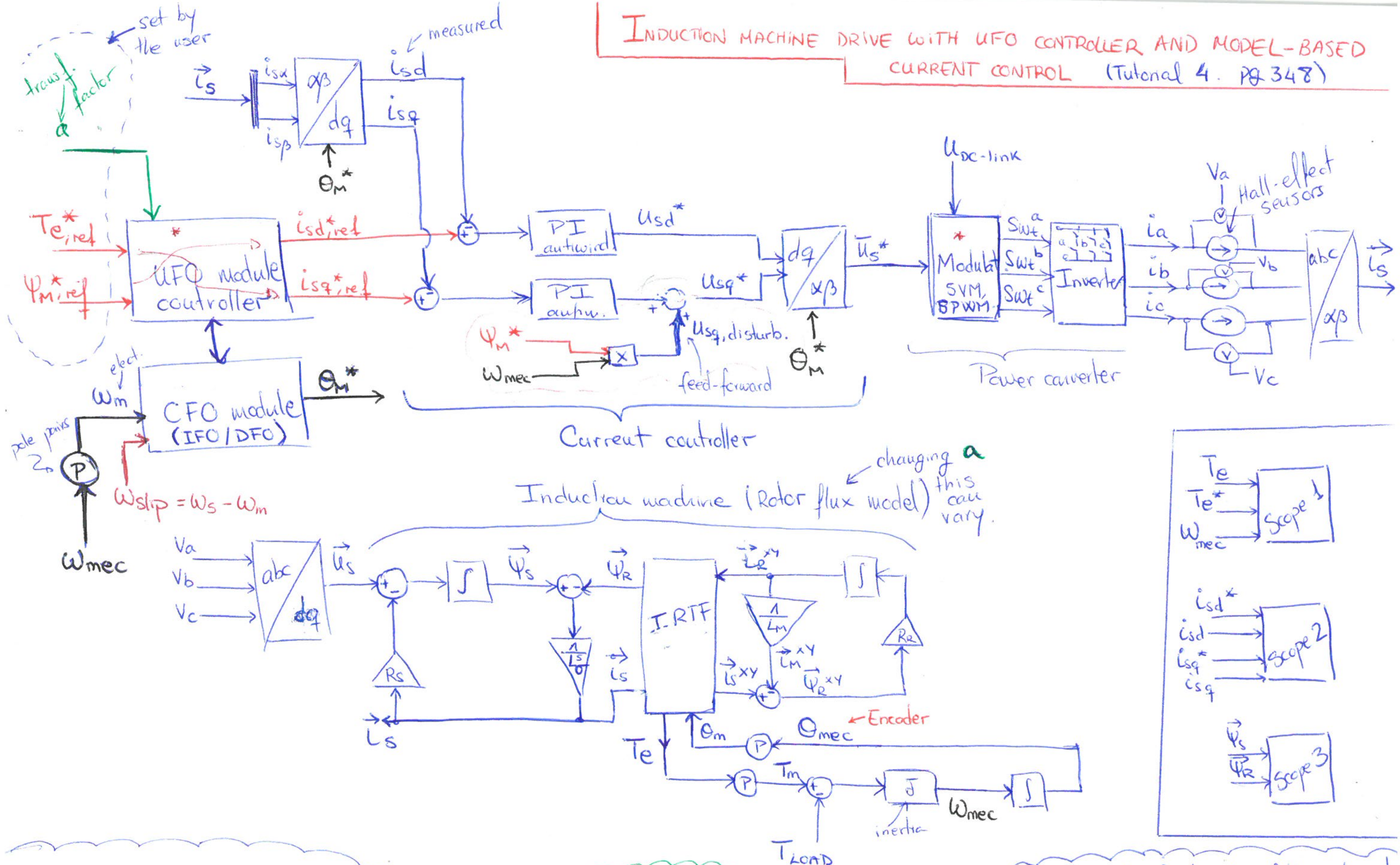
INDUCTION MACHINE DRIVE WITH UFO CONTROLLER AND MODEL-BASED CURRENT CONTROL (Tutorial 4. pg 348)



*NOTE: Sec. 3.3.5 - Tutorial 5 there's a simplified approach in which the inverter's switching action isn't shown.

$a = 1$ ← original 5 param. model

$\frac{L_m}{L_r} \le a \le \frac{L_s}{L_m}$

rotor flux model → ; ← stator flux

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*NOTE: A first order filter placed after T_e^* to limit torque variation to realistic values. Also one after ω^*

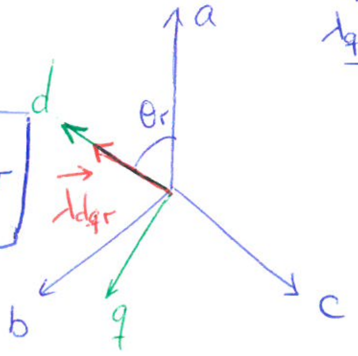
FOC of IM

• ROTOR FLUX ORIENTED CONTROL – INDIRECT METHOD

* Force the d-axis to be aligned with $\vec{\lambda}_{dqr}$ → $|\vec{\lambda}_{dqr}| = \lambda_{dr} \underset{\substack{\uparrow \\ \underline{\lambda_{qr} = 0 !}}}{=} \overset{\substack{\text{constant !} \\ \downarrow}}{\lambda_r}$

* This simplifies the torque equation :

$$\boxed{T = \frac{3}{2} p \cdot \frac{L_m}{L_r} \cdot \mathrm{Im}\left(\vec{i}_{dqs}\, \lambda^*_{dqr}\right) \simeq \frac{3}{2} p \frac{L_m}{L_r} i_{qs} \cdot \lambda_r}$$



* Current controllers $\left\{ \begin{array}{l} i_d \rightarrow \text{rotor flux linkage} \\ i_q \rightarrow \text{torque} \rightarrow \text{speed} \end{array} \right.$

* The indirect method uses sensors (encoder) to obtain the rotor flux position

* Methodology $\left\{ \begin{array}{l} \text{1) Allow } \bar{\lambda}_r \text{ and } \vec{i}_s \text{ in the final equation.} \\ \text{2) Get rid of the rotor currents (using rotor equations).} \\ \text{3) Normally, we don't need stator voltage eq.} \end{array} \right.$

* The IM equations are :

$$\begin{aligned} u_{qs} &= R_s i_{qs} + p \lambda_{qs} + \omega_e \lambda_{ds} \\ u_{ds} &= R_s i_{ds} + p \lambda_{ds} - \omega_e \lambda_{qs} \\ \\ \lambda_{qs} &= L_{ls} i_{qs} + L_m (i_{qs} + i_{qr}) \\ \lambda_{ds} &= L_{ls} i_{ds} + L_m (i_{ds} + i_{dr}) \end{aligned} \qquad \Bigg\| \qquad \begin{aligned} u_{qr} &= R_r i_{qr} + p \lambda_{qr} + (\overset{\text{synchronous speed}}{\omega_e} - \overset{\text{rotor speed}}{\omega_r}) \lambda_{dr} \\ u_{dr} &= R_r i_{dr} + p \lambda_{dr} - (\omega_e - \omega_r) \lambda_{qr} \\ \\ \lambda_{qr} &= L_{lr} i_{qr} + L_m (i_{qs} + i_{qr}) \\ \lambda_{dr} &= L_{lr} i_{dr} + L_m (i_{ds} + i_{dr}) \end{aligned}$$

→ Rotor equations : Knowing that $\lambda_{qr} = 0$ and $u_{qr} = u_{dr} = 0$ (short-circuited)

$$\left\{ \begin{array}{ll} \text{q-axis} : & 0 = R_r i_{qr} + \underbrace{s\omega_e}_{} \lambda_{dr} \; ① \; ; \quad 0 = \underbrace{L_r}_{} i_{qr} + L_m i_{qs} \; ② \\ & \text{slip speed} \rightarrow s\omega_e = \omega_e - \omega_r \qquad L_r = L_{lr} + L_m \\ \text{d-axis} : & 0 = R_r i_{dr} + p \cdot \lambda_{dr} \; ③ \; ; \quad \lambda_{dr} = L_r i_{dr} + L_m i_{ds} \; ④ \end{array} \right.$$

② $i_{qr} = \frac{-L_m}{L_r} \cdot i_{qs} \xrightarrow{\text{subs.}}$ ① $0 = R_r \left(-\frac{L_m}{L_r} i_{qs} \right) + s\omega_e \lambda_{dr} \Rightarrow$

$\Rightarrow s\omega_e = L_m \cdot \frac{R_r}{L_r} i_{qs} \cdot \frac{1}{\lambda_r} \rightarrow \boxed{s\omega_e = \frac{L_m}{\tau_r} \frac{i_{qs}}{\lambda_{dr}}}$ (*)

rotor time const $(\tau_r^{-1}) \rightarrow \frac{R_r}{L_r}$ $\qquad \tau_r = \frac{L_r}{R_r}$

③ $i_{dr} = -\frac{1}{R_r}\cdot p\lambda_{dr}$ $\xrightarrow{\text{subs.}}$ ④ $\lambda_{dr} = L_r\left(-\frac{1}{R_r}p\lambda_{dr}\right) + L_m i_{ds}$

$$(*)\ \boxed{\lambda_{dr} = \frac{L_m i_{ds}}{1+\tau_r\cdot \underset{s\text{ (Laplace domain)}}{p}}} \leftarrow \text{Low-pass filter}$$

(*) Gives the relation between stator d-axis current and rotor flux. It's observed that to keep λ_{dr} = constant → i_{ds} = constant

In s.s, from ③ it's obtained that $i_{dr} = 0$ → ④ $\boxed{\lambda_r = L_m\cdot i_{ds}}$

And the slip speed found in (*) is reduced to → $\boxed{S\omega_e = \frac{1}{\tau_r}\frac{i_{qs}^*}{i_{ds}^*}}$

It's common to apply this simplification and ALSO usually the commanded currents are used instead of the measured currents because they are more stable.

The rotor flux position is obtained from $S\omega_e$: $\omega_e = S\omega_e + \omega_r$

Adjusting the slip angular freq. indirectly controls the position → $\boxed{\theta_e = \int S\omega_e + \omega_r}$

* Therefore,

- $i_{qs,ref}$ is determined directlly from the torque demand (PI speed controller)
- $i_{ds,ref}$ is determined from the desired rotor flux level (rated)
- The location of the rotating reference frame needs the rotor flux angle θ_e

How $i_{ds,ref}$ is calculated? We have to write the eq. in vector form first

① $\vec{u}_{dqs} = R_s\vec{i}_{dqs} + j\omega_e\lambda_{dqs}$

② $0 = \overline{u}_{dqr} = R_r\vec{i}_{dqr} + j(\omega_e - \omega_r)\vec{\lambda}_{dqr}$

③ $\overline{\lambda}_{dqs} = L_{ls}\vec{i}_{dqs} + L_m(\vec{i}_{dqs} + \vec{i}_{dqr})$

④ $\overline{\lambda}_{dqr} = L_{lr}\vec{i}_{dqr} + L_m(\vec{i}_{dqr} + \vec{i}_{dqs})$

→① We obtain $\vec{\lambda}_{dqs}$ because we know all the other parameters:

$\overline{u}_{dqs} = U_{pk,rated} = U_{RMS}\cdot\sqrt{2}$; $\vec{i}_{dqs} = I_{pk,rated}\,e^{-j\alpha}$ where α = PF angle

④ $\vec{i}_{dqr} = \frac{\vec{\lambda}_{dqr} - L_m\vec{i}_{dqs}}{L_r}$ $\xrightarrow{\text{subs.}}$ ③ $\vec{\lambda}_{dqs} = L_s\vec{i}_{dqs} + \frac{L_m}{L_r}\cdot(\overline{\lambda}_{dqr} - L_m\vec{i}_{dqs})$

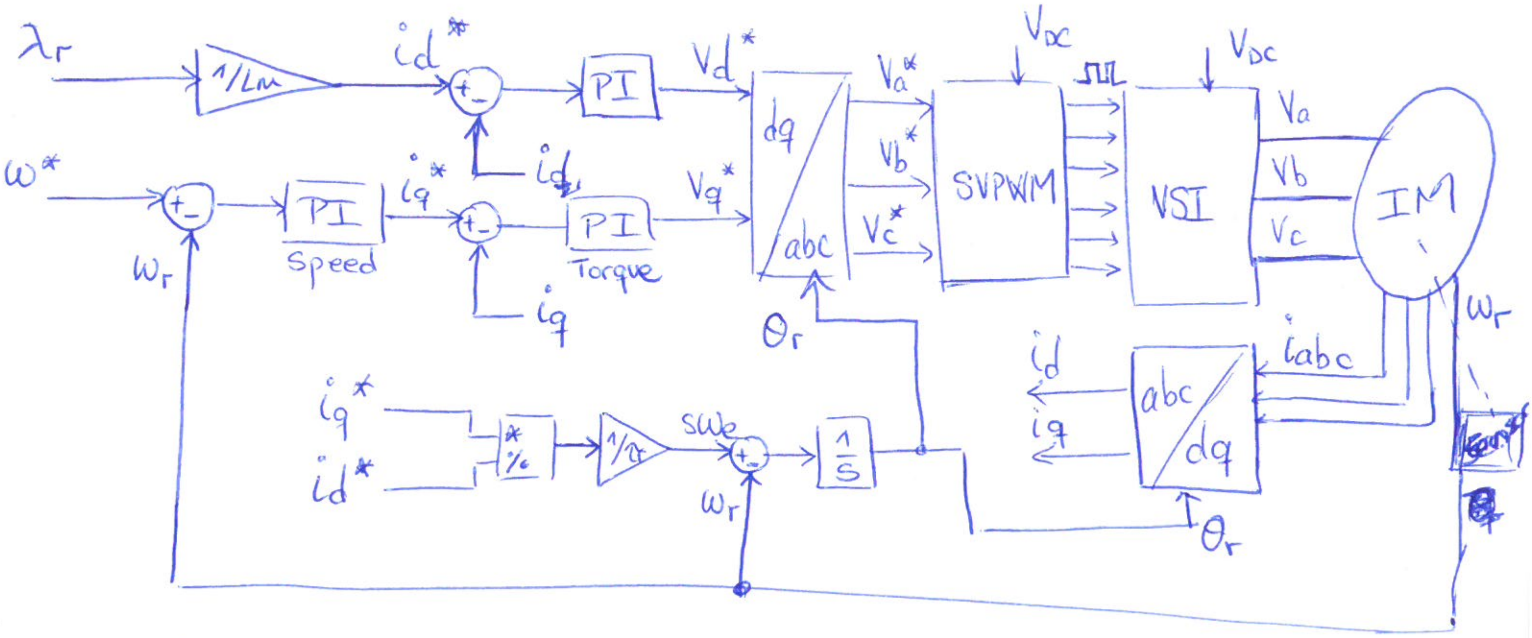
$= \left(L_s - \frac{L_m^2}{L_r}\right)\vec{i}_{dqs} + \frac{L_m}{L_r}\lambda_{dqr} =$

$= \sigma L_s\cdot\vec{i}_{dqs} + \frac{L_m}{L_r}\vec{\lambda}_{dqr}$ (*)

From (*) we get $\vec{\lambda}_{dqr}$ → $|\vec{\lambda}_{dqr}| = \lambda_r$

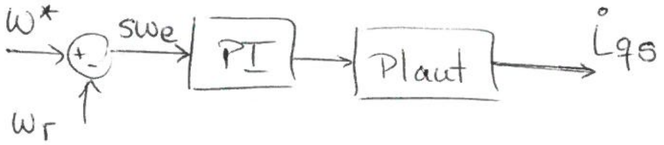
Thus, using eq. $\lambda_r = L_m\cdot i_{ds}$ then i_{ds} is determined!

BLOCK DIAGRAM FOC



- PI Speed controller : the reference speed (ω^*) is compared with the rotor's mechanical speed (ω_r) which is obtained from a sensor in the rotor. Here, the error (slip speed) should be zero.

$$G_c(s) = K_P + \frac{K_I}{s}$$



To obtain the transfer function for the plant we'll use the previous obtained equation (*).

$$G_P(s) = \frac{i_{qs}}{s\omega_e} \leftarrow \text{need to find this relationship.}$$

$$s\omega_e = \frac{L_m}{\tau_r}\frac{i_{qs}}{\lambda_{dr}} \underset{(*)}{\overset{\uparrow}{=}} \frac{\cancel{L_m}}{\tau_r} \cdot \frac{i_{qs}}{\left(\frac{L_m i_{ds}}{1+s\tau_r}\right)} = \frac{1}{\tau_r i_{ds}} \cdot (1+s\tau_r)\, i_{qs}$$

$$G_P(s) = \frac{i_{qs}}{s\omega_e} = \frac{\tau_r \cdot i_{ds}}{(1+s\tau_r)} \longrightarrow G_{OL}(s) = G_c(s)G_P(s) = \left(K_P + \frac{K_I}{s}\right)\left(\frac{\tau_r i_{ds}}{(1+s\tau_r)}\right)$$

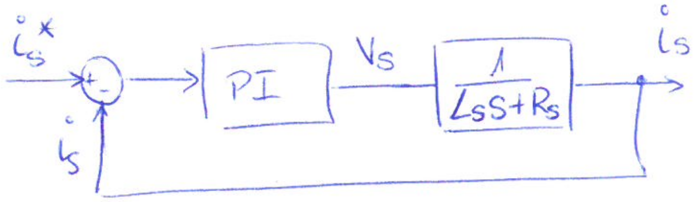
Equating to a first order low pass filter :

$$\frac{\omega_c}{s+\omega_c} = \frac{(\tau_r i_{ds})(sK_P+K_I)}{s(1+s\tau_r)} \longrightarrow \tau_r\omega_c s^2 + \omega_c s \pm \tau_r i_{ds}(sK_P+K_I)(s+\omega_c)$$

$$\frac{\tau_r\omega_c}{\tau_r i_{ds}} \cdot s^2 + \frac{\omega_c}{\tau_r i_{ds}} = s^2 K_p + (K_i + \omega_c K_p)s + \omega_c K_i \quad \left\{ \begin{array}{l} K_P = \frac{\omega_c}{i_{ds}} \\ K_i = \left(\frac{1}{\tau_r} - \omega_c\right) K_P \end{array} \right.$$

$\omega_c \equiv$ bandwidth speed controller

- Current controller : the inner loop current controllers should be faster than the speed control



$$G_c(s) = K_p + K_I/s \quad ; \quad G_p(s) = \frac{1}{Ls+R} \rightarrow G_{OL}(s) = \left(K_p + \frac{K_I}{s}\right)\left(\frac{1}{Ls+R}\right)$$

$$G_{CL}(s) = \frac{G_{OL}(s)}{1+G_{OL}(s)\underset{1}{\underbrace{H(s)}}} = \frac{(K_p+K_I/s)(1/Ls+R)}{1+(K_p+K_I/s)(1/Ls+R)} = \frac{W_{cc}}{s+W_{cc}}$$

$W_{cc} \equiv$ bandwidth current controller

$$\text{Equaling terms} \rightarrow \begin{cases} K_p = L_s \cdot W_{cc} \\ K_I = R_s \cdot W_{cc} \end{cases}$$

NOTE : In page ③ there's a more detailed calculation and we get the same result with a different approach.

It's necessary to select a proper bandwidth of speed and current control loops

* Calculation of λ_r and $i_d^* = i_{d,ref}$

① $$\vec{u}_{dqs} = R_s \vec{i}_{dqs} + jW_e \vec{\lambda}_{dqs} \rightarrow j \cdot (2\pi \cdot \overset{f_n}{58}) \vec{\lambda}_{dqs} = \frac{\overset{U_n}{\overbrace{224}}}{\sqrt{3}} \cdot \sqrt{2} - 0{,}0025 \cdot \overset{I_n}{\overbrace{189}} \cdot \sqrt{2}\, e^{-j\alpha}$$

$$364{,}42 j \vec{\lambda}_{dqs} = 19{,}6 - 0{,}6682 \cdot (\cos 40{,}53^\circ - j \sin 40{,}53^\circ)$$

PF = $\cos\alpha = 0{,}76$

$\alpha = 0{,}7075$ rad $= 40{,}53^\circ$

$$364{,}42 j \vec{\lambda}_{dqs} = 19{,}0921 + 0{,}4342 j$$

$$\vec{\lambda}_{dqs} = 0{,}0012 - 0{,}0524 j \rightarrow |\vec{\lambda}_{dqs}| = 0{,}0524$$

③ $$\vec{\lambda}_{dqs} = \sigma \cdot L_s \vec{i}_{dqs} + \frac{L_m}{L_r} \vec{\lambda}_{dqr} \rightarrow \vec{\lambda}_{dqr} = \frac{L_r}{L_m} \cdot \left(\vec{\lambda}_{dqs} - \underset{L_s - \frac{L_m^2}{L_r}}{\underbrace{\sigma}} L_s \cdot \vec{i}_{dqs}\right)$$

$$\boxed{|\vec{\lambda}_{dqr}| = \lambda_{dr} = \lambda_r}$$

$$\vec{\lambda}_{dqr} = 0{,}0013 - 0{,}0567 j \Rightarrow \boxed{\lambda_r = 0{,}0567 \text{ Wb}}$$

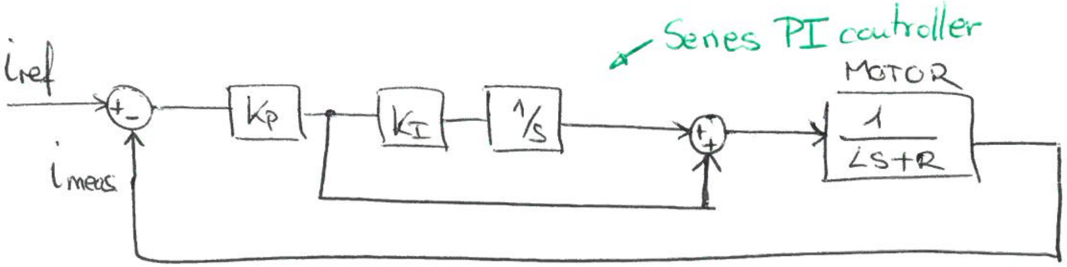
④ $$\lambda_r = L_m \cdot i_{ds} \rightarrow i_{ds} = \frac{0{,}0567}{L_m} = \underline{149{,}2287 \text{ A}} = i_{d,ref}$$

③

PI current controllers

$\rightarrow G_p(s) = \frac{1}{Ls+R}$; $G_c(s) = \frac{K_P \cdot K_I \cdot \left(1+\frac{s}{K_I}\right)}{s} = K_P + \frac{K_I}{s}$

$$G_{OL}(s) = \frac{K_P K_I (1+ s/K_I)}{s}\left(\frac{1}{Ls+R}\right) \longrightarrow G_{CL}(s) = \frac{G_{OL}(s)}{1+G_{OL}(s)} = \frac{1+s/K_I}{\left(\frac{L}{K_P K_I}\right)s^2 + \left(\frac{R}{K_P K_I}+\frac{1}{K_I}\right)s + 1}$$



Second order TF. Kp and KI must be selected avoiding complex poles to maintain stable control. Choose the poles that aren't situated near jω axis to AVOID HIGH RESONANT SPIKES!

Denominator of $G_{CL}(s)$ can be factorised:

$$G_{CL}(s)_{den.} = \left(\frac{L}{K_P K_I}\right)s^2 + \left(\frac{R}{K_P K_I}+\frac{1}{K_I}\right)s + 1 = (1+Cs)(1+Ds) = 1 + (C+D)s + CDs^2$$

real numbers!

Equating terms: $\left\{\begin{array}{l} \frac{L}{K_P K_I} = C\cdot D \ (*) \\ \frac{R}{K_P K_I} + \frac{1}{K_I} = C+D \end{array}\right\} \xrightarrow{\text{Solving}} G_{CL}(s) = \frac{1+s/K_I}{\left(1+\underbrace{\left(\frac{R}{K_P K_I}\right)}_{C}s\right)\left(1+\underbrace{\left(\frac{1}{K_I}\right)}_{D}s\right)}$ (*)

* This way, $(1+Ds) = \left(1+\frac{s}{K_I}\right)$ cancels out the zero of the original TF and thus it's possible to get a closed-loop system with no zeroes and only one real (non complex) pole. → NO resonant responses

* Substituting $C = \frac{R}{K_P K_I}$ and $D = \frac{1}{K_I}$ in (*) $\rightarrow \frac{L}{\cancel{K_P K_I}} = \frac{R}{\cancel{K_P K_I}} \cdot \frac{1}{K_I} \longrightarrow \boxed{K_I = \frac{R}{L}}$ ← Gives the freq at which the controller's zero occu

↑ Corresponds to the Pole of the plant

(*) $G_{CL}(s) = \frac{1}{1+\left(\frac{R}{K_P \cdot R}\right)s} = \frac{1}{1+\frac{L}{K_P}\cdot s} \Rightarrow \boxed{K_P = L \cdot \text{Bandwidth}} = K_{P\,real}$ ← Kp sets the BW of the current controller

$\boxed{K_{I\,real} = K_I \cdot K_P}$

Bandwidth = ω_c

$G_{CL}(s) = \frac{1}{\ } = \frac{1}{1+s\cdot\frac{1}{\ }}$