

Written examination in the course

Optimisation Theory and Stochastic Processes

Thursday June 7th 2012

kl. 9 - 13 (4 hours)

All usual helping aids are allowed, i.e. books, notes, calculator, computer etc. All communication equipment and computer communication protocols must be turned off.

The questions should be answered in English.

REMEMBER to write your study number and page number on all sheets handed in.

The set consists of seven exercises. The total weighting for each of the exercises is stated in percentage. Sub-questions in each exercise have equal weight. You need 50 % in order to pass the exam.

It should be clear from the solution, which methods are used, and there should be a sufficient number of intermediate calculations, so the line of thought is clear.



Written recommunion in the course

Optimisation Theory and Stochastic Processes

Library June 7th 2012

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Exercise 1: (10 %)

The following optimization problem is considered:

Minimize
$$f(\mathbf{x}) = (x_1 - 1)^2 + (x_2 - 1)^2$$

Subject to $h(\mathbf{x}) = x_1 + x_2 - 4 = 0$ (1)

- a) Set up the Lagrangian function and find point(s) satisfying the KKT necessary conditions.
- b) Check if the point(s) is an optimum point using the graphical method (make a simple sketch).

Exercise 2: (15 %)

We will consider gradient-based minimization of the following unconstrained function:

$$f(\mathbf{x}) = (1 - x_1)^2 + (x_2 - 2)^2 + 2 \cdot x_1 \tag{2}$$

The starting point is: $\mathbf{x}^{(0)} = [3 \ 1]^T$.

- a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.
- b) Determine the search direction for the first iteration of Newton's method for the function.

Exercise 3: (13 %)

An optimisation problem is given as:

minimise
$$f(\mathbf{x}) = -x_1^2 + 3x_2^2 + x_1x_2 - 3$$
 (3)

Subject to the constraints:

$$g_{1}(\mathbf{x}) = \frac{1}{x_{1}} - 2x_{2} \leq 0$$

$$g_{2}(\mathbf{x}) = x_{1} - 2x_{2}^{3} \leq 0$$

$$x_{i} \geq 0 \quad \forall \quad x_{i} = \{1, 2\}$$

$$(4)$$

- a) Linearise the problem at the point $(x_1, x_2) = (1, 1)$, and write up the linearised subproblem. Note that there is no need to do a normalisation of the problem!
- b) Solve the linearised sub-problem using tableaus and the basic steps of the Simplex method

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Exercise 4: (12 %)

A multi-objective optimisation problem is formulated as:

minimise
$$f_1(\mathbf{x}) = (x_1 - 4)^2 + (x_2 - 2)^2$$

 $f_2(\mathbf{x}) = (x_1 - 4)^2 + (x_2 - 8)^2$ (5)

Subject to the constraints:

$$g_1(\mathbf{x}) = x_2 - 7 \leq 0 g_2(\mathbf{x}) = -x_1 - x_2 + 8 \leq 0$$
 (6)

The two contour curves along with the constraints are shown in figure 1.

- a) Illustrate the Pareto optimal points in figure 1 (the page should be handed in with the solution).
- b) Sketch the Pareto front in the criterion space. The sketch should be based on function values from the contour plot. A coordinate system may be found in figure 2.
- c) Indicate where the utopia point is located for the given problem.
- d) A weighting method is used in solving the problem (i.e. minimise $U = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x})$), with $w_1 = w_2 = 1$. What is the solution to this problem give both function value U, and design variables x_1 and x_2 . **Hint:** Use the results from questions a) and b).

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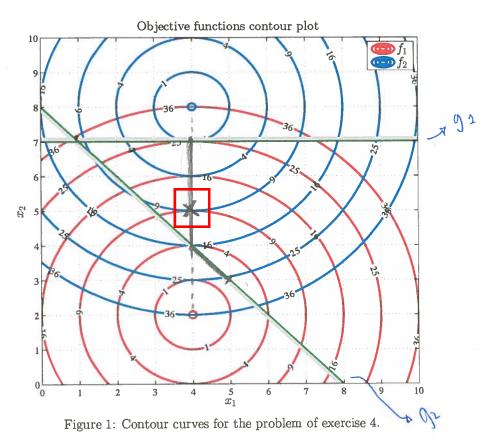
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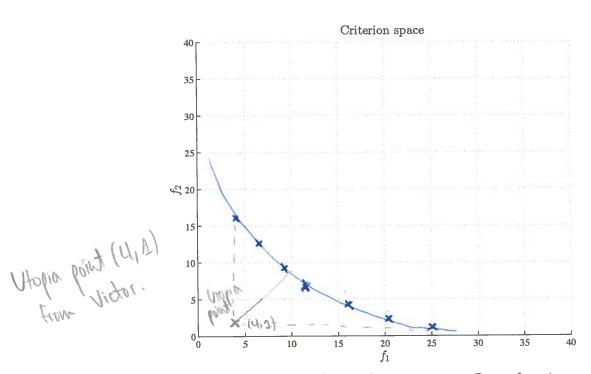
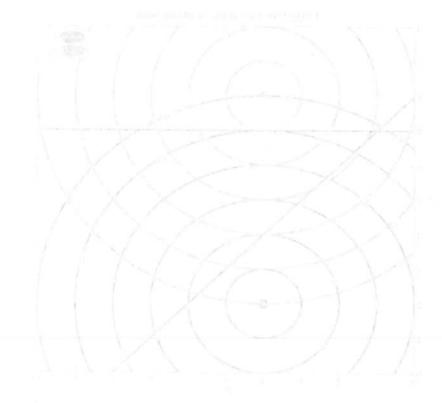


Figure 2: Coordinate system for plotting the criterion space Pareto front in exercise 4.

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Exercise 5: Question 1: (points 25%)

A system with an **input** u and an **output** y and zero initial conditions is described by the following differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 5\frac{du}{dt} + u,\tag{7}$$

a) Derive the impulse response of the system.

From now on, we assume: u is a wide-sense stationary, zero-mean Gaussian random process with σ_u^2 .

- b) Determine the autocorrelation function R_{yy} .
- c) Derive the power spectrum S_{yy} .

Assume now that the system is replaced by a square law detector; that is, a nonlinear system without memory. In other words, from now on our system is described by:

$$y = u^2. (8)$$

- d) Verify that the output of the system is no longer Gaussian.
- e) Determine the autocorrelation function Ryy of the output and its variance.

Hint

For zero-mean Gaussian random variables x_1, x_2, x_3, x_4 , the following equality holds: $E[x_1x_2x_3x_4] = E[x_1x_2] E[x_3x_4] + E[x_2x_3] E[x_1x_4] + E[x_1x_3] E[x_2x_4]$.

Question 6: (points 15%)

In a digital communication system, consider a source whose output under hypothesis H_1 is a constant voltage of value m, while its output under H_0 is zero. The received signal is corrupted by N, an additive white Gaussian noise of zero mean, and variance σ^2 .

- a) Find the probability density function of the output under both hypotheses.
- b) Calculate the log-likelihood function.
- c) Find the MAP decision rule for the following a priori probability distributions: $P[H_0] = P[H_1] = 0.5$

Question 7: (points 10%)

We wish to observe a variable Y of a system. But the observation X is actually

$$X = 0.9Y + W$$

where W is a normal random variable which is independent of Y. The variances $\sigma_W = \sigma_Y = 1$ and the mean values $\mu_W = \mu_Y = 0.2$ are given.

Find the best linear estimator of Y

$$\hat{Y} = a + bX \tag{9}$$

Examples is Openition in (points 15 a)

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$$m = \frac{m_s}{4 \cdot m} \hat{q} = -m_s = \frac{2 \hat{q}}{m_s} \hat{q} = -\frac{2 \hat{q}}{m_s} \hat{q} = \frac{2 \hat{q}}{m_s} \hat{q}$$

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system with an input u and an output y and zero initial onditions is described by the following differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 5\frac{du}{dt} + u$$

y Derive the impulse of the system:

Laplace:

$$y(s^2 + 2s + 3) = u(5s + 1)$$

$$\frac{y}{u} = \frac{5s+1}{s^2+2s+3} = H(s)$$

$$h(t) = \int_{-\infty}^{\infty} \left\{ H(s) \right\} = 5e^{-t} \left(\cos(\sqrt{2}t) - 2\sqrt{2} \cdot \sin(\sqrt{2}t) \right)$$

b) Determine the autocorrelation Ryy.

$$R_{yy} = \int_{-1}^{1} \{S_{yy}\}$$

Sun = Ou since U is a zero-mean white noise random sequence. Slide 30 lect. 4.

$$= O_u^2 \frac{(5S+1)^2}{(s^2+2S+3)^2}$$

$$R_{yy} = \int_{-1}^{-1} \left\{ O_u^2 \frac{(5s+1)^2}{(s^2+2s+3)^2} \right\}$$

$$= \frac{1}{4} \sigma_u^2 e^{-t} \left(34 + \cos(\sqrt{2}t) - \sin(\sqrt{2}t) \sqrt{2}(-33 + 40t) \right)$$

C/ Determine the power spectrum Syy

$$S_{yy} = O_u^2 \frac{(5S+1)^2}{(s^2+2S+3)^2}$$

N is an additive white Gaussian noise of zero mean and

a) Find the probability density function of Y of under bothhypother $f(N) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2\sigma^2} ||\mathbf{N}|^2\right\}$

$$f(y|H_1) = f(N) \Big|_{N=\sqrt{2\pi}} = \sqrt{\frac{1}{2\pi}} \exp \left\{-\frac{1}{2\sigma_y^2} (Y-m)^2\right\}$$

$$f(y|H_0) = f(N)|_{N=Y} = \frac{1}{\sqrt{2\pi} \sigma_y} exp\left(-\frac{1}{2\sigma_y^2}(Y)^2\right)$$

b) Calculate the log-likelyhood function: $L(y) = \frac{f(y|H_0)}{f(y|H_0)} = \frac{1}{\sqrt{2\pi}} \frac{$ 1/2/1 Oy exp {- 2 0y2 y2}

$$= \exp\left\{-\frac{1}{2\sigma_{y}^{2}} \left[y^{2} - (y-m)^{2}\right]\right\}$$

$$l(y) = ln\left[L(y)\right] = -\frac{1}{2\sigma_{y}^{2}} \left[y^{2} - (y-m)^{2}\right]$$

c) Find the MAP decision rule for the following a priori probability distributions: $P[H_0] = P[H_1] = 0.5$ $\frac{1}{y} = 0.5$

$$l(y)$$

$$\begin{cases}
\frac{P[H_0]}{P[H_1]} = 0
\end{cases}$$

$$-\frac{1}{2 \sigma_{y}^{2}} \left[y^{2} - \left(y - m \right)^{2} \right]$$

$$= y^{2} + m^{2} - 2my$$
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$$m^{2}-2my$$

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Je wish to observe a variable Y of a system. But the observation is actually: X = 0,9 Y + W where W is a normal random variable which is independent of 4. Ow = Oy = 1, Mw = My = 0.2 How Find the best linear estimator of Y $\hat{Y} = a + bX$ $\hat{Y} = h_0 + \sum_{m=1}^{M} h_m X(m)$ $h_o = a = \mu_y - (h)^T \mu_X$ slide 16, lect. 7 h = b = (Zxx) Zxy $\Sigma_{xx} = E[(X-\mu_x)(X-\mu_x)^T] = \sigma_x^2 = 0.9^2 \sigma_y^2 + \sigma_w^2$ $Z_{XY} = E[(X-\mu_X)(Y-\mu_Y)]$ where X = 0.9Y+W and $\mu_X = 0.9\mu_Y+H$ $= \mp \left[\left((0,9Y + W) - (0,9\mu_Y + \mu_W) \right) (Y - \mu_Y) \right]$ = E [(0,9 (Y-My)+W-Mw) (Y-My)] = 0,9 E[(Y-My)] + E[(W-MW)(Y-M7)] = 0? Why? from solution, lect. 7. = 0,9 oy2 $h = \sum_{xx} \cdot \sum_{xy} = \frac{0.9 \, \sigma_y^2}{\sigma_x^2} = \frac{0.9 \, \sigma_y^2}{0.9^2 \sigma_y^2 + \sigma_w^2} = \frac{0.9}{0.9^2 + 1} =$ h = my - (h) mx = my - (h) mx (0,9 my + mw) $= 0.2 - 40 (0.9 \cdot 0.2 + 0.2) = 40.2 = 0.011$

EXAM 2012

Exercise 1:

the following optimization problem is considered:

LECTUREZ/4, 45

Minimize
$$f(x) = (x_1 - 1)^2 + (x_2 - 1)^2$$

Subject to $h(x) = x_1 + x_2 - y = 0$.

a) Set up the Lagrangian function and find point(s) satisfying the kext necessary conditions.

b) Chack if the points) is an optimum using the graphical method (simple states)

The Lagrangian is firmulated as,

Next step is to check the KKT necessary conditions:

$$\frac{\partial L}{\partial x_{1}} = 2 \cdot (x_{1} - 1) + V = 0$$

$$\frac{\partial L}{\partial x_{2}} = 2 \cdot (x_{1} - 1) + V$$

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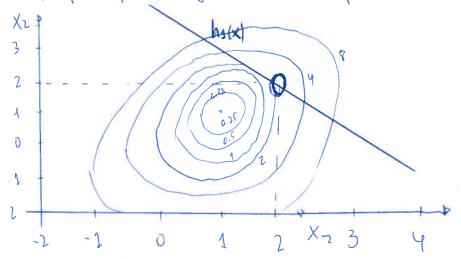
$$\frac{\partial L}{\partial x_{1}} = \frac{2 \cdot (x_{1} - 1) + V}{2}$$

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$$\frac{\partial L}{\partial x_{1}} = \frac{2 \cdot (x_{1} - 1) + V}$$

After solving (by hand or viving the MATLAB script), the following solution is obtained $X_3=Z$ $X_2=Z$ v=-Z

Therefore, the objective is minimised to: $f(x^*) = (2-1)^2 + (2-1)^2 = 1^2 + 1^2 = 2$ -o $f(x^*) = 2$. To check if the solution is indeed an optimum point, we need to solve graphically. Using MATLAB script:



We can observe that the optimum point is also a GLOBAL MINIMUM! I

Exercise 2:

LECTURE 3 - Ex 10.52

We will consider a gradient-bosed minimization of the following unconstrained function: $\{(x_1, x_2) = (1 - x_1)^2 + (x_2 - 2)^2 + 2 \cdot x_1 = x_1^2 + x_2^2 - 4x_2 + 5.$

The starting point is: $x^{(0)} = \begin{bmatrix} 3\\1 \end{bmatrix} \rightarrow x^{(0)} = (3,1)$

a) Complete the 1st iteration of the steepest descent method for the function. The 10 line search problem should be solved analytically.

1) Determine the search direction for the 1st iteration of Newton's method.

The procedure to apply the steepest descent method is shown in Arora p 43?

1) Colculate the gradient:

$$\nabla f(x) = C(x) = \begin{bmatrix} 2 \cdot x_1 \\ 2 \cdot x_2 - 4 \end{bmatrix}$$

Evaluated at the starting point $x^{(0)}$ $C(x^{0}) = \begin{bmatrix} 2.3 \\ 2.1 - 4 \end{bmatrix} = \begin{bmatrix} 67 \\ -2 \end{bmatrix} \rightarrow C^{0} = \begin{bmatrix} 67 \\ -2 \end{bmatrix}$

EXAM 2012

Exercise 2 (continuation):

") The direction of the steepest derent can be computed from the gradient:

$$d^{(0)} = -C(x^{(0)}) = -C^{(0)} = \begin{bmatrix} -6 \\ 2 \end{bmatrix} \qquad \forall \quad d^{(0)} = (-6, 2)$$

Next step is to compute the step size (analytically): 1st, we transformed the problem into a 1-D line search problem:

$$f(\alpha) = f(x^{(0)} + \alpha \cdot d^{(0)})$$
 where $x^{(0)} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $d^{(0)} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

$$f(\alpha) = (3-6\alpha)^2 + (1+2\alpha)^2 - 4\cdot(1+2\alpha) + 5$$

The optimal size is determined by differentiating:

$$f'(\alpha) = \frac{\partial f(\alpha)}{\partial \alpha} = 80 \alpha - 40$$
 $f'(\alpha) = 0 \rightarrow 80 \alpha = 40 \rightarrow \alpha = \frac{4}{8} = \frac{1}{2} = 0.5$

Therefore, the new design can be computed as,

$$x^{(4)} = x^{(0)} + \alpha \cdot d^{(0)} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + 0.5 \begin{bmatrix} -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Being,
$$x^{(1)} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \rightarrow x^{(1)} = (0,2)$$

The process may be repeated until the optimum is found.

b) 1st iteration of Newton's method: - Determine search direction:

To determine the search direction for Wenton's nethod

Using the previous gradient and function walnations for the stanting point,

$$C^{(0)} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \qquad f(x^{\circ}) = 11 \qquad x^{(0)} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

We need to colculate the Hessian:

and we should evaluate the eigen valuer.

$$|A - \lambda I| = \begin{vmatrix} 2 - \lambda & 0 \\ 0 & 2 - \lambda \end{vmatrix} = \begin{bmatrix} (2 - \lambda) \cdot (2 - \lambda) - 0 \end{bmatrix} = 4 - 2\lambda - 2\lambda + \lambda^{2}$$

 $\lambda^{2} - 4\lambda + 4 = 0 - \lambda = \frac{4 + \sqrt{16 + 4 + 4}}{2}$
 $\lambda = 2$ $\lambda = 2$ $\lambda = 2$ $\lambda = 2$

Both eigen volver are positive $\chi \leq 0$ N.S.D. $\chi_{1,2} \geq 0$ P.D.

Calculating the search direction, we need the inverse of the Hessian.

$$H^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\times^{(0)} + \alpha \cdot d^{(0)} \uparrow$$

the search direction is then,

$$d^{(0)} = -H^{-1} \cdot C^{(0)} = -\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} -3 \\ +1 \end{bmatrix}$$

$$d^{(0)} = \begin{bmatrix} -3 \\ +1 \end{bmatrix}$$

The depent condition is checked, $c^{(0)} \cdot d^{(0)} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -18 \\ -2 \end{bmatrix} \quad \text{a -20} < 0 \quad \text{v}$

EXAM 2012

LECTURE 6-12.15 Exercise 3: An optimisation problem is given as:

Minimise
$$f(x) = -X_1^2 + 3 \times 2^2 + X_1 \cdot \times 2 - 3$$

Subject to:
$$g_1(x) = \frac{1}{x_1} - 2x_2 \le 0$$

 $g_2(x) = x_1 - 2x_2^3 \le 0$
 $x_1 \ge 0 + x_1 - \{1, 2\}$

- a) linearise the problem at the point $x^* = (x_1, x_2) = (2, 1)$ and write up the linearised subproblem. (Note that there's no weed to normalize)
- b) Solve the linearised sub-problem using Tableans and the boxic steps of the Simpler method.
- a) Evaluate cost function at start point $f(x^{\circ}) = 0.$

Compute the gradient:

$$\nabla f(x) = C(x) = \begin{bmatrix} -2x_1 + x_2 \\ 6x_2 + x_1 \end{bmatrix}$$

Evoluated at
$$x^{(0)}$$

$$C^{(0)} = \begin{bmatrix} -2.1 + 1 \\ 6.1 + 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

$$C^{(0)} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

the gradients of the constraints are:

$$\nabla g_{2}(x_{1}, x_{2}) = \begin{bmatrix} -1 \\ -2 \end{bmatrix}_{5}^{5} \begin{bmatrix} -1 \\ -2 \end{bmatrix}_{5}^{5}$$

and do - 1 mid do = 7.

Which yields motrix A and vector b:

$$A = \begin{bmatrix} -1/x^{12} & 1 & -1 & 0 \\ -2 & -6x^{12} & 0 & -1 \end{bmatrix} \quad \text{and} \quad k = \begin{bmatrix} -1 + 2 \cdot 1 \\ -1 + 2 \cdot 1^{3} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

The linearised subproblem may how be rewritten as: $\int_{-\infty}^{\infty} dz = x_1 - x_2$ Minimise $\bar{f} = [-1, 7] \cdot [dz] = -dz + 7 \cdot dz$ $\bar{f} = c^{-1} \cdot d$

Subject to the constraints:

Apply simple directly subject At. dels

$$\begin{bmatrix}
-1/x_1^2 & -2 \\
1 & -6x_2^2 \\
-1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
d_1 \\
-d_1
\end{bmatrix} = \begin{bmatrix}
-d_1fx_1^2 & -2d_2 \\
d_1 & -6x_2^2 & d_2
\end{bmatrix}$$

$$\begin{bmatrix}
d_1 \\
-d_1
\end{bmatrix}$$
we can finish here.

Written in terms of the original variables may be formulated as $(d=\times-\times^{(\circ)})$

Minimise
$$f(x_1, x_2) = f(x^{(0)}) + c^T \cdot d = 0 + [-1 \ 7] \cdot \begin{bmatrix} x_1 - 1 \\ x_2 - 1 \end{bmatrix}$$

Subject to the constraints:

$$\bar{q}_{1}(X_{11}X_{2}) = \frac{1}{X_{2}} - 2X_{2} \leq 0$$
 $\bar{q}_{2}(X_{11}X_{2}) = X_{1} - 2X_{2}^{3} \leq 0$
 $\bar{q}_{3}(X_{11}X_{2}) = -X_{2} \leq 0$
 $\bar{q}_{4}(X_{11}X_{2}) = -X_{2} \leq 0$

Exercise 3 (continuation)

A) Solve the linearised subproblem using tableaus and the boxic steps of the simplex method.

Minuse:
$$\{(X_1, X_2) = -X_1 + 7X_2 - 6\}$$

Subject to: $g_1 = \frac{1}{X_1} - 2X_2 \le 0$
 $g_2 = X_1 - 2X_2^3 \le 0$
 $g_3 = -X_1 \le 0$

Written os a standard linear programming produn:
Minerius $f(X_1, X_2) = -X_3 + 7 - X_2 - 6$

gu= - X2 < 0.

Subject to:
$$1/x_1 - 2x_2 + x_3 \le 0$$

 $x_1 - 2x_2^3 + x_4 \le 0$
 $-x_1 \le 0$
 $-x_2 \le 0$
 $-x_3 \le 0$
 $-x_4 \le 0$.

Or written in a natrix form.

Subject to:
$$A \cdot x = b$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad c = \begin{bmatrix} -1 \\ 7 \\ 0 \\ 0 \end{bmatrix} \quad A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The initial tableau may now be set up as:

Basic van & | X1 X2 X3 X4 & Ratio bilaij

....

91

EXAM 2012 |

Exercise 4:

A multi-objective optimisation problem formulated as: minimise $f_1(x) = (x_1-y)^2 + (x_2-z)^2$ $f_2(x) = (x_1-y)^2 + (x_2-8)^2$

Subject to the constraints:

$$g_1(x) = x_2 - 7 \le 0$$

 $g_2(x) = -x_1 - x_2 + 8 \le 0$.

The 2 countour waves along with the constraints are shown in Fig J.

a) Illustrate the Pareto optival points in Figure 1.

- b) Sketch the Pareto front in the criterion space. The sketch should be brosed on function values from the contour plot. A coordinate system is given.
- iwns A) A weighting nethod is used in solving the problem (W= w, filelalus file with w= w= 1. What is the solution to this problem?

 Give both function value V, and design variables X1 and K2.

a) Drawn in Figure 1. It will goe inside the danger space (constraints)

c) Utopia point - When to and to are minimum.

Exercise 5: A system with an input "u" and an output "y" and zero initial conditions is described by the following differential equation.

a) Derive the impulse response of the system.

newsiting the differential equation:

Applying LAPLACE,

$$y(s^2 + 2s + 3) = u(5s + 1)$$

$$H(s) = \frac{V(s)}{V(s)} = \frac{5s + 1}{s^2 + 2s + 3}$$

o Chick in MAPLE.

In time domain:

From MATLAB - 5. e. (cos (Jz.t) - 2. Vz. sih (Tzt))= Use TLAPLACE (F)!

b) u is WSS, Zero-wear Gaussian random prown with Ju?.

Auto correlation Ryy:

R47 = 4 - Ju? e- (34. f. cos (Jz. t) - sin (Jz. t)) - Vz. (-33 + 40t)

c) Detunine the power spectrum Suy.

From the previous section:

Now the system is replaced by a square laws detector that is a nonlinear system without memory. Our system is described by:

d) Verify that the output of the system is no longer Garssian.

e) Determine the autocorrelation function very of the output and its vanions.

EXAM 2012

Exercise 6: In a digital communication system, consider a source whouse output under hypothesis Hz is a constant voltage of value in, while its output under Ho is ELVO.

the received signal is corrupted by M, an additive white Garman noise

of zero mean and variance C2.

a) Find the probability dansity function (pdf) of the output under both hypotheses.

b) Calculate the log-likelihood function

c) Find the MAR decision rule for the following a priori probability distributions: P[Ho] = P[Ho] = 0.5.

(a)
$$H_1: Y = M + N$$

 $H_0 = Y = N$.
 $f(N) = \frac{1}{\sqrt{2\pi \cdot 5}} \cdot e$
 $f(Y|H_1) = f(N)|_{N = Y = M} = \frac{1}{\sqrt{2\pi \cdot 5}} \cdot \frac{(Y - M)^2}{\sqrt{2\pi \cdot 5}} \cdot \frac{1}{\sqrt{2\pi \cdot$

V22-V12 = ZV24+ZV1.4

Exercise 7

We wish to observe a variable Y of a system. But the observation X is actually: X=0.9.4 + W.

where wis a normal random variable which is independent of Y. The varion or Gw = Gy = 1 and the mean values $\mu w = \mu y = 0.7$ are gien. Find the best linear estimator of Y:

$$\sum_{k=1}^{\infty} x_{k} = 0.9 \cdot \sqrt{3}$$

$$= \sum_{k=1}^{\infty} x_{k}^{-1} \cdot \sum_{k=1}^{\infty} \frac{0.9 \cdot \sqrt{3}^{2}}{\sqrt{3}^{2}} = \frac{0.9 \cdot \sqrt{3}^{2}}{\sqrt{3}$$

$$h^{\circ} = hy - (h^{-})^{T} \mu x = \mu y - (h^{-})^{T} \cdot (0.9 \mu y + \mu w) = 0.2 - 0.4972 \cdot (0.9 \cdot 0.7 + 0.72)$$

 $h^{\circ} = 0.2 - 0.186936 = 0.011064$ $h^{\circ} = 0.011$

Possible exam quartions:

· Modity Example - Quad 112) - Steeper Derent to introduce the Conjugate Gradient Holland (made with Steepert derent)

W1-f1(x) = W2 f2(x)

IMPORTANT:

· lecture 4 - Slide 12

$$2X_{1} - V - 4 = 0$$

$$2X_{1} - V + 2 = 0$$

$$-X_{1} - X_{2} + 4 = 0$$

$$-X_{1} - X_{2} + 4 = 0$$

$$2X_{1} - 2X_{1} + 4 + 2 = 0$$

$$2X_{1} - 2X_{1} + 6 = 0$$

$$2X_{1} - 2X_{1} + 6 = 0$$

$$2X_{1} - 2X_{1} - 6$$

$$2X_{2} - 2X_{1} - 6$$

$$2X_{3} - 2X_{1} - 7$$

$$X_{1} = X_{1} - 3$$

X1=-7[2] V=7-(-7)-4

1 X1

$$(2-X_1)^2 = 2^2 + X_1^2 - 2 \cdot 2 \times x = X_1^4 - 4 \times 1 + 4 \times 1$$

$$(X_1-2)^2 = X_1^2 + 4 + 4 \times 1$$

LECTURE 1(7ex): 2-1/2-3/2-5/2.14/7.20/3.1/3.2 LECTURE 2: (5 x) 4.8 | 4.70 | 4.72 | 4.45 | 4.67 LECTURE 3 (Suercises): 10.4/10.72/10.32/10.42/10.52 LECTURE 4(50): 10.67/11.9/11.10/11.21/11.22 (alra 11.3) LECTURE 5: 8.4 8 38 8.44. (MATLAB: 8.55 | 8.57 (8.59) LECTURE 6: 5.14 | 12.7 | 12.15 | 12.26 m cylinder volume = 150 m³ LECTURE 7: 12.51 (14 holes vol. = 150m3), 12.52/13.11. Simulated Amedia ex. LECTURE 8 - M.A | M.A | M.Z | M.Y. 400 + 200

Stochartic:

LECTURE 1:

LECTURE 7:

LECTURE 3:

LECTURE U:

LECTURE S:

LECTURE 6:

LECTURE 7:

LECTURE 8:



Written examination in the course

Optimisation Theory and Stochastic Processes

Tuesday June 11th 2013

kl. 9 - 13 (4 hours)

All usual helping aids are allowed, i.e. books, notes, calculator, computer etc. All communication equipment and computer communication protocols must be turned off.

The questions should be answered in English.

REMEMBER to write your study number and page number on all sheets handed in.

The set consists of seven exercises. The total weighting for each of the exercises is stated in percentage. You need 50 % in order to pass the exam.

It should be clear from the solution, which methods are used, and there should be a sufficient number of intermediate calculations, so the line of thought is clear.

Exercise 1: (10 %)

The following optimisation problem is considered:

Minimise
$$f(\mathbf{x}) = (2 - x_1)^2 + (x_2 + 1)^2$$

Subject to $h(\mathbf{x}) = -x_1 - x_2 + 4 = 0$ (1)

- a) Set up the Lagrangian function and find point(s) satisfying the KKT necessary conditions.
- b) Check if the point(s) is an optimum point using the graphical method (make a simple sketch).

Exercise 2: (15 %)

We will consider gradient-based minimisation of the following unconstrained function:

$$f(\mathbf{x}) = 10(x_1^2 - x_2) + x_1^2 - 2x_1 + 5 \tag{2}$$

The starting point is: $\mathbf{x}^{(0)} = [-1 \ 3]^T$.

- a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.
- b) Can Newton's method be applied for determining the search direction in iteration 1? If yes, then determine the search direction.

If no, then state an alternative robust method for determining the search direction.

Exercise 3: (18 %)

An optimisation problem is given as:

minimise
$$f(\mathbf{x}) = (x_1 - 1)^2 + 2(x_2 - 1)^2 - x_1 x_2$$
 (3)

Subject to the constraints:

$$g_{1}(\mathbf{x}) = 5 - x_{1} - x_{2} \leq 0$$

$$g_{2}(\mathbf{x}) = x_{1}^{2} - x_{2} - 36 \leq 0$$

$$g_{3}(\mathbf{x}) = \frac{x_{1}^{2}}{4} - x_{1} + x_{2} - 8 \leq 0$$

$$g_{4}(\mathbf{x}) = -x_{1} \leq 0$$

$$g_{5}(\mathbf{x}) = -x_{2} \leq 0$$

$$(4)$$

- a) Complete one iteration of the Sequential Linear Programming (SLP) method for the above problem, where you solve step 4 graphically. Use a starting point of $\mathbf{x}^{(0)} = (5,5)$ and 20% move limits. As a help the contour plot of the linearised objective function for $\mathbf{x}^{(0)} = (5,5)$ is shown in figure. The page may be handed in with the solution.
- b) Describe in words, if there are any limitations in using the SLP-method and/or if there are any type of problems, for which the SLP-method is unsuited.

Question 5: (points 25%)

A system with an **input** u and an **output** y and zero initial conditions is described by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2\frac{du(t)}{dt} + 5u(t),\tag{6}$$

a) Derive the impulse response of the system.

From now on, we assume: u is a wide-sense stationary, white zero-mean Gaussian random process with $\sigma_u^2 = 0.5$.

- b) Determine the autocorrelation function R_{yy} .
- c) Derive the power spectrum S_{yy} .

Assume that the system is replaced by a continuous-time integrator. In other words, from now on our system is described by:

$$u(t) = \frac{dy(t)}{dt} \tag{7}$$

- (d) Find the mean value of the output(μ_y).
- e) Determine the autocorrelation function of the output (R_{yy}) .

Question 6: (points 15%)

In a communication system, consider a source whose output under hypothesis H_0 is a constant voltage of value v_1 , while its output under H_1 is a constant voltage of value v_2 . The received signal is corrupted by N, an additive white Gaussian noise of zero mean, and variance $\sigma^2 = 1$.

- a) Find the probability density function of the output under both hypotheses.
- b) Calculate the log-likelihood function.
- c) Find the MAP decision rule for the following a priori probability distributions: $P[H_0] = 0.4$ and $P[H_1] = 0.6$

Question 3:(points 10%)

We wish to observe a variable Y of a system. But the observation X is actually $X = k_1Y + k_2W$ where W is a normal random variable which is independent of Y. Note that k_1 and k_2 are some known constants. The variances $\sigma_W = \sigma_Y = 1$ and the mean values $\mu_W = \mu_Y = 0.2$ are given.

Find the best linear estimator of Y

$$\hat{Y} = a + bX \tag{8}$$

in terms of k_1 and k_2 .



Written examination in the course

Optimisation Theory and Stochastic Processes

Wednesday June 1th 2011

kl. 8.30 - 11.30 (3 hours)

All usual helping aids are allowed, i.e. books, notes, calculator, computer etc. All communication equipment and computer communication protocols must be turned off.

The questions should be answered in English.

REMEMBER to write your study number and page number on all sheets handed in.

The set consists of 6 exercises. The total weighting for each of the exercises is stated in percentage. Sub-questions in each exercise have equal weight.

It should be clear from the solution, which methods are used, and there should be a sufficient number of intermediate calculations, so the line of thought is clear.



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Optimization Throny and Studenstir Protesses

Wednesday June 10b 2011

M. 8.30 - 11.30 (3 hours)

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Exercise 1: (10 %)

a) Find stationary points for the following function:

$$f(\mathbf{x}) = -2 \cdot x_1^2 + 3 \cdot x_1 \cdot x_2 - 2 \cdot x_2^2 + 2 \tag{1}$$

b) Determine the local minimum, local maximum, or inflection (saddle) points for the function.

LIECTURE 2 (4.22...)

Exercise 2: (20 %)

We will consider gradient based minimization of the following unconstrained function:

~ LECTUPE 3 (10.57..)

$$f(\mathbf{x}) = 2 \cdot x_1^2 + x_2^2 + 2 \cdot x_1 \cdot x_2 - 4 \cdot x_2 \tag{2}$$

The starting point is: $\mathbf{x}^{(0)} = [1 \ 1]^T$.

- a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.
- b) Determine the search direction for the first iteration of the modified Newton's method for the function.

Exercise 3: (10 %)

Solve the following linear optimisation problem using the basic steps of the Simplex method and tableau's:

$$minimise f(\mathbf{x}) = -3x_1 + 2x_2 (3)$$

Subject to the constraints:

$$g_{1}(\mathbf{x}) = -\frac{x_{1}}{2} + x_{2} \leq 2$$

$$g_{2}(\mathbf{x}) = x_{1} + x_{2} \leq 3$$

$$x_{i} \geq 0 \quad \forall \quad x_{i} = \{1, 2\}$$

$$(4)$$

Exercise 4: (10 %)

A multi-objective optimisation problem is formulated as:

minimise
$$f_1(\mathbf{x}) = (x_1 - 3)^2 + (x_2 - 2)^2$$

 $f_2(\mathbf{x}) = (x_1 - 6)^2 + (x_2 - 5)^2$ (5)

The two contour curves are shown in figure 1.

- a) Illustrate the Pareto optimal points in figure 1 (the page should be handed in with the solution).
- b) Sketch the Pareto front in the criterion space. The sketch may be based on function values from the contour plot. A coordinate system may be found in figure 2.

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Exercise 25 (20 5%)

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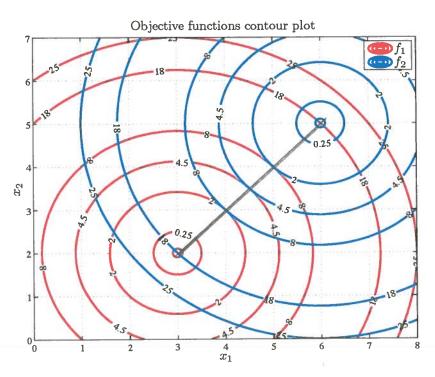


Figure 1: Contour curves for the problem of exercise 4.

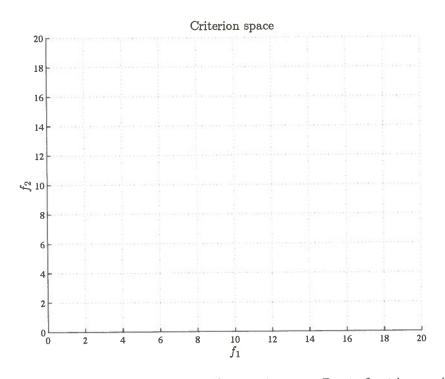
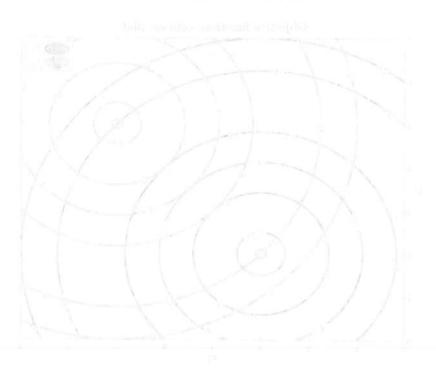


Figure 2: Coordinate system for plotting the criterion space Pareto front in exercise 4.

Page to be handed in with the solution!



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Exercise 5: Linear systems and ARMA(1,1) process (30 %)

Let us consider the following linear system:

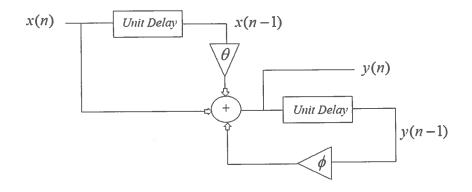


Figure 3: System for exercise 5.

the input-output relationship of which is given by:

$$y(n) = \phi y(n-1) + x(n) + \theta x(n-1)$$
 (6)

- a) Derive the impulse response h(n) of the linear system for initialization x(n) = y(n) = 0, n < 0.
- b) Determine the range of values of ϕ and θ for which the linear system is stable.

From now on, we assume that $\phi = \theta = 0.5$.

c) Derive the transfer function H(f) of the linear system.

Let us now assume that the input sequence is a white noise sequence $\{X(n)\}$ with unit variance, i.e. E[X(n)] = 0, $R_{XX}(k) = [X(n)X(n+k)] = \sigma_X^2 \delta(k)$ with $\sigma_X^2 = 1$.

- d) Show that $\{Y(n)\}$ is wide-sense stationary only if E[Y(n)] = 0 for any n.
- e) Derive the power spectrum $S_{YY}(f)$ of $\{Y(n)\}$.
- f) What is the interpretation of surface under graph of $S_{YY}(f)$, i. e. the quantity $\int_{-0.5}^{+0.5} S_{YY}(f) df$.

Notice: you do not have to calculate this integral, but only specify the manner it can be interpreted.

Work assemble in University come and ARMAGLI process (30 %).

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Exercise 6: Detection of a Gaussian random signal in background noise (20 %)

Detection of a Gaussian random signal in background noise:

 H_0 : Only Gaussian noise W is present

 H_1 : A random Gaussian signal X plus Gaussian noise W is present.

More specifically the received signal under both hypotheses reads:

$$H_0: Y = W \tag{7}$$

$$H_1: Y = X + W \tag{8}$$

Where

A. W is a zero-mean Gaussian random variable with variance $\sigma_W^2 = 1$, i.e. $W \sim N(0,1)$.

B. X is a zero-mean Gaussian random variable with σ_X^2 , i.e. $X \sim N(0, \sigma_X^2)$.

We further assume that X and W are independent. The signal to noise ratio is:

$$\eta = \left(\frac{\sigma_X}{\sigma_W}\right)^2 = 3\tag{9}$$

a) Find the probability density function of Y under both hypotheses, i. e. $f(y|H_0)$ and $f(y|H_1)$.

Hint: Notice that the variance of the sum of two independent random variables equals to the sum of their individual variances.

b) Show that $L(y) = \frac{f(y|H_1)}{f(y|H_0)}$ is given by:

$$L(y) = \frac{1}{\sqrt{1+\eta}} \exp\left(\frac{1}{2} \frac{\eta}{(1+\eta)} \left| \frac{y}{\sigma_W} \right|^2\right) = \frac{1}{2} \exp\left(\frac{3}{8} |y|^2\right)$$
(10)

c) Calculate the log-likelihood function: $l(y) = \ln \left(\frac{f(Y|H_1)}{f(Y|H_0)} \right)$

d) Find the MAP decision rule for the following a priori probability distributions: $P[H_0] = P[H_1] = \frac{1}{2}$.

Occurse & Describer of a Gaussian random signal in its Aground notes (2)

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EXAM 2011

Exercise 1:

a) Find stationary points for the following function:

- b) Octerrano the local minimum, local maximum or inflection (saddle) points.
- a) we need to compute the gradient:

$$\nabla f(x) = C(x) = \begin{bmatrix} -4 \times 1 + 3 \times 1 \\ -4 \times 2 + 3 \times 1 \end{bmatrix}$$

Stationary points or
$$C(x)=0$$
 or $\begin{bmatrix} -4x_2+3x_2\\ -4x_2+3x_1 \end{bmatrix} = \begin{bmatrix} 0\\ 0 \end{bmatrix}$

this yields:
$$x^* = \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
 $x^* = (0,0)$

the temption has I stationary point.

To determine the local minimum maximum and inflection, the H(x)= [3 -4] Hessian is readed

And the eigen-volue should be calculated:

Both eigh relies are vegative, which means the Hessian is Negative Definite

Los So then, we have a COCAL MAKIMUM. How to know if it's global

Exercise 7:

we will consider gradient based minimization of the following: $f(x) = 2x_1^2 + x_2^2 + 2 \cdot x_1 \cdot x_2 - 4x_2$

Start point is x'01=[1 1]T.

- a) (amplete the 1st iteration of the steepest descent method for the function. the 1D live search problem should be solved analytically.
- b) Determine the search direction for the 1st iteration of the modified howton's nethod.

The gradient:
$$\nabla f(x) = c(x) = \begin{bmatrix} u \times 1 + 2 \times 2 \\ 2 \times 2 + 2 \times 1 - 4 \end{bmatrix}$$

the direction of the steepest descent can be computed os:

$$d^{(0)} = -C(X^{0}) = -C^{(0)} = \begin{bmatrix} -6 \\ -6 \end{bmatrix}$$
 $-6 d^{(0)} = (-6,0)$

Next step is to compute the step size, having $x^{(1)} = x^{*} + a \cdot d^{(0)}$ $f(\alpha) = f(x^{(0)} + a \cdot d^{(0)}) = \text{where } x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } d^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$f(\alpha) = 2 - (1 - 6\alpha)^2 + 1^2 + 2 \cdot 1 \cdot (1 - 6\alpha) - 4 \cdot 1$$

$$f(\alpha) = 2 \cdot (1 + 36\alpha^2 - 12\alpha) + 1 + 2 - 12\alpha - 4 = 72\alpha^2 - 36\alpha + 1$$

New dosign, $X^{(1)} = X^{(0)} + a \cdot d^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.25 \begin{bmatrix} -6 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}$ $X^{(1)} = (-0.5, 1)$

which gives the following cost territion value: [==-3,5]

EXAM 2011

Exercise 2 (continuation):

b) Search direction - Modified Newton's method.

To determine search direction with modified Newton's method:

Using the previous gradient,
$$C(x) = \begin{bmatrix} 4x_1 + 2x_2 \\ 2x_2 + 2x_1 - 4 \end{bmatrix}$$

and at x° , $C^{\circ} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$ $f^{\circ} = 1$. $f^{\circ} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

We need to calculate the Hessian:

$$H(x) = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} - 0 + 1 = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

And the eigen values of H(x) are:

the search direction is then,

$$d^{(0)} = -H^{-1} \cdot C^{(0)} = -\begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} d^{(0)} = (-3, 3) \\ 0 \end{bmatrix}$$

the derient condition is checked, $c^{(0)} \cdot d^{(0)} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 3 \end{bmatrix} = \begin{bmatrix} -18 \\ 0 \end{bmatrix} = -18 < 0 \text{ percent}!$

Exercise 3:

Solve the following linear optimisation problem using the losic steps of the Simplex nethod and tableau's:

$$g_3(x) = \frac{x_1}{2} + x_1 \le 2 + x_3$$

 $g_2(x) = x_1 + x_2 \le 3 + x_4$
 $g_3(x) = -x_2 \le 0$
 $g_4(x) = -x_2 \le 0$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \quad C = \begin{bmatrix} -3 \\ Z \end{bmatrix}$$

EXAM 2011

Exercise 4: A multi objective optimisation problem is formulated as:

Minimise $f_1(X_1, X_2) = (X_1 - 3)^2 + (X_2 - 2)^2$ $f_2(X_1, X_2) = (X_1 - 6)^2 + (X_2 - 5)^2$

The 2 contour curves are shown in Figure 2 (attached paper) a) Illustrate the bueto optimal points in Figure 1.

b) Shetch the Pareto Front in the criterion space. The sketch may be based on femcion rollies from the countour plot.