2013 opg 2

Exercise 1: (10 %)

The following optimisation problem is considered:

$$\begin{aligned} \text{Minimise} \quad f(\mathbf{x}) &= (2-x_1)^2 + (x_2+1)^2 \\ \text{Subject to} \quad h(\mathbf{x}) &= -x_1 - x_2 + 4 = 0 \end{aligned} \tag{1}$$

a) Set up the Lagrangian function and find point(s) satisfying the KKT necessary conditions.

b) Check if the point(s) is an optimum point using the graphical method (make a simple sketch).

Exercise 2: (15 %)

We will consider gradient-based minimisation of the following unconstrained function:

$$f(\mathbf{x}) = 10(x_1^2 - x_2) + x_1^2 - 2x_1 + 5 \tag{2}$$

The starting point is: $\mathbf{x}^{(0)} = [\ -1 \quad 3\ ]^T$.

a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.

b) Can Newton's method be applied for determining the search direction in iteration 1?
If yes, then determine the search direction.
If no, then state an alternative robust method for determining the search direction.

Exercise 3: (18 %)

An optimisation problem is given as:

$$\text{minimise} \qquad f(\mathbf{x}) = (x_1 - 1)^2 + 2\,(x_2 - 1)^2 - x_1 x_2 \tag{3}$$

Subject to the constraints:

$$\begin{aligned} g_1(\mathbf{x}) &= 5 - x_1 - x_2 \leq 0 \\ g_2(\mathbf{x}) &= x_1^2 - x_2 - 36 \leq 0 \\ g_3(\mathbf{x}) &= \tfrac{x_1^2}{4} - x_1 + x_2 - 8 \leq 0 \\ g_4(\mathbf{x}) &= -x_1 \leq 0 \\ g_5(\mathbf{x}) &= -x_2 \leq 0 \end{aligned} \tag{4}$$

a) Complete one iteration of the Sequential Linear Programming (SLP) method for the above problem, where you solve step 4 graphically. Use a starting point of $\mathbf{x}^{(0)} = (5,5)$ and 20% move limits. As a help the contour plot of the linearised objective function for $\mathbf{x}^{(0)} = (5,5)$ is shown in figure . The page may be handed in with the solution.

b) Describe in words, if there are any limitations in using the SLP-method and/or if there are any type of problems, for which the SLP-method is unsuited.

Optimisation Theory - Exam B

3 - An optimisation problem:

Minimise $f(x) = (x_1 - 1)^2 + 2(x_2 - 1)^2 - x_1 x_2$

Subject $g_1(x) = 5 - x_1 - x_2 \leq 0$

$g_2(x) = x_1^2 - x_2 - 36 \leq 0$

$g_3(x) = \frac{x_1^2}{4} - x_1 + x_2 - 8 \leq 0$

$g_4(x) = -x_1 \leq 0$

$g_5(x) = -x_2 \leq 0$

- One iteration of the Sequential Linear programming (SLP) method:

- Starting Point

$$x^{(0)} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

Book, p. 508 - Evaluate Cost and constraint problem:

$f(x^{(0)}) = f(5,5) = (5-1)^2 + 2(5-1)^2 - 5 \cdot 5 = 23$

$g_1(x^{(0)}) = g(5,5) = 5 - 5 - 5 = -5 \leq 0$ (inactive)

$g_2(x^{(0)}) = = 5^2 - 5 - 36 = -16 \leq 0$ (inactive)

$g_3(x^{(0)}) = = \frac{5^2}{4} - 5 + 5 - 8 = -1{,}75 \leq 0$ (inactive)

$g_4(x^{(0)}) = = -5 \leq 0$ (inactive)

$g_5(x^{(0)}) = = -5 \leq 0$ (inactive)

All constraints are inactive

↳ Within the feasiable region.

Optimisation Theory

- Gradients:

$$c = \nabla f = \begin{bmatrix} 2x_1 - x_2 - 2 \\ -x_1 + 4x_2 - 4 \end{bmatrix} \Rightarrow c^{(0)} = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$$

$$\nabla g_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\nabla g_2 = \begin{bmatrix} 2x_1 \\ -1 \end{bmatrix} \Rightarrow \nabla g_2^{(0)} = \begin{bmatrix} 10 \\ -1 \end{bmatrix}$$

$$\nabla g_3 = \begin{bmatrix} \frac{1}{2}x_1 - 1 \\ 1 \end{bmatrix} \Rightarrow \nabla g_3^{(0)} = \begin{bmatrix} 1{,}5 \\ 1 \end{bmatrix}$$

$$\nabla g_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\nabla g_5 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

- Move limits, specified to 20%

x_1: $-0{,}2 \cdot 5 \leq d_1 \leq 0{,}2 \cdot 5 \Rightarrow -1 \leq d_1 \leq 1$

x_2: $-0{,}2 \cdot 5 \leq d_2 \leq 0{,}2 \cdot 5 \Rightarrow -1 \leq d_2 \leq 1$

- Linearised sub-problem

minimise $f = c^T d \Rightarrow \begin{bmatrix} 0 & 11 \end{bmatrix}^T \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

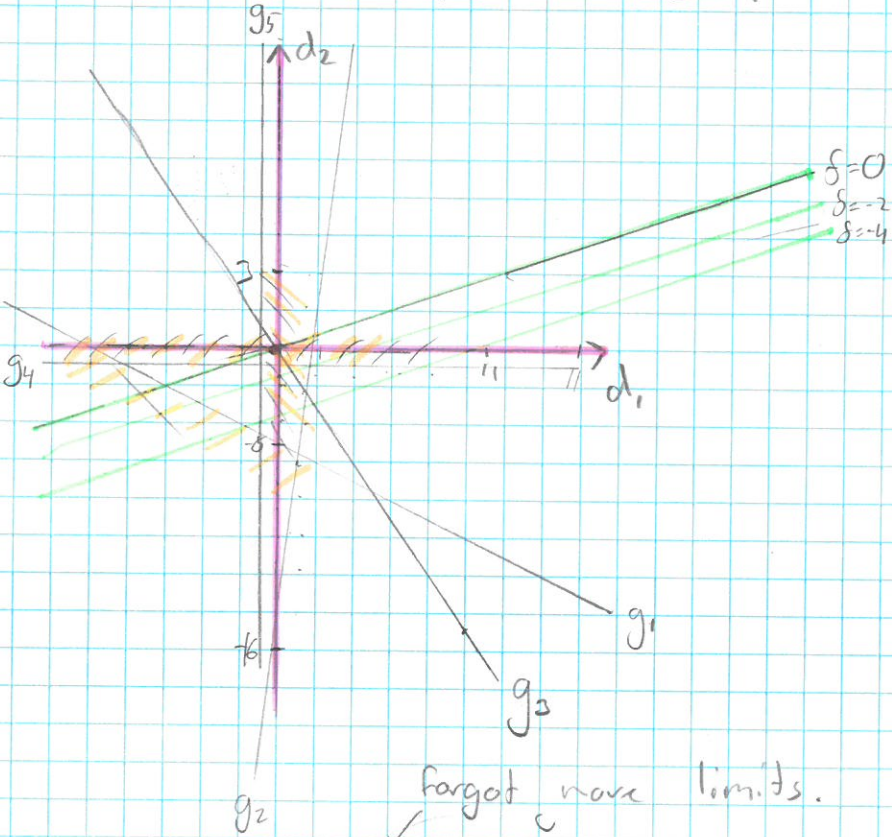
Subject to $A^T d \leq b$

$$A = \begin{bmatrix} -1 & 10 & 1{,}5 & -1 & 0 \\ -1 & -1 & 1 & 0 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} 5 \\ 16 \\ 1{,}75 \\ 5 \\ 5 \end{bmatrix}, \quad d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

Optimisation Theory - Exam 3

- Solve problem graphically:



$f = 3d_1 + 11 d_2$

$d_2 = \frac{3}{11} d_1$

$g_1 = -d_1 - d_2 \leq 5$

$d_2 = -d_1 - 5$

$g_2 = 10 d_1 - d_2 \leq 16$

$d_2 = 10 d_1 - 16$

$g_3 = 1{,}5 d_1 + d_2$

$d_2 = -1{,}5 d_1$

forgot move limits.

- $\cancel{d = \begin{bmatrix} -1 \\ -4{,}5 \end{bmatrix}}$, by Matlab: $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

- Check for convergence:

$$\|d\| = \sqrt{(-1)^2 + (-4{,}5)^2} > \varepsilon \rightarrow \text{Not converges}$$

- Calculate the new design variables

$$x^{(1)} = x^{(0)} + d$$

$$\cancel{= \begin{bmatrix} 5 \\ 5 \end{bmatrix} + \begin{bmatrix} -1 \\ -4{,}5 \end{bmatrix} = \begin{bmatrix} 4 \\ 0{,}5 \end{bmatrix}}, = \begin{bmatrix} 5 \\ 5 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

It will work in most cases, but as no descent condition is defined, it cannot be seen how progress is. Thus, it can yield cycling between points, so it is important to check the move-limit → Not constant for iterations.