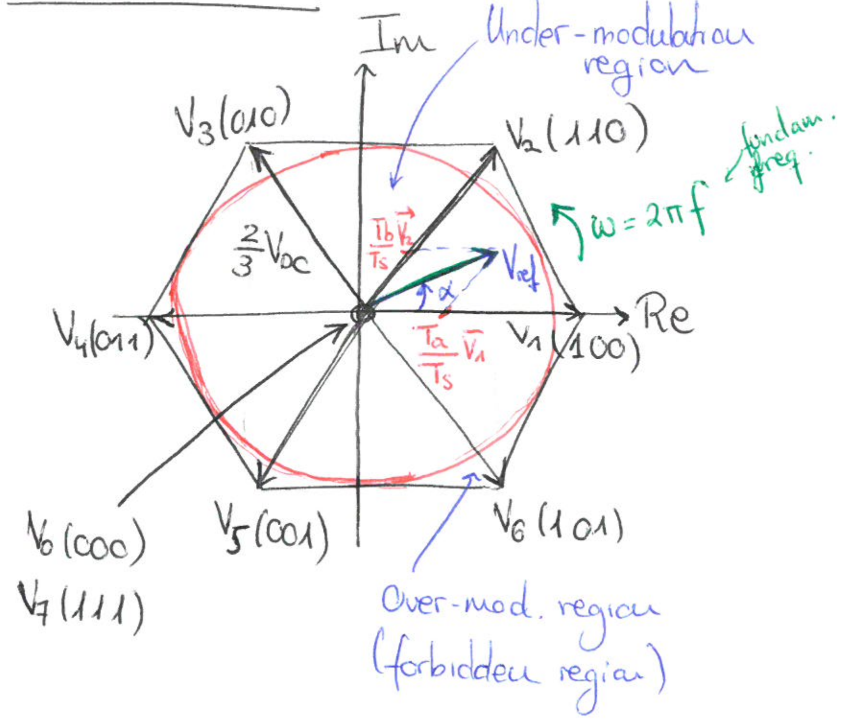
SVPWM

"Matlab/Simulink Implementation..."

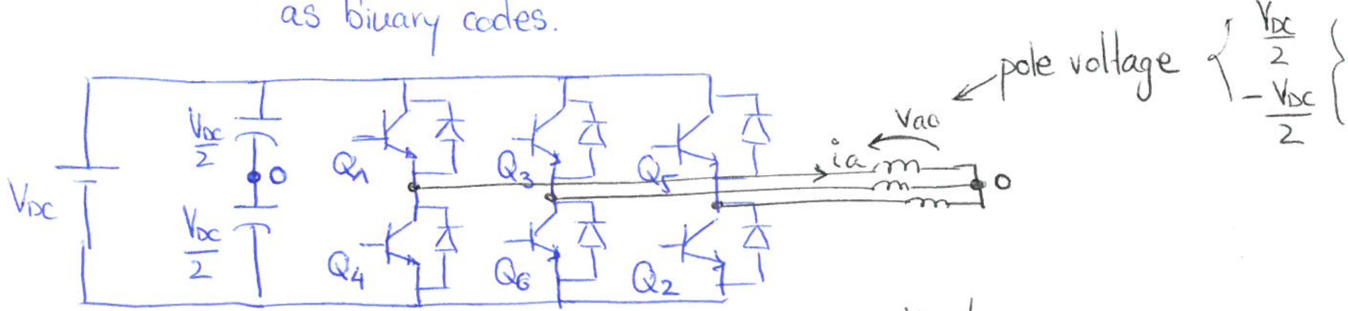


* Space vector:

(1) $\vec{V}(t) = \frac{2}{3}\left[V_a(t)e^{j0} + V_b(t)e^{j\frac{2\pi}{3}} + V_c(t)e^{-j\frac{2\pi}{3}}\right]$

It maintains its magnitude at any given time. As time increases the angle of the space vector (α) increases, causing the vector to rotate with frequency equal to fundamental freq.

* VSI → 8 switching states { Six active states (1-6); Two zero states (0 and 7) }
  ↳ Represented as binary codes.



Example: Q_1, Q_6 and Q_2 closed → $\begin{cases} V_{ao} = V_{DC}/2 \\ V_{bo} = -V_{DC}/2 \\ V_{co} = -V_{DC}/2 \end{cases}$

State denoted as 100 and according to (1) → $\vec{V}(t) = \frac{2}{3}\cdot\left[V_{DC}e^{j0}\right]$

In each switching state the vector identification uses a '0' to represent the negative phase voltage and a '1' the positive.

$$\vec{V}_K = \begin{cases} \frac{2}{3}V_{DC}\cdot e^{j(k-1)\frac{\pi}{3}} & \text{if } K=1,2,3,4,5,6 \\ 0 & \text{if } K=0,7 \end{cases}$$

Space vector	Sw. state	On-state sw.	Vector definition
V_0	000	S4, S6, S2	$\vec{V_0} = 0$
V_1	100	S1, S6, S2	$\vec{V_1} = 2/3\, V_{DC}\, e^{j0}$
V_2	110	S1, S3, S2	$\vec{V_2} = 2/3\, V_{DC}\, e^{j\pi/3}$
V_3	010	S4, S3, S2	$\vec{V_3} = 2/3\, V_{DC}\, e^{j2\pi/3}$
V_4	011	S4, S3, S5	$\vec{V_4} = 2/3\, V_{DC}\, e^{j\pi}$
V_5	001	S4, S6, S5	$\vec{V_5} = 2/3\, V_{DC}\, e^{j4\pi/3}$
V_6	101	S1, S6, S5	$\vec{V_6} = 2/3 \cdot V_{DC}\, e^{j5\pi/3}$
V_7	111	S1, S3, S5	$\vec{V_7} = 2/3\, V_{DC} \cdot e^{j2\pi}$

1) Calculate angle θ and $|\vec{V}_{ref}|$ based on input voltage components.

2) Calculate modulation index and determine if it's in overmodulation region

3) Find sector in wich $\vec{V}_{ref}$ lies, and the adjacent space vectors $\vec{V}_k$ and $\vec{V}_{k+1}$ based on sector angle θ.

4) Find T_a, T_b and T_c based on T_s and θ.

5) Determine modulation times for the different switching states.

$\boxed{1}$ In SVPWM, the 3-ph output voltage vector is represented by a reference vector that rotates at an angular velocity of $\omega = 2\pi f$.

$$\begin{cases} V_a(t) = V_{ref}\cos(\omega t) \\ V_b(t) = V_{ref}\cos(\omega t - 2\pi/3) \\ V_c(t) = V_{ref}\cos(\omega t + 2\pi/3) \\ V_a(t) + V_b(t) + V_c(t) = 0 \end{cases}$$

The 3-ph voltages can be represented in a two-dimensional $\alpha\beta$ ref-frame. The magnitude of each active vector is $\frac{2}{3}V_{DC}$ and they are $60°$ apart.

$$\vec{V}_{ref} = \frac{2}{3}\left(V_a + V_b e^{j2\pi/3} + V_c e^{-j2\pi/3}\right)$$

The magnitude of the reference vector is:

$$|\vec{V}_{ref}| = \sqrt{V_\alpha^2 + V_\beta^2} \quad ; \quad \theta = \tan^{-1}\left(\frac{V_\beta}{V_\alpha}\right)$$

$\boxed{2}$ Linear region $\rightarrow$ $\vec{V}_{ref}$ always remains within the hexagon.

Applying Fourier analysis the fundamental voltage amplitude is:

$$V_{max-sixstep} = \frac{4}{\pi}\left[\int_0^{\pi/3} \frac{V_{dc}}{3}\sin\theta\, d\theta + \int_{\pi/3}^{\pi/2} \frac{2}{3}V_{DC}\sin\theta\, d\theta\right] = \frac{2V_{DC}}{\pi}$$

The ratio between $\vec{V}_{ref}$ and the fundamental peak value of the square phase voltage wave $\left(\frac{2V_{DC}}{\pi}\right)$ is called modulation index. This MI indicates the mode of operation $\rightarrow$ $\boxed{MI = \frac{\vec{V}_{ref}}{V_{max-sixstep}}}$

The max. modulation index is obtained when $\overline{V}_{ref}$ equals the radius of the inscribed circle:

$$MI_{max} = \frac{\frac{2}{3}V_{DC}\cdot\cos(\pi/6)}{\frac{2V_{DC}}{\pi}} \simeq 0{,}907$$

3 Necessary to determine the sector in which $\vec{V}_{ref}$ lies in order to determine switching time and sequence.
Depending on V_α and V_β, the angle of the reference vector is used to determine the sector.

Sector	Degrees
1	$0 < \theta \leq 60°$
2	$60° < \theta \leq 120°$
3	$120° < \theta \leq 180°$
4	$180° < \theta \leq 240°$
5	$240° < \theta \leq 300°$
6	$300° < \theta \leq 360°$

4 Ta, Tb and To

Modulation vector $\vec{V}_{ref}$ is mapped onto two adjacent vectors:

$$\vec{V}_K = \frac{2}{3} V_{DC} \cdot e^{j(K-1)\frac{\pi}{3}} = \frac{2}{3} V_{DC} \cdot \left[\cos(K-1)\frac{\pi}{3} + j\sin(K-1)\frac{\pi}{3}\right]$$

$$\vec{V}_{K+1} = \frac{2}{3} V_{DC} \cdot e^{jK\pi/3} = \frac{2}{3} V_{DC} \cdot \left[\cos\frac{K\pi}{3} + j\sin\frac{K\pi}{3}\right]$$

Due to symmetry in the patterns in the six sectors the integration can be carried out only half of the PWM period $\left(\frac{T_S}{2}\right)$:

$$\int_0^{T_S/2} \vec{V}_{ref}\, dt = \int_0^{T_0/4} \vec{V}_0\, dt + \int_{T_0/4}^{T_0/4+T_a} \vec{V}_K \cdot dt + \int_{T_0/4+T_a}^{T_0/4+T_a+T_b} \vec{V}_{K+1}\, dt + \int_{T_0/4+T_a+T_b}^{T_S/2} \vec{V}_7\, dt$$

$$\downarrow \quad \int_0^{T_0/4} \vec{V}_0\, dt = \int_{T_0/4+T_a+T_b}^{T_S/2} \vec{V}_7\, dt = 0 \leftarrow$$ Zero voltages at null states!

$$\begin{cases} \vec{V}_{ref}\dfrac{T_S}{2} = \vec{V}_K \cdot T_a + \vec{V}_{K+1} T_b \\ \vec{V}_{ref} = V_\alpha + jV_\beta \end{cases}$$

Ta and Tb denote the required on-time of $\vec{V}_K$ and $\vec{V}_{K+1}$ during each sample period. K is the sector number denoting $\vec{V}_{ref}$ location.

$$\downarrow$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \frac{T_S}{2} = \frac{2V_{DC}}{3} \left(T_a \begin{bmatrix} \cos\frac{(K-1)\pi}{3} \\ \sin\frac{(K-1)\pi}{3} \end{bmatrix} + T_b \cdot \begin{bmatrix} \cos\frac{K\pi}{3} \\ \sin\frac{K\pi}{3} \end{bmatrix} \right) \longrightarrow$$

$$\begin{bmatrix} T_a \\ T_b \end{bmatrix} = \frac{\sqrt{3}\,T_s}{2V_{DC}} \begin{bmatrix} \sin k\pi/3 & -\cos\frac{k\pi}{3} \\ -\sin\frac{(k-1)\pi}{3} & \cos\frac{(k-1)\pi}{3} \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$$

$$\downarrow \quad \vec{V}_{ref} = |\vec{V}_{ref}|\, e^{j\omega t} = |\vec{V}_{ref}|(\cos\omega t + j\sin\omega t)$$

$$\begin{bmatrix} T_a \\ T_b \end{bmatrix} = \frac{\sqrt{3}\,T_s \cdot |\vec{V}_{ref}|}{2\,V_{DC}} \cdot \underbrace{\begin{bmatrix} \sin k\pi/3 & -\cos k\pi/3 \\ -\sin\frac{(k-1)\pi}{3} & \cos\frac{(k-1)\pi}{3} \end{bmatrix}}_{*} \begin{bmatrix} \cos n\omega T_s \\ \sin n\omega T_s \end{bmatrix}$$

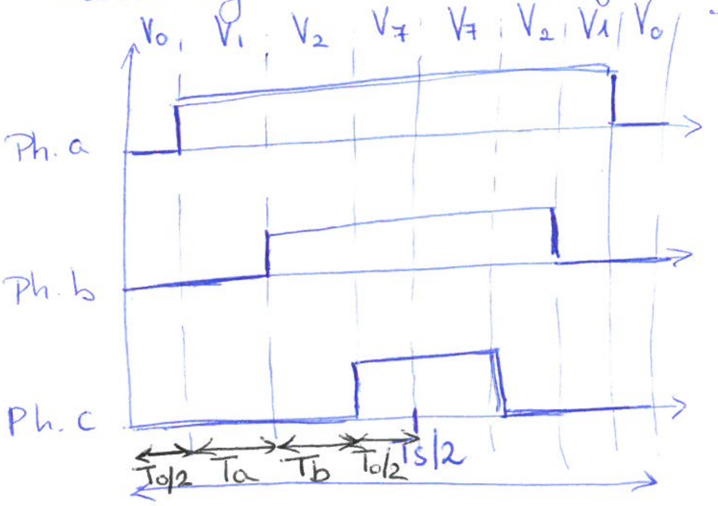
$$\downarrow \quad MI = \frac{|\vec{V}_{ref}|}{\frac{2V_{DC}}{\pi}} \; ; \; |\vec{V}_{ref}| = \frac{MI\, 2V_{DC}}{\pi}$$

$$\begin{bmatrix} T_a \\ T_b \end{bmatrix} = \frac{MI\sqrt{3}\,T_s}{\pi} \begin{bmatrix} & * & \end{bmatrix} \begin{bmatrix} \cos n\omega T_s \\ \sin n\omega T_s \end{bmatrix} \quad (*)$$

Since the sum of $2T_a$ and $2T_b$ (due to symmetry) should be less than or equal to T_s, the inverter has to stay in the zero stage for the rest of the period:

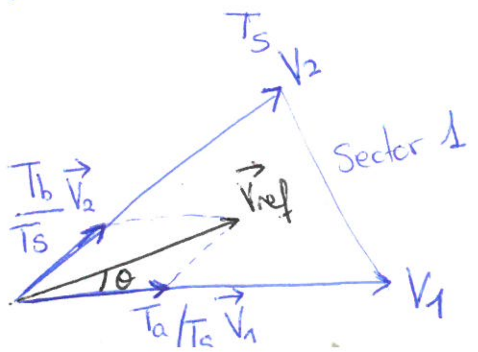
$$T_s = T_o + 2(T_a + T_b) \longrightarrow \boxed{T_o = T_s - 2(T_a + T_b)}$$

Switching times are arranged symmetrical around the center of T_s.



$\vec{V}_7$ is placed at the center of T_s and the $\vec{V}_0$ at the start and end and the total period for a zero vector is divided equally among the two zero vectors.

The symmetrical PWM signal is preferred because it has the lowest harmonic distortion (THD).



(*) $$T_a = \frac{\sqrt{3}\cdot MI\cdot T_s}{\pi} \cdot \sin\left(k\pi/3 - \overbrace{n\omega T_s}^{\theta}\right)$$

$$T_b = \frac{\sqrt{3}\cdot MI\cdot T_s}{\pi} \cdot \sin\Big(\underbrace{\underbrace{n\omega T_s}_{\theta} - \frac{(k-1)\pi}{3}}_{\beta}\Big)$$

sector

5 Determination of Switching times for each transistor switch

→ Necessary to arrange switching sequence so that f_s of each inverter leg is minimized.
Two minimize switch. losses, only 2 adjacent active vectors and two zero vectors are used in a sector. This means that each switching period starts withe one zero vector and ends with one zero vector.

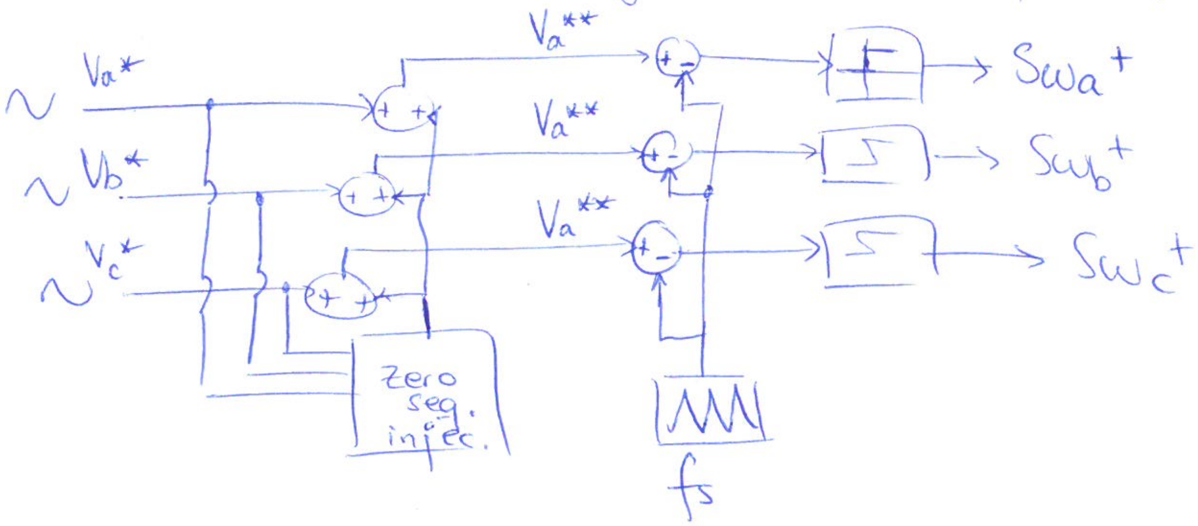
→ Therefore, the switching cycle of the output voltage is double the sampling time, and the two output voltage waveforms become symmetrical

→ In order to reduce the swith. loss it's required that at each time only one bridge arm is switched.

Sector	Switching seq.
1	$V_0 - V_1 - V_2 - V_7 - V_2 - V_1 - V_0$
2	$V_0 - V_3 - V_2 - V_7 - V_2 - V_3 - V_0$
3	$V_0 - V_3 - V_4 - V_7 - V_4 - V_3 - V_0$
4	$V_0 - V_5 - V_4 - V_7 - V_4 - V_5 - V_0$
5	$V_0 - V_5 - V_6 - V_7 - V_6 - V_5 - V_0$
6	$V_0 - V_1 - V_6 - V_7 - V_6 - V_1 - V_0$

Modulation

* Zero sequence injection (chapter 2. pg 32 Rik.) "Simple analytical ... (paper)"



- Improve waveform quality
- Reduce switching losses
- It doesn't affect inverter's L-L voltage but influences switching freq. characteristics.

fundamental freq.

$$V_a^{**} = V_a^* + V_o = V_{1m}^* \cos(w_e t) + V_o$$

$$V_b^{**} = V_b^* + V_o = V_{1m}^* \cos\left(w_e t - \frac{2\pi}{3}\right) + V_o$$

$$V_c^{**} = V_c^* + V_o = V_{1m}^* \cos\left(w_e t + \frac{2\pi}{3}\right) + V_o$$

↳ $\boxed{\text{SVPWM}}$ → Zero seq. calculation using minimum magnitude test.

Compares magnitudes of V_a^*, V_b^* and V_c^* and selects signal with minimum magnitude. Scaling this signal by $\frac{1}{2}$ the zero sequence signal is found.

Ex. $|V_a^*| \leq |V_b^*|, |V_c^*| \longrightarrow V_o = \frac{1}{2} \cdot V_a^*$

* According to Rik:

$$U_0^* = -\frac{1}{2}\left[\max\{U_a^*, U_b^*, U_c^*\} + \min\{U_a^*, U_b^*, U_c^*\}\right]$$

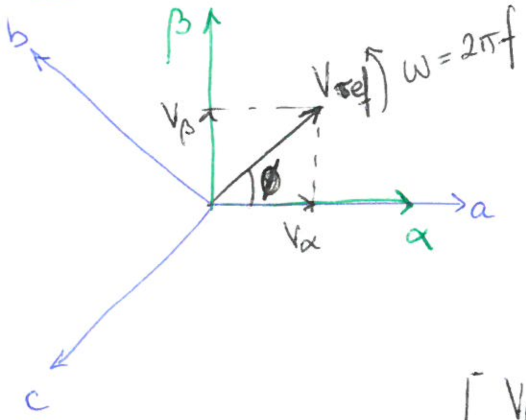
- This approach, known as pulse centering unit, SYMMETRIZES the max and min average voltage references with respect to the time axis

- Possible to increase the space vector amplitude from $|U^*| = \sqrt{\frac{3}{8}}\, U_{DC}$ to $|\bar{U}^*| = \sqrt{\frac{1}{2}}\, V_{DC}$ $\rightarrow$ increase $\simeq 15\%$

$$U^* = \sqrt{\frac{2}{3}}\left(U_a^* + U_b^* e^{j120°} + U_c^* e^{j240°}\right) - \sqrt{\frac{2}{3}}\, U_0 \cdot \underbrace{\left(1 + e^{j120°} + e^{j240°}\right)}_{0}$$

- Pulse centering unit important to maximize the linear operating region of the inverter. ($\simeq 15\%$) allowing the inverter to be operated with higher ~~reference~~ voltages and consequently lower currents.

SVPWM



$$\begin{cases} V_{AN} = V_m \cdot \sin(2\pi f t) \\ V_{BN} = V_m \cdot \sin(2\pi f t - 2\pi/3) \\ V_{CN} = V_m \cdot \sin(2\pi f t + 2\pi/3) \end{cases}$$

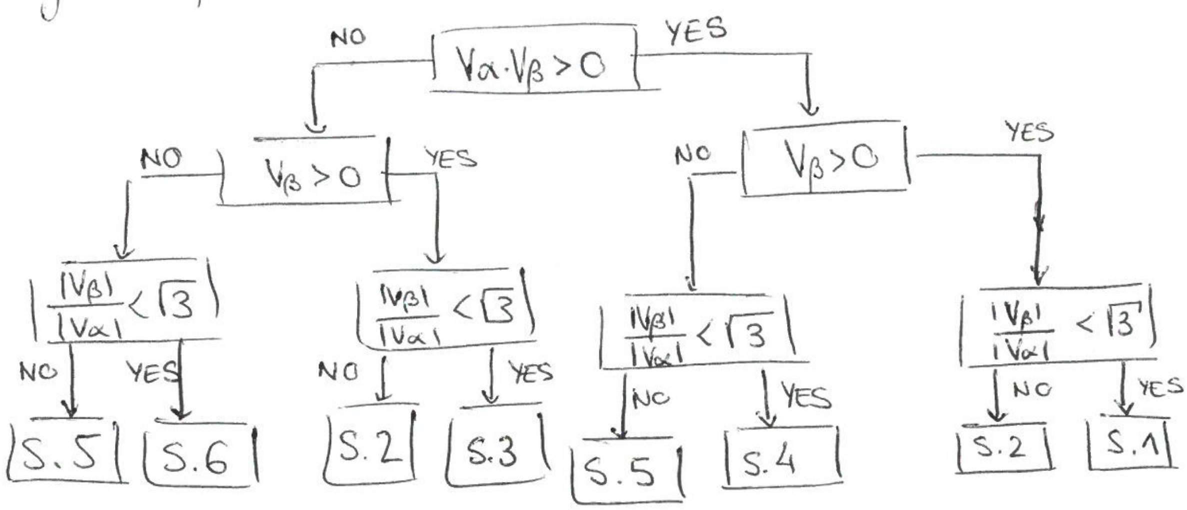
Controlled according to the rotation of the space vector Vref.

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_{AO} \\ V_{BO} \\ V_{CO} \end{bmatrix}$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix}$$

$$\boxed{V_{ref} = \sqrt{V_\alpha^2 + V_\beta^2}} \quad ; \quad \boxed{\phi = \tan^{-1}\left(\frac{V_\beta}{V_\alpha}\right)}$$

* Algorithm for sector determination:



* $V_{ref} = \frac{T_K}{T_S} \cdot V_K + \frac{T_{K+1}}{T_S} \cdot V_{K+1}$, $K = 1, 2, \ldots, 6$ / $T_K + T_{K+1} \leq T_S$

$V_0 = 0$

$V_{ref} \cdot T_S = T_K \cdot V_K + T_{K+1} \cdot V_{K+1} + T_0 V_0$; $T_0 = T_S - T_K - T_{K+1}$

$$\begin{cases} \theta = \phi - \frac{k-1}{3}\pi \\ V_{ref} \cdot \sin\left(\frac{\pi}{3} - \theta\right) = \frac{T_K}{T_S} \cdot V_K \cdot \sin(\pi/3) \\ V_{ref} \cdot \sin(\theta) = \frac{T_{K+1}}{T_S} \cdot V_{K+1} \cdot \sin\left(\frac{\pi}{3}\right) \end{cases} \longrightarrow \begin{cases} T_K = \frac{\sqrt{3} \cdot T_S \cdot V_{ref}}{V_{DC}} \cdot \sin\left(\frac{k}{3}\pi - \phi\right) \\ T_{K+1} = \frac{\sqrt{3} \cdot T_S \cdot V_{ref}}{V_{DC}} \cdot \sin\left(\phi - \frac{k-1}{3}\pi\right) \end{cases}$$