2017

AALBORG UNIVERSITY
DENMARK

Written examination in the course

Optimisation Theory and

Modern Reliability from a Practical Approach

Friday June 9th 2017

kl. 9 - 13 (4 hours)

All usual helping aids are allowed, i.e. books, notes, calculator, computer etc. All communication equipment and computer communication protocols must be turned off.

The questions should be answered in English.

REMEMBER to write your study number and page number on all sheets handed in.

The set consists of eight exercises. The total weighting for each of the exercises is stated in percentage. You need 50 % in order to pass the exam.

It should be clear from the solution, which methods are used, and there should be a sufficient number of intermediate calculations, so the line of thought is clear.

Exercise 1: (10 %)

A company wants to maximize its day-to-day profit from manufacturing woodburning stoves. Formulate the profit maximization problem as a standard optimization problem based on the following information:

- The company produces two stoves: 1) Type A and 2) Type B.
- The company has a total of 5000 man hours available per day. It takes 20 man hours to produce a stove of Type A and 24 man hours to produce a stove of Type B.
- The daily cost of producing Type A is a fixed cost of 500 Euro and a further 300 Euro per stove. The daily cost of producing Type B is a fixed cost of 900 Euro and a further 400 Euro per stove.
- To maintain market shares the company needs to produce at least 50 stoves of Type A every day and at least 30 stoves of Type B every day.
- The company sells the stoves at the following prices: 500 Euro per stove of type A and 600 Euro per stove of type B.

Note: you should only formulate the problem, not solve it.

Exercise 2: (15 %)

We will consider gradient-based minimisation of the following unconstrained function:

$$f(\mathbf{x}) = (x_1 - 1)^2 + 2x_2^2 \qquad (1)$$

The starting point is: $\mathbf{x}^{(0)} = [\ 3 \quad 1\ ]^T$.

a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.

b) Complete the first iteration of the modified Newton's method for the function. The 1D line search problem should be solved analytically.

Exercise 3: (10 %)

Solve the following linear optimisation problem using the basic steps of the Simplex method and tableau's:

$$\text{minimise} \qquad f(\mathbf{x}) = -5x_1 - 2x_2$$

Subject to the constraints:

$$\begin{aligned} g_1(\mathbf{x}) &= -x_1 + x_2 \leq 10 \\ g_2(\mathbf{x}) &= 2x_1 - x_2 \leq 20 \\ & x_i \geq 0 \quad \forall \quad x_i = \{1, 2\} \end{aligned}$$

Exercise 4: (15 %)

The following multi-objective optimisation problem is considered:

$$\text{minimise} \qquad \begin{aligned} f_1(\mathbf{x}) &= (x_1 - 5)^2 + (x_2 - 15)^2 \\ f_2(\mathbf{x}) &= (x_1 - 15)^2 + (x_2 - 5)^2 \end{aligned}$$

Subject to the constraint:

$$g_1(\mathbf{x}) = x_1 \leq 10$$

Figure 1 (next page) shows the contour curves for the objective functions.

a) Draw the constraint in the design space (i.e. objective functions contour plot).

b) Draw the Pareto optimal points in the design space.

c) Sketch the Pareto front in the Criterion space (Figure 2 may be used).

d) Determine the objective function values of the utopia point.

e) Assume that the multi-objective problem is solved as single objective problem, $U(\boldsymbol{x})$, using the weighting method with $w_1 = 2$ and 1. Determine analytically the minimum objective function value $U(\mathbf{x}^*)$, and the optimum set of design variables $\mathbf{x}^*$.

f) Determine whether the found optimum is a global optimum or not - justify the answer.

Exercise 5: (14 %)

8 paper clips (no. 1, no. 2, ..., to no. 8) are tested to study the wear out behavior with a bending angle of 180°. Each test stops at a maximum of 60 times of bending regardless if the paper clip fails or not. The clips no. 7 and no. 8 still survive after 60 times of bending. The cycles to failure for clips no. 1 to no. 6 are recorded as: 15, 50, 40, 10, 30, and 60.

a) State the relationship between accumulated failure and time in Weibull distribution.

b) Arrange these time-to-failure numbers by using the median ranking method.

c) Plot the ranking number as listed in b) and the cycles to failure in the attached Weibull paper on next page. Please find out the values for β and η, respectively.

d) Explain how to identify the values of β and η in the Weibull plot.

Appendix I - Median rank table

sample size = n

failure rank = i

n

i	1	2	3	4	5	6	7	8	9	10
1	.5000	.2929	.2063	.1591	.1294	.1091	.0943	.0830	.0741	.0670
2		.7071	.5000	.3864	.3147	.2655	.2295	.2021	.1806	.1632
3			.7937	.6136	.5000	.4218	.3648	.3213	.2871	.2594
4				.8409	.6853	.5782	.5000	.4404	.3935	.3557
5					.8706	.7345	.6352	.5596	.5000	.4519
6						.8906	.7705	.6787	.6065	.5481
7							.9057	.7979	.7129	.6443
8								.9170	.8194	.7406
9									.9259	.8368
10										.9330

Exercise 6: (11 %)

The accelerated testing of a type of electronic device is performed under 4 different voltage stress levels. The four respective Weibull plots of the time-to-failure data based on median rank are shown in the figure below.

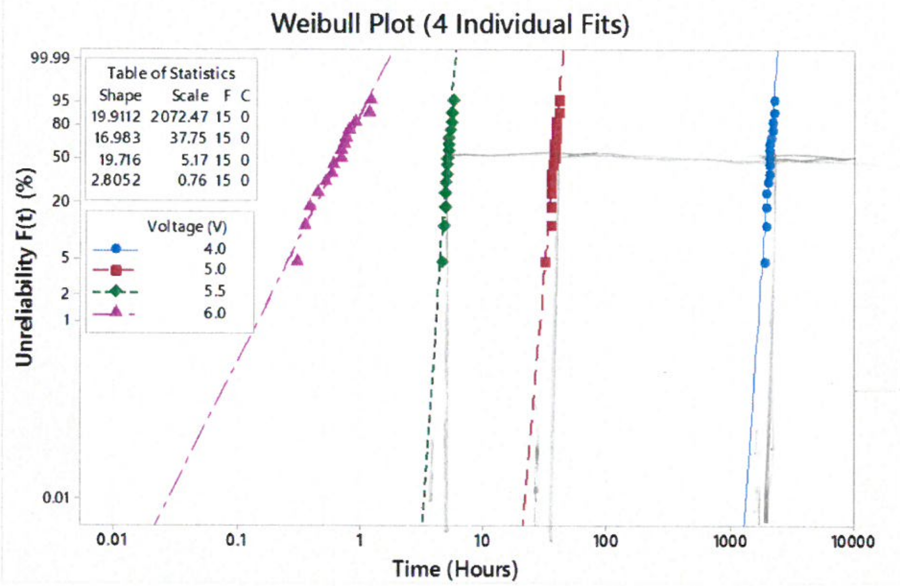

Figure 3: Weibull plots of a type of electronic device under four different voltage stress levels.

a) The slope of the Weibull curve of the results from 6 V stress level testing is significantly different from the other three curves. Please explain the possible reason.

b) Based on the information provided in the above figure, derive the relationships between B5 lifetime of the electronic device with 50% conference level and the voltage levels.

Exercise 7: (12 %)

Two components from two different manufacturers M1 and M2 are in "active redundancy" connection. Manufacturer M1 provides a Weibull reliability function with $\beta_1 = 0.5$ and $\eta_1 = 3000$ h. Manufacturer M2 provides an exponentially distributed reliability function with $\text{MTBF}_2 = 2500$ h.

a) Sketch up an RBD schematic of the overall system;

b) Calculate the reliability of each component after 2000 h;

c) Work out the overall reliability function.

Exercise 8: (13 %)

The 1-hour mission profile of a train shuttle is reported in the following figure, where the motor temperature is plotted against time. The same profile is repeated 20 hours per day, 7 days per week.

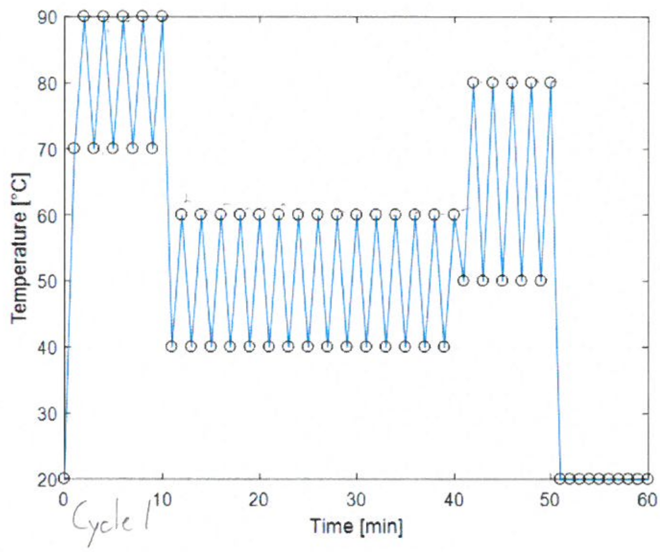

Figure 4: 1-hour mission profile of a train shuttle

The motor lifetime has been evaluated and modeled as Arrhenius lifetime model, i.e. taking into account temperature average (Tavg) and variation (Delta T), as shown in the following chart.

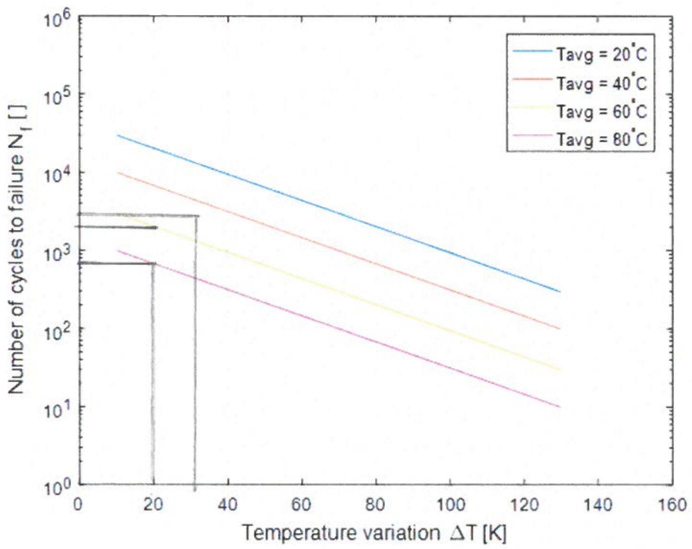

Figure 5: Arrhenius motor lifetime model

a) Identify and list in a table the three different stress levels (Tavg, DeltaT) occurring in the given mission profile;

b) Identify such levels as points in the Arrhenius chart and complete the above table with the number of cycles to failure Nf for each of them;

c) Calculate the accumulated damage (in %) within 1 hour;

d) Calculate the expected lifetime.