AALBORG UNIVERSITY
DENMARK

Written examination in the course

Optimisation Theory and

Modern Reliability from a Practical Approach

Friday June 10th 2016

kl. 9 - 13 (4 hours)

All usual helping aids are allowed, i.e. books, notes, calculator, computer etc. All communication equipment and computer communication protocols must be turned off.

The questions should be answered in English.

REMEMBER to write your study number and page number on all sheets handed in.

The set consists of eight exercises. The total weighting for each of the exercises is stated in percentage. You need 50 % in order to pass the exam.

It should be clear from the solution, which methods are used, and there should be a sufficient number of intermediate calculations, so the line of thought is clear.

Exercise 1: (10 %)

The following optimisation problem is considered:

$$\begin{aligned} \text{Minimise} \quad f(\mathbf{x}) &= (3 - x_1)^2 + (x_2 + 2)^2 \\ \text{Subject to} \quad h(\mathbf{x}) &= -x_1 - x_2 + 2 = 0 \end{aligned} \tag{1}$$

a) Set up the Lagrangian function and find point(s) satisfying the KKT necessary conditions.

b) Check if the point(s) is an optimum point using the graphical method (make a simple sketch).

Exercise 2: (15 %)

We will consider gradient-based minimisation of the following unconstrained function:

$$f(\mathbf{x}) = \frac{3}{2}x_1^2 + x_1^2 - 6x_1 + x_1 x_2 + \frac{1}{2}x_2^2 + 3 \tag{2}$$

The starting point is: $\mathbf{x}^{(0)} = [\ 1 \quad 1\ ]^T$.

a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.

b) Can Newton's method be applied for determining the search direction in iteration 1?
If yes, then determine the search direction.
If no, then state an alternative robust method for determining the search direction.

Exercise 3: (9 %)

Solve the following problem by setting up the solution tree and using the *Local Minimization Branch & Bound Method*:

$$\text{minimise} \qquad f(\mathbf{x}) = -2x_1 - 3x_2$$

Subject to:

$$\begin{aligned} g_1(\mathbf{x}) &= 0.4x_1 + x_2 - 8 &\leq 0 \\ g_2(\mathbf{x}) &= x_1 + x_2 - 9.8 &\leq 0 \\ g_3(\mathbf{x}) &= 3x_1 - x_2 - 9 &\leq 0 \end{aligned}$$

Both x_1 and x_2 should be integer values

As a help for solving the problem the objective function contours and constraints are plotted in figure 1.

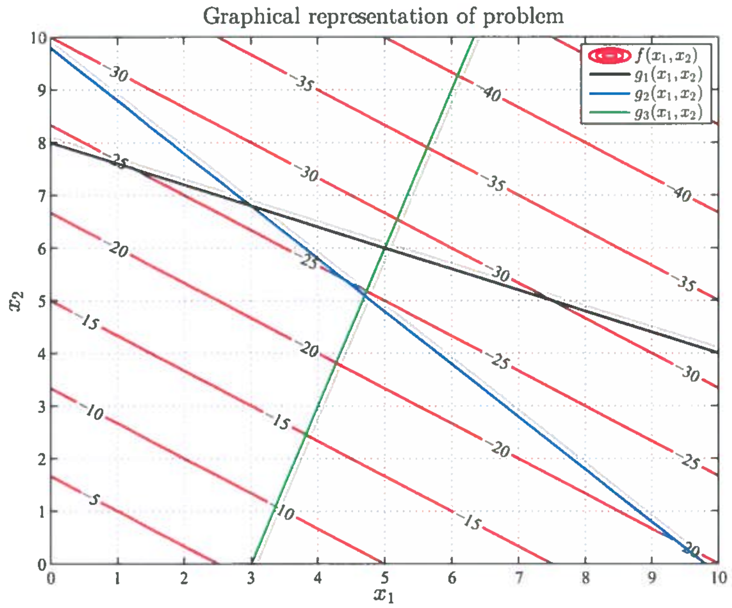


Figure 1: Graphical representation of problem in exercise 3.

Exercise 4: (8 %)

Solve the following linear optimisation problem using the basic steps of the Simplex method and tableau's:

$$\text{minimise} \qquad f(\mathbf{x}) = -5x_1 - 2x_2$$

Subject to the constraints:

$$\begin{aligned} g_1(\mathbf{x}) &= 4x_1 + 3x_2 &\leq& \quad 27 \\ g_2(\mathbf{x}) &= x_1 - 2x_2 &\leq& \quad 4 \\ & \quad x_i \geq 0 & \forall & \quad x_i = \{1, 2\} \end{aligned}$$

Exercise 5: (8 %)

The following multi-objective optimisation problem is considered:

$$\text{minimise} \qquad \begin{aligned} f_1(\mathbf{x}) &= (x_1-2)^2+(x_2-6)^2+5 \\ f_2(\mathbf{x}) &= (x_1-7)^2+(x_2-1)^2+8 \end{aligned}$$

Figure 2 shows the Pareto optimal points in the design space and figure 3 shows the Pareto optimal set in the criterion space.

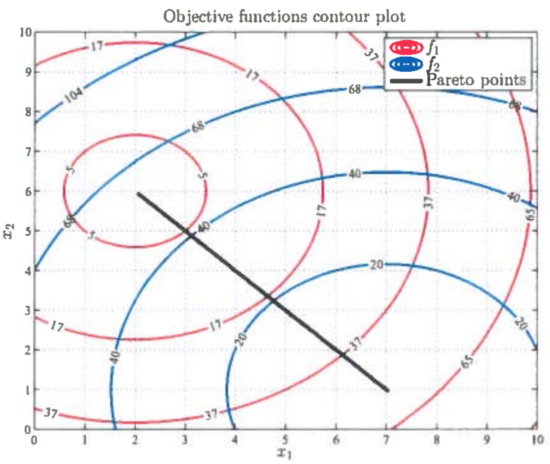


Figure 2: Contour curves and Pareto optimal points.

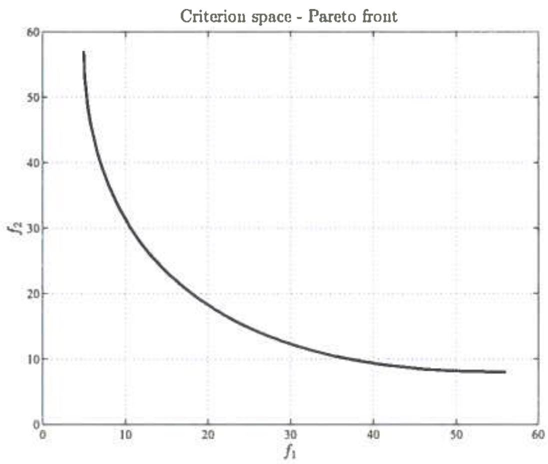


Figure 3: Pareto set in criterion space.

a) Determine the objective function values of the utopia point.

b) Assume that the multi-objective problem is solved as single objective problem, $U(\boldsymbol{x})$, using the weighting method with $w_1 = w_2 = 1$. Determine the minimum objective function value $U(\mathbf{x}^*)$, and the optimum set of design variables $\mathbf{x}^*$.

Exercise 6 (16%)

8 of paper clips are tested with 180 º bending angle, and the cycles to failure for each clip are recorded as: 20, 50, 40, 10, 30, 80, 70, and 60.

1) *State the relationship between accumulated failure and time in Weibull distribution.*
2) *Arrange these time-to-failure numbers by using median ranking method.*
3) *Plot the ranking number as listed in (b) and the cycles to failure in the attached Weibull paper below. Please find out the values for β and η, respectively.*
4) *Explain how to identify the values of β and η in the Weibull plot.*

Appendix I – Median rank table

sample size = n
failure rank = i

i	n = 1	2	3	4	5	6	7	8	9	10
1	.5000	.2929	.2063	.1591	.1294	.1091	.0943	.0830	.0741	.0670
2		.7071	.5000	.3864	.3147	.2655	.2295	.2021	.1806	.1632
3			.7937	.6136	.5000	.4218	.3648	.3213	.2871	.2594
4				.8409	.6853	.5782	.5000	.4404	.3935	.3557
5					.8706	.7345	.6352	.5596	.5000	.4519
6						.8906	.7705	.6787	.6065	.5481
7							.9057	.7979	.7129	.6443
8								.9170	.8194	.7406
9									.9259	.8368
10										.9330

Exercise 7 (17%)

A mining drill head was tested in the laboratory to determine its expected lifetime. The following data were collected:

Stress level ($\times 10^6$ N)	1.0	2.0	3.0	3.5
Mean cycles to failure ($\times 10^3$)	20.3	7.5	1.8	0.35

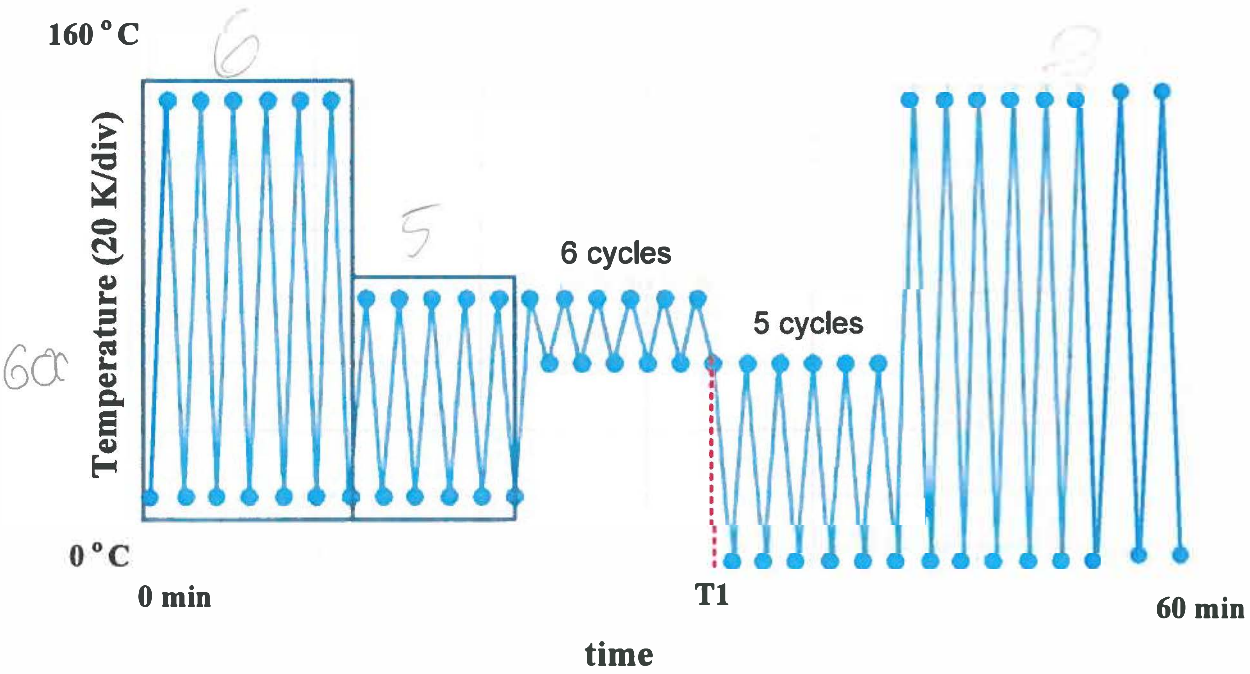
The head will operate in the drill with the following most stressful levels, respectively:

Proportion of cycles	0.8	0.15	0.04	0.01

a) In case of 100 cycles per hour, what will be the expected time to failure in service?
b) what will it be in case of 10 cycles per hour?

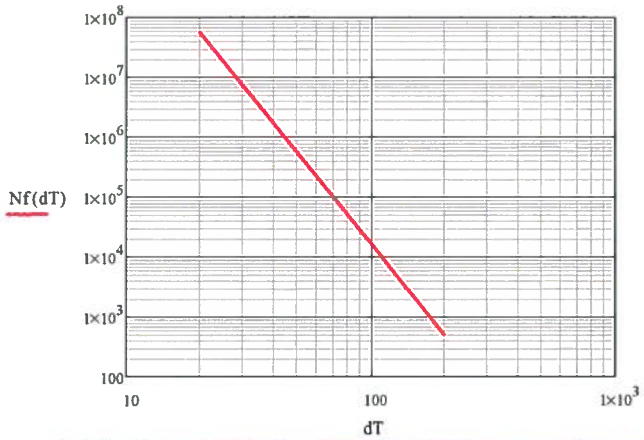
Exercise 8 (17%)

It has been found that the thermal cycle or temperature excursion is the main cause of failure for a component. The 1 hour thermal profiles of this component in the real-field operation has been recorded and shown in the following figure, where many thermal cycles at different levels of ΔT can be identified. The y-axis is the temperature of the component with the scale of 20 degree per div, and x-axis is the time with scale of 10 minute per div.



The lifetime of this component has been tested and modeled as Coffin-Masson lifetime model, as shown in the following function and table.

Coffin-Masson lifetime model of component



Coffin-Masson Model:

$$N_f = A \cdot \Delta T^{\alpha}$$

where:

$$A = 1.99 \times 10^{14},\ \alpha = -5.039$$

ΔT	Nf	ΔT	Nf	ΔT	Nf
10	NA	70	$1.00*10^5$	130	$4.43*10^3$
20	$5.53*10^7$	80	$5.12*10^4$	140	$3.05*10^3$
30	$7.17*10^6$	90	$2.83*10^4$	150	$2.16*10^3$
40	$1.68*10^6$	100	$1.66*10^4$	160	$1.56*10^3$
50	$5.47*10^5$	110	$1.03*10^4$	170	$1.15*10^3$
60	$2.18*10^5$	120	$6.63*10^3$	180	860

Nf: the number of thermal cycles to failure.

1) *List the Rainflow counting table which summarizes the number of thermal cycles at different stress levels.*
2) *Calculate the damage (in %) of component at time T1.*
3) *Calculate the damage (in %) of component within this 60 minutes or 1 hour.*
4) *Assuming the component will experience the repeated thermal profiles of this 1 hour until the failure, calculate how many hours the component can survive.*