MSc Course in

Optimization Theory and Modern Reliability from a Practical Approach

Solutions to Sample exam assignments (Reliability part), by

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Task No. 1

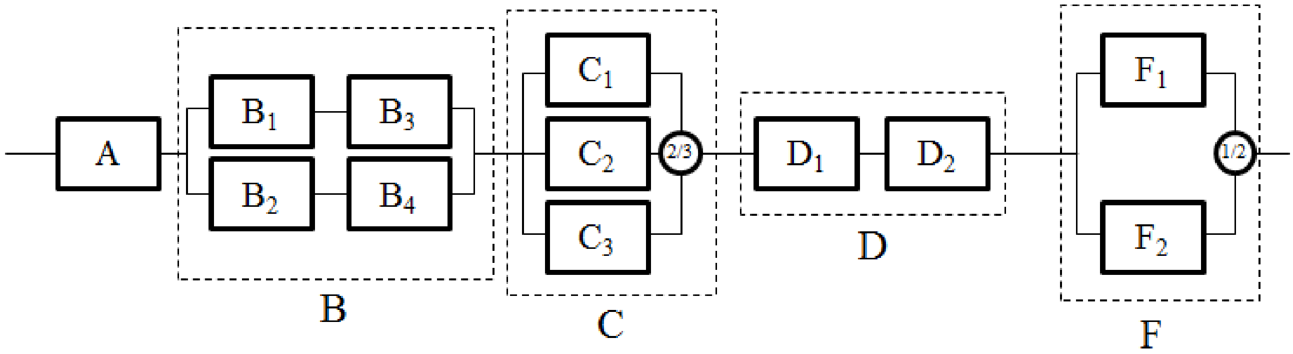
Define the normal life (in terms of number of cycles) as L_{normal}, and the life at the given severe condition is L_{severe}, the given exponent constant m = 4.75, L_{normal} = 3,000 cycles, the temperature variation in normal operation ΔT_{normal} = 90°C and in severe condition ΔT_{severe} = 125°C. Therefore,

$$\frac{L_{severe}}{L_{normal}} = \left(\frac{\Delta T_{severe}}{\Delta T_{normal}}\right)^{-m}$$

$$L_{severe} = L_{normal} \times \left(\frac{\Delta T_{severe}}{\Delta T_{normal}}\right)^{-m} = 3000\ cycles \times \left(\frac{125°C}{90°C}\right)^{-4.75} \cong 630\ cycles$$

Task No. 2

1) The RBD can be drawn as follows:



Note that there is no reference to blocks E (transformer) and G (output filter) as they are supposed to have infinite lifetime, and H (LED) as its malfunction does not affect the overall system reliability.

2) For ease of calculation, it is convenient to work out the reliability of single blocks first, and then compose with each other. Referring to Figure 2.11 of the book, "Shapes of common failure distributions, reliability and hazard rate functions" and using Microsoft Excel, one obtains for blocks A, B, C, D and F:

	A	B	C	D
1				
2			**1 month**	**1 year**
3	Block	Formula	720 hours	8760 hours
4	R_A	EXP(-20/1000000*D$3)	0,986	0,839
5	$R_{B1\text{-}4}$	EXP(-((D$3/30000)^1,5))	0,996	0,854
6	$R_{C1\text{-}3}$	1-NORMDIST(D$3;20000;3000;TRUE)	1,000	1,000
7	$R_{D1\text{-}2}$	1-LOGNORMDIST(D$3;9;4)	0,727	0,492
8	$R_{F1\text{-}2}$	EXP(-175/1000000*D$3)	0,882	0,216

Working out the reliability of complex blocks one obtains the following formulae:

a) For block B, it is the parallel of blocks in series:

$$\mathrm{R_B} = 1 - (1 - \mathrm{R_{B13}}) \cdot (1 - \mathrm{R_{B24}}) = 1 - (1 - \mathrm{R_{B1}R_{B3}}) \cdot (1 - \mathrm{R_{B2}R_{B4}}) = 1 - \left(1 - \mathrm{R_{B1-4}}^2\right)^2$$

b) For block C, it is the redundancy m-out-of-n:

$$R_C = 1 - \sum_{i=0}^{2-1} \binom{3}{i} R_{C13}{}^{i} (1 - R_{C1\text{-}3})^{3-i} = 1 - [(1 - R_{C1\text{-}3})^3 + 3 \cdot R_{C1\text{-}3}(1 - R_{C1\text{-}3})^2]$$

c) For block D, it is the series of two blocks:

$$R_D = R_{D1}R_{D2}$$

d) For block F, it is the parallel of 2 blocks:

$$R_F = 1 - (1 - R_{F1-2})^2$$

Using Excel once again, one obtains:

	1 month	**1 year**
	720 hours	8760 hours
R_A	0,99	0,84
R_B	1,00	0,93
R_C	1,00	1,00
R_D	0,53	0,24
R_F	0,99	0,39
$R_A \times R_B \times R_C \times R_D \times R_E \times R_F$	**0,51**	**0,07**

In conclusion, one obtains:

a) R(t = 1 month) = 51%

b) R(t = 1 year) = 7%

Task No. 3

From the given data [$\Delta\sigma$ = 6.0MPa – (– 6.0 MPa) = 12.0 MPa; $\Delta\sigma_{elastic}$ = 0; m = 3.5; CTF =10.000] we can calculate B_0 rearranging (1) in (2):

$$\mathrm{CTF} = B_0(\Delta\sigma - \Delta\sigma_{\text{elastic}})^{-m} \tag{1}$$

$$B_0 = \frac{CTF}{(\Delta\sigma - \Delta\sigma_{elastic})^{-m}} = \frac{10.000}{(12.0-0)^{-3.5}} = 5.99 \cdot 10^7 \tag{2}$$

a)

$$CTF = 5.99 \cdot 10^7 \cdot \big(2.5\,MPa - (-2.5\,MPa)\big)^{-3.5} \cong 210\,000\,cycles$$

b)

From theory in case of stress bias:

$$\mathrm{CTF} = B_0 \left[\frac{\Delta\sigma}{1-(\sigma_{\mathrm{mean}}/\sigma_{\mathrm{TS}})} - \Delta\sigma_{\mathrm{elastic}}\right]^{-m} \tag{3}$$

Using (3) one can obtain:

$$CTF = 5.99 \cdot 10^7 \cdot \left[\frac{2.5\,MPa - (-2.5\,MPa)}{1 - \frac{1.8\,MPa}{7.4\,MPa}} - 0\right]^{-3.5} \cong 81.000\,cycles$$

Task No. 4

The expression of Miner's rule for a reference period (e.g. 1 year) is the following:

$$TCL = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{1}$$

Where TCL is the "total consumed lifetime", k is the number of different stress levels, N_i is the number of cycles to failure at i-th stress level and n_i is the number of cycles performed in reality at i-th stress level.

For a reference period, e.g. 1000 cycles, using Miner's rule it can be easily concluded:

$$TCL|_{1000\,cycles} = \frac{0.5\cdot1000}{10.9\cdot10^6} + \frac{0.3\cdot1000}{4.5\cdot10^6} + \frac{0.15\cdot1000}{0.76\cdot10^6} + \frac{0.05\cdot1000}{0.23\cdot10^6} = (4{,}58 + 6{,}66 + 19{,}7 + 21{,}7)\cdot10^{-5}$$
$$= 0{,}000526 = 0{,}0526\% \tag{2}$$

The number of cycles to failure is the number of cycles to get a 100% consumed lifetime, hence:

$$\mathrm{CTF} = \frac{100\%}{0{,}0526\%|_{1000\,cycles}} = 1899 \times 1000\ \mathrm{cycles} = 1{,}899 \cdot 10^6 cycles \tag{3}$$

a) In case of 100 cycles per hour, one obtains:

$$TTF = \frac{1{,}899 \cdot 10^6 cycles}{100\,cycles/hour} = 18990\,hours\ (\cong 2.1\,years)$$

b) In case of 1000 cycles per hour, one obtains:

$$TTF = \frac{1{,}899 \cdot 10^6 cycles}{1000\,cycles/hour} = 1899\,hours\ (\cong 79\,days)$$

Task No. 5

1) $F(t) = 1 - \exp[-(\frac{t}{\eta})^{\beta}]$
2) See Table 1 in the attached Weibull paper on next page.
3) β=1.6 (1.2-3); η=50 (20-100). These ranges can be acceptable due to the error of the curve fitting.
4) β is obtained by using parallel translation of the fit line to the β-nomogram in the Weibull plot. As shown in Weibull expression, regardless of the variation of β, F(t) becomes 63.2% if t equals η. That is the reason why η refers to the cycles when failure function reaches 63.2%.

Task No. 6

Exponential

$$R(300) = \exp\left(-\frac{300}{1000}\right) = 0.7408$$

Weibull:

a. $\beta = 0.5$

$$\mathrm{R}(300) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] = \exp\left[-\left(\frac{300}{1000}\right)^{0.5}\right] = 0.5783$$

b. $\beta = 1.0$

$$\mathrm{R}(300) = \exp\left[-\left(\frac{300}{1000}\right)^{1.0}\right] = 0.7408$$

c. $\beta = 3.0$

$$\mathrm{R}(300) = \exp\left[-\left(\frac{300}{1000}\right)^{3.0}\right] = 0.9734$$

The higher the Weibull slope the higher the reliability at time before the characteristic life ($t < \eta$). The opposite is true if $t > \eta$.

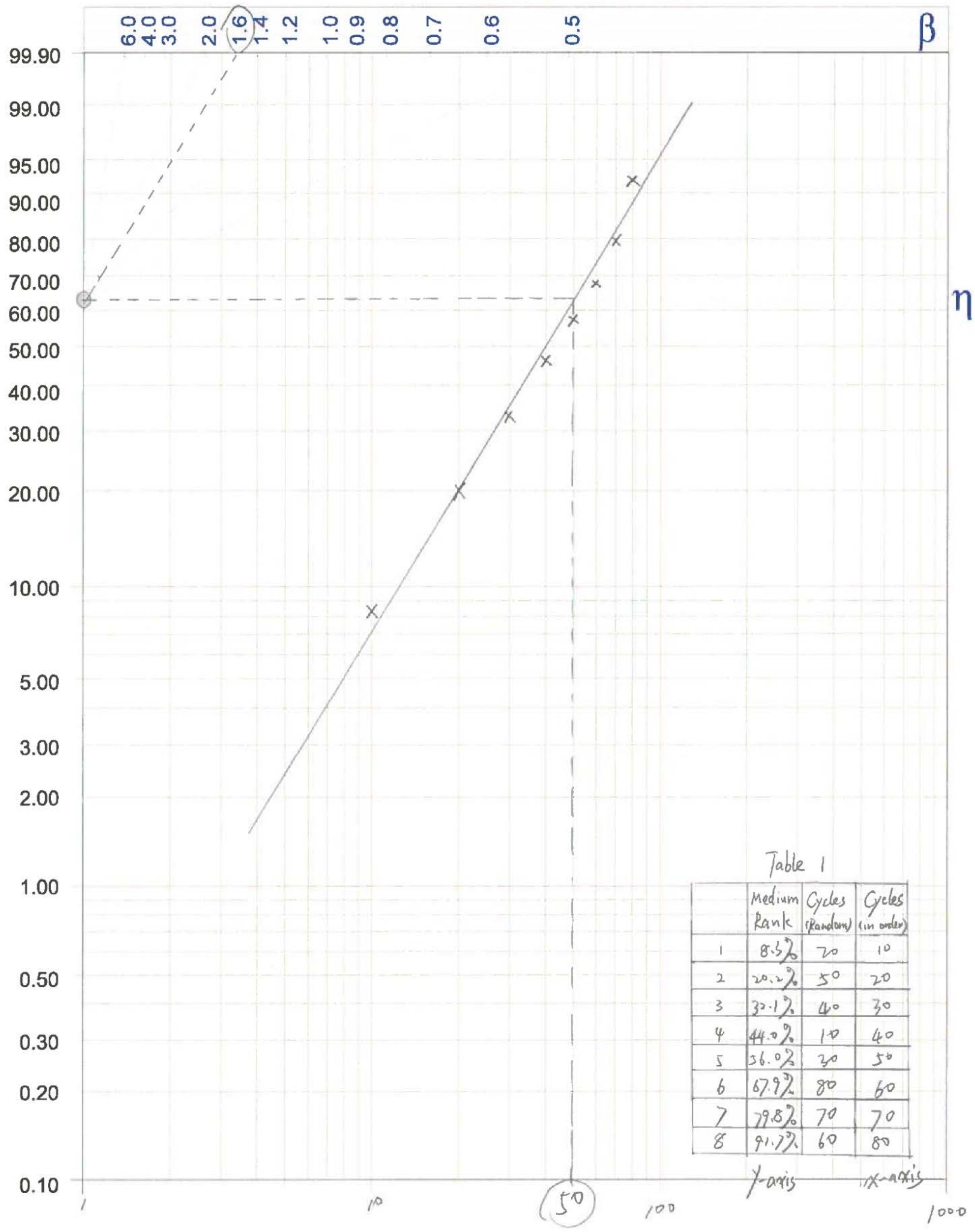


Table 1

	Medium Rank	Cycles (Random)	Cycles (in order)
1	8.3%	20	10
2	20.2%	50	20
3	32.1%	40	30
4	44.0%	10	40
5	56.0%	30	50
6	67.9%	80	60
7	79.8%	70	70
8	91.7%	60	80
	Y-axis		X-axis