AALBORG UNIVERSITY
DENMARK

Written examination in the course

Optimisation Theory and

Modern Reliability from a Practical Approach

Monday June 8th 2015

kl. 9 - 13 (4 hours)

All usual helping aids are allowed, i.e. books, notes, calculator, computer etc. All communication equipment and computer communication protocols must be turned off.

The questions should be answered in English.

REMEMBER to write your study number and page number on all sheets handed in.

The set consists of eight exercises. The total weighting for each of the exercises is stated in percentage. You need 50 % in order to pass the exam.

It should be clear from the solution, which methods are used, and there should be a sufficient number of intermediate calculations, so the line of thought is clear.

$H(x) = (X_1 \cdot 20) + (X_2 \cdot 24) = 5000$

$A = (X_1 \cdot 150) + 1000$ $B \geq 30$

$B = (X_2 \cdot 225) + 1500$ $A \geq 30$

✓ **Exercise 1: (10 %)**

A company wants to maximize its day-to-day profit from manufacturing electrical motors. Formulate the profit maximization problem as a standard optimization problem based on the following information:

- The company produces two motors: 1) Type A and 2) Type B.
- To maintain market shares the company needs to produce at least 30 motors of Type A every day and at least 30 motors of Type B every day.
- The daily cost of producing Type A is a fixed cost of 1000 Euro and a further 150 Euro per motor. The daily cost of producing Type B is a fixed cost of 1500 Euro and a further 225 Euro per motor.
- The company has a total of 5000 man hours available per day. It takes 20 man hours to produce a motor of Type A and 24 man hours to produce a motor of Type B.
- The company sells the motors at the following prices: 300 Euro per motor of type A and 400 Euro per motor of type B.

$f(X) = (X_1 \cdot 300) + (X_2 \cdot 400) - (X_1 \cdot 150) - (X_2 \cdot 225) - 1000 - 1500$

Note: you should only formulate the problem, not solve it.

✓ **Exercise 2: (15 %)**

We will consider gradient-based minimisation of the following unconstrained function:

$$f(\mathbf{x}) = 2(x_1^2 - x_2) + x_1^2 - 2x_1 + 5$$

The starting point is: $\mathbf{x}^{(0)} = [\ -1 \quad 3\ ]^T$.

a) Complete the first iteration of the steepest descent method for the function. The 1D line search problem should be solved analytically.

b) Can Newton's method be applied for determining the search direction in iteration 1?
If yes, then determine the search direction.
If no, then state an alternative robust method for determining the search direction.

✓ **Exercise 3: (9 %)**

An optimisation problem is formulated as:

$$\text{minimise} \qquad f(\mathbf{x}) = 2x_1^2 + 6x_2^2 - 3x_1x_2 - x_1 + 2x_2$$

Subject to:

$$\begin{aligned} g_1(\mathbf{x}) &= x_1 - 12 &\leq 0 \\ g_2(\mathbf{x}) &= 7x_1 - 2x_2 - 4 &\leq 0 \\ g_3(\mathbf{x}) &= x_1 + x_2 - 20 &\leq 0 \end{aligned}$$

The initial starting point for an optimisation is $\mathbf{x}^{(0)} = (0.5, 0)$, for which a search direction is determined to $\mathbf{d}^{(0)} = [\ -1 \quad -0.5\ ]^T$ and the vector of Lagrange multipliers for the constraints is $\mathbf{u} = [\ 0 \quad 0 \quad 0\ ]^T$. Let $R_0 = 1$ and $\gamma = 0.5$. Choose the trial step according to the sequence $t_0 = 1$, $t_1 = \frac{1}{2}$, $t_2 = \frac{1}{4}$, $t_3 = \frac{1}{8}$...

Calculate the step size using the inexact line search procedure (approximate step size procedure) and determine the new design variables, $\mathbf{x}^{(1)}$, for the next iteration.

Exercise 4: (9 %)

Solve the following problem by setting up the solution tree and using the *Local Minimization Branch & Bound Method*:

$$\text{minimise} \qquad f(\mathbf{x}) = -6x_1 - 3x_2$$

Subject to:

$$\begin{aligned} g_1(\mathbf{x}) &= 0.5x_1 + x_2 - 11 \leq 0 \\ g_2(\mathbf{x}) &= 2.5x_1 + x_2 - 20 \leq 0 \\ g_3(\mathbf{x}) &= 2x_1 - x_2 - 10 \leq 0 \end{aligned}$$

Both x_1 and x_2 should be integer values

As a help for solving the problem the objective function contours and constraints are plotted in figure 1.

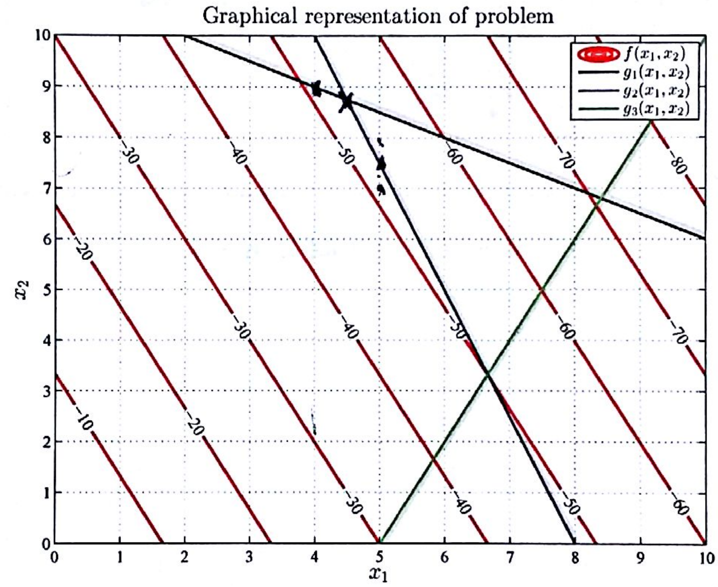


Figure 1: Graphical representation of problem in exercise 4.

Exercise 5: (7 %)

Solve the following linear optimisation problem using the basic steps of the Simplex method and tableau's:

$$\text{minimise} \qquad f(\mathbf{x}) = 3x_1 - 2x_2$$

Subject to the constraints:

$$\begin{aligned} g_1(\mathbf{x}) &= -2x_1 + x_2 \leq 5 \\ g_2(\mathbf{x}) &= x_1 + 4x_2 \leq 3 \\ & x_i \geq 0 \quad \forall \quad x_i = \{1, 2\} \end{aligned}$$

Exercise 6: (16 %)

8 paper clips are tested with 180° bending angle, and the cycles to failure for each clip are recorded as: 20, 50, 40, 10, 30, 80, 70, and 60.

1) Arrange these time-to-failure numbers by using the median ranking method.

2) Plot the ranking number as listed in (b) and the cycles to failure in the attached Weibull paper below. Please find out the values for β and η, respectively.

3) Explain how to identify the values of β and η in the Weibull plot.

4) State the relationship between accumulated failure and time in the Weibull distribution.

sample size = n
failure rank = i

i \ n	1	2	3	4	5	6	7	8	9	10
1	.5000	.2929	.2063	.1591	.1294	.1091	.0943	.0830	.0741	.0670
2		.7071	.5000	.3864	.3147	.2655	.2295	.2021	.1806	.1632
3			.7937	6136	5000	4218	3648	3213	2871	.2594
4				8409	6853	5782	.5000	4404	3935	.3557
5					8706	.7345	6352	5596	.5000	.4519
6						8906	.7705	6787	.6065	5481
7							.9057	.7979	.7129	.6443
8								9170	.8194	.7406
9									.9259	.8368
10										.9330

Figure 2: Median rank table

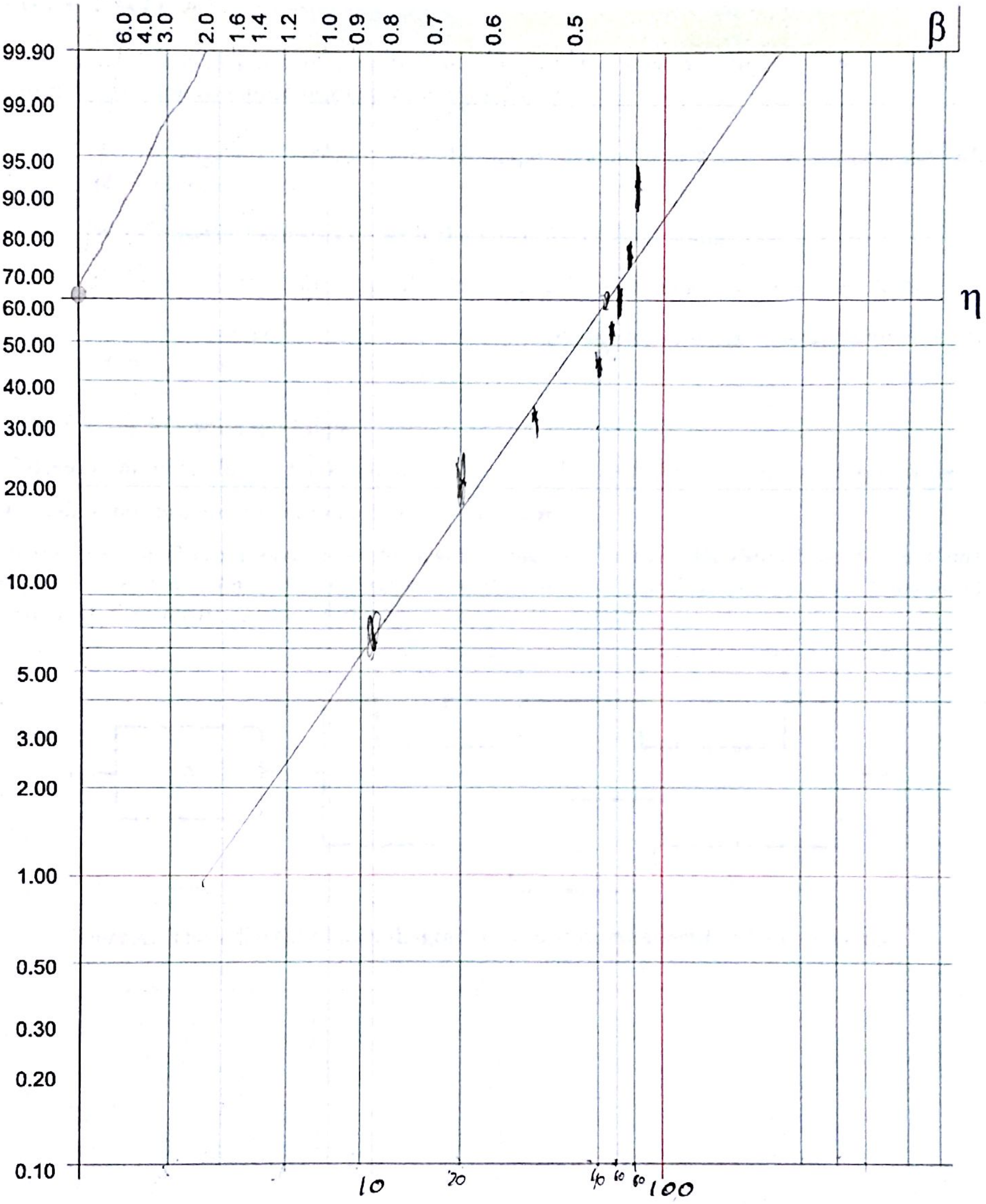


Figure 3: Weibull paper

Exercise 7: (17 %)

The reliability block diagram of a system is shown in the following figure. The probability distributions of time to failure of the four elements are:

> A – 2- parameter Weibull, with the shape parameter of 2 and the characteristic life of 350 hours.
>
> B – Constant hazard rate, with the mean life of 1000 hours
>
> C – Normal, with the mean of 420 hours and the standard deviation of 133 hours
>
> D – 2-parameter Weibull, with the shape parameter of 4 and the characteristic life of 800 hours.

Please solve the following questions:

1) Calculate the reliability of the elements A, B, C, and D at 200 hours mission, respectively.

2) Calculate the system reliability at 200 hours mission

3) If the element D is removed from the system, what will be the reliability of the new system? Compare the system reliability with and without the element D and comment on the function of the element D in terms of reliability performance of the system.

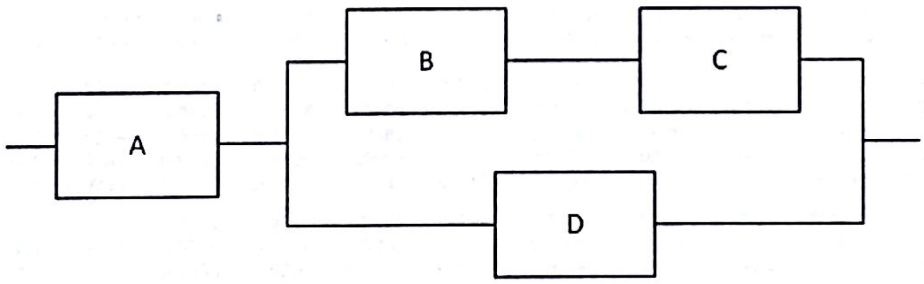
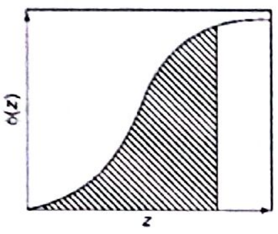


Figure 4: The reliability block diagram of a system composed of four elements.

Appendix 2 - The Standard Cumulative Normal Distribution Function



$$\Phi(z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{z} \exp\left(\frac{-x^2}{2}\right) dx$$

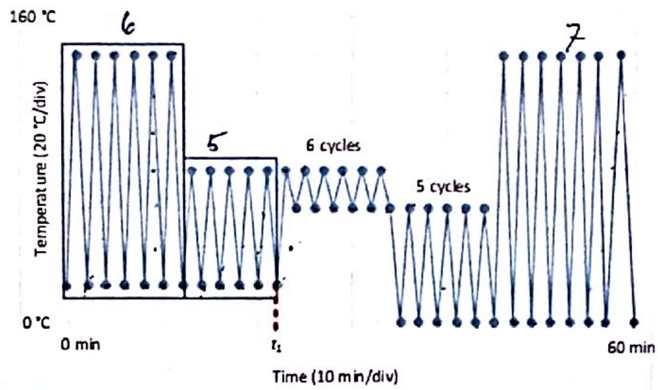
for $0.00 \le z \le 4.00$

$$1 - \Phi(z) = \Phi(-z)$$

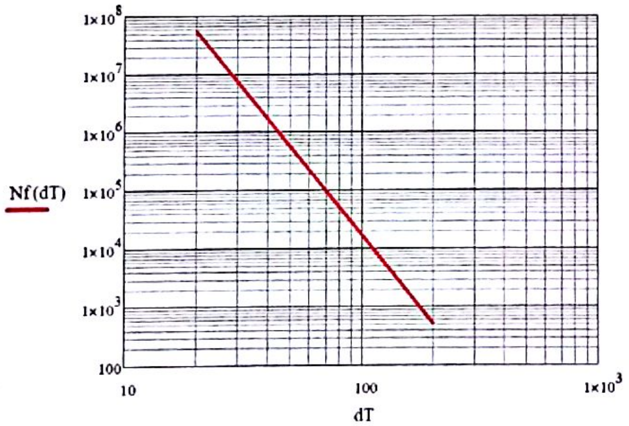
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6985	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9440
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	$0.9^{2}010$	$0.9^{2}061$	$0.9^{2}035$	$0.9^{2}086$	$0.9^{2}111$	$0.9^{2}134$	$0.9^{2}158$
2.4	$0.9^{2}180$	$0.9^{2}202$	$0.9^{2}224$	$0.9^{2}245$	$0.9^{2}266$	$0.9^{2}286$	$0.9^{2}305$	$0.9^{2}324$	$0.9^{2}343$	$0.9^{2}361$
2.5	$0.9^{2}379$	$0.9^{2}396$	$0.9^{2}413$	$0.9^{2}430$	$0.9^{2}446$	$0.9^{2}461$	$0.9^{2}477$	$0.9^{2}492$	$0.9^{2}506$	$0.9^{2}520$
2.6	$0.9^{2}534$	$0.9^{2}547$	$0.9^{2}560$	$0.9^{2}573$	$0.9^{2}586$	$0.9^{2}598$	$0.9^{2}609$	$0.9^{2}621$	$0.9^{2}632$	$0.9^{2}643$
2.7	$0.9^{2}653$	$0.9^{2}664$	$0.9^{2}674$	$0.9^{2}683$	$0.9^{2}693$	$0.9^{2}702$	$0.9^{2}711$	$0.9^{2}720$	$0.9^{2}728$	$0.9^{2}737$
2.8	$0.9^{2}745$	$0.9^{2}752$	$0.9^{2}760$	$0.9^{2}767$	$0.9^{2}774$	$0.9^{2}781$	$0.9^{2}788$	$0.9^{2}795$	$0.9^{2}801$	$0.9^{2}807$
2.9	$0.9^{2}813$	$0.9^{2}819$	$0.9^{2}825$	$0.9^{2}831$	$0.9^{2}836$	$0.9^{2}841$	$0.9^{2}846$	$0.9^{2}851$	$0.9^{2}856$	$0.9^{2}861$
3.0	$0.9^{2}865$	$0.9^{2}869$	$0.9^{2}874$	$0.9^{2}878$	$0.9^{2}882$	$0.9^{2}886$	$0.9^{2}889$	$0.9^{2}893$	$0.9^{2}897$	$0.9^{2}900$
3.1	$0.9^{3}032$	$0.9^{3}065$	$0.9^{3}096$	$0.9^{3}126$	$0.9^{3}155$	$0.9^{3}184$	$0.9^{3}211$	$0.9^{3}238$	$0.9^{3}264$	$0.9^{3}289$
3.2	$0.9^{3}313$	$0.9^{3}336$	$0.9^{3}359$	$0.9^{3}381$	$0.9^{3}402$	$0.9^{3}423$	$0.9^{3}443$	$0.9^{3}462$	$0.9^{3}481$	$0.9^{3}499$
3.3	$0.9^{3}517$	$0.9^{3}534$	$0.9^{3}550$	$0.9^{3}566$	$0.9^{3}581$	$0.9^{3}596$	$0.9^{3}610$	$0.9^{3}624$	$0.9^{3}638$	$0.9^{3}651$
3.4	$0.9^{3}663$	$0.9^{3}675$	$0.9^{3}687$	$0.9^{3}698$	$0.9^{3}709$	$0.9^{3}720$	$0.9^{3}730$	$0.9^{3}740$	$0.9^{3}749$	$0.9^{3}759$
3.5	$0.9^{3}767$	$0.9^{3}776$	$0.9^{3}784$	$0.9^{3}792$	$0.9^{3}800$	$0.9^{3}807$	$0.9^{3}815$	$0.9^{3}822$	$0.9^{3}822$	$0.9^{3}835$
3.6	$0.9^{3}841$	$0.9^{3}847$	$0.9^{3}853$	$0.9^{3}858$	$0.9^{3}864$	$0.9^{3}869$	$0.9^{3}874$	$0.9^{3}879$	$0.9^{3}883$	$0.9^{3}888$
3.7	$0.9^{3}892$	$0.9^{3}896$	$0.9^{4}004$	$0.9^{4}043$	$0.9^{4}116$	$0.9^{4}116$	$0.9^{4}150$	$0.9^{4}184$	$0.9^{4}216$	$0.9^{4}257$
3.8	$0.9^{4}277$	$0.9^{4}305$	$0.9^{4}333$	$0.9^{4}359$	$0.9^{4}385$	$0.9^{4}409$	$0.9^{4}433$	$0.9^{4}456$	$0.9^{4}478$	$0.9^{4}499$
3.9	$0.9^{4}519$	$0.9^{4}539$	$0.9^{4}557$	$0.9^{4}575$	$0.9^{4}593$	$0.9^{4}609$	$0.9^{4}625$	$0.9^{4}641$	$0.9^{4}655$	$0.9^{4}670$

Exercise 8: (17 %)

It has been found that the thermal cycle or temperature excursion is the main cause of failure for a component. The 1 hour thermal profiles of this component in the real-field operation has been recorded and shown in the following figure, where many thermal cycles at different levels of ΔT can be identified. The y-axis is the temperature of the component with the scale of 20°C per div, and x-axis is the time with scale of 10 minutes per div.



The lifetime of this component has been tested and modelled as the Coffin-Masson lifetime model, as shown in the following function and table.



Coffin-Masson Model:

$$N_f = A \times \Delta T^{\alpha}$$

where:

$A = 1.99\times10^{14}$, $\alpha = -5.039$

Figure 5: Coffin-Masson lifetime model of component

Table 1: The cycle-to-failure of the component under different thermal cycles.

ΔT (°C)	N_f	ΔT (°C)	N_f	ΔT (°C)	N_f
10	N/A	70	1.00×10^5	130	4.43×10^3
20	5.53×10^7	80	5.12×10^4	140	3.05×10^3
30	7.17×10^6	90	2.83×10^4	150	2.16×10^3
40	1.68×10^6	100	1.66×10^4	160	1.56×10^3
50	5.47×10^5	110	1.03×10^4	170	1.15×10^3
60	2.18×10^5	120	6.63×10^3	180	8.60×10^2

ΔT – Amplitude of the thermal cycle, N_f - the number of thermal cycles to failure.

Questions:

1) *List the Rainflow counting table which summarizes the number of thermal cycles at different stress levels.*

2) *Calculate the damage (in %) of component at time t_1 as shown in the above thermal profiles.*

3) *Calculate the damage (in %) of component within this 1 hour.*

4) *Assuming the component will experience the repeated thermal profiles of this 1 hour until the failure, calculate how many hours the component can survive.*