Proof by Induction for The Recursive Function
pop(n)

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$$pop(n) = \begin{cases} 0, & \text{if } n = 0. \\ 1, & \text{if } n = 1. \\ \lfloor \frac{5 \cdot pop(n-1)}{2} \rfloor - pop(n-2), & \text{if } n \geq 2. \end{cases}$$

The following proof shows that the function *pop* as defined above recursively, is equal to 2^{n-1} for all $n \in \mathbb{W}$.

Base Cases:

$$pop(0) = 0, \quad 2^{\text{n-1}} = 0$$
$$pop(1) = 1, \quad 2^0 = 1$$
$$pop(2) = 2, \quad 2^1 = 2$$

Inductive Step: Assume $pop(n) = 2^{n-1}$ for some $n \in \mathbb{W}$, we will show that this implies $pop(n + 1) = 2^n$.

$$pop(n + 1) = \lfloor \frac{5 \cdot pop(n)}{2} \rfloor - pop(n - 1)$$
$$\overset{\text{IH}}{=} \lfloor \frac{5 \cdot 2^{n-1}}{2} \rfloor - 2^{n-2}$$
$$= 5 \cdot 2^{n-2} - 2^{n-2}$$
$$= 4 \cdot 2^{n-2}$$
$$= 2^n$$

Thus, it is shown that $pop(n) = 2^{n-1}$ for all $n \in \mathbb{W}$.