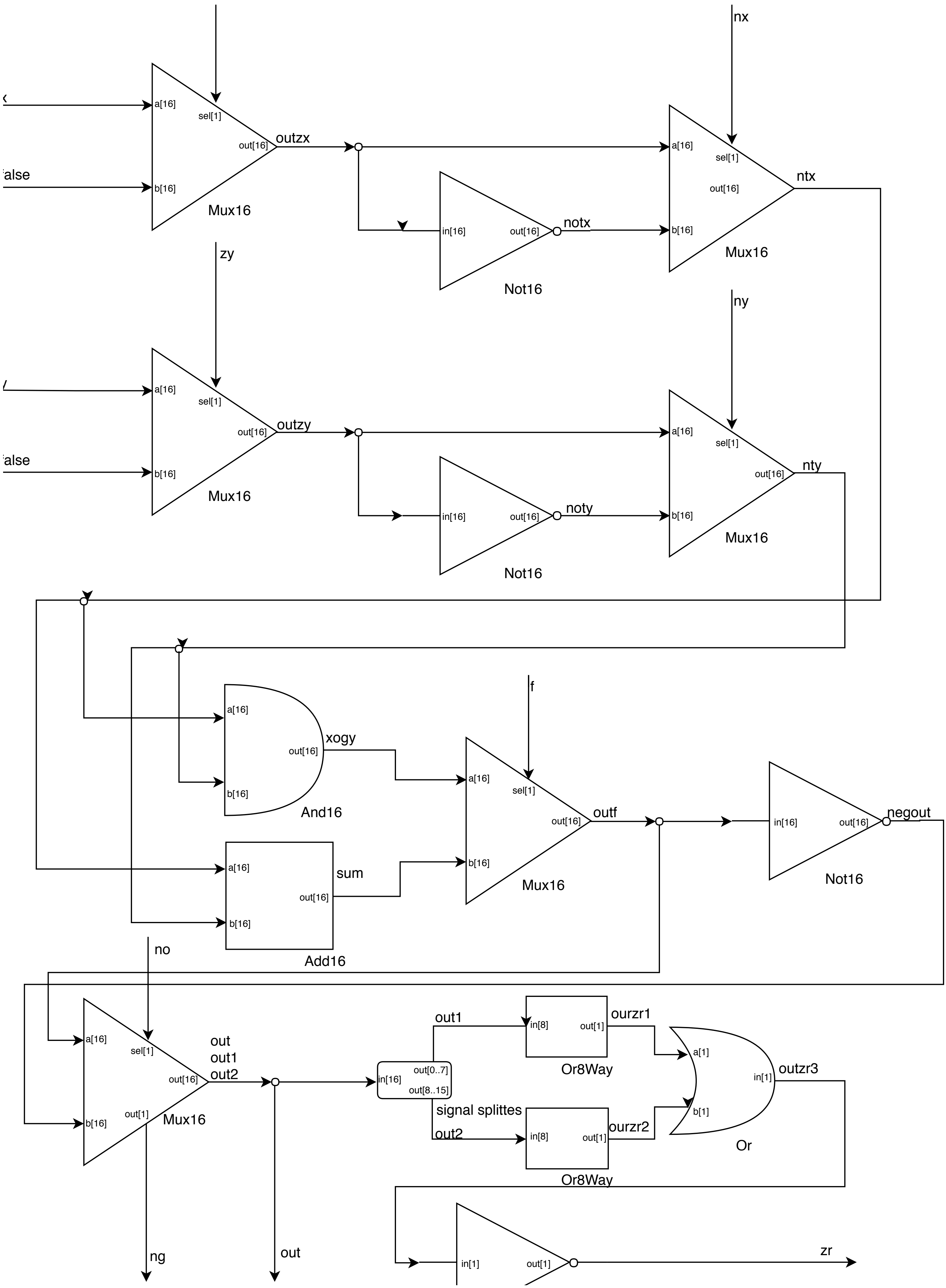
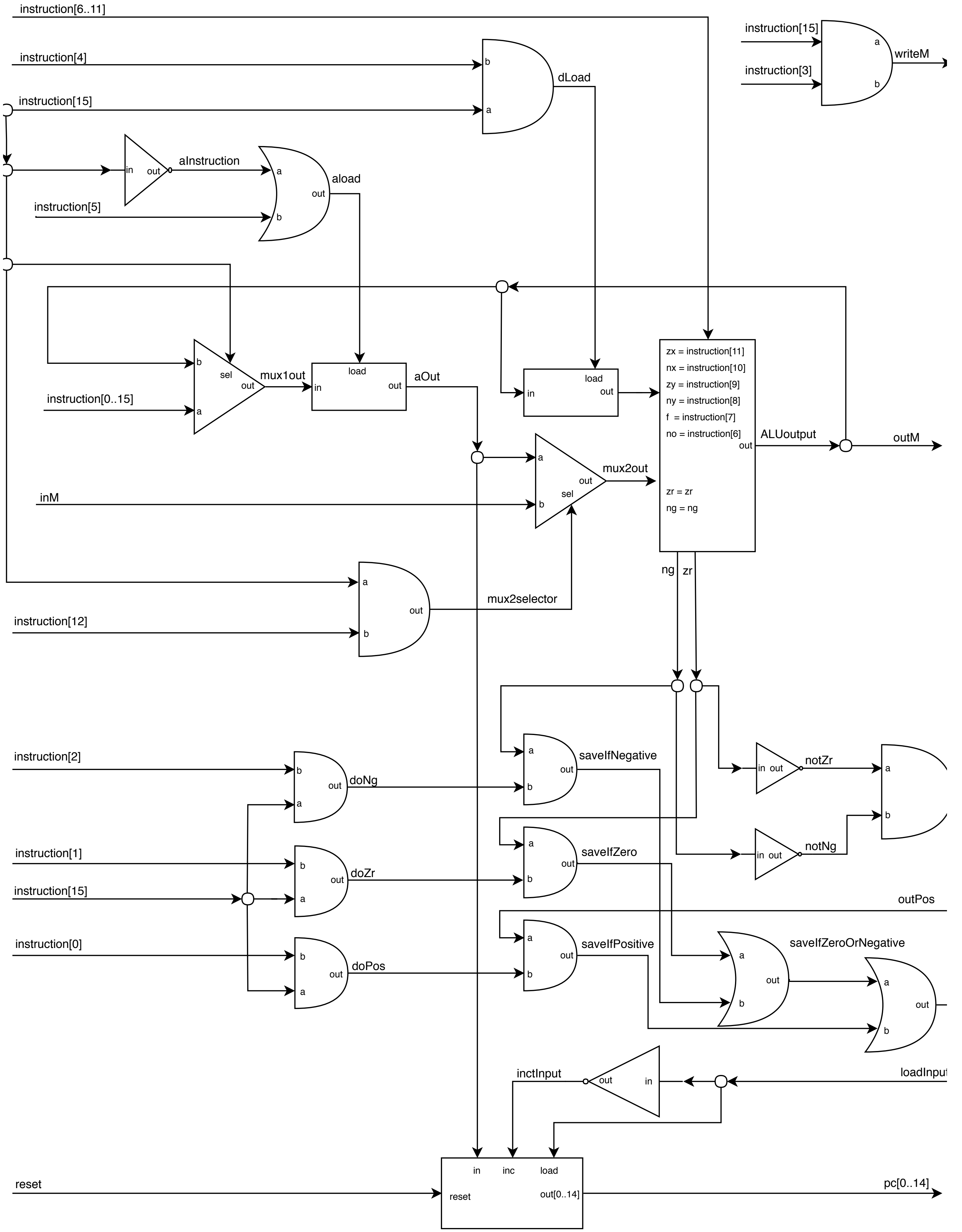
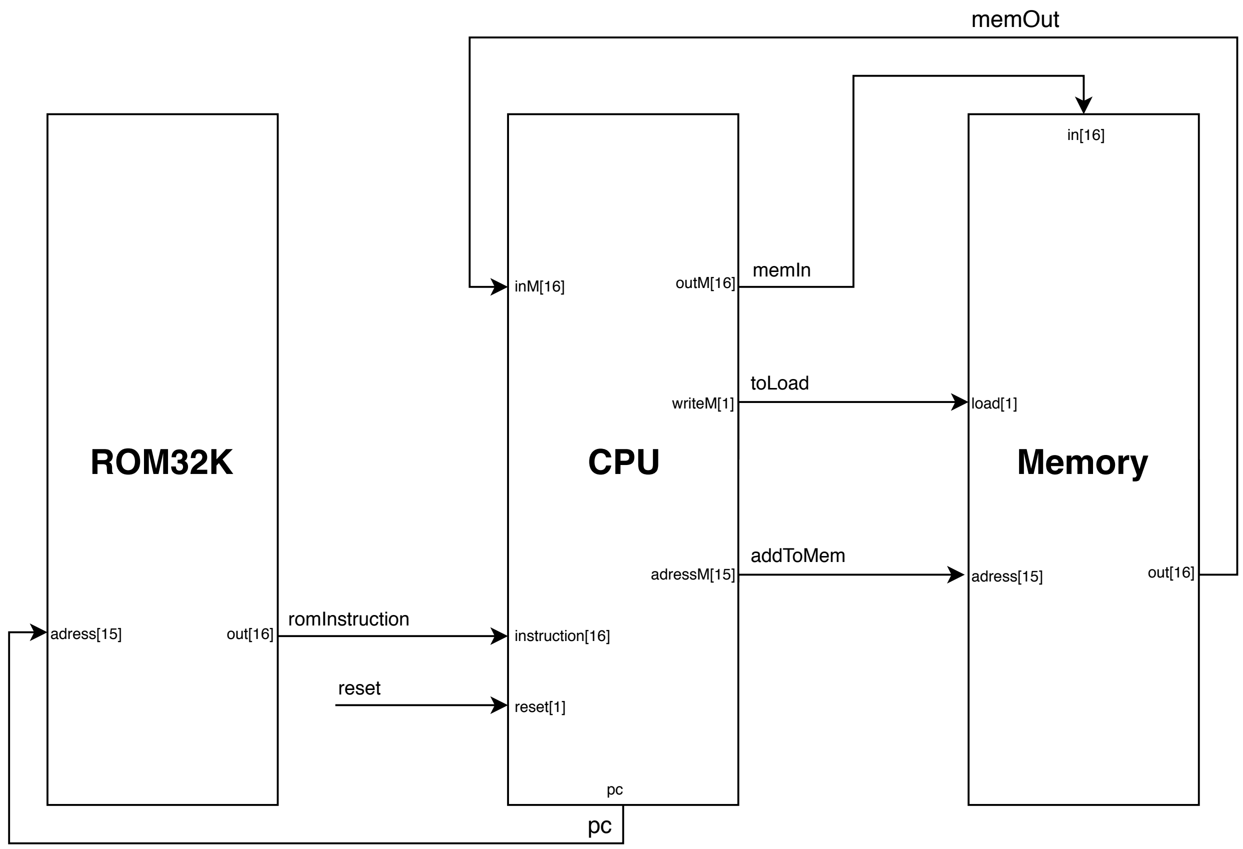


nx
a[16]
sel[1]
out[16]
b[16]
outzx
Mux16
in[16]
notx
Not16
ntx
zy
ny
outzy
noty
nty
f
xogy
And16
sum
Add16
outf
negout
Not16
no
out
out1
out2
Mux16
out[1]
out[0..7]
out[8..15]
signal splittes
in[8]
ourzr1
ourzr2
Or8Way
a[1]
b[1]
in[1]
outzr3
Or
ng
zr


instruction[6..11]
instruction[4]
instruction[15]
dLoad
writeM
instruction[3]
aInstruction
aload
instruction[5]
mux1out
aOut
instruction[0..15]
zx = instruction[11]
nx = instruction[10]
zy = instruction[9]
ny = instruction[8]
f = instruction[7]
no = instruction[6]
zr = zr
ng = ng
ALUoutput
outM
mux2out
inM
mux2selector
instruction[12]
ng
zr
instruction[2]
doNg
saveIfNegative
notZr
notNg
instruction[1]
saveIfZero
doZr
instruction[15]
outPos
instruction[0]
saveIfPositive
saveIfZeroOrNegative
doPos
inctInput
loadInput
reset
pc[0..14]



memOut
in[16]
ROM32K
CPU
Memory
inM[16]
outM[16]
memIn
toLoad
writeM[1]
load[1]
addToMem
adressM[15]
adress[15]
out[16]
adress[15]
out[16]
romInstruction
instruction[16]
reset
reset[1]
pc
pc

Proof by Induction for The Recursive Function *pop(n)*

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$$pop(n) = \begin{cases} 0, & \text{if } n = 0. \\ 1, & \text{if } n = 1. \\ \lfloor \frac{5 \cdot pop(n-1)}{2} \rfloor - pop(n-2), & \text{if } n \geq 2. \end{cases}$$

The following proof shows that the function *pop* as defined above recursively, is equal to 2^{n-1} for all $n \in \mathbb{W}$.

Base Cases:

$$\begin{aligned} pop(0) &= 0, \qquad 2^{\text{n-1}} = 0 \\ pop(1) &= 1, \qquad 2^{0} = 1 \\ pop(2) &= 2, \qquad 2^{1} = 2 \end{aligned}$$

Inductive Step: Assume $pop(n) = 2^{n-1}$ for some $n \in \mathbb{W}$, we will show that this implies $pop(n+1) = 2^{n}$.

$$\begin{aligned} pop(n+1) &= \lfloor \frac{5 \cdot pop(n)}{2} \rfloor - pop(n-1) \\ &\overset{\text{IH}}{=} \quad \lfloor \frac{5 \cdot 2^{n-1}}{2} \rfloor - 2^{n-2} \\ &= 5 \cdot 2^{n-2} - 2^{n-2} \\ &= 4 \cdot 2^{n-2} \\ &= 2^{n} \end{aligned}$$

Thus, it is shown that $pop(n) = 2^{n-1}$ for all $n \in \mathbb{W}$.