Modelling of Electricity Futures

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Nicolaj Hans Nielsen, s184335

Danmarks
Tekniske Universitet

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A | Introduction

The energy market is in a unprecedented stage of distress with energy commodities prices on record highs, bailout on major utilities, government initiatives to support citizens, and exceptional volatilities caused by a storm of political events, historical droughts, and a generation stack in poor conditions. To survive in this enviroment it is very important to understand the constituents of the energy markets, the mechanism of the liberalized energy market, and how future and forward markets can be used to hedge energy production. We will focus on the German market and futures traded on EEX. Specifically, we consider the dynamics of the quarterly German baseload contracts in 2022 and how they evolve simultaneously, the interlinkage with related energy futures contracts to get broad sense of the dynamics of baseload futures contracts. Subsequently, we study the highly liquid front month contracts leading up to the 2021 energy crunch and make a separate study of the market in 2022 with record high prices droughts and the Russian invasion of Ukraine. The focus will for the most part be the dynamics of the futures contracts, however, as they are written on the underlying spot prices, we will briefly cover how we can construct a risk neutral futures price based on a model of the spot prices. This will serve as an introduction the next course on stochastic modelling of electricity prices which is to be conducted in autumn 2022.

B | The German Energy System - History and Outlook

The energy system has been on a long journey over the last two decades. It used to be a highly monopolized industry with only a few major players and a generation stack of mainly thermal energy supply. This changed dramatically under EU energy initiative and the green agenda. In the following, we will briefly cover the founding ideas for this transformation, market design, participants, and how this introduced a liquid, granular forward market in Germany. The section is primarily based on [1].

B.1 European Union Initiative

A core principal in EU policy is to create free, effective and competitive markets across all member states. The monopolized energy markets across Europe needed to be reformed to align with these principals. A growing global agenda on reducing green house emissions also meant that sustainable energy generation became a vital part of the EU energy policy. The EU energy policy triangle in fig. B.1 summarizes three constituent parts of what is also known as the energy trilemma.

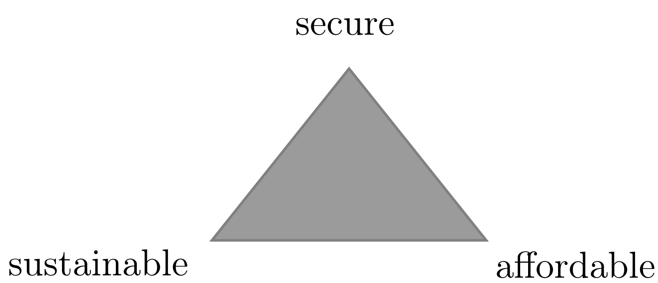


Figure B.1 – The EU energy policy triangle, greatly inspired by figure 4 in [1].

- **secure**: A stable energy grid is essential for a prosperous society and economy.
- **affordable**: Citizens need to heat and power their homes. If the price of energy increases, it would take up a greater part of their available capital and potentially compromise their well-being. For businesses, it would increase production cost and make them unable to compete on global markets
- **sustainable**: To reduce green house gas emissions, it is vital to ensure that the energy supply chain is as green as possible hence it includes promotion of renewable energy.

A range of reforms were introduced over the following decades which ultimately lead to the *liberalization* of the energy markets. To enable this, the reform separated the psychical flow of electricity and the commercial flow of electricity. The metaphor for this idea is an 'electricity lake' where costumers and producers are all connected to one lake. Where the exact location of in- and outflow of electricity does not matter - what matters is that the level of the lakes remains constant. Physically, the electricity still comes from the nearest producer but commercially, the consumer can freely choose the supplier.

B.2 Market Design

The new market design created three means by which electricity is traded in Germany:

- **Exchange**: The energy can be traded through exchanges. These both include spot markets facilitated by EPEX spot, and derivative contracts which are traded on exchanges such as *InterContinental Exchange*, ICE, and *European Energy Exchange*, EEX
- **OTC through brokerage platform** here the buyers and sellers submit bids anonymously and the broker closes the deals. This could e.g. be on *Trayports* platform Joule.
- **OTC directly** this would often involve large contracts involving utilities and enterprises. In the end, these contract would often still be cleared through e.g. EEX to reduce the counterparty risk using the exchange as a central counterparty. OTC trades could happen by means of a *power purchase agreements*, PPA.

In figure B.2, we a graphical representation of the market design is included and we will mostly focus only on the wholesale market.
After the markets were introduced, it took some time for the markets to be fully efficient. The old major players had a substantial advantage with long lasting OTC contracts. However, over time an increasing amount of players started to enter the open exchange markets which eventually became the benchmark for pricing electricity. Now, nearly 50% of the German energy consumption is now happening in what is known as the EPEX spot exchange day-ahead market [1]. This is one exchange that focuses only on trading energy with delivery the following day. As indicated in figure B.2, there exists multiple markets for trading of different temporal granularities hence, we will briefly go through the most important and traded exchanges and granularities.

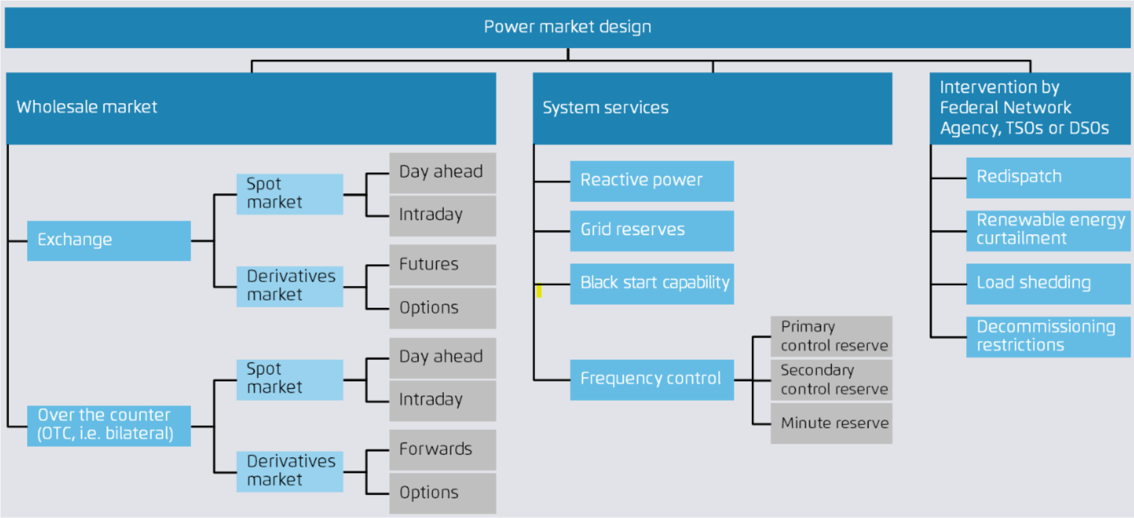


Figure B.2 – Available energy markets, direct replica of figure 9 in [1].

- **Spot market** is energy traded with delivery no later than two days after trading and is divided into:
  - Day ahead: which is energy traded for each hour of the following day with market closing at 12:00.
  - Intraday: Is traded on 5-min, 15-min, 30-min and hourly contracts and closes 5 minute prior to the delivery start. It allows participants to fine-tune production

  It is important to stress the day-ahead market which accounts for 50% of the consumed energy whereas the intraday is around 10%

- **Derivatives Market** This is often split into forward and futures contracts.
  - Futures are settled daily hence there will be a continues transfer of money between the parties on their accounts on the exchange.
  - Forwards: would be cleared at the end of the delivery period hence there will be a greater counterparty credit exposure.

  Most derivatives are cash settled to allow more participants in the market, however, there also exists futures with actual physical delivery. The contracts are traded in EUR/MWh over various delivery periods such as on specific day, weekend, week, month, quarter, season and year. We will get back to these much more in chapter C.

The statistical analysis will focus solely on the futures contracts, however, these are written on the underlying day-ahead spot price hence to understand the dynamics of the futures, we first need to understand the drivers of the day-ahead spot price.

B.2.1 Auction Based Market Clearing

The day-ahead market is cleared by means of the merit order principal. Here the suppliers will have to submit bids on energy production and volume. The suppliers can submit multiple bids with different volume and prices. Likewise, the buyers will submit offers to cover their needed amount of energy. At 12:00, the EPEX auction closes for each hour and the generation stack is constructed by their merit order and the *market clearing price* is determined as the intersection of the supply and demand curve. All bids below the intersection will obtain the same *market clearing price* whereas the supply and demand bids and offers above, will loose the auction. A demonstration of this principal can be seen in fig. B.3.

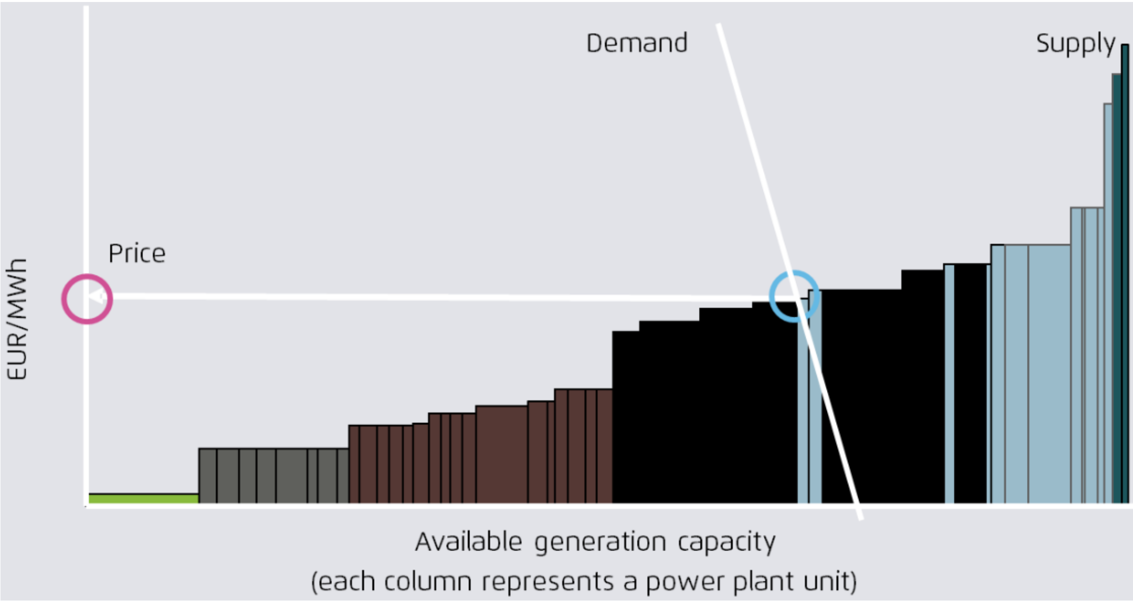


Figure B.3 – Merit order principal and market clearing price, direct replica of figure 11 in [1].

In fig. B.3, the colors are indicative of the technology in the supply stack e.g. green energy, lignite, coal, and gas. The bars increases with the fuel cost used by the technology in question. This is an intended effect of the merit order principal as it encourages the market participants to offer actual marginal cost of producing one unit more of electricity. This auction favors renewable energy generation as the marginal cost of producing one unit more of energy is zero as no fuel is used. On the other hand, the market clearing price could be very high if a gas generator will have to ramp up production for a specific hour and would use an additional feedstock of gas.

B.2.2 Merit Order Effect and Sustainable Energy

The merit order principal initially seem to benefit the sustainable energy source. An unfortunate implication of this principal is evident when renewable energy generators becomes a substantial part of the generation stack. If meteorological variable are favorable for one specific technology e.g. wind, then all wind generators would produce energy which would make the price plummet and the generators would scarcely obtain any profits. This effect is sometimes termed price cannibalism, however, this effect has not been a wide problem until recently due to the feed-in tariff offered by governments to foster installment of renewable energy assets. In Germany, this initiative was introduced in 2012 and sometimes termed the market premium model [1]. In this model, the renewable generators would rely directly on the day-ahead prices quoted on the newly created exchanges. This made the traded volume increase on the exchange and encouraged even more players to enter this more transparent market. With the market premium model, the renewable energy generators would be paid a fixed price say 9 EUR/MWh. When they generated a volume of energy, they would sell it on the day-ahead exchange and obtain some market clearing prices; say 7 EUR/MWh. The government would then only pay the additional 2 EUR/MWh. Now a substantial part of renewable energy was now traded exclusively on the exchange. This meant that they would pay a fixed price for the energy produced which would create a safe environment for investors to allocate capital to sustainable energy projects. However, when the feed-in tariffs started expire the renewable generators were much more exposed to merchant risk hence hedging in futures markets is became increasingly popular.

B.2.3 Liberalization and Thermal Producers

After the liberalization, there initially were no major changes and prices were stable which was also partly due to an overcapacity in the market which had always been there when the generation was nationalized [1]. However, in the early 2000s, old inefficient power plants capacity were decommissioned as a result of overcapacity, low energy prices, more competition on the market, and introduction of CO_2-prices in the EU. The period was short-lived as already in 2008, the lack of overcapacity was a major reason why prices started to surge. Two key reason was the increase in CO_2-prices and commodity prices until 2008. After the collapse of Lehman Brothers, the global recession started which lead to a drastic down-turn in economic activity and energy demand; in turn with a depressing effect on commodity prices and energy prices where lowering. With an increasing built-out of renewable energy sources, the thermal generation was pushed further out the generation stack which meant more oil, gas and coal fired power plans where decommissioned. One can see this directly in the electricity production fig. B.4.
As energy pricing started to have short-term marginal costs as the main driver, the industry has also seen a much greater focus on increasing the efficiency of power plants and their ability to dispatch.

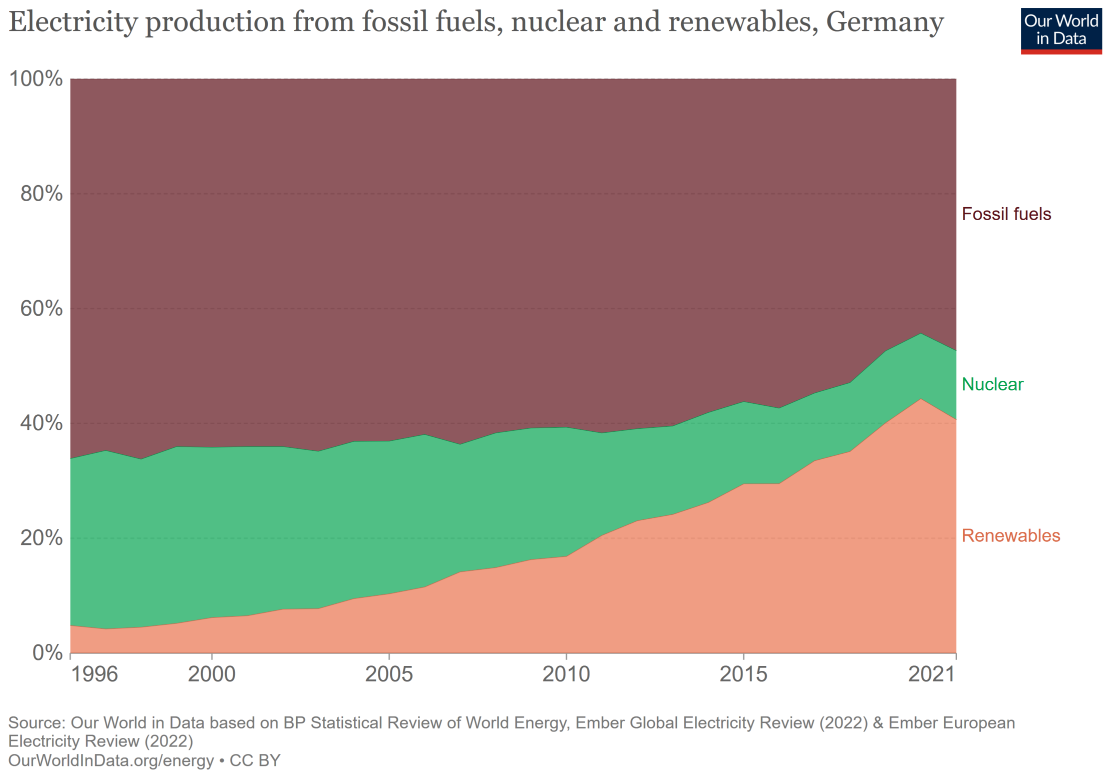


Figure B.4 – The change in generation stack by technology taken from [2]. We see that the amount of energy from fossil fuels takes up a smaller part and the renewable energy takes up a greater part. As a side note, we see a significant drop in the nuclear power generation which especially followed after the German government decision to phase our nuclear power after the disaster in Fukushima.

Due to the merit order principal, we know that it is the marginal cost that is the main driver of the prices and in Germany coal has been a substantial part of the generation stack. This we can see more directly of we compare the gross generation by the type of fuel used, fig. B.5.

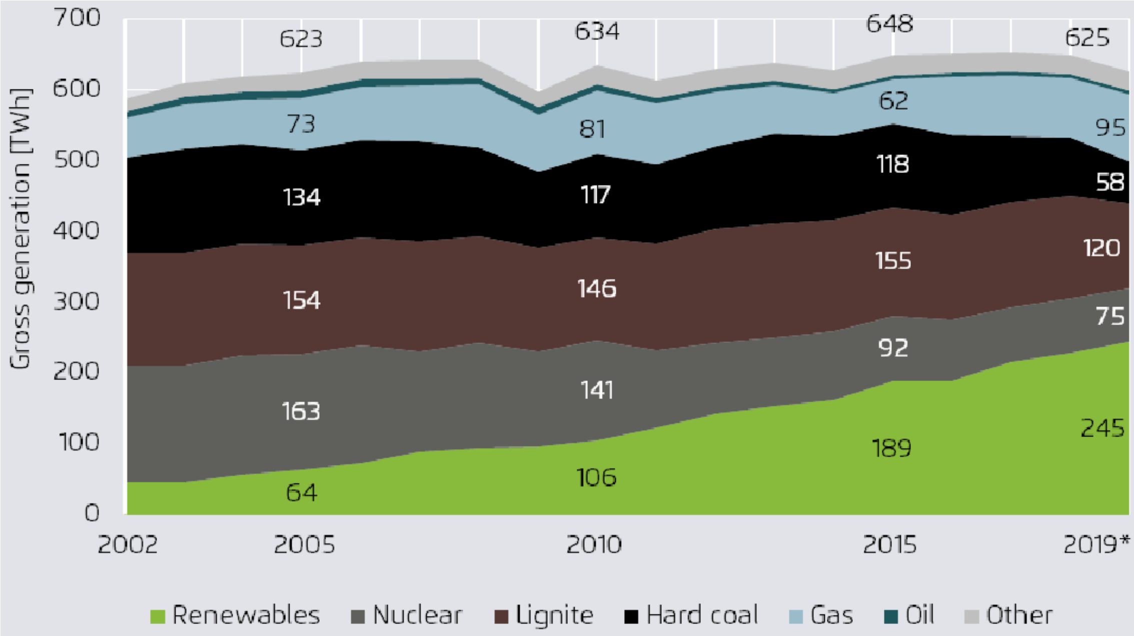


Figure B.5 – The gross generation of electricity by source, a direct replica of figure 14 [1]

In fig. B.5, we see that the total share of coal has shrunk but the hard coal generating more so than the lignite. The main reason for this is that Germany still has large lignite coal mines and is in fact the largest lignite producer in the world [3]. The proximity of the mines and power plans makes the fuel cheap which is also the main reason it is still one of the only profitable means of generation using fossil fuels [3] even tough the pay a high CO_2 price. A recent review on the price determining technology for the German market [4] suggested that indeed still gas and lignite of determines the price of electricity across most hours as seen in fig. B.6. In our analysis, we will also see that of late the gas futures are the main determinant of the baseload futures contracts in Germany. In fig. B.6, the price setting technology in 2020 is displayed. Note that the German market imports a substantial amount of electricity from neighbouring markets which can also be seen.

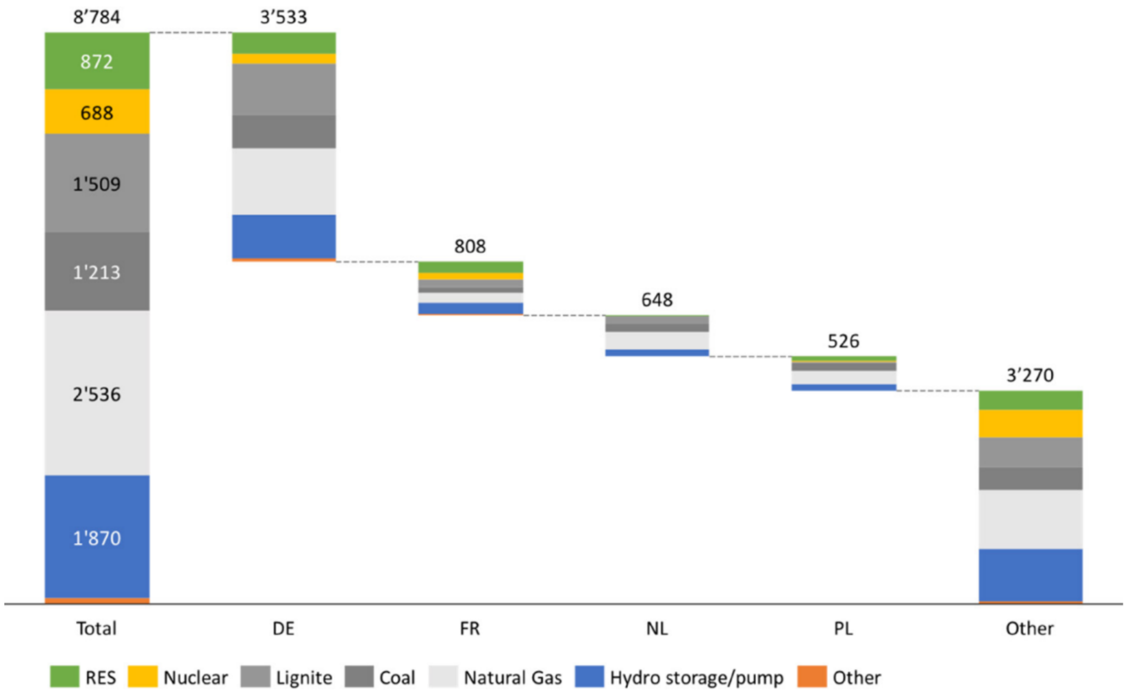


Figure B.6 – Electricity production by technology in 2020 in the German market, a direct replica of figure 3 [4].

As a small note, we will mention that as seen in fig. B.4, the nuclear capacity is declining following the decision of decommissioning of nuclear capacity in the aftermath of the Fukushima disaster. With the market design of favoring high dispatchability, the nuclear power does also not fit in very well, however, it is a non-CO_2 emitting source that delivers a solid baseload.
A key take-away from the above is the the price determining factor for electricity is the fuel cost and price of CO_2. This means the industry is still highly exposed to commodity prices, availability and imports of these commodities.

B.3 Market Participants

After the liberalization of the energy system, new players entered the opened, traded energy markets. This is also in part because the futures started to be cash settled which meant that participants without generating capabilities could also trade the contract *if they wanted the exposure or simple wanted to speculate. In the following, we will describe the main players that emerged and how they affect the generation.
buyers, sellers, speculators and arbitrageurs. The former two are obvious while the latter two have different objectives.

- **buyers** these could be utilities and large energy intensive industries. These would like to get the energy as cheap as possible, try to hedge price spikes in period of high spot-market volatility, and could have a preference for green energy.
- **sellers** would like to sell their production at a premium. With the high penetration of renewable energy, the intermittent production of renewable energy has lead to a high volatility on the spot market. To ensure a stable cash-flow, the producers would accept lower profits for a fixed cash-flow by hedging in forward markets.

- **The speculators** have no interest in any physical delivery of the commodity but instead seek only to make a profit by e.g. buying when prices are low and sell the electricity again when the prices are high. This would often involve taking a greater risk on the traded market in return for a larger profit. In this way, these participants take on the risk the producers are not willing to take and in return they expect a premium. In this way, they also provide liquidity in the market because they make it easier for market participants to find a trading partner. They are often more present in derivative trading with purely financial settlement.
- **The arbitrageurs** are interested in a risk-free profit and they can obtain this by buying a contract in one market at a low price and selling it in another market at a higher price. This is called *local arbitrage*. *Temporal arbitrage* is also possible where the trader would buy a yearly contract at a low price and sell the sum of the quarterly contract to lock in a profit. With these players in the market, we can be sure that the markets are aligned properly both on a temporal and spatial scale.

Though the participants have widely different insensitive for participating, they all provide a vital function to ensure a open, effective, and competitive market.
A potential future actor that is on the door step of the market are the the actors offering flexibility. This could be batteries, EVs and new effective demand responses strategies that would profit from the volatile price movements with the simple strategy of buying energy when the price is low.

C The Fundamental of the Forward Curve

C.1 Fundamentals of Properties of the Forward Curve

In the following, we will cover some fundamental properties of a standard forward curve. Consider a generic forward figure of the forward curves prices quoted on 01-10-2021, dd-mm-yyyy:

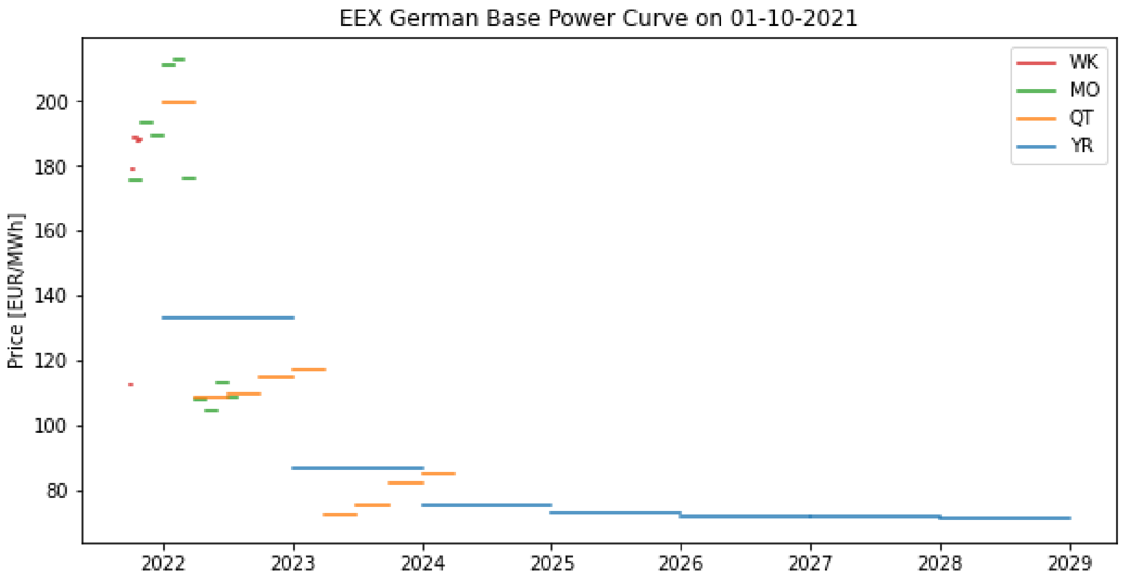


Figure C.1 – Closing price of EEX base load power futures quoted on 01-10-2021

In the following, we will describe and digest the information we obtain from this one instance of a forward curve fig. H.1.

C.1.1 A Stylus Example of a Forward Curve

- Granularity: Each of the lines in fig. H.1 indicates a contract with a duration corresponding to the length of the line and a price corresponding to the level of the line. Notice, that contracts overlap as they can be traded on different granularities; yearly, quarterly, monthly and weekly.

- Cascading: Notice that the contracts go from annual to quarterly, monthly and weekly when we get closer to the time to maturity. This is called cascading and is something the exchanges e.g. EEX do. In practice they take longer contract maturities and replace them by equivalent positions in shorter contract maturities instead of immediate settlement.

- Overall movement: The price is initially increasing and then it decreases drastically after about one year into the future. This is very common dynamics for these forward curves and we will dive into the reasons behind.

- Front-end and back-end: Related to the bullet point above, it is common to refer to the frontend and backend of the curve. In fig. C.2, we have depicted roughly where one would make the cut for this curve, however, there is no exact definition. Usually the liquidity and traded volumes are much higher in the frontend hence effectively much more information is priced in. However, if new information arrives, the prices will fluctuate dramatically hence the frontend is also much more volatile.

- Temporal arbitrage: As mentioned in section B.3, we see an efficient market with no temporal arbitrage possibility. Consider for instance year 2022 and only the quarterly and yearly contracts as in fig. C.3a. Here we see that the sum of the quarterly contracts is equal to the yearly contract. The arbitrageurs in the market constantly insure this as they would always try to make a little extra money by e.g. buying the yearly contract at a low price and selling each of the quarterly contracts at a higher price.

- Seasonality: In fig. H.1 and in fig. C.3a, we see a strong seasonal component which is most obvious for the traded quarterly contracts. Q1 is always traded higher than the Q2, Q3 and Q4. This is because the spring is usually much more uncertain as gas storages are shrinking after a winter, temperatures could still drop, the sun is still only present a few hours a day, and the water reservoir in Norway might not be available due to later snow-melting. This introduces a great level of uncertainties that could push up prices in that period which the consumers want to hedge. However, this of cause comes with a premium.

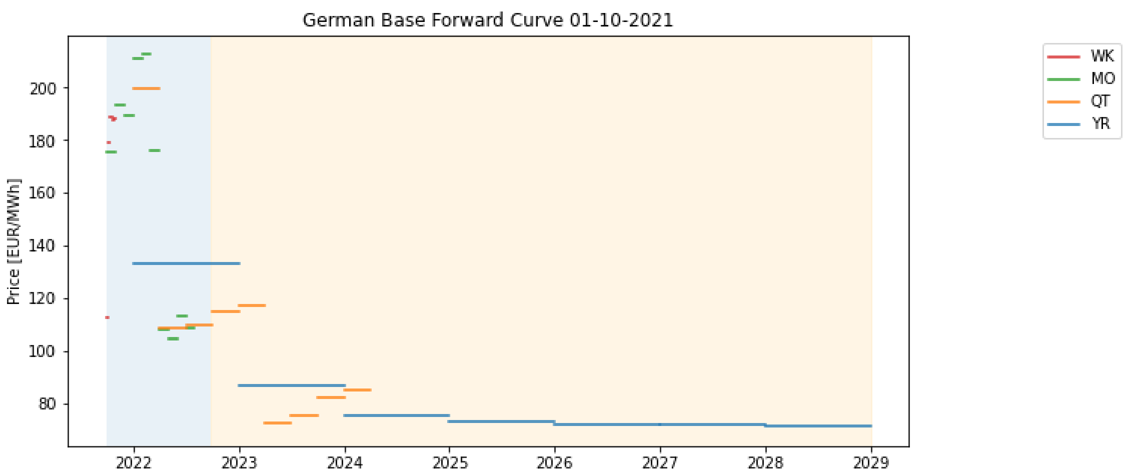
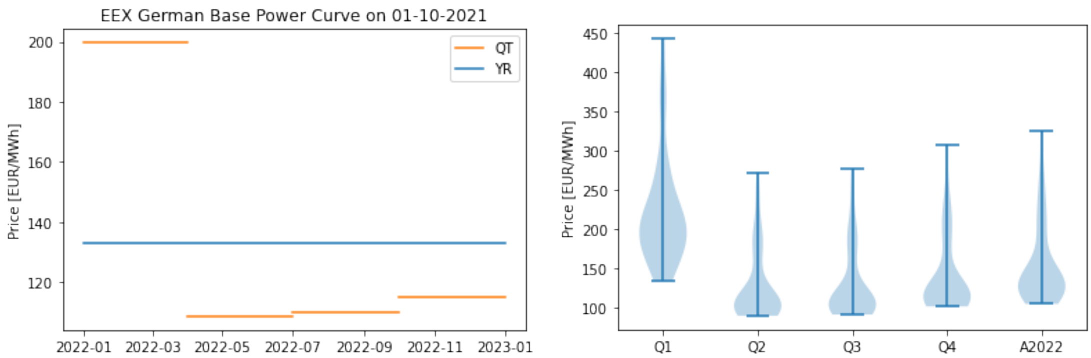


Figure C.2 – A rough annotation of the split between front- and backend of the forward curve.



(a) A focus on temporal arbitrage in the year 2022 for curve depicted in fig. H.1.

(b) The distribution of prices in the traded period from 01-10-2021 to 29-12-2021.

Figure C.3 – Quarterly fundamentals and variability with the yearly contract. In this case, the contracts with delivery in 2022.

From the initial description of the forward curve above, we see that there are multiple sources of variability. In the following, we will consider these forms of variability.

C.2 Variability

The variability in the forward curve originated from multiple interesting sources, market designs, and market participants. In the following, we will consider some of them and use contracts with delivery within 2022 as an example. These are observed over the trading period from 01-10-2021 to 29-12-2021.

C.2.1 Temporal Hierarchical Variability

In fig. H.1 and fig. H.1, we saw that the sum of the quarterly must be equal to the yearly contract. This means we have a natural system of nested observations. For a start, consider first the variation in price for the quarterly contracts in fig. C.3b. Here we see that Q1 varies much more and at a higher level than the Q2, Q3, and Q4. In fig. C.4, we see the distribution of the difference between the quarterly and yearly contracts i.e. $p_{Q_i} - p_A$ for each of quarter, $i = 1, 2, 3, 4$. This underpins the fact that Q1 trades above the yearly contract over the trading period. The Q1-A difference has a large dispersion and is much more positively skewed. The Q2-A and Q3-A is concentrated heavily around their mean while the Q4-A has a slightly higher price and larger dispersion than the Q2-A and Q3-A dispersion. This is because Q2 and Q3 usually have higher temperatures which lowers drastically the demand for gas for heating and electricity generation which in turn lowers the price.

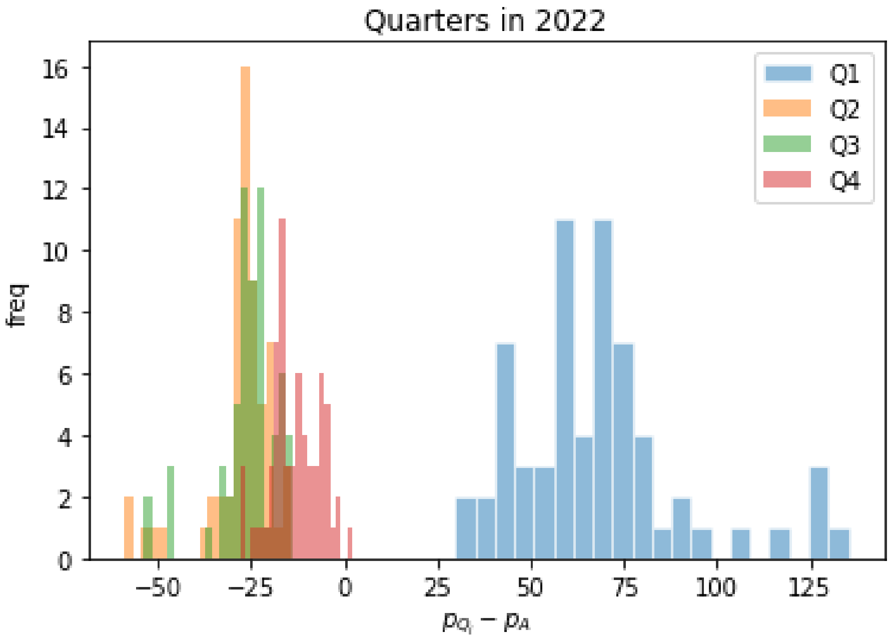
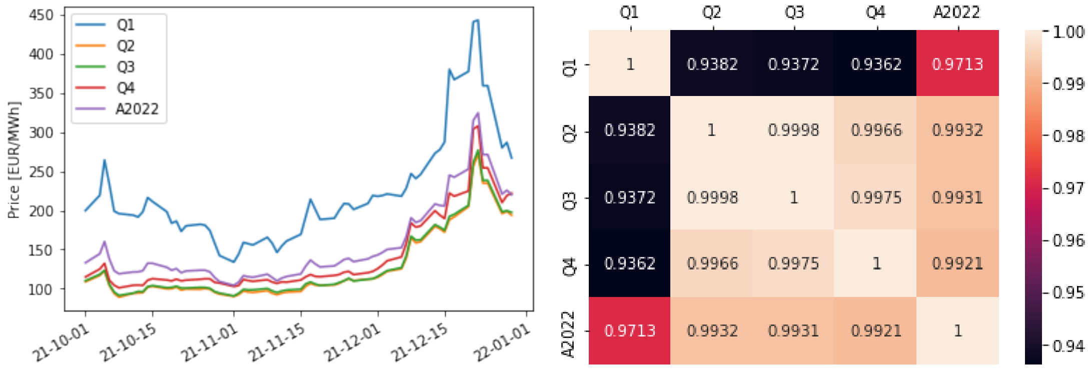


Figure C.4 – The difference between the yearly and quarterly contract $p_{Q_i} - p_A$.

C.2.2 Variability Caused by Time to Delivery

Time to delivery is most correct but often used interchangeably with time to maturity. It is very common that the variation of the price increases with time to delivery as more participants starts to trade and as information is available. At some point, all available information is prices into the contract which means that the variability will tend to decrease again. In fig. C.5a we see the price dynamics of the contracts over the trading period. We see a high degree of fluctuations as the time to maturity, 29-12-2021, approaches. However, when we get very close to the start of delivery, the price drops again in this instance of the forward curve. An interesting point here is that the other contracts co-varies. Indeed, the correlation matrix indicates a very strong linear relationship, fig. C.5b. Therefore, even though the time to delivery for the Q2, Q3, and Q4 is far away, they are still affected so much by the time to delivery of the Q1 and A2022 contract. This also strongly underpins this existence of the *temporal hierarchical variability*. However, as indicated in fig. C.5b, the Q1 contract is the contract with lowest correlation with the other contracts.



(a) The price dynamics in the trading period. **(b)** The correlation of the quarterly and yearly contracts.

Figure C.5 – Quarterly and annually variability close to the start of delivery for the Q1 and A2022 contract.

C.2.3 Spatial Variability

The EEX contracts are traded as one zone across all of Germany hence there is no explicit zonal of nodal pricing. However, it means that a substantial amount of people trade the product which makes it highly liquid and serves as a benchmark for the rest of EU. With the data available, we choose to also include the dutch EEX power futures as it is a neighbouring market and could give an idea about a high-level spatial variability. In fig. C.6, we see that the contracts co-varies highly. The correlation in the trading period is 0.9936 hence the markets are highly correlated. This is also quite intuitive as the Dutch market can be the price determining factor for the German market as seen in fig. B.6. As both markets often has gas generation as marginal producer [4], they would both be correlated highly with the gas prices at the most traded gas hub in Europe, TTF. This in turn will also make the covariation stronger.

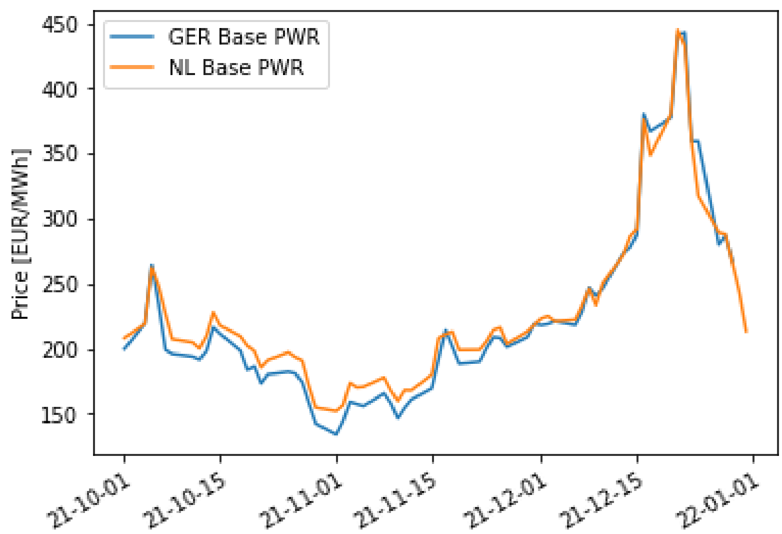


Figure C.6 – The Q1, 2022 contracts of Dutch Base Power contracts and German Base Power contracts

There exists more EEX power futures for e.g. French and Italian power and the differentials are also often traded, see [5].

C.2.4 Demand

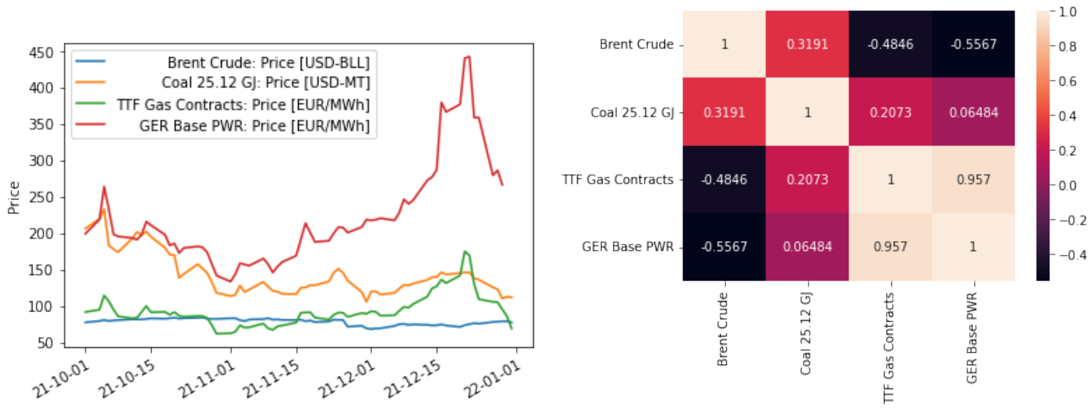
The demand is of cause a very important factor. One can see the demand prices in directly as the large Q1 contract. Here the temperatures are low on average but a mild spring might make demand fall drastically but on the flip side, the spring with many cold days would make the spot price fluctuate greatly which is undesirable for energy consumers. They would like to hedge the price risk by buying this forward contract.

C.2.5 Meteorological Variables

As already mention, the temperature a very important meteorological variable for the demand. However, on the supply side, the wind, solar and precipitation is very important variables for renewable energy generation. The spot prices, day-ahead and week prices would often change drastically when new information about wind and solar is available. As these forecasts are also quite often uncertain, participants will react differently to these signals. The contracts with longer duration would often rely on historical averages which makes them less susceptible to these information shifts and the information would often be priced-in long time in advance as it is available to all participants. We stress that one could make an analysis only on these variables, however, we will focus more on the fuels used by the marginal producers in this project.

C.2.6 Fuel Linked Variability

In section B.2.1, we described how the auction based market clearing would obtain just one market clearing price determined by the marginal producer. In B.2.1, we noticed the distribution of hours where fuels and related technology is the price determining fuel. Therefore, we could imagine that the correlation with the base-load contracts should be related. In fig. C.7a, we see that there seems to be a noticeable co-variation between the TTF gas contracts and the base load contract. It is quite interesting to note that there seems to be a great correlation between Q1 2022 coal and the base-load until close to trading in Novemeber. Then suddenly, the contracts starts to diverge and the coal future is not experiencing any large spike in price towards the end of the trading period. This is also reflected in the correlation matrix over the period which can be seen in fig. C.7b.



(a) The price dynamics in the trading period for the Q1 contract in 2022.

(b) The correlation of the Q1 2022 contracts and the fuels.

Figure C.7 – Co-variability of the Q1, 2022 contract for the base load and fuel contracts.

One could imagine that the coal future would be correlated more with the base load contract fig. C.7a. There are multiple reasons for this:

- The Q1 contract is always more uncertain period with large price spikes which means that dispatchable generators as gas would be favored.
- The Q1 contract should be prices according to the aggregated information over the entire period hence though coal might be the determining technology for some hours, gas would be the price determining technology in the expensive hours.
- The 2021-2022 energy crunch had its peak around the end of 2021 [6]. Though the futures are traded energy in the future, the price is still affected by the condition and frame of mind among the market participants at the day they are traded.

C.2.7 Peakload contracts

On the exchange, peakload contracts are also traded. These accounts for the energy traded in the hours from 8-20 which would often be the ones would high demand hence these are traded at a slightly higher price as seen in fig. C.8. The two prices overall follow the dynamics very closely which is also reflected in the correlation which is 0.9927. In the analysis above, we focused mostly on the base-load, however, we could have done the same with the peak load. The spread between the base and peak load is also sometimes used as an indicator. In [7], they analyse in great detail the German base peak spread. They find that a key driver is the gas-to-coal prices that determines the base peak spreads. Under normal circumstances, coal provides the baseload while gas would be more prevalent in the the peak hours due to the dispatchability. For renewable generator, the spread is sometimes used to proxy hedging price risk. For renewable generators, the volume weighted price is often very different from the average price as wind and solar energy brings down the price significantly and the intermittency of the generating assets means they cannot plan to run in the lucrative peak hours with large profit margins. Instead, they can trade the spread to hedge the production they think will occur in the lucrative hours.

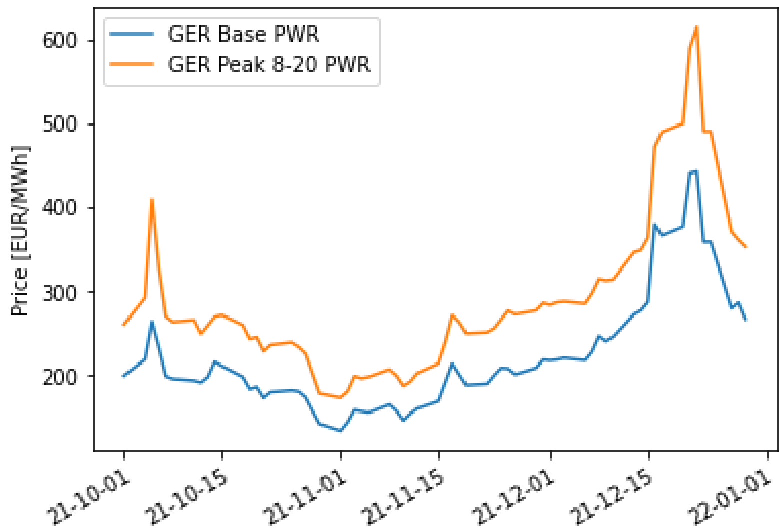


Figure C.8 – The Q1, 2022 contracts of German peak and base load contracts

C.2.8 Financial Spread Trading

As introduce in the sections above, the electricity market is highly depend on related commodities. Indeed, one can even trade the relations between the commodities using, *Financial Energy Spreads*. For gas-fired assets, the differential between the price of electricity and price of the needed equivalent gas is called the *spark spread*. For coal-fired plants the differential is called *dark spreads*. It can be seen as a theoretical measure of the profitability of the given technology under current market conditions is heavily used in the market and creates a natural link between the input fuels and the produced price of electricity. In the following, we will cover the most important energy spreads to better understand how these markets are trading.

Spark Spreads

The spark spreads is a theoretical price difference for a gas-fired powerplant to sell an amount of electricity, having bought the equivalent feedstock of gas. The quoted Platts [8] spreads in Germany assumes the gas-to-power energy conversion factor is 3.412141, an efficiency of 60%, and TTF price assessments as input. In enssens, the spread is calculated as:

$$p_{bss} = p_{base} - \frac{p_{TTF}}{\alpha_{gas}}. \tag{C.2.1}$$

In eq. (C.2.1), p_{bss} is the price of the baseload spark spread, p_{base} is the baseload price, p_{TTF} is the TTF price, and α_{gas} is the efficiency factor which is assumed to be 0.6 for the German market [8]. A very related spark spread is the peakload spark spread which would have the formula

$$p_{pss} = p_{peak} - \frac{p_{TTF}}{\alpha_{gas}}, \tag{C.2.2}$$

where p_{pss} is the price of the peakload spark spread, and p_{peak} is the peakload price. The peakload spark spreads is available for gas because gas-fired power plants often will be the dispatchable technology used in peak hours to make supply meet demand, whereas, e.g.

coal will be in the generation stack and provide energy most of the day hence there does not exits a peakload dark spread.

Clean Spark Spreads

The clean spark spread takes the gas emissions intensity into account and the price of $\mathrm{CO_2}$. The formula is

$$p_{bcss} = p_{peak} - \frac{p_{TTF} p_{\mathrm{CO_2}} \beta_{gas}}{\alpha_{gas}}, \tag{C.2.3}$$

where p_{bcss} is the price of the baseload clean spark spread, $p_{\mathrm{CO_2}}$ is the EUA assessment price, and the emissions intensity is assumed 0.053952 $\frac{\mathrm{tCO_2e}}{\mathrm{MMBtu}}$ [8].

Dark Spreads

The dark spreads is a measure of the price difference between an amount of coal and the equivalent price of electricity. In the Platts quotes, the coal-to-power energy conversion is 6.978 [8] for one metric ton of coal to be converted to one MWh. For Germany, the efficiency of the coal fleet is assumed to be 45%.
The formula for the dark spread is:

$$p_{ds} = p_{base} - \frac{p_{coal}}{\gamma_{cp}\alpha_{coal}} \tag{C.2.4}$$

here p_{ds} is the quoted price of the dark spread, p_{coal} is the coal price, γ_{cp} is the coal-to-power conversion factor, and α_{coal} is the fuel efficiency factor of coal.

Clean Dark Spread

For the clean dark spread we include the emissions intensity factor and emissions price to the dark spread formula from eq. (C.2.4):

$$p_{cds} = p_{ds} - \frac{p_{\mathrm{CO_2}} \beta_{coal}}{\alpha_{coal}} \tag{C.2.5}$$

where p_{cds} is the clean dark spread, p_{ds} is the dark spread defined in eq. (C.2.4), β_{coal} is the coal emissions intensity which assumed to be 0.757 $\frac{\mathrm{tCO_2e}}{\mathrm{MWhe}}$ in Germany [8], and α_{coal} is the fuel efficient factor of coal which is still assumed to be 45% for Germany.

Energy Spreads Effect on the Market

These explicit formulas are traded on the market, and creates a strong bond between interlinked market. Generators use the spread as a mean of hedging their profits. Indeed, the hedge is only theoretical and their generation is not hedged directly hence it is only a *proxy hedge* but it is still very useful. In [9], they show that with the introduction of more renewable assets, the spark and dark spreads fall because the energy prices are lowered. On the flip side, they also introduce more volatility, however, in the period studied, they noticed only minor increases in the dark and spark spreads when volatility increased. Indeed, it was also noticed that the spark spreads are not easy to model because each input follows its own dynamics and calls for different models for each commodity.

C.3 Modelling approaches

We will consider two methods to extract the information contained in the forward markets [10]:

1. The fundamental modelling approach: here we consider driving factors for the electricity markets as futures gas, oil, nearby markets, and coal contracts. [11] [12].

2. The dynamical model: here we model the relationship between historic spot and futures prices. [13]

In both cases, it is important how the price series are considered. In the following, we will briefly cover how we construct the price series.

C.3.1 Construction of Price Series

In the following, we will cover the different modelling choices available when we work with prices series of futures contracts. First we will cover how we can construct long series of e.g. monthly contracts even though they are only traded over a short period and secondly we will cover how log returns can be an effective series to model instead of the prices series directly.

Fixed Contract Series or Rolling Contract Series

As discussed above, the most liquid contracts are the contracts with shortest time to maturity as participants will have more information about the expected spot prices in the period and they will also be much more volatile. This means that often the series are modelled on rolling basis e.g. the first month ahead would be M1, the second month M2, the first quarter Q1, and the second quarter Q2. From a statistical point of view, this also means that we will have more data to work where dynamics is happening. Consider the 22-April contract. If we fix our attention to this alone then with 2-3 months to delivery, the participant will start to actively traded. However, the exchange only quoted closing prices ones a day and only on working days hence we will have around 40 data points and mostly with very different volatility. Therefore, it is useful to instead use the rolling contract series.

Log Returns

In the sections below, we will work with the prices series directly e.g. for baseload p_{base}. Here we will explore how we can use tool from time series analysis to account for autocorrelation and create models that focus on the impact of granularity and gas prices directly. However, it is also very common in modelling of energy prices to use the log returns which would corresponds to the difference between the log of the prices [14]. In the following, let r be the return and p_t be the price today and p_{t-1} be the price yesterday:

$$\begin{aligned} r_t &= \frac{p_t - p_{t-1}}{p_{t-1}} \\ &= \frac{p_t}{p_{t-1}} - \frac{p_{t-1}}{p_{t-1}} \end{aligned} \tag{C.3.1}$$

which we can write as $1 + r_t = \frac{p_t}{p_{t-1}}$ and then define the log returns as:

$$\log(1 + r_t) = \log p_t - \log p_{t-1} \tag{C.3.2}$$

Modelling the prices series has useful properties:

- The series is likely to be more stationary by using the difference and the variance stabilizing log-transformation.
- The represent continuously compounded prices changes.
- They are scaled by the price level at a point. This means that we have more information about the simultanously day-to-day variation in prices across granularities and commodities.
- A linear model of the log returns on the price series would make the coefficients be interpreted as elasticities. E.g. we would be able to make statements as 'If the prices of gas goes up 20% tomorrow, we would probably have a 10 % increase in the baseload power price'. This could be more useful than a model directly on the price levels as we would only make statements as 'If the price of gas becomes above 400 EUR, then we would see baseload power price of 500 EUR'

In the statistical analysis below, it would be stated explicitly which series we model, and the difference will be compared.

D Statistical Analysis

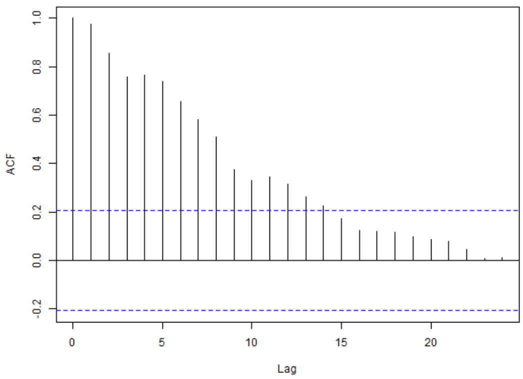
In this chapter, we will consider the evidence of the market effects described in chapter C. The chapter is split sections that each focus on a different topic. Initially, we will consider the raw price series and the statistical properties of quarterly prices for electricity futures. Then we will consider how the quarterly contract varies with the yearly contracts and construct a statistical model. Then we will consider the front M1 contract and the statistical properties in the period before and after the 2021 energy crunch.

D.1 Quarterly Variation in 2022

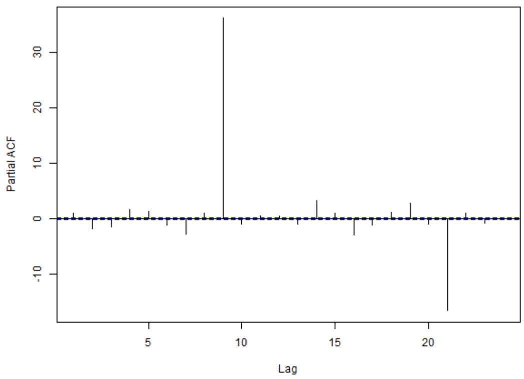
In this section, we will analyse how the quarterly futures contract varies. We focus on the contracts with delivery in year 2022 traded over the period from 01-10-2021 to 29-12-2021 hence the initial exploitative analysis is already provided in chapter C.

D.1.1 Auto-correlation and Trading days

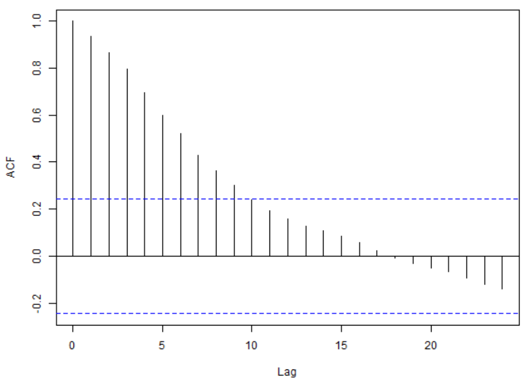
First, we will consider the autocorrelation in the data. As the exchange is only open on business days, then there will be a substantial amount of gaps in the data from e.g. weekends. Initially, we considered the non-trading days as instances of missing data and made AIC and PACF function pass these days i.e. the correlation between Fridays and Mondays were not considered. However, we discovered that this cluttered the PACF, as seen in figure fig. D.1b. Therefore, we considered the series as one consecutive series without any missing data. This made the signal much stronger as seen for Q1 2022 in fig. D.0c and fig. D.0d.



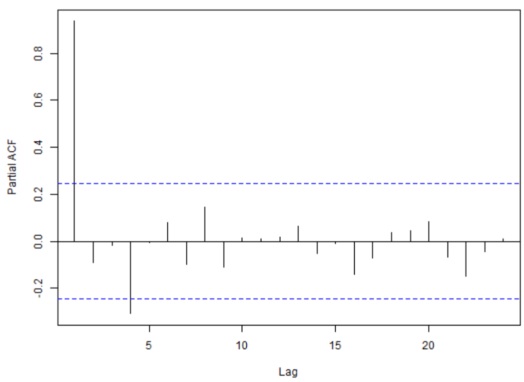
(a) ACF with non-traded days encoded as missing values.



(b) PACF with non-traded days encoded as missing values.



(c) ACF with data treated as consecutive series.



(d) PACF with data treated as consecutive series.

Figure D.0 – Comparison of the auto-correlation for the Q1-2022 contract over the trading period. We consider price series as either a time series with gaps on non-trading days or one consecutive time series.

From fig. D.0, we see that it would simplify the statistical analysis drastically if we considered the series as one consecutive time series. The same is seen for all of the quarterly contracts in 2022 as can be seen in fig. H.2 and fig. H.3

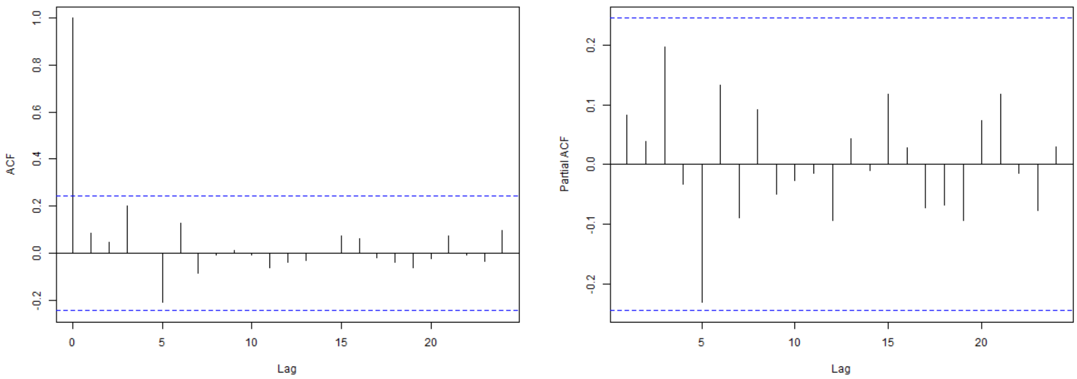
In fig. D.0, there is clear signal in the time series which we identify directly using the rules introduced in [15] and intuition from [16]. The time series seems to follow an AR(1) process. We see this as the autocorrelation is significant for a large number of lags and the PACF has a great spike at lag 1 which means that the subsequent higher order lags are mostly explained by the lag-1 autocorrelation. One could also difference the times series ones which would also make the series more stationary.

D.1.2 AR(1) Model

The AR(1) model is a model of the type:

$$Y_{j+1} = \mu + \phi_1 Y_j \tag{D.1.1}$$

In this case Y_j would be price of a specific quarter e.g. Q1-2022. When we fit the AR(1) model for the Q1 2022 contract, we obtain the following ACF and PACF, fig. D.1.

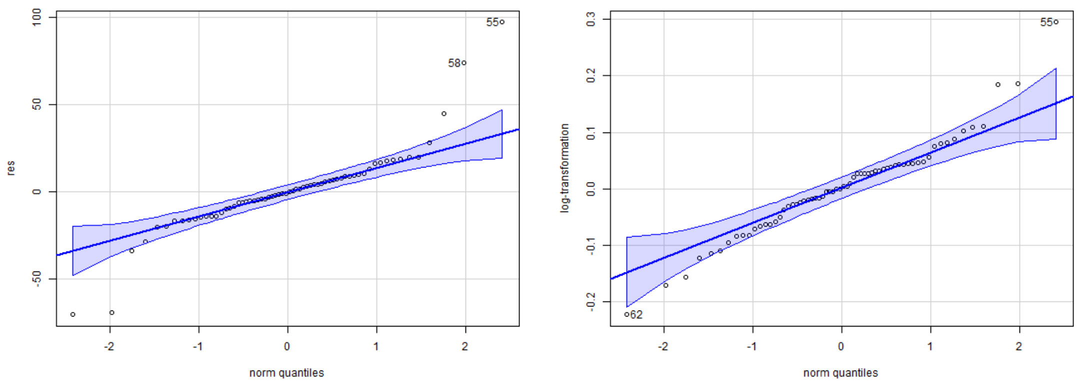


(e) ACF for the AR(1) model. **(f)** PACF for the AR(1) model.

Figure D.1 – Autocorrelation considered for the fitted AR(1) model for the Q1-2022 contract in the trading period.

In fig. D.1, we see that a substantial part of the signal is captures by this simple model. We see the same ACF and PACF for the remaining of the quarters as can be seen in fig. H.4 and fig. H.5.

We will now assess the assumptions of normality of the residuals for the for model. In fig. D.2, we see that there are some problems in ends of the qq-plot. As these forward prices are all positive, we can do a simple log-transformation of the prices to stabilize the variance. When we do this, the qq-plot is improved slightly towards each end.



(a) qq-plot for the AR(1) model. **(b)** qq-plot for the AR(1) model with log prices.

Figure D.2 – qq-plots for the fitted AR(1) model with and without a log transformation of the prices.

We fitted an AR(1) model for each of the quarters hence if we let $\phi_{1,Qi}$ and μ_i denote the model parameters for quarter $i = 1, 2, 3, 4$ we obtained:

	2.5 %	β	97.5 %
μ_{Q1}	154.756	225.929	297.103
$\phi_{1,Q1}$	0.855	0.932	1.008
μ_{Q2}	74.516	138.901	203.287
$\phi_{1,Q2}$ 0.918	0.968	1.019	
μ_{Q3}	75.565	140.989	206.413
$\phi_{1,Q3}$	0.916	0.968	1.019
μ_{Q3}	83.692	152.856	222.019
$\phi_{1,Q3}$	0.897	0.957	1.020

Table D.1 – Parameter estimates for AR(1) models for each quarter.

D.1.3 Linear Regression Models

After this initial analysis with the AR model, we notice that we could describe the data with relatively simple model if we include the autocorrelation. In the following, we will formulate simple linear models with intercepts only. It serves as a way for us to introduce different variance structures of the residuals which we will use for subsequent analysis. Consider, therefore, a very simple model with intercepts for each of the quarterly contracts,

$$Y_{i,j} = \beta_{Q1}\mathbb{1}_{j=Q_1} + \beta_{Q2}\mathbb{1}_{j=Q_2} + \beta_{Q3}\mathbb{1}_{j=Q_3} + \beta_{Q4}\mathbb{1}_{j=Q_4} + \epsilon_{i,j}, \tag{D.1.2}$$

with $\epsilon_{i,j} \sim \mathcal{N}\left(0, \sigma^2\right)$. In the formulation above $Y_{i,j}$ is the price for quarter $i = 1, 2, 3, 4$ over the time points $j = 1, 2, \ldots, N$. We will present the results further below.

Accounting for Autocorrelation

We know that the data has autocorrelation hence we would have to specify this for us to ensure that the estimate of the model parameters is not bias incorrectly. We can specify this using generalized least squares, *gls*, from the package *nlme* from [17]. This makes us able to formulate models in the light of correlated errors and heteroscedastic errors. Initially, we will assume that all quarters share the same AR(1) correlation structure with the same correlation coefficient hence $\mathrm{Cor}(\epsilon_{i,j}, \epsilon_{i,s}) = \rho^{|i-s|}$. If we introduce $\epsilon_i^\intercal = [\epsilon_{i,1}, \epsilon_{i,2}, \ldots, \epsilon_{i,N}]$ for each quarter $i = 1, 2, \ldots, 4$ and trading days N, then we explicitly assume:

$$\epsilon_i \sim \mathcal{N}\left(0, \Lambda_i\right) \qquad \Lambda_i = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{bmatrix}. \tag{D.1.3}$$

This way of factorizing the covariance matrix into a variance parameter and correlation parameter is common in the *gls* framework [17]. We specify this using the argument `correlation=corAR1(form=bus_days_to_maturity|quarter)` as introduced on p. 235, [17].

Accounting for Quarterly Specific Autocorrelation

One could imagine that the variance could be different for each quarter. We can impose this using the argument `weights=varIndent(form= form=bus_days_to_maturity|quarter)` which would amount to the assumed distribution:

$$\epsilon_i \sim \mathcal{N}(0, \Lambda_i) \qquad \Lambda_i = \sigma^2 \delta_i^2 \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{bmatrix}. \tag{D.1.4}$$

To ensure *identifiability*, we impose some restriction such that $\delta_1 = 1$ and $\delta_2, \delta_3, \delta_4$ are the ratio between the standard deviation of the first quarter and the subsequent quarters i.e. $\delta_2 = \frac{\sigma_2^2}{\sigma_1^2}$.

D.1.4 Test of Significance of the Linear Models

We will consider the fit for each of the linear regression models introduced above.

The Initial Simpel model

When we fit the initial model, we obtain a model with one shared variance $\epsilon_{i,j} \sim \mathcal{N}\left(0, \sigma^2\right)$. Here we obtain:

	2.5 %	β	97.5 %
β_{Q1}	209.3381	222.7519	236.1656
β_{Q2}	113.6962	127.1100	140.5238
β_{Q3}	115.6122	129.0259	142.4397
β_{Q4}	128.3830	141.7967	155.2105
σ	50.11791	54.48808	59.69968

Table D.2 – Parameter estimates for eq. (D.1.2).

with all coefficients very significant and AIC on 2756.76 and BIC on 2774.411.

Accounting for Autocorrelation

Then we introduce the autocorrelation structure and test for significance. As these models are nested, we can use an ANOVA test. The AR(1) correlation structure is very significance as seen below.

Df	χ^2_{obs}	p-value
1	617.58	0

Table D.3 – Likelihood Ratio Test after the introduction of the AR(1) correlation structure on the errors.

Accounting for Quarterly Specific Autocorrelation

We now introduce a different variance for each quarters. This models is also a direct extension hence we can use an LRT test to test for significance:

Df	χ^2_{obs}	p-value
1	617.58	0

Table D.4 – Likelihood Ratio Test after introducing difference variances for each quarter to the already established AR(1) structure.

Indeed, this structure is also significant.

Df	χ^2_{obs}	p-value
3	47.51	0

Table D.5 – Likelihood Ratio Test after introducing difference variances for each quarter to the already established AR(1) structure.

After the introduction of these structure the parameter estimates are now:

	2.5 %	β	97.5 %
β_{Q1}	-55.115	230.682	516.478
β_{Q2}	7.716	145.164	282.613
β_{Q3}	6.0352	147.536	289.037
β_{Q4}	-19.817	161.731	343.280
σ	23.725	167.328	1180.130

Table D.6 – Coefficient estimates.

	2.5 %	β	97.5 %
ϕ	0.589	0.990	0.9998

Table D.7 – Correlation estimates.

	2.5 %	β	97.5 %
δ_2	0.3756	0.4809	0.6157
δ_3	0.3867	0.4951	0.6339
δ_4	0.4962	0.6352	0.8132

Table D.8 – Correlation estimates.

Note: for the model to be identifiable, we imposed $\delta_1 = 1$.
From the above, we make the following observations:

- **Insignificant Coefficients** As zero is included in β_{Q1} and β_{Q4}, these are now insignificant with a significance level of 5%. This shows how important it is to take the correlation structure into account as we would otherwise think we have more information than we actually have.

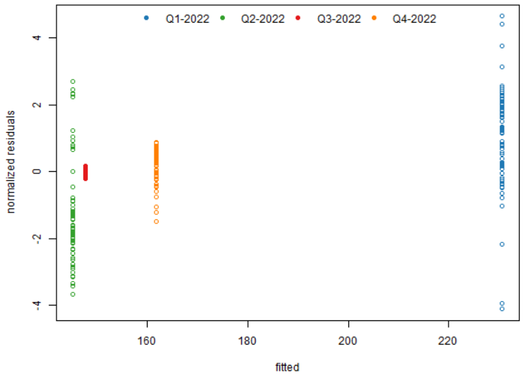
- **The estimates are close to the AR(1)** when we compare the coefficients and autocorrelation with the AR(1) obtained in table D.1, we see they are almost equal. The difference could be partially described by the fact that the REML is used and because all quarters share the same autocorrelation in the model above.

- **The observation that $\rho \approx 1$:** The auto-correlation is very close to 1 which indicates that the model assumes that the errors resembles that close to a random walk.

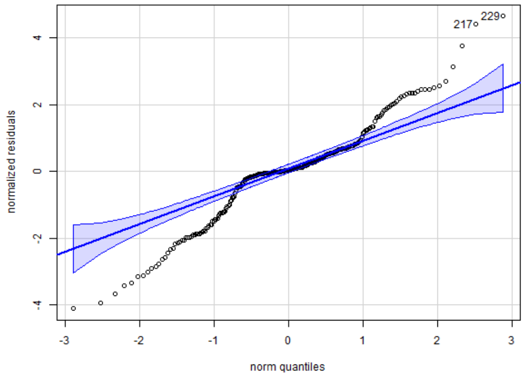
Each quarter has its own covariance matrix, however, as $\Lambda_i \in \mathcal{M}_{(N\times N)}(\mathbb{R}^+)$ and $N = 64$ hence the matrices are quite huge but can be constructed directly from the estimated parameters and the covariance structure from eq. (D.1.4).

Residual Diagnostics for the Final Model

In the following, we will consider the residuals of the final model where we account for auto-correlation for each for quarters. Remember that mean value structure is very simple with intercepts only hence residuals would probably be quite poor, however, if the covariance structure is correct, then the residuals need not be unsatisfactory.



(a) The normalized residuals with the fitted values. They look like this because we only have intercepts in the model.



(b) The qqplot of the errors that clearly shows non-normality.

At first notice fig. D.3a reminds us that the mean value specification is still just intercepts hence the residuals only can be seen on the estimated intercepts. We see that this very simple model seems to fit quarter 3 slightly better as the deviance in the residuals is small. In fig. D.3b, we see that the normalized residuals are far from normal which also shows that the model is misspecified. As we only assume an intercept, the residuals would still have autocorrelation as the series will stay above and below the intercepts for some periods of time which means that we still see a strong auto-correlation in fig. D.3 the ACF plot below.

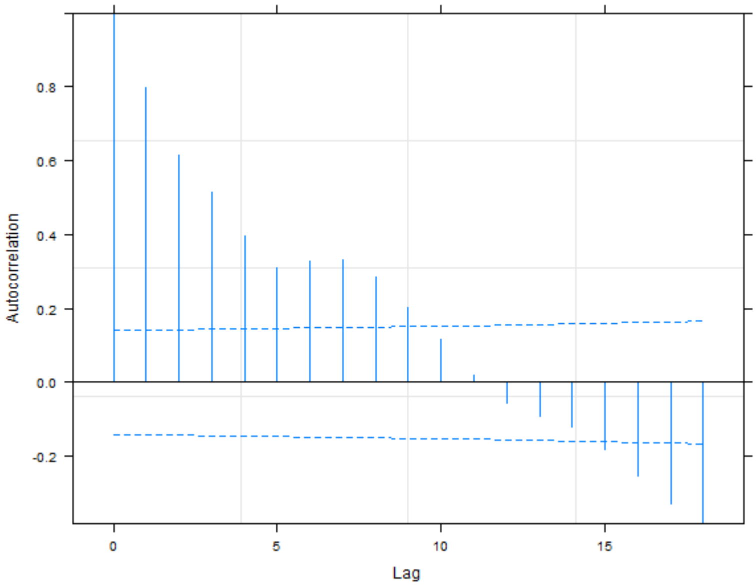


Figure D.3 – The ACF for the final model with AR(1) correlation structure and independent variance for each quarter.

D.1.5 Understanding the Quarterly-Yearly Variation

In this section, we will devote our attention to the relationship between the yearly and quarterly contracts. We already know that they co-vary from section C.2.1, however, it will be interesting to see how. For risk management purposes, a model that can predict the quarters given the yearly contracts would also be useful as quarters are not traded much on the back-end of the curve, but risk management, it is useful to have a *marked-to-market*, *MTM*, value of quarterly contracts as well. Therefore, it will be useful to break-down the contracts to finer granularity.
We will first start simple and use a linear model, eq. (D.1.5).

$$Y_{i,j} = \beta_{Q1}\mathbb{1}_{j=Q_1} + \beta_{Q2}\mathbb{1}_{j=Q_2} + \beta_{Q3}\mathbb{1}_{j=Q_3} + \beta_{Q4}\mathbb{1}_{j=Q_4} + Y_{A2022,j} + \epsilon_{i,j}. \tag{D.1.5}$$

with $\epsilon_{i,j} \sim \mathcal{N}\left(0, \sigma^2\right)$ and explicitly, we still have $Y_{i,j}$ is the price for quarter $i = 1, 2, 3, 4$ over the time points $j = 1, 2, \ldots, N$.
When fit this, all the parameters are significant and we obtain an AIC of 1940.7. As we see below:

	2.5 %	β	97.5 %
β_{Q1}	61.788	67.689	73.59
β_{Q2}	-33.854	-27.953	-22.051
β_{Q3}	-31.938	-26.037	-20.136
β_{Q4}	-19.167	-13.266	-7.365
β_{A2022}	0.97	1.001	1.033

Table D.9 – Coefficient estimates for eq. (D.1.5).

However, we obtain quite some issues in the residuals:

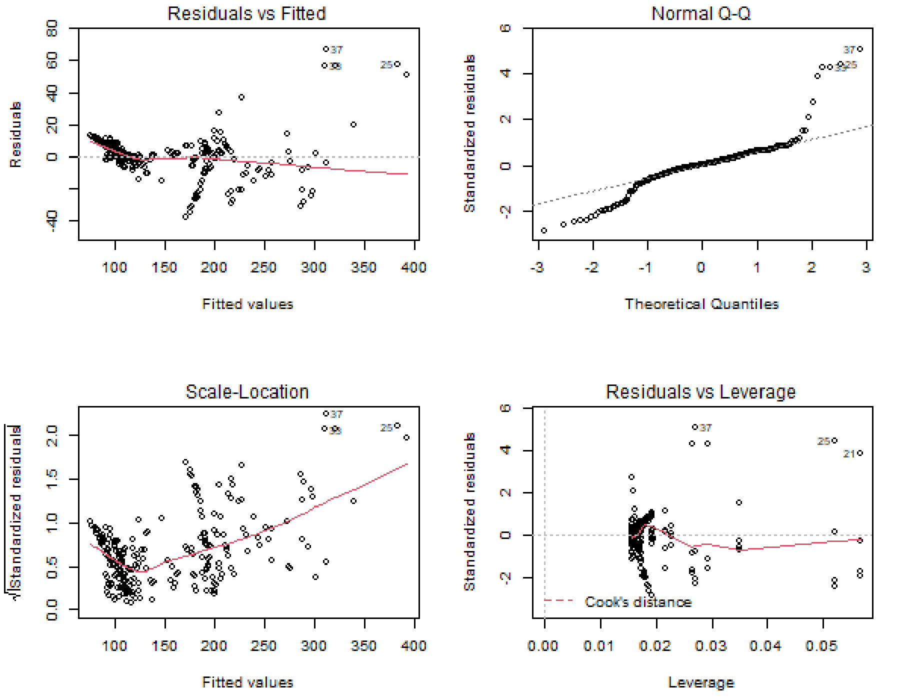
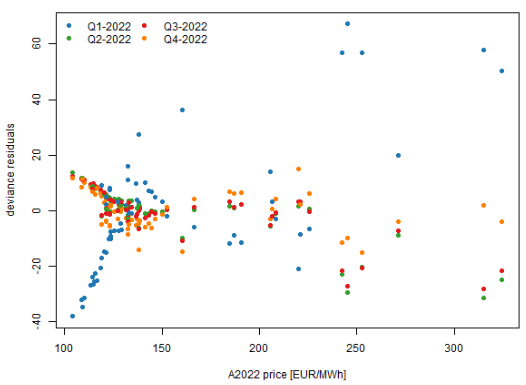
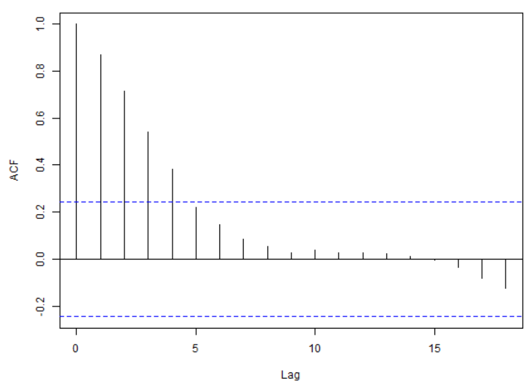


Figure D.4 – Residual diagnostics for the simple model in eq. (D.1.5)

In fig. D.4, we see that the variance seems to grow with the fitted values hence according to section 3.10, [18] we should do a weighted analysis. Additionally, in fig. D.5a, we see that the deviance residuals are increasing as a function of the yearly price hence we will also consider a log transformation. Notice, also there is some variation the model cannot describe when the price is below 150 EUR/MWh. In fig. D.5b, we still see a strong autocorrelation that we should try to better account for.



(a) The deviance residuals for the quarterly prices as a function of the yearly price.



(b) The ACF for the simple linear model for Q1-2022.

Figure D.5 – Deviance residuals and ACF for the model introduced in eq. (D.1.5)

Therefore, we introduce a log-transformation and account for autocorrelation using the correlation specified in eq. (D.1.4) hence also with a different variance parameter for each of the quarters. Hence we specify the model:

$$\log Y_{i,j} = \beta_{Q1}\mathbb{1}_{j=Q_1} + \beta_{Q2}\mathbb{1}_{j=Q_2} + \beta_{Q3}\mathbb{1}_{j=Q_3} + \beta_{Q4}\mathbb{1}_{j=Q_4} + \beta_{A2022}\log\left(Y_{A2022,j}\right) + \epsilon_{i,j}. \tag{D.1.6}$$

In fig. D.6a, we see that the the normalized residuals as a function of the fitted values and there seems to be some systematic variance that we do not capture. Likewise in fig. D.6b, the variation in the yearly price cannot describe the variation in each of the quarterly prices, especially, for the Q1-2022 contracts. This indicates that there is some trading activity on the individual quarterly contract and there is not just a fixed offset from the yearly to each of the quarterly contracts. This is also seen in the ACF fig. D.5c where there is still a strong auto-correlation. The qq-plot in fig. D.5d also indicates deficiencies as there is a very systematic variation towards the mid and deficiencies in the ends.
We see a huge drop in AIC to 1379.437, the intercepts for Q1 is the only significant term while Q2 is close to significant. The yearly price is, however, still very significant. This we can also see directly in the parameter confidence intervals:

	2.5 %	β	97.5 %
β_{Q1}	0.351	0.653	0.954
β_{Q2}	-0.095	0.141	0.377
β_{Q3}	-0.078	0.157	0.391
β_{Q4}	-0.005	0.243	0.49
β_{A2022}	0.898	0.937	0.975

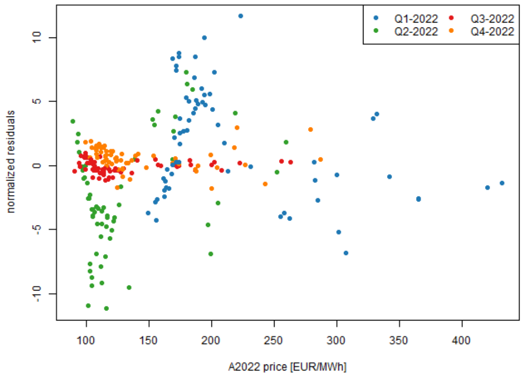
Table D.10 – Coefficient estimates for eq. (D.1.6).

	2.5 %	β	97.5 %
ϕ 0.705	0.976	0.998	

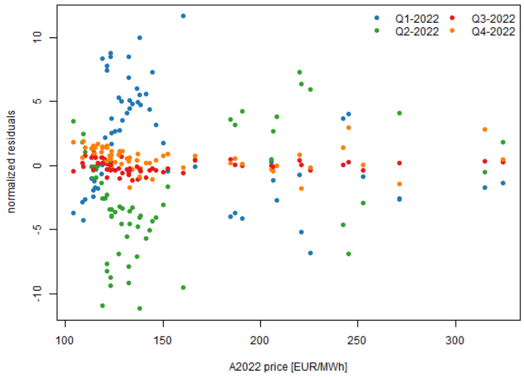
Table D.11 – Correlation estimates

	2.5 %	β	97.5 %
σ 0.041	0.155	0.587	
δ_1 0.440	0.568	0.733	
δ_1 0.429	0.554	0.715	
δ_1 0.511	0.656	0.841	

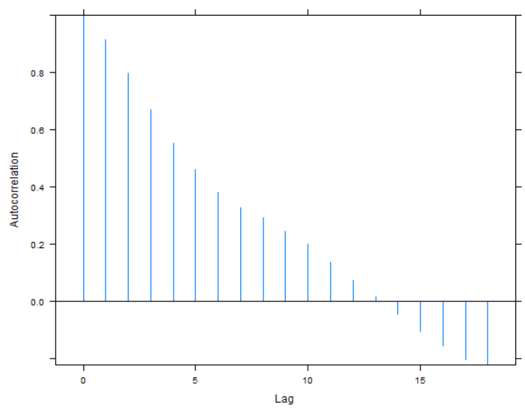
Table D.12 – Correlation estimates



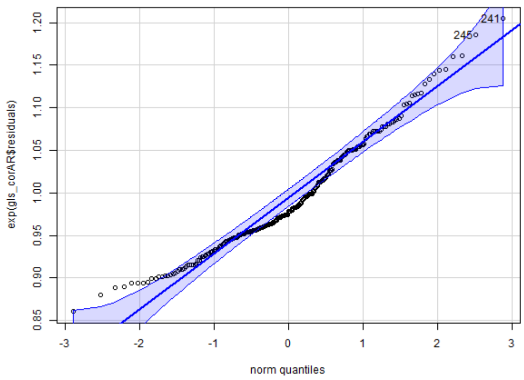
(a) The normalized residuals and fitted values.



(b) The residuals as a function of the price of the yearly contract.



(c) ACF for fitted model.



(d) The qq-plot of the response residuals.

Figure D.5 – Residual diagnostics for the model fitted in eq. (D.1.6)

D.2 Understanding the Cross-Commodity Variation

In this section, we will explore how the commodities co-vary and to initially limit the scope, we will use only the Q1 2022 contract traded in Q4 of 2021 hence we will consider the price series seen fig. C.7a with relevant correlation matrix C.7b. Indeed, this is the period where we enter the energy crunch hence the price is increasing drastically towards the end of the period. It also seems that coal price follow the gas closely in the beginning but towards the end it is only gas contract that seems to closely follow the baseload price. Therefore, we will start only with gas price.

D.2.1 German Baseload and TTF gas interlink

Initially, we will consider a simple linear model where we try to account for autocorrelation. In the following Y_i^{base} is the Q1-2022 baseload price and X^{TTF} is the TTF gas price.

$$Y_i^{base} = \alpha + \beta_{TTF} X^{TTF} + \epsilon_i \tag{D.2.1}$$

with $\mathrm{Cor}(\epsilon_{i,j}, \epsilon_{i,s}) = \rho^{|i-s|}$ hence we will have a structure similar to that of eq. (D.1.3):

$$\epsilon_i \sim \mathcal{N}(0, \Lambda) \qquad \Lambda = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{bmatrix}. \tag{D.2.2}$$

We discovered very fast that the intercept was insignificant hence we removed that to obtain:

$$Y_i^{base} = \beta_{TTF} X_i^{TTF} + \epsilon_i \tag{D.2.3}$$

with which we obtain the following parameters:

	2.5 %	β	97.5 %
β_{TTF}	2.121	2.315	2.509
ϕ	0.5155	0.821	0.941
σ	16.494	27.742	46.660

Table D.13 – Parameter estimates for eq. (D.2.3).

In fig. D.6, relevant residuals are presented. We see that the normalized residuals scales with the fitted values and the TTF gas price. We should do a weighted analysis possibly introducing a log transformation. The qq-plot also indicates few datapoints that seems of. In the time series plot we see a couple of days towards the end in the 2021 energy crunch where the model is off.

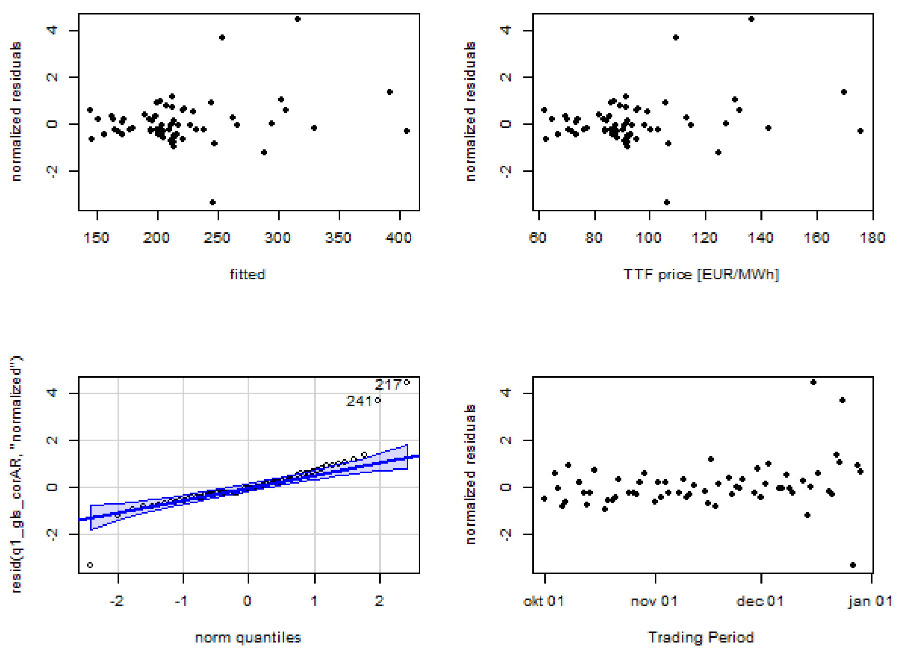


Figure D.6

Introducing a log transformation We now perform a weighted analysis using a log transformation of the baseload and price and gas price i.e. we test the model:

$$\log Y_i^{base} = \alpha + \beta_{TTF} \log \left(X^{TTF} \right) + \epsilon_i \tag{D.2.4}$$

Note that we introduced the intercept again that is very significant now. In fig. D.7, we see that the residuals are very close to that of fig. D.6. Apart from minor changes in the QQ-plot, the transformation did not resolve any problems. Indeed the energy crunch made the market go into a state is has not been before. We will now try to use the log returns to see if it make the model better.

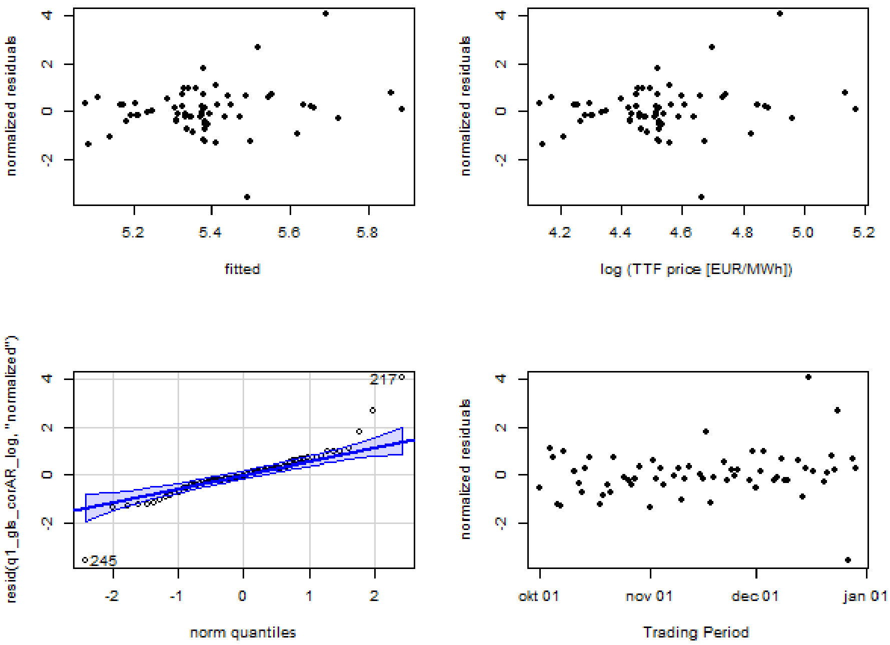


Figure D.7

D.2.2 Log Returns on the Baseload and TTF

Initially, we will first consider the distribution of the log returns for the Q1-2022 contract for German baseload and the TTF contract:

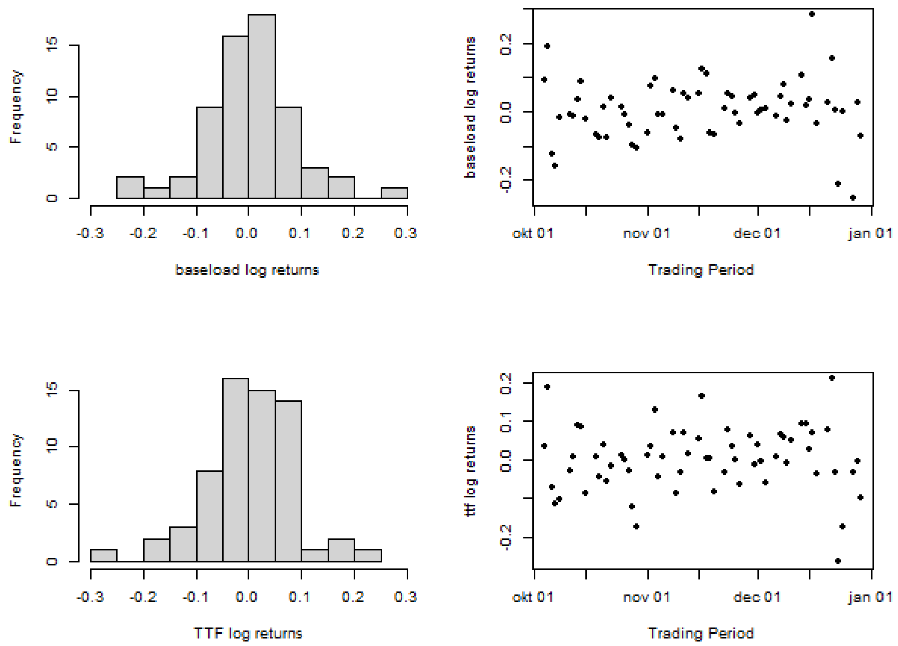


Figure D.8 – Distribution and Time Series of the log returns on TTF and baseload Q1-2022 contracts.

In fig. D.8, we see that the series are not perfectly normal, however, the dynamics is much simpler now. The summary statistics can be seen in table D.14 where we see that indeed the mean is close to 0, the standard deviation is really small. On the other hand, the skewness is negative in both cases with TTF with most skewness. The kurtosis is also larger than the that of a perfectly normal that would have a kurtosis of 3.

	μ	σ	skew	kurt
Baseload Germany	0.0046	0.0856	-0.0208	4.8469
Gas TTF	0.0006	0.0836	-0.2835	4.0616

Table D.14 – Summary Statistics for log returns of Baseload and TTF.

In terms of autocorrelation we see in fig. D.9 that for the log return of the base load contract, there is nothing while there seems to be a bit of signal in first lag of TTF.

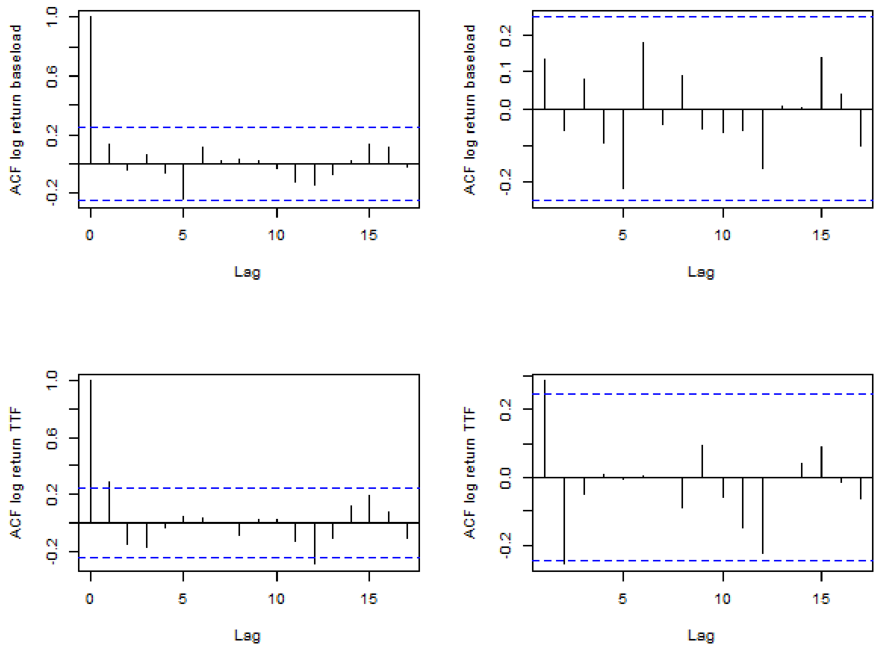


Figure D.9 – ACF and PACF for the log returns on baseload and TTF Q1-2022 contracts.

When we consider the daily log returns plotted against each other it becomes very evident that a simple model can be considered, fig. D.10:

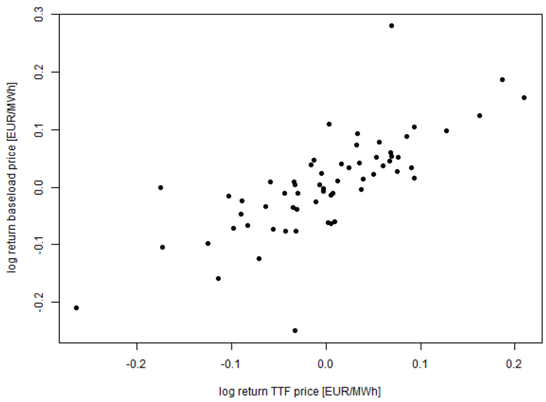


Figure D.10 – The relation between daily log return of baseload and TTF price.

Log Return Model Formulation Therefore, we will start very simply and build a linear model using the TTF log returns as inputs. In the following, we will let Υ_i denote the log return on day i, then we define the model:

$$\Upsilon_i^{base} = \beta_{TTF}\Upsilon_i^{TTF} \tag{D.2.5}$$

Note there is no intercept as it turned out insignificant. As a start, we obtain the following residuals, fig. D.11:

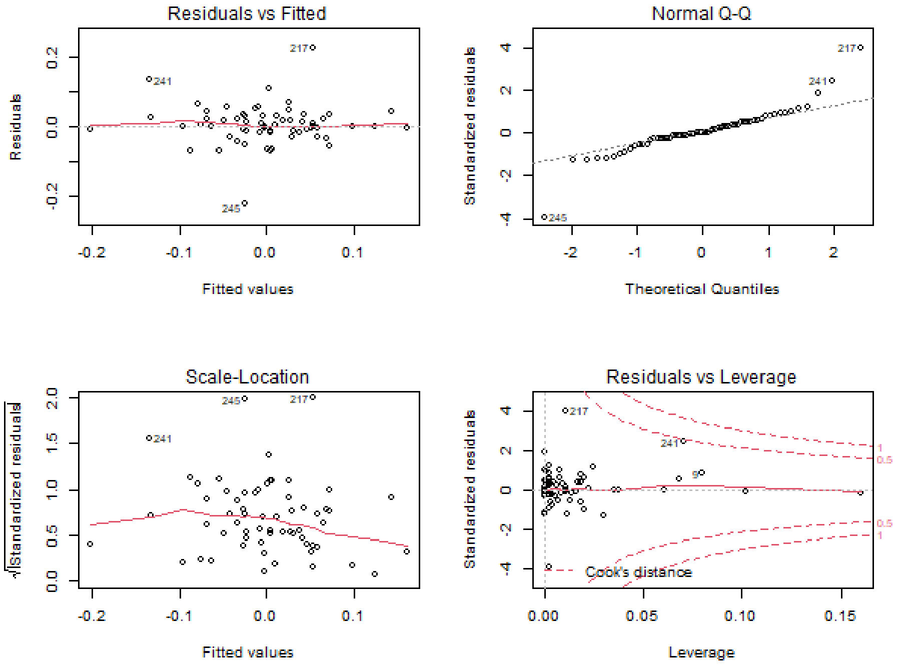


Figure D.11 – Residuals for the model specified in eq. (D.2.5).

The residuals have improved drastically with only minor issues in the scale-location plot and a few outliers in the qq-plot. Indeed, it seems that we have three data points that could be classifies as slight outliers. We will investigate these. Below the time series with the 3 datapoints indicated in red:

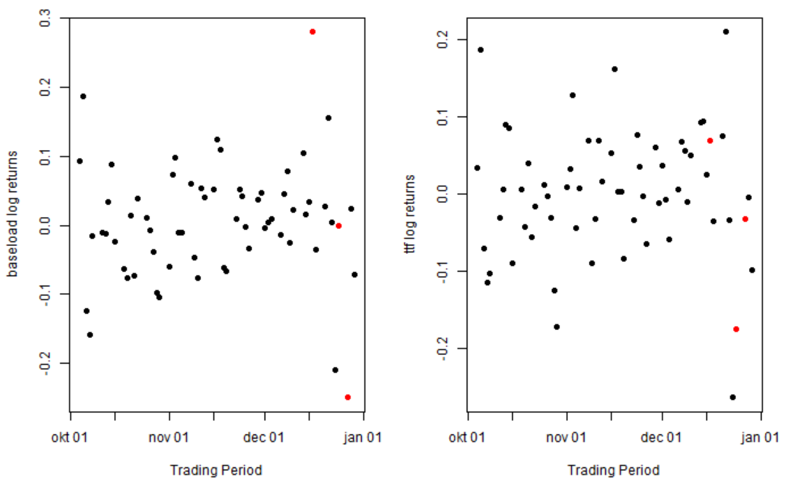


Figure D.12 – Time series with the outliers plotted in red.

It seems that on these three days, there was a - as the model also indicates - a discrepancy between the log returns on baseload and TTF.
Since the residuals look reasonably, we now consider the model parameter:

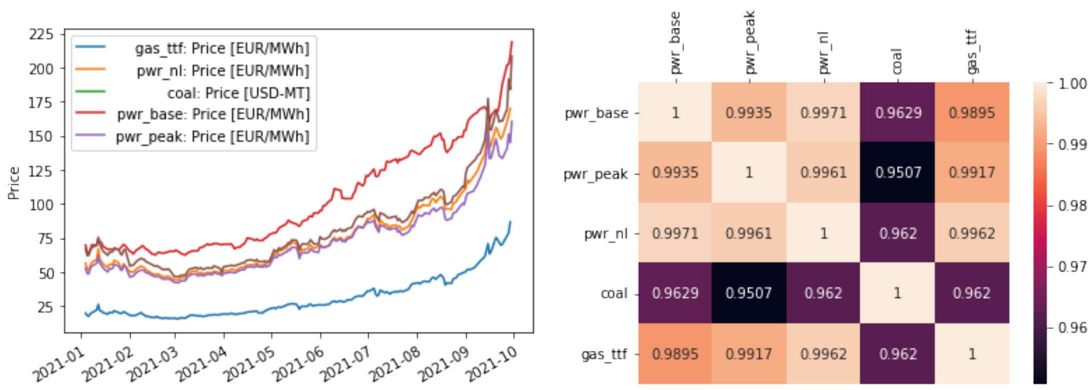
	2.5 %	β	97.5 %
β_{TTF}	0.592	0.765	0.938

Table D.15 – Parameter estimates for eq. (D.2.5).

We can now utilize that we have used log-returns to make and make an interpretation of the result. The above mean value of 0.765 mean that for a 1% increase in the TTF Q1-2022 gas price, we will on average see a 0.765% increase in the German baseload power price for the contract in Q1-2022.
We see that it is much simpler to model the log returns hence we will utilize this in the following analysis of the front month contracts.

D.3 M1 Contracts in the Trading Period from Q1-Q3 2021

In the next section, we will consider the M1 contract in the period from the beginning of 2021 to the start of Q4-2021 i.e. from 1st of January 2021 to 1st of October 2021. We choose to look at the M1 contract instead to consider a different granularity. In the analysis we will will focus strongly on the link between TTF and the baseload contracts as it commonly said that gas determines the price. Below the M1 rolling contracts can be seen:

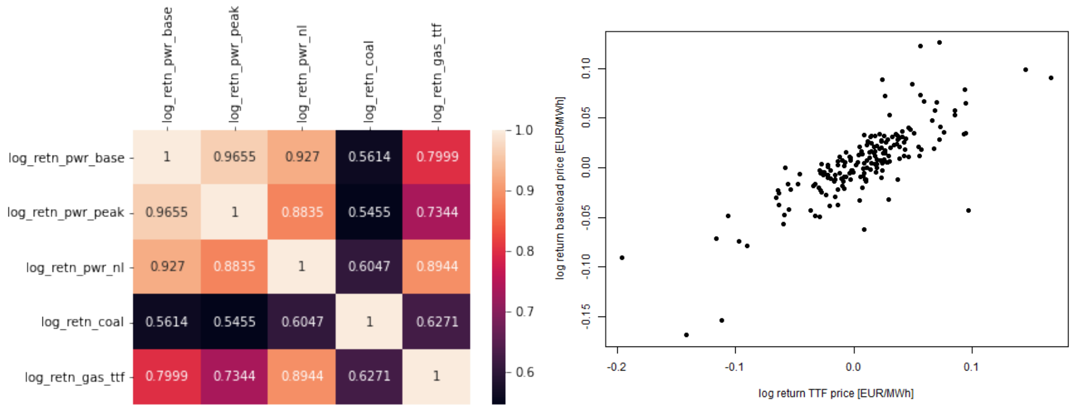


(a) The price dynamics in the trading period for the rolling M1 in 2021 Q1-Q3.

(b) The correlation of the Q1 2022 contracts and the fuels.

Figure D.13 – Co-variability of the rolling M1 contracts in 2021.

In fig. D.13a, we see how the prices are very correlated and all tends to increase towards Q4-2022 following the opening of the society. In fig. D.13b, we see a strong correlation between all prices which indeed stipulates how intertwined the markets are.
We will now consider the log returns and their correlation. Indeed, the log returns are also highly correlated as we see in fig. D.14a. However, we see that the correlation is smaller especially for coal. This tells us that much of the same information is kept in all variables. Therefore, we take the TTF gas prices and baseload contracts and first consider only the linear relationship between the two. This can be seen in fig. D.14b.



(a) Log return correlation for M1 contract.

(b) Log returns of baseload against TTF M1 contracts.

Figure D.14 – Co-variability log returns on the rolling M1 contracts in 2021.

To understand the distribution of the individual prices series, we depict them below. Here we see that log-returns seem more normal and the series also seems more stationary. Indeed, in the ends of each trading period, we see some non-stationarity we should perhaps look at further.

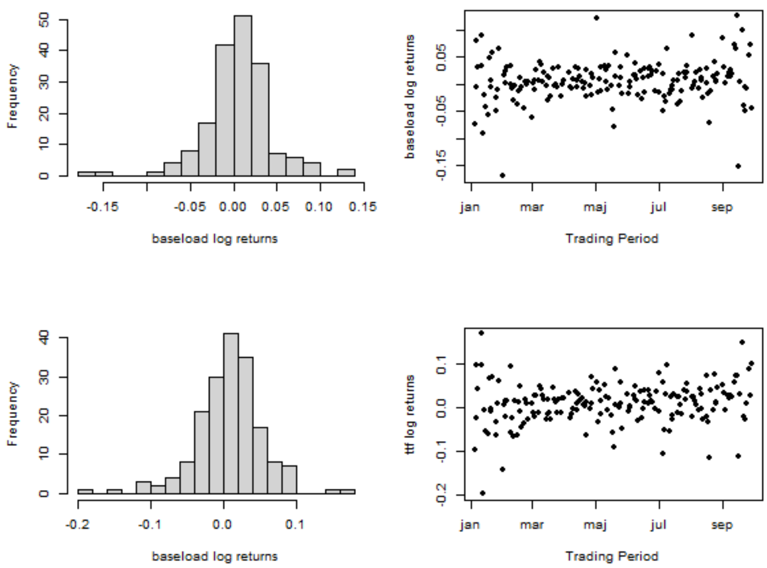


Figure D.15 – Summary figures of log returns on M1 TTF and baseload contracts.

D.3.1 TTF and Baseload M1 Contracts Model for 2021

Initially, we will start with the very simple model as in eq. (D.2.5).

$$\Upsilon_i^{base} = \beta_{TTF}\Upsilon_i^{TTF} \tag{D.3.1}$$

where Υ_i^{base} are the log return for closing prices for baseload M1 contracts and Υ_i^{TTF} are the log returns on the TTF M1 contracts in the trading period in Q1-Q3 in 2021. The diagnostics plot for this simple model are presented below. Indeed, we see that the model is not perfect. The plots with the fitted values and residuals shows slight epecially for very negative values of the fitted values. In the QQ-plot, we also see that we have issues in the ends.

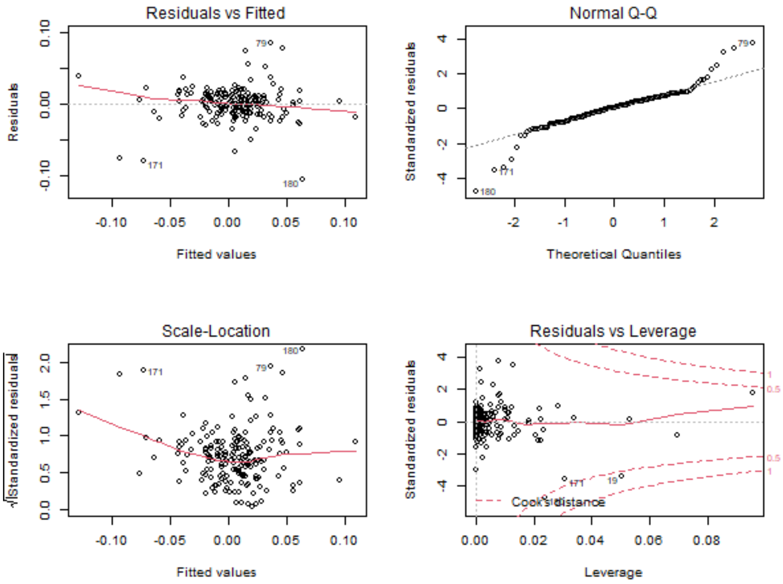


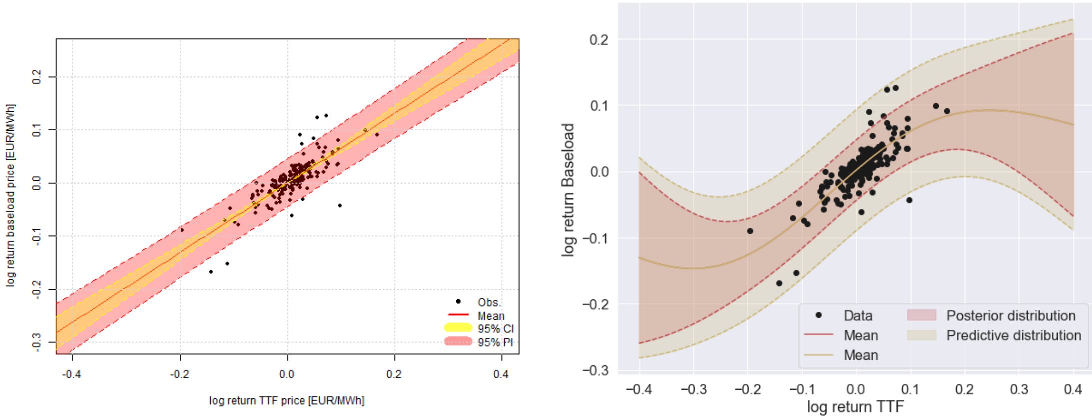
Figure D.16 – Diagnostic figures of log returns on M1 TTF and baseload contracts for the model specified in eq. (D.3.1).

The parameter estimate for this model can be seen below. The parameter value of 0.654 tells us that for a 1% increase in the TTF M1 price, the baseload German M1 contract would increase with 0.653%

	2.5 %	β	97.5 %
β_{TTF}	0.582	0.654	0.725

Table D.16 – Parameter estimates for eq. (D.3.1).

In fig. D.17a, we show the confidence and prediction intervals of the model. We see that it seems like the model contains most of the data in the span of the prediction interval. However, there are definitely some days where the model relationship doesn't seem to be exactly linear especially for days with large changes - both negative and positive.



(a) Confidence and prediction intervals for the computed model.

(b) Bayesian Regression of the M1 contract in the same period.

Figure D.17 – Uncertainty measures for the M1 contracts in 2021.

Bayesian Regression To have a better estimate of the uncertainty, we also tried a Bayesian regression as can be seen in fig. D.17b. Indeed, it seems useful but not necessarily better to describe the market as it assumes no relationship between the factors outside the datasupport. Therefore, we will not introduce this modelling framework rigorously.

Inclusion of Related Commodities

In fig. D.14b, we see there is a high correlation for the input variables hence spurious results will appear if we try to fit a model and indeed the linear changes are almost identical. However, we will try to add coal as it is the least correlated of the input variables hence we fit eq. (D.3.2).

$$\Upsilon_i^{base} = \beta_{TTF}\Upsilon_i^{TTF} + \beta_{COAL}\Upsilon_i^{COAL} \tag{D.3.2}$$

When we do this, we obtain:

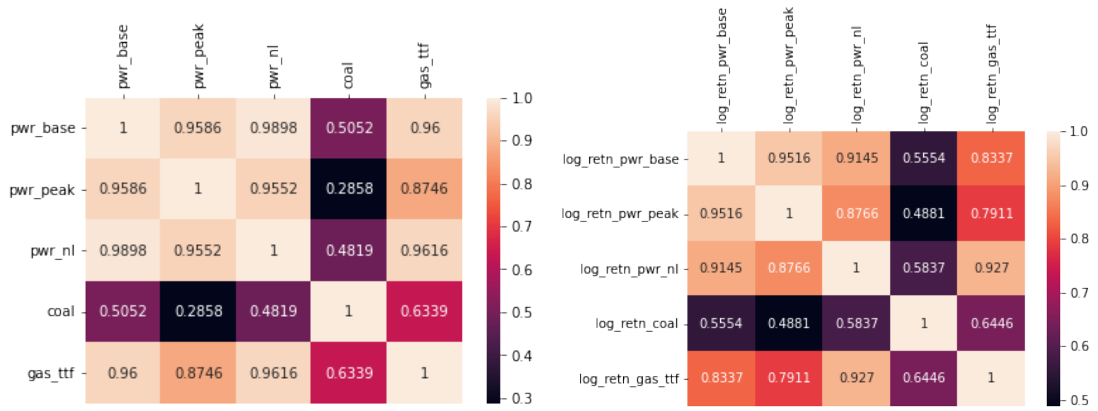
	2.5 %	β	97.5 %
β_{TTF}	0.510	0.602	0.694
β_{COAL}	-0.024	0.162	0.347

Table D.17 – Parameter estimates for eq. (D.3.2).

Indeed from eq. (D.3.2) and the model output, we can see that the coal parameter is insignificant with a 0.95% level hence we should not include it in the model.

D.4 M1 Contracts in the Trading Period from Q4-2021 to 15-08-2022

In the following, we will consider the M1 contracts in the period from Q4-2021 to the 15-08-2022. Below are some is a figures with the correlation with the prices directly and the log returns. In fig. D.19, we see the time series of prices. Here we see that the series is very non-stationary. This is of cause due to the volatile market, however, some of the jumps around month end is also due to the rolling framework where discontinuities might happen when we go from e.g. the March-22 contract to the April-22 contract.



(a) The correlation of the M1 contracts over the trading period.

(b) The correlation of the log returns for the M1 contracts over the trading period.

Figure D.18 – Co-variability of the rolling M1 contracts in 2021-Q4 to 15-08-2022.

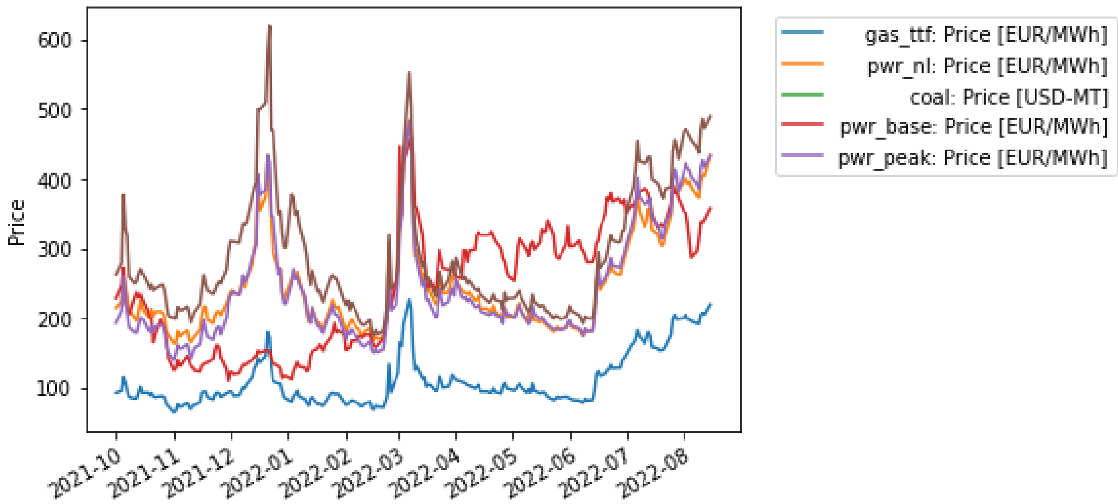


Figure D.19 – The price dynamics in the trading period for the rolling M1 in 2021 Q4 to 15-08-2022.

As above, we will try to make the most simple linear regression of on the log returns. Hence we will use the model:

$$\Upsilon_i^{base} = \beta_{TTF}\Upsilon_i^{TTF} \tag{D.4.1}$$

When we introduce this model, we get the following diagnostics:

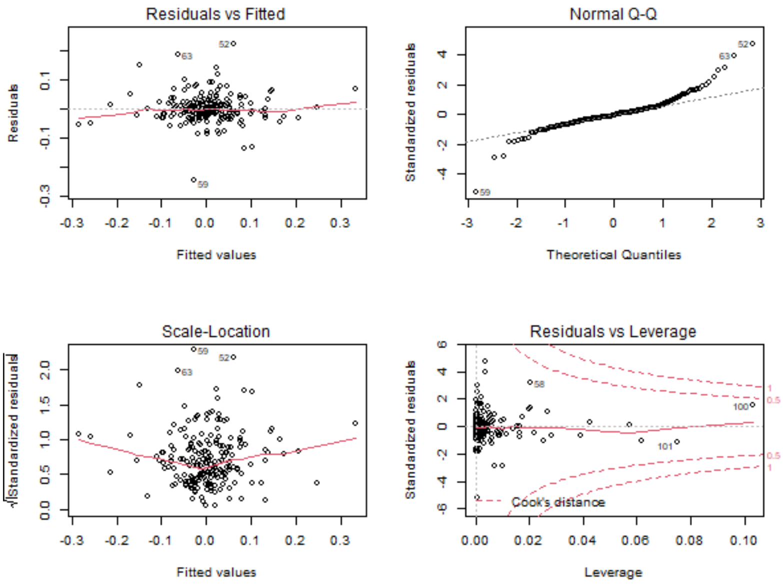


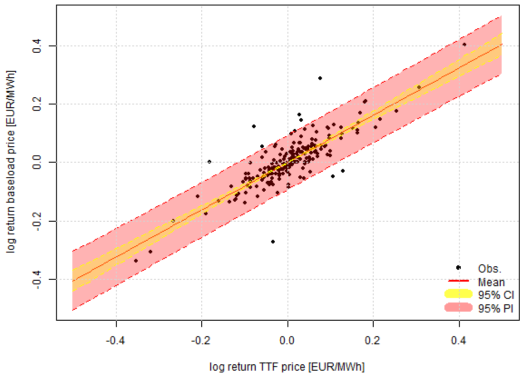
Figure D.20 – Diagnostic figures of log returns on M1 TTF and baseload contracts for the model specified in eq. (D.4.1).

The parameter estimate for this model can be seen below. Here the parameter value of 0.809 tells us that for a 1% increase in the TTF M1 price, the baseload German M1 contract would increase with 0.809%.

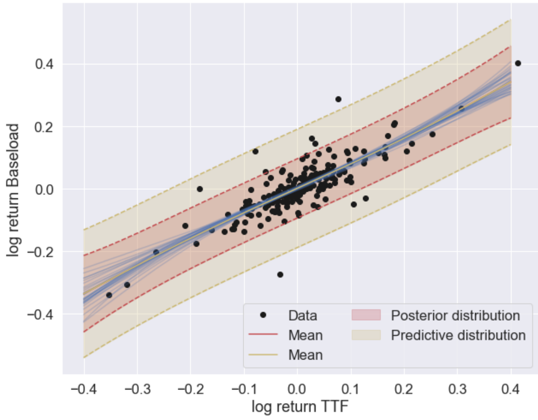
	2.5 %	β	97.5 %
β_{TTF}	0.737	0.809	0.882

Table D.18 – Parameter estimates for eq. (D.4.1).

It is interesting that there seems to be a much stronger correlation between the two in this period. This is also very evident when we depict the final model in fig. D.21a. Even for very large and small log returns from TTF, the return on the baseload is also large. This is also stipulated in the predictive intervals of the bayesian regression in fig. D.21b.



(a) Confidence and prediction intervals for the computed model.



(b) Bayesian Regression of the M1 contract in the same period.

Figure D.21 – Uncertainty measures for the M1 contracts in from Q4 2021 to 15-08-2022

E | Dynamical model

In the following, we will make a brief introduction to the modelling of futures contracts and introduce a mathematical framework to be used for at dynamical model of the EEX baseload futures contract. We will cover only the basic theory relevant for this section and refer [19] and [20] for a more detailed introduction.

E.1 Introduction to the EEX German Baseload Futures Contract

The EEX exchange traded baseload futures contracts are cash settled against the realized hourly EPEX spot prices in the span of the contract period. For instance, the July contract will give the buyer a stream of money in return for a fixed forward price. This stream of money the buyer would retrieve would amount to the sum of the hourly spot prices over the period,

$$\sum_{i=1}^{31\times 24} S(i) \tag{E.1.1}$$

where $S(i)$ is the spot price for day i in July. If we let T_1 be the first hour in July and T_2 be the last hour in July, then we can introduce the forward price of the contract, $F(t, T_1, T_2)$, where $t < T_1$ is the time the contract was entered. The stream of money the buyer should pay in return is equal to

$$(T_2 - T_1) F(t, T_1, T_2). \tag{E.1.2}$$

Indeed, $F(t, T_1, T_2)$ is traded in EUR/MWh for the German baseload EEX contract. Therefore, the buyer of the contract would obtain a profit, π of

$$\pi = \sum_{i=1}^{31\times 24} S(i) - (T_2 - T_1) F(t, T_1, T_2). \tag{E.1.3}$$

Therefore, we see that the forward price should constitute a market consensus price of the average spot price over the period. Indeed, spikes in spot prices due to outages, sudden demand spikes, gas storage and political risk should be information that is prices into the forward contracts.

As the futures contracts are traded continuously it is relevant to introduce a continuous-time modelling approach for the spot prices as well. After all, electricity is a commodity that is generated and consumed in infinitesimal instances. Therefore, let $S(t)$ be a continuous stochastic process which we consider to be the unobservable instantaneous spot price i.e. price of electricity in $[t, \mathrm{d}t)$ on a complete filtered probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, P\right)$.

If we can construct a model for $S(t)$, then we can integrate this over e.g. a specific hour to get the hourly spot price which we could compare with the realized spot price. If we assume a continuous spot price model, we can introduce the continuous equivalent of eq. (E.1.3),

$$\pi_t = \int_{T_1}^{T_2} S(u)\mathrm{d}u - (T_2 - T_1)\, F\left(t, T_1, T_2\right). \tag{E.1.4}$$

where π_t would be the anticipated profit at time $t \in \{\mathbb{R} :\ 0 < t < T_1\}$. If we consider he futures to be settled at the end of the delivery period, the futures prices would be defined as in (1.12) [19]

$$F\left(t, T_1, T_2\right) = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(u)\,\mathrm{d}u|\mathcal{F}_t\right] \tag{E.1.5}$$

where $\mathbb{E}_Q\left[\cdot|\mathcal{F}_t\right]$ is the conditional expectation under the pricing measure $\mathbb{Q}$ and the historical filtration $\mathcal{F}_t = \sigma\left(S_u :\ u \leq t\right)$ as noted in [19] [20] [21] there will be multiple candidates to the measure $\mathbb{Q}$ as long as they are equivalent to the real world measure $\mathbb{P}$. Therefore, $\mathbb{Q}$ need not to be a martingale measure as is otherwise often assumed for many traded assets classes in mathematical finance [22]. This is because unlike shares and bonds, you cannot hold the underlying electricity that gave rise to the realized spot prices. Therefore, we cannot use no-arbitrage argument and subsequent stipulation of $\mathbb{Q}$ to be a martingal measure. We refer to [19] [20] [21] for further details. Instead, we focus our attention to a stochastic spot price model we can use to calculate the conditional expectation that is key in pricing the forward contract.

E.2 Spot Price Modelling

The dynamics of spot prices is characterized by three important components

1. strong mean reversion component
2. spikes caused by e.g. outages or favorable winds
3. a strong seasonal component

Based on these characteristics, we introduce the following model for the spot price, S_t

$$S_t = \Lambda\left(t\right) + X_t + Y_t \tag{E.2.1}$$

where $\Lambda(t)$ is a deterministic function to capture the seasonal component in the spot price, X_t is an Ornstein-Uhlenbeck process defined by the SDE

$$\mathrm{d}X_t = \alpha X_t \mathrm{d}t + \sigma \mathrm{d}W_t, \tag{E.2.2}$$

with the mean reversion parameter $\alpha \in \mathbb{R}^+$, $\sigma \in \mathbb{R}^+$ is the volatility, and W_t is a standard Brownian motion. X_t will constitute the long time dynamics of the spot prices while Y_t is to represent the short spikes with

$$\mathrm{d}Y_t = -\beta Y_t \mathrm{d}t + dL_t \tag{E.2.3}$$

with the mean reversion parameter $\beta \in \mathbb{R}^+$ and L_t is a square integrable Lévy process.

F | Discussion

This project has been a research in electricity futures markets and how the price dynamics could be modelled. There exists a great amount of litterateur on the topic and a great variety of questions can be posed and the outcome of this analysis is just a collection of possible analysis. Initially, we considered the prices directly which turned out to be much harder than anticipated. It was not trivial capture inter-quarter variability as the participants started to trade the quarters very differently and the correlation structures could very quickly become quite complex. Even with the yearly price, the model of the quarterly contracts was not simple which tells us that the participants indeed change their view on how the quarters should be traded relative to each other. The analysis turned out much easier with log-returns and when we introduced the TTF gas prices, a simple linear model could be used. This tells us just how important the gas market is for the electricity prices. We could have made a more in-depth analysis of related commodities such as the dutch, peak and even French power contracts but we choose to only focus on gas. Indeed, if the input series are to correlated, then it would be hard to understand from which series the signal is coming. In the end, they are all strongly related and traded even with financial differentials so all participants will ensure that the contracts have priced in all information and arbitrager would use any opportunity on a mispriced related contracts. This makes the statistical analysis more complicated.

One could question why the TTF gas prices are used instead of the German hub, *Trading Hub Europe*, THE. This was lunched on the 01-10-2021 hence in the mid of the period considered. Therefore, we used instead the TTF prices which is the most liquid gas hub in Europa.

The main focus of this study was statistical models and properties of the futures prices but of cause the spot prices are just as important. Therefore, the chapter on the dynamical model was introduced to lay out the groundwork to understand the relation and fit a dynamical model in the next course.

G | Conclusion

This project serves as an explorative analysis of the dynamics of baseload contracts in Germany. To better understand the mechanics of the market, we covered how the futures and spot market were constructed when the markets were liberalized, how political incentives affects the market, the development of the generation stack, market participants, granularities of contracts, forward curves construction, and related commodity futures. In our statistical analysis we covered different aspects of the futures market We found that quarterly contracts from the same year covaries but Q1 contracts tends to have a much larger variance than Q2-Q4 and the best model to describe this was a linear model with a correlation matrix that accounted for the autocorrelation for each of the quarters. We found a strong, very significant link between the German baseload and the corresponding TTF contracts both for yearly and monthly contracts. Using log returns made the modelling much simpler and we were able to make some interesting conclusion about the dynamics of the M1 contracts before the 2021 energy crunch and after the energy crunch. Before the energy crunch a 1% increase in the TTF M1 contract would lead to a 0.653% increase in the baseload contract whereas after the energy crunch the 1% increase would lead to a 0.809% increase hence the market is much more dependent on gas. We considered many different modelling framework and more advanced models structures and gained a great understanding of the statistical properties. In the end, we presented a dynamical model and link between the spot and futures market that will be the building block for our future studies in the next time to come.

H | Appendix

H.1 Average Quarterly Distribution of Wind in Europe

In the figure below, a comparison of the yearly to quarterly average wind speeds are compared. All averages are calculated 100m above the surface.

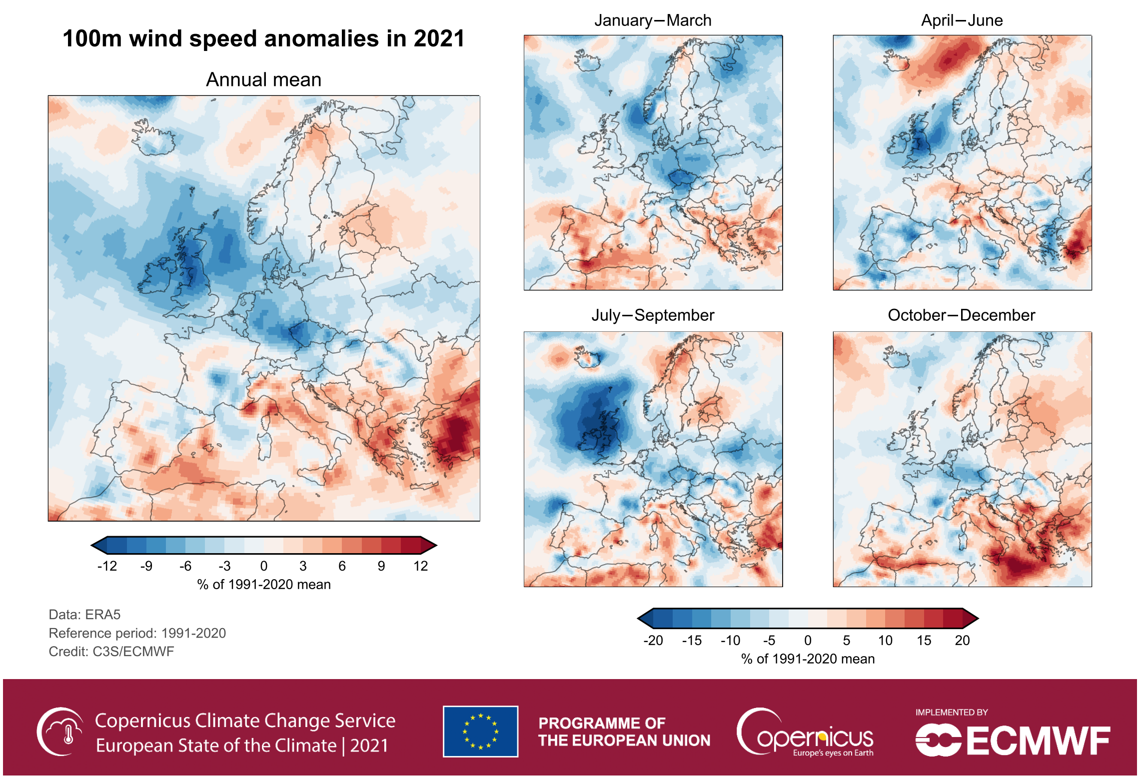


Figure H.1

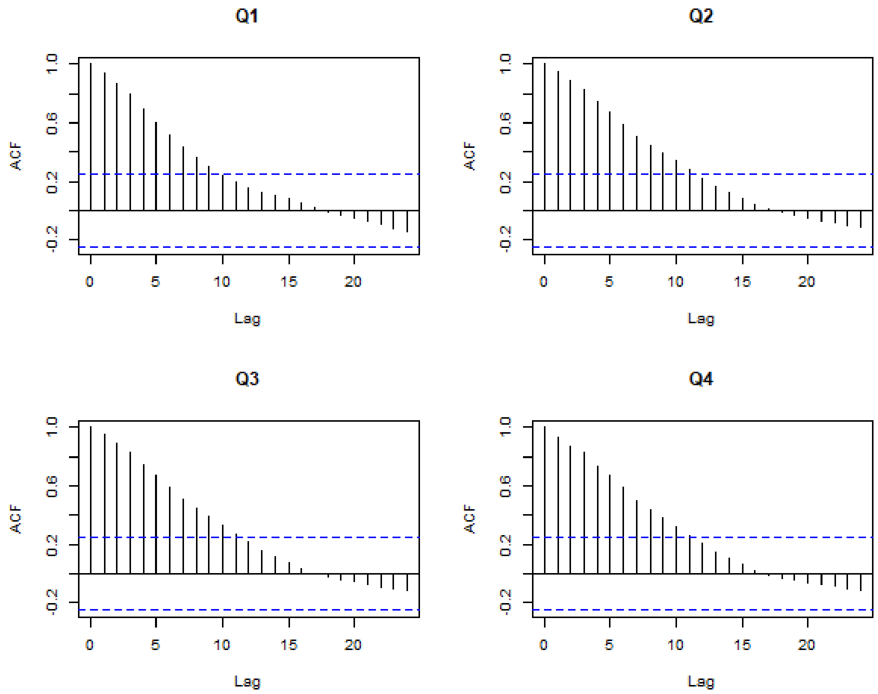


Figure H.2 – The ACF for each of the 2022 quarterly contracts where the prices are considered as one consecutive time series and non-traded days not considered as days of missing values.

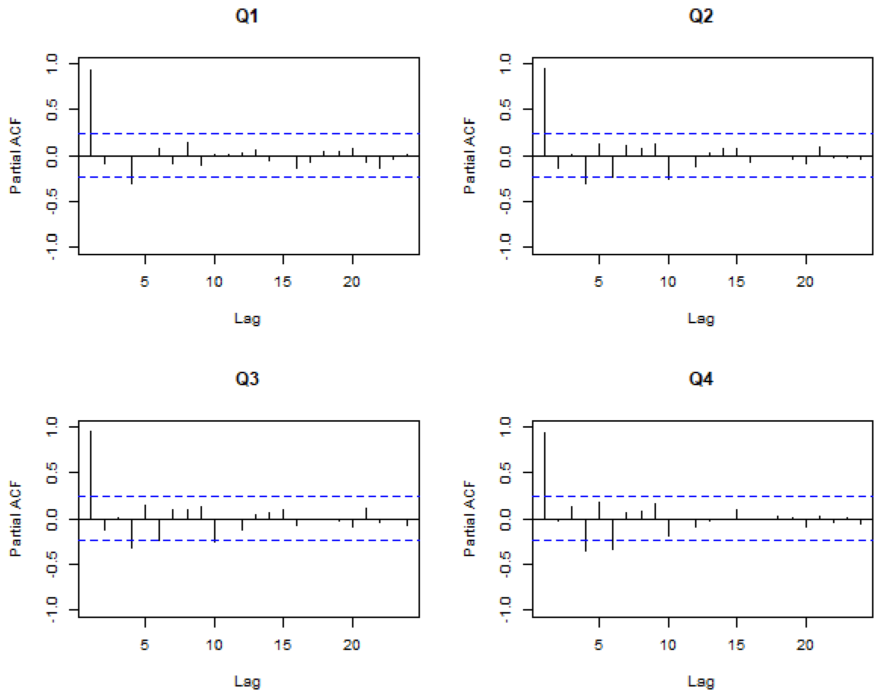


Figure H.3 – The PACF for each of the 2022 quarterly contracts where the prices are considered as one consecutive time series and non-traded days not considered as days of missing values.

H.1.1 AR(1)

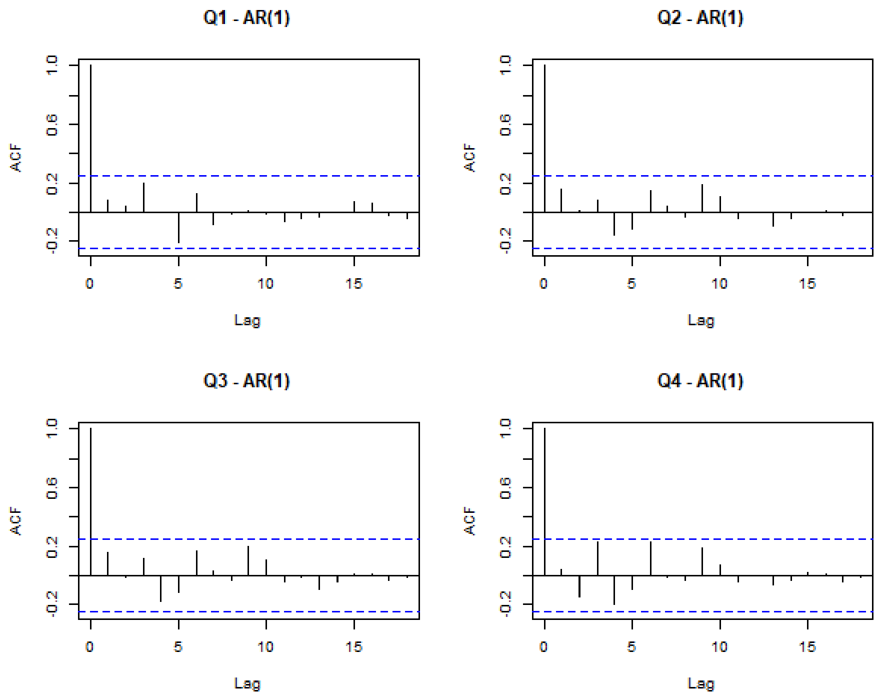


Figure H.4 – The ACF for each of the 2022 quarterly contracts for an AR(1) model.

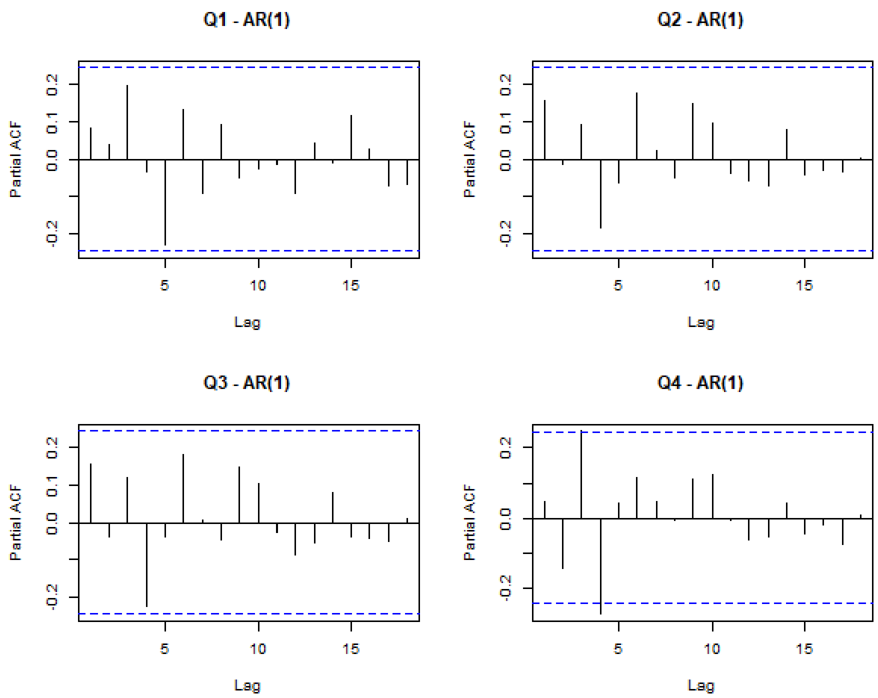


Figure H.5 – The PACF for each of the 2022 quarterly contracts for an AR(1) model.

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