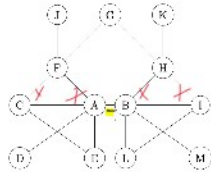




Lab 2: Neighborhoods

Van Roy, Peter Duróczy, Balint Gillurd, Xavier
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1 Bridges & neighborhood overlapping



In the above graph,

1. What edges are bridges? Which ones are local bridges?
2. Using neighborhood overlapping compute the strength of edges A-F, C-E, A-B.
3. If we were to remove edges C-E, A-F, B-H, H-I, what could we say about A-B?
4. Show that for any bridge, the neighborhood overlapping is always 0.

$$AF: \frac{1(1)}{6(1+3+4)} = \frac{1}{14}$$

$$CE: \frac{2(1)}{3(3+4)} = \frac{2}{21}$$

$$AB: \frac{0}{4} = 0$$

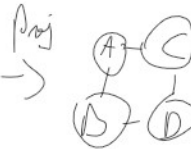
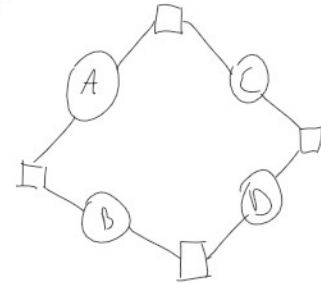
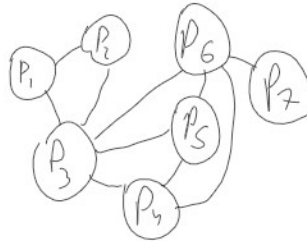
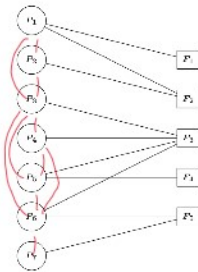
$$\text{overlapping}(A,B) = \frac{|A \cap B|}{|A \cup B|}$$

2 Affiliation graphs

Build an affiliation graph such that two people A and B be a local bridge in the graph projected onto people (draw it). Now colour the edges you would remove in that graph to make A-B a bridge rather than a local bridge.

3 Focuses

Given $G = (P, F)$ an affiliation graph where P denotes the set of people and F their focus. G is said to be *projected onto people* if there exists an edge between people $\{A, B\} \in P$ if they share a common interest (focus).

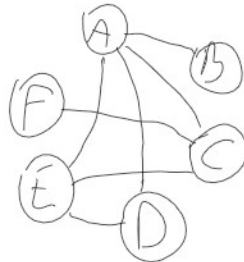
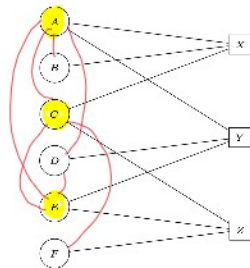


Considering the above graph:

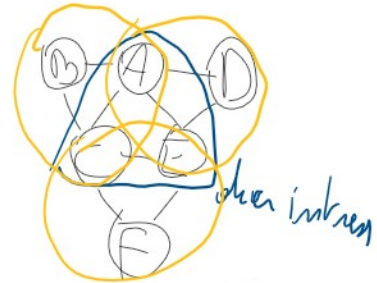
1. Draw the corresponding graph projected onto people.
2. If possible, draw an other graph having the same projection onto people.

4 Projected Graph (again)

1. Draw the graph corresponding to the projection onto people of the graph here below.
2. In the resulting graph, explain the qualitative difference between triangles A-E-C and the others.



AEC est un peu comme un centre du graphe car ils relient tout

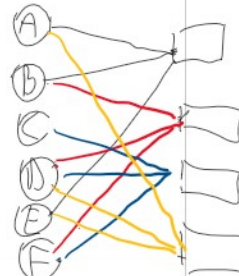
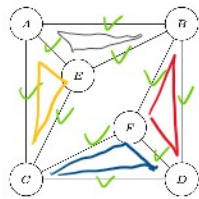


other interest

Same interest

5 Minimal focus

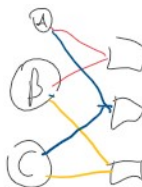
Considering the graph G below, draw an affiliation graph that admits G as a projection onto the people, having a minimal number of focuses.



6 Complete projected graph

What is the minimum number of focus necessary to build an affiliation graph s.t. its projection onto the people be the complete graph if there are no single focus shared by everyone?

3



1 Definitions

1.1 Bridge

An edge that joins two nodes A and B in a graph is called a *bridge* if deleting the edge would cause A and B to lie in two different components.

1.2 Local bridge

We say that an edge joining two nodes A and B in a graph is a *local bridge* if its endpoints A and B have no friends in common – in other words, deleting the edge would increase the distance between A and B to a value strictly more than 2. Note, the *span* of a local bridge is the distance its endpoints would be from each other if the edge were deleted.

1.3 Neighborhood overlap

The neighborhood overlap of an edge connecting A and B to be the ratio

$$\frac{\text{number of nodes who are neighbors of both A and B}}{\text{number of nodes who are neighbors of at least one of A or B}}$$

where in the denominator we do not count A or B themselves (even though A is a neighbor of B and B is a neighbor of A).

1.4 Affiliation network

An affiliation network is a *bipartite graph* that shows which individuals are affiliated with which groups or activities. We say that a graph is bipartite if its nodes can be divided into two sets in such a way that every edge connects a node in one set to a node in the other set.

1.5 Projection of affiliation network onto people

G is said to be *projected onto people* if G contains the people of the affiliation graph as nodes and there exists an edge between people $\{A, B\} \in P$ if they share a common interest (focus).