Exercise Session 2: Bond Valuation

Exercise 1:
Consider a Zero-Coupon Bond with maturity 10 years, with YTM 8% and that will be paid back, at maturity, at its nominal value. Which is the value of this bond? ↳ 100

Exercise 2:
Consider a Bond with maturity 7 years and with coupons paid annually at the coupon rate of 5%. This bond can be paid back at maturity at its nominal value, that is 100 euros.
Which is the price of this bond knowing that the expected borrowing rate from the market is 6%?
↳ annuity (yearly)
↳ YTM

Exercise 3:
A financial analyst investigates the possibility to buy a bond traded on the market with maturity 4 years and annual coupons.
Every year, the bond pays a 8% coupon. → 8% of VN = 8 €
The financial analyst forecasts three different expected borrowing rates from the market (yield to maturity – YTM) for the bond:
-Scenario 1: 8,62%;
-Scenario 2: 8%;
-Scenario 3: 7,25%

Determine the price of the bond under each scenario.

Exercise 4:
↳ face value (100 $)
Consider two bonds traded at par with maturity of 2 and 10 years respectively.
The coupon rate is equal to 4% with respect to par.

1) Which is the YTM of these two bonds? → YTM = coupon if purchased a bond at its par value.
2) What will be the impact on the price of the bonds if the YTM changes by ±1%?
3) Which bond is more sensible to changes in yields? **Why**?

$P = \sum DCF$

NV = what you receive at maturity

5% of NV = you receive 5€ each year.

Exercise 5:

A firm wishes to sell new bonds on the market in order to finance a large project. These new obligations have a maturity of 8 years and they are paid back at expiration at their nominal value. The coupon rate amounts to 5% of the nominal value. The rating is BBB- which implies a spread of 200bp with respect to risk-free rates. Here are attached the maturity rates of Government bonds (rated AAA):

5% de 100 = 5 = C

spread = risky rate − risk free rate

1 year	3 years	5 years	6 years	7 years	8 years	10 years
0.8%	1.25%	2.1%	2.4%	2.6%	2.8%	3.05%

YTM risk free rate

1) Which is the price of this bond at the day of issuance?
2) After two years, the spread linked to the rating BBB- increases by 60bp with respect to AAA. Which is then the rate of return of the bond? Assume that the coupons are not reinvested

260 → 2,6%

YTM = coupon + spread 2,6

1bp = 0,0001

Exercise 6:

Consider a corporate bond with maturity 2 years and 165 days rated BB+ and that is paid back at expiration at its nominal value. The coupons are annual and the coupon rate is 7%. The spread is 250bp and the interest rate on Government bonds (rated AAA) with the same maturity is 4%.

1) Which is the amount of accrued interests?
2) Which is the *dirty price*?
3) This obligation is over or undervalued knowing that is exchanged at 99,5% of its nominal value?

EXERCISE SESSION 2

EXERCISE N°1 :

Which is the value of this bond at T_0 = PV ?

$$PV = \frac{VN}{(1+YTM)^t} \rightarrow \frac{100}{(1+0,08)^{10}} = 46,32€$$ the price that I will pay for this bond

EXERCISE N°2 :

Which is the price of this bond knowing that expected borrowing rate from the market is 6% (YTM) ?

C = 5% of NV(100)

$$PV = C.\left[\frac{1-(1+YTM)^{-t}}{YTM}\right] + \frac{100}{(1+YTM)^t}$$

$$\hookrightarrow 5.\left[\frac{1-(1+0,06)^{-7}}{0,06}\right] + \frac{100}{(1+0,06)^7} = 94,41€$$

Here we have an annuity → series of payments made at equal intervals

EXERCISE N°3 :

Determine the price of the bond under each scenario (≠ YTM)

a) $YTM = 8,62\% \rightarrow 8.\left[\frac{1-(1+0,0862)^{-4}}{0,0862}\right] + \frac{100}{(1+0,0862)^4} = 97,97€$

b) YTM = 8% → 100€ c) YTM = 7,25% → 102,53€

YTM = coupon return + price appreciation net

$8,62 = 8\% + 0,62$] ≥ 0 the price today is higher than what we will receive

$8 = 8\% + 0$

$7,25 = 8\% + (-0,75)$] < 0 the price today is lower.

EXERCISE N°4 :

1) Which is the YTM of these two bonds ?

YTM = coupon because the bonds are purchased at their par value (face value)

YTM of these bonds = 4%.

2) What will be the impact on the price if YTM changes by ± 1%?

$4 \to 3\% \Rightarrow 4.\left[\frac{1-\frac{1}{(1+0{,}03)^2}}{0{,}03}\right] + \frac{100}{(1+0{,}03)^2} = 101{,}91\,€ > 100$ here the price is higher when the YTM is lower

$4 \to 5\%$ → the price is lower when the YTM is higher

3) Which bond is more sensible to changes in yields?

Maturity = 2 years (3% et 5%)

$101{,}91 - 98{,}14 = 3{,}77\,€$

Maturity = 10 years (3% et 5%)

$108{,}54 - 92{,}28 = 16{,}26\,€$

The one with the 10 years Maturity is more sensible to changes in yields
↳ the longer the Maturity, the more sensitive

EXERCISE N°5:

Spread = 200 bp → 2%

$YTM_{risky} = YTM_{risk\ free\ (AAA)} + spread$ (spread ↳ compensation for risk)

$= 2{,}8 + 2$

$= 4{,}8\%$

$YTM = 5\% + \underbrace{\text{appreciation}}_{(-0{,}2)} \ (<0)$

4,8 ↑ ; ↳ at 8 years

1) Which is the price of this bond?

$PV = C \cdot \left[\frac{1-(1+YTM)^{-t}}{YTM}\right] + \frac{100}{(1+YTM)^t}$ (C: constant CF)

$= 5\left[\frac{1-(1+0{,}048)^{-8}}{0{,}048}\right] + \frac{100}{(1+0{,}048)^8}$

$= 101{,}31\,€ > 100$ because negative price appreciation

↳ we are at 2 years, so 6 years left to discount.

2) After 2 years (year 6 (2,4%)) ; spread increased by 60 bp ⇒ 200 + 60 → 2,6%

(il reste 6 ans) Car maturité 8 ans ici elle sera de 6 ans en maturity

$YTM = 2{,}4\% + 2{,}6\% = 5\%$ $\quad P_0 = 101{,}31\,€$

the price at which we can sell the bond ↙ $PV_2^* = 5\left[\frac{1-(1+0{,}05)^{-6}}{0{,}05}\right] + \frac{100}{(1+0{,}05)^6} = 100$ be YTM = Coupon rate 5% = 5

the rate of return : $r = \frac{\overbrace{100}^{\text{coupon 5\% de 100 (year 1)}} + \overbrace{5}^{\text{year 2}} + 5 - 101{,}3}{101{,}3} = 8{,}58\%$

$r = \frac{(\underbrace{C_1 + C_2 + P_2^*}_{\text{what we get}}) - \underbrace{P_0}_{101{,}3}}{P_0}$

BB+ => risky.

EXERCISE N°6 : Spread 250 → 2,5% YTM = 4%.

1) Which is the amount of accrued interests?

↳ Measure

$$Ai = \frac{7}{C} \cdot \frac{365 - 165}{365} = 3,835 \Rightarrow C \cdot \frac{\text{trading date} - \text{date of coupon}}{\text{number days between 2 coupons}}$$

if I sell this bond I will receive 3,84€ and the buyer will pay CP + Ai

2) Which is the dirty price?

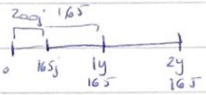
↳ $\sum$ DCF

$$\frac{C}{(1+YTM)^t} + \frac{C}{(1+YTM)^t} + \dots + \frac{100+C}{(1+YTM)^t}$$

Here the YTM is 4% + 2,5% = 6,5%

↳ spread

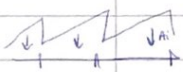
risk free rate



$\frac{165}{365}$ car discounted CF on ne veut pas le point

$$\frac{7}{(1+0,065)^{\frac{165}{365}}} + \frac{7}{(1+0,065)^{\frac{165}{365}+1}} + \frac{100+7}{(1+0,065)^{\frac{165}{365}+2}} = 104,88$$

in years



And so, the clean price is 104,88 − 3,84 = 101,04

DP − Ai = stable price

↳ on retire le Ai, pour un prix stable

3) This obligation is over or undervalued knowing that is exchanged at 99,5% of its nominal value?

Undervalued, so we should buy it. It is like a bond premium that is exchanged like a discount bond

99,5 < 104,88

↳ everyone will buy it, so it will reaches until the fair price (101,04).