Correctif des APEs de graph theory, game theory, and networks
Q5 - LINFO1115

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Informations importantes Ce document est grandement inspiré de l'excellent cours donné par Peter Van Roy à l'EPL (École Polytechnique de Louvain), faculté de l'UCL (Université Catholique de Louvain). Il est écrit par les auteurs susnommés avec l'aide de tous les autres étudiants, la vôtre est donc la bienvenue. Il y a toujours moyen de l'améliorer, surtout si le cours change car le correctif doit alors être mis à jour en conséquence. On peut retrouver le code source et un lien vers la dernière version du pdf à l'adresse suivante

`https://github.com/Gp2mv3/Syntheses`.

On y trouve aussi le contenu du `README` qui contient de plus amples informations, vous êtes invités à le lire.

Il y est indiqué que les questions, signalements d'erreurs, suggestions d'améliorations ou quelque discussion que ce soit relative au projet sont à spécifier de préférence à l'adresse suivante

`https://github.com/Gp2mv3/Syntheses/issues`.

Ça permet à tout le monde de les voir, les commenter et agir en conséquence. Vous êtes d'ailleurs invités à participer aux discussions.

Vous trouverez aussi des informations dans le wiki

`https://github.com/Gp2mv3/Syntheses/wiki`

comme le statut des documents pour chaque cours

`https://github.com/Gp2mv3/Syntheses/wiki/Status`

Vous pouvez d'ailleurs remarquer qu'il en manque encore beaucoup, votre aide est la bienvenue.

Pour contribuer au bug tracker et au wiki, il vous suffira de créer un compte sur GitHub. Pour interagir avec le code des documents, il vous faudra installer LaTeX. Pour interagir directement avec le code sur GitHub, vous devrez utiliser `git`. Si cela pose problème, nous sommes évidemment ouverts à des contributeurs envoyant leurs changements par mail (à l'adresse contact.epldrive@gmail.com) ou par n'importe quel autre moyen.

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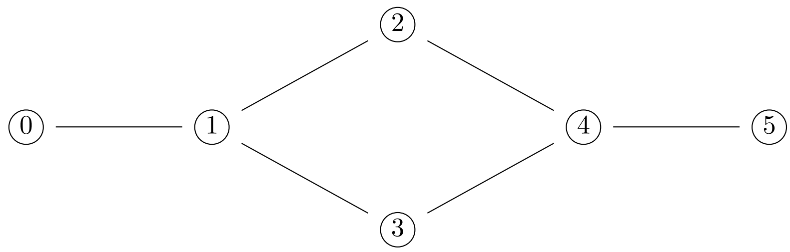
1 Lab 1 : Introduction to Graph Theory and Social Networks

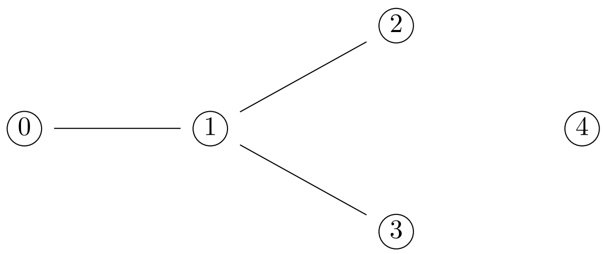
Exercice 1.1

Definitions

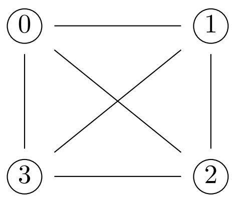
1. What is a connected graph ? Provide an example.
   A **connected graph** is a graph such as for any pair of node a, v belonging to the graph there exist a path connecting a to b.



2. Give an example for a graph that is not connected.



3. What is a complete graph ? Give an example
   A **complete graph** is a graph such as for any pair of node a, v belonging to the graph there exist an edge connecting a to b.



4. Is it always the case that a complete graph is connected ?
   By definition, yes, since there an edge connecting each pair of node, there's neccessarily a path.
5. How many edges holds a complete graph with 3 vertices (a.k.a nodes) ? And for 5 vertices ? How about N ?
   — 3 edges for 3 nodes
   — 10 edges for 5 nodes
   — $n * \frac{n-1}{2}$

Exercice 1.2

Components

1. What is a graph component ? Give an example.
   A **component**is a subset of node that is connected such as there is no superset that is connected. (maximal connected parts of a graph)

2. Given the graph $G = (V, E)$ having k components. Prove that :

$$|E| \leq \frac{(|V| - k + 1) * (|V| - k)}{2}$$

left as homework. If somebody as a solution, dont hesitate to send it

Exercice 1.3

Degree of a vertex

he degree of a vertex v is the number of edges connected to v . Given the function $deg(v)$ yielding the degree of v and a graph $G = (V, E)$, does the following proposition hold ?

$$\sum_{v}^{V} deg(v) \in \{2a | a \in \mathbb{N}\}$$

Since every edge is conected to 2 nodes ; it count 2 times in the degree's of the node of the graph. The sum an then only be a multiple of 2, "a" in this case being the number of edges.

Exercice 1.4

Shaking hands

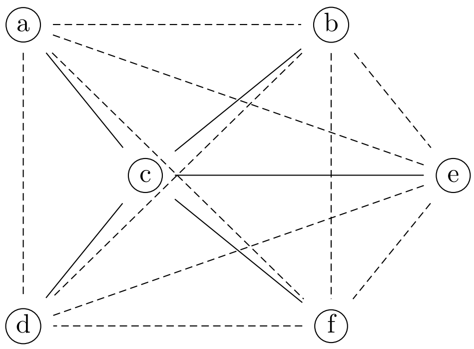
n people attend the same meeting. Among them, some greet one another shaking hands. Show that, at the end of the meeting, at least two people shook hands with the same number of different people.

Let's use the pigeonhole principle. Since there is n people, we have a maximum of n-1 edges going from each node. Because of this, it is impossible that there is not at least 2 people with the same number of connection, since there is less possibilities for this number than there is people attending the meeting

Exercice 1.5

Triadic closure

What is strongtriadic closure ? Given the graph below, what will be the final version of the graph assuming strong triadic closure (solid stroke = strong connection, dashed = weak) ?



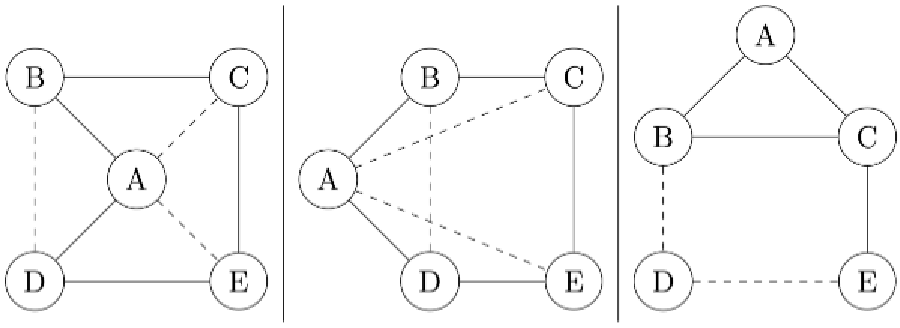
(note : all dashed lines here are the added lines from the exercise

note2 : we can see that it become a complete graph)

Exercice 1.6

Triadic closure (again)

In the graphs below, what are the nodes fulfilling the strong triadic closure property ? What nodes do not respect that property ?



Attention there was a lot of debate about this exercises in both tp, consider these solution with a grain of salt.

from the book *"We say that a node A violates the Strong Triadic Closure property if it has strong ties to two other nodes B and C, and there is no edge at all (either a strong or weak tie) between B and C. We say that a node A satisfies the Strong Triadic Closure property if it does not violate it."(p.49)*

From this definition, only c violate the strong triadic closure in all of thoses graphs

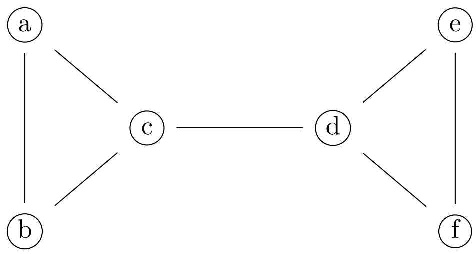
(note : graph 1 and 2 are exactly the sames, just drawn in different ways)

Exercice 1.7

Bridges

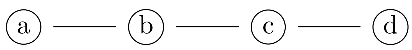
A bridge is an edge between two vertices A and B iff the removal of that edges splits the graph in two disconnected components

1. Show a graph featuring a bridge.



bridge between c and d

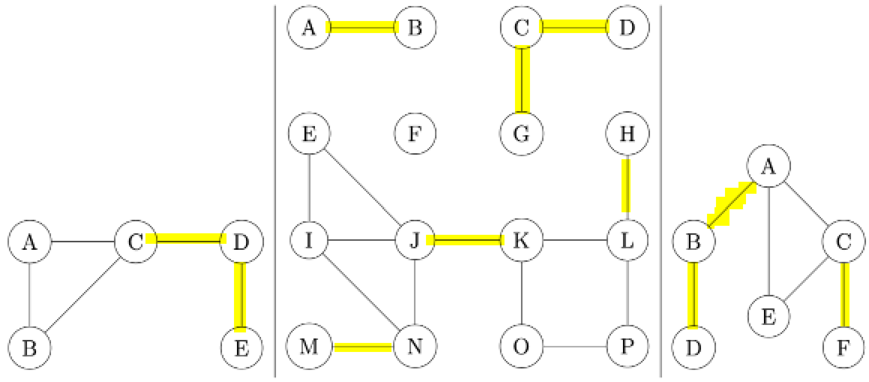
2. Show a graph such that every edge is a bridge.



Exercice 1.8

More Bridges

In the graphs below, show the edges that are bridges.



note : bridges are the highlighted edges