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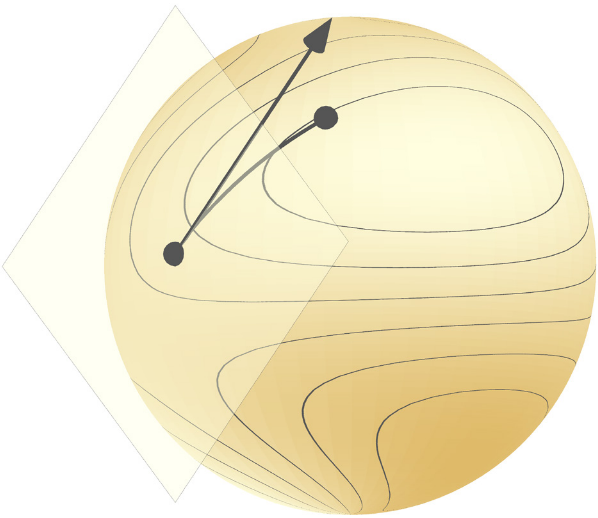
Geodesic convexity: why, and what we need



Spring 2023

Optimization on manifolds, MATH 512 @ EPFL

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Why optimizers love convexity

In a linear space $\mathcal{E}$, a minimization problem

$$\min_{x \in S} f(x)$$

is convex if the search space S and the cost function f are convex.

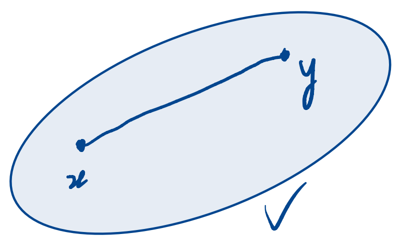
This is a fruitful notion because:

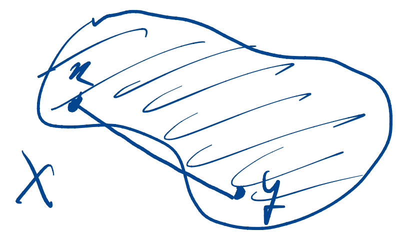
1. Local minima are global minima.
2. This comes up in applications and it's easy to spot.
3. We have good algorithms.

Convexity in linear spaces

Def.: A set $S \subseteq \mathbf{R}^n$ is convex if

$$x, y \in S \Rightarrow (1-t)x + ty \in S \text{ for all } t \in [0,1].$$

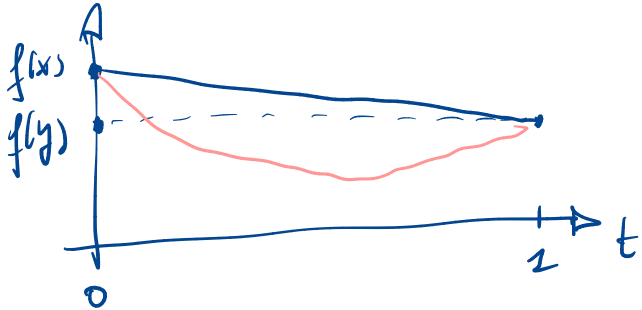




Def.: A function $f: S \to \mathbf{R}$ is convex if its domain S is convex and

$$f\big((1-t)x + ty\big) \leq (1-t)f(x) + tf(y)$$

for all $x, y \in S$ and $t \in [0,1]$.



Our goal is to extend these notions to optimization on manifolds.

What will it take?

Look ahead: the key fact "local min $\Rightarrow$ global min" will be preserved.