RT2 Statistic Assignment

Ierardi Ambra 4843302, Bravi Nicolas 6402442

Master Degree in Robotics Engineering, UNIGE

Supervised by Carmine Recchiuto

Year of Study: 1st

Abstract

This paper reports the process of a statistical test performed on two different python algorithms which aim to group a set of golden boxes in a predefined arena developed on ROS, where the layout of the tokens was set each time a in random way. The measured characteristic is the time spent on completing this task. In order to perform the **pooled T test**, the mean value and the standard deviation of the two samples were computed, as well as the pooled standard deviation and, eventually, the *t* value, which has been compared with the table value corresponding to the number of degrees of freedom of this problem, with the desired significance level.

Considering the results, it is possible to state that the two samples belong to different populations.

1 Introduction

The aim of this project is to perform a statistical analysis of two different algorithms used on the first assignment of Research Track 1 course. In detail, each student had to develop a python script which makes an automated robot in a simulated environment move with the goal to bring all the boxes called token in an arena. The robot grabs them, transports them to the center of the arena and releases them. The process continues until all tokens have not been moved into the arena. In this paper we are going to analyse the algorithms of the students Ambra Ierardi and Nicolas Bravi, testing both many times in random environments, composed of 6 golden tokens placed randomly in order to order to test if the proposed approaches are quite similar or there is a big difference in terms of time.

2 Materials

The software evaluated in this assignment are two python scripts performing the collection of a set of golden tokens in an environment provided by Student Robotics, which was modified in order to test each time a random new disposition of the boxes. The randomly disposition of the 6 tokens in the environment has been set following a criterion. The center of the environment is the origin of the map and we talk about positions through polar coordinates. At the beginning of the simulation, each box can be placed in the environment with a random radius in the interval $(2.0, 2.2)$ and with a random angle in the interval $(0, 2\pi)$. An example of randomly placement is shown in the figure 1.
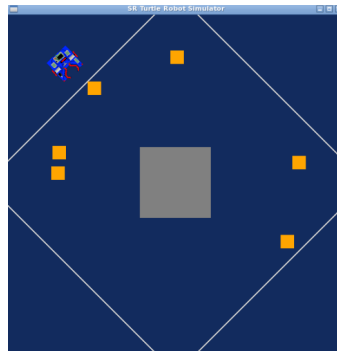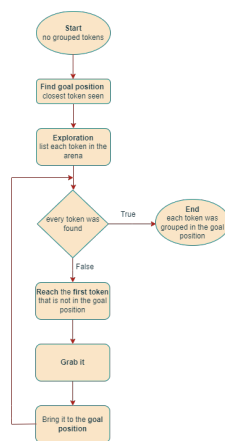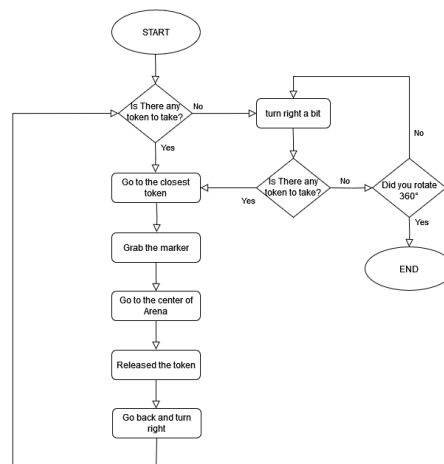
Figure 1: Random placement of the tokens

The two algorithms were implemented respectively following these two flow-charts:

(a) Ambra Ier-ardi's flowchart

(b) Nicols Bravi's flowchart

Figure 2: Flowcharts of the python scripts

These two algorithms use different implementations: the one corresponding to the left graph first performs an exploration in order to record all the tokens' codes, then groups all of them near the closest token; while the second one looks for new tokens: until it finds a token in the arena which has not been brought to the center of the arena, it keeps moving.

Both algorithms have been run 30 times, with a different environment for each simulation. The time intervals are shown in Table 1.

Index	Time Ambra	Time Nicolas
1	106	43
2	141	47
3	108	40
4	114	44
5	109	45
6	104	44
7	106	53
8	110	45
9	137	47
10	119	44
11	107	43
12	108	44
13	105	42
14	109	51
15	110	41
16	104	50
17	115	43
18	100	46
19	104	48
20	117	44
21	135	44
22	113	46
23	91	49
24	109	40
25	116	43
26	105	44
27	114	42
28	113	50
29	108	52
30	114	46

Table 1: Simulation time Ambra and Nicolas

3 Implementation

In this section, it is described the implementation of the statistic of this assignment. First of all we have computed the mean and the standard deviation s of the times in the Table 1, with the following formula:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{1}$$

In this case, we know both the populations and the **pooled T-test** fits our data. It is a probabilistic assessment used for independent samples, which is the current case since the two algorithms were implemented individually, without comparing the two methods, and the environment is each time generated randomly. Also, this kind of test shall be used in case the populations from which the two samples are obtained are not known, and it is required to compare the two groups of measures just on the results. To be more accurate, it was performed a two-tailed pooled T-test, with null hypothesis $H0$ corresponding to the two mean values of the original populations to be equal, which means that the two populations are coinciding, while the alternative hypothesis $H1$ corresponds to prove the opposite.

$$H_0 : \mu_A = \mu_B \leftrightarrow \mu_A - \mu_B = 0$$
$$H_1 : \mu_A \neq \mu_B \leftrightarrow \mu_A - \mu_B \neq 0$$

The pooled, estimated variance of the sampling distribution of the difference of means, called $\hat{\sigma}^2_{pooled}$, is given by the following formula.

$$\hat{\sigma}^2_{pooled} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \tag{2}$$

Once computed the pooled variance, we have eventually to compute the *estimated pooled standard error* that is:

$$\hat{\sigma}_{\bar{x}_1 - \bar{x}_2} = \sqrt{\hat{\sigma}^2_{\text{pooled}} \left(\frac{1}{N_1} + \frac{1}{N_2} \right)} \tag{3}$$

The equation to retrieve the T value is the following one:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\hat{\sigma}_{\bar{x}_1 - \bar{x}_2}} \tag{4}$$

In order to try to prove $H0$, we find the appropriate t value in the t-Table after choosing the level of significance. Given that we want to understand if the populations are equal or not, we will be going for a two-tailed test. If this value is bigger than the obtained with the Formula 4, the Null Hypothesis can be confirmed. The DOF is given by $N1 + N2 - 2$.

4 Results

As first thing, we computed the mean and the standard deviation for the tests of both algorithms. Ambra's algorithm got a mean of $m1 = 111.1$ and a std of $s1 = 10.23$, while Nicolas' one got a mean of $m2 = 45.33$ and a std of $s2 = 3.36$. In addiction, we have computed the histograms in order to understand the distribution for both algorithms, shown in the figures below.
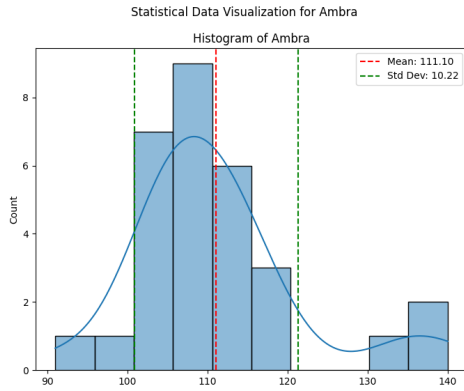
(a) Ambra's distribution time
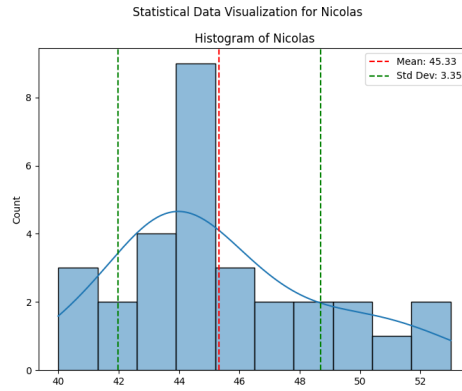
(b) Nicolas's distribution time

As mentioned before, the null hypothesis $H0$ is that the two populations are coinciding, while the alternative hypothesis $H1$ corresponds to the opposite.

Using the formula 2, we obtained that $\hat{\sigma}^2_{pooled} = 57.97$ and with the formula 3, we got $\hat{\sigma}_{\bar{x}_1 - \bar{x}_2} = 1.97$.

$$\hat{\sigma}^2_{pooled} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{29(10.23)^2 + 29(3.36)^2}{58} = 57.97$$

$$\hat{\sigma}_{\bar{x}_1 - \bar{x}_2} = \sqrt{\hat{\sigma}^2_{pooled} \left(\frac{1}{N_1} + \frac{1}{N_2} \right)} = \sqrt{57.97 \cdot \frac{1}{15}} = 1.97$$

After these results, it is possible to get the $t_{\bar{x}_1 - \bar{x}_2}$ with formula 4 value and to compare it with the value on the t-Table.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\hat{\sigma}_{\bar{x}_1 - \bar{x}_2}} = \frac{111.1 - 45.33}{1.97} = 33.39$$

So, the $t_{\bar{x}_1 - \bar{x}_2}$ value is 33.39. As mentioned before, we have $DOF = 58$ and we choose the level of significance at α=0,05: in this way we take the risk of

rejecting the null hypothesis when it is true with a probability equal to the 5%, a reasonable value. For a two-tailed test, we obtained that the t value is $|t| = 2.001$. It means that the null hypothesis $H0$ is a wrong assumption because $|t| < 33.39$, so we reject this hypothesis. Thus, we can say that the alternative hypothesis $H1$ is true: the populations are different.

5 Conclusion

Since the computed value is greater than the t value in the table, the hypothesis H_0 is rejected, in favour of the alternative hypothesis H_1, meaning that the two samples belong to different original populations, and their mean values will be $\mu_A \neq \mu_B$.