An Introduction to Transfer Learning and Domain Adaptation

Machine Learning course

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Semester 2

Transfer Learning

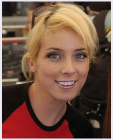
Definition [Pan, TL-IJCAI'13 tutorial]

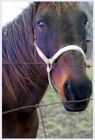
Ability of a system to recognize and apply knowledge and skills learned in previous domains/tasks to novel domains/tasks

Domain adaptation: The Learning distribution is different from the Testing distribution.

An example

- We have **labeled** images from a **Web image** corpus
- Is there a `Person` in **unlabeled** images from a **Video** corpus ?



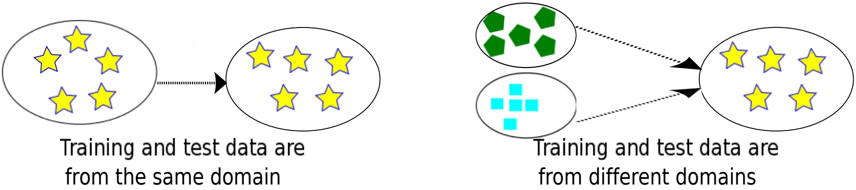


?

Outline

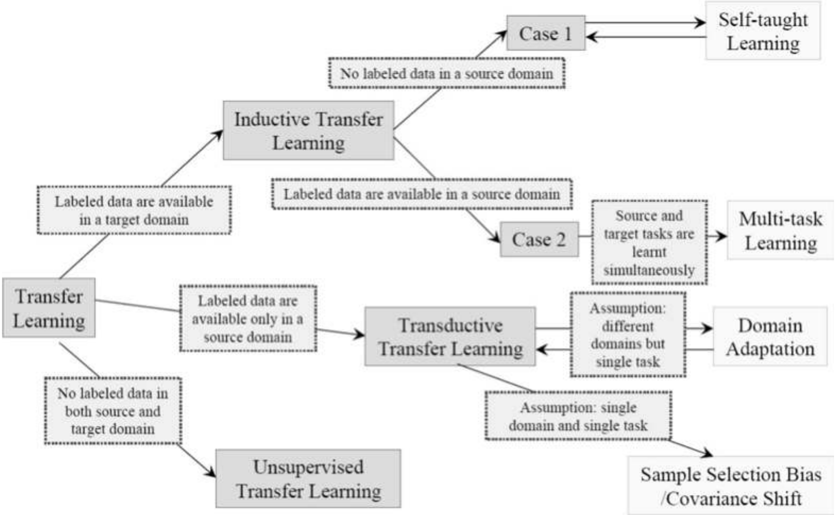
Introduction

Settings



- Domains are modeled as probability distributions over an instance space
- Tasks associated to a domain (classification, regression, clustering, ...)
- **Objective:** From source to target
  $\Rightarrow$ Improve a target predictive function in the target domain using knowledge from the source domain

A Taxonomy of Transfer Learning



"A survey on Transfer Learning" [Pan and Yang, TKDE 2010]

In this presentation

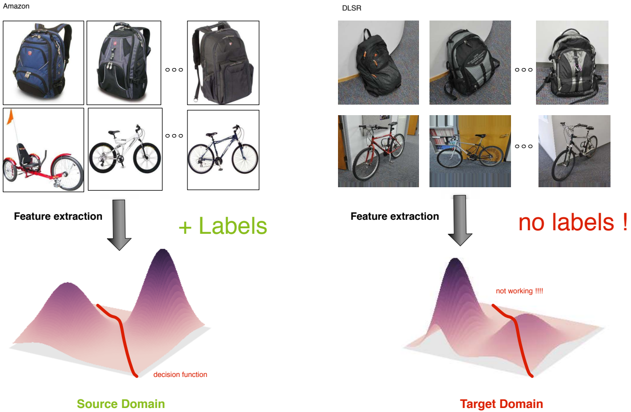
- We will make a focus on **domain adaptation**
- We will consider classification tasks

⇒ How can we learn, using labeled data from a source distribution, a low-error classifier for another related target distribution?

⇒ "Hot topic" - a lot of research works on it - tutorials at ICML 2010, CVPR 2012, Interspeech 2012, workshops at ICCV 2013, NIPS 2013,ECML 2014

⇒ An important subject: we want to be able to transfer some learned knowledge
⇒ Motivating examples

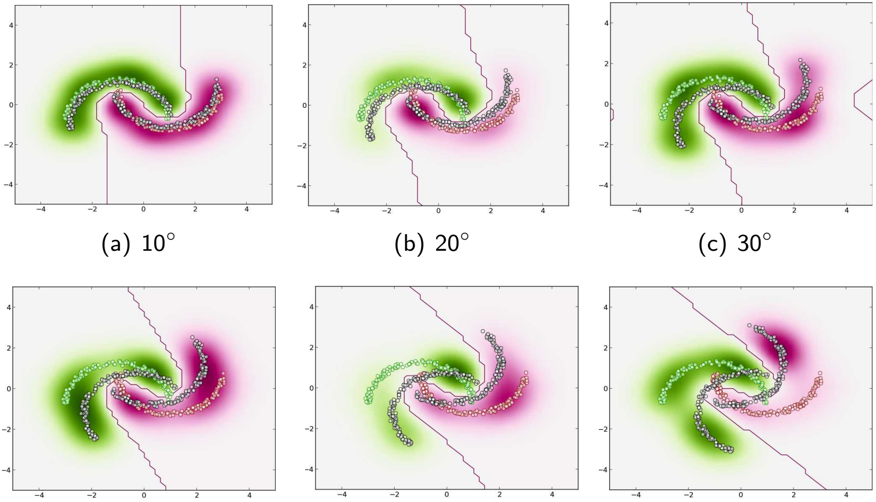
Unsupervised domain adaptation problem



Problems

- Labels only available in the **source domain**, and classification is conducted in the **target domain**.
- Classifier trained on the source domain data performs badly in the target domain

A toy problem: Inter-twinning moons



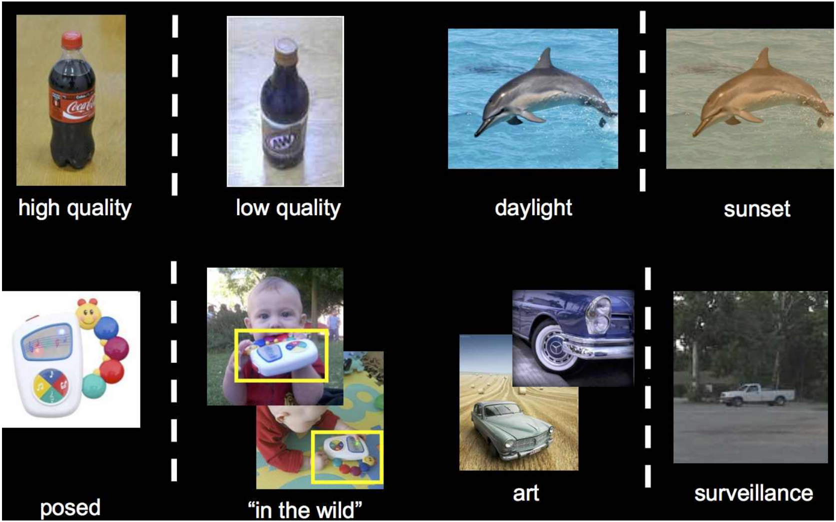
(a) 10° (b) 20° (c) 30°

(d) 40° (e) 50° (f) 70°

Intuition and motivation from a CV perspective

- "Can we train classifiers with Flickr photos, as they have already been collected and annotated, and hope the classifiers still work well on mobile camera images?" [Gonq et al., CVPR 2012]
- "object classifiers optimized on benchmark dataset often exhibit significant degradation in recognition accuracy when evaluated on another one" [Gonq et al.,ICML 2013, Torralba et al., CVPR 2011, Perronnin et al., CVPR 2010]
- "Hot topic" -Visual domain adaptation [Tutorial CVPR'12, ICCV'13]

Hard to predict what will change in the new domain



[Xu,Saenko,Tsang, Domain Transfer Tutorial - CVPR'12]

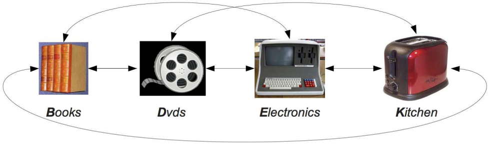
Natural Language Processing

Text are represented by "words" (Bag of Words)

- Part of Speech Tagging: Adapt a tagger learned from medical papers to a journal (Wall Street Journal) - Newsgroup

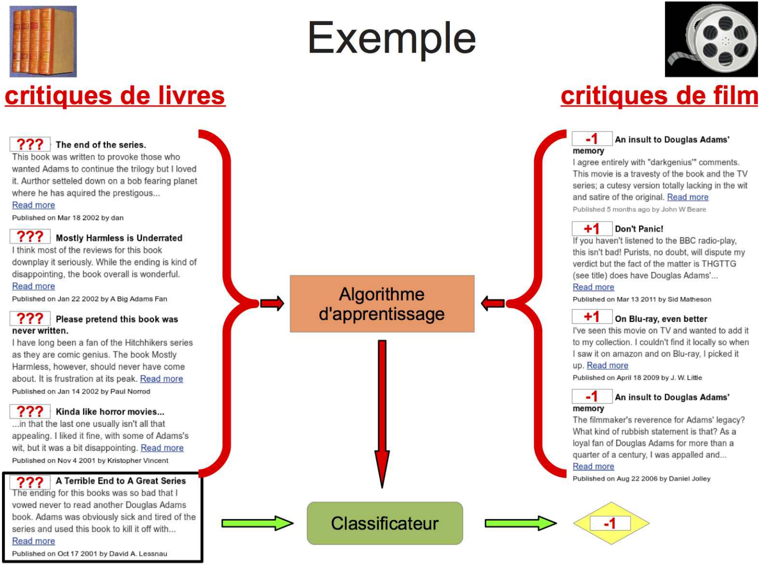
Biomedical	WSJ
the **signal** *required* to	**investment** *required*
stimulatory **signal** *from*	**buyouts** *from* buyers
essential **signal** *for*	to **jail** *for* violating

- Spam detection: Adapt a classifier from one mailbox to another



- Sentiment analysis:

Domain Adaptation for sentiment analysis



Domain Adaptation for sentiment analysis - ex [Pan-IJCAI'13 tutorial]

	Electronics	Video games
✓	(1) Compact; easy to operate; very good picture quality; looks sharp!	(2) A very good game! It is action packed and full of excitement. I am very much hooked on this game.
✓	(3) I purchased this unit from Circuit City and I was very excited about the quality of the picture. It is really nice and sharp.	(4) Very realistic shooting action and good plots. We played this and were hooked.
✗	(5) It is also quite blurry in very dark settings. I will never_buy HP again.	(6) It is so boring. I am extremely unhappy and will probably never_buy UbiSoft again.

- Source specific: *compact, sharp, blurry*.
- Target specific: *hooked, realistic, boring*.
- Domain independent: *good, excited, nice*, *never_buy, unhappy*.

Other applications

- Speech recognition [Tutorial at Interspeech'12]
- Medecine
- Biology
- Time series
- Wifi localization
- ...

Notations

Notations

- $X \subseteq \mathbb{R}^d$ input space, $Y = \{-1, +1\}$ output space
- P_S **source** domain: distribution over $X \times Y$
  D_S marginal distribution over X
- P_T **target** domain: different distribution over $X \times Y$
  D_T marginal distribution over X
- $\mathcal{H} \subseteq Y^X$: hypothesis class

Expected error of a hypothesis $h : X \to Y$

- $R_{P_S}(h) = \underset{(\mathbf{x}^s, y^s) \sim P_S}{\mathbf{E}} \mathbf{I}\left[h(\mathbf{x}^s) \neq y^s\right]$ **source** domain error
- $R_{P_T}(h) = \underset{(\mathbf{x}^t, y^t) \sim P_T}{\mathbf{E}} \mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]$ **target** domain error

Domain Adaptation: find $h \in \mathcal{H}$ with R_{P_T} **small** from data $\sim D_T$ and P_S

Classical result in supervised learning

Empirical error

- $R_S = \{(\mathbf{x}_i^s, y_i^s)\}_{i=1}^{m_s} \sim (P_S)^{m_s}$ a labeled sample drawn i.i.d. from P_S
- Associated **empirical error** of an hypothesis h:

$$R_S(h) = \frac{1}{m_s} \sum_{i=1}^{m_s} \mathbf{I}\left[h(\mathbf{x}_i^s) \neq y_i^s\right]$$

Classical PAC result: From the same distribution

$$R_{P_S}(h) \leq R_S(h) + O\left(\frac{complexity(h)}{\sqrt{m_s}}\right)$$

$\Rightarrow$ Occam razor principle

What about R_{P_T} if we have no or very few labeled data? $\rightarrow$ try to make use of source information

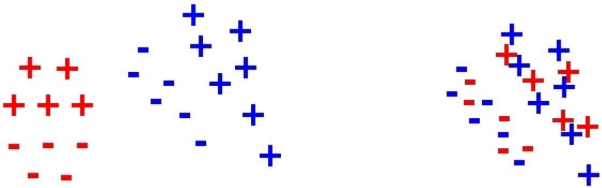
- **Labeled** Source Sample

  $S = \{(\mathbf{x}_i, y_i)\}_{i=1}^{m_s}$ **Source** sample drawn i.i.d. from P_S

- **Unlabeled** Target Sample

  $T = \{\mathbf{x}_j\}_{j=1}^{m_t}$ **Target** Sample drawn i.i.d. from D_T
  optionnal: a few labeled target examples

If h is learned from **source** domain, how does it perform on **target** domain?



Domain Adaptation
Setting

- **Labeled** Source Sample

  $S = \{(\mathbf{x}_i, y_i)\}_{i=1}^{m_s}$ **Source** sample drawn i.i.d. from P_S

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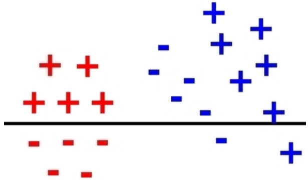
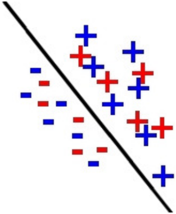
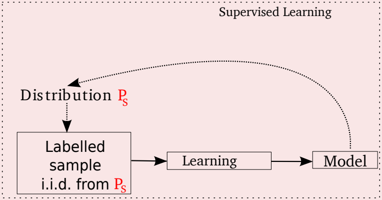




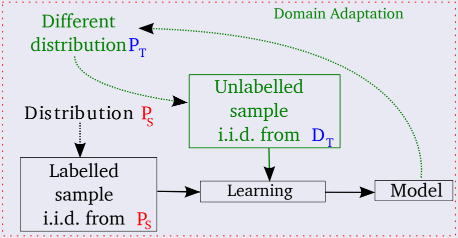
If the domains are close a good classifier on source is also good one target

Illustration settings

Classic supervised learning



Domain adaptation



A bit of vocabulary

Unsupervised Transfer Learning

No labels

Unsupervised DA

Presence of source labels, no target labels

Semi-supervised DA

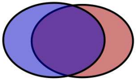
Presence of source labels, few target labels and a lot of unlabeled data

$\neq$ Semi-supervised learning

No distribution shift, few labeled data and a lot of unlabeled data from the same domain

Some key points

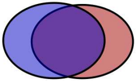
- Estimating of the distribution shift



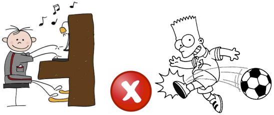
- Deriving generalization guarantees
  $R_{P_T}(h) \leq ? R_{P_S}(h) ? + ?$

Some key points

- Estimating of the distribution shift



- Deriving generalization guarantees
  $R_{P_T}(h) \leq ?R_{P_S}(h)?+?$
- Characterizing when the adaptation is possible



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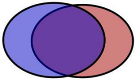

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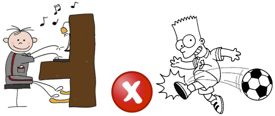
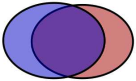


- Defining algorithms
  Underlying idea: Try to move closer the two distributions

Some key points

- Estimating of the distribution shift



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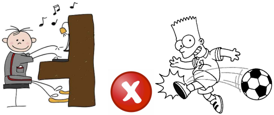
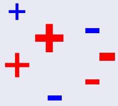


- Defining algorithms
  Underlying idea: Try to move closer the two distributions
- Applying model selection principle
  How to tune hyperparameters with no labeled information from target

3 main classes of algorithms

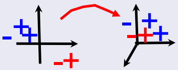
Reweighting/Instance-based methods

Correct a sample bias by reweighting source labeled data: source instances close to target instances are more important.



Feature-based methods/Find new representation spaces

Find a common space where source and target are close (projection, new features, etc)



Ajustement/Iterative methods

Modify the model by incorporating pseudo-labeled information

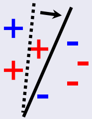
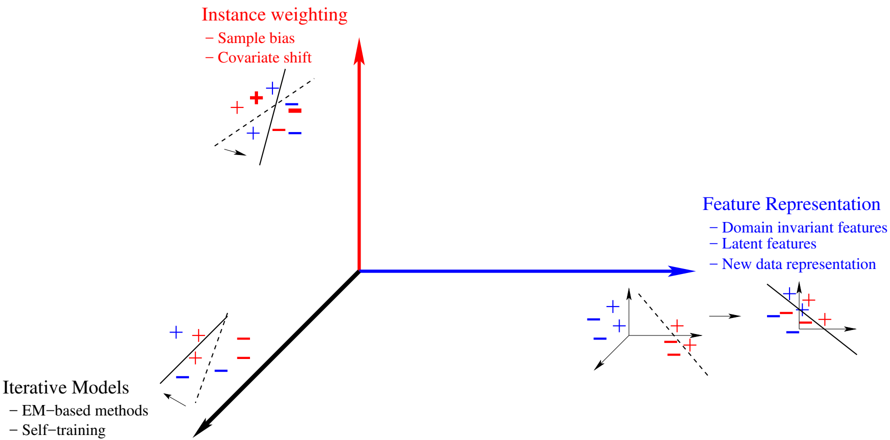


Illustration of the main methods



Reweighting/Instance based Methods

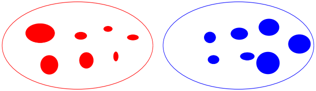
Context

Motivation

- Domains share the **same** support (*i.e.* bag of words)
- Distribution shift is caused by **sampling bias/shift between marginals**

Idea

Reweight or **select** instances to reduce the discrepancy between source and target domains.



A first analysis

$$R_{P_T}(h) = \mathop{\mathbf{E}}_{(\mathbf{x}^t, y^t) \sim P_T} \mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]$$

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\begin{aligned}
R_{P_T}(h) &= \mathop{\mathbf{E}}_{(\mathbf{x}^t,y^t)\sim P_T} \mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right] \\
&= \mathop{\mathbf{E}}_{(\mathbf{x}^t,y^t)\sim P_T} \frac{P_S(\mathbf{x}^t, y^t)}{P_S(\mathbf{x}^t, y^t)} \mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]
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&= \sum_{(\mathbf{x}^t, y^t)} P_T(\mathbf{x}^t, y^t) \frac{P_S(\mathbf{x}^t, y^t)}{P_S(\mathbf{x}^t, y^t)} \mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]
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&= \mathop{\mathbf{E}}_{(\mathbf{x}^t,y^t)\sim P_S} \frac{P_T(\mathbf{x}^t, y^t)}{P_S(\mathbf{x}^t, y^t)} \mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]
\end{aligned}
$$

Covariate shift [Shimodaira,'00]

$\Rightarrow$ Assume similar tasks, $P_S(y|\mathbf{x}) = P_T(y|\mathbf{x})$, then:

$$= \mathop{\mathbf{E}}_{(\mathbf{x}^t,y^t)\sim P_S} \frac{D_T(\mathbf{x}^t)P_T(y^t|\mathbf{x}^t)}{D_S(\mathbf{x}^t)P_S(y^t|\mathbf{x}^t)} \mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]$$

$$= \mathop{\mathbf{E}}_{(\mathbf{x}^t,y^t)\sim P_S} \frac{D_T(\mathbf{x}^t)}{D_S(\mathbf{x}^t)} \mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]$$

$$= \mathop{\mathbf{E}}_{(\mathbf{x}^t)\sim D_S} \frac{D_T(\mathbf{x}^t)}{D_S(\mathbf{x}^t)} \mathop{\mathbf{E}}_{y^t\sim P_S(y^t|\mathbf{x}^t)} \}\mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]$$

$\Rightarrow$ **weighted error** on the source domain: $\omega(x^t) = \frac{D_T(\mathbf{x}^t)}{D_S(\mathbf{x}^t)}$

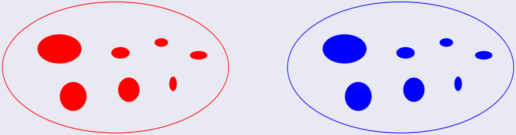
Idea reweight labeled source data according to an estimate of $\omega(x^t)$:

$$\mathop{\mathbf{E}}_{(\mathbf{x}^t,y^t)\sim P_S} \omega(\mathbf{x}^t)\mathbf{I}\left[h(\mathbf{x}^t) \neq y^t\right]$$

Illustration

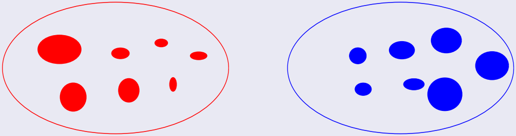
No Bias

$D_S(\mathbf{x}) = D_T(\mathbf{x}) \Rightarrow \omega(\mathbf{x}) = 1$



With Bias

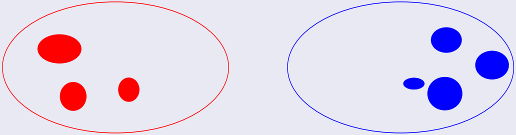
$D_S(\mathbf{x}) \neq D_T(\mathbf{x}) \Rightarrow \omega(\mathbf{x}) \neq 1$



Difficult case

No shared support

$\exists \mathbf{x}, D_S(\mathbf{x}) = 0$ and $D_T(\mathbf{x}) \neq 0$



Shared support

$D_S(\mathbf{x}) = 0$ if and only if $D_T(\mathbf{x}) = 0$

Intuition: the quality of the adaptation depends on the magnitude on the weights

How to deal with the sample selection bias?

Setting

A source sample $S = \{(\mathbf{x}_i^s, y_i^s)\}_{i=1}^{m_s}$ and a target sample $T = \{\mathbf{x}_j^t\}_{j=1}^{m_t}$

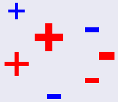
Estimate new weights without using labels

$$\hat{\omega}(\mathbf{x}_i^s) = \frac{\hat{Pr}_T(\mathbf{x}_i^s)}{\hat{Pr}_S(\mathbf{x}_i^s)}$$

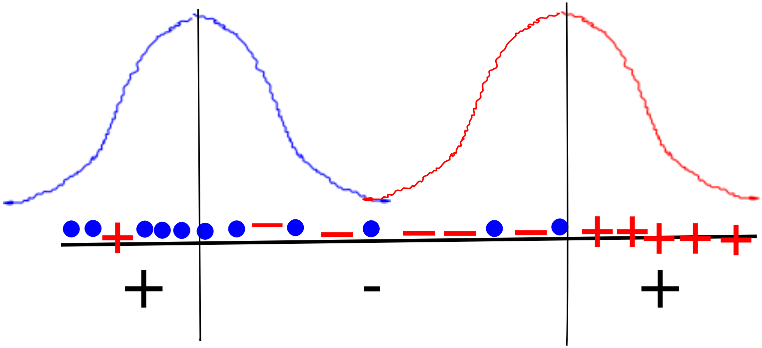
Learn a classifier on the classifier w.r.t. $\hat{\omega}$

$$\sum_{(\mathbf{x}_i^s, y_i^s) \in S} \hat{\omega}(\mathbf{x}_i^s) I[h(\mathbf{x}_i^s) \neq y_i^s]$$

(Other losses: margin-based hinge-loss, least-square)



Illustration



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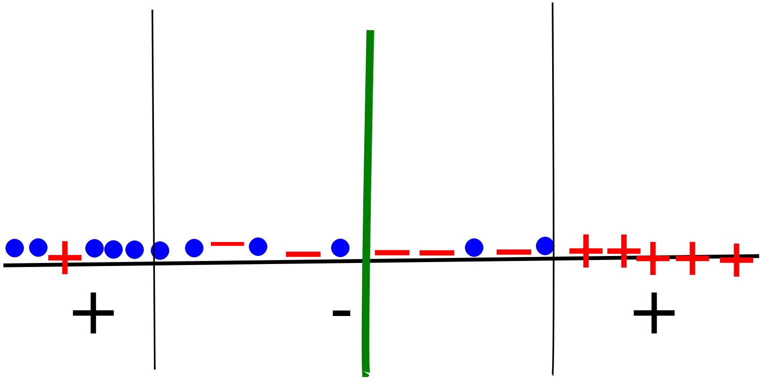

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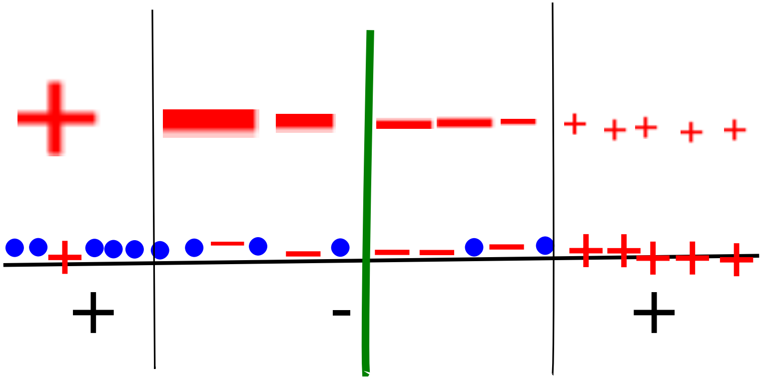

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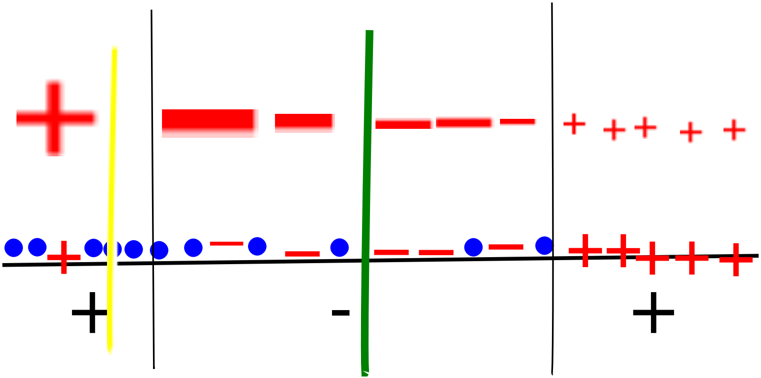

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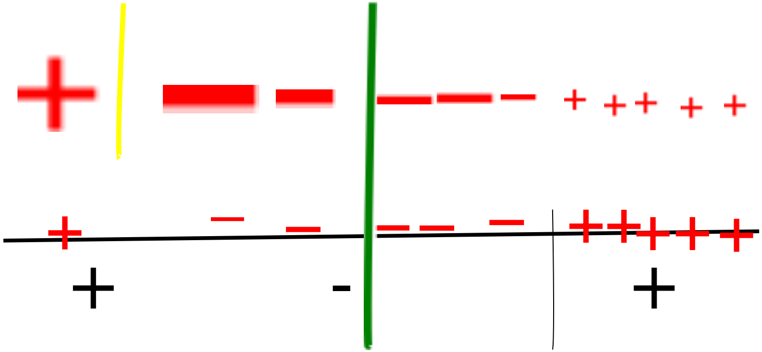


Some existing approaches (1/2)

Density estimators

Build density estimators for source and target domains and estimate the ratio between them - Ex [Sugiyama et al.,NIPS'07]:

- $\hat{\omega}(\mathbf{x}) = \sum_{l=1}^{b} \alpha_l \psi_l(\mathbf{x})$
- Learning: $\text{argmin}_\alpha KL(\hat{\omega} D_S, D_T)$

Learn the weights discriminatively [Bickel et al.,ICML'07]

- Assume $\frac{D_T(\mathbf{x}_i)}{D_S(\mathbf{x}_i)} \propto \frac{1}{p(q=1|\mathbf{x},\theta)}$
- Label source with label 1, target with label 0 and train a classifier $(\hat{\theta})$ to classify examples 1 or 0 (e.g. with logistic regression)
- Compute the new weights $\hat{\omega}(\mathbf{x}_i^s) = \dfrac{1}{p(q=1|\mathbf{x}_i^s;\hat{\theta})}$

Some existing approaches (2/2)

Kernel Mean Matching [Huang et al.,NIPS'06]

- Maximum Mean Discrepancy
  $\text{MMD}(S, T) = \|\frac{1}{m_S}\sum_{i=1}^{m_s}\phi(\mathbf{x}_i^s) - \frac{1}{m_t}\sum_{j=1}^{m_t}\phi(\mathbf{x}_j^t)\|_H$
- $\min_\beta \| \dfrac{1}{m_S}\displaystyle\sum_{i=1}^{m_s}\beta(\mathbf{x}_i^s)\phi(\mathbf{x}_i^s) - \dfrac{1}{m_S}\displaystyle\sum_{j=1}^{m_t}\phi(\mathbf{x}_j^t)\|_H$
  s.t. $\beta(\mathbf{x}_i^s) \in [0, B]$ and $|\frac{1}{m_S}\sum_{i=1}^{m_s}\beta(\mathbf{x}_i^s) - 1| < \epsilon$
- $\min_\beta \dfrac{1}{2}\beta^T \mathbf{K}_T \beta - \boldsymbol{\kappa}_{S,T}^T$ s.t. $\beta_i \in [0, B]$ and $|\sum_{i=1}^{m_s}\beta(\mathbf{x}_i^s) - m_s| < m_s\epsilon$

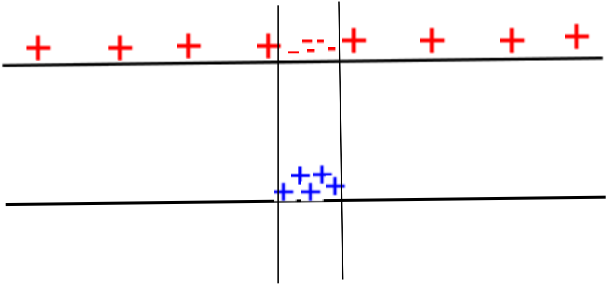
Guarantees [Gretton et al.,2008] - Under covariate shift assumptions

$$|R_{P_T}(h) - \text{weighted}(R_S(h))| < \sqrt{\frac{O(1/\delta) + O(\max_{\mathbf{x}} \omega(\mathbf{x})^2)}{m_s}} + C\epsilon \text{ and}$$

$$\|\frac{1}{m_S}\sum_{i=1}^{m_s}\omega(\mathbf{x}_i^s)\phi(\mathbf{x}_i^s) - \frac{1}{m_t}\sum_{j=1}^{m_t}\phi(\mathbf{x}_j^t)\| \leq O((1/\delta)\sqrt{\omega_{\max}^2/m_s + 1/m_t}$$

Bad news

- DA is hard, even under covariate shift [Ben-David et al.,ALT'12]
  $\Rightarrow$ To learn a classifier the number of examples depend on $|\mathcal{H}|$ (finite) or exponentially on the dimension of X
- Covariate shift assumption may fail: Tasks are not similar in general
  $P_S(y|\mathbf{x}) \neq P_T(y|\mathbf{x})$



- We did not consider the hypothesis space.
- Can define a general theory about DA?

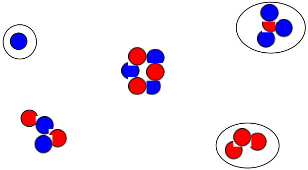
Theoretical frameworks for Domain Adaptation

A keypoint: estimating the distribution shift

First idea: Total variation measure

$d_{L_1}(D_S, D_T) = \sup_{B \subseteq X} |D_S(B) - D_T(B)|$

Subset of points maximizing the divergence



But:

- Not computable in general, and thus not estimable from finite samples

A keypoint: estimating the distribution shift

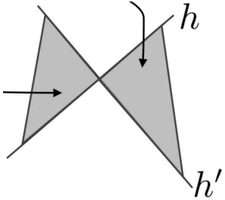
First idea: Total variation measure

$d_{L_1}(D_S, D_T) = \sup_{B \subseteq X} |D_S(B) - D_T(B)|$

Subset of points maximizing the divergence

But:

- Not computable in general, and thus not estimable from finite samples
- Not related to the hypothesis class
- Do not characterize the difficulty of the problem for $\mathcal{H}$



The $\mathcal{H}\Delta\mathcal{H}$-divergence [Ben-David et al.,NIPS'06;MLJ'10]

Definition

$$d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) = \sup_{(h,h')\in\mathcal{H}^2} \left| R_{D_T}(h,h') - R_{D_S}(h,h') \right|$$

$$= \sup_{(h,h')\in\mathcal{H}^2} \left| \mathop{\mathbf{E}}_{\mathbf{x}^t\sim D_T} \mathbf{I}\left[h(\mathbf{x}^t) \neq h'(\mathbf{x}^t)\right] - \mathop{\mathbf{E}}_{\mathbf{x}^s\sim D_S} \mathbf{I}\left[h(\mathbf{x}^s) \neq h'(\mathbf{x}^s)\right] \right|$$

The $\mathcal{H}\Delta\mathcal{H}$-divergence [Ben-David et al.,NIPS'06;MLJ'10]

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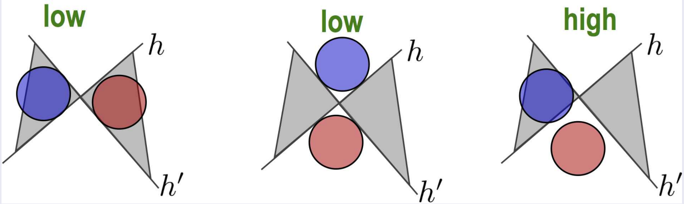
Illustration with **only 2 hypothesis in $\mathcal{H}$** h and h'



Note: With a larger $\mathcal{H}$, the distance will be **high** since we can easily find two hypothesis able to **distinguish** the two domains

Computable from samples

Consider two samples S, T of size m from D_S and D_T

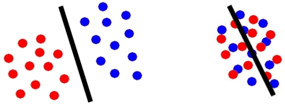
$$d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) \leq d_{\mathcal{H}\Delta\mathcal{H}}(S, T) + O(\text{complexity}(\mathcal{H})\sqrt{\frac{\log(m)}{m}})$$

complexity($\mathcal{H}$): VC-dimension [Ben-david et al.,06;'10], Rademacher [Mansour et al.,'09]

Empirical estimation

$$\hat{d}_{\mathcal{H}\Delta\mathcal{H}}(S, T) = 2\left(1 - min_{h\in\mathcal{H}}\left[\frac{1}{m}\sum_{\mathbf{x}:h(\mathbf{x})=-1} I[\mathbf{x} \in S] + \frac{1}{m}\sum_{\mathbf{x}:h(\mathbf{x})=1} I[\mathbf{x} \in T]\right]\right)$$

$\Rightarrow$ Already seen: label source examples as -1, target ones as +1 and try to learn a classifier in $\mathcal{H}$ minimizing the associated empirical error



Going to a generalization bound

Preliminaries

- $R_{P_T}(h, h') = \underset{(\mathbf{x},y)\sim P_S}{\mathbf{E}} I[h(\mathbf{x}) \neq h'(\mathbf{x})] = \underset{\mathbf{x}\sim D_T}{\mathbf{E}} I[h(\mathbf{x}) \neq h'(\mathbf{x})]$
  R_{P_T} (R_{P_S}) fulfills the triangle inequality
- $|R_{P_T}(h, h') - R_{P_S}(h, h')| \leq \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T)$
  since $d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) = 2 \sup_{(h,h')\in\mathcal{H}^2} \left| R_{D_T}(h, h') - R_{D_S}(h, h') \right|$
- $h^*_S = \mathrm{argmin}_{h\in\mathcal{H}} R_{P_S}(h)$: best on source
- $h^*_T = \mathrm{argmin}_{h\in\mathcal{H}} R_{P_T}(h)$: best on target

Ideal joint hypothesis

$h^* = \mathrm{argmin}_{h\in\mathcal{H}} R_{P_S}(h) + R_{P_T}(h)$; $\lambda = R_{P_S}(h^*) + R_{P_T}(h^*)$

A first bound

$$R_{P_T}(h) \leq$$

A first bound

$$R_{P_T}(h) \leq R_{P_T}(h^*) + R_{P_T}(h, h^*)$$

A first bound

$$
\begin{aligned}
R_{P_T}(h) &\leq R_{P_T}(h^*) + R_{P_T}(h, h^*) \\
&\leq R_{P_T}(h^*) + R_{P_S}(h, h^*) + R_{P_T}(h, h^*) - R_{P_S}(h, h^*)
\end{aligned}
$$

A first bound

$$
\begin{aligned}
R_{P_T}(h) &\leq R_{P_T}(h^*) + R_{P_T}(h, h^*) \\
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&\leq R_{P_T}(h^*) + R_{P_S}(h, h^*) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T)
\end{aligned}
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\begin{aligned}
R_{P_T}(h) &\leq R_{P_T}(h^*) + R_{P_T}(h, h^*) \\
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&\leq R_{P_T}(h^*) + R_{P_S}(h) + R_{P_S}(h^*) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T)
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R_{P_T}(h) &\leq R_{P_T}(h^*) + R_{P_T}(h, h^*) \\
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&\leq R_{P_S}(h) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + \lambda
\end{aligned}
$$

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\begin{aligned}
R_{P_T}(h) &\leq R_{P_T}(h^*) + R_{P_T}(h, h^*) \\
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&\leq R_{P_S}(h) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + \lambda \\
&\leq R_S(h) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(S, T) + O(\text{complexity}(\mathcal{H})\sqrt{\frac{\log(m)}{m}}) + \lambda
\end{aligned}
$$

Main theoretical bound

Theorem [Ben-David et al.,MLJ'10,NIPS'06]

Let $\mathcal{H}$ a symmetric hypothesis space. If D_S and D_T are respectively the marginal distributions of source and target instances, then for all $\delta \in (0,1]$, with probability at least $1-\delta$:

$$\forall h \in \mathcal{H}, \quad R_{P_T}(h) \leq R_{P_S}(h) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + \lambda$$

Formalizes a natural approach: Move closer the two distributions while ensuring a low error on the source domain.

Justifies many algorithms:

- reweighting methods,
- feature-based methods,
- adjusting/iterative methods.

Another analysis [Mansour et al.,COLT'09]

$$R_{P_T}(h) \leq$$

Another analysis [Mansour et al.,COLT'09]

$$R_{P_T}(h) \leq R_{P_T}(h, h_S^*) + R_{P_T}(h_S^*, h_T^*) + R_{P_T}(h_T^*)$$

Another analysis [Mansour et al.,COLT'09]

$$
\begin{aligned}
R_{P_T}(h) &\leq R_{P_T}(h, h^*_S) + R_{P_T}(h^*_S, h^*_T) + R_{P_T}(h^*_T) \\
&= R_{P_T}(h, h^*_S) + \nu
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&= R_{P_T}(h, h_S^*) + \nu\\
&\leq R_{P_S}(h, h_S^*) + R_{P_T}(h, h_S^*) - R_{P_S}(h, h_S^*) + \nu
\end{aligned}
$$

Another analysis [Mansour et al.,COLT'09]

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&\leq R_{P_S}(h, h_S^*) + R_{P_T}(h, h_S^*) - R_{P_S}(h, h_S^*) + \nu \\
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&\leq R_{P_S}(h, h_S^*) + R_{P_T}(h, h_S^*) - R_{P_S}(h, h_S^*) + \nu \\
&\leq R_{P_S}(h, h_S^*) + |R_{P_T}(h, h_S^*) - R_{P_S}(h, h_S^*)| + \nu \\
&\leq R_{P_S}(h, h_S^*)) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + \nu
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&\leq R_{P_S}(h, h_S^*) + R_{P_T}(h, h_S^*) - R_{P_S}(h, h_S^*) + \nu \\
&\leq R_{P_S}(h, h_S^*) + |R_{P_T}(h, h_S^*) - R_{P_S}(h, h_S^*)| + \nu \\
&\leq R_{P_S}(h, h_S^*)) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + \nu \\
&(\leq R_{P_S}(h) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + R_{P_S}(h_S^*) + \nu)
\end{aligned}
$$

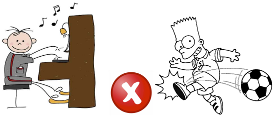
if h_S^* is not the true labeling function

Another analysis [Mansour et al.,COLT'09]

$$
\begin{aligned}
R_{P_T}(h) &\leq R_{P_T}(h, h^*_S) + R_{P_T}(h^*_S, h^*_T) + R_{P_T}(h^*_T) \\
&= R_{P_T}(h, h^*_S) + \nu \\
&\leq R_{P_S}(h, h^*_S) + R_{P_T}(h, h^*_S) - R_{P_S}(h, h^*_S) + \nu \\
&\leq R_{P_S}(h, h^*_S) + |R_{P_T}(h, h^*_S) - R_{P_S}(h, h^*_S)| + \nu \\
&\leq R_{P_S}(h, h^*_S)) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + \nu \\
&(\leq R_{P_S}(h) + \frac{1}{2} d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T) + R_{P_S}(h^*_S) + \nu) \\
&\qquad \text{if } h^*_S \text{ is not the true labeling function}
\end{aligned}
$$

- This analysis can lead to smaller when adaptation is possible
- Leads to the same type of bound, just the constant changes .....

Characterization of the possibility of domain adaptation



Constants characterize when adaptation is possible

- $\lambda = R_{P_S}(h^*) + R_{P_T}(h^*),\ h^* = \mathrm{argmin}_{h \in \mathcal{H}} R_{P_S}(h) + RPT(h)$
  There must exist an ideal joint hypothesis with small error
- $\nu = R_{P_T}(h^*_S, h^*_T) + R_{P_T}(h^*_T)$
  there must exist a very good hypothesis on the target and the best hypothesis on source must be close to the best on target w.r.t to D_T

Other settings

Discrepancy [Mansour et al.,COLT'09]

- instead of the 0-1 loss, more general loss functions ℓ (*i.e.* L_p norms) $\text{disc}^{\ell}(D_S, D_T) = \sup_{h,h' \in \mathcal{H}} |R^{\ell}_{D_S}(h, h') - R^{\ell}_{D_T}(h, h')|$
- This discrepancy can be minimized and used as a reweighting method ([Mansour et al.,COLT'09] - polynomial for L_2 norm for example)

Using some target labeled data

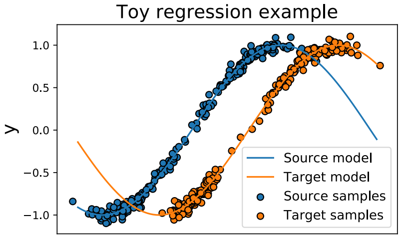
- Weighting the empirical source and target risks [Ben David et al.,2010]
- Using a divergence taking into account target labels [Zhang et al.,NIPS'12] (a divergence must take into account marginals over X and Y, the λ constant counts for Y)

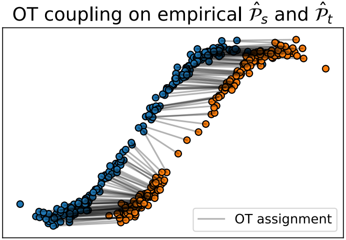
Other settings

Averaged quantities [Germain et al.,ICML'13]

- Consider a probability distribution ρ (posterior) over $\mathcal{H}$ to learn and the following risk: $\mathop{\mathbf{E}}_{h\sim\rho} R_{P_T}(h)$
- Definition of an averaged distance
  $$\mathrm{dis}(D_S, D_T) = \left| \mathop{\mathbf{E}}_{h,h'\sim\rho^2} [R_{D_T}(h,h') - R_{D_S}(h,h')] \right|$$
- Similar generalization bound
  $$\mathop{\mathbf{E}}_{h\sim\rho} R_{P_T}(h) \leq \mathop{\mathbf{E}}_{h\sim\rho} R_{P_S}(h) + \mathrm{dis}(D_S, D_T) + \lambda_{\rho^*}$$
- Estimation from samples controlled by PAC-Bayesian theory
- Bound tighter without a supremum

Other settings





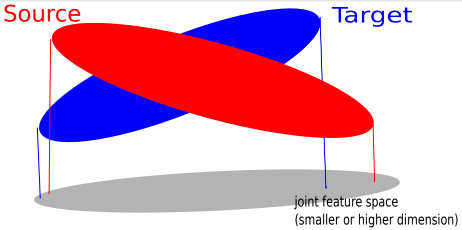
Alignment with optimal transport [Courty et al., '14-'16]

- Find an alignment that minimizes the cost of transportation between source and target
- Optimal transport (Wasserstein distance)
  $W(P_s, P_t) = \min_\gamma \int_{\Omega_s \times \Omega_t} c(x_s, x_t)\gamma(x_s, x_t) dx_s dx_t$
  such that $\int_{\Omega_t} \gamma(x_s, x_t) dx_t = P_s$ and $\int_{\Omega_s} \gamma(x_s, x_t) dx_s = P_t$, where c is a distance/cost function (*i.e.* euclidean distance).

Feature/Projection based Approaches

Idea

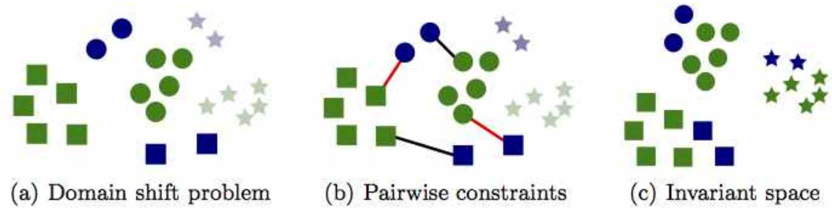
- Change the feature representation X to better represent shared characteristics between the two domains
  - some features are domain-specific,
  - others are generalizable
  - or there exist mappings from the original space
- $\Rightarrow$ Make source and target domain explicitely similar
- $\Rightarrow$ Learn a new feature space by embedding or projection



Metric Learning [Kulis et al.,'11;Saenko et al.,'10]

- Mahalanobis: $d^2_{\mathbf{W}}(\mathbf{x},\mathbf{x}') = (\mathbf{x}-\mathbf{x}')^T\mathbf{W}(\mathbf{x}-\mathbf{x}')$
- PSD matrix $\mathbf{W} = L^T L$,
  L projection space of dimension $\mathbb{R}^{rank(\mathbf{W})\times d}$
  $(L\mathbf{x} - L\mathbf{x}')^T(L\mathbf{x} - L\mathbf{x}')$
- Pair-wise constraints: source ex. $(\mathbf{x}^s_i, y^s_i)$ and target $(\mathbf{x}^t_j, y^t_j)$
  - $d^2_{\mathbf{W}}(\mathbf{x}^s_i,\mathbf{x}^t_j) \leq u$ if $y^s_i = y^t_j$ (source and target must be similar)
  - $d^2_{\mathbf{W}}(\mathbf{x}^s_i,\mathbf{x}^t_j) \geq l$ if $y^s_i \neq y^t_j$ (source and target must be dissimilar)
  - Require some target labels

Metric Learning [Kulis et al.,CVPR'11;Saenko et al.,ECCV'10]



[Saenko et al.,ECCV'10]

Formulation (based on ITML [Davis et al.,ICML'07])

$$\min_{\mathbf{W}\succeq 0} \quad Tr(\mathbf{W}) - \log\det\mathbf{W}$$

$$\text{s.t.} \quad d^2_{\mathbf{W}}(\mathbf{x}^s_i, \mathbf{x}^t_j) \leq u, \forall(\mathbf{x}^s_i, \mathbf{x}^t_j) \in \mathcal{S}imilarSet$$

$$d^2_{\mathbf{W}}(\mathbf{x}^s_i, \mathbf{x}^t_j) \geq l, \forall(\mathbf{x}^s_i, \mathbf{x}^t_j) \in \mathcal{D}issimilarSet$$

$\Rightarrow$ Can be kernelized

(Simple) Feature augmentation [Daume III et al.,'07;'10]

- $\phi(\mathbf{x}) =< \mathbf{x}, \mathbf{x}, 0 >$ for source instances
- $\phi(\mathbf{x}) =< \mathbf{x}, 0, \mathbf{x} >$ for target instances
- $\Rightarrow$ Share some relevant features and not irrelevant ones (*e.g.* in text sentiment analysis: find shared words)
  $\Rightarrow$ a way to allow the existency of the ideal joint hypothesis h^*

Learn in the new space ϕ

- Require target labels
- Bound: $R_{D_T} \leq$
  $\frac{1}{2}(R_T + R_S) + O(complexity) + (\frac{1}{m_s} + \frac{1}{m_t})O(\frac{1}{\delta}) + O(d_{\mathcal{H}\Delta\mathcal{H}}(D_S, D_T))$
- Kernelized and semi-supervised versions [add: $(+1, < 0, \mathbf{x}, -\mathbf{x} >)$ and $(-1, < 0, \mathbf{x}, -\mathbf{x} >)$ to learning sample]

Find latent spaces - Structural Correspondence Learning [Blitzer et al.,'07]

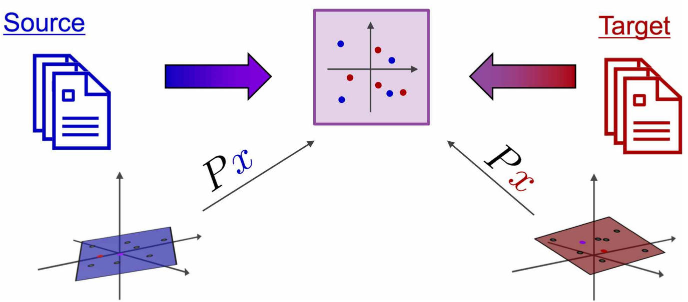
Identify shared features

Domains	Negative	Positive
Books	plot <num>_pages predictable reading_this page_<num>	reader grisham engaging must_read fascinating
Kitchen	the_plastic poorly_designed leaking awkward_to defective	excellent_product espresso are_perfect years_now a_breeze
Pivot features	weak don't_waste awful	and_easy loved_it a_wonderful a_must highly_recommended

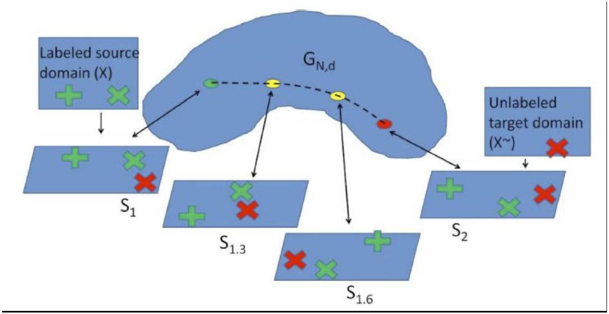
- Sentiment analysis - Bag of words (bigrams)
- Choose K pivot features (frequent words in both domains, highly correlated with labels)
- Learn K classifiers to predict pivot features from remaining features
- For each feature add K new features
- Represents source and target data with these features

Find latent spaces - Structural Correspondence Learning [Blitzer et al.,'07]

- Apply PCA source+target new features to get a low rank latent representation
- Learn a classifier in the new projection space defined by PCA

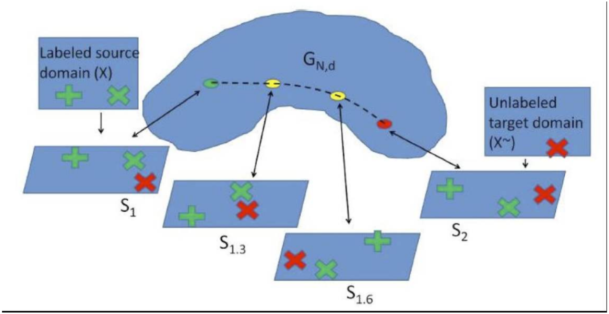


Manifold-based methods



- Assume $X \subseteq \mathbb{R}^N$
- Apply PCA on source data $\Rightarrow$ matrix $\mathbf{S}_1$ of rank d
- Apply PCA on target data $\Rightarrow$ matrix $\mathbf{S}_2$ of rank d
- Geodesic path on the Grassman manifold $\mathbb{G}_{N,d}$ (d-dimensional vector subspaces $\subset \mathbb{R}^N$) between $\mathbf{S}_1$ and $\mathbf{S}_2$

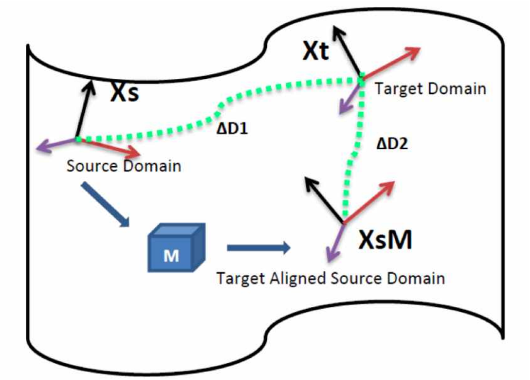
Manifold-based methods



[Gopalan et al.,'10]

- Use of an exponential flow $\psi(t') = \mathbf{Q}\exp(t'\mathbf{B})\mathbf{J}$ with $\mathbf{Q}$ $N \times N$ matrix with determinant 1 s.t. $\mathbf{Q}^T\mathbf{S}_1 = \mathbf{J}$ and $\mathbf{J}^T = [\mathbf{I}_d \mathbf{0}_{N-d,d}]$ intermediate subspaces are obtained by computing $\mathbf{B}$ (skew block-diagonal matrix) and varying t' between 0 and 1
- Take a collection S' of l subspaces between $\mathbf{S}_1$ and $\mathbf{S}_2$ on the manifold
- Project the data on $\mathbf{S'}$ and learn in that new space

A simpler approach - Subspace alignment [Fernando et al.,ICCV'13]



- Move closer PCA-based representations
- Totally unsupervised

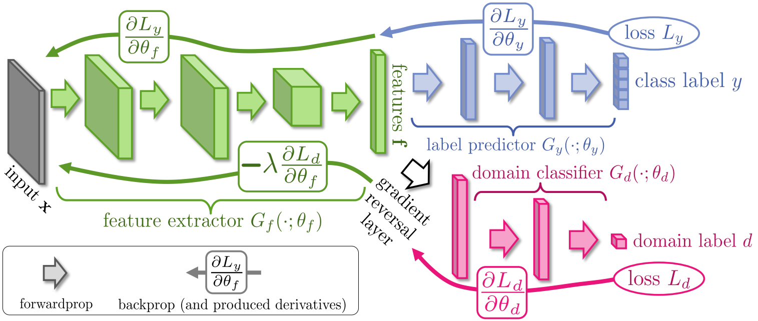
Subspace alignment algorithm

Algorithm 1: Subspace alignment DA algorithm

Data: Source data S, Target data T, Source labels Y_S, Subspace dimension d
Result: Predicted target labels Y_T
$\mathbf{S}_1 \leftarrow PCA(S, d)$ (source subspace defined by the first d eigenvectors) ;
$\mathbf{S}_2 \leftarrow PCA(T, d)$ (target subspace defined by the first d eigenvectors);
$\mathbf{X}_a \leftarrow \mathbf{S}_1\mathbf{S}_1{}'\mathbf{S}_2$ (operator for aligning the source subspace to the target one);
$\mathbf{S}_a = S\mathbf{X}_a$ (new source data in the aligned space);
$\mathbf{T}_T = T\mathbf{S}_2$ (new target data in the aligned space);
$Y_T \leftarrow Classifier(\mathbf{S}_a, \mathbf{T}_T, Y_S)$;

- $\mathbf{M}^* = \mathbf{S}_1{}'\mathbf{S}_2$ corresponds to the "subspace alignment matrix":
  $\mathbf{M}^* = \mathrm{argmin}_{\mathbf{M}} \ \|\mathbf{S}_1\mathbf{M} - \mathbf{S}_2\|$
- $X_a = \mathbf{S}_1\mathbf{S}_1{}'\mathbf{S}_2 = \mathbf{S}_1\mathbf{M}^*$ projects the source data to the target subspace
- A natural similarity: $\mathrm{Sim}(\mathbf{x}_s, \mathbf{x}_t) = \mathbf{x}_s\mathbf{S}_1\mathbf{M}^*\mathbf{S}_1{}'\mathbf{x}_t' = \mathbf{x}_s\mathbf{A}\mathbf{x}_t'$

Deep Learning - adversarial strategy



Idea of adversarial Learning [Ganin et al., 2015, 2016]

- Find a representation where source and target cannot be discriminated
- while ensuring a good performance on source.

Feature-based method

- Feature-based approaches are very popular
  Many other (SVM/kernel-based, MKL, deep learning [Glorot et al.,ICML'11], ...) methods not covered here,
- Subspace-based methods/Deep learning $\Rightarrow$ "hot topic"
- Embed similarity map: define feature as similarity to landmarks points
  - labeled source instances distributed similarly to the target domain

[Grauman,VisDA-WS_ICCV'13] $\rightarrow$ subsampling: work with instances facilitating adaptation
or use distances to headphones as a representation
$< k(\cdot,\mathbf{x}_1), k(\cdot,\mathbf{x}_2), k(\cdot,\mathbf{x}_3), k(\cdot,\mathbf{x}_4), k(\cdot,\mathbf{x}_5), \ldots >, \ldots$

Adjusting/Iterative methods

Principle

- Integrate some information about the target samples iteratively
  $\Rightarrow$ use of pseudo-labels
- "Move" closer distributions
  $\Rightarrow$ Remove/add some instances $\Rightarrow$ take into account a divergence measure
- Repeat the process until convergence or no remaining instances

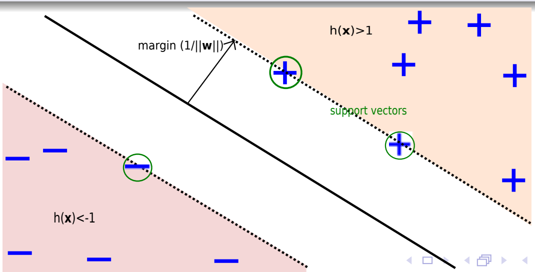
A brief recap on SVM

- Learning sample $LS = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$
- Learn a classifier $h(\mathbf{x}) = \langle \mathbf{w}, \mathbf{x} \rangle + b$

$$\text{Formulation: } \min_{\mathbf{w},b,\boldsymbol{\xi}} \quad \frac{1}{2}\|\mathbf{w}\|_2^2 + C\sum_{i=1}^n \xi_i$$

$$\text{subject to} \quad \ell_i(\langle \mathbf{w}, \mathbf{x_i} \rangle + b) \geq 1 - \xi_i, \quad 1 \leq i \leq n$$

$$\boldsymbol{\xi} \succeq 0$$

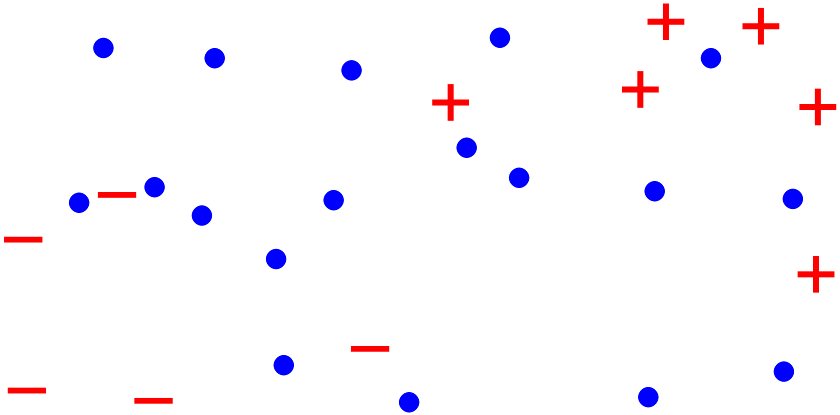


DASVM principle

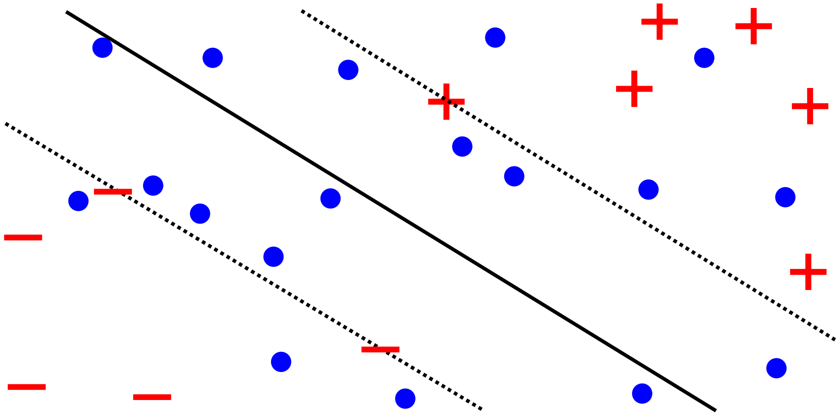
1. $LS = S$
2. Learn a classifier h^0 from the learning sample LS
3. Repeat until stopping criterion
   - Select the first k target examples $\mathbf{x}^t$ s.t. $0 < h(\mathbf{x}^t) < 1$ with highest margin and affect the pseudo-label -1
   - Select the first k target examples $\mathbf{x}^t$ s.t. $-1 < h(\mathbf{x}^t) < 0$ with highest margin and affect them the pseudo-label $+1$
   - Add these $2k$ examples (pseudo-labeled) to LS
   - Remove from LS the first k positive and k negative source instances with highest margin
4. Output the last classifier

Algorithm stops when the number of selected instances at each step downs to a threshold.

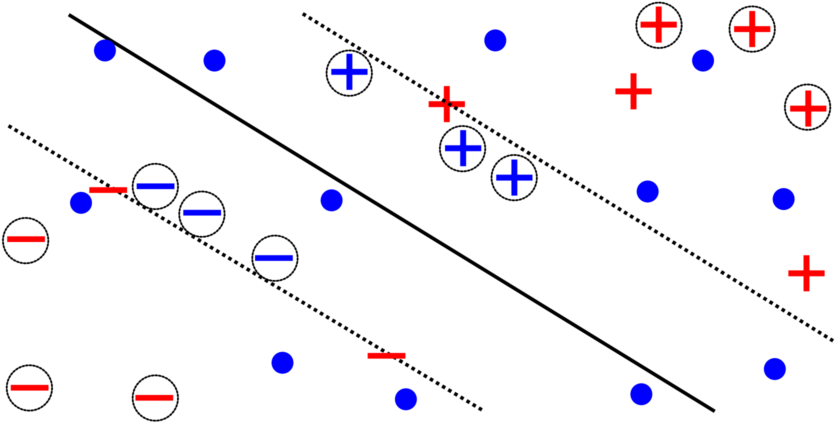
DASVM - graphical illustration



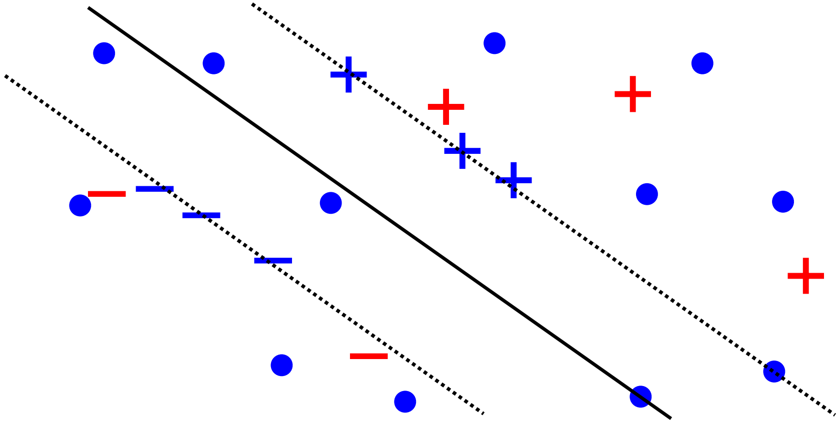
DASVM - graphical illustration



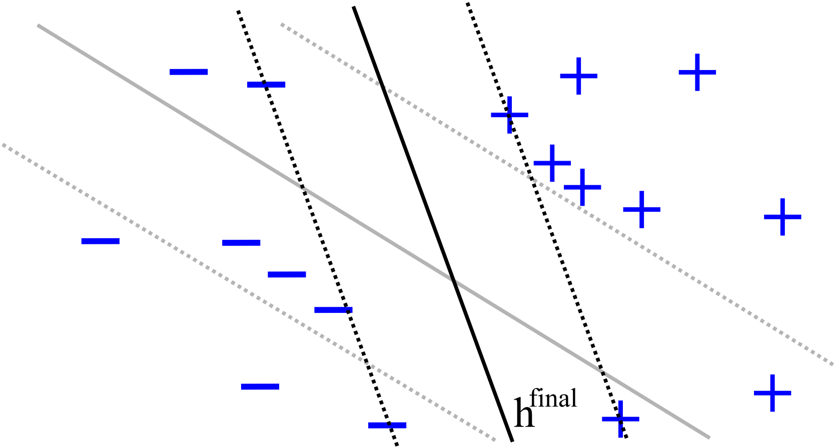
DASVM - graphical illustration



DASVM - graphical illustration



DASVM - graphical illustration



Convergence - theoretical guarantees

- What we need?: At each step pseudo-labels on target are sufficiently reasonable.
- $\Rightarrow$ Can be tackled with a notion of weak classifier

Weak classifier on a single domain

An hypothesis h^n, learned at iteration n, is a weak learner over a labeled sample S if it performs a bit better than a random guessing: $\exists \gamma_n \in]0; \frac{1}{2}$

$$R_S(h^n) = \hat{Pr}_{(\mathbf{x}_i^s, y_i) \sim S^n}[h^n(\mathbf{x}_i) \neq y_i] = \frac{1}{2} - \gamma_n$$

A notion of weak learner for controlling pseudo-labels

Self labeling weak learner

A classifier $h^{(i)}$ learned at iteration over a current learning sample LS^i is self labeling weak learner w.r.t. a set SL^j of $2k$ pseudo-labeled examples introduced at step j if its true error over SL^j is strictly better than random guessing:

$$R_{SL^j}(h^{(i)}) = Pr_{\mathbf{x}_i^t \in SL^j}[h(\mathbf{x}_i^t) \neq y_i^t] < \frac{1}{2}$$

A first necessary condition

Theorem

Let $h^{(i)}$ a weak learner output at iteration i from $LS^{(i)}$, let $\tilde{R}_{LS^i}(h^{(i)}) = \frac{1}{2} - \gamma^i_{LS}$ the associated empirical error.
Let $R^{(i)}_T = \frac{1}{2} - \gamma^{(i)}_T$ the true empirical error over T.

$$h^{(i)} \text{ is a self-labeling weak learner if } \gamma^i_{LS} > 0$$

$\Rightarrow$ $h^{(i)}$ will be able to correctly classify (w.r.t. their unknown true label) more than k pseudo-labeled target examples among $2k$ if at least half of them have been correctly pseudo-labeled.

A second result

Theorem

Let S a labeled source sample of m_S instances and T a target sample of $m_t \geq m_s$ unlabeled instances. Let $\mathcal{A}$ an iterative labeling algorithm using $2k$ examples at each step. $\mathcal{A}$ is able to perform an adaptation if

- $\gamma_S^{(i)} \geq \gamma_T^{(i)}, \forall i = 1 \ldots \frac{m_s}{2k}$
- $\gamma_S^{\max} > \sqrt{\frac{\gamma_T^{(0)}}{2}}$

$\Rightarrow$ $h^{(i)}$ has to perform sufficiently well on the data it has been learned from
$\Rightarrow$ $\mathcal{A}$ the final classifier has to work better on T than a classifier learned only from source data.

A simple illustration

Task: handwritten digit recognition. P_1 scaling problem between 1 and 0 and P_2 rotation problem between 5 and 7.

Iteration	P_1			P_2		
	$\gamma_S^{(i)}$	$\gamma_T^{(i-1)}$	1 - $\hat{\epsilon}_T^{(i)}$	$\gamma_S^{(i)}$	$\gamma_T^{(i-1)}$	1 - $\hat{\epsilon}_T^{(i)}$
1	0.5	0	0.585	0.50	-0.1	0.32
2	0.475	0.085	0.75	0.50	-0.18	0.285
3	0.48	0.25	0.73	0.50	-0.215	0.285
4	0.49	0.23	0.795	0.50	-0.215	0.24
5	0.49	0.295	0.875	0.50	-0.26	0.18
6	0.49	0.375	0.94	0.50	-0.32	0.205
7	0.49	0.44	0.94	0.50	-0.295	0.19
8	0.49	0.44	0.94	0.50	-0.31	0.12
9	0.49	0.44	0.94	0.50	-0.38	0.145
10	0.495	0.44	0.985	0.50	-0.355	0.115
11	0.5	0.485	0.99	0.495	-0.385	0.115

For P_1, we can check $\gamma_S^{(i)} \geq \gamma_T^{(i)}, \forall i = 1 \ldots \frac{m_s}{2k}$, and $\gamma_S^{\max} > \sqrt{\frac{\gamma_T^{(0)}}{2}}$.

Interpretation summary

- $h^{(i)}$ must work well on T
- $h^{(i)}$ must work well on S
- $\mathcal{A}$ works better than a non adaptation process

$\Rightarrow$ Necessary and reasonable conditions
$\Rightarrow$ Condition on T hard to check in practise
$\Rightarrow$ Boosting

Ensemble Methods and Boosting

Definition

Ensemble methods infer a set of classifiers $h_1, \ldots, h_N$ whose individual decisions are combined in some way to classify new examples.

Necessary conditions for an ensemble method to be efficient

- the individual classifiers are better than random guessing.
- they are diverse, *i.e.* they make different errors on new data points.

AdaBoost

- Learns step by step **weak** binary classifiers.
- **Optimizes a convex loss** by increasing the weights of misclassified examples.
- Builds a **convex combination** of the weak classifiers.

AdaBoost

Data: A learning sample S, a number of iterations N, a weak learner L
Result: A global hypothesis H_N
for $i = 1$ **to** m **do** $D_1(\mathbf{x_i}) = 1/m$;
for $t = 1$ **to** T **do**

$h_n = \textsc{Learn}(S, \mathbf{D}_n)$;

$\hat{\epsilon}_n = \sum_{\mathbf{x_i}\ s.t.\ y_i \neq h_n(\mathbf{x_i})} D_n(\mathbf{x_i})$;

$\alpha_n = \frac{1}{2} \ln \frac{1-\hat{\epsilon}_n}{\hat{\epsilon}_n}$;

for $i = 1$ **to** m **do**

$D_{t+1}(\mathbf{x_i}) = D_n(\mathbf{x_i}) \exp\left(-\alpha_t y_i h_t(\mathbf{x_i})\right)/Z_n$;

/* Z_n is a normalization coefficient*/

end

end

$f(\mathbf{x}) = \sum_{t=1}^{T} \alpha_n h_n(\mathbf{x})$;
$H_N(\mathbf{x}) = \text{sign}\left(f(\mathbf{x})\right)$;

Theoretical result on the empirical error

Theorem

Upper bound on the empirical error of the final classifier H_N

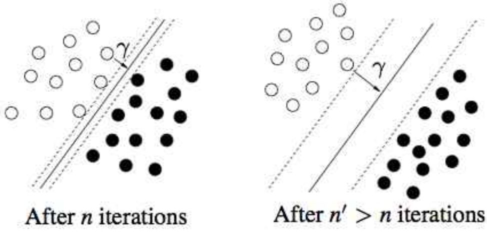
$$\hat{\epsilon}_{H_N} \leq \prod_{t=1}^{T} Z_n \leq exp(-2\sum_{t=1}^{T}\gamma_n^2)$$

where $\hat{\epsilon}_n = \frac{1}{2} - \gamma_n$ (weak hypothesis).
$\hat{\epsilon}_{H_N}$ is optimized with $\alpha_n = \frac{1}{2}\ln\frac{1-\hat{\epsilon}_n}{\hat{\epsilon}_n}$.

Meaning

This theorem means that the empirical error **exponentially decreases towards 0** with the number T of iterations.

Explanation in terms of margins of the training examples



Theorem

$\forall \gamma > 0$, with probability $1 - \delta$, any classifier ensemble H_T satisfies:

$$\epsilon_{H_T} \leq \mathbb{E}_{\mathbf{x} \in S}[margin(\mathbf{x}) \leq \gamma] + \mathcal{O}\left(\sqrt{\frac{d_h}{m}\frac{log^2(m/d_h)}{\gamma^2} + log(1/\delta)}\right),$$

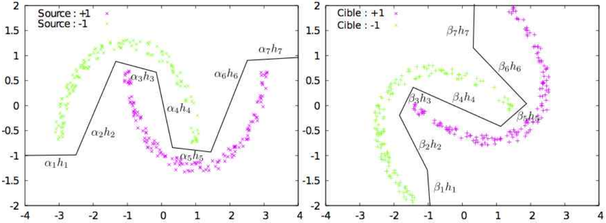
where $\mathbb{E}_{\mathbf{x} \in S}[margin(\mathbf{x}] \leq \gamma]$ exponentially decreases towards 0 with T.

$\Rightarrow$ apply this idea on source and target (pseudo-labels)

Idea for domain adaptation

Double weighting strategy

- Keep the same weak hypothesis for both domains $h_1, \ldots, h_n, \ldots, h_N$
- Learn two functions
  - Source domain: $F_S^N = \sum_{n=1}^N \alpha_n h_n(\mathbf{x})$
  - Target domain: $F_T^N = \sum_{n=1}^N \beta_n h_n(\mathbf{x})$
- β_n depends on the (pseudo-)margin of the examples and a divergence measure



A notion of weak DA learner

Weak DA learner

A classifier h_n learned at iteration n from a S and T and a divergence $g_n \in [0,1]$ between S and T and $f_{DA}(h_n(\mathbf{x}_i)) = |h_n(\mathbf{x}_i)| - \lambda g_n$, is a weak DA learner for T if:

1. h_n is a weak learner for S.
2. $\hat{L}_n = \mathbb{E}_{\mathbf{x}_i \sim T}[|f_{DA}(h_n(\mathbf{x}_i))| \leq \gamma] < \frac{\gamma}{\gamma + \max(\gamma, \lambda g_n)}$

- f_{DA}: obtaining high margin with small divergence
- if $\max(\gamma, \lambda g_n) = \gamma$: divergence is small, we are close to a semi-supervised setting
- if $\max(\gamma, \lambda g_n) = \lambda g_n$: divergence is high and the reweighting scheme requires a specific attention to the divergence.

Algorithm SLDAB

SLDAB

Data: Learning sample S, Nb of iterations N, unlabeled sample T, $\gamma \in [0,1]$, $\lambda \in [0,1]$
Result: Source and target hypothesis H_S, H_T
foreach $\forall(\mathbf{x}_i^s, y_i^s) \in S,\ \mathbf{x}_j^t \in T$ **do** $D_1^S(\mathbf{x}_i^s) = 1/m_s$; $D_1^T(\mathbf{x}_j^t) = 1/m_t$;
for $n = 1$ **to** N **do**

   Learn h_n to produce a **weak DA** learner; compute g_n;

   $\alpha_n = \frac{1}{2} \ln \frac{1-\hat{\epsilon}_{S^n}(h^n)}{\hat{\epsilon}_{S^n}(h^n)}$; $\beta_n = \frac{1}{\gamma + \max(\gamma, \lambda g_n)} \ln \frac{\gamma W_n^+}{\max(\gamma, \lambda g_n) W_n^-}$;

   for $(\mathbf{x}_i^s, y_i^s) \in S$ **do** $D_{n+1}(\mathbf{x}_i) = D_n(\mathbf{x}_i^s) \exp\left(-\alpha_n y_i^s \mathrm{sign}(h_n(\mathbf{x}_i^s))\right)/Z_S^n$;

   for $\mathbf{x}_j^t \in T$ **do**

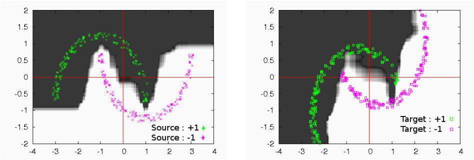
      $D_T^{n+1}(\mathbf{x}_j) = D_T^n(\mathbf{x}_j^t) \exp\left(-\beta_n y_j^n f_{DA}(h_n(\mathbf{x}_j^t))\right)/Z_T^n$;
      where $y_j^n = -\mathrm{sign}(f_{DA}(h_n(\mathbf{x})))$ if $|f_{DA}(h_n(\mathbf{x}))| > \gamma$ and $y_j^n = -\mathrm{sign}(f_{DA}(h_n(\mathbf{x})))$ otherwise;

   end

end

$F_S^N(\mathbf{x}^s) = \sum_{n=1}^N \alpha_n \mathrm{sign}(h_n(\mathbf{x}^s))$;
$H_T^N(\mathbf{x}^t) = \sum_{n=1}^N \beta_n \mathrm{sign}(h_n(\mathbf{x}^t))$;

Conclusion



Theoretical results

- Convergence of the empirical losses (exponentially fast to 0) $\Rightarrow$ (pseudo-)margin increases
- No generalization bound over the true error on P_T

Difficult aspects

- Defining divergence g_n: avoid degenerate cases
- Finding/Learning weak DA learned: need to take into account both source error and divergence information

Model selection?

Some existing method, not really admitted as state of the art but used sometimes in unsupervised DA.

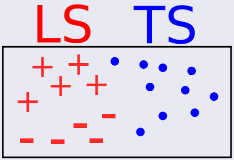
Otherwise, some people use a small set of target instances for validation.

Reverse validation [Zhong et aL.,ECML'10;Bruzzone et al.,PAMI'10]

Reverse classifier h^r

Reverse validation [Zhong et aL.,ECML'10;Bruzzone et al.,PAMI'10]

Reverse classifier h^r



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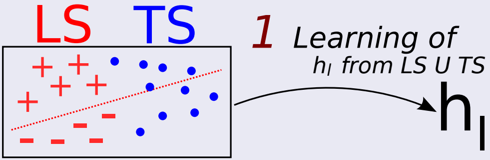
Reverse classifier h^r



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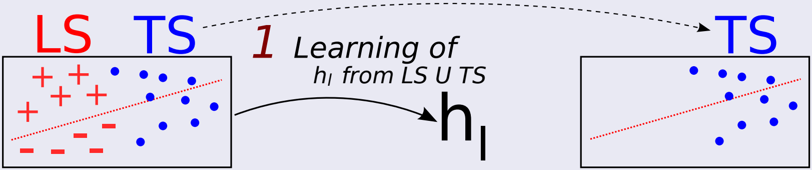

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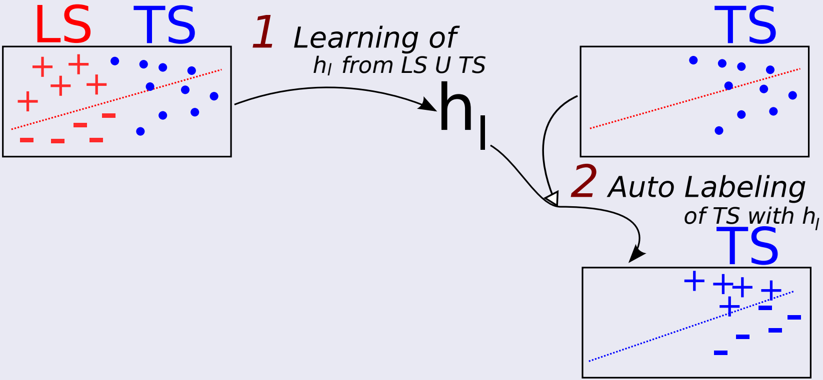
Reverse classifier h^r



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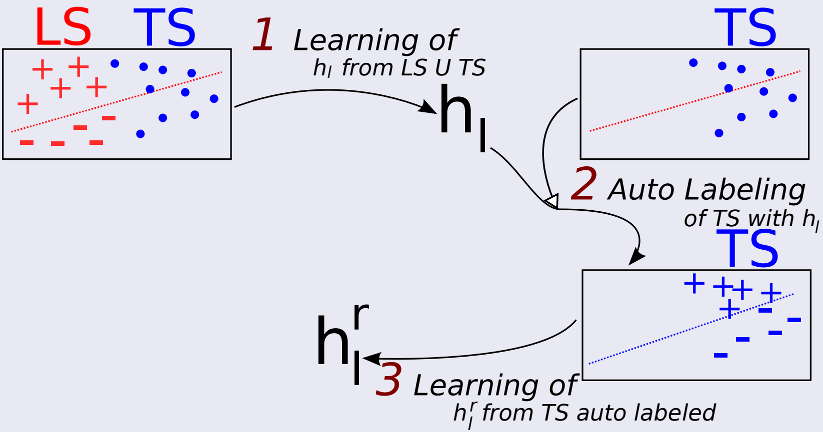
Reverse classifier h^r



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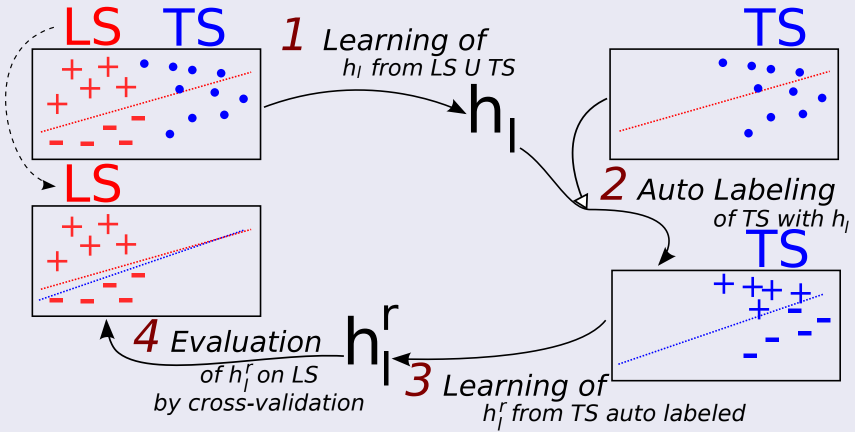

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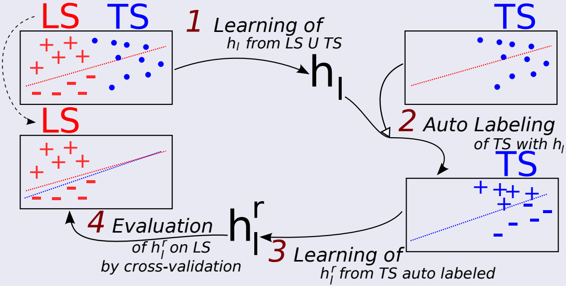
Reverse classifier h^r



- Two domains are related $\Rightarrow$ h_l^r performs well on the source domain
- Used with target labels to have an estimation of R_{P_T}
- Used to heuristically estimate theoretical constants of adaptability (λ) [Morvant et al.,ICDM'11;KAIS'12]

Conclusion

Conclusion

- Very active domains - Lots of methods (Sometimes difficult to follow)
  Approaches not covered here: probabilistic-based, bayesian, deep learning methods, etc.
- Same idea: Moving closer the distributions while ensuring good accuracy on labeled data
- Can we imagine general efficient frameworks
  $\Rightarrow$ probably No: DA is difficult [Ben-David et al.,ALT'12]
  $\Rightarrow$ Choose a method in function of the task/data
- Importance of data preparation
- Importance of divergence measures (there exist other frameworks)
- Transfer learning: transfer (the parameters of) a model to another task (initialization/regularization) can make learning faster.

Perspectives

- Understanding negative transfer
- Model selection
- Heterogeneous data
- Large scale
- Links with multi-tasks and multi-source learning, lifelong learning, concept drift, etc.
- $\Rightarrow$ A large room for more research