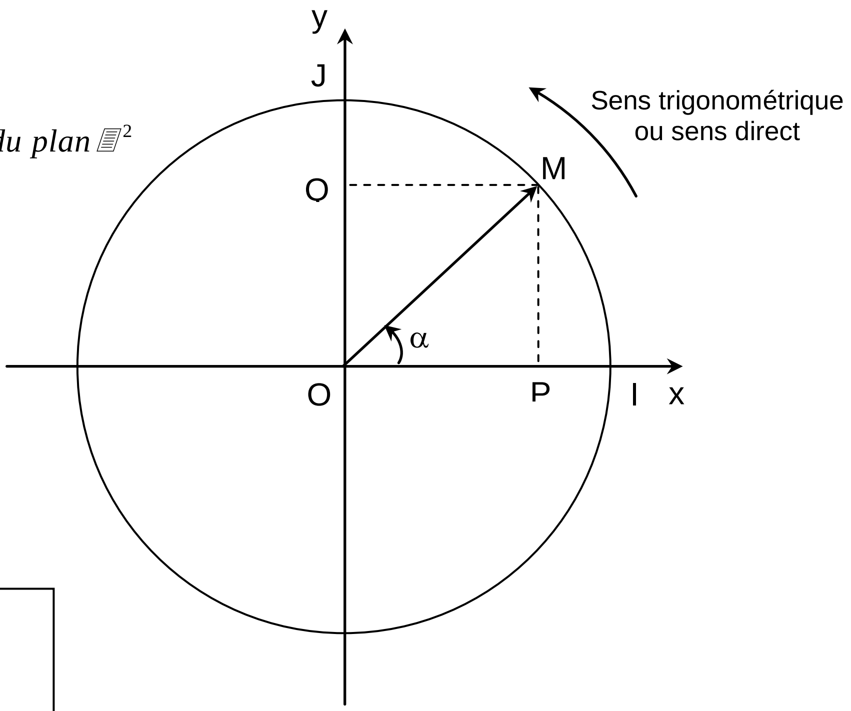
TRIGONOMETRIE

Le cercle trigonométrique :



$\left(O,\vec{i},\vec{j}\right)$ *repère orthonormal du plan* $\mathbb{R}^2$

$\overrightarrow{OI} = \vec{i}$

$\overrightarrow{OJ} = \vec{j}$

$\left\|\overrightarrow{OI}\right\| = \left\|\overrightarrow{OJ}\right\| = r = 1\,(si\ r = 1)$

Rappels :

$OM = \left\|\overrightarrow{OM}\right\| = r$

$$\boxed{\begin{array}{l} cos\,\alpha = \dfrac{OP}{OM} = OP \\[2em] sin\,\alpha = \dfrac{MP}{OM} = \dfrac{OQ}{OM} = OQ \\[2em] tan\,\alpha = \dfrac{OQ}{OP} = \dfrac{sin\,\alpha}{cos\,\alpha}\ , \alpha \neq \dfrac{\pi}{2}\,[\pi] \end{array}}$$

$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM} \quad = \overrightarrow{OP} + \overrightarrow{OQ}$ *(relation de Chasles)*

$\overrightarrow{OP} = \left\|\overrightarrow{OP}\right\|\vec{i}$

$\overrightarrow{OQ} = \left\|\overrightarrow{OQ}\right\|\vec{j}$

$\boxed{\overrightarrow{OM} = r\,cos(\alpha)\,\vec{i} + r\,sin(\alpha)\,\vec{j}}$; *Si le cercle est unitaire, r = 1* $\Rightarrow \boxed{\overrightarrow{OM} = cos\,\alpha\,\vec{i} + sin\,\alpha\,\vec{j}}$

On note : $\boxed{\overrightarrow{OM}\begin{pmatrix} cos\,\alpha \\ sin\,\alpha \end{pmatrix} ou\ \overrightarrow{OM}(cos\,\alpha ; sin\,\alpha)}$

$OP^2 + OQ^2 = OM^2$ *(théorème de Pythagore)*

$\left\|\overrightarrow{OM}\right\| = r = 1 = \sqrt{cos^2\,\alpha + sin^2\,\alpha}$

$\Leftrightarrow \boxed{cos^2(\alpha) + sin^2(\alpha) = 1}$ *(relation fondamentale de la trigonométrie)*

Valeurs remarquables :

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\mathbf{1}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	$\mathbf{1}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$\mathbf{-1}$
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	$\mathbf{1}$	$\sqrt{3}$		$-\sqrt{3}$	$\mathbf{-1}$	$-\frac{\sqrt{3}}{3}$	0

$\cos(-\theta) = \cos\theta$	$\sin(-\theta) = -\sin\theta$	$\tan(-\theta) = -\tan\theta$	$\mathrm{cot}an(-\theta) = -\mathrm{cot}an\theta$
$\cos(\theta+\pi) = -\cos\theta$	$\sin(\theta+\pi) = -\sin\theta$	$\tan(\theta+\pi) = \tan\theta$	$\mathrm{cot}an(\theta+\pi) = \mathrm{cot}an\theta$
$\cos(\pi-\theta) = -\cos\theta$	$\sin(\pi-\theta) = \sin\theta$	$\tan(\pi-\theta) = -\tan\theta$	$\mathrm{cot}an(\pi-\theta) = -\mathrm{cot}an\theta$
$\cos(\theta+\frac{\pi}{2}) = -\sin\theta$	$\sin(\theta+\frac{\pi}{2}) = \cos\theta$	$\tan(\theta+\frac{\pi}{2}) = -\mathrm{cot}an\theta$	$\mathrm{cot}an(\theta+\frac{\pi}{2}) = -\tan\theta$
$\cos(\frac{\pi}{2}-\theta) = \sin\theta$	$\sin(\frac{\pi}{2}-\theta) = \cos\theta$	$\tan(\frac{\pi}{2}-\theta) = \mathrm{cot}an\theta$	$\mathrm{cot}an(\frac{\pi}{2}-\theta) = \tan\theta$

$$cot\,an(\theta) = \frac{1}{tan(\theta)} = \frac{cos(\theta)}{sin(\theta)}$$

$\forall (a,b) \in \mathbb{R}^2,$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$
$$\sin(a+b) = \sin a\cos b + \sin b\cos a$$
$$\sin(a-b) = \sin a\cos b - \sin b\cos a$$

$$\cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$$

$$\sin 2a = 2\sin a\cos a$$

$$\cos^2 a = \frac{1+\cos 2a}{2} \qquad \sin^2 a = \frac{1-\cos 2a}{2}$$

$\forall \alpha \in \mathbb{R},\quad -1 \leq cos\alpha \leq 1$

$\qquad -1 \leq sin\alpha \leq 1$

Equationstrigonométriques :

$$\forall(\alpha,\beta)\in\mathbb{R}^2, \sin(\alpha)=\sin(\beta)$$
$$\Leftrightarrow \alpha=\beta+2k\pi,\ k\in\mathbb{Z}$$
$$ou\ \alpha=\pi-\beta+2k\pi,\ k\in\mathbb{Z}$$

$$\forall(\alpha,\beta)\in\mathbb{R}^2, \cos(\alpha)=\cos(\beta)$$
$$\Leftrightarrow \alpha=\beta+2k\pi,\ k\in\mathbb{Z}$$
$$ou\ \alpha=-\beta+2k\pi,\ k\in\mathbb{Z}$$

$$\forall(\alpha,\beta)\in\mathbb{R}^2, \tan(\alpha)=\tan(\beta)$$
$$\Leftrightarrow \alpha=\beta+k\pi,\ k\in\mathbb{Z} \quad ; \quad \alpha\in\mathbb{R}\setminus\left\{\frac{\pi}{2}+k\pi\right\}$$
$$\beta\in\mathbb{R}\setminus\left\{\frac{\pi}{2}+k\pi\right\}$$