FORMULAIRE DE TRIGONOMETRIE

$\forall (a,b) \in \mathbb{R}^2,$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$
$\cos(a-b) = \cos a \cos b + \sin a \sin b$
$\sin(a+b) = \sin a \cos b + \sin b \cos a$
$\sin(a-b) = \sin a \cos b - \sin b \cos a$

$\cos^2 a + \sin^2 a = 1$
$\cos 3a = 4\cos^3 a - 3\cos a$
$\sin 3a = -4\sin^3 a + 3\sin a$

$\cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$
$\sin 2a = 2\sin a \cos a$

$$\cos^2 a = \frac{1+\cos 2a}{2} \qquad \sin^2 a = \frac{1-\cos 2a}{2} \qquad \tan a = \frac{\sin a}{\cos a},\ a \neq \frac{\pi}{2}[2\pi]$$

$$\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\sin p - \sin q = 2\sin\frac{p-q}{2}\cos\frac{p+q}{2}$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\tan 2a = \frac{2\tan a}{1-\tan^2 a}$$

$$\cos 2a = \frac{1-\tan^2 a}{1+\tan^2 a}$$

$$\sin 2a = \frac{2\tan a}{1+\tan^2 a}$$

$$\forall \theta \neq 0[\pi],\ \cot an\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\forall \theta \in \mathbb{R},\ 1+\cos\theta = 2\cos^2\frac{\theta}{2} \quad ; \quad 1-\cos\theta = 2\sin^2\frac{\theta}{2}$$

$$\forall \theta \neq \frac{\pi}{2}[\pi],\ 1+\tan^2\theta = \frac{1}{\cos^2\theta}$$

Formules de Moivre et d'Euler :

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} \qquad \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$\forall \theta \neq 0[\pi],\ 1+\cot an^2\theta = \frac{1}{\sin^2\theta}$$

$$\forall \theta \in \mathbb{R}, \forall n \in \mathbb{N}, (\cos\theta + i\sin\theta)^n = (e^{i\theta})^n = e^{ni\theta} = \cos n\theta + i\sin n\theta$$

Tangente de l'angle moitié :

$$\forall \theta \neq \pi[2\pi] \wedge \theta \neq \frac{\pi}{2}[\pi],\ en\ posant\ u = \tan\frac{\theta}{2}, \qquad \tan\theta = \frac{2u}{1-u^2}$$

$$\forall \theta \neq \pi[2\pi],\ en\ posant\ u = \tan\frac{\theta}{2}, \qquad \sin\theta = \frac{2u}{1+u^2}, \quad \cos\theta = \frac{1-u^2}{1+u^2}$$

Valeurs remarquables :

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\mathbf{1}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	$\mathbf{1}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$\mathbf{-1}$
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	$\mathbf{1}$	$\sqrt{3}$		$-\sqrt{3}$	$\mathbf{-1}$	$-\frac{\sqrt{3}}{3}$	0

$\cos(-\theta)=\cos\theta$	$\sin(-\theta)=-\sin\theta$	$\tan(-\theta)=-\tan\theta$	$\text{cotan}(-\theta)=-\text{cotan}\,\theta$
$\cos(\theta+\pi)=-\cos\theta$	$\sin(\theta+\pi)=-\sin\theta$	$\tan(\theta+\pi)=\tan\theta$	$\text{cotan}(\theta+\pi)=\text{cotan}\,\theta$
$\cos(\pi-\theta)=-\cos\theta$	$\sin(\pi-\theta)=\sin\theta$	$\tan(\pi-\theta)=-\tan\theta$	$\text{cotan}(\pi-\theta)=-\text{cotan}\,\theta$
$\cos(\theta+\frac{\pi}{2})=-\sin\theta$	$\sin(\theta+\frac{\pi}{2})=\cos\theta$	$\tan(\theta+\frac{\pi}{2})=-\text{cotan}\,\theta$	$\text{cotan}(\theta+\frac{\pi}{2})=-\tan\theta$
$\cos(\frac{\pi}{2}-\theta)=\sin\theta$	$\sin(\frac{\pi}{2}-\theta)=\cos\theta$	$\tan(\frac{\pi}{2}-\theta)=\text{cotan}\,\theta$	$\text{cotan}(\frac{\pi}{2}-\theta)=\tan\theta$

Fonctions hyperboliques :

$$\forall x\in\mathbb{R},\quad chx=\frac{e^x+e^{-x}}{2}\quad;\quad shx=\frac{e^x-e^{-x}}{2}\quad;\quad thx=\frac{shx}{chx}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\forall x\in\mathbb{R},\quad chx+shx=e^x\quad;\quad chx-shx=e^{-x}$$

$$\forall x\in\mathbb{R},\ ch(-x)=ch(x)\quad;\quad sh(-x)=-sh(x)\quad;\quad th(-x)=-th(x)$$

$$\forall x\in\mathbb{R},\quad ch^2x-sh^2x=1$$

$$\forall x\in\mathbb{R},\ chx>0$$

$$\forall (a,b)\in\mathbb{R}^2,$$

$$\forall x\in\mathbb{R},\ ch(ix)=\cos x\quad;\quad sh(ix)=i\sin x$$

$$ch(a+b)=chachb+shashb$$
$$ch(a-b)=chachb-shashb$$
$$sh(a+b)=shachb+chashb$$
$$sh(a-b)=shachb-chashb$$

$$th(a+b)=\frac{tha+thb}{1+thathb}\qquad th(a-b)=\frac{tha-thb}{1-thathb}$$

$\forall x \in \mathbb{R},\ sh2x = 2shxchx$

$\forall x \in \mathbb{R},\ ch2x = ch^2x + sh^2x = 2ch^2x - 1 = 1 + 2sh^2x$

$En\ posant\ u = th\frac{x}{2},\quad chx = \frac{1+u^2}{1-u^2} \quad ; \quad shx = \frac{2u}{1-u^2}$

Fonctions hyperboliques réciproques :

$\forall x \in [1 + \infty[,\ Argchx = \ln\left(x + \sqrt{x^2 - 1}\right)$ $\qquad$ $\forall x \in]-1\,1[,\ Argthx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

$\forall x \in \mathbb{R},\ Argshx = \ln\left(x + \sqrt{x^2 + 1}\right)$

Fonctions trigonométriques réciproques :

$\forall x \in [-1\,1],\ \forall y \in \left[-\frac{\pi}{2}\ \frac{\pi}{2}\right],\ y = Arc\sin x \Leftrightarrow \sin y = x$ $\qquad$ $\forall x \in]-1\,1[,\ Arc\sin(-x) = -Arc\sin x$

$\forall x \in [-1\,1],\ \forall y \in [0\ \pi],\ y = Arc\cos x \Leftrightarrow \cos y = x$

$\forall x \in \mathbb{R}, \forall y \in \left]-\frac{\pi}{2}\ \frac{\pi}{2}\right[,\ y = Arc\tan x \Leftrightarrow \tan y = x$ $\qquad$ $\forall x \in \mathbb{R},\ Arc\tan(-x) = -Arc\tan x$

Dérivées des fonctions trigonométriques :

$\forall x \in \mathbb{R},\ \cos' x = -\sin x \quad ; \quad \sin' x = \cos x$

$\forall x \in \left]-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right[,\ \tan' x = \tan^2 x + 1 = \frac{1}{\cos^2 x} \quad , \quad k \in \mathbb{Z}$

$\forall x \in]\,0 + 2k\pi, \pi + 2k\pi[,\ \cot an' x = \frac{-1}{\sin^2 x} = -1 - \cot an^2 x \quad , \quad k \in \mathbb{Z}$

$\forall x \in]-1\,1[, Arc\sin' x = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\forall x \in]-1\,1[, Arc\cos' x = \frac{-1}{\sqrt{1-x^2}}$

$\forall x \in \mathbb{R},\ Arc\tan' x = \frac{1}{1+x^2}$

$\forall x \in \mathbb{R},\ sh' x = chx \quad ; \quad ch' x = shx \quad ; \quad th' x = \frac{1}{ch^2x} = 1 - th^2x$