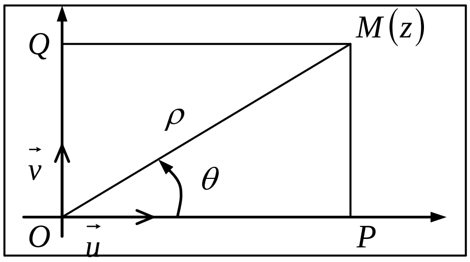
NOMBRES COMPLEXES

$Forme\,algébrique : z = x + iy$

$Forme\,trigonométrique : z = \rho(\cos\theta + i\sin\theta) = \rho e^{i\theta} \ , \quad \rho > 0$

$x = \rho\cos\theta = \mathrm{Re}(z) = \overline{OP} \quad ; \quad y = \rho\sin\theta = \mathrm{Im}\,(z) = \overline{OQ}$

$\overrightarrow{OM} = x\vec{u} + y\vec{v} \quad ; \quad OM = \rho = |z| = \sqrt{x^2 + y^2} \geq 0 \,; \tan\theta \equiv \dfrac{y}{x}\,[\pi]$



$Opérations\,algébriques$

$z + z' = (x + iy) + (x' + iy') = (x + x') + (y + y')$

$zz' = (x + iy)(x' + iy') = (xx' - yy') + i(xy' + x'y)$

$Conjugué$

$z = x + iy = \rho e^{i\theta} \quad ; \quad \overline{z} = x - iy = \rho e^{-i\theta}$

$x = \dfrac{1}{2}(z + \overline{z}) \quad ; \quad y = \dfrac{1}{2i}(z - \overline{z}) \quad ; \quad \overline{z + z'} = \overline{z} + \overline{z}'$

$z\overline{z} = x^2 + y^2 = |x|^2 \quad ; \quad \overline{zz'} = \overline{z}\overline{z}'$

$\dfrac{1}{z} = \dfrac{\overline{z}}{z\overline{z}} = \dfrac{x}{x^2 + y^2} + i\dfrac{-y}{x^2 + y^2} = \dfrac{1}{\rho}e^{-i\vartheta}$

$Cas\ particuliers\ remarquables$

$e^{i0} = 1 \quad ; \quad e^{i\frac{\pi}{2}} = i$

$e^{-i\frac{\pi}{2}} = -i \quad ; \quad e^{i\pi} = -1$

$i^2 = -1$

$Module\,et\ argument\ d'un\ produit, d'un\ quotient$

$zz' = (\rho e^{i\theta})(\rho' e^{i\theta'}) = \rho\rho' e^{i(\theta + \theta')}$

$|zz'| = |z||z'|$

$\dfrac{z}{z'} = \dfrac{\rho e^{i\theta}}{\rho' e^{i\theta'}} = \dfrac{\rho}{\rho'}e^{i(\theta - \theta')}$

$\left|\dfrac{z}{z'}\right| = \dfrac{|z|}{|z'|}$

$z^n = (\rho e^{i\theta})^n = \rho^n e^{in\theta}$

$Formule\,d'Euler$

$\forall\theta \in \mathbb{R}, \begin{cases} \cos\theta = \dfrac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right) \\ \sin\theta = \dfrac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right) \end{cases}$

$Formule\,de\ Moivre$

$\forall\theta \in \mathbb{R}, \forall n \in \mathbb{N}, (\cos\theta + i\sin\theta)^n = (e^{i\theta})^n = e^{in\theta} = \cos n\theta + i\sin n\theta$

$Arg\left(\dfrac{z_c - z_a}{z_b - z_a}\right) = \left(\overrightarrow{AB}, \overrightarrow{AC}\right)$

$\forall(\rho, \rho') \in \mathbb{R}_+^{*2}, \quad \forall(\theta, \theta') \in \mathbb{R}, \quad arg(zz') = arg(z) + arg(z') \quad [2\pi]$

$arg\left(\dfrac{z}{z'}\right) = arg(z) - arg(z') \quad [2\pi]$

$arg\left(\dfrac{1}{z}\right) = -arg(z)$

$\forall z \in \mathbb{C}, \quad \forall n \in \mathbb{N}, \quad arg(z^n) = n\,arg(z) \quad [2\pi]$

$Attention : en\ général, arg(z + z') \neq arg(z) + arg(z')$