Surrogate models and Gaussian Process regression – lecture 3/5

Kriging with trend and/or noisy observations

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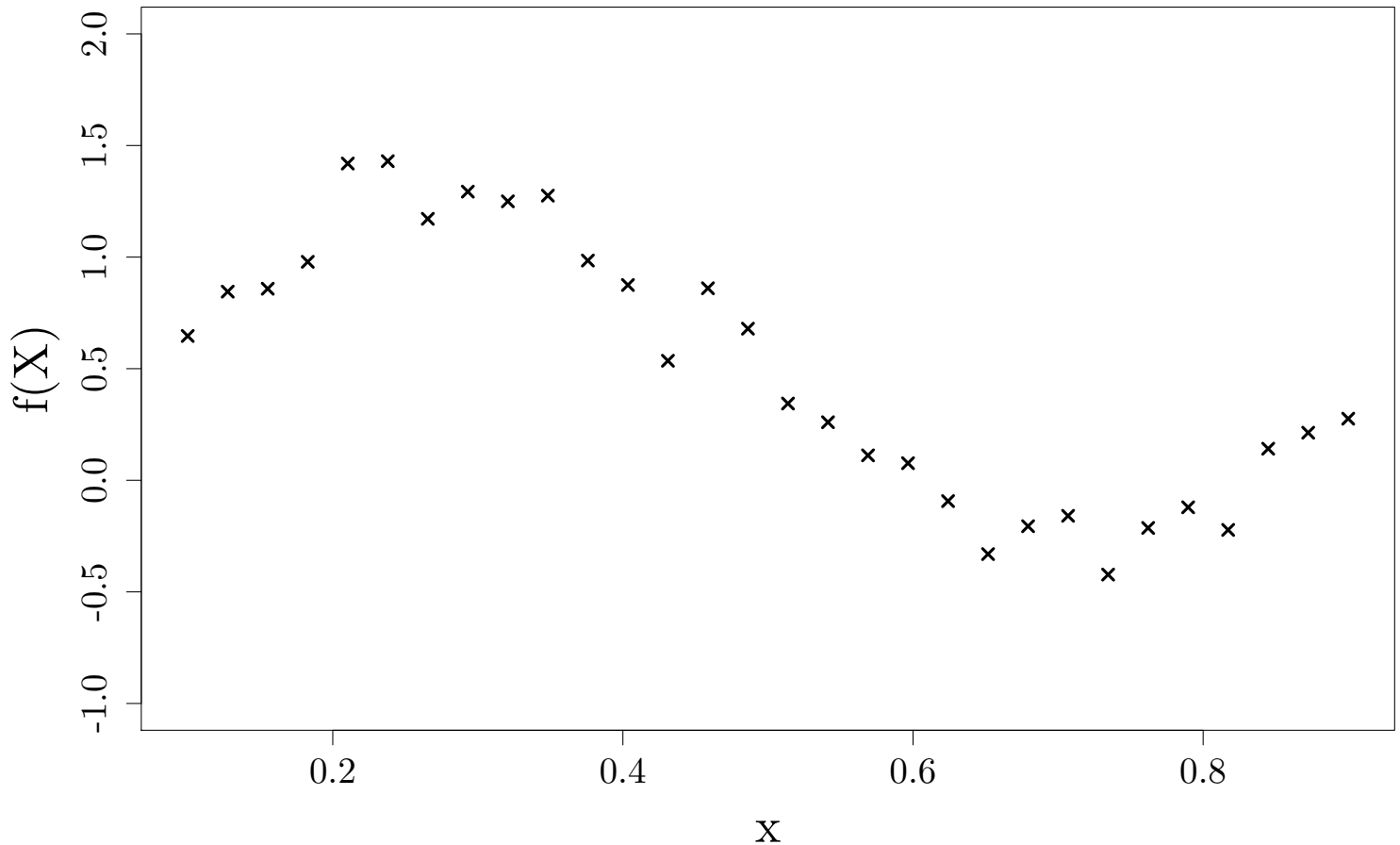
Approximation

We are not always interested in models that interpolate the data.
For example, if there is some observation noise: $F = f(X) + \varepsilon$.

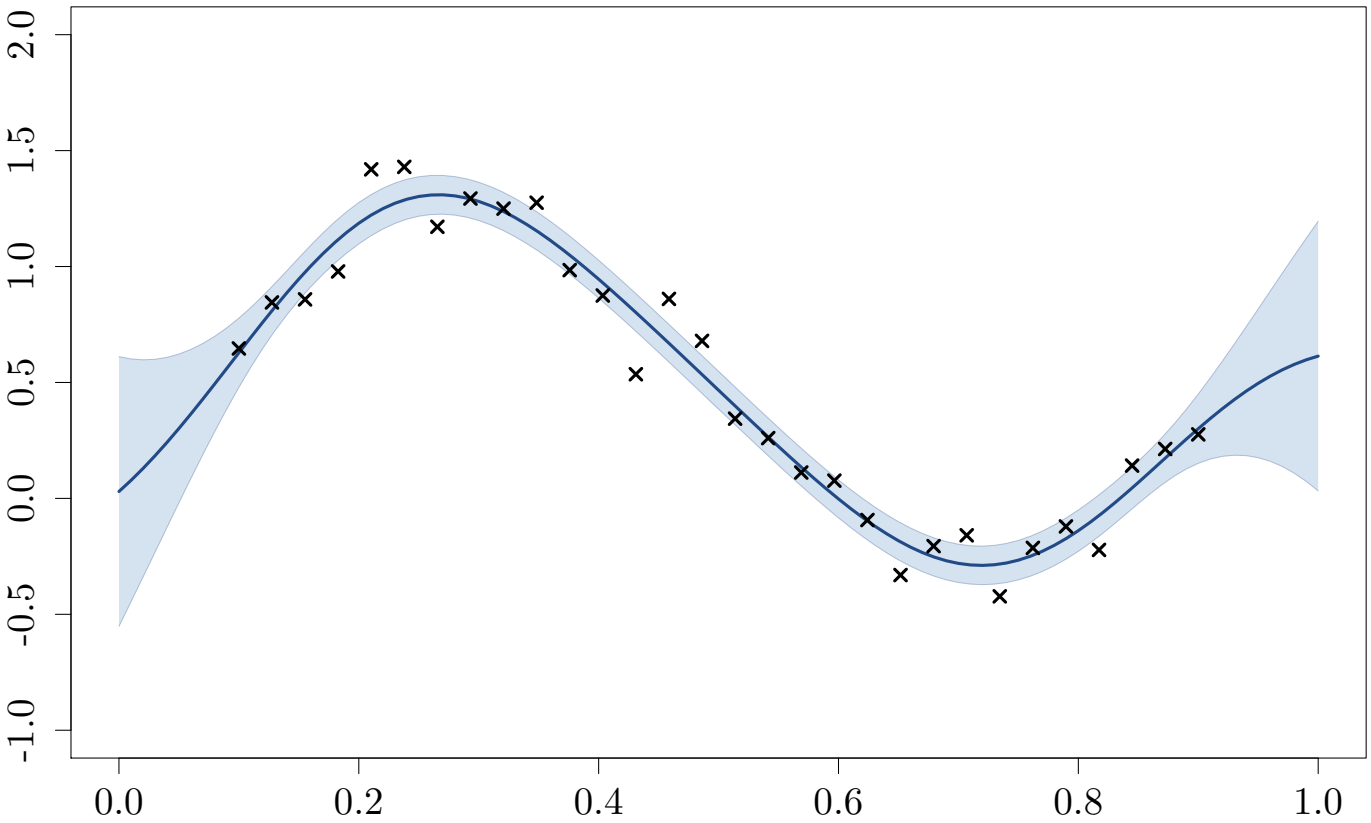


Let N be a process $\mathcal{N}(0, n)$ that represent the observation noise. The expressions of GPR with noise are

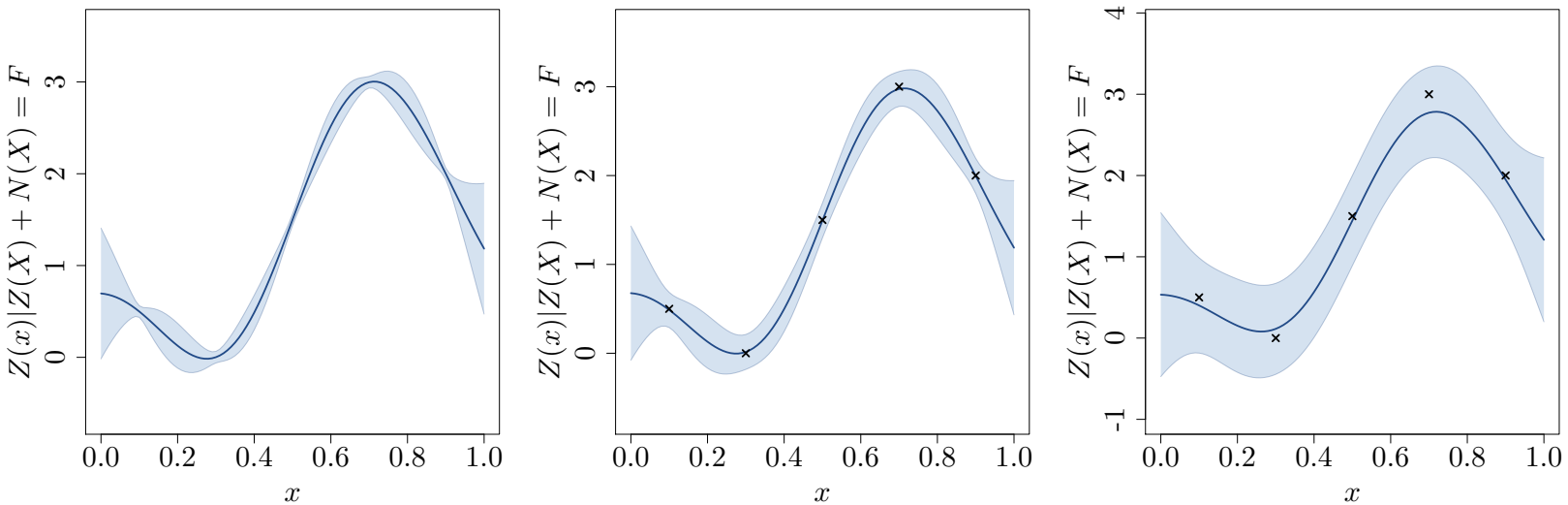
$$\begin{aligned} m(x) &= \mathrm{E}[Z(x)|Z(X) + N(X)=F] \\ &= k(x, X)(k(X, X) + n(X, X))^{-1} F \end{aligned}$$

$$\begin{aligned} c(x, y) &= \mathrm{cov}[Z(x), Z(y)|Z(X) + N(X)=F] \\ &= k(x, y) - k(x, X)(k(X, X) + n(X, X))^{-1} k(X, y) \end{aligned}$$

We obtain the following model



Influence of observation noise τ^2 (for $n(x, y) = \tau^2 \delta_{x,y}$):



The values of τ^2 are respectively 0.001, 0.01 and 0.1.

In practice, τ^2 can be estimated with Maximum Likelihood.