situation where an outside point belonging to one normal is actually an interior point in another part of the surface or that a supposedly interior point is so far away from its associated surface point that it is actually outside the surface at another place. The associated function values that s should attain are chosen to be proportional to the signed distance of the point from the surface.

Another possible way of adding off-surface points is based on the following fact. Suppose that x is a point which should be added. If x_j denotes its nearest neighbor in X and if X is a sufficiently dense sample of $\mathcal{S}$, then x_j comes close to the projection of x onto $\mathcal{S}$. Hence if x_j is approximately equal to x then the latter is a point of $\mathcal{S}$ itself. Otherwise, if the angle between the line through x_j and x on the one hand and the normal η_j (pointing outwards) on the other hand is less than 90 degrees then the point is outside the surface; if the angle is greater than 90 degrees then it is inside the surface.

After augmenting our initial data set by off-surface points, we are now back to a classical interpolation or approximation problem.

1.2 Fluid–structure interaction in aeroelasticity

Aereolasticity is the science that studies, among other things, the behavior of an elastic aircraft during flight. This behavior is influenced by the interaction between the deformations of the elastic structure caused by the fluid flow, and the impact that the aerodynamic forces would have on a rigid structural framework. To model these different aspects in a physically correct manner, different models have been developed, adapted to the specific problems.

The related aeroelastic problem can be described in a coupled-field formulation, where the interaction between the fluid and structural models is limited to the exchange of boundary conditions. This *loose* coupling has the advantage that each component of the coupled problem can be handled as an isolated entity. However, the challenging task is to reconcile the benefits of this isolated view with a realistic treatment of the new physical effects arising from the interaction.

Let us make this more precise. Suppose at first that we are interested only in computing the flow field around a given aircraft. This can be modeled mathematically by the Navier–Stokes or the Euler equations, which can be solved numerically using for example a finite-volume code. Such a solver requires a detailed model of the aircraft and its surroundings. In particular, the surface of the aircraft has to be rendered with a very high resolution, as indicated in the right-hand part of Figure 1.3. Let us suppose that our solver has computed a solution, which consists of a velocity field and a pressure distribution. For the time being, we are not interested in the problem of how such a solution can be computed. For us, it is crucial that the pressure distribution creates loads on the aircraft, which might and probably will lead to a deformation. So the next step is to compute the deformation from the loads or forces acting on the aircraft.

Obviously, though, a model having a fine resolution of the surface of the aircraft is not necessary for describing its structure; this might even impede the numerical stability. Hence,

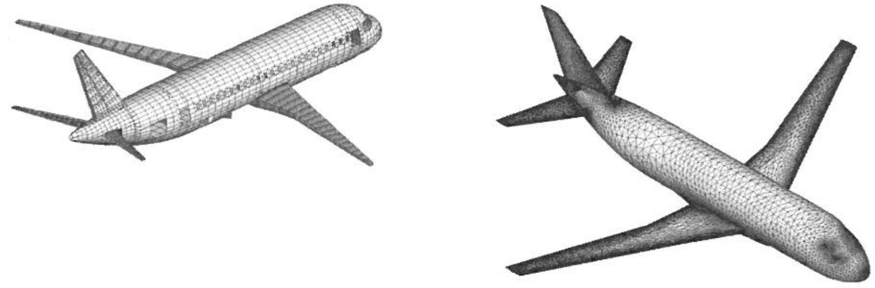

Fig. 1.3 The structural and aerodynamical model of a modern aircraft.

another model is required which is better suited to describing the structural deformation, for example the one shown in Figure 1.3 on the left. Again, along with the model comes a partial differential equation, this time from elasticity theory, which can again be solved, for example by finite elements. But before this can be done, the loads have to be transferred from one mesh to the other in a physically reasonable way. If this has been done and the deformation has been computed then we are confronted with another coupling problem. This time, the deformations have to be transferred from the structural to the aerodynamical model. If all these problems can be solved we can start to iterate the process until we find a steady state, which presumably exists.

Since we have the aerodynamical model, the structural model, and the coupling problem, one usually speaks in this context of a *three-field formulation*. As we said earlier, here we are interested only in the coupling process, which can be described as a scattered data approximation problem, as follows. Suppose that X denotes the nodes of the structural mesh and Y the nodes of the aerodynamical mesh (neither actually has to be a mesh). To transfer the deformations $u(x_j) \in \mathbb{R}^3$ from X to Y we need to find a vector-valued interpolant $s_{u,X}$ satisfying $s_{u,X}(x_j) = u(x_j)$. Then the deformations of Y are given simply by $s_{u,X}(y_j)$, $y_j \in Y$. Conversely, if $f(y_j) \in \mathbb{R}$ denotes the load at $y_j \in Y$ then we need another function $s_{f,Y}$ to interpolate f in Y. The loads on the mesh X are again simply given by evaluation at X. A few more things have to be said. First of all, if the loads are constant or if the displacements come from a linear transformation, this situation should be recovered exactly, which means that our interpolation process has to be exact for linear polynomials. Furthermore, certain physical entities such as energy and work should be conserved. This means at least that

$$\sum_{y\in Y} f(y) = \sum_{x\in X} s_{f,Y}(x)$$

and

$$\sum_{y\in Y} f(y) s_{u,X}(y) = \sum_{x\in X} s_{f,Y}(x) u(x),$$