Universidad Del Quindío

Programa de Economía

Ejercicio (1) - Microeconomía I

Recordemos cómo se plantea un problema de maximización del beneficio, en este caso para el consumidor.

$$\max U(x, y)$$

s.a

$$g(x,y) = m$$

Entonces

$$\mathcal{L} = U(x,y) - \lambda (g(x,y) - m)$$

$$[x] \partial_x U = \lambda \partial_x q(x, y)$$

$$[y] \qquad \partial_y U = \lambda \partial_y g(x, y)$$

$$[\lambda] \quad g(x,y) = m$$

Mi problema:

$$U = x^{\alpha} y^{\beta}$$
$$m = p_x x + p_y y$$

1. Armemos el langrangiano:

$$\mathcal{L} = x^{\alpha} y^{\beta} - \lambda [p_x x + p_u y - m]$$

2. C.P.O:

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha x^{\alpha - 1} y^{\beta} - \lambda p_x = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = \beta x^{\alpha} y^{\beta - 1} - \lambda p_y = 0$$

3. Igualemos λ :

$$\frac{\alpha x^{\alpha-1}y^{\beta}}{p_x} = \frac{\beta x^{\alpha}y^{\beta-1}}{p_y}$$

$$\frac{p_x}{p_y} = \frac{\alpha y}{\beta x}]E.E.F$$

4. Despejamos y:

$$y = \frac{\beta p_x x}{\alpha p_y}$$

5. Reemplazamos y en la restricción presupuestaria:

$$m = p_x x + p_y \left(\frac{\beta p_x x}{\alpha p_y}\right)$$
$$m = p_x x + \left(\frac{\beta p_x x}{\alpha}\right)$$
$$m = p_x x \left(1 + \frac{\beta}{\alpha}\right)$$
$$m = p_x x \left(\frac{\alpha + \beta}{\alpha}\right)$$

6. Óptimo x:

$$x^* = \frac{m\alpha}{(\alpha + \beta)p_x}$$

7. Despejamos x:

$$x = \frac{\alpha p_y y}{\beta p_x}$$

8. Reemplazamos x en la restricción presupuestaria:

$$m = p_x(\frac{\alpha p_y y}{\beta p_x}) + p_y y$$

$$m = (\frac{\alpha p_y y}{\beta}) + p_y y$$

$$m = p_y y(1 + \frac{\alpha}{\beta})$$

$$m = p_y y(\frac{\beta + \alpha}{\beta})$$

9. Despejamos y:

$$y^* = \frac{m\beta}{(\beta + \alpha)p_y}$$