# Control of prosumer Networks

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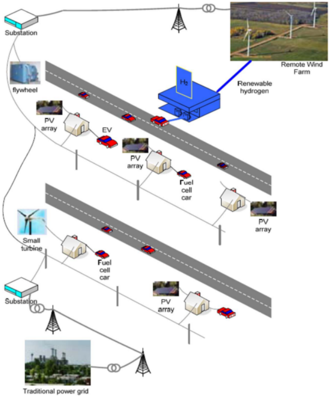
# Global introduction

- PhD in SAMOVAR
- Background : Telecom, computer science, complex systems
- Smart grid : new and cross domains
- Complex system approach of prosumer managment in the smart grid
- Prosumer = PROducer + conSUMER
- Chapter 1 : Forming stable aggregations of prosumers for energy markets
- Chapter 2 : Control of prosumer networks



# Outline

# Scenario

- Prosumers use DER
- Production and Consumption are stochastic and succeptible to change
- Generators and loads are not fixed
- Synchronization to $\Omega = 50Hz$ is necessary for stability
- Production / Consumption imbalance $==>$ desynchronization
- Restaure synchrony by injecting or withdrawing power from the grid
- Use of storage (fixed, V2G)
- How to place these elements ?

# Kuramoto Model

- Widely used model to study synchronization in complex networks
- Nodes = oscillators with phase angles $\theta_i$ and frequencies $\dot{\theta}_i$
- Nodes have a natural frequency $\omega_i$
- Interconnected through a network $G = (V, E)$ of N nodes
- Each node's frequency is affected by its neighbors (K is the coupling strength) :

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{i=1}^{N} sin(\theta_j - \theta_i)$$

What is synchronization ?

- Synchronization : $\forall i,\ \dot{\theta}_i = \omega_{SYNC}$
- Phase cohesiveness : $\forall (i,j),\ \exists \gamma \in [0, \frac{\pi}{2}[,\ |\theta_i - \theta_j| \leq \gamma$

# Synchronization in Kuramoto Model

- Can we predict wether a given system will synchronize or not ?
- The network synchronizes if $\|L^{\dagger}\omega\|_{\infty,E} \leq sin(\gamma)$
- Where $L^{\dagger}$ is the pseudo-inverse of the network Laplacian L
- $\omega = \{\omega_1, \omega_2, ..., \omega_N\}$
- $\|x\|_{\infty,E} = max_{i,j}\|x_i - x_j\|$ such that edge $(i,j)$ in E
- In our case : $K \geq \frac{N\|L^{\dagger}\omega\|_{\infty,E}}{sin\gamma}$

# Synchronization in Kuramoto Model

# Model power grid with Kuramoto

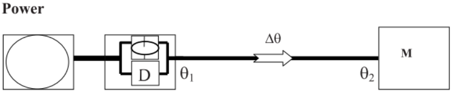



**Fig. 1.** Equivalent diagram of generator and machine connected by a transmission line. The turbine consists of a flywheel and dissipation $D$.

- OBJ : Achieve synchronization at $\Omega = 50Hz$
- Rewrite $P_{source} = P_{diss} + P_{acc} + P_{trans}$ in terms of frequencies and phase angles :
  - $P_{diss} = K_D \dot{\theta}_i^2$
  - $P_{acc} = \frac{1}{2} I \frac{d}{dt} \dot{\theta}_i^2$
  - $P_{trans}(i \rightarrow j) = -P_{ij}^{MAX} sin(\theta_j - \theta_i)$
- Dynamics is expressed in terms of the deviations from $\Omega$
- $\ddot{\theta}_i \sim \psi_i - \alpha_i \dot{\theta}_i + \sum_j K_{ij} g_{ij} sin(\theta_j - \theta_i)$
- Where :
  - $\alpha = \frac{2K_D}{I}$ : dissipation
  - $K_{ij} = \frac{P_{ij}^{MAX}}{I\Omega}$ : coupling
  - $\psi_i = \left[ \frac{P_{S,i}}{I\Omega} - \frac{K_D \Omega}{I} \right]$ : power distribution
  - $g_{ij}$ : adjacency matrix

# In matrix form

- Small angle differences : $sin(\theta_j - \theta_i) \sim \theta_j - \theta_i$
- Dynamics can be written in matrix form :

$$\begin{pmatrix} \theta_1(t+\Delta t) \\ \theta_2(t+\Delta t) \\ \vdots \\ \theta_N(t+\Delta t) \\ \dot{\theta}_1(t+\Delta t) \\ \dot{\theta}_2(t+\Delta t) \\ \vdots \\ \dot{\theta}_N(t+\Delta t) \\ 1 \end{pmatrix} = \begin{pmatrix} I & I\Delta t & 0 \\ -KL\Delta t & (1-\alpha\Delta t)I & \Psi\Delta t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_1(t) \\ \theta_2(t) \\ \vdots \\ \theta_N(t) \\ \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \vdots \\ \dot{\theta}_N(t) \\ 1 \end{pmatrix}$$

Let $Y(t) = \{\theta_1, \theta_2, ..., \theta_N, \dot{\theta}_1, \dot{\theta}_2, ..., \dot{\theta}_N, 1\}$ be the state vector of the system at time t :

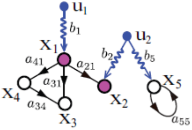
$$Y(t+\Delta t) = AY(t)$$

# Control

# Control

$$\mathbf{A}=\begin{bmatrix}0&0&0&0&0\\a_{21}&0&0&0&0\\a_{31}&0&0&0&0\\a_{41}&0&0&a_{34}&0\\0&0&0&0&a_{55}\end{bmatrix};\ \mathbf{B}=\begin{bmatrix}b_1&0\\0&b_2\\0&0\\0&0\\0&b_5\end{bmatrix}$$

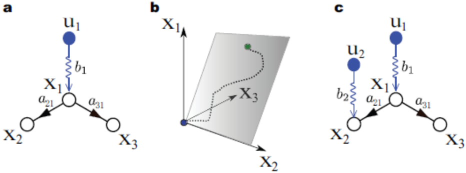
$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}$$




$G(\mathbf{A},\mathbf{B})$

- B indicates which nodes are drivers
- $u(t)$ are the control inputs

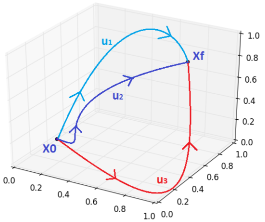
$(A, B)$ is controllable if it can be steered from any initial state $x_0$ to any final state $x_f$ with a sequence of inputs $u(t)$




- Can we determine if a system $(A, B)$ is controllable ?
- What is the minimum set to fully control the network ?

# Optimal control

- There might exists multiple sequences of inputs $u(t)$ that drives the system $(A, B)$ from $x_0$ to $x_f$




- Optimal control : finding the one that minimizes some cost function
- Here we are concerned with the control energy : $\mathcal{E} = \int_{t_0}^{t_f} \|u(t)\|^2 dt$
- The smallest this energy, the less stress on the storage devices
- It has been shown that :
  - the control input sequence that minimizes the control energy can be written as $u^\star(t) = B^T (A^T)^{t_f - t - 1} W^{-1}(t_f) \left[x_f - A^{t_f} x_0\right]$
  - Where the gramian matrix $W(t) = \sum_{k=0}^{t} A^k B B^T (A^T)^k$
  - The control energy is then $\mathcal{E}_{min} = \left[x_f - A^{t_f} x_0\right]^T W^{-1}(t_f) \left[x_f - A^{t_f} x_0\right]$

# Gramian

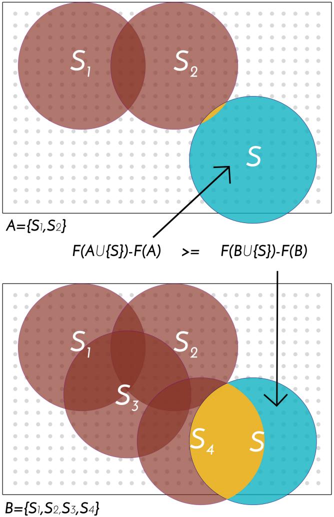
- Control energy depends on $W^{-1}$
- $W$ depends on A and B, but not on $x_0$ and $x_f$
- $W$ can be used to obtain average information :
  - $rank[W]$ gives the dimension of the controllable subspace
  - $Tr[W^{-1}]$ gives the average control energy needed
- Useful for us since the prosumers change a lot

PROBLEMS :

- We still do not know how to find B (the driver nodes)
- We have constraints on line capacities, battery capacities and charge/discharge rates

# Submodular Set Functions

- Let $F : 2^V \longrightarrow \Re$ be a set function
- F is submodular if for all sets $A, B \subset V$ such that $A \subseteq B$, and for all element $x \in V\ \ B$ :
  $F(A \cup \{x\}) - F(A) \geq F(B \cup \{x\}) - F(B)$
- Diminishing returns
- Greedy heuristic with worst case guarantee : $\frac{F(S_{greedy})}{F(S_{opt})} \geq 63\%$




<u>Greedy algorithm:</u>

Start with $\mathcal{A} = \emptyset$

For i = 1 to k

$s^* \leftarrow \arg\max_s F(\mathcal{A} \cup \{s\})$

$\mathcal{A} \leftarrow \mathcal{A} \cup \{s^*\}$

# Gramian and Submodularity

What is the link between submodular set functions and controllability ?

- Let $B_S$ be matrix B when driver set is S
- $W_S$ : Gramain of system $(A, B_S)$
  - $F_{trace} : S \longrightarrow Tr[W_S]$ is modular
  - $F_{traceInv} : S \longrightarrow Tr[W_S^{-1}]$ is submodular
  - $F_{rank} : S \longrightarrow rank[W_S]$ is submodular

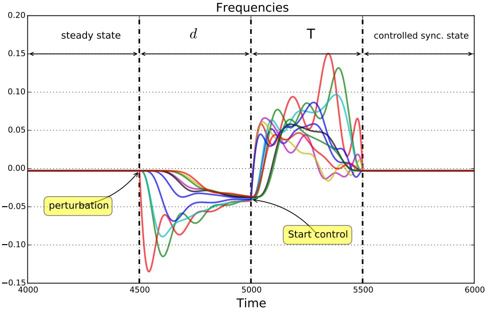
# Some results

# Combining the pieces

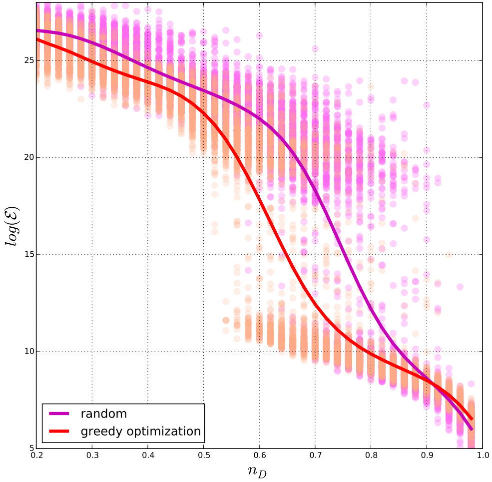
- System state at t : $Y(t) = \{\theta_1, \theta_2, ..., \theta_N, \dot{\theta}_1, \dot{\theta}_2, ..., \dot{\theta}_N, 1\}$
- Dynamics : $Y(t + \Delta t) = AY(t) + Bu(t)$
- Energy : $\mathcal{E}_{min} = [x_f - A^{t_f} x_0]^T \, W^{-1}(t_f) \, [x_f - A^{t_f} x_0]$
- Submodular function $F(S) = Tr[W_S^{-1}]$

Constraints :

- Flow constraints
- Synchronization constraint
- Balance constraint
- Battery levels constraints
- Charge/discharge rate constraints

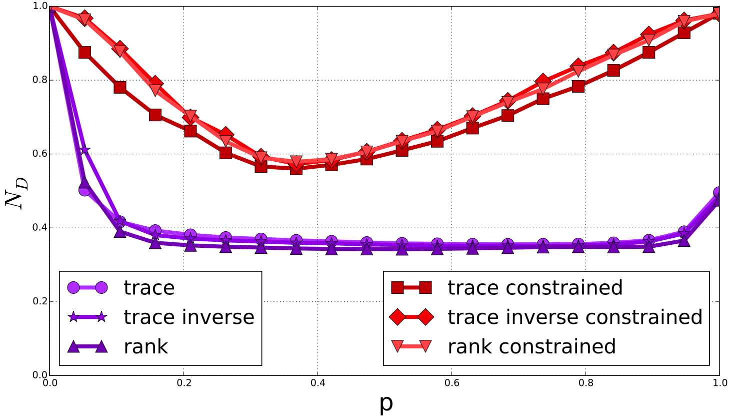


Are we doing better than random ?



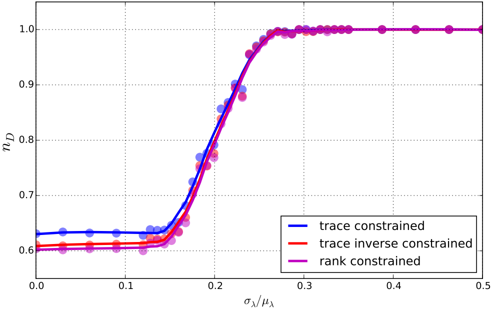

- Random scale-free topologies (200 nodes)
- Random power distributions and line capacities

How does the topology impact the size of the driver set ?




- Random erdos-renyi topologies (200 nodes)
- Random power distributions and line capacities

How does the capacity distribution of the batteries impact the size of the driver set ?




- Random erdos-renyi topologies (200 nodes)
- Random power distributions and line capacities
- Battery capacities are drawn from : $\mathcal{N}(\mu_\lambda, \sigma_\lambda)$

European transmission power grid (1494 nodes and 2196 edges over 25 countries)