

Control of prosumer Networks

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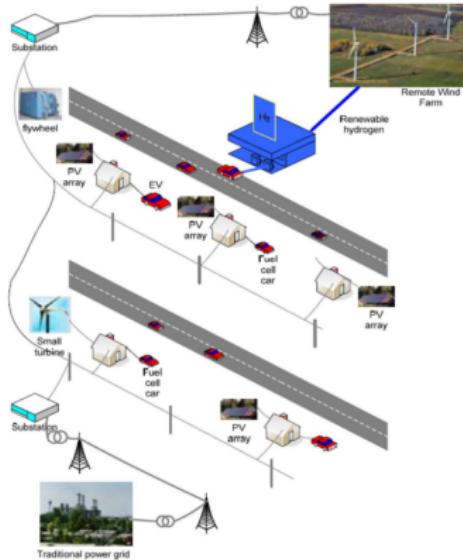
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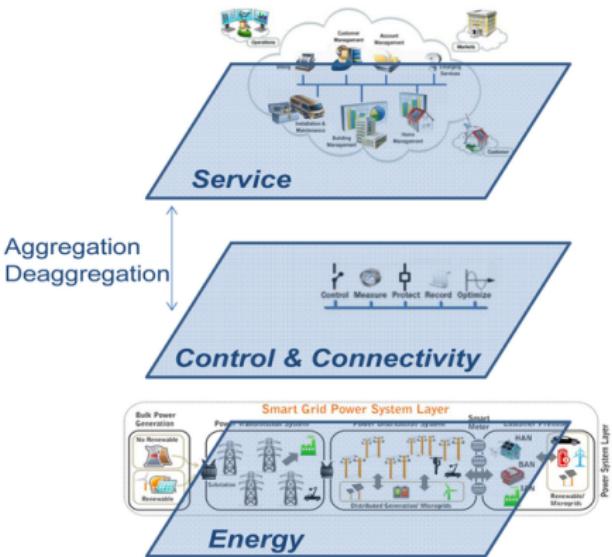
Global introduction

- PhD in SAMOVAR
- Background : Telecom, computer science, complex systems
- Smart grid : new and cross disciplines
- Complex system approach of prosumer management in the smart grid
- Prosumer = PROducer + conSUMER



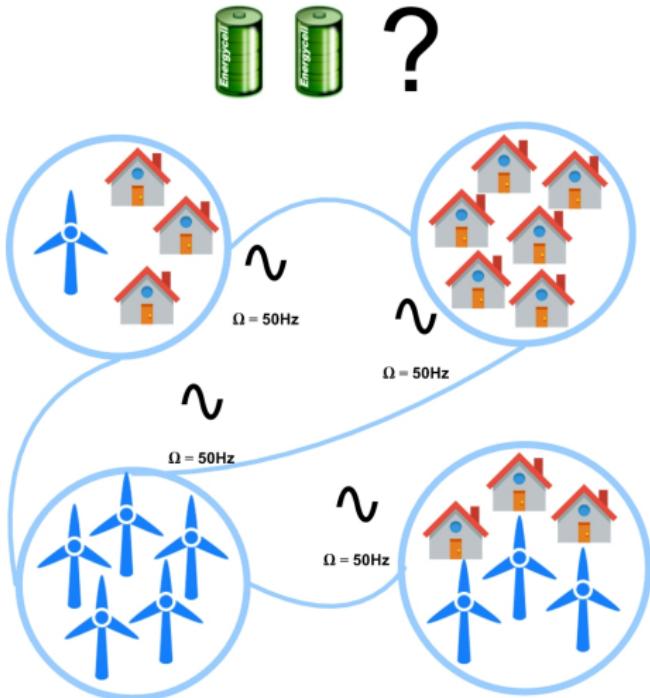
Smart Grid as a complex system

- Multiple layers
- Decentralized production
- Extensive use of renewables
- Stochasticity
- Use of different storage technologies
- Electric vehicles
- Complex applications :
 - ▶ Dynamic pricing
 - ▶ Demand Side Management
 - ▶ V2G...



Scenario

- Smart grid with decentralized production
- Production and Consumption are stochastic
- Generators and loads are not fixed
- Synchronization at $\Omega = 50\text{Hz}$
- Production / Consumption imbalance ==> desynchronization
- Restaure synchrony by injecting or withdrawing power from the grid
- Use of storage (fixed, V2G)
- How to place these elements ?



Outline

① Kuramoto Model

② Control

③ submodular set functions

④ Some results

Simple Kuramoto Model

- Widely used model to study synchronization in complex networks
- Nodes = oscillators with phase angles θ_i and frequencies $\dot{\theta}_i$
- Nodes have a natural frequency ω_i
- Interconnected through a network $G = (V, E)$ of N nodes
- Each node's frequency is affected by its neighbors (K is the coupling strength) :

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i)$$

What is synchronization ?

- Synchronization : $\forall i, \dot{\theta}_i = \omega_{SYNC}$
- Phase cohesiveness : $\forall (i, j), \exists \gamma \in [0, \frac{\pi}{2}[, |\theta_i - \theta_j| \leq \gamma$

Synchronization in Kuramoto Model

- Can we predict whether a given system will synchronize or not ?
- The network synchronizes if $\|L^\dagger \omega\|_{\infty, E} \leq \sin(\gamma)$ ¹
- Where L^\dagger is the pseudo-inverse of the network Laplacian L
- $\omega = \{\omega_1, \omega_2, \dots, \omega_N\}$
- $\|x\|_{\infty, E} = \max_{i,j} \|x_i - x_j\|$ such that edge (i,j) in E
- In our case : $K \geq \frac{N\|L^\dagger \omega\|_{\infty, E}}{\sin \gamma}$

Synchronization in Kuramoto Model

Model power grid with Kuramoto

Power

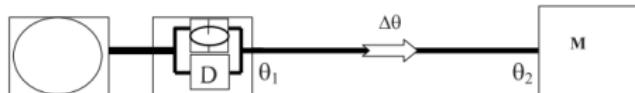


Fig. 1. Equivalent diagram of generator and machine connected by a transmission line. The turbine consists of a flywheel and dissipation D .

- OBJ : Achieve synchronization at $\Omega = 50Hz$
- Rewrite $P_{source} = P_{diss} + P_{acc} + P_{trans}$ in terms of frequencies and phase angles :
 - ▶ $P_{diss} = K_D \dot{\theta}_i^2$
 - ▶ $P_{acc} = \frac{1}{2} I \frac{d}{dt} \dot{\theta}_i^2$
 - ▶ $P_{trans}(i \rightarrow j) = -P_{ij}^{MAX} \sin(\theta_j - \theta_i)$

- Dynamics is expressed in terms of the deviations from Ω (see²)
- $\ddot{\theta}_i \sim \psi_i - \alpha_i \dot{\theta}_i + \sum_j K_{ij} \sin(\theta_j - \theta_i)$
- Where :
 - ▶ $\alpha = \frac{2K_D}{I}$: dissipation
 - ▶ $K_{ij} = \frac{P_{ij}^{MAX}}{I\Omega}$: coupling
 - ▶ $\psi_i = \left[\frac{P_{S,i}}{I\Omega} - \frac{K_D \Omega}{I} \right]$: power distribution
- Small angle differences : $\sin(\theta_j - \theta_i) \sim \theta_j - \theta_i$
- Dynamics can be written in matrix form : $Y(t + \Delta t) = AY(t)$

2. Filatrella2008.

1 Kuramoto Model

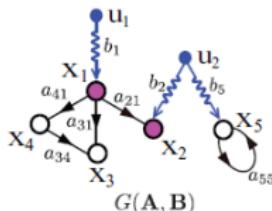
2 Control

3 submodular set functions

4 Some results

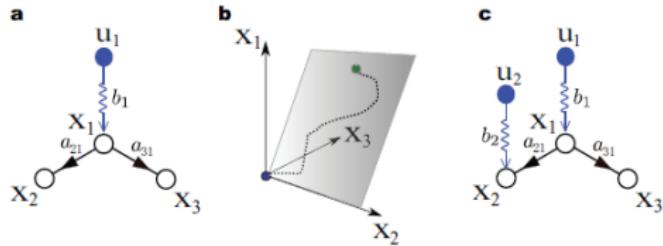
Control

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 \\ a_{31} & 0 & 0 & 0 & 0 \\ a_{41} & 0 & 0 & a_{34} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix}; B = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & b_5 \end{bmatrix}$$
$$\dot{x} = Ax + Bu$$



(A, B) is controllable if it can be steered from any initial state x_0 to any final state x_f with a sequence of inputs $u(t)$

- B indicates which nodes are drivers
- $u(t)$ are the control inputs



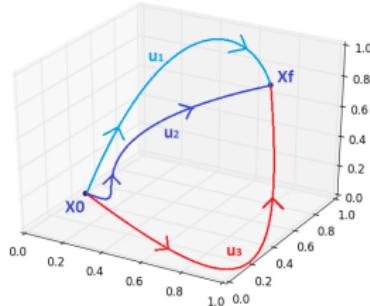
- Can we determine if a system (A, B) is controllable?
- What is the minimum set to fully control the network?

More information in³

3. Liu2015.

Optimal control

- There might exist multiple sequences of inputs $u(t)$ that drives the system (A, B) from x_0 to x_f



- Optimal control : finding the one that minimizes some cost function
- Here we are concerned with the control energy : $\mathcal{E} = \int_{t_0}^{t_f} \|u(t)\|^2 dt$
- The smaller this energy, the less stress on the storage devices
- It has been shown that :
 - ▶ the control input sequence that minimizes the control energy can be written as $u^*(t) = B^T (A^T)^{t_f - t - 1} W^{-1}(t_f) [x_f - A^{t_f} x_0]^4$
 - ▶ Where the gramian matrix $W(t) = \sum_{k=0}^t A^k B B^T (A^T)^k$
 - ▶ The control energy is then $\mathcal{E}_{min} = [x_f - A^{t_f} x_0]^T W^{-1}(t_f) [x_f - A^{t_f} x_0]$

- Control energy depends on W^{-1}
- W depends on A and B, but not on x_0 and x_f
- W can be used to obtain average information :
 - ▶ $\text{rank}[W]$ gives the dimension of the controllable subspace
 - ▶ $\text{Tr}[W^{-1}]$ gives information about average control energy
- Useful for us since the prosumers change a lot
- Goal : good performances (low control energy) on average

PROBLEMS :

- We still do not know how to find B (the driver nodes)
- We have constraints on line capacities, battery capacities and charge/discharge rates

1 Kuramoto Model

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Submodular set functions

- Let $F : 2^V \rightarrow \mathbb{R}$ be a set function
- F is submodular if for all sets $A, B \subset V$ such that $A \subseteq B$, and for all element $x \in V \setminus B$:
$$F(A \cup \{x\}) - F(A) \geq F(B \cup \{x\}) - F(B)$$
- Diminishing returns
- Greedy heuristic with worst case guarantee : $\frac{F(S_{greedy})}{F(S_{opt})} \geq 63\%$

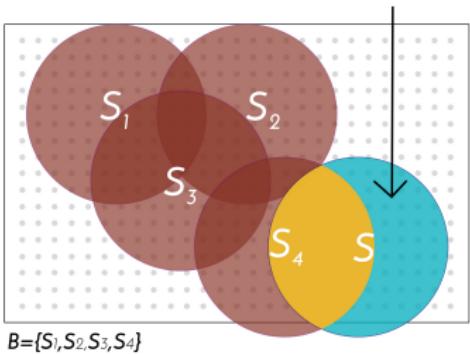
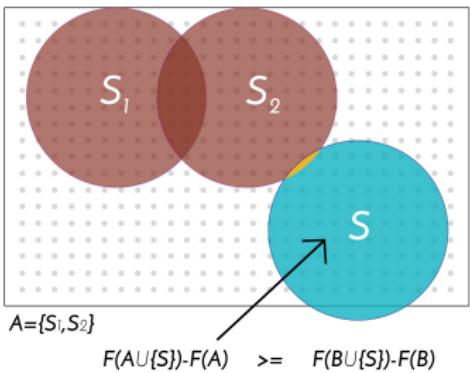
Greedy algorithm:

Start with $\mathcal{A} = \emptyset$

For $i = 1$ to k

$$s^* \leftarrow \arg \max_s F(\mathcal{A} \cup \{s\})$$

$$\mathcal{A} \leftarrow \mathcal{A} \cup \{s^*\}$$



What is the link between submodular set functions and controllability ?

- Let B_S be matrix B when driver set is S
- W_S : Gramian of system (A, B_S)
 - ▶ $F_{trace} : S \rightarrow \text{Tr}[W_S]$ is modular
 - ▶ $F_{traceInv} : S \rightarrow \text{Tr}[W_S^{-1}]$ is submodular
 - ▶ $F_{rank} : S \rightarrow \text{rank}[W_S]$ is submodular

More information and demonstrations in⁵

More information about submodularity and maximization of submodular set functions in⁶

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- 5. Summers2014.
 - 6. Krause2014.

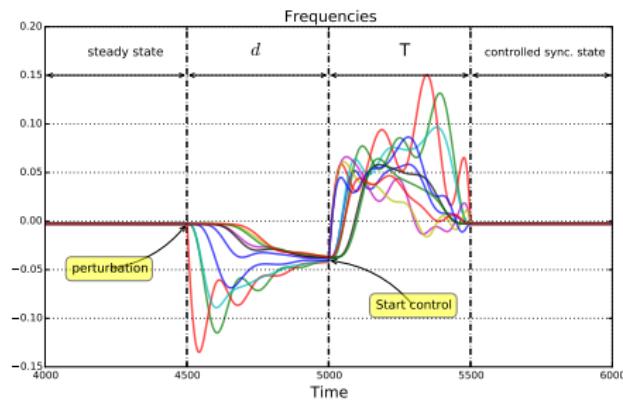
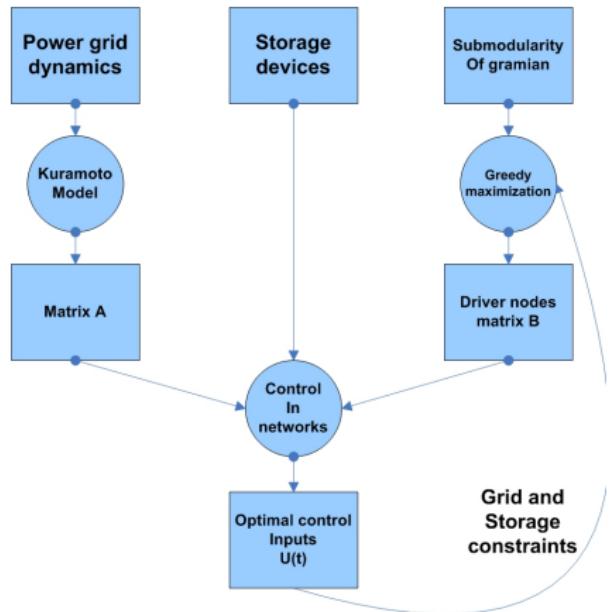
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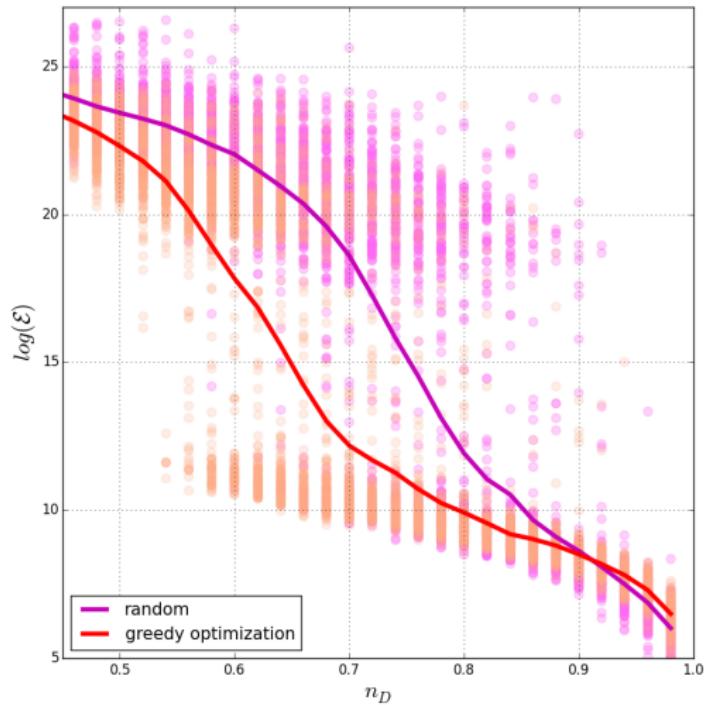
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Combining the pieces



Control energy

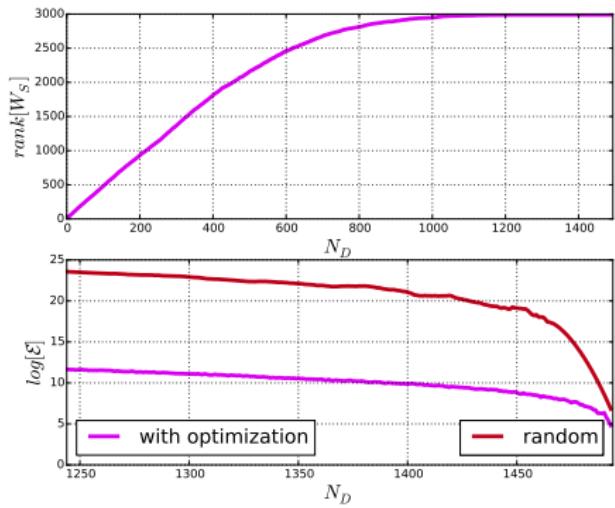
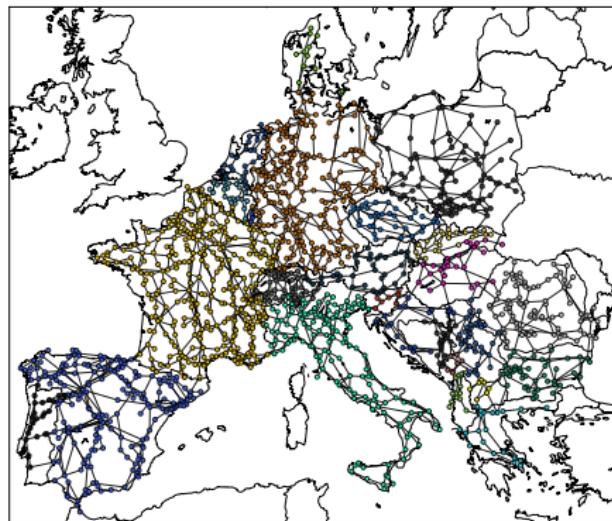
Are we doing better than random ?



- Random scale-free topologies (200 nodes)
- Random power distributions and line capacities

real topology

European transmission power grid (1494 nodes and 2196 edges over 25 countries)⁷

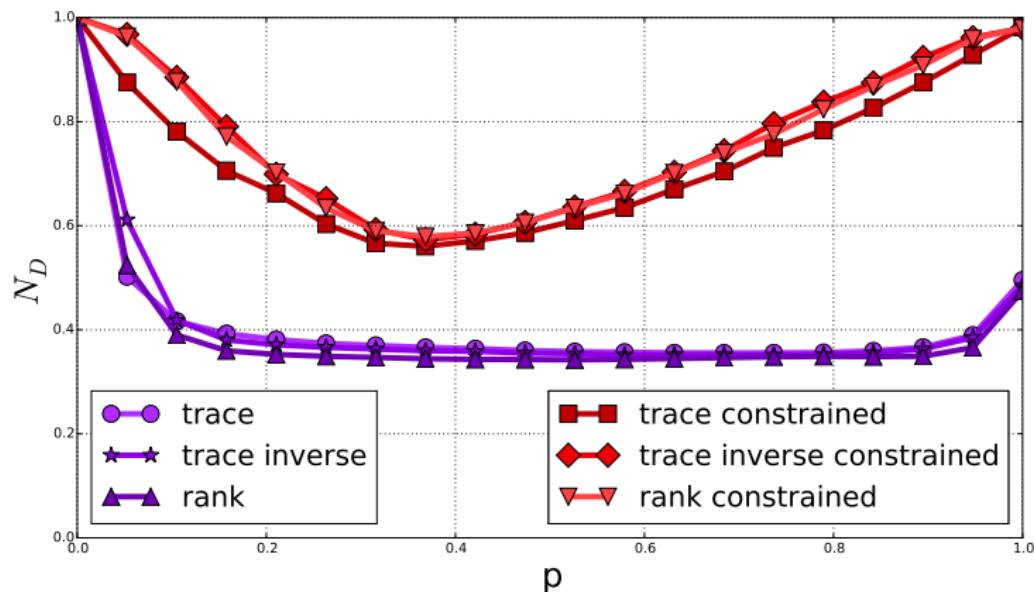


Questions ?

Thank you for your attention. Questions ?

Control and topology

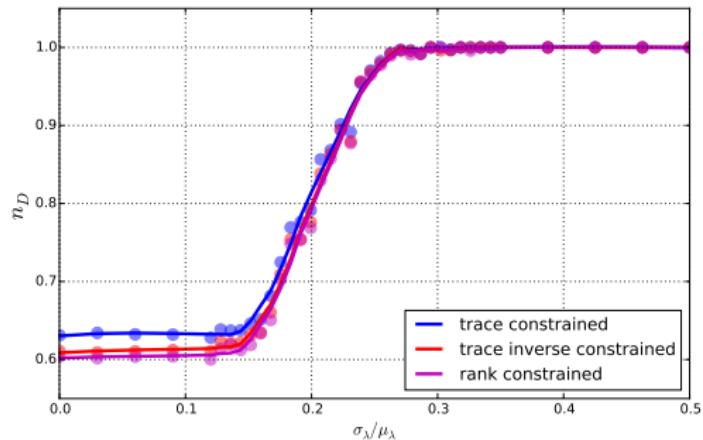
How does the topology impact the size of the driver set?



- Random erdos-renyi topologies (200 nodes)
- Random power distributions and line capacities

Control and capacity distribution

How does the capacity distribution of the batteries impact the size of the driver set?



- Random erdos-renyi topologies (200 nodes)
- Random power distributions and line capacities
- Battery capacities are drawn from : $\mathcal{N}(\mu_\lambda, \sigma_\lambda)$