Econometric Methods

Assignment 1

Due 20/2 before class

Consider the following model

$$y_i = \begin{cases} 1 & \text{if } y_i^* > 0 \\ 0 & \text{if } y_i^* < 0 \end{cases},$$

where y_i^* is a latent variable given by

$$y_i^* = \theta_0 + \theta_1 x_i + u_i$$

and where

$$u_i |\, x_i \sim N(0, 1).$$

Note that here x_i is scalar. The above model is the probit model. In what follows, we will generate samples of n iid observations from the probit model, where $x_i \sim N(1, 1)$ and $\theta = (1, 1)'$.

(a) Write down the objective function and its gradient for the (conditional) log-likelihood function (multiplied by $-1/n$).

(b) Estimate the model using the code online, namely the file "Assignment1_a_i.R". [Note that you'll have to complete the file as well as the functions that the file is calling.] Report $\hat{\theta} = (\hat{\theta}_0, \hat{\theta}_1)'$. Here, only use the objective function that you coded up.

(c) Redo (b) while also providing the gradient to the optimization algorithm. [Hint: use the library "numDeriv" to check that you coded the gradient correctly.]

(d) Write down the nonlinear least squares (NLS) objective function and its gradient for the probit model.

(e) Estimate the model by NLS providing only the objective function to the algorithm and report $\hat{\theta}$.

(f) Estimate the model by NLS providing the objective function and its gradient to the algorithm and report $\hat{\theta}$. [Hint: use the library "numDeriv" to check that you coded the gradient correctly.]

The probit model implies that

$$E[y_i - \Phi(x_i'\theta)|x] = 0.$$

This in turn implies that

$$E[(y_i - \Phi(x_i'\theta))g(x_i)] = 0 \tag{1}$$

for any function $g(x)$.

(g) Write down the GMM objective function and its gradient based on (1) with $g(x) = x$. (Hint: Note that given $g(x) = x$, the model is "just-identified", i.e., there are as many equations as parameters, such that the choice of $\hat{W}$ does not matter. Hence, you may choose $\hat{W} = I_2$.)

(h) Estimate the model by GMM providing only the objective function to the algorithm and report $\hat{\theta}$.

(i) Estimate the model by NLS providing the objective function and its gradient to the algorithm and report $\hat{\theta}$. [Hint: use the library "numDeriv" to check that you coded the gradient correctly.]

Now, the Monte Carlo part starts. From now on, use the file "Assignment1_j.R" and feel free to copy+paste any lines you may need from the file "Assignment1_a_i.R". Please use the estimator that does NOT use the gradient provided by the user. Throughout the Monte Carlo part the number of iterations equals 1000.

(j) Report the frequencies (or estimated probabilities) over 1000 Monte Carlo simulations with which the three estimators lie within a neighborhood of 0.1 for $n = 300$ and $n = 500$. Comment on what you find.

(k) Plot a histogram for $\hat{\theta}_2$ for all three estimators using n = 300. What do you find? Are your findings conformable with the Central Limit Theorem (CLT)?

(l) Report the 2×2 variance matrices of the three estimators over the 1000 Monte Carlo simulations for $n = 300$.

(m) Report the average estimates over the 1000 Monte Carlo simulations (using $n = 300$) of $(\hat{J}_1(\hat{\theta}))^{-1}/n$ and $(\hat{J}_2(\hat{\theta}))^{-1}/n$ (i.e., of two different estimates of the finite-sample variance of ML). You may use the hessian spit out by R's optim function as your $\hat{J}_2(\hat{\theta})$. What do you find? How do these (average) estimates compare to your finding in (l)?

(n) Write down Σ as defined in Theorem 4 (see slides) for the NLS estimator.

(o) Report the average estimate over the 1000 Monte Carlo simulations (using $n = 300$) of $\hat{H}^{-1}\hat{\Sigma}\hat{H}^{-1}/n$ (i.e., the estimate of the finite-sample variance of NLS). You may use the hessian spit out by R's optim function as your $\hat{H}$. The estimator $\hat{\Sigma}$ should be based on the Σ you obtained in (n); replace expected values by sample averages and θ_0 by $\hat{\theta}$. What do you find? How does this (average) estimate compare to your finding in (l)?

(p) Derive G and Ω as defined in Theorem 7 (see slides) for the GMM estimator.

(q) Show that in the case at hand, the asymptotic variance of the GMM estimator simplifies to

$$G^{-1}\Omega G^{-1}.$$

(r) Report the average estimate over the 1000 Monte Carlo simulations (using $n = 300$) of $\hat{G}^{-1}\hat{\Omega}\hat{G}^{-1}/n$ (i.e., the estimate of the finite-sample variance of GMM). The estimators $\hat{G}$ and $\hat{\Sigma}$ should be based on the G and Ω you obtained in (p); replace expected values by sample averages and θ_0 by $\hat{\theta}$. What do you find? How does this (average) estimate compare to your finding in (l)?