Linear Quadratic Regulator

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1 Introduction

A Linear Quadratic Regulator (LQR) is an optimal control algorithm used in control theory. The method involves calculating a control signal that minimizes a cost function, typically composed of state error and control effort.

2 Algorithm Steps

1. **System Definition**

   We start with a linear system of the form:

   $$\dot{x} = Ax + Bu$$
   $$y = Cx + Du$$

   where x is the state vector, u is the control input vector, A and B are system matrices, and y is the output.

2. **Cost Function Definition**

   The cost function to minimize is:

   $$J = \int_0^\infty (x^T Q x + u^T R u)\, dt$$

   where Q and R are weight matrices that decide the importance of state error and control effort.

3. **Solving the Riccati Equation**

   The solution to the LQR problem is given by the Riccati equation:

   $$A^T P A - P - A^T P B (B^T P B + R)^{-1} B^T P A + Q = 0$$

4. **Iterative Solution**

   Iteratively solve the Riccati equation until P converges within some small tolerance ε. Then compute the feedback gain:

   $$K = (B^T P B + R)^{-1} B^T P A$$

5. **Control Signal Calculation**

   Calculate the control signal as:

$$u = -Kx$$

6. **System Control**

   Apply the control signal u to the system. The control is designed to minimize the cost function while driving the system towards the desired state.