Tarea

$$(M+m)\ddot{x} + m\,l\ddot{\theta} = u$$

$$(I + m\,l^2)\ddot{\theta} + m\,l\ddot{x} = mg\,l\theta$$

$$\ddot{x} = \frac{u - ml\ddot{\theta}}{(M+m)}$$

$$(I + ml^2)\ddot{\theta} + \frac{ml\,u - m^2 l^2 \ddot{\theta}}{(M+m)} = mgl\theta$$

$$\ddot{\theta}\left((I+ml^2) - \frac{m^2 l^2}{M+m}\right) = mgl\theta - \frac{ml\,u}{M+m}$$

$$\ddot{\theta}\left(\frac{MI + Im + Mml^2 + m^2 l^2 - m^2 l^2}{M+m}\right) = \frac{mgl\theta(M+m) - ml\,u}{M+m}$$

$$\ddot{\theta} = \frac{mgl\theta(M+m) - ml\,u}{MI + Im + Mml^2}$$

$$\ddot{\theta} = \frac{mgl(M+m)}{MI + Im + mMl^2}\theta - \frac{ml\,u}{MI + Im + mMl^2}$$

$$\ddot{x} = \frac{u}{M+m} - \frac{ml}{M+m}\left(\frac{mgl(M+m)}{MI + Im + mMl^2}\theta - \frac{ml}{MI + Im + mMl^2}u\right)$$

$$\ddot{x} = \frac{u}{M+m} + \frac{m^2 l^2 u}{(M+m)(MI + Im + Mml^2)} - \frac{m^2 gl^2(M+m)}{(M+m)(MI + Im + mMl^2)}\theta$$

$$\ddot{x} = u\left(\frac{1}{(M+m)} + \frac{m^2 l^2}{(M+m)(MI + Im + Mml^2)}\right) - \frac{m^2 gl^2(M+m)}{(M+m)(MI + Im + mMl^2)}\theta$$

$$\ddot{x} = u\left(\frac{MI + Im + Mml^2 + m^2 l^2}{(M+m)(MI + Im + Mml^2)}\right) - \frac{m^2 gl^2(M+m)}{(M+m)(MI + Im + mMl^2)}\theta$$

$$\ddot{X} = u\left(\frac{I(M+m) + m\ell^2(M+m)}{(M+m)(MI+Im+Mm\ell^2)}\right) - \left(\frac{m^2g\ell^2(M+m)}{(M+m)(MI+Im+Mm\ell^2)}\right)\theta$$

$$\ddot{X} = u\left(\frac{I+m\ell^2}{MI+Im+Mm\ell^2}\right) - \frac{m^2g\ell^2(M+m)}{(M+m)(MI+Im+Mm\ell^2)}\theta$$

$$X = q_1 \qquad\qquad q_3 = \theta$$

$$\dot{X} = \dot{q}_1 = q_2 \qquad q_4 = \dot{q}_3 = \dot{\theta}$$

$$\dot{q}_2 = \ddot{q}_1 = \ddot{X} \qquad \dot{q}_4 = \ddot{q}_3 = \ddot{\theta}$$

$$
\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & \dfrac{mgl(M+m)}{MI+Im+Mm\ell^2} & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & \dfrac{-m^2g\ell^2}{MI+I_m+Mm\ell^2} & 0
\end{bmatrix}
\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} +
\begin{bmatrix}
0 \\
\dfrac{m\ell}{MI+Im+Mm\ell^2} \\
0 \\
\dfrac{I+m\ell^2}{MI+Im+Mm\ell^2}
\end{bmatrix} [u]
$$

$$
\begin{bmatrix} X \\ \theta \end{bmatrix} =
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + [0] u
$$