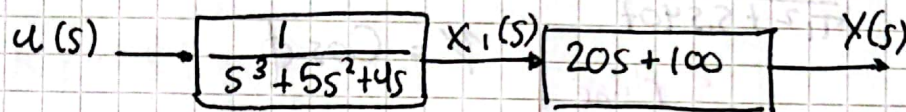Tarea 5

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)} \qquad \begin{cases} O.S = 9.5\% \\ ts = 0,74 \, sg \end{cases}$$

$$G(s) = \frac{20s + 100}{s^3 + 5s^2 + 4s}$$

$$u(s) \longrightarrow \boxed{\frac{1}{s^3 + 5s^2 + 4s}} \xrightarrow{X_1(s)} \boxed{20s + 100} \xrightarrow{X(s)}$$

$$\frac{X_1(s)}{u(s)} = \frac{1}{s^3 + 5s^2 + 4s}$$

$$x_1 = x_1$$
$$x_2 = \dot{x}_1$$
$$x_3 = \dot{x}_2 = \ddot{x}_1$$

$$u = \dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u \qquad \dot{x}_3 = \dddot{x}_1$$

$$\boxed{\dot{x}_3 = -5x_3 - 4x_2 + u} \quad ①$$

$$Y(s) = (20s + 100) X_1(s)$$

$$= 20\dot{x}_1 + 100x \quad \Rightarrow \quad \boxed{y = 20x_2 + 100x_1} \quad ②$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\% \, O.S = e^{-\left(3\pi / \sqrt{1 - y^2}\right)} \times 100$$

$$\ln(0.095) = \ln\left(e^{-\left(3\pi / \sqrt{1 - y^2}\right)} \right)$$

$$-2.3539 = \frac{-\frac{7}{3}\pi}{\sqrt{1-\frac{7}{3}^2}}$$

$$5.5407(1-\frac{7}{3}^2) = \frac{7}{3}^2\pi^2$$

$$5.5407 = \frac{7}{3}^2(\pi^2 + 5.5407)$$

$$\frac{7}{3}^2 = \frac{5.5407}{\pi^2 + 5.5407} \Rightarrow \frac{7}{3} = 0.5996$$

$$\frac{7}{3} = \cos\phi$$

$$\cos^{-1}(\frac{7}{3}) = \phi$$

$$\phi = 53.16°$$

$$s = \sigma + j\omega_d$$
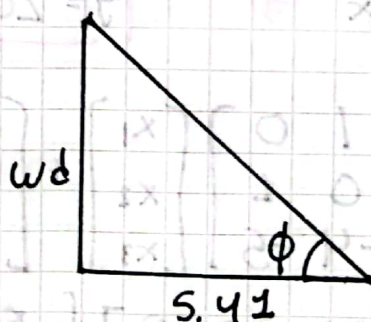$$= \frac{7}{3}\omega_n$$

S-plane

wd

-5.41

ϕ

$ts = 0.74$

$$ts = \frac{4}{\sigma}$$

$$\sigma = \frac{4}{0.74} = 5.405$$

$$5.0405 = \frac{7}{3}\omega_n$$

$$\frac{5.0405}{0.5996} = \omega_n$$

$$9.02 = \omega_n$$

wd

ϕ

5.41

$$\tan\phi = \frac{wd}{5.41}$$

$$\omega \approx 7.214$$

$\ddot{x_3}$ $1/5$ $\dot{x_3}$ $1/5$ $\dot{x_2}$ $1/5$ $\dot{x_1}$ 100 y

u -5 -4 20