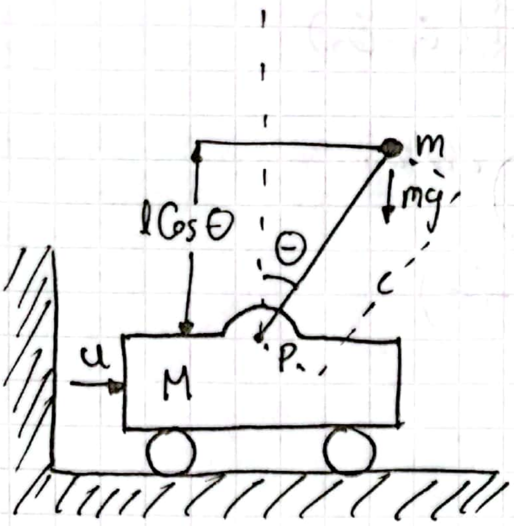
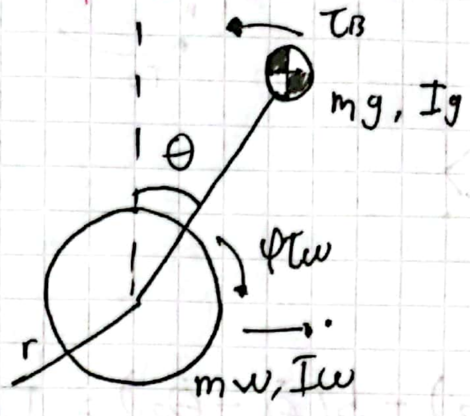
Tarea pendulo





eje X

u: Fuerza motor

$F = u - b\dot{x}$ b = fricción

M : masa del carro $x_p = x + l\,sen(\theta)$

x : Posición del carro

m = masa del pendulo

$F = ma$

$u - b\dot{x} = ma$

$$u - b\dot{x} = M\frac{d^2x}{dt^2} + m\frac{d^2x_P}{dt^2}$$

$$u - b\frac{dx}{dt} = (M+m)\frac{d^2x}{dt^2} + ml\frac{d^2}{dt^2}(sen\,\theta)$$

$$u - b\frac{dx}{dt} = (M+m)\frac{d^2x}{dt^2} + ml(-\dot{\theta}^2 sen\,\theta + Cos(\theta)\ddot{\theta})$$

$$u - b\dot{x} = (M+m)\ddot{x} - ml\,sen\,\theta\,\dot{\theta}^2 + ml\,Cos\,\theta\,\ddot{\theta}$$

Eje y

$$F = mg\,Sen\,\theta$$

$$x_p = x + l\,sen\,\theta$$

$$y_p = l\,Cos\,\theta$$

$$mg\,sen\,\theta = m\,Cos\,\theta\,\frac{d^2}{dt^2}x_p - m\,sen\,\theta\,\frac{d^2}{dt^2}y_p$$

$$mg\,sen\,\theta = m\,Cos\,\theta\,\frac{d^2}{dt^2}(x + l\,sen\,\theta) - m\,sen\,\theta\,\frac{d^2}{dt^2}(l\,Cos\,\theta)$$

$$mg\,sen\,\theta = m\,Cos\,\theta(\ddot{x} + l\,Cos\,\theta\,\ddot{\theta} - l\dot{\theta}^2 sen\,\theta) - ml\,sen\,\theta(-sen\,\theta\,\ddot{\theta} - Cos(\theta)\dot{\theta}^2)$$

$$mg\,sen\,\theta = m\ddot{x}\,cos\,\theta + ml\,Cos^2\theta\,\ddot{\theta} - ml\,sen\,\theta\,Cos\,\theta\,\dot{\theta}^2 + ml\,sen^2\theta\,\ddot{\theta} + ml\,sen\,\theta\,Cos\,\theta\,\dot{\theta}^2$$

$$g\,\cancel{sen}\,sen\,\theta = m\ddot{x}\,Cos\,\theta + ml\ddot{\theta}$$

$$g\,sen\,\theta = \ddot{x}\,Cos\,\theta + l\ddot{\theta}$$

linealizar

Podemos aproximar $sen\,\theta \approx \theta$, $Cos\,\theta \approx 1$

Para x

$$u - b\dot{x} = (M+m)\ddot{x} - ml\dot{\theta}^2 sen\,\theta + ml\,Cos\,\theta\,\ddot{\theta}$$

$$u - b\dot{x} = (M+m)\ddot{x} - ml\theta\dot{\theta}^2 + ml\ddot{\theta}$$

asumimos la velocidad angular cercana a 0

$u - b\dot{x} = (M+m)\ddot{x} + ml\ddot{\theta}$ 1.

Eje y

$g \operatorname{sen}\theta = \ddot{x}\cos\theta + l\ddot{\theta}$

$g\theta = \ddot{x} + l\ddot{\theta}$ 2.

Espacio de estados

$x_1 = \theta$ $\quad x_3 = x$

$x_2 = \dot{\theta}$ $\quad x_4 = \dot{x}$

$$\begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]u$$

$u - b\dot{x} = M\ddot{x} + m\ddot{x} + ml\ddot{\theta}$

$u - b\dot{x} = M\ddot{x} + m(\ddot{x} + l\ddot{\theta})$ 2 en 1

$M\ddot{x} = u - b\dot{x} - mg\theta$

$\ddot{x} = \frac{u}{M} - \frac{b\dot{x}}{M} - \frac{mg\theta}{M} \Rightarrow \dot{x}_4 = \frac{1}{M}u - \frac{b}{M}x_4 - \frac{mg}{M}x_1$

$g\theta = \frac{u}{M} - \frac{b}{M}\dot{x} - \frac{mg\theta}{M} + l\ddot{\theta}$

$l\ddot{\theta} = g\theta + \frac{b}{M}\dot{x} - \frac{1}{M}u + \frac{mg}{M}\theta$

$\ddot{\theta} = g\frac{M+m}{Ml}\theta - \frac{1}{Ml}u + \frac{b}{Ml}\dot{x} \Rightarrow \dot{x}_2 = g\frac{M+m}{Ml}x_1 - \frac{1}{Ml}u + \frac{b}{Ml}x_4$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ g\frac{M+m}{Ml} & 0 & 0 & \frac{b}{Ml} \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & -\frac{b}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} [u]$$