Computing Cup-Products in integral cohomology of Hilbert schemes of points on K3 surfaces

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Simon Kapfer

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The following changements were made due to the suggestions:

- Corrected the typos and changed the wording as suggested.
- Definition 1.1: Improved the verbalisation.
- Definition 1.2: Added definitions of the Ring of Symmetric Functions, monomial symmetric functions and power sums. Dropped the example, as suggested. Left the defition of the $\psi_{\lambda\mu}$ since they are needed in Theorem 1.7.
- Definition 1.6: Corrected B to $B \otimes B$, as suggested. Erased the questionable $B(\Delta(1), \Delta(1))$. Added a definition of a general adjoint map.
- Added references for Fogarty and Nakajima result. Corrected the degree of $\mathfrak{q}_l(\beta)$.
- Proof of Lemma 1.9: The original proof and the suggested correction were both wrong (maybe because of the confusion with the notation of partitions?). The necessary corrections were made.
- Definition 1.10: Replaced the old incomprehensible description of $A\{S_n\}$ by a longer one, more close to Lehn and Sorger's paper.
- Theorem 1.11: Changed the formulation a bit.
- Theorem 1.12: Wrote an addendum to the statement of the theorem, which is already mentioned in the original Li–Qin–Wang version.
- Remark 1.14: I admit that the inequality in the second line was confusing, though correct. Replaced it by a clearer formulation, I hope.
- Example 1.15 (5) is now Example 1.16. In the appendix it is shown how to get the results of Example 1.15 using the code. Example 1.16 can be computed by hand, as shown in the proof.

- Theorem 1.17: This theorem is new.
- Remark 2.1: Left this remark unchanged. The rank of $H^6(S^{[n]})$ stabilizes to 2876, not 2300, e. g. as shown by Göttsche's formula. Indeed, 2300 is the rank of $Sym^3 H^2(S^{[n]})$.
- Proposition 2.4: Changed the rank of the second quotient to 254, as suggested. The freeness result is now an instance of Theorem 1.17.
- Proof of Proposition 2.5. Lemma 1.9 says $\|v\| \geq \frac{3}{2}$ which is equivalent to $\|v\| \geq 2$.
- Appendix: following the suggestions, I added two more subsections describing the usage of the code (including Example 1.15) and the rough structure of the program.
- Bibliography: Changed reference 1. Added pages and number of reference 3. Added references 2 and 4. Various improvements in the formatting.