

Taller 1 (Pregunta teórica)

5. (Theoretical) Show that the $D^4 f$ operator is given by:

$$D^4 f(x_j) \cong \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2}))}{h^4} \quad (3.33)$$

For this operator, what is the order ($\mathcal{O}(h^k)$) of the approximation?

Tenemos la definición del libro

$$f'(x_j) = \frac{f(x_{j+1}) - f(x_{j-1}))}{2h} - \underbrace{\frac{h^2}{6} f'''(x)}_{\mathcal{O}(h^2)}$$

$$f'(x_j) \approx \frac{f(x_{j+1}) - f(x_{j-1}))}{2h}$$

El orden de nuestra aproximación es h^2

Aplicando el concepto de derivada central

$$f''(x_j) \approx \frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1}))}{h^2}$$

encontramos $f^{(4)}(x)$ usando la anterior definición de derivada central

$$f^{(4)}(f''(x_j)) = \frac{f(x_{j+2}) - 2f(x_{j+1}) + f(x_j)}{h^2} - 2 \frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1}))}{h^2} + \frac{f(x_j) - 2f(x_{j-1}) + f(x_{j-2}))}{h^2}$$

$$f^{(4)}(x_j) = \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2}))}{h^4}$$

La derivada tiene orden $\mathcal{O}(h^4)$