Computational Neuroscience

Synchronous States in Homogeneous Populations of LIF Neurons

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0 Exercice 0

0.1 Question 0.1 and 0.2

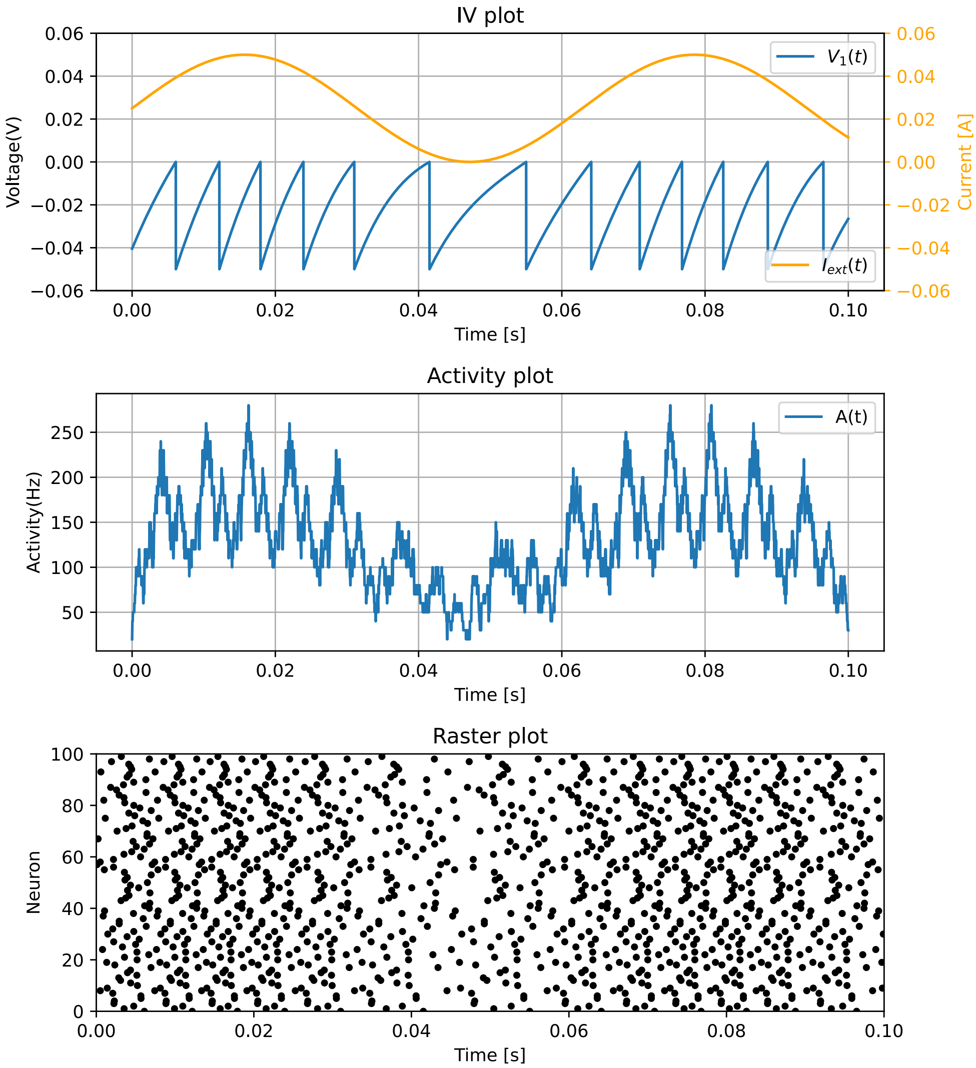


Figure 1: (a) Voltage and external current evolution across time, (b) Low-pass filter activity using $\frac{1}{\tau_A}\mathbb{1}_{[\frac{-\tau_A}{2},\frac{\tau_A}{2}]}$, (c) Raster plot of the neuron spikes for a non synchronous population

<u>How would the system behave as $\to \infty$?</u>

In this simulation, we are using a periodically changing current which in term induces a periodically alternating voltage in the neurons. As we can see on the raster plot, the behavior does not significantly change between the end and the beginning of the simulation. We can in fact deduce that the alternating current will not lead to a synchrony of spike firing or a convergence to a specific behavior. As the neurons interact in a noiseless manner, independently from each other, the change in their voltage is only determined by the alternating current and the shape of the activity will depend on the initial voltages Vi. The voltage simply periodically follows the oscillating input current.

1 Exercice 1

1.1 Question 1 : Implementation of the interaction

See implementation in the notebook

1.2 Question 2 : Spontaneous dynamics of the system (I = 0 mA)

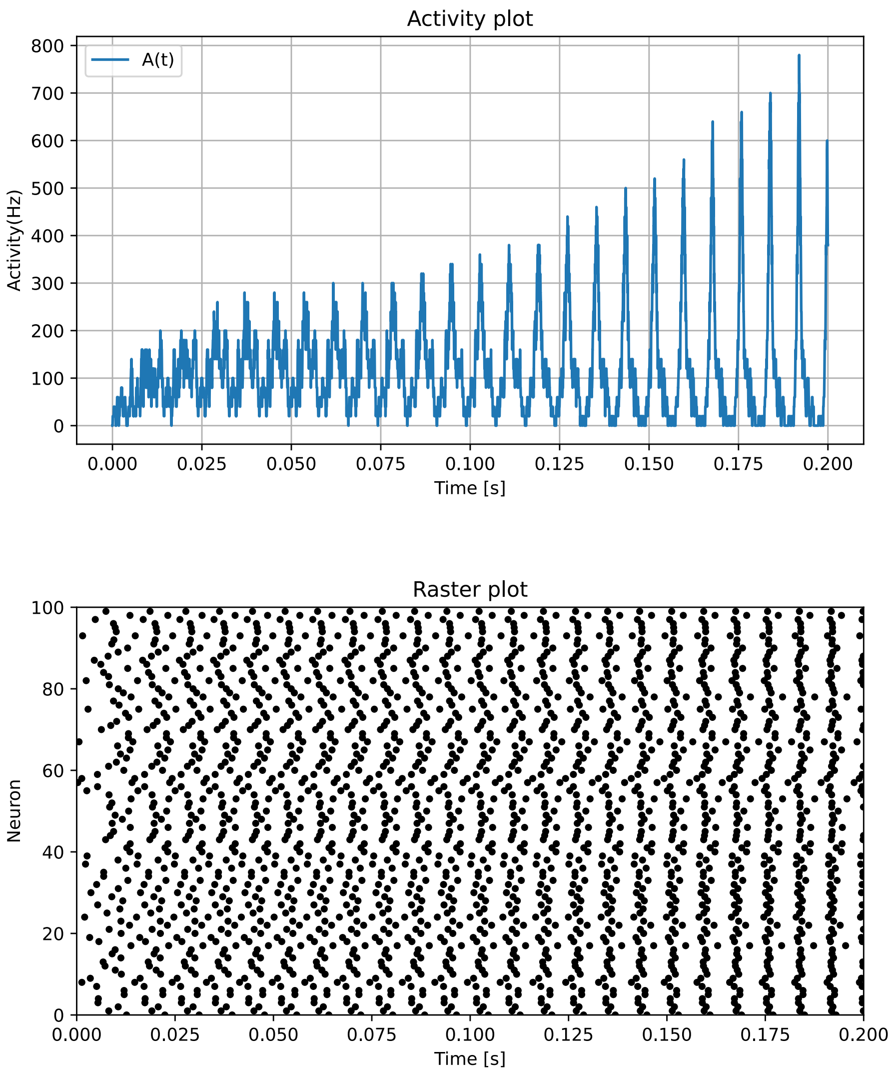
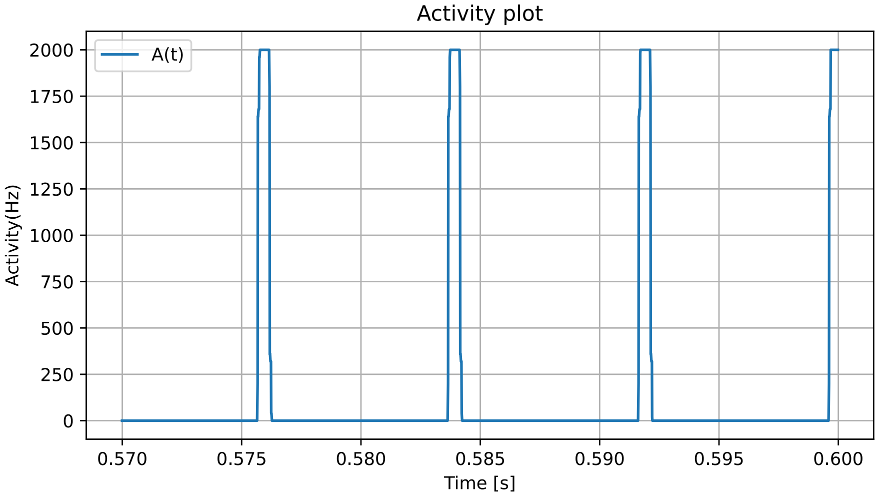


Figure 2: (a) Low pass filtered activity of the synchronous population for $t \in [0 - 200]$ ms, (b) Raster plot of the neuron spikes



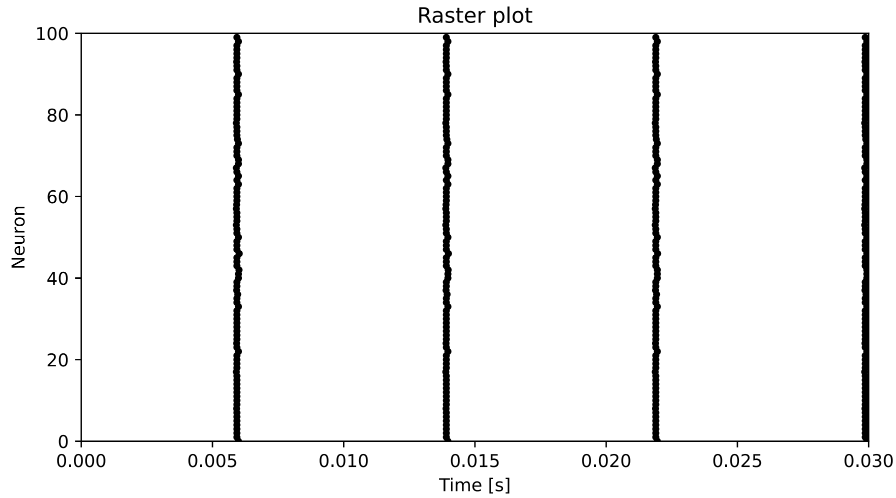


Figure 3: (a) Low pass filtered activity of the synchronous population for $t \in [30 - T_f]$ ms and $I_0 = 0$ mA, (b) Raster plot of the neuron spikes

Why is the activity composed of sharp oscillations/peaks?

As we can on the first 200 ms, the activity starts in a non synchronous state with small peaks and high noise. Each peak corresponds to the moment in time (rather an interval as we are using a rect filter $\frac{1}{\tau_A}\mathbb{1}_{[\frac{-\tau_A}{2},\frac{\tau_A}{2}]}$)where several neurons fired together. As the simulations evolves, the network synchronizes and the peaks increase in size and the noise is reduced as the neurons fire together.

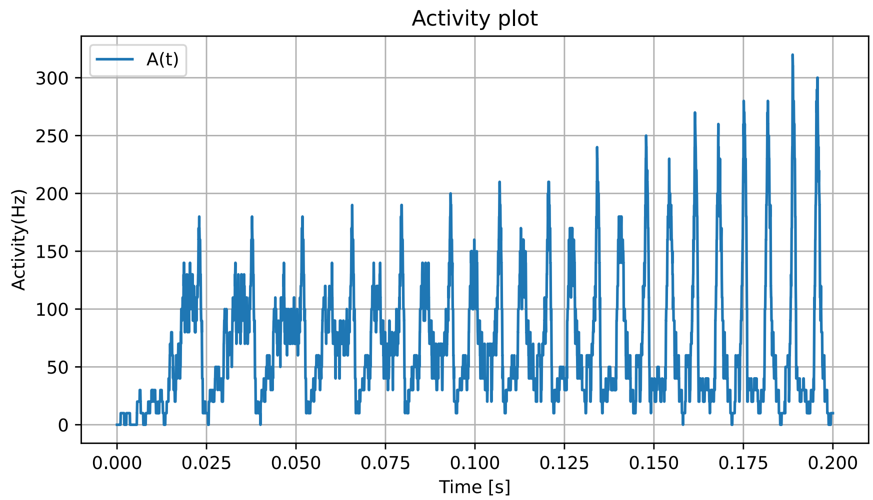
Period between 2 peaks ?

Using the last 30 ms where nearly all the neurons are synchronous, we estimate the period between two peaks at $7.98ms$

How would the network behave as $\rightarrow +\infty$?

As $t \rightarrow +\infty$, the network should approach perfect synchrony which results in nearly simultaneous firing as shown by vertically and periodically aligned columns in the raster plot and rectangle pulse of $2e3Hz$ activity plots. We can assume that for t very large and due to the noiseless nature of the system, all neurons will fire simultaneously which would correspond to perfectly aligned columns in the raster plot.

1.3 Question 3 : Spontaneous dynamics of the system (I=-0.25 mA)



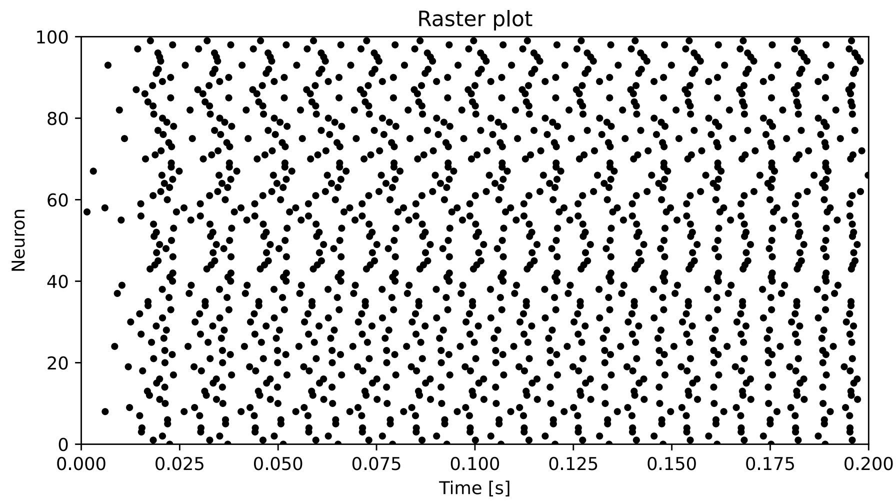
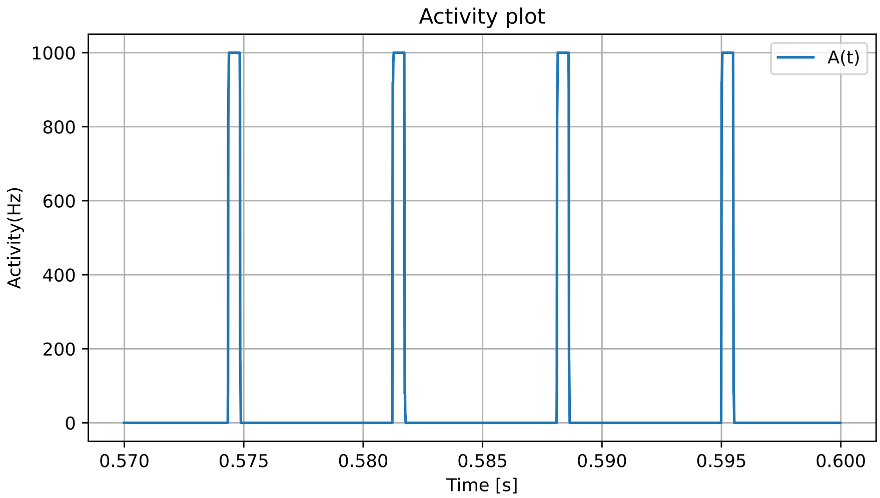


Figure 4: (a) Low pass filtered activity of the synchronous population for $t \in [0 - 200]$ ms and $I_0 = -0.25$ mA, (b) Raster plot of the neuron spikes



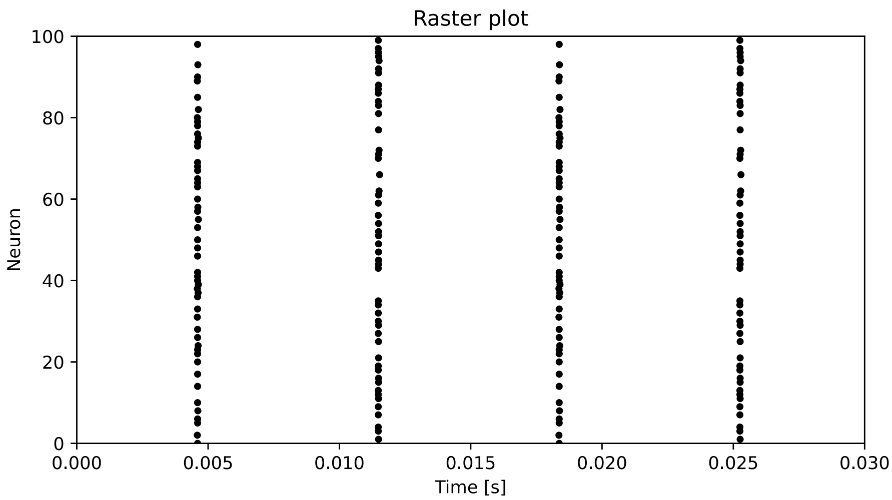


Figure 5: (a) Low pass filtered activity of the synchronous population for $t \in [30 - T_f]$ ms and $I_0 = -0.25$ mA, (b) Raster plot of the neuron spikes

What does the system converge to now ? What is different from the synchronous state observed previously?

By decreasing the constant external current the synchrony is still preserved at large t but the amplitude of the rectangle pulses on the activity plot is decreased by half, meaning less neurons fire at the same time. We can also observe periodical fluctuations in the amplitude of the activity peaks, under some specific initial states (Vi) , oscillating below and above $1e3Hz$. This could mean that there is more than one group of neurons firing at the same time, at least two one with period T and the other with period $T/2$, here $T = 13.77ms$

What is the corresponding value of vrest, for which the dynamics would be the same without any external input?

If $V_{rest} = 27.5mV$ then the dynamics would be the same as if there was no external current

2 Exercice 2

2.1 Question 1

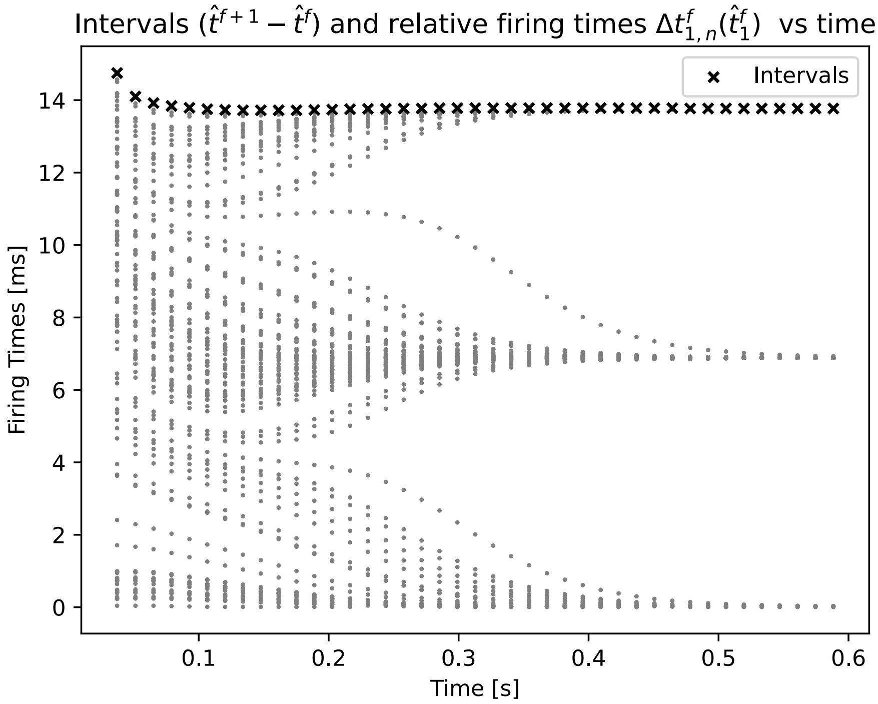


Figure 6: (a) Relative firing times $\Delta t^f_{1,f}(\hat{t}^f_1)$ for $n = 2, ..., N$ in grey and inter-spike intervals of the reference neuron $\Delta t^f_{1,n}(\hat{t}^f_1)$ in black

How does the synchronisation look like in this representation?

As the synchronization in a raster plot can be seen as columns on the network activity, here synchronization can be deduced from the convergence of the trajectories to specific relative firing times.

How do you read the number of clusters in the synchronous state?

We can see that after synchronisation of the network, there are three clusters of neurons. Ones which fire very closely to neuron 1, other that fire after 7 ms and finally ones that fire after 14 ms. The first and last group of neurons is basically one same cluster as shifting a periodic signal by its period gives you the same signal, i.e. eventhough that the corresponding firing times are shifted T it results in neurons firing at the same time. Interestingly, the second group of neurons has a period of $T/2$ as observed on 2 peak amplitudes of the activity plot of Q 2.

2.2 Question 2

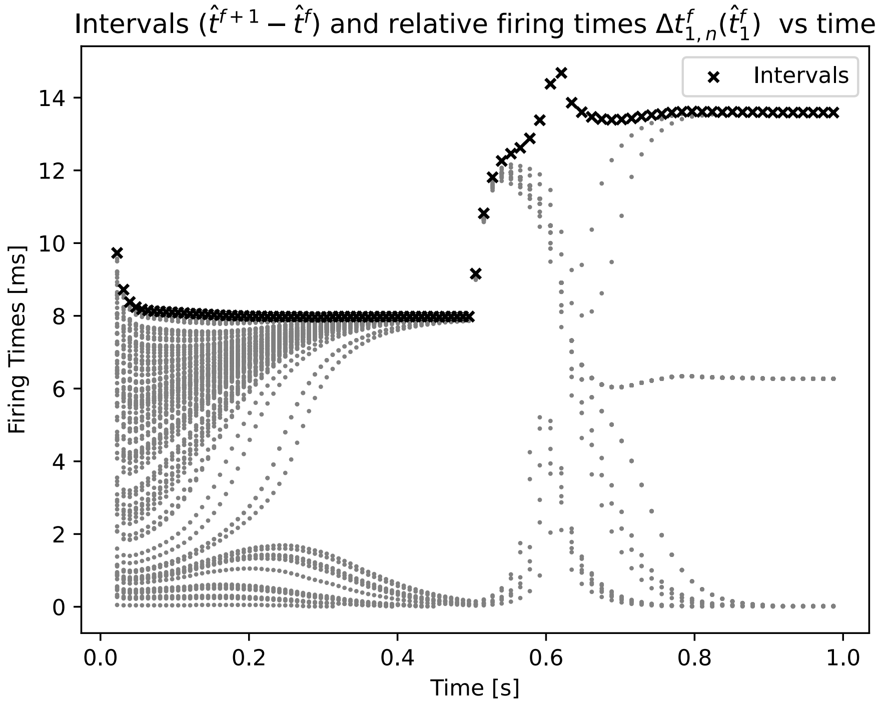


Figure 7: (a) Relative firing times in grey and inter-spike intervals of the reference neuron in black, here $I_{ext}(t) = I_0 \cdot \mathbb{1}_{t>T_{step}}$

<u>Why does the system switch from one state to another at $t = T_{step}$?</u>

If we perform a phase-plane analysis or a similar tool to analyze the period of the network, introducing a step current induces a shift in the corresponding null-clines which results in a change of the fixed points and eventually their stability. Here we introduce a negative current at $t = T_{step}$ which results in a change in the number of fixed points $1 \to 2$ as well as their values. The network converges to 2 stable fixed states instead of one, one with a relative firing time period of 14ms and another with 7ms. (Remember that the state at 0ms is just the state at 14ms shifted by a full period)

2.3 Question 3

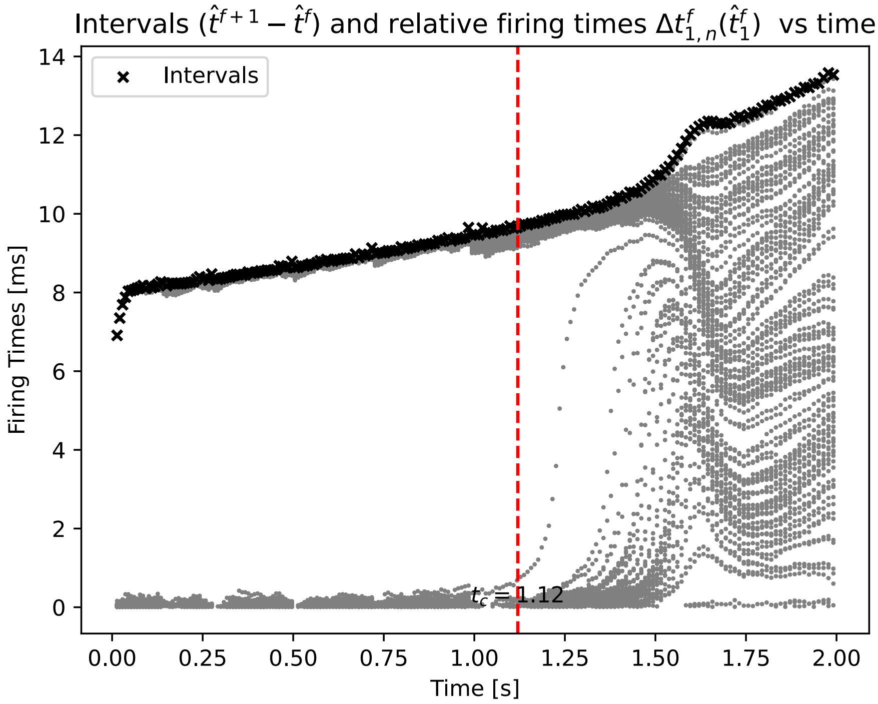


Figure 8: (a) Relative firing times in grey and inter-spike intervals of the reference neuron in black, here $I_{ext}(t) = I_0 \cdot \frac{t}{T_f}$

<u>Estimate on the plot the time tc at which neurons start to get away from the synchrony, and thus the critical current $I_c = I_{ext}(t_c)$. What is the corresponding critical value of v_{rest}?</u>

The current which induces a change in stability is $I_c = -0.14$ mA and the corresponding resting voltage is 22 mV $= V_{rest} + R \cdot I_c$

3 Exercice 3

3.1 Dilation factor

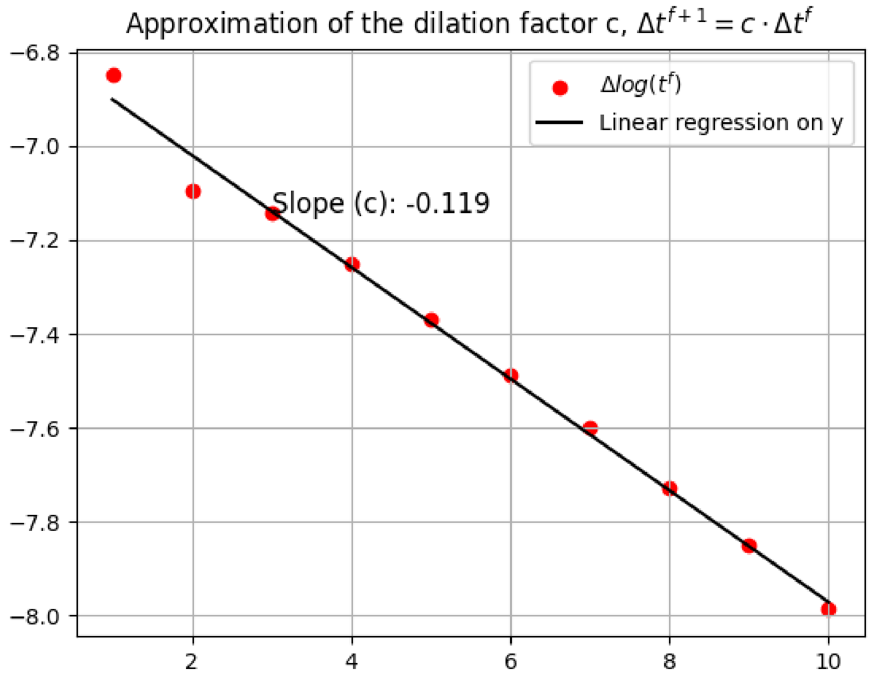


Figure 9: Dilation factor estimation $\Delta t^{f+1} = c \cdot \Delta t^f$, in red $log(\Delta t^f)$ and in black it's linear approximation

Using a linear regression on the red dots we estimate the dilation factor at -0.119 (slope of the black curve)

3.2 Question 2

What is the condition on c so that the synchronous state is stable?

As $\Delta t^{f+1} = (c)^f \cdot \Delta t^0$, if $|c| < 1, lim_{f\to\infty} c^f = lim_{f\to\infty} \Delta t^f = 0$ which implies a stable synchronicity of the network

How would c change if we inject an external current Iext = I0 = 0.25mA ?

We would expect c to increase. See below for the explanation

Can you explain better what you observed in Qus2 and 3?

If the current decreases, it's harder for the system to reach perfect synchrony as the neuronal activity decreases (see 2.2). As c represents the ratio between two consecutive time steps, it needs to be bigger as it takes longer to reestablish synchronicity when one neuron goes out of sync which is the case here or in 2.3 with the addition of noise.

3.3 Region of attraction

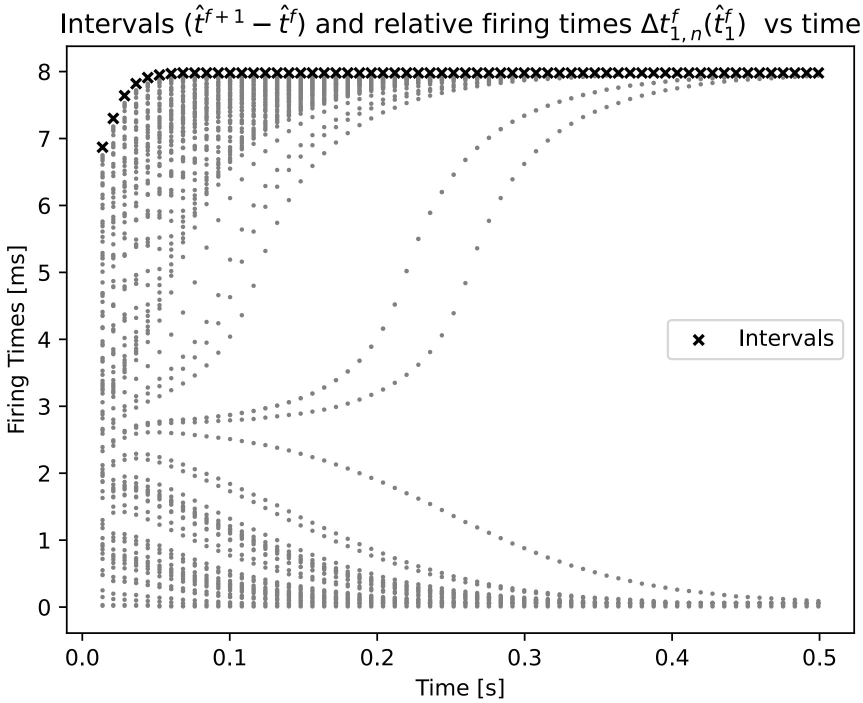


Figure 10: Relative firing times of the non-synchronous population with respect to the synchronous one

3.4 Question 4

What is the link between a synchronous population of interacting neurons in which one neuron gets perturbed, and a population of N non-interacting neurons receiving the input from the synchronous population?

The synchronous population will send its synchronous current to the non-synchronous population. The non-synchronous network will synchronize with the input of the synchronous population. We use the epsilon values obtained from the simulation of the synchronous network which encapsulates the periodic time component of the voltage dynamics. This allows us to produce a periodic stimulus of the non-synchronous network which induces synchrony.

Evaluate the time interval around a synchronous firing that delimits its region of attraction. What happens if a neuron fires at the boundary of two regions?

We can see that all the neurons either synchronize with $\hat{t}^k$ or $\hat{t}^{k+1}$, the region of attraction to the peaks is $T = 8ms$ centered around the peak. We can also assume that if a neuron fires at the limit of the two regions, it will stay in the saddle point. Further analysis, especially using phase-plane tools is necessary to determine the existence of the fixed point and its stability

4 Bonus

We chose to work with networks of size $50, 10, 5$. We first simulated a synchronous neuronal network with the specified number of neurons for $600ms$ at the end of which the neurons reached convergence as shown in the previous exercises. After running the first simulation we extracted its spiking times and instantiated a new synchronous network using the last values of ϵ_a, ϵ_b as well as neurons' voltages. We then induced a $10mV$ perturbation on a random neuron of the new network and started the simulation. This allowed us then to calculate the synchronicity delays and the dilation factor to analyze the stability. The networks take different times to reach stability but they all stabilize. In fact, we found that the period does not vary between the different neuron network sizes, it is actually dependent on the external current as seen during this project meaning that all networks of 50,10 and 5 neurons all stabilize at the same period, they just take different times to do so. Below you can find the activity plots, relative firing times and dilation factor logarithmic approximation for the 3 neuron sizes

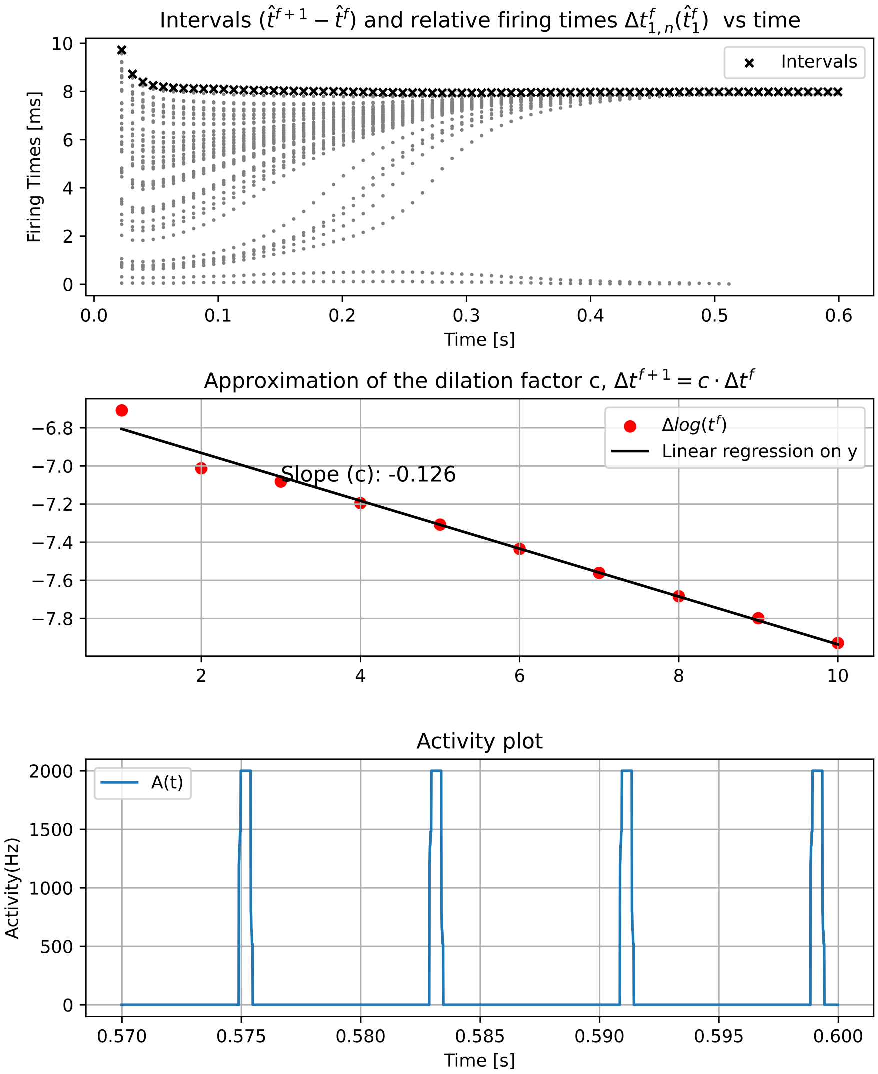


Figure 11: (a) Relative firing times of a 50 neuron synchronous network, (b) Linear approximation of the dilation factor, (c) Activity plot

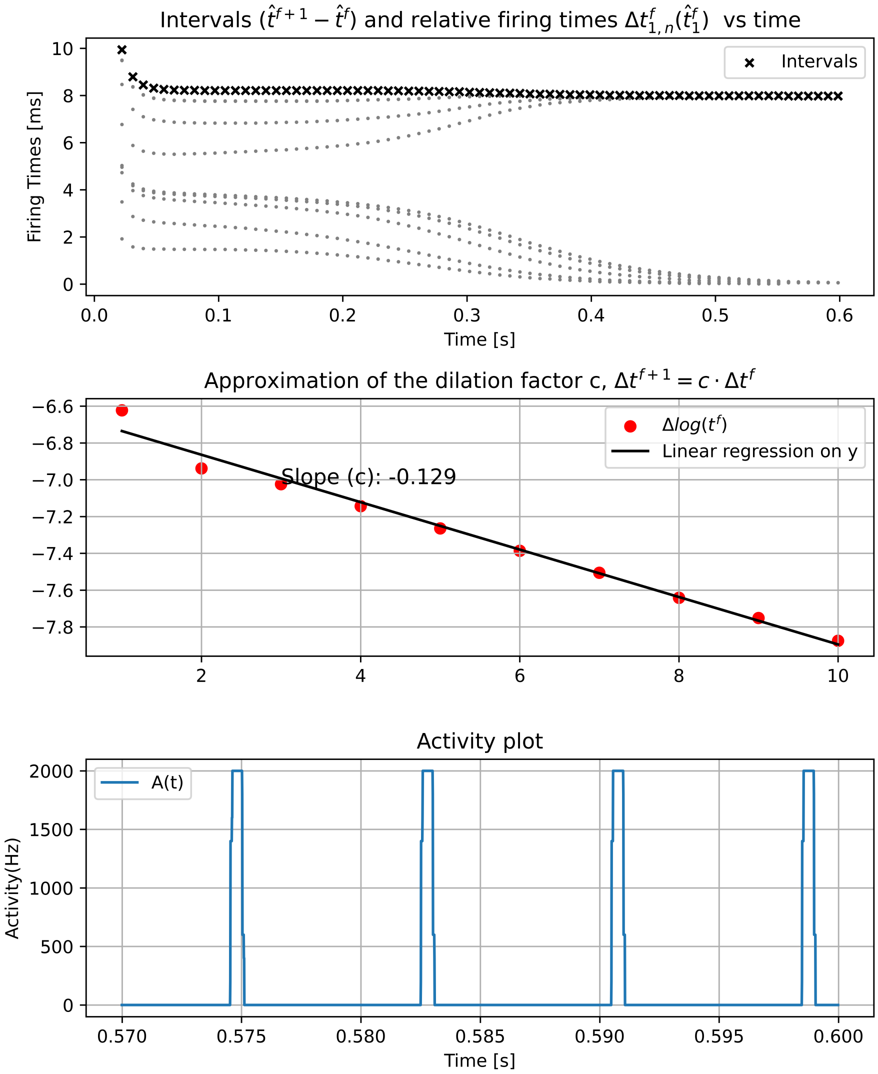


Figure 12: (a) Relative firing times of a 10 neuron synchronous network, (b) Linear approximation of the dilation factor, (c) Activity plot

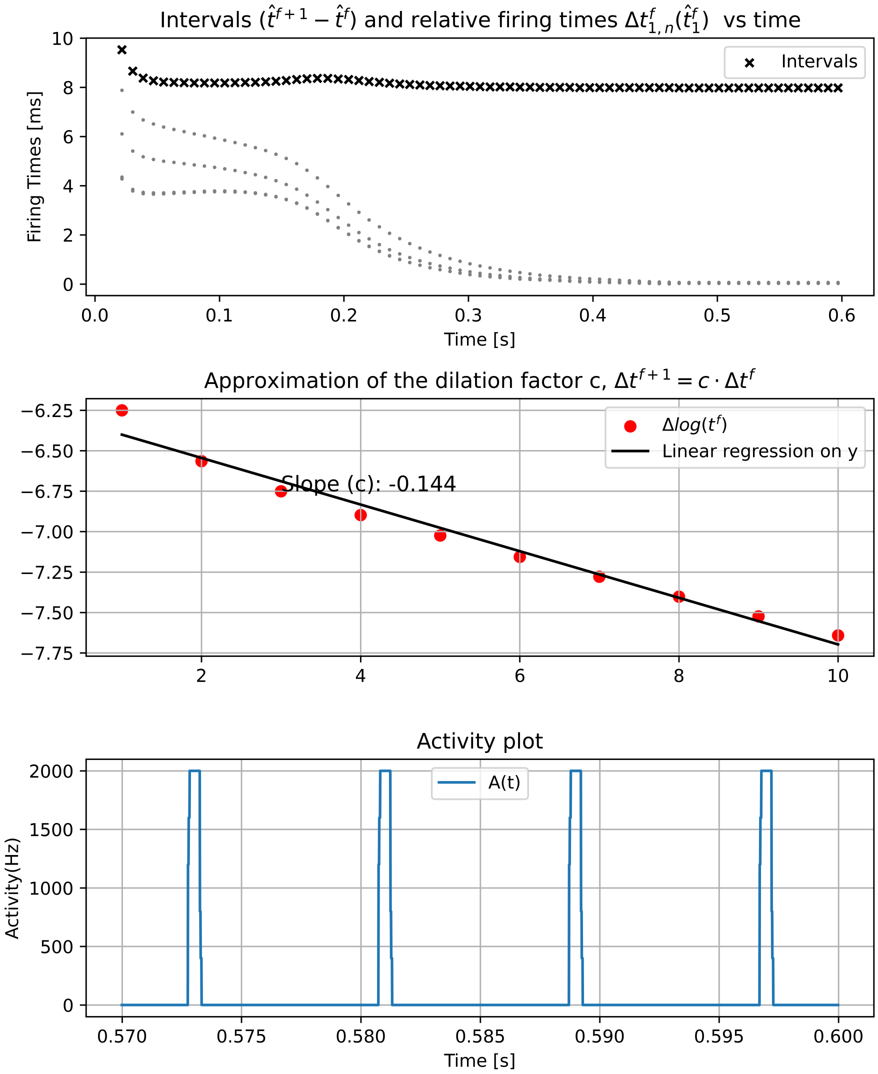


Figure 13: (a) Relative firing times of a 5 neuron synchronous network, (b) Linear approximation of the dilation factor, (c) Activity plot