LABORATORY REPORT

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Modeling of Dynamical Systems

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Introduction

A clinic recently experienced a power outage, impacting its services. While most of the services have backup support systems in place, the energy support systems for the organ bank and the medication storage room were significantly damaged, prompting ongoing repair work. The responsible teams are understandably concerned and eager to make prompt decisions. This urgency is due to the vulnerability of certain medications, such as insulin and biologics, to changes in temperature, along with the sensitivity of transplant organs and tissues to temperature fluctuations. Given the absence of the cooling system, the overall thermal transmission coefficient for both rooms falls within the range of 0.001 to 0.1. You have been informed that the current temperatures is 2°C in thea medication room and 1°C in the organ bank.

Implementation

To provide an optimal solution to this problem, we decided to create a code in the Python programming language using the provided data. This program takes the given information and inputs it into a modeling equation that explains the temperature change in each of the rooms. This way, we are presented with the time it would take for each of them to return to a temperature that allows both the organs and the medications to not be negatively affected and can be stored correctly.

The functions involved in this process are *initial_conditions*, *observe*, and *update_system*. Where the first one is responsible for receiving the initial values necessary for the system's operation.

The *observe* function uses the data that was previously processed in the *initial_conditions* function and places it within a list, which is then utilized by *update_system* to start generating temperature values within the rooms for each moment intime. It's worth mentioning that the code also includes a function that plots these results in order to have a better analysis of the outcomes.

Another analysis that is of great interest is the equilibrium point of this system, which would indicate at what point the temperature of the rooms becomes constant, referring to the fact that it does not grow or decrease regardless of the time that passes unless, of course, some external event to the system occurs that can generate an impact on it.

For this, a code was developed which takes the equation of the system to be modeled and converts the target variables into constants. It then proceeds to iterate a number '*t*' of times until it manages to obtain a value for this variable.

Analysis

The following difference equation was obtained to model the problem posed:

$$T_t = T_{t-1} - ACT_{t-1}$$

Where T_{t-n} the temperature for the medicine room and the organ bank, A is the heat transmission and C is the coefficient of cooling. From this equation, the characteristic value for the best and worst cases was obtained, for this it was assumed that $T_t = \lambda^t$, therefore, the following characteristic equation is obtained:

$\lambda^t = \lambda^{t-1} - AC\lambda^{t-1}$, proceed to clear λ

$$\frac{\lambda^t}{\lambda^{t-1}} = 1 - AC$$

$$\frac{1}{\lambda^{-1}} = 1 - AC$$

$$\lambda = 1 - AC$$

Now to find the equilibrium point, algebraic methods were applied where $T_{t-n} = T_{cte}$, in such a way that the following equation is obtained:

$$T_{cte} = T_{cte} - ACT_{cte}$$

In this way, it is necessary to know the time it will take both the medication room and the organ bank to reach a temperature of 4° in the best and worst cases. It must be kept in mind that the temperature can be in the range of 2° to 4° since for both places the items found have different ideal temperatures, which midpoint is in the range already mentioned. Although the equation clearly models the decrease in temperature, it does not really change the time it takes to get from one temperature to another and the rate that this will take.

For the best of cases which when $A = 0.1$, is had for the medicine room reaches a temperature of 22 ° C when 2340 minutes pass as shown in figure 1.

On the other hand, the organ bank takes about 400 minutes to reach room temperature of 22°C, as shown in Figure 2.

In the worst case when $A = 0.001$, the time it takes for the medicine room to reach 22°C exceeds 2 months, due to Python limitations it was not possible to visualize the final time it would take.

The organ bank for the worst case takes much longer to reach 22°C, exceeding 2 months, as shown in figure 4.

Results

As can be seen in the Analysis section, you must replace A and C For the values given in the statement of the problem we have the following results.

For the medicine room in the best and worst case we have:

$$A = 0.1,\ C = 0.01$$

$$\lambda = 1 - (0.1)(0.01) = 0.999$$

$$A = 0.001,\ C = 0.01$$

$$\lambda = 1 - (0.001)(0.01) = 0.99999$$

For the organ bank for the best and worst cases we have:

$$A = 0.1,\ C = 0.2$$

$$\lambda = 1 - 0.1 \cdot 0.2 = 0.98$$

$$A = 0.001,\ C = 0.2$$

$$\lambda = 1 - 0.001 \cdot 0.2 = 0.9998$$

On the other hand, to obtain the equilibrium point of the equation proposed in Analysis, we cleared T_{cte} from this, following the following algebraic process.

$$T_{cte} - T_{cte} = -ACT_{cte}$$

$$0 = -ACT_{cte}$$

$$0 = T_{cte}$$

Thus, it can be seen that the equilibrium point is 0.

Using the same values for $A \text{ and } C$ in both cases the following results were obtained.

In the best of cases, as mentioned in the previous section, we have the following graph for the case of the medicine room.

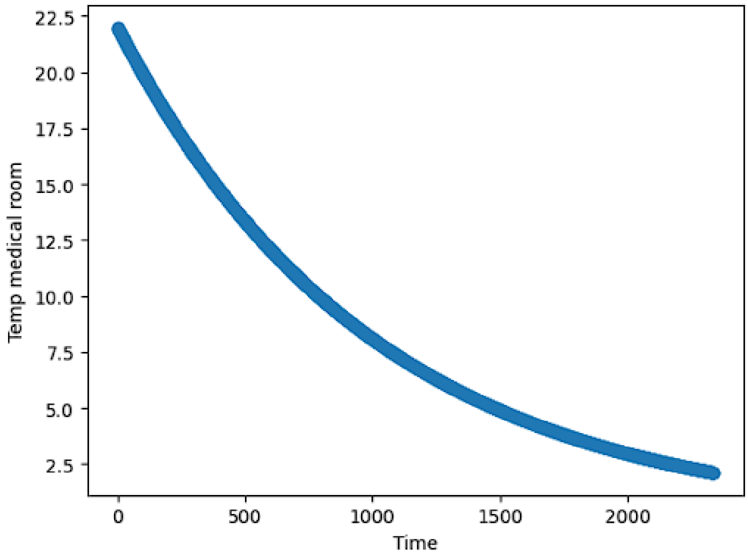

Figure 1

For the organ bank it can be clearly visualized that its rate of change is much faster than the medicine room.

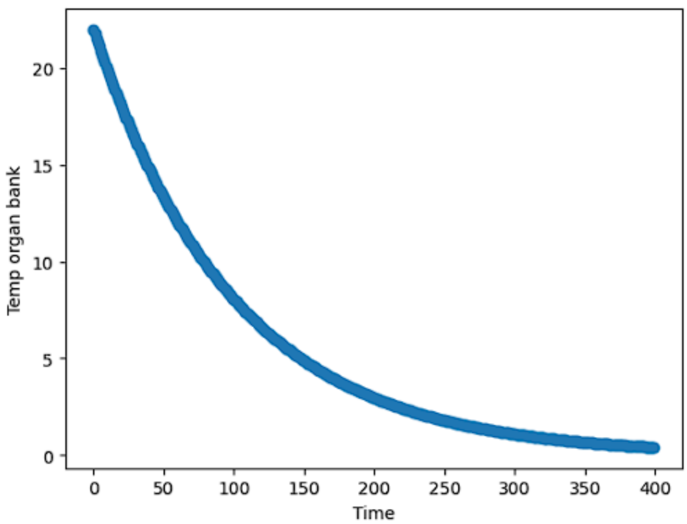

Figure 2

Next, the results are displayed in the worst case, where the medicine room after two months has not been able to reach the required point.

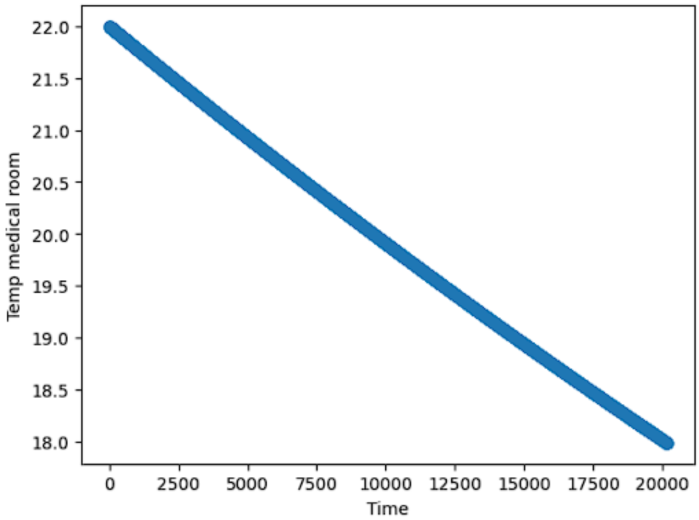

Figure 3

As in the case of the medicine room, the organ bank is not able to change its temperature quickly either, taking more than two months.

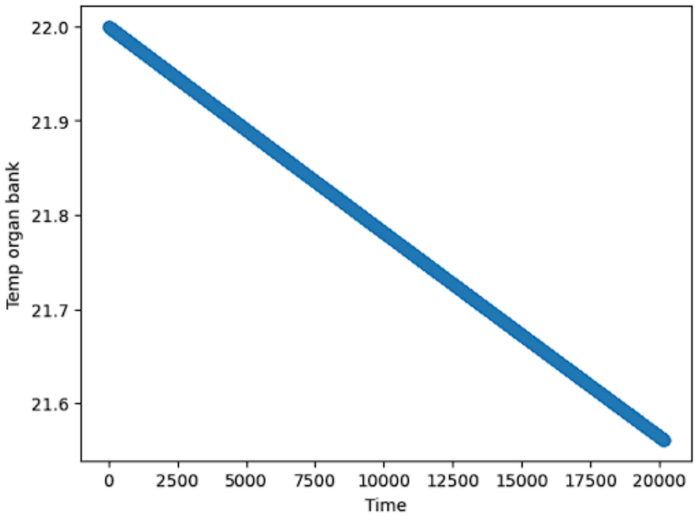

Figure 4

Conclusions

- The temperature change for both graphs resembles an exponential function. Additionally, the growth and decline in the organ bank are much faster than in the medication room. Furthermore, in the best-case scenario, the time it takes for the organ bank is less than 10 hours, unlike the medication room, which takes more than a day.

- Even though the model was created for temperature decrease, the time it takes to reach the ambient temperature of 22°C remains the same as if it had been modeled for temperature increase.

- The fact that the equilibrium point is 0 makes sense, as the maximum point to which the temperature can decrease is absolute zero "theoretically.

- While in the best-case scenario, the graph is exponential for both. It can be observed that for the worst-case scenario, the graph is linear for both as well.