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## TOPICAL REVIEW

# The plasma–sheath boundary region

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**Abstract**
In this review an attempt is made to give a broad coverage of the problem of joining plasma and sheath over a wide range of physical conditions. We go back to the earliest works quoting them, where appropriate, to understand what those who introduced the various terms associated with the structure of the plasma–sheath had in mind. We try to bring out the essence of the insights that have been gained subsequently, by quoting from the literature selectively, indicating how misunderstandings have arisen. In order to make it accessible to the generality of those currently working in low temperature plasmas we have sought to avoid mathematical complexity but retain physical insight, quoting from published work where appropriate. Nevertheless, in clarifying my own ideas I have found it necessary to do additional original work in order to give a consistent picture. In this way I have sought to bring together work in the late 1920s, the 1960s, and now mindful of the commercial importance of plasma processing, work over the past 15 years that adds to the general understanding.

## 1. Motivation

This review has been stimulated by controversy following the work of Godyak and Sternberg (1990a, b, 2002) and by a recent publication by Slemrod (2002) showing that the results obtained by Riemann (1997) in his treatment of a collisional generationless plasma–collisionless sheath represent a solution to that problem that in a mathematical sense has well-defined properties. It follows by analogy that the solutions given by Su and Lam (1963), Lam (1965), and Franklin and Ockendon (1970) for similar related problems have the same properties.

Thus, it is opportune to survey the large body of work that there has been since Langmuir introduced the terms 'sheath' (1923) and 'plasma' (1928).

In what follows it is necessary to define some terms—by 'generationless' we mean that the creation rate of charged particles is negligibly small, by 'collisionless' we mean that the motion of the ions is essentially inertial. These or their opposites combined with 'plasma' and 'sheath' constitute the territory we seek to describe.

For simplicity we will, in general, deal here with the one-dimensional situation, but we note that for a probe immersed in a plasma it is perfectly possible to envisage a situation that is both collisionless and generationless.

## 2. Historical introduction

We begin by quoting relevant passages from Langmuir, giving the original two 'definitions'.

(1) 'Let us now assume that the plane electrode be charged to a negative potential of 100 volts. Electrons will therefore be prevented from approaching close to the electrode, whereas positive ions will be drawn towards it. There will therefore be a layer of gas near the electrode where there are positive ions but no electrons, and in this region there will therefore be a positive ion space charge. The outer edge of this sheath of ions will have a potential of $-1$ and the positive ions pass through this outer edge with a velocity corresponding to 2 volts', Langmuir (1923).

We note that 100 V is large compared to $kT_e/e$ for a typical gas discharge, and the implication that the sheath is a region of effectively zero electron density, i.e. it contains only positive ions.

(2) 'Except near the electrodes where there are sheaths containing very few electrons, the ionized gas contains ions and electrons in about equal numbers so that the resultant space charge is very small. We shall use the name plasma to describe this region containing balanced charges of ions and electrons' Langmuir (1928).

Thus, there is an apparent significant difference in electron density in 'plasma' and 'sheath'. However where they meet electron and ion densities must be comparable.

The next significant development is usually described as the criterion attributed to Bohm (1949), namely that the ions leaving the plasma and entering the sheath are required to have a speed $v \geqslant c_s$ where $c_s^2 = kT_e/M$, $T_e$ is the electron temperature and $M$ the ion mass. But the basic concept is there in the passage quoted above, and Langmuir (1932) was even more explicit. The value of the 'wall potential' used by Bohm, no doubt influenced by earlier work, was also $\sim$100 V.

In their groundbreaking paper Tonks and Langmuir (1929) introduced two new concepts the 'sheath edge' and the 'plasma balance equation'. The first represented the 'point' where plasma ended and sheath began, and they identified it with a point inside the singularity for the plasma solution where for the full plasma–sheath solution the difference in electron and ion densities reached 'a certain fractional part'. The second represents the fact that in the steady state the volume generation rate of charged particles must equal the wall loss rate in the absence of volume recombination. For their 'sheath edge' it is clear from their figure 6 that they had in mind $\sim$1% as being a certain fractional part.

The plasma equations obtained by setting electron and ion densities equal have infinite spatial derivatives in all the potential, field and charged particle densities at a certain point, near where Tonks and Langmuir had put their 'sheath edge'. Whereas the collisionless generationless sheath equations have the property that at large distances from the wall, all physical quantities have zero spatial derivatives when the criterion is satisfied.

The scale lengths of plasma and sheath are fundamentally different, and this was made explicit by Caruso and Cavaliere (1962), but they did not take the further step of considering how to join plasma and sheath for finite plasma size $L$ and finite Debye length $\lambda_D$.

The mathematical tool that allowed a suitable joining of plasma and sheath, was the method of matched asymptotic expansions, which had been developed primarily to deal with the structure of boundary layers in fluid mechanics and in aerodynamics (Van Dyke 1964). The first application of it to plasma boundaries was by Su and Lam (1963), and Cohen (1963). This was followed rapidly by Lam (1965, 1967), and Su (1967)—all concerned with electrostatic probes in plasmas of varying degrees of collisionality. Blank (1968) dealt with the positive column in a collisional active plasma, and Franklin and Ockendon (1970) gave the corresponding treatment for a collisionless active plasma, in both the fluid approximation and the free-fall approximation.

What emerged was that there was a fundamental difference in the structure depending on whether the space–charge sheath was collisionless or collisional. In the collision-dominated case, the plasma and collisional sheath joined smoothly without the need for a transition layer. But in all cases when the sheath was collisionless, there was the structure of plasma–transition layer–collisionless sheath, with the scale of the transition layer varying with the nature of the model used to describe the plasma.

This has been confirmed by subsequent work summarized by Riemann (1991), and more recently in Riemann (1997), Slemrod and Sternberg (2001), Riemann (2000), Benilov and Franklin (2002), and Riemann (2003). There is now effectively a full coverage of parameter space in terms of the ordering of the quantities, distance from the plasma centre to the wall, $L$, ion mean free path, $\lambda_i$, and Debye length, $\lambda_D$, using the method. Of course $\lambda_D \ll L$, for one to be able to talk of a plasma, but $\lambda_i$ ranges from the largest to the smallest length, and so in terms of the method, which depends on expansions in terms of a small parameter, that varies from treatment to treatment quoted above, as they range from $\lambda_i \gg L$ to $\lambda_i \gg \lambda_D$.

In recent years, considerable effort has gone into generalizing the Bohm criterion to take into account the velocity distributions of both electrons and ions, the effect of several species of both positive and negative particles, and the inclusion of ion temperatures. These we shall review below.

The increase in computing power has led to other treatments solving the full plasma–sheath equations again over a wide range of parameters (Franklin and Snell 2000, 2001). Also simulations have been carried out, though generally over a lesser range (Gozadinos *et al* 2001).

Thus, a comprehensive description of the structure of the plasma–sheath can now be given, but it must be recognized that for radio-frequency (rf) generated plasmas a fully self-consistent treatment including generation is not yet available. However, we will attempt to relate the rf situation to the dc, indicating what is generally agreed, and what remains to be done.

## 3. Plasmas—active and quiescent

Historically, the first plasmas studied were gas discharges where the positive column provided examples of steady state plasmas over a wide range of the similarity variable pressure $\times$ plasma size—from 0.1 to $10^3$ Pa m in a number of electropositive gases. In general, ionization occurred by electron impact in the volume, the volume generation being balanced by losses to the wall. Such plasmas have number density distributions that are a function of position with a central maximum and decreasing monotonically towards the wall. Such a plasma is described as active.

More recently, efforts have been made to create uniform plasmas by separating the generation region, and thus the plasma being studied is 'generationless'. Examples of devices producing such plasmas are Q-machines, thermally produced plasmas, and multipolar machines or 'bucket sources'. Such plasmas are described as quiescent. Typically, these devices operate at 0.1 Pa m or less, and the presence of magnetic fields in some cases is over-riding in terms of the description of the plasma and its boundary regions.

## 4. Why is the Bohm criterion so universal?

To answer this question we will concentrate on establishing the fact that in a simplistic fluid model the plasma equations are singular where the criterion is satisfied, regardless of the collision model for the ions and regardless of the generation mechanism for the charged particles.

We take the equations:

$$\frac{\mathrm{d}(n_i v_i)}{\mathrm{d}x} = G(n_e, x) \qquad \text{(generation)} \qquad (1)$$

$$\frac{\mathrm{d}(n_\mathrm{i}v_\mathrm{i}^2)}{\mathrm{d}x} + C(n_\mathrm{i}, v_\mathrm{i}, x) = \frac{n_\mathrm{i}eE}{M} \qquad \text{(ion momentum)} \quad (2)$$

$$n_\mathrm{e} = n_{\mathrm{e}0} \exp\left(\frac{eV}{kT_\mathrm{e}}\right) \qquad \text{(Boltzmann relation)} \quad (3)$$

and using the plasma aproximation $n_\mathrm{e} = n_\mathrm{i} = n$, find that with $E = -\mathrm{d}V/\mathrm{d}x$, and $n_{\mathrm{e}0}$ the electron density where $V = 0$,

$$\frac{\mathrm{d}n}{\mathrm{d}x} = \frac{C + 2v_\mathrm{i}G}{v_\mathrm{i}^2 - (kT_\mathrm{e}/M)}$$

and

$$\frac{\mathrm{d}v_\mathrm{i}}{\mathrm{d}x} = \frac{Cv_\mathrm{i} + G(v_\mathrm{i}^2 + kT_\mathrm{e}/M)}{n(kT_\mathrm{e}/M - v_\mathrm{i}^2)}$$

Thus, both equations are singular, regardless of the functional form of $G$ and $C$ the functions describing generation and collisions, provided that the numerators are everywhere positive, and that is certainly so if both $C$ and $G \geqslant 0$, and one of them is $>0$. Equations of this form with $C = 0$ and for all geometries were given in Rosa and Allen (1970), and for generation other than electron impact in Johnson *et al* (1978).

It follows that there cannot be a collisionally modified Bohm criterion as has been suggested (Godyak and Sternberg 1990a, b).

In this simplified fluid model due to Woods (1965) we have neglected the stress tensor.

We add here for completeness, an example of a situation where the Bohm criterion is not relevant. That is, in plasma diodes where electrons and ions are generated in equal numbers at the hot boundaries, the gas being an alkali metal vapour (Fang *et al* 1969, Phelps and Allen 1976, Braithwaite and Allen 1981).

We also note that in high current discharges it may be necessary to take into account the fact that the neutral density is not constant but recycling of ions from the walls leads to density depletion in the centre (Allen and Thonemann 1954, Allen and Coville 1961, Franklin 1963). The Bohm criterion still applies as noted above even though $G$ is a function of $x$.

## 5. Collisionless sheath boundary conditions

The usual conditions that are applied at large distances from the wall in a collisionless sheath are that the electric field and the net space charge $\rho$ become vanishingly small. The potential thus becomes constant at the 'plasma potential' and the electron and ion densities become equal.

In some circumstances, it turns out to be more convenient to examine $\mathrm{d}\rho/\mathrm{d}V$ and treat it as the vanishingly small quantity.

Examples where this has proved particularly useful, relate to sheaths adjacent to longitudinally uniform plasmas. The case of a double sheath between two plasmas was treated by Andrews and Allen (1971). The structure of the double sheath associated with a hot cathode was described by Prewett and Allen (1976), who obtained the expression for the equivalent of the Bohm criterion for this case. This agreed with that given by Pak and Emeleus (1971) for ion waves in a beam-generated plasma, in the limit $\omega \to 0$.

The expansion of a plasma into a vacuum proved to be another case where this boundary condition was relevant (Wickens and Allen 1979).

These examples show how in some situations it is both convenient and insightful to use the potential as the independent variable.

## 6. The structure of collisionless plasma and collisionless sheath

In an earlier section we demonstrated how universal the Bohm criterion is.

The essential problem in joining plasma and sheath under collisionless conditions is how to pass beyond the singularity in the plasma solution where $v_\mathrm{i}^2 = kT_\mathrm{e}/M$. This is coupled with the fact that the generationless, collisionless sheath solution is such that all spatial derivatives go to zero at distances that are many Debye lengths from the wall. The two solutions are disjoint and it is necessary to introduce another region to achieve smooth joining and it is this transition layer that has been the subject of Lam (1965), Franklin and Ockendon (1970), Riemann (1977, 1991, 1997), and Slemrod and Sternberg (2001).

The various treatments have subtle differences depending on the plasma model, but they do have one feature in common, namely that there is a transition layer and it has a scale length $\lambda_{\mathrm{D}0}^p L^{1-p}$ where $1 > p > 0$, with $\lambda_{\mathrm{D}0}$ the central Debye length, and thus intermediate in size between plasma and sheath.

For the particular case of the collisionless fluid ion model we show the structure of the ion and electron spatial density distributions near the wall obtained by integrating the basic equations (1)–(3) above with $G = Zn_\mathrm{e}$, where $Z$ is the ionization rate, $C = 0$, and also taking into account Poisson's equation $\mathrm{d}^2V/\mathrm{d}x^2 = e(n_\mathrm{e} - n_\mathrm{i})/\varepsilon_0$.

The central boundary conditions are $n_\mathrm{e} = n_{\mathrm{e}0}$, $v_\mathrm{i} = 0$, $V = 0$, $E = 0$, and these when fed into equations (1)–(3) plus Poisson's equation lead to the requirement in plane geometry that $n_{\mathrm{i}0}/n_{\mathrm{e}0} = 1 + (n_{\mathrm{e}0}/n_{\mathrm{i}0})^2(\lambda_{\mathrm{D}0}Z/c_\mathrm{s})^2$, while at the wall we require that the ion directed flux and the electron random flux should be equal. A convenient parameter is $\alpha = \lambda_{\mathrm{D}0}Z/c_\mathrm{s}$ and we introduce $X = xZ/c_\mathrm{s}$, so that the eigenvalue of the problem is the value of $X$ at the wall $X_\mathrm{w} \equiv LZ/c_\mathrm{s}$.

The method of matched asymptotic approximations gives this eigenvalue in plane geometry as—$X_\mathrm{w} = (\pi/2 - 1)\times [1 + K_1(\alpha/X_\mathrm{w})^{4/5} + K_2(\alpha/X_\mathrm{w}) + \cdots]$ where the expansion parameter is $\alpha/X_\mathrm{w} \equiv \lambda_{\mathrm{D}0}/L$. $K_1$ is a universal constant $\sim$4.4 . . ., and $K_2$ depends on the ion mass because of the wall boundary condition. However, this method does not give a solution that is 'uniformly valid' throughout all three regions, and that was recognized in Franklin and Ockendon (1970).

Nevertheless, we can use the results from it to obtain estimates of the electric field at different locations in the plasma since the potential is normalized to $kT_\mathrm{e}/e$ throughout and thus the fields in the different regions scale to this quantity divided by their extent. Furthermore, we can estimate the values of the field at the 'point' where the plasma and transition layer merge by estimating the field in mid transition layer to be $\sim kT_\mathrm{e}/e\lambda_{\mathrm{Ds}}^{4/5}L^{1/5}$ while that at the boundary between transition layer and sheath it is $\sim kT_\mathrm{e}/e\lambda_{\mathrm{Ds}}$. This points to the field at the 'plasma edge' being $\sim kT_\mathrm{e}/e\lambda_{\mathrm{Ds}}^{3/5}L^{2/5}$ the value advanced by Riemann (1997).

Displayed in figure 1 are the computed solutions of the full plasma–sheath equations, without any approximation, for

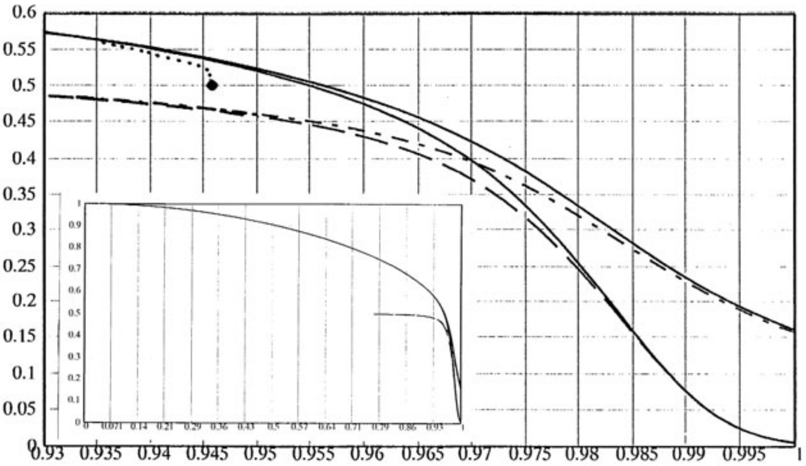


**Figure 1.** The ion and electron density distributions for a collisionless plasma–collisionless sheath for $\alpha \equiv \lambda_{D0} Z/c_s = 0.002$ showing (a) the plasma solution ($\cdots\cdots$), (b) the full computed solutions (——), and (c) the sheath solutions (- - - -) obtained by integrating inwards from the wall. The asymptotic behaviour is clearly seen and the problem of identifying a simple single plasma–sheath boundary manifested. In order to make the structure visible we show only 7% of the total dimension, i.e. $0.93 < x/L < 1$, but we give the full solutions for $0 < x/L < 1$ as an inset.

parameters $\alpha = 0.002$ and $m/M = 1.36 \times 10^{-5}$ corresponding to argon. Also in the same figure is the plasma solution corresponding to $\alpha = 0$, and the sheath solutions obtained by taking the wall values from the integration of the full equations, and then, with appropriate rescaling, integrating the sheath equations inwards from the wall. This shows that the full solution breaks away from the plasma solution near to, but shortly before, the singular point. That was the Tonks and Langmuir 'sheath edge' and which, with present insights, is more appropriately named the 'plasma edge'. After the transition layer the full solution becomes asymptotic to the sheath solution and one might then talk of a 'sheath edge' in this vicinity. This is where the sheath solution and the transition region solution, echoing the words of Tonks and Langmuir, are such that the electron and ion densities of the two solutions are once again within 'a certain fractional part'. It is important to note that it is within the transition layer that the Bohm criterion is satisfied, and that this 'point' cannot be located with precision. This fact is clearly demonstrated in figure 4.11 of Franklin (1976) where a relatively large value of $\alpha$ (0.0144) was chosen to make it visible on a diagram.

Furthermore, for this floating wall situation, the electron density at the wall is 1.46% of its central value in hydrogen, 0.23% in argon and 0.095% in mercury; i.e. the electron density while small, is not vanishingly small.

For such a small value of $\alpha$ as given above (0.002), the analytical intermediate solution is effectively indistinguishable from the full computed solution, and this is borne out by the relative values of the eigenvalue obtained analytically and computationally, viz 0.6030, cf 0.6033 for argon and $\alpha = 0.002$.

It has been suggested that neglect of volume ionization gives rise to the problem of simply joining the plasma and sheath solution, usually described as patching, at some suitably selected point. But in the intermediate region, while the relative magnitude of the terms concerned is reflected in the rescaling in the analytic method, none is neglected, and it is for this reason that the asymptotic behaviour shown in figure 1 is as it is.

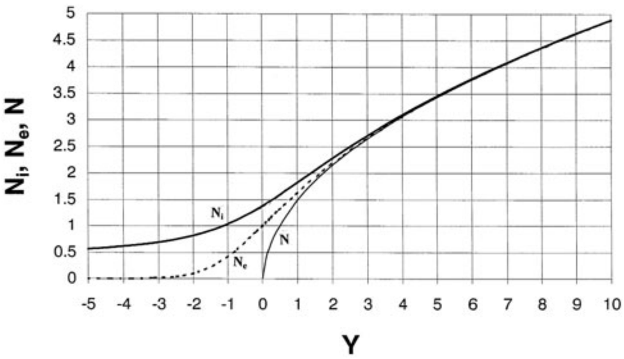


**Figure 2.** The plasma density $N$, the electron density $N_e$, and the ion density $N_i$ near the plasma edge for a collisional plasma–collisional sheath for the constant ion mean free path case. The spatial coordinate $Y \equiv (x_p - x)/\lambda_{D0}^{4/5} L^{1/5}$. Thus, the plasma solution is zero at $x = x_p \cdot N \equiv n/n_0\ (L/\lambda_{D0})^{4/5}$. The extent of the sheath is determined by the wall boundary conditions, scales as $\lambda_{D0}^{4/5} L^{1/5}$, and the sheath solutions join the plasma smoothly after a few scale lengths.

We discuss in a later section the magnitude of the errors introduced when one introduces the expediency of simply patching.

## 7. The structure of collisional plasma–collisional sheath

In order to emphasize the difference between the collisionless case and the collision-dominated we show in figure 2 the structure of the region near the wall for the collisional situation with constant ion mean free path (Benilov and Franklin 2002).

It is clear that the 'plasma solution' which varies parabolically near $N = 0$, and the 'sheath solution', join smoothly. $N$ the normalized density here is $(n/n_0)(L/\lambda_D)^{4/5}$. There is no need for an intermediate layer.

The scaling of the collisional sheath is similar to that of the collisionless transition layer in terms of the parameters involved, but that has to do with the fact that the density varies parabolically near the 'plasma edge' in both cases, rather than any deeper physical reason.

If the ion motion model is of constant collision frequency then the density varies linearly and the sheath dimension is of order $\lambda_{D0}^{2/3} L^{1/3}$ (Blank 1968).

## 8. When is a sheath thick?

In the situation of the positive column at low pressure with a floating wall the structure of the plasma–sheath is in the notation used by Lam (1967)—plasma, transition layer, thin electron sheath.

If the 'wall' is an electrode whose potential is determined by other external variables—this could equally well be a negatively biased probe—then the structure has additionally what Lam called a thick ion sheath. For such a situation, the Child–Langmuir description is a good approximation of that region, and this yields an expression for the extent of the thick ion sheath—$2^{5/4}/3(eV_s/kT_e)^{3/4}\lambda_{Ds}$ or $\sim 25\lambda_{Ds}$ for $V_s$ the bias voltage $100kT_e/e$. For such a situation, the thick ion sheath may well be more extensive than the transition layer.

This specific case is the subject of the asymptotic treatment of Benilov (2000) which contains additional insights, effectively identifying the scale of the thin electron sheath.

When the sheath is thick the ions strike the 'wall' with energies such that secondary electron emission may occur and this adds a further degree of complication. Such a situation was analysed by Prewett and Allen (1976), though the specific case they covered was an electron emitting cathode adjacent to a plasma. This has been followed by further work by Benilov and Coulombe (2001) in relation to high pressure lamps.

## 9. Patching plasma and sheath

There have been many attempts over the years to modify the plasma and sheath solutions to enable them to be simply joined. Basically what is envisaged is that the plasma solution is taken to be valid up to some specific point and the generationless sheath solution is joined to it and then integrated to the wall. An interesting example is to be found in Allis and Buchsbaum (1967)—historically they were post-Bohm, but without explicit recognition of the transition region, they were unable to achieve their object of a smooth joining of plasma and sheath.

It appeared that the problem had been satisfactorily resolved with the work of Franklin and Ockendon (1970), and Riemann (1977, 1991). However, Godyak and Sternberg (1990a, b) intervened asserting that the transition was not necessary, and that the mathematical ideas behind it introduced unphysical complexity.

This has caused a considerable degree of confusion especially since that paper also advanced the concept of a collisionally modified Bohm criterion.

As we have indicated above the plasma solution is singular where the Bohm criterion is satisfied regardless of the collisionality.

What happens as the plasma becomes collisional is that the density at the 'plasma edge' falls, the corresponding Debye length increases, and at some point ions undergo at least one collision in traversing the 'transition region' and they consequently do not pass through the Bohm speed. Then the structure of 'plasma–transition layer–collisionless sheath' gives way to that of 'collisional plasma–collisional sheath' as described for the probe situation by Su (1967) and Blank (1968) for the active positive column.

**Table 1.** Normalized wall values of the electron density, ion density, ion flux, ion speed, potential, field, and physical size, obtained by computing the full plasma–sheath equations and by patching. The gas is hydrogen and the parameter $\alpha = Z\lambda_{D0}/c_s = 0.002$. The difference in percentage terms is indicated as the error. The effect of ignoring the transition layer is evident. The wall values for the patched case correspond to those given in figure 3. $\Lambda_w$ is the eigenvalue $ZL_w/c_s$.

| | $N_{ew}$ | $N_{iw}$ | $\text{Flux}_w$ | $U_w$ | $\Phi_w$ | $E_w$ | $\Lambda_w$ |
|---|---|---|---|---|---|---|---|
| Computed | 0.0298 | 0.1989 | 0.5102 | 2.566 | 3.513 | 411.5 | 0.5999 |
| Patched | 0.0292 | 0.1935 | 0.5000 | 2.584 | 3.536 | 534.5 | 0.5776 |
| Error % | 2.1 | 2.8 | 2.0 | 0.7 | 0.6 | 29.9 | 3.9 |

This picture has been confirmed by recent extensive computations carried out by Franklin and Snell (2000, 2001).

In order to be specific we give in table 1 the error in using patching, for the case of a floating wall and $\alpha = 0.002$, the gas chosen was hydrogen, and the 'patch point' at the point where $E = kT_e/e\lambda_{Ds}$. It is seen that the error is tolerable in some quantities, but not in others. Again, it must be emphasized that these are wall values.

Since the wall potential is determined by flux equality, it shows the least error, the ion speed at the wall $U_w$ is likewise little in error. However, the fact that the transition layer has been 'excised' affects the densities and the ion flux because of ionization occurring in the transition layer and the total dimension is significantly in error. But the field at the wall is the quantity that is most seriously in error because the potential difference has been forced to occur over a significantly shorter distance.

We give as figures 3(*a*)–(*c*) the spatial variations in the volume of the charged particle densities, ion speed and potential, and finally, the field. The gradient of the field, or net space charge, as seen in figure 3(*c*), is zero on the sheath side of the patch point but $\sim kT_e/e\lambda_{D0}^2$ on the plasma side. It is far from continuous, and so the idea that patching can give smooth solutions is not well-founded.

## 10. When is the ion motion inertial and collisional?

The simplified ion fluid equation of motion that we have been using is

$$\frac{v_i\,\mathrm{d}v_i}{\mathrm{d}x} + \left(\frac{Zn_e}{n_i}\right)v_i + \nu_i v_i = \frac{eE}{M} \qquad (4)$$

and the first term on the left-hand side represents 'inertia' and the second and third 'friction', the fluid speed $v_i$ is continously 'renormalized' to ensure that momentum is conserved and the ions can be described by a single speed. We can therefore talk in terms of inertial motion or collisional, depending on which terms dominate. This section and the subsequent one will concentrate on what can be learned from this ion fluid equation of motion.

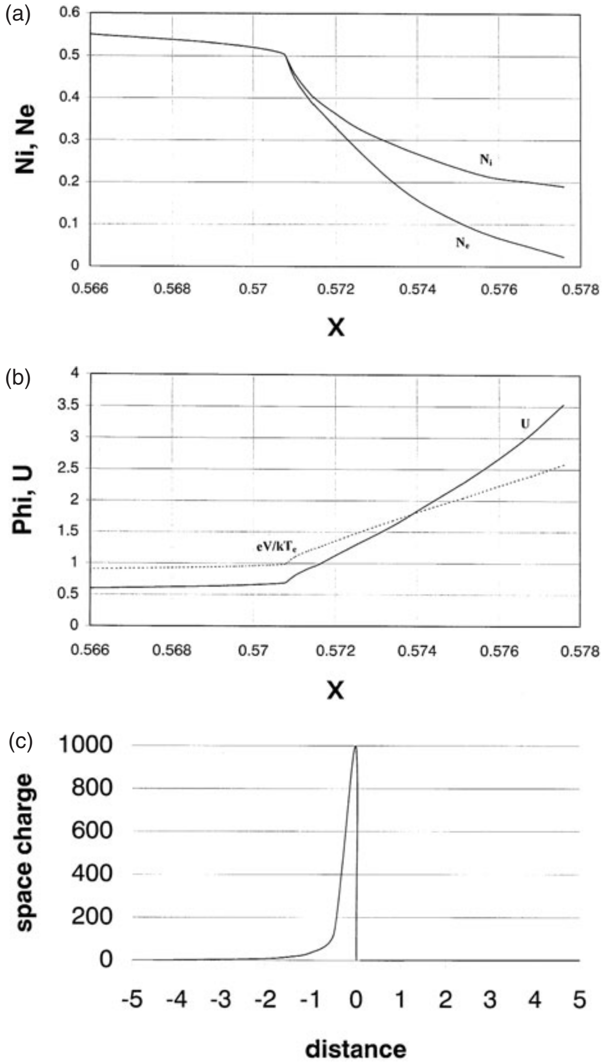


**Figure 3.** Patched 'solutions' showing (*a*) charged particle densities (*b*) ion speed and field, and (*c*) the field gradient or space charge. While (*a*) and (*b*) are continuous, (*c*) is discontinuous at the patch point.

In the plasma $n_e = n_i$, and thus we are comparing $U\,\mathrm{d}U/\mathrm{d}X$ with $(1+\delta_i)U$, where $U = v_i/c_s$, $\delta_i = \nu_i/Z$ and $X = xZ/c_s$.

From the plasma solution, i.e. (1)–(3) with $n_e = n_i$ we can derive expressions for both of these quantities and identify a value of $U = U^*$ where they are equal. This effectively marks the transition from 'collisional' motion to inertial, and we follow the ion motion thereafter asking whether, within the plasma, once the ions move out of collisional equilibrium with the field, they ever return. $U^*$ can readily be shown to be given by $[\delta_i/2(1+\delta_i)]^{1/2}$, and the 'singular point' occurs at $U = 1$.

But, in general, $X = -U/(1+\delta_i) + (2+\delta_i)/(1+\delta_i)^{3/2} \times \tan^{-1} U(1+\delta_i)^{1/2}$ so that we can readily determine the number of collisions made between disequilibrium setting in and the 'plasma edge' being reached. It is given by $\delta_i(X_w - X^*)$, and we show this function in figure 4. Clearly if the ions move out of equilibrium within the plasma, they continue in that mode since the number of collisions is always less than one, regardless of the collisionality.

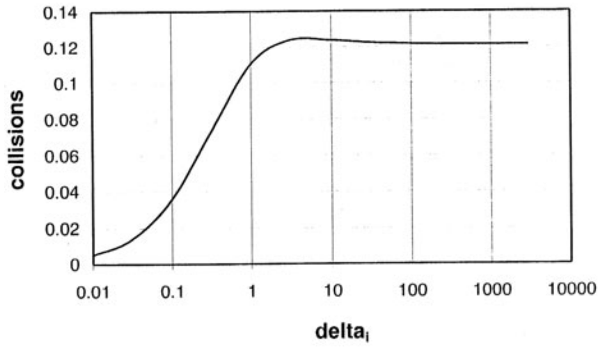


**Figure 4.** The number of collisions after disequilibrium sets in, and before the 'plasma edge' over the full range of collisionality for the plasma solution. Note that it is always less than one.

However, this raises another pertinent question as to whether the sheath can be sufficiently long and collisional that the ions once again return to collisional equilibrium with the field in a thick ion sheath. It is well-known from sheath solutions, be they collisionless or collisional (see, e.g. Cobine (1941)) that the ion speed increases monotonically as does the electric field and the potential. And it is perfectly possible for ions to be travelling in collisional equilibrium with the field in the sheath and their speed exceed the ion sound speed for the plasma from which they derived, though their motion at this stage is not linked in any way to the ion sound speed. Experimental data on ion drift shows this to be true for $T_e$ of the order of a few electronvolts (McDaniel 1964, Brown 1966).

In order to gain further enlightenment, we solved the full plasma–sheath equations for situations where the sheath length is comparable to the plasma dimension, plotting the ratio of the inertial term to the electric field, i.e. the ratio of the first term in (4) to the right-hand side, for a wide range of values of $\delta_i$. What is immediately obvious from figure 5 is that there is a significant 'break point' in the solutions near the plasma–sheath 'boundary', and we will examine that further later. But it is also true that for values of $\delta_i \geqslant 100$, while there is some influence on the ion motion near the 'boundary', the ions do not move significantly out of equilibrium as the electron density goes to zero, and soon return to that condition. However, the rapid change in field in this region means that there is an influence on the ion motion that is still discernible at quite high collisionalities.

We then sought to characterize the point of maximum disequilibrium by determining the electric field at that point. The results are shown in figure 6 where it is seen that, over a wide range of collisionality, the field is given by $eE\lambda_{D0}/kT_e \sim 1$. This is reminiscent of the value proposed by Godyak and Sternberg (1990a, b) for patching of plasma and sheath. But it is important to note that here we have the central plasma Debye length, whereas the Debye length at the plasma edge $\lambda_{Ds}$ tends to infinity as $\lambda_{D0}\delta_i^{1/4}$ as $\delta_i$ becomes large. Thus, we are not simply recovering by another route $eE\lambda_{Ds}/kT_e = 1$.

However, on the basis of this evidence it is possible to conclude that in situations where there is a thick ion sheath, it may be possible to obtain solutions of sufficient accuracy for some purposes by patching plasma and sheath. This

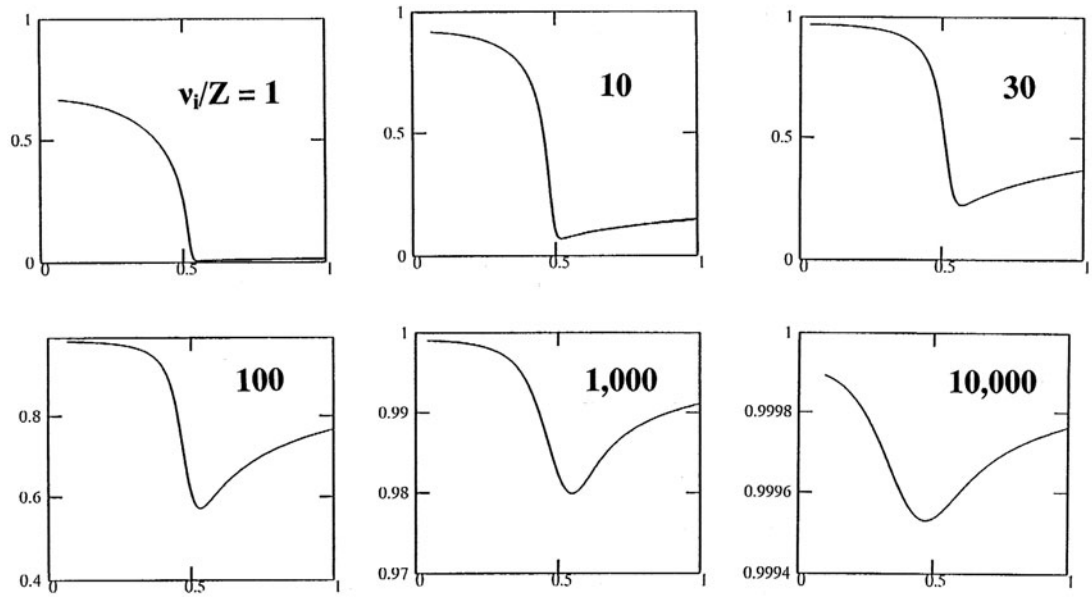


**Figure 5.** Values of $1 - R$ where $R$ is the ratio of the inertial term in the positive ion fluid momentum equation to the electric field for collisionality $\nu_i/Z = 1$, 10, 30, 100, 1000, and 10 000. $R \equiv Mv(\mathrm{d}v/\mathrm{d}x)/eE$. In the first three cases, the Bohm criterion is satisfied and the ions remain in disequilibrium even though the thick ion sheath is comparable in size with the plasma. In the second three, inertia never dominates and for 10 000 the sheath edge is scarcely detectable, i.e. the ions are everywhere in collisional equilibrium. $\alpha = 0.002$.

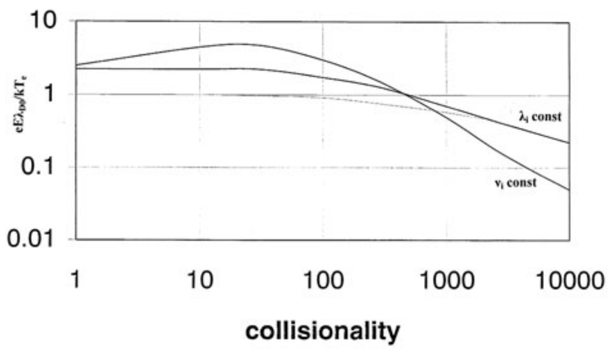


**Figure 6.** The value of the quantity $eE\lambda_{D0}/kT_e$ at the point of maximum disequilibrium over a wide range of collisionality $1 < \nu_i/Z < 1000$ showing that this quantity varies by less than a factor of 2 over the range.

clearly requires the thick ion sheath to be much larger than the combined extent of the transition layer and the 'thin electron sheath' when the plasma is collisionless, and very many Debye lengths long when collisional. Furthermore, it does not negate the existence of the detailed structure found at low collisionality (Franklin and Ockendon 1970, Riemann 1991, 1997, Slemrod and Sternberg 2001, Kaganovich 2002a). And the changes in the physical quantities in these regions should be properly taken into account, before patching.

Thus, unless one recognizes that the 'plasma edge' and the 'sheath edge' are at different spatial positions albeit only a few Debye lengths apart, there will be problems arising from the different conceptions of what constitutes 'plasma' and what 'sheath'. It is for this reason that the recognition of a transition layer as necessary to achieve a smooth variation of all physical quantities is so important, and why we have pointed up the conceptual differences between quotations (1) and (2).

## 11. Determining the return to collisional equilibrium

In this section we examine in greater detail the situation in the sheath when it is so extensive that the ions do return to collisional equilibrium.

This involves solving the ion equation of motion in the situation where there is no generation because the electron density has gone to zero and thus the relevant equations are—$v_i\,\mathrm{d}v_i/\mathrm{d}x + \nu_i v_i = eE/M$, or $v_i\,\mathrm{d}v_i/\mathrm{d}x + v_i^2/\lambda_i = eE/M$, and $\mathrm{d}E/\mathrm{d}x = n_i e/\varepsilon_0$, $n_i v_i = \text{constant} = n_s v_s$. Not surprisingly it turns out to be easier, and mathematically more elegant, to treat this problem in time in the constant collision frequency case.

We give that case first, substituting $\mathrm{d}/\mathrm{d}t$ for $v_i\,\mathrm{d}/\mathrm{d}x$. The combined differential equations reduce to the relatively simple form

$$v_i'' + \nu_i v_i' = \frac{n_{is} v_{is} e^2}{\varepsilon_0 M} = \omega_{pis}^2 v_{is} \tag{5}$$

where $'$ denotes differentiation with respect to time. This can readily be integrated to give—$v_i = v_{is} + \omega_{pis}^2 v_{is} t/\nu_i + (v_{is}\omega_{pis}^2/\nu_i^2 - c_s^2/\lambda_{D0}\nu_i)\cdot[\mathrm{e}^{-\nu_i t} - 1]$ and integrating this again with respect to time allows one to relate $v_i$ and $E$ to distance in the thick sheath. Clearly, the motion settles down to equilibrium with a time constant $1/\nu_i$, and does so from above if $v_{is} > c_s^2\nu_i/\lambda_{D0}\omega_{pis}^2$ or $v_{is} > \lambda_{Ds}^2\nu_i/\lambda_{D0}$, and otherwise from below. Indeed the long time behaviour is $v_i \sim \omega_{pis}^2 v_{is} t/\nu_i$ as expected from (5) when $v_i \ll v_i'\nu_i$. Since then the inertial term is no longer of significance in determining the ion motion.

The constant ion mean free path case has correspondingly $v_i\,\mathrm{d}v_i/\mathrm{d}x + v_i^2/\lambda_i = eE/M, n_i v_i = n_{is}v_{is}$ and $\mathrm{d}E/\mathrm{d}x = n_i e/\varepsilon_0$. This gives $v_i'' + 2v_i v_i'/\lambda_i = \omega_{pis}^2 v_{is}$ with a corresponding large $t$ solution $v_i = v_{is}\lambda_i/(\lambda_i + 2v_{is}t) + \omega_{pis}(v_{is}t\lambda_i)^{1/2} - \lambda_i/4t + \cdots$ for $t \gg \lambda_i/v_{is}$.

The constant collision frequency model allows a smooth transition as it increases to the case considered by Schottky (1924) with constant ion mobility. However, the constant ion mean free path is more accurate when charge exchange collisions are the dominant collisions that the ions undergo.

Of course there are physical limitations to these descriptions but they are not significant until the ion energies reach 10–20 keV when the effective ion collision cross-section decreases with energy, and thus there is the possibility of ion runaway (Massey and Burhop 1952).

## 12. The stress equation for collisionless plasma and for collisionless sheath

If one takes the Poisson's equation for a collisionless, generationless sheath and integrates it, one derives a stress equation that describes the physical situation.

Thus, we begin with $\mathrm{d}^2V/\mathrm{d}x^2 = e(n_\mathrm{e} - n_\mathrm{i})/\varepsilon_0$ and then substituting $n_\mathrm{e} = n_\mathrm{s}\exp(eV/kT_\mathrm{e})$ and $n_\mathrm{i}v_\mathrm{i} = n_\mathrm{s}v_\mathrm{s}$, with $v_\mathrm{s}^2 = kT_\mathrm{e}/M$ and $v_\mathrm{i}^2 = v_\mathrm{s}^2 - eV/M$, obtain an equation which reads—$2n_\mathrm{s}kT_\mathrm{e} = n_\mathrm{s}kT_\mathrm{e}\exp(eV/kT_\mathrm{e}) + n_\mathrm{s}kT_\mathrm{e}(1\text{–}2\,\mathrm{eV}/Mv_\mathrm{s}^2)^{1/2} - \varepsilon_0/2(\mathrm{d}V/\mathrm{d}x)^2$.

This is the stress balance equation for a collisionless sheath (Allen 1974) and holds throughout the sheath from the plasma to the wall.

However, it does not give credence, or validity, to the suggestion that the point where $\varepsilon_0(\mathrm{d}V/\mathrm{d}x)^2 = n_\mathrm{s}kT_\mathrm{e}$ has the significance suggested by Godyak and Sternberg. The three terms on the right-hand side are such that the first goes rapidly to zero, while the others grow at a rate that is limited by the left-hand side. Thus, on this basis, the so-called 'Godyak point' is not significant. Throughout the sheath once the electron term has gone to zero, $\varepsilon_0(\mathrm{d}V/\mathrm{d}x)^2$ and $n_\mathrm{s}kT_\mathrm{e}$ may be of similar magnitude, but where they are equal is of no significance.

Equally, one can compare the magnitudes of the terms $n_\mathrm{e}kT_\mathrm{e}$, $n_\mathrm{i}Mv_\mathrm{i}^2$, and $\varepsilon_0E^2/2$ in the plasma solution. The ratio of the first two is simply $U^2$ and that of the first and last can be shown to be $(\alpha^2/2N_\mathrm{e})(U/(1-U^2))^2$. Thus, the electric stress within the plasma is of significance only within a Deybe length or two of the boundary, i.e. where we have placed the 'plasma edge'.

## 13. Criteria for the development of a sheath

Here we give comments stimulated by the recent work of Valentini (2000) who sought to characterize space charge separation and how and why it occurred.

His standpoint then was to ask the question 'What conditions must be fulfilled for the electron density to fall more rapidly than the ion density while still essentially within the plasma?'.

In order to answer the question he posed, it is necessary once again to separate the equations involved into a hierarchy. Thus, we maintain an electron distribution given by $n_\mathrm{e} = n_{\mathrm{e}0}\exp(eV/kT_\mathrm{e})$, but describe the ions by their equations of motion and of generation, viz $v_\mathrm{i}\,\mathrm{d}v_\mathrm{i}/\mathrm{d}x + v_\mathrm{i}\nu_\mathrm{i} + v_\mathrm{i}Zn_\mathrm{e}/n_\mathrm{i} = eE/M$, $\mathrm{d}(n_\mathrm{i}v_\mathrm{i})/\mathrm{d}x = n_\mathrm{e}Z$, and then ask under what conditions is $|\mathrm{d}n_\mathrm{e}/n_\mathrm{e}\mathrm{d}x| > |\mathrm{d}n_\mathrm{i}/n_\mathrm{i}\mathrm{d}x|$, or rather, $|\mathrm{d}n_\mathrm{e}/n_\mathrm{e}\mathrm{d}x| > |\mathrm{d}n_\mathrm{i}/n_\mathrm{i}\mathrm{d}x|$. This, when we take into account the fact that $v_\mathrm{i}$ is monotonically increasing, i.e. $\mathrm{d}v_\mathrm{i}/\mathrm{d}x > 0$ gives the two conditions

$$\frac{eE}{M}\left(\nu_\mathrm{i} + \frac{Zn_\mathrm{e}}{n_\mathrm{i}}\right) > v_\mathrm{i} > \frac{eE(1 - v_\mathrm{i}^2/c_\mathrm{s}^2)}{M(\nu_\mathrm{i} + 2Zn_\mathrm{e}/n_\mathrm{i})}$$

We conclude that a sheath always develops if there are collisions, and also if $Mv_\mathrm{i}^2 > kT_\mathrm{e}$ at some point. This is, in itself, a significant insight. It demonstrates that there are two regimes namely the collision-dominated, and the collisionless.

The latter is characterized by the Bohm criterion, but the former is one where the Bohm criterion has no significance.

## 14. Generalizations of the Bohm criterion

The descriptions given above have all been limited by their assumption that the ions behave as a simplified 'fluid', i.e. have a common speed, and the electrons obey a Boltzmann relation. These restrictions have been relaxed by Harrison and Thompson (1959) who showed that for an ion distribution function $f_\mathrm{i}(v_\mathrm{i})$ the corresponding expression was $\int_0^\infty f_\mathrm{i}(v_\mathrm{i})/Mv_\mathrm{i}^2\,\mathrm{d}v_\mathrm{i} \geqslant \int_0^\infty f_\mathrm{i}(v_\mathrm{i})/kT_\mathrm{e}\,\mathrm{d}v_\mathrm{i}$.

The further generalization to allow the electrons to have a non-Maxwellian distribution was given by Riemann (1995), and can be written as

$$\frac{\int_0^\infty f_\mathrm{i}(v_\mathrm{i})/Mv_\mathrm{i}^2\,\mathrm{d}v_\mathrm{i}}{\int_0^\infty f_\mathrm{i}(v_\mathrm{i})\,\mathrm{d}v_\mathrm{i}} + \frac{\int_{-\infty}^\infty (1/mv_\mathrm{e})\partial f_\mathrm{e}(v_\mathrm{e})/\partial v_\mathrm{e}\,\mathrm{d}v_\mathrm{e}}{\int_{-\infty}^\infty f_\mathrm{e}(v_\mathrm{e})\,\mathrm{d}v_\mathrm{e}} = 0$$

These expressions place requirements on the behaviour of both $f_\mathrm{i}$ and $f_\mathrm{e}$ near $v_\mathrm{i} = 0$ and $v_\mathrm{e} = 0$.

Other generalizations can be found in situations where there are more than one species of charged particle. At low pressures when one has two negatively charged species obeying Boltzmann distributions with different temperatures $T_\mathrm{e}$ and $T_\mathrm{n}$, Braithwaite and Allen (1988), considering the plane sheath equations for an electronegative plasma, and also the probe-sheath equations, showed that the analogous expression was

$$Mv_\mathrm{i}^2 = \frac{kT_\mathrm{e}T_\mathrm{n}(n_\mathrm{e} + n_\mathrm{n})}{n_\mathrm{e}T_\mathrm{n} + n_\mathrm{n}T_\mathrm{e}} \tag{6}$$

Equally, they showed that for a bi-Maxwellian electron distribution the expression was (6), with $T_\mathrm{h}$ replacing $T_\mathrm{e}$ and $T_\mathrm{c}$ replacing $T_\mathrm{n}$, because if the negative particles obey their own Boltzmann relation they are independent of mass. Franklin and Snell (1992), considering the active plasma equations, showed that they were singular where (6) is satisfied. These expressions can be rewritten in terms of density ratios and temperature ratios, and the density ratios in particular, may well be local values rather than quantities that are constant throughout the plasma. Further subtleties of such plasmas will be considered in a later section.

The case of two positive ion species was considered by Tokar (1994), but had been examined earlier (Cooke 1981). Designating the species 1 and 2, the sheath equations allow one to obtain $(n_1 + n_2)/kT_\mathrm{e} = n_1/M_1v_1^2 + n_2/M_2v_2^2$. The same expression can be shown to correspond to a singularity in the plasma equations (Franklin 2000), but again the densities are local quantities and so the position where this composite Bohm criterion is satisfied will not in general coincide with either species satisfying its own Bohm speed. That occurs for collisionless active plasmas generated by electron impact (Franklin 2000), but not for collisional (Franklin 2001). Further expressions for multi-species cases have been given by Valentini (1988) and Benilov (1996).

Other workers have examined the ion distribution near the wall at a level of detail where it is possible to distinguish those ions that have made a collision in the sheath region and those that have fallen through the full plasma–wall potential. In this connection we mention the experimental and theoretical work of Riemann *et al* (1992), the analytical/computational kinetic treatments of Vasenkov and Shigal (2002a, b) and of Riemann (2003).

We conclude this section by noting that some workers have linked the Bohm criterion with limits in the propagation of ion waves. The simplest observation is that the long wavelength dispersion is $\omega/k = v_D \pm c_s$ in a uniform plasma and thus the zero frequency, i.e. dc limit coincides with $v_D = c_s$. More detailed consideration has been given by Stangeby and Allen (1970) and Allen (1976), where the concepts of a Mach surface and sonic flow are introduced and discussed. For rf generated plasmas, Riemann (1992) and Allen and Skorik (1993) have considered the propagation of ion waves and their limits.

## 15. Can the Bohm criterion be satisfied at more than one point in a plasma?

The fact that this is so has been known for some time, and could have been deduced in connection with electronegative plasmas by Boyd and Thompson (1959), but it was not made explicit until Braithwaite and Allen (1988). They pointed out that for certain values of density ratios and temperature ratios there was a choice to be made because the criterion in terms of local variables was apparently satisfied for three different sets of parameters. But the significance of the ratios had already been appreciated in Wickens and Allen (1979) for two electron temperature plasmas, $T_{eh}$ and $T_{ec}$ where the ratio of $T_{eh}/T_{ec} = 5 + \sqrt{24} \approx 10$ being a critical value appeared for the first time.

The same ambiguity was noted for an active electronegative plasma in Franklin and Snell (1992). Sato and Miyawaki (1992) treated the situation by introducing a double layer between two different plasmas, but their treatment though imaginative was not fully self-consistent. And it was the work of Kono (1999) on probes in electronegative plasmas that really brought to attention the fact that at low pressures, where Boltzmann relations apply for all negative species, there can exist separate 'plasma' regions each terminating in its own Bohm criterion. The colder (heavier) negative species is the more confined. In the region between the two different criteria being satisfied, there exists a space charge region which has been called a 'quasi-plasma'. It is essentially a hot negative species-positive ion plasma, but because particles enter it under conditions determined by the inner plasma, it is not in equilibrium, and this gives rise to spatial oscillations in the variables concerned. The wavelength of the oscillations is of the order of the local Debye length, but their magnitude and extent depends on the model employed. For instance, the free-fall (Tonks–Langmuir) model imposes monotonicity of potential and then they are almost tantamount to a double layer. This, as noted above, was anticipated in the model described by Sato and Miyawaki (1992). A collisionless fluid model gives them 'free rein', while it is known that collisions, diffusion and ion trapping can all contribute to damping the oscillations (Kono (2003) and references therein).

A treatment of this problem giving the insights that matched asymptotic approximations can give, albeit for a particular model of ion motion, was carried out by Benilov and Franklin (1999).

Simulations have revealed the same phenomenon, but to date there have been no experiments under the necessary conditions of low collisionality, significant temperature ratio, and a significant proportion of the colder (heavier) negative species to test theory.

## 16. Non-local plasmas

One of the most significant developments in the understanding of gas discharges in the late twentieth century has been that of treating the positive column as a situation where all physical quantities, not only the charged particle densities, vary within the spatial dimension of the plasma.

Much of this development was due to Tsendin (1974) who concentrated on the electron energy distribution and its variation with position. The revision of basic ideas in this connection has been considerably assisted by Ingold (1997) and his attention to detail.

But since our concern here is with the boundary regions we concentrate on recent results revealing the nature of the electron energy distributions near the wall. In particular we refer to the work of Uhrlandt (2002) which shows that, even with this refinement, the electron energy distribution near the wall is closely Maxwellian.

Thus, it seems probable that an equivalent Bohm criterion could if necessary, be called into play. However, methods of solution for non-local treatments do not require the criterion to be invoked in order to solve the problem since Poisson's equation is retained throughout.

But they do indicate why in so many real plasmas, experimentalists relying on the visual aspect, may have judged the sheath to be more extensive than a simple generationless collisionless sheath is.

So one question for the future is 'When is the Bohm criterion a useful concept?'

## 17. Rf plasma–sheaths

The situation in the positive column plasma–sheath with the electric field that determines the basic plasma parameters being orthogonal to the radius makes it have a degree of simplicity which does not necessarily apply when the plasma is generated by rf fields. The case where the plasma is generated between parallel plate electrodes is usually described as capacitively coupled, and that we will treat briefly.

The fact that the driving field is alternating introduces a new element in that there are time variations, which have to be taken into account. These include the driving frequency, $\omega$, the electron plasma frequency, $\omega_{pe}$, the ion plasma frequency, $\omega_{pi}$, the ion collision frequency, $\nu_i$, and the ion sheath transit time $\tau_i$. Now $\tau_i \sim \lambda_{Ds}/c_s \sim 1/\omega_{pi}$, which reduces the number of parameters. In the most usual situation $\omega_{pe} > \omega > \omega_{pi}$ though the inequalities may not be strong, but this enables one to construct a model which separates out the motion of the charged particles so that the ions are described by their time averaged behaviour while the electrons respond to the driving field. This has the effect that the sheath expands and contracts with the driving field. Furthermore, the rf fields supply the energy that causes ionization and thus one has to consider the mechanisms by which the electrons gain their energy.

One such mechanism that is operative at higher pressures is Ohmic heating deriving from the electrons colliding with the background gas, but at lower pressures the sheath dynamics of the electrons is an important heating mechanism, usually referred to as stochastic heating. Early work (Lieberman 1988) suggested that this heating might be analogous to one mechanism believed to be of significance in the acceleration of cosmic rays, however the 'hard wall' model (Godyak 1976) on which this was based has recently undergone significant revision (Gozadinos *et al* 2001a, b), although the need had been recognized for some time (Skorik *et al* 1992).

The problem of describing how the electric field penetrates the plasma has not yet been solved in a totally self-consistent manner. Early work was done by Riemann (1992) and Allen and Skorik (1993) and progress is currently being made (Kaganovich 2002b, Slemrod 2003).

It has been established for some time that the Bohm criterion is still relevant at the plasma edge, even though it is time-varying in position (Riemann 1995).

A further factor which arises is that the sheath acts as a rectifier and thus there can be at one or both electrodes depending on the driving field arrangements, a significant dc bias voltage and the sheath is many Debye lengths long, though the electrons sweep in and out of this sheath, so that it is not a simple replication of the dc sheath.

This, in turn, leads to ions striking the electrodes with energies sufficient to give rise to secondary emission of electrons, indeed one mode of such discharges known as the gamma mode (Raizer 1997) relies on secondary emission to generate the plasma.

The principal parameter describing the oscillating sheath, in what has come to be called the 'moving step model' because the electron density is taken to go to zero on a scale shorter than others under consideration, has been given a different symbol by different workers, but once one recognizes that they are related, then it is possible to understand and compare the underlying physics. The quantity $H$ used by Lieberman (1988), $\rho$ used by Godyak and Sternberg (1990a, b) and $\kappa$ used by Benilov (2003) are simply related by $\pi^2 H^2 = \pi\kappa = \rho^4$ and $\rho = (J_0/J_i)\cdot(\omega_{pi}/\omega)$ that is the ratio of the oscillating rf current to the dc ion current through the sheath times the ratio of the ion plasma frequency to the driving frequency, and so it has a relatively simple physical interpretation. This quantity, which is proportional to the ratio of the sheath oscillation amplitude to the Debye length, is usually greater than one, and the treatment of Benilov is notable because it solves the problem to higher order using the method of matched approximations and thus is able to bridge $\kappa \sim 1$. A different approach is that of Skorik reported in Allen (1995).

Slemrod (2003) takes the problem one stage further and seeks to give a description of the whole plasma–sheath. He recovers results that link directly to the fluid model for the dc case as given in Franklin and Ockendon (1970), but it has to be recognized that the generation rate is treated as a separately determined constant and independent of position.

Kaganovich (2002b) has sought to give a description of the penetration of the rf field into the plasma, but introduces an arbitrary parameter being the ratio of the ion density in the sheath to that in the bulk plasma.

Thus, the treatments available to date are not fully self-consistent and it is for this reason that much use has been made of simulations in this area, however we will not attempt to review them here, basically because of the present heavy computational cost in carrying out simulations over a sufficient range of parameters to give sufficient physical insight to link them back into more simple treatments.

## 18. Experimental work

It is intrinsically difficult to make accurate measurements of the relevant quantities in the plasma–sheath region. Introducing any material physical probe causes a disturbance on the scale of interest and thus it is necessary to try to devise means that are not so intrusive.

The first technique that we are aware of was the use of electron beams to measure the electric field, and this was used in relation to the hot cathode–plasma sheath by Crawford and Cannara (1965), and Goldan (1970) in a more conventional plasma–sheath situation. Goldan obtained good agreement with the full Tonks–Langmuir (1929) description covering the region from plasma to within 2.2 mm = $4.4\lambda_D$ of the wall. In the hot cathode case, theory followed experiment, but good agreement was found by Prewett and Allen (1976), who developed the relevant theory.

The technique of laser-induced fluorescence (LIF) allows the possibility of measuring ion velocity distributions in the plasma–sheath boundary region and from the ion mean speed to deduce the ion density. This has been applied using argon ions by Goeckner *et al* (1992) and Severn *et al* (2003). These latter have made comparison with the theoretical description of Riemann (1997) for the case of a single ion species. And for two positive ion species, comparison can be made with its extension (Franklin 2003).

It should be noted that good agreement with Riemann (1997), which described a quiescent plasma–sheath for a single ion species, had been reported earlier using a combination of more conventional probe techniques by Oksuz and Hershkowitz (2002).

A recent diagnostic of the plasma–sheath region has arisen in connection with dusty plasmas, or more correctly, in situations where there is dust in plasmas, when the motion of dust particles can be used to effectively diagnose the potential distribution, we mention in this connection the work reported in Tomme *et al* (2000). This brings into perspective the relatively small differences that exist so far as the potential variation with distance is concerned, and it is approximately parabolic in the many treatments we have described.

The LIF technique has been used with effect in rf discharges to give both space and time resolution by Czarnetzki *et al* (1999) and a more general discussion of its use in plasma diagnostics is given in Freegarde and Hancock (1997).

## 19. Conclusions

The structure of the boundary region between 'plasma' and 'sheath' is more complicated than Langmuir envisaged when he introduced the two terms. Interestingly 'plasma' became a universal term, whereas 'sheath' got translated into other languages.

The structure varies with pressure in a significant way, particularly according as the Debye length associated with the edge of the plasma $\lambda_{Ds}$ is greater than or less than the ion mean free path $\lambda_i$. There are also other subtleties associated with the nature of the plasma itself.

The Bohm criterion has proved useful in understanding structure, and provides a check for more complicated treatments than the monoenergetic ion model that he used, as it often applies in some limit. Though, in spatial terms, precisely where it is met, when $\lambda_D$ and $\lambda_i$ are finite, is not easy to locate. However, the fact that most practical plasmas are such that $\lambda_D/L$ is a small number, and that in the limit of $\lambda_D/L \to 0$ it survives as a singularity in the plasma equation for so many different models of plasmas, has given it an enduring significance, particularly once its generalized forms are appreciated.

Practising plasma physicists need to ask themselves the question 'How much of the detail of the structure is needed in my particular application?'.

This paper is intended to assist in answering that question, and to indicate where to look for help.

## Acknowledgments

I would like to thank those with whom I have interacted over the years in developing the understanding of the plasma boundary regions. Some of them are contributors to this volume, some have predeceased me, and some as research students were demanding at the time.

I have tried to make the description as comprehensive and up-to-date as possible, and am grateful for early sight of some of the work referred to.

I acknowledge several fruitful discussions with Professor J E Allen.

*Note added in proof* A recent paper by Sternberg and Godyak (2003) recovers all of the results given for plane geometry and the fluid model for a collisionless plasma by Franklin and Ockendon (1970) and therefore represents a considerable retraction on their part.

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