Petersson Inner Product of Binary Theta Series

A computational approach

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September 17th, 2016

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Mobius transformations

Let $\mathcal{H}$ be the Poincarre upper-half plane. Recall that $\mathrm{GL}_2(\mathbb{R})_+$ acts on $\mathcal{H}$ via Mobius transformations :

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} z = \frac{az+b}{cz+d}.$$

Definition

Let $N \geq 1$ and define the Hecke subgroup of level N as

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z}) | c \equiv 0 \pmod{N} \right\}.$$

Level N modular forms with characters

Definition

Let $N \geq 1$ and $k \geq 0$ be integers and let χ be a Dirichlet character mod N. A modular form of weight k, level N and character χ is a holomorphic function

$$f : \mathcal{H} \longrightarrow \mathbb{C}$$

such that

$$f(\gamma z) = \chi(d)(cz + d)^{-k} f(z)$$

for all $z \in \mathcal{H}$ and all $\gamma \in \Gamma_0(N)$, which satisfies certain growth conditions at the cusps. The $\mathbb{C}$-vector-space of such modular forms is denoted

$$M_k(\Gamma_0(N), \chi).$$

q-expansion of modular forms

Every modular form f has a Taylor (or Fourrier) expansion at infinity, called its q-expansion :

$$f(z) = \sum_{n=0}^{\infty} a_n q^n,$$

where $q = exp(2\pi iz)$.

Example

Let $k \geq 4$ be an even integer and define

$$G_k(z) = \sum_{m,n} \frac{1}{(mz+n)^k}.$$

After renormalisation, its q-expansion is

$$E_k(z) = \frac{B_k}{2k} + \sum_{n=1}^{\infty} \sigma_{k-1}(n) q^n.$$