Limited Stability. A property of a system characterized by stability when the input signal falls within a particular range and by instability when the signal falls outside this range.

Linear Varying-Parameter Network. A linear network in which one or more parameters vary with time.

Nonlinear Distortion. Distortion caused by a deviation from a desired linear relationship between specified measures of the output and input of a system.

Note—The related measures need not be output and input values of the same quantity; e.g., in a linear detector the desired relation is between the output signal voltage and the input modulation envelope.

Nonlinear Network (Circuit). A network (circuit) not specifiable by linear differential equations with time as the independent variable.

***N*th Harmonic.** The harmonic of frequency N times that of the fundamental component.

Phase Distortion. See **Phase-Frequency Distortion.**

Phase-Frequency Distortion. Distortion due to lack of direct proportionality of phase shift to frequency over the frequency range required for transmission.

Note 1—*Delay Distortion* is a special case.

Note 2—This definition includes the case of a linear phase-frequency relation with the zero frequency intercept differing from an integral multiple of π.

Waveform-Amplitude Distortion. *Nonlinear Distortion* (q.v.) in the special case where the desired relationship is direct proportionality between input and output.

Note—Also sometimes called *Amplitude Distortion.*

RC Active Filters*

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Summary—**Inductorless filters are attractive for numerous practical reasons. Passive RC filters, however, suffer from the defects of high in-band loss and poor economy of elements. These defects are overcome in active RC filters in which amplifying elements supply power to the filter in addition to that applied by the signal. A class of active filters is described in which one active component, a transistor negative-impedance converter, is employed. Simple unbalanced network configurations are obtained in which the number of capacitors in the RC circuits is equal to the total number of reactive elements in the corresponding LC filter. The ultimate limit in performance in this class of active filters is the drift in the converter. The drift in input impedance in converters employing Darlington's compound transistors is only a few tenths of a per cent of the load impedance for a wide range of loads. Such stability is more than adequate for many practical filter applications. The theory of this type of active RC filters is discussed and experimental tests are reported on low-pass, high-pass and band-pass filters.**

Introduction

THE USE of only resistive or capacitive elements in filters is attractive on the grounds that usually these elements are cheaper, simpler, and more nearly ideal elements than are inductors. Passive RC filters, though they have been applied for some purposes, suffer from two defects: they introduce loss in the pass band, and, because of restrictions on impedance functions realizable with R's and C's only, the network complexity of RC filters to meet a given filter specification is ordinarily much greater than that of an equivalent RLC filter. These defects may be overcome by using active elements in addition to R's and C's. One type of active RC filter employing a stabilized amplifier as the active element has been proposed by Dietzold.[1] Bangert of Bell Telephone laboratories has successfully built several filters of this type employing transistor amplifiers. In this paper a different kind of filter is described in which the active element is a negative-impedance converter using transistors. A negative-impedance converter[2] is an active four-pole which presents at either of its terminal pairs the negative of the impedance connected to the other terminal pair. A negative-impedance converter has voltage-current relationships for its terminal pairs exactly like those of an ideal one-to-one transformer except for a polarity reversal in the voltage transformation ratio. RC filters employing negative-impedance converters (called simply *converters* hereafter) can be constructed to provide characteristics corresponding to those of the usual types of RLC filters using only as many capacitors in the RC circuit as the sum of reactive elements in the RLC circuit. Moreover, simple network configurations are required; unbalanced forms giving a common ground connection can be obtained. The pass-band losses are reduced by the active portion; in fact, in some circuits gains are obtained.

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[1] R. L. Dietzold, U. S. Patent 2,549,065, April 17, 1951.

[2] J. G. Linvill, "Transistor negative-impedance converters," Proc. I.R.E., vol. 41, pp. 725–729; June, 1953.

Theory of Operation and Design

The transfer functions of filters employing lumped elements are rational functions of frequency. Accordingly one writes as the transfer impedance of a filter

$$Z_T(p) = \frac{N(p)}{D(p)} \cdot \tag{1}$$

The denominator polynomial has zeros at complex frequencies which are the natural frequencies of the circuit. In passive RC circuits these zeros are restricted to the negative real axis of the complex frequency plane, and this constraint seriously limits the quality of approximation to an ideal filter characteristic which one can make using polynomials of a limited degree for N and D. Active RC circuits can have natural frequencies anywhere in the left-half plane, the same restriction applying to passive RLC filters. For the active RC filter of Fig. 1, one obtains for the transfer impedance by a straightforward analysis,

$$\left.\frac{E_2}{I_1}\right|_{I_2=0} = Z_{21} = \frac{Z_{12a}Z_{12b}}{Z_{22a} - Z_{11b}} \cdot \tag{2}$$

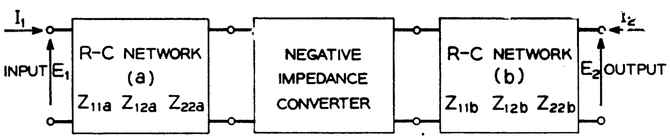

Fig. 1—RC active filter.

In (2), Z_{12a} and Z_{12b} are the transter impedances of networks a and b; Z_{22a} and Z_{11b} are their driving-point impedances from the terminal pairs connected to the negative-impedance converter.

The zeros of N of (1) are associated with the structure of the filter, not with its natural frequencies. For instance, ladder networks have zeros of transfer impedance at frequencies where shunt elements become short-circuits or frequencies where series elements become open-circuits. Thus, an RC ladder with three shunt capacitors with resistances in the series arms possesses three zeros of transfer impedance at infinity (where the capacitors are short circuits), irrespective of the element values or natural frequencies. Lattices, bridged-T networks and twin-T networks possess zeros of transmission at frequencies where a bridge-like balance occurs, independent of the location of natural frequencies of the complete network.

The principle of design of an RC active filter is simply this: for a transfer impedance (1) prescribed within a constant multiplier, the zeros of $D(p)$ are selected as the natural frequencies of the complete structure of Fig. 1. From this, by a method to be described, are obtained the driving-point impedances of structures a and b, Z_{22a} and Z_{11b}. Lastly, in the realization of the RC circuits, the structure form is selected to provide zeros of transmission at required frequencies, zeros of $N(p)$.

To illustrate the procedure it is convenient to describe the design of an active RC filter with a low-pass Butterworth characteristic and an attenuation level of 18 db/octave, cut-off occurring at 1,000 cps. For such a characteristic, the poles of the transfer impedance (zeros of $D(p)$) are known to fall on a semi-circle as shown in Fig. 2(a). The three zeros of transfer impedance all fall at infinity and $N(p)$ is a constant. The natural frequencies of the network to be designed must be at the complex frequencies noted on Fig. 2(a). If one breaks any loop of a network and determines the impedance between the terminals thus created, it is zero at the natural frequencies of the network. For Fig. 1 with attenuation on the loop at the input of the converter, natural frequencies occur where

$$Z_{22a} - Z_{11b} = 0. \tag{3}$$

The poles of the function of (3), must, by the nature of RC networks, fall singly on the negative real axis of the frequency plane. One can now pick three points on the negative real axis [σ_1, σ_2, σ_3, see Fig. 2(b)] and identify these as the poles of $Z_{22a} - Z_{11b}$. Hence, one has

$$Z_{22a} - Z_{11b} = \frac{D(p)}{(p - \sigma_1)(p - \sigma_2)(p - \sigma_3)} \cdot \tag{4}$$

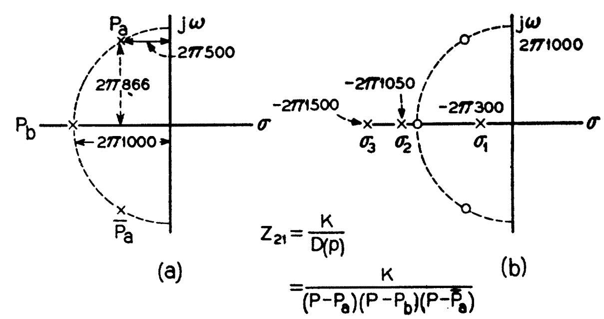

Fig. 2—Frequency-plane distributions of poles and zeros.

The selection of σ_1, σ_2, and σ_3 is arbitrary as far as the transfer impedance of the filter is concerned (of course, none of these three values should coincide with any zero of $D(p)$. As will be seen, the driving-point impedance and other characteristics are influenced by the selection. In expanding $Z_{22a} - Z_{11b}$ in partial fractions one finds that the residues are always real, but may be positive or negative. It is well known that any function with simple poles on the negative real axis and positive real residues in those poles is the driving-point impedance of an RC network. Hence in the partial fraction expansion of (4), if one groups together all terms with positive residues and groups all terms with negative residues, the first group should be associated with Z_{22a}, the second group with Z_{11b}. Finally, one makes a Cauer[3] synthesis

[3] E. A. Guillemin, "Communication Networks," vol. II, p. 213, John Wiley & Sons, New York, N. Y.; 1935.

of RC ladders with capacitor shunt elements and the filter synthesis is complete. For the critical-frequency distributions shown in Fig. 2, the network form will be that shown in Fig. 3(a). That this is the correct form of the filter can be understood by observing from the distribution of poles of $Z_{22a}-Z_{11b}$ in Fig. 2(b) that the residues in the poles at σ_1 and σ_2 are positive, while the residue in the pole at σ_3 is negative. The constant K and the partial fractions associated with the poles at σ_1 and σ_2 are accordingly identified with Z_{22a} and the partial fraction associated with the pole at σ_3 is identified with $-Z_{11b}$. The network synthesizes to realize Z_{22a} and Z_{11b} accordingly require two and one capacitors, respectively, as indicated in Fig. 3(a), which gives element values for a filter of this type. The plot of the measured transfer characteristic is shown in Fig. 3(b), along with a few calculated points of the characteristic.

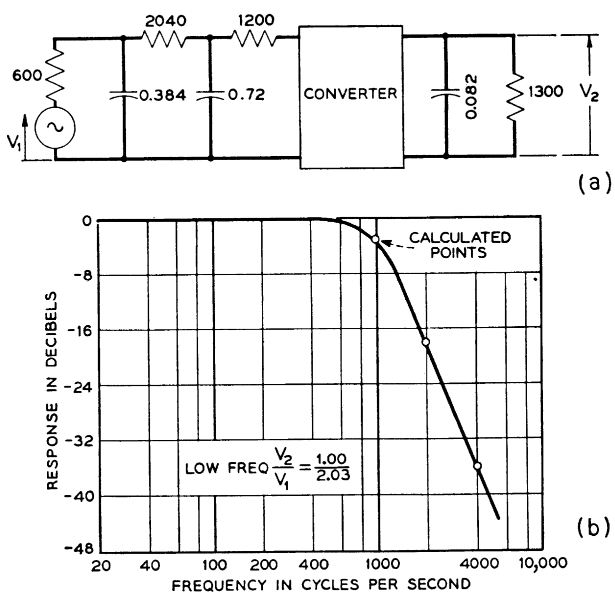

Fig. 3—Network and characteristic of active filter with Butterworth characteristic.

The synthesis of active filters employing RC ladder networks and providing high-pass or band-pass characteristics follows precisely the same pattern illustrated by the low-pass design except for the fact that the RC structures have capacitors in the series arms for the high-pass case in both series and shunt arms in the band-pass case. Fig. 4 illustrates typical pole-zero locations for high- and band-pass filters along with corresponding network configurations which are obtained.

The filters illustrated so far are all of the ladder type, and all of the zeros of transmission occur at zero or infinite frequency. With simple RC ladders, it is impossible to obtain zeros of transmission at a real frequency between zero and infinity. However, internal zeros of transmission are produced with RC lattice structures, or unbalanced equivalents to the lattice, bridged-T, or twin-T structures. The synthesis of filters to provide internal points of infinite attenuation proceeds as has been described for the ladder types

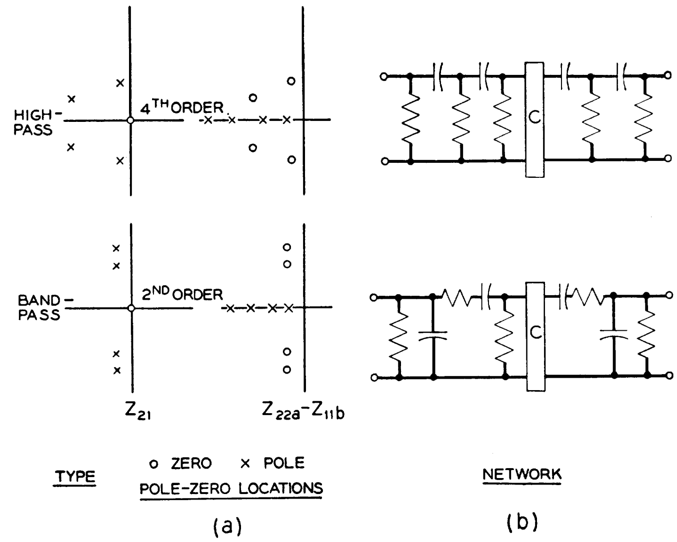

Fig. 4—Pole-zero locations and network configurations of H-P and B-P filters.

through the specification of transfer impedance, the selection of $Z_{22a}-Z_{11b}$, and the evaluation of Z_{22a} and Z_{11b}. At this point the driving-point impedances of the RC networks have been selected, and the frequencies at which these networks should introduce zeros of transmission are known. For the lattice structure (Fig. 5), the driving-point impedance is

$$Z_{11} = Z_{22} = \frac{Z_a + Z_b}{2} = \frac{n(p)}{d(p)} \cdot \tag{5}$$

Further, the transfer impedance is

$$Z_{12} = \frac{Z_b - Z_a}{2} = \frac{t(p)}{d(p)} \cdot \tag{6}$$

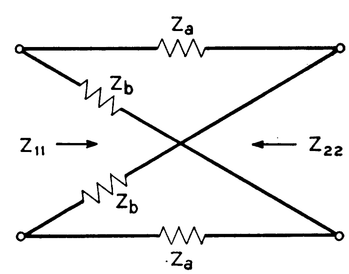

Fig. 5—The lattice structure.

Knowing the driving-point impedance for each of the RC networks, Z_{22a} or Z_{11b}, one can associate with each of networks a and b the appropriate number of factors of $N(p)$ [see (2)], thus obtaining the transfer impedance of each within a constant multiplier. For the case in which an RC network is to be a lattice, Z_a and Z_b can be found through (5) and (6) as follows: Z_{12} is specified with an undetermined constant multiplier and then Z_{11} is added to Z_{12} to obtain Z_b, Z_{12} subtracted from Z_{11} to find Z_a. Finally, the largest constant multiplier of Z_{12} is selected to permit Z_a and Z_b to still be realized as RC networks.[4,5] Instead of employing the physical networks in the lattice form it is ordinarily preferable to seek by well-known means an unbalanced equivalent network.

The technique of designing networks in the general lattice form and then subsequently finding an unbalanced equivalent is a familiar one, but it suffers from the defect that it is difficult, if not impossible, to predict the nature of the unbalanced equivalent circuit from the form of the lattice. An alternate, less general but possibly more practical technique, is to constrain the nature of the lattice to a form which has a practical equivalent (a twin-T requiring only three capacitors and three resistors, for instance) and carry through the synthesis on this basis. This technique is satisfactory for the RC filter case because inherent in the synthesis method employed is a latitude in the choice of the poles of $Z_{22a}-Z_{11b}$, and this latitude can be exchanged for a constraint in the form of the unbalanced equivalent. A later example will illustrate this in detail.

The synthesis procedure described in the foregoing to obtain an RC active filter with prescribed pole and zero locations for its transfer impedance is complete, but it does not lead to a unique network because of the latitude in the choice of poles of $Z_{22a}-Z_{11b}$. Different networks can be found which have the same poles and zeros of transfer impedance and hence the same filtering properties, but their driving-point impedances will be different and the effects of imperfections in the converter will differ from one to the other. The driving-point impedances are important for some applications and it is also important to build networks whose properties as filters are reasonably independent of changes in the characteristics of the converter.

Simple relationships between driving-point impedance and transfer impedance have been determined, but an explicit technique has not been found in which both are prescribed. Further, simple relationships express the imperfections of the filter characteristics in terms of the imperfections in the converter properties.

Driving-Point Impedance of RC Active Filters

The driving-point and transfer impedances of the active circuit shown in Fig. 1 are easily written in terms of the four-pole parameters of the RC networks. One obtains:

$$Z_{21} \text{ (Input to Output)} = \underset{(I_2=0)}{\frac{E_2}{I_1}} = \frac{Z_{12a}Z_{12b}}{Z_{22a}-Z_{11b}}, \tag{7}$$

$$Z_{12} \text{ (Output to Input)} = \underset{(I_1=0)}{\frac{E_1}{I_2}} = \frac{-Z_{12a}Z_{12b}}{Z_{22a}-Z_{11b}} = -Z_{21}, \tag{8}$$

$$Z_{11} = \underset{(I_2=0)}{\frac{E_1}{I_1}} = Z_{11a} - \frac{Z_{12a}^2}{Z_{22a}-Z_{11b}}, \quad \text{and} \tag{9}$$

$$Z_{22} = \underset{(I_1=0)}{\frac{E_2}{I_2}} = Z_{22b} - \frac{Z_{12b}^2}{Z_{11b}-Z_{22a}}. \tag{10}$$

Some interesting conclusions can be drawn on the basis of the relationships indicated for the driving-point and transfer functions. In the first place, the filter obeys reciprocity in magnitude only as the transfer impedance exhibits a change in sign as the input and output terminal pairs are reversed.

The second conclusion has to do with the relationship between driving-point and transfer impedance. One recalls that passive two-element-kind four-poles obey what is referred to as the residue condition.[6] This condition simply requires that the residue of transfer impedance in any pole of transfer impedance not exceed the geometric mean of the residues of the driving-point impedances in that pole. In a sense, it amounts to stating a limit on the size of transfer impedance for a given level of driving-point impedance. For the more general active RC networks employed here a related similar relationship can be stated. The product of residues in a pole of Z_{12} and Z_{21} is equal to the product of residues of Z_{11} and Z_{22} in that pole. This means that the level of transfer impedance is, in a sense, inflexibly related to driving-point impedance. The proof is direct. Suppose that the impedance functions have a pole of p_1; this means that $Z_{22a}-Z_{11b}$ is zero there. The residues of the functions in that pole are given by setting p equal to p_1 in the following expressions:

$$\begin{aligned}
&\text{for } Z_{11}, \text{ the residue } r_{11p_1} = \frac{-Z_{12a}^2(p-p_1)}{(Z_{22a}-Z_{11b})}; \\
&\text{for } Z_{22}, \text{ the residue } r_{22p_1} = \frac{-Z_{12b}^2(p-p_1)}{(Z_{11b}-Z_{22a})}; \\
&\text{for } Z_{21}, \text{ the residue } r_{21p_1} = \frac{Z_{12a}Z_{12b}(p-p_1)}{(Z_{22a}-Z_{11b})}; \\
&\text{for } Z_{12}, \text{ the residue } r_{12p_1} = \frac{Z_{12a}Z_{12b}(p-p_1)}{Z_{11b}-Z_{22a}}.
\end{aligned} \tag{11}$$

From the above it is obtained directly that

$$r_{11p_1}r_{22p_1} = r_{12p_1}r_{21p_1}. \tag{12}$$

Observe from (7), (8), (9) and (10) that Z_{12}, Z_{21}, Z_{11} and Z_{22} have poles at zeros of $Z_{22a}-Z_{11b}$ and that there are no other poles of Z_{21} or Z_{12}. However, Z_{11} and Z_{22} have as additional poles, respectively, the frequencies which are poles of Z_{11a} or Z_{22b} at which the residue condition for the passive RC structures is fulfilled with the "greater than" sign. Ladder networks of the forms illustrated for

[4] E. A. Guillemin, M.I.T. Radiation Laboratory Report No. 43, Cambridge, Mass.; October 11, 1944.
[5] J. L. Bower, and P. F. Ordung, "The synthesis of resistor-capacitor networks," Proc. I.R.E., vol. 38, pp. 263–269; March, 1950.
[6] Guillemin, *op. cit.*, p. 217.

the active filters fulfill the residue condition with the equals' sign in all poles of their open-circuit driving-point impedance. However, the lattice structure ordinarily fulfills the residue condition at some poles of its open-circuit driving-point impedance with the "greater than" sign.

Effect of Inaccuracies in the Converter

The foregoing analysis has assumed perfect negative-impedance converters. Physical converters are not perfect and the effect of this imperfection must be assessed. The principal imperfection is described by indicating that the input impedance is not the negative of the load impedance but is that quantity plus an increment, ΔZ (see Fig. 6). Imperfections in a converter are of two sorts. The first and most serious is a drift with time or temperature or other environmental condition; the second arises because the conversion factor is not -1, or there are parasitic impedances not shown in the circuit.

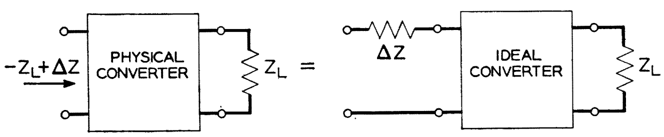

Fig. 6—Representation of parasitic impedance in a converter.

The first-mentioned imperfection puts an upper limit on the operation of the device once specifications are set regarding stability of characteristics with time. The second imperfection is something which can be compensated for within the limit of one's patience. In the following the first kind of imperfection only will be considered. The quality of a converter may be characterized for filter-synthesis problems by a statement like the following: ΔZ drifts less than a per cent for all magnitudes of Z_L between A and B ohms. Once such a characterization has been made, the stability of characteristics of filters employing the converter can be assessed. The filter characteristic is influenced in that a corresponding drift in natural frequencies of the complete circuit occurs for a drift in converter performance. The zeros of transmission, stated in (1), are not affected. A drift in the natural frequencies can be translated into a change in the characteristic. For instance, a motion of a natural frequency toward the $j\omega$ axis to the fraction f of the original displacement from the $j\omega$ axis causes a maximum change in the amplitude characteristic of 20 log $1/f$ db. Thus a motion of 10 per cent results in a maximum change of 0.9 db in the characteristic.

As pointed out earlier, natural frequencies occur where $Z_{22a}-Z_{11b}$ (Fig. 1) equals zero. If the input impedance of the converter changes from $-Z_{11b}$ by ΔZ, the natural frequencies shift until $Z_{22a}-Z_{11b}+\Delta Z$ is zero. Thus a natural frequency at p_a moves approximately

$$\Delta p_a = -\left.\frac{\Delta Z}{\dfrac{d(Z_{22a}-Z_{11b})}{dp}}\right|_{p=p_a} \tag{13}$$

On the basis of (13) one observes that poles of $Z_{22a}-Z_{11b}$ should be selected with a view to making $Z_{22a}-Z_{11b}$ possess the largest possible rate of change at the natural frequencies of the filter.

The value of $d(Z_{22a}-Z_{11b})/dp$ at a zero of $Z_{22a}-Z_{11b}$ is simply expressed in terms of the pole and zero locations. For the pole-zero distribution of Fig. 2(b),

$$\left.\frac{d(Z_{22a}-Z_{11b})}{dp}\right|_{p_a} = K\frac{(p_a-\bar{p}_a)(p_a-p_b)}{(p_a-\sigma_1)(p_a-\sigma_2)(p_a-\sigma_3)}, \tag{14}$$

where K is the value approached by $Z_{22a}-Z_{11b}$ as p approaches infinity. In general, $d(Z_{22a}-Z_{11b})/dp$ at a zero of $Z_{22a}-Z_{11b}$ is K times the product of vectors from other zeros of $Z_{22a}-Z_{11b}$ to p_a, divided by the product of vectors from the poles of $Z_{22a}-Z_{11b}$ to p_a.

For the design illustrated in Fig. 3 an evaluation based on the above relationship shows that for the natural frequency at $(-500+j866)2\pi$

$$\left.\begin{aligned} &\frac{d(Z_{22a}-Z_{11b})}{dp} = .275 \angle -51.4^\circ, \\ &\text{while at } (-2\pi)1{,}000 \\ &\frac{d(Z_{22a}-Z_{11b})}{dp} = 10.9. \end{aligned}\right\} \tag{15}$$

Using these values, one can determine the sensitivity of the filter characteristic to changes in the converter. If the converter were to drift in such a manner that its input impedance exhibits a magnitude of change of 13 ohms (1 per cent of the load resistance), the position of p_a would change by 13/0.275 or 47.4 units. If the argument of the 13-ohm change were of the correct value to move p_a directly toward the $j\omega$ axis, the transmission of the filter is multiplied by the factor 3,187/3,140 at 866 cycles, an increase of 0.08 db. At other frequencies the effect of the motion of p_a would be less than this. The pole at p_b is moved also, but by a much smaller amount. A change of 13 ohms at the input of the converter moves p_b by 13/10.6 or 1.23 units. If p_b is moved toward the $j\omega$ axis by this amount, at low frequencies the transmission is multiplied by the factor 6,281.2/6,280, which is less than a 0.02 per cent change.

Transistor Converters Used in Active Filters

The transistor negative-impedance converters employed for the filter application are generally similar to those which have been described elsewhere in the literature.[7] The circuit configuration is shown in Fig. 7. The operation of the unbalanced converter can be qualitatively explained by approximating the properties of transistors by statements that the emitter current all flows out from the collector and that the emitter potential always follows the base potential. For a qualitative explanation one can neglect the shunting effects of the emitter and collector resistors of the upper transistor

[7] Linvill, *ibid.*

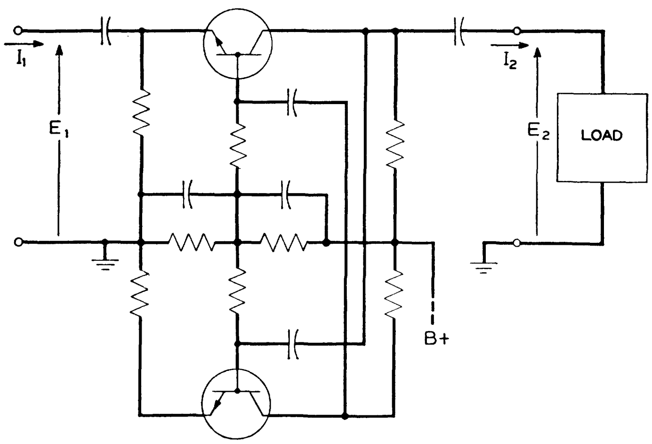

Fig. 7—A transistor converter.

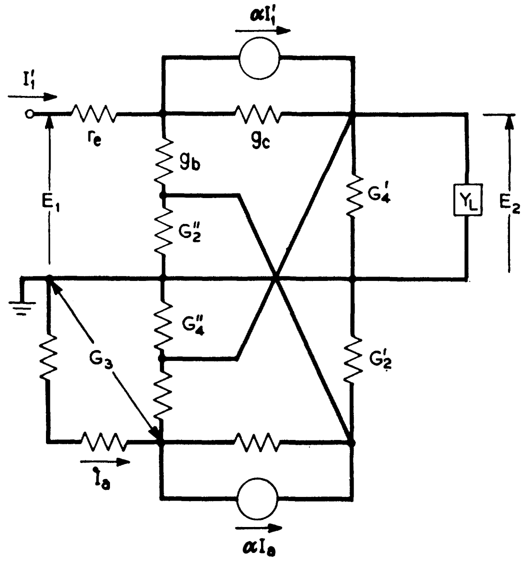

Fig. 8—Equivalent circuit of unbalanced converter.

in Fig. 7, along with the resistors connected to the bases of both transistors and the voltage drops across the coupling and by-pass condensers. A current I_1 injected into the upper emitter flows out through the output terminal to the connected load. The voltage E_2 developed across the load by virtue of the current fed into it is transferred by a coupling condenser to the base of the lower transistor and results in a current in the resistor being connected to its emitter, since the emitter potential follows that of the base. The same current flows through the resistor connected to the collector and if these two resistors are of equal size, the incremental voltage from collector to ground is $-E_2$. This voltage is fed to the base of the upper transistor and thence to the emitter of the upper transistor. Thus E_1 is simply $-E_2$. The idealization of the circuit indicated in Fig. 7 exhibits the properties that its input current equals the output current, while its input voltage is the negative of the output voltage. These properties characterize a negative-impedance converter.

The equivalent circuit, shown in Fig. 8, which better approximates the converter in the frequency regions where coupling condensers are effective, can be readily analyzed. When the collector conductance is taken to be zero, one finds

$$\frac{E_1}{I_1'} = \frac{-\alpha^2 G_3}{G_2 + G_2''} \, \frac{1}{Y_L + G_4' + G_4'' + G_3(1-\alpha)\left(1 + \dfrac{G_4' + G_4'' + Y_L}{g_b}\right)} + r_e + (1-\alpha)\frac{G_2' + G_2'' + g_b}{(G_2' + G_2'')g_b}, \tag{16}$$

and

$$\frac{E_2}{I_1'} = \frac{\alpha}{Y_L + g_b + G_4' + G_4'' - \dfrac{g_b^2}{g_b + G_3(1-\alpha)}}. \tag{17}$$

For perfect conversion E_1/I_1' should be simply $-(1/Y_L)$. The multiplying factor, $-\alpha^2 G_3/(G_2+G_2'')$, should be -1; this value can be obtained by the proper choice of G_3. If the multiplying factor is -1, one still has as imperfections two parasitic impedances, one in parallel with Y_L and a second in series with the converter at its input, as illustrated in Fig. 9. Merrill[8] has pointed out

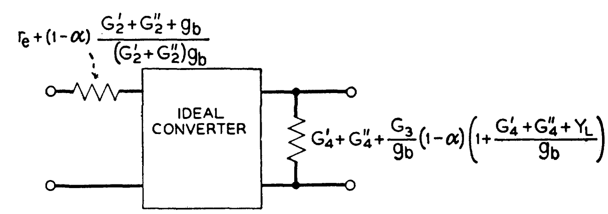

Fig. 9—Representation of converter of Fig. 8.

that one can cancel the effect of parasitic elements on the performance of a converter as illustrated in Fig. 10. With the parasitics shown in Fig. 9, a perfect compensation is impossible, one of the inherent parasitics being in parallel and dependent upon Y_L, the other being in series. However, a satisfactory compensation can be effected with the arrangement shown in Fig. 11. By proper choice of the settings of the variable resistors one obtains a converter which converts at 1,000 cps any resistance from 100 to 10,000 ohms to the negative of itself within a few per cent. However, because of the phase shift in alpha with frequency, it is not possible to obtain

[8] J. L. Merrill, "Theory of the negative impedance converter," *Bell Sys. Tech. Jour.*, vol. 30, pp. 88–109; Jan., 1951.

accurate -1 to 1 conversion at frequencies higher than a few thousand cycles. The imperfections in the converter, if they are static ones, can be compensated for in the RC networks associated with the accessible terminals of the converter. The only problem in this connection becomes some loss of simplicity in the design and alignment of the filter. The use of Darlington's connection of compound transistors, as will be described presently, effects a large improvement in the conversion properties.

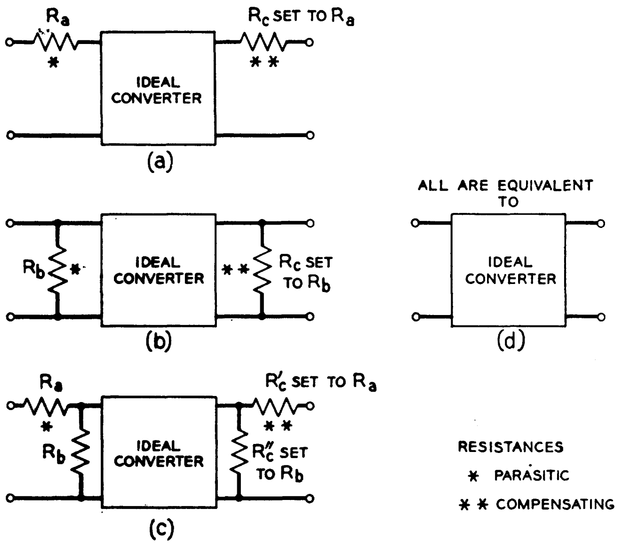

Fig. 10—Merrill's technique for compensating a converter for parasitic impedances.

A more significant characteristic than the value of the conversion ratio is the constancy of the ratio with time, temperature and other environmental conditions. The converter shown in Fig. 11 has been tested with the circuit shown in Fig. 12, which is a balancing arrangement whereby one can accurately measure the input impedance of the terminated converter. For a conversion of

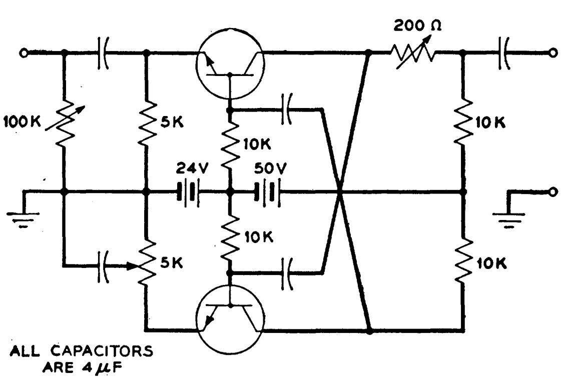

Fig. 11—Unbalanced converter with compensating variable resistors.

-1 to 1 a balance is obtained when Z_b equals Z_L. Typical converters of the type shown in Fig. 11 exhibit changes with time for the same supply voltage of about 1 per cent, for impedance levels in the hundreds or thousands of ohms.

Converters which are more stable with -1 to 1 conversion more nearly obtained can be constructed using four transistors as in the configuration shown in Fig. 13. The transistor arrangement shown is substantially that of the compound transistor suggested by Darlington.

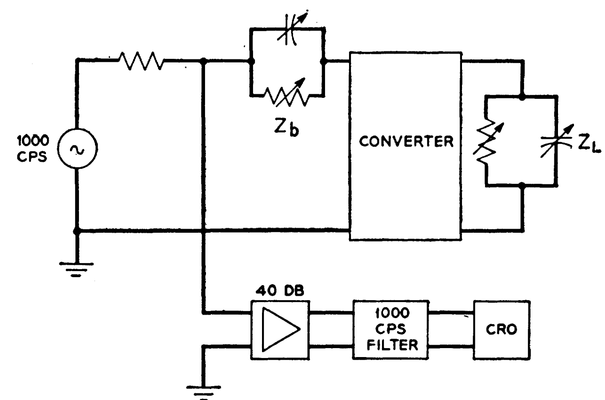

Fig. 12—Circuit for measurement of conversion factor.

The basis of the desirability of the unit in the present circuit is that the equivalent alpha of the compound unit is much more nearly equal to one. A simple expression indicating this fact is

$$1 - \alpha_{\text{compound}} = (1 - \alpha_1)(1 - \alpha_2). \tag{18}$$

For example, using units with alphas of 0.98, one has a compound transistor with the value of alpha of 0.9996.

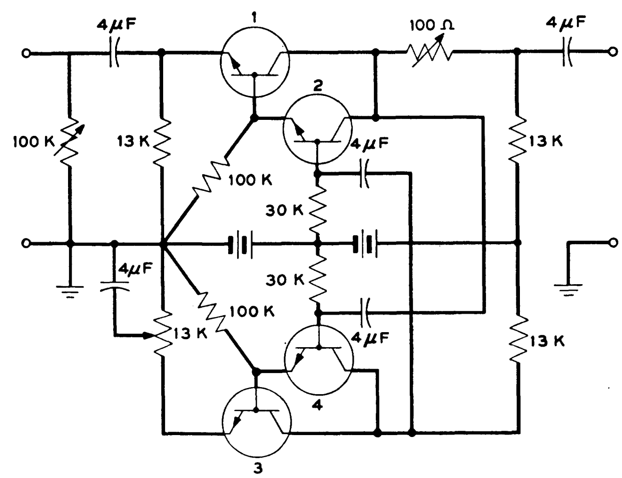

Fig. 13—Converter using four transistors.

Further, changes in the value of alpha for the two transistors effect a far less significant change in the value of alpha of the compound unit than for the individual units. The conversion ratio can be held with the compound transistor through a much larger frequency range to nearly -1 to 1, in spite of the phase shift in alpha. For instance, at 10 per cent of the alpha cut-off frequency, the phase shift of alpha in a single transistor is approximately 5.7 degrees, while for the compound transistor it is only 0.1 degree. Converters using compound transistors can be adjusted to give conversion ratios of -1 to 1 within about 1 per cent for impedance levels between 100 and 10,000 ohms and for frequencies beyond 10 kilocycles. The stability of converters with the compound transistor is also much better. With the circuit shown in Fig. 13, converters for loads in the thousands of ohms at 1,000 cps have maintained their conversion factor constant to within a few tenths of a per cent.

Most of the filters which have been built in the laboratory have been built with the converter as a separate unit. Actually this is not necessary; the biasing elements of the converter can frequently be identified with certain elements of the filter. Converters which operate down to dc can be obtained by replacing the cross-coupling condensers with Zener diodes and omitting the coupling condensers at the input and output terminals. Such converters have operated successfully in the laboratory.

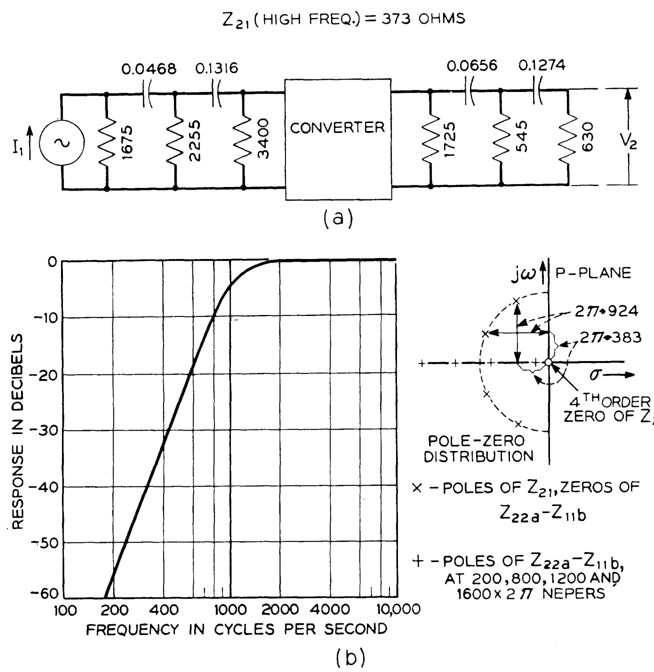

Fig. 14—High-pass active filter.

Details of Experimental Filter Designs

In Fig. 14 is shown the pole and zero distributions of transfer-impedance function and function $Z_{22a}-Z_{11b}$, the network configuration, and measured characteristics of a high-pass filter of Butterworth type.

the distribution of poles and zeros for the transfer impedance was selected to give a desirable pass-band characteristic and the point of high attenuation at 2,000 cycles.[9] Next the three-condenser three-resistor twin-T network was decided upon as a simple configuration to

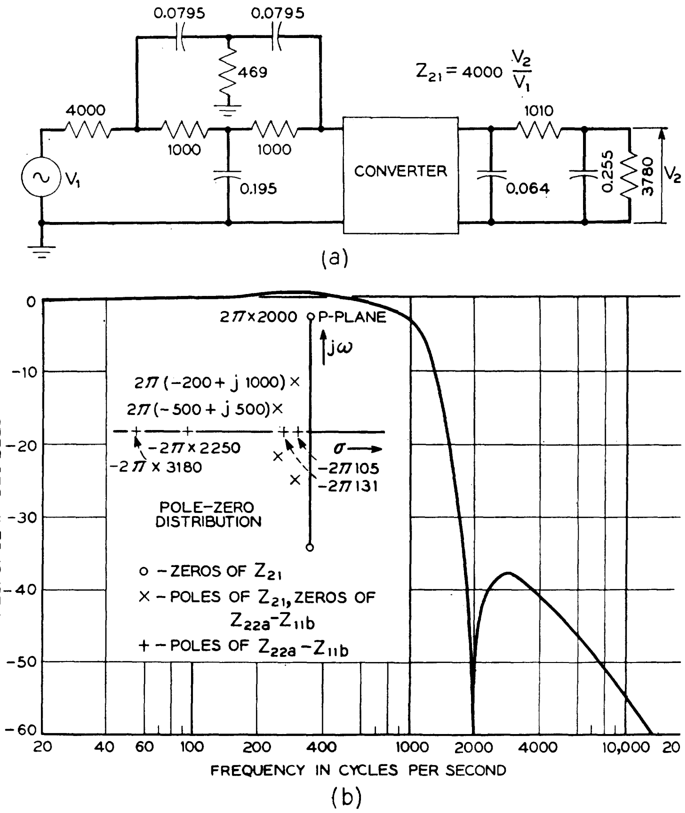

Fig. 15—Low-pass active filter with point of infinite rejection.

give the point of infinite attenuation. It is in this point that the technique differs from that suggested previously, where unbalanced forms are determined as the equivalents of a symmetrical lattice which is first obtained. The twin-T structure selected to provide infinite attenuation at 2,000 cps is analyzed to determine Z_{22a}, obtaining

$$Z_{22a} = 235 + \frac{555}{p + 2\pi\ 105} + \frac{957}{p + 2\pi\ 2250}. \tag{19}$$

At this point $Z_{22a}-Z_{11b}$ can be expressed as

$$Z_{22a} - Z_{11b} = \frac{K[p + 2\pi(200 + j1000)][p + 2\pi(200 - j1000)][p + 2\pi(500 + j500)][p + 2\pi(500 - j500)]}{(p + 2\pi\ 105)(p + a)(p + 2\pi\ 2250)(p + b)} \tag{20}$$

In Fig. 15 are shown the corresponding characteristics for a low-pass filter with a single point of infinite attenuation at twice the cut-off frequency. In connection with the design of this filter, techniques were employed which differ from those described earlier. At the outset,

where K, a and b must be determined in such a manner that Z_{11b} is a driving-point impedance and the residues in the poles of -2π 105 and -2π 2250 are those of Z_{22a}.

[9] J. G. Linvill, "The Selection of Network Functions to Approximate Prescribed Frequency Characteristics," Tech. Report No. 145, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass.; March, 1950.

An infinite number of choices will work, so arbitrarily select a suitable value of K and then solve for a and b explicitly. Thus the design illustrated in Fig. 15 is obtained.

As explained below, band-pass filters impose more severe requirements on the stability of negative-impedance converters for a given permissible drift in their transmission than do low-pass or high-pass filters of comparable complexity. Moreover, as the ratio of mid-band frequency to bandwidth increases, the requirements on the converter become greater as they do with an increase in the complexity of the filter.

The requirements on the stability of converters can be discussed in a semi-quantitative manner by considering the pole-zero distribution as shown in Fig. 16 for the simple function $Z_{22a}-Z_{11b}$ which might be applied for a filter. From the discussion of this simple case, it is possible to obtain salient facts which can easily be extended to apply to other more complicated situations.

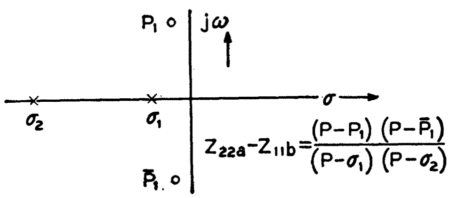

Fig. 16—Distribution of poles and zeros of $Z_{22a}-Z_{11b}$.

Equation (13) illustrates that the stability of characteristics as far as any particular natural frequency is concerned is directly proportional to the derivative of $Z_{22a}-Z_{11b}$ at that natural frequency. Further, the influence of a given shift in a natural frequency on the transmission characteristic is inversely proportional to the displacement of the natural frequency from the $j\omega$ or real-frequency axis. It is simple to interpret these statements in terms of the pole-zero distribution shown in Fig. 16. The derivative of $Z_{22a}-Z_{11b}$ at p_1 is found by applying (14). It is quite clear that moving the point p_1 much closer to the real-frequency axis does not influence the derivative significantly. Hence the motion of the natural frequency at p_1 associated with a small change ΔZ in the input impedance of the converter is nearly constant as this natural frequency is brought toward the axis. Since the effect of a shift in a natural frequency of the transmission characteristic increases as the natural frequency moves toward the real-frequency axis (as does the Q of the filter) it is clear that drift in the converter influences high-Q filters more than it does less selective ones. One recalls that the selection of the poles of $Z_{22a}-Z_{11b}$ is arbitrary and the question arises as to what represents a satisfactory spacing of the poles at σ_1 and σ_2. Referring to Fig. 16, consider leaving σ_1 fixed and moving σ_2. One can determine the location of σ_2 which minimizes the change in natural frequency at p_1 resulting from a change in the converter. Suppose the conversion factor of the converter changes from -1 by ΔC. Through (13) one can write

$$\Delta p_1 = \left.\frac{-\Delta C Z_{11b}}{\dfrac{d(Z_{22a}-Z_{11b})}{dp}}\right|_{p_1}. \tag{21}$$

But

$$-Z_{11b} = \frac{K\dfrac{(\sigma_2-p_1)(\sigma_2-\bar{p}_1)}{(\sigma_2-\sigma_1)}}{p-\sigma_2}, \tag{22}$$

and by (14)

$$\left.\frac{d(Z_{22a}-Z_{11b})}{dp}\right|_{p_1} = K\frac{(p_1-\bar{p}_1)}{(p_1-\sigma_1)(p_1-\sigma_2)}. \tag{23}$$

Thus

$$\Delta p_1 = \Delta C\frac{(p_1-\sigma_2)(\bar{p}_1-\sigma_2)(p_1-\sigma_1)}{(\sigma_2-\sigma_1)(p_1-\bar{p}_1)}. \tag{24}$$

From (24) one observes that placing the point σ_2 far out makes the first two factors of the numerator very large placing the point σ_2 close to σ_1 makes the first term of the denominator very small. Between these undesirable extremes lies the optimum. It lies at the point where a small motion of σ_2 toward σ_1 results in a fractional decrease of $|\sigma_2-\sigma_1|$ which is just twice the fractional decrease of $|\sigma_2-p_1|$.

To illustrate that increases in the complication of a band-pass filter increase the demands on the stability of the converter, consider Fig. 17, which shows the pole-zero distribution for a filter with a 12 db/octave cut-off characteristics at high and low frequencies. As one considers the derivative at p_1, observe that the addition of p_2, $\bar{p}_2$, σ_3 and σ_4 decreases the derivative in that one new factor, p_2-p_1, is much smaller than the other three new factors and it appears in the numerator of the expression analogous to that of (14). The value of Z_{11b} at p_1 is not decreased as is the derivative, the short vector p_2-p_1 not appearing in the calculation of residues. To illustrate numerically, using Fig. 17, the values of

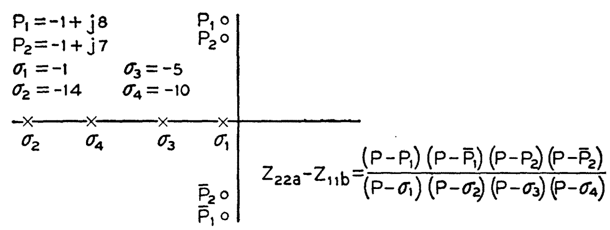

Fig. 17—Distribution of poles and zeros of $Z_{22a}-Z_{11b}$ for band-pass filter.

$$\left|\frac{\dfrac{d(Z_{22a}-Z_{11b})}{dp}}{Z_{11b}}\right|_{p=p_1}$$

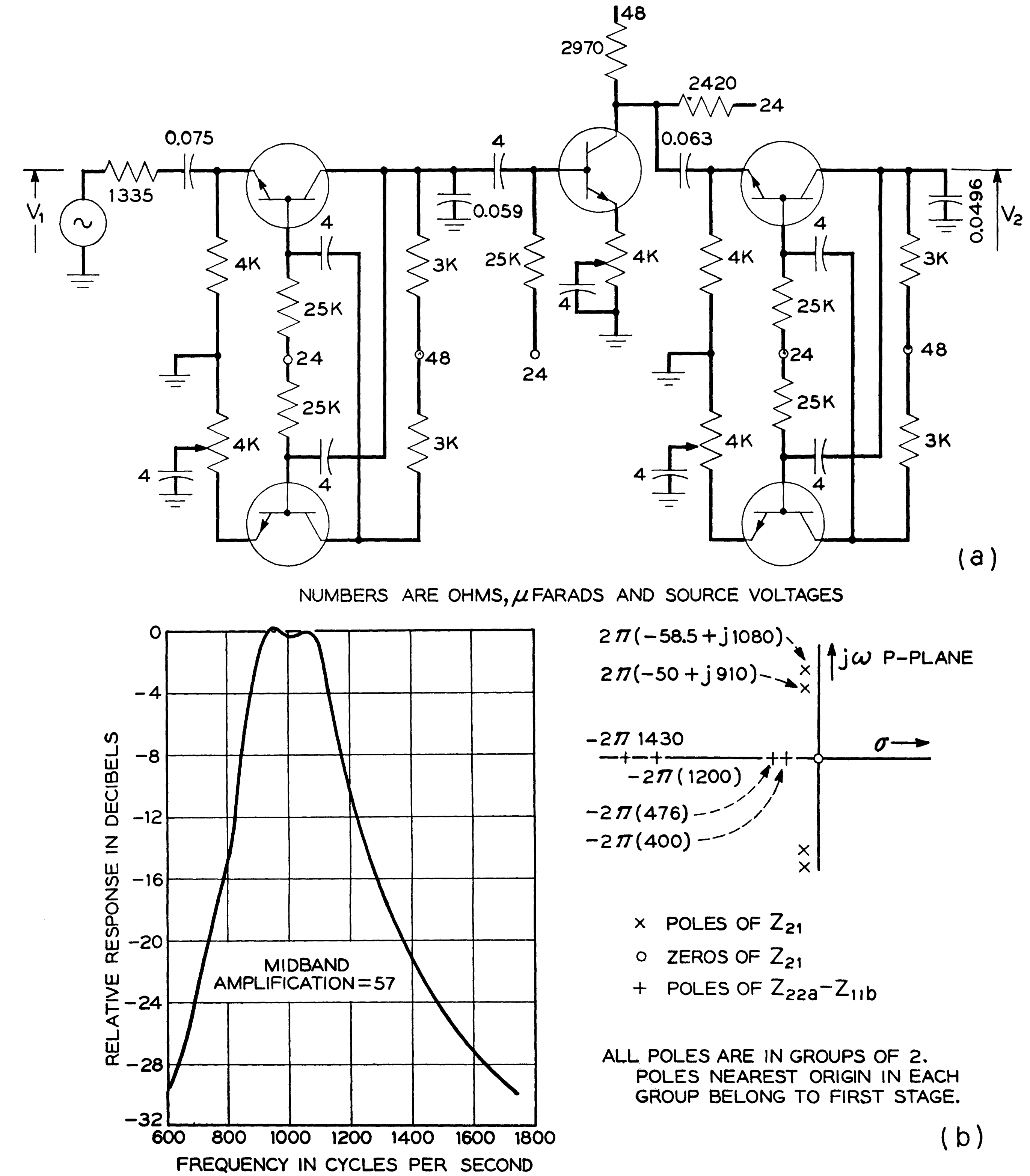

Fig. 18—Two-stage active band-pass filter.

before and after p_2, $\bar{p}_2$, σ_3 and σ_4 are added, amount to 0.112 and 0.00172, respectively.

One can cascade filters isolating one stage from the other in a simple manner with transistor amplifiers. Circuit above is a band-pass filter constructed in this manner employing only R's and C's. This filter achieves considerable gain in addition to its filtering properties. Calculations based on (14) reveal that a converter whose input impedance drifts no more than $\frac{1}{2}$ per cent will make possible a filter characteristic which drifts less than 1 db at every frequency.