Thus $K = j\mu\omega INA$ is the effective magnetic current. If the electric current varies as a step function of time according to $I_0u(t)$ [where $u(t) = 0$ for $t < 0$, $= 1$ for $t > 0$], we know that its Laplace transform $I(s) = I_0 s$. On replacing $j\omega$ by s in the time-harmonic solution, it is now a simple matter to obtain the Laplace transform of the magnetic potentials. Thus, for the step-function excitation, we have

$$U^e(s) = \frac{ANI_0}{4\pi}\left\{\frac{1}{sR} + \frac{1}{s}\sum_{n=0}^{\infty}\frac{n}{n+1}\frac{a^{2n+1}}{l^{n+1}r^{n+1}} \times \left(1 - \frac{2n+1}{2n+1+\alpha s}\right)\mathrm{P}_n(\cos\theta)\right\} \quad (14)$$

and

$$U^i(s) = \frac{ANI_0}{4\pi s}\sum_{n=0}^{\infty}\frac{r^n}{l^{n+1}}\left(\frac{2n+1}{2n+1+\alpha s}\right)\mathrm{P}_n(\cos\theta) \quad (15)$$

where $\alpha = \sigma\mu a d$. The corresponding time functions are obtained simply by inverting the above elementary transforms, giving

$$U^e(t) = \frac{ANI_0}{4\pi}\left\{\frac{1}{R} + \sum_{n=0}^{\infty}\frac{n}{n+1}\frac{a^{2n+1}}{l^{n+1}r^{n+1}} \times e^{-(2n+1)t/\alpha}\mathrm{P}_n(\cos\theta)\right\}u(t) \quad (16)$$

and

$$U^i(t) = \frac{ANI_0}{4\pi}\left[\sum_{n=0}^{\infty}\frac{r^n}{l^{n+1}}\{1 - e^{-(2n+1)t/\alpha}\}\mathrm{P}_n(\cos\theta)\right]u(t) \quad . \quad . \quad . \quad . \quad (17)$$

The corresponding transient magnetic fields are obtained from

$$H_r(t) = -\partial U(t)/\partial r \text{ and } H_\theta = -\partial U(t)/(r\partial\theta) \quad . \quad (18)$$

which need not be written out.

The expression for the external potential has an interesting interpretation. We see that the summation term in eqn. 16, which is the secondary field, can be written in the equivalent form

$$\sum_{n=0}^{\infty}\frac{n}{n+1}\frac{\{a_{eff}(t)\}^{2n+1}}{l^{n+1}r^{n+1}}\mathrm{P}_n(\cos\theta)u(t)$$

where $a_{eff}(t) = a\exp(-t/\alpha)$. Thus, in the external region, the transient response can be described in terms of the static solution for a perfectly conducting sphere of effective radius a_{eff} which varies with time in an exponential manner. Physically, we may attribute this to the decreasing induced eddy currents following the initial application of the primary step-function magnetic field.

To interpret the internal field, we observe immediately, in eqn. 17, that

$$(r^n/l^{n+1})\exp\{-(2n+1)t/\alpha\} = r_{eff}^n/l_{eff}^{n+1} \quad . \quad . \quad (19)$$

if $r_{eff} = r\exp(-t/\alpha)$ and $l_{eff} = l\exp(+t/\alpha)$. Thus, using eqn. 6, we find that

$$U^i(t) = \frac{ANI_0}{4\pi}\left\{\frac{1}{R} - \frac{1}{R_{eff}(t)}\right\}u(t) \quad . \quad . \quad . \quad . \quad (20)$$

where $R = (r^2 + l^2 - 2rl\cos\theta)^{1/2}$

and $R_{eff} = (r_{eff}^2 + l_{eff}^2 - 2r_{eff}l_{eff}\cos\theta)^{1/2}$
$= (r^2e^{-2t/\alpha} + l^2e^{2t/\alpha} - 2rt\cos\theta)^{1/2}$

We see that, at the initial instant $t = 0$, $R_{eff} = R$, and no field exists internally. However, for $t > 0$, the curly-bracket term in eqn. 20 is not zero. In fact, as $t \to \infty$, $1/R_{eff}$ vanishes. This corresponds to the recession of the virtual source to infinity on the polar axis. Here the relaxation time $\alpha = \sigma\mu a d$ plays an important role.

The extreme simplicity of the transient response of the spherical shell is remarkable. The corresponding steady-state solution, although not excessively complicated, requires summation of the spherical harmonics and manipulation of complex quantities.

The present quasistatic solution for a magnetic pole is readily adapted to a more complicated situation. For example, an arbitrarily oriented magnetic dipole is simply treated as the suitable superposition of two separated magnetic poles of opposite sign.[3]

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HIGH-Q FACTOR CIRCUIT WITH REDUCED SENSITIVITY

An active network is suggested that uses one differential-input operational amplifier and is suitable for realising high-Q factor second-order bandpass functions. Its Q sensitivity to component values is found to be less than the optimum Q sensitivities of the corresponding networks of Huelsman and Antoniou. The network has been used to realise a sixth-order bandpass filter in three cascaded stages. Experimental results are given.

The lowpass–bandpass transformation of an all-pole filter function gives a function which is the product of second-order terms of the following form:

$$F(s) = \frac{Hs}{s^2 + \beta s + \gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where H, β and γ are real and positive constants. In high-selectivity filters, these terms will be of high Q factor ($Q = \sqrt{\gamma}/\beta$). If active networks are to be used for their realisation, these networks must have low sensitivity of the

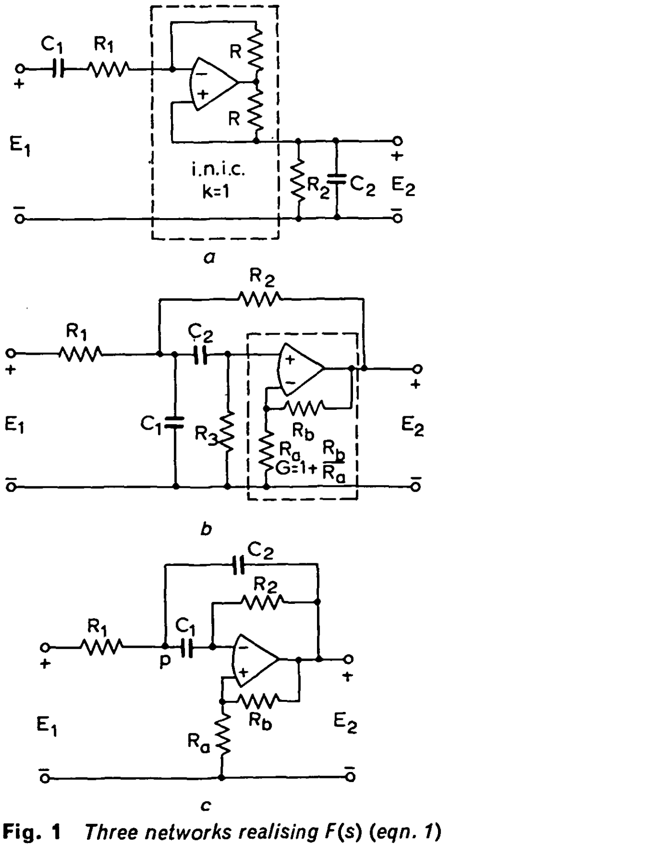

Fig. 1 *Three networks realising F(s) (eqn. 1)*

a Huelsman's network using an i.n.i.c.
b Antoniou's network using an amplifier with positive gain G
c Proposed network

Q factor to component values. Two networks with optimum sensitivity are shown in Figs. 1*a* (Reference 1) and *b* (Reference 2). The purpose of this letter is to describe an alternative circuit, which is suitable for the realisation of $F(s)$ and has lower Q sensitivity than the other two networks.

The network is shown in Fig. 1*c*. Assuming that the operational amplifier has infinite gain, infinite input impedance and zero output impedance, the network transfer function is as follows:

$$\frac{E_1}{E_2} = -\frac{hs}{s^2 + \left(\frac{C_1 + C_2}{R_2 C_1 C_2} - K\frac{1}{R_1 C_2}\right)s + \frac{1}{R_1 R_2 C_1 C_2}} \quad . \quad . \quad . \quad . \quad (2)$$

where $K = \frac{R_a}{R_b}$

$$h = \frac{l + K}{R_1 C_2}$$

$F(s)$ can be realised by this network as the voltage ratio $-E_2/E_1$, to within a constant multiplier, if we rewrite it as follows:

$$F(s) = \frac{Hs}{s^2 + \{(n+1)\beta - n\beta\}s + \gamma} \quad . \quad . \quad . \quad . \quad (3)$$

(where n is real and positive) and match coefficients of equal powers of s in eqns. 2 and 3. The sensitivity of the circuit Q factor to variations in the component values is obviously affected by the choice of n, which has to be as low as possible for low sensitivity of the Q factor.

Let $r = \frac{R_1}{R_2}$ and $q = \frac{C_1}{C_2}$

Coefficient matching between eqns. 2 and 3 and simple manipulations result in the following equations:

$$n = \sqrt{\frac{r}{q}}(q+1)Q - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$K = \frac{n}{Q}\sqrt{\frac{r}{q}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$R_2 C_1 = \sqrt{\frac{q}{\gamma r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$R_1 C_2 = \sqrt{\frac{r}{\gamma q}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$R_2 C_2 = \frac{1}{\sqrt{(\gamma r q)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The value of n in eqn. 4 can be shown to be minimum for any r, when

$$q = 1$$

i.e. when

$$C_1 = C_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Then $n = 2Q\sqrt{r} - 1$ . . . . (10)

Thus, depending on r, n can be (within the practical limitations) as small as it is desired.

Since eqns. 4–9 are not sufficient to give unique values of the components, the designer has to select r and one of R_1, R_2 or C_1 as well as R_a or R_b, taking into consideration the operational-amplifier specifications. Selection of C_1, however, has the advantage that all capacitors in a cascade connection of similar circuits may be of the same value, and this can be a standard one.

The sensitivities of the Q factor and $\omega_0 = \sqrt{\gamma}$ to variations in component values are given in Tables 1 and 2, respectively. These Tables also give the corresponding sensitivities of the networks in Figs. 1*a* and *b*. It should be noted that the Q factor sensitivity of the network in Fig. 1*b* is given by $S_G^Q = 2Q - 1 + k$, where, in practice, $k \ll 2Q - 1$.

It can be seen that the proposed network, when $r < 1$, has Q sensitivities to component values lower than the other two networks. There is no difference for $S_{x_i}^{\omega_0}$ except for R_1 and R_2 of the network in Fig. 1*b*, which, however, uses one component (R_3) more than the other two. The lower Q sensitivity is due to a lower n, since the design of the network in Fig. 1*c* does not require the decomposition of the denominator of $F(s)$ into positive and negative RC driving-

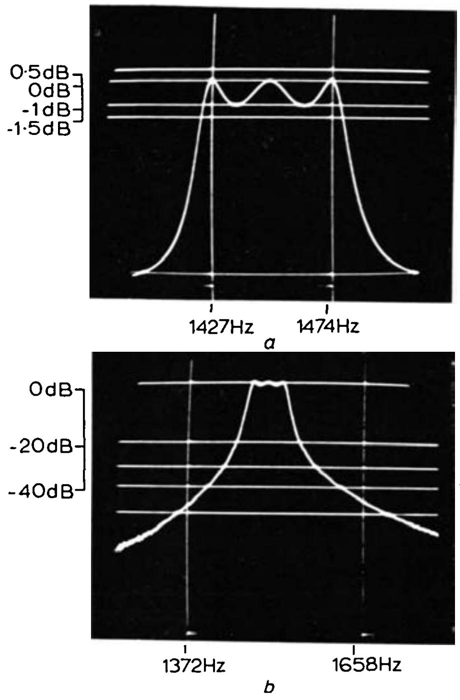

Fig. 2 *Frequency response of suggested network realising F(s) (eqn. 11) in three cascaded stages:*

a Linear gain and frequency scales
b Logarithmic gain and linear frequency scales

Table 1

	$S_{x_i}^Q$		
x_i	Fig. 1*a*	Fig. 1*b*	Fig. 1*c*
R_1	$-(Q - \frac{1}{2})$	$\frac{1}{2}(Q - \frac{1}{2})$	$-(2Q\sqrt{r} + \frac{3}{2})$
R_2	$Q - \frac{1}{2}$	$-(Q - \frac{3}{4})$	$2Q\sqrt{r} + \frac{3}{2}$
R_3	0	$Q - \frac{1}{2}$	0
C_1	$Q - \frac{1}{2}$	$-(Q - \frac{1}{2})$	$Q\sqrt{r} - \frac{1}{2}$
C_2	$-(Q - \frac{1}{2})$	$Q - \frac{1}{2}$	$-(Q\sqrt{r} - \frac{1}{2})$
K or G	$2Q - 1$	$2Q - 1$	$2Q\sqrt{r} - 1$
A	$2Q/A$*	$8Q/A$	$Q/A\sqrt{r}$

A is the actual gain of operational amplifier
* Not very accurate, since K varies with load

Table 2

	$S_{x_i}^{\omega_0}$		
x_i	Fig. 1*a*	Fig. 1*b*	Fig. 1*c*
R_1	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$
R_2	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$
R_3	0	$-\frac{1}{2}$	0
C_1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
C_2	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
K or G	0	0	0

point admittances. The minimum possible value of n for the other two networks is $2Q-1$.

Some important features of the proposed network are as follows:

(a) Q factor, and hence bandwidth, can be varied independently of ω_0 by varying K
(b) Successive stages can be cascaded without the need for isolating stages
(c) All capacitors in all cascaded stages can be designed to be of the same value.

The form of the transfer function of the network remains unchanged if R_1 is interchanged with C_1, and R_2 is interchanged with C_2. However, two reasons make the circuit in Fig. 1c preferable: (i) It provides for nonzero source impedance, and (ii) if it is desired to reduce the output voltage at the centre frequency, this can be achieved in Fig. 1c by connecting a resistance from node P to earth and arranging that the Thévenin impedance looking back from P to the source supplying E_1 is still R_1.

The circuit was used to realise the following bandpass function with 1 dB ripple in the passband denormalised to a centre frequency $f_0 = 1500$ Hz and an impedance level of $10^5\,\Omega$:

$$F(s) = \frac{0{\cdot}010076s}{s^2+0{\cdot}010076s+1{\cdot}03939} \times \frac{0{\cdot}019767s}{s^2+0{\cdot}019767s+1} \times \frac{0{\cdot}00969s}{s^2+0{\cdot}00969s+0{\cdot}962099} \quad . \quad . \quad . \quad . \quad (11)$$

r was selected to be 1/49 in all stages. The three stages were built using Fairchild μA741 operational amplifiers, polystyrene capacitors and resistors of various types. Resistance R_a was made variable in all stages for accurate adjustment. The response of the adjusted filter is shown in Fig. 2.

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DIRECTLY COUPLED ACTIVE CIRCULATORS

A method is described for making multiport circulators with active elements which are directly coupled; each port requires one differential amplifier.

Circulators using active elements, which are coupled by capacitors or transformers, have been described in the past few years.[1–4] Recently, the realisation of circulators with differential-input operational amplifiers has been discussed by Keen, Glover and Harris.[5] In this letter, we describe a general method[6] for making multiport active circulators which are directly coupled; the circuit of Keen *et al.*[5] may be regarded as a particularly useful example of the method. Besides the well known uses of circulators for separating forward and reverse signal paths,[7] it is possible, with circulators and capacitors, to realise floating inductances, or to synthetise filters and allpass networks requiring floating inductances.[8]

An essential property of a correctly terminated circulator is that a signal applied at one port appears only at the next port in the sequence. The present method[6] is illustrated in Fig. 1, which shows two stages of a terminated circulator. The differential amplifiers are assumed to have a very high input impedance, negligible output impedance and a finite gain, G. The output voltage of one amplifier is applied to both input terminals of the next amplifier through two separate potential dividers,[4] each dividing in the same ratio r, so that the differential input signal is zero; consequently no signal is passed on to the following stage. At each stage, one of the two earthed impedances may be removed to create a port. If the last stage feeds the first, a multiport circulator results, and each differential amplifier provides one port between either the positive or the negative input and earth.

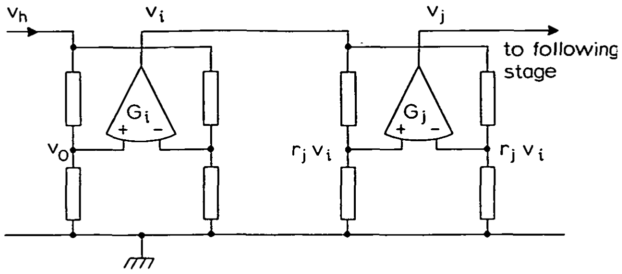

Fig. 1 *Two stages of terminated circulator*

If, in Fig. 1, the voltage v_h is assumed to be zero, and a voltage v_0 is applied to the positive input of amplifier i, the voltage appearing at both inputs of amplifier j will be $G_i r_j v_0$. The voltage gain from port i (at the positive input of amplifier i) to port j (at either input of amplifier j), when port j is correctly terminated, is therefore $G_i r_j$, which is usually chosen to be unity. The correct termination at any port is that impedance which forms a balanced bridge with the other impedances of that stage. The input impedance at any port, with the circulator correctly terminated, is, in general, not equal to the correct termination at the port. The input impedance and the correct termination may be made equal, either by choosing $r = \frac{1}{2}$, or by connecting an internal impedance in series or in parallel with the port.

Circuit using transistors: Fig. 2 shows one stage of a circulator using transistors. The differential amplifier consists

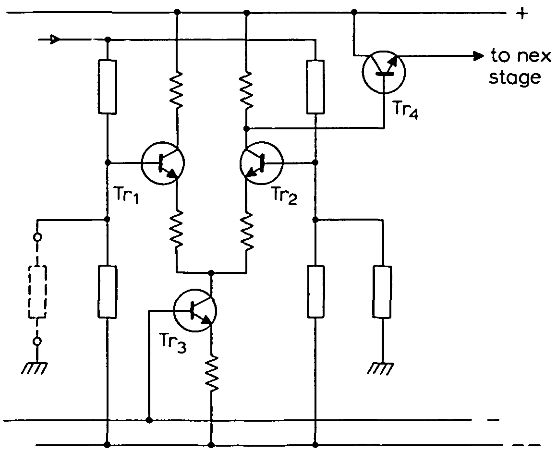

Fig. 2 *One stage of circulator using transistors*

of a long-tailed pair, Tr_1 and Tr_2, with emitter-feedback resistors to make the voltage gain insensitive to transistor parameters, and to produce a high input resistance. Emitter current is supplied by a constant-current generator, Tr_3, and a common-collector stage, Tr_4, is used to reduce the loading effect of the potential dividers of the next stage on the output of the long-tailed pair.

The potential dividers are arranged to make the d.c. potential at the port zero, and to give the required dividing ratio. To satisfy both these conditions simultaneously, each potential divider requires three impedances, which would