RC Active Networks Using Current Inversion Type Negative Impedance Converters*

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Introduction

RC FILTERS are attractive for their compactness and cheapness. However, the characteristic function realized by passive RC networks has many restrictions compared with that of ordinary LCR networks. To overcome this defect, active elements such as vacuum tubes or transistors are used in addition to R's and C's. One type of RC active filter, which has been widely used, is composed of passive RC networks and a feedback amplifier. In 1954, Linvill proposed a new type of RC active filter using a negative impedance converter (called simply converter hereafter) as an active element.[1] When the characteristic function is of polynomial type, this type of active filter can be designed on the basis of the input impedance like an ordinary LCR filter. In the case of a characteristic function which has attenuation poles at finite frequencies, however, Linvill's circuit needs twin-T circuits.

This paper describes a different type of RC active network employing two RC networks and a current-inversion type converter. By this method, fractional characteristic functions of filters and phase shifters are easily realized using the synthesis of RC two-terminal impedances. The current-inversion type converter, which is necessary in this method, is derived by the modification of conventional El type converter.[2] This converter can omit coupling capacitors, by the use of complementary transistors.

Theory of Active RC Networks Using Current-Inversion Type Converter

The circuit of Fig. 1 is considered. The A parameter of this circuit is

$$A = \frac{N(p)}{D(p)} = \frac{E_1}{E_2}\bigg|_{I_2=0} = -\frac{y_{22} - Y_{22}}{y_{12} - Y_{12}}. \tag{1}$$

In (1), the activity of the converter acts on the denominator as well as on the numerator. Therefore, one can realize the fractional characteristics without the difficulty of Linvill's method, where the denominator is the product of admittances. In the circuit of Fig. 1, the converter

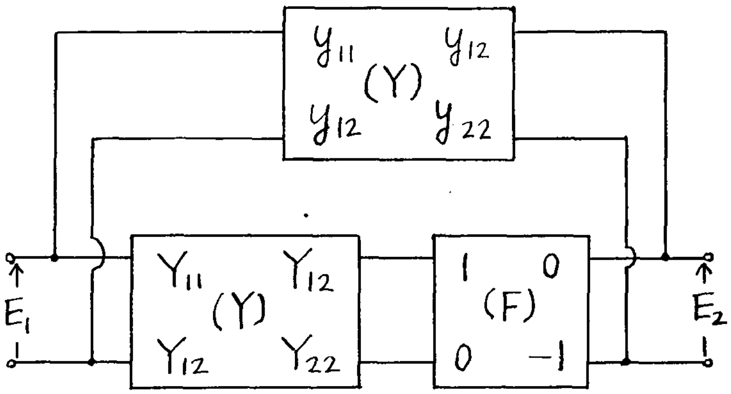

Fig. 1.

must be the current-inversion type. If the converter were the ordinary voltage-inversion type, (1) becomes $-(y_{22} - Y_{22})/(y_{12} + Y_{12})$. That is, the activity of converter does not act on the denominator. Using (1), one can realize $N(p)$ and $D(p)$ by the use of $y_{22} - Y_{22}$ and $y_{12} - Y_{12}$, respectively.

However, Y_{12} and Y_{22} cannot be synthesized separately. To solve this problem, one may use inverse L circuits as each RC network. The circuit configuration is shown in Fig. 2. The A parameter of the circuit of Fig. 2 is

$$A = \frac{y_a - Y_a + y_b - Y_b}{y_a - Y_a}. \tag{2}$$

The design method is as follows. At first, (1) is modified to the form:

$$S(p) = \frac{D(p) + N(p) - D(p)}{D(p)}. \tag{3}$$

One now picks $n - 1$ points as σ_i on negative real axis, and forms the equation [where n is the highest order of $D(p)$ or $N(p)$]:

$$K(p) = \prod_{i=1}^{n-1} (p - \sigma_i). \tag{4}$$

From (2)–(4), one obtains

$$y_a - Y_a = \frac{D(p)}{K(p)} = k_\infty p + k_0 + \sum_{i=1}^{n-1} \frac{k_i p}{p - \sigma_i}$$

$$y_b - Y_b = \frac{N(p) - D(p)}{K(p)} = k'_\infty p + k'_0 + \sum_{i=1}^{n-1} \frac{k'_i p}{p - \sigma_i}. \tag{5}$$

In (5), the partial fractions with positive residues are realized by y_a and y_b, those of negative residues by Y_a and Y_b, respectively. If $N(p) - D(p)$ or $D(p)$ has a nega-

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[1] J. G. Linvill, "RC active filters," Proc. IRE, vol. 42, pp. 555–564; March, 1954.

[2] J. L. Merrill, "Theory of the negative impedance converter," *Bell Sys. Tech. J.*, vol. 30, pp. 88–109; January, 1956.

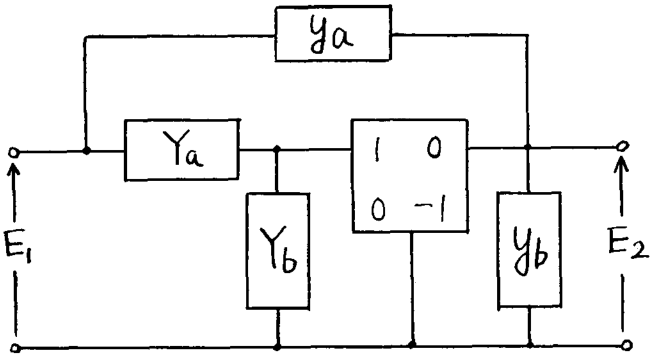

Fig. 2.

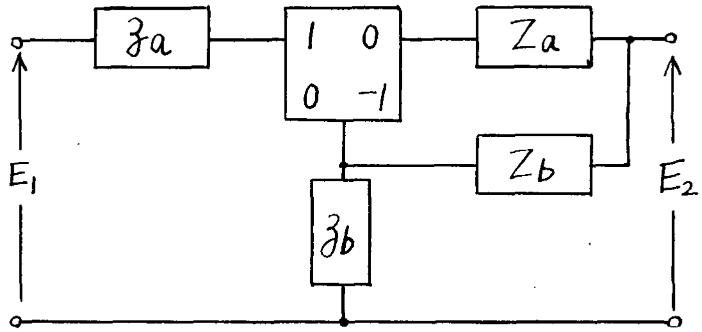

Fig. 3.

tive real zero, one may save the number of circuit elements by the coincidence of this zero and one of σ_i's.

In the construction of this network, one needs a low-impedance signal source, such as a cathode follower, to drive this circuit, since it has the parallel connection of y_a and Y_a at the input terminals. At the output terminal, if y_b has a parallel resistor, one may use this resistor as the output resistance. So this circuit may be called a OR circuit.

As a dual of the above parallel network configuration, the series type circuit is shown in Fig. 3. The A parameter of the F matrix of this circuit is

$$A = \frac{z_a - Z_a + z_b - Z_b}{z_b - Z_b}. \tag{6}$$

Eq. (6) has the same form as (2), so the design method is almost identical to that described above. One needs n points as σ_i here, since (6) is based on RC impedance. One of the σ_i can be made zero by the nature of RC impedance. The circuit of Fig. 3 has z_a in series with the input terminal. If z_a has a series resistor, one may use it as the input resistance. This circuit may be called $R - \infty$ circuit. Since the return path of the converter in Fig. 3 cannot be grounded directly, the parallel-type configuration may be more practical than the series-type in ordinary cases.

Design Examples of Filters and Phase Shifters

A low-pass characteristic function which has three zeros and one finite pole is

$$S(p) = \frac{0.8p^3 + 1.64p^2 + 1.6p + 1}{0.36p^2 + 1}. \tag{7}$$

One may put $\sigma_1 = -1$, $\sigma_2 = -0.5$. From (5)

$$y_a - Y_a = \frac{0.36p^2 + 1}{(p+1)(p+0.5)} = 2 + \frac{2.72p}{p+1} - \frac{4.36p}{p+0.5}$$

$$y_b - Y_b = \frac{0.8p^3 + 1.28p^2 + 1.6p}{(p+1)(p+0.5)}$$

$$= 0.8p + \frac{2.32p}{p+0.5} - \frac{2.24p}{p+1}. \tag{8}$$

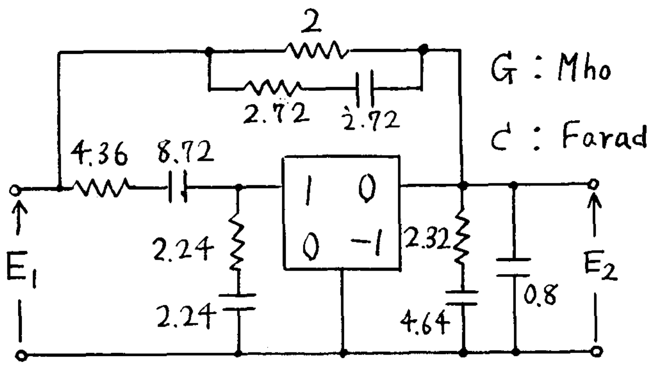

Fig. 4.

The circuit is shown in Fig. 4. In this case y_b has no parallel resistance, since the characteristic of (7) has no loss at $p = 0$. To make y_b have a parallel resistor, one may choose a characteristic with some loss at $p = 0$.

If, in (7) one puts $p \rightarrow 1/p$, the characteristic function of a high-pass filter is obtained:

$$S(p) = \frac{p^3 + 1.6p^2 + 1.64p + 0.8}{p^3 + 0.36p}. \tag{9}$$

The circuit is shown in Fig. 5. The σ_i's are same as in the low-pass case.

The next equation is the characteristic function of a band-pass filter, which has a 3-db deviation in the pass band, and 18-db attenuation in attenuation band. The ratio of bandwidth to center frequency $\Delta f/f_0$ is 0.6.

$$S(p) = \frac{p^4 + 0.839p^3 + 2.402p^2 + 0.839p + 1}{0.124p^4 + 0.650p^2 + 0.124}. \tag{10}$$

One may put $\sigma_1 = -2$, $\sigma_2 = -1$, $\sigma_3 = -0.5$; the circuit of Fig. 6 is obtained.

The characteristic function of the phase shifter is also realized by the same method. As an example,

$$S(p) = \frac{p^3 + 2p^2 + 2p + 1}{-p^3 + 2p^2 - 2p + 1}. \tag{11}$$

The circuit of Fig. 7 satisfies (11); σ_i's are the same as in the low-pass case.

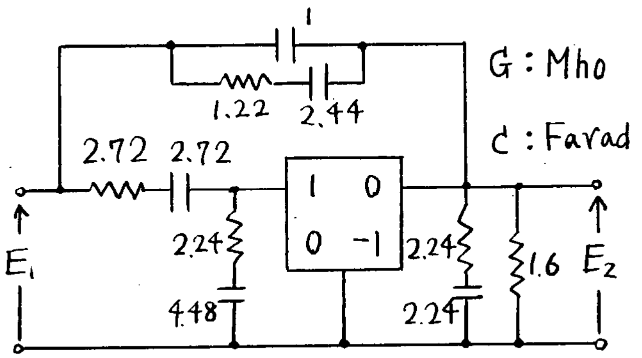

Fig. 5.

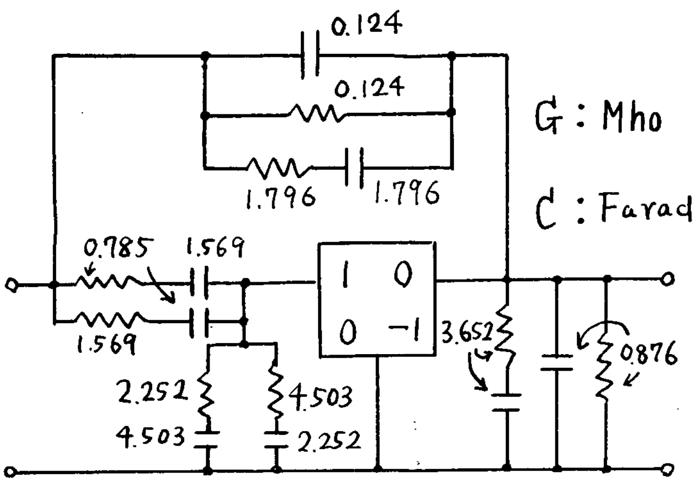

Fig. 6.

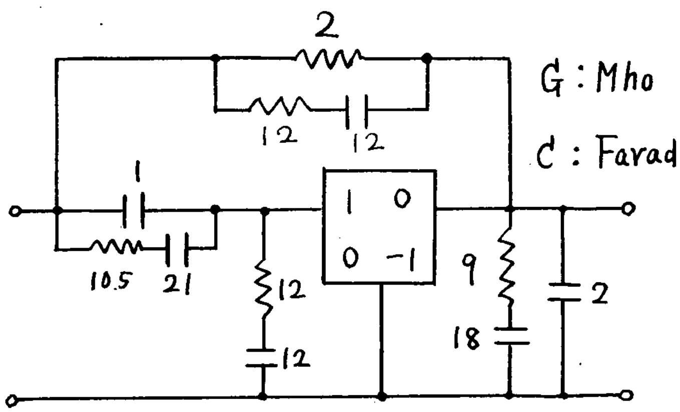

Fig. 7.

Current-Inversion Type Negative Impedance Converters

The converter needed here must be the current-inversion type, which is represented by

$$\begin{pmatrix} e_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} e_2 \\ i_2 \end{pmatrix}. \tag{12}$$

Moreover, one needs a converter of unbalanced form. The El type converter of Fig. 8 is easily modified to current-inversion type by the exchange of output terminals

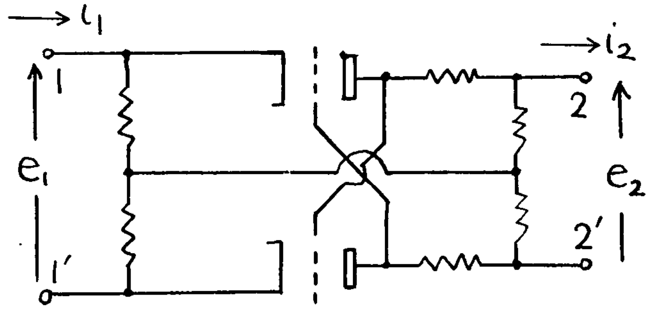

Fig. 8.

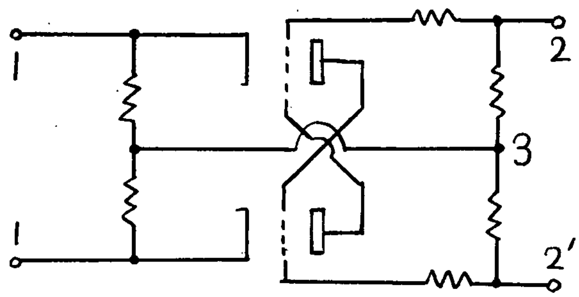

Fig. 9.

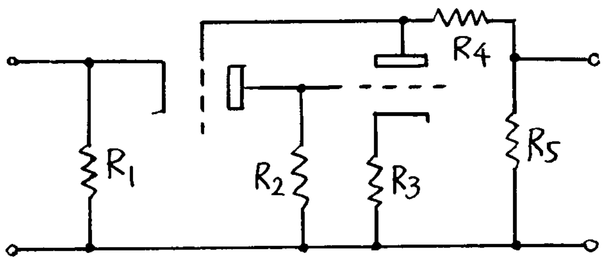

Fig. 10.

$2 - 2'$. Since this process is equivalent to connecting an ideal transformer of $1: -1$ ratio at the output terminals, the current-inversion type converter of balanced form shown in Fig. 9 is obtained. To make this converter in unbalanced form, one may use point 3 in Fig. 9 as the common terminal. The final circuit is shown in Fig. 10.

If one chooses the values of each resistor in Fig. 10 as

$$\left.\begin{array}{ll} R_1 = (\mu/\mu + 1)^2 R_2 & R_3 = (\mu R_2 - v_p)/(\mu + 1) \\ R_4 = (R_2 + v_p)/\mu & R_5 = R_2 \end{array}\right\} \tag{13}$$

then the F matrix of this circuit is represented by

$$(F) = \begin{pmatrix} \mu/\mu + 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{14}$$

In (13), μ and v_p represent the amplification factor and anode resistance of the tubes, respectively. In the calculation of the above formulas, the approximation $(R_2 + v_p)/\mu^2 R_2 \ll 1$ is necessary. The above circuits are easily transistorized by the use of the vacuum tube triode—junction transistor analogy. Fig. 11 shows the tran-

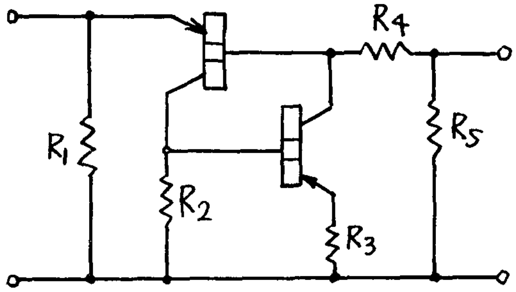

Fig. 11.

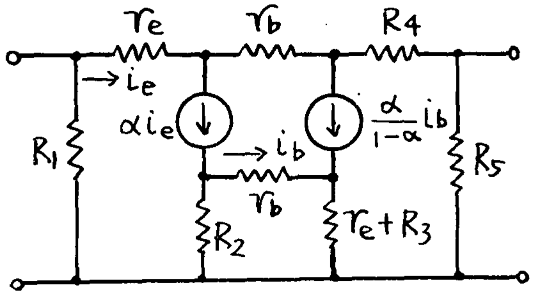

Fig. 12.

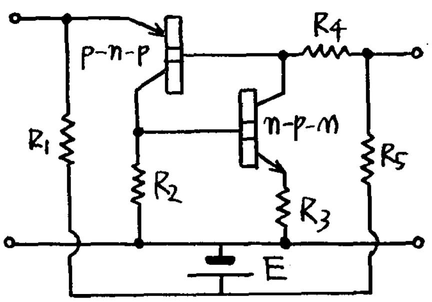

Fig. 13.

sistorized current-inversion type converter in unbalanced form. If one neglects the collector resistance v_c of the transistors, the equivalent circuit is shown in Fig. 12. The condition to realize ideal converter action is as follows:

$$\left.\begin{aligned} R_1 &= R_5 \qquad R_4 = v_e + (1-\alpha)v_b \\ R_2 &= \frac{(2-\alpha)\{v_e + R_3 + (1-\alpha)v_b\}}{3\alpha - 2} \end{aligned}\right\}. \tag{15}$$

In Fig. 11, R_3 is inserted to stabilize the conversion ratio by the current feedback action. This circuit can be direct coupled by the use of complementary transistors. The actual direct coupled circuit which contains a power source is shown in Fig. 13. This circuit operates on dc signals, so it will be useful in the very low-frequency band.

The Selection of σ_i's

The reasonable way to choose σ_i's has not been found as yet. Therefore, the cut and try method is necessary for several values of each σ_i. The stability of the characteristics against variation of the active element is most important in active RC networks. Hence, one must select the σ_i's to give better stability.

If the current conversion ratio of the converter drifts from -1 to $-1/(1+\delta)$, the circuit transfer function of (2) will be varied as

$$\begin{aligned} A &= \frac{y_a + y_b - (1+\delta)(Y_a + Y_b)}{y_a - (1+\delta)Y_a} \\ &= \frac{N(p) - \delta(Y_a + Y_b)K(p)}{D(p) - \delta Y_a K(p)}. \end{aligned} \tag{16}$$

In (16), one may put,

$$\left.\begin{aligned} N(p) &= \sum a_i p^i \\ (Y_a + Y_b)K(p) &= \sum b_i p^i. \end{aligned}\right\} \tag{17}$$

In (17), the ratio of b_i to a_i is related to the stability of the characteristics directly. The σ_i's must be chosen to give this ratio as small as possible. This ratio is also used to estimate the delicacy of the adjustment of the actual circuit to get the desired characteristic. When the ratio is about 1, the adjustment of the circuit is fairly easy. When it becomes $5 \sim 10$, the adjustment is rather difficult. For the band-pass filter of Fig. 6, the ratio b_1/a_1 becomes 10.8. Hence, the realization of this filter seems to be very difficult.

Experimental Results

As experimental results, high-pass and band-pass filters will be described. The actual circuit which possesses the high-pass characteristic of (9) is shown in Fig. 14,

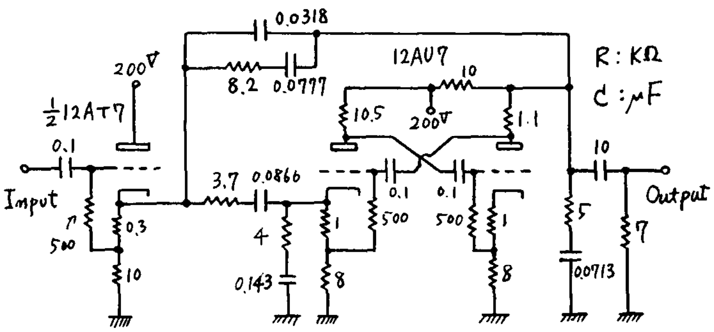

Fig. 14.

where f_0 is 0.5 kc. The characteristic obtained is shown in Fig. 15. Figs. 16 and 17 show the circuit and characteristic of the band-pass filter of (10). In this case, the ratio b_1/a_1 is in excess of 10, and the adjustment of this filter is rather critical.

In the adjustments of these networks, making the conversion ratio of the converter -1 is most important. After this control, the attenuation poles may be placed at prescribed points by adjusting the elements of y_a and

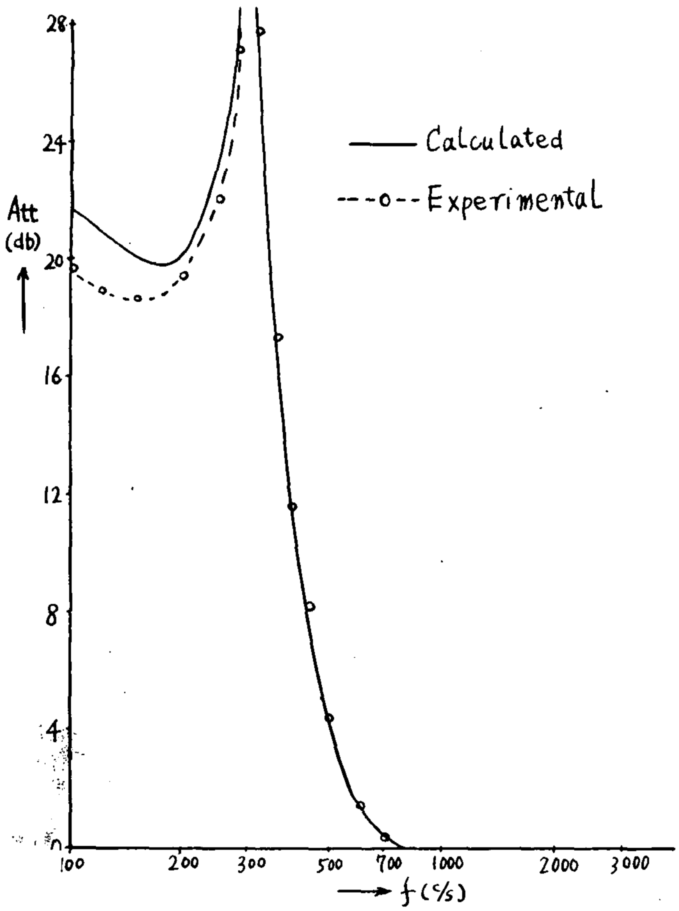

Fig. 15.

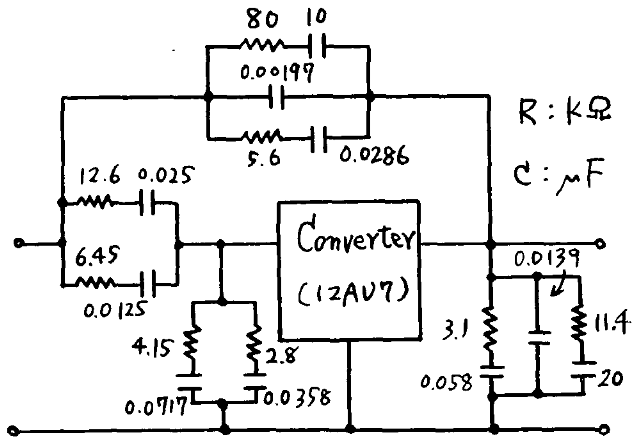

Fig. 16.

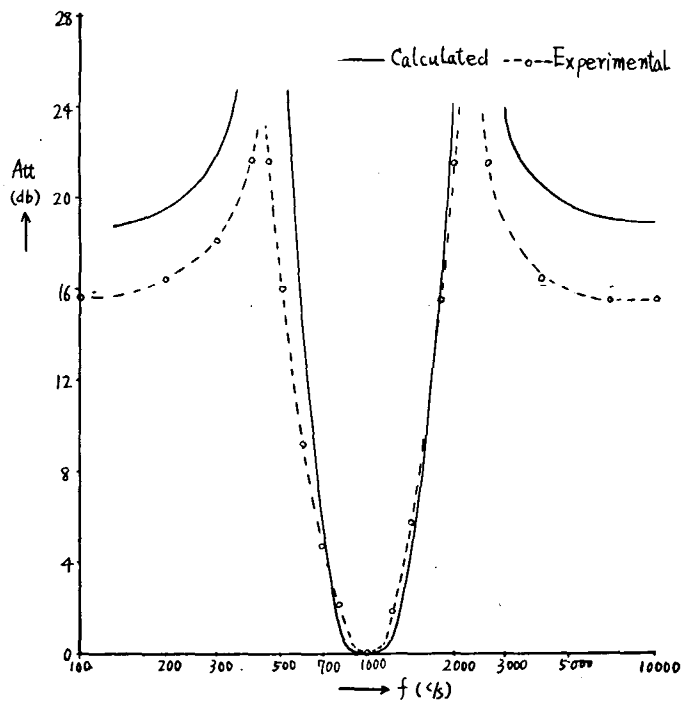

Fig. 17.

Y_a. Finally, the pass band characteristic is controlled by the adjustment of y_b and Y_b.

Conclusion

The above discussion shows one possible synthesis method for active RC networks. Since in this method, only the synthesis of RC two-terminal impedances is necessary, design may be simpler than for ordinary LCR networks.

The circuit configuration is based on the parallel or series connection of two networks; therefore input and output resistance cannot exist simultaneously. To remove this defect, one needs to devise different circuit configurations. In general, the selection of the σ_i's is the important unsolved problem in the practical design of active RC networks.

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