R_o. It also depends on the corner frequency ω_1 of the open-loop gain, the dc open-loop gain A_0, and the feedback factor β. The value of inductance increases for increasing closed-loop gain. Reference [9] discusses in detail the effect of the finite frequency-dependent output impedance on single-amplifier active RC networks.

4) In active RC realizations using more than one OA such as the biquad [7], the effect of the output impedance will be to add additional poles and zeros, thus affecting the performance of the filter. This effect is under detailed investigation.

5) Since the above inductance is also proportional to R_o, adding a resistor at the output of the OA results in a higher value of inductance which is also adjustable. This may be used to simulate inductance. Theoretical and experimental aspects of such simulation of inductance are under investigation and will be reported shortly.

Acknowledgment

The authors wish to thank Prof. B. N. Narayana Iyengar, Department of Electrical Engineering, Indian Institute of Science, for providing the laboratory facilities and for his continuous encouragement. They also wish to thank Prof. J. Vithayathil for useful discussions and his critical appreciation of the work.

References

[1] G. Nieu, "Operational amplifiers act as universal gain elements," *Electron. Des.*, vol. 16, Jan. 18, 1968.
[2] G. S. Moschytz, "The operational amplifier in linear active networks," *IEEE Spectrum*, vol. 7, pp. 42–50, Jan. 1970.
[3] H. W. Bode, *Network Analysis and Feedback Amplifier Design*. Princeton, N.J.: Van Nostrand, 1945.
[4] E. Renschler, "The MC 1539 operational amplifier and its applications," Motorola Semiconductor Products Inc., Rep. AN-439.
[5] *Linear Integrated Circuits*, Signetics Corp., vol. I, 1972.
[6] R. Tarmy and M. S. Ghausi, "Very high-Q insensitive active RC networks," *IEEE Trans. Circuit Theory*, vol. CT-17, pp. 338–366, Aug. 1970.
[7] L. C. Thomas, "The biquad: Part I—Some practical design considerations," *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 350–357, May 1971.
—, "The biquad: Part II—A multipurpose active filtering system," *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 358–361, May 1971.
[8] A. Budak and D. M. Petrela, "Frequency limitations of active filters using operational amplifiers," *IEEE Trans. Circuit Theory*, vol. CT-19, pp. 322–328, July 1972.
[9] K. Soundararajan and K. Ramakrishna, "Effect of nonideal operational amplifiers on active filters," *IEEE Trans. Circuit Theory*, to be published.

A Versatile Active *RC* Building Block with Inherent Compensation for the Finite Bandwidth of the Amplifier

DAG ÅKERBERG AND KARE MOSSBERG

***Abstract*—An earlier paper by the authors describes an easily trimmed universal building block for active *RC* filters which possesses the valuable characteristic that, with suitable design, the *Q*-value can be made approximately independent of the gain-bandwidth product of the operational amplifiers. This makes the filter usable for high frequencies, while at the same time the dependence of the *Q*-value on temperature variations in the operational amplifiers is drastically reduced. Design formulas are presented, as well as comparative measurements which verify the theory. The building block is shown to have excellent characteristics both as a universal second-order building block and as a standard block for active ladder synthesis of bandpass filters.**

Manuscript received September 5, 1972; revised April 13, 1973. This work was supported in part by the Swedish Board for Technical Development and in part by the L. M. Ericsson Telephone Company.
D. Åkerberg (M'72) is with Svenska Radio AB, Stockholm, Sweden.
K. Mossberg is with the Department of Telecommunication Theory, Royal Institute of Technology, Stockholm, Sweden.

Introduction

ACTIVE RC building blocks of the type "three operational amplifiers in a ring" are widely known, from many points of view, as the best second-order networks available. Such building blocks are made commercially by several firms. Compared with other networks [2], these networks have low sensitivity to variations in active and passive components. The

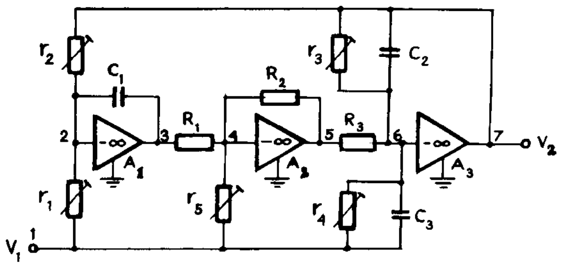

Fig. 1. Building block realizing a general biquadratic transfer function.

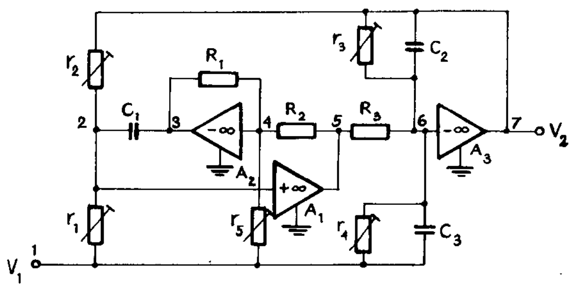

Fig. 2. Modified amplifier connection of Fig. 1 giving the same transfer function, but with improved stability.

best known of them is probably that reported by Kerwin *et al.* [3]. The variant earlier described by the authors [1],[1] and shown in Figs. 1 and 2, has, in addition, the valuable characteristic that frequency and attenuation—ω and σ in (4) *both* for poles and zeros—can be trimmed independently with four resistances, one for each parameter.

In practice, as is known, the amplifiers have a finite frequency-dependent gain

$$A \simeq -\frac{1}{A_0^{-1} + \dfrac{s}{\omega_1}} \tag{1}$$

where A_0 is the dc gain and ω_1 is the gain-bandwidth product in radians per second.

The frequency dependence of the amplifiers limits the usability of the filters at high frequencies, as the Q-value for the type of network under consideration becomes frequency dependent [2] according to the formula

$$Q_p = Q_{0p} \frac{1}{1 - k\, Q_{0p} \dfrac{\omega_{0p}}{\omega_1}} \tag{2}$$

where ω_{0p} is the pole frequency of the filter and Q_{0p} is the Q-value for ideal operational amplifiers; k is typically 4.

The Q-sensitivity with respect to the gain-bandwidth product ω_1 will be

$$S_{\omega_1}^{Q_p} = -kQ_{0p} \frac{\omega_{0p}}{\omega_1}. \tag{3}$$

Tarmy and Ghausi [4], using three operational amplifiers with dual outputs and inputs, have designed a network for which $k = 0$. Arbitrarily situated zeros, however, are not realized so simply with this network. However, $k = 0$ can also be obtained with our network by modification of the connection of the amplifiers in the manner earlier indicated [1], and is described as follows.

We present, accordingly, a unique building block for active filters, consisting of three single-ended operational amplifiers, which realizes arbitrarily situated poles and zeros. The frequency and attenuation of the poles and zeros can be trimmed independently with a resistance for each parameter. The Q-value is nearly independent of the limited bandwidth of the amplifiers and of variations in the bandwidth caused by temperature changes.

Calculation of Sensitivity and Design Formulas for Compensation of the Effect of the Limited Bandwidth of the Amplifiers

Fig. 1 shows the universal building block with traditional amplifier connections. Fig. 2 shows the network with modified amplifier connections achieved through nullator-norator equivalents, as described in [1].

The ideal transfer function, i.e., with infinite gain of the amplifiers, will be the same for the two networks:

$$\frac{V_2}{V_1} = K \frac{s^2 + 2\sigma_0 s + \omega_0^2}{s^2 + 2\sigma_p s + \omega_p^2} = -\frac{C_3}{C_2} \cdot \frac{s^2 + s\left(\dfrac{1}{r_4} - \dfrac{R_2}{r_5 R_3}\right)\dfrac{1}{C_3} + \dfrac{R_2}{r_1 R_1 R_3 C_1 C_3}}{s^2 + s\dfrac{1}{r_3 C_2} + \dfrac{R_2}{r_2 R_1 R_3 C_1 C_2}} \tag{4}$$

$$Q_p = \frac{\omega_p}{2\sigma_p}. \tag{5}$$

If the transfer function for frequency-dependent gains A_1, A_2, and A_3 is calculated according to (1), a fifth-order polynomial is obtained in s in the denominator. In [2] it is shown that, for high Q-values, the denominator can be approximated with good accuracy to a second-order polynomial by disregarding the s^5 and s^4 terms and substituting for s^3 the term

$$s^3 = -s\omega_{0p}^2. \tag{6}$$

For pole frequencies above 1 kHz, the gains for typical amplifiers can be approximated by

$$A_1 = -\frac{\omega_{11}}{s}, \quad A_2 = -\frac{\omega_{12}}{s}, \quad A_3 = -\frac{\omega_{13}}{s}. \tag{7}$$

Then, both for Figs. 1 and 2, the real pole frequency ω_p (ω_{0p} is the ideal value) for the typical values $R_1 = R_2 = R_3 = r_1 = r_2$, r_3 and $r_4 >> R_1$, and $C_1 = C_2$ will be

$$\omega_p \simeq \omega_{0p} \left(1 + \frac{2\omega_{0p}}{\omega_{11}} + \frac{\omega_{0p}}{\omega_{13}}\right)^{-1/2} \tag{8}$$

which gives the sensitivity

$$S_{\omega_1}^{\omega_p} \simeq \tfrac{3}{2} \frac{\omega_{0p}}{\omega_1}, \qquad \text{if } \omega_{11} = \omega_{13} = \omega_1. \tag{9}$$

The real Q-value for the network in Fig. 1 (Q_{0P} is the ideal value) will be

[1] Amplifier A_1 in [1, fig. 2] is turned the wrong way.

$$Q_p \simeq Q_{0p} \cdot \left[1 - Q_{0p}\,\omega_{0p}\left(\frac{1 + R_2/R_1 + R_2/r_5}{\omega_{12}} + \frac{1}{\omega_{11}} + \frac{1 + C_3/C_2}{\omega_{13}}\right)\right]^{-1}. \tag{10}$$

With $\omega_{11} = \omega_{12} = \omega_{13} = \omega_1$ we obtain

$$Q_p \simeq Q_{0p}\left[1 - kQ_{0p}\frac{\omega_{0p}}{\omega_1}\right]^{-1} \tag{2}$$

where

$$k = 3 + \frac{R_2}{R_1} + \frac{R_2}{r_5} + \frac{C_3}{C_2} > 3. \tag{11}$$

With $r_5 = \infty$, $C_3 = 0$, and $R_2 = R_1$, we get $k = 4$, which gives a typical sensitivity for conventional triple operational-amplifier networks:

$$S_{\omega_1}^{Q_p} = -4Q_{0p}\frac{\omega_{0p}}{\omega_1}. \tag{12}$$

As appears from (11), the sensitivity may be considerably greater in the event of unsuitable design.

For the network in Fig. 2,

$$Q_p \simeq Q_{0p}\left[1 + Q_{0p}\,\omega_{0p} \cdot \left(\frac{1 + R_1/R_2 + R_1/r_5}{\omega_{12}} - \frac{R_2/R_1}{\omega_{11}} - \frac{1 + C_3/C_2}{\omega_{13}}\right)\right]^{-1}. \tag{13}$$

The interesting point about (13) is that the terms in this case have different signs, so that—with suitable design of the *RC* components—the effects on Q_p of the various operational amplifiers can compensate for one another. This also results in a temperature compensation, as one may reckon that the gain-bandwidth products of the three operational amplifiers (monolithic) have practically the same temperature coefficient. With $\omega_{11} = \omega_{12} = \omega_{13} = \omega_1$, we get, according to (2),

$$k = R_2/R_1 + C_3/C_2 - R_1/R_2 - R_1/r_5. \tag{14}$$

Compensation is obtained for $k = 0$, in which case $S_{\omega_1}^{Q_p} = 0$.

The ordinary case without zeros, i.e., with $r_4 = r_5 = \infty$ and $C_3 = 0$, gives

$$k = 0, \quad \text{for } R_1 = R_2. \tag{15}$$

Another common case is zeros on the $j\omega$-axis, i.e., $r_4 = r_5 = \infty$, which gives

$$k = 0, \quad \text{for } \frac{C_3}{C_2} = \frac{R_1}{R_2} - \frac{R_2}{R_1}. \tag{16}$$

A suitable value of R_1/R_2 thus gives zero sensitivity. However, to obtain a signal level as uniform as possible in the network, R_1 must be equal to R_2, which conflicts with minimization of the sensitivity according to (16). If, however, a resistor R_{in} is placed between the amplifier input—node 4—and earth in accordance with Fig. 3, then ($r_5 = \infty$)

$$k = \frac{C_3}{C_2} + \frac{R_2}{R_1} - \frac{R_1}{R_2} - \frac{R_1}{R_{\text{in}}} \tag{17}$$

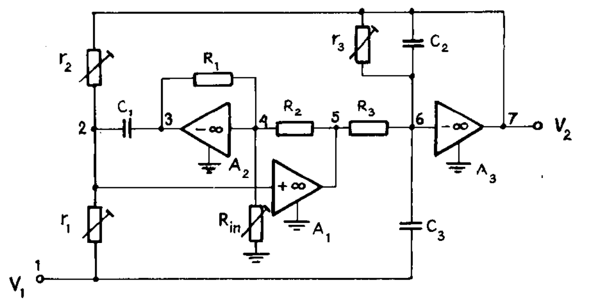

Fig. 3. Compensation by R_{in} providing equal peak voltages at nodes 3, 5, and 7.

and

$$k = 0, \quad \text{for } R_1 = R_2 \text{ and } \frac{C_3}{C_2} = \frac{R_1}{R_{\text{in}}}. \tag{18}$$

If one follows the given formulas for the design of $k = 0$, one gets for the network with the modified amplifier connection a Q-value with considerably smaller frequency dependence than for the ordinary network (Fig. 1), even if the amplifiers do not have exactly the same bandwidth.

If one wishes to compensate more exactly for the differences in bandwidth of the amplifiers, the network can be trimmed with R_{in} up to the limit for self-oscillation with r_3 disconnected.

Comment

The network in Fig. 1 can be compensated for by connecting a small capacitor c across R_1 or R_3, as proposed by Thomas [5]. If $R_1 = R_2 = R_3 = r_2 = R$, $C_1 = C_2 = C$ (suitable standard design), and $r_5 = r_4 = \infty$, then, for $\omega_{11} = \omega_{12} = \omega_{13} = \omega_1$,

$$k = 4 + \frac{C_3}{C} - \frac{c\omega_1}{C\omega_{0p}} \tag{19}$$

and

$$k = 0, \quad \text{for } c = C\left(4 + \frac{C_3}{C}\right)\frac{\omega_{0p}}{\omega_1}. \tag{20}$$

As c and ω_1 differ in their temperature dependence, however, this compensation provides no temperature compensation, which is provided by the network in Fig. 2.

RESULTS OF MEASUREMENTS

The results shown below are some of those obtained from more extensive measurements and calculations reported in [6].

Fig. 4 shows the drastic difference in temperature stability between the networks in Figs. 1 and 2. Both networks are designed for $Q_{0p} = 50$, $f_{0p} = \omega_{0p}/2\pi = 10$ kHz, and with a zero on the $j\omega$-axis at 7.5 kHz, which gives $R_1 = R_2 = R_3 = r_2 = 10$ kΩ, $r_1 = 18$ kΩ, $r_3 = 500$ kΩ, $C_1 = C_2 = C_3 = 1.5$ nF, and $r_4 = r_5 = \infty$. The amplifiers used are the μA 741 with $f_1 = \omega_1/2\pi \simeq 500$ kHz. The ordinary network (Fig. 1), without compensation according to (20), would be unconditionally unstable according to (2) and (11) for pole frequencies:

$$f_{0p} > \frac{f_1}{5Q_{0p}} = 2 \text{ kHz}. \tag{21}$$

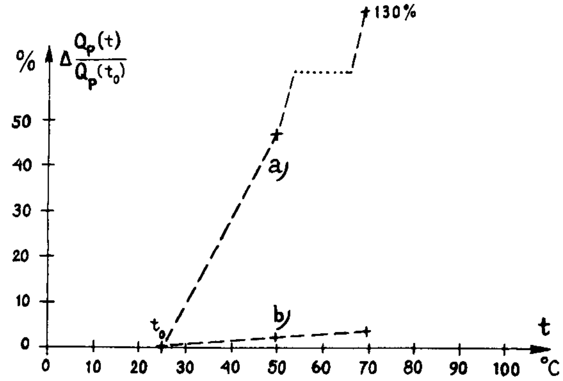

Fig. 4. Temperature dependence of networks with a pole pair at 10 kHz, $Q_p(t_0) = 50$, and a zero pair on the $j\omega$-axis at 7.5 kHz. a) Network according to Fig. 1 and compensated with a capacitor c according to (20). b) Network and compensation according to Fig. 3 and (18).

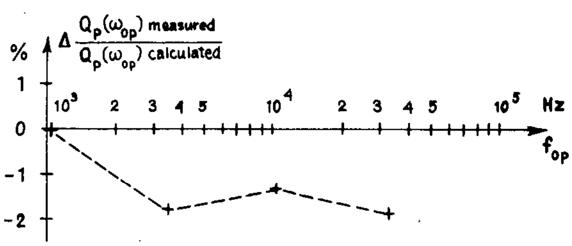

Fig. 5. Change in Q-value in percent when $f_{0p} (= \omega_{0p}/2\pi)$ is changed from 1 to 30 kHz by changing the capacitors.

The network has therefore been compensated for according to (20) for $f_{0p} = 10$ kHz, which gives

$$c \simeq 150 \text{ pF}.$$

The network with modified amplifier connection has been compensated for as shown in Fig. 3 [(18)], which ideally would give

$$R_{\text{in}} \simeq 10 \text{ k}\Omega.$$

Measurement of the bandwidth of the operational amplifiers, however, showed that the choice should be

$$R_{\text{in}} = 7.9 \text{ k}\Omega.$$

Fig. 5 shows how good the compensation can be for the network in Fig. 3. On variation of the pole frequency (replacement of C_1, C_2, and C_3) from 1 to 30 kHz, Q_p varied by less than 2 percent.

Comment

The network in Fig. 2 requires stricter compensation of the operational amplifiers A_1 and A_2 than the network in Fig. 1 to be stable from the high-frequency aspect. This is because A_1 and A_2 in Fig. 2 form part of a closed loop without any of the amplifiers being fed back with a capacitor which short-circuits at high frequencies (cf., Fig. 1). Practical tests show that most makes of the amplifier type 741, e.g., Fairchild, SG, and SGS, are sufficiently compensated for the network in Fig. 2. The Texas Instruments amplifier functions very well in Fig. 1, but in Fig. 2 it produced throughout a high-frequency oscillation of about 1 MHz on node 3 and 5. The high-frequency oscillation is limited in amplitude by the slew rate of the amplifiers, and does not appear to disturb the function of the filter to any great extent. The high-frequency oscillation is generally not observable on the output–node 7–as it is heavily damped by C_2.

At high Q-values and high pole frequencies, the networks often have a stable and an unstable mode. In their unstable mode the filter self-oscillates with maximum amplitude at a frequency close to the pole frequency, probably due to nonlinear effects in the amplifiers. In this case, the network in Fig. 2 performs better than that in Fig. 1. Self-oscillation may arise when the main voltage is switched on, or in the case of too high an input signal to the filter. By connecting in Fig. 2 a couple of silicon diodes in parallel, but with opposite polarity, across the input of amplifier A_2 and/or A_1, a temporarily too high input signal can be prevented without affecting the normal function of the filter. Practical trials show that the unstable mode is entirely eliminated in the network in Fig. 2 with μA 741 amplifiers at, for example, $Q = 50$, $f_p = 10$ kHz, in which a couple of silicon diodes have been connected across the input of A_2. For further details on this point, see [2] and [6].

The usual method of realizing active filters is the cascade connection of second-order networks. It is known, however [7], [8], that no cascade connection can compete with resistively matched double-terminated LC networks with regard to the sensitivity to variations in passive components. This applies especially to higher order bandpass filters. Solutions for active RC bandpass filters must therefore be sought among structures which simulate currents and voltages in a resistively terminated LC network [9]. We have developed a couple of such methods using the network of Fig. 2 as a building block with excellent performance [7].

Acknowledgment

The authors wish to thank Prof. L. H. Zetterberg for his encouragement and support in this work, and K. Engström for his practical design and measurements.

References

[1] D. Åkerberg and K. Mossberg, "Low sensitive easily trimmed standard building block for active RC-filters," *Electron. Lett.*, vol. 5, pp. 528–529, 1969.

[2] D. Åkerberg, "Comparison of methods for active RC synthesis. Part III–Measurements and extended theory," Telecommunication Theory, Royal Inst. Technol., Stockholm, Sweden, Tech. Rep. 31, Sept. 1970.

[3] W. J. Kerwin, L. P. Huelsman, and R. W. Newcomb, "State-variable synthesis for insensitive integrated circuit transfer function," *IEEE J. Solid-State Circuits*, vol. SC-2, pp. 87–92, Sept. 1967.

[4] R. Tarmy and M. S. Ghausi, "Very high Q insensitive active RC networks," *IEEE Trans. Circuit Theory*, vol. CT-17, pp. 358–367, Aug. 1970.

[5] L. C. Thomas, "The Biquad: Part I–Some practical design considerations," *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 350–357, May 1971.

[6] K. Engström, "DIG-nät med kommersiella operationsförstärkare," Telecommunication Theory, Royal Inst. Technol., Stockholm, Sweden, Tech. Rep. 42, Jan. 1971.

[7] D. Åkerberg and K. Mossberg, "Comparison of methods for active RC synthesis. Part IV–High order networks," Telecommunication Theory, Royal Inst. Technol., Stockholm, Sweden, Tech. Rep. 47.

[8] H. J. Orchard, "Inductorless filters," *Electron. Lett.*, vol. 2, pp. 224–225, June 1966.

[9] F. E. J. Girling and E. F. Good, "The leapfrog or active ladder synthesis," *Wireless World*, pp. 341–345, July 1970.