Parcial #2

① Ejemplo 3,3 libro de Ogata:

- Función de transferencia: $G(s) = \frac{Y(s)}{U(s)} = \frac{bs+k}{ms^2+bs+k}$

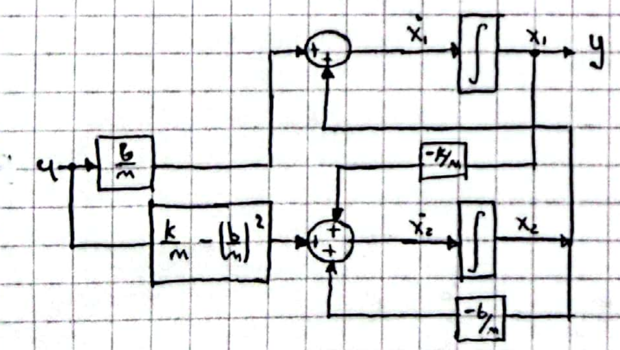
- Espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b/m \\ \frac{k}{m} - \left(\frac{b}{m}\right)^2 \end{bmatrix} u$$

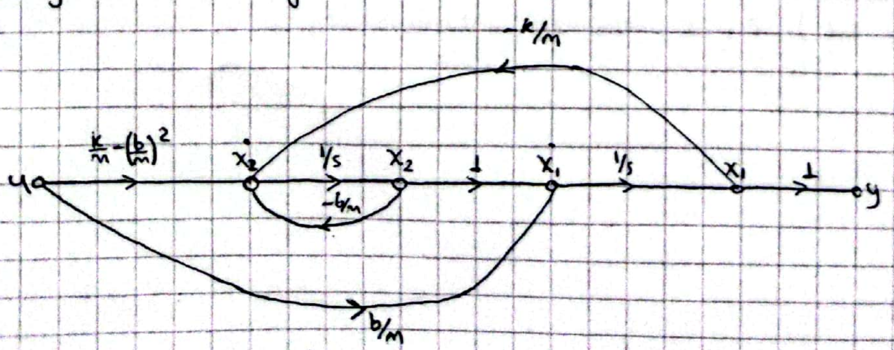
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- $\dot{x}_1 = x_2 + \frac{b}{m} u$
- $\dot{x}_2 = -\frac{k}{m} x_1 - \frac{b}{m} x_2 + \left[\frac{k}{m} - \left(\frac{b}{m}\right)^2\right] u$

- Diagrama de bloques



- Diagrama de flujo



1.1 Ejercicio A-3,9 libro de ogata.

• Función de transferencia: $G(s) = \frac{Y(s)}{U(s)} = \frac{k}{Js^2 + Bs}$

$\rightarrow Y(s)(Js^2 + Bs) = kU(s)$

• Transformada inversa

$$J\ddot{y} + B\dot{y} = ku$$

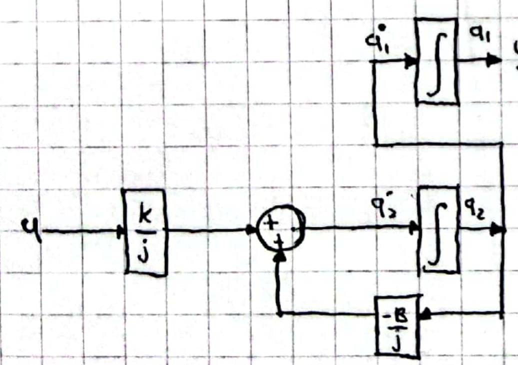
$q_1 = y$

$q_2 = \dot{y} = \dot{q}_1 \rightarrow \dot{q}_2 = -\frac{B}{J}q_2 + \frac{k}{J}u$

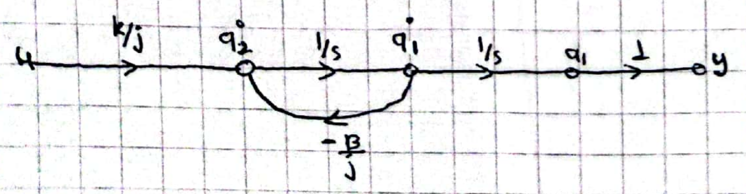
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k}{J} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

• Diagrama de bloques.



• Diagrama de flujo.



1.2 Ejercicio 2,23 libro Norman Nise.

- Función de transferencia: $G(s) = \frac{\Theta_L(s)}{E_a(s)} = \frac{0,0417}{s^2 + s1,667}$

- $\Theta_L(s)(s^2 + 1,667s) = 0,0417 \cdot E_a(s)$

$$\ddot{\Theta}_L + 1,667\,\dot{\Theta}_L = 0,0417\,E_a$$

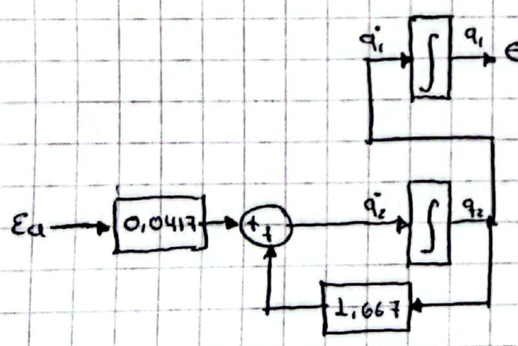
$q_1 = \Theta_L$

$q_2 = \dot{\Theta}_L = \dot{q}_1 \longrightarrow \dot{q}_2 = 0,0417 \cdot E_a - 1,667 \cdot q_2$

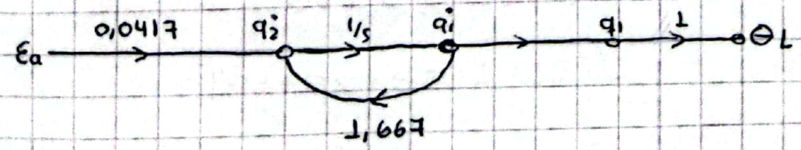
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1,667 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0,0417 \end{bmatrix} E_a$$

$$\Theta_L = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

- Diagrama de bloque.



- Diagrama de flujo.



1,3 Diferencias entre planteamientos ejercicios 2 y 3 (1,1 y 1,2)

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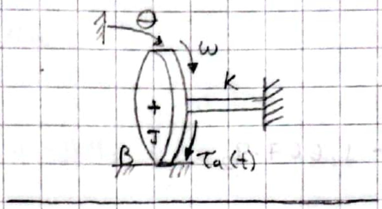
② Para el sistema rotacional en la figura, determine:

a. Representación en espacio de estados

b. Diagrama de bloques y flujo de señal

c. Función de transferencia



- $\sum F = Ja = J\ddot{\theta}$

$$\tau - k\theta - b\dot{\theta} = J\ddot{\theta} \rightarrow \ddot{\theta} = \frac{\tau}{J} - \frac{k}{J}\theta - \frac{b}{J}\dot{\theta}$$

$q_1 = \theta$

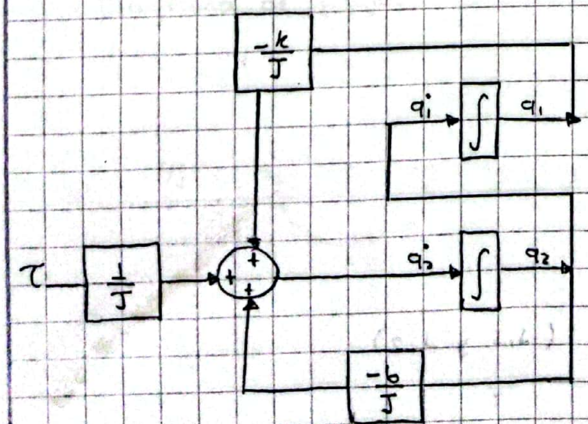
$q_2 = \dot{\theta} = \dot{q}_1$

$$\dot{q}_2 = \frac{\tau}{J} - \frac{k}{J}q_1 - \frac{b}{J}q_2$$

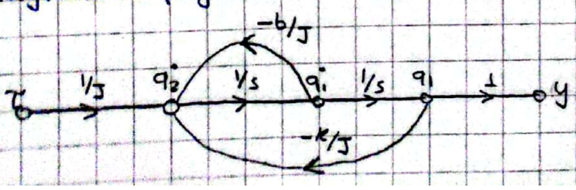
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{J} & -\frac{b}{J} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} \tau$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

- Diagrama de bloques



- Diagrama de flujo



- Función de transferencia

$\tau - k\theta - b\dot{\theta} = J\ddot{\theta}$

$\tau = J\ddot{\theta} + k\theta + b\dot{\theta}$

$\mathcal{L}[\tau] = \mathcal{L}[J\ddot{\theta} + k\theta + b\dot{\theta}]$

$\tau(s) = Js^2\theta(s) + bs\theta(s) + k\theta(s)$

$\tau(s) = \theta(s)[Js^2 + bs + k]$

$$\frac{\theta(s)}{\tau(s)} = \frac{1}{Js^2 + bs + k} = G(s)$$

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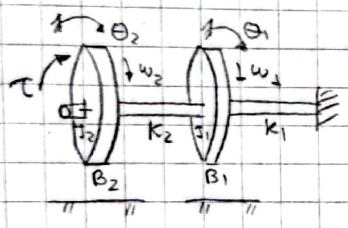
3. Para el sistema rotacional en la figura, asuma $\theta_2 > \theta_1$ y determine

a. La función de transferencia relacionando θ_2 y τa.

b. La representación en espacio de estados

c. Diagrama de bloques y flujo de señal todo en término de θ_2



* Para el disco 2

$$\Sigma F = J_2 \ddot{\theta}_2$$

$$\tau - B_2\dot{\theta}_2 - k_2(\theta_2 - \theta_1) = J_2 \ddot{\theta}_2$$

* Para el disco 1

$$\Sigma F = J_1 \ddot{\theta}_1$$

$$-B_1\dot{\theta}_1 + k_2(\theta_2 - \theta_1) - k_1\theta_1 = J_1\ddot{\theta}_1$$

- $\ddot{\theta}_2 = \frac{\tau}{J_2} - \frac{b_2}{J}\dot{\theta}_2 - \frac{k_2}{J_2}\theta_2 + \frac{k_2\theta_1}{J_2}$

- $\ddot{\theta}_1 = \frac{k_2\theta_2}{J_1} - \left(\frac{k_2 + k_1}{J_1}\right)\theta_1 - \frac{b_1}{J_1}\dot{\theta}_1$

• Función de transferencia

Disco 2 • $\tau - B_2\dot{\theta}_2 - k_2\theta_2 + k_2\theta_1 = J_2\ddot{\theta}_2$

$$\tau = J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + k_2\theta_2 - k_2\theta_1 \rightarrow \mathcal{L}[\tau] = \mathcal{L}[J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + k_2\theta_2 - k_2\theta_1]$$

$$① \quad \tau(s) = J_2 s^2 \theta_2(s) + B_2 s\theta_2(s) + k_2\theta_2(s) - k_2\theta_1(s)$$

Disco 1 • $-B_1\dot{\theta}_1 + k_2(\theta_2 - \theta_1) - k_1\theta_1 = J_1\ddot{\theta}_1 \rightarrow J_1\ddot{\theta}_1 + k_2\theta_1 + k_1\theta_1 + B_1\dot{\theta}_1 = k_2\theta_2$

Laplace: $J_1 s^2\theta(s) + k_2\theta_1(s) + k_1\theta_1(s) + B_1 s\theta_1(s) = k_2\theta_2(s)$

$$② \quad \theta_1(s) = \frac{k_2\theta_2(s)}{J_1 s^2 + B_1 s + (k_1 + k_2)}$$

• Remplazamos ② en ①

$$\tau(s) = J_2 s^2\theta_2(s) + B_2 s\theta_2(s) + k_2\theta_2(s) - \frac{k_2^2\theta_2(s)}{J_1 s^2 + B_1 s + (k_1 + k_2)}$$

- Representación en espacio de estados en términos de Θ_2

  - Con función de transferencia multiplicamos y hacemos laplace inversa

$$\Theta_2(s)\,(J_2s^2+B_2s+k_2)(J_1s^2+B_1s+(k_1+k_2))-k_2^2 = \tau(s)\,(J_1s^2+B_1s+(k_1+k_2))$$

$$J_1J_2\ddddot{\Theta}_2+(J_2b_1+J_1b_2)\dddot{\Theta}_2+(J_2k_1+J_2k_2+J_1k_2+b_1b_2)\ddot{\Theta}_2+(b_2k_1+b_2k_2+b_1k_2)\dot{\Theta}_2+\dots+$$

$$+\Theta_2k_1k_2 = J_1\ddot{\tau}+b_1\dot{\tau}+(k_1+k_2)\tau$$

$$\cdot\frac{1}{J_1\cdot J_2}\rightarrow \ddddot{\Theta}_2+\left(\frac{b_1}{J_1}+\frac{b_2}{J_2}\right)\dddot{\Theta}_2+\left(\frac{k_1}{J_1}+\frac{k_2}{J_1}+\frac{k_2}{J_2}+\frac{b_1b_2}{J_1J_2}\right)\ddot{\Theta}_2+\left(\frac{b_2k_1}{J_1J_2}+\frac{b_2k_2}{J_1J_2}+\frac{b_1k_2}{J_1J_2}\right)\dot{\Theta}_2+$$

$$\left(\frac{k_1k_2}{J_1J_2}\right)\Theta_2=\frac{\ddot{\tau}}{J_2}+\frac{b_1}{J_1J_2}\dot{\tau}+\frac{(k_1+k_2)}{J_1J_2}\tau$$

- Para los coeficientes

$$a_1=\frac{b_1}{J_1}+\frac{b_2}{J_2}$$

$$a_2=\frac{k_1+k_2}{J_1}+\frac{k_2}{J_2}+\frac{b_1b_2}{J_1J_2}$$

$$a_3=\frac{b_2k_1}{J_1J_2}+\frac{b_2k_2}{J_1J_2}+\frac{b_1k_2}{J_1J_2}$$

$$a_4=\frac{k_1k_2}{J_1J_2}$$

$$b_0=0$$

$$b_1=0$$

$$b_2=\frac{1}{J_2}$$

$$b_3=\frac{b_1}{J_1J_2}$$

$$b_4=\frac{(k_1+k_2)}{J_1J_2}$$

- $B_0 = b_0 = 0$

$$B_1 = b_1 - a_1B_0 = 0$$

$$B_2 = b_2 - a_1B_1 - a_2B_0 = \frac{1}{J_2}$$

$$B_3 = b_3 - a_1B_2 - a_2B_1 - a_3B_0 = \frac{b_1}{J_1J_2} - \left(\frac{b_1}{J_1}+\frac{b_2}{J_2}\right)\frac{1}{J_2}$$

$$B_4 = b_4 - a_1B_3 - a_2B_2 = \frac{k_1+k_2}{J_1J_2} - \left[\frac{b_1}{J_1}+\frac{b_2}{J_2}\right]\left[\frac{b_1}{J_1J_2}-\frac{1}{J_2}\left(\frac{b_1}{J_1}+\frac{b_2}{J_2}\right)\right]-\frac{1}{J_2}\left[\frac{k_1+k_2}{J_1}+\frac{k_2}{J_2}+\frac{b_1b_2}{J_1J_2}\right]$$

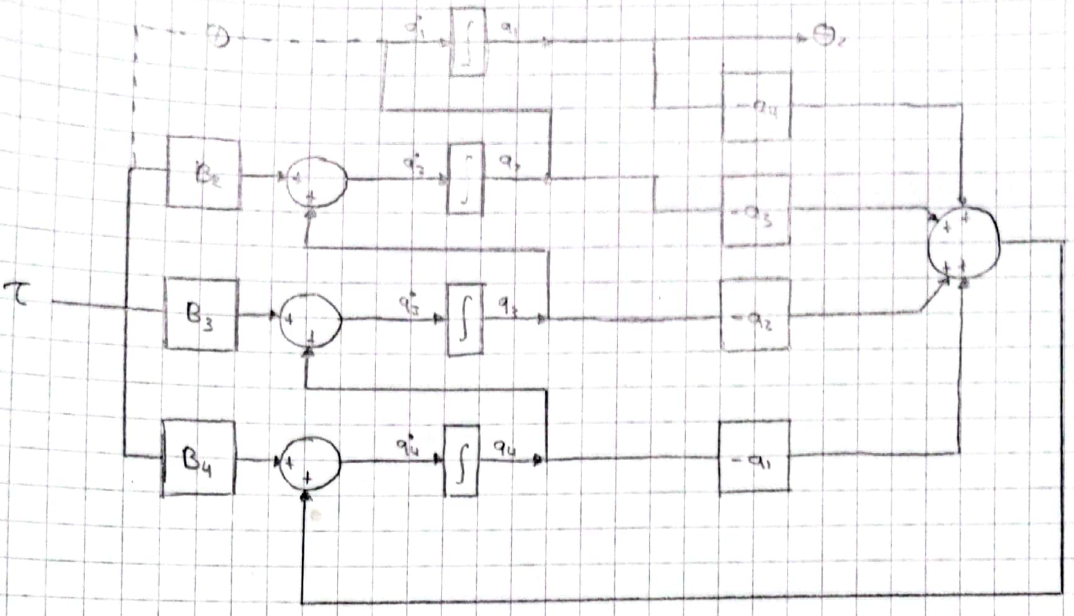
- $q_1=\Theta_2$
  $q_2=\dot{\Theta}_2=\dot{q}_1$
  $q_3=\ddot{\Theta}_2=\dot{q}_2$
  $q_4=\dddot{\Theta}_2=\dot{q}_3$

$$\longrightarrow \quad \dot{q}_1=q_2+B_1u \qquad \dot{q}_2=q_3+B_2u \qquad \dot{q}_3=q_4+B_3u$$

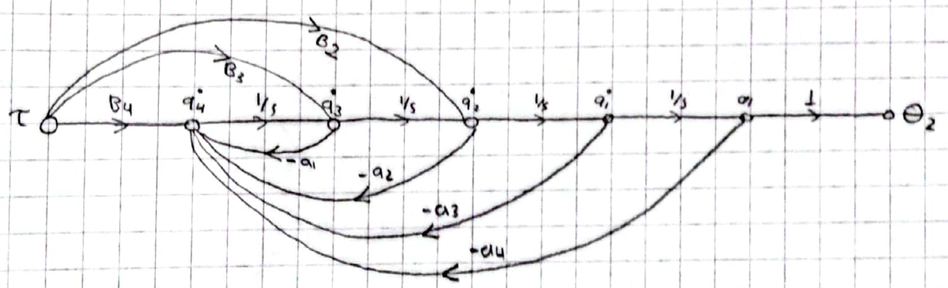
$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-a_4&-a_3&-a_2&-a_1\end{bmatrix}\cdot\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}+\begin{bmatrix}B_1\\B_2\\B_3\\B_4\end{bmatrix}\tau$$

$$\Theta_2=\begin{bmatrix}1&0&0&0\end{bmatrix}\begin{bmatrix}q_1\\q_2\\q_3\\ \dots\end{bmatrix}$$

- Diagrama de bloque



- Diagrama de flujo de señal



④ Punto 3 considerando $k_1 = 0$

$$\text{Con } k_1 = 0 \longrightarrow J_1\ddot{\theta}_1 = k_2(\theta_2 - \theta_1) - B_1\dot{\theta}_1 \quad ①$$

$$J_2\ddot{\theta}_2 = \tau_a - k_2(\theta_2 - \theta_1) - B_2\dot{\theta}_2 \quad ②$$

- Función de transferencia

$$\frac{\theta_2(s)}{\tau(s)} = \frac{J_1 s^2 + B_1 s + k_2}{(J_2 s^2 + B_2 s + k_2)\ (J_1 s^2 + B_1 s + k_2)}$$

- Para coeficientes

$$C_1 = \frac{b_1}{J_1} + \frac{b_2}{J_2}$$

$$C_2 = \frac{k_2}{J_1} + \frac{k_2}{J_2} + \frac{b_1 b_2}{J_1 J_2}$$

$$C_3 = \frac{b_2 k_2}{J_1 J_2} + \frac{b_1 k_2}{J_1 J_2}$$

$$C_4 = 0$$

$$D_0 = 0$$

$$D_1 = 0$$

$$D_2 = \frac{1}{J_2}$$

$$D_3 = \frac{b_1}{J_1 J_2}$$

$$D_4 = \frac{k_2}{J_1 J_2}$$

$$E_0 = D_0 = 0$$

$$E_1 = D_1 - C_1 D_0$$

$$E_2 = D_2 - C_1 E_1 - C_2 E_0 = \frac{1}{J_2}$$

$$E_3 = D_3 - C_1 E_2 - C_2 E_1 - C_3 E_0 = \frac{b_1}{J_1 J_2} - \left(\frac{b_1}{J_1} + \frac{b_2}{J_2}\right)\frac{1}{J_2}$$

$$E_4 = D_4 - C_1 E_3 - C_2 E_2 =$$

$$\frac{k_2}{J_1 J_2} - \left[\frac{b_1}{J_1} + \frac{b_2}{J_2}\right]\left[\frac{b_1}{J_1 J_2} - \frac{1}{J_2}\left(\frac{b_1}{J_1} + \frac{b_2}{J_2}\right) - \frac{1}{J_2}\left(\frac{k_2}{J_1} + \frac{k_2}{J_2} + \frac{b_1 b_2}{J_1 J_2}\right)\right]$$

- Representación variable de estados.

$$q_1 = \theta_2$$
$$q_2 = \dot{\theta}_2 = \dot{q}_1 \longrightarrow \dot{q}_1 = q_2 + E_1 u$$
$$q_3 = \ddot{\theta}_2 = \dot{q}_2 \qquad \dot{q}_2 = q_3 + E_2 u$$
$$q_4 = \dddot{\theta}_2 = \dot{q}_3 \qquad \dot{q}_3 = q_4 + E_3 u$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -C_3 & -C_2 & -C_1 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} \tau$$

$$\theta_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

- Diagrama de bloques.