Notes of Optical design and instrumentation

Wyant College of Optical Sciences
University of Arizona

Nicolás Hernández Alegría

Preface

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Contents

List of Figures

List of Tables

Listings

Part I

Introduction to Geometrical Optics principles

Chapter 1

Concepts of optics

1.1 Stops and pupils

1.1.1 Aperture stop

The **aperture stop** is a physical/real surface that limits the cone of light entering and exiting the optical system.

- The **entrance pupil** (EP) is the image of the stop in the object space.
- The **exit pupil** (XP) is the image of the spot in the image space.

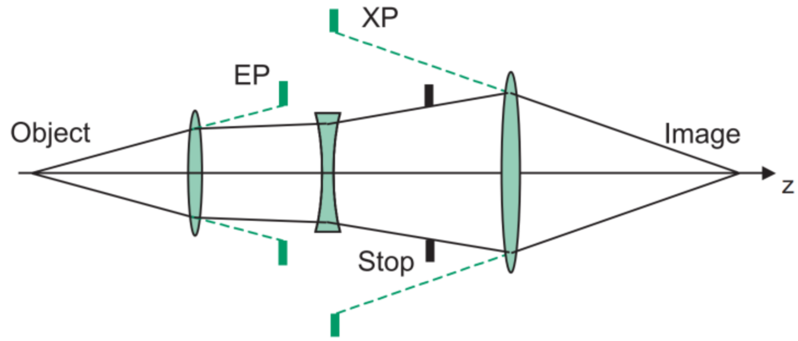


Figure 1.1 The stop limits the cone of light, and its image in object (image) space creates the entrance (exit) pupil.

There is a stop or pupil in each optical space. Intermediate pupils are formed in other spaces. There are two methods to determine which aperture in a system serves as the system stop:

a) Image each potential stop into object space. The pupil with the **smallest** angular size corresponds to the stop. The same can be done in image space.

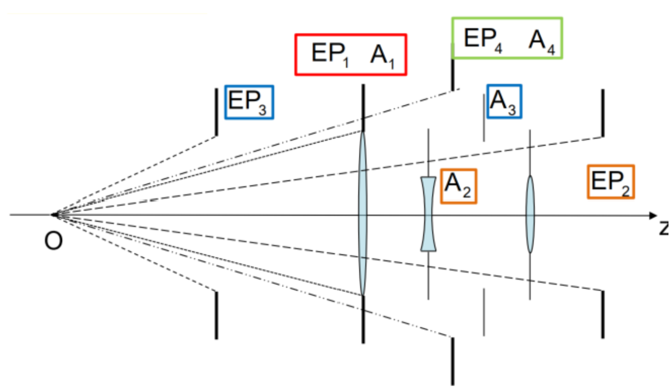


Figure 1.2 The smallest angular size corrsponds to the stop in object space. Same for image space.

b) Trace a ray through the system from the axial object point with arbitrary initial angle. At each potential stop, determine the ratio of the aperture radio a_k to the ray height at that surface $\tilde{y}_k$.

$$\text{Aperture stop} = \min \left\{ \left| \frac{a_k}{\tilde{y}_k} \right| \right\}. \tag{1.1}$$

The pupils are the image of the stop and do not change position or size with an off-axis object. Intermediate pupils are formed in each optical space for multi-element systems. If there are N elements, there are $N + 1$ pupils (including the stop).

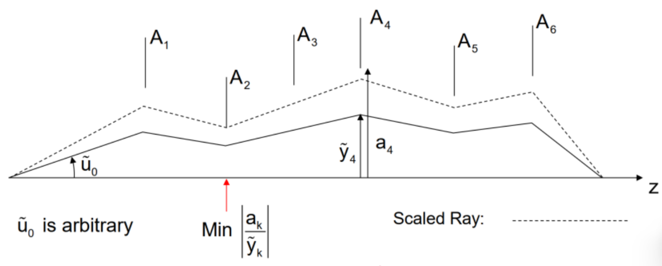


Figure 1.3 The minimum slope value corresponds to the aperture stop.

> When designing a system, is usually critical that the stop surface does not change over a range of possible object positions that the system will be used with.

1.1.2 Marginal and Chief rays

Rays confined to the yz-plane are called **meridional rays**. There are two special meridional rays that define properties of the object, images and pupils:

- The **marginal ray** travels from the base of the object to the edge of EP . It defines image locations and pupil sizes.
- The **chief ray** travels from the edge of the object to the center of the EP. It defines image heights and pupil locations.

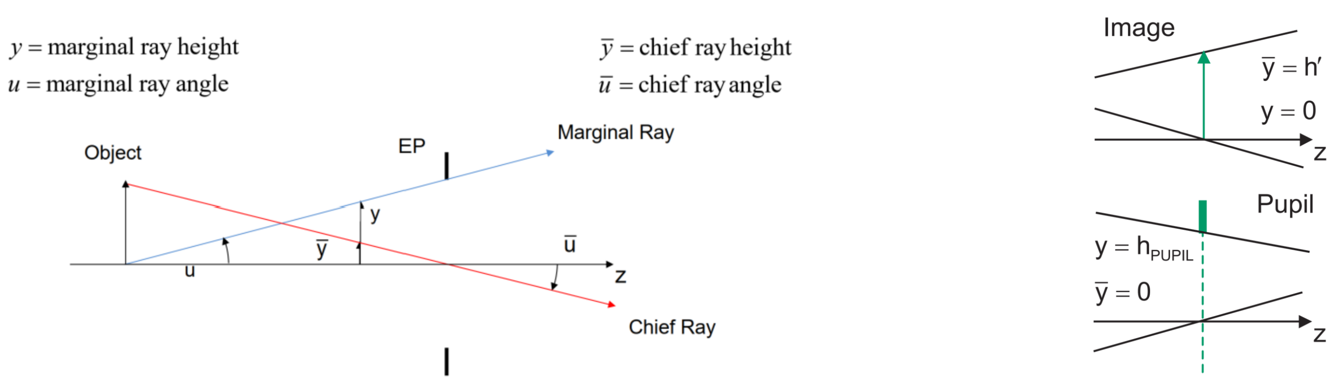


Figure 1.4 The

The heights of the marginal ray and the chief ray can be evaluated at any z in any optical space. When the marginal ray crosses the axis, an image is located, and the size of the image is given by the chief ray height. Whenever the chief ray crosses the axis, a pupil or stop is located, and the pupil radius is given by the marginal ray height. Intermediate images and pupils are often virtual.

1.1.3 Pupil locations

By raytrace

Once you know which surface is the stop, you have the information to determine the location of EP and XP. The **pupil locations** can be found by tracing a paraxial ray starting at the center of the stop and is back/forward propagated. The intersections of this ray with the axis in object and image space determine the locations of EP and XP.

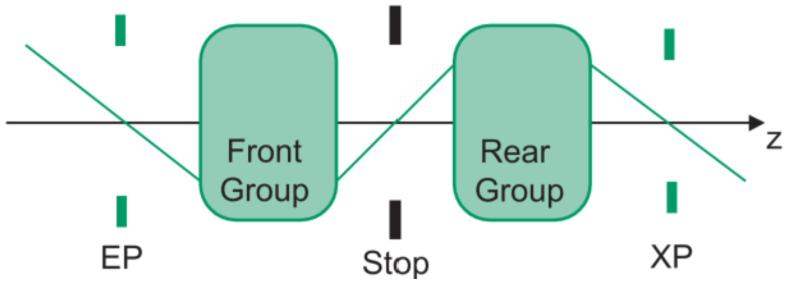
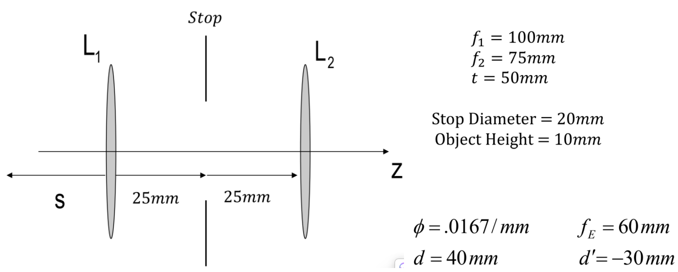


Figure 1.5 The

This ray become the chief ray when it is scaled to the object or image size. The marginal ray gives the pupil sizes.

Ejemplo 1.1 **Pupil location by paraxial raytrace**



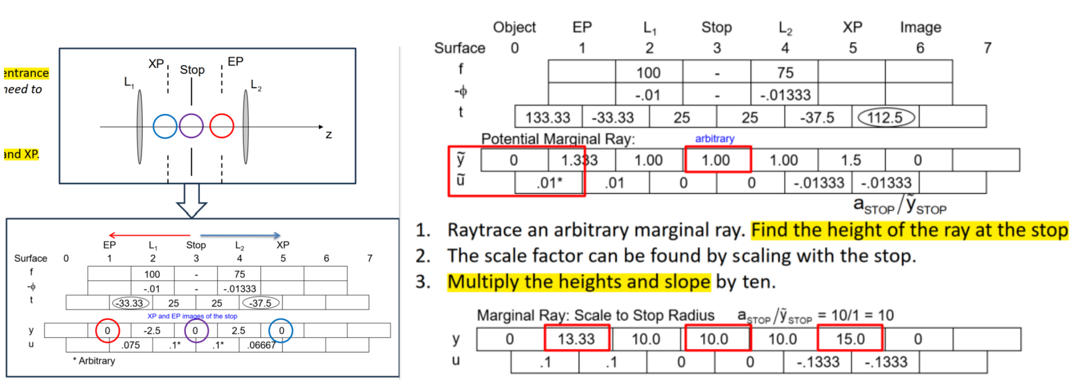
Solution

The stop is a real object for the formation of both EP and XP. There is a ray that has a height of 0 at the EP, stop and XP. We first set $y = 0$ at the stop, and then with arbitrary angle

EP we set $y = 0$ for the EP and solve for the distance.

XP we set $y = 0$ for the XP and solve for the distance.

We used a potential chief to find pupil locations. For the pupil sizes, we find the true marginal ray scaling a potential maginal ray. Remember that the chief ray was for pupil locations, now with the marginal ray we find the pupil sizes. We can also use it to find the image location.



Finally, the height of the EP is $13.33\ mm$, the stop $10\ mm$, the XP $15\ mm$.

By Gaussian imagery

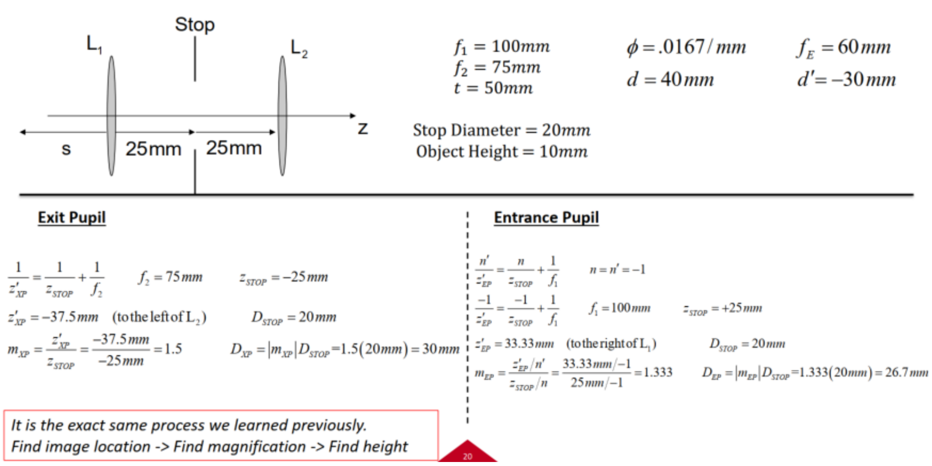
We treat each group independently, considering the stop as our object propagating in the direction of te given group. For EP, the object propagates from right to left, so we flip the sign of the refractive index (as in reflection).

Figure 1.6

$$\text{For XP} \quad \frac{n'}{z'_{XP}} = \frac{n}{Z_{stop}} + \frac{1}{f_{RG}}, \quad m_{XP} = \frac{z'_{XP}}{z_{stop}}, \quad D_{XP} = |m_{XP}| D_{stop} \tag{1.2}$$

$$\text{For EP} \quad \frac{n'}{Z'_{EP}} = \frac{n}{Z_{stop}} + \frac{1}{f_{FG}}, \quad m_{EP} = \frac{z'_{EP}}{z_{stop}}, \quad D_{EP} = |m_{EP}| D_{stop} \quad (n = n' = -1) \tag{1.3}$$

Ejemplo 1.2 — Pupil locations by Gaussian imagery

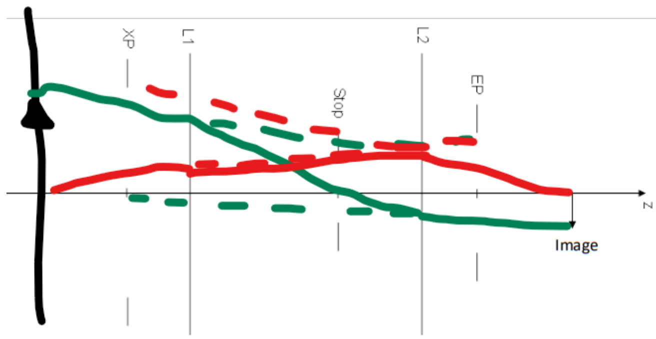


EP,STOP,XP are invariant to object location

Changing the object location does not change the position of the EP, stop, and XP.

Ejemplo 1.3 — Raytrace of chief and marginal rays

The following diagram illustrate the trace by hand of the chief and marginal ray by definition.



1.1.4 Lagrange invariant

The linearity of paraxial optics provides a relationship between the heights and angles of any two ray s propagating through the system. The **Lagrande invariant** Ξ is formed with the paraxial marginal and chief rays:

$$\text{Lagrange invariant} \qquad \Xi = n\bar{u}y - nu\bar{y} = \bar{\omega}y - \omega\bar{y}\,. \tag{1.4}$$

It is invariant for refraction and transferm and it can be evaluated at any z in any optical space. The Lagrange invariant is particularly simple at images or objects ($y = 0$) and pupils ($\bar{y} = 0$):

$$\text{Image/Object} \qquad y = 0, \quad \Xi = -nu\bar{y} = -\omega\bar{y} \tag{1.5}$$

$$\text{Pupils} \qquad \bar{y} = 0, \quad \Xi = n\bar{u}y = \bar{\omega}y \tag{1.6}$$

If two rays other than the marginal and chief are used, the more general **optical invariant** I is formed.

Given two rays, a third ray can be formed as a linear combination of the two rays. The coefficients are the ratios of the pair-wise invariantas of the values for the three rays at some initial z. The expressions are valid for any z:

$$y_3 = Ay_1 + By_2, \quad u_3 = Au_1 + Bu_2 \tag{1.7}$$

$$A = I_{32}/I_{12}, \quad B = I_{13}/I_{12}, \quad I_{ij} = nu_iy_j - nu_jy_i. \tag{1.8}$$

Changing the Lagrange invariant of a system **scales** the optical system. Doubling the invariant while maintaining the same object and image sizes and pupil diameters haves all of the axial distances (and the focal length).

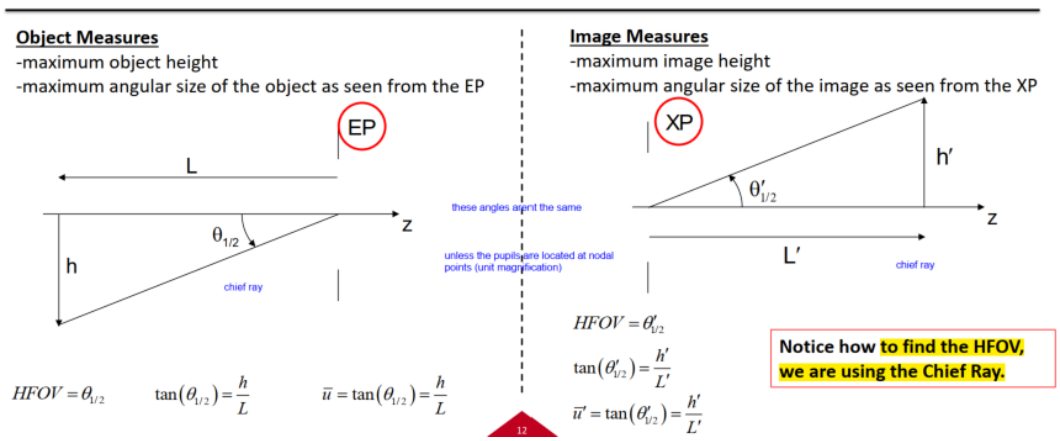
The **throughout**, **etedue** of $A\Omega$ **product** in **radiometry** and **radiative transfer** are relalted to the square of the Lagrange invariant:

$$n^2 A\Omega = \pi^2 \Xi^2. \tag{1.9}$$

1.1.5 Field of view

We revisit again the concept of FOV but know using the EP and XP.

- **Field of view FOV** diameter of the object/image.
- **Half field of view HFOV** radius of the object/image.



1.1.6 Numerical aperture and F-number

In an optical space of index n_k, the **numerical aperture** N_A describes the axial cone of light in terms of the real marginal angle U_k:

$$\text{Numerical aperture} \qquad \boxed{NA = n_k|sinU_k| \approx n_k|u_k|}\,. \tag{1.10}$$

The **F-number** $f/\#$ describes the image-space cone of light for an object **at infinity**:

$$\text{F-number} \qquad \boxed{f/\# = \frac{f_E}{D_{EP}}}\,. \tag{1.11}$$

While the $f/\#$ is an image-space, infinite-conjugate measure, the approximate relationship between NA and $f/\#$ allows and $f/\#$ to be defined for other optical spaces and conjugates. As a result, an $f/\#$ can be defined for any cone of light. This $f/\#$ is called **working F-number** $f/\#_W$. This previous relationship becomes a definition

$$\text{Working F-number} \qquad f/\#_W = \frac{1}{2NA} \approx \frac{1}{2n|u|} = (1-m)f/\#. \tag{1.12}$$

Fast optical system have small numeric values for the $f/\#$. Most lenses with adjustable stops have $f/\#$ of **f-stops** labeled in increments of $\sqrt{2}$. The usual progression is:

$$f/1.4, \quad f/2, \quad f/2.8, \quad f/4, \quad f/5.6, \quad f/8, \quad f/11, \quad f/16, \quad f/22, \quad \text{etc.}$$

Each stop changes the area of the EP (light colelction ability) by a factor of 2.

The Lagrange invariant relates the magnification between two pupils to the chief ray angles at the pupils.

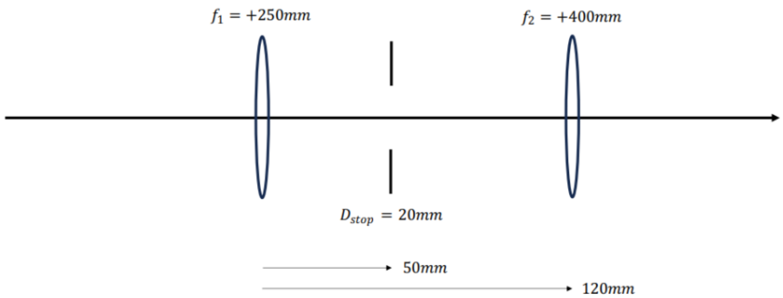
$$\Xi = n\bar{u}y_{pupil} = n'\bar{u}'y'_{pupil}, \quad m_{pupil} = \frac{y'_{pupil}}{y_{pupil}} = \frac{n\bar{u}}{n'\bar{u}'} = \frac{\bar{\omega}}{\bar{\omega}'}. \tag{1.13}$$

Use of working F-number (left)

The most common use of the working F-number is to describe the image-forming cone for a finite conjugate optical system. This is the cone formed by the XP and the axial image point.

Ejemplo 1.4 **Determination of stop and pupil**

Determine the location and size of the pupils for th efollowing the system in air.



Solution

a) We trace the chief ray denoted as CR, and a potential marginal ray MR with unitary height at the stop.

		Object space	EP		L_1		Stop		L_2		XP	Image space
	$C/R/f$				250				400			
	t			$z_{EP} = -62.5$		50		70		$z_{XP} = -84.8$		
	n	1	1	1	1	1	1	1	1	1	1	1
	$-\phi$				-0.004				-0.0025			
	t/n			$\tau_{EP} = -62.5$		50		70		$\tau_{XP} = -84.8$		
	y		0		-5		0		7		0	
CR	nu			0.08		0.1		0.1		0.0825		
	u			0.08		0.1		0.1		0.0825		
	y		$R_{EP} = 1.25$		1		1		1		$R_{XP} = 1.21$	
MR	nu			0.004		0		0		-0.0025		
	u					0		0				

Table 1.1 Raytrace, with CR=Chief ray, MR=Marginal ray.

Due to the diameter of the stop is $R_{stop} = 10\ mm$, we scale the potential marginal ray to give the true marginal ray and therefore obtain the radius of the pupils:

$$\begin{array}{l} R_{EP} = (10)(1.25) = 12.5\ mm \\ R_{XP} = (10)(1.21) = 12.1\ mm \end{array} \implies \begin{array}{l} D_{EP} = 2R_{EP} = 25.0\ mm \\ D_{XP} = 2R_{XP} = 24.2\ mm \end{array}$$

b) For Gaussian imagery, we see the stop as the object for the front group and rear group. For the EP, we have a backward propagation that is maganed with the flip of the sign in the refrractive indices.

$$\frac{-1}{z_{EP}} = \frac{-1}{Z_{\text{stop}}} + \frac{1}{250} \longrightarrow z_{EP} = 62.5\ mm.$$

This entrance pupil is to the right of the lens L_1. The magnification is:

$$m_{EP} = \frac{z_{EP}}{z_{\text{stop}}} = \frac{R_{EP}}{R_{\text{stop}}} = -1.25.$$

The diameter of the entrance pupil is therefore:

$$D_{EP} = 2R_{EP} = 2[|m_{EP}|R_{\text{stop}}] = 25\ mm.$$

For the rear group, we have analogously:

$$\frac{1}{z_{XP}} = \frac{1}{Z_{\text{stop}}} + \frac{1}{400} \longrightarrow z_{XP} = -84.848\ mm.$$

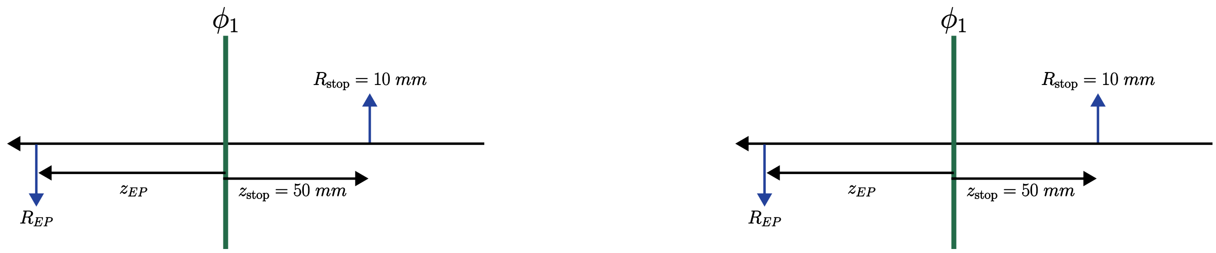
The exit pupil is then to the left of the lens L_2. The magnification in this case is

$$m_{XP} = \frac{z_{XP}}{z_{\text{stop}}} = \frac{R_{XP}}{R_{\text{stop}}} = 1.21.$$

The diameter of the exit pupil is:

$$D_{XP} = 2R_{XP} = 2[|m_{XP}|R_{\text{stop}}] = 24.2\ mm.$$

The illustration of each case is illustrated in the figure 1.7.



(a) EP for front group

(b) XP for rear group

Figure 1.7 With Gaussian imagery, the computation of the pupils is based on the stop as the object of two optical systems.

Using either method, the result is the same and is shown in figure 1.8

1.2 Vignetting

1.2.1 Ray bundles

The **ray bundle** for an **on-axis** object is a rotationally symmetric spindle made up of section of right circular cones. Each cone section if bounded by the pupil and the object/image in that optical space. At any z, the cross section of the bundle is circular, and the radius of the bundle is the marginal ray value.

For an **off-axis** object point, the ray bundle skews, and is comprised of section of skew circular cones wwhich are still defined by the same elements. The cross section of the ray bundle at any z remains circular with a radius equal to the radius of the axial bundle. The off-axis bundle is centered about the chief ray height.

The maximum radial extent of the ray bundle at any z is:

$$\text{Maximum radial extent} \qquad |y_{max}| = |y| + |\bar{y}|. \tag{1.14}$$

1.2.2 Vignetting

The **vignetting** occurs when other apertures in the system (others than the stop) block a proportion of an off-axis ray bundle. For no vignetting, each aperture radius a must equal or exceed the maximum height of the ray bundle at the aperture.

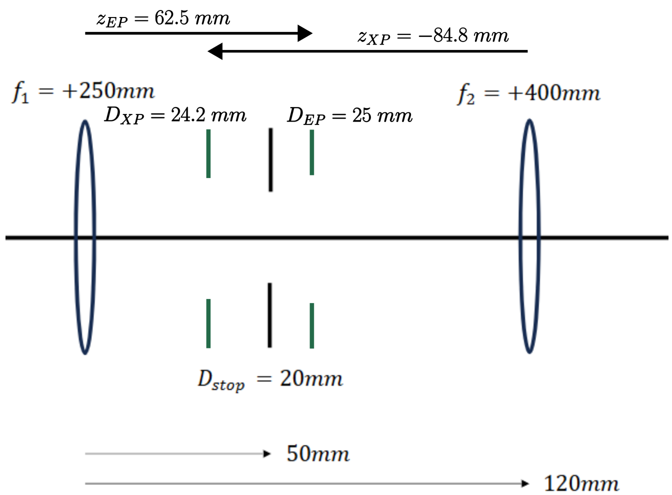


Figure 1.8 Illustration of the stop and pupil in the optical system.

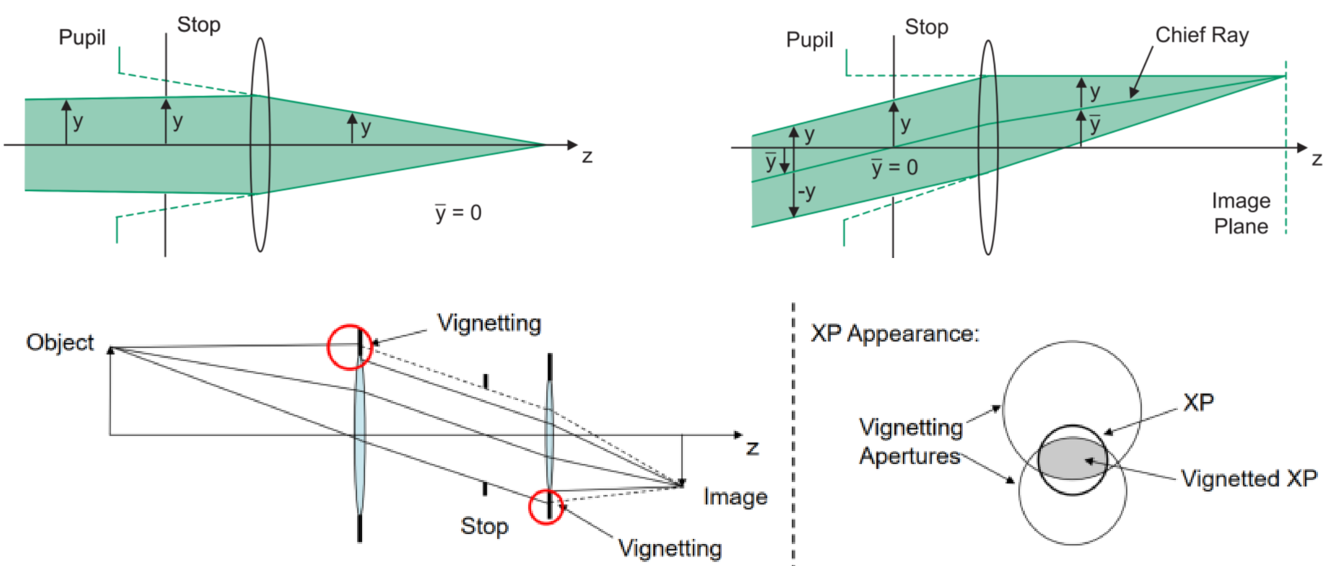


Figure 1.1 The ray bundle is clipped and the beam is no longer circular.

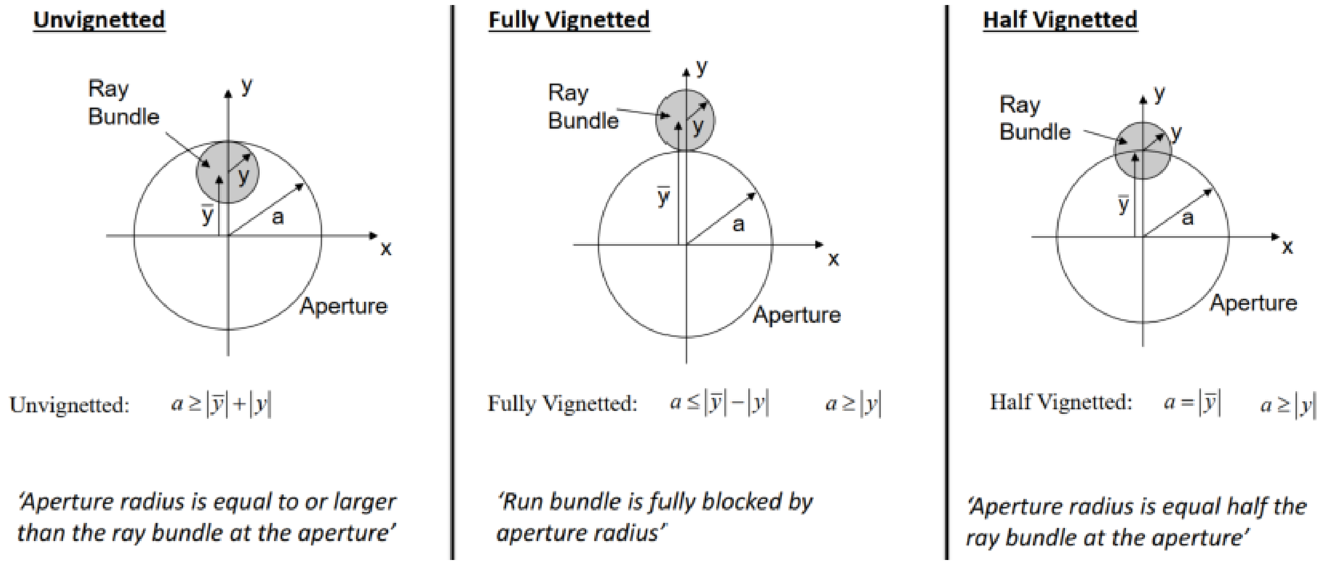
The maximum FOV supported by the system occurs when an aperture completely blocks the ray bundle from the object point.

We can have three conditions of vignetting, depending on the proportion of clip of the light beam.

The vignetting conditions are used in two different manners:

- For a given set of apertures, the FOV that the system will suppport with a prescribed amount of vignetting can be determined. A different chief ray defined each FOV.
- For a given FOC and vignetting condition, the required aperture diameters can be determined.

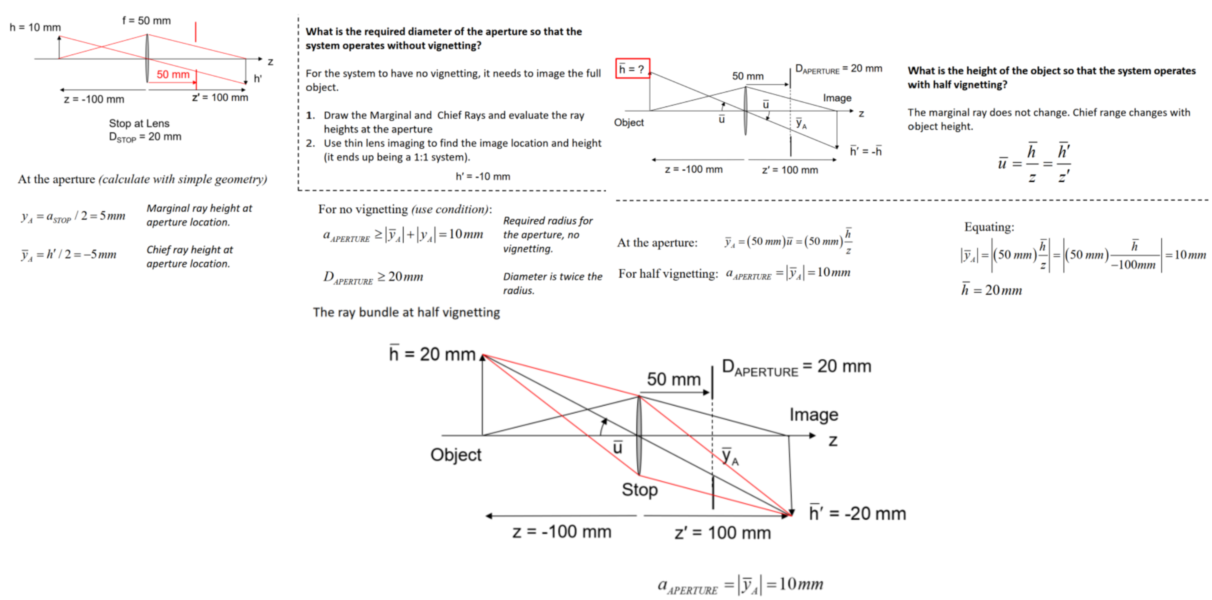
A system with vignetting will have an image that has full irradiance or brightness out to a radius corresponding to the unvignetted FOV limit. The irradiance will then begin to fall off, going to about halr at the half-vignetted FOV, and decreasing to zero at the fully vignetting FOV. This fully vignetted FOV is the absolute maximum possible.



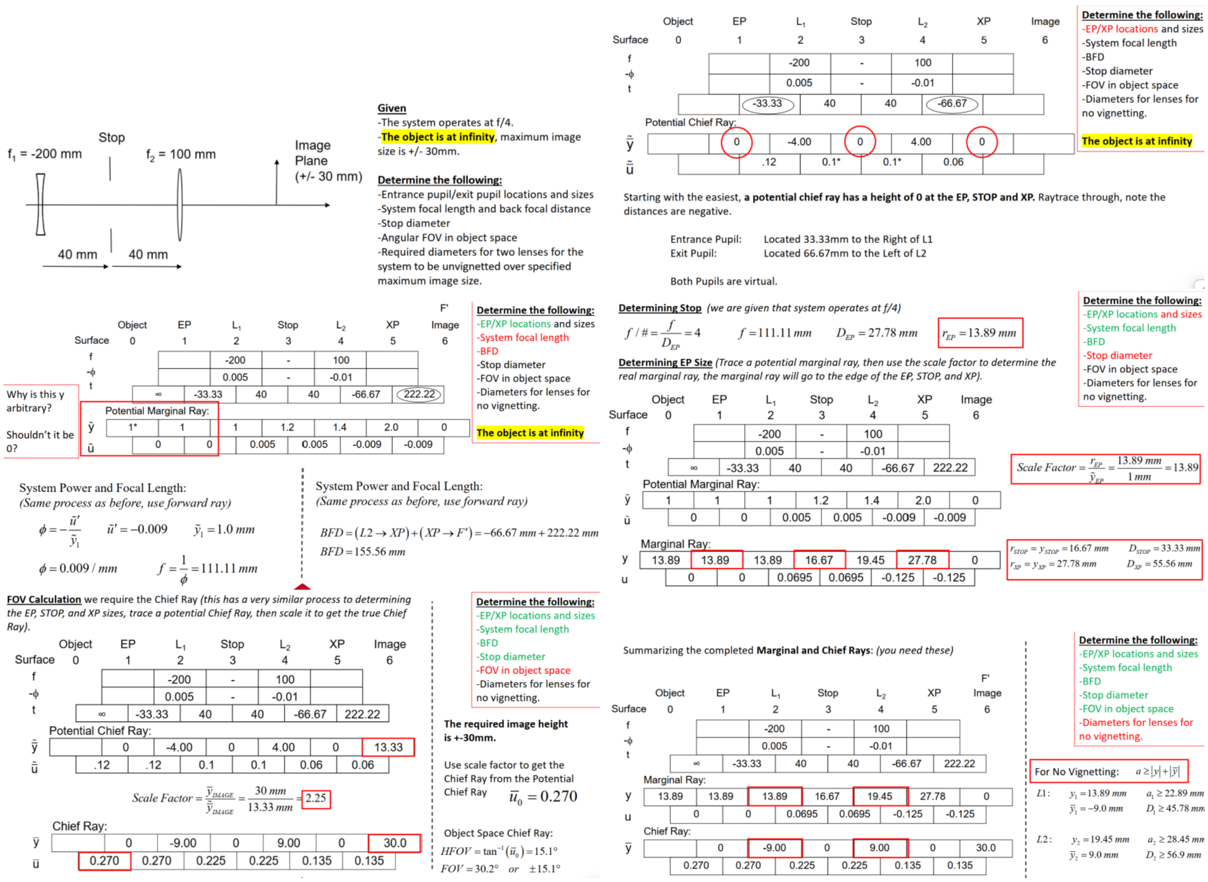
The diameter of the aperture stop is very important design parameter for an optical system as it controls five separate performance aspects of the system:

- The system FOV determined by vignetting.
- The radiometric or photometric speed of the system or its light collection ability.
- The depth of focus and depth of field of the system.
- The amount of aberrations degrading image quality.
- The diffraction-based performance of the system.

Ejemplo 1.5



Ejemplo 1.6 **Vignetting with paraxial raytrace**



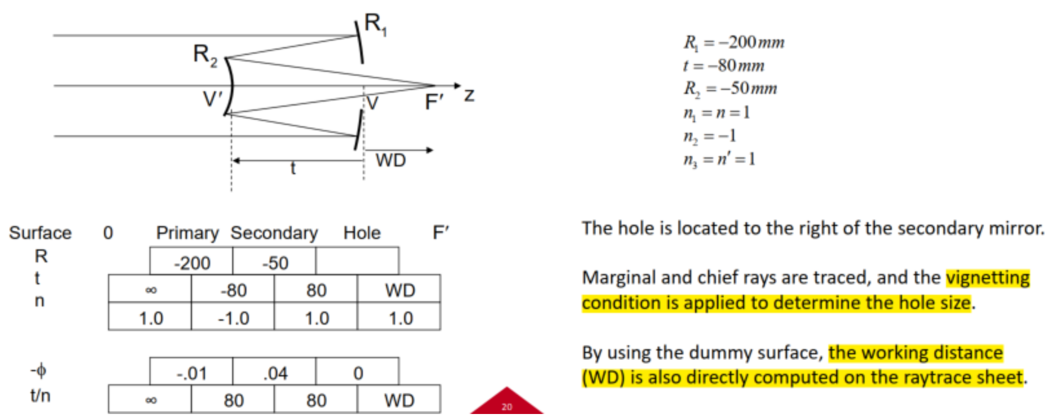
In general,

> **Key points in solving problems**
>
> - Trace the potential chief ray (CR) to know the locations of the pupils (and image size).
> - Trace the potential marginal ray (MR) to determine image location and pupil sizes.
> - If the MR comes parallel, then it can be used to obtain the first-order properties.
> - The F-number gives us the real size of EP so we can scale the MR.
> - The image size allows us to get the real CR.
> - The HFOV is determined with the incident angle $\bar{u}$ at EP in the real CR: $\text{HFOV} = \tan^{-1}\bar{u}$.
> - The vignetting is found by looking at $y, \bar{y}$ in the real MR and CR and applying the criteria.
> - We can arbitrarily define a dummy surface to our convenience.
> - EP and XP are dummy surfaces (w location defined) of zero-power.

1.2.3 Dummy surfaces

In a raytrace, a zero-power surface can be inserted at any location to examine the ray properties.

An example of its application is the following Cassegrain objective, where we require to find the size of the hole. For that, we place a dummy surface **at the hole**.



1.3 Radiative transfer

1.3.1 Radiometry

Radiometry characterizes the propagation of radiant energy through an optical system. The basic unit is the watt W. Radiometric terminology and units are: There are some assumptions:

Quantity	Symbol	Units	Units description
Energy	Q	J	
Flux	Φ	W	Power
Intensity	I	W/sr	Power per unit solid angle
Irradiance	E	W/m^2	Incident power per unit area
Exitance	M	W/m^2	Exiting power per unit area
Radiance	L	W/m^2sr	Power per unit projectedarea per unit solid angle

- The source is **incoherent**, meaning that scenes are collection of independently point sources, no interference.
- Objects and images on-axis and perpendicular to the optical axis, so that the projected area equals the area.

The solid angle Ω equals the surface area of the unit sphere in a given vecinity. The units are 4π steradians (sr).

$$d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin\theta d\theta d\phi}{r^2} = \sin\theta d\theta d\phi \xrightarrow{\int} \boxed{\Omega_{\text{sphere}} = 4\pi \sin^2(\theta_0/2)}\,.$$

In optics, its common to approximate the solid angle of a sphere to the section of a cone:

$$\Omega \approx \frac{\pi r_0^2}{d^2} \approx \pi \sin^2\theta_0 \approx \pi\theta_0^2 \quad \text{(small angle approximation)}.$$

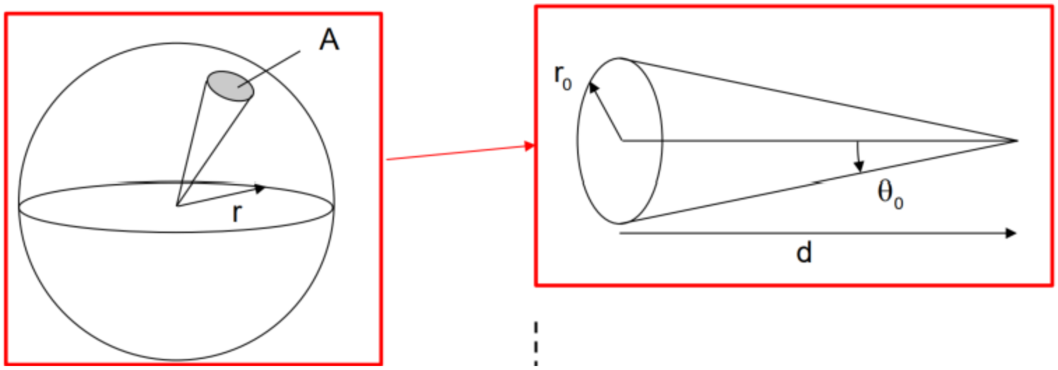


Figure 1.1 The solid angle of a sphere can be approximated to the solid angle of a cone.

1.3.2 Radiative transfer

Radiative transfer uses first-order geometrical principles to determine the amount of light from an object that reaches an image or detector.

Exitance and irradiance are related by the **reflectance** of the surface ρ:

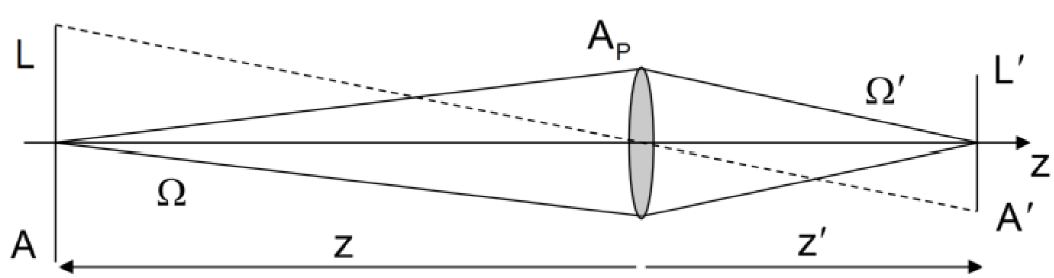
$$M = \rho E. \tag{1.15}$$

For average scenes, $\rho = 18\%$. Exposures are often set using this value, that is, we expose the print to that average so that the prin reflectance ends up with 18%.

The irradiance of a **Lambertian source** (perfectly diffuse surface) is constant. The intensity falls off with the apparaent source size or the **orjected area** (**Lambert's law**). The exitance of a Lambertian source is related to its radiance by π.

Lambertian source
$$L(\theta,\phi) = \text{constant} \qquad I = I_0 \cos\theta$$
$$M = \pi L \qquad \pi L = \rho E$$

We now analyze the optical power from an object that reaches the image in an optical system.



In air, the radiance and the $A\Omega$ **product** or **throughput** are conserved, and the flux collected by the lens Φ is transferred to the image area A':

$$\Phi = L(\text{object area})(\text{solid angle projection in lens}) = LA_p\Omega = LA\frac{\pi D_p^2}{4z^2} \overset{A'=m^2A}{\longrightarrow} \Phi = \frac{\pi L A' D_p^2}{4m^2z^2}.$$

Using gaussian equations and f-number equations, the image plane irradiance E' is

Camera equation
$$E' = \frac{\Phi}{A'} = \frac{\pi L}{4(f/\#_W)^2} \to \pi L(\text{NA})^2, \quad L = \frac{\rho E_0}{\pi}. \tag{1.16}$$

Spectral dependence

Spectral dependence can also be added.

$$\begin{aligned} E_0(\lambda) &= \text{Object irradiance} \\ \rho(\lambda) &= \text{Object reflectance} \\ L(\lambda) &= \text{Object radiance} \end{aligned} \quad \longrightarrow \quad \begin{aligned} L(\lambda) &= \frac{M(\lambda)}{\pi} = \frac{\rho(\lambda)E_0(\lambda)}{\pi} \\ E'(\lambda) &= \frac{\rho(\lambda)E_0(\lambda)}{4(1-m)^2(f/\#)^2} \end{aligned}$$

This can be integrated over all wavelengths for total irradiance

$$E' = \frac{1}{4(1-m)^2(f/\#)^2}\int_{\lambda_1}^{\lambda_2} \rho(\lambda)E_0(\lambda)\, d\lambda.$$

Exposure

Most detectors respond to energy per unit area rathen than power per unit area. Multiplying the image irradiance by the exposure time gives the expoure (J/m^2):

$$\text{Exposure} \qquad H = E'\Delta t\,. \tag{1.17}$$

The mean solar constant is $1368\ W/m^2$ outside the atmosphere of the earth, and the solar irradiance on the surface is about $1000\ W/m^2$.

1.3.3 Photometry

Photometry is the subset of radiometry that deals with visual measurements, and luminous power is measured in **lumens** lm. The lumen is a watt wighted to the visual **photopic response**. This peak response occurs at $555\ nm$, where the conversion is $683\ lm/W$. The dark adapted or **scotopic response** peaks at $507\ nm$ with $1700\ lm/W$.

(a) Photometric terminology

Luminous power	Φ_V	lm
Luminous intensity	I_V	lm/sr
Illuminance	E_V	lm/m^2
Luminous exitance	M_V	lm/m^2
Luminance	L_V	$lm/m^2 sr$
Exposure	H_V	$lm\ s/m^2$
candela (cd)	I_V	lm/sr
lux (lx)	E_V	lm/m^2
foot-candle (fc)		lm/ft^2
	$1fc = 10.76\ lx$	
foot-lambert (fL)	L_V	$\frac{1}{\pi}cd/ft^2$
nit (nt)		$= cd/m^2$
	$1fL = 3.426\ nt$	
lux-second (lx s)	H_V	$lm\ s/m^2$

(b) Luminous photopic efficacy

λ (nm)	lm/W
400	0.3
420	2.7
440	15.7
460	41.0
480	95.0
500	221
520	485
540	652
560	680
580	594
600	425
620	260
640	120
660	41.7
680	11.6
700	2.8
720	0.7

(c) Typical illuminance levels

Sunny day	$10^5\ lx$	Moonlit night	$10^{-1}\ lx$	Overcast day	$10^3\ lx$	Starry night	$10^{-3}\ lx$
Inerior	$10^2\ lx$	Desk lighting	$10^3\ lx$				

The candela (cd) is the fundamental SI unit for luminous intensity.

$A\Omega$ product

Recall the flux hrough a system is

$$\Phi = LA\Omega.$$

The $A\Omega$ product appears to be the geometric portion, while L would be related to the source characteristics. In an object or an image plane,

$$\begin{array}{rllll} \text{object/pupil plane} & A = \pi\bar{y}^2, & \theta = u, & A\Omega = \pi^2\bar{y}^2u^2 = \pi^2\chi^2/n^2, & \chi = n\bar{y}u \\ \text{pupil plane} & A = \pi y^2, & \theta = \bar{u}, & A\Omega = \pi^2 y^2\bar{u}^2 = \pi^2\chi^2/n^2, & \chi = ny\bar{u} \end{array}$$

In the general situation when the index is not unity, the **basic throughout** $n^2A\Omega$ and the **basic radiance** L/n^2 are invariant. Since throughput is based on areas, the basic throughput is proportional to the Lagrange invariant squared:

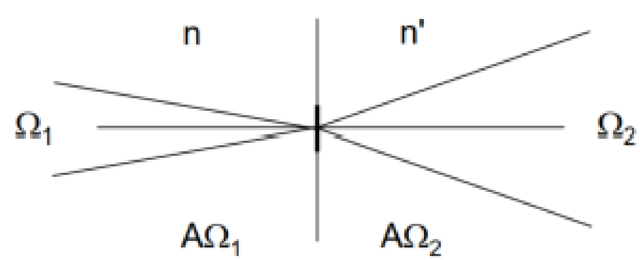
$$n^2A\Omega = \pi^2\chi^2. \tag{1.18}$$

For a lossless optical system, the flux through the system is constant. For $n = 1$, we have

$$\Phi = L_1A_1\Omega_1 = L_2A_2\Omega_1 = \cdots = \text{constant}.$$

Since $A\Omega$ is also constant, the radiance L must also be constant. This allow us to relate different portions of the optical system as the flux is conserved.

However, if the index of refraction is not unity and changes ,the radiance is no longer conserved. It will change at each interface as the solid angle will change. We then have that



$$A\Omega_1 \neq A\Omega_2 \wedge \Omega = L_1A\Omega_1 = L_2A\Omega_2 \implies L_1 \neq L_2.$$

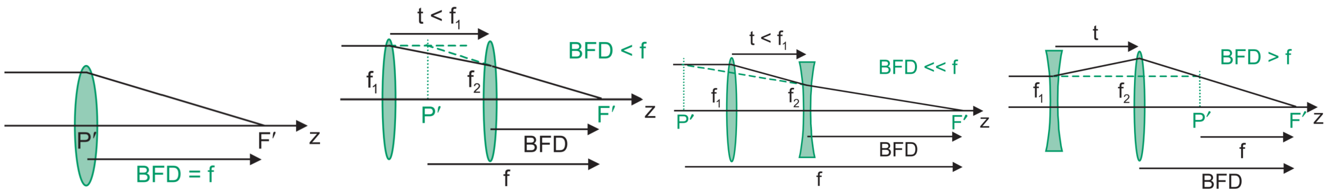
The flux is still constant. In fact, $L/n^2 = \text{constant}$ as well as $n^2A\Omega = \text{constant}$.

1.4 Objectives

1.4.1 Type of objectives

Objectives are lens element combinations used to image distance objects. To classify them, separated group of lens elements are modeled sa thin lenses.

- **Simple objective** consists of a positive thin lens.



(a) Simple objective **(b)** Petzval objective **(c)** Telephoto objective **(d)** Reverse Telephoto

Figure 1.1 Different type of objectives.

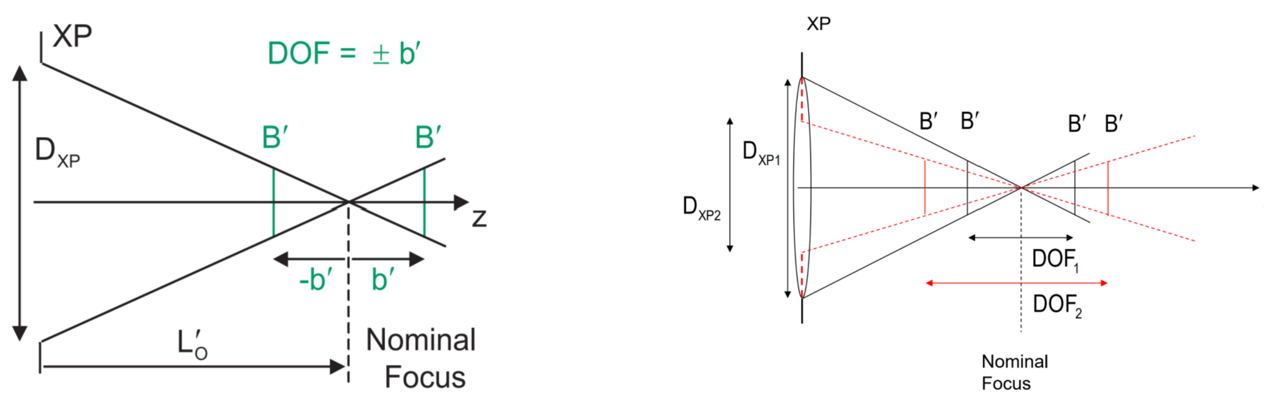
- **Collimator** is a reversed simple objective. It creates a collimated beam from a source at the system focal point.
- **Petzval objective** consists of two separated positive groups of elements. The rear principal plane is located between the groups.
- **Telephoto objective** produces a system focal length longer than the BFD. It consists of a positive element followed by a negative group.
- **Reverse telephoto objective** consists of a negative group followed a positive group. Used to produce a system with BFD larger than the system focal length.

1.4.2 Depth of focus and field

There is often some allowable image blut that defines the performance requirement of an optical system. No diffraction or aberrations are included.

Depth of focus

The **depth of focus** DOF describes the amount the detector can be shifted from the nominal image position (focal plane) for a fixed object before the resulting blur exceed the blur diameter criterion B'.



(a) Depth of focus **(b)** Depth of focus vs F-number

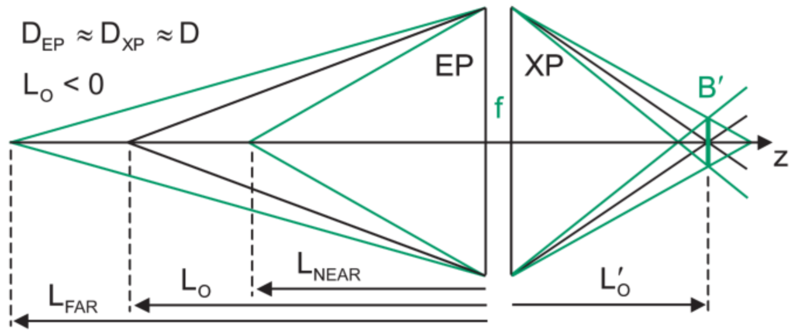
The $\pm b'$ transverse distance from the nominal point is used to define the DOF as the whole distance is $2b'$:

$$b' = \frac{B'L'_0}{D_{XP}} \approx \frac{B'z'}{D_{EP}}, \quad \text{DOF} \approx \pm b' \approx \pm B' f/\#_W \approx \frac{B'}{2\text{NA}}. \tag{1.19}$$

The figure illustrates how the DOF is changed as the diameter of the lens is varied; they are inversely proportional.

Depth of field

The **depth of field** is the maximum distance, from L_{near} to L_{far}, the object can move before exceed the aceptable blur B' at the fixed image plane. The following relations are given



$$L_{\text{far}} \approx \frac{L_0 f D}{fD + L_0 B'}, \quad L_{\text{near}} \approx \frac{L_0 f D}{fD - L_0 B'}. \tag{1.20}$$

Al objects positiones between these distances will produces images on the detector that have geometrical blurs less than the blur criterion B'.

1.4.3 Hyperfocal distance

When the far point of the depth of field L_{far} is at infinity, the optical system is focused at the **hyperfocal distance** L_H, and all objects from L_{near} to infinity meet the image plane blur criterion.

$$L_{\text{far}} = \infty \Longrightarrow fD + L_H B' = 0.$$

Solving for L_H,

$$\text{Hyperfocal distance} \qquad \boxed{L_H = -\frac{fD}{B'} = -\frac{f^2}{(f/\#)B'}}. \tag{1.21}$$

Substituting of L_H in L_{near} yields

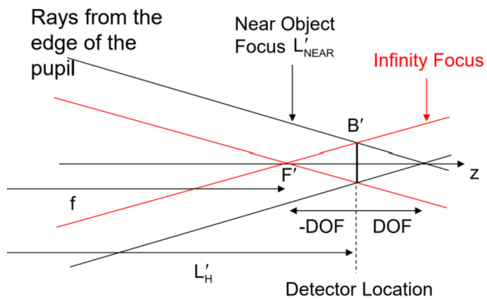
$$L_{\text{near}} \approx -\frac{fD}{2B'} = \frac{L_H}{2}. \tag{1.22}$$

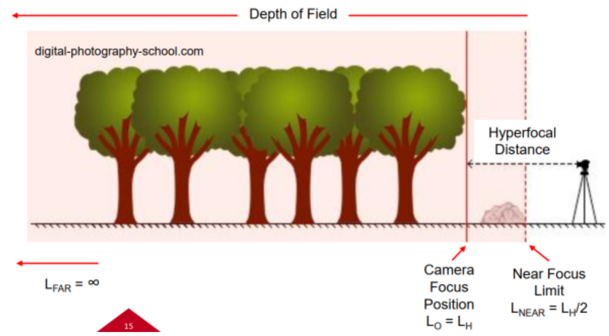
Hyperfocal distance and depth of

The relation between the depth of focus and the hyperfocal distance where the detector is placed is, by the thin-lens equations,

$$L'_H \approx f + B'f = f + \text{DOF}. \tag{1.23}$$

- Focusing at the hyperfocal distance **ensures** that any greater distance meets the blur criteria.
- As $f/\#$ increases (lens stopped down), the hyperfocal distance moves closer to the lens.





You take a picture with a film camera. You want to apply that picture to a larger print. **Given the specifications, what is the f/#?**

System Specifications

Film size:	24 x 36 mm
Print size:	100 x 150 mm
Maximum blur on print:	0.15 mm
Focal length:	38 mm
Near focus:	1200 mm

We image onto film. Notice the blur criteria is for the PRINT.

$f/\# = f/Dep$

We are given the focal length; we need to find the diameter EP.

$L_H = -\frac{fD}{B'}$ $\quad D = -\frac{L_H B'}{f}$

Use relationship for hyperfocal distance to isolate D. Need to determine L_H and B′.

$L_H = 2L_{NEAR} = -2400mm$

L_H is easy, use relationship with L_{NEAR}. Be careful with signs!

Print Magnification $\approx$ 4X

$B' = \frac{.15mm}{4} = .038mm$

Our maximum blur on the PRINT is 4mm, we need to calculate the maximum blur on the FILM.

After plugging in everything, *you will find D = 2.44mm and f/15.5.*

Ejemplo 1.7 Fixed-focus camera

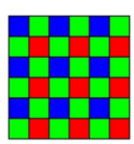
Ejemplo 1.8 Fixed-focus digital camera

You take a picture with your phone.
Given the specifications, what is the f/#?

System Specifications

Number of Pixels =	3264 x 2488 (8MP)
Pixel Size =	1.4 µm
Near focus =	1200 mm
Focal length =	4.8 mm

Image blur is twice the pixel size. **Why?**



$f/\# = f/D$

$L_H = -\frac{fD}{B'}$ $\quad D = -\frac{L_H B'}{f}$

$L_H = 2L_{NEAR} = -2400mm$

$B' = 2 * Pixel\ Size = 2.8\mu m$

Same process as before, the key difference is HOW the blur is calculated.

$D = 1.4mm$

$f/\# = f/D = f/2.9$

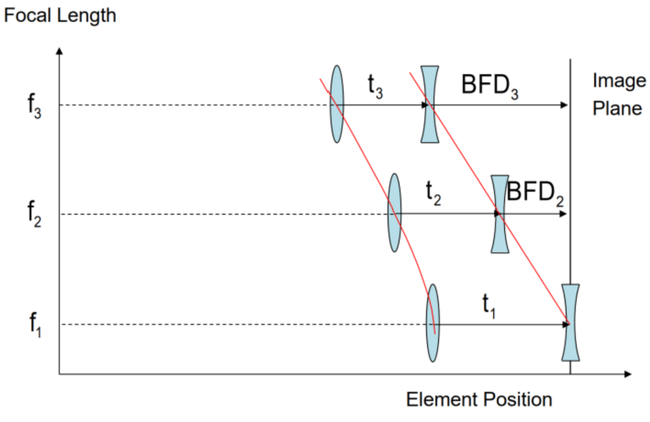
1.4.4 Zoom lenses

A **zoom lens** is a variable focal length objective with a **fixed** image plane. The simplest system is composed of two groups with powers ϕ_1 and ϕ_2 where both the focal length f and the BFD vary with the element spacing t.

$$\phi = \frac{1}{f} = \phi_1 + \phi_2 - \phi_1\phi_2 t \quad \mathrm{BFD} = f + d' = f - \frac{\phi_1}{\phi}t.$$

To vary the focal length with a fixed image plane, we:

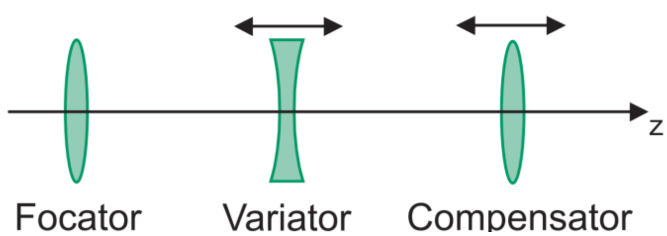
- Move the lens L_1 a distance $L = t + \text{BFD}$ from the image plane.
- Displace lens L_2 a distance t from L_1.



(a) Telephoto zoom (limited by BFD) **(b)** Reverse telephoto zoom (commonly used)

As the separation approaches the sum of the individual focal lengths ($f_1 + f_2$), the system becomes afocal ($f \to \infty$).

A mechanical cam provides the complicated lens motion required for these **mechanical compoensated** zoom lenses. A common three group configuration uses a fixed front element and moving second and third groups.



Ejemplo 1.9 **Digital camera**

You are using a digital camera to take a picture of a house against a distant mountain backdrop. The camera is focused on the house. You have a wide selection of camera lenses and 16:9 (width = 12.48mm, height = 7.02 mm) CCD detectors at your disposal. The CCD detectors available to you have pixels with sizes ranging between 3 µm and 6 µm, however, cameras with pixel sizes of exactly 3 µm and 6 µm are not available. You want to make sure that you can take a high-quality picture of the house and the mountains and therefore, want to achieve a diffraction-limited image based on the pixel pitch of the CCD detector you selected. The location allows you to place the camera anywhere between 10 and 30 meters (not exactly 10 nor 30 m) away from the house. Assume that the lens is a thin lens with the stop at the lens.

a) What are the hyperfocal distance and acceptable blur for your camera system?
b) Determine the focal length, f, and the diameter, D, of the lens.
c) How far behind the lens must the detector be located? What is the maximum size (height and width) of the house in meters you would be able to image on the detector?
d) What is the horizontal and vertical Field of View of this photographic system (in degrees)?
e) What is the f/# of this camera? What is the closest object that will be considered to be in focus?

f) It is an overcast day (outdoor illuminance=103 lux=103 lm/m2). The reflectance of the house is r = 0.5 and the mountains have an average reflectance of r=0.18. What are the image plane illuminances for the house and the mountains individually?

Solution

a) CCD pixel pitch is $4\ \mu m$. And camera at $20\ m$ away from the house.
b) The hyperfocal distance is $L_H = 20\ mm$. The blur is $B' \approx 4\ \mu m \to f/\# = 4$.
c) The hyperfocal idstance can be related to focal length:

$$|L_H| = \frac{f^2}{B' f/\#} \longrightarrow f = \sqrt{L_H B' f/\#} = \sqrt{20 \cdot 4\ \mu m \cdot 4} = 17.889\ mm.$$

The diameter is then:

$$D = \frac{f}{f/\#} = \frac{17.889\ mm}{4} = 4.472\ mm.$$

We can think os a single thin lens that images an object at $-20\ mm$ to L_H. The magnification is:

$$m = \frac{L'_H}{L_H} = fracz'z = 1 \cdot 1^{-3}.$$

The field of view for each dimension is:

$$\text{HFOV}_H = \tan^{-1} \frac{7.02}{L'_H} =$$
$$\text{HFOV}_W = \tan^{-1} \frac{12.48}{L'_H} =$$

d) The near distance is half of the hyperfocal distance

$$L_{\text{near}} = \frac{L_H}{2} = 10\ mm.$$

e) For the house,

$$E' = \frac{M}{4(f/\#_W)^2} = \frac{\rho E}{4(f/\#_W)^2} = \frac{\rho E}{4(f/\#)^2(1-m)^2}.$$

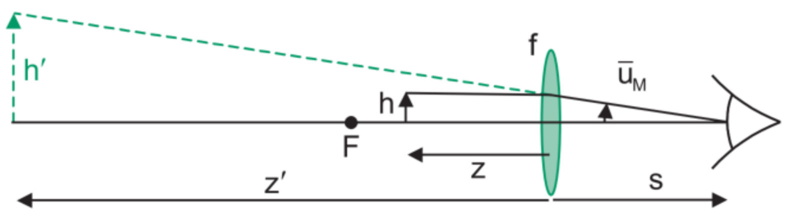
For the mountain (located at infinity):

$$E' = \frac{\rho E}{4(f/\#)^2}.$$

1.5 Magnifiers and Telescopes

1.5.1 Magnifiers

The largest image magnification possible with the unaided eye occurs when the object is placed at the **near point** of the eye, by convention, $250\ mm$ or $10\ in$. A **magnifier** is a single lens that provides an enlarged **erect virtual image** of a nearby object for visual observation. The object must be placed within the fron focal length of the lens.



The **magnifying power** MP is defined as (stop at the eye):

$$\begin{aligned} \text{MP} &= \frac{\text{Angular size of the image (with lens)}}{\text{Angular size of the object at the near point}} \\ &= \frac{\bar{u}_M}{\bar{u}_U} = \frac{h'/(z'-s)}{h/d_{NP}}, \quad d_{NP} = -250\ mm \\ &= \frac{250\ mm(z'-f)}{f(z'-s)} \approx \frac{250\ mm}{f}. \end{aligned}$$

The approximation relation is the most common definition of the MP. It assumes that the lens is close to the eye and the image is presented to a relaxed eye ($z' = \infty$).

The angular substense θ of the image h' at the eye is

$$\theta = \frac{h\text{MP}}{250\ mm}. \tag{1.24}$$

The resolution of the human eye is about $1\ arcmin$, or $(1/60)^\circ$. In order to resolve an object of size h, th required MP is then

$$\text{MP} \geq \frac{0.075\ mm}{h}.$$

Magnifiers up t about 25X are practical; 10X is common.

1.5.2 Telescopes

Telescopes are afocal systems used for visual observation of distant objects. The image through the telescope subtends an angle θ' different from the angle subtend by the object θ. The magnifying power of a telescope is:

$$\text{MP} = \frac{\theta'}{\theta} = \begin{cases} |\text{MP}| > 1, & \text{Telescope magnifies} \\ |\text{MP}| < 1, & \text{Telescope minifies} \end{cases}. \tag{1.25}$$

In Keplerian and Galilean telescope, the lateral magnification m is given by:

$$\text{Lateral magnification of telescope} \qquad m = \frac{1}{\text{MP}} = -\frac{f_{\text{EYE}}}{f_{\text{OBJ}}}. \tag{1.26}$$

It is important to notice the reciprocal relation between the magnification and the magnifying power. For instance an image smaller than the object, also have the image much closer, so that the apparent size is much larger.

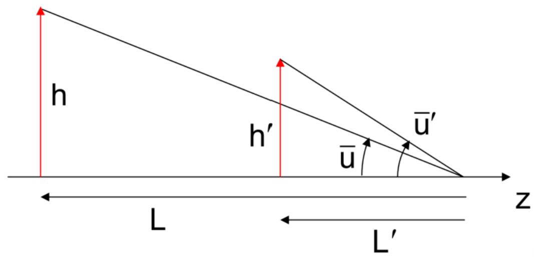


Figure 1.1 In this case, $m < 1$ while also MP > 1: height and distance are important for these quantities.

Keplerian telescope

A Keplerian telescope or astronomical telescope consists of an objective that creates an **aerial image** (real image in the air) followed by a magnifier separated by $f_1 + f_2$. The system stop is usually at or near the objective lens.

The Keplearian has a negative MP: MP < 1 and the image presented to the eye is inverted and reverted (rotated $180°$). The eye should be placed at the real XP to couple the eye to the telescope and see the entire FOV, if not vignetting may occur. The XP position from the last surface is called the **eye relief** ER. The magnification of the telescope related the diameters of EP and XP:

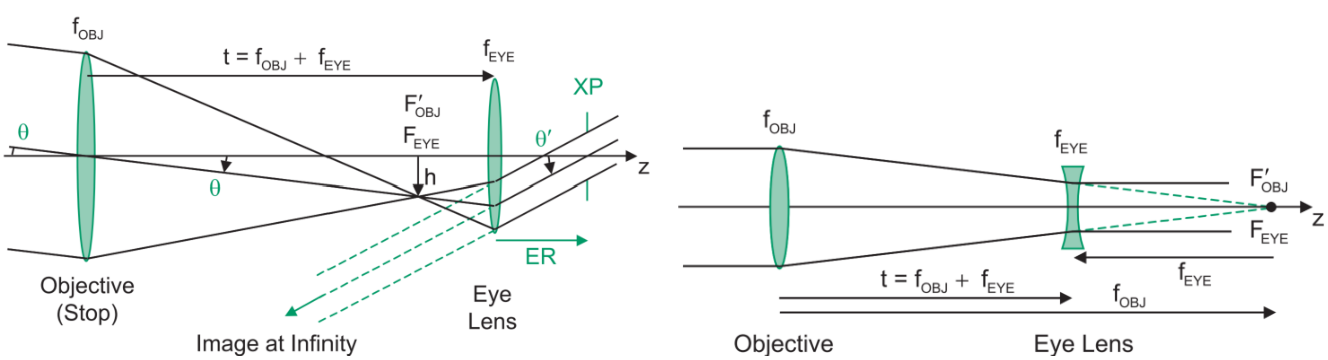
$$\mathrm{ER} = z' = (1 - m) f_{\mathrm{EYE}}, \quad D_{\mathrm{XP}} = |m| D_{\mathrm{EP}}. \tag{1.27}$$

The XP of a visual instrument is also known as the **eye circle** or the **Ramsden circle**.

Galilean telescope

The Galilean telescope uses a positive lens followed by a negative lens to obtain an erect image and a positive MP MP > 1. In this case, the XP is internal o virtual and not accesible to the eye. The FOV of the system is therefore small. There is no intermediate image plane.

For a given $|\mathrm{MP}|$, the Galilean telescope is shorter and the FOV smaller than the corresponding Keplerian telescope.



(a) Keplerian telescope

(b) Galilean telescope

We describe several important points:

- Usually in telescope, the objective is the stop so that this lens is also the EP. Keplerian have real XP to the right of the eye lens.

- A **reversed Galilean telescope** provides a minified erect image $0 < \mathrm{MP} < 1$ and the eye is usually the system stop.
- **Binoculars** are a pair of parallel telescope for each eye.
- The specification provided on telescope and binoculars is of the form

$$\mathrm{AXB} \Longrightarrow A = |\mathrm{MP}|, \quad \text{and} \quad B = \text{Objective diameter in mm.} \tag{1.28}$$

Ejemplo 1.10 **Eye relief of a Keplerian telescope**

A 5X Keplerian telescope is constructed out of two thin lenses. The separation between the two lenses is 120 mm, and the diameter of the objective lens is 25 mm. The system stop is at the objective. Determine the eye relief and the diameter of the exit pupil for this telescope.

$$\boxed{D_{XP} = |m| D_{EP} = \frac{D_{EP}}{|MP|} = \frac{f_{EYE}}{f_{OBJ}} D_{EP} \qquad ER = z' = (1-m) f_{EYE}}$$

Exit Pupil Diameter

$$D_{EP} = D_{STOP} = 25mm$$

$$D_{XP} = \frac{D_{EP}}{|MP|} = \frac{25mm}{5} = 5mm$$

We can use this same information to find the focal lengths

$$MP = -5 = -\frac{f_{OBJ}}{f_{EYE}} \qquad f_{OBJ} = 5f_{EYE} \qquad t = f_{OBJ} + f_{EYE} = 6f_{EYE} = 120mm$$

$$f_{EYE} = 20mm \qquad f_{OBJ} = 100mm$$

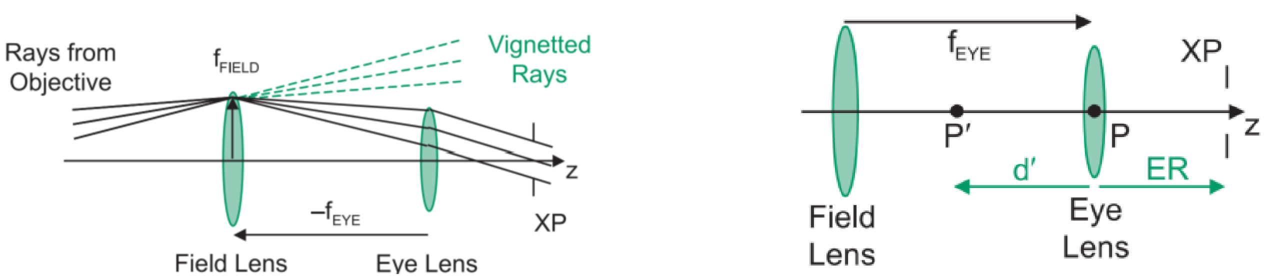
Determining Eye Relief Location *image stop through lens, or use equation*

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_{EYE}} \qquad z = -\left(f_{OBJ} + f_{EYE}\right) = -t = -120mm$$

$$z' = ER = 24mm$$

1.5.3 Field lenses

The FOV of the Keplerian telescope is limited by vignetting at the eye lens. A **field lens** placed at the intermediate image plane increases the FOV by bending the ray bundle into the aperture of the eye lens.



(a) Field lens **(b)** Reduction of the ER

This combination is called an **eyepiece**. If we assume the separation between them is equal to the focal length of the eye lens, then the overall power is:

$$\phi = \phi_{\mathrm{EYE}} + \phi_{\mathrm{FIELD}} - \phi_{\mathrm{EYE}}\phi_{\mathrm{FIELD}} \cdot \frac{1}{\phi_{\mathrm{EYE}}} = \phi_{\mathrm{EYE}}.$$

Thus, the eyepiece maintain its focal length and therefore its MP. The shifts of the principal planes are:

$$d = \frac{\phi_{\mathrm{EYE}}}{\phi_{\mathrm{EYE}}} \frac{1}{\phi_{\mathrm{EYE}}} = f_{\mathrm{EYE}}, \quad \text{and} \quad d' = -\frac{\phi_{\mathrm{FIELD}}}{\phi_{\mathrm{EYE}}} \frac{1}{\phi_{\mathrm{EYE}}} = -\frac{f_{\mathrm{EYE}}^2}{f_{\mathrm{FIELD}}}. \tag{1.29}$$

To summarize, the inclusion of the field lens:

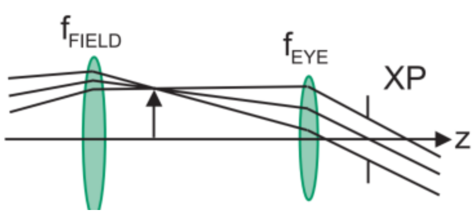
- Decrease vigneting incresing in the same way the FOV.
- Does not change the MP of the telescope or the size of the XP.
- The front principal plane is unchanged, but the rear principal plane and ER are shifted by d'.
- The field lens is usually also displaced from the image plane to minimize the inclusion of information of the field lens to the image through defocus.
- MP of the eyepiece is the same as the MP of a magnifier.

1.5.4 Eyepieces

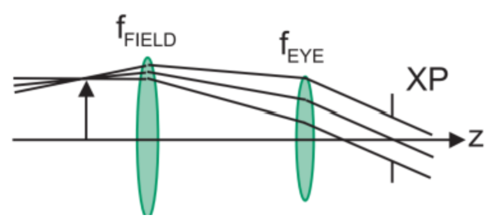
An **eyepiece** or **ocular** is the combination of the field lens and the eye lens. A simple eye piece does not have a field lens. A compound eyepiece has both of them. A field stop can be placed at the intermediate image plane to restrict the system FOV. This aperture serves to correct the vignetting.

Two special eyepiece configuations displace the field lens from the intermediate image:

- **Huygens eyepiece** has the intermediate image between the two elements.
- **Ramsden eyepiece** has intermediate image in front of the field lens. This configuration has about 50% more ER than the Huygens eyepiece.
- **Kellner eyepiece** replaces the singlet eye lens of the Ramsden eyepiece with a doublet for color correciton.



(a) Huygens eyepiece



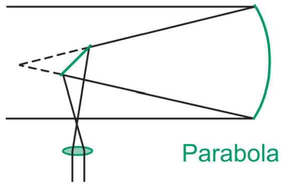
(b) Ramsden eyepiece

Hand-held instruments should have $15-20\ mm$ of eye relief. Microscopes may have as little as $2-3\ mm$. Other systems, such as riflescopes, shoud have a very long eye relief. The human eye pupil diameter varies from $2-8\ mm$, with a diameter of about $4\ mm$ under ordinary lighting conditions. When overfiled, the eye becomes the system stop.

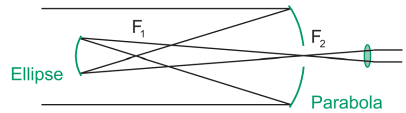
1.5.5 Mirror-based Telescopes

The imaging properties of conic surfaces are used in the design of **mirror-based telescopes**.

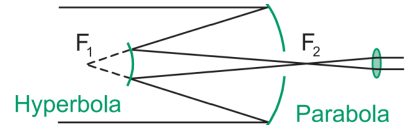
- **Newtonian telescope** uses a parabola with a fold flat. It is analogous to a Keplerian refracting telescope.
- **Gregorian telescope** uses a parabola followed by an ellipse to relay the intermediate image. It prooduces an erect image.
- **Cassegrain telescope** uses a parabola combined with a hyperbolic secondary mirror to reduce the system length. It is equivalent to a telephoto objective. The two conic surfaces correct the spherical aberration.
- **Ritchey-Chretien telescope** is identical to Cassegrain telescope in layout, except that it uses two hyperbolic mirrors to correct coma as well as spherical aberration.



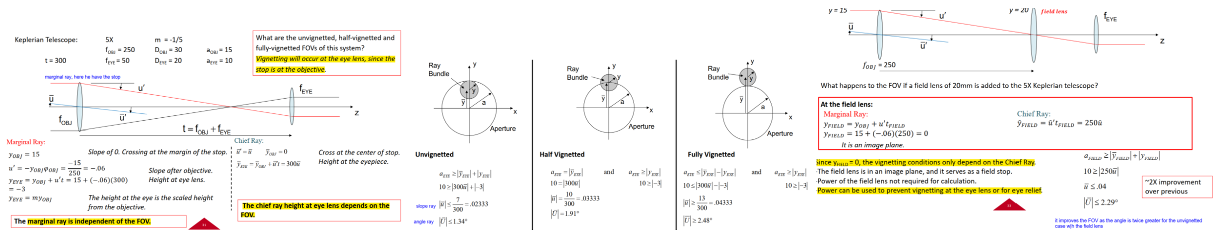
(a) Newtonian

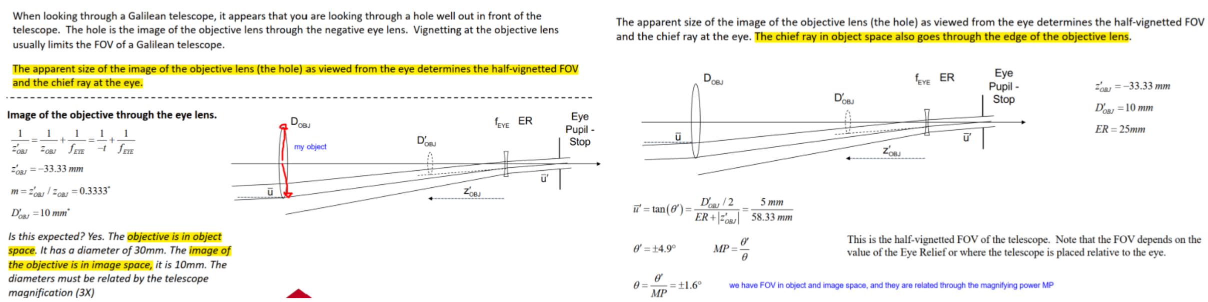


(b) Gregorian



(c) Cassegrain





Ejemplo 1.11 **Vignetting in a Keplerian telescope**

Ejemplo 1.12 **Vignetting in a Galilean telescope**

1.6 Relays and Microscopes

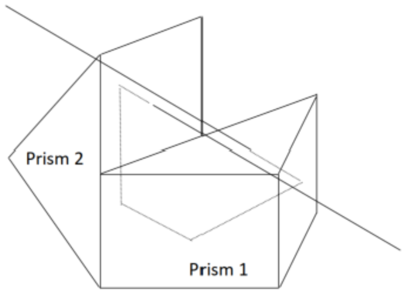
1.6.1 Prisms and relay lenses

For many applications, it is important that the image have the same orientation as the object. The commonly used methods for image erection is to use prism, and relay sistems.

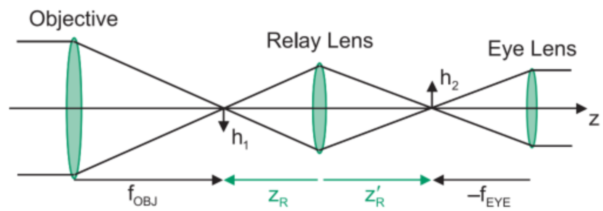
On the one hand, prisms as porro or Pechan-roof prism may help to erect the image. It is important that the ray bundle must be sized so the entrance and exit faces do not vignette the FOV. Unfolding the prism make possible to treat it as a plane parallel plate of glass.

On the other hand, the **relay lens** consists of an additional lens that takes the image of the objective (object for this lens) and image it to the right. This image is the object for the eyepiece. The net MP of the relayed Keplerian telescope in the figure is positive and equals to the product of the magnification of the relay and the MP of the original Keplerian telescope:

$$\text{MP} = m_R \text{MP}_K = -\frac{z'_R f_{\text{OBJ}}}{z_R f_{\text{EYE}}}, \quad \text{where} \quad m_R = \frac{z'_R}{z_R}. \tag{1.30}$$



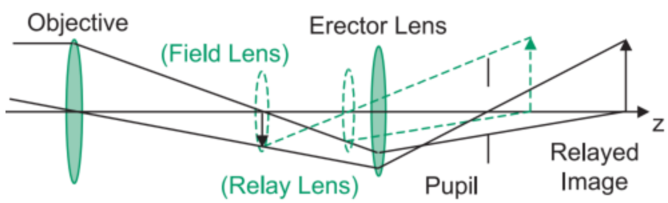
(a) Using a prism



(b) Using a relay lens

Multiple relay lenses can be used to transfer the imager over long distances, such as in periscoopes, andoescopes and borescopes. Field lenses can also be added at the intermediate lenses.

The functions of a field lens and a relay lens ca be combined into a single **erector lens**. This lens will require a diameter larger than the replaced field or relay lenses. The relayed image and pupil are shifter from their original positions.



Ejemplo 1.13 **Keplerian telescope with relay lens**

A 30X relayed Keplerian telescope is constructed out of three thin lenses in air. The relay lens of the telescope operates with a magnification of 1.5. The focal length of the objective lens is 400 mm, and the overall telescope length is 500 mm.

a) Determine the design of the telescope.
b) Assuming that the system stop is located at the objective lens, determine the eye relief of the telescope (distance from the eye lens to the XP).

Solution

a) The magnification of the relay lens and the total magnification are:

$$m_R = \frac{z'_R}{z_R} = -1.5, \quad \text{MP} = m_R \text{MP}_K = (-1.5)(-\frac{f_1}{f_2}) = 30 \longrightarrow f_2 = \frac{400 \cdot 1.5}{30} = 20\ mm.$$

Therefore, the remaining space is $80\ mm$ which is distribtued for the relay lens.

$$-z_R + z'_R = 80\ mm.$$

Using it with the relay magnification we have that

$$z_R = -32\ mm, \quad \text{and} \quad z'_R = 48\ mm.$$

The focal length of the relay lens is obtained with the thin-lens equation:

$$\frac{1}{z'_R} = \frac{1}{z_R} + \frac{1}{f_R} \longrightarrow f_R = 19.2\ mm.$$

b) The eye relief is located at the image position in image space of the stop. The relay lens take that object located at $z = -(32 + 400) = -432\ mm$.

$$z' = \frac{1}{\frac{1}{-432} + \frac{1}{19.2}} = 20.093\ mm.$$

This image is located at $z = -(20 + 48 - 20.093) = -47.907\ mm$ to the eye lens and is seen as the object, which is imaged to:

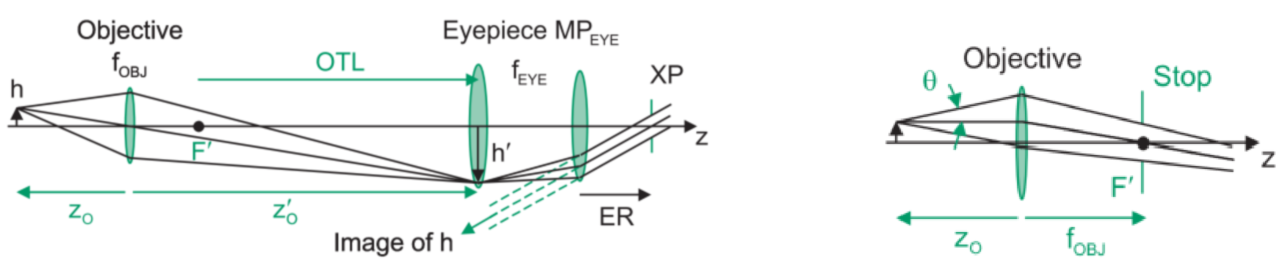
$$z'_{\text{XP}} = \text{ER} = \frac{1}{\frac{1}{-47.907} + \frac{1}{20}} = 34.333\ mm.$$

1.6.2 Microscopes

Definition

A **microscope** is a sophisticated magnifier consisting of an objective plus an eyepiece. The **visual magnification** is the product of the objective magnification and the eyepiece MP:

$$\text{Visual magnification} \qquad m_V = m_{\text{OBJ}}\text{MP}_{\text{EYE}} = \frac{z'_0}{z_0}\frac{250\ mm}{f_{\text{EYE}}}. \tag{1.31}$$



(a) Structure **(b)** Numerical aperture

The **optical tube length** OTL of a microscope is defined as the distance from the rear focal point of the objective to the front focal point of the eyepiece (intermediate image). Standard values are $160\ mm$ and $125\ mm$. The OTL is a Newtonian mage distance:

$$m_{\text{OBJ}} = -\frac{\text{OTL}}{f_{\text{OBJ}}}. \tag{1.32}$$

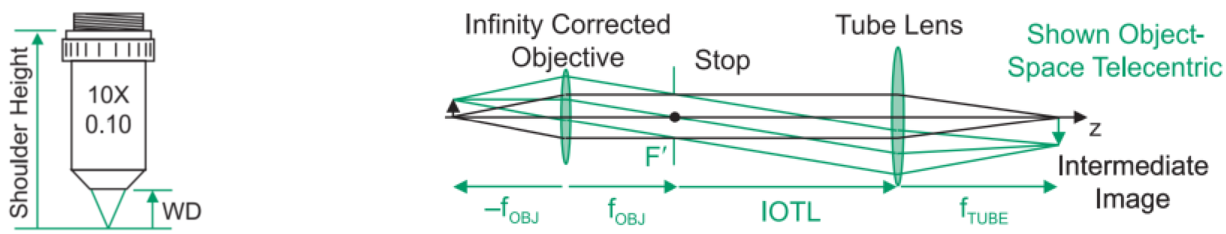
The NA of a microscope objective is defined in object space by the half-angle of the accepted input ray bundle. Along with the objective magnification, the NA is inscribed on the objective barrel:

$$\text{NA} = n \sin\theta. \tag{1.33}$$

Microscope objective are often telecentric in object space. The stop is placed at the rear focal point of the objective so that the magnification does not change with defocus.

Microscope terminology

- The **working distance** WD is the distance from the object to the first element of the objective; can be less than $1\ mm$ for high-power objectives.
- The **mechanical tube length** is the separation between the shoulder of the threaded mount of the objetctive an the end of the tube into which the eyepiece is inserted.
- A set of **parfocal objectives** have different magnifications, but the same **shoulder height** and the same shoulder-to-intermediate image distance.
- **Biological objectives** are aberration corrected assuming a cover glass between the object and the objective.
- Research-graded microscopes are usually designed using **infinity corrected objectives**. The object plane is the front focal plane of the objective, and a collimated beam results for each point.



The magnification of the objective-tube lens combination is

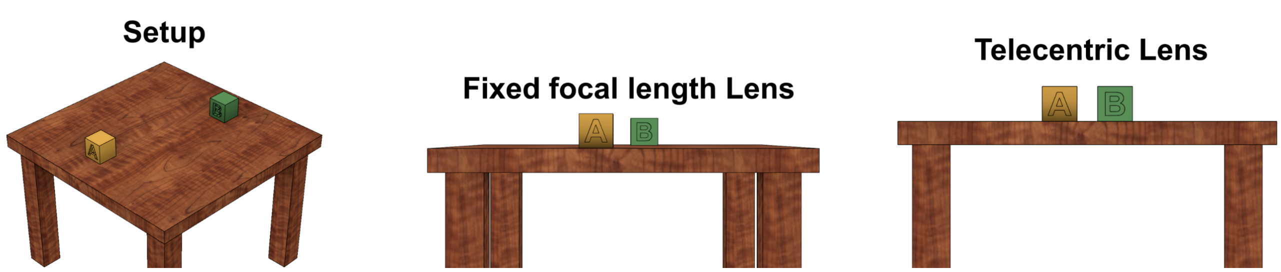
$$m_{\mathrm{OBJ}} = -\frac{f_{\mathrm{TUBE}}}{f_{\mathrm{OBJ}}}. \tag{1.34}$$

If the objective is object-space telecentric and f_{TUBE} equals the infinite optical tube length IOTL, the combination is afocal and double telecentric. This is a useful feature when using reticles in the eyepiece.

1.7 Telecentric systems

1.7.1 Telecentricity

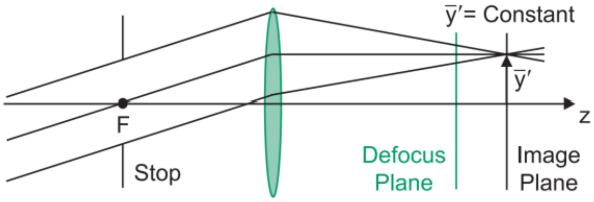
In a **telecentric system**, the EP and/or the XP are located at **infinity**. **Telecentricity** in object or image space requires that the chief ray be parallel to the axis in that space. Consequently, the apparaent system magnification is **constant** even if the object or image plane is displaced. The image will be blurred, but of the correct size or magnification.



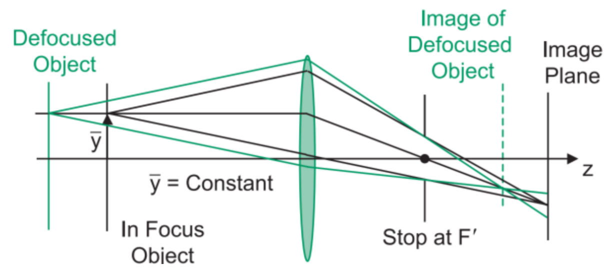
1.7.2 Single telecenctric systems

When the stop is located at the front focal plane of a focal system, the XP is at infinity, and the system is **image-space telecentric**. Defocus of the image plane or detector will not change the image height.

On the other hand, placing the stop at the rear focal plane puts the EP at infinity and forms an **object-space telecentric** system. Tthe blur from the defocused object is centered about the chief ray and the image height at the nomial image is constant.



(a) Image-space telecentric



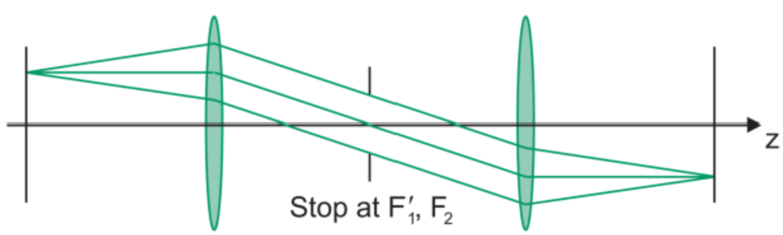
(b) Object-space telecentric

> The telecentric system must be unvignetted over the entire object FOV, otherwise the blut spot will no longer be symmetric and the centroid will shift.

Object-space telecentric systems are almost used at finite conjugated. The maximum object size is limited to approximately the radius of the objective lens due to vignetting considerations. This system doesnt have a $f/\#$ because the EP is at infinity and is infinite in size.

1.7.3 Double telecentric systems

An afocal system is made **double telecentric** by placing the system stop at the common focal point. The chief ray is parallel to the axis in object and image space, and both the EP and XP are located at infinity. All double telecentric systems must be afocal.



Since the ray bundle is centered on the chief ray, this condition guarantees that height of the blur forming the image is independent of axial object/image shifts.

> **FOV in telecentric systems**
>
> Defining te angular FOV relative to the EP or XP is impossible if the system is telecentric in that particular optical space because that pupil is in infinity. The object height or image height can be used instead.
>
> A second method is to measure the angular size of the object relative to the front nodal point N. Angular sizes of the object and image are equal when viewed from te respective nodal points. But

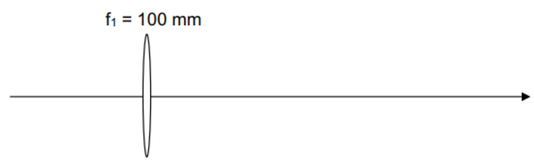
for afocal systems (like telecentric) fails as they do not have these points.

The choice of the method is of little consequence when the object is distant.

Ejemplo 1.14 **Double telecentric system**

A double telecentric system is constructed out of two thin lenses in air. It has a magnification of |1/10|. The focal length of the first lens of the system is 100 mm. An object of size $\pm 100\ mm$ is located 500 mm to the left of the first lens in the system.

a) Provide a layout of the system showing the second lens, spacings and the stop.
b) What is the focal length of the second lens?
c) Where is the image plane (location relative to the second lens element)?
d) What is the size and orientation of the image?



Solution

a) Second lens should be $f_1 + f_2$ to the right of L_1 while the stop in the intermediate image.
b) Magnification is

$$m = -\frac{f_2}{f_1} = -\frac{1}{10} = -\frac{f_2}{100\ mm} \longrightarrow f_2 = 10\ mm.$$

The length between the lenses is therefore $L = f_1 + f_2 = 110\ mm$.

c) The longitudinal magnification is

$$\bar{m} = m^2 = 0.01.$$

Assuming the pair of conjugates from the front and rear focal lengths, the depth of the object is:

$$\Delta z = -500 - (-100) = -400\ mm, \quad \text{and} \quad \Delta z' = \bar{m}\Delta z = -4\ mm.$$

The object is located $500\ mm$ to the left of the first lens and the image is $6\ mm$ to the right of the second lens.

d) The size of the image is

$$h' = mh = (-0.1)(\pm 100) = \mp 10\ mm,$$

which is inverted.

1.7.4 Imaging with an afocal system

An afocal system consists of two focal system sharing a common focal point. If the stop is located at this location, the system becomes telecentric. Because the magnification is constant, the cardinal points are not defined.

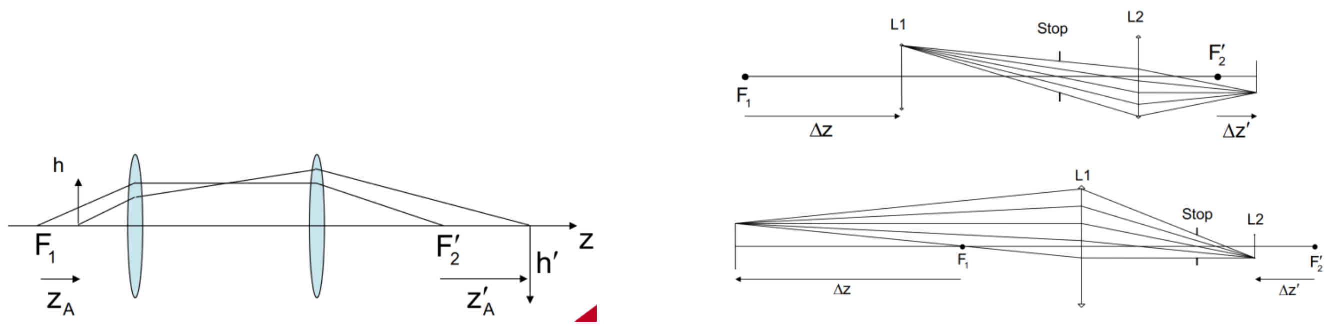
However, we can pick any pair of conjugate planes coupled with the longitudinal magnification:

$$\bar{m} = m^2 = \frac{\Delta z'}{\Delta z} = \frac{z'_A}{z_A}. \tag{1.35}$$

A usual pair is the front focal point of the first system with the rear focal point of the second system. Another is either the object is located at the first lens, or the image is at te second lens; these two situations image as if the other lens as not present.

Once selected, any other pair can be found using the longitudinal magnification. An axial shift in object space results in an image plane shift given by

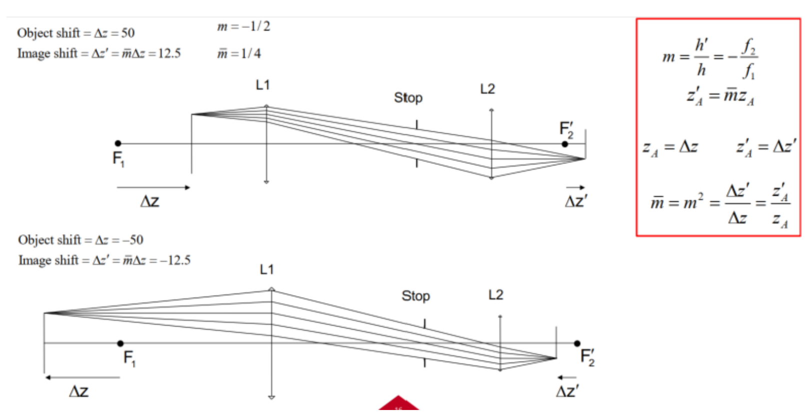
$$z'_A = \bar{m} z_A. \tag{1.36}$$



(a) Focal points conjugate pair

(b) First/second lens conjugate pair

Ejemplo 1.15 Imaging with afocal system



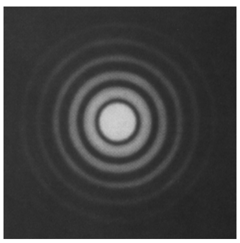
1.8 Stop and image quality

1.8.1 Diffraction-limited

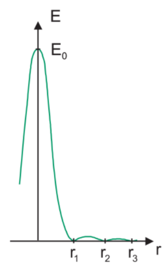
Because light is a wave, it does not focus to a perfect point image. Diffraction from the aperture limit the size of the image spot. For circular aperture, we get an **Airy disk** pattern.

$$\text{Airy disk equation} \qquad E = E_0 \left[\frac{2J_1(\pi r/\lambda f/\#_W)}{\pi r/\lambda f/\#_W} \right]^2, \tag{1.37}$$

where r is the radial coordinate, J_1 is the first-kind Bessel function, and $f/\#_W$ the image space working.



(a) Airy disk pattern



(b) Airy disk profile

	Radius r	Peak E	Energy in Ring (%)
Central maximum	0	$1.0\, E_0$	83.9
First zero r_1	$1.22\lambda f/\#_W$	0.0	
First ring	$1.64\lambda f/\#_W$	$0.017\, E_0$	7.1
Second zero r_2	$2.24\lambda f/\#_W$	0.0	
Second ring	$2.66\lambda f/\#_W$	$0.0041\, E_0$	2.8
Third zero r_3	$3.24\lambda f/\#_W$	0.0	
Third ring	$3.70\lambda f/\#_W$	$0.0016\, E_0$	1.5
Fourth zero r_4	$4.24\lambda f/\#_W$	0.0	

(c) Airy zeros

The diameter of the Airy disk to the first zero is:

$$D = 2.44\lambda \cdot f/\#_W. \tag{1.38}$$

The **Rayleight resolution criterion** states that two point objects can be resolved if the peak of one falls on the first zero on the other:

$$\text{Resolution} = 1.22\lambda \cdot f/\#_W. \tag{1.39}$$

The **angular resolution** is found by dividing by the focal length (or image distance):

$$\text{Angular resolution} = \alpha = 1.22\lambda/D_{\text{EP}}. \tag{1.40}$$

1.8.2 Spherical aberration

Spherical aberration SA causes the power or focal length of the system to vary with pupil radius. For a singlet, the power of the lens increases quadratically with pupil radius; the focal length decrease quadratically.

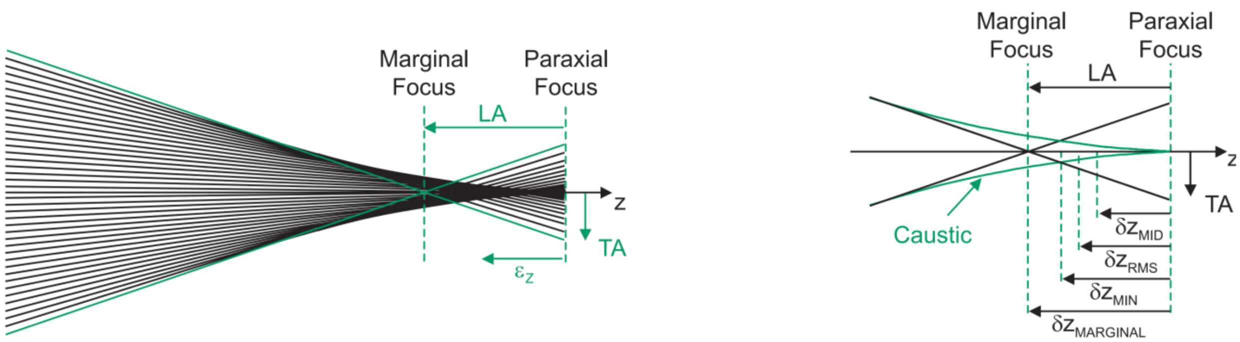
The image plane can be shifted from paraxial focus to obtain better image quality in the presence of SA. There are different focus criteria as seen in the figure.

In first-order geometrical optics, each point on the object plane corresponds to a point on the image plane. However, in real life we are not so lucky.

The spot size scales as the cube of the entrance pupil diameter.

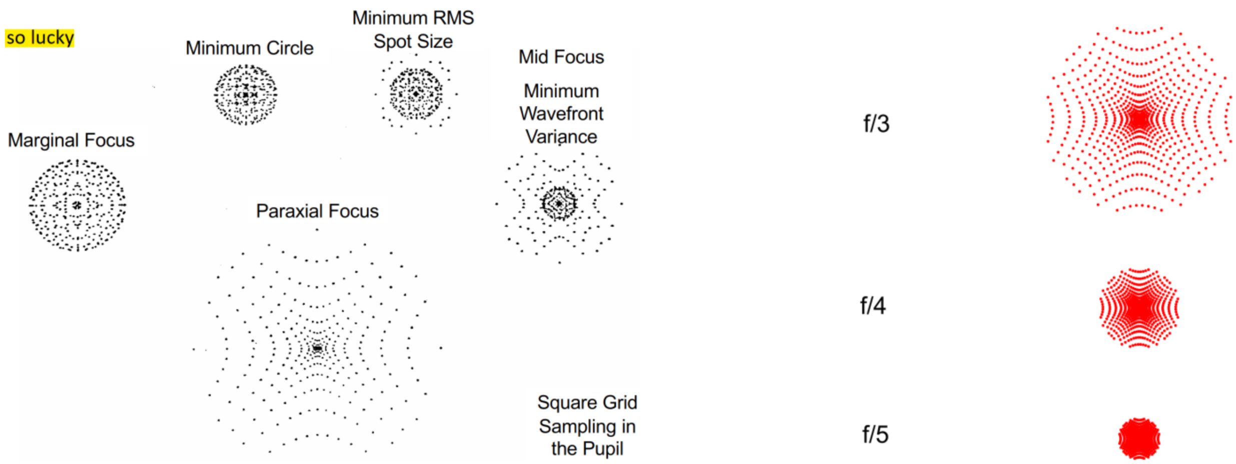
1.8.3 Lens bending and minimum spherical aberration

Bending the lens of orientation does not change the power, but its aberration do change. The minimum SA occurs when the ray is bent the same at both surfaces. This is directly analogous to the angle of



(a) Spherical aberration **(b)** Image plane criteria

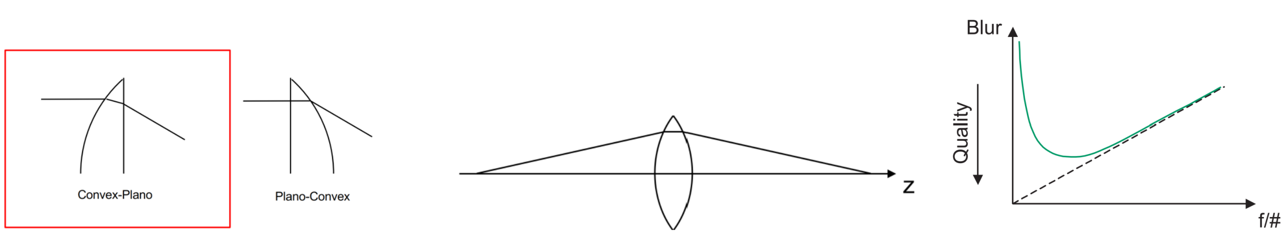
Figure 1.2 Spherical aberration produces different image plane criteria. LA and Ta stand for longitudinal and transverse aberration, respectively.



(a) Spot diagram of focus criteria **(b)** Spot size versus F-number

minimum deviation for prisms. For an object at infinity and $n = 1.5$, the corect lens shape is approximately convex-plano. At finite conjugates, a biconvex lens is used. A trick to further minimize spherical aberration in finite conjugates is to split the biconvex into two plano-convex lenses and then flip each of the lenses.

With large apertures, aberrations and depth of field errors are dominant. With small apertures, diffraction dominates with a linear dependence of blur with $f/\#$.

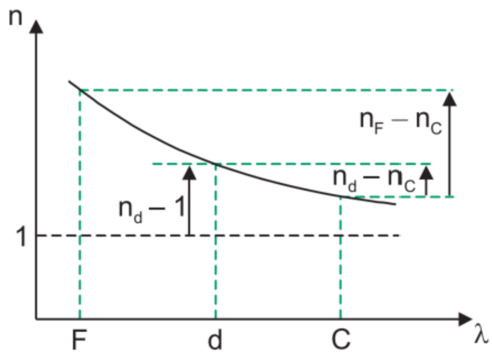


(a) Infinite conjugate **(b)** Finite conjugate **(c)** Blur vs F-number

1.9 Materials

1.9.1 Dispersion

Index of refration is commonly measured and reported at the specific wavelengths of elemental spectral lines. Over the visible spectrum, the **dispersion** of the index for optical glass is about 0.5% (low dispersion) to 1.5% (high dispersion) of the mean value of the index.



F	(H)	486.1 nm
d	(He)	587.6 nm
C	(H)	656.3 nm
I	(Hg)	365.0 nm
h	(Hg)	404.7 nm
F′	(Cd)	480.0 nm
g	(Hg)	435.8 nm
e	(Hg)	546.1 nm
D	(Na)	589.3 nm
C′	(Cd)	643.8 nm
r	(He)	706.5 nm
t	(Hg)	1014.0 nm

Figure 1.1 For visible applications, the F, d, and C lines are usually used.

We define some useful quantities:

$$\text{Refractivity} = n_d - 1, \quad \text{Principal dispersion} = n_F - n_C, \quad \text{Partial dispersion} = n_d - n_C. \tag{1.41}$$

The **Abbe number** is the single number used to characterize the disperion of the index of an optical material:

$$\text{Abbe number} \qquad \boxed{\nu = V = \frac{n_d - 1}{n_F - n_C}} \tag{1.42}$$

Typical values of the Abbe number for optical glass range from 25 to 65. Low ν-values indicate high dispersion.

Relative partial dispersion ratio or P-value gives the fraction of the total index change that occurs between the d and C wavelengths $n_d - n_C$:

$$P = P_{d,C} = \frac{n_d - n_C}{n_F - n_C}. \tag{1.43}$$

Due to flattening of the dispersion, $P_{d,C} < 0.5$. P-values can also be defined for other sets of wavelengths:

$$\text{Relative partial dispersion ratio} \qquad \boxed{P_{X,Y} = \frac{n_X - n_Y}{n_F - n_C}}. \tag{1.44}$$

1.9.2 Optical glass

Glass map

The **glass map** plots index of refraction versus Abbe number. By traddition, the Abbe number increases to the left, so that dispersion increases to the right. The **glass line** is the locus of ordinary optical glasses based on silicon dioxide.

The green line at $\nu \sim 50 - 55$ separates the glasses into crown glass (low dispersion) and flint flass (high dispersion).

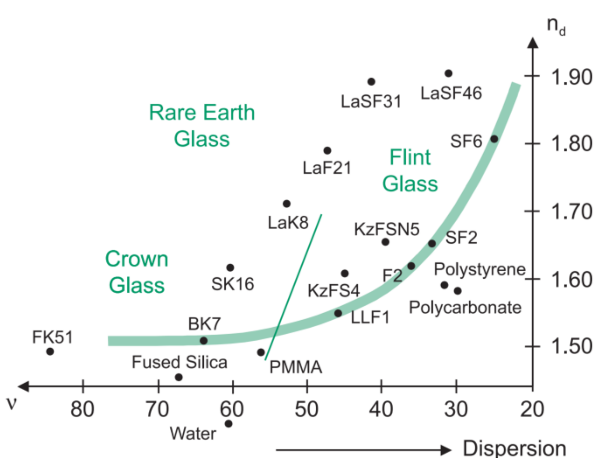
The addition of lead oxide increases the dispersion and the index and moves the glass up the glass line. To increase the index without changing the dispersion, barium oxide is added.

Glass away from the glass line are softer ans more difficult to polish. Low index glasses are less dense and generally have better blue transmission.

Glass code

The six-digit **glass code** specifies the index and the Abbe number:

$$abcdef \Longrightarrow n_d = 1.abc, \quad \nu = de.f \tag{1.45}$$



(a) Glass map

Material	Code	n_d	n_F	n_C	ν	P
N-FK51*	487845	1.48656	1.49056	1.48480	84.5	0.306
N-BK7	517642	1.51680	1.52238	1.51432	64.2	0.308
LLF1	548458	1.54814	1.55655	1.54457	45.8	0.298
N-KzFS4	613445	1.61336	1.62300	1.60922	44.5	0.301
N-F2	620364	1.62005	1.63208	1.61506	36.4	0.294
N-SK16	620603	1.62041	1.62756	1.61727	60.3	0.305
SF2	648339	1.64769	1.66123	1.64210	33.9	0.292
KzFSN5	654396	1.65412	1.66571	1.64920	39.6	0.298
N-LaK8	713538	1.71300	1.72222	1.70897	53.8	0.304
N-LaF21	788475	1.78800	1.79960	1.78301	47.5	0.301
N-SF6	805254	1.80518	1.82783	1.79608	25.4	0.287
N-LaSF31	881410	1.88067	1.89576	1.87429	41.0	0.297
N-LaSF46	901316	1.90138	1.92156	1.89307	31.6	0.292
Fused Silica	458678	1.45847	1.46313	1.45637	67.8	0.311
PMMA	492574	1.492	1.498	1.489	≈55	≈0.33
Polycarbonate	585299	1.585	1.600	1.580	≈30	≈0.25
Polystyrene	590311	1.590	1.604	1.585	≈31	≈0.26
Water	333560	1.333	1.337	1.331	≈60	≈0.33

(b) Glass code

The properties of an individual sample, especially for the plastic material and water, can vary from these catalog values. The measured indices of the actual glass should be used in final design for precision systems. The listed indices are measured relative to air ($n \approx 1.0003$), and the indices should be corrected for use in vacuum. The glass catalog lists other material properties important for a design such as **thermal expansion coefficient**, **teperature coefficient of refractive index**, **internal transmission**, etc.

This page is blank intentionally

This page is blank intentionally