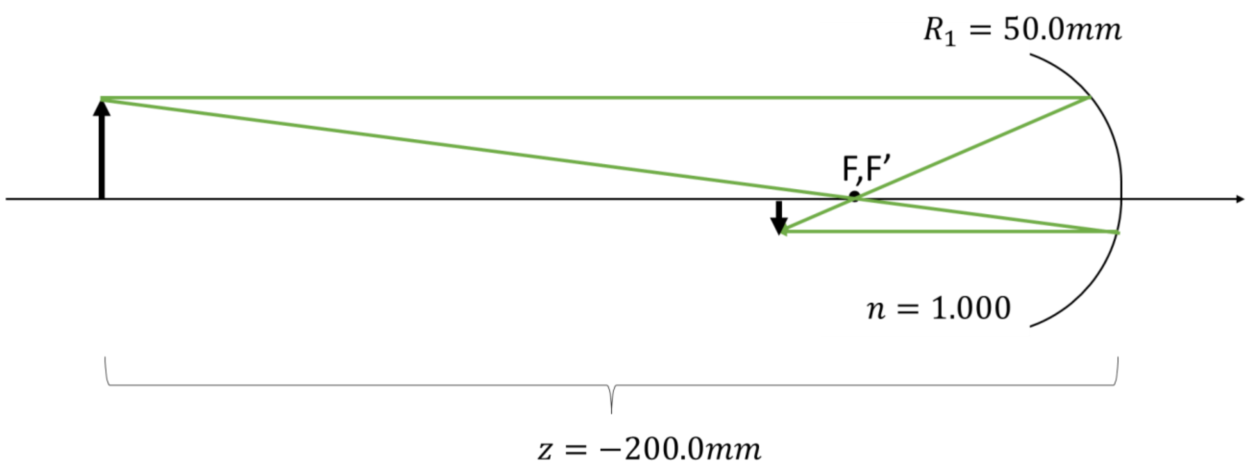
# Homework 7 Solutions

## 1. Gaussian Equations: Refracting or Reflecting Surface

An optical system in air, $n = 1.00$, consists of a single concave reflecting surface. The magnitude of the radius of curvature of the surface is 50mm. An object with a height of 10mm from the optical axis is placed 200mm to the left of the mirror.

a. Using a ray diagram, determine approximately the location and size of the resulting image through this system.




b. Using Gaussian equations determine the image location and size.

$n = 1.0, \qquad n' = -1.0$

$$\varphi = \frac{n' - n}{R} = \frac{-1.0 - 1.0}{-50mm} = 0.040mm^{-1}, \qquad f = 25mm$$

$$\frac{n'}{z'} = \frac{n}{z} + \varphi = \frac{1.0}{-200mm} + 0.040mm^{-1} = 0.035mm^{-1} \rightarrow z' = \frac{-1.0}{0.035mm^{-1}} = -28.57mm$$

$$m = \frac{z'/n'}{z/n} = \frac{-28.57/-1.0}{-200/1.0} = -0.143$$

$$h' = mh = -0.143 \cdot 10mm = -1.43mm$$

As expected from our ray diagram, our image is minimized and inverted while being located a little further from the surface than the focal plane.

c. Repeat part b with the surface now being refractive. The curvature, object height and positions are kept the same. The light begins in air, $n = 1.00$, then refracts through a medium with a refractive index of $n = 1.5$.

$$n = 1.0, \qquad n' = 1.5$$

$$\varphi = \frac{n' - n}{R} = \frac{1.5 - 1.0}{-50mm} = -0.010mm^{-1}$$

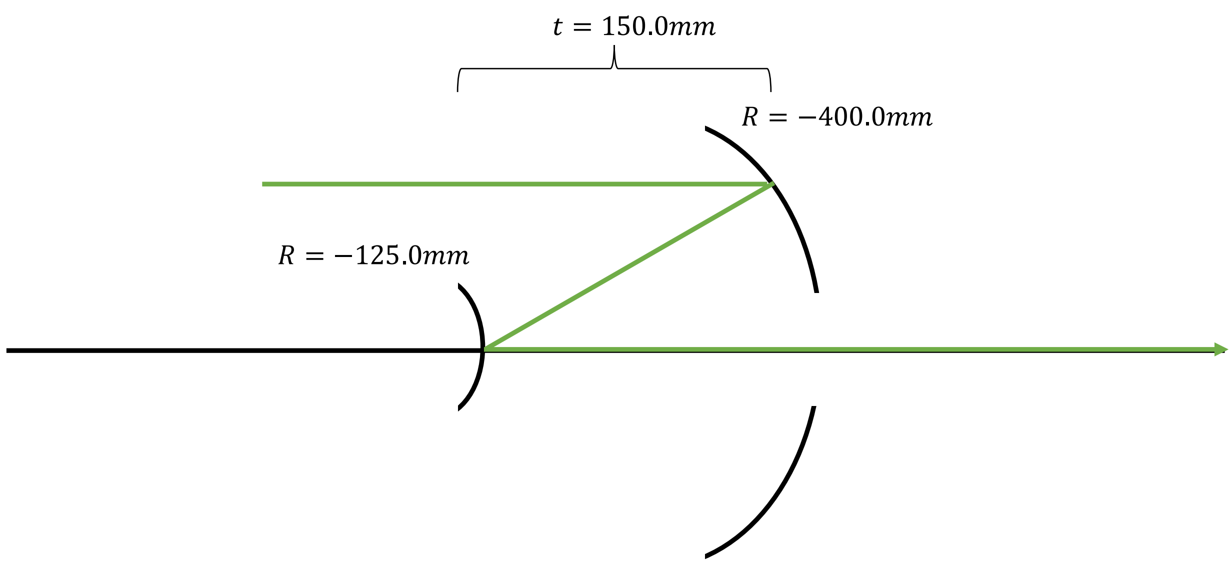
$$\frac{n'}{z'} = \frac{n}{z} + \varphi = \frac{1.0}{-200mm} - 0.010mm^{-1} = -0.015mm^{-1} \rightarrow z' = \frac{1.5}{-0.015mm^{-1}} = -100mm$$

$$m = \frac{z'/n'}{z/n} = \frac{-100/1.5}{-200/1.0} = +0.75$$

$$h' = mh = 0.75 \cdot 10mm = 7.5mm$$

## 2. Gaussian Reduction: Cassegrain Telescope

A Cassegrain telescope has a primary mirror with radius of curvature $R_1 = -400.0mm$, a secondary mirror with radius of curvature $R_2 = -125.0mm$ and thickness $t = 150.0mm$. You should use Gaussian reduction for this question.




a. Determine the power and effective focal length of this system.

Note that the distance travelled from the primary mirror to the secondary mirror is in a negative direction. Also note that when light is reflected, the refractive index is considered negative.

<table>
<tr><td></td><td colspan="2">$R_1 = -400.0mm$</td><td colspan="2">$R_2 = -125.0mm$</td><td></td></tr>
<tr><td colspan="2">$object\ distance$</td><td colspan="2">$t = -150.0mm$</td><td colspan="2">$image\ distance$</td></tr>
<tr><td colspan="2">$n_0 = 1.00$</td><td colspan="2">$n_1 = -1.00$</td><td colspan="2">$n_2 = 1.00$</td></tr>
</table>

| | | |
|---|---|---|
| | $\varphi_1 = \dfrac{(-1.00)-(1.00)}{-400.00}$ $= +0.005mm^{-1}$ | $R_2 = \dfrac{(1.00)-(-1.00)}{-125.00}$ $= -0.016mm^{-1}$ |
| $object\ distance$ | $\tau = \dfrac{-150.0mm}{-1.00} = 150.0mm$ | $image\ distance$ |

Combining the two surfaces

$$\varphi_{12} = \varphi_1 + \varphi_2 - \varphi_1 \cdot \varphi_2 \cdot \tau_1 = +0.005 - 0.016 - (+0.005)\cdot(-0.016)\cdot(150.0)$$

$$= 0.001mm^{-1}$$

$$f_E = \frac{1}{\varphi} = 1000mm$$

b. What is the distance between the primary mirror (the rightmost element) and the rear focal plane F'? This is the Working Distance.

$$d' = n_2\delta'_{12} = -\frac{\varphi_1}{\varphi_{12}}\tau_1 = -\frac{+0.005mm^{-1}}{+0.001mm^{-1}}\cdot 150.0mm = -750.0mm$$

With $V_1$ being the vertex of the primary mirror and $V_2$ being the vertex of the secondary mirror.

$$d' = -750.0 = -300.0\ (from\ passing\ back\ through\ the\ thickness\ twice) + V_2P'$$

$$V_2P' = -450.0mm$$

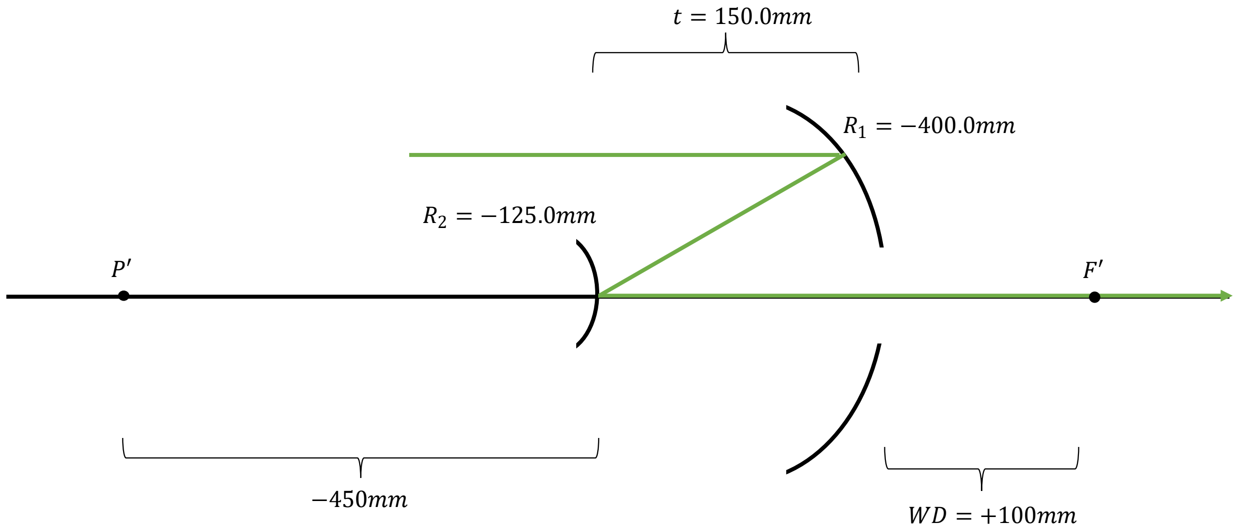
$$f'_R = 1000mm$$

$$f'_R = +1000 = 450.0\ (from\ P'V_2)$$

$$+\ 450.0\ (from\ passing\ through\ the\ thickness\ three\ times) + V_1F'$$

$$Working\ Distance = V_1F = 100.0mm$$

c. Illustrate the P' and F' plane locations on a diagram (include distances on this diagram).


$t = 150.0mm$
$R_1 = -400.0mm$
$R_2 = -125.0mm$
$P'$
$F'$
$-450mm$
$WD = +100mm$