Assignment 5

OPTI 502 Optical Design and Instrumentation I

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Exercise 1

a) In this case, the object is virtual and the image will be real and erected.

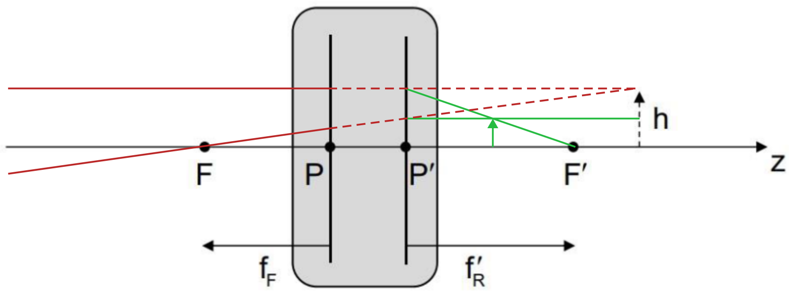

Figure 1

b) The object is virtual, and therefore the image will be real and erected.

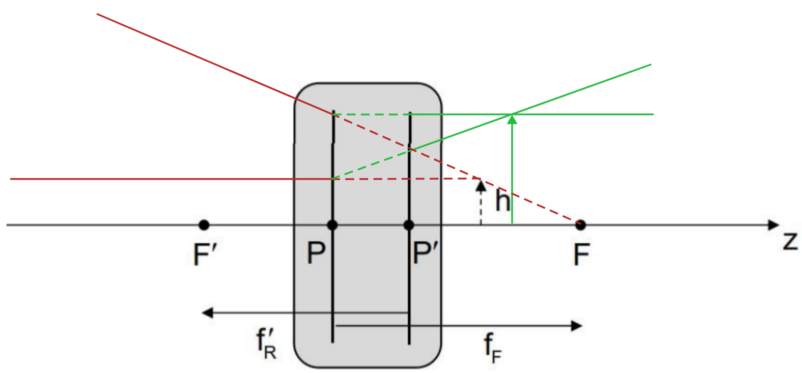

Figure 2

c) The object is virtual, and the image will be virtual and inverted.

Exercise 2

a) For a single refracting surface, we have that:

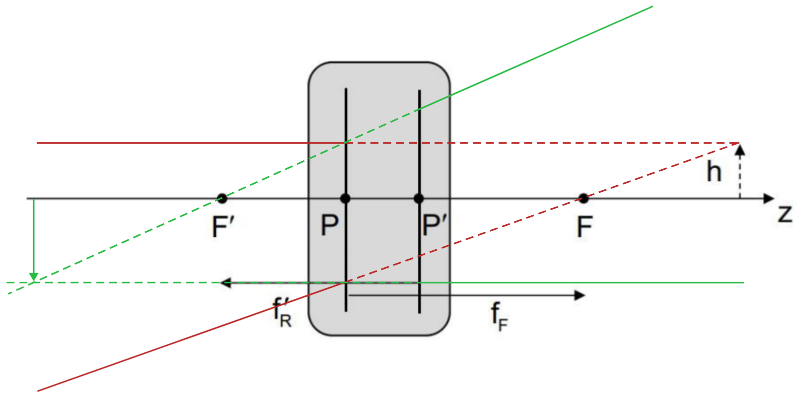

Figure 3

- Both nodal points are located at the center of curvature CC.
- Front and real principal planes are located at the vertex.
- The reduced thickness of the surface is the focal length of its thin lens representation.

We illustrate these quantities along with the vertex and the focal lengths in the following figure. We

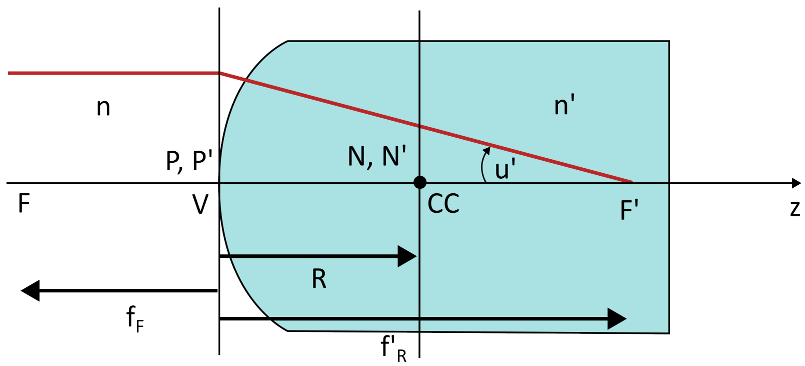

Figure 4: Illustration of cardinal point for a single refractive surface.

illustrate also some quantities of this surface:

$$C = \frac{1}{R} = 100\ m^{-1}, \quad \phi = (n' - n)C = 33.3\ m^{-1}, \quad f_E = \frac{1}{\phi} = 30\ mm,$$
$$f_F = -nf_E = -30\ mm, \quad f'_R = n'f_E = 40\ mm.$$

b) We use the following equation:

$$\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} \longrightarrow z' = \frac{n'zf_E}{nf_E + z}.$$

Replacing the physical values and the EFL:

$$z' = \frac{(1.333)(30)(100)}{(1)(30) + 100} = +30.7615\ mm.$$

Its height is determined by the magnification:

$$m = \frac{h'}{h} = \frac{z'/n'}{z/n} = \frac{30.7615/1.333}{100/1} = 0.231 \longrightarrow h' = mh = (0.231)(10\ mm) = 2.31\ mm.$$

c) The cube is divided in equal part by the optical axis, yielding a height of $h = 5\ mm$. Its last side is located 100 mm from the principal planes, as all their sides have the same sizes. Its first face is located 110 mm from the principal plane. Now, the area of one side is 100 mm^2. We want to find the equivalent (area of volume) of its image. We can address this problem by considering two lines at each side of the cube as two independent objects with (z_1, h_1) and (z_2, h_2). Then, we do imaging of both to get (z_1', h_1') and (z_2', h_2'). The difference between positions and heights allow us to construct the image dimension.

$$\begin{aligned} z_1 :\quad z_1' &= \frac{(1.333)(30)(110)}{(1)(30)+110} = +31.42\ mm \\ z_2 :\quad z_2' &= \frac{(1.333)(30)(100)}{(1)(30)+100} = +30.76\ mm \end{aligned}$$

We do the same for the magnification to compute the corresponding heights:

$$\begin{aligned} m_1 &= \frac{31.42/1.333}{110/1} = 0.214 \longrightarrow h_1' = m_1 h_1 = (0.214)(5) = 1.07\ mm \\ m_2 &= \frac{30.76/1.333}{100/1} = 0.231 \longrightarrow h_2' = m_2 h_2 = (0.231)(5) = 1.16\ mm. \end{aligned}$$

The other dimension is demagnified by the magnification m_1. The area, would be the integration from z_1' to z_2' with the respective height which can be used to create a linear function (interpolation). However, we will assum the mean value between them to consider it as a constant value. Then, the are is:

$$A = \Delta z' \Delta h' = 2(z_2' - z_1')\left(\frac{h_1' + h_2'}{2}\right) = 1.472\ mm^2.$$

And the volume is this area multiplied by the remaining dimension:

$$\Delta x \cdot A = (2m_1 5) \cdot A = (2 \cdot 1.07)(1.472) = 3.15\ mm^3.$$

Exercise 3

For a two positive lens system, we use Gaussian reduction to reduce the effect to a single thin lens. We first compute the overall optical power with the power of individual lenses:

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t = \frac{1}{40} + \frac{1}{40} - \frac{1}{40}\frac{1}{40} \cdot 20 = 0.038 mm^{-1} \longrightarrow f_E = \frac{1}{\phi} = 26.67\ mm.$$

The front and real focal lengths are:

$$f_F = -n_1 f_E = (1)(26.67\ mm) = -26.67\ mm, \quad \text{and} \quad f_R' = n_3 f_E = (1)(26.67\ mm) = 26.67\ mm.$$

Then the distances d and d', corresponding to the shift from the front (rear) principal planes P, P' of the equivalent system with respect to f_F, f'_R are given by

$$d = \frac{\phi_2}{\phi} t = \frac{0.025}{0.038} 20 = 13.158\ mm, \quad \text{and} \quad d' = -\frac{\phi_1}{\phi} t = -\frac{0.025}{0.038} 20 = -13.158\ mm.$$

The front (back) focal distances are then: The FFD and BFD are therefore,

$$\begin{aligned} \text{FFD} &= f_F + d = -26.67\ mm + 13.158\ mm = -13.512\ mm. \\ \text{BFD} &= f'_R + d' = 26.67\ mm - 13.512\ mm = 13.512\ mm. \end{aligned}$$

The reduction process and the quantities obtained are illustrated in figure 5.

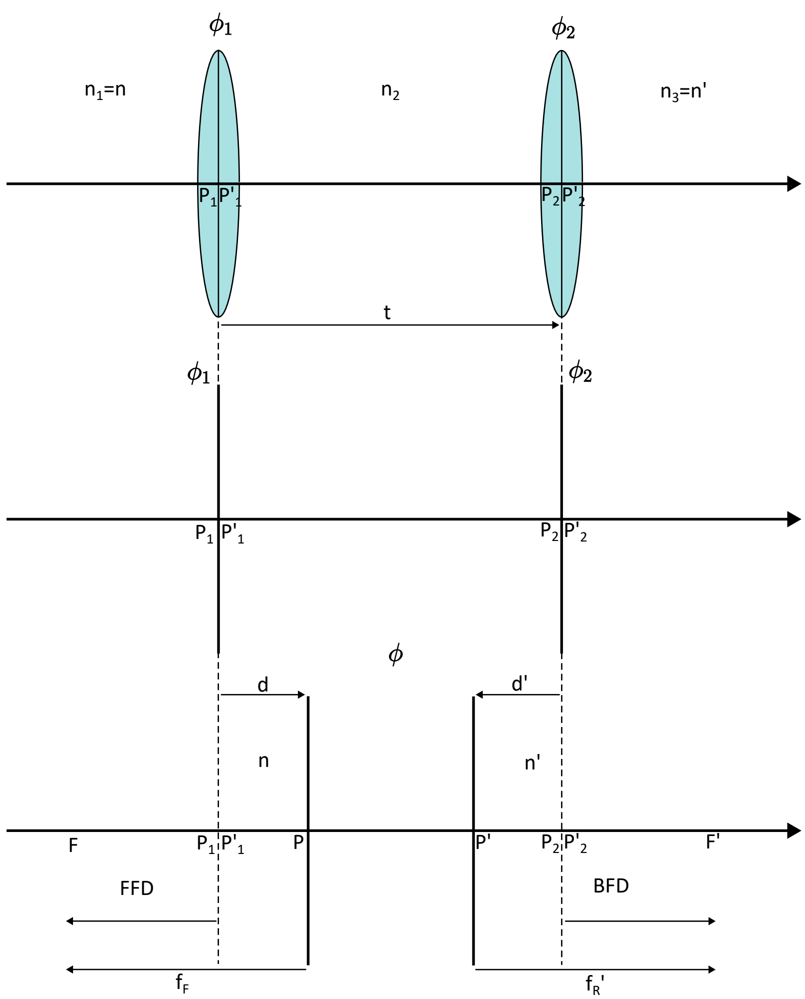

Figure 5: Gaussian reduction for two positive lenses.

Exercise 4

In this case we have three surface, each with their correspond surface curvature C and index of refraction n.

- **Gaussian reduction** The optical power of each surface is:

$$\phi_1 = \frac{n_1 - n_0}{R_1} = \frac{1.336 - 1}{7.8\ mm} = 0.043\ mm^{-1},$$
$$\phi_2 = \frac{n_2 - n_1}{R_2} = \frac{1.413 - 1.336}{10\ mm} = 0.007\ mm^{-1},$$
$$\phi_3 = \frac{n_3 - n_2}{R_3} = \frac{1.336 - 1.413}{-6\ mm} = 0.013\ mm^{-1}.$$

Now, we combine surface 1 with 2:

$$\phi_{12} = \phi_1 + \phi_2 - \phi_1\phi_2\tau_1 = 0.043 + 0.007 + 0.043 \cdot 0.007 \cdot \frac{3.6}{1.336} = 0.046\ mm^{-1}.$$

The shift of the principal plane are given by

$$\delta_{12} = \frac{\phi_2}{\phi_{12}}\tau_1 = \frac{0.007}{0.046} \cdot \frac{3.6}{1.336} = 0.410\ mm \longrightarrow d_{12} = \delta_{12}.$$
$$\delta'_{12} = -\frac{\phi_1}{\phi_{12}}\tau_1 = -\frac{0.043}{0.046} \cdot \frac{3.6}{1.336} = -2.519\ mm \longrightarrow d'_{12} = n_2\delta'_{12} = -3.560\ mm.$$

We can see that the front principal plane is displaced from V_1 to the left, while the rear principal plane is shifted to the right of V_2. In addition, the distance d'_{12} considered the index n_2 as it belong to that space. The distance of propagation through the index n_2 must be adjusted due to the shift of the rear principal plane:

$$\tau_{12} = \frac{t_2 - d'_{12}}{n_3} = \tau_2 - \delta'_{12} = 3.6 + 2.519 = 6.119\ mm.$$

Now, we compute the total optical power considering the reduction and the third surface:

$$\phi = \phi_{12} + \phi_3 - \phi_{12}\phi_3\tau_{12} = 0.046 + 0.013 - (0.046)(0.013)(6.119) = 0.055\ mm^{-1}.$$

The shifts are:

$$d_{123} = \delta_{123} = \frac{\phi_3}{\phi}\tau_{12} = \frac{0.013}{0.055} = 0.236\ mm$$
$$d'_{123} = n_3\delta'_{123} = -n_3\frac{\phi_{12}}{\phi}\tau_{12} = -(1.336)\frac{0.046}{0.055} = -1.117\ mm.$$

The total shift from the first surface is the sum of individual fron shift computed, while for the last surface is just the shift computed in the last reduction:

$$d = d_{12} + d_{123} = 0.410 + 0.236 = 0.646\ mm$$
$$d' = d'_{123} = -1.117\ mm.$$

The front (rear) focal lengths are then

$$f_E = \frac{1}{\phi} = 18.18\ mm \longrightarrow \begin{array}{l} f_F = -n_0 f_E = -(1)(18.18) = -18.182\ mm \\ f'_R = n_3 f_E = (1.336)(18.18) = 24.289\ mm. \end{array}$$

Finally, the FFD and BFD are:

$$\text{FFD} = f_F + d_{123} = -18.182 + 0.646 = -17.536\ mm$$
$$\text{BFD} = f'_R + d'_{123} = 24.289 - 1.117 = 23.172\ mm.$$

The reduction process is shown in figure 6. The green quantities are the equivalent of the final reduction.

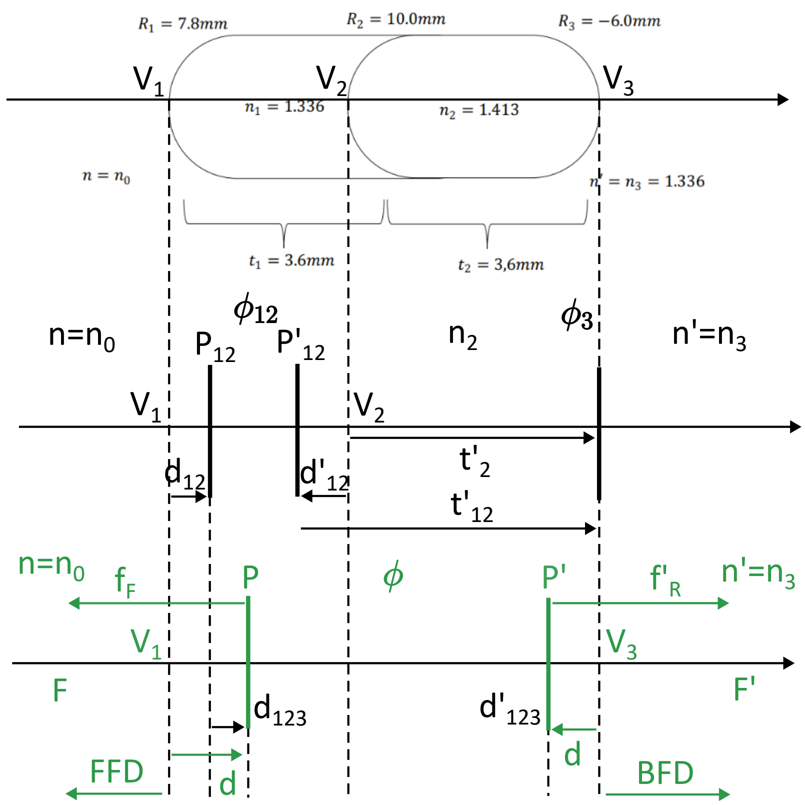

Figure 6: Gaussian reduction for the three-surfaces object.

- **Ray tracing** For the ray tracing, we will fill the ynu spreadsheet. We see the object space, three surfaces, two intermediate spaces, and the image space. Therefore, the table will have five columns. We will trace two rays, one from left to right and other in opposite direction in order to find the front and real focal lengths.

	Object space	Space 1	Surface 1	Space 2	Surface 2	Space 3	Surface 3	Space 4	Image space
C			0.128		0.1		-0.167		
t		15.79		3.6		3.6		12.864	
n		1		1.336		1.413		1.336	
$-\phi$			-0.043		-0.007		-0.013		
t/n		15.79		2.695		2.547		12.864	
y	1	1	1		0.884		0.759		0
nu	0	0		-0.043		-0.0492		-0.059	
u	0	0						-0.044	
y	0		0.916		0.967		1	1	1
nu		0.058		0.019		0.013		0	0
u		0.058						0	0

Exercise 5

For this problem, we must consider the reflection which translates to a negative index and a negativa reduced thickness. We will asume that oter media is air ($n = 1$).

	Object space	Space 1	Surface 1	Space 2	Surface 2	Space 3	Surface 3	Space 4	Image space
C			-0.01		-0.006		-0.01		
t		127.667		10		-10		-25.459	
n		1		1.5		-1.5		-1	
$-\phi$			0.005		-0.018		-0.025		
t/n		127.667		6.667		6.667		25.459	
y	1	1	1		1.033		0.942		0
nu	0	0		0.005		-0.0136		-0.037	
u	0	0						-0.037	
y	0		0.766		0.833		1	1	1
nu		0.006		0.010		0.025		0	0
u		0.006						0	0