Assignment 3

OPTI 502 Optical Design and Instrumentation I

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1 Exercise 1

(a) i. The ray diagram is illustrated in figure 1. We have traced three rays:

- Parallel to the optical axis from the object, then it is refracted with direction to the fron focal length.
- Direct to the back focal length: it is refracted so that it becomes parallel to the optical axis.
- The chief ray, which maintain its direction through its propagation.

The intersection of these three rays producte the image. We can see that the image is to the left of the lens, but to the right of the object. Therefore, it will be a virtual image and demagnified.

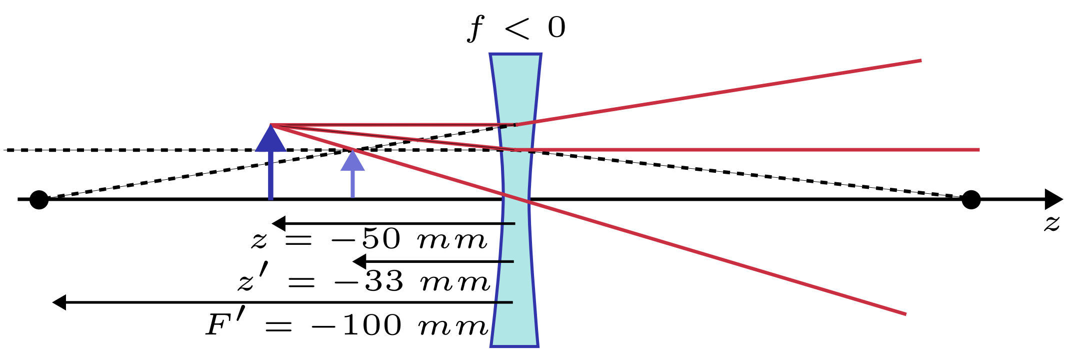


Figure 1: Ray diagram of the problem, the position of the image is z' and is located to the right of the object. Dashed lines correspond to virtual rays.

ii. Using the thin lens equation, considering that $F' = -100\ mm$ and $z = -50\ mm$ provides

$$\frac{1}{z'} = \frac{1}{F'} + \frac{1}{z}$$
$$\frac{1}{z'} = \frac{1}{-100} + \frac{1}{-50}$$
$$z' = \frac{(-100)(-50)}{-150} = -33.333\ mm.$$

Because $z' < 0$, the image is **virtual** and will be to the left of the lens. Its magnification is:

$$m = \frac{z'}{z} = \frac{-33.333}{-50} = 0.667.$$

The image is then erected ($\text{sgn}(m) = 1$), and demagnified ($|m| < 1$) making it smaller than the object.

(b) The Galilean telescope is composed of a positive lens and a negative to the right (figure 2). In order to make the system afocal, is necessary that the concave lens be located so that F_2 is at the back focal plane of the convex lens, that is, $F'_1 = F_2$.

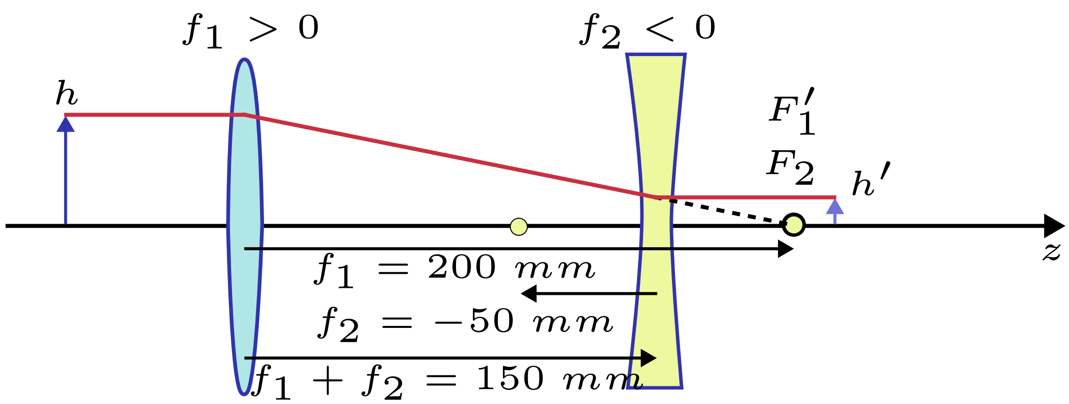


Figure 2: Galilean telescope illustration.

i. The distance between the two lenses t is computed if we plot the focal length of each one as illustrated in the figure. We see that f_1 is the total distance, shich can be thought as the sum of f_1 and t. Then, equating them and solving gives the distance t:

$$\begin{aligned} f_1 &= t - f_2 \\ 200 &= t - (-50) \longrightarrow t = 200 - 50 = 150\ mm. \end{aligned}$$

Lets remember that an afocal system does not have focal points, that is, incoming rays parallel to the optical axis will produce an image ray also parallel to the optical axis.
On the other hand, the magnification is the ratio of the heights h'/h, or in this case $-f_2/f_1$:

$$m = \frac{-f_2}{f_1} = \frac{50}{200} = \frac{1}{4}.$$

Therefore, the image will be demagnified but erected; there will be no parity change.

ii. Figure 2 also shows the ray diagram that begins parallel to the optical axis. Then, it is refracted to be converged at point F'_1. However, before reaching the point the negative lens refracts again the ray so that it goes back to its initial direction parallel to the optical axis. This effect is because F_2 is located at F'_1 so that the second lens see an object converging to its focal length.

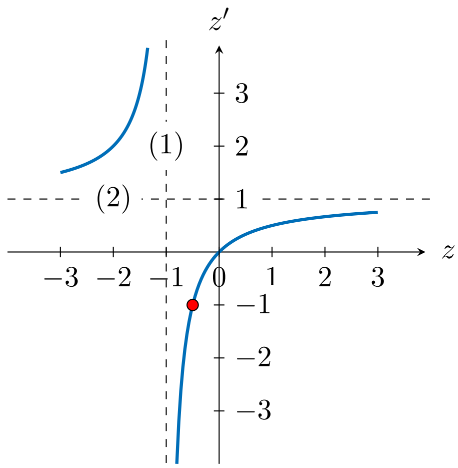
2 Exercise 2

(a) What we are asked to plot is the position z' of the image when the position of the object is varied in the range $z \in [-300, 300]\ mm$. To plot it as a function, we will use the rearranged thin lens equation

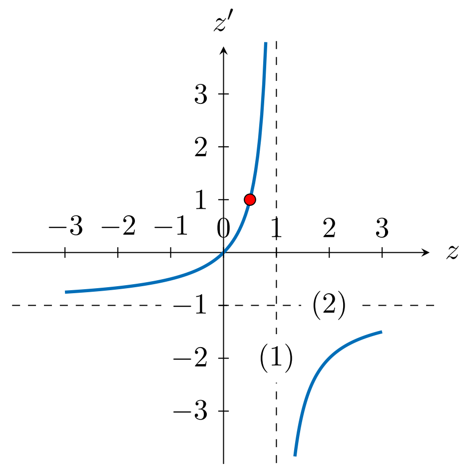
$$z' = \frac{zf}{z+f}. \quad (1)$$

i. For a positive lens $f = +100\ mm$, we have figure 3a. We can see that the curve has two asymptotas which corresponds to:
   - **(1)** Object located at $z = -1$, generating a real image at $z' = \infty$.
   - **(2)** Real image located at $z' = 1$, which is due to an object at $z = -\infty$.

ii. For $f = -100\ mm$, we have the figure 3b, where the same behavior is obtained as in th eprevious case. In this case, the asymptotes are displaced so that:
   - **(1)** Object located at $z = 1$, generating a real image at $z' = \infty$.
   - **(2)** Virtual image located at $z' = -1$, which is due to an object at $z = -\infty$.



(a) $f = +100\ mm$



(b) $f = -100\ mm$

Figure 3: Plot $z' - z$. Both axis were normalized by 100 mm for clarity.

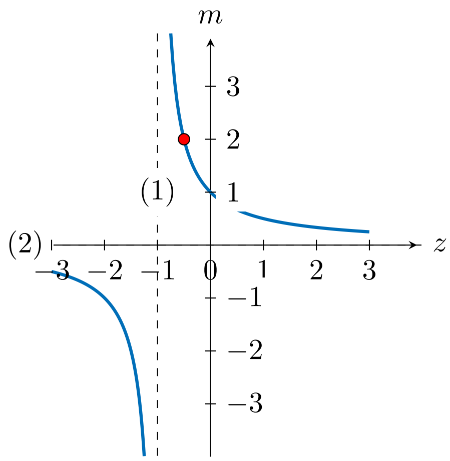
(b) The magnification can be described as a function of the object distance z'. We use the magnification equation $m = z'/z$ and the equation (1):

$$m = \frac{z'}{z} = \frac{f}{z+f}. \quad (2)$$

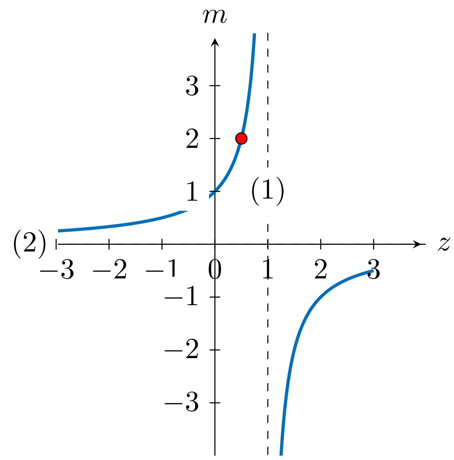
i. For a positive lens $f = +100\ mm$, we have figure 4a. We can see that the curve has two asymptotas which corresponds to:
   - **(1)** Real object located at $z = -1$, generating a real image with $m = \infty$.
   - **(2)** Real image with $m = 0$, which is due to an real object at $z = -\infty$.

ii. For $f = -100\ mm$, we have the figure 4b, where the same behavior is obtained as in th eprevious case. In this case, the asymptotes are displaced so that:
   - **(1)** Virtual object located at $z = 1$, generating a real image with $m = \infty$.
   - **(2)** Virtual image located with $m = 0$, which is due to a real object at $z = -\infty$.

(c) For a magnification $m+ = 2$, the distances of the object and image can be obtained by looking the figures 4a and 4b, respectively.



(a) $f = +100\ mm$



(b) $f = -100\ mm$

Figure 4: Plot $m - z$. Horizontal axis was normalized by $100\ mm$ for clarity.

i. For a positive lens $f = +100\ mm$, this magnification is obtained by looking at figure 4a (red circle), indicating that $z < 0$. Then, we solve the magnification equation (2)

$$\begin{aligned} m = \frac{f}{z+f} = 2 \longrightarrow f &= 2z + 2f \\ -f &= 2z \\ z &= -f/2. \end{aligned}$$

Substituting the image position in $m = z'/z$ give the object position:

$$m = \frac{z'}{z} = \frac{z'}{-f/2} = 2 \longrightarrow z' = -f.$$

Thus, both the object and image are located to the left of the lens, the object is real but the image is virtual and to the left of the object.

ii. In the case of a negative lens $f = -100\ mm$, by looking figure 4b we know that the object must be to the right of the negative lens, as $z' > 0$. By t=doing an analogus procedure as before, we found that $z = f/2$ and $z' = f$. However, after verifying with a ray diagram we concluded that is **not possible** to obtain a magnified image by either considering a real or virtual object. So, is not possible to meet a +2 magnified image even when the plot is showing the contrary.