

# **Notes of Optical design and instrumentation**

Wyant College of Optical Sciences  
University of Arizona

Nicolás Hernández Alegría

# Preface

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## **Part I**

# **Introduction to Geometrical Optics principles**

# Chapter 1

# Applications

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## 1.1 Thin prisms and dispersing prisms

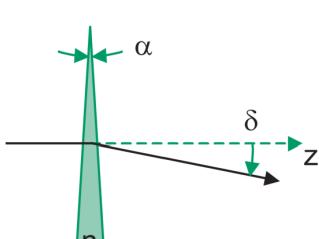
### 1.1.1 Thin prisms

**Thin prisms** introduce small angular beam deviations  $\delta$  that is approximately independent of the incident angle:

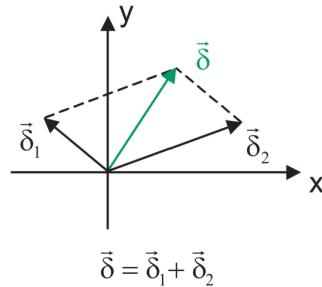
$$\delta \approx -(n - 1)\alpha. \quad (1.1)$$

The deviation is measured in prism diopters. A prisms of 1 diopter deviates a beam by 1 cm at 1 m. The beam deviation is parallel to a principal section of the prism and towards the thick end of the prism. The magnitude and direction of the deviation defines a vector perpendicular to the optical axis (xy plane). The net deviation vector for a series of thin prisms is then the vector sum of the component vectors:

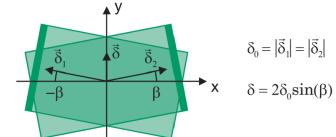
$$\delta = \delta_1 + \delta_2.$$



(a) Thin prism



(b) Beam deviation



(c) Risley prism

### Proof of the deviation

We tip the prism by  $\theta$  so that the front face is perpendicular to the input ray (no refraction):

$$\begin{aligned} n\theta &= \theta' \\ \delta = -\alpha, \delta' = \delta - \alpha &\longrightarrow -n\alpha = \delta - \alpha \quad . \\ \delta &\approx -(n - 1)\alpha \end{aligned} \quad (1.2)$$

### 1.1.2 Risley prism

A **Risley prism** consists of a pair of identical, but opposing, thin prisms. The prisms are counter-rotated by  $\pm\beta$  to obtain a variable net deviation in a fixed direction. The Risley prism allows the fine angular alignment for an optical system by adjusting the prism orientation  $\beta$ .

### 1.1.3 Thin prism dispersion

Thin prism

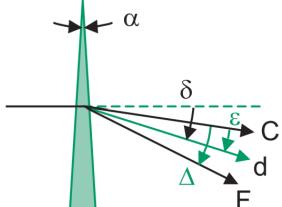
The **dispersion of a thin prism**  $\Delta$  measures the total angular spread from  $C$  to  $F$  light, and the **secondary dispersion**  $\epsilon$  gives the spread from the  $C$  to  $d$  wavelengths. The results depend on the index  $n_d$ ,

Abbe number  $\nu$  and partial dispersion ratio  $P$  of the glass:

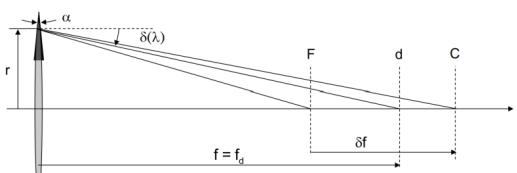
$$\text{Deviation } \delta = -(n_d - 1)\alpha \quad (1.3)$$

$$\text{Dispersion } \Delta = -(n_F - n_C)\alpha, \quad \Delta = \frac{\delta}{\nu} \quad (1.4)$$

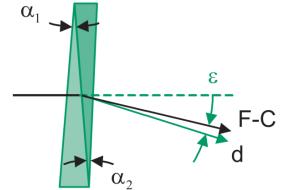
$$\text{Secondary dispersion } \varepsilon = -(n_d - n_C)\alpha, \quad \varepsilon = P\Delta = P\frac{\delta}{\nu}. \quad (1.5)$$



(a) Thin prism dispersion



(b) Edge of a thin prism



(c) Achromatic thin prism

### How does the difference in focal length is related with the Abbe number?

$$\delta f = f_C - f_F = -\frac{r}{\delta_C} + \frac{r}{\delta_F} = -r \frac{\delta_F - \delta_C}{\delta_F \delta_C} \approx -r \frac{\delta_F - \delta_C}{\delta_d^2}.$$

$$\frac{\delta f}{f_d} = \frac{-r \frac{\delta_F - \delta_C}{\delta_d^2}}{-\frac{r}{\delta_d}} = \frac{\delta_F - \delta_C}{\delta_d} = \frac{-i(n_F - 1)\alpha + (n_C - 1)\alpha}{-(n_d - 1)\alpha} = \frac{n_F - n_C}{n_d - 1} = \frac{1}{\nu} \rightarrow \frac{\delta f}{f_d} = \frac{1}{\nu}.$$

An inverted prism deviates a ray up and has a negative vertex angle  $\alpha$ .

Deviations and dispersions adds.

$$\delta = \sum_i \delta_i, \quad \Delta = \sum_i \Delta_i, \quad \varepsilon = \sum_i \varepsilon_i. \quad (1.6)$$

### Achromatic thin prism

An **achromatic thin prism** or **achromatic wedge** provides deviation without dispersion. Opposite prisms made from two different glasses ( $n_{d1}, \nu_1, P_1$ ) and ( $n_{d2}, \nu_2, P_2$ ) are combined to force the dispersion between the  $F$  and  $C$  wavelengths to be zero. A deviation of  $\delta$  is maintained for  $d$  light:

$$\text{Achromatic relations} \quad \frac{\alpha_1}{\delta} = \frac{1}{\nu_2 - \nu_1} \frac{\nu_1}{n_{d1} - 1}, \quad \frac{\alpha_2}{\delta} = -\frac{1}{\nu_2 - \nu_1} \frac{\nu_2}{n_{d2} - 1}. \quad (1.7)$$

### Proof of the above relations

We force the dispersion of  $F$  and  $C$  to be zero:

$$\Delta = \Delta_1 + \Delta_2 = \frac{\delta_1}{\nu_1} + \frac{\delta_2}{\nu_2} = 0 \rightarrow \delta_2 = -\frac{\nu_2}{\nu_1} \delta_1.$$

A deviation  $\delta$  for d light is maintained:

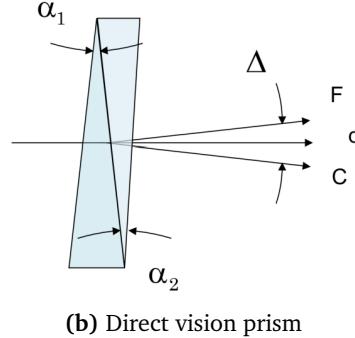
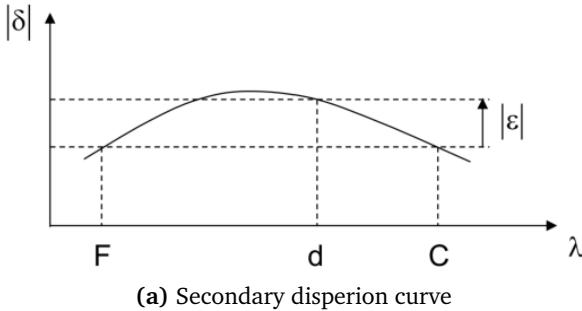
$$\delta = \delta_1 + \delta_2 = \delta_1 - \frac{\nu_2}{\nu_1} \delta_1 = (\nu_1 - \nu_2) \frac{\delta_1}{\nu_1} = -(\nu_1 - \nu_2) \frac{(n_{d1} - 1)\alpha_1}{\nu_1}.$$

Doing  $\alpha_1/\delta$  and  $\alpha_2/\delta$  yields the above results.

The high-dispersion prism is inverted to obtain an opposing deviation. While the F and C wavelengths are corrected, a residual secondary dispersion remains. For most glass pairs, d light will be bent more than the F and C wavelengths:

$$\text{Secondary aberration} \quad \frac{\varepsilon}{\delta} = \frac{P_2 - P_1}{\nu_2 - \nu_1} = \frac{\Delta P}{\Delta\nu}. \quad (1.8)$$

For most glasses,  $\frac{\varepsilon}{\delta} > 0$ . The shape of the curve of  $|\delta| - \lambda$  is concave and the maximum dispersion does not occur at d light. The achromatic thin prism has about 40 less secondary dispersion compared to a simple thin prism.



## Direct vision prism

A **direct vision prism** uses opposing prisms to provide dispersion without deviation of the d light.

We first set the total dispersion to zero:  $\delta = \delta_1 + \delta_2 = 0$ . Then,

$$\Delta = \Delta_1 + \Delta_2 = \frac{\delta_1}{\nu_1} + \frac{\delta_2}{\nu_2} = -\frac{\nu_1 - \nu_2}{\nu_1 \nu_2} \delta_1 \rightarrow \frac{\alpha_1}{\Delta} = \left( \frac{\nu_1 \nu_2}{\nu_1 - \nu_2} \right) \left( \frac{1}{n_{s1} - 1} \right) \wedge \frac{\alpha_2}{\Delta} = -\left( \frac{\nu_1 \nu_2}{\nu_1 - \nu_2} \right) \left( \frac{1}{n_{d2} - 1} \right).$$

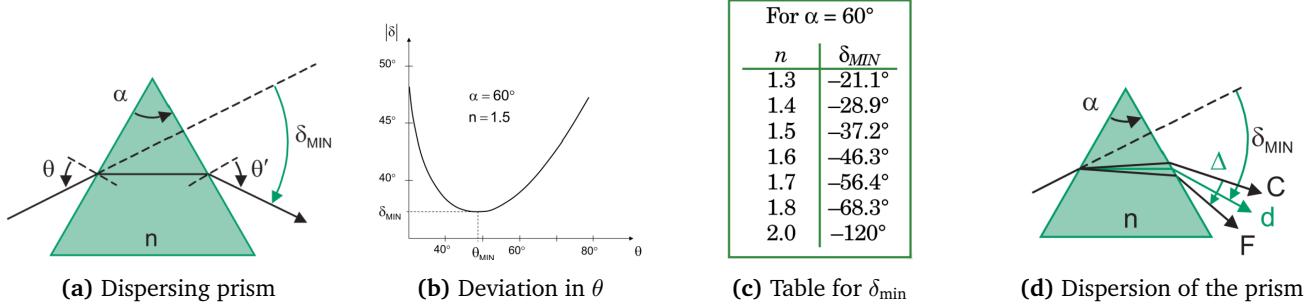
### 1.1.4 Dispersing prism

The total deviation  $\delta$  in the **dispersion prism** is the sum of the deviations at the two surfaces:

$$\text{Total deviation} \quad \delta = \alpha - \sin^{-1}[\sqrt{n^2 - \sin^2 \theta}] \sin \alpha - \cos \alpha \sin \theta] - \theta.$$

There is a minimum deviation angle  $\delta_{\min}$ , at which the ray path through the prism is symmetric  $\theta' = -\theta$ . The ray is bent an equal amount at each surface. The deviation is negative for the orientation of the prism in the figure. The **angle of minimum deviation** is

$$\delta_{\min} = \alpha - 2 \sin^{-1}[n \sin(\alpha/2)]. \quad (1.9)$$



The following table shows  $\delta_{\min}$  for several  $n$ .

The measurement of the index depends only on  $\delta_{\min}$  and the prism apex angle  $\alpha$ :

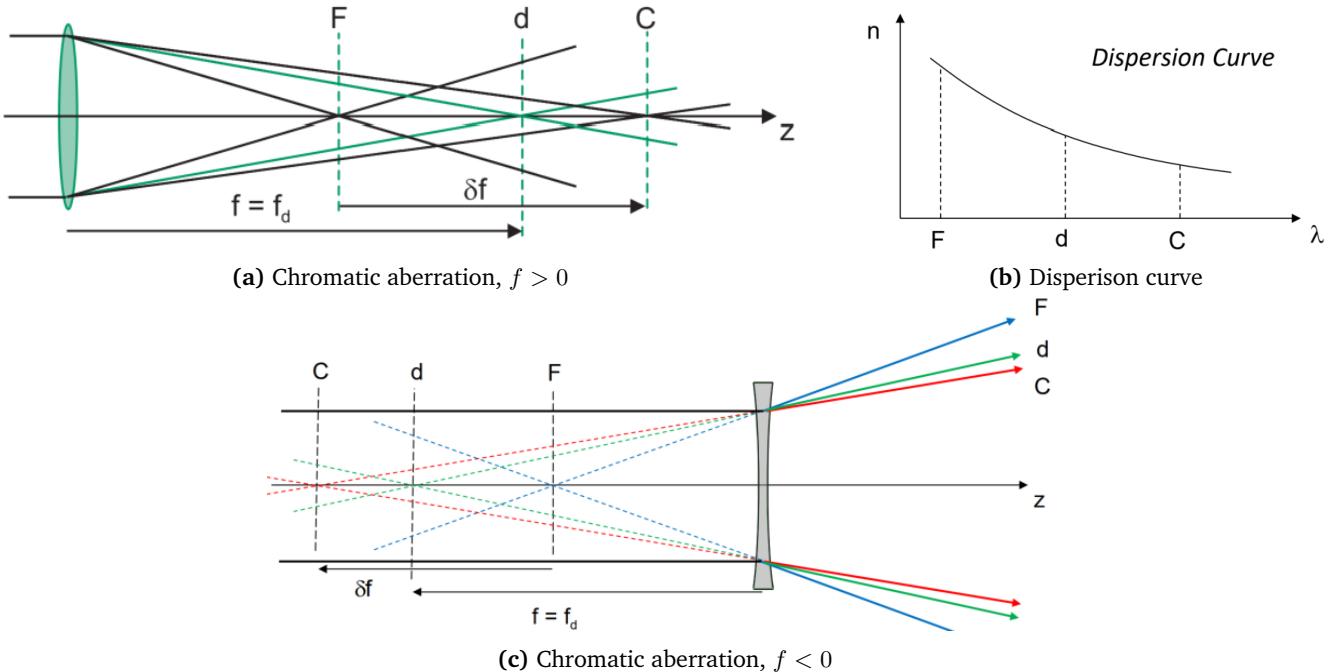
$$n = \frac{\sin \frac{\alpha - \delta_{\min}}{2}}{\sin(\alpha/2)}. \quad (1.10)$$

Prism spectrometers can obtain accuracies of one part in  $10^6$ .

## 1.2 Chromatic effects

### 1.2.1 Chromatic aberration

**Axial chromatic aberration** or **axial color** is a variation of the system focal length with wavelength. This aberration derives from the dispersion of the glass as the index changes with wavelength  $n(\lambda)$ .



Because of the higher index for F light, blue light is bent more and therefore the blue focus is closest to the lens.

### How much does the focal length change for the F and C wavelengths?

We look at the difference in power between these two wavelengths:

$$\delta\phi = \phi_F - \phi_C = (n_F - 1)(C_1 - C_2) - (n_C - 1)(C_1 - C_2) = (n_F - n_C)(C_1 - C_2)$$

$$\delta\phi = \underbrace{\frac{n_F - n_C}{n_d - 1}}_{1/\nu} \underbrace{(n_d - 1)(C_1 - C_2)}_{\phi_d} = \frac{\phi_d}{\nu}.$$

Similarly, for the focal length:

$$\delta f = f_C - f_F = \frac{1}{\phi_C} - \frac{1}{\phi_F} = \frac{\phi_F - \phi_C}{\phi_C \phi_F} = \frac{\delta\phi}{\phi_C \phi_F} \approx \frac{\delta\phi}{\phi_d^2} = \frac{\phi_d}{\nu \phi_d^2} = \frac{f_d}{\nu}.$$

The foci of F, d and C are not evenly spaced due to the shape of the dispersion curve. The relative order of the foci is reversed for a negative lens.

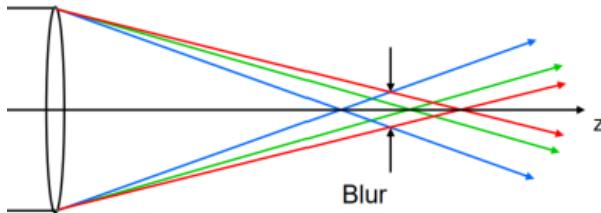
$$+ \text{lens chromatic aberration} \quad \delta f_{CF} = f_C - f_F, \quad \delta\phi_{FC} = \phi_F - \phi_C, \quad \frac{\delta f_{CF}}{f_d} = \frac{\delta\phi_{FC}}{\phi_d} = \frac{1}{\nu} \quad (1.11)$$

$$- \text{lens chromatic aberration} \quad \text{same, with } f_d < 0, \quad \phi_d < 0, \quad \delta f_{CF} < 0, \quad \delta\phi_{FC} < 0. \quad (1.12)$$

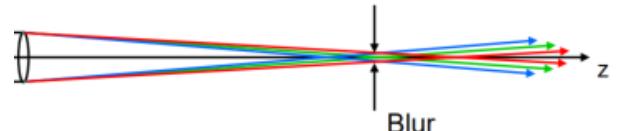
### 1.2.2 Type of chromatic aberrations

#### Longitudinal chromatic aberration

The blur associated with the chromatic aberration of the objective lens limits the performance of an objective. To reduce the blur, a small diameter objective lens is required. The blur is then proportional to the lens diameter.



(a) Axial longitudinal chromatic aberration



(b) Small diameter to reduce aberration

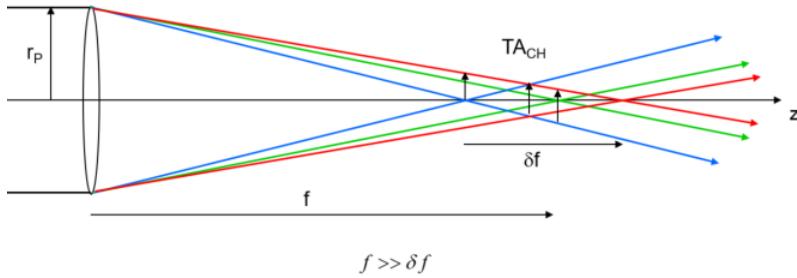
#### Transverse axial chromatic aberration

**Transverse axial chromatic aberration** measures the image blur size due to axial chromatic aberration. It depends only on the glass and the pupil radius  $r_P$  (stop at the lens):

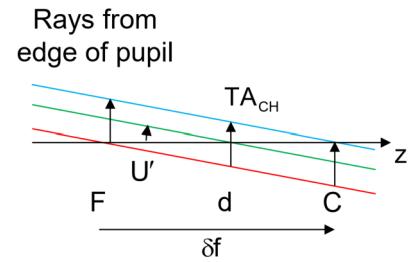
$$\text{TA}_{CH} = \frac{r_P}{\nu}. \quad (1.13)$$

#### Lateral chromatic aberration

**Lateral chromatic aberration** or **lateral color** is caused by dispersion of the chief ray. The edge of the lens behaves like a thin prism. Off-axis image points will exhibit a radial color smear. The blur length



(a) Transverse axial chromatic aberration



(b) Color swapped, should be blue first

increases linearly with the image height. Each color has a different lateral magnification.

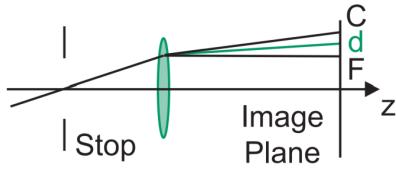
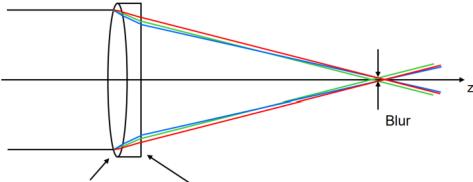


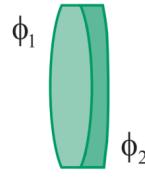
Figure 1.4 Lateral chromatic aberration

### 1.2.3 Achromatic doublet

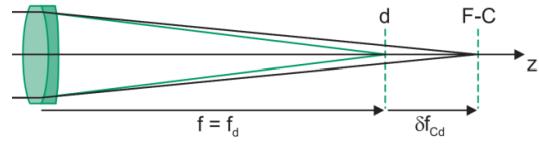
The thin lens **achromatic doublet** corrects longitudinal chromatic aberration by combining a positive element with a negative one. Two different glasses ( $\nu_1, P_1$ ) and ( $\nu_2, P_2$ ) are used.



(a) Achromatic doublet



(b) Composition



(c) Secondary aberration

#### How do we design the individual powers of the achromatic doublet?

$$\phi = \phi_1 + \phi_2 \implies \delta\phi_{FC} = \delta\phi_{FC1} + \delta\phi_{FC2} = \frac{\phi_1}{\nu_1} + \frac{\phi_2}{\nu_2} = \phi_F - \phi_C = 0 \implies \frac{\phi_1}{\nu_1} = -\frac{\phi_2}{\nu_2}.$$

$$\phi = \phi_2 - \frac{\nu_1}{\nu_2} \phi_2 = \frac{\nu_2 - \nu_1}{\nu_2} \phi_2 \implies \frac{\phi_2}{\phi} = -\frac{\nu_2}{\nu_1 - \nu_2} \wedge \frac{\phi_1}{\phi} = \frac{\nu_1}{\nu_1 - \nu_2}.$$

All we have done is to force the axial focus for F and C light. However, the d line can focus at a different location. This is known as **secondary chromatic aberration**.

#### Ejemplo 1.1

Design a 160 mm focal length thin-lens achromatic doublet using the following glasses. Provide the focal lengths

#### Design of an achromatic doublet

and indices of refraction of the two thin lenses.

Glass 1: Fused Silica, 458678, Glass 2: SF6, 805254.

## Solution

Glass 1:  $n_1 = 1.458$ ,  $\nu_1 = 67.8$ , Glass 2:  $n_2 = 1.805$ ,  $\nu_2 = 25.4$ .

$$\frac{1}{f_2} = -\frac{\nu_2}{\nu_1 - \nu_2} \frac{1}{f} = -\frac{25.4}{67.8 - 25.4} \frac{1}{160} = -0.00374 \text{ mm}^{-1} \rightarrow f_2 = -267.380 \text{ mm}$$

$$\frac{1}{f_1} = \frac{\nu_1}{\nu_1 - \nu_2} \frac{1}{f} = \frac{67.8}{67.8 - 25.4} \frac{1}{160} = 0.0010 \text{ mm}^{-1} \rightarrow f_1 = 100 \text{ mm}.$$


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## 1.3 Illumination systems

### 1.3.1 Illumination systems and types

The illumination system provides light for the optical system. Important considerations are the amount of light, the uniformity, and the angular spread of the light as seen by the object.

A **projector** is the general term for an imaging system that also provides the illumination for the object.

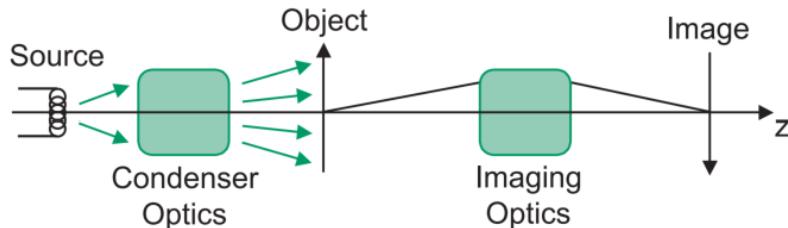


Figure 1.1 Projector as the illumination system.

There are three basic classifications of illumination systems:

- **Diffuse illumination** Light with a large angular spread is incident on the object. There is no attempt to image the source into the imaging system. It provides uniform illumination but is light inefficient.

No source coupling

- **Specular illumination** The light source is imaged by the condenser optics into the EP of the imaging optics. As it is good light efficient, is used for most optical systems with an integral light source.

Source to pupil coupling

- **Critical illumination** The light source is imaged directly onto the object.

Source to object coupling

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