Assignment 1

OPTI 502 Optical Design and Instrumentation I

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1 Exercise 1

2 Exercise 2

For the derivation of the law of reflection, we are going to use the following scheme illustrated in figure 1. An incident ray with path length L_1 hits a plane mirror with an angle θ_1. A reflection is obtained with an angle θ_2 which propagates with optical legnth L_2.

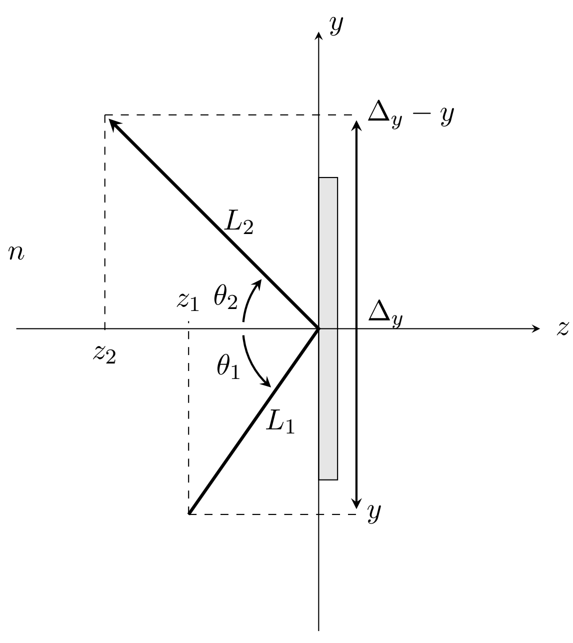

Figure 1: Scheme used for the derivation of the law of reflection.

This figure has the right-hand convention so that the quantities listed have the following properties:

$$\theta_1 > 0, \quad \theta_2 < 0, \quad z_1 > 0, \quad z_2 < 0, \quad y > 0, \quad (\Delta_y - y) > 0.$$

The position coordinates were evaluated in terms of theirs direction. As both z_1 and y_1 intend to go to the positive quadrant, they are considered positive. The same idea was applied to the L_2 ray.

To begin with, the optical path length is the sum of the terms nL_1 and nL_2:

$$\mathrm{OPL}(y) = nL_1 + nL_2 = n\left[\sqrt{z_1^2 + y^2} + \sqrt{(-z_2)^2 + (\Delta_y - y)^2}\right] = n\left[\sqrt{z_1^2 + y^2} + \sqrt{z_2^2 + (\Delta_y - y)^2}\right].$$

where n is factored out as the ray remains in the same medium.

To apply the fermat principle, we set $d\mathrm{OPL}/dy = 0$:

$$\frac{d\mathrm{OPL}}{dy} = n\frac{d}{dy}\left[\sqrt{z_1^2 + y^2} + \sqrt{z_2^2 + (\Delta_y - y)^2}\right] = n\left[\frac{y}{\sqrt{z_1^2 + y^2}} + \frac{\Delta_y - y}{\sqrt{z_2^2 + (\Delta y - y)^2}}\right] = 0$$

The last result can be reduced by substituting back the definition of L_1 and L_2:

$$\frac{d\mathrm{OPL}}{dy} = \frac{y}{L_1} + \frac{\Delta_y - y}{L_2} = \sin\theta_1 + \sin\theta_2 = 0$$

Finally, using the odd property $f(x) = -f(x)$ of the sine function and the last result, we obtain the law of refraction:

$$\begin{aligned}
\sin\theta_1 &= -\sin\theta_2 \\
\sin\theta_1 &= \sin(-\theta_2) / \sin^{-1}(\cdot) \\
\theta_1 &= -\theta_2.
\end{aligned}$$