Notes of Optical design and instrumentation

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Preface

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Introduction to Optics

1.1 Introduction

1.1.1 Light propagation

Geometrical optics is the study of light in the limit of short wavelenegths. We treat light as propagating rays. Geometrical optics usually ignores interferences, diffraction, polarization and quantum effects.

It often includes:

- Reflection, refraction
- Optical design
- Imaging properties
- Aberrations
- Radiometry

Light is a self-propagating EM wave where electric and magnetic fields are perpendicular or transverse to direction of propagation. In a vacuum, light propagates at the speed of light c, which is

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792458 \times 10^8 \quad (m/s). \tag{1.1}$$

The **wavelength** λ is the distance between two peaks or two valleys on the wave.

A **wavefront** is a surface of constant propagtion time from the source. It begins from a point source in spherical form, and as it propagates away, a given solid arc tend to behaves a a planar wavefront.

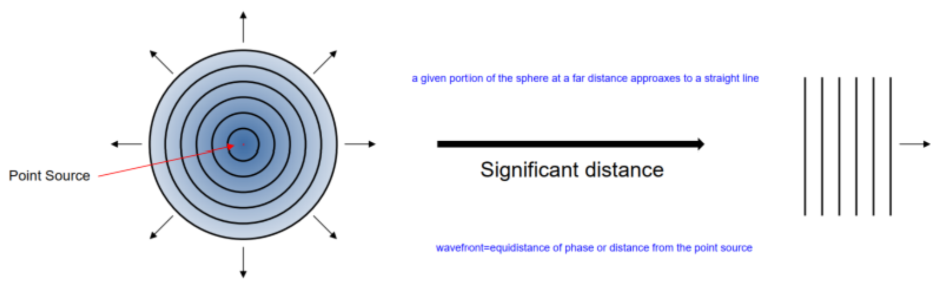


Figure 1.1 We can treat the wavefront as planar when assuming a distant object.

The time for one wavelength to pass is knwon as the **period** T:

$$T = \frac{\lambda}{V} \quad (s), \tag{1.2}$$

where V is the velocity of propagation. The number of wavelengths to pass in one second is the **frequency** ν:

$$\nu = \frac{1}{T} \quad (s^{-1})(Hz). \tag{1.3}$$

1.1.2 Sign convention

We define the sign convention for which the light propagates. It allows us to keep track of physical quantities and multiple reflections when analyzing complex optical systems.

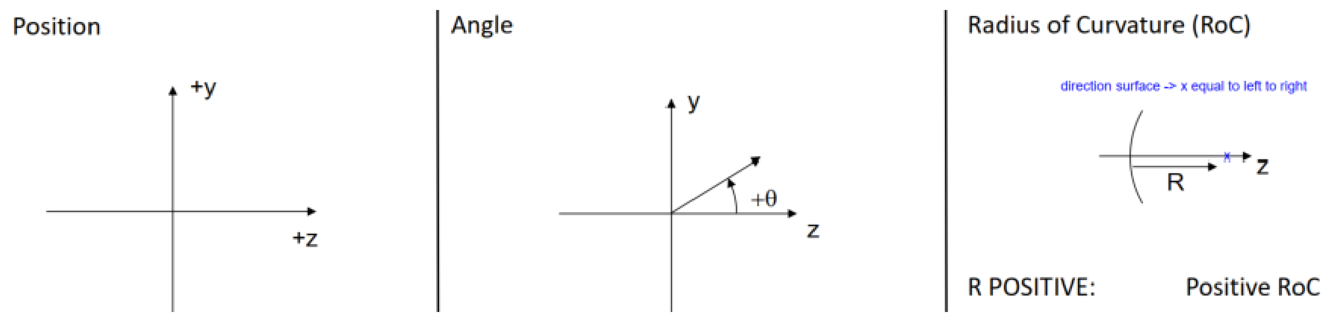


Figure 1.2

1.1.3 Electromagnetic spectrum

The light can be of various wavelengths (frequencies) which translates to the color of the light. The range of the wavelengths is called the **electromagnetic spectrum**. The **index of refraction** tells how much the

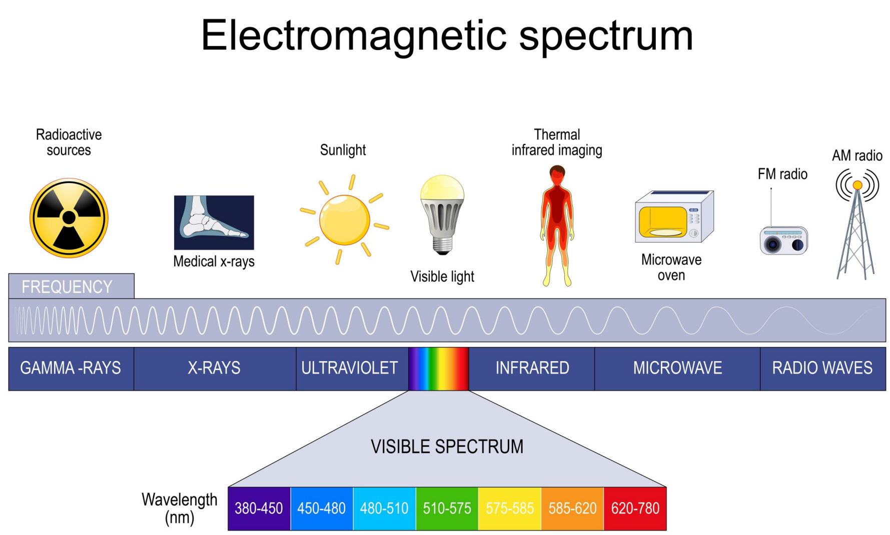


Figure 1.3

light is slown down in a medium with respecto to vacuum.

$$\text{Index of refraction} \qquad n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{V} \geq 1\,. \tag{1.4}$$

From one medium to another, the frequency remains unchanged but only the wavelength is modified. The index of refraction is a function of the wavelength and of the temperature.

Vaccum equal to air

In geometrical optics, vacuum and air are used interchangable as the index of refraction of air is $n \approx 1$.

1.1.4 Optical path length

The **optical path length** (OPL) is the equivalent distance in vacuum that light would cover in the same time as it takes to cross the actual medium.

$$\text{Optical path length} \qquad \text{OPL} = \int_a^b \boldsymbol{n}(s) \cdot d\boldsymbol{s}\,. \tag{1.5}$$

When the medium is homogeneous, the index n reduces to a constant value. Consequently, the ray travels in **straight lines**.

Fermat's principle states that the path taken by the light from one point to another is the path for which the OPL is stationary:

$$\text{Fermat's principle} \qquad \frac{d\text{OPL}}{d\text{path}} = 0\,. \tag{1.6}$$

1.1.5 Snell's laws of reflection and refraction

Snell's law can be obtained from Fermat's postulate. They governs the dynamics of the ray when passing thourgh an interface of different index of refractions:

$$\text{Snell's laws} \qquad \begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ \theta_1 &= -\theta_2 \end{aligned} \tag{1.7}$$

The angles are measured relative to the surface normal.

(a) Refraction **(b)** Reflection

Figure 1.4 In refraction and reflection, the angles are taken respective to the surface normal.

> The reflection is equal to refraction with a negative index: $n = -n$.

1.1.6 Total internal reflection (TIR)

Total internal reflection occurs when the light propagating from a medium n_1 to another n_2, with $n1 > n_2$, exceed a critical incident angle

$$\text{Total internal reflection} \qquad \theta_i > \theta_c = \sin^{-1}\frac{n_2}{n_1}\,. \tag{1.8}$$

Under this condition, 100% of the light is reflected into n_1, and no refracted light is present.

1.2 Mirrors and prisms

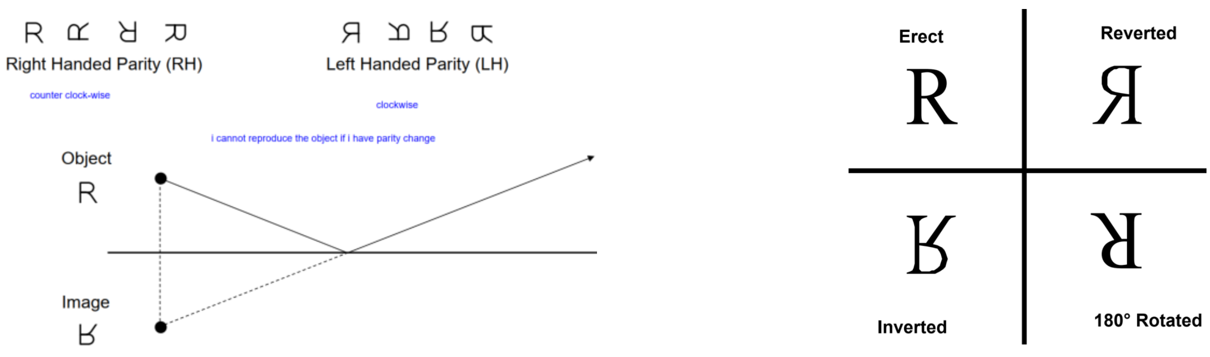
1.2.1 Parallel mirrors

Plane mirrors are used to:

- Produce a deviation
- Fold the optical path
- Change image parity

1.2.2 Parity change

A reflection from the plane mirror will cause a parity change in the image. An inversion (reversion) is a



(a) (b)

parity change about the horizontal (vertical) line, whereas a 180° rotation has no parity change and is rotated about the optical axis. An inversion and a reversion is equivalent to a 180° rotation.

Parity change

Only an **odd** number of reflections changes parity.

Parity is determined by looking back agains the propagation towards the obejct. Compare looking directly the objet vs at the reflection.

A lens adds inversion and reversion to the object, so that the image has no parity change, only rotation.

1.2.3 Non-parallel plane mirror

The **dihedral line** is the line of intersection of two non-parallel plane mirrors. The ray is deviated twice the angle between the mirrors.

$$\gamma = 2\alpha = \begin{cases} \text{Input-Output rays cross,} & \alpha < 90^\circ \\ \text{Input-Output rays diverge,} & \alpha > 90^\circ \\ \text{Input-Output rays anti-parallel,} & \alpha = 90^\circ \end{cases} \tag{1.9}$$

There are several mirrors,

- **Roof mirror** Two plane mirrors with a dihedral angle of 90°. It is used to insert two reflection in the propagation. The presence of this mirror is indicated by a "V".

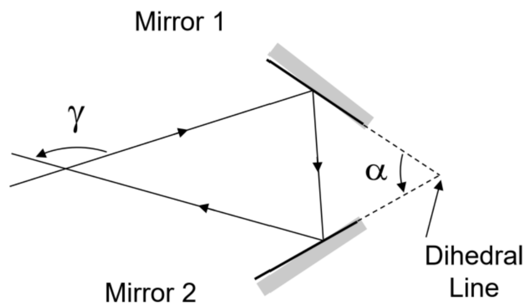


Figure 1.2

1.2.4 Prisms and tunnel diagrams

Prisms can be considered systems of plane mirrors. The reflection may be due to TIR, or by reflective coating.

A **tunnel diagram** **unfolds** the optical path at each reflection so that the ray is maintained straight through the propagation in the prism.

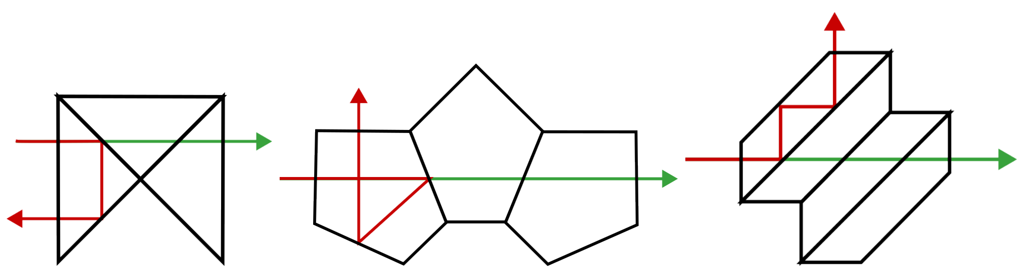


Figure 1.3

1.2.5 Reduced thickness

The **reduced thickness** is the vacuum equivalent distance of the medium that has the same propagation effect.

$$\text{Reduced thickness} \qquad \boxed{\tau = \frac{t}{n}}. \tag{1.10}$$

Expressing all distances in τ is equivalent to propagates the light in only vacuum (or air). This quantity is implicitly in the optical propagation and will be present in equations. When a reflection takes place, both n and t are negative, but τ remains positive.

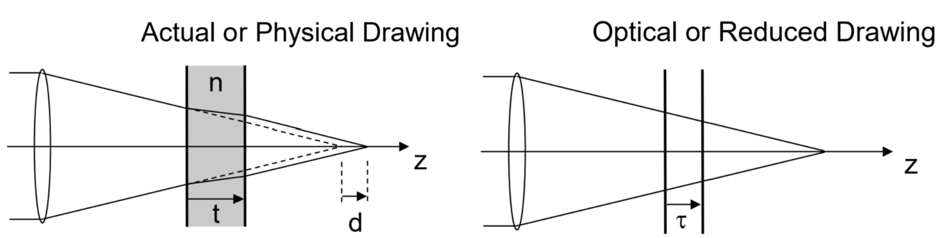


Figure 1.4 Reduced thickness is the vacuum (air) equivalent distance.

Tunnel diagrams are afected by the reduced thickness along the propagation distance. If the total distance is L, then the reduced is L/n.

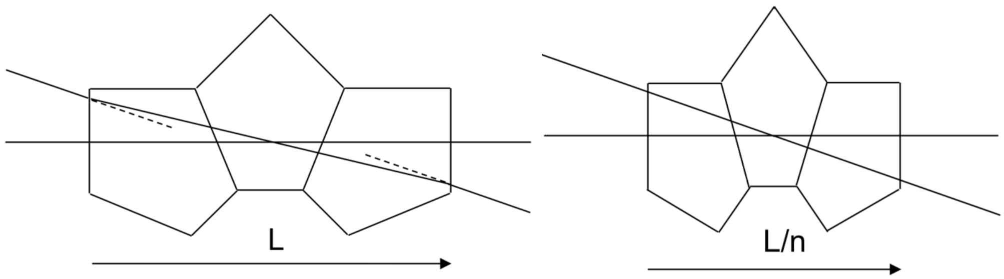


Figure 1.5 The diagram is only shortened along the direction of the propagation.

In a plate parallel plate (PPP), the beam is shifted horizontally a distance proportional to τ when is placed perpendicular ot the optical axis:

$$d = \frac{n-1}{n}t. \tag{1.11}$$

If it is disposed with an angle θ, then the ray will be shifted vertically

$$D \approx -t\theta\frac{n-1}{n}. \tag{1.12}$$

Ejemplo 1.1 **Reduced thickness and aparent distance**

a) The fish is $500\ mm$ beneath the surface of the water ($n = 1.33$). For the cat observing in air, how far below the water's surface does the fish appear to be?
In this case, we have

$$d_{\text{total}} = \frac{500\ mm}{1.33} = 375.94\ mm.$$

The fish appears to be $377\ mm$ below the surface of the water.

b) The cat is $500\ mm$ above the surface of the water. For the fish observing in water, how far abothe the water's surface does the cat appear to be?
The total distance is the sum of the air thickness in terms of the water and the thickness of the water:

$$d_{\text{total}} = 1.33 \cdot 500\ mm + 500\ mm = 665\ mm + 500\ mm = 1165\ mm.$$

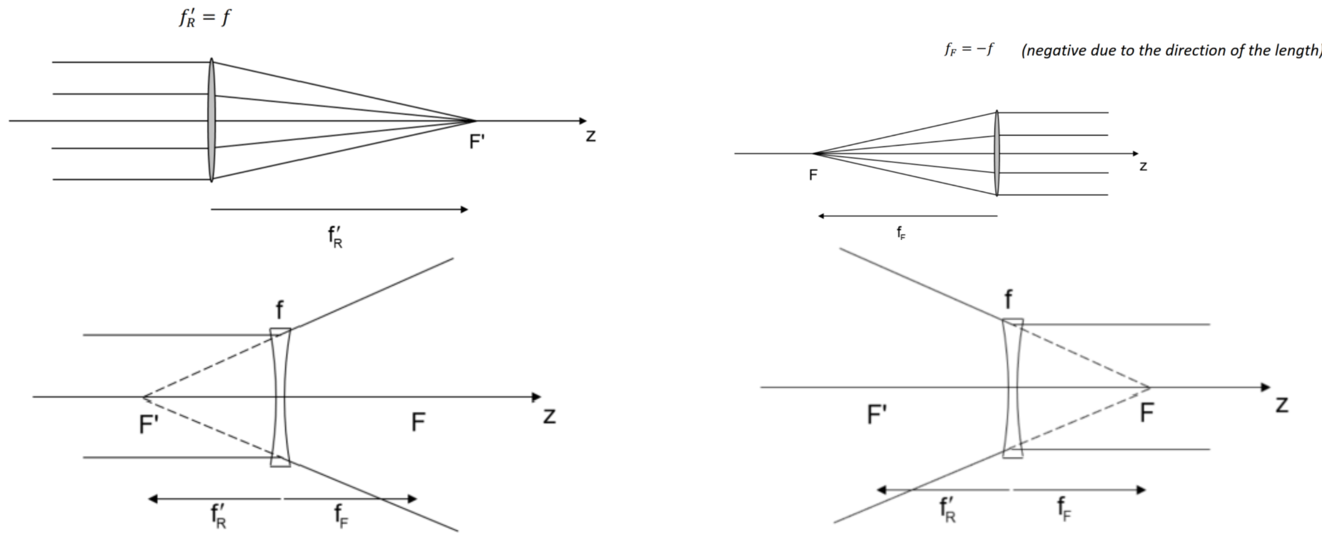
The cat appears to be $665\ mm$ above the surface of the water.

c) Several months later the cat return to watch the fish again. This time, there is a $100\ mm$ thick layer of ice ($n = 1.31$) on the surface. The fish is still a total physical distance of $500\ mm$ below the surface. Repeat parts a) and b).
In this case, we assume that the thick layer of ice has **replaced** $100\ m$ of the water while the distance of air remains the same.

- For the part a), the distance would be:

$$d_{\text{total}} = (\frac{100\ mm}{1.31} + \frac{400\ mm}{1.33}) + 500\ mm = 377\ mm + 500\ mm = 877\ mm.$$

The fish appears to be $377\ mm$ below the surface of the ice.



(a) Rear focal length f'_R **(b)** Front focal length f_F

- For part b), the total equivalent distance is the distance of the water, plus the equivalent distance in water of the ice and air:

$$d_{\text{total}} = 1.33 \cdot (\frac{100\ mm}{1.31} + 500\ mm) + 400\ mm = 767\ mm + 400\ mm = 1166.53\ mm.$$

  The cat appears to be $767\ mm$ above the water, that is, below the air and the ice. We computed first the reduced thickness of ice in order to then convert it to the equivalent in water.

1.3 Thin lens Imaging

1.3.1 Introduction

A **thin lens** is an idealization of an optical system with:

- Zero thickness τ.
- Refracting power ϕ.
- Characterized by its focal length f.

An object at infiniity is imaged to the **rear focal point** F', whereas an object ar the **front focal point** F is iamged to infinity. The respective distances from the center of the thin lens to F is f_F and to F' is f'_R.

Positive vs negative thin lenses

A positive lens has a positive focal length $f > 0$, a positive $f'_R > 0$ but a negative $f_F < 0$. In the negative lens, **all** is opposite.

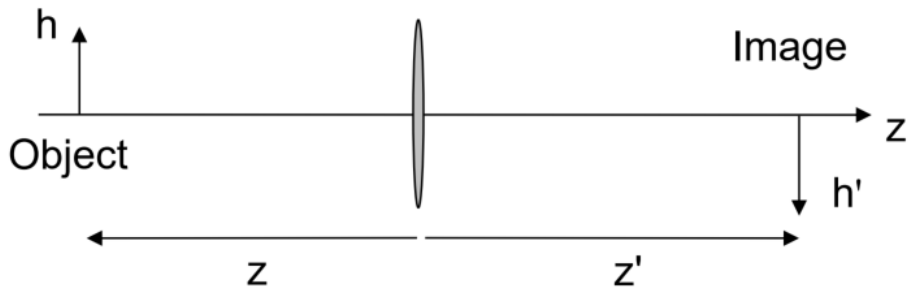


Figure 1.2 Imaging scheme

These points and lenghts are purely geometric properties of the lens.

Real rays are rays that are physically present, they can be touched. On the other hand, **virtual rays** are rays that are projection of real rays, and cannot be touched. Both type of rays are useful for imaging.

1.3.2 Imaging relationships

The imaging property of a thin lens relates the position of the object with that of the image.

The **thin lens equation** is

$$\text{Thin lens equation} \qquad \boxed{\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}}. \tag{1.13}$$

The **transverse magnification** is the ratio of the heights:

$$\text{Transverse magnitication} \qquad \boxed{m = \frac{h'}{h} = \frac{z'}{z}}. \tag{1.14}$$

These two equations are the most fundamental for imaging. They are used extensivelty through geometrical optics.

Intersecting at elast 2 rays is enough to map a point from object to image. The following are the trivial rays used:

- Parallel ray from the object, emerges (diverges) through the rear focal point.
- Ray from object throught the front focal point, emerging parallel (antiparallel).
- A ray that goes directly from the object through the center of the lens which is not refracted.

Ejemplo 1.2 **Imaging with a negative lens**

The ray diagram is illustrated in figure **??**. We have traced three rays:

- Parallel to the optical axis from the object, then it is refracted with direction to F'.
- Direct to F: it is refracted so that it becomes parallel to the optical axis.
- The chief ray, which maintain its direction through its propagation.

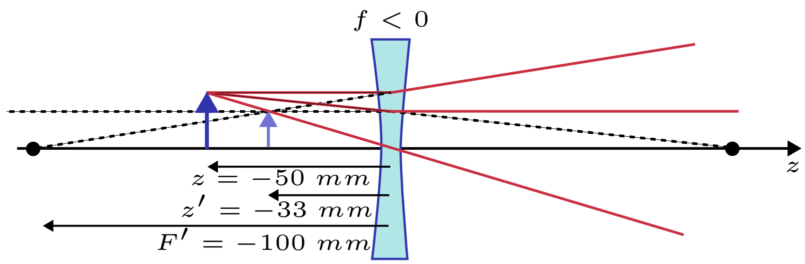


Figure 1.3 Ray diagram of the problem, the position of the image is z' and is located to the right of the object. Dashed lines correspond to virtual rays.

The intersection of these three rays produces the image. We can see that the image is to the left of the lens, but to the right of the object. Therefore, it will be a virtual image and demagnified. Using the thin lens equation, considering that $F' = -100\ mm$ and $z = -50\ mm$ provides

$$\frac{1}{z'} = \frac{1}{F'} + \frac{1}{z}$$
$$\frac{1}{z'} = \frac{1}{-100} + \frac{1}{-50}$$
$$z' = \frac{(-100)(-50)}{-150} = -33.333\ mm.$$

Because $z' < 0$, the image is **virtual** and will be to the left of the lens. Its magnification is:

$$m = \frac{z'}{z} = \frac{-33.333}{-50} = 0.667.$$

The image is then erected (sgn$(m) = 1$), and demagnified ($|m| < 1$) making it smaller than the object.

1.3.3 Optical spaces

Any optical object creates two optical spaces: the object space and the image space. Each optical space extends from $-\infty$ to ∞ and has an associated index of refraction. The connection of both spaces is through the optical object. A **real object** is to the left of the object while a **virtual object** is to the right. A

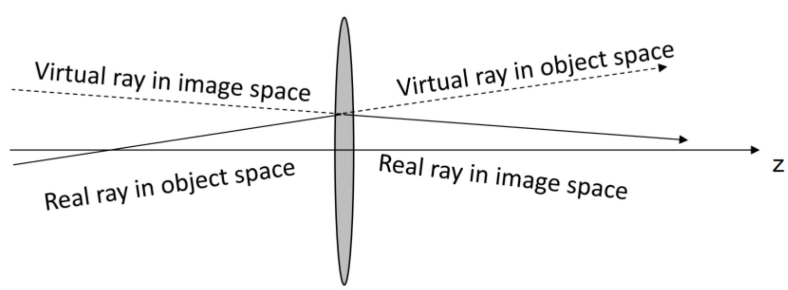
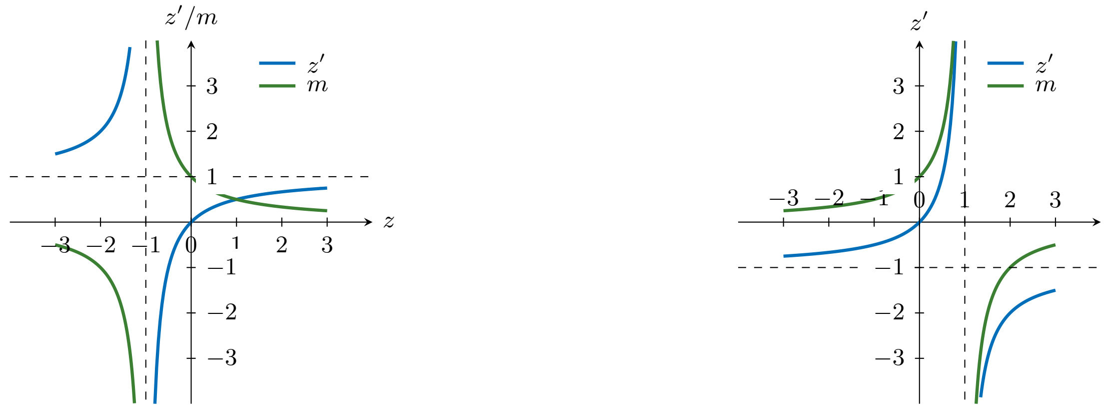


Figure 1.4

real image is to the right of the object and a **virtual image** to the left. In an optical space with negative index, left and right are reversed in these descriptions.

A thin lens creates two optical spaces:

- **Object space** contains the object and the front focal point F.
- **Image space** contains the image and the rear focal point F'.



(a) $f > 0$ **(b)** $f < 0$

Figure 1.5 Plot of z' and m for positive and negative lenses. We can directly see when an object (image) is real or virtual.

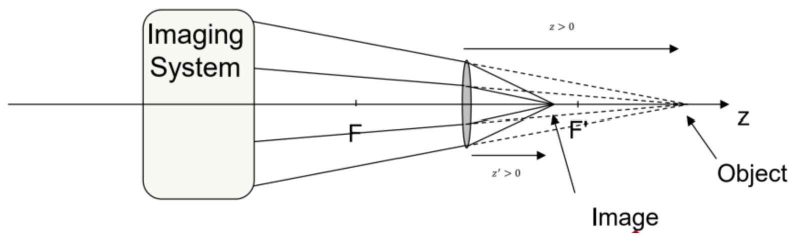


Figure 1.6 A virtual object is the projection of the image of a previous optical system.

Virtual objects occur when an image is projected into the lens by a previous optical system.

1.3.4 Object-image approximations

We can make further appproximations for extreme situations:

- **Distant object** When the magnitude of the object distance z is more then a few times the magnitude of the system focal length, the image distance z' is approximately equal to the real focal length.

$$|z| \gg |f| \Longrightarrow a)\ z' \approx f \quad b)\ L = z' - z \approx f - z \approx -z \quad c)\ m = \frac{z'}{z} \approx \frac{f}{z}. \tag{1.15}$$

- **Distant image** Similarly,

$$|z'| \gg |f| \Longrightarrow a)\ z \approx -f \quad b)\ L = z' - z \approx z' + f \approx z' \quad c)\ m = \frac{z'}{z} \approx -\frac{z'}{f}. \tag{1.16}$$

This fraction error of these approximations is about $|f|/|z|$ so they are very useful for distant object/image from over $4f$.

1.3.5 Field of view

The half **field of view** (HFOV) is often expressed as:

- maximum object height h.
- maximum image height h'.

- maximum angular size of the object as seen from the optical system $\theta_{1/2}$.
- maximum angular size of the image as seen from the optical system $\theta'_{1/2}$.

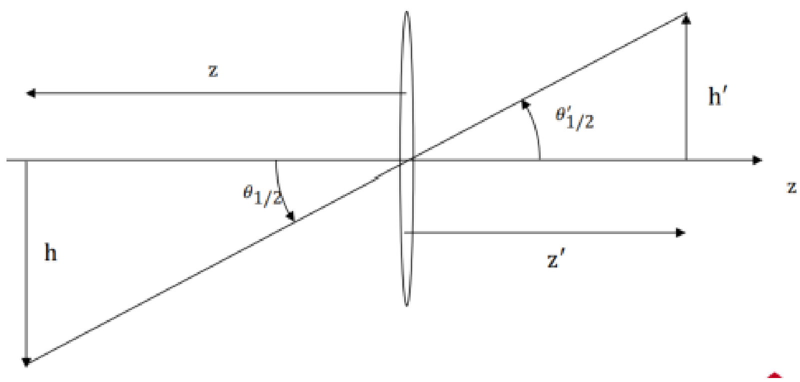


Figure 1.7

$$\text{FOV} = 2\text{HFOV}, \quad \text{HFOV} = \theta_{1/2} = \theta'_{1/2}, \quad \tan\theta_{1/2} = \frac{h}{z} = \frac{h'}{z'}. \tag{1.17}$$

In many situations, the FOV is determines by the detector size, which impose the maximum spatial dimensions.

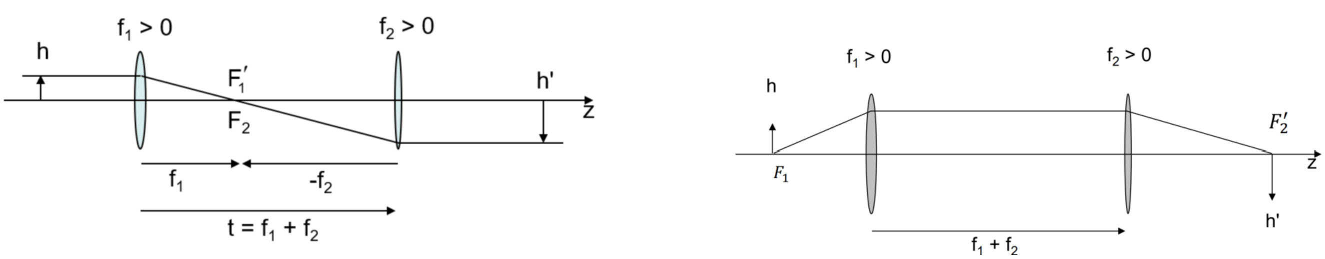
1.3.6 Afocal systems

An **afocal system** does not have focal points. Parallel rays will produce parallel images. The only change is in the transverse magnification.

$$m = \frac{h'}{h} = \frac{-f_2}{f_1}. \tag{1.18}$$

1.4 Imaging and paraxial optics

An optical system is a collection of optical elements. The first-order properties of the system is charectized by a **single** focal length, or magnification. First-order optics is the optics of perfect imaging systems: no aberrations, where the object is **mapped** to its image.



(a) Keplerian telescope ($m < 0$). **(b)** Galilean telescope ($m > 0$).

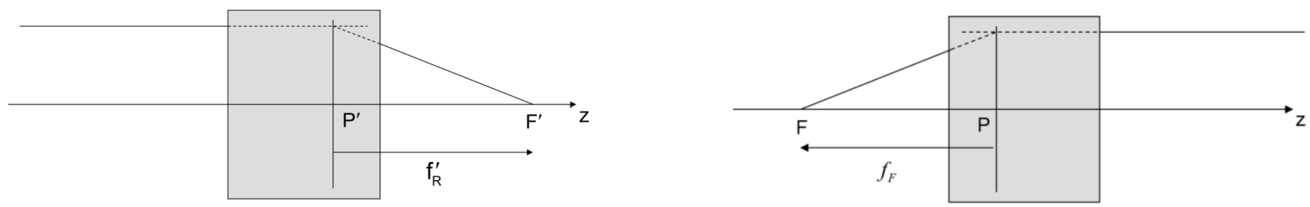
A small number of system properties can completely define and determine the mapping of first-order imaging properties. These are known as the **cardinal points** of the imaging system.

Each time a rafecting or reflecting surface is encountered, a new optical space is entered. In general, if a system coantains N surfaces, there will be $N+1$ optical spaces. The first space is calles the **object space** and the last **image space**.

1.4.1 General system

A black box is a convenient way to treat the optical system and analyze the refractions.

- An infinite object from left is effectively refracted by the system at the **rear principal plane** P'. The distance from P' to the **rear focal point** F' is the **rear focal length** f'_R.
- An object starting at F is effectively refracted by the system at the **front principal plane** P. The distance from P to the **front focal point** F is the **front focal length** f_F.



(a) Refraction at P' ($f'_R > 0$)

(b) Refraction at P ($f_F < 0$)

The system can now be trated as a thin lens, with the difference that object and image distances are from their respective principal planes.

1.4.2 Paraxial optics

The first-order properties of the system can be found using **paraxial rays**.

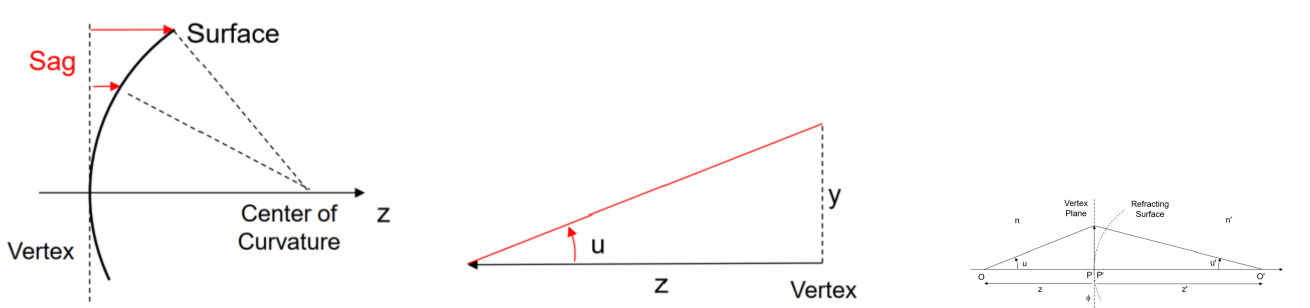
Paraxial ray

a) Rays are nearly parallel to the optical axis.

b) The amount a ray is bent at surfaces is assumed to be small: $\cos u \approx \cos u'$.

c) The sag of surfaces is ignored: $|\text{sag}| \ll |R|, |z|, |z'|$. Rays refracts at the vertex.

d) Rays are traced using the slopes of the rays instead of ray angles: $u = y/-z$.



(a) Sag is ignored

(b) Ray slopes instead of ray angles

(c) Ray slopes instead of ray angles

Single refractive surface

Single refracting surface are the fundamental object from which all other are composed of. They are defined by two refractive indices at the object and image space n and n', respectively, and the curvature of the surface:

$$\text{Radius of curvature} \qquad \boxed{R = \frac{1}{C}}\,. \tag{1.19}$$

The **optical power** ϕ is a measure of the benting power of the surface:

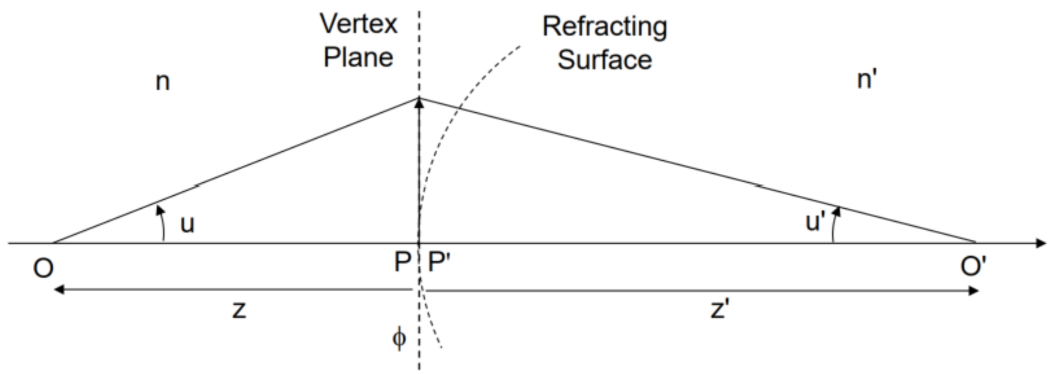


Figure 1.3 SIngle refracting surface.

$$\text{Optical power} \qquad \boxed{\phi = (n' - n)C \quad (m^{-1})(\text{diopters})}\,. \tag{1.20}$$

Then, the **paraxial raytrace equation** describes how the ray will travel after hitting the refracting surface:

$$\text{Paraxial raytrace equation} \qquad \boxed{n'u' = nu - y\phi}\,. \tag{1.21}$$

There are others useful equations, illustrated as follows

$$\frac{n'}{z'} = \frac{n}{z} + \phi, \quad f_E = \frac{1}{\phi}, \quad f_F = -nf_E, \quad f_R' = n'f_E. \tag{1.22}$$

A reflective surface is a special case with $n' = -n$.

With all variables defined, the scheme is illustrated as follows:

The focal length f_E is not a physical distance, but the front and real focal lengths are physical distances.

1.5 Gaussian imagery

Gaussian optics is a system of treating imaging as a mapping from object into image space. It is a special case of a **collinear transformation** applied to rotationally symmetric systems, and it maps points to

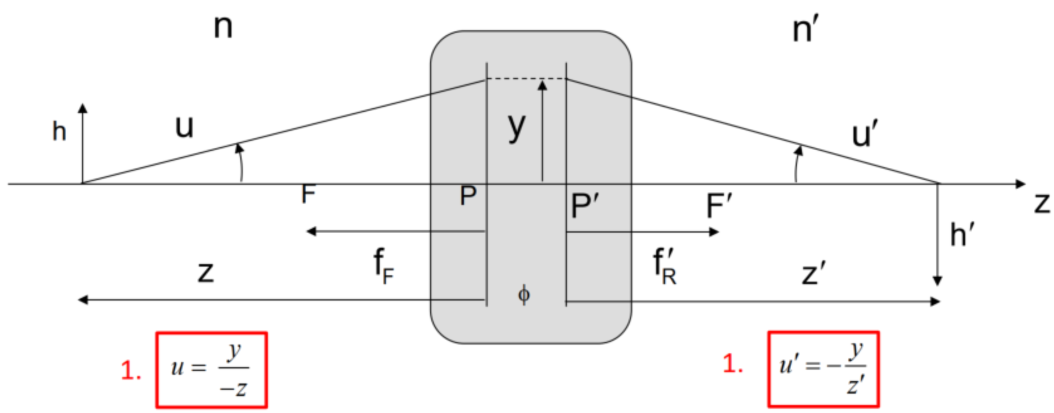
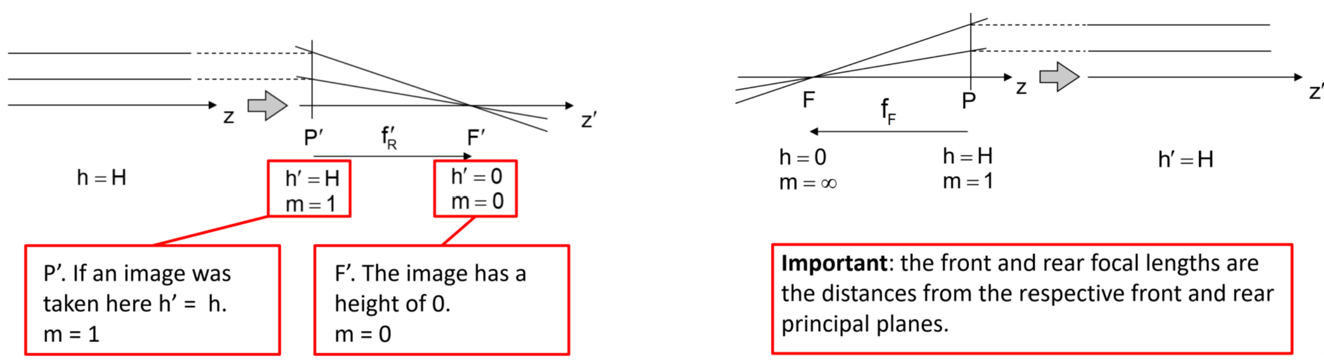


Figure 1.4 General paraxial system, with parameters defined previously.

points, lines to lines, and places to planes. The corresponding object and image elements are called **conjugate elements**.

The cardinal points and planes completely describes the focal mapping. They are defined by specific magnifications:

$$\begin{array}{lll} F & \text{Front focal point/plane} & m = \infty \\ F' & \text{Rear focal point/plane} & m = 0 \\ P & \text{Front principal plane} & m = 1 \\ P' & \text{Rear principal plane} & m = 1 \end{array} \qquad m = \frac{h'}{h}.$$



(a) Rear cardinal point/plane

(b) Front cardinal point/plane

Also, there exists **nodal points** N and N' that define the location of unit angular magnification for a focal system. A ray passing through one is mapped to a ray passsing through the other having the same angle.

$$z_{PN} = z'_{PN} = f_F + f'_R = (n' - n)f_E, \quad m_N = -\frac{f_F}{f'_R} = \frac{n}{n'}. \tag{1.23}$$

- Both nodal points of a single surface are located at the center of curvature of the surface: $z_{PN} = z'_{PN} = R$.
- If $n = n'$, then $z_{PN} = z'_{PN} = 0$ and the nodal points are coincident with the respective principal planes.
- The angular substense of an image seen from N' equals to the one seen from N: $m = h'/h = z'_N/z_N$.

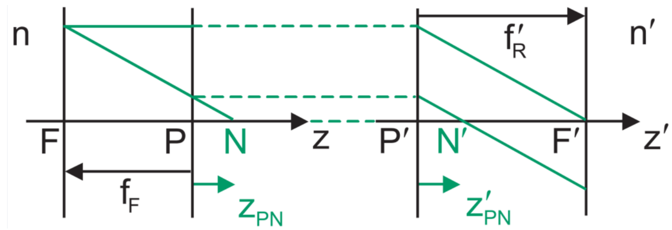
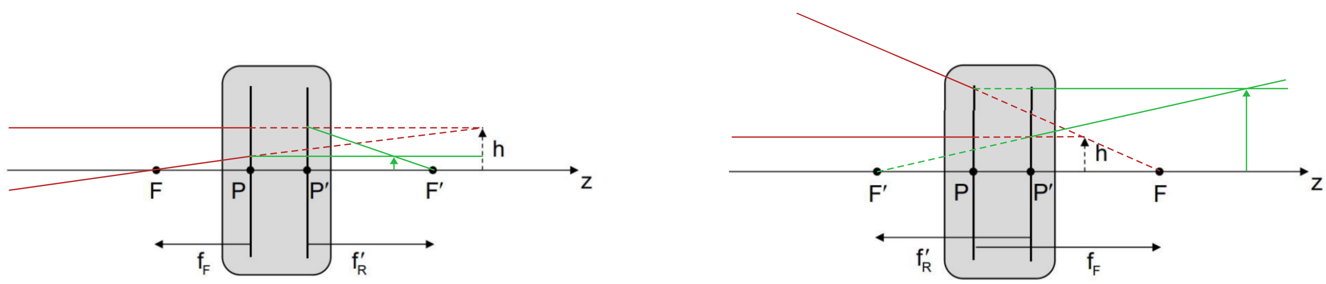


Figure 1.2

1.5.1 Representation of an optical system

An optical system can be represented as a set of principal planes and a set of focal points.



(a) Positive system **(b)** Negative system

> Remember that refraction for F must happen at P while for F' at P'.

1.5.2 Newtonian equations

Newtonian equations measure object and image distances form the **focal planes**

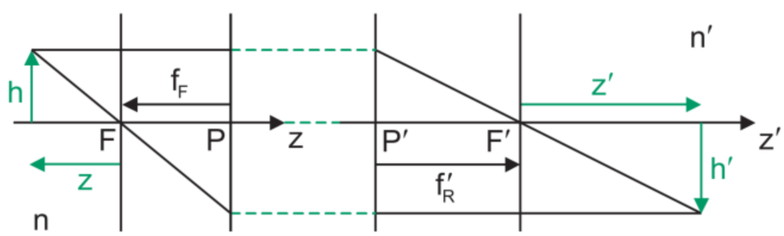


Figure 1.4 Newtonian equations.

$$z = -\frac{f_F}{m} \;\Big|\; z' = -m f'_R \;\Big|\; zz' = f_F f'_R \;\Big|\; \frac{z}{n} = \frac{f_E}{m} \;\Big|\; \frac{z'}{n'} = -m f_E \;\Big|\; \frac{z}{n}\frac{z'}{n'} = -f_E^2$$

1.5.3 Gaussian equations

Gaussian equations measure object and image distances from the **principal planes**.

A ray angle multiplied by the refractive indesx of its optical space is called **optical angle**:

$$\text{Optical angle} \qquad \boxed{\omega = nu \quad (-)}\,. \tag{1.24}$$

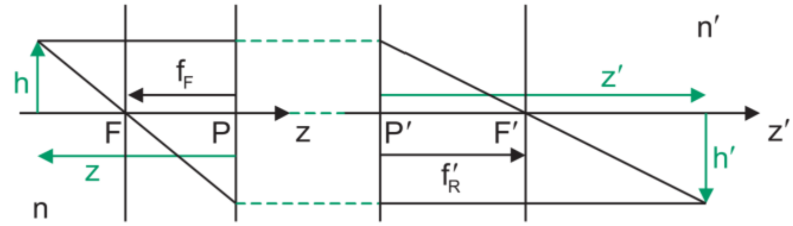


Figure 1.5 Gaussian equations.

$z = -\dfrac{(1-m)}{m} f_F$	$z' = (1-m) f'_R$	$m = -\dfrac{z'}{z}\dfrac{f_F}{f'_R}$	$\dfrac{f'_R}{z'} + \dfrac{f_F}{z} = 1$	$\dfrac{z}{n} = \dfrac{(1-m)}{m} f_E$
$\dfrac{z'}{n'} = (1-m) f_E$	$m = \dfrac{z'/n'}{z/n}$	$\dfrac{n'}{z'} = \dfrac{n}{z} + \dfrac{1}{f_E}$		

1.5.4 Longitudinal magnification

The **longitudinal magnification** relates the distances between pairs of conjugate planes.

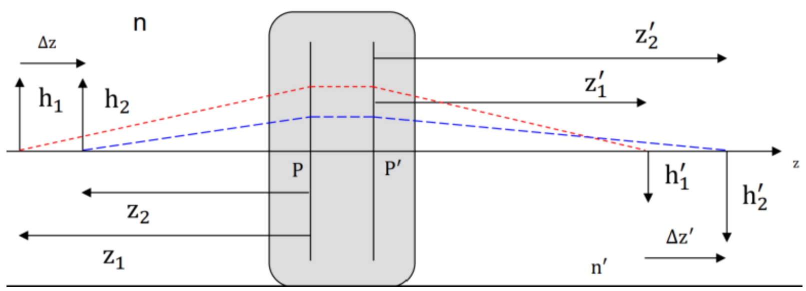


Figure 1.6 Longitudinal magnification allows you to have the thickness of the object or image.

$$\Delta z = z_2 - z_1, \quad \Delta z' = z'_2 - z'_1, \quad m_1 = \frac{h'_1}{h_1}, \quad m_2 = \frac{h'_2}{h_2}, \quad \frac{\Delta z'}{\Delta z} = -\frac{f'_R}{f_F} m_1 m_2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m_1 m_2 \tag{1.25}$$

As the plane separation approaches zero, $m_1 \approx m_2 \approx m$ and the local magnification $\bar{m}$ is obtained:

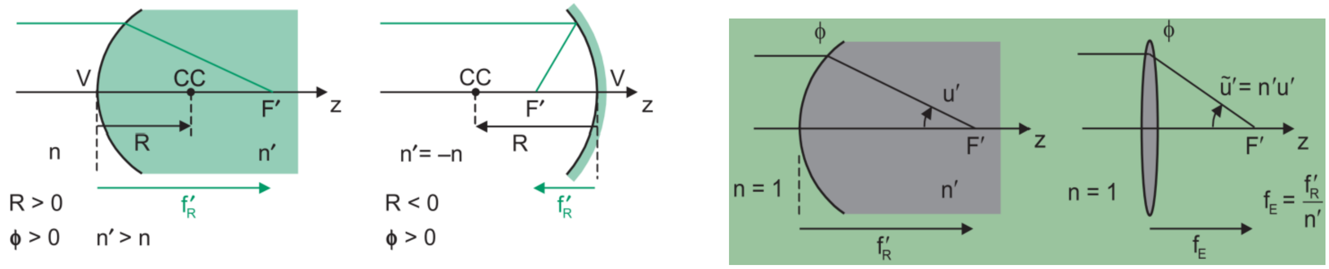
$$\bar{m} = \frac{n'}{n} m^2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m^2. \tag{1.26}$$

1.5.5 Gaussian properties of a single refracting surface

The radius of curvature R is defined to be the distance from its vertex to the center of curvature CC. The front and rear principal planes are coincident and located at the surface vertex. In addition, both nodal points are located at the center of the curvature (CC) of the optical surface.

> The use of reduced distances and optical angles allows a system to be represented as an air-equivalent system with thin lenses of the same power ϕ.

Ejemplo 1.3 **Gaussian imagery of a single refractive surface**



(a) Single surface **(b)** Thin lens reduction

a) For a single refracting surface, we have that:

- Both nodal points are located at the center of curvature CC.
- Front and real principal planes are located at the vertex.
- The reduced thickness of the surface is the focal length of its thin lens representation.

We illustrate these quantities along with the vertex and the focal lengths in the following figure. We illustrate

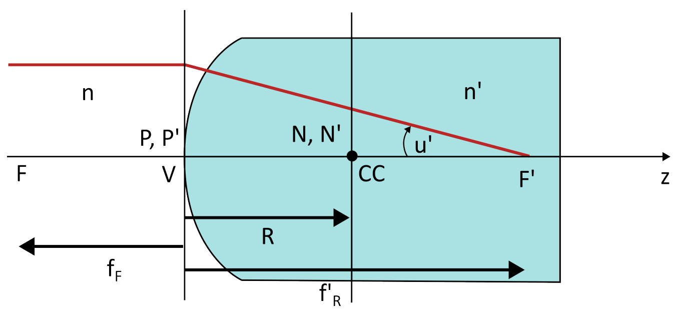


Figure 1.8 Illustration of cardinal point for a single refractive surface.

also some quantities of this surface:

$$C = \frac{1}{R} = 100\ m^{-1}, \quad \phi = (n' - n)C = 33.3\ m^{-1}, \quad f_E = \frac{1}{\phi} = 30\ mm,$$
$$f_F = -nf_E = -30\ mm, \quad f'_R = n' f_E = 40\ mm.$$

b) We use the following equation:

$$\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} \longrightarrow z' = \frac{n' z f_E}{n f_E + z}.$$

Replacing the physical values and the EFL:

$$z' = \frac{(1.333)(30)(-100)}{(1)(30) - 100} = +57.129\ mm.$$

Its height is determined by the magnification:

$$m = \frac{h'}{h} = \frac{z'/n'}{z/n} = \frac{57.129/1.333}{-100/1} = -0.429 \longrightarrow h' = mh = (-0.429)(10\ mm) = -4.29\ mm.$$

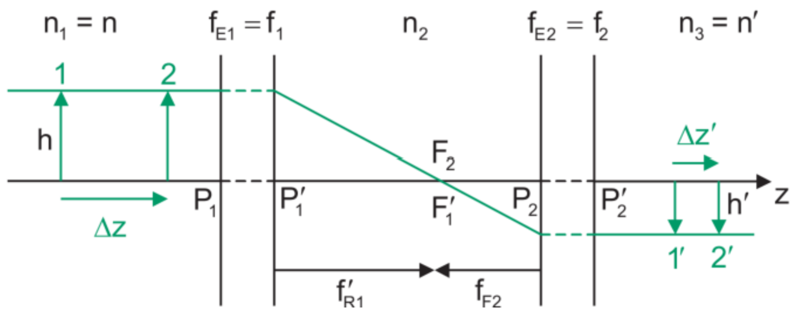


Figure 1.9 Generalized afocal system.

1.5.6 Generalized afocal systems

An afocal system is formed by the combination of two focal systems. The rear focal point of the first one is coincident with the front focal points of the second system. Common afocal systems are telescopes, binoculars, and beam expanders. The transverse and longitudinal magnification are constant. Due to this, the cardinal points are not defined, and the Gaussian and Newton equations **cannot** be used to determine cnojugate planes. However, any pair of conjugate planes coupled with $\bar{m}$ can be employed.

$$m = \frac{f_{F2}}{f'_{R1}} = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1}, \quad \bar{m} = \frac{n'}{n}m^2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m^2. \tag{1.27}$$

1.6 Object image relationship

1.7 Gaussian reduction

Gaussian reduction is the process that combine multiple elements two at a time into a single equivalent focal system. The system is defined by its Gaussian properties which include:

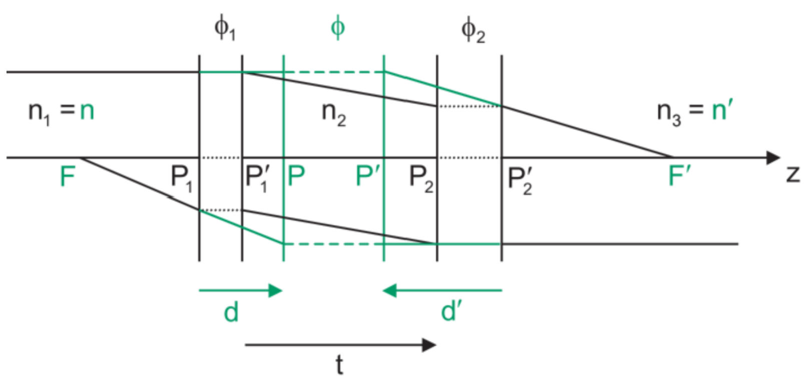


Figure 1.1 Gaussian reduction scheme.

- Power of the overall system.
- Front and rear focal lengths of the overall system.
- Principal planes of the overall system.

Each surface is represented by its principal planes and optical power. When two surface 1 and 2 are combined, the overall power is:

$$\text{Overall power} \qquad \phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau, \quad \tau = \frac{t}{n_2}. \tag{1.28}$$

As each surface has its principal planes at the vertex, we use only the vertex to move posteriors PPs. The reduction then **shifts** the principal planes of the equivalent system, P_{12}, P'_{12}, with respect to V_1 and V_2, respectively, by the following amount:

$$\text{Shifting distance from } P_1 \text{ and } P'_2 \qquad \frac{d}{n} = \frac{\phi_2}{\phi}\tau, \quad \frac{d'}{n'} = -\frac{\phi_1}{\phi}\tau \tag{1.29}$$

Now, P_{12} lives in the object space of the equivalent system (left to surface 1) whereas P' lives in the object space of the system (right to surface 2). If further reductions take place, then the shift of P'_{12} **must be considered** in the new distance from P'_{12} to P_3 to create τ_{123}.

1.7.1 Vertex distances

The **surface vertices** are the mechanical datums in a system and are often the reference locations for the cardinal points. The **back focal distance** (BFD) and **front focal distance** (FFD) are the distance measured from the back (front) vertex to the back (front) focal point F' (F).

$$\text{Distances} \qquad \text{BFD} = f'_R + d', \quad \text{FFD} = f_F + d \tag{1.30}$$

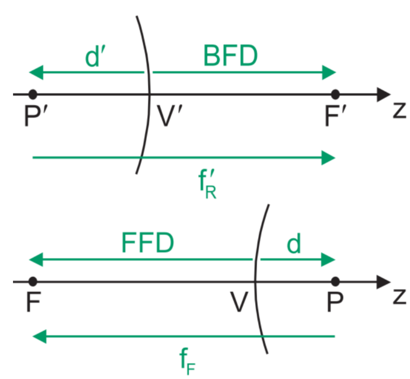


Figure 1.2 Vertex distances are used to define BFD and FFD.

The utility of Gaussian reduction is that the imaging properties of any combination of optical elements can be represented by a system power of focal length, a pair of principal planes and a pair of focal points.

Imaging with the final system

Once the single ϕ is obtained, we can do imaging with the generalized thin lens equation, considering n the object space and n' the image space:

$$\text{Generalized thin lens} \qquad \frac{n'}{z'} = \frac{n}{z} + \phi\,. \tag{1.31}$$

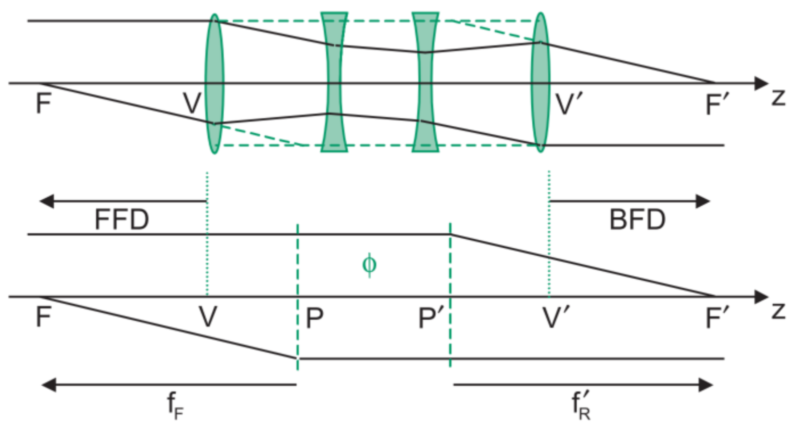
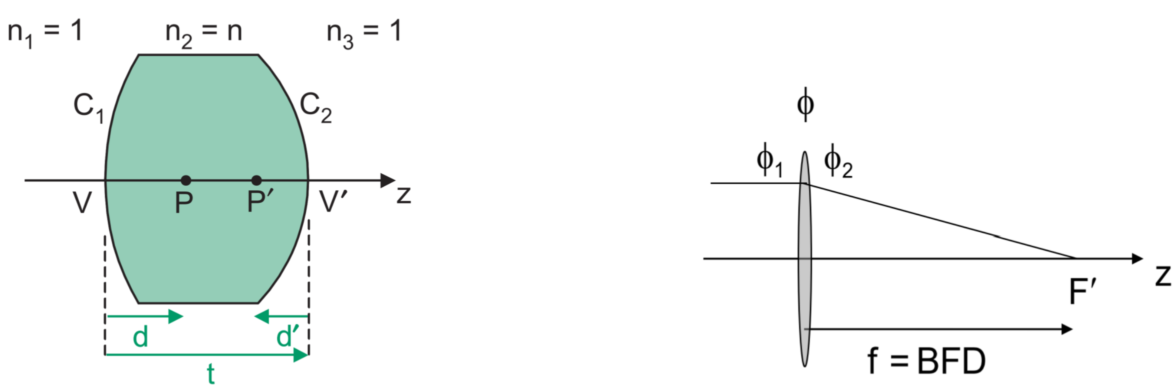


Figure 1.3 All reduces to 5 cardinal points.

Remember that z must be relative to the final front principal plane P and z' to the final rear principal plane P'. So probably we will need some conversion to give the correct distances.

1.7.2 Thick and thin lenses

The **thick lens** is composed ot two refractive surfaces with a thicknes between them. The overall power



(a) Thick lens

(b) Thin lens

in term of curvature is:

$$\phi_{\text{thick}} = (n-1)[C_1 - C_2 + (n-1)C_1C_2\tau], \quad d = \frac{\phi_2}{\phi}\tau, \quad d' = -\frac{\phi_1}{\phi}\tau. \tag{1.32}$$

In this case, the nodal points are coincident with the principal planes.

The **thin lens** approximation is obtained for $t \to 0$, which reduces the overall power to

$$\phi_{\text{thin}} = (n-1)(C_1 - C_2), \quad d = d' = 0, \quad \text{BFD} = f. \tag{1.33}$$

This idealized element can be considered as a singel refracting surface separating two spaces. The principal planes and nodal points are located at the lens (middle).

Ejemplo 1.4 **Gaussian reduction of two lenses**

For a two positive lens system, we use Gaussian reduction to reduce the effect to a single thin lens. We first compute the overall optical power with the power of individual lenses:

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t = \frac{1}{40} + \frac{1}{40} - \frac{1}{40}\frac{1}{40} \cdot 20 = 0.038mm^{-1} \longrightarrow f_E = \frac{1}{\phi} = 26.67\ mm.$$

The front and real focal lengths are:

$$f_F = -n_1 f_E = (1)(26.67\ mm) = -26.67\ mm, \quad \text{and} \quad f'_R = n_3 f_E = (1)(26.67\ mm) = 26.67\ mm.$$

Then the distances d and d', corresponding to the shift from the front (rear) principal planes P, P' of the equivalent system with respect to f_F, f'_R are given by

$$d = \frac{\phi_2}{\phi} t = \frac{0.025}{0.038} 20 = 13.158\ mm, \quad \text{and} \quad d' = -\frac{\phi_1}{\phi} t = -\frac{0.025}{0.038} 20 = -13.158\ mm.$$

The front (back) focal distances are then: The FFD and BFD are therefore,

$$\text{FFD} = f_F + d = -26.67\ mm + 13.158\ mm = -13.512\ mm.$$
$$\text{BFD} = f'_R + d' = 26.67\ mm - 13.512\ mm = 13.512\ mm.$$

The reduction process and the quantities obtained are illustrated in figure 1.5. The nodal points are coincident with

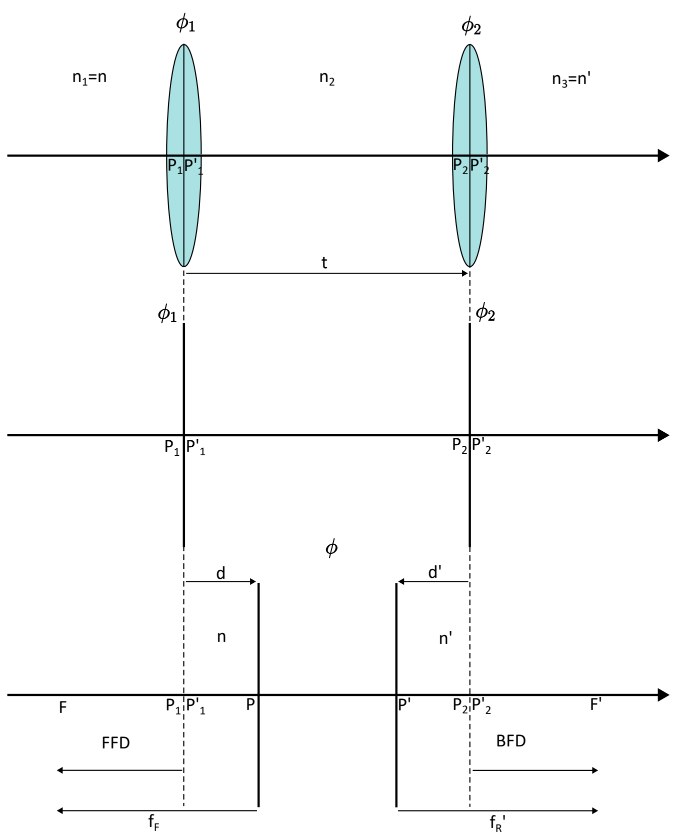


Figure 1.5 Gaussian reduction for two positive lenses.

the principal planes.

1.8 Paraxial raytrace

1.8.1 Introduction

Paraxial optics is a method of determining the first-order properties of an optical system that assumess all ray angles are small. It follows the same assumptions of **paraxial optics** regime seen.

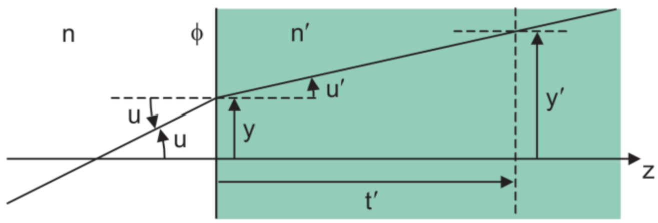


Figure 1.1 A paraxial raytrace is linear with respect to ray angle and heights.

It composes of iterative **refraction** and **Transfer** processes. These type of raytrace are called **YNU raytrace**:

$$\text{Object} \longrightarrow \text{Image} \quad \begin{cases} \text{Refraction (reflection)} & \boxed{n'u' = nu - y\phi \qquad \omega' = \omega - y\phi} \\ \text{Transfer} & \boxed{y' = y + u't' \qquad y' = y + \omega'\tau'} \end{cases} \tag{1.34}$$

$$\text{Image} \longrightarrow \text{Object} \quad \begin{cases} \text{Refraction (reflection)} & \boxed{nu = n'u' + y\phi \qquad \omega = \omega' + y\phi} \\ \text{Transfer} & \boxed{y = y' - u't' \qquad y = y' - \omega'\tau'} \end{cases} \tag{1.35}$$

1.8.2 Procedure

The procedure is always the same:

- You set the optical propoerties of the system.
- **Rear cardinal points** Trace a forward ray from object to image, and at the image space, you look for t that satisfies $y = 0$ to get the BFD.

$$\phi = -\frac{\omega'_k}{y_1}, \quad f_E = \frac{1}{\phi}, \quad f'_R = n' f_E, \quad \text{BFD} = -\frac{y_k}{u'_k}, \quad d' = \text{BFD} - f'_R. \tag{1.36}$$

- **Front cardinal points** Trace a backward ray from image to object, and at object space, you look for t that satisfies $y = 0$ to het the FFD.

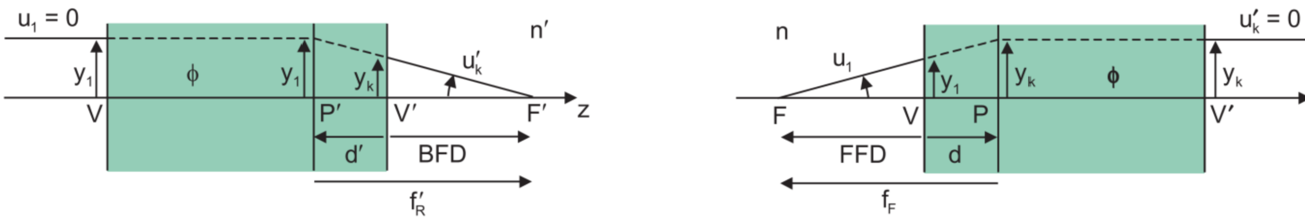
$$\phi = \frac{\omega_1}{y_k}, \quad f_E = \frac{1}{\phi}, \quad f_F = -n f_E, \quad \text{FFD} = -\frac{y_1}{u_1}, \quad d = \text{FFD} - f_F. \tag{1.37}$$

Imaging with the final system

Once the single ϕ is obtained, we can do imaging with the generalized thin lens equation, considering n the object space and n' the image space:

$$\text{Generalized thin lens} \qquad \boxed{\frac{n'}{z'} = \frac{n}{z} + \phi}\,. \tag{1.38}$$

Remember that z must be relative to the final front principal plane P and z' to the final rear principal plane P'. So probably we will need some conversion to give the correct distances.

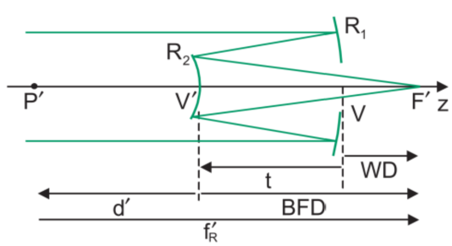


(a) Finding rear cardinal points **(b)** Finding front cardinal points

Once you have the rays traced, you construct a reduced table, where

- List out all the surfaces: the first part of the big table. Parameters vertically in-line with the surface are associated with optical surfaces. Parameters sandwiched between refer to the optical spaces.
- Another box below contains the information about the rays traced to find the cardinal points.

Ejemplo 1.5 Cassegrain telescope



$$R_1 = -200 \text{ mm}$$
$$t = -80 \text{ mm}$$
$$R_2 = -50 \text{ mm}$$
$$n_1 = n = 1$$
$$n_2 = -1$$
$$n_3 = n' = 1$$
$$C_1 = -0.005\,\text{mm}^{-1}$$
$$C_2 = -0.02\,\text{mm}^{-1}$$

Paraxial raytrace: $\omega' = \omega - y\phi \qquad y' = y + \omega'\tau'$

Surface	Object		V		V′		F′
C			-0.005		-0.02		
t		∞		-80		BFD	
n		1.0		-1.0		1.0	
−ϕ			-0.01		0.04		
t/n		∞		80		100	
y	1.0		1.0		0.2		0.0
nu		0.0		-0.01		-0.002	
u		0.0		0.01		-0.002	

The analysis of the raytrace results:

$$\phi = -\frac{n'u_2'}{y_1} = -\frac{-0.002}{1.0} = 0.002\,\text{mm}^{-1} \qquad f_R' = f_E = \frac{1}{\phi} = 500\,\text{mm}$$

$$BFD = -\frac{y_2}{u_2'} = -\frac{0.2}{-0.002} = 100\,\text{mm}$$

$$d' = BFD - f_R' = BFD - f_E = -400\,\text{mm}$$

$$WD = BFD + t = 20\,\text{mm}$$

Gaussian reduction:

$$\phi_1 = (n_2 - n)C_1 = 0.01\,\text{mm}^{-1}$$

$$\phi_2 = (n' - n_2)C_2 = -0.04\,\text{mm}^{-1} \qquad \tau = \frac{t}{n_2} = 80\,\text{mm}$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau = 0.002\,\text{mm}^{-1} \qquad f_R' = f_E = \frac{1}{\phi} = 500\,\text{mm}$$

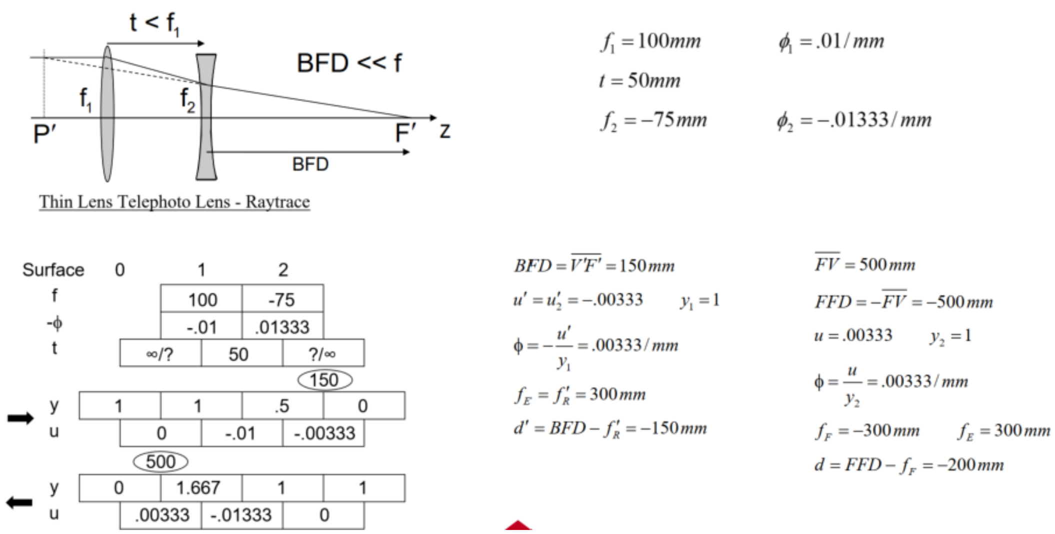
$$d' = -n'\frac{\phi_1}{\phi}\tau = -400\,\text{mm}$$

$$BFD = f_R' + d' = f_E + d' = 100\,\text{mm}$$

$$WD = BFD + t = 20\,\text{mm}$$

Ejemplo 1.6 Thin lens

Ejemplo 1.7



In this case we have three surface, each with their correspond surface curvature C and index of refraction n.

- **Gaussian reduction** The optical power of each surface is:

$$\phi_1 = \frac{n_1 - n_0}{R_1} = \frac{1.336 - 1}{7.8\ mm} = 0.043\ mm^{-1},$$
$$\phi_2 = \frac{n_2 - n_1}{R_2} = \frac{1.413 - 1.336}{10\ mm} = 0.008\ mm^{-1},$$
$$\phi_3 = \frac{n_3 - n_2}{R_3} = \frac{1.336 - 1.413}{-6\ mm} = 0.013\ mm^{-1}.$$

Now, we combine surface 1 with 2:

$$\phi_{12} = \phi_1 + \phi_2 - \phi_1\phi_2\tau_1 = 0.043 + 0.008 - 0.043 \cdot 0.008 \cdot \frac{3.6}{1.336} = 0.050\ mm^{-1}.$$

The shift of the principal plane are given by

$$\delta_{12} = \frac{\phi_2}{\phi_{12}}\tau_1 = \frac{0.008}{0.050} \cdot \frac{3.6}{1.336} = 0.431\ mm \longrightarrow d_{12} = \delta_{12}.$$
$$\delta'_{12} = -\frac{\phi_1}{\phi_{12}}\tau_1 = -\frac{0.043}{0.050} \cdot \frac{3.6}{1.336} = -2.317\ mm \longrightarrow d'_{12} = n_2\delta'_{12} = -3.274\ mm.$$

We can see that the front principal plane is displaced from V_1 to the left, while the rear principal plane is shifted to the right of V_2. In addition, the distance d'_{12} considered the index n_2 as it belong to that space. The distance of propagation through the index n_2 must be adjusted due to the shift of the rear principal plane:

$$\tau_{12} = \frac{t_2 - d'_{12}}{n_3} = \tau_2 - \delta'_{12} = \frac{3.6}{1.413} + 2.317 = 4.865\ mm.$$

Now, we compute the total optical power considering the reduction and the third surface:

$$\phi = \phi_{12} + \phi_3 - \phi_{12}\phi_3\tau_{12} = 0.050 + 0.013 - (0.046)(0.013)(4.865) = 0.060\ mm^{-1}.$$

The shifts are:

$$d_{123} = n_0\delta_{123} = \frac{\phi_3}{\phi}\tau_{12} = \frac{0.013}{0.060} \cdot 4.865 = 1.054\ mm$$
$$d'_{123} = n_3\delta'_{123} = -n_3\frac{\phi_{12}}{\phi}\tau_{12} = -(1.336)\frac{0.050}{0.060} \cdot 4.865 = -5.416\ mm.$$

The total shift from the first surface is the sum of individual fron shift computed, while for the last surface is just the shift computed in the last reduction:

$$d = d_{12} + d_{123} = 0.431 + 1.054 = 1.485\ mm$$
$$d' = d'_{123} = -5.416\ mm.$$

The front (rear) focal lengths are then

$$f_E = \frac{1}{\phi} = 16.667\ mm \longrightarrow \begin{array}{l} f_F = -n_0 f_E = -(1)(16.667) = -16.667\ mm \\ f'_R = n_3 f_E = (1.336)(16.667) = 22.267\ mm. \end{array}$$

Finally, the FFD and BFD are:

$$\text{FFD} = f_F + d_{123} = -16.667 + 1.054 = 15.613\ mm$$
$$\text{BFD} = f'_R + d'_{123} = 22.267 - 5.416 = 16.851\ mm.$$

The reduction process is shown in figure 1.3. The green quantities are the equivalent of the final reduction.

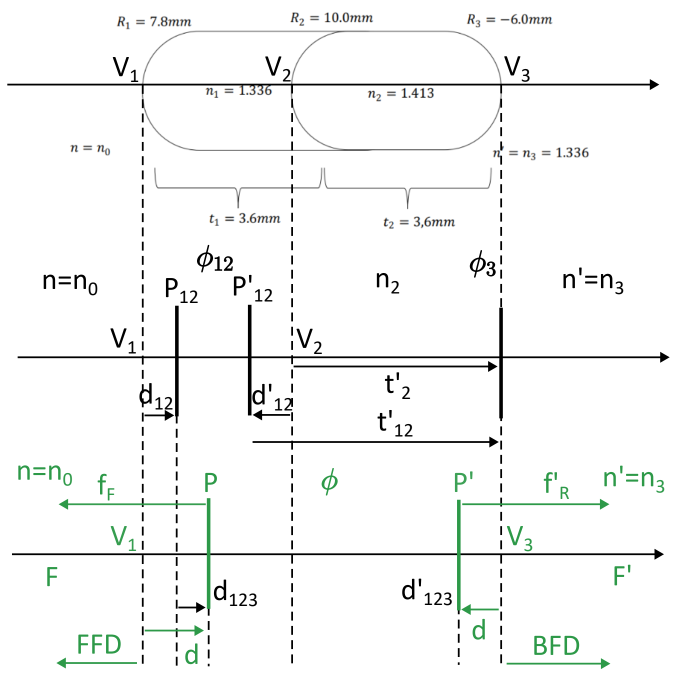


Figure 1.3 Gaussian reduction for the three-surfaces object.

- **Ray tracing** For the ray tracing, we will fill the ynu spreadsheet. We will trace two rays, one from left to right and other in opposite direction in order to find the front and real focal lengths.

We must compare the t in blue with the FFD and the red t with the BFD. The differences are due to the approximation in intermediate computations. We can see that both methods yield the same answer, despite that ynu raytracing is way faster than Gaussian reduction.

The effective focal length is defined considering the magnification nu divided by the input ray:

$$f'_E = \frac{1}{\phi'} = -\frac{y_1}{nu'} = \frac{1}{0.060} = 16.667\ mm \longrightarrow f'_R = n_3 f'_E = 22.267\ mm.$$

Similarly,

$$f'_E = \frac{1}{\phi} = \frac{y_2}{nu} = \frac{1}{0.060} = 16.667\ mm \longrightarrow f_F = -n_0 f_E = -16.667\ mm.$$

	Object space	Space 1	Surface 1	Space 2	Surface 2	Space 3	Surface 3	Space 4	Image space
C			0.128		0.1		-0.167		
t		15.167		3.6		3.6		16.856	
n		1		1.336		1.413		1.336	
$-\phi$			-0.043		-0.008		-0.013		
t/n		15.167		2.695		2.548		12.617	
y	1	1	1		0.884		0.757		0
nu	0	0		-0.043		-0.05		-0.060	
u	0	0						-0.045	
y	0		0.910		0.967		1	1	1
nu		0.060		0.021		0.013		0	0
u		0.060						0	0

The focal lengths match exactly as the ones computed by Gaussian reduction. We can also compute the principal planes shifts, but we will not do it as we already know the answer.

1.8.3 Table worksheet

Surface	0	1	2	3	4	5	6	7
C								
t								
n								
$-\phi$								
t/n								
y								
nu								
u								
y								
nu								
u								

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Chapter 2

Concepts of optics

2.1 Stops and pupils

2.1.1 Aperture stop

The **aperture stop** is a physical/real surface that limits the cone of light entering and exiting the optical system.

- The **entrance pupil** (EP) is the image of the stop in the object space.
- The **exit pupil** (XP) is the image of the spot in the image space.

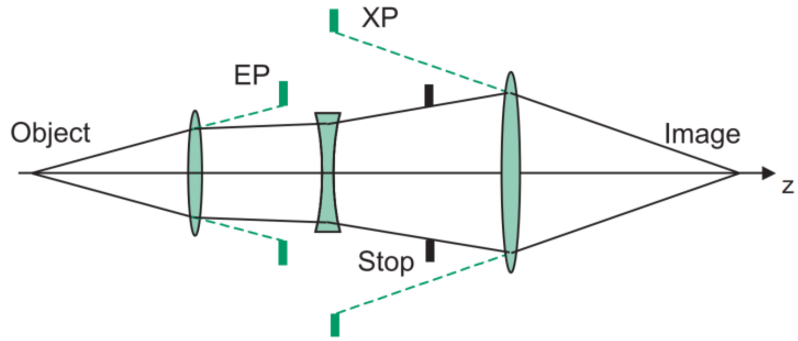


Figure 2.1 The stop limits the cone of light, and its image in object (image) space creates the entrance (exit) pupil.

There is a stop or pupil in each optical space. Intermediate pupils are formed in other spaces. There are two methods to determine which aperture in a system serves as the system stop:

a) Image each potential stop into object space. The pupil with the **smallest** angular size corresponds to the stop. The same can be done in image space.

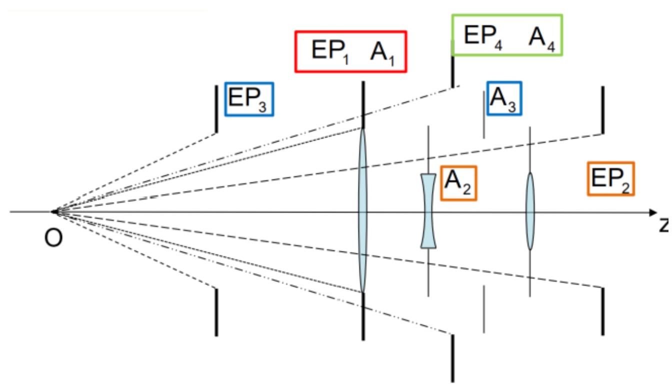


Figure 2.2 The smallest angular size corrsponds to the stop in object space. Same for image space.

b) Trace a ray through the system from the axial object point with arbitrary initial angle. At each potential stop, determine the ratio of the aperture radio a_k to the ray height at that surface $\tilde{y}_k$.

$$\text{Aperture stop} = \min\left\{\left|\frac{a_k}{\tilde{y}_k}\right|\right\}. \tag{2.1}$$

The pupils are the image of the stop and do not change position or size with an off-axis object. Intermediate pupils are formed in each optical space for multi-element systems. If there are N elements, there are $N+1$ pupils (including the stop).

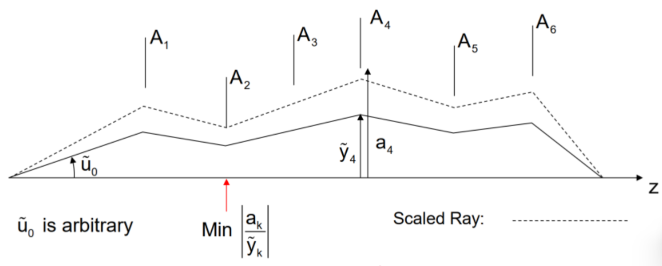


Figure 2.3 The minimum slope value corresponds to the aperture stop.

When designing a system, is usually critical that the stop surface does not change over a range of possible object positions that the system will be used with.

2.1.2 Marginal and Chief rays

Rays confined to the yz-plane are called **meridional rays**. There are two special meridional rays that define properties of the object, images and pupils:

- The **marginal ray** travels from the base of the object to the edge of EP . It defines image locations and pupil sizes.
- The **chief ray** travels from the edge of the object to the center of the EP. It defines image heights and pupil locations.

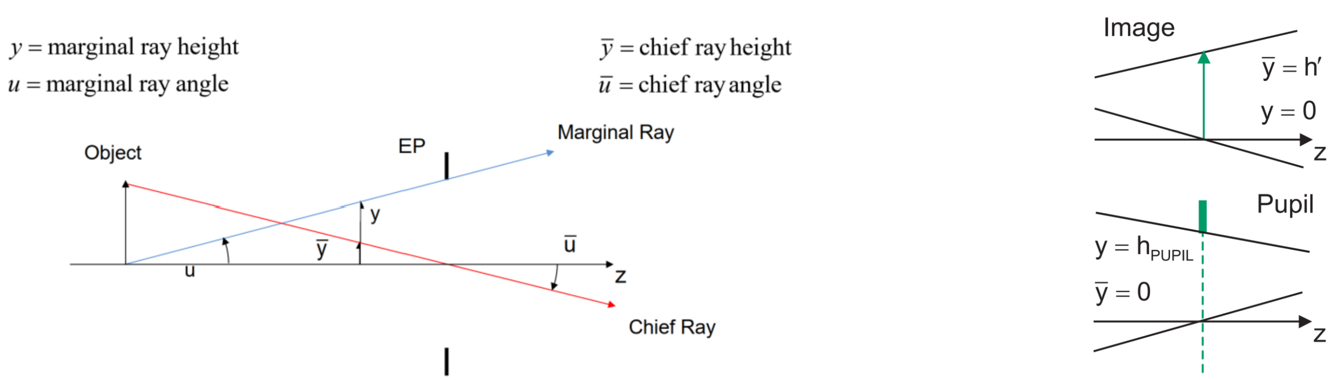


Figure 2.4 The

The heights of the marginal ray and the chief ray can be evaluated at any z in any optical space. When the marginal ray crosses the axis, an image is located, and the size of the image is given by the chief ray height. Whenever the chief ray crosses the axis, a pupil or stop is located, and the pupil radius is given by the marginal ray height. Intermediate images and pupils are often virtual.

2.1.3 Pupil locations

By raytrace

Once you know which surface is the stop, you have the information to determine the location of EP and XP. The **pupil locations** can be found by tracing a paraxial ray starting at the center of the stop and is back/forward propagated. The intersections of this ray with the axis in object and image space determine the locations of EP and XP.

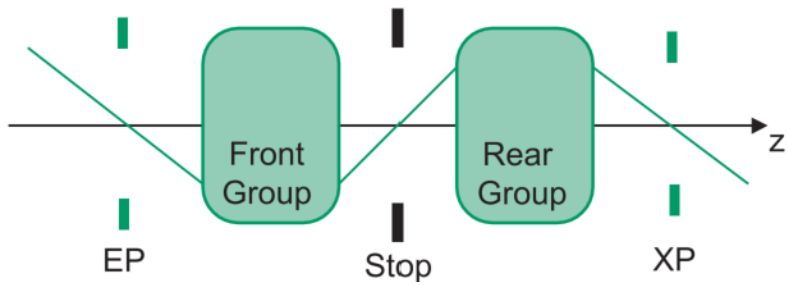
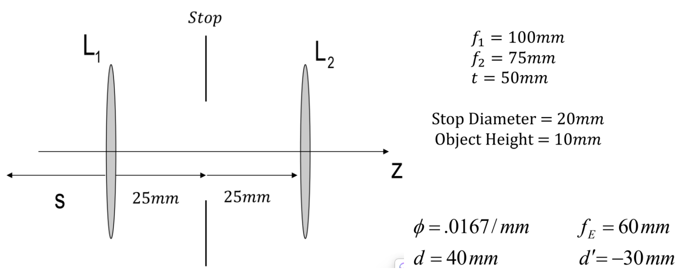


Figure 2.5 The

This ray become the chief ray when it is scaled to the object or image size. The marginal ray gives the pupil sizes.

Ejemplo 2.1 **Pupil location by paraxial raytrace**



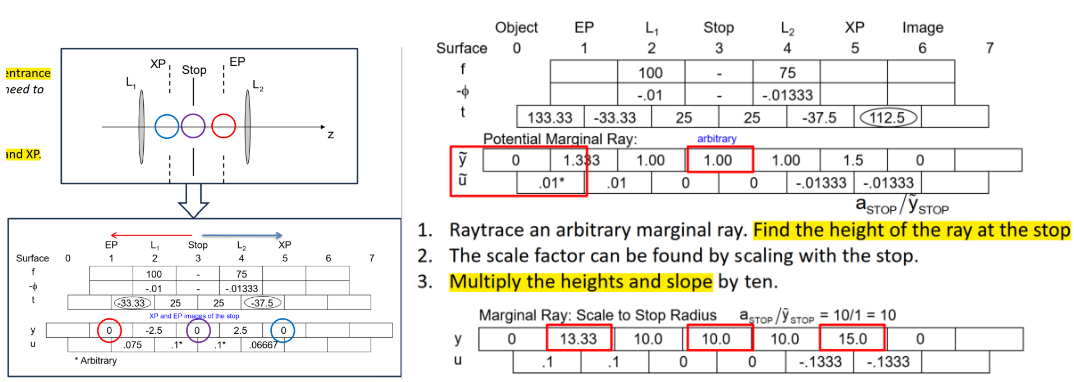
Solution

The stop is a real object for the formation of both EP and XP. There is a ray that has a height of 0 at the EP, stop and XP. We first set $y = 0$ at the stop, and then with arbitrary angle

EP we set $y = 0$ for the EP and solve for the distance.

XP we set $y = 0$ for the XP and solve for the distance.

We used a potential chief to find pupil locations. For the pupil sizes, we find the true marginal ray scaling a potential maginal ray. Remember that the chief ray was for pupil locations, now with the marginal ray we find the pupil sizes. We can also use it to find the image location.



f			100	-	75			
-ϕ			-.01	-	-.01333			
t		133.33	-33.33	25	25	-37.5	112.5	
ỹ	0	1.333	1.00	1.00	1.00	1.5	0	
ũ		.01*	.01	0	0	-.01333	-.01333	

$a_{STOP}/\bar{y}_{STOP} = 10/1 = 10$

y	0	13.33	10.0	10.0	10.0	15.0	0	
u		.1	.1	0	0	-.1333	-.1333	

Finally, the height of the EP is $13.33\ mm$, the stop $10\ mm$, the XP $15\ mm$.

By Gaussian imagery

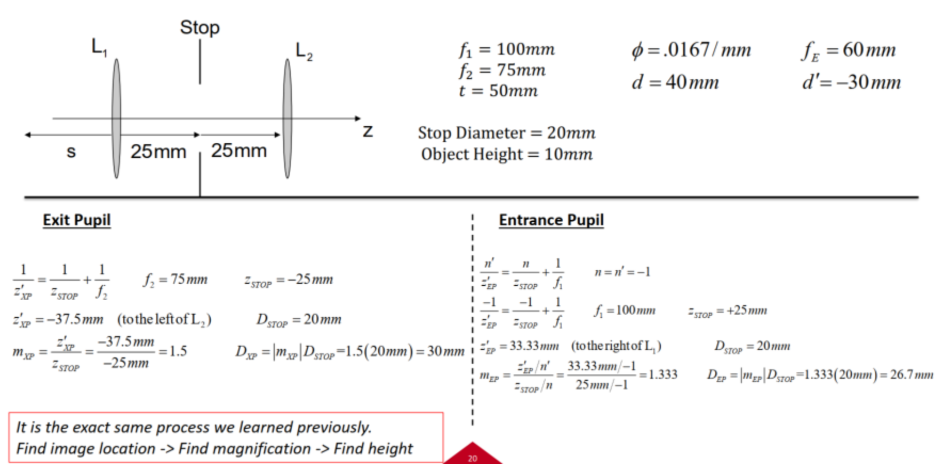
We treat each group independently, considering the stop as our object propagating in the direction of te given group. For EP, the object propagates from right to left, so we flip the sign of the refractive index (as in reflection).

Figure 2.6

$$\text{For XP} \quad \frac{n'}{z'_{XP}} = \frac{n}{Z_{stop}} + \frac{1}{f_{RG}}, \quad m_{XP} = \frac{z'_{XP}}{z_{stop}}, \quad D_{XP} = |m_{XP}| D_{stop} \tag{2.2}$$

$$\text{For EP} \quad \frac{n'}{Z'_{EP}} = \frac{n}{Z_{stop}} + \frac{1}{f_{FG}}, \quad m_{EP} = \frac{z'_{EP}}{z_{stop}}, \quad D_{EP} = |m_{EP}| D_{stop} \quad (n = n' = -1) \tag{2.3}$$

Ejemplo 2.2 **Pupil locations by Gaussian imagery**

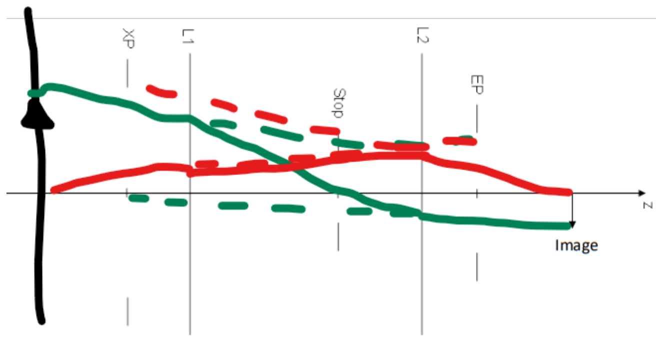


EP,STOP,XP are invariant to object location

Changing the object location does not change the position of the EP, stop, and XP.

Ejemplo 2.3 **Raytrace of chief and marginal rays**

The following diagram illustrate the trace by hand of the chief and marginal ray by definition.



2.1.4 Lagrange invariant

The linearity of paraxial optics provides a relationship between the heights and angles of any two ray s propagating through the system. The **Lagrande invariant** Ξ is formed with the paraxial marginal and chief rays:

$$\text{Lagrange invariant} \qquad \Xi = n\bar{u}y - nu\bar{y} = \bar{\omega}y - \omega\bar{y}\,. \tag{2.4}$$

It is invariant for refraction and transferm and it can be evaluated at any z in any optical space. The Lagrange invariant is particularly simple at images or objects ($y = 0$) and pupils ($\bar{y} = 0$):

$$\text{Image/Object} \qquad y = 0, \quad \Xi = -nu\bar{y} = -\omega\bar{y} \tag{2.5}$$

$$\text{Pupils} \qquad \bar{y} = 0, \quad \Xi = n\bar{u}y = \bar{\omega}y \tag{2.6}$$

If two rays other than the marginal and chief are used, the more general **optical invariant** I is formed.

Given two rays, a third ray can be formed as a linear combination of the two rays. The coefficients are the ratios of the pair-wise invariantas of the values for the three rays at some initial z. The expressions are valid for any z:

$$y_3 = Ay_1 + By_2, \quad u_3 = Au_1 + Bu_2 \tag{2.7}$$

$$A = I_{32}/I_{12}, \quad B = I_{13}/I_{12}, \quad I_{ij} = nu_iy_j - nu_jy_i. \tag{2.8}$$

Changing the Lagrange invariant of a system **scales** the optical system. Doubling the invariant while maintaining the same object and image sizes and pupil diameters haves all of the axial distances (and the focal length).

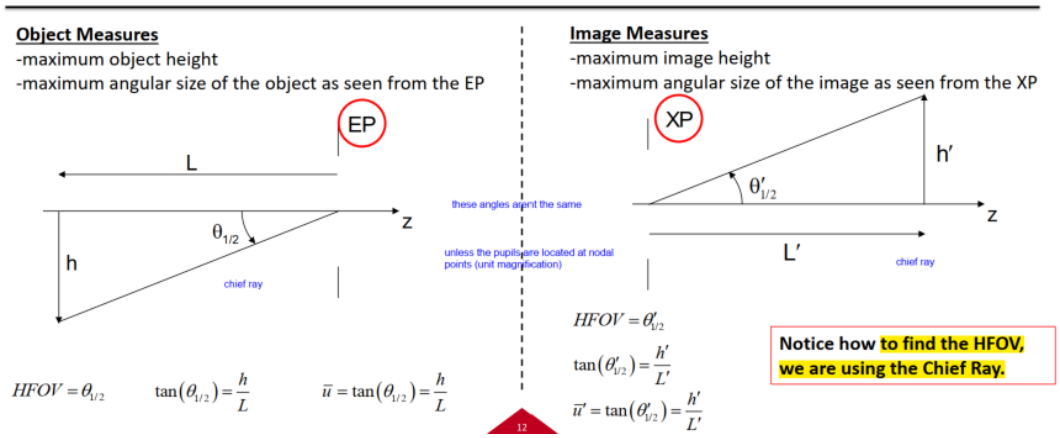
The **throughout**, **etedue** of $A\Omega$ **product** in **radiometry** and **radiative transfer** are relalted to the square of the Lagrange invariant:

$$n^2 A\Omega = \pi^2\Xi^2. \tag{2.9}$$

2.1.5 Field of view

We revisit again the concept of FOV but know using the EP and XP.

- **Field of view FOV** diameter of the object/image.
- **Half field of view HFOV** radius of the object/image.



2.1.6 Numerical aperture and F-number

In an optical space of index n_k, the **numerical aperture** N_A describes the axial cone of light in terms of the real marginal angle U_k:

$$\text{Numerical aperture} \qquad \boxed{NA = n_k|sinU_k| \approx n_k|u_k|}\,. \tag{2.10}$$

The **F-number** $f/\#$ describes the image-space cone of light for an object **at infinity**:

$$\text{F-number} \qquad \boxed{f/\# = \frac{f_E}{D_{EP}}}\,. \tag{2.11}$$

While the $f/\#$ is an image-space, infinite-conjugate measure, the approximate relationship between NA and $f/\#$ allows and $f/\#$ to be defined for other optical spaces and conjugates. As a result, an $f/\#$ can be defined for any cone of light. This $f/\#$ is called **working F-number** $f/\#_W$. This previous relationship becomes a definition

$$\text{Working F-number} \qquad f/\#_W = \frac{1}{2NA} \approx \frac{1}{2n|u|} = (1-m)f/\#. \tag{2.12}$$

Fast optical system have small numeric values for the $f/\#$. Most lenses with adjustable stops have $f/\#$ of **f-stops** labeled in increments of $\sqrt{2}$. The usual progression is:

$$f/1.4, \quad f/2, \quad f/2.8, \quad f/4, \quad f/5.6, \quad f/8, \quad f/11, \quad f/16, \quad f/22, \quad \text{etc.}$$

Each stop changes the area of the EP (light colelction ability) by a factor of 2.

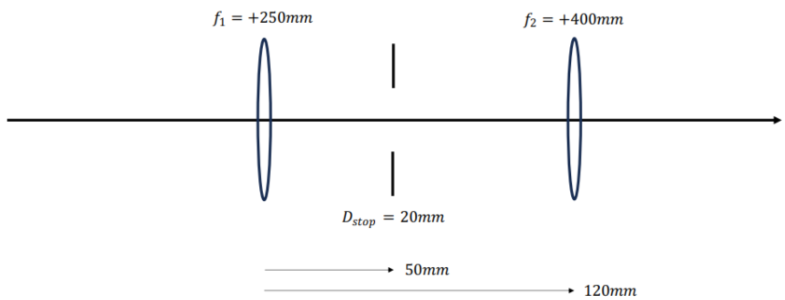
> The Lagrange invariant relates the magnification between two pupils to the chief ray angles at the pupils.
>
> $$\Xi = n\bar{u}y_{pupil} = n'\bar{u}'y'_{pupil}, \quad m_{pupil} = \frac{y'_{pupil}}{y_{pupil}} = \frac{n\bar{u}}{n'\bar{u}'} = \frac{\bar{\omega}}{\bar{\omega}'}. \tag{2.13}$$

Use of working F-number (left)

The most common use of the working F-number is to describe the image-forming cone for a finite conjugate optical system. This is the cone formed by the XP and the axial image point.

Ejemplo 2.4 — Determination of stop and pupil

Determine the location and size of the pupils for th efollowing the system in air.



Solution

a) We trace the chief ray denoted as CR, and a potential marginal ray MR with unitary height at the stop.

		Object space	EP		L_1		Stop		L_2		XP	Image space
	$C/R/f$				250				400			
	t			$z_{EP} = -62.5$		50		70		$z_{XP} = -84.8$		
	n	1	1	1	1	1	1	1	1	1	1	1
	$-\phi$				-0.004				-0.0025			
	t/n			$\tau_{EP} = -62.5$		50		70		$\tau_{XP} = -84.8$		
	y		0		-5		0		7		0	
CR	nu			0.08		0.1		0.1		0.0825		
	u			0.08		0.1		0.1		0.0825		
	y		$R_{EP} = 1.25$		1		1		1		$R_{XP} = 1.21$	
MR	nu			0.004		0		0		-0.0025		
	u					0		0				

Table 2.1 Raytrace, with CR=Chief ray, MR=Marginal ray.

Due to the diameter of the stop is $R_{stop} = 10\ mm$, we scale the potential marginal ray to give the true marginal ray and therefore obtain the radius of the pupils:

$$\begin{array}{l} R_{EP} = (10)(1.25) = 12.5\ mm \\ R_{XP} = (10)(1.21) = 12.1\ mm \end{array} \implies \begin{array}{l} D_{EP} = 2R_{EP} = 25.0\ mm \\ D_{XP} = 2R_{XP} = 24.2\ mm \end{array}$$

b) For Gaussian imagery, we see the stop as the object for the front group and rear group. For the EP, we have a backward propagation that is maganed with the flip of the sign in the refrractive indices.

$$\frac{-1}{z_{EP}} = \frac{-1}{Z_{\text{stop}}} + \frac{1}{250} \longrightarrow z_{EP} = 62.5\ mm.$$

This entrance pupil is to the right of the lens L_1. The magnification is:

$$m_{EP} = \frac{z_{EP}}{z_{\text{stop}}} = \frac{R_{EP}}{R_{\text{stop}}} = -1.25.$$

The diameter of the entrance pupil is therefore:

$$D_{EP} = 2R_{EP} = 2[|m_{EP}|R_{\text{stop}}] = 25\ mm.$$

For the rear group, we have analogously:

$$\frac{1}{z_{XP}} = \frac{1}{Z_{\text{stop}}} + \frac{1}{400} \longrightarrow z_{XP} = -84.848\ mm.$$

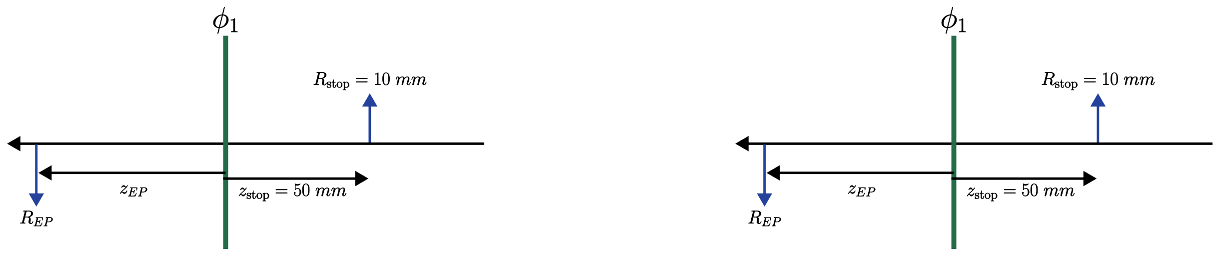
The exit pupil is then to the left of the lens L_2. The magnification in this case is

$$m_{XP} = \frac{z_{XP}}{z_{\text{stop}}} = \frac{R_{XP}}{R_{\text{stop}}} = 1.21.$$

The diameter of the exit pupil is:

$$D_{XP} = 2R_{XP} = 2[|m_{XP}|R_{\text{stop}}] = 24.2\ mm.$$

The illustration of each case is illustrated in the figure 2.7.



(a) EP for front group **(b)** XP for rear group

Figure 2.7 With Gaussian imagery, the computation of the pupils is based on the stop as the object of two optical systems.

Using either method, the result is the same and is shown in figure 2.8

2.2 Vignetting

2.2.1 Ray bundles

The **ray bundle** for an **on-axis** object is a rotationally symmetric spindle made up of section of right circular cones. Each cone section if bounded by the pupil and the object/image in that optical space. At any z, the cross section of the bundle is circular, and the radius of the bundle is the marginal ray value.

For an **off-axis** object point, the ray bundle skews, and is comprised of section of skew circular cones wwhich are still defined by the same elements. The cross section of the ray bundle at any z remains circular with a radius equal to the radius of the axial bundle. The off-axis bundle is centered about the chief ray height.

The maximum radial extent of the ray bundle at any z is:

$$\text{Maximum radial extent} \qquad |y_{max}| = |y| + |\bar{y}|. \tag{2.14}$$

2.2.2 Vignetting

The **vignetting** occurs when other apertures in the system (others than the stop) block a proportion of an off-axis ray bundle. For no vignetting, each aperture radius a must equal or exceed the maximum height of the ray bundle at the aperture.

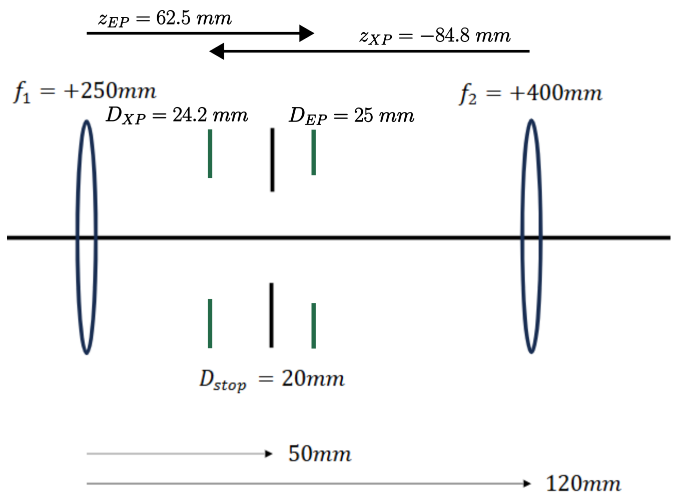


Figure 2.8 Illustration of the stop and pupil in the optical system.

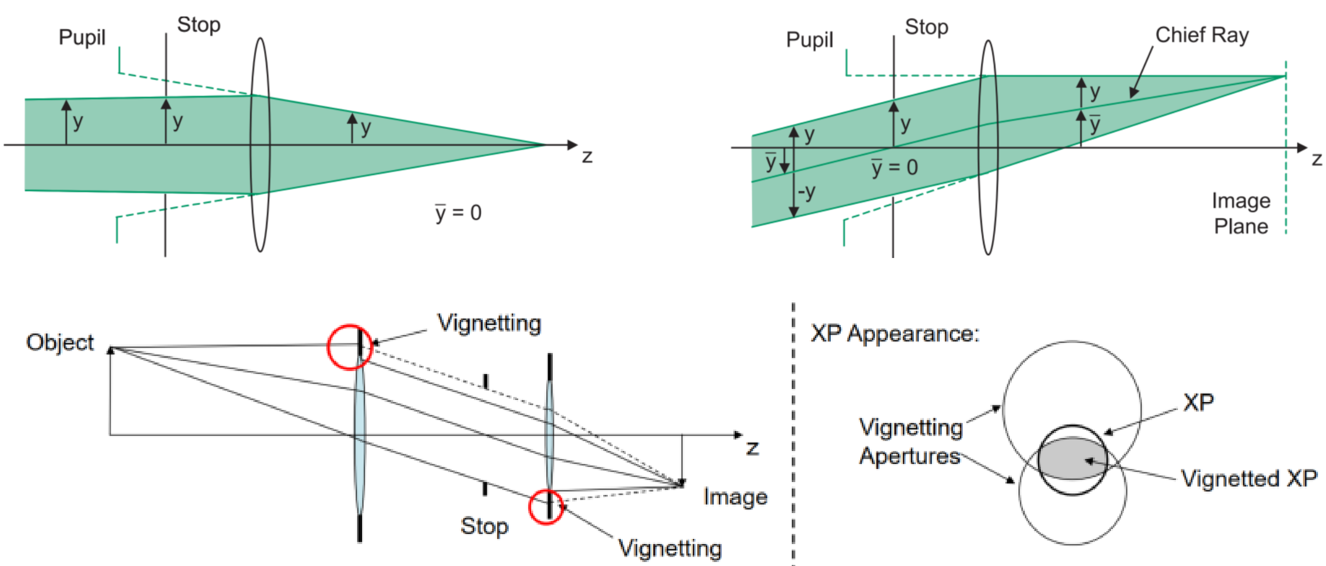


Figure 2.1 The ray bundle is clipped and the beam is no longer circular.

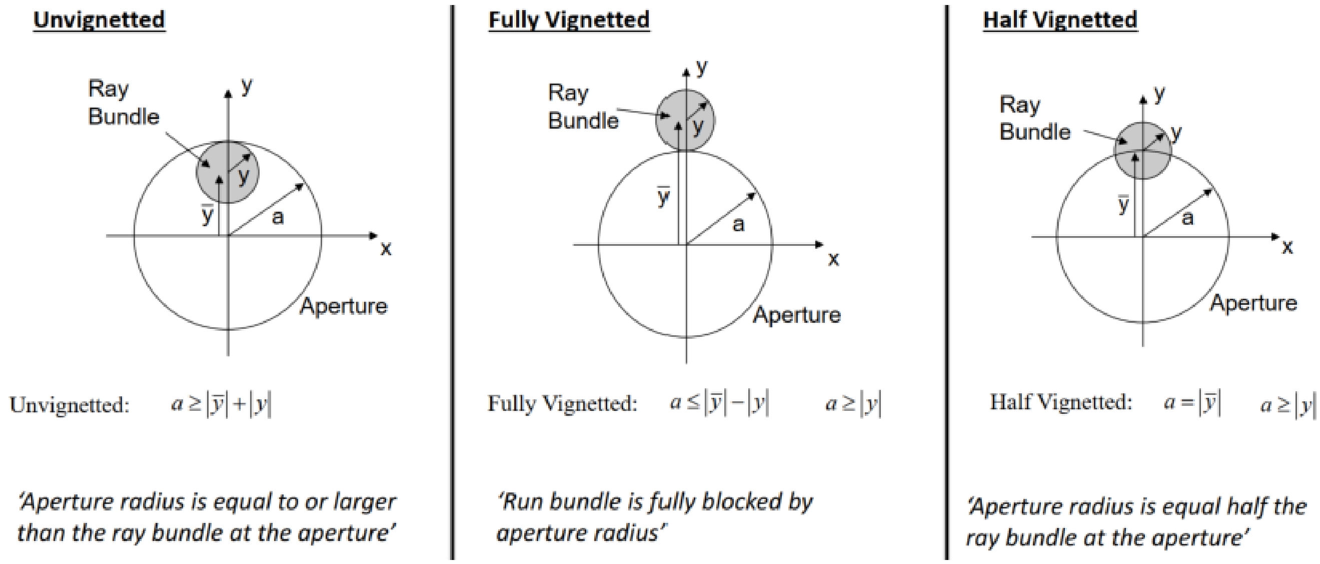
The maximum FOV supported by the system occurs when an aperture completely blocks the ray bundle from the object point.

We can have three conditions of vignetting, depending on the proportion of clip of the light beam.

The vignetting conditions are used in two different manners:

- For a given set of apertures, the FOV that the system will suppport with a prescribed amount of vignetting can be determined. A different chief ray defined each FOV.
- For a given FOC and vignetting condition, the required aperture diameters can be determined.

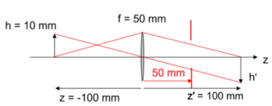
A system with vignetting will have an image that has full irradiance or brightness out to a radius corresponding to the unvignetted FOV limit. The irradiance will then begin to fall off, going to about halr at the half-vignetted FOV, and decreasing to zero at the fully vignetting FOV. This fully vignetted FOV is the absolute maximum possible.



The diameter of the aperture stop is very important design parameter for an optical system as it controls five separate performance aspects of the system:

- The system FOV determined by vignetting.
- The radiometric or photometric speed of the system or its light collection ability.
- The depth of focus and depth of field of the system.
- The amount of aberrations degrading image quality.
- The diffraction-based performance of the system.

Ejemplo 2.5



Stop at Lens
D_{STOP} = 20 mm

At the aperture *(calculate with simple geometry)*

$y_A = a_{STOP}/2 = 5mm$ *Marginal ray height at aperture location.*

$\bar{y}_A = h'/2 = -5mm$ *Chief ray height at aperture location.*

What is the required diameter of the aperture so that the system operates without vignetting?

For the system to have no vignetting, it needs to image the full object.

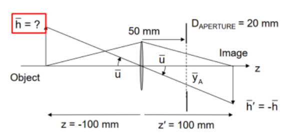
1. Draw the Marginal and Chief Rays and evaluate the ray heights at the aperture
2. Use thin lens imaging to find the image location and height (it ends up being a 1:1 system).

h' = -10 mm

For no vignetting *(use condition)*:

$a_{APERTURE} \geq |\bar{y}_A| + |y_A| = 10mm$ *Required radius for the aperture, no vignetting.*

$D_{APERTURE} \geq 20mm$ *Diameter is twice the radius.*



What is the height of the object so that the system operates with half vignetting?

The marginal ray does not change. Chief range changes with object height.

$$\bar{u} = \frac{\bar{h}}{z} = \frac{\bar{h}'}{z'}$$

At the aperture: $\bar{y}_A = (50\ mm)\bar{u} = (50\ mm)\frac{\bar{h}}{z}$

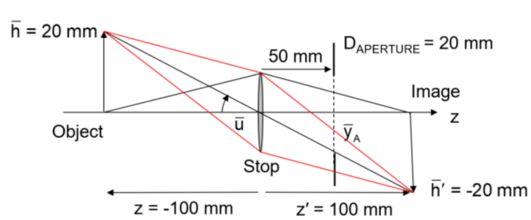
For half vignetting: $a_{APERTURE} = |\bar{y}_A| = 10mm$

Equating:

$$|\bar{y}_A| = \left|(50\ mm)\frac{\bar{h}}{z}\right| = \left|(50\ mm)\frac{\bar{h}}{-100mm}\right| = 10mm$$

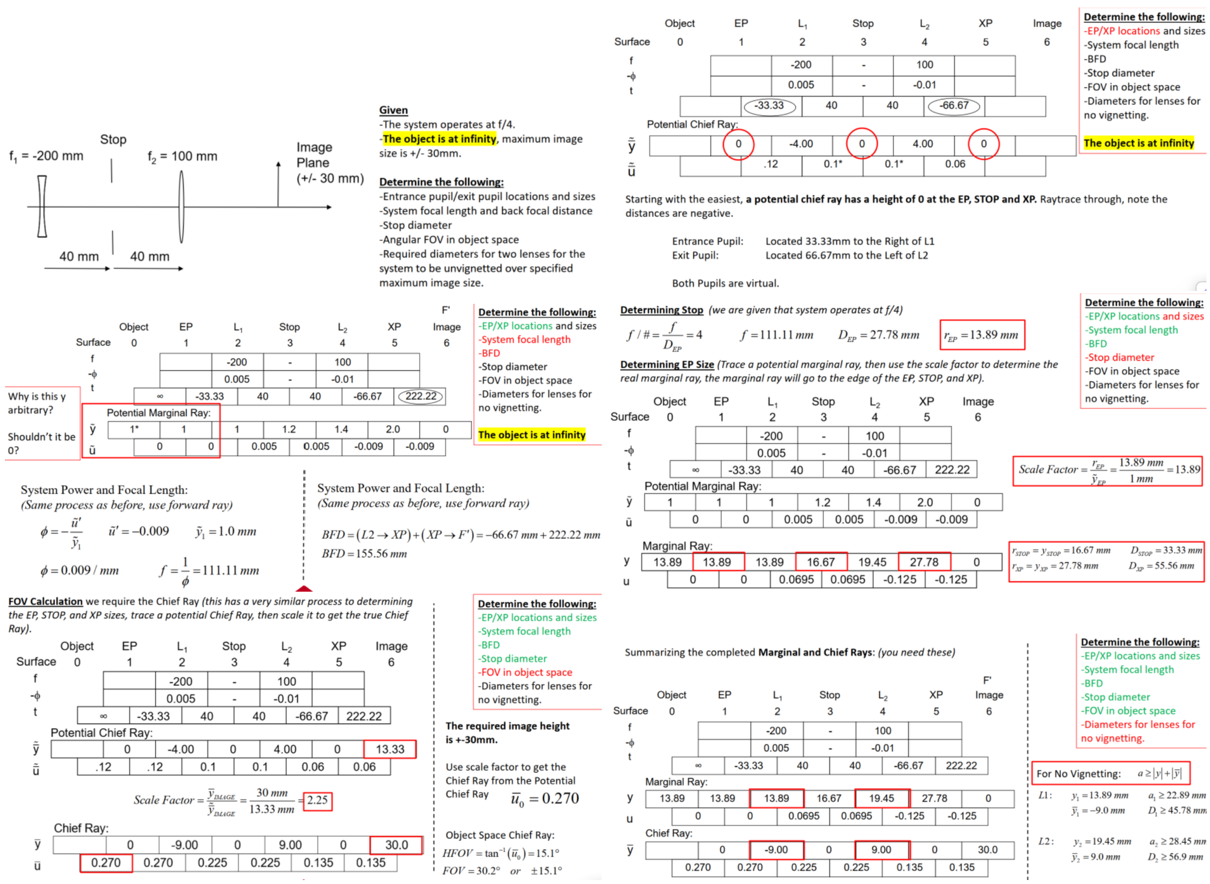
$$\bar{h} = 20mm$$

The ray bundle at half vignetting



$$a_{APERTURE} = |\bar{y}_A| = 10mm$$

Ejemplo 2.6 **Vignetting with paraxial raytrace**



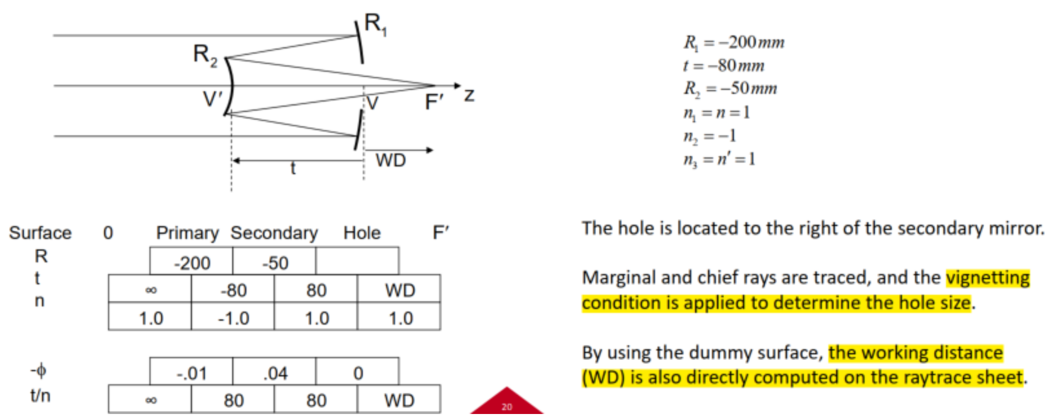
In general,

> **Key points in solving problems**
>
> - Trace the potential chief ray (CR) to know the locations of the pupils (and image size).
> - Trace the potential marginal ray (MR) to determine image location and pupil sizes.
> - If the MR comes parallel, then it can be used to obtain the first-order properties.
> - The F-number gives us the real size of EP so we can scale the MR.
> - The image size allows us to get the real CR.
> - The HFOV is determined with the incident angle $\bar{u}$ at EP in the real CR: $\text{HFOV} = \tan^{-1}\bar{u}$.
> - The vignetting is found by looking at $y, \bar{y}$ in the real MR and CR and applying the criteria.
> - We can arbitrarily define a dummy surface to our convenience.
> - EP and XP are dummy surfaces (w location defined) of zero-power.

2.2.3 Dummy surfaces

In a raytrace, a zero-power surface can be inserted at any location to examine the ray properties.

An example of its application is the following Cassegrain objective, where we require to find the size of the hole. For that, we place a dummy surface **at the hole**.



2.3 Radiative transfer

2.3.1 Radiometry

Radiometry characterizes the propagation of radiant energy through an optical system. The basic unit is the watt W. Radiometric terminology and units are: There are some assumptions:

Quantity	Symbol	Units	Units description
Energy	Q	J	
Flux	Φ	W	Power
Intensity	I	W/sr	Power per unit solid angle
Irradiance	E	W/m^2	Incident power per unit area
Exitance	M	W/m^2	Exiting power per unit area
Radiance	L	$W/m^2 sr$	Power per unit projectedarea per unit solid angle

- The source is **incoherent**, meaning that scenes are collection of independently point sources, no interference.
- Objects and images on-axis and perpendicular to the optical axis, so that the projected area equals the area.

The solid angle Ω equals the surface area of the unit sphere in a given vecinity. The units are 4π steradians (sr).

$$d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin\theta d\theta d\phi}{r^2} = \sin\theta d\theta d\phi \xrightarrow{\int} \boxed{\Omega_{\text{sphere}} = 4\pi \sin^2(\theta_0/2)}\,.$$

In optics, its common to approximate the solid angle of a sphere to the section of a cone:

$$\Omega \approx \frac{\pi r_0^2}{d^2} \approx \pi \sin^2\theta_0 \approx \pi\theta_0^2 \quad (\text{small angle approximation}).$$

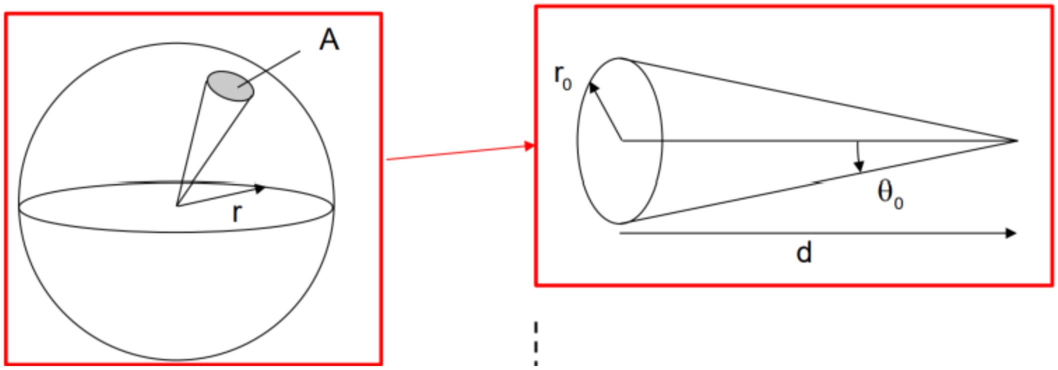


Figure 2.1 The solid angle of a sphere can be approximated to the solid angle of a cone.

2.3.2 Radiative transfer

Radiative transfer uses first-order geometrical principles to determine the amount of light from an object that reaches an image or detector.

Exitance and irradiance are related by the **reflectance** of the surface ρ:

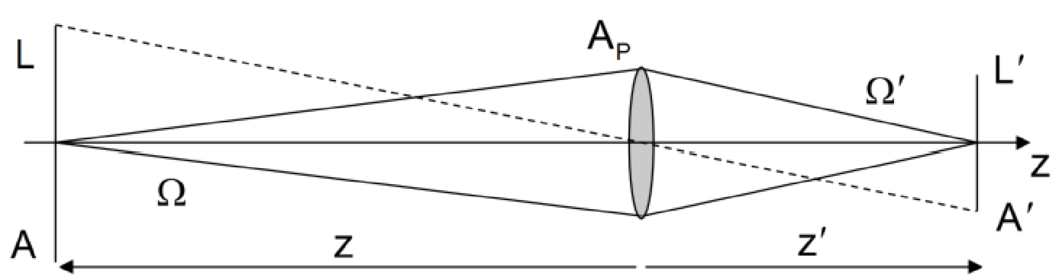
$$M = \rho E. \tag{2.15}$$

For average scenes, $\rho = 18\%$. Exposures are often set using this value, that is, we expose the print to that average so that the prin reflectance ends up with 18%.

The irradiance of a **Lambertian source** (perfectly diffuse surface) is constant. The intensity falls off with the apparaent source size or the **orjected area** (**Lambert's law**). The exitance of a Lambertian source is related to its radiance by π.

Lambertian source	$L(\theta,\phi) = \text{constant}$	$I = I_0 \cos\theta$
	$M = \pi L$	$\pi L = \rho E$

We now analyze the optical power from an object that reaches the image in an optical system.



In air, the radiance and the **$A\Omega$ product** or **throughput** are conserved, and the flux collected by the lens Φ is transferred to the image area A':

$$\Phi = L(\text{object area})(\text{solid angle projection in lens}) = LA_p\Omega = LA\frac{\pi D_p^2}{4z^2} \overset{A'=m^2A}{\longrightarrow} \Phi = \frac{\pi LA'D_p^2}{4m^2z^2}.$$

Using gaussian equations and f-number equations, the image plane irradiance E' is

$$\text{Camera equation} \qquad E' = \frac{\Phi}{A'} = \frac{\pi L}{4(f/\#_W)^2} \to \pi L(\text{NA})^2, \quad L = \frac{\rho E_0}{\pi}. \tag{2.16}$$

Spectral dependence

Spectral dependence can also be added.

$$\begin{array}{l} E_0(\lambda) = \text{Object irradiance} \\ \rho(\lambda) = \text{Object reflectance} \\ L(\lambda) = \text{Object radiance} \end{array} \longrightarrow \begin{array}{l} L(\lambda) = \frac{M(\lambda)}{\pi} = \frac{\rho(\lambda)E_0(\lambda)}{\pi} \\ E'(\lambda) = \frac{\rho(\lambda)E_0(\lambda)}{4(1-m)^2(f/\#)^2} \end{array}$$

This can be integrated over all wavelengths for total irradiance

$$E' = \frac{1}{4(1-m)^2(f/\#)^2} \int_{\lambda_1}^{\lambda_2} \rho(\lambda)E_0(\lambda)\, d\lambda.$$

Exposure

Most detectors respond to energy per unit area rathen than power per unit area. Multiplying the image irradiance by the exposure time gives the expoure (J/m^2):

$$\text{Exposure} \qquad H = E'\Delta t\,. \tag{2.17}$$

The mean solar constant is $1368\ W/m^2$ outside the atmosphere of the earth, and the solar irradiance on the surface is about $1000\ W/m^2$.

2.3.3 Photometry

Photometry is the subset of radiometry that deals with visual measurements, and luminous power is measured in **lumens** lm. The lumen is a watt wighted to the visual **photopic response**. This peak response occurs at $555\ nm$, where the conversion is $683\ lm/W$. The dark adapted or **scotopic response** peaks at $507\ nm$ with $1700\ lm/W$.

(a) Photometric terminology

Luminous power	Φ_V	lm
Luminous intensity	I_V	lm/sr
Illuminance	E_V	lm/m^2
Luminous exitance	M_V	lm/m^2
Luminance	L_V	$lm/m^2 sr$
Exposure	H_V	$lm\ s/m^2$
candela (cd)	I_V	lm/sr
lux (lx)	E_V	lm/m^2
foot-candle (fc)		lm/ft^2
	$1fc = 10.76\ lx$	
foot-lambert (fL)	L_V	$\frac{1}{\pi}cd/ft^2$
nit (nt)		$= cd/m^2$
	$1fL = 3.426\ nt$	
lux-second (lx s)	H_V	$lm\ s/m^2$

(b) Luminous photopic efficacy

$\lambda\ (nm)$	lm/W
400	0.3
420	2.7
440	15.7
460	41.0
480	95.0
500	221
520	485
540	652
560	680
580	594
600	425
620	260
640	120
660	41.7
680	11.6
700	2.8
720	0.7

(c) Typical illuminance levels

Sunny day	$10^5\ lx$	Moonlit night	$10^{-1}\ lx$	Overcast day	$10^3\ lx$	Starry night	$10^{-3}\ lx$
Inerior	$10^2\ lx$	Desk lighting	$10^3\ lx$				

The candela (cd) is the fundamental SI unit for luminous intensity.

$A\Omega$ product

Recall the flux hrough a system is

$$\Phi = LA\Omega.$$

The $A\Omega$ product appears to be the geometric portion, while L would be related to the source characteristics. In an object or an image plane,

$$\begin{array}{rllll} \text{object/pupil plane} & A = \pi\bar{y}^2, & \theta = u, & A\Omega = \pi^2\bar{y}^2u^2 = \pi^2\chi^2/n^2, & \chi = n\bar{y}u \\ \text{pupil plane} & A = \pi y^2, & \theta = \bar{u}, & A\Omega = \pi^2 y^2\bar{u}^2 = \pi^2\chi^2/n^2, & \chi = ny\bar{u} \end{array}$$

In the general situation when the index is not unity, the **basic throughout** $n^2A\Omega$ and the **basic radiance** L/n^2 are invariant. Since throughput is based on areas, the basic throughput is proportional to the Lagrange invariant squared:

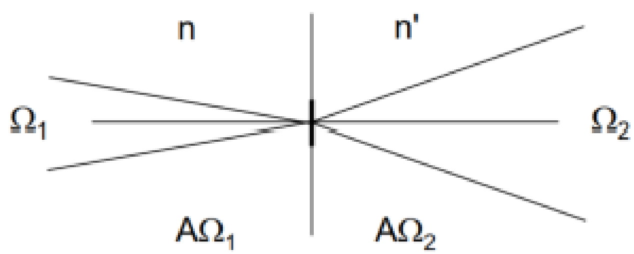
$$n^2A\Omega = \pi^2\chi^2. \tag{2.18}$$

For a lossless optical system, the flux through the system is constant. For $n = 1$, we have

$$\Phi = L_1A_1\Omega_1 = L_2A_2\Omega_1 = \cdots = \text{constant}.$$

Since $A\Omega$ is also constant, the radiance L must also be constant. This allow us to relate different portions of the optical system as the flux is conserved.

However, if the index of refraction is not unity and changes ,the radiance is no longer conserved. It will change at each interface as the solid angle will change. We then have that



$$A\Omega_1 \neq A\Omega_2 \wedge \Omega = L_1A\Omega_1 = L_2A\Omega_2 \implies L_1 \neq L_2.$$

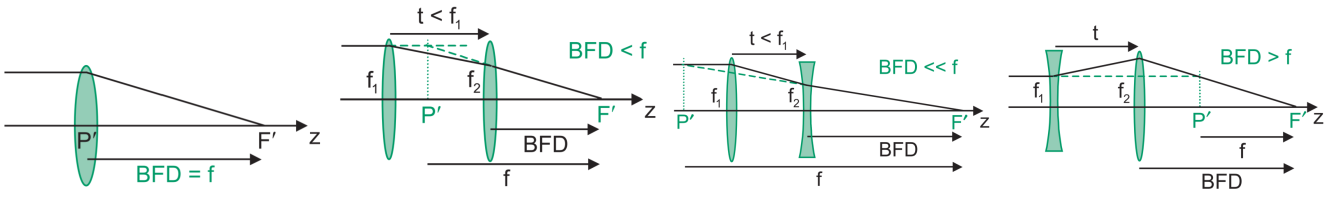
The flux is still constant. In fact, $L/n^2 =$ constant as well as $n^2A\Omega =$ constant.

2.4 Objectives

2.4.1 Type of objectives

Objectives are lens element combinations used to image distance objects. To classify them, separated group of lens elements are modeled sa thin lenses.

- **Simple objective** consists of a positive thin lens.



(a) Simple objective **(b)** Petzval objective **(c)** Telephoto objective **(d)** Reverse Telephoto

Figure 2.1 Different type of objectives.

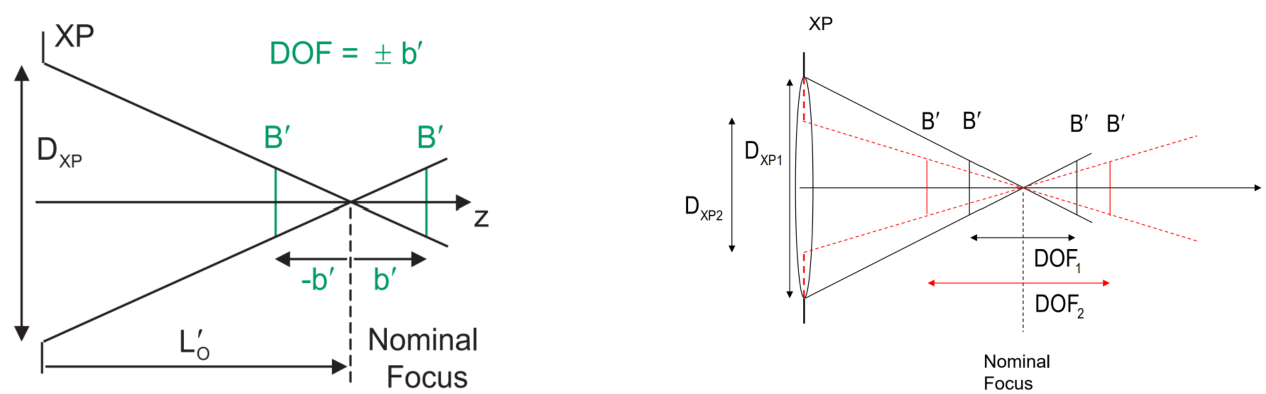
- **Collimator** is a reversed simple objective. It creates a collimated beam from a source at the system focal point.
- **Petzval objective** consists of two separated positive groups of elements. The rear principal plane is located between the groups.
- **Telephoto objective** produces a system focal length longer than the BFD. It consists of a positive element followed by a negative group.
- **Reverse telephoto objective** consists of a negative group followed a positive group. Used to produce a system with BFD larger than the system focal length.

2.4.2 Depth of focus and field

There is often some allowable image blut that defines the performance requirement of an optical system. No diffraction or aberrations are included.

Depth of focus

The **depth of focus** DOF describes the amount the detector can be shifted from the nominal image position (focal plane) for a fixed object before the resulting blur exceed the blur diameter criterion B'.



(a) Depth of focus **(b)** Depth of focus vs F-number

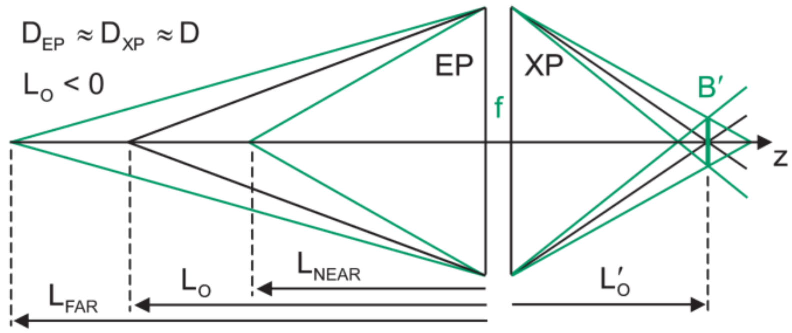
The $\pm b'$ transverse distance from the nominal point is used to define the DOF as the whole distance is $2b'$:

$$b' = \frac{B'L'_0}{D_{XP}} \approx \frac{B'z'}{D_{EP}}, \quad \text{DOF} \approx \pm b' \approx \pm B' f/\#_W \approx \frac{B'}{2\text{NA}}. \tag{2.19}$$

The figure illustrates how the DOF is changed as the diameter of the lens is varied; they are inversely proportional.

Depth of field

The **depth of field** is the maximum distance, from L_{near} to L_{far}, the object can move before exceed the aceptable blur B' at the fixed image plane. The following relations are given



$$L_{\text{far}} \approx \frac{L_0 f D}{f D + L_0 B'}, \quad L_{\text{near}} \approx \frac{L_0 f D}{f D - L_0 B'}. \tag{2.20}$$

Al objects positiones between these distances will produces images on the detector that have geometrical blurs less than the blur criterion B'.

2.4.3 Hyperfocal distance

When the far point of the depth of field L_{far} is at infinity, the optical system is focused at the **hyperfocal distance** L_H, and all objects from L_{near} to infinity meet the image plane blur criterion.

$$L_{\text{far}} = \infty \implies f D + L_H B' = 0.$$

Solving for L_H,

$$\text{Hyperfocal distance} \qquad \boxed{L_H = -\frac{f D}{B'} = -\frac{f^2}{(f/\#) B'}}. \tag{2.21}$$

Substituting of L_H in L_{near} yields

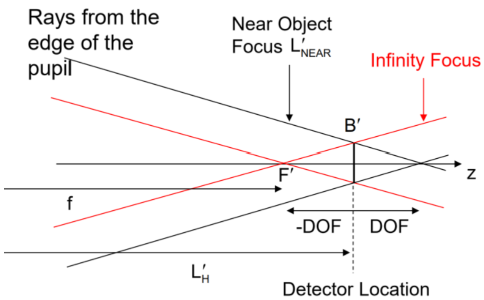
$$L_{\text{near}} \approx -\frac{f D}{2 B'} = \frac{L_H}{2}. \tag{2.22}$$

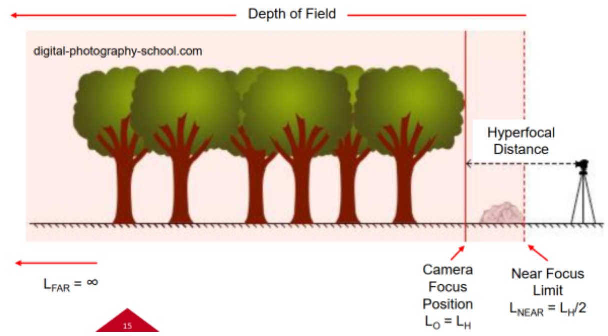
Hyperfocal distance and depth of

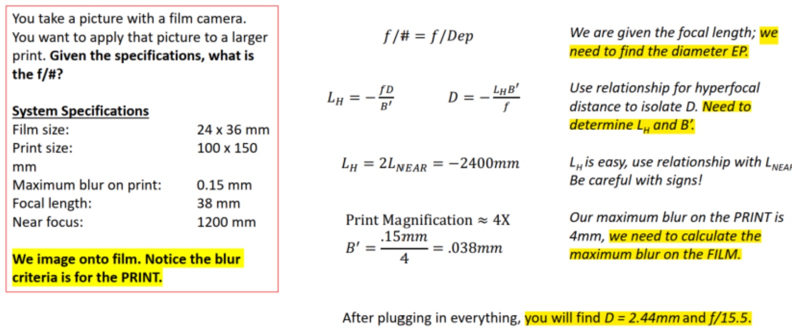
The relation between the depth of focus and the hyperfocal distance where the detector is placed is, by the thin-lens equations,

$$L'_H \approx f + B' f = f + \text{DOF}. \tag{2.23}$$

- Focusing at the hyperfocal distance **ensures** that any greater distance meets the blur criteria.
- As $f/\#$ increases (lens stopped down), the hyperfocal distance moves closer to the lens.

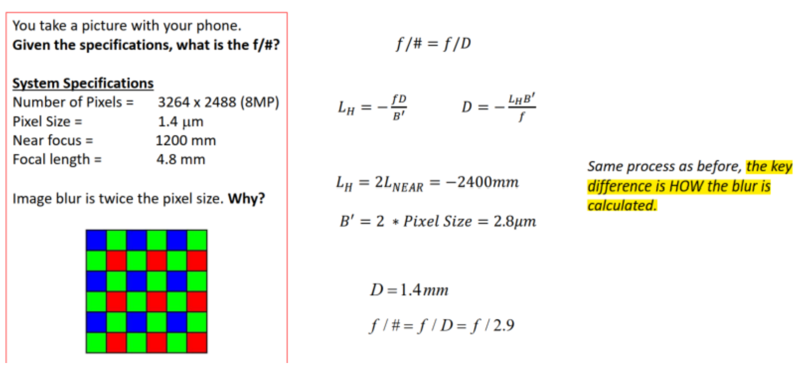






Ejemplo 2.7 Fixed-focus camera

Ejemplo 2.8 Fixed-focus digital camera



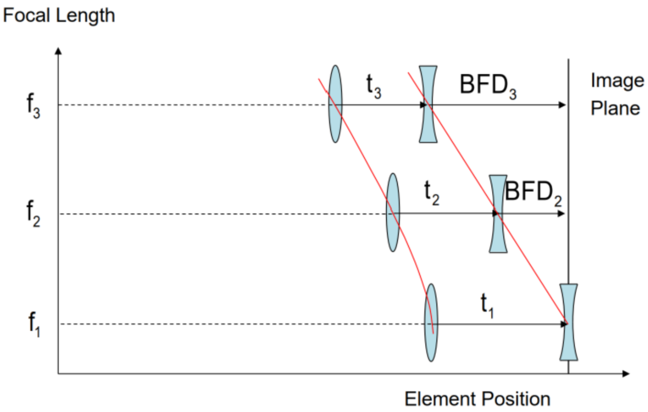
2.4.4 Zoom lenses

A **zoom lens** is a variable focal length objective with a **fixed** image plane. The simplest system is composed of two groups with powers ϕ_1 and ϕ_2 where both the focal length f and the BFD vary with the element spacing t.

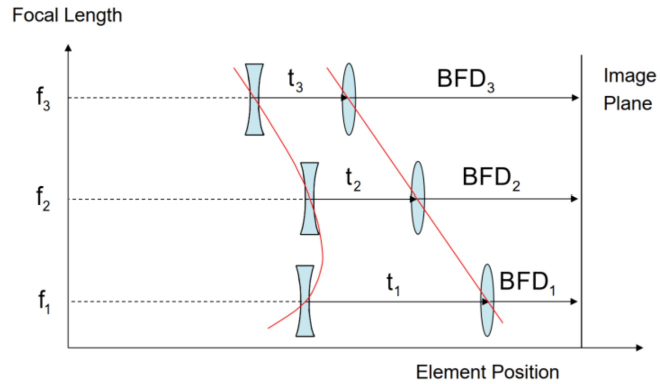
$$\phi = \frac{1}{f} = \phi_1 + \phi_2 - \phi_1\phi_2 t \quad \text{BFD} = f + d' = f - \frac{\phi_1}{\phi} t.$$

To vary the focal length with a fixed image plane, we:

- Move the lens L_1 a distance $L = t + \text{BFD}$ from the image plane.
- Displace lens L_2 a distance t from L_1.



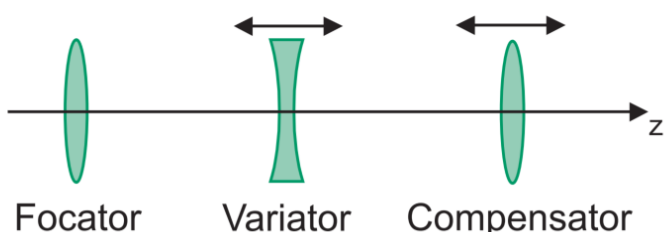
(a) Telephoto zoom (limited by BFD)



(b) Reverse telephoto zoom (commonly used)

As the separation approaches the sum of the individual focal lengths ($f_1 + f_2$), the system becomes afocal ($f \to \infty$).

A mechanical cam provides the complicated lens motion required for these **mechanical compoensated** zoom lenses. A common three group configuration uses a fixed front element and moving second and third groups.



Ejemplo 2.9 **Digital camera**

You are using a digital camera to take a picture of a house against a distant mountain backdrop. The camera is focused on the house. You have a wide selection of camera lenses and 16:9 (width = 12.48mm, height = 7.02 mm) CCD detectors at your disposal. The CCD detectors available to you have pixels with sizes ranging between 3 µm and 6 µm, however, cameras with pixel sizes of exactly 3 µm and 6 µm are not available. You want to make sure that you can take a high-quality picture of the house and the mountains and therefore, want to achieve a diffraction-limited image based on the pixel pitch of the CCD detector you selected. The location allows you to place the camera anywhere between 10 and 30 meters (not exactly 10 nor 30 m) away from the house. Assume that the lens is a thin lens with the stop at the lens.

a) What are the hyperfocal distance and acceptable blur for your camera system?
b) Determine the focal length, f, and the diameter, D, of the lens.
c) How far behind the lens must the detector be located? What is the maximum size (height and width) of the house in meters you would be able to image on the detector?
d) What is the horizontal and vertical Field of View of this photographic system (in degrees)?
e) What is the f/# of this camera? What is the closest object that will be considered to be in focus?

f) It is an overcast day (outdoor illuminance=103 lux=103 lm/m2). The reflectance of the house is r = 0.5 and the mountains have an average reflectance of r=0.18. What are the image plane illuminances for the house and the mountains individually?

Solution

a) CCD pixel pitch is $4\ \mu m$. And camera at $20\ m$ away from the house.

b) The hyperfocal distance is $L_H = 20\ mm$. The blur is $B' \approx 4\ \mu m \rightarrow f/\# = 4$.

c) The hyperfocal idstance can be related to focal length:

$$|L_H| = \frac{f^2}{B' f/\#} \longrightarrow f = \sqrt{L_H B' f/\#} = \sqrt{20 \cdot 4\ \mu m \cdot 4} = 17.889\ mm.$$

The diameter is then:

$$D = \frac{f}{f/\#} = \frac{17.889\ mm}{4} = 4.472\ mm.$$

We can think os a single thin lens that images an object at $-20\ mm$ to L_H. The magnification is:

$$m = \frac{L'_H}{L_H} = fracz'z = 1 \cdot 1^{-3}.$$

The field of view for each dimension is:

$$\text{HFOV}_H = \tan^{-1}\frac{7.02}{L'_H} =$$
$$\text{HFOV}_W = \tan^{-1}\frac{12.48}{L'_H} =$$

d) The near distance is half of the hyperfocal distance

$$L_{\text{near}} = \frac{L_H}{2} = 10\ mm.$$

e) For the house,

$$E' = \frac{M}{4(f/\#_W)^2} = \frac{\rho E}{4(f/\#_W)^2} = \frac{\rho E}{4(f/\#)^2(1-m)^2}.$$

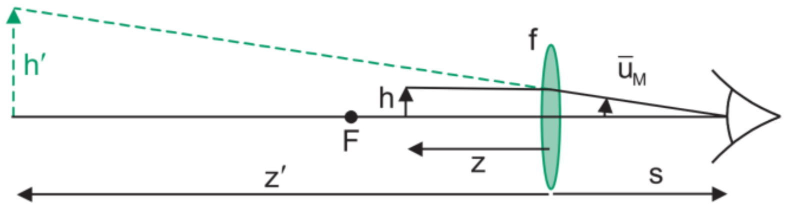
For the mountain (located at infinity):

$$E' = \frac{\rho E}{4(f/\#)^2}.$$

2.5 Magnifiers and Telescopes

2.5.1 Magnifiers

The largest image magnification possible with the unaided eye occurs when the object is placed at the **near point** of the eye, by convention, $250\ mm$ or $10\ in$. A **magnifier** is a single lens that provides an enlarged **erect virtual image** of a nearby object for visual observation. The object must be placed within the fron focal length of the lens.



The **magnifying power** MP is defined as (stop at the eye):

$$\begin{aligned}\text{MP} &= \frac{\text{Angular size of the image (with lens)}}{\text{Angular size of the object at the near point}} \\ &= \frac{\bar{u}_M}{\bar{u}_U} = \frac{h'/(z'-s)}{h/d_{NP}}, \quad d_{NP} = -250\ mm \\ &= \frac{250\ mm(z'-f)}{f(z'-s)} \approx \frac{250\ mm}{f}.\end{aligned}$$

The approximation relation is the most common definition of the MP. It assumes that the lens is close to the eye and the image is presented to a relaxed eye ($z' = \infty$).

The angular substense θ of the image h' at the eye is

$$\theta = \frac{h\text{MP}}{250\ mm}. \tag{2.24}$$

The resolution of the human eye is about $1\ arcmin$, or $(1/60)^\circ$. In order to resolve an object of size h, th required MP is then

$$\text{MP} \geq \frac{0.075\ mm}{h}.$$

Magnifiers up t about 25X are practical; 10X is common.

2.5.2 Telescopes

Telescopes are afocal systems used for visual observation of distant objects. The image through the telescope subtends an angle θ' different from the angle subtend by the object θ. The magnifying power of a telescope is:

$$\text{MP} = \frac{\theta'}{\theta} = \begin{cases} |\text{MP}| > 1, & \text{Telescope magnifies} \\ |\text{MP}| < 1, & \text{Telescope minifies} \end{cases}. \tag{2.25}$$

In Keplerian and Galilean telescope, the lateral magnification m is given by:

$$\text{Lateral magnification of telescope} \qquad m = \frac{1}{\text{MP}} = -\frac{f_{\text{EYE}}}{f_{\text{OBJ}}}. \tag{2.26}$$

It is important to notice the reciprocal relation between the magnification and the magnifying power. For instance an image smaller than the object, also have the image much closer, so that the apparent size is much larger.

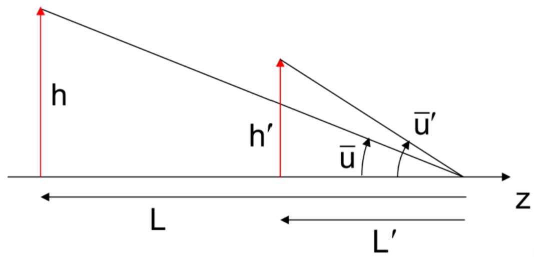


Figure 2.1 In this case, $m < 1$ while also MP > 1: height and distance are important for these quantities.

Keplerian telescope

A Keplerian telescope or astronomical telescope consists of an objective that creates an **aerial image** (real image in the air) followed by a magnifier separated by $f_1 + f_2$. The system stop is usually at or near the objective lens.

The Keplearian has a negative MP: MP < 1 and the image presented to the eye is inverted and reverted (rotated $180°$). The eye should be placed at the real XP to couple the eye to the telescope and see the entire FOV, if not vignetting may occur. The XP position from the last surface is called the **eye relief** ER. The magnification of the telescope related the diameters of EP and XP:

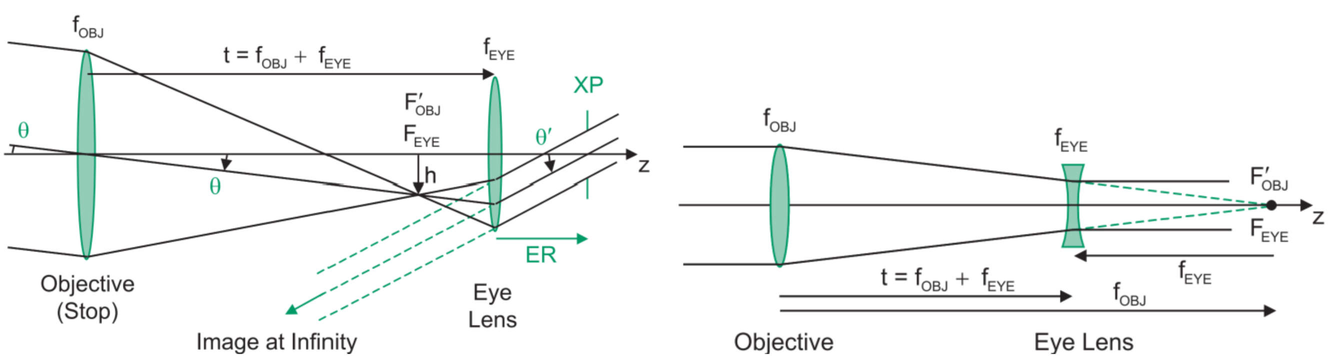
$$\mathrm{ER} = z' = (1 - m) f_{\mathrm{EYE}}, \quad D_{\mathrm{XP}} = |m| D_{\mathrm{EP}}. \tag{2.27}$$

The XP of a visual instrument is also known as the **eye circle** or the **Ramsden circle**.

Galilean telescope

The Galilean telescope uses a positive lens followed by a negative lens to obtain an erect image and a positive MP MP > 1. In this case, the XP is internal o virtual and not accesible to the eye. The FOV of the system is therefore small. There is no intermediate image plane.

> For a given |MP|, the Galilean telescope is shorter and the FOV smaller than the corresponding Keplerian telescope.



(a) Keplerian telescope **(b)** Galilean telescope

We describe several important points:

- Usually in telescope, the objective is the stop so that this lens is also the EP. Keplerian have real XP to the right of the eye lens.

- A **reversed Galilean telescope** provides a minified erect image $0 < \mathrm{MP} < 1$ and the eye is usually the system stop.
- **Binoculars** are a pair of parallel telescope for each eye.
- The specification provided on telescope and binoculars is of the form

$$\mathrm{AXB} \implies A = |\mathrm{MP}|, \quad \text{and} \quad B = \text{Objective diameter in mm.} \tag{2.28}$$

Ejemplo 2.10 **Eye relief of a Keplerian telescope**

A 5X Keplerian telescope is constructed out of two thin lenses. The separation between the two lenses is 120 mm, and the diameter of the objective lens is 25 mm. The system stop is at the objective. Determine the eye relief and the diameter of the exit pupil for this telescope.

$$\boxed{D_{XP} = |m| D_{EP} = \frac{D_{EP}}{|MP|} = \frac{f_{EYE}}{f_{OBJ}} D_{EP} \qquad ER = z' = (1-m) f_{EYE}}$$

Exit Pupil Diameter

$$D_{EP} = D_{STOP} = 25mm$$

$$D_{XP} = \frac{D_{EP}}{|MP|} = \frac{25mm}{5} = 5mm$$

We can use this same information to find the focal lengths

$$MP = -5 = -\frac{f_{OBJ}}{f_{EYE}} \qquad f_{OBJ} = 5 f_{EYE} \qquad t = f_{OBJ} + f_{EYE} = 6 f_{EYE} = 120mm$$

$$f_{EYE} = 20mm \qquad f_{OBJ} = 100mm$$

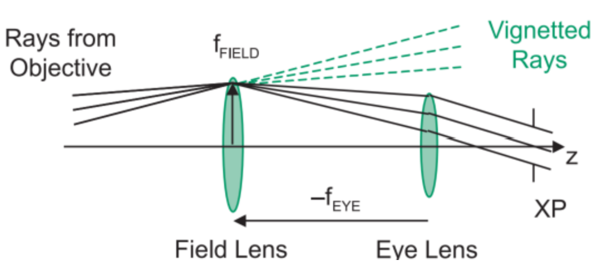
Determining Eye Relief Location *image stop through lens, or use equation*

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_{EYE}} \qquad z = -(f_{OBJ} + f_{EYE}) = -t = -120mm$$

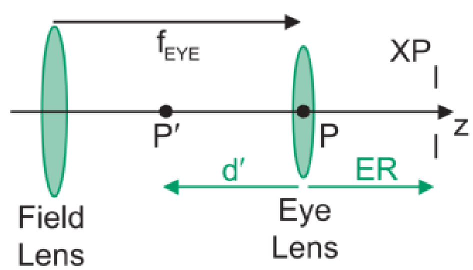
$$z' = ER = 24mm$$

2.5.3 Field lenses

The FOV of the Keplerian telescope is limited by vignetting at the eye lens. A **field lens** placed at the intermediate image plane increases the FOV by bending the ray bundle into the aperture of the eye lens.



(a) Field lens



(b) Reduction of the ER

This combination is called an **eyepiece**. If we assume the separation between them is equal to the focal length of the eye lens, then the overall power is:

$$\phi = \phi_{\mathrm{EYE}} + \phi_{\mathrm{FIELD}} - \phi_{\mathrm{EYE}}\phi_{\mathrm{FIELD}} \cdot \frac{1}{\phi_{\mathrm{EYE}}} = \phi_{\mathrm{EYE}}.$$

Thus, the eyepiece maintain its focal length and therefore its MP. The shifts of the principal planes are:

$$d = \frac{\phi_{\mathrm{EYE}}}{\phi_{\mathrm{EYE}}} \frac{1}{\phi_{\mathrm{EYE}}} = f_{\mathrm{EYE}}, \quad \text{and} \quad d' = -\frac{\phi_{\mathrm{FIELD}}}{\phi_{\mathrm{EYE}}} \frac{1}{\phi_{\mathrm{EYE}}} = -\frac{f_{\mathrm{EYE}}^2}{f_{\mathrm{FIELD}}}. \tag{2.29}$$

To summarize, the inclusion of the field lens:

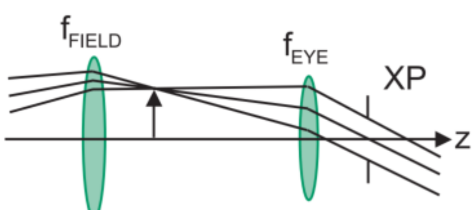
- Decrease vigneting incresing in the same way the FOV.
- Does not change the MP of the telescope or the size of the XP.
- The front principal plane is unchanged, but the rear principal plane and ER are shifted by d'.
- The field lens is usually also displaced from the image plane to minimize the inclusion of information of the field lens to the image through defocus.
- MP of the eyepiece is the same as the MP of a magnifier.

2.5.4 Eyepieces

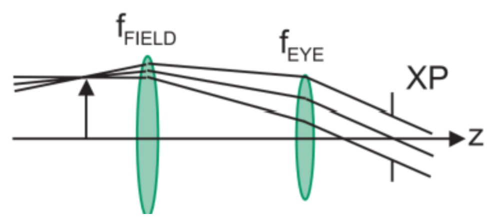
An **eyepiece** or **ocular** is the combination of the field lens and the eye lens. A simple eye piece does not have a field lens. A compound eyepiece has both of them. A field stop can be placed at the intermediate image plane to restrict the system FOV. This aperture serves to correct the vignetting.

Two special eyepiece configuations displace the field lens from the intermediate image:

- **Huygens eyepiece** has the intermediate image between the two elements.
- **Ramsden eyepiece** has intermediate image in front of the field lens. This configuration has about 50% more ER than the Huygens eyepiece.
- **Kellner eyepiece** replaces the singlet eye lens of the Ramsden eyepiece with a doublet for color correciton.



(a) Huygens eyepiece



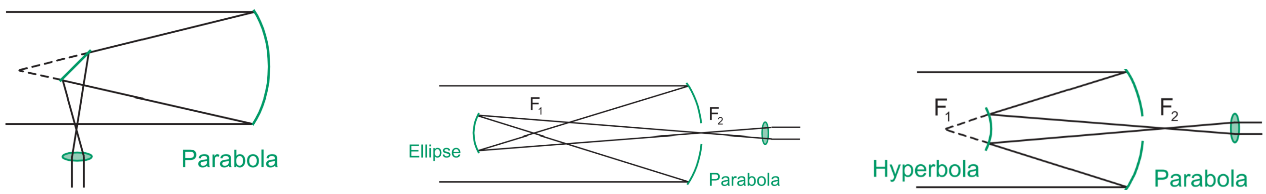
(b) Ramsden eyepiece

Hand-held instruments should have $15-20\ mm$ of eye relief. Microscopes may have as little as $2-3\ mm$. Other systems, such as riflescopes, shoud have a very long eye relief. The human eye pupil diameter varies from $2-8\ mm$, with a diameter of about $4\ mm$ under ordinary lighting conditions. When overfiled, the eye becomes the system stop.

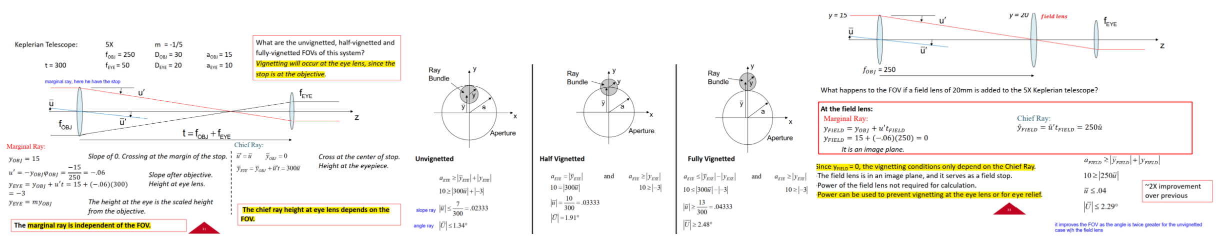
2.5.5 Mirror-based Telescopes

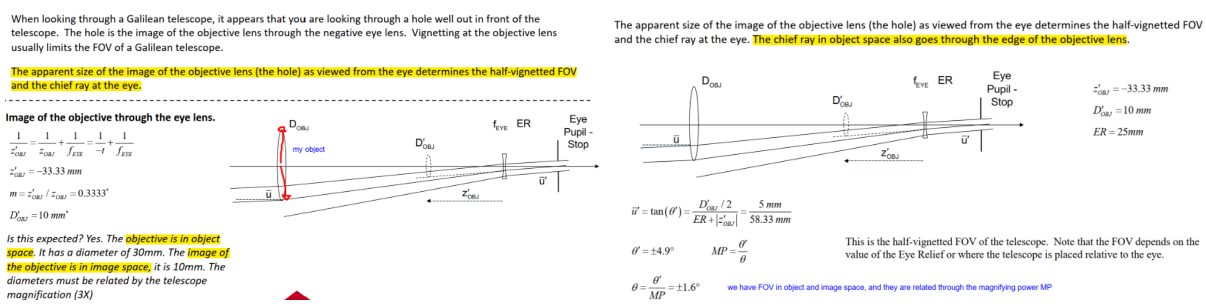
The imaging properties of conic surfaces are used in the design of **mirror-based telescopes**.

- **Newtonian telescope** uses a parabola with a fold flat. It is analogous to a Keplerian refracting telescope.
- **Gregorian telescope** uses a parabola followed by an ellipse to relay the intermediate image. It prooduces an erect image.
- **Cassegrain telescope** uses a parabola combined with a hyperbolic secondary mirror to reduce the system length. It is equivalent to a telephoto objective. The two conic surfaces correct the spherical aberration.
- **Ritchey-Chretien telescope** is identical to Cassegrain telescope in layout, except that it uses two hyperbolic mirrors to correct coma as well as spherical aberration.



(a) Newtonian **(b)** Gregorian **(c)** Cassegrain





Ejemplo 2.11 Vignetting in a Keplerian telescope

Ejemplo 2.12 Vignetting in a Galilean telescope

2.6 Relays and Microscopes

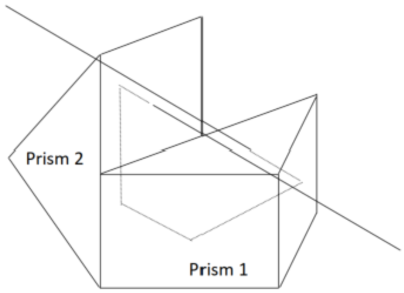
2.6.1 Prisms and relay lenses

For many applications, it is important that the image have the same orientation as the object. The commonly used methods for image erection is to use prism, and relay sistems.

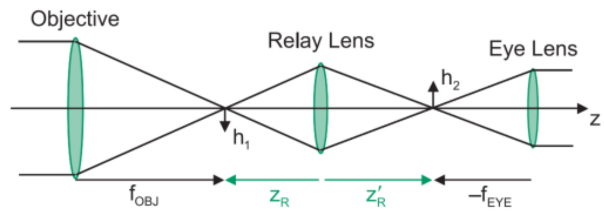
On the one hand, prisms as porro or Pechan-roof prism may help to erect the image. It is important that the ray bundle must be sized so the entrance and exit faces do not vignette the FOV. Unfolding the prism make possible to treat it as a plane parallel plate of glass.

On the other hand, the **relay lens** consists of an additional lens that takes the image of the objective (object for this lens) and image it to the right. This image is the object for the eyepiece. The net MP of the relayed Keplerian telescope in the figure is positive and equals to the product of the magnification of the relay and the MP of the original Keplerian telescope:

$$\text{MP} = m_R \text{MP}_K = -\frac{z'_R f_{\text{OBJ}}}{z_R f_{\text{EYE}}}, \quad \text{where} \quad m_R = \frac{z'_R}{z_R}. \tag{2.30}$$



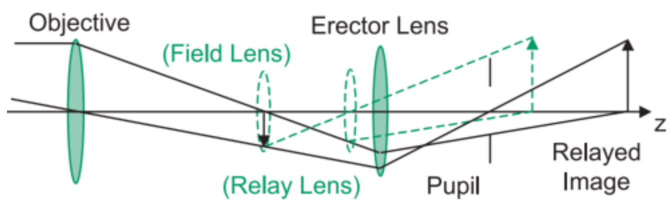
(a) Using a prism



(b) Using a relay lens

Multiple relay lenses can be used to transfer the imager over long distances, such as in periscoopes, andoescopes and borescopes. Field lenses can also be added at the intermediate lenses.

The functions of a field lens and a relay lens ca be combined into a single **erector lens**. This lens will require a diameter larger than the replaced field or relay lenses. The relayed image and pupil are shifter from their original positions.



Ejemplo 2.13 **Keplerian telescope with relay lens**

A 30X relayed Keplerian telescope is constructed out of three thin lenses in air. The relay lens of the telescope operates with a magnification of 1.5. The focal length of the objective lens is 400 mm, and the overall telescope length is 500 mm.

a) Determine the design of the telescope.
b) Assuming that the system stop is located at the objective lens, determine the eye relief of the telescope (distance from the eye lens to the XP).

Solution

a) The magnification of the relay lens and the total magnification are:

$$m_R = \frac{z'_R}{z_R} = -1.5, \quad \text{MP} = m_R \text{MP}_K = (-1.5)(-\frac{f_1}{f_2}) = 30 \longrightarrow f_2 = \frac{400 \cdot 1.5}{30} = 20\ mm.$$

Therefore, the remaining space is $80\ mm$ which is distribtued for the relay lens.

$$-z_R + z'_R = 80\ mm.$$

Using it with the relay magnification we have that

$$z_R = -32\ mm, \quad \text{and} \quad z'_R = 48\ mm.$$

The focal length of the relay lens is obtained with the thin-lens equation:

$$\frac{1}{z'_R} = \frac{1}{z_R} + \frac{1}{f_R} \longrightarrow f_R = 19.2\ mm.$$

b) The eye relief is located at the image position in image space of the stop. The relay lens take that object located at $z = -(32 + 400) = -432\ mm$.

$$z' = \frac{1}{\frac{1}{-432} + \frac{1}{19.2}} = 20.093\ mm.$$

This image is located at $z = -(20 + 48 - 20.093) = -47.907\ mm$ to the eye lens and is seen as the object, which is imaged to:

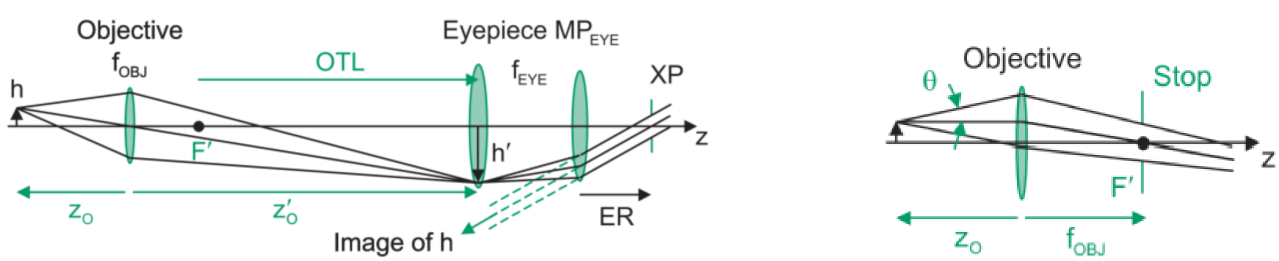
$$z'_{\mathrm{XP}} = \mathrm{ER} = \frac{1}{\frac{1}{-47.907} + \frac{1}{20}} = 34.333\ mm.$$

2.6.2 Microscopes

Definition

A **microscope** is a sophisticated magnifier consisting of an objective plus an eyepiece. The **visual magnification** is the product of the objective magnification and the eyepiece MP:

$$\text{Visual magnification} \qquad \boxed{m_V = m_{\mathrm{OBJ}}\mathrm{MP}_{\mathrm{EYE}} = \frac{z'_0}{z_0}\frac{250\ mm}{f_{\mathrm{EYE}}}}. \tag{2.31}$$



(a) Structure **(b)** Numerical aperture

The **optical tube length** OTL of a microscope is defined as the distance from the rear focal point of the objective to the front focal point of the eyepiece (intermediate image). Standard values are $160\ mm$ and $125\ mm$. The OTL is a Newtonian mage distance:

$$m_{\mathrm{OBJ}} = -\frac{\mathrm{OTL}}{f_{\mathrm{OBJ}}}. \tag{2.32}$$

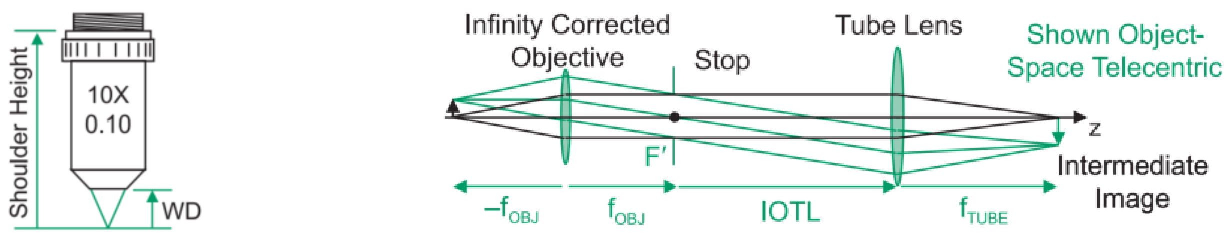
The NA of a microscope objective is defined in object space by the half-angle of the accepted input ray bundle. Along with the objective magnification, the NA is inscribed on the objective barrel:

$$\mathrm{NA} = n \sin\theta. \tag{2.33}$$

Microscope objective are often telecentric in object space. The stop is placed at the rear focal point of the objective so that the magnification does not change with defocus.

Microscope terminology

- The **working distance** WD is the distance from the object to the first element of the objective; can be less than $1\ mm$ for high-power objectives.
- The **mechanical tube length** is the separation between the shoulder of the threaded mount of the objetctive an the end of the tube into which the eyepiece is inserted.
- A set of **parfocal objectives** have different magnifications, but the same **shoulder height** and the same shoulder-to-intermediate image distance.
- **Biological objectives** are aberration corrected assuming a cover glass between the object and the objective.
- Research-graded microscopes are usually designed using **infinity corrected objectives**. The object plane is the front focal plane of the objective, and a collimated beam results for each point.



The magnification of the objective-tube lens combination is

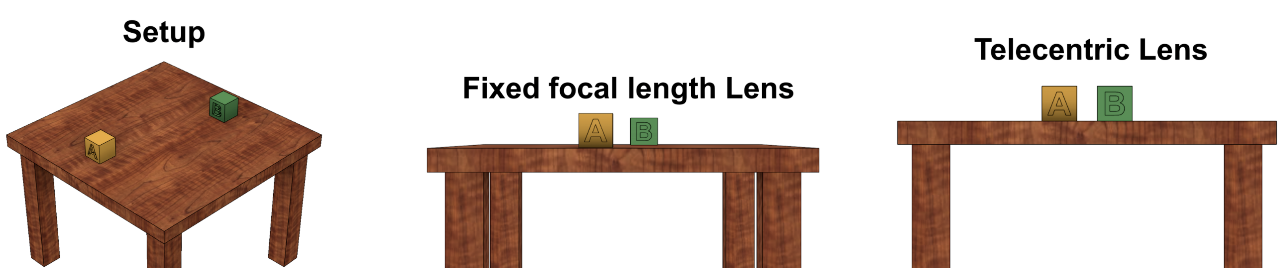
$$m_{\mathrm{OBJ}} = -\frac{f_{\mathrm{TUBE}}}{f_{\mathrm{OBJ}}}. \tag{2.34}$$

If the objective is object-space telecentric and f_{TUBE} equals the infinite optical tube length IOTL, the combination is afocal and double telecentric. This is a useful feature when using reticles in the eyepiece.

2.7 Telecentric systems

2.7.1 Telecentricity

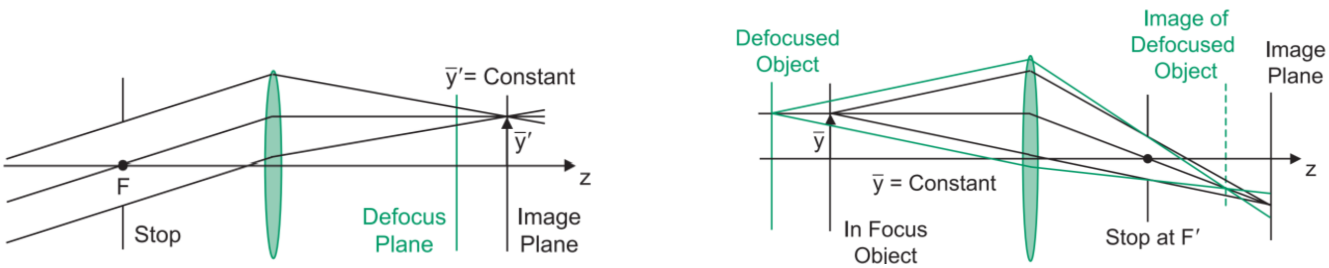
In a **telecentric system**, the EP and/or the XP are located at **infinity**. **Telecentricity** in object or image space requires that the chief ray be parallel to the axis in that space. Consequently, the apparaent system magnification is **constant** even if the object or image plane is displaced. The image will be blurred, but of the correct size or magnification.



2.7.2 Single telecenctric systems

When the stop is located at the front focal plane of a focal system, the XP is at infinity, and the system is **image-space telecentric**. Defocus of the image plane or detector will not change the image height.

On the other hand, placing the stop at the rear focal plane puts the EP at infinity and forms an **object-space telecentric** system. Tthe blur from the defocused object is centered about the chief ray and the image height at the nomial image is constant.



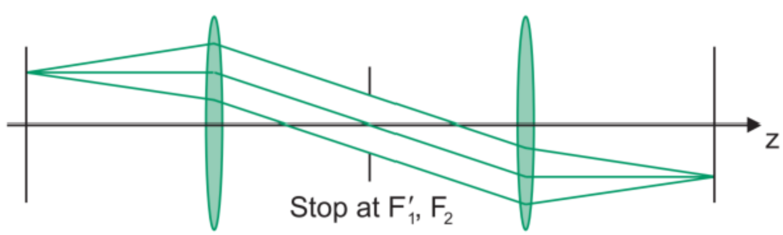
(a) Image-space telecentric **(b)** Object-space telecentric

> The telecentric system must be unvignetted over the entire object FOV, otherwise the blut spot will no longer be symmetric and the centroid will shift.

Object-space telecentric systems are almost used at finite conjugated. The maximum object size is limited to approximately the radius of the objective lens due to vignetting considerations. This system doesnt have a $f/\#$ because the EP is at infinity and is infinite in size.

2.7.3 Double telecentric systems

An afocal system is made **double telecentric** by placing the system stop at the common focal point. The chief ray is parallel to the axis in object and image space, and both the EP and XP are located at infinity. All double telecentric systems must be afocal.



Since the ray bundle is centered on the chief ray, this condition guarantees that height of the blur forming the image is independent of axial object/image shifts.

> **FOV in telecentric systems**
>
> Defining te angular FOV relative to the EP or XP is impossible if the system is telecentric in that particular optical space because that pupil is in infinity. The object height or image height can be used instead.
>
> A second method is to measure the angular size of the object relative to the front nodal point N. Angular sizes of the object and image are equal when viewed from te respective nodal points. But

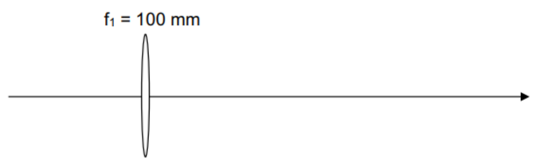
for afocal systems (like telecentric) fails as they do not have these points.

The choice of the method is of little consequence when the object is distant.

Ejemplo 2.14 **Double telecentric system**

A double telecentric system is constructed out of two thin lenses in air. It has a magnification of |1/10|. The focal length of the first lens of the system is 100 mm. An object of size $\pm 100\ mm$ is located 500 mm to the left of the first lens in the system.

a) Provide a layout of the system showing the second lens, spacings and the stop.
b) What is the focal length of the second lens?
c) Where is the image plane (location relative to the second lens element)?
d) What is the size and orientation of the image?



Solution

a) Second lens should be $f_1 + f_2$ to the right of L_1 while the stop in the intermediate image.
b) Magnification is

$$m = -\frac{f_2}{f_1} = -\frac{1}{10} = -\frac{f_2}{100\ mm} \longrightarrow f_2 = 10\ mm.$$

The length between the lenses is therefore $L = f_1 + f_2 = 110\ mm$.

c) The longitudinal magnification is

$$\bar{m} = m^2 = 0.01.$$

Assuming the pair of conjugates from the front and rear focal lengths, the depth of the object is:

$$\Delta z = -500 - (-100) = -400\ mm, \quad \text{and} \quad \Delta z' = \bar{m}\Delta z = -4\ mm.$$

The object is located $500\ mm$ to the left of the first lens and the image is $6\ mm$ to the right of the second lens.

d) The size of the image is

$$h' = mh = (-0.1)(\pm 100) = \mp 10\ mm,$$

which is inverted.

2.7.4 Imaging with an afocal system

An afocal system consists of two focal system sharing a common focal point. If the stop is located at this location, the system becomes telecentric. Because the magnification is constant, the cardinal points are not defined.

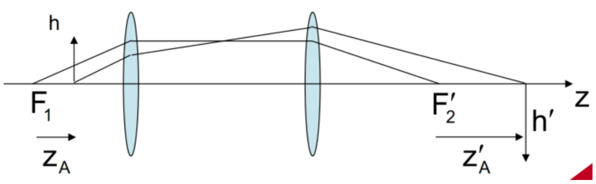
However, we can pick any pair of conjugate planes coupled with the longitudinal magnification:

$$\bar{m} = m^2 = \frac{\Delta z'}{\Delta z} = \frac{z'_A}{z_A}. \tag{2.35}$$

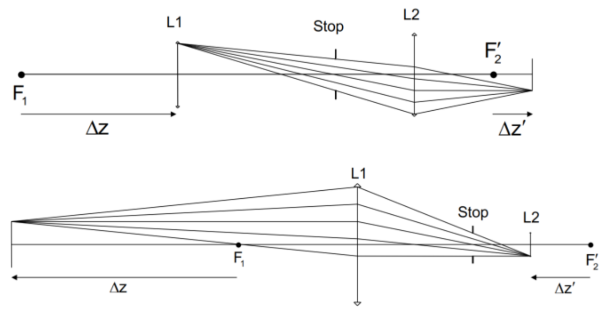
A usual pair is the front focal point of the first system with the rear focal point of the second system. Another is either the object is located at the first lens, or the image is at te second lens; these two situations image as if the other lens as not present.

Once selected, any other pair can be found using the longitudinal magnification. An axial shift in object space results in an image plane shift given by

$$z'_A = \bar{m} z_A. \tag{2.36}$$

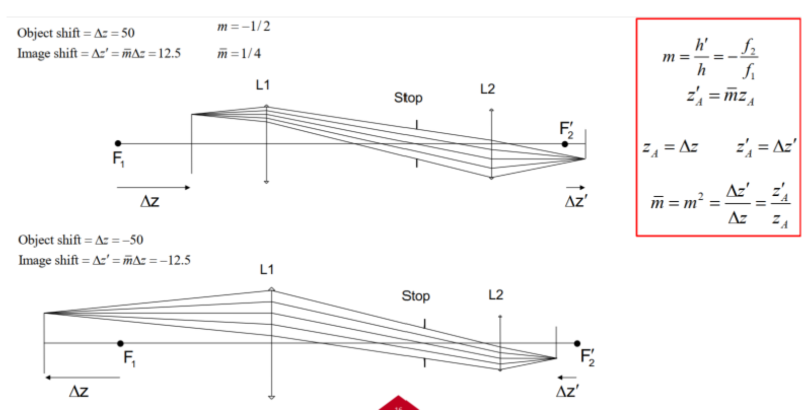


(a) Focal points conjugate pair



(b) First/second lens conjugate pair

Ejemplo 2.15 Imaging with afocal system



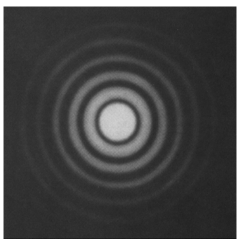
2.8 Stop and image quality

2.8.1 Diffraction-limited

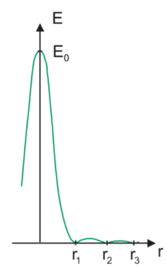
Because light is a wave, it does not focus to a perfect point image. Diffraction from the aperture limit the size of the image spot. For circular aperture, we get an **Airy disk** pattern.

$$\text{Airy disk equation} \qquad E = E_0 \left[\frac{2J_1(\pi r/\lambda f/\#_W)}{\pi r/\lambda f/\#_W} \right]^2, \tag{2.37}$$

where r is the radial coordinate, J_1 is the first-kind Bessel function, and $f/\#_W$ the image space working.



(a) Airy disk pattern



(b) Airy disk profile

	Radius r	Peak E	Energy in Ring (%)
Central maximum	0	$1.0\,E_0$	83.9
First zero r_1	$1.22\lambda f/\#_W$	0.0	
First ring	$1.64\lambda f/\#_W$	$0.017\,E_0$	7.1
Second zero r_2	$2.24\lambda f/\#_W$	0.0	
Second ring	$2.66\lambda f/\#_W$	$0.0041\,E_0$	2.8
Third zero r_3	$3.24\lambda f/\#_W$	0.0	
Third ring	$3.70\lambda f/\#_W$	$0.0016\,E_0$	1.5
Fourth zero r_4	$4.24\lambda f/\#_W$	0.0	

(c) Airy zeros

The diameter of the Airy disk to the first zero is:

$$D = 2.44\lambda \cdot f/\#_W. \tag{2.38}$$

The **Rayleight resolution criterion** states that two point objects can be resolved if the peak of one falls on the first zero on the other:

$$\text{Resolution} = 1.22\lambda \cdot f/\#_W. \tag{2.39}$$

The **angular resolution** is found by dividing by the focal length (or image distance):

$$\text{Angular resolution} = \alpha = 1.22\lambda/D_{\text{EP}}. \tag{2.40}$$

2.8.2 Spherical aberration

Spherical aberration SA causes the power or focal length of the system to vary with pupil radius. For a singlet, the power of the lens increases quadratically with pupil radius; the focal length decrease quadratically.

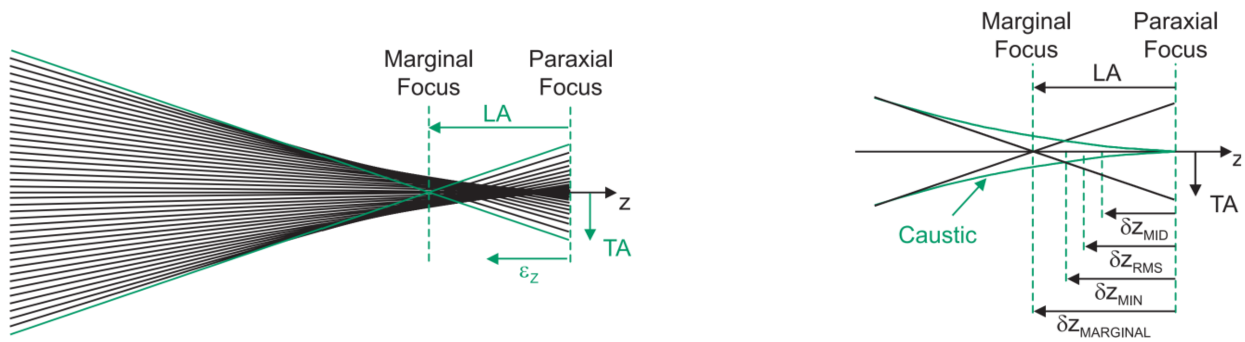
The image plane can be shifted from paraxial focus to obtain better image quality in the presence of SA. There are different focus criteria as seen in the figure.

In first-order geometrical optics, each point on the object plane corresponds to a point on the image plane. However, in real life we are not so lucky.

The spot size scales as the cube of the entrance pupil diameter.

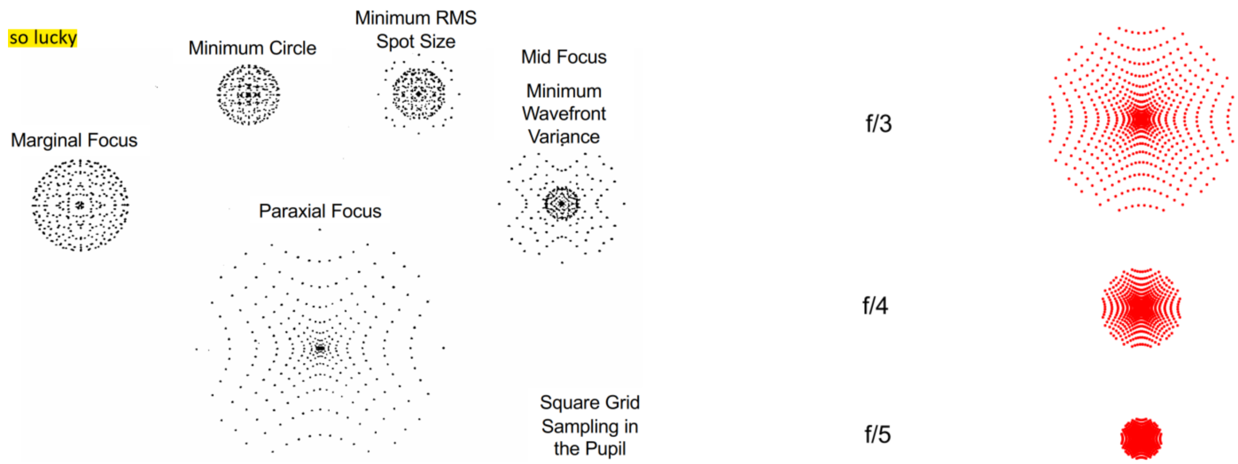
2.8.3 Lens bending and minimum spherical aberration

Bending the lens of orientation does not change the power, but its aberration do change. The minimum SA occurs when the ray is bent the same at both surfaces. This is directly analogous to the angle of



(a) Spherical aberration **(b)** Image plane criteria

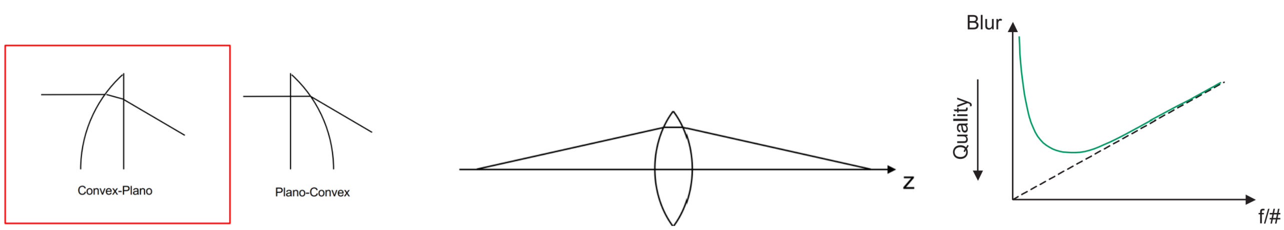
Figure 2.2 Spherical aberration produces different image plane criteria. LA and Ta stand for longitudinal and transverse aberration, respectively.



(a) Spot diagram of focus criteria **(b)** Spot size versus F-number

minimum deviation for prisms. For an object at infinity and $n = 1.5$, the corect lens shape is approximately convex-plano. At finite conjugates, a biconvex lens is used. A trick to further minimize spherical aberration in finite conjugates is to split the biconvex into two plano-convex lenses and then flip each of the lenses.

With large apertures, aberrations and depth of field errors are dominant. With small apertures, diffraction dominates with a linear dependence of blur with $f/\#$.

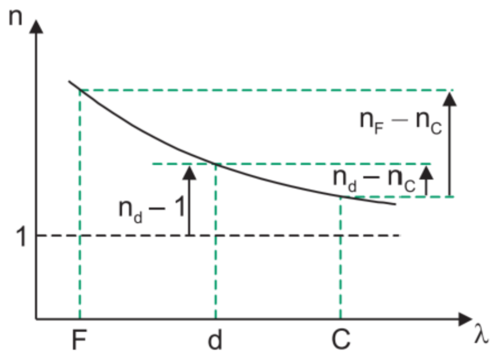


(a) Infinite conjugate **(b)** Finite conjugate **(c)** Blur vs F-number

2.9 Materials

2.9.1 Dispersion

Index of refration is commonly measured and reported at the specific wavelengths of elemental spectral lines. Over the visible spectrum, the **dispersion** of the index for optical glass is about 0.5% (low dispersion) to 1.5% (high dispersion) of the mean value of the index.



F	(H)	486.1 nm
d	(He)	587.6 nm
C	(H)	656.3 nm
I	(Hg)	365.0 nm
h	(Hg)	404.7 nm
F′	(Cd)	480.0 nm
g	(Hg)	435.8 nm
e	(Hg)	546.1 nm
D	(Na)	589.3 nm
C′	(Cd)	643.8 nm
r	(He)	706.5 nm
t	(Hg)	1014.0 nm

Figure 2.1 For visible applications, the F, d, and C lines are usually used.

We define some useful quantities:

$$\text{Refractivity} = n_d - 1, \quad \text{Principal dispersion} = n_F - n_C, \quad \text{Partial dispersion} = n_d - n_C. \tag{2.41}$$

The **Abbe number** is the single number used to characterize the disperion of the index of an optical material:

$$\text{Abbe number} \qquad \boxed{\nu = V = \frac{n_d - 1}{n_F - n_C}} \tag{2.42}$$

Typical values of the Abbe number for optical glass range from 25 to 65. Low ν-values indicate high dispersion.

Relative partial dispersion ratio or P-value gives the fraction of the total index change that occurs between the d and C wavelengths $n_d - n_C$:

$$P = P_{d,C} = \frac{n_d - n_C}{n_F - n_C}. \tag{2.43}$$

Due to flattening of the dispersion, $P_{d,C} < 0.5$. P-values can also be defined for other sets of wavelengths:

$$\text{Relative partial dispersion ratio} \qquad \boxed{P_{X,Y} = \frac{n_X - n_Y}{n_F - n_C}}. \tag{2.44}$$

2.9.2 Optical glass

Glass map

The **glass map** plots index of refraction versus Abbe number. By traddition, the Abbe number increases to the left, so that dispersion increases to the right. The **glass line** is the locus of ordinary optical glasses based on silicon dioxide.

The green line at $\nu \sim 50 - 55$ separates the glasses into crown glass (low dispersion) and flint flass (high dispersion).

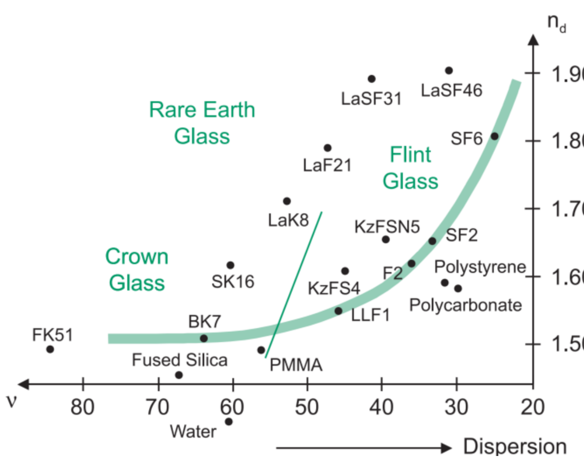
The addition of lead oxide increases the dispersion and the index and moves the glass up the glass line. To increase the index without changing the dispersion, barium oxide is added.

Glass away from the glass line are softer ans more difficult to polish. Low index glasses are less dense and generally have better blue transmission.

Glass code

The six-digit **glass code** specifies the index and the Abbe number:

$$abcdef \implies n_d = 1.abc, \quad \nu = de.f \tag{2.45}$$



(a) Glass map

Material	Code	n_d	n_F	n_C	ν	*P*
N-FK51*	487845	1.48656	1.49056	1.48480	84.5	0.306
N-BK7	517642	1.51680	1.52238	1.51432	64.2	0.308
LLF1	548458	1.54814	1.55655	1.54457	45.8	0.298
N-KzFS4	613445	1.61336	1.62300	1.60922	44.5	0.301
N-F2	620364	1.62005	1.63208	1.61506	36.4	0.294
N-SK16	620603	1.62041	1.62756	1.61727	60.3	0.305
SF2	648339	1.64769	1.66123	1.64210	33.9	0.292
KzFSN5	654396	1.65412	1.66571	1.64920	39.6	0.298
N-LaK8	713538	1.71300	1.72222	1.70897	53.8	0.304
N-LaF21	788475	1.78800	1.79960	1.78301	47.5	0.301
N-SF6	805254	1.80518	1.82783	1.79608	25.4	0.287
N-LaSF31	881410	1.88067	1.89576	1.87429	41.0	0.297
N-LaSF46	901316	1.90138	1.92156	1.89307	31.6	0.292
Fused Silica	458678	1.45847	1.46313	1.45637	67.8	0.311
PMMA	492574	1.492	1.498	1.489	≈55	≈0.33
Polycarbonate	585299	1.585	1.600	1.580	≈30	≈0.25
Polystyrene	590311	1.590	1.604	1.585	≈31	≈0.26
Water	333560	1.333	1.337	1.331	≈60	≈0.33

(b) Glass code

The properties of an individual sample, especially for the plastic material and water, can vary from these catalog values. The measured indices of the actual glass should be used in final design for precision systems. The listed indices are measured relative to air ($n \approx 1.0003$), and the indices should be corrected for use in vacuum. The glass catalog lists other material properties important for a design such as **thermal expansion coefficient**, **teperature coefficient of refractive index**, **internal transmission**, etc.

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