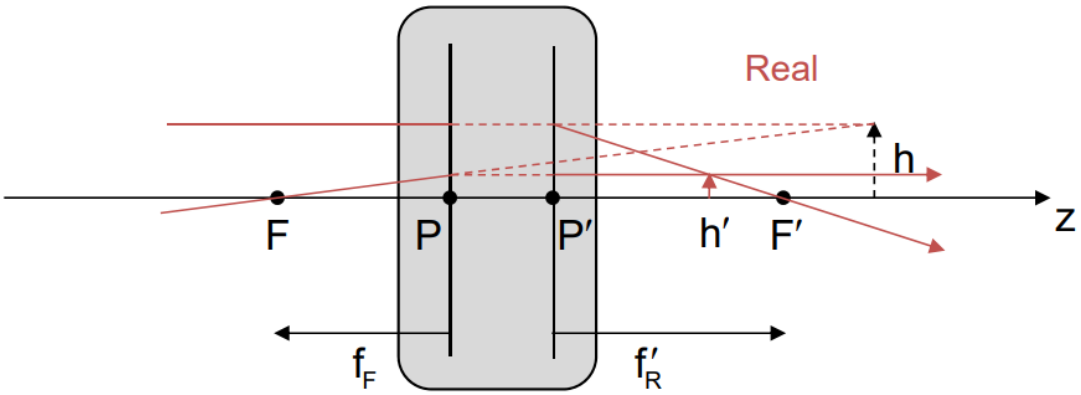
Homework 5 Solutions

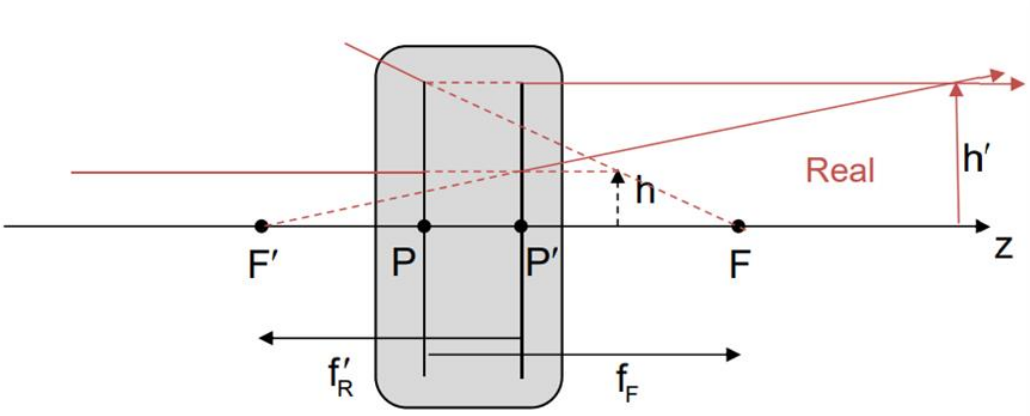
1. Virtual Images: Optical System Ray Diagrams

For the following optical systems, complete the ray diagram. Indicate whether the image is real or virtual and where it is approximately imaged.

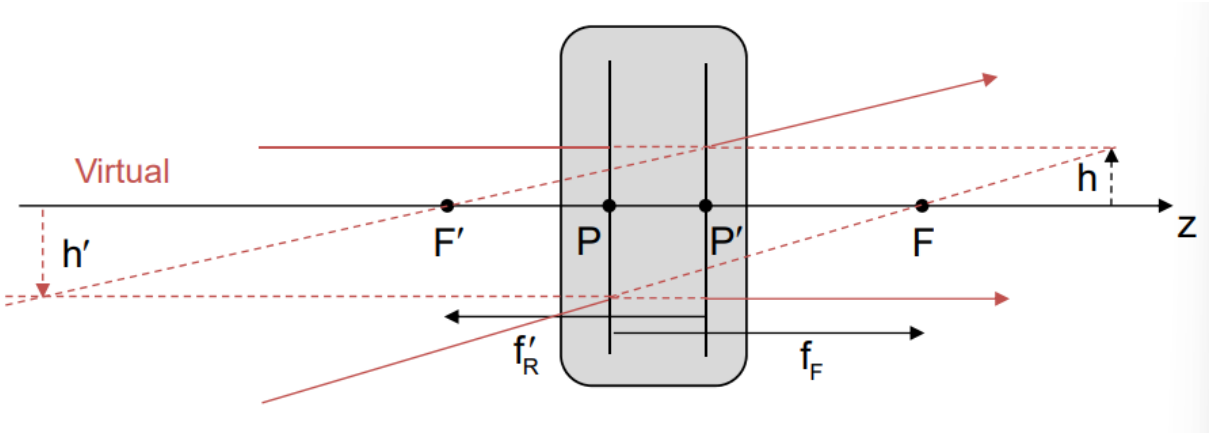
(a) Virtual image beyond (to the right of) the F' plane for a positive focal length system.



(a) Virtual image between the P' and F plane for a negative focal length system.



(a) Virtual image beyond (to the right of) the F plane for a negative focal length system.

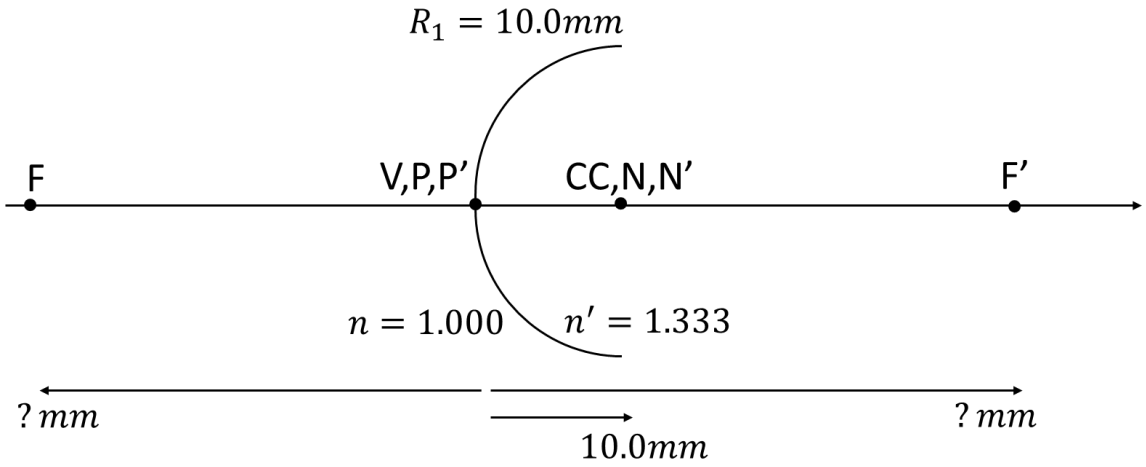


2. Gaussian Imaging: Refracting Surface

Consider a refracting surface with radius of curvature +10.0mm. Rays begin in air ($n = 1.000$), hit the refracting surface, then continue through a medium ($n' = 1.333$). Use Gaussian imaging for this question.

(a) Calculate and illustrate the exact positions of the following planes for this system: F, F', P, P', N, N', V, CC.

Even without calculating anything, we know the Gaussian properties of a single refracting surface. The surface vertex V shares its plane with the front and rear principal planes P and P' while the centre of curvature CC shares its plane with both nodal points N and N'.



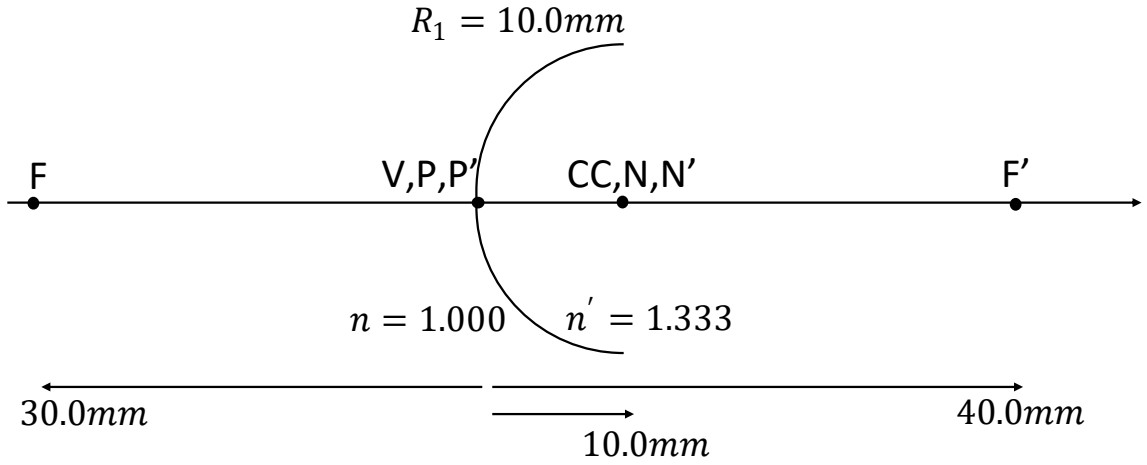
$$\varphi = (n' - n) \cdot \frac{1}{R} = (1.333 - 1.000) \cdot \frac{1}{10.0mm} = 0.0333\ mm^{-1}$$

$$f_E = \frac{1}{\varphi} = +30.0\ mm$$

$$f_F = -\frac{n}{\varphi} = -30.0\ mm$$

$$f_R' = \frac{n'}{\varphi} = +40.0\ mm$$

$$z_{PN} = z_{PN}' = R = +10.0mm$$



(a) A real 10.0mm height object is now placed <u>100mm in front (to the left) of the principal plane P</u>. Where is the image located and what is its height?

$$\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} \ \rightarrow \ z' = \frac{n'}{\frac{n}{z} + \frac{1}{f_E}}$$

$$z' = \frac{1.333}{\frac{1}{-100\ mm} + \frac{1}{+30.0\ mm}} = +57.13\ mm$$

$$m = \frac{z' \cdot n}{z \cdot n'} \ \rightarrow \ h' = m \cdot h = \frac{z' \cdot n}{z \cdot n'} \cdot h$$

$$h' = \frac{+57.13\ mm \cdot 1.000}{-100\ mm \cdot 1.333} \cdot 10\ mm = -4.29\ mm$$

The image is located 57.13mm to the right of P'. Its image height is 4.29mm.

(a) A 10.0mm cube is now placed with the centre of the cube being 105mm from the principal plane and centred on the optical axis. What are the physical (not air equivalent) dimensions of the image of the cube? Do not use an approximation for longitudinal magnification.

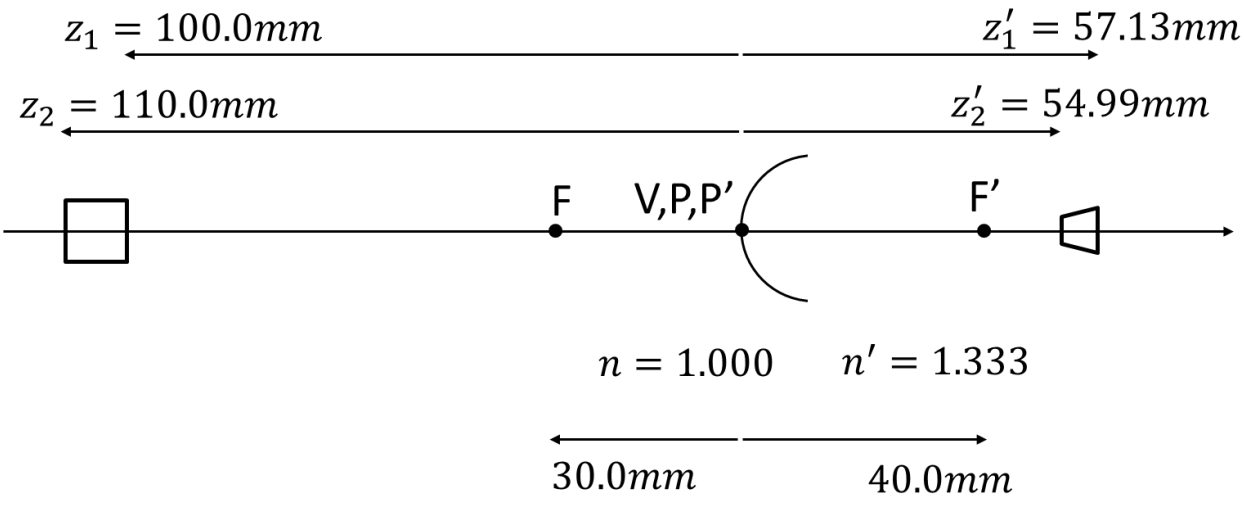
As calculated in the previous part:

$z_1' = 57.13mm, \qquad h' = -4.29mm$

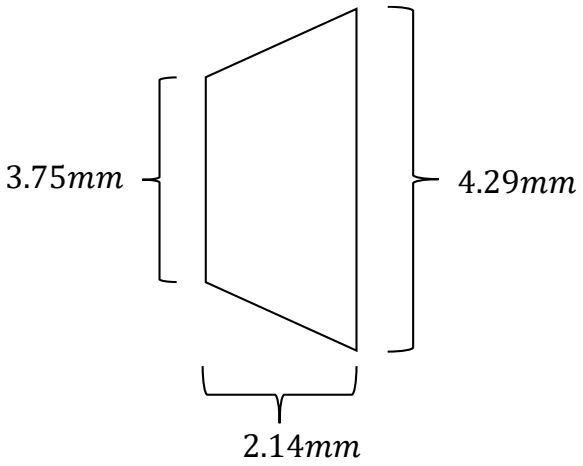
The second side, further from the refractive surface, is 110mm away.

$$z_2' = \frac{1.333}{\frac{1}{-110\ mm} + \frac{1}{+30.0\ mm}} = +54.99\ mm$$

$$h_2' = \frac{54.99mm \cdot 1.000}{-110mm \cdot 1.333} \cdot 10mm = -3.75mm$$



$\Delta z = 57.13 - 54.99 = 2.14mm$


3.75mm
4.29mm
2.14mm