# Assignment 1

## OPTI 502 Optical Design and Instrumentation I


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## 1 Exercise 1

The sign conventions studied in the class was the following:

1. Right-Above directions are positive, in the $zy$ plane.
2. Counter clock-wise angles are positive.
3. Radius of curvature to its radius, in this direction, is positive.

The scheme given is reproduced here for its analysis.

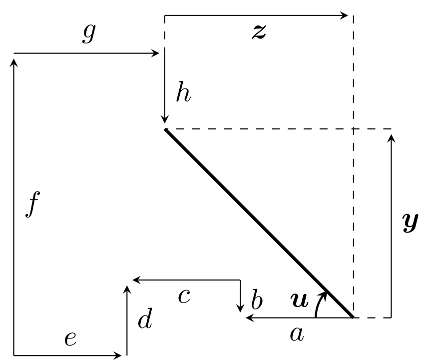


Figure 1: Original scheme.

a) The tangent of the angle $\boldsymbol{u}$ is computed analyzing the directions of arrows $\boldsymbol{z}$ and $\boldsymbol{y}$. Both are positively defined, but the angle direction is clockwise, so that $u < 0$. Consequently, we plug a minus sign into the fraction $y/z$.

$$\tan(-\boldsymbol{u}) = -\tan\boldsymbol{u} = -\frac{\boldsymbol{y}}{\boldsymbol{z}}. \tag{1}$$

If we wish to develop a little the expression we can use the others rays to reexpress $\boldsymbol{z}$ and $\boldsymbol{y}$. On the one hand, we have

$$\boldsymbol{z} + g = e - c - a \longrightarrow \boldsymbol{z} = e - c - a - g. \tag{2}$$

Then we do the same for $\boldsymbol{y}$:

$$f = (d + b) + \boldsymbol{y} - h \longrightarrow \boldsymbol{y} = f + h - b - d.$$

Therefore,

$$\tan \boldsymbol{u} = -\frac{\boldsymbol{y}}{\boldsymbol{z}} = -\frac{f + h - b - d}{e - c - a - g}. \tag{3}$$

b) We have already expressed the directed distance $z$ in equation (2), but we are going to explain the derivation. To obtain $\boldsymbol{z}$, we calculate the total length of the diagram as $g + \boldsymbol{z}$. This result is equated to a positive oriented length that is obtained by summing the rays at the bottom: $e - c - a$. Once the equation is constructed, we can solve for $\boldsymbol{z}$ obtaining in that way the equation (2).

## 2 Exercise 2

For the derivation of the law of reflection, we are going to use the following scheme illustrated in figure 2. An incident ray with path length $L_1$ hits a plane mirror with an angle $\theta_1$. A reflection is obtained with an angle $\theta_2$ which propagates with optical length $L_2$.

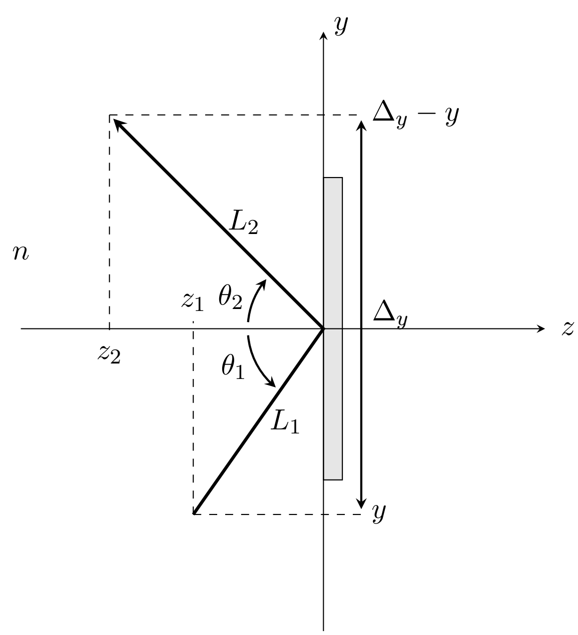


Figure 2: Scheme used for the derivation of the law of reflection.

This figure has the right-hand convention so that the quantities listed have the following properties:

$$\theta_1 > 0, \quad \theta_2 < 0, \quad z_1 > 0, \quad z_2 < 0, \quad y > 0, \quad (\Delta_y - y) > 0.$$

The position coordinates were evaluated in terms of theirs direction. As both $z_1$ and $y_1$ intend to go to the positive quadrant, they are considered positive. The same idea was applied to the $L_2$ ray.

To begin with, the optical path length is the sum of the terms $nL_1$ and $nL_2$:

$$\text{OPL}(y) = nL_1 + nL_2 = n\left[\sqrt{z_1^2 + y^2} + \sqrt{(-z_2)^2 + (\Delta_y - y)^2}\right] = n\left[\sqrt{z_1^2 + y^2} + \sqrt{z_2^2 + (\Delta_y - y)^2}\right].$$

where $n$ is factored out as the ray remains in the same medium, and the squares correspond to the geometrical distance using pytagoras theorem.

To apply the fermat principle, we set $d\text{OPL}/dy = 0$:

$$\frac{d\text{OPL}}{dy} = n\frac{d}{dy}\left[\sqrt{z_1^2 + y^2} + \sqrt{z_2^2 + (\Delta_y - y)^2}\right] = n\left[\frac{y}{\sqrt{z_1^2 + y^2}} + \frac{\Delta_y - y}{\sqrt{z_2^2 + (\Delta y - y)^2}}\right] = 0$$

The last result can be reduced by substituting back the definition of $L_1$ and $L_2$:

$$\frac{d\text{OPL}}{dy} = \frac{y}{L_1} + \frac{\Delta_y - y}{L_2} = \sin\theta_1 + \sin\theta_2 = 0$$

Finally, using the odd property $f(x) = -f(x)$ of the sine function and the last result, we obtain the law of refraction:

$$\begin{aligned}
\sin\theta_1 &= -\sin\theta_2 \\
\sin\theta_1 &= \sin(-\theta_2) / \sin^{-1}(\cdot) \\
\theta_1 &= -\theta_2.
\end{aligned}$$