Notes of Optical design and instrumentation

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Preface

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Introduction to Geometrical Optics principles

Chapter 1

Concepts of optics

1.1 Stops and pupils

1.1.1 Aperture stop

The **aperture stop** is a physical/real surface that limits the cone of light entering and exiting the optical system.

- The **entrance pupil** (EP) is the image of the stop in the object space.
- The **exit pupil** (XP) is the image of the spot in the image space.

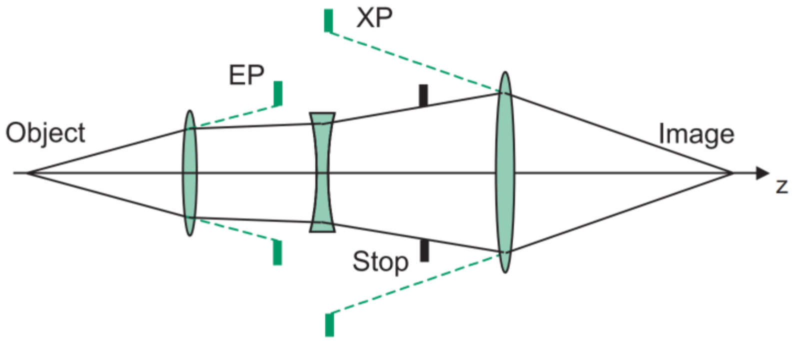


Figure 1.1 The stop limits the cone of light, and its image in object (image) space creates the entrance (exit) pupil.

There is a stop or pupil in each optical space. Intermediate pupils are formed in other spaces. There are two methods to determine which aperture in a system serves as the system stop:

a) Image each potential stop into object space. The pupil with the **smallest** angular size corresponds to the stop. The same can be done in image space.

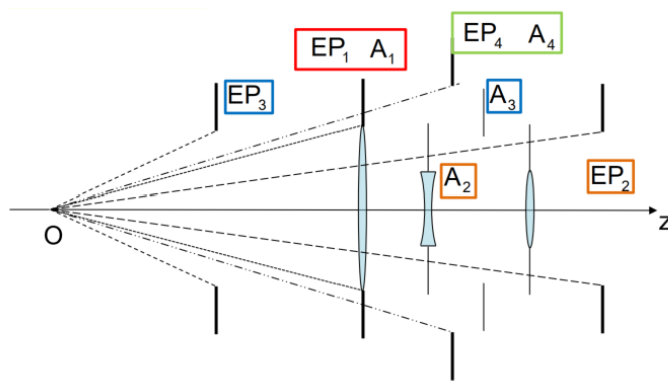


Figure 1.2 The smallest angular size corrsponds to the stop in object space. Same for image space.

b) Trace a ray through the system from the axial object point with arbitrary initial angle. At each potential stop, determine the ratio of the aperture radio a_k to the ray height at that surface $\tilde{y}_k$.

$$\text{Aperture stop} = \min\left\{\left|\frac{a_k}{\tilde{y}_k}\right|\right\}. \tag{1.1}$$

The pupils are the image of the stop and do not change position or size with an off-axis object. Intermediate pupils are formed in each optical space for multi-element systems. If there are N elements, there are $N + 1$ pupils (including the stop).

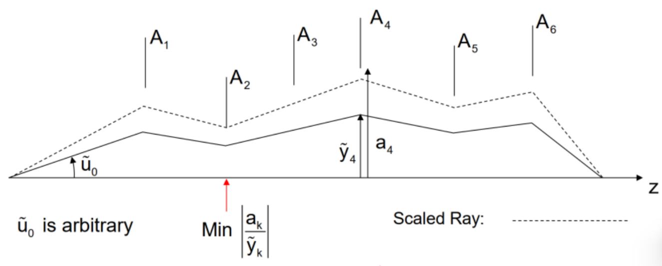


Figure 1.3 The minimum slope value corresponds to the aperture stop.

> When designing a system, is usually critical that the stop surface does not change over a range of possible object positions that the system will be used with.

1.1.2 Marginal and Chief rays

Rays confined to the yz-plane are called **meridional rays**. There are two special meridional rays that define properties of the object, images and pupils:

- The **marginal ray** travels from the base of the object to the edge of EP . It defines image locations and pupil sizes.
- The **chief ray** travels from the edge of the object to the center of the EP. It defines image heights and pupil locations.

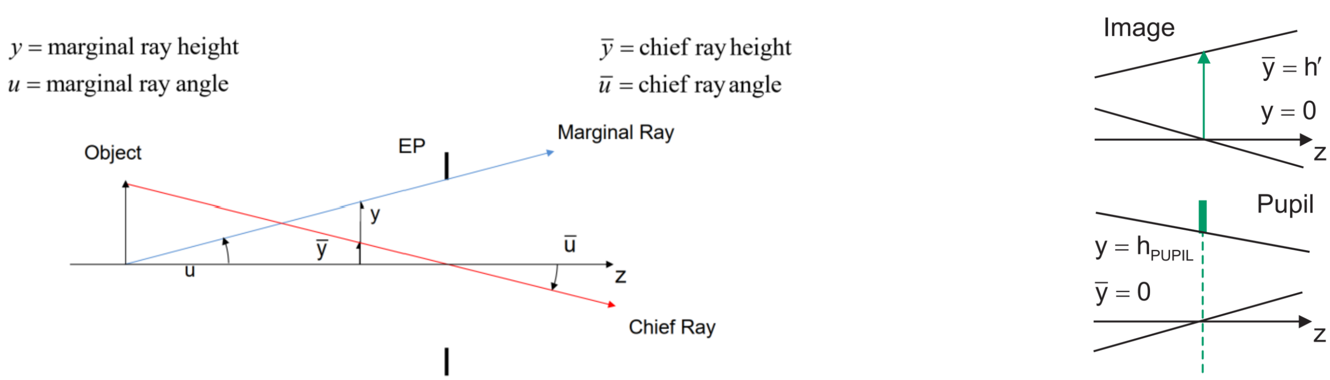


Figure 1.4 The

The heights of the marginal ray and the chief ray can be evaluated at any z in any optical space. When the marginal ray crosses the axis, an image is located, and the size of the image is given by the chief ray height. Whenever the chief ray crosses the axis, a pupil or stop is located, and the pupil radius is given by the marginal ray height. Intermediate images and pupils are often virtual.

1.1.3 Pupil locations

By raytrace

Once you know which surface is the stop, you have the information to determine the location of EP and XP. The **pupil locations** can be found by tracing a paraxial ray starting at the center of the stop and is back/forward propagated. The intersections of this ray with the axis in object and image space determine the locations of EP and XP.

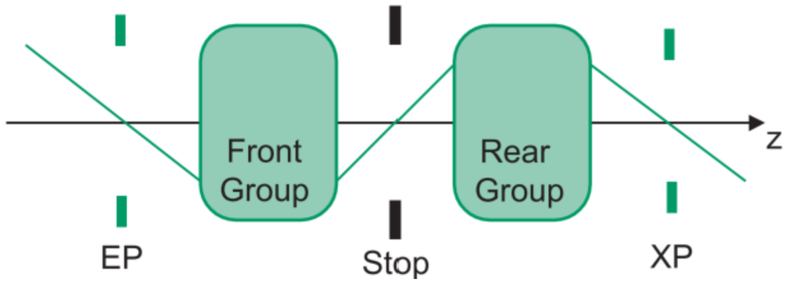
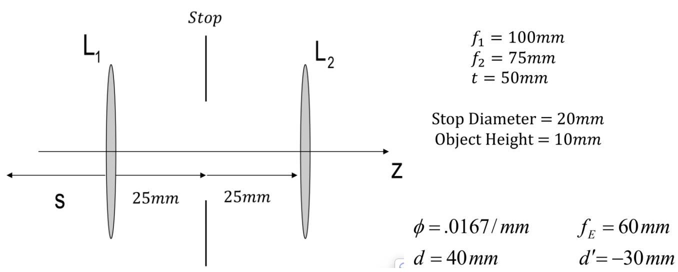


Figure 1.5 The

This ray become the chief ray when it is scaled to the object or image size. The marginal ray gives the pupil sizes.

Ejemplo 1.1 — Pupil location by paraxial raytrace



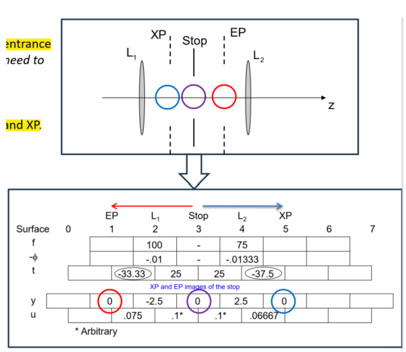
Solution

The stop is a real object for the formation of both EP and XP. There is a ray that has a height of 0 at the EP, stop and XP. We first set $y = 0$ at the stop, and then with arbitrary angle

EP we set $y = 0$ for the EP and solve for the distance.

XP we set $y = 0$ for the XP and solve for the distance.

We used a potential chief to find pupil locations. For the pupil sizes, we find the true marginal ray scaling a potential maginal ray. Remember that the chief ray was for pupil locations, now with the marginal ray we find the pupil sizes. We can also use it to find the image location.



	Object	EP	L_1	Stop	L_2	XP	Image	
Surface	0	1	2	3	4	5	6	7
f			100	-	75			
-ϕ			-.01	-	-.01333			
t	133.33	-33.33	25	25	-37.5	112.5		

Potential Marginal Ray: arbitrary

$\tilde{y}$	0	1.333	1.00	1.00	1.00	1.5	0	
$\tilde{u}$	.01*	.01	0	0	-.01333	-.01333		

$a_{STOP}/\tilde{y}_{STOP}$

1. Raytrace an arbitrary marginal ray. Find the height of the ray at the stop
2. The scale factor can be found by scaling with the stop.
3. Multiply the heights and slope by ten.

Marginal Ray: Scale to Stop Radius $a_{STOP}/\tilde{y}_{STOP} = 10/1 = 10$

y	0	13.33	10.0	10.0	10.0	15.0	0	
u	.1	.1	0	0	-.1333	-.1333		

Finally, the height of the EP is $13.33\ mm$, the stop $10\ mm$, the XP $15\ mm$.

By Gaussian imagery

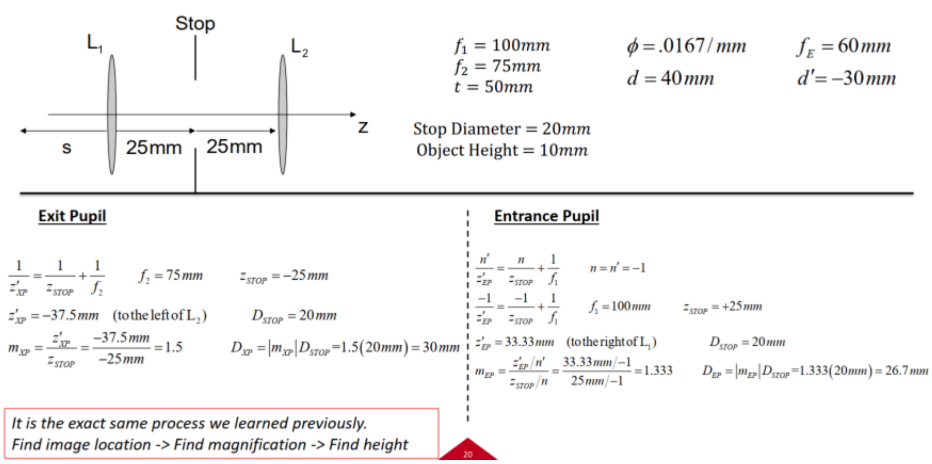
We treat each group independently, considering the stop as our object propagating in the direction of te given group. For EP, the object propagates from right to left, so we flip the sign of the refractive index (as in reflection).

Figure 1.6

$$\text{For XP}\quad \frac{n'}{z'_{XP}} = \frac{n}{Z_{stop}} + \frac{1}{f_{RG}}, \quad m_{XP} = \frac{z'_{XP}}{z_{stop}}, \quad D_{XP} = |m_{XP}| D_{stop} \tag{1.2}$$

$$\text{For EP}\quad \frac{n'}{Z'_{EP}} = \frac{n}{Z_{stop}} + \frac{1}{f_{FG}}, \quad m_{EP} = \frac{z'_{EP}}{z_{stop}}, \quad D_{EP} = |m_{EP}| D_{stop} \quad (n = n' = -1) \tag{1.3}$$

Ejemplo 1.2 **Pupil locations by Gaussian imagery**



EP,STOP,XP are invariant to object location

Changing the object location does not change the position of the EP, stop, and XP.

1.1.4 Lagrange invariant

The linearity of paraxial optics provides a relationship between the heights and angles of any two ray s propagating through the system. The **Lagrande invariant** Ξ is formed with the paraxial marginal and chief rays:

$$\text{Lagrange invariant} \quad \Xi = n\bar{u}y - nu\bar{y} = \bar{\omega}y - \omega\bar{y}\,. \tag{1.4}$$

It is invariant for refraction and transferm and it can be evaluated at any z in any optical space. The Lagrange invariant is particularly simple at images or objects ($y = 0$) and pupils ($\bar{y} = 0$):

$$\text{Image/Object} \qquad y = 0, \quad \Xi = -nu\bar{y} = -\omega\bar{y} \tag{1.5}$$
$$\text{Pupils} \qquad \bar{y} = 0, \quad \Xi = n\bar{u}y = \bar{\omega}y \tag{1.6}$$

If two rays other than the marginal and chief are used, the more general **optical invariant** I is formed.

Given two rays, a third ray can be formed as a linear combination of the two rays. The coefficients are the ratios of the pair-wise invariantas of the values for the three rays at some initial z. The expressions are valid for any z:

$$y_3 = Ay_1 + By_2, \quad u_3 = Au_1 + Bu_2 \tag{1.7}$$
$$A = I_{32}/I_{12}, \quad B = I_{13}/I_{12}, \quad I_{ij} = nu_iy_j - nu_jy_i. \tag{1.8}$$

Changing the Lagrange invariant of a system **scales** the optical system. Doubling the invariant while maintaining the same object and image sizes and pupil diameters haves all of the axial distances (and the focal length).

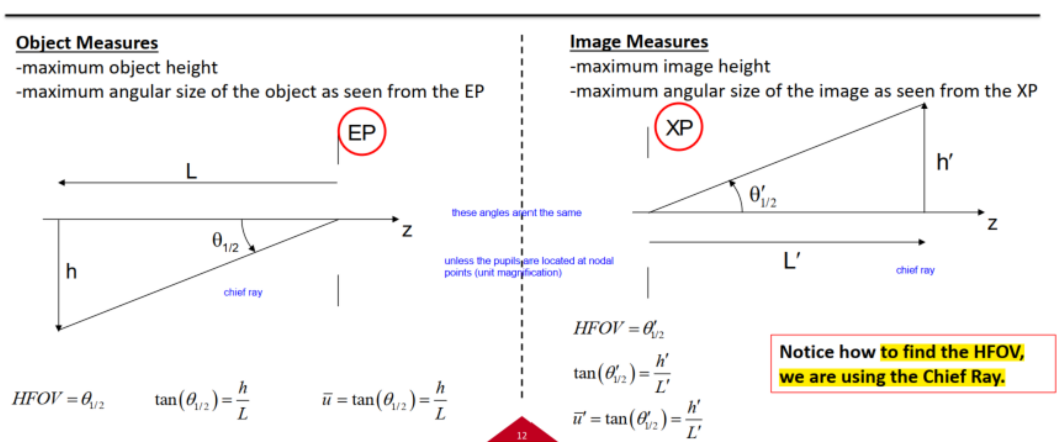
The **throughout**, **etedue** of $A\Omega$ **product** in **radiometry** and **radiative transfer** are relalted to the square of the Lagrange invariant:

$$n^2 A\Omega = \pi^2 \Xi^2. \tag{1.9}$$

1.1.5 Field of view

We revisit again the concept of FOV but know using the EP and XP.

- **Field of view FOV** diameter of the object/image.
- **Half field of view HFOV** radius of the object/image.



1.1.6 Numerical aperture and F-number

In an optical space of index n_k, the **numerical aperture** N_A describes the axial cone of light in terms of the real marginal angle U_k:

$$\text{Numerical aperture} \qquad NA = n_k|sinU_k| \approx n_k|u_k|\,. \tag{1.10}$$

The **F-number** $f/\#$ describes the image-space cone of light for an object **at infinity**:

$$\text{F-number} \qquad \boxed{f/\# = \frac{f_E}{D_{EP}}}. \tag{1.11}$$

While the $f/\#$ is an image-space, infinite-conjugate measure, the approximate relationship between NA and $f/\#$ allows and $f/\#$ to be defined for other optical spaces and conjugates. As a result, an $f/\#$ can be defined for any cone of light. This $f/\#$ is called **working F-number** $f/\#_W$. This previous relationship becomes a definition

$$\text{Working F-number} \qquad f/\#_W = \frac{1}{2NA} \approx \frac{1}{2n|u|} = (1-m)f/\#. \tag{1.12}$$

Fast optical system have small numeric values for the $f/\#$. Most lenses with adjustable stops have $f/\#$ of **f-stops** labeled in increments of $\sqrt{2}$. The usual progression is:

$$f/1.4, \quad f/2, \quad f/2.8, \quad f/4, \quad f/5.6, \quad f/8, \quad f/11, \quad f/16, \quad f/22, \quad \text{etc.}$$

Each stop changes the area of the EP (light colelction ability) by a factor of 2.

The Lagrange invariant relates the magnification between two pupils to the chief ray angles at the pupils.

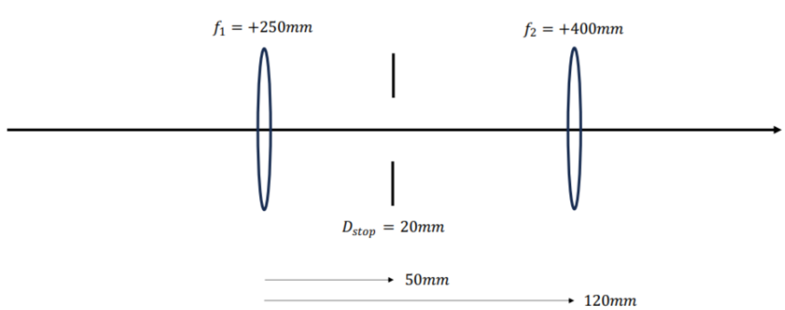
$$\Xi = n\bar{u}y_{pupil} = n'\bar{u}'y'_{pupil}, \quad m_{pupil} = \frac{y'_{pupil}}{y_{pupil}} = \frac{n\bar{u}}{n'\bar{u}'} = \frac{\bar{\omega}}{\bar{\omega}'}. \tag{1.13}$$

Use of working F-number (left)

The most common use of the working F-number is to describe the image-forming cone for a finite conjugate optical system. This is the cone formed by the XP and the axial image point.

Ejemplo 1.3 **Determination of stop and pupil**

Determine the location and size of the pupils for th efollowing the system in air.



Solution

a) We trace the chief ray denoted as CR, and a potential marginal ray MR with unitary height at the stop.

		Object space	EP		L_1		Stop		L_2		XP	Image space
	$C/R/f$				250				400			
	t			$z_{EP} = -62.5$		50		70		$z_{XP} = -84.8$		
	n	1	1	1	1	1	1	1	1	1	1	1
	$-\phi$				-0.004				-0.0025			
	t/n			$\tau_{EP} = -62.5$		50		70		$\tau_{XP} = -84.8$		
	y		0		-5		0		7		0	
CR	nu			0.08		0.1		0.1		0.0825		
	u			0.08		0.1		0.1		0.0825		
	y		$R_{EP} = 1.25$		1		1		1		$R_{XP} = 1.21$	
MR	nu			0.004		0		0		-0.0025		
	u					0		0				

Table 1.1 Raytrace, with CR=Chief ray, MR=Marginal ray.

Due to the diameter of the stop is $R_{stop} = 10\ mm$, we scale the potential marginal ray to give the true marginal ray and therefore obtain the radius of the pupils:

$$\begin{matrix} R_{EP} = (10)(1.25) = 12.5\ mm \\ R_{XP} = (10)(1.21) = 12.1\ mm \end{matrix} \implies \begin{matrix} D_{EP} = 2R_{EP} = 25.0\ mm \\ D_{XP} = 2R_{XP} = 24.2\ mm \end{matrix}$$

b) For Gaussian imagery, we see the stop as the object for the front group and rear group. For the EP, we have a backward propagation that is maganed with the flip of the sign in the refrractive indices.

$$\frac{-1}{z_{EP}} = \frac{-1}{Z_{\text{stop}}} + \frac{1}{250} \longrightarrow z_{EP} = 62.5\ mm.$$

This entrance pupil is to the right of the lens L_1. The magnification is:

$$m_{EP} = \frac{z_{EP}}{z_{\text{stop}}} = \frac{R_{EP}}{R_{\text{stop}}} = -1.25.$$

The diameter of the entrance pupil is therefore:

$$D_{EP} = 2R_{EP} = 2[|m_{EP}|R_{\text{stop}}] = 25\ mm.$$

For the rear group, we have analogously:

$$\frac{1}{z_{XP}} = \frac{1}{Z_{\text{stop}}} + \frac{1}{400} \longrightarrow z_{XP} = -84.848\ mm.$$

The exit pupil is then to the left of the lens L_2. The magnification in this case is

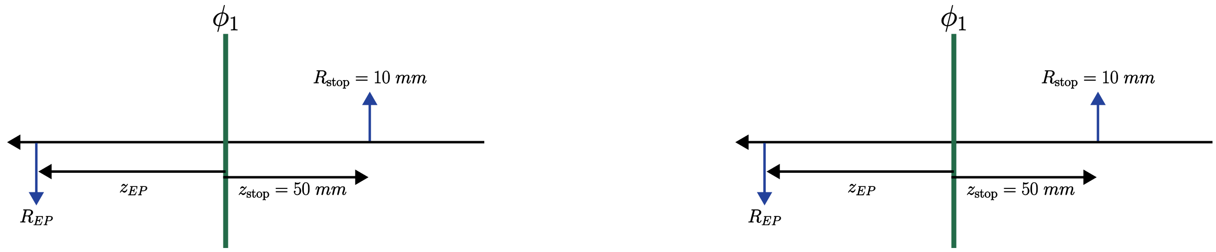
$$m_{XP} = \frac{z_{XP}}{z_{\text{stop}}} = \frac{R_{XP}}{R_{\text{stop}}} = 1.21.$$

The diameter of the exit pupil is:

$$D_{XP} = 2R_{XP} = 2[|m_{XP}|R_{\text{stop}}] = 24.2\ mm.$$

The illustration of each case is illustrated in the figure 1.7.

Using either method, the result is the same and is shown in figure 1.8



(a) EP for front group **(b)** XP for rear group

Figure 1.7 With Gaussian imagery, the computation of the pupils is based on the stop as the object of two optical systems.

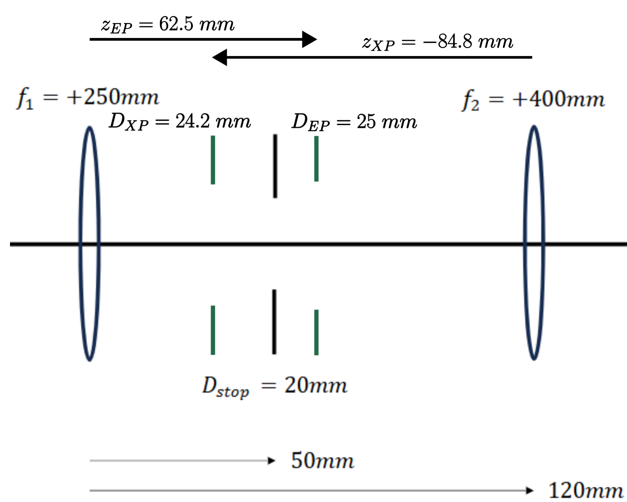
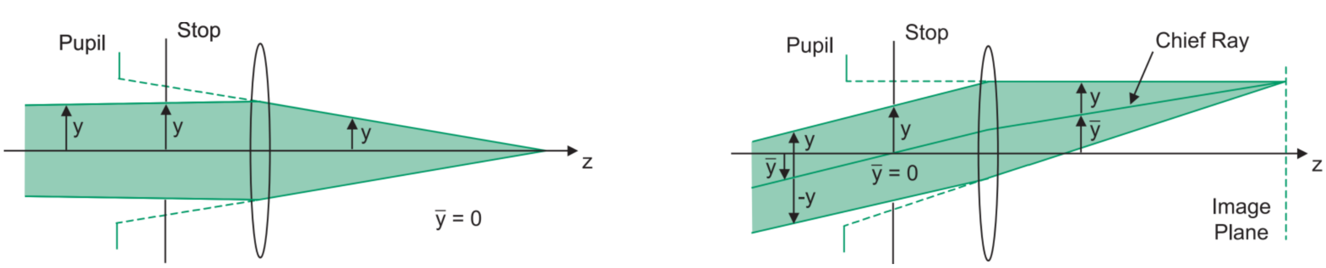


Figure 1.8 Illustration of the stop and pupil in the optical system.

1.2 Vignetting

1.2.1 Ray bundles

The **ray bundle** for an **on-axis** object is a rotationally symmetric spindle made up of section of right circular cones. Each cone section if bounded by the pupil and the object/image in that optical space. At



any z, the cross section of the bundle is circular, and the radius of the bundle is the marginal ray value.

For an **off-axis** object point, the ray bundle skews, and is comprised of section of skew circular cones wwhich are still defined by the same elements. The cross section of the ray bundle at any z remains circular with a radius equal to the radius of the axial bundle. The off-axis bundle is centered about the chief ray height.

The maximum radial extent of the ray bundle at any z is:

$$\text{Maximum radial extent} \qquad |y_{max}| = |y| + |\bar{y}|. \tag{1.14}$$

1.2.2 Vignetting

The **vignetting** occurs when other apertures in the system (others than the stop) block a proportion of an off-axis ray bundle. For no vignetting, each aperture radius a must equal or exceed the maximum height of the ray bundle at the aperture.

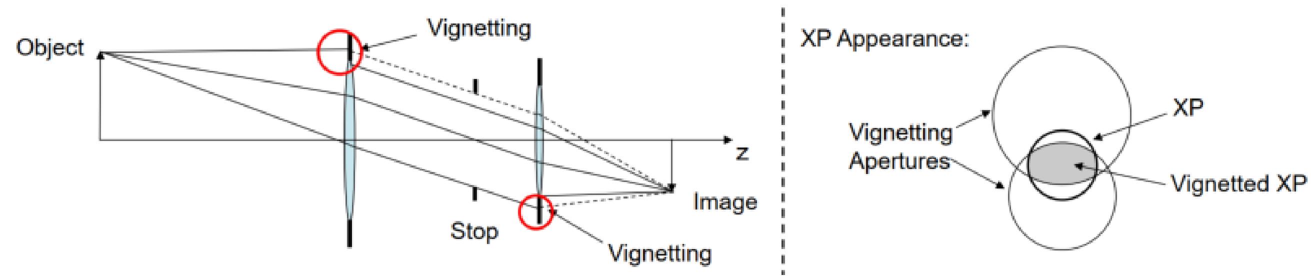
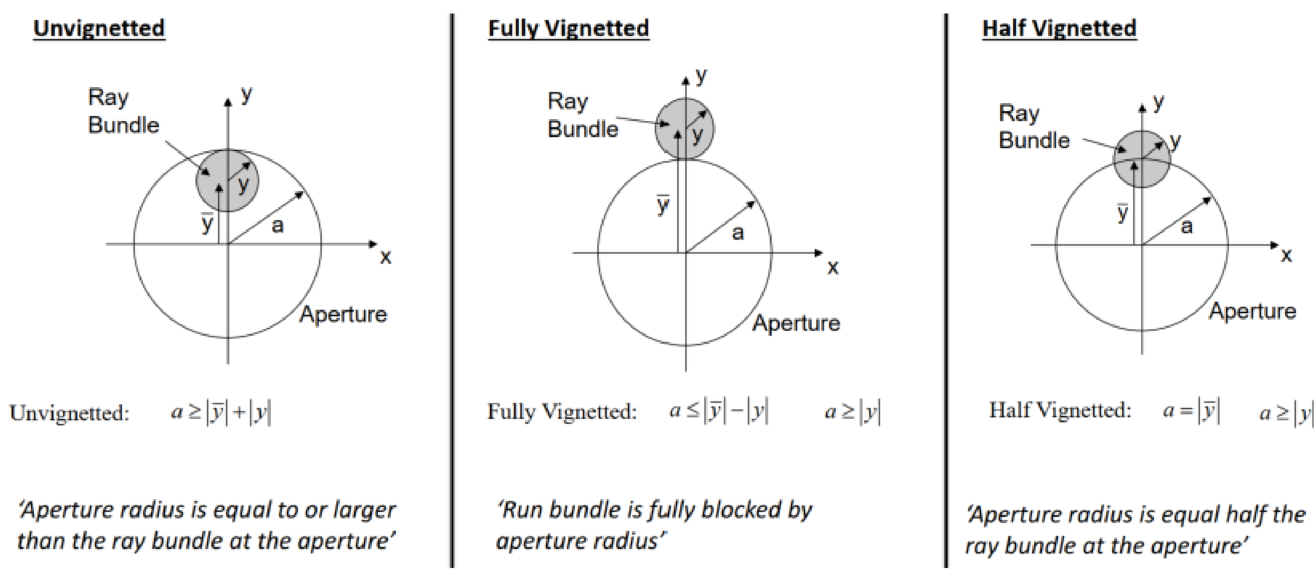


Figure 1.1 The ray bundle is clipped and the beam is no longer circular.

The maximum FOV supported by the system occurs when an aperture completely blocks the ray bundle from the object point.

We can have three conditions of vignetting, depending on the proportion of clip of the light beam.



The vignetting conditions are used in two different manners:

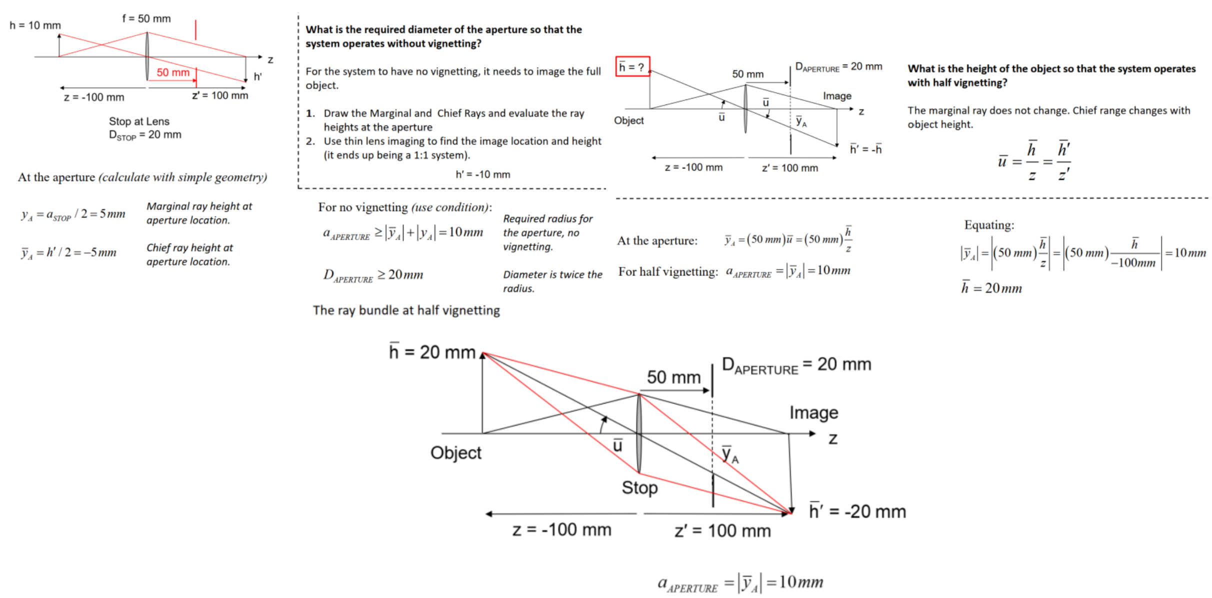
- For a given set of apertures, the FOV that the system will suppport with a prescribed amount of vignetting can be determined. A different chief ray defined each FOV.
- For a given FOC and vignetting condition, the required aperture diameters can be determined.

A system with vignetting will have an image that has full irradiance or brightness out to a radius corresponding to the unvignetted FOV limit. The irradiance will then begin to fall off, going to about halr at the half-vignetted FOV, and decreasing to zero at the fully vignetting FOV. This fully vignetted FOV is the absolute maximum possible.

The diameter of the aperture stop is very important design parameter for an optical system as it controls five separate performance aspects of the system:

- The system FOV determined by vignetting.
- The radiometric or photometric speed of the system or its light collection ability.
- The depth of focus and depth of field of the system.
- The amount of aberrations degrading image quality.
- The diffraction-based performance of the system.

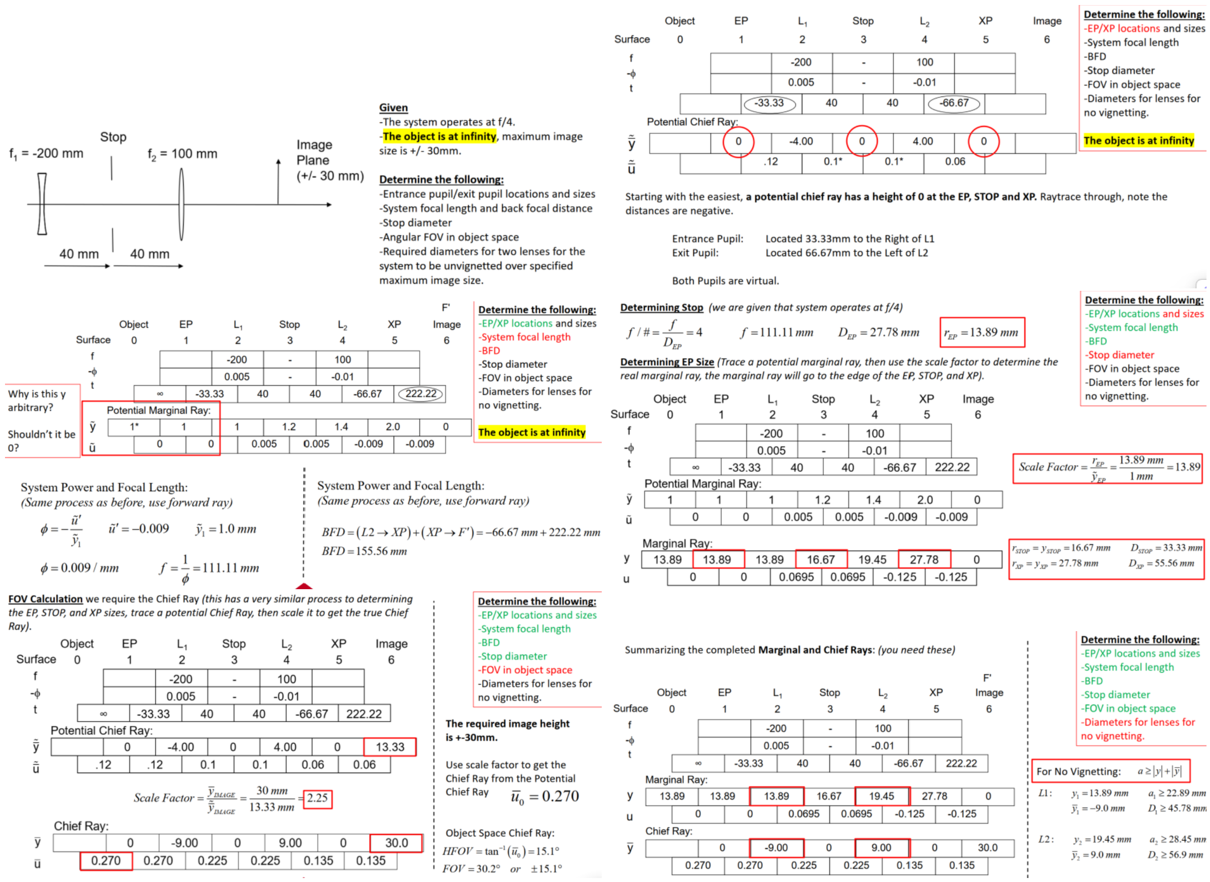
Ejemplo 1.4



Ejemplo 1.5 — Vignetting with paraxial raytrace

In general,

> **Key points in solving problems**
>
> - Trace the potential chief ray (CR) to know the locations of the pupils (and image size).
> - Trace the potential marginal ray (MR) to determine image location and pupil sizes.
> - If the MR comes parallel, then it can be used to obtain the first-order properties.
> - The F-number gives us the real size of EP so we can scale the MR.
> - The image size allows us to get the real CR.



- The HFOV is determined with the incident angle $\bar{u}$ at EP in the real CR: HFOV $= \tan^{-1}\bar{u}$.
- The vignetting is found by looking at $y, \bar{y}$ in the real MR and CR and applying the criteria.
- We can arbitrarily define a dummy surface to our convenience.
- EP and XP are dummy surfaces (w location defined) of zero-power.

1.2.3 Dummy surfaces

In a raytrace, a zero-power surface can be inserted at any location to examine the ray properties.

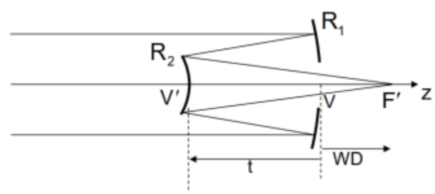
An example of its application is the following Cassegrain objective, where we require to find the size of the hole. For that, we place a dummy surface **at the hole**.

1.3 Radiative transfer

1.3.1 Radiometry

Radiometry characterizes the propagation of radiant energy through an optical system. The basic unit is the watt W. Radiometric terminology and units are: There are some assumptions:

- The source is **incoherent**, meaning that scenes are collection of independently point sources, no interference.



$R_1 = -200\,mm$
$t = -80\,mm$
$R_2 = -50\,mm$
$n_1 = n = 1$
$n_2 = -1$
$n_3 = n' = 1$

Surface	0	Primary	Secondary	Hole	F'
R		-200	-50		
t	∞	-80	80	WD	
n	1.0	-1.0	1.0	1.0	
-φ		-.01	.04	0	
t/n	∞	80	80	WD	

The hole is located to the right of the secondary mirror.

Marginal and chief rays are traced, and the vignetting condition is applied to determine the hole size.

By using the dummy surface, the working distance (WD) is also directly computed on the raytrace sheet.

Quantity	Symbol	Units	Units description
Energy	Q	J	
Flux	Φ	W	Power
Intensity	I	W/sr	Power per unit solid angle
Irradiance	E	W/m^2	Incident power per unit area
Exitance	M	W/m^2	Exiting power per unit area
Radiance	L	W/m^2sr	Power per unit projectedarea per unit solid angle

- Objects and images on-axis and perpendicular to the optical axis, so that the projected area equals the area.

The solid angle Ω equals the surface area of the unit sphere in a given vecinity. The units are 4π steradians (sr).

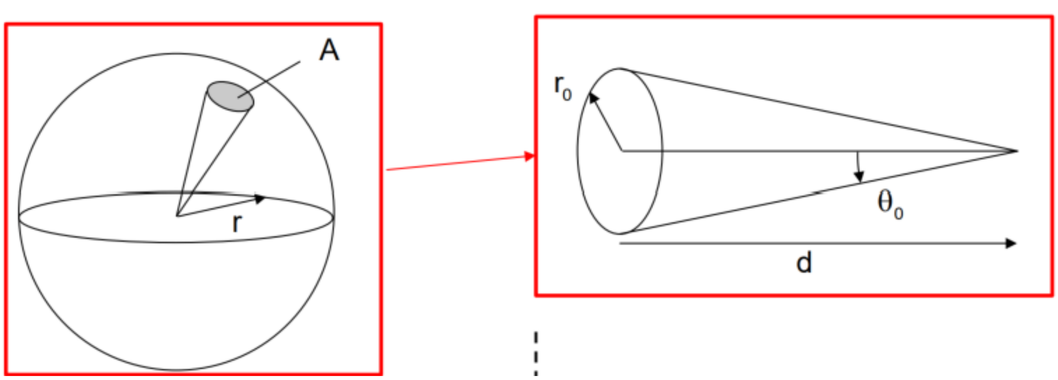


Figure 1.1 The solid angle of a sphere can be approximated to the solid angle of a cone.

$$d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin\theta d\theta d\phi}{r^2} = \sin\theta d\theta d\phi \xrightarrow{\int} \boxed{\Omega_{\text{sphere}} = 4\pi \sin^2(\theta_0/2)}\,.$$

In optics, its common to approximate the solid angle of a sphere to the section of a cone:

$$\Omega \approx \frac{\pi r_0^2}{d^2} \approx \pi \sin^2\theta_0 \approx \pi\theta_0^2 \quad \text{(small angle approximation)}.$$

1.3.2 Radiative transfer

Radiative transfer uses first-order geometrical principles to determine the amount of light from an object that reaches an image or detector.

Exitance and irradiance are related by the **reflectance** of the surface ρ:

$$M = \rho E. \tag{1.15}$$

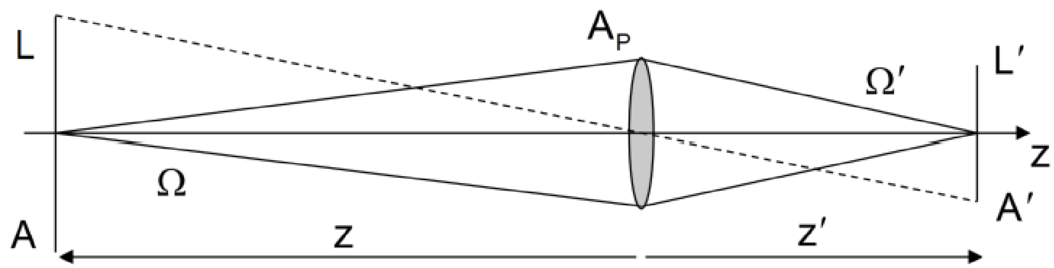
For average scenes, $\rho = 18\%$. Exposures are often set using this value, that is, we expose the print to that average so that the prin reflectance ends up with 18%.

The irradiance of a **Lambertian source** (perfectly diffuse surface) is constant. The intensity falls off with the apparaent source size or the **orjected area** (**Lambert's law**). The exitance of a Lambertian source is related to its radiance by π.

Lambertian source

$$\begin{array}{ll} L(\theta,\phi) = \text{constant} & I = I_0 \cos\theta \\ M = \pi L & \pi L = \rho E \end{array}$$

We now analyze the optical power from an object that reaches the image in an optical system.



In air, the radiance and the **$A\Omega$ product** or **throughput** are conserved, and the flux collected by the lens Φ is transferred to the image area A':

$$\Phi = L(\text{object area})(\text{solid angle projection in lens}) = LA_p\Omega = LA\frac{\pi D_p^2}{4z^2} \xrightarrow{A'=m^2A} \Phi = \frac{\pi L A' D_p^2}{4m^2z^2}.$$

Using gaussian equations and f-number equations, the image plane irradiance E' is

Camera equation

$$E' = \frac{\Phi}{A'} = \frac{\pi L}{4(f/\#_W)^2} \to \pi L(\text{NA})^2, \quad L = \frac{\rho E_0}{\pi}. \tag{1.16}$$

Spectral dependence

Spectral dependence can also be added.

$$\begin{array}{l} E_0(\lambda) = \text{Object irradiance} \\ \rho(\lambda) = \text{Object reflectance} \\ L(\lambda) = \text{Object radiance} \end{array} \longrightarrow \begin{array}{l} L(\lambda) = \frac{M(\lambda)}{\pi} = \frac{\rho(\lambda)E_0(\lambda)}{\pi} \\ E'(\lambda) = \frac{\rho(\lambda)E_0(\lambda)}{4(1-m)^2(f/\#)^2} \end{array}$$

This can be integrated over all wavelengths for total irradiance

$$E' = \frac{1}{4(1-m)^2(f/\#)^2}\int_{\lambda_1}^{\lambda_2} \rho(\lambda)E_0(\lambda)\, d\lambda.$$

Exposure

Most detectors respond to energy per unit area rathen than power per unit area. Multiplying the image irradiance by the exposure time gives the expoure (J/m^2):

$$\text{Exposure} \qquad H = E'\Delta t\,. \tag{1.17}$$

The mean solar constant is $1368\ W/m^2$ outside the atmosphere of the earth, and the solar irradiance on the surface is about $1000\ W/m^2$.

1.3.3 Photometry

Photometry is the subset of radiometry that deals with visual measurements, and luminous power is measured in **lumens** lm. The lumen is a watt wighted to the visual **photopic response**. This peak response occurs at $555\ nm$, where the conversion is $683\ lm/W$. The dark adapted or **scotopic response** peaks at $507\ nm$ with $1700\ lm/W$.

(a) Photometric terminology

Luminous power	Φ_V	lm
Luminous intensity	I_V	lm/sr
Illuminance	E_V	lm/m^2
Luminous exitance	M_V	lm/m^2
Luminance	L_V	$lm/m^2 sr$
Exposure	H_V	$lm\ s/m^2$
candela (cd)	I_V	lm/sr
lux (lx)	E_V	lm/m^2
foot-candle (fc)		lm/ft^2
	$1fc = 10.76\ lx$	
foot-lambert (fL)	L_V	$\frac{1}{\pi}cd/ft^2$
nit (nt)		$= cd/m^2$
	$1fL = 3.426\ nt$	
lux-second (lx s)	H_V	$lm\ s/m^2$

(b) Luminous photopic efficacy

$\lambda\ (nm)$	lm/W
400	0.3
420	2.7
440	15.7
460	41.0
480	95.0
500	221
520	485
540	652
560	680
580	594
600	425
620	260
640	120
660	41.7
680	11.6
700	2.8
720	0.7

(c) Typical illuminance levels

Sunny day	$10^5\ lx$	Moonlit night	$10^{-1}\ lx$	Overcast day	$10^3\ lx$	Starry night	$10^{-3}\ lx$
Inerior	$10^2\ lx$	Desk lighting	$10^3\ lx$				

The candela (cd) is the fundamental SI unit for luminous intensity.

$A\Omega$ product

Recall the flux hrough a system is

$$\Phi = LA\Omega.$$

The $A\Omega$ product appears to be the geometric portion, while L would be related to the source characteristics. In an object or an image plane,

$$\begin{aligned} \text{object/pupil plane} \qquad & A = \pi\bar{y}^2, \quad \theta = u, \quad A\Omega = \pi^2\bar{y}^2u^2 = \pi^2\chi^2/n^2, \quad \chi = n\bar{y}u \\ \text{pupil plane} \qquad & A = \pi y^2, \quad \theta = \bar{u}, \quad A\Omega = \pi^2 y^2\bar{u}^2 = \pi^2\chi^2/n^2, \quad \chi = ny\bar{u} \end{aligned}$$

In the general situation when the index is not unity, the **basic throughout** $n^2 A\Omega$ and the **basic radiance** L/n^2 are invariant. Since throughput is based on areas, the basic throughput is proportional to the Lagrange invariant squared:

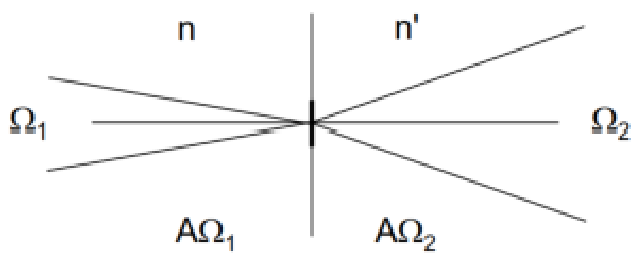
$$n^2 A\Omega = \pi^2 \chi^2. \tag{1.18}$$

For a lossless optical system, the flux through the system is constant. For $n = 1$, we have

$$\Phi = L_1 A_1 \Omega_1 = L_2 A_2 \Omega_1 = \cdots = \text{constant}.$$

Since $A\Omega$ is also constant, the radiance L must also be constant. This allow us to relate different portions of the optical system as the flux is conserved.

However, if the index of refraction is not unity and changes ,the radiance is no longer conserved. It will change at each interface as the solid angle will change. We then have that



$$A\Omega_1 \neq A\Omega_2 \wedge \Omega = L_1 A\Omega_1 = L_2 A\Omega_2 \Longrightarrow L_1 \neq L_2.$$

The flux is still constant. In fact, $L/n^2 =$ constant as well as $n^2 A\Omega =$ constant.

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