Notes of Optical design and instrumentation

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Preface

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Introduction to Geometrical Optics principles

Chapter 1

Introduction to Optics

1.1 Introduction

1.1.1 Light propagation

Geometrical optics is the study of light in the limit of short wavelenegths. We treat light as propagating rays. Geometrical optics usually ignores interferences, diffraction, polarization and quantum effects.

It often includes:

- Reflection, refraction
- Optical design
- Imaging properties
- Aberrations
- Radiometry

Light is a self-propagating EM wave where electric and magnetic fields are perpendicular or transverse to direction of propagation. In a vacuum, light propagates at the speed of light c, which is

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792458 \times 10^8 \quad (m/s). \tag{1.1}$$

The **wavelength** λ is the distance between two peaks or two valleys on the wave.

A **wavefront** is a surface of constant propagtion time from the source. It begins from a point source in spherical form, and as it propagates away, a given solid arc tend to behaves a a planar wavefront.

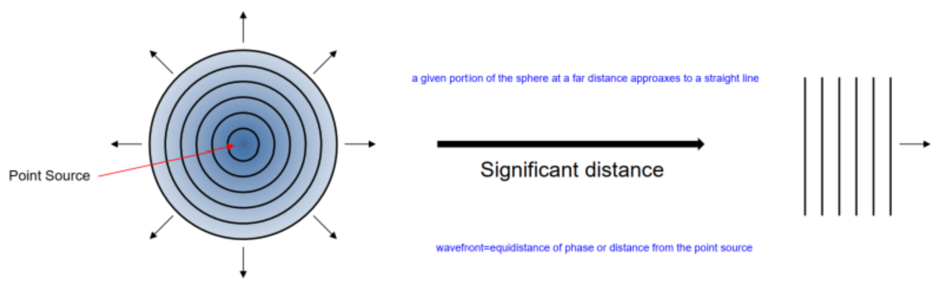


Figure 1.1 We can treat the wavefront as planar when assuming a distant object.

The time for one wavelength to pass is knwon as the **period** T:

$$T = \frac{\lambda}{V} \quad (s), \tag{1.2}$$

where V is the velocity of propagation. The number of wavelengths to pass in one second is the **frequency** ν:

$$\nu = \frac{1}{T} \quad (s^{-1})(Hz). \tag{1.3}$$

1.1.2 Sign convention

We define the sign convention for which the light propagates. It allows us to keep track of physical quantities and multiple reflections when analyzing complex optical systems.

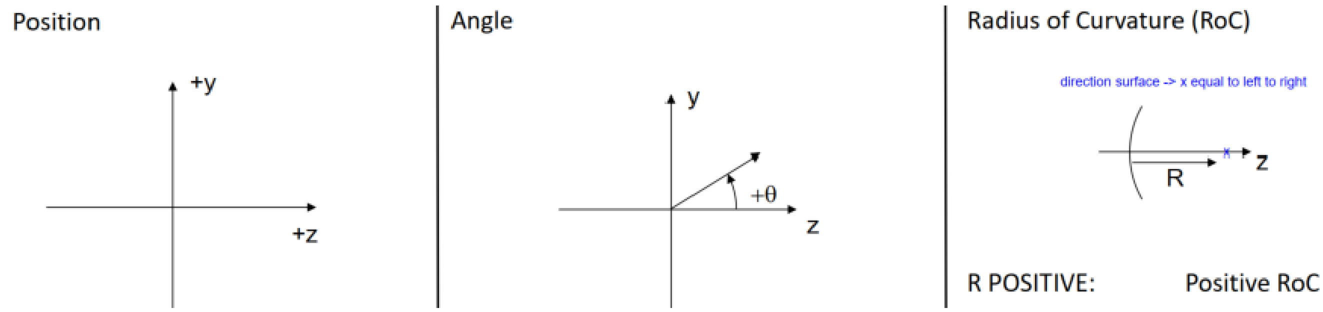


Figure 1.2

1.1.3 Electromagnetic spectrum

The light can be of various wavelengths (frequencies) which translates to the color of the light. The range of the wavelengths is called the **electromagnetic spectrum**. The **index of refraction** tells how much the

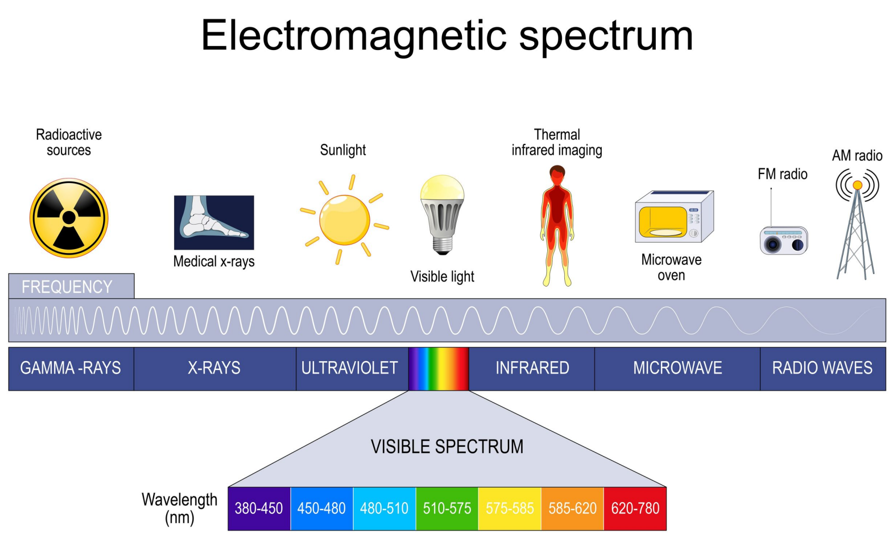


Figure 1.3

light is slown down in a medium with respecto to vacuum.

$$\text{Index of refraction} \qquad n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{V} \geq 1\,. \tag{1.4}$$

From one medium to another, the frequency remains unchanged but only the wavelength is modified. The index of refraction is a function of the wavelength and of the temperature.

Vaccum equal to air

In geometrical optics, vacuum and air are used interchangable as the index of refraction of air is $n \approx 1$.

1.1.4 Optical path length

The **optical path length** (OPL) is the equivalent distance in vacuum that light would cover in the same time as it takes to cross the actual medium.

$$\text{Optical path length} \qquad \text{OPL} = \int_a^b \boldsymbol{n}(s) \cdot d\boldsymbol{s}\,. \tag{1.5}$$

When the medium is homogeneous, the index n reduces to a constant value. Consequently, the ray travels in **straight lines**.

Fermat's principle states that the path taken by the light from one point to another is the path for which the OPL is stationary:

$$\text{Fermat's principle} \qquad \frac{d\text{OPL}}{d\text{path}} = 0\,. \tag{1.6}$$

1.1.5 Snell's laws of reflection and refraction

Snell's law can be obtained from Fermat's postulate. They governs the dynamics of the ray when passing thourgh an interface of different index of refractions:

$$\text{Snell's laws} \qquad \begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ \theta_1 &= -\theta_2 \end{aligned} \tag{1.7}$$

The angles are measured relative to the surface normal.

(a) Refraction **(b)** Reflection

Figure 1.4 In refraction and reflection, the angles are taken respective to the surface normal.

> The reflection is equal to refraction with a negative index: $n = -n$.

1.1.6 Total internal reflection (TIR)

Total internal reflection occurs when the light propagating from a medium n_1 to another n_2, with $n1 > n_2$, exceed a critical incident angle

$$\text{Total internal reflection} \qquad \theta_i > \theta_c = \sin^{-1}\frac{n_2}{n_1}\,. \tag{1.8}$$

Under this condition, 100% of the light is reflected into n_1, and no refracted light is present.

1.2 Mirrors and prisms

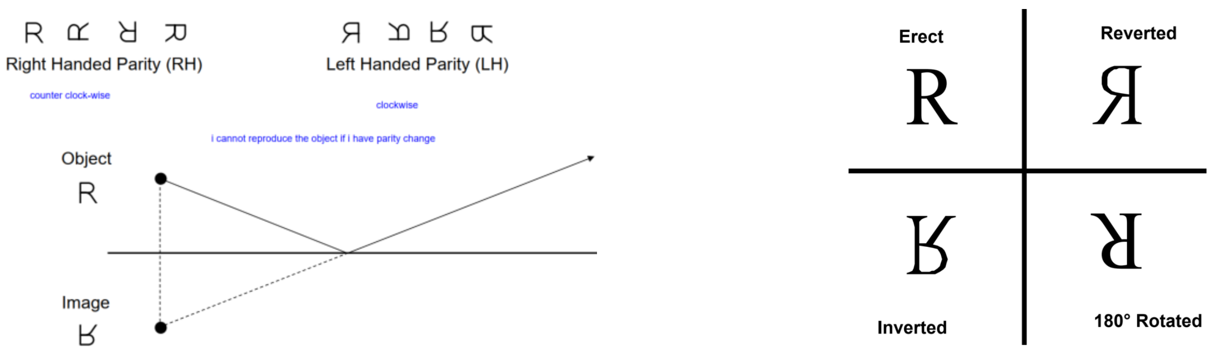
1.2.1 Parallel mirrors

Plane mirrors are used to:

- Produce a deviation
- Fold the optical path
- Change image parity

1.2.2 Parity change

A reflection from the plane mirror will cause a parity change in the image. An inversion (reversion) is a



(a) (b)

parity change about the horizontal (vertical) line, whereas a $180°$ rotation has no parity change and is rotated about the optical axis. An inversion and a reversion is equivalent to a $180°$ rotation.

Parity change

Only an **odd** number of reflections changes parity.

Parity is determined by looking back agains the propagation towards the obejct. Compare looking directly the objet vs at the reflection.

A lens adds inversion and reversion to the object, so that the image has no parity change, only rotation.

1.2.3 Non-parallel plane mirror

The **dihedral line** is the line of intersection of two non-parallel plane mirrors. The ray is deviated twice the angle between the mirrors.

$$\gamma = 2\alpha = \begin{cases} \text{Input-Output rays cross,} & \alpha < 90° \\ \text{Input-Output rays diverge,} & \alpha > 90° \\ \text{Input-Output rays anti-parallel,} & \alpha = 90° \end{cases} \tag{1.9}$$

There are several mirrors,

- **Roof mirror** Two plane mirrors with a dihedral angle of $90°$. It is used to insert two reflection in the propagation. The presence of this mirror is indicated by a "V".

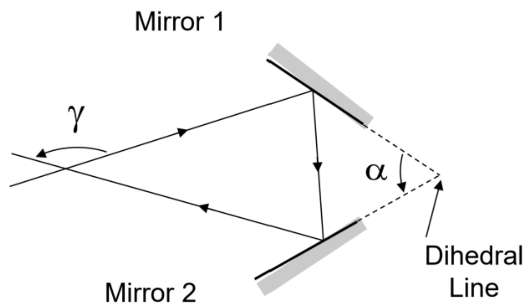


Figure 1.2

1.2.4 Prisms and tunnel diagrams

Prisms can be considered systems of plane mirrors. The reflection may be due to TIR, or by reflective coating.

A **tunnel diagram** **unfolds** the optical path at each reflection so that the ray is maintained straight through the propagation in the prism.

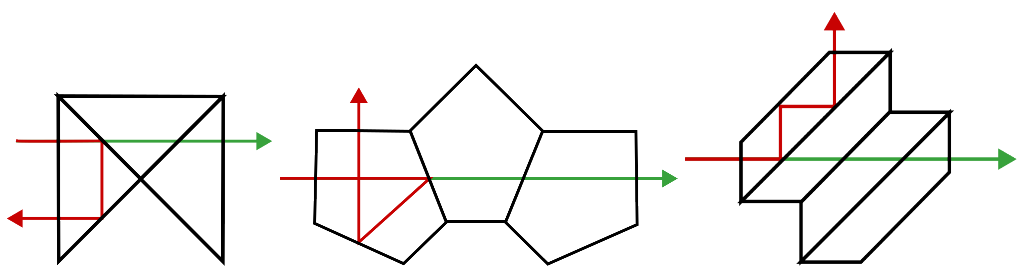


Figure 1.3

1.2.5 Reduced thickness

The **reduced thickness** is the vacuum equivalent distance of the medium that has the same propagation effect.

$$\text{Reduced thickness} \qquad \boxed{\tau = \frac{t}{n}}. \tag{1.10}$$

Expressing all distances in τ is equivalent to propagates the light in only vacuum (or air). This quantity is implicitly in the optical propagation and will be present in equations. When a reflection takes place, both n and t are negative, but τ remains positive.

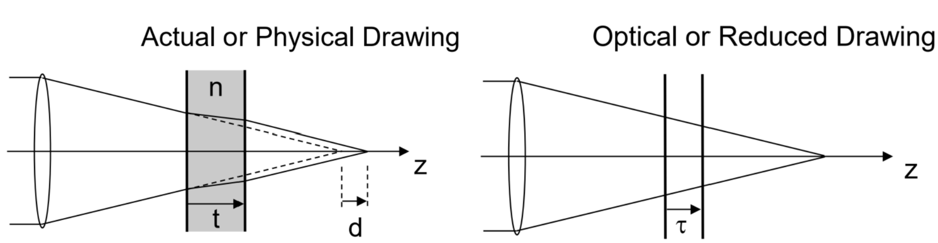


Figure 1.4 Reduced thickness is the vacuum (air) equivalent distance.

Tunnel diagrams are afected by the reduced thickness along the propagation distance. If the total distance is L, then the reduced is L/n.

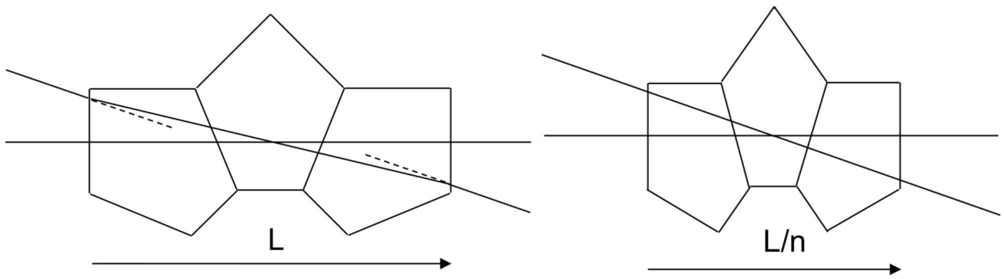


Figure 1.5 The diagram is only shortened along the direction of the propagation.

In a plate parallel plate (PPP), the beam is shifted horizontally a distance proportional to τ when is placed perpendicular ot the optical axis:

$$d = \frac{n-1}{n}t. \tag{1.11}$$

If it is disposed with an angle θ, then the ray will be shifted vertically

$$D \approx -t\theta\frac{n-1}{n}. \tag{1.12}$$

Ejemplo 1.1 **Reduced thickness and aparent distance**

a) The fish is $500\ mm$ beneath the surface of the water ($n = 1.33$). For the cat observing in air, how far below the water's surface does the fish appear to be?
In this case, we have

$$d_{\text{total}} = \frac{500\ mm}{1.33} = 375.94\ mm.$$

The fish appears to be $377\ mm$ below the surface of the water.

b) The cat is $500\ mm$ above the surface of the water. For the fish observing in water, how far abothe the water's surface does the cat appear to be?
The total distance is the sum of the air thickness in terms of the water and the thickness of the water:

$$d_{\text{total}} = 1.33 \cdot 500\ mm + 500\ mm = 665\ mm + 500\ mm = 1165\ mm.$$

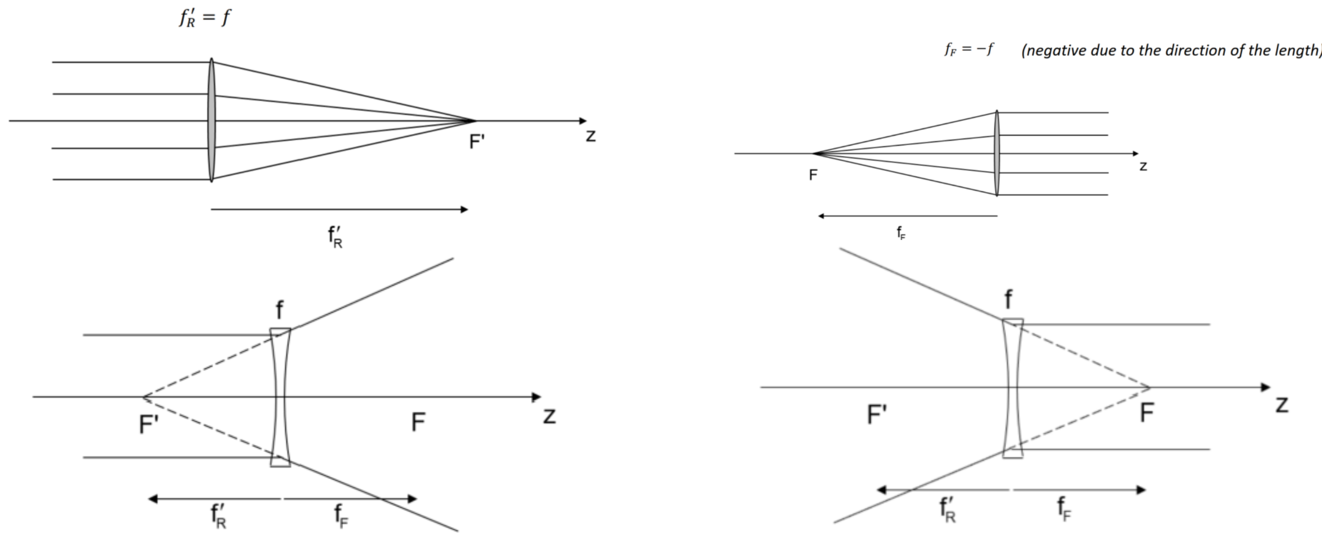
The cat appears to be $665\ mm$ above the surface of the water.

c) Several months later the cat return to watch the fish again. This time, there is a $100\ mm$ thick layer of ice ($n = 1.31$) on the surface. The fish is still a total physical distance of $500\ mm$ below the surface. Repeat parts a) and b).
In this case, we assume that the thick layer of ice has **replaced** $100\ m$ of the water while the distance of air remains the same.

- For the part a), the distance would be:

$$d_{\text{total}} = (\frac{100\ mm}{1.31} + \frac{400\ mm}{1.33}) + 500\ mm = 377\ mm + 500\ mm = 877\ mm.$$

The fish appears to be $377\ mm$ below the surface of the ice.



(a) Rear focal length f'_R **(b)** Front focal length f_F

- For part b), the total equivalent distance is the distance of the water, plus the equivalent distance in water of the ice and air:

$$d_{\text{total}} = 1.33 \cdot (\frac{100\ mm}{1.31} + 500\ mm) + 400\ mm = 767\ mm + 400\ mm = 1166.53\ mm.$$

The cat appears to be $767\ mm$ above the water, that is, below the air and the ice. We computed first the reduced thickness of ice in order to then convert it to the equivalent in water.

1.3 Thin lens Imaging

1.3.1 Introduction

A **thin lens** is an idealization of an optical system with:

- Zero thickness τ.
- Refracting power ϕ.
- Characterized by its focal length f.

An object at infiniity is imaged to the **rear focal point** F', whereas an object ar the **front focal point** F is iamged to infinity. The respective distances from the center of the thin lens to F is f_F and to F' is f'_R.

Positive vs negative thin lenses

A positive lens has a positive focal length $f > 0$, a positive $f'_R > 0$ but a negative $f_F < 0$. In the negative lens, **all** is opposite.

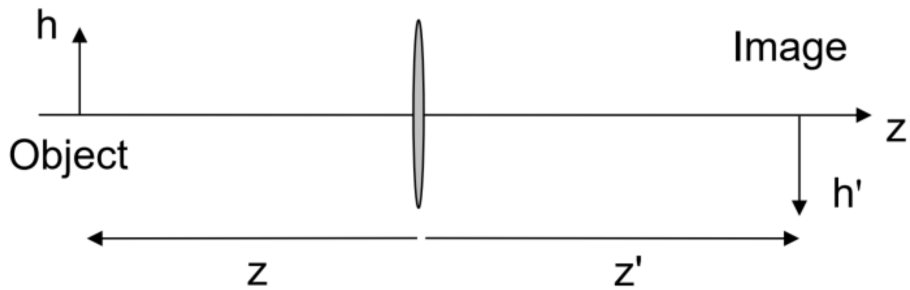


Figure 1.2 Imaging scheme

These points and lenghts are purely geometric properties of the lens.

Real rays are rays that are physically present, they can be touched. On the other hand, **virtual rays** are rays that are projection of real rays, and cannot be touched. Both type of rays are useful for imaging.

1.3.2 Imaging relationships

The imaging property of a thin lens relates the position of the object with that of the image.

The **thin lens equation** is

$$\text{Thin lens equation} \qquad \frac{1}{z'} = \frac{1}{z} + \frac{1}{f}. \tag{1.13}$$

The **transverse magnification** is the ratio of the heights:

$$\text{Transverse magnitication} \qquad m = \frac{h'}{h} = \frac{z'}{z}. \tag{1.14}$$

These two equations are the most fundamental for imaging. They are used extensivelty through geometrical optics.

Intersecting at elast 2 rays is enough to map a point from object to image. The following are the trivial rays used:

- Parallel ray from the object, emerges (diverges) through the rear focal point.
- Ray from object throught the front focal point, emerging parallel (antiparallel).
- A ray that goes directly from the object through the center of the lens which is not refracted.

Ejemplo 1.2 **Imaging with a negative lens**

The ray diagram is illustrated in figure **??**. We have traced three rays:

- Parallel to the optical axis from the object, then it is refracted with direction to F'.
- Direct to F: it is refracted so that it becomes parallel to the optical axis.
- The chief ray, which maintain its direction through its propagation.

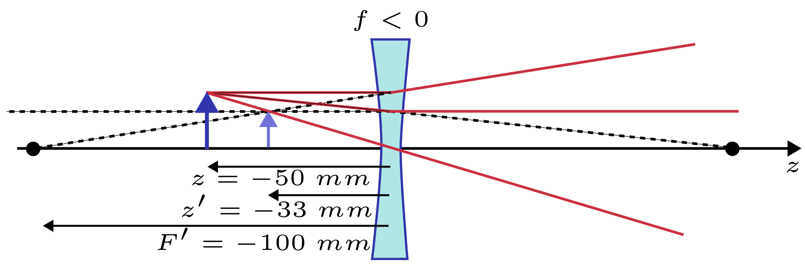


Figure 1.3 Ray diagram of the problem, the position of the image is z' and is located to the right of the object. Dashed lines correspond to virtual rays.

The intersection of these three rays produces the image. We can see that the image is to the left of the lens, but to the right of the object. Therefore, it will be a virtual image and demagnified. Using the thin lens equation, considering that $F' = -100\ mm$ and $z = -50\ mm$ provides

$$\begin{aligned} \frac{1}{z'} &= \frac{1}{F'} + \frac{1}{z} \\ \frac{1}{z'} &= \frac{1}{-100} + \frac{1}{-50} \\ z' &= \frac{(-100)(-50)}{-150} = -33.333\ mm. \end{aligned}$$

Because $z' < 0$, the image is **virtual** and will be to the left of the lens. Its magnification is:

$$m = \frac{z'}{z} = \frac{-33.333}{-50} = 0.667.$$

The image is then erected ($\text{sgn}(m) = 1$), and demagnified ($|m| < 1$) making it smaller than the object.

1.3.3 Optical spaces

Any optical object creates two optical spaces: the object space and the image space. Each optical space extends from $-\infty$ to ∞ and has an associated index of refraction. The connection of both spaces is through the optical object. A **real object** is to the left of the object while a **virtual object** is to the right. A

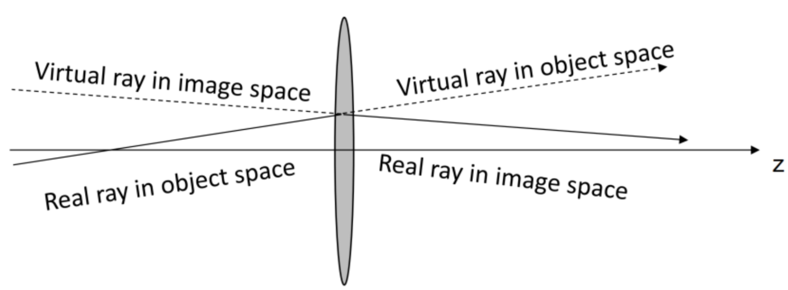
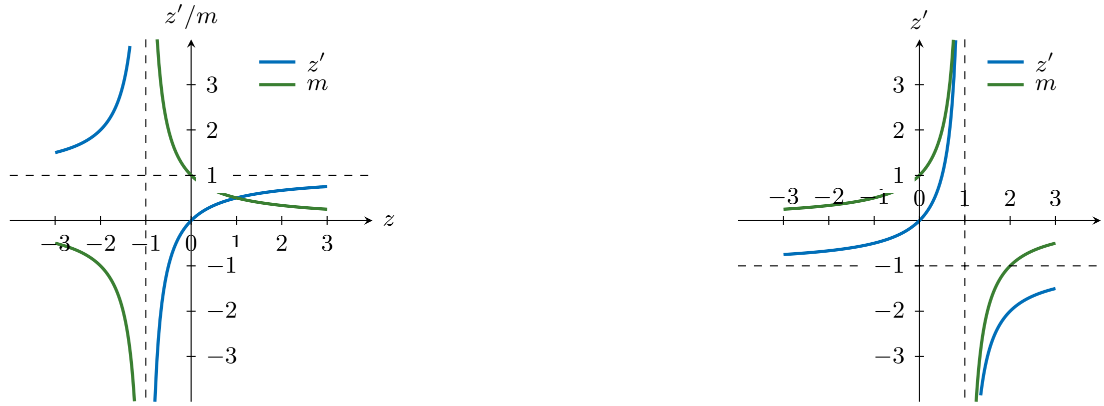


Figure 1.4

real image is to the right of the object and a **virtual image** to the left. In an optical space with negative index, left and right are reversed in these descriptions.

A thin lens creates two optical spaces:

- **Object space** contains the object and the front focal point F.
- **Image space** contains the image and the rear focal point F'.



(a) $f > 0$ **(b)** $f < 0$

Figure 1.5 Plot of z' and m for positive and negative lenses. We can directly see when an object (image) is real or virtual.

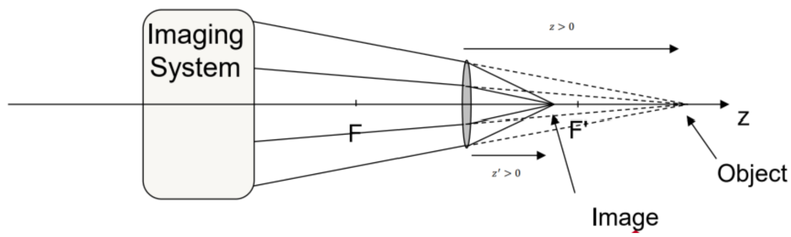


Figure 1.6 A virtual object is the projection of the image of a previous optical system.

Virtual objects occur when an image is projected into the lens by a previous optical system.

1.3.4 Object-image approximations

We can make further appproximations for extreme situations:

- **Distant object** When the magnitude of the object distance z is more then a few times the magnitude of the system focal length, the image distance z' is approximately equal to the real focal length.

$$|z| \gg |f| \Longrightarrow a)\ z' \approx f \quad b)\ L = z' - z \approx f - z \approx -z \quad c)\ m = \frac{z'}{z} \approx \frac{f}{z}. \tag{1.15}$$

- **Distant image** Similarly,

$$|z'| \gg |f| \Longrightarrow a)\ z \approx -f \quad b)\ L = z' - z \approx z' + f \approx z' \quad c)\ m = \frac{z'}{z} \approx -\frac{z'}{f}. \tag{1.16}$$

This fraction error of these approximations is about $|f|/|z|$ so they are very useful for distant object/image from over $4f$.

1.3.5 Field of view

The half **field of view** (HFOV) is often expressed as:

- maximum object height h.
- maximum image height h'.

- maximum angular size of the object as seen from the optical system $\theta_{1/2}$.
- maximum angular size of the image as seen from the optical system $\theta'_{1/2}$.

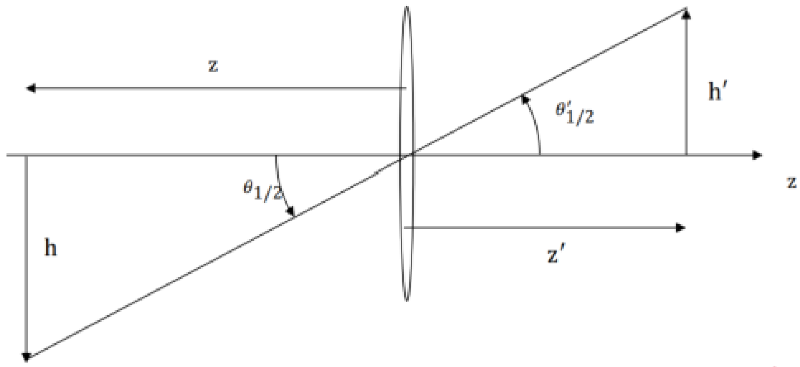


Figure 1.7

$$\text{FOV} = 2\text{HFOV}, \quad \text{HFOV} = \theta_{1/2} = \theta'_{1/2}, \quad \tan\theta_{1/2} = \frac{h}{z} = \frac{h'}{z'}. \tag{1.17}$$

In many situations, the FOV is determines by the detector size, which impose the maximum spatial dimensions.

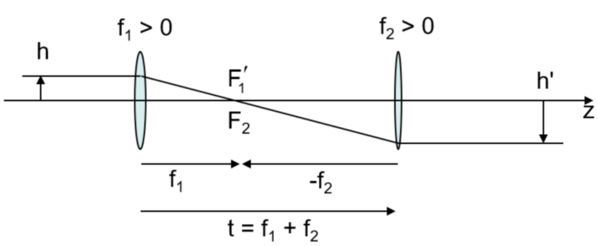
1.3.6 Afocal systems

An **afocal system** does not have focal points. Parallel rays will produce parallel images. The only change is in the transverse magnification.

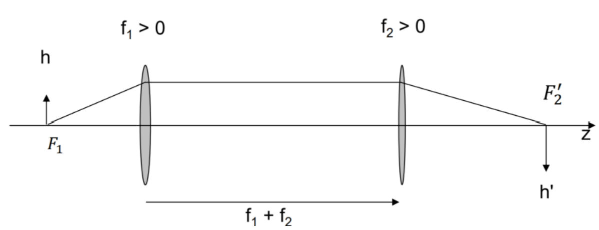
$$m = \frac{h'}{h} = \frac{-f_2}{f_1}. \tag{1.18}$$

1.4 Imaging and paraxial optics

An optical system is a collection of optical elements. The first-order properties of the system is charectized by a **single** focal length, or magnification. First-order optics is the optics of perfect imaging systems: no aberrations, where the object is **mapped** to its image.



(a) Keplerian telescope ($m < 0$).



(b) Galilean telescope ($m > 0$).

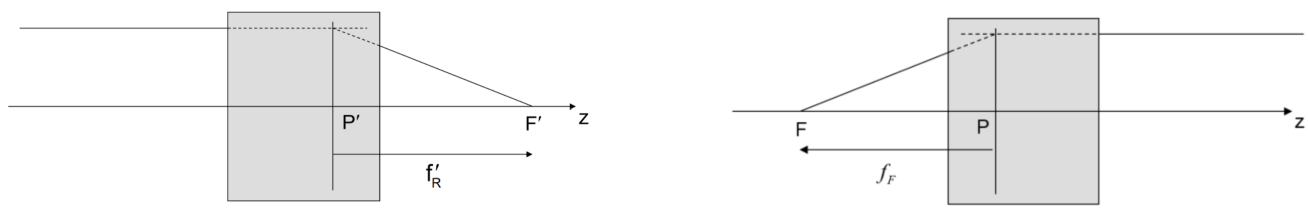
A small number of system properties can completely define and determine the mapping of first-order imaging properties. These are known as the **cardinal points** of the imaging system.

Each time a rafecting or reflecting surface is encountered, a new optical space is entered. In general, if a system coantains N surfaces, there will be $N+1$ optical spaces. The first space is calles the **object space** and the last **image space**.

1.4.1 General system

A black box is a convenient way to treat the optical system and analyze the refractions.

- An infinite object from left is effectively refracted by the system at the **rear principal plane** P'. The distance from P' to the **rear focal point** F' is the **rear focal length** f'_R.
- An object starting at F is effectively refracted by the system at the **front principal plane** P. The distance from P to the **front focal point** F is the **front focal length** f_F.



(a) Refraction at P' ($f'_R > 0$)

(b) Refraction at P ($f_F < 0$)

The system can now be trated as a thin lens, with the difference that object and image distances are from their respective principal planes.

1.4.2 Paraxial optics

The first-order properties of the system can be found using **paraxial rays**.

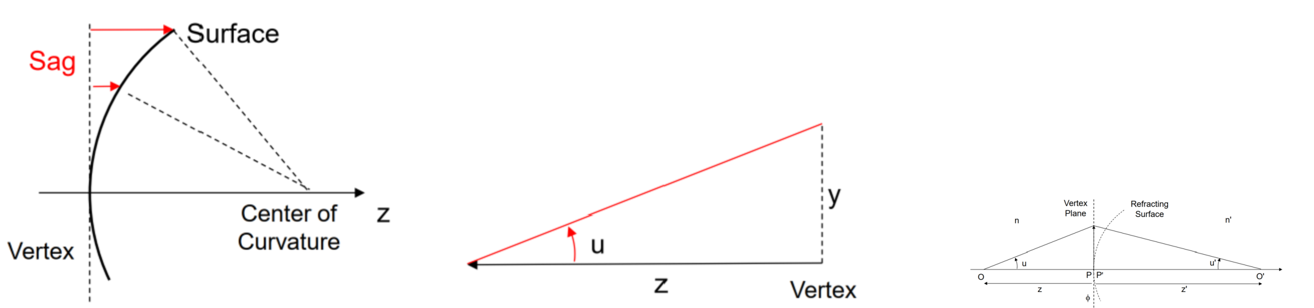
Paraxial ray

a) Rays are nearly parallel to the optical axis.

b) The amount a ray is bent at surfaces is assumed to be small: $\cos u \approx \cos u'$.

c) The sag of surfaces is ignored: $|\text{sag}| \ll |R|, |z|, |z'|$. Rays refracts at the vertex.

d) Rays are traced using the slopes of the rays instead of ray angles: $u = y / -z$.



(a) Sag is ignored

(b) Ray slopes instead of ray angles

(c) Ray slopes instead of ray angles

Single refractive surface

Single refracting surface are the fundamental object from which all other are composed of. They are defined by two refractive indices at the object and image space n and n', respectively, and the curvature of the surface:

$$\text{Radius of curvature} \qquad \boxed{R = \frac{1}{C}}\,. \tag{1.19}$$

The **optical power** ϕ is a measure of the benting power of the surface:

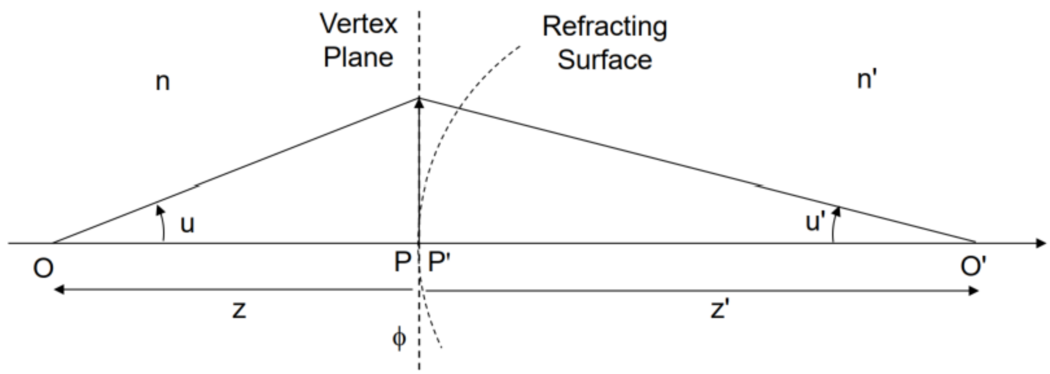


Figure 1.3 SIngle refracting surface.

$$\text{Optical power} \qquad \boxed{\phi = (n' - n)C \quad (m^{-1})(\text{diopters})}\,. \tag{1.20}$$

Then, the **paraxial raytrace equation** describes how the ray will travel after hitting the refracting surface:

$$\text{Paraxial raytrace equation} \qquad \boxed{n'u' = nu - y\phi}\,. \tag{1.21}$$

There are others useful equations, illustrated as follows

$$\frac{n'}{z'} = \frac{n}{z} + \phi, \quad f_E = \frac{1}{\phi}, \quad f_F = -nf_E, \quad f'_R = n'f_E. \tag{1.22}$$

A reflective surface is a special case with $n' = -n$.

With all variables defined, the scheme is illustrated as follows:

The focal length f_E is not a physical distance, but the front and real focal lengths are physical distances.

1.5 Gaussian imagery

Gaussian optics is a system of treating imaging as a mapping from object into image space. It is a special case of a **collinear transformation** applied to rotationally symmetric systems, and it maps points to

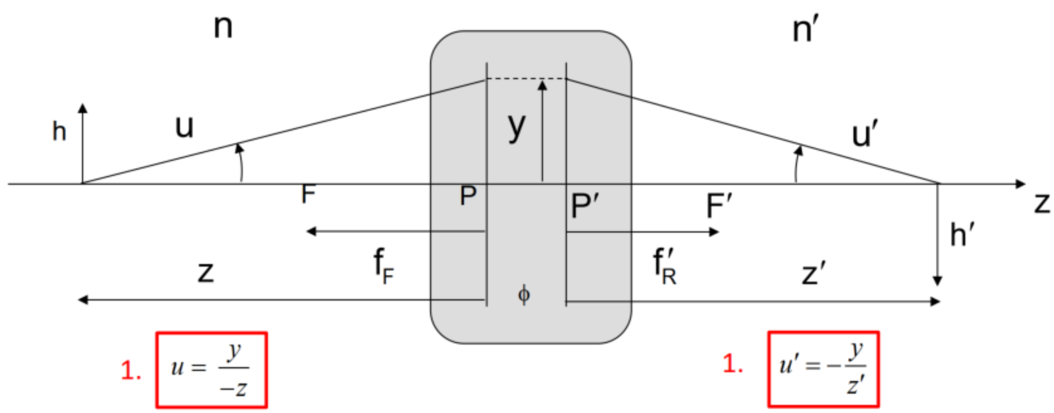


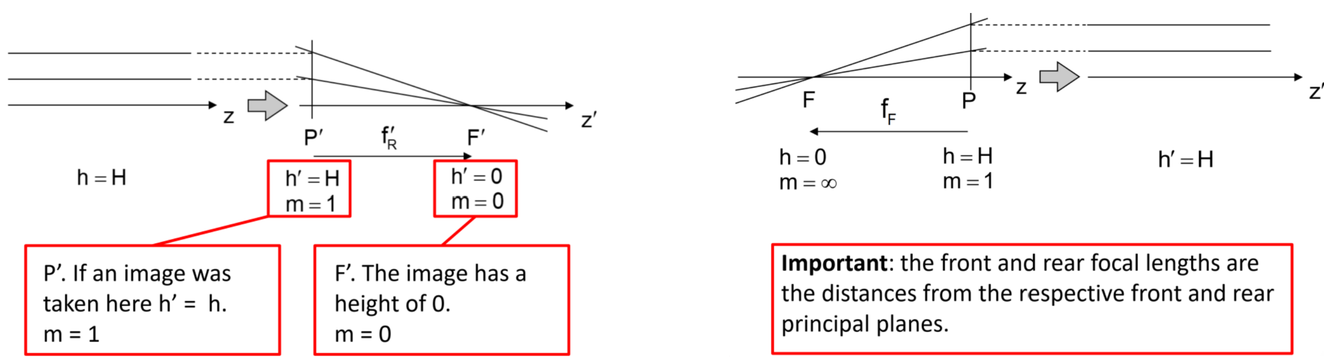
Figure 1.4 General paraxial system, with parameters defined previously.

points, lines to lines, and places to planes. The corresponding object and image elements are called **conjugate elements**.

The cardinal points and planes completely describes the focal mapping. They are defined by specific magnifications:

F	Front focal point/plane	$m = \infty$
F'	Rear focal point/plane	$m = 0$
P	Front principal plane	$m = 1$
P'	Rear principal plane	$m = 1$

$$m = \frac{h'}{h}.$$



(a) Rear cardinal point/plane **(b)** Front cardinal point/plane

Also, there exists **nodal points** N and N' that define the location of unit angular magnification for a focal system. A ray passing through one is mapped to a ray passsing through the other having the same angle.

$$z_{PN} = z'_{PN} = f_F + f'_R = (n' - n)f_E, \quad m_N = -\frac{f_F}{f'_R} = \frac{n}{n'}. \tag{1.23}$$

- Both nodal points of a single surface are located at the center of curvature of the surface: $z_{PN} = z'_{PN} = R$.
- If $n = n'$, then $z_{PN} = z'_{PN} = 0$ and the nodal points are coincident with the respective principal planes.
- The angular substense of an image seen from N' equals to the one seen from N: $m = h'/h = z'_N/z_N$.

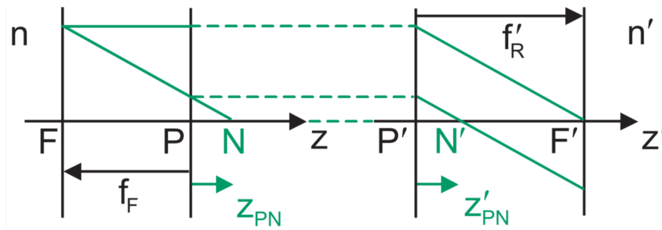
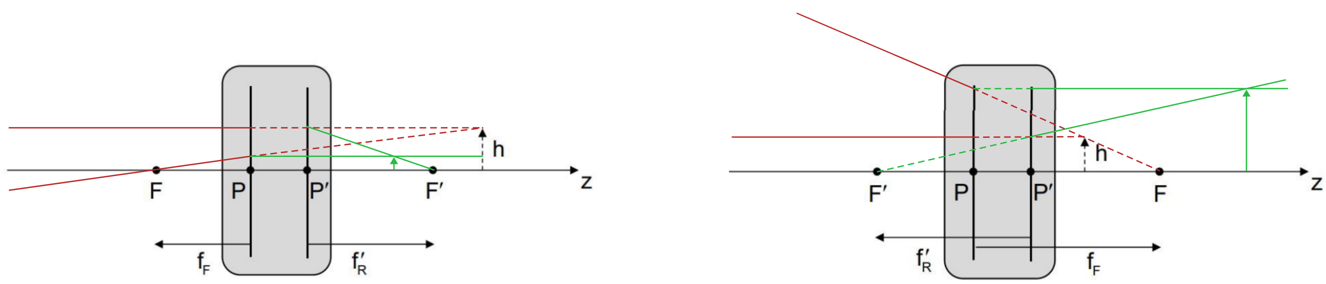


Figure 1.2

1.5.1 Representation of an optical system

An optical system can be represented as a set of principal planes and a set of focal points.



(a) Positive system **(b)** Negative system

Remember that refraction for F must happen at P while for F' at P'.

1.5.2 Newtonian equations

Newtonian equations measure object and image distances form the **focal planes**

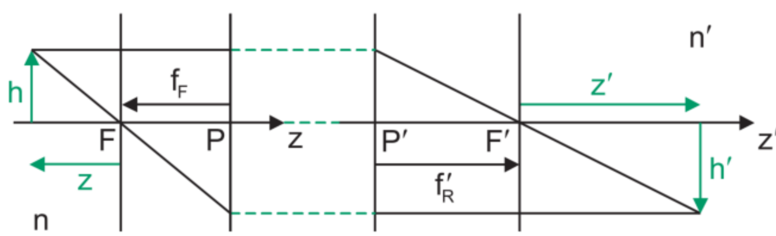


Figure 1.4 Newtonian equations.

$$z = -\frac{f_F}{m} \;\Big|\; z' = -m f'_R \;\Big|\; zz' = f_F f'_R \;\Big|\; \frac{z}{n} = \frac{f_E}{m} \;\Big|\; \frac{z'}{n'} = -m f_E \;\Big|\; \frac{z}{n}\frac{z'}{n'} = -f_E^2$$

1.5.3 Gaussian equations

Gaussian equations measure object and image distances from the **principal planes**.

A ray angle multiplied by the refractive indesx of its optical space is called **optical angle**:

$$\text{Optical angle} \qquad \omega = nu \quad (-)\,. \tag{1.24}$$

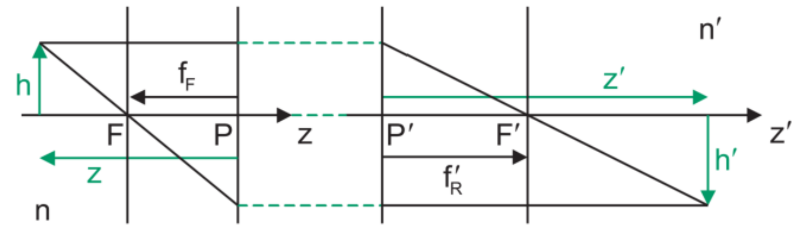


Figure 1.5 Gaussian equations.

$$z = -\frac{(1-m)}{m} f_F \quad \frac{z'}{n'} = (1-m) f_E \quad \Big| \quad z' = (1-m) f'_R \quad m = \frac{z'/n'}{z/n} \quad \Big| \quad m = -\frac{z'}{z}\frac{f_F}{f'_R} \quad \frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} \quad \Big| \quad \frac{f'_R}{z'} + \frac{f_F}{z} = 1 \quad \Big| \quad \frac{z}{n} = \frac{(1-m)}{m} f_E$$

1.5.4 Longitudinal magnification

The **longitudinal magnification** relates the distances between pairs of conjugate planes.

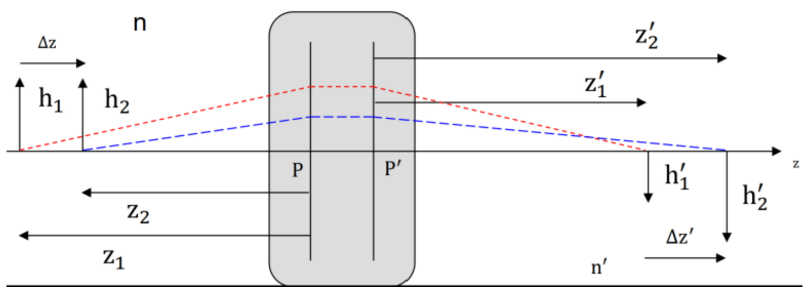


Figure 1.6 Longitudinal magnification allows you to have the thickness of the object or image.

$$\Delta z = z_2 - z_1, \quad \Delta z' = z'_2 - z'_1, \quad m_1 = \frac{h'_1}{h_1}, \quad m_2 = \frac{h'_2}{h_2}, \quad \frac{\Delta z'}{\Delta z} = -\frac{f'_R}{f_F} m_1 m_2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m_1 m_2 \quad (1.25)$$

As the plane separation approaches zero, $m_1 \approx m_2 \approx m$ and the local magnification $\bar{m}$ is obtained:

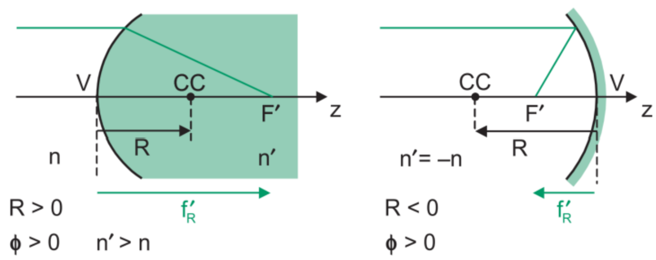
$$\bar{m} = \frac{n'}{n} m^2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m^2. \quad (1.26)$$

1.5.5 Gaussian properties of a single refracting surface

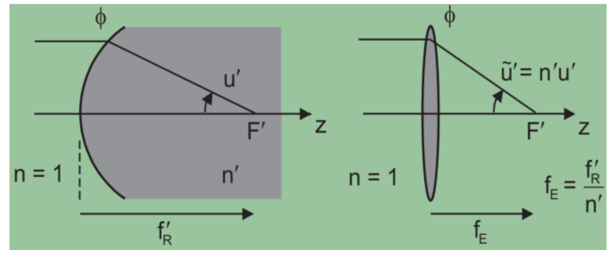
The radius of curvature R is defined to be the distance from its vertex to the center of curvature CC. The front and rear principal planes are coincident and located at the surface vertex. In addition, both nodal points are located at the center of the curvature (CC) of the optical surface.

> The use of reduced distances and optical angles allows a system to be represented as an air-equivalent system with thin lenses of the same power ϕ.

Ejemplo 1.3 **Gaussian imagery of a single refractive surface**



(a) Single surface



(b) Thin lens reduction

a) For a single refracting surface, we have that:

- Both nodal points are located at the center of curvature CC.
- Front and real principal planes are located at the vertex.
- The reduced thickness of the surface is the focal length of its thin lens representation.

We illustrate these quantities along with the vertex and the focal lengths in the following figure. We illustrate

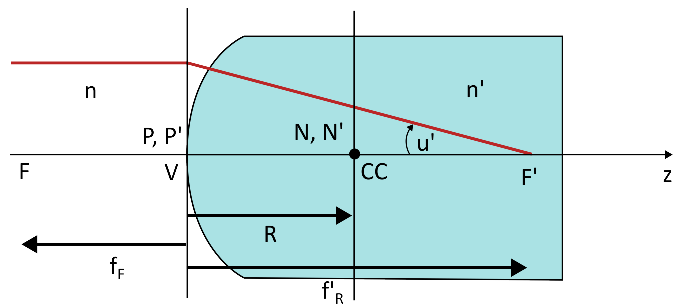


Figure 1.8 Illustration of cardinal point for a single refractive surface.

also some quantities of this surface:

$$C = \frac{1}{R} = 100\ m^{-1}, \quad \phi = (n' - n)C = 33.3\ m^{-1}, \quad f_E = \frac{1}{\phi} = 30\ mm,$$
$$f_F = -nf_E = -30\ mm, \quad f'_R = n'f_E = 40\ mm.$$

b) We use the following equation:

$$\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} \longrightarrow z' = \frac{n'zf_E}{nf_E + z}.$$

Replacing the physical values and the EFL:

$$z' = \frac{(1.333)(30)(-100)}{(1)(30) - 100} = +57.129\ mm.$$

Its height is determined by the magnification:

$$m = \frac{h'}{h} = \frac{z'/n'}{z/n} = \frac{57.129/1.333}{-100/1} = -0.429 \longrightarrow h' = mh = (-0.429)(10\ mm) = -4.29\ mm.$$

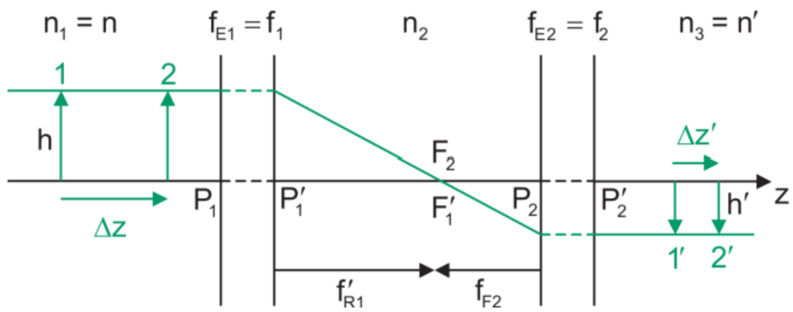


Figure 1.9 Generalized afocal system.

1.5.6 Generalized afocal systems

An afocal system is formed by the combination of two focal systems. The rear focal point of the first one is coincident with the front focal points of the second system. Common afocal systems are telescopes, binoculars, and beam expanders. The transverse and longitudinal magnification are constant. Due to this, the cardinal points are not defined, and the Gaussian and Newton equations **cannot** be used to determine cnojugate planes. However, any pair of conjugate planes coupled with $\bar{m}$ can be employed.

$$m = \frac{f_{F2}}{f'_{R1}} = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1}, \quad \bar{m} = \frac{n'}{n}m^2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m^2. \tag{1.27}$$

1.6 Object image relationship

1.7 Gaussian reduction

Gaussian reduction is the process that combinar multiple elements two at a time into a single equivalent focal system. The system is defined by its Gaussian properties which include:

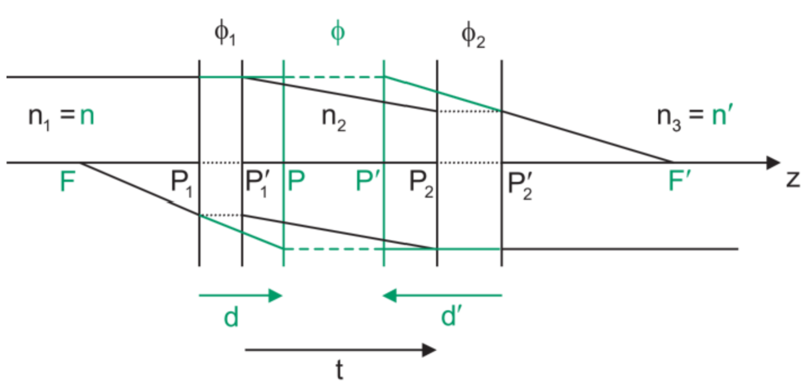


Figure 1.1 Gaussian reduction scheme.

- Power of the overall system.
- Front and rear focal lengths of the overall system.
- Principal planes of the overall system.

The principal equation for Gaussian reduction are:

$$\text{Overall power} \qquad \phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau, \quad \tau = \frac{t}{n_2}. \tag{1.28}$$

The new principal planes P, P' will be **shifted** from the front principal plane of the left system P_1 and the rear principal plane of the second system P_2', by the following amount:

$$\text{Shifting distance from } P_1 \text{ and } P_2' \qquad \frac{d}{n} = \frac{\phi_2}{\phi}\tau, \quad \frac{d'}{n'} = -\frac{\phi_1}{\phi}\tau \tag{1.29}$$

Recall that P is in the object space (n) and P' is in the image space n'. Therefore, d occurs in the system object space n, whereas d; occurs in the system image space n'.

1.7.1 Vertex distances

The **surface vertices** are the mechanical datums in a system and are often the reference locations for the cardinal points. The **back focal distance** (BFD) and **front focal distance** (FFD) are the distance measured from the back (front) vertex to the back (front) focal point F' (F).

$$\text{Distances} \qquad \text{BFD} = f_R' + d', \quad \text{FFD} = f_F + d \tag{1.30}$$

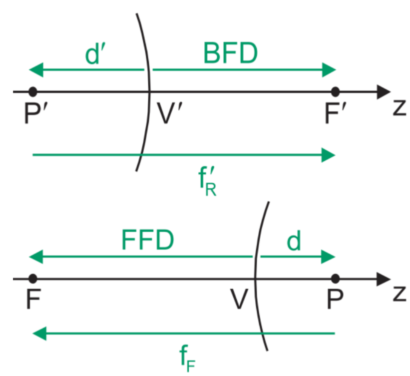


Figure 1.2 Vertex distances are used to define BFD and FFD.

The utility of Gaussian reduction is that the imaging properties of any combination of optical elements can be represented by a system power of focal length, a pair of principal planes and a pair of focal points.

1.7.2 Thick and thin lenses

The **thick lens** is composed ot two refractive surfaces with a thicknes between them. The overall power in term of curvature is:

$$\phi_{\text{thick}} = (n-1)[C_1 - C_2 + (n-1)C_1C_2\tau], \quad d = \frac{\phi_2}{\phi}\tau, \quad d' = -\frac{\phi_1}{\phi}\tau. \tag{1.31}$$

In this case, the nodal points are coincident with the principal planes.

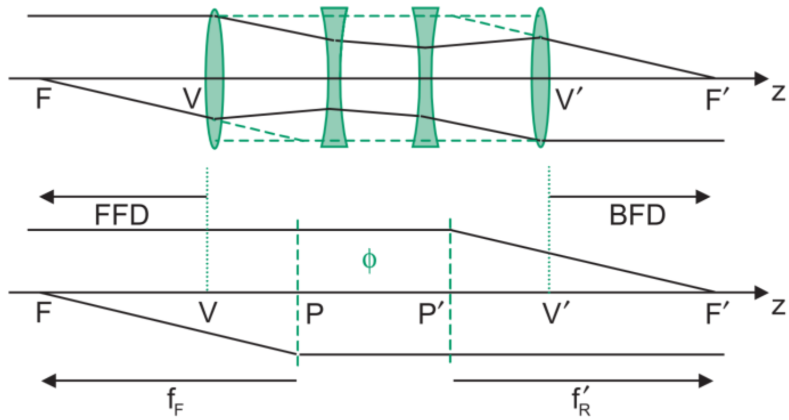
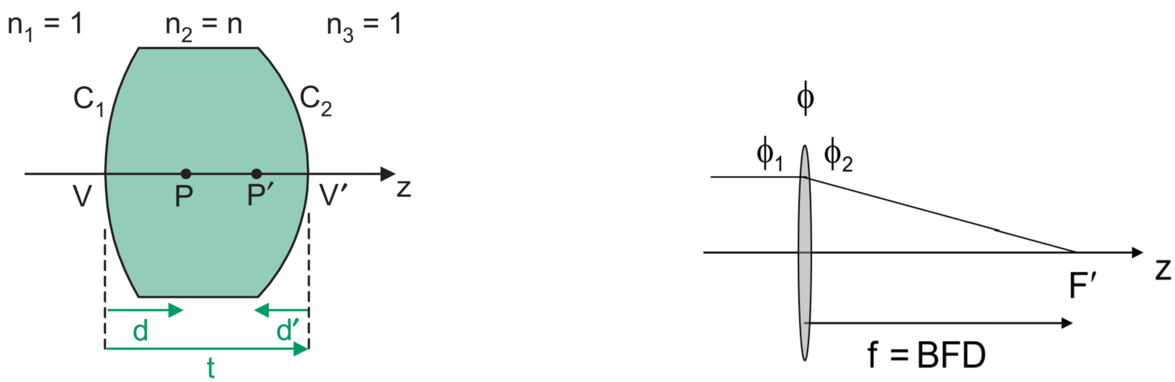


Figure 1.3 All reduces to 5 cardinal points.



(a) Thick lens **(b)** Thin lens

The **thin lens** approximation is obtained for $t \to 0$, which reduces the overall power to

$$\phi_{\text{thin}} = (n-1)(C_1 - C_2), \quad d = d' = 0, \quad \text{BFD} = f. \tag{1.32}$$

This idealized element can be considered as a singel refracting surface separating two spaces. The principal planes and nodal points are located at the lens (middle).

Ejemplo 1.4 Gaussian reduction of two lenses

For a two positive lens system, we use Gaussian reduction to reduce the effect to a single thin lens. We first compute the overall optical power with the power of individual lenses:

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t = \frac{1}{40} + \frac{1}{40} - \frac{1}{40}\frac{1}{40} \cdot 20 = 0.038mm^{-1} \longrightarrow f_E = \frac{1}{\phi} = 26.67\ mm.$$

The front and real focal lengths are:

$$f_F = -n_1 f_E = (1)(26.67\ mm) = -26.67\ mm, \quad \text{and} \quad f'_R = n_3 f_E = (1)(26.67\ mm) = 26.67\ mm.$$

Then the distances d and d', corresponding to the shift from the front (rear) principal planes P, P' of the equivalent system with respect to f_F, f'_R are given by

$$d = \frac{\phi_2}{\phi} t = \frac{0.025}{0.038} 20 = 13.158\ mm, \quad \text{and} \quad d' = -\frac{\phi_1}{\phi} t = -\frac{0.025}{0.038} 20 = -13.158\ mm.$$

The front (back) focal distances are then: The FFD and BFD are therefore,

$$\text{FFD} = f_F + d = -26.67\ mm + 13.158\ mm = -13.512\ mm.$$
$$\text{BFD} = f'_R + d' = 26.67\ mm - 13.512\ mm = 13.512\ mm.$$

The reduction process and the quantities obtained are illustrated in figure 1.5. The nodal points are coincident with

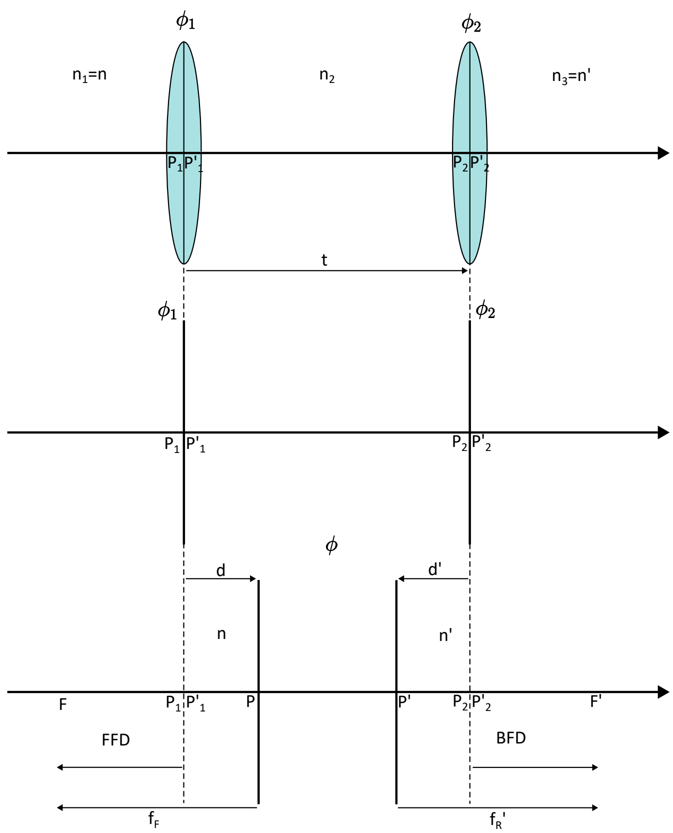


Figure 1.5 Gaussian reduction for two positive lenses.

the principal planes.

1.8 Paraxial raytrace

1.8.1 Introduction

Paraxial optics is a method of determining the first-order properties of an optical system that assumess all ray angles are small. It follows the same assumptions of **paraxial optics** regime seen.

It composes of iterative **refraction** and **Transfer** processes. These type of raytrace are called **YNU ray-**

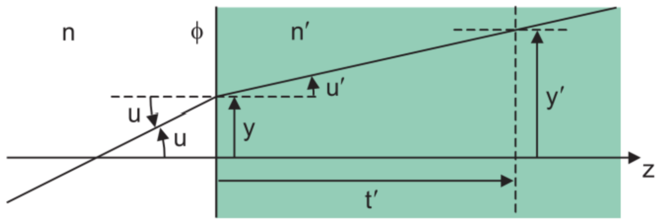


Figure 1.1 A paraxial raytrace is linear with respect to ray angle and heights.

trace:

$$\text{Object} \longrightarrow \text{Image} \quad \begin{cases} \text{Refraction (reflection)} & n'u' = nu - y\phi \qquad \omega' = \omega - y\phi \\ \text{Transfer} & y' = y + u't' \qquad y' = y + \omega'\tau' \end{cases} \tag{1.33}$$

$$\text{Image} \longrightarrow \text{Object} \quad \begin{cases} \text{Refraction (reflection)} & nu = n'u' + y\phi \qquad \omega = \omega' + y\phi \\ \text{Transfer} & y = y' - u't' \qquad y = y' - \omega'\tau' \end{cases} \tag{1.34}$$

1.8.2 Procedure

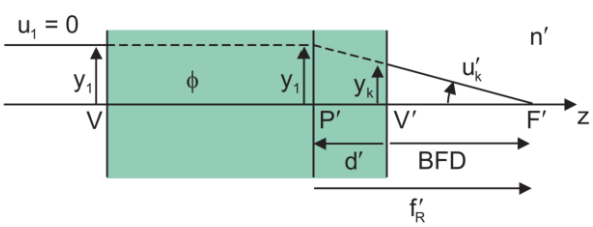
The procedure is always the same:

- You set the optical propoerties of the system.
- **Rear cardinal points** Trace a forward ray from object to image, and at the image space, you look for t that satisfies $y = 0$ to get the BFD.

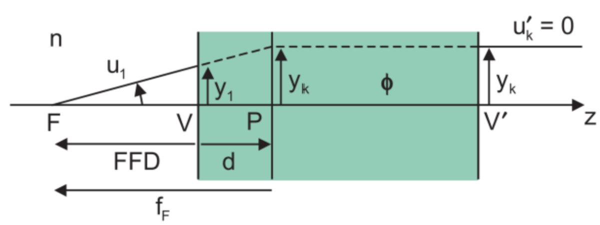
$$\phi = -\frac{\omega'_k}{y_1}, \quad f_E = \frac{1}{\phi}, \quad f'_R = n' f_E, \quad \text{BFD} = -\frac{y_k}{u'_k}, \quad d' = \text{BFD} - f'_R. \tag{1.35}$$

- **Front cardinal points** Trace a backward ray from image to object, and at object space, you look for t that satisfies $y = 0$ to het the FFD.

$$\phi = \frac{\omega_1}{y_k}, \quad f_E = \frac{1}{\phi}, \quad f_F = -n f_E, \quad \text{FFD} = -\frac{y_1}{u_1}, \quad d = \text{FFD} - f_F. \tag{1.36}$$



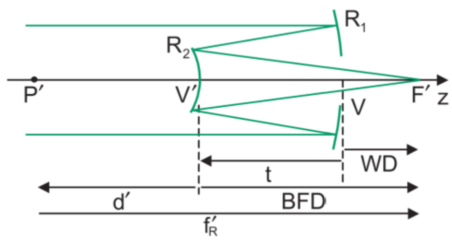
(a) Finding rear cardinal points



(b) Finding front cardinal points

Once you have the rays traced, you construct a reduced table, where

- List out all the surfaces: the first part of the big table. Parameters vertically in-line with the surface are associated with optical surfaces. Parameters sandwiched between refer to the optical spaces.
- Another box below contains the information about the rays traced to find the cardinal points.



$R_1 = -200$ mm
$t = -80$ mm
$R_2 = -50$ mm
$n_1 = n = 1$
$n_2 = -1$
$n_3 = n' = 1$
$C_1 = -0.005\ \text{mm}^{-1}$
$C_2 = -0.02\ \text{mm}^{-1}$

Paraxial raytrace: $\omega' = \omega - y\phi \qquad y' = y + \omega'\tau'$

Surface	Object	V	V′	F′
C		-0.005	-0.02	
t	∞	-80	BFD	
n	1.0	-1.0	1.0	
$-\phi$		-0.01	0.04	
t/n	∞	80	100	
y	1.0	1.0	0.2	0.0
nu	0.0	-0.01	-0.002	
u	0.0	0.01	-0.002	

The analysis of the raytrace results:

$$\phi = -\frac{n'u_2'}{y_1} = -\frac{-0.002}{1.0} = 0.002\ \text{mm}^{-1} \qquad f_R' = f_E = \frac{1}{\phi} = 500\ \text{mm}$$

$$BFD = -\frac{y_2}{u_2'} = -\frac{0.2}{-0.002} = 100\ \text{mm}$$

$$d' = BFD - f_R' = BFD - f_E = -400\ \text{mm}$$

$$WD = BFD + t = 20\ \text{mm}$$

Gaussian reduction:

$$\phi_1 = (n_2 - n)C_1 = 0.01\ \text{mm}^{-1}$$

$$\phi_2 = (n' - n_2)C_2 = -0.04\ \text{mm}^{-1} \qquad \tau = \frac{t}{n_2} = 80\ \text{mm}$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau = 0.002\ \text{mm}^{-1} \qquad f_R' = f_E = \frac{1}{\phi} = 500\ \text{mm}$$

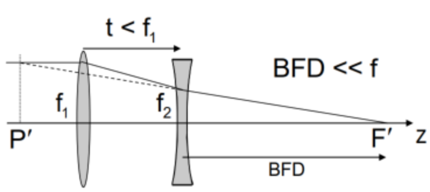
$$d' = -n'\frac{\phi_1}{\phi}\tau = -400\ \text{mm}$$

$$BFD = f_R' + d' = f_E + d' = 100\ \text{mm}$$

$$WD = BFD + t = 20\ \text{mm}$$

Ejemplo 1.5 Cassegrain telescope

Ejemplo 1.6 Thin lens



Thin Lens Telephoto Lens - Raytrace

$f_1 = 100mm \qquad \phi_1 = .01/mm$

$t = 50mm$

$f_2 = -75mm \qquad \phi_2 = -.01333/mm$

Surface	0	1	2	
f		100	-75	
-φ		-.01	.01333	
t	∞/?	50	?/∞	
			150	
➡ y	1	1	.5	0
u	0	-.01	-.00333	
	500			
⬅ y	0	1.667	1	1
u	.00333	-.01333	0	

$BFD = \overline{V'F'} = 150mm$

$u' = u_2' = -.00333 \qquad y_1 = 1$

$\phi = -\frac{u'}{y_1} = .00333/mm$

$f_E = f_R' = 300mm$

$d' = BFD - f_R' = -150mm$

$\overline{FV} = 500mm$

$FFD = -\overline{FV} = -500mm$

$u = .00333 \qquad y_2 = 1$

$\phi = -\frac{u}{y_2} = .00333/mm$

$f_F = -300mm \qquad f_E = 300mm$

$d = FFD - f_F = -200mm$

Ejemplo 1.7

In this case we have three surface, each with their correspond surface curvature C and index of refraction n.

- **Gaussian reduction** The optical power of each surface is:

$$\phi_1 = \frac{n_1 - n_0}{R_1} = \frac{1.336 - 1}{7.8\ mm} = 0.043\ mm^{-1},$$
$$\phi_2 = \frac{n_2 - n_1}{R_2} = \frac{1.413 - 1.336}{10\ mm} = 0.008\ mm^{-1},$$
$$\phi_3 = \frac{n_3 - n_2}{R_3} = \frac{1.336 - 1.413}{-6\ mm} = 0.013\ mm^{-1}.$$

Now, we combine surface 1 with 2:

$$\phi_{12} = \phi_1 + \phi_2 - \phi_1\phi_2\tau_1 = 0.043 + 0.008 - 0.043 \cdot 0.008 \cdot \frac{3.6}{1.336} = 0.050\ mm^{-1}.$$

The shift of the principal plane are given by

$$\delta_{12} = \frac{\phi_2}{\phi_{12}}\tau_1 = \frac{0.008}{0.050} \cdot \frac{3.6}{1.336} = 0.431\ mm \longrightarrow d_{12} = \delta_{12}.$$
$$\delta'_{12} = -\frac{\phi_1}{\phi_{12}}\tau_1 = -\frac{0.043}{0.050} \cdot \frac{3.6}{1.336} = -2.317\ mm \longrightarrow d'_{12} = n_2\delta'_{12} = -3.274\ mm.$$

We can see that the front principal plane is displaced from V_1 to the left, while the rear principal plane is shifted to the right of V_2. In addition, the distance d'_{12} considered the index n_2 as it belong to that space. The distance of propagation through the index n_2 must be adjusted due to the shift of the rear principal plane:

$$\tau_{12} = \frac{t_2 - d'_{12}}{n_3} = \tau_2 - \delta'_{12} = \frac{3.6}{1.413} + 2.317 = 4.865\ mm.$$

Now, we compute the total optical power considering the reduction and the third surface:

$$\phi = \phi_{12} + \phi_3 - \phi_{12}\phi_3\tau_{12} = 0.050 + 0.013 - (0.046)(0.013)(4.865) = 0.060\ mm^{-1}.$$

The shifts are:

$$d_{123} = n_0\delta_{123} = \frac{\phi_3}{\phi}\tau_{12} = \frac{0.013}{0.060} \cdot 4.865 = 1.054\ mm$$
$$d'_{123} = n_3\delta'_{123} = -n_3\frac{\phi_{12}}{\phi}\tau_{12} = -(1.336)\frac{0.050}{0.060} \cdot 4.865 = -5.416\ mm.$$

The total shift from the first surface is the sum of individual fron shift computed, while for the last surface is just the shift computed in the last reduction:

$$d = d_{12} + d_{123} = 0.431 + 1.054 = 1.485\ mm$$
$$d' = d'_{123} = -5.416\ mm.$$

The front (rear) focal lengths are then

$$f_E = \frac{1}{\phi} = 16.667\ mm \longrightarrow \begin{array}{l} f_F = -n_0 f_E = -(1)(16.667) = -16.667\ mm \\ f'_R = n_3 f_E = (1.336)(16.667) = 22.267\ mm. \end{array}$$

Finally, the FFD and BFD are:

$$\text{FFD} = f_F + d_{123} = -16.667 + 1.054 = 15.613\ mm$$
$$\text{BFD} = f'_R + d'_{123} = 22.267 - 5.416 = 16.851\ mm.$$

The reduction process is shown in figure 1.3. The green quantities are the equivalent of the final reduction.

- **Ray tracing** For the ray tracing, we will fill the ynu spreadsheet. We will trace two rays, one from left to right and other in opposite direction in order to find the front and real focal lengths.

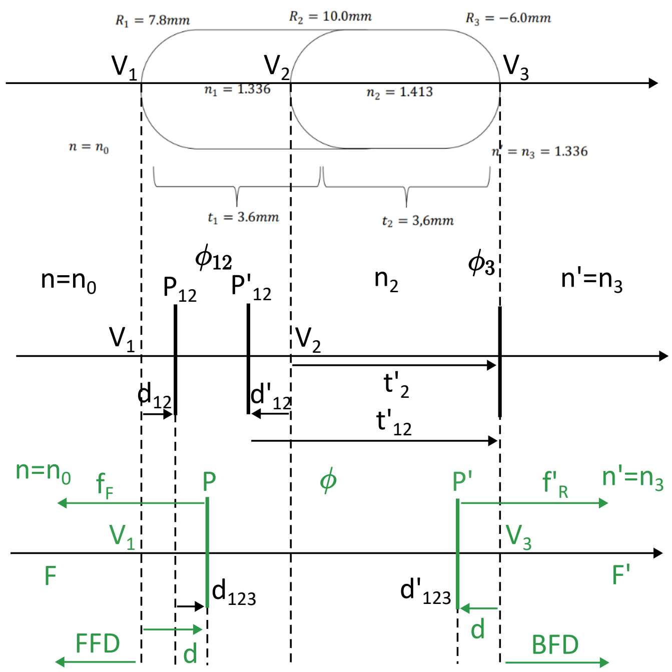


Figure 1.3 Gaussian reduction for the three-surfaces object.

	Object space	Space 1	Surface 1	Space 2	Surface 2	Space 3	Surface 3	Space 4	Image space
C			0.128		0.1		-0.167		
t		15.167		3.6		3.6		16.856	
n		1		1.336		1.413		1.336	
$-\phi$			-0.043		-0.008		-0.013		
t/n		15.167		2.695		2.548		12.617	
y	1	1	1		0.884		0.757		0
nu	0	0		-0.043		-0.05		-0.060	
u	0	0						-0.045	
y	0		0.910		0.967		1	1	1
nu		0.060		0.021		0.013		0	0
u		0.060						0	0

We must compare the t in blue with the FFD and the red t with the BFD. The differences are due to the approximation in intermediate computations. We can see that both methods yield the same answer, despite that ynu raytracing is way faster than Gaussian reduction.

The effective focal length is defined considering the magnification nu divided by the input ray:

$$f'_E = \frac{1}{\phi'} = -\frac{y_1}{nu'} = \frac{1}{0.060} = 16.667\ mm \longrightarrow f'_R = n_3 f'_E = 22.267\ mm.$$

Similarly,

$$f'_E = \frac{1}{\phi} = \frac{y_2}{nu} = \frac{1}{0.060} = 16.667\ mm \longrightarrow f_F = -n_0 f_E = -16.667\ mm.$$

The focal lengths match exactly as the ones computed by Gaussian reduction. We can also compute the principal planes shifts, but we will not do it as we already know the answer.

1.8.3 Table worksheet

Surface	0	1	2	3	4	5	6	7
C								
t								
n								
$-\phi$								
t/n								
y								
nu								
u								
y								
nu								
u								

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