Notes of Optical design and instrumentation

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Preface

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Introduction to Geometrical Optics principles

Chapter 1

Applications

1.1 Thin prisms and dispersing prisms

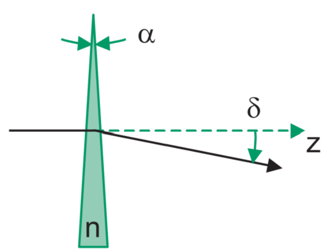
1.1.1 Thin prisms

Thin prisms introduce small angular beam deviations δ that is approximately independen of the incident angle:

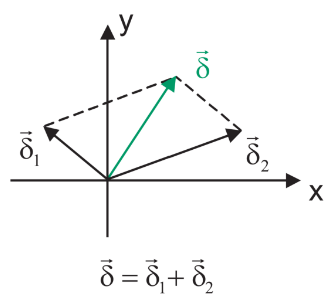
$$\delta \approx -(n-1)\alpha. \tag{1.1}$$

The deviation is measured in prism diopters. A prisms of 1 diopter deviates a beam by $1\ cm$ at $1\ m$. The beam deviation is parallel to a principal section of the prism and towards the thick end of the prism. De magnitude and direction of the deviation defines a vector perpendicular to the optical axis (xy plane). The net deviation vector for a series of thin prisms is then the vector sum of the copmonent vectors:

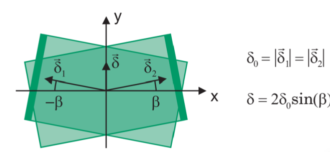
$$\boldsymbol{\delta} = \boldsymbol{\delta}_1 + \boldsymbol{\delta}_2.$$



(a) Thin prism



(b) Beam deviation



(c) Risley prism

1.1.2 Risley prism

A **Risley prism** consists of a pair of identical, but opposing, thin prisms. The prisms are counter-rotated by $\pm\beta$ to obtain a variable net deviation in a fixed direction. The Risley prism allows the fine angular alignment for an optical system by adjusting the prism orientation β.

1.1.3 Thin prism dispersion

Thin prism

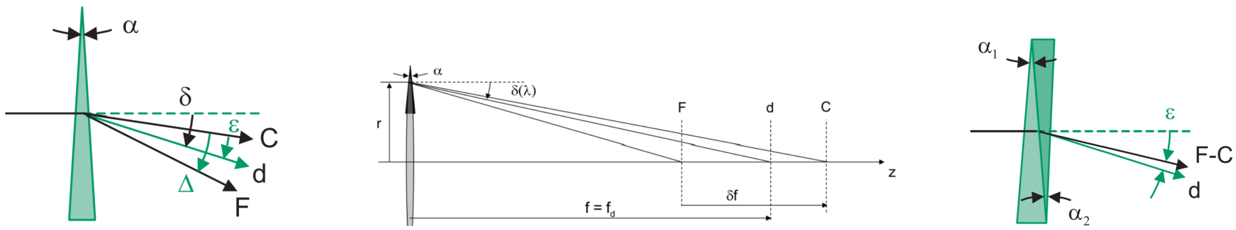
The **dispersion of a thin prism** Δ measures the total angular spread from C to F light, and the **secondary dispersion** ϵ gives the spread from the C to d wavelengths. The results depend on the index n_d, Abbe number ν and partial dispersion ratio P of the glass:

$$\text{Deviation} \quad \delta = -(n_d - 1)\alpha \tag{1.2}$$

$$\text{Dispersion} \quad \Delta = -(n_F - n_C)\alpha, \quad \Delta = \frac{\delta}{\nu} \tag{1.3}$$

$$\text{Secondary dispersion} \quad \varepsilon = -(n_d - n_C)\alpha, \quad \varepsilon = P\Delta = P\frac{\delta}{\nu}. \tag{1.4}$$

An inverted prism deviates a ray up and has a negative vertex angle α.



(a) Thin prism dispersion **(b)** Edge of a thin prism **(c)** Achromatic thin prism

Deviations and dispersions adds.

$$\delta = \sum_i \delta_i, \quad \Delta = \sum_i \Delta_i, \quad \varepsilon = \sum_i \varepsilon_i. \tag{1.5}$$

Achromatic thin prism

An **achromatic thin prism** or **achromatic wedge** provides deviation without dispersion. Opposite prisms made from two different glasses (n_{d1}, ν_1, P_1) and (n_{d2}, ν_2, P_2) are combined to force the dispersion between the F and C wavelengths to be zero. A deviation of δ is maintained for d light:

$$\Delta = \Delta_1 + \Delta_2 = \longrightarrow \Delta_1 = \frac{\delta_1}{\nu_1}, \ \Delta_2 = \frac{\delta_2}{\nu_2} \longrightarrow \delta_2 = -\frac{\nu_2}{\nu_1}\delta_1.$$
$$\delta = \delta_1 + \delta_2 = \delta_1 - \frac{\nu_2}{\nu_1}\delta_1 = (\nu_1 - \nu_2)\frac{\delta_1}{\nu_1} = -(\nu_1 - \nu_2)\frac{(n_{d1} - 1)\alpha_1}{\nu_1}$$

Therefore,

$$\frac{\alpha_1}{\delta} = \frac{1}{\nu_2 - \nu_1}\frac{\nu_1}{n_{d1} - 1} \tag{1.6}$$
$$\frac{\alpha_2}{\delta} = -\frac{1}{\nu_2 - \nu_1}\frac{\nu_2}{n_{d2} - 1}. \tag{1.7}$$

The high-dispersion prism is inverted to obtain an opposing deviation. While the F and C wavelegths are corrected, a residual secondary dispersion remains. For most glass pairs, d light will be bent more than the F and C wavelengths:

$$\frac{\varepsilon}{\delta} = \frac{P_2 - P_1}{\nu_2 - \nu_1} = \frac{\Delta P}{\Delta \nu}. \tag{1.8}$$

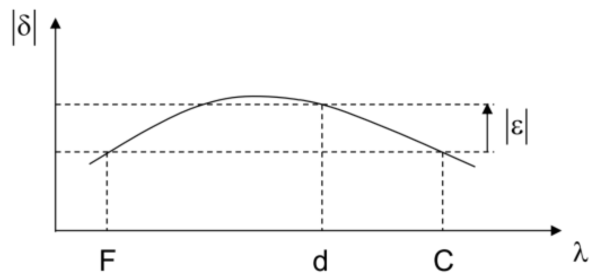
For most glases, $\frac{\varepsilon}{\delta} > 0$. The shape of the curve of $|\delta| - \lambda$ is concave and the maxium dispersion does not occur at d ligth. The achromatic thin prism has about 40 less secondary dispersion compared to a simple thin prism.

Direct vision prism

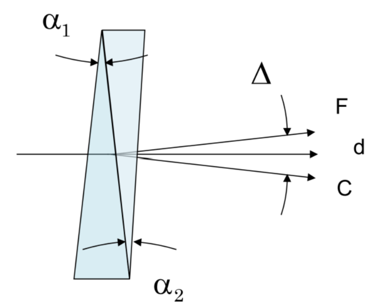
A **direct vision prism** uses opposing prisms to provide dispersion without deviation of the d light.

We first set the total dispersion to zero: $\delta = \delta_1 + \delta_2 = 0$. Then,

$$\Delta = \Delta_1 + \Delta_2 = \frac{\delta_1}{\nu_1} + \frac{\delta_2}{\nu_2} = -\frac{\nu_1 - \nu_2}{\nu_1\nu_2}\delta_1 \longrightarrow \frac{\alpha_1}{\Delta} = \left(\frac{\nu_1\nu_2}{\nu_1 - \nu_2}\right)\left(\frac{1}{n_{s1} - 1}\right) \wedge \frac{\alpha_2}{\Delta} = -\left(\frac{\nu_1\nu_2}{\nu_1 - \nu_2}\right)\left(\frac{1}{n_{d2} - 1}\right).$$



(a) Secondary disperion curve



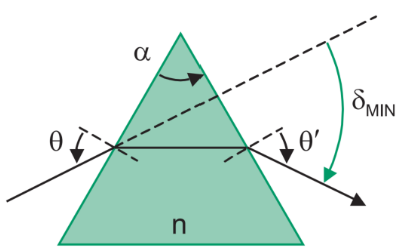
(b) Direct vision prism

1.1.4 Dispersing prism

At minimum deviation, the ray path through a **dispersion prism** is symetric $\theta' = -\theta$. The ray is bent an equal amount at each surface. The deviation is negative for this prims orientation. The **angle of minimum deviation** is

$$\delta_{\text{min}} = \alpha - 2\sin^{-1}[n\sin(\alpha/2)]. \tag{1.9}$$

The following table shows δ_{min} for several n.

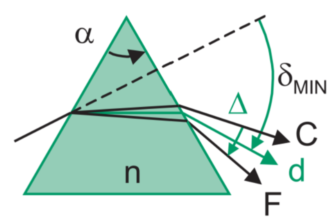


(a) Dispersing prism

For $\alpha = 60°$

n	δ_{MIN}
1.3	–21.1°
1.4	–28.9°
1.5	–37.2°
1.6	–46.3°
1.7	–56.4°
1.8	–68.3°
2.0	–120°

(b) Table for δ_{min}



(c) Dispersion of the prism

The measurement of the index depends only on δ_{min} and the prism apex angle α:

$$n = \frac{\sin\dfrac{\alpha - \delta_{\text{min}}}{2}}{\sin(\alpha/2)}. \tag{1.10}$$

Prism spectrometers can obtain accuracies of one part in 10^6.

In general, the total deviation δ is the sum od the deviations at the two surfaces:

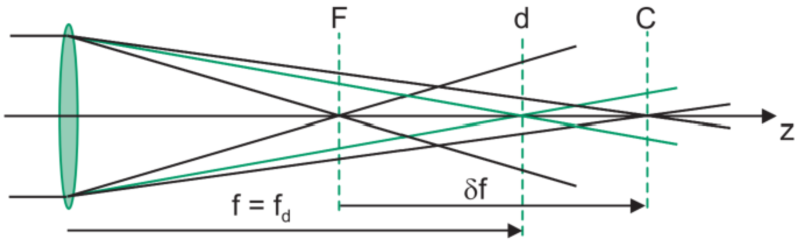
$$\text{Total deviation} \qquad \boxed{\delta = \alpha - \sin^{-1}[\sqrt{n^2 - \sin^2\theta}\sin\alpha - \cos\alpha\sin\theta] - \theta}\,.$$

1.2 Chromatic effects

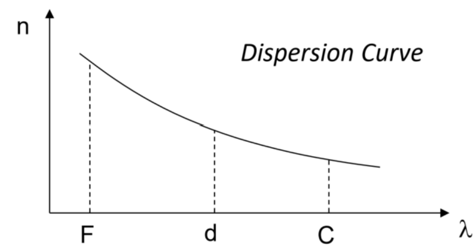
1.2.1 Chromatic aberration

Axial chromatic aberration or **axial color** is a variation of the system focal lenfgth with wavelenegth. This aberration derives from the dispersion of the glass as the index changes with wavelength $n(\lambda)$.

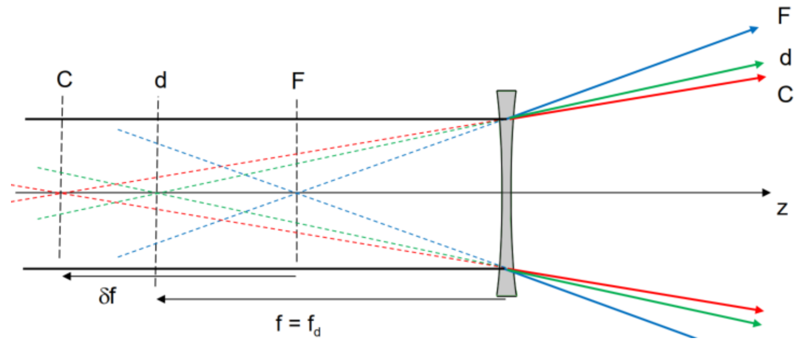
Because of the higher index for F light, blue light is bent more and therefore the blue focus is closest to the lens.



(a) Chromatic aberration, $f > 0$



(b) Dispersion curve



(c) Chromatic aberration, $f < 0$

How much does the focal length change for the F and C wavelengths?

We look at the difference in power between these two wavelengths:

$$\delta\phi = \phi_F - \phi_C = (n_F - 1)(C_1 - C_2) - (n_C - 1)(C_1 - C_2) = (n_F - n_C)(C_1 - C_2)$$

$$\delta\phi = \underbrace{\frac{n_F - n_C}{n_d - 1}}_{1/\nu}\underbrace{(n_d - 1)(C_1 - C_2)}_{\phi_d} = \frac{\phi_d}{\nu}.$$

Similarly, for the focal length:

$$\delta f = f_C - f_F = \frac{1}{\phi_C} - \frac{1}{\phi_F} = \frac{\phi_F - \phi_C}{\phi_C\phi_F} = \frac{\delta\phi}{\phi_C\phi_F} \approx \frac{\delta\phi}{\phi_d^2} = \frac{\phi_d}{\nu\phi_d^2} = \frac{f_d}{\nu}.$$

The foci of F, d and C are not evenly spaced due to the shape of the dispersion curve. The relative order of the foci is reversed for a negative lens.

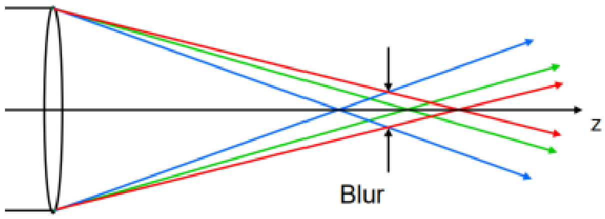
+ lens chromatic aberration $$\delta f_{CF} = f_C - f_F, \quad \delta\phi_{FC} = \phi_F - \phi_C, \quad \frac{\delta f_{CF}}{f_d} = \frac{\delta\phi_{FC}}{\phi_d} = \frac{1}{\nu} \tag{1.11}$$

- lens chromatic aberration $$\text{same, with } f_d < 0, \quad \phi_d < 0, \quad \delta f_{CF} < 0, \quad \delta\phi_{FC} < 0\,. \tag{1.12}$$

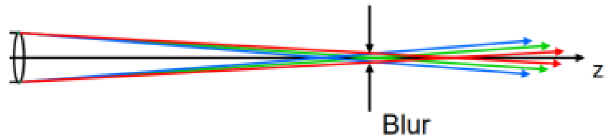
1.2.2 Type of chromatic aberrations

Longitudinal chromatic aberration

The blur associated wit hthe chromatic aberration of the objective lens limits the performance of an objective. To reduce the blut, a small diameter objective lens is required. The blur is then proportional to the lens diameter.



(a) Axial longitudinal chromatic aberration

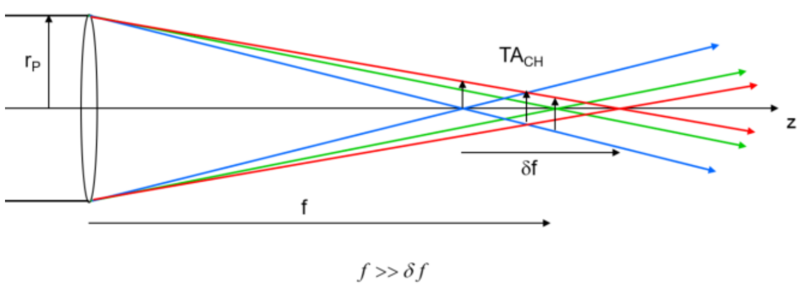


(b) Small diameter to reduce aberration

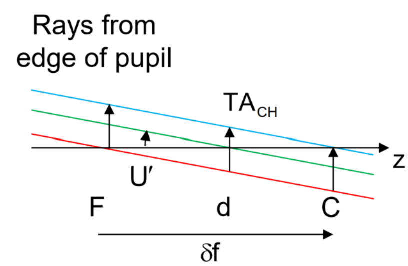
Transverse axial chromatic aberration

Transverse axial chromatic aberration measures the image blur size due to axial chromatic aberration. It depends only on the glass and the pupil radius r_P (stop at the lens):

$$\mathrm{TA}_{CH} = \frac{r_P}{\nu}. \tag{1.13}$$



(a) Transverse axial chromatic aberration



(b) Color swaped, should be blue first

Lateral chromatic aberration

Lateral chromatic aberration or **lateral color** is caused by dispersion of the chief ray. The edge of the lens behaves like a thin prism. Off-axis image points will exhibit a radial color smear. The blur length increases linearly with the image height. Each color has a different lateral magnification.

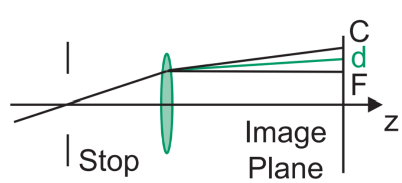
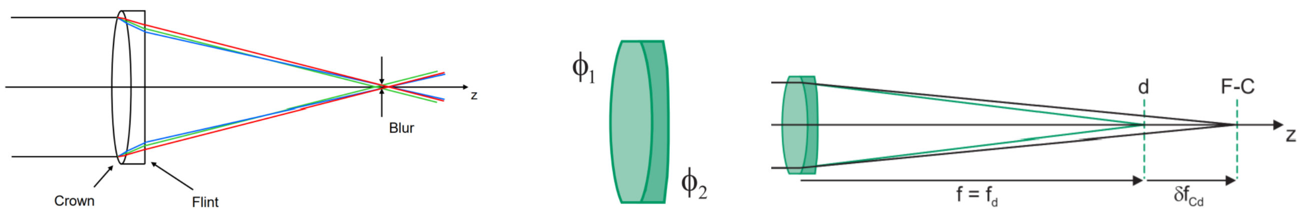


Figure 1.4 Lateral chromatic aberration

1.2.3 Achromatic doublet

The thin lens **achromatic doublet** corrects longtiduinal chromatic aberration by combining a positive element with a negatvie one. Two different glasses (ν_1, P_1) and (ν_2, P_2) are used.

How do we design the individual powers of the achromatic doublet?



(a) Achromatic doublet **(b)** Composition **(c)** Secondary aberration

$$\phi = \phi_1 + \phi_2 \Longrightarrow \delta\phi_{FC} = \delta\phi_{FC1} + \delta\phi_{FC2} = \frac{\phi_1}{\nu_1} + \frac{\phi_2}{\nu_2} = \phi_F - \phi_C = 0 \longrightarrow \frac{\phi_1}{\nu_1} = -\frac{\phi_2}{\nu_2}.$$

$$\phi = \phi_2 - \frac{\nu_1}{\nu_2}\phi_2 = \frac{\nu_2 - \nu_1}{\nu_2}\phi_2 \longrightarrow= \boxed{\frac{\phi_2}{\phi} = -\frac{\nu_2}{\nu_1 - \nu_2} \wedge \frac{\phi_1}{\phi} = \frac{\nu_1}{\nu_1 - \nu_2}}.$$

All we have done is to force the axial focus for F and C light. However, the d line can focus at a different location. This is known as **secondary chromatic aberration**.

Ejemplo 1.1 **Design of an achromatic doublet**

Design a $160\ mm$ focal length thin-lens ahcromatic doublet using the following glases. Provide the focal lengths and indices of refraction of the two thin lenses.

Glass 1: Fused Silica, 458678, Glass 2: SF6, 805254.

Solution

Glass 1: $n_1 = 1458,\ \nu_1 = 67.8,$ Glass 2: $n_2 = 1.805,\ \nu_2 = 25.4.$

$$\frac{1}{f_2} = -\frac{\nu_2}{\nu_1 - \nu_2}\frac{1}{f} = -\frac{25.4}{67.8 - 25.4}\frac{1}{160} = -0.00374\ mm^{-1} \longrightarrow f_2 = -267.380\ mm$$
$$\frac{1}{f_1} = \frac{\nu_1}{\nu_1 - \nu_2}\frac{1}{f} = \frac{67.8}{67.8 - 25.4}\frac{1}{160} = 0.0010\ mm^{-1} \longrightarrow f_1 = 100\ mm.$$

1.3 Illumination systems

Bibliography

Mathematics

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