

# **Notes of Optical design and instrumentation**

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# Preface

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## **Part I**

# **Introduction to Geometrical Optics principles**

# Chapter 1

# Introduction to Optics

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## 1.1 Introduction

### 1.1.1 Light propagation

Geometrical optics is the study of light in the limit of short wavelengths. We treat light as propagating rays. Geometrical optics usually ignores interferences, diffraction, polarization and quantum effects.

It often includes:

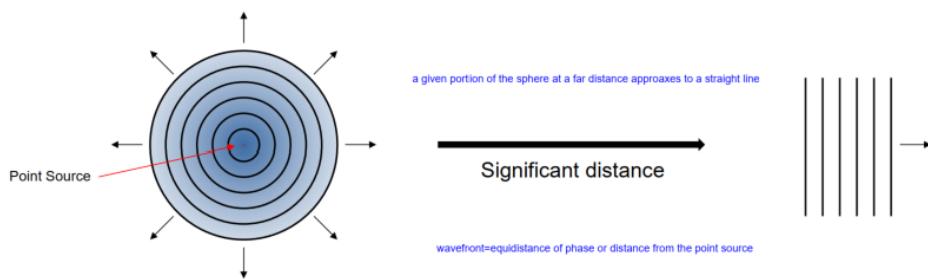
- Reflection, refraction
- Optical design
- Imaging properties
- Aberrations
- Radiometry

Light is a self-propagating EM wave where electric and magnetic fields are perpendicular or transverse to direction of propagation. In a vacuum, light propagates at the speed of light  $c$ , which is

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792458 \times 10^8 \text{ (m/s).} \quad (1.1)$$

The **wavelength**  $\lambda$  is the distance between two peaks or two valleys on the wave.

A **wavefront** is a surface of constant propagation time from the source. It begins from a point source in spherical form, and as it propagates away, a given solid arc tends to behave as a planar wavefront.



**Figure 1.1** We can treat the wavefront as planar when assuming a distant object.

The time for one wavelength to pass is known as the **period**  $T$ :

$$T = \frac{\lambda}{V} \quad (\text{s}), \quad (1.2)$$

where  $V$  is the velocity of propagation. The number of wavelengths to pass in one second is the **frequency**  $\nu$ :

$$\nu = \frac{1}{T} \quad (\text{s}^{-1})(\text{Hz}). \quad (1.3)$$

### 1.1.2 Sign convention

We define the sign convention for which the light propagates. It allows us to keep track of physical quantities and multiple reflections when analyzing complex optical systems.

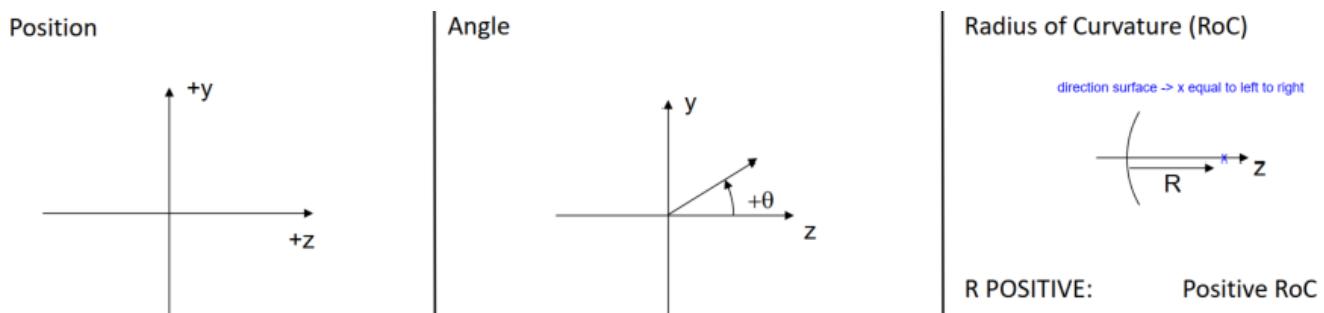


Figure 1.2

### 1.1.3 Electromagnetic spectrum

The light can be of various wavelengths (frequencies) which translates to the color of the light. The range of the wavelengths is called the **electromagnetic spectrum**. The **index of refraction** tells how much the

## Electromagnetic spectrum

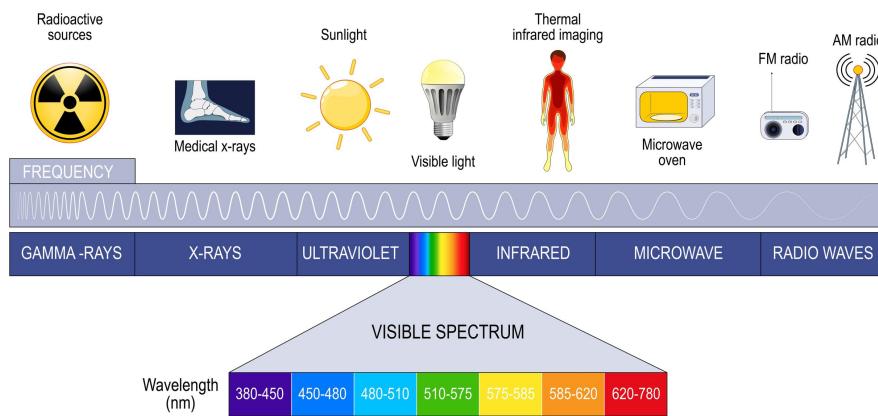


Figure 1.3

light is slowed down in a medium with respect to vacuum.

$$\text{Index of refraction} \quad n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{V} \geq 1 . \quad (1.4)$$

From one medium to another, the frequency remains unchanged but only the wavelength is modified. The index of refraction is a function of the wavelength and of the temperature.

### Vaccum equal to air

In geometrical optics, vacuum and air are used interchangeable as the index of refraction of air is  $n \approx 1$ .

### 1.1.4 Optical path length

The **optical path length** (OPL) is the equivalent distance in vacuum that light would cover in the same time as it takes to cross the actual medium.

$$\text{Optical path length} \quad \text{OPL} = \int_a^b \mathbf{n}(s) \cdot d\mathbf{s} . \quad (1.5)$$

When the medium is homogeneous, the index  $n$  reduces to a constant value. Consequently, the ray travels in **straight lines**.

**Fermat's principle** states that the path taken by the light from one point to another is the path for which the OPL is stationary:

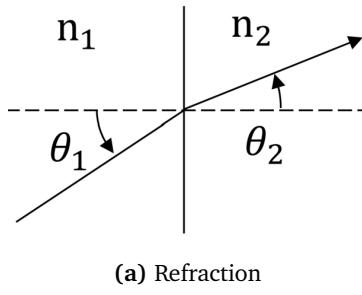
$$\text{Fermat's principle} \quad \frac{d\text{OPL}}{d\text{path}} = 0 . \quad (1.6)$$

### 1.1.5 Snell's laws of reflection and refraction

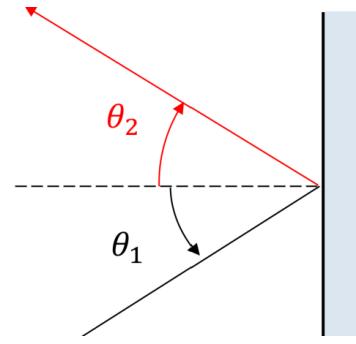
Snell's law can be obtained from Fermat's postulate. They governs the dynamics of the ray when passing through an interface of different index of refractions:

$$\text{Snell's laws} \quad \begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ \theta_1 &= -\theta_2 \end{aligned} \quad (1.7)$$

The angles are measured relative to the surface normal.



(a) Refraction



(b) Reflection

**Figure 1.4** In refraction and reflection, the angles are taken respective to the surface normal.

The reflection is equal to refraction with a negative index:  $n = -n$ .

### 1.1.6 Total internal reflection (TIR)

**Total internal reflection** occurs when the light propagating from a medium  $n_1$  to another  $n_2$ , with  $n_1 > n_2$ , exceed a critical incident angle

$$\text{Total internal reflection} \quad \theta_i > \theta_c = \sin^{-1} \frac{n_2}{n_1} . \quad (1.8)$$

Under this condition, 100% of the light is reflected into  $n_1$ , and no refracted light is present.

## 1.2 Mirrors and prisms

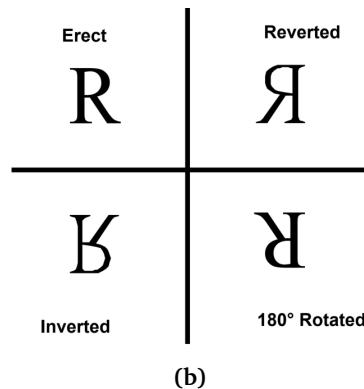
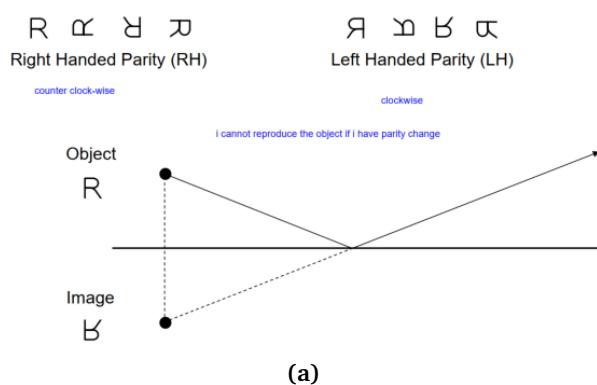
### 1.2.1 Parallel mirrors

Plane mirrors are used to:

- Produce a deviation
- Fold the optical path
- Change image parity

### 1.2.2 Parity change

A reflection from the plane mirror will cause a parity change in the image. An inversion (reversion) is a



parity change about the horizontal (vertical) line, whereas a 180° rotation has no parity change and is rotated about the optical axis. An inversion and a reversion is equivalent to a 180° rotation.

#### Parity change

Only an **odd** number of reflections changes parity.

Parity is determined by looking back against the propagation towards the object. Compare looking directly at the object vs at the reflection.

A lens adds inversion and reversion to the object, so that the image has no parity change, only rotation.

### 1.2.3 Non-parallel plane mirror

The **dihedral line** is the line of intersection of two non-parallel plane mirrors. The ray is deviated twice the angle between the mirrors.

$$\gamma = 2\alpha = \begin{cases} \text{Input-Output rays cross,} & \alpha < 90^\circ \\ \text{Input-Output rays diverge,} & \alpha > 90^\circ \\ \text{Input-Output rays anti-parallel,} & \alpha = 90^\circ \end{cases} \quad (1.9)$$

There are several mirrors,

- **Roof mirror** Two plane mirrors with a dihedral angle of 90°. It is used to insert two reflection in the propagation. The presence of this mirror is indicated by a "V".

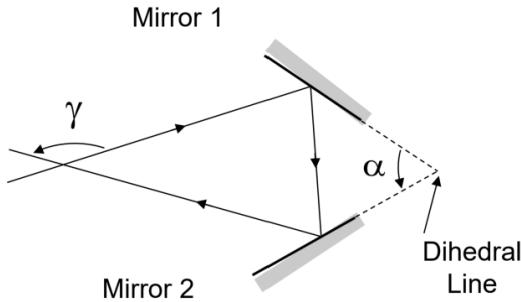


Figure 1.2

### 1.2.4 Prisms and tunnel diagrams

**Prisms** can be considered systems of plane mirrors. The reflection may be due to TIR, or by reflective coating.

A **tunnel diagram** unfolds the optical path at each reflection so that the ray is maintained straight through the propagation in the prism.

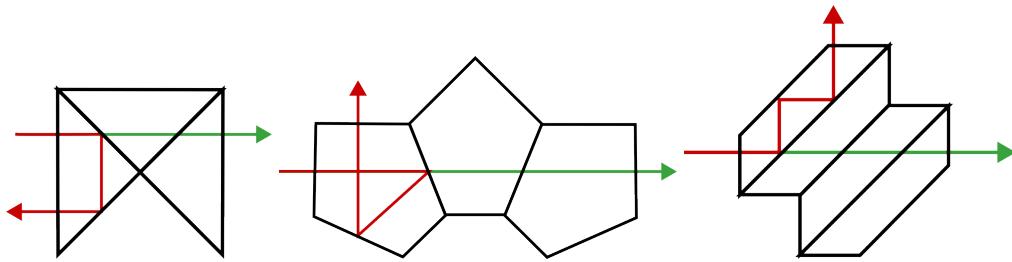


Figure 1.3

### 1.2.5 Reduced thickness

The **reduced thickness** is the vacuum equivalent distance of the medium that has the same propagation effect.

$$\text{Reduced thickness} \quad \tau = \frac{t}{n}. \quad (1.10)$$

Expressing all distances in  $\tau$  is equivalent to propagating the light in only vacuum (or air). This quantity is implicitly in the optical propagation and will be present in equations. When a reflection takes place, both  $n$  and  $t$  are negative, but  $\tau$  remains positive.

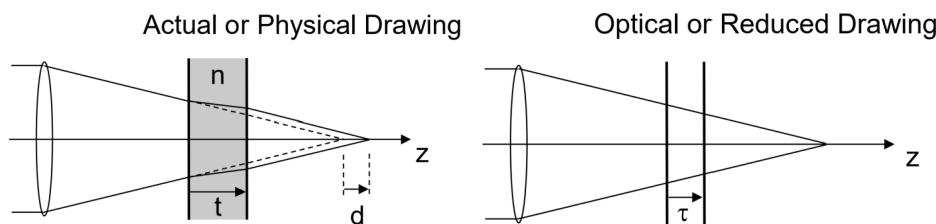
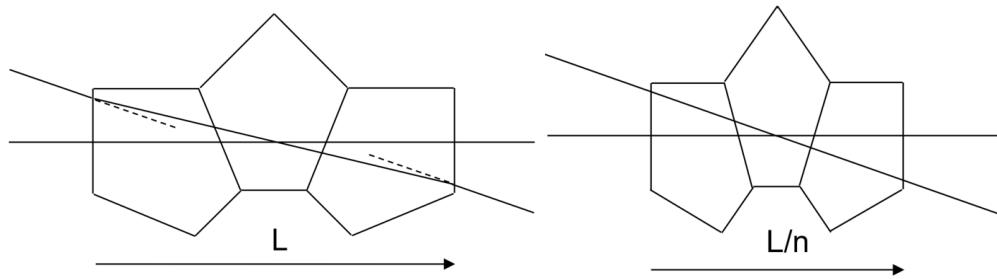


Figure 1.4 Reduced thickness is the vacuum (air) equivalent distance.

Tunnel diagrams are affected by the reduced thickness along the propagation distance. If the total distance is  $L$ , then the reduced is  $L/n$ .



**Figure 1.5** The diagram is only shortened along the direction of the propagation.

In a plate parallel plate (PPP), the beam is shifted horizontally a distance proportional to  $\tau$  when is placed perpendicular ot the optical axis:

$$d = \frac{n - 1}{n} t. \quad (1.11)$$

If it is disposed with an angle  $\theta$ , then the ray will be shifted vertically

$$D \approx -t\theta \frac{n - 1}{n}. \quad (1.12)$$

### Ejemplo 1.1

### Reduced thickness and apparent distance

- a) The fish is 500 mm beneath the surface of the water ( $n = 1.33$ ). For the cat observing in air, how far below the water's surface does the fish appear to be?  
In this case, we have

$$d_{\text{total}} = \frac{500 \text{ mm}}{1.33} = 375.94 \text{ mm.}$$

The fish appears to be 377 mm below the surface of the water.

- b) The cat is 500 mm above the surface of the water. For the fish observing in water, how far abothe the water's surface does the cat appear to be?

The total distance is the sum of the air thickness in terms of the water and the thickness of the water:

$$d_{\text{total}} = 1.33 \cdot 500 \text{ mm} + 500 \text{ mm} = 665 \text{ mm} + 500 \text{ mm} = 1165 \text{ mm.}$$

The cat appears to be 665 mm above the surface of the water.

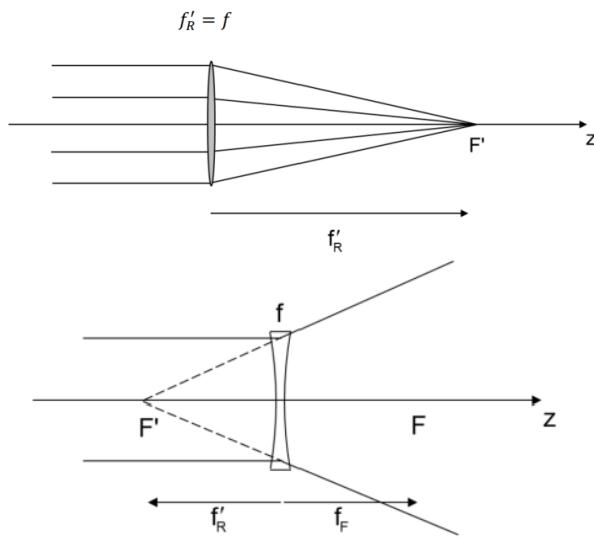
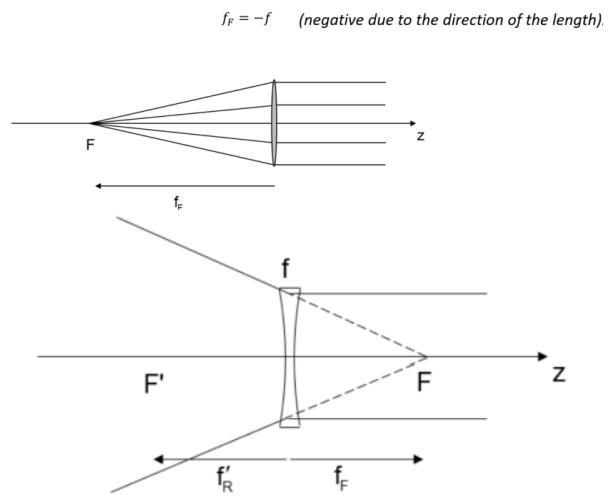
- c) Several months later the cat return to watch the fish again. This time, there is a 100 mm thick layer of ice ( $n = 1.31$ ) on the surface. The fish is still a total physical distance of 500 mm below the surface. Repeat parts a) and b).

In this case, we assume that the thick layer of ice has **replaced** 100 m of the water while the distance of air remains the same.

- For the part a), the distance would be:

$$d_{\text{total}} = \left( \frac{100 \text{ mm}}{1.31} + \frac{400 \text{ mm}}{1.33} \right) + 500 \text{ mm} = 377 \text{ mm} + 500 \text{ mm} = 877 \text{ mm.}$$

The fish appears to be 377 mm below the surface of the ice.

(a) Rear focal length  $f'_R$ (b) Front focal length  $f_F$ 

- For part b), the total equivalent distance is the distance of the water, plus the equivalent distance in water of the ice and air:

$$d_{\text{total}} = 1.33 \cdot \left( \frac{100 \text{ mm}}{1.31} + 500 \text{ mm} \right) + 400 \text{ mm} = 767 \text{ mm} + 400 \text{ mm} = 1166.53 \text{ mm}.$$

The cat appears to be 767 mm above the water, that is, below the air and the ice. We computed first the reduced thickness of ice in order to then convert it to the equivalent in water.

## 1.3 Thin lens Imaging

### 1.3.1 Introduction

A **thin lens** is an idealization of an optical system with:

- Zero thickness  $\tau$ .
- Refracting power  $\phi$ .
- Characterized by its focal length  $f$ .

An object at infinity is imaged to the **rear focal point**  $F'$ , whereas an object at the **front focal point**  $F$  is imaged to infinity. The respective distances from the center of the thin lens to  $F$  is  $f_F$  and to  $F'$  is  $f'_R$ .

#### Positive vs negative thin lenses

A positive lens has a positive focal length  $f > 0$ , a positive  $f'_R > 0$  but a negative  $f_F < 0$ . In the negative lens, **all** is opposite.

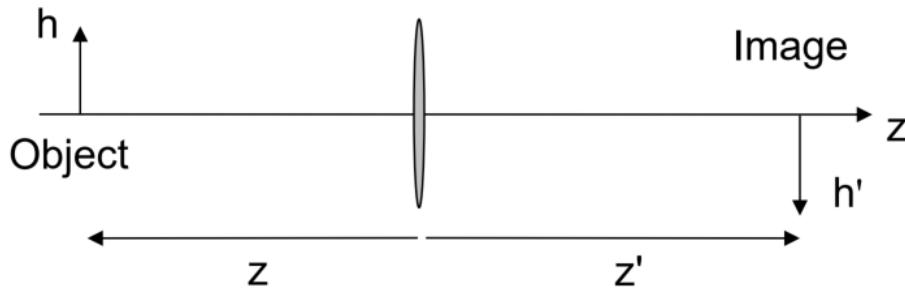


Figure 1.2 Imaging scheme

These points and lengths are purely geometric properties of the lens.

**Real rays** are rays that are physically present, they can be touched. On the other hand, **virtual rays** are rays that are projection of real rays, and cannot be touched. Both type of rays are useful for imaging.

### 1.3.2 Imaging relationships

The imaging property of a thin lens relates the position of the object with that of the image.

The **thin lens equation** is

$$\text{Thin lens equation} \quad \frac{1}{z'} = \frac{1}{z} + \frac{1}{f}. \quad (1.13)$$

The **transverse magnification** is the ratio of the heights:

$$\text{Transverse magnification} \quad m = \frac{h'}{h} = \frac{z'}{z}. \quad (1.14)$$

These two equations are the most fundamental for imaging. They are used extensively through geometrical optics.

Intersecting at least 2 rays is enough to map a point from object to image. The following are the trivial rays used:

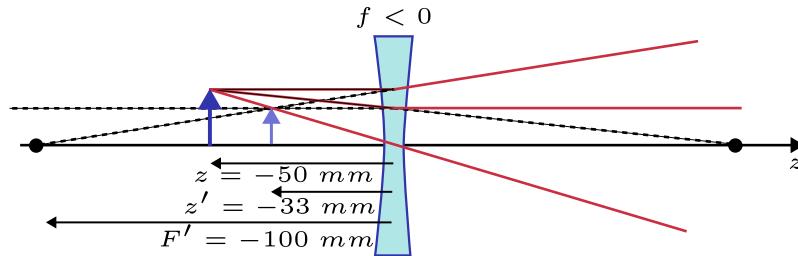
- Parallel ray from the object, emerges (diverges) through the rear focal point.
- Ray from object through the front focal point, emerging parallel (antiparallel).
- A ray that goes directly from the object through the center of the lens which is not refracted.

### Ejemplo 1.2

### Imaging with a negative lens

The ray diagram is illustrated in figure ???. We have traced three rays:

- Parallel to the optical axis from the object, then it is refracted with direction to  $F'$ .
- Direct to  $F$ : it is refracted so that it becomes parallel to the optical axis.
- The chief ray, which maintains its direction through its propagation.



**Figure 1.3** Ray diagram of the problem, the position of the image is  $z'$  and is located to the right of the object. Dashed lines correspond to virtual rays.

The intersection of these three rays produces the image. We can see that the image is to the left of the lens, but to the right of the object. Therefore, it will be a virtual image and demagnified. Using the thin lens equation, considering that  $F' = -100 \text{ mm}$  and  $z = -50 \text{ mm}$  provides

$$\begin{aligned}\frac{1}{z'} &= \frac{1}{F'} + \frac{1}{z} \\ \frac{1}{z'} &= \frac{1}{-100} + \frac{1}{-50} \\ z' &= \frac{(-100)(-50)}{-150} = -33.333 \text{ mm}.\end{aligned}$$

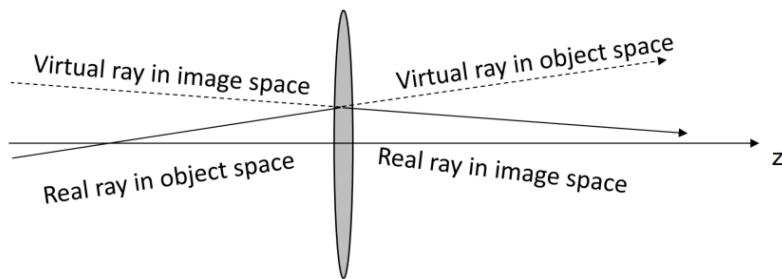
Because  $z' < 0$ , the image is **virtual** and will be to the left of the lens. Its magnification is:

$$m = \frac{z'}{z} = \frac{-33.333}{-50} = 0.667.$$

The image is then erected ( $\text{sgn}(m) = 1$ ), and demagnified ( $|m| < 1$ ) making it smaller than the object.

### 1.3.3 Optical spaces

Any optical object creates two optical spaces: the object space and the image space. Each optical space extends from  $-\infty$  to  $\infty$  and has an associated index of refraction. The connection of both spaces is through the optical object. A **real object** is to the left of the object while a **virtual object** is to the right. A

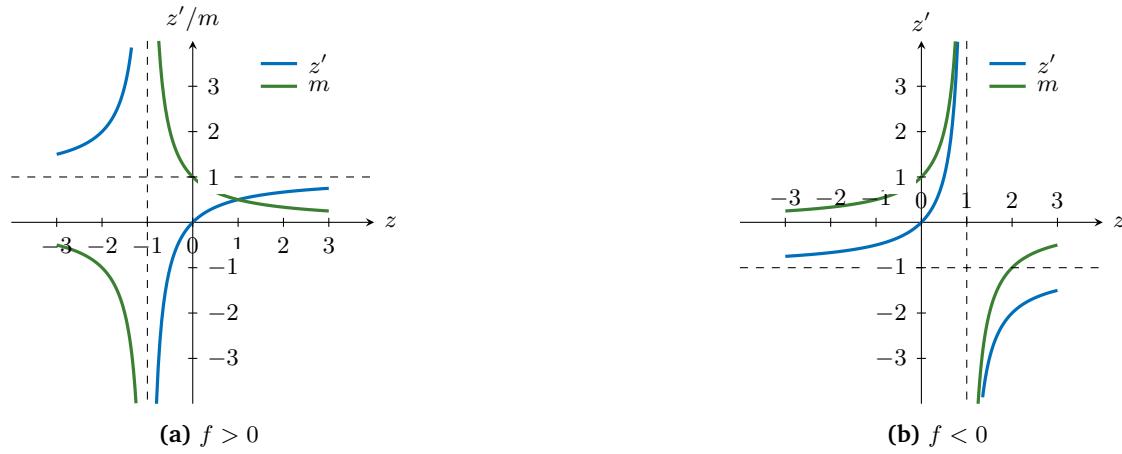


**Figure 1.4**

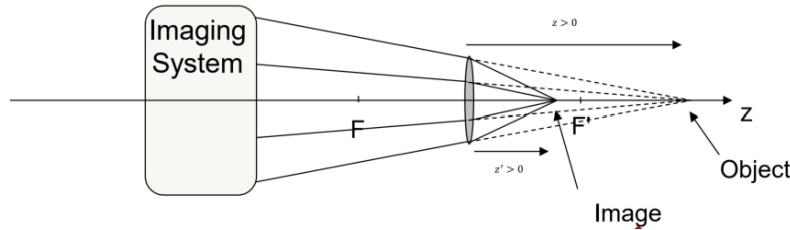
**real image** is to the right of the object and a **virtual image** to the left. In an optical space with negative index, left and right are reversed in these descriptions.

A thin lens creates two optical spaces:

- **Object space** contains the object and the front focal point  $F$ .
- **Image space** contains the image and the rear focal point  $F'$ .



**Figure 1.5** Plot of  $z'$  and  $m$  for positive and negative lenses. We can directly see when an object (image) is real or virtual.



**Figure 1.6** A virtual object is the projection of the image of a previous optical system.

Virtual objects occur when an image is projected into the lens by a previous optical system.

### 1.3.4 Object-image approximations

We can make further appproximations for extreme situations:

- **Distant object** When the magnitude of the object distance  $z$  is more then a few times the magnitude of the system focal length, the image distance  $z'$  is approximately equal to the real focal length.

$$|z| \gg |f| \implies a) z' \approx f \quad b) L = z' - z \approx f - z \approx -z \quad c) m = \frac{z'}{z} \approx \frac{f}{z}. \quad (1.15)$$

- **Distant image** Similarly,

$$|z'| \gg |f| \implies a) z \approx -f \quad b) L = z' - z \approx z' + f \approx z' \quad c) m = \frac{z'}{z} \approx -\frac{z'}{f}. \quad (1.16)$$

This fraction error of these approximations is about  $|f|/|z|$  so they are very useful for distant object/image from over  $4f$ .

### 1.3.5 Field of view

The half **field of view** (HFOV) is often expressed as:

- maximum object height  $h$ .
- maximum image height  $h'$ .

- maximum angular size of the object as seen from the optical system  $\theta_{1/2}$ .
- maximum angular size of the image as seen from the optical system  $\theta'_{1/2}$ .

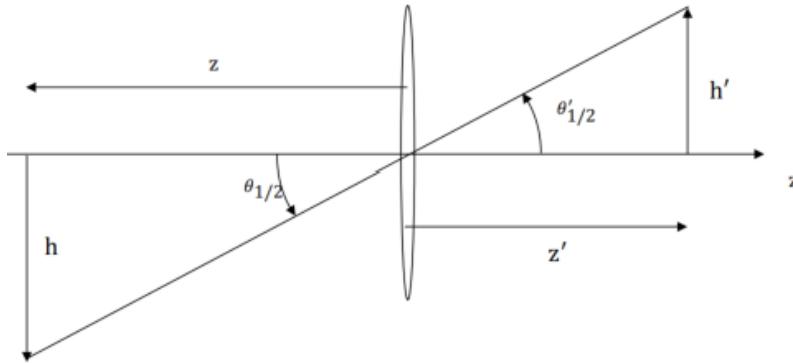


Figure 1.7

$$\text{FOV} = 2\text{HFOV}, \quad \text{HFOV} = \theta_{1/2} = \theta'_{1/2}, \quad \tan \theta_{1/2} = \frac{h}{z} = \frac{h'}{z'}. \quad (1.17)$$

In many situations, the FOV is determined by the detector size, which impose the maximum spatial dimensions.

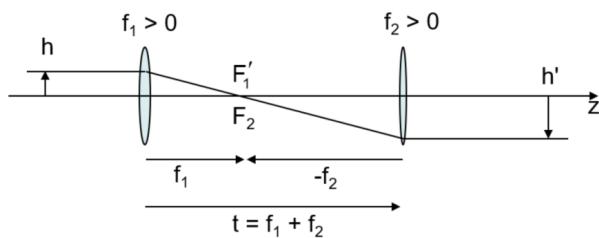
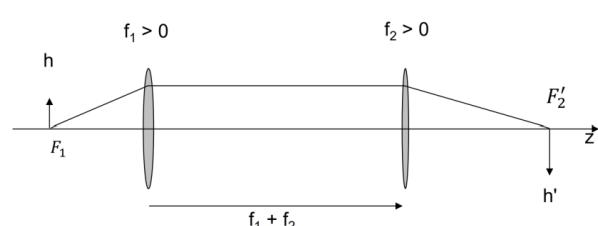
### 1.3.6 Afocal systems

An **afocal system** does not have focal points. Parallel rays will produce parallel images. The only change is in the transverse magnification.

$$m = \frac{h'}{h} = \frac{-f_2}{f_1}. \quad (1.18)$$

## 1.4 Imaging and paraxial optics

An optical system is a collection of optical elements. The first-order properties of the system is characterized by a **single** focal length, or magnification. First-order optics is the optics of perfect imaging systems: no aberrations, where the object is **mapped** to its image.

(a) Keplerian telescope ( $m < 0$ ).(b) Galilean telescope ( $m > 0$ ).

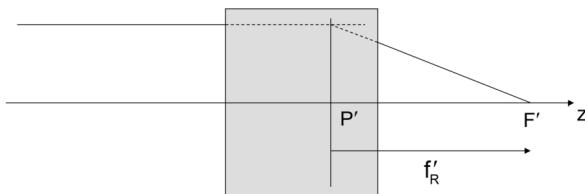
A small number of system properties can completely define and determine the mapping of first-order imaging properties. These are known as the **cardinal points** of the imaging system.

Each time a refracting or reflecting surface is encountered, a new optical space is entered. In general, if a system contains  $N$  surfaces, there will be  $N + 1$  optical spaces. The first space is called the **object space** and the last **image space**.

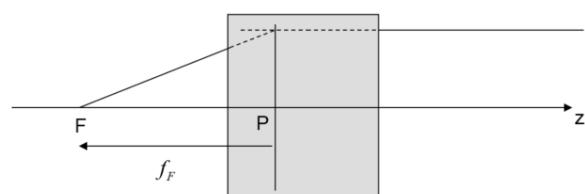
### 1.4.1 General system

A black box is a convenient way to treat the optical system and analyze the refractions.

- An infinite object from left is effectively refracted by the system at the **rear principal plane**  $P'$ . The distance from  $P'$  to the **rear focal point**  $F'$  is the **rear focal length**  $f'_R$ .
- An object starting at  $F$  is effectively refracted by the system at the **front principal plane**  $P$ . The distance from  $P$  to the **front focal point**  $F$  is the **front focal length**  $f_F$ .



(a) Refraction at  $P'$  ( $f'_R > 0$ )



(b) Refraction at  $P$  ( $f_F < 0$ )

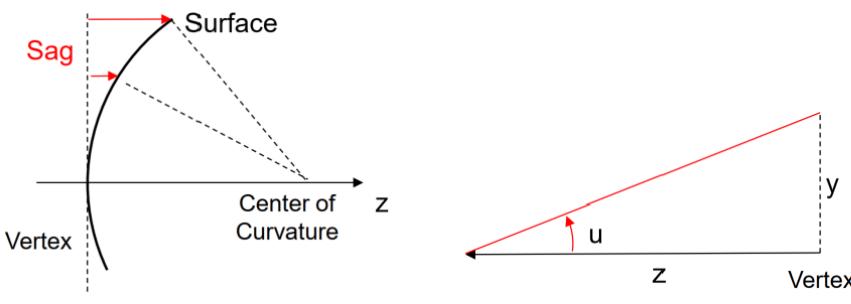
The system can now be treated as a thin lens, with the difference that object and image distances are from their respective principal planes.

### 1.4.2 Paraxial optics

The first-order properties of the system can be found using **paraxial rays**.

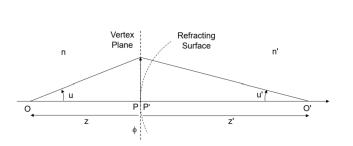
#### Paraxial ray

- Rays are nearly parallel to the optical axis.
- The amount a ray is bent at surfaces is assumed to be small:  $\cos u \approx \cos u'$ .
- The sag of surfaces is ignored:  $|\text{sag}| \ll |R|, |z|, |z'|$ . Rays refract at the vertex.
- Rays are traced using the slopes of the rays instead of ray angles:  $u = y/z$ .



(a) Sag is ignored

(b) Ray slopes instead of ray angles



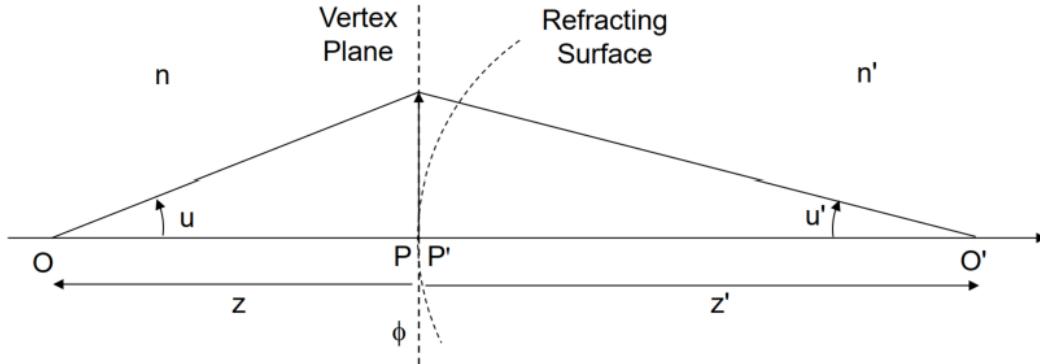
(c) Ray slopes instead of ray angles

## Single refractive surface

Single refracting surfaces are the fundamental objects from which all other are composed of. They are defined by two refractive indices at the object and image space  $n$  and  $n'$ , respectively, and the curvature of the surface:

$$\text{Radius of curvature} \quad R = \frac{1}{C}. \quad (1.19)$$

The **optical power**  $\phi$  is a measure of the bending power of the surface:



**Figure 1.3** Single refracting surface.

$$\text{Optical power} \quad \phi = (n' - n)C \quad (m^{-1}) \text{(diopters)}. \quad (1.20)$$

Then, the **paraxial raytrace equation** describes how the ray will travel after hitting the refracting surface:

$$\text{Paraxial raytrace equation} \quad n'u' = nu - y\phi. \quad (1.21)$$

There are other useful equations, illustrated as follows

$$\frac{n'}{z'} = \frac{n}{z} + \phi, \quad f_E = \frac{1}{\phi}, \quad f_F = -nf_E, \quad f_R' = n'f_E. \quad (1.22)$$

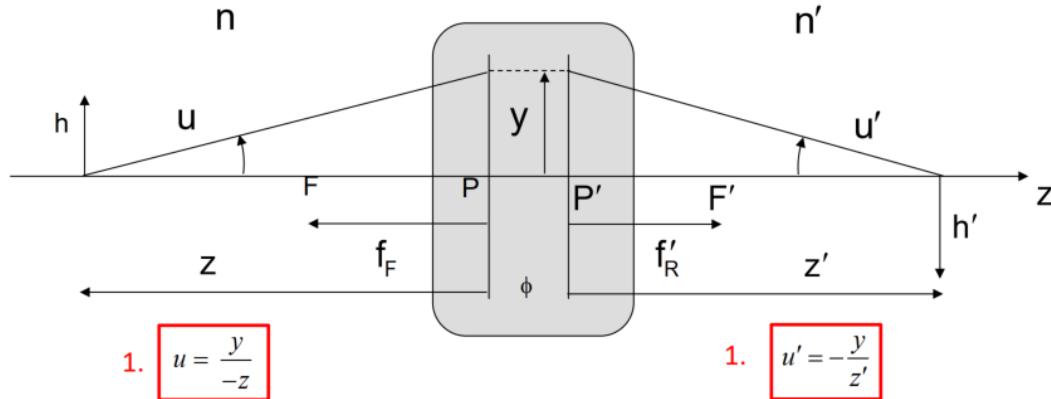
A reflective surface is a special case with  $n' = -n$ .

With all variables defined, the scheme is illustrated as follows:

The focal length  $f_E$  is not a physical distance, but the front and real focal lengths are physical distances.

## 1.5 Gaussian imagery

Gaussian optics is a system of treating imaging as a mapping from object into image space. It is a special case of a **collinear transformation** applied to rotationally symmetric systems, and it maps points to

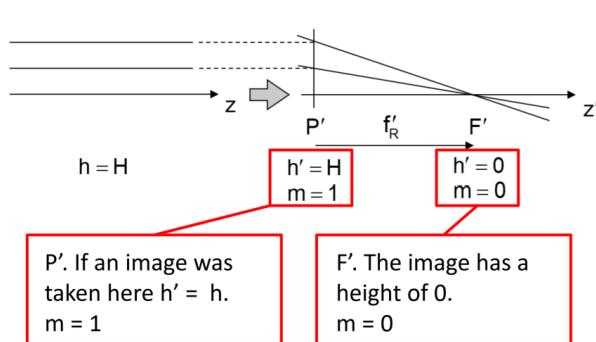


**Figure 1.4** General paraxial system, with parameters defined previously.

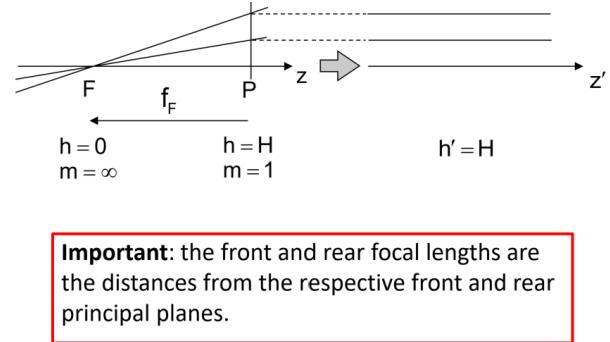
points, lines to lines, and places to planes. The corresponding object and image elements are called **conjugate elements**.

The cardinal points and planes completely describes the focal mapping. They are defined by specific magnifications:

$F$	Front focal point/plane	$m = \infty$
$F'$	Rear focal point/plane	$m = 0$
$P$	Front principal plane	$m = 1$
$P'$	Rear principal plane	$m = 1$



(a) Rear cardinal point/plane



(b) Front cardinal point/plane

Also, there exists **nodal points**  $N$  and  $N'$  that define the location of unit angular magnification for a focal system. A ray passing through one is mapped to a ray passing through the other having the same angle.

$$z_{PN} = z'_{PN} = f_F + f'_R = (n' - n)f_E, \quad m_N = -\frac{f_F}{f'_R} = \frac{n}{n'}. \quad (1.23)$$

- Both nodal points of a single surface are located at the center of curvature of the surface:  $z_{PN} = z'_{PN} = R$ .
- If  $n = n'$ , then  $z_{PN} = z'_{PN} = 0$  and the nodal points are coincident with the respective principal planes.
- The angular subtense of an image seen from  $N'$  equals to the one seen from  $N$ :  $m = h'/h = z'_N/z_N$ .

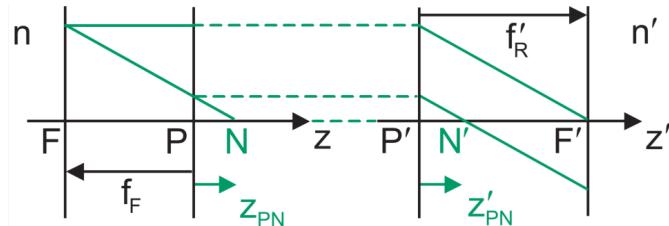
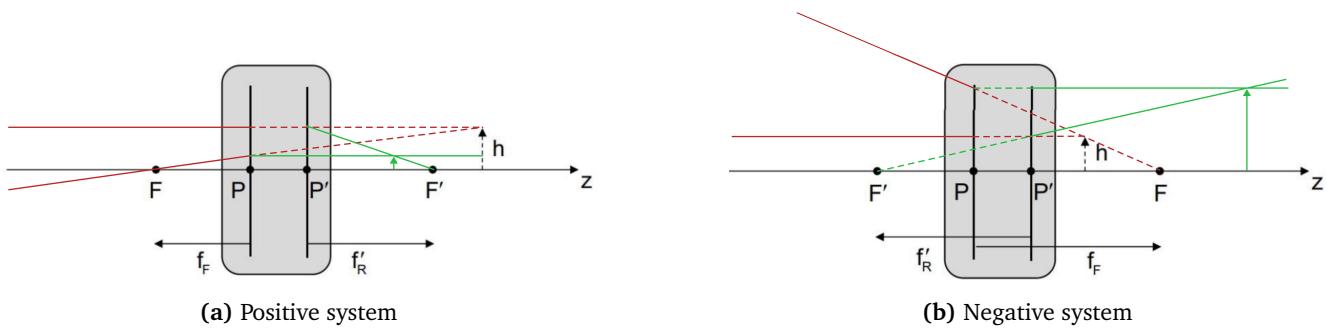


Figure 1.2

### 1.5.1 Representation of an optical system

An optical system can be represented as a set of principal planes and a set of focal points.



Remember that refraction for  $F$  must happen at  $P$  while for  $F'$  at  $P'$ .

### 1.5.2 Newtonian equations

**Newtonian equations** measure object and image distances from the **focal planes**

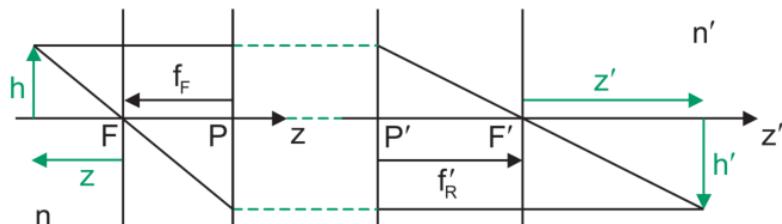


Figure 1.4 Newtonian equations.

$$z = -\frac{f_F}{m} \quad | \quad z' = -mf'_R \quad | \quad zz' = f_F f'_R \quad | \quad \frac{z}{n} = \frac{f_E}{m} \quad | \quad \frac{z'}{n'} = -mf_E \quad | \quad \frac{z z'}{n n'} = -f_E^2$$

### 1.5.3 Gaussian equations

**Gaussian equations** measure object and image distances from the **principal planes**.

A ray angle multiplied by the refractive index of its optical space is called **optical angle**:

$$\text{Optical angle} \quad \omega = nu \quad (-) . \quad (1.24)$$

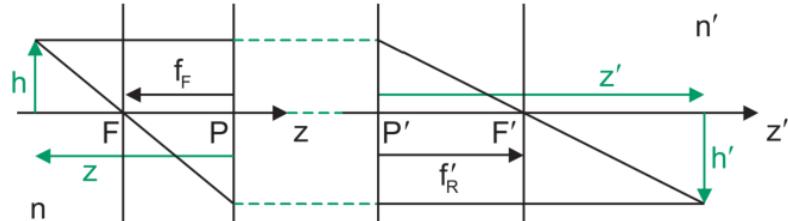


Figure 1.5 Gaussian equations.

$$\begin{array}{l|l|l|l|l} z = -\frac{(1-m)}{m}f_F & z' = (1-m)f'_R & m = -\frac{z'}{z} \frac{f_F}{f'_R} & \frac{f'_R}{z'} + \frac{f_F}{z} = 1 & \frac{z}{n} = \frac{(1-m)}{m}f_E \\ \hline \frac{z'}{n'} = (1-m)f_E & m = \frac{z'/n'}{z/n} & \frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} & & \end{array}$$

### 1.5.4 Longitudinal magnification

The **longitudinal magnification** relates the distances between pairs of conjugate planes.

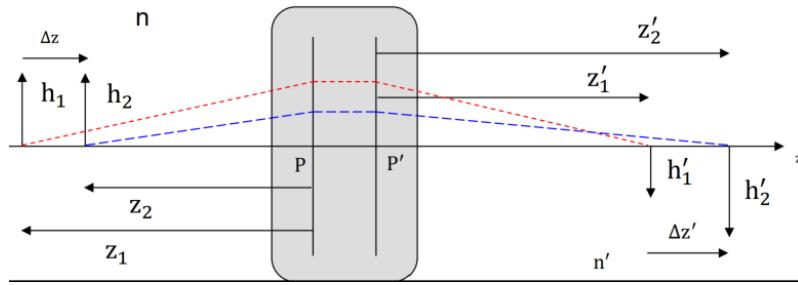


Figure 1.6 Longitudinal magnification allows you to have the thickness of the object or image.

$$\Delta z = z_2 - z_1, \quad \Delta z' = z'_2 - z'_1, \quad m_1 = \frac{h'_1}{h_1}, \quad m_2 = \frac{h'_2}{h_2}, \quad \frac{\Delta z'}{\Delta z} = -\frac{f'_R}{f_F} m_1 m_2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m_1 m_2 \quad (1.25)$$

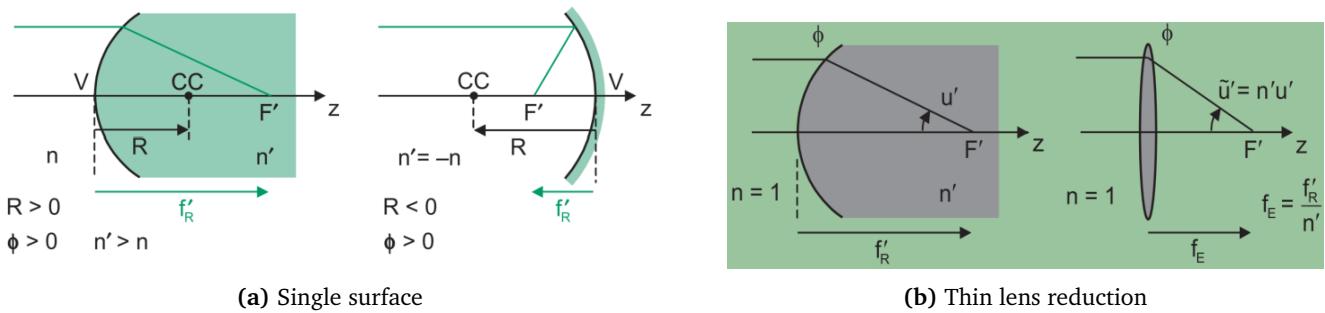
As the plane separation approaches zero,  $m_1 \approx m_2 \approx m$  and the local magnification  $\bar{m}$  is obtained:

$$\bar{m} = \frac{n'}{n} m^2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m^2. \quad (1.26)$$

### 1.5.5 Gaussian properties of a single refracting surface

The radius of curvature  $R$  is defined to be the distance from its vertex to the center of curvature CC. The front and rear principal planes are coincident and located at the surface vertex. In addition, both nodal points are located at the center of the curvature (CC) of the optical surface.

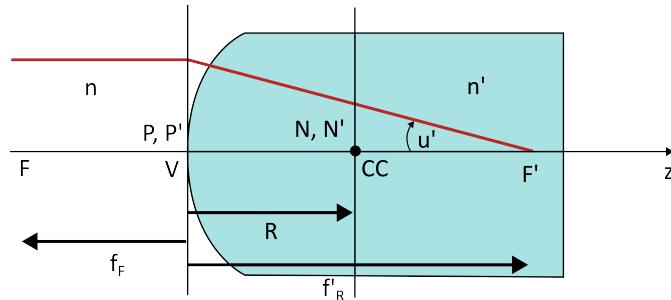
The use of reduced distances and optical angles allows a system to be represented as an air-equivalent system with thin lenses of the same power  $\phi$ .



- a) For a single refracting surface, we have that:

- Both nodal points are located at the center of curvature CC.
  - Front and real principal planes are located at the vortex.
  - The reduced thickness of the surface is the focal length of its thin lens representation.

We illustrate these quantities along with the vertex and the focal lengths in the following figure. We illustrate



**Figure 1.8** Illustration of cardinal point for a single refractive surface.

also some quantities of this surface:

$$C = \frac{1}{R} = 100 \text{ m}^{-1}, \quad \phi = (n' - n)C = 33.3 \text{ m}^{-1}, \quad f_E = \frac{1}{\phi} = 30 \text{ mm},$$

$$f_F = -nf_E = -30 \text{ mm}, \quad f'_R = n'f_E = 40 \text{ mm}.$$

- b) We use the following equation:

$$\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} \longrightarrow z' = \frac{n'zf_E}{nf_E+z}.$$

Replacing the physical values and the EFL:

$$z' = \frac{(1.333)(30)(-100)}{(1)(30) - 100} = +57.129 \text{ mm.}$$

Its height is determined by the magnification:

$$m = \frac{h'}{h} = \frac{z'/n'}{z/n} = \frac{57.129/1.333}{-100/1} = -0.429 \longrightarrow h' = mh = (-0.429)(10\ mm) = -4.29\ mm.$$

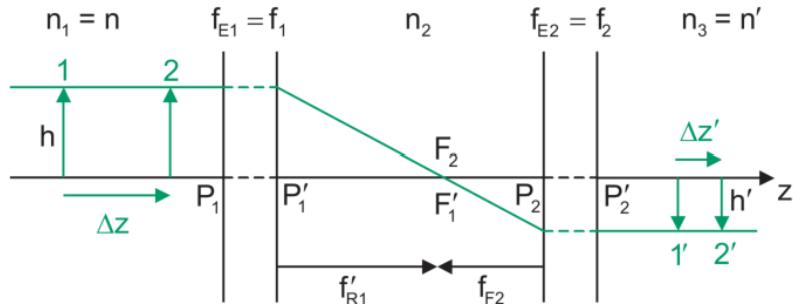


Figure 1.9 Generalized afocal system.

### 1.5.6 Generalized afocal systems

An afocal system is formed by the combination of two focal systems. The rear focal point of the first one is coincident with the front focal points of the second system. Common afocal systems are telescopes, binoculars, and beam expanders. The transverse and longitudinal magnification are constant. Due to this, the cardinal points are not defined, and the Gaussian and Newton equations **cannot** be used to determine conjugate planes. However, any pair of conjugate planes coupled with  $\bar{m}$  can be employed.

$$m = \frac{f_{F2}}{f'_{R1}} = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1}, \quad \bar{m} = \frac{n'}{n} m^2, \quad \frac{\Delta z'/n'}{\Delta z/n} = m^2. \quad (1.27)$$

## 1.6 Object image relationship

### 1.7 Gaussian reduction

**Gaussian reduction** is the process that combine multiple elements two at a time into a single equivalent focal system. The system is defined by its Gaussian properties which include:

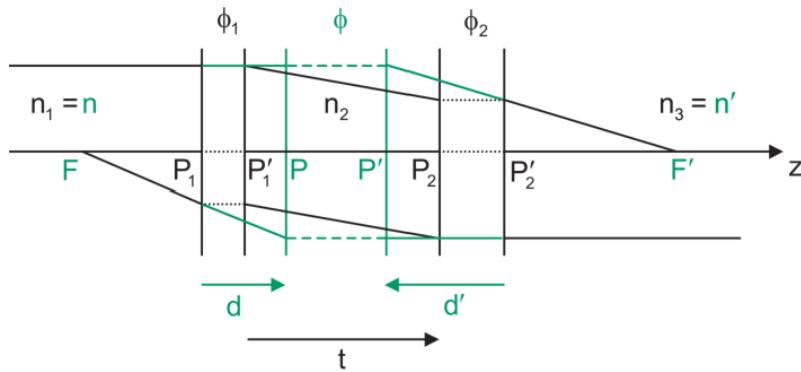


Figure 1.1 Gaussian reduction scheme.

- Power of the overall system.
- Front and rear focal lengths of the overall system.
- Principal planes of the overall system.

Each surface is represented by its principal planes and optical power. When two surface 1 and 2 are combined, the overall power is:

$$\text{Overall power} \quad \phi = \phi_1 + \phi_2 - \phi_1 \phi_2 \tau, \quad \tau = \frac{t}{n_2}. \quad (1.28)$$

As each surface has its principal planes at the vertex, we use only the vertex to move posteriors PPs. The reduction then **shifts** the principal planes of the equivalent system,  $P_{12}, P'_{12}$ , with respect to  $V_1$  and  $V_2$ , respectively, by the following amount:

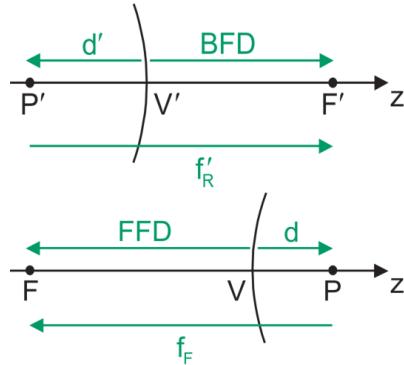
$$\text{Shifting distance from } P_1 \text{ and } P'_1 \quad \frac{d}{n} = \frac{\phi_2}{\phi} \tau, \quad \frac{d'}{n'} = -\frac{\phi_1}{\phi} \tau \quad (1.29)$$

Now,  $P_{12}$  lives in the object space of the equivalent system (left to surface 1) whereas  $P'$  lives in the object space of the system (right to surface 2). If further reductions take place, then the shift of  $P'_{12}$  **must** be considered in the new distance from  $P'_{12}$  to  $P_3$  to create  $\tau_{123}$ .

### 1.7.1 Vertex distances

The **surface vertices** are the mechanical datums in a system and are often the reference locations for the cardinal points. The **back focal distance** (BFD) and **front focal distance** (FFD) are the distance measured from the back (front) vertex to the back (front) focal point  $F'$  ( $F$ ).

$$\text{Distances} \quad \text{BFD} = f'_R + d', \quad \text{FFD} = f_F + d \quad (1.30)$$



**Figure 1.2** Vertex distances are used to define BFD and FFD.

The utility of Gaussian reduction is that the imaging properties of any combination of optical elements can be represented by a system power of focal length, a pair of principal planes and a pair of focal points.

#### Imaging with the final system

Once the single  $\phi$  is obtained, we can do imaging with the generalized thin lens equation, considering  $n$  the object space and  $n'$  the image space:

$$\text{Generalized thin lens} \quad \frac{n'}{z'} = \frac{n}{z} + \phi. \quad (1.31)$$

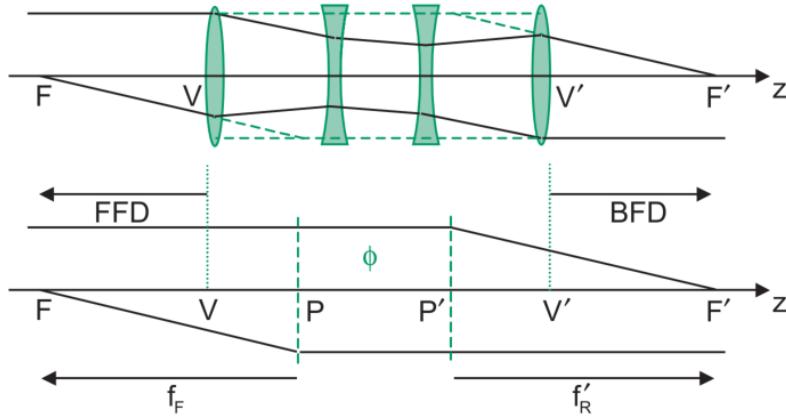
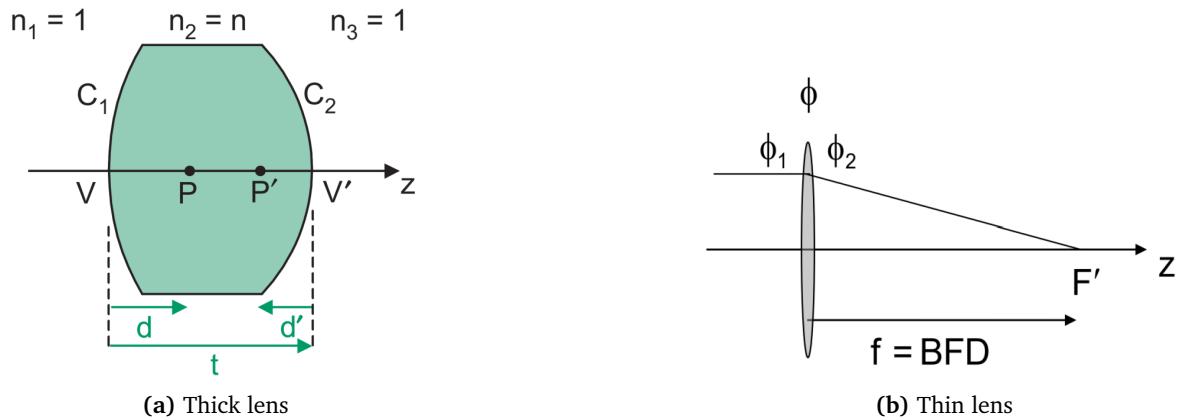


Figure 1.3 All reduces to 5 cardinal points.

Remember that  $z$  must be relative to the final front principal plane  $P$  and  $z'$  to the final rear principal plane  $P'$ . So probably we will need some conversion to give the correct distances.

### 1.7.2 Thick and thin lenses

The **thick lens** is composed of two refractive surfaces with a thickness between them. The overall power



in term of curvature is:

$$\phi_{\text{thick}} = (n - 1)[C_1 - C_2 + (n - 1)C_1C_2\tau], \quad d = \frac{\phi_2}{\phi}\tau, \quad d' = -\frac{\phi_1}{\phi}\tau. \quad (1.32)$$

In this case, the nodal points are coincident with the principal planes.

The **thin lens** approximation is obtained for  $t \rightarrow 0$ , which reduces the overall power to

$$\phi_{\text{thin}} = (n - 1)(C_1 - C_2), \quad d = d' = 0, \quad \text{BFD} = f. \quad (1.33)$$

This idealized element can be considered as a singel refracting surface separating two spaces. The principal planes and nodal points are located at the lens (middle).

For a two positive lens system, we use Gaussian reduction to reduce the effect to a single thin lens. We first compute the overall optical power with the power of individual lenses:

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t = \frac{1}{40} + \frac{1}{40} - \frac{1}{40} \frac{1}{40} \cdot 20 = 0.038 \text{ mm}^{-1} \longrightarrow f_E = \frac{1}{\phi} = 26.67 \text{ mm.}$$

The front and real focal lengths are:

$$f_F = -n_1 f_E = (1)(26.67 \text{ mm}) = -26.67 \text{ mm}, \quad \text{and} \quad f'_R = n_3 f_E = (1)(26.67 \text{ mm}) = 26.67 \text{ mm.}$$

Then the distances  $d$  and  $d'$ , corresponding to the shift from the front (rear) principal planes  $P, P'$  of the equivalent system with respect to  $f_F, f'_R$  are given by

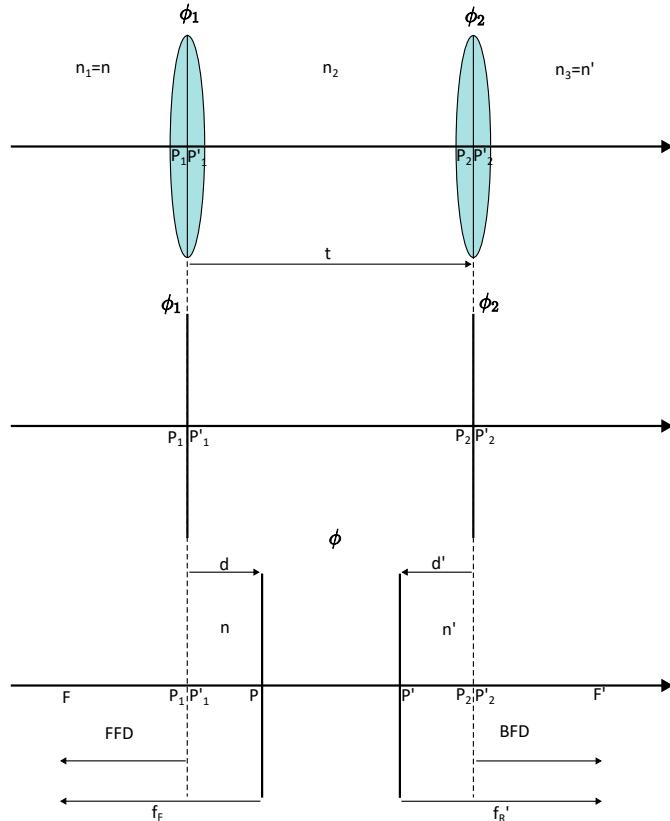
$$d = \frac{\phi_2}{\phi} t = \frac{0.025}{0.038} 20 = 13.158 \text{ mm}, \quad \text{and} \quad d' = -\frac{\phi_1}{\phi} t = -\frac{0.025}{0.038} 20 = -13.158 \text{ mm.}$$

The front (back) focal distances are then: The FFD and BFD are therefore,

$$\text{FFD} = f_F + d = -26.67 \text{ mm} + 13.158 \text{ mm} = -13.512 \text{ mm.}$$

$$\text{BFD} = f'_R + d' = 26.67 \text{ mm} - 13.512 \text{ mm} = 13.512 \text{ mm.}$$

The reduction process and the quantities obtained are illustrated in figure 1.5. The nodal points are coincident with



**Figure 1.5** Gaussian reduction for two positive lenses.

the principal planes.

## 1.8 Paraxial raytrace

### 1.8.1 Introduction

**Paraxial optics** is a method of determining the first-order properties of an optical system that assumes all ray angles are small. It follows the same assumptions of **paraxial optics** regime seen.

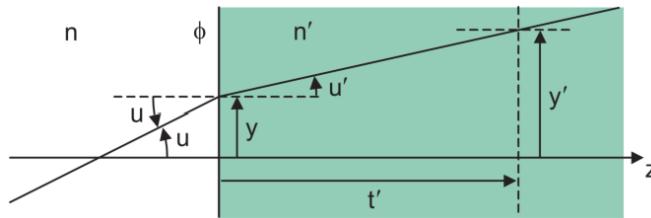


Figure 1.1 A paraxial raytrace is linear with respect to ray angle and heights.

It composes of iterative **refraction** and **Transfer** processes. These type of raytrace are called **YNU raytrace**:

$$\text{Object} \rightarrow \text{Image} \quad \left\{ \begin{array}{ll} \text{Refraction (reflection)} & n'u' = nu - y\phi \quad \omega' = \omega - y\phi \\ \text{Transfer} & y' = y + u't' \quad y' = y + \omega't' \end{array} \right. \quad (1.34)$$

$$\text{Image} \rightarrow \text{Object} \quad \left\{ \begin{array}{ll} \text{Refraction (reflection)} & nu = n'u' + y\phi \quad \omega = \omega' + y\phi \\ \text{Transfer} & y = y' - u't' \quad y = y' - \omega't' \end{array} \right. \quad (1.35)$$

### 1.8.2 Procedure

The procedure is always the same:

- You set the optical properties of the system.
- **Rear cardinal points** Trace a forward ray from object to image, and at the image space, you look for  $t$  that satisfies  $y = 0$  to get the BFD.

$$\phi = -\frac{\omega'_k}{y_1}, \quad f_E = \frac{1}{\phi}, \quad f'_R = n'f_E, \quad \text{BFD} = -\frac{y_k}{u'_k}, \quad d' = \text{BFD} - f'_R. \quad (1.36)$$

- **Front cardinal points** Trace a backward ray from image to object, and at object space, you look for  $t$  that satisfies  $y = 0$  to get the FFD.

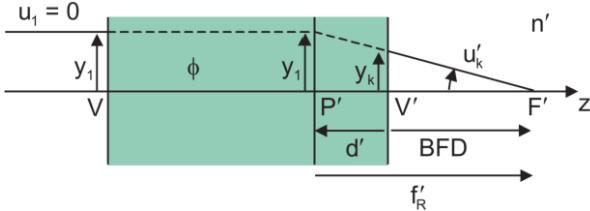
$$\phi = \frac{\omega_1}{y_k}, \quad f_E = \frac{1}{\phi}, \quad f_F = -nf_E, \quad \text{FFD} = -\frac{y_1}{u_1}, \quad d = \text{FFD} - f_F. \quad (1.37)$$

#### Imaging with the final system

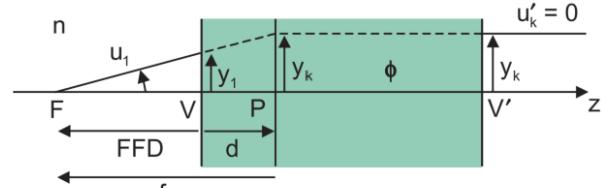
Once the single  $\phi$  is obtained, we can do imaging with the generalized thin lens equation, considering  $n$  the object space and  $n'$  the image space:

$$\text{Generalized thin lens} \quad \frac{n'}{z'} = \frac{n}{z} + \phi. \quad (1.38)$$

Remember that  $z$  must be relative to the final front principal plane  $P$  and  $z'$  to the final rear principal plane  $P'$ . So probably we will need some conversion to give the correct distances.



(a) Finding rear cardinal points

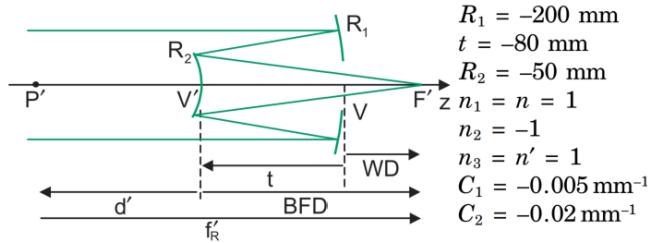


(b) Finding front cardinal points

Once you have the rays traced, you construct a reduced table, where

- List out all the surfaces: the first part of the big table. Parameters vertically in-line with the surface are associated with optical surfaces. Parameters sandwiched between refer to the optical spaces.
- Another box below contains the information about the rays traced to find the cardinal points.

### Ejemplo 1.5



$$\text{Paraxial raytrace: } \omega' = \omega - y\phi \quad y' = y + \omega'\tau'$$

Surface	Object	V	V'	F'
C		-0.005	-0.02	
t	$\infty$		-80	BFD
n	1.0		-1.0	1.0
$-\phi$		-0.01	0.04	
t/n	$\infty$		80	100
y	1.0	1.0	= 0.2	0.0
nu	0.0	=	-0.01	-0.002
u	0.0		0.01	-0.002

The analysis of the raytrace results:

$$\phi = -\frac{n'u'_2}{y_1} = -\frac{-0.002}{1.0} = 0.002 \text{ mm}^{-1} \quad f'_R = f_E = \frac{1}{\phi} = 500 \text{ mm}$$

$$BFD = -\frac{y_2}{u'_2} = -\frac{0.2}{-0.002} = 100 \text{ mm}$$

$$d' = BFD - f'_R = BFD - f_E = -400 \text{ mm}$$

$$WD = BFD + t = 20 \text{ mm}$$

**Gaussian reduction:**

$$\phi_1 = (n_2 - n)C_1 = 0.01 \text{ mm}^{-1}$$

$$\phi_2 = (n' - n_2)C_2 = -0.04 \text{ mm}^{-1} \quad \tau = \frac{t}{n_2} = 80 \text{ mm}$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau = 0.002 \text{ mm}^{-1} \quad f'_R = f_E = \frac{1}{\phi} = 500 \text{ mm}$$

$$d' = -n'\frac{\phi_1}{\phi}\tau = -400 \text{ mm}$$

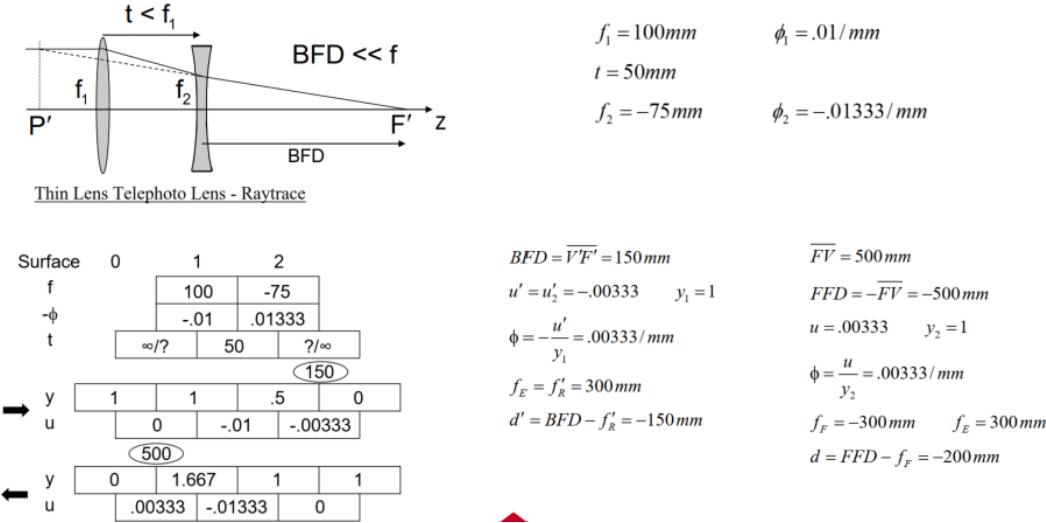
$$BFD = f'_R + d' = f_E + d' = 100 \text{ mm}$$

$$WD = BFD + t = 20 \text{ mm}$$

### Ejemplo 1.6

**Thin lens**

### Ejemplo 1.7



In this case we have three surface, each with their correspond surface curvature  $C$  and index of refraction  $n$ .

- **Gaussian reduction** The optical power of each surface is:

$$\begin{aligned}\phi_1 &= \frac{n_1 - n_0}{R_1} = \frac{1.336 - 1}{7.8 \text{ mm}} = 0.043 \text{ mm}^{-1}, \\ \phi_2 &= \frac{n_2 - n_1}{R_2} = \frac{1.413 - 1.336}{10 \text{ mm}} = 0.008 \text{ mm}^{-1}, \\ \phi_3 &= \frac{n_3 - n_2}{R_3} = \frac{1.336 - 1.413}{-6 \text{ mm}} = 0.013 \text{ mm}^{-1}.\end{aligned}$$

Now, we combine surface 1 with 2:

$$\phi_{12} = \phi_1 + \phi_2 - \phi_1 \phi_2 \tau_1 = 0.043 + 0.008 - 0.043 \cdot 0.008 \cdot \frac{3.6}{1.336} = 0.050 \text{ mm}^{-1}.$$

The shift of the principal plane are given by

$$\begin{aligned}\delta_{12} &= \frac{\phi_2}{\phi_{12}} \tau_1 = \frac{0.008}{0.050} \cdot \frac{3.6}{1.336} = 0.431 \text{ mm} \rightarrow d_{12} = \delta_{12}. \\ \delta'_{12} &= -\frac{\phi_1}{\phi_{12}} \tau_1 = -\frac{0.043}{0.050} \cdot \frac{3.6}{1.336} = -2.317 \text{ mm} \rightarrow d'_{12} = n_2 \delta'_{12} = -3.274 \text{ mm}.\end{aligned}$$

We can see that the front principal plane is displaced from  $V_1$  to the left, while the rear principal plane is shifted to the right of  $V_2$ . In addition, the distance  $d'_{12}$  considered the index  $n_2$  as it belong to that space. The distance of propagation through the index  $n_2$  must be adjusted due to the shift of the rear principal plane:

$$\tau_{12} = \frac{t_2 - d'_{12}}{n_3} = \tau_2 - \delta'_{12} = \frac{3.6}{1.413} + 2.317 = 4.865 \text{ mm}.$$

Now, we compute the total optical power considering the reduction and the third surface:

$$\phi = \phi_{12} + \phi_3 - \phi_{12} \phi_3 \tau_{12} = 0.050 + 0.013 - (0.046)(0.013)(4.865) = 0.060 \text{ mm}^{-1}.$$

The shifts are:

$$\begin{aligned}d_{123} &= n_0 \delta_{123} = \frac{\phi_3}{\phi} \tau_{12} = \frac{0.013}{0.060} \cdot 4.865 = 1.054 \text{ mm} \\ d'_{123} &= n_3 \delta'_{123} = -n_3 \frac{\phi_{12}}{\phi} \tau_{12} = -(1.336) \frac{0.050}{0.060} \cdot 4.865 = -5.416 \text{ mm}.\end{aligned}$$

The total shift from the first surface is the sum of individual front shift computed, while for the last surface is just the shift computed in the last reduction:

$$d = d_{12} + d_{123} = 0.431 + 1.054 = 1.485 \text{ mm}$$

$$d' = d'_{123} = -5.416 \text{ mm.}$$

The front (rear) focal lengths are then

$$f_E = \frac{1}{\phi} = 16.667 \text{ mm} \longrightarrow f_F = -n_0 f_E = -(1)(16.667) = -16.667 \text{ mm}$$

$$f'_R = n_3 f_E = (1.336)(16.667) = 22.267 \text{ mm.}$$

Finally, the FFD and BFD are:

$$\text{FFD} = f_F + d_{123} = -16.667 + 1.054 = 15.613 \text{ mm}$$

$$\text{BFD} = f'_R + d'_{123} = 22.267 - 5.416 = 16.851 \text{ mm.}$$

The reduction process is shown in figure 1.3. The green quantities are the equivalent of the final reduction.

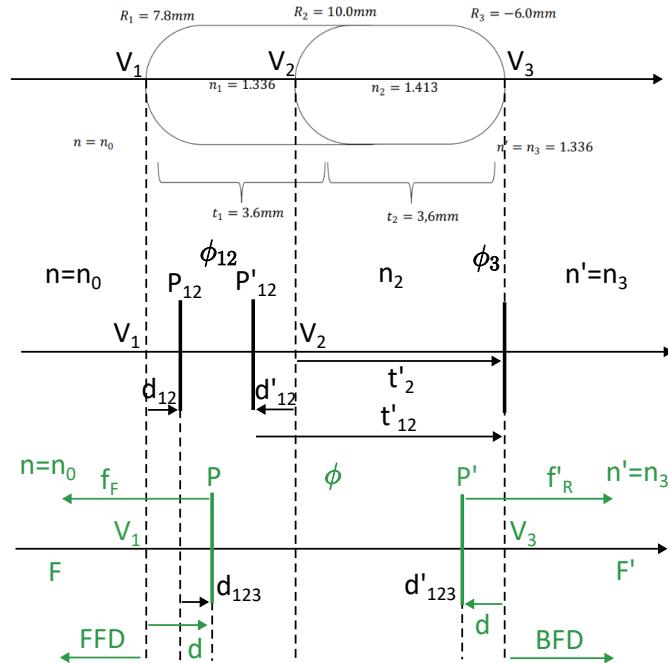


Figure 1.3 Gaussian reduction for the three-surfaces object.

- Ray tracing** For the ray tracing, we will fill the ynu spreadsheet. We will trace two rays, one from left to right and other in opposite direction in order to find the front and real focal lengths.

We must compare the  $t$  in blue with the FFD and the red  $t$  with the BFD. The differences are due to the approximation in intermediate computations. We can see that both methods yield the same answer, despite that ynu raytracing is way faster than Gaussian reduction.

The effective focal length is defined considering the magnification  $nu$  divided by the input ray:

$$f'_E = \frac{1}{\phi'} = -\frac{y_1}{nu'} = \frac{1}{0.060} = 16.667 \text{ mm} \longrightarrow f'_R = n_3 f'_E = 22.267 \text{ mm.}$$

Similarly,

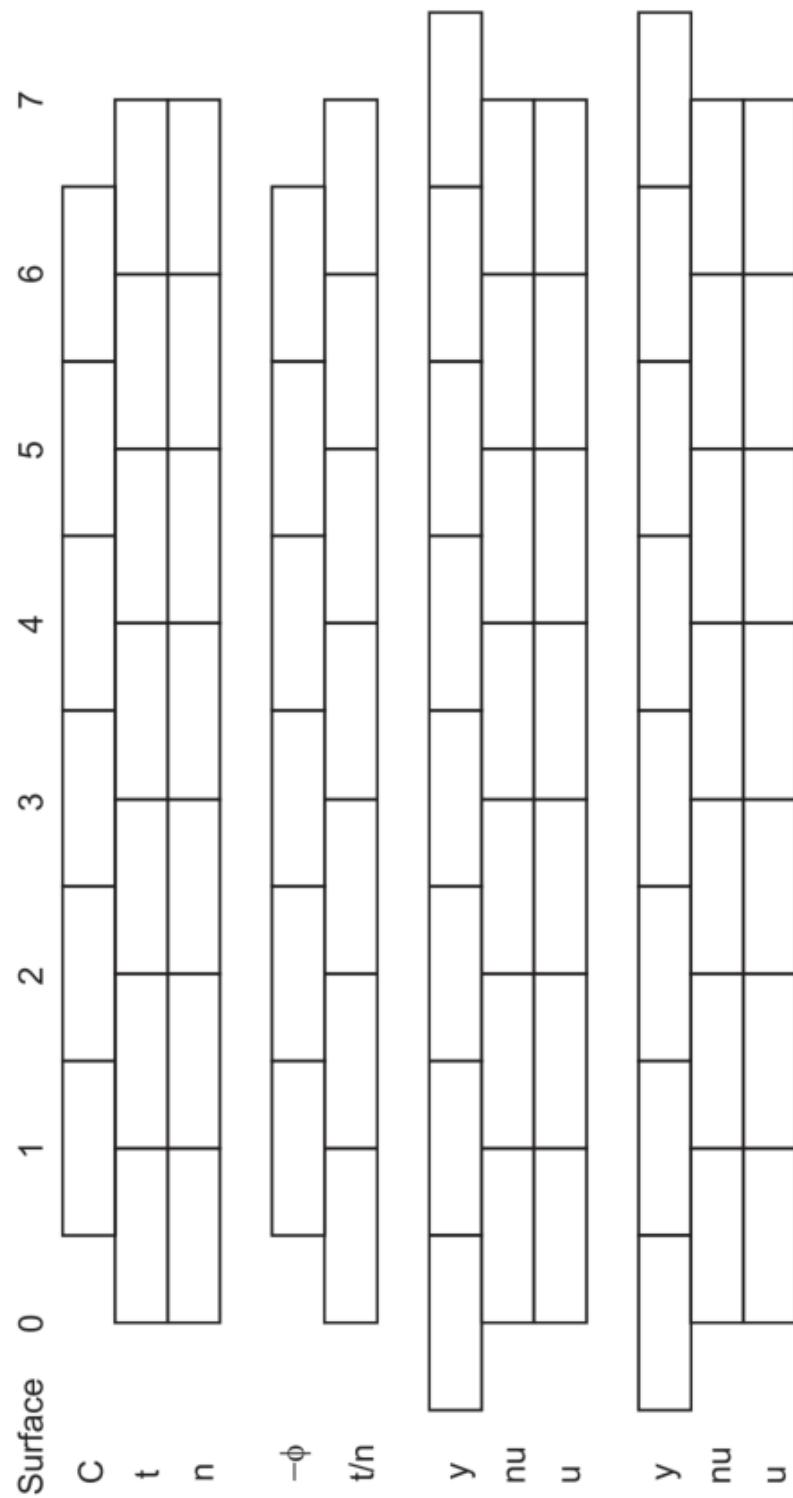
$$f'_E = \frac{1}{\phi} = \frac{y_2}{nu} = \frac{1}{0.060} = 16.667 \text{ mm} \longrightarrow f_F = -n_0 f_E = -16.667 \text{ mm.}$$

	Object space	Space 1	Surface 1	Space 2	Surface 2	Space 3	Surface 3	Space 4	Image space
$C$			0.128		0.1		-0.167		
$t$		15.167		3.6		3.6		16.856	
$n$	1			1.336		1.413		1.336	
$-\phi$			-0.043		-0.008		-0.013		
$t/n$		15.167		2.695		2.548		12.617	
$y$	1	1	1		0.884		0.757		0
$nu$	0	0		-0.043		-0.05		-0.060	
$u$	0	0						-0.045	
$y$	0		0.910		0.967		1	1	1
$nu$		0.060		0.021		0.013		0	0
$u$		0.060						0	0

The focal lengths match exactly as the ones computed by Gaussian reduction. We can also compute the principal planes shifts, but we will not do it as we already know the answer.

---

### 1.8.3 Table worksheet



## Bibliography

### Mathematics

- [1] Daniel Fleisch. *A student's guide to Maxwell's equations*. Cambridge University Press, 2008.
- [2] Gregory J Gbur. *Mathematical methods for optical physics and engineering*. Cambridge University Press, 2011.
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## Chapter 2

# Concepts of optics

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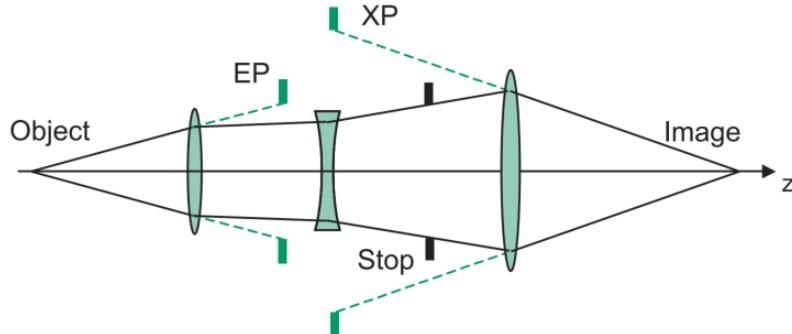
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## 2.1 Stops and pupils

### 2.1.1 Aperture stop

The **aperture stop** is a physical/real surface that limits the cone of light entering and exiting the optical system.

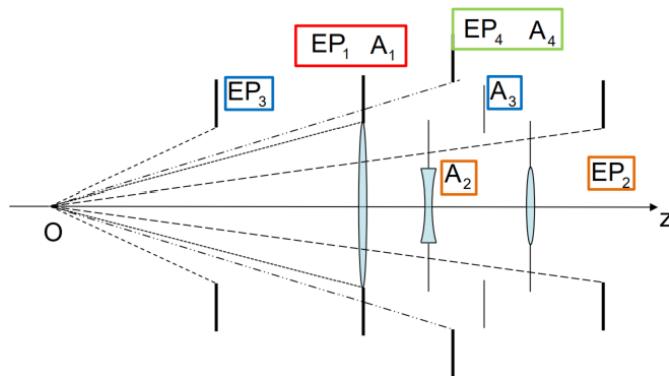
- The **entrance pupil** (EP) is the image of the stop in the object space.
- The **exit pupil** (XP) is the image of the spot in the image space.



**Figure 2.1** The stop limits the cone of light, and its image in object (image) space creates the entrance (exit) pupil.

There is a stop or pupil in each optical space. Intermediate pupils are formed in other spaces. There are two methods to determine which aperture in a system serves as the system stop:

- a) Image each potential stop into object space. The pupil with the **smallest** angular size corresponds to the stop. The same can be done in image space.

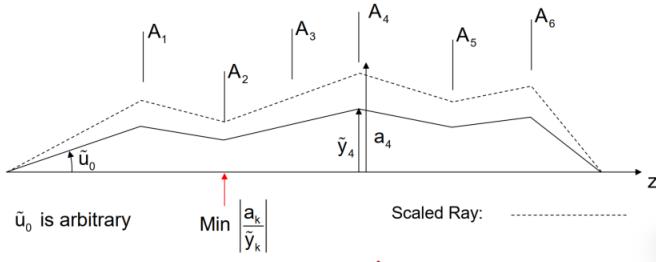


**Figure 2.2** The smallest angular size corresponds to the stop in object space. Same for image space.

- b) Trace a ray through the system from the axial object point with arbitrary initial angle. At each potential stop, determine the ratio of the aperture radio  $a_k$  to the ray height at that surface  $\tilde{y}_k$ .

$$\text{Aperture stop} = \min \left\{ \left| \frac{a_k}{\tilde{y}_k} \right| \right\}. \quad (2.1)$$

The pupils are the image of the stop and do not change position or size with an off-axis object. Intermediate pupils are formed in each optical space for multi-element systems. If there are  $N$  elements, there are  $N + 1$  pupils (including the stop).



**Figure 2.3** The minimum slope value corresponds to the aperture stop.

When designing a system, it is usually critical that the stop surface does not change over a range of possible object positions that the system will be used with.

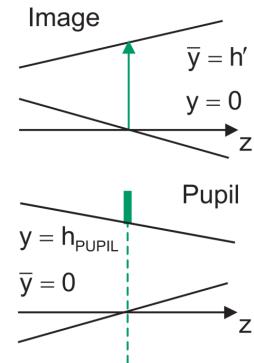
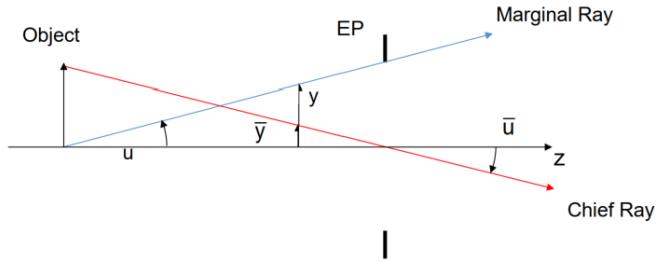
### 2.1.2 Marginal and Chief rays

Rays confined to the  $yz$ -plane are called **meridional rays**. There are two special meridional rays that define properties of the object, images and pupils:

- The **marginal ray** travels from the base of the object to the edge of EP. It defines image locations and pupil sizes.
- The **chief ray** travels from the edge of the object to the center of the EP. It defines image heights and pupil locations.

$y$  = marginal ray height  
 $u$  = marginal ray angle

$\bar{y}$  = chief ray height  
 $\bar{u}$  = chief ray angle



**Figure 2.4** The

The heights of the marginal ray and the chief ray can be evaluated at any  $z$  in any optical space. When the marginal ray crosses the axis, an image is located, and the size of the image is given by the chief ray height. Whenever the chief ray crosses the axis, a pupil or stop is located, and the pupil radius is given by the marginal ray height. Intermediate images and pupils are often virtual.

### 2.1.3 Pupil locations

By raytrace

Once you know which surface is the stop, you have the information to determine the location of EP and XP. The **pupil locations** can be found by tracing a paraxial ray starting at the center of the stop and is back/forward propagated. The intersections of this ray with the axis in object and image space determine the locations of EP and XP.

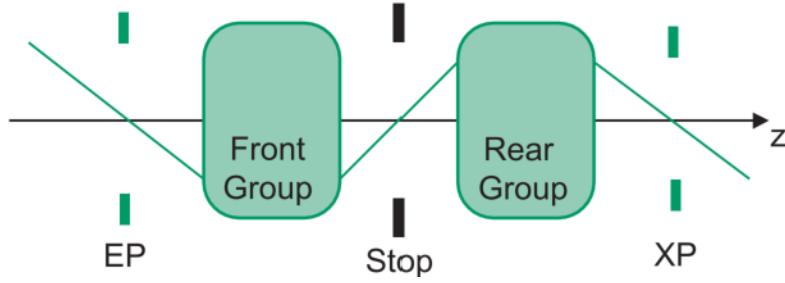
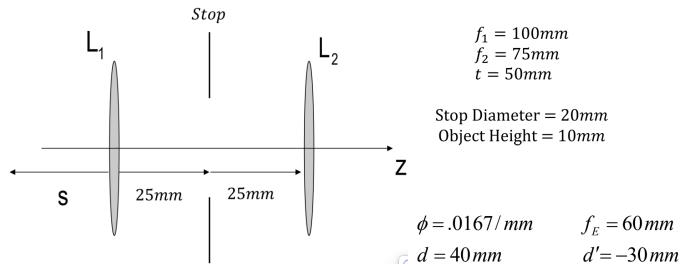


Figure 2.5 The

This ray become the chief ray when it is scaled to the object or image size. The marginal ray gives the pupil sizes.

### Ejemplo 2.1

### Pupil location by paraxial raytrace



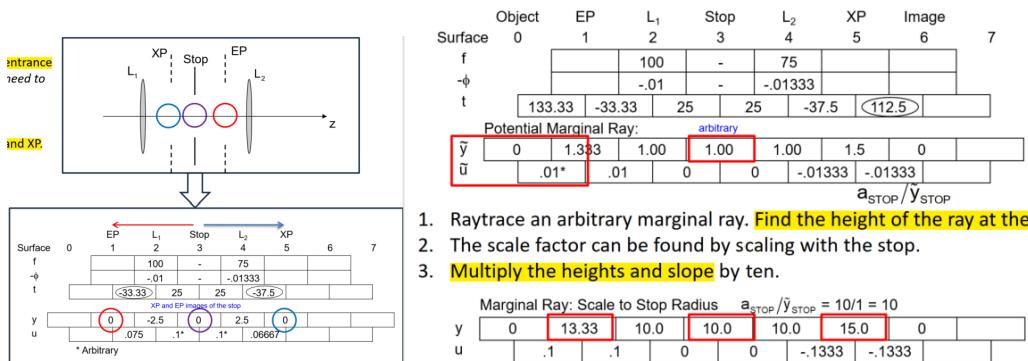
### Solution

The stop is a real object for the formation of both EP and XP. There is a ray that has a height of 0 at the EP, stop and XP. We first set  $y = 0$  at the stop, and then with arbitrary angle

EP we set  $y = 0$  for the EP and solve for the distance.

XP we set  $y = 0$  for the XP and solve for the distance.

We used a potential chief to find pupil locations. For the pupil sizes, we find the true marginal ray scaling a potential marginal ray. Remember that the chief ray was for pupil locations, now with the marginal ray we find the pupil sizes. We can also use it to find the image location.



Finally, the height of the EP is 13.33 mm, the stop 10 mm, the XP 15 mm.

## By Gaussian imagery

We treat each group independently, considering the stop as our object propagating in the direction of the given group. For EP, the object propagates from right to left, so we flip the sign of the refractive index (as in reflection).

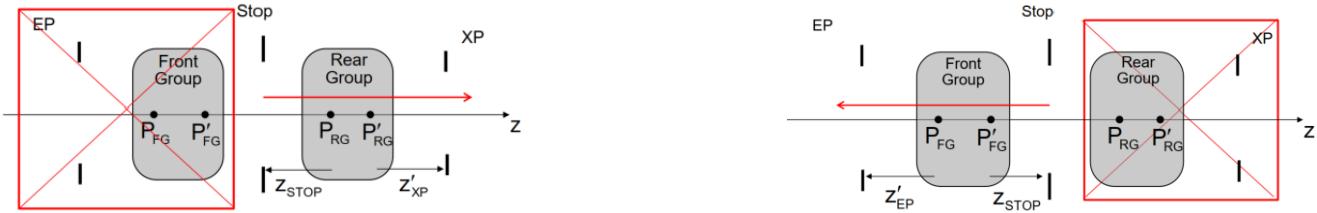


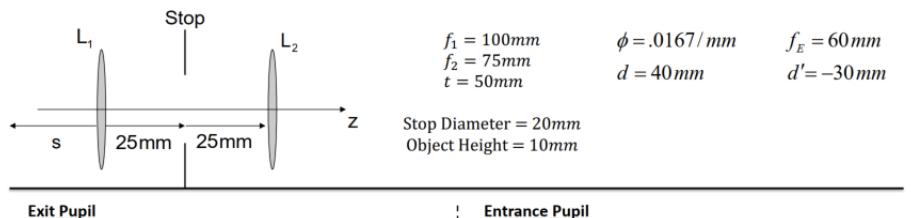
Figure 2.6

$$\text{For XP} \quad \frac{n'}{z'_{XP}} = \frac{n}{Z_{stop}} + \frac{1}{f_{RG}}, \quad m_{XP} = \frac{z'_{XP}}{z_{stop}}, \quad D_{XP} = |m_{XP}|D_{stop} \quad (2.2)$$

$$\text{For EP} \quad \frac{n'}{Z'_{EP}} = \frac{n}{Z_{stop}} + \frac{1}{f_{FG}}, \quad m_{EP} = \frac{z'_{EP}}{z_{stop}}, \quad D_{EP} = |m_{EP}|D_{stop} \quad (n = n' = -1) \quad (2.3)$$

## Ejemplo 2.2

## Pupil locations by Gaussian imagery



$\frac{1}{z'_{XP}} = \frac{1}{z_{STOP}} + \frac{1}{f_2} \quad f_2 = 75\text{mm} \quad z_{STOP} = -25\text{mm}$ $z'_{XP} = -37.5\text{mm} \quad (\text{to the left of L}_2)$ $m_{XP} = \frac{z'_{XP}}{z_{STOP}} = \frac{-37.5\text{mm}}{-25\text{mm}} = 1.5 \quad D_{XP} =  m_{XP} D_{stop} = 1.5(20\text{mm}) = 30\text{mm}$	$\frac{n'}{z'_{EP}} = \frac{n}{z_{STOP}} + \frac{1}{f_1} \quad n = n' = -1$ $\frac{-1}{z'_{EP}} = \frac{-1}{z_{STOP}} + \frac{1}{f_1} \quad f_1 = 100\text{mm} \quad z_{STOP} = +25\text{mm}$ $z'_{EP} = 33.33\text{mm} \quad (\text{to the right of L}_1)$ $m_{EP} = \frac{z'_{EP}}{z_{STOP}/n} = \frac{33.33\text{mm}/-1}{25\text{mm}/-1} = 1.333 \quad D_{EP} =  m_{EP} D_{stop} = 1.333(20\text{mm}) = 26.7\text{mm}$
--	---

It is the exact same process we learned previously.  
Find image location -> Find magnification -> Find height

20

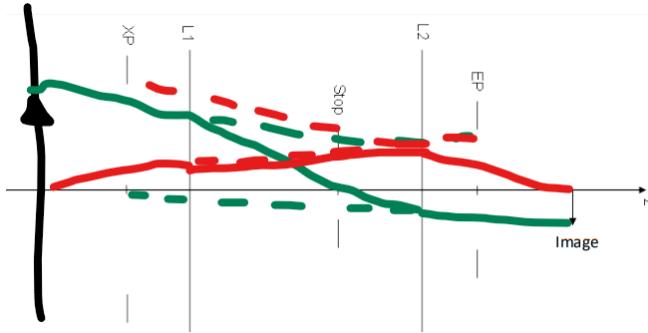
## EP,STOP,XP are invariant to object location

Changing the object location does not change the position of the EP, stop, and XP.

## Ejemplo 2.3

## Raytrace of chief and marginal rays

The following diagram illustrate the trace by hand of the chief and marginal ray by definition.



### 2.1.4 Lagrange invariant

The linearity of paraxial optics provides a relationship between the heights and angles of any two rays propagating through the system. The **Lagrange invariant**  $\Xi$  is formed with the paraxial marginal and chief rays:

$$\text{Lagrange invariant} \quad \Xi = n\bar{y} - nu\bar{y} = \bar{\omega}y - \omega\bar{y}. \quad (2.4)$$

It is invariant for refraction and transference and it can be evaluated at any  $z$  in any optical space. The Lagrange invariant is particularly simple at images or objects ( $y = 0$ ) and pupils ( $\bar{y} = 0$ ):

$$\text{Image/Object} \quad y = 0, \quad \Xi = -nu\bar{y} = -\omega\bar{y} \quad (2.5)$$

$$\text{Pupils} \quad \bar{y} = 0, \quad \Xi = n\bar{y} = \bar{\omega}y \quad (2.6)$$

If two rays other than the marginal and chief are used, the more general **optical invariant**  $I$  is formed.

Given two rays, a third ray can be formed as a linear combination of the two rays. The coefficients are the ratios of the pair-wise invariants of the values for the three rays at some initial  $z$ . The expressions are valid for any  $z$ :

$$y_3 = Ay_1 + By_2, \quad u_3 = Au_1 + Bu_2 \quad (2.7)$$

$$A = I_{32}/I_{12}, \quad B = I_{13}/I_{12}, \quad I_{ij} = nu_i y_j - nu_j y_i. \quad (2.8)$$

Changing the Lagrange invariant of a system **scales** the optical system. Doubling the invariant while maintaining the same object and image sizes and pupil diameters halves all of the axial distances (and the focal length).

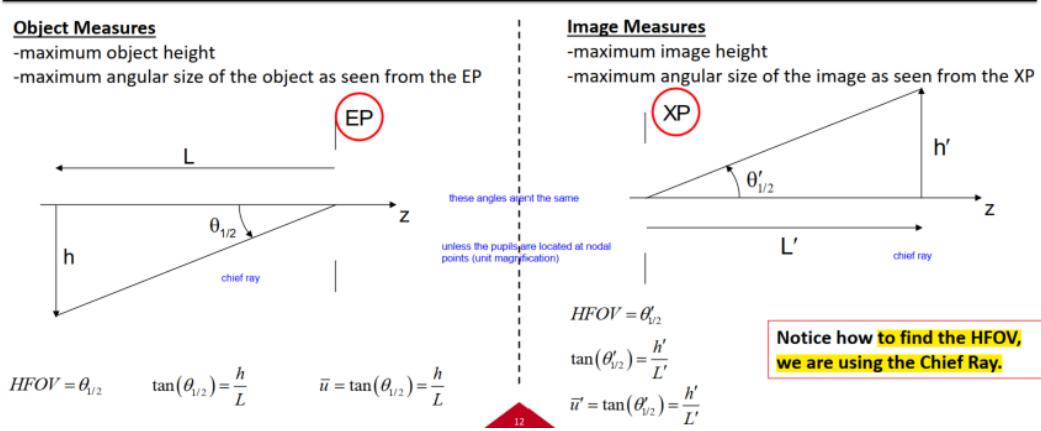
The **throughput**, **etendue** of  $A\Omega$  product in **radiometry** and **radiative transfer** are related to the square of the Lagrange invariant:

$$n^2 A\Omega = \pi^2 \Xi^2. \quad (2.9)$$

### 2.1.5 Field of view

We revisit again the concept of FOV but now using the EP and XP.

- **Field of view** FOV diameter of the object/image.
- **Half field of view** HFOV radius of the object/image.



## 2.1.6 Numerical aperture and F-number

In an optical space of index  $n_k$ , the **numerical aperture**  $NA$  describes the axial cone of light in terms of the real marginal angle  $U_k$ :

$$\text{Numerical aperture} \quad NA = n_k |\sin U_k| \approx n_k |u_k|. \quad (2.10)$$

The **F-number**  $f/\#$  describes the image-space cone of light for an object **at infinity**:

$$\text{F-number} \quad f/\# = \frac{f_E}{D_{EP}}. \quad (2.11)$$

While the  $f/\#$  is an image-space, infinite-conjugate measure, the approximate relationship between NA and  $f/\#$  allows and  $f/\#$  to be defined for other optical spaces and conjugates. As a result, an  $f/\#$  can be defined for any cone of light. This  $f/\#$  is called **working F-number**  $f/\#_W$ . This previous relationship becomes a definition

$$\text{Working F-number} \quad f/\#_W = \frac{1}{2NA} \approx \frac{1}{2n|u|} = (1-m)f/\#. \quad (2.12)$$

Fast optical system have small numeric values for the  $f/\#$ . Most lenses with adjustable stops have  $f/\#$  of **f-stops** labeled in increments of  $\sqrt{2}$ . The usual progression is:

$$f/1.4, \quad f/2, \quad f/2.8, \quad f/4, \quad f/5.6, \quad f/8, \quad f/11, \quad f/16, \quad f/22, \quad \text{etc.}$$

Each stop changes the area of the EP (light collection ability) by a factor of 2.

The Lagrange invariant relates the magnification between two pupils to the chief ray angles at the pupils.

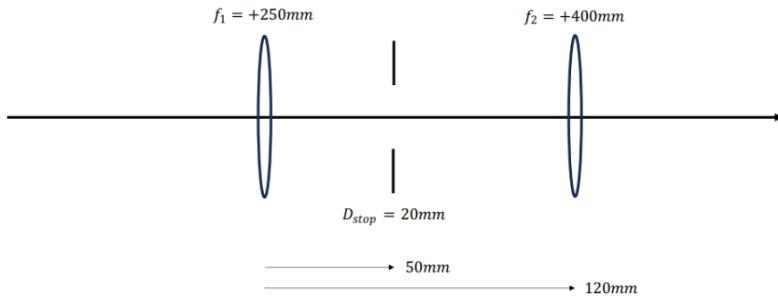
$$\Xi = n\bar{u}y_{pupil} = n'\bar{u}'y'_{pupil}, \quad m_{pupil} = \frac{y'_{pupil}}{y_{pupil}} = \frac{n\bar{u}}{n'\bar{u}'} = \frac{\bar{\omega}}{\bar{\omega'}}. \quad (2.13)$$

### Use of working F-number (left)

The most common use of the working F-number is to describe the image-forming cone for a finite conjugate optical system. This is the cone formed by the XP and the axial image point.

**Ejemplo 2.4****Determination of stop and pupil**

Determine the location and size of the pupils for the following system in air.

**Solution**

- a) We trace the chief ray denoted as CR, and a potential marginal ray MR with unitary height at the stop.

	Object space	EP		$L_1$		Stop	$L_2$		XP	Image space
$C/R/f$				250			400			
$t$	1	1	$z_{EP} = -62.5$	1	50	1	70	1	$z_{XP} = -84.8$	
$n$			1	1	1	1	1	1	1	1
$-\phi$				-0.004			-0.0025			
$t/n$			$\tau_{EP} = -62.5$		50		70		$\tau_{XP} = -84.8$	
$y$		0		-5		0	7		0	
$nu$			0.08		0.1		0.1		0.0825	
$u$			0.08		0.1		0.1		0.0825	
$MR$			$R_{EP} = 1.25$	1		1	1		$R_{XP} = 1.21$	
$y$				0.004		0	0		-0.0025	
$nu$						0				
$u$						0				

**Table 2.1** Raytrace, with CR=Chief ray, MR=Marginal ray.

Due to the diameter of the stop is  $R_{stop} = 10 \text{ mm}$ , we scale the potential marginal ray to give the true marginal ray and therefore obtain the radius of the pupils:

$$\begin{aligned} R_{EP} &= (10)(1.25) = 12.5 \text{ mm} \implies D_{EP} = 2R_{EP} = 25.0 \text{ mm} \\ R_{XP} &= (10)(1.21) = 12.1 \text{ mm} \implies D_{XP} = 2R_{XP} = 24.2 \text{ mm} \end{aligned}$$

- b) For Gaussian imagery, we see the stop as the object for the front group and rear group. For the EP, we have a backward propagation that is managed with the flip of the sign in the refractive indices.

$$\frac{-1}{z_{EP}} = \frac{-1}{Z_{stop}} + \frac{1}{250} \implies z_{EP} = 62.5 \text{ mm.}$$

This entrance pupil is to the right of the lens  $L_1$ . The magnification is:

$$m_{EP} = \frac{z_{EP}}{z_{stop}} = \frac{R_{EP}}{R_{stop}} = -1.25.$$

The diameter of the entrance pupil is therefore:

$$D_{EP} = 2R_{EP} = 2[|m_{EP}|R_{stop}] = 25 \text{ mm.}$$

For the rear group, we have analogously:

$$\frac{1}{z_{XP}} = \frac{1}{Z_{\text{stop}}} + \frac{1}{400} \rightarrow z_{XP} = -84.848 \text{ mm.}$$

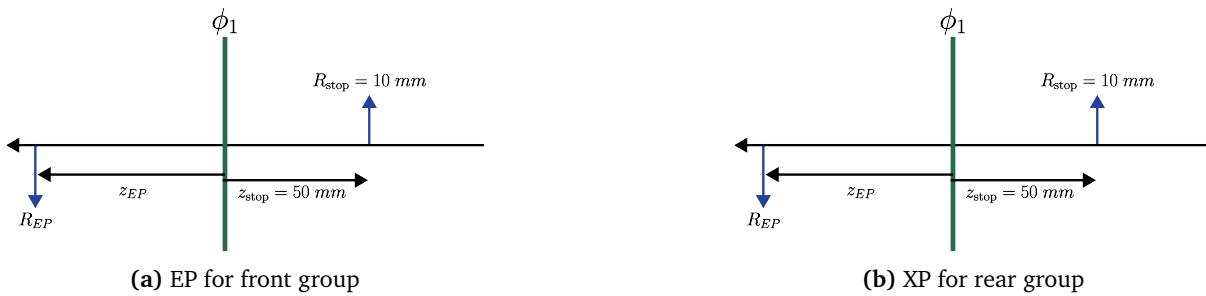
The exit pupil is then to the left of the lens  $L_2$ . The magnification in this case is

$$m_{XP} = \frac{z_{XP}}{z_{\text{stop}}} = \frac{R_{XP}}{R_{\text{stop}}} = 1.21.$$

The diameter of the exit pupil is:

$$D_{XP} = 2R_{XP} = 2[m_{XP}|R_{\text{stop}}] = 24.2 \text{ mm.}$$

The illustration of each case is illustrated in the figure 2.7.



**Figure 2.7** With Gaussian imagery, the computation of the pupils is based on the stop as the object of two optical systems.

Using either method, the result is the same and is shown in figure 2.8

---

## 2.2 Vignetting

### 2.2.1 Ray bundles

The **ray bundle** for an **on-axis** object is a rotationally symmetric spindle made up of section of right circular cones. Each cone section if bounded by the pupil and the object/image in that optical space. At any  $z$ , the cross section of the bundle is circular, and the radius of the bundle is the marginal ray value.

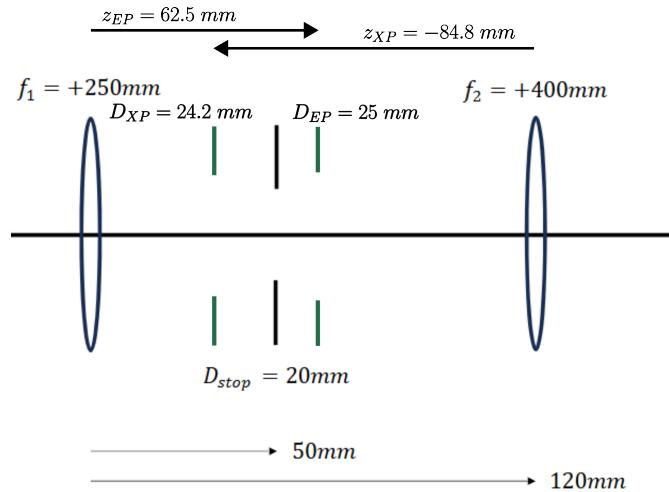
For an **off-axis** object point, the ray bundle skews, and is comprised of section of skew circular cones which are still defined by the same elements. The cross section of the ray bundle at any  $z$  remains circular with a radius equal to the radius of the axial bundle. The off-axis bundle is centered about the chief ray height.

The maximum radial extent of the ray bundle at any  $z$  is:

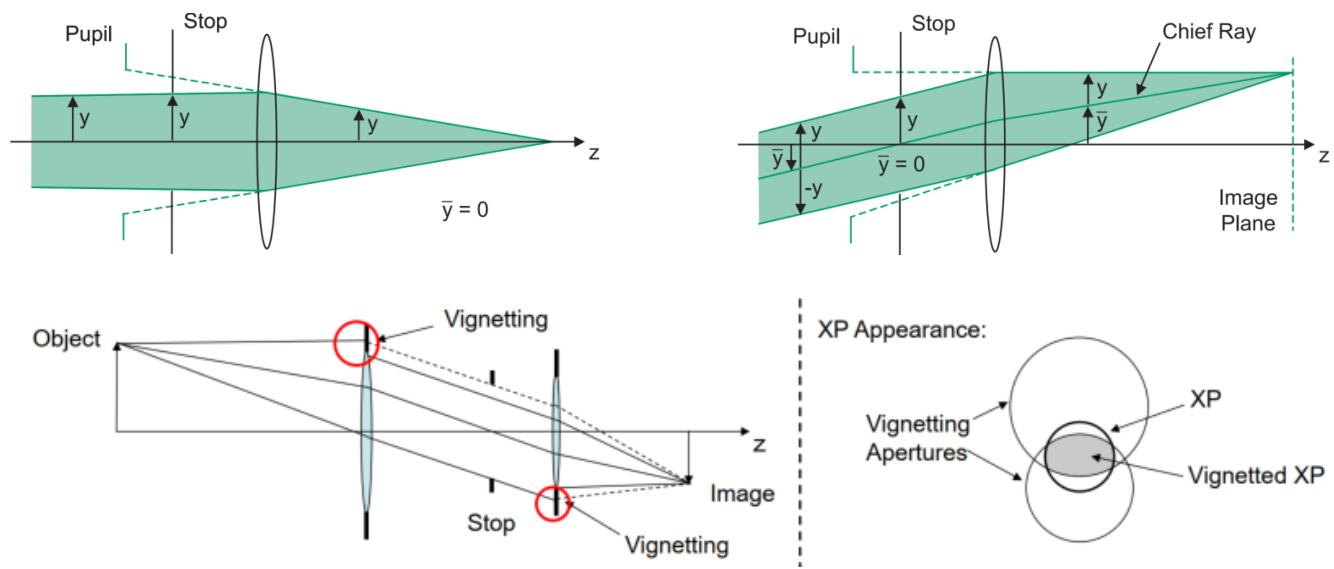
$$\text{Maximum radial extent } |y_{\max}| = |y| + |\bar{y}|. \quad (2.14)$$

### 2.2.2 Vignetting

The **vignetting** occurs when other apertures in the system (others than the stop) block a proportion of an off-axis ray bundle. For no vignetting, each aperture radius  $a$  must equal or exceed the maximum height of the ray bundle at the aperture.



**Figure 2.8** Illustration of the stop and pupil in the optical system.



**Figure 2.1** The ray bundle is clipped and the beam is no longer circular.

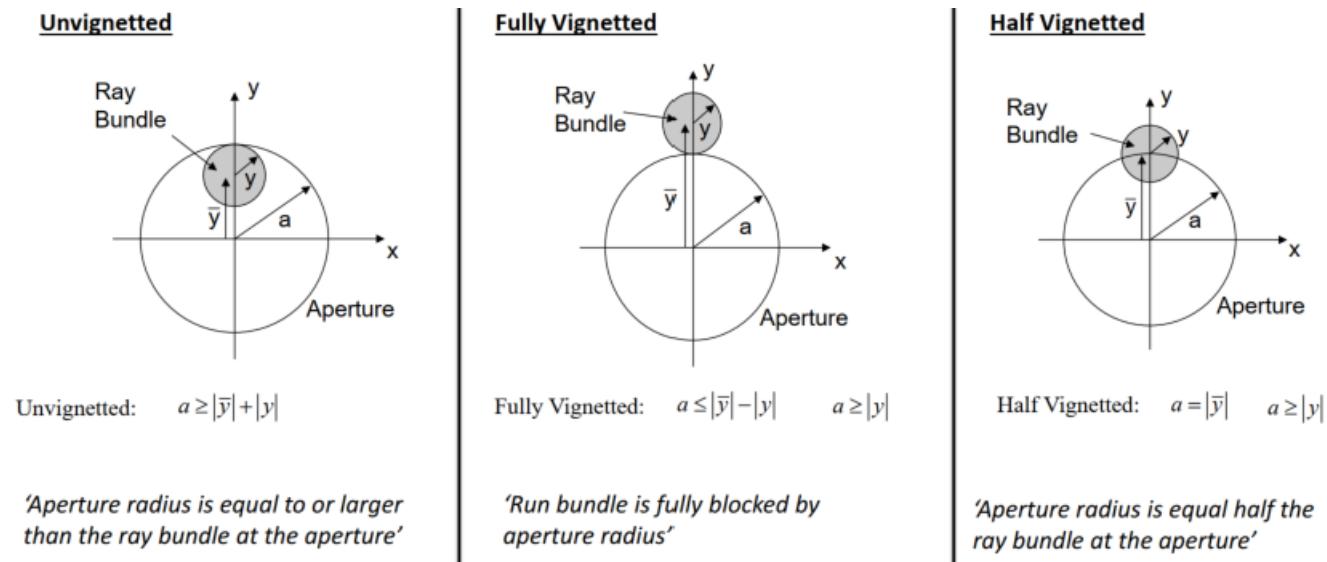
The maximum FOV supported by the system occurs when an aperture completely blocks the ray bundle from the object point.

We can have three conditions of vignetting, depending on the proportion of clip of the light beam.

The vignetting conditions are used in two different manners:

- For a given set of apertures, the FOV that the system will support with a prescribed amount of vignetting can be determined. A different chief ray defined each FOV.
- For a given FOC and vignetting condition, the required aperture diameters can be determined.

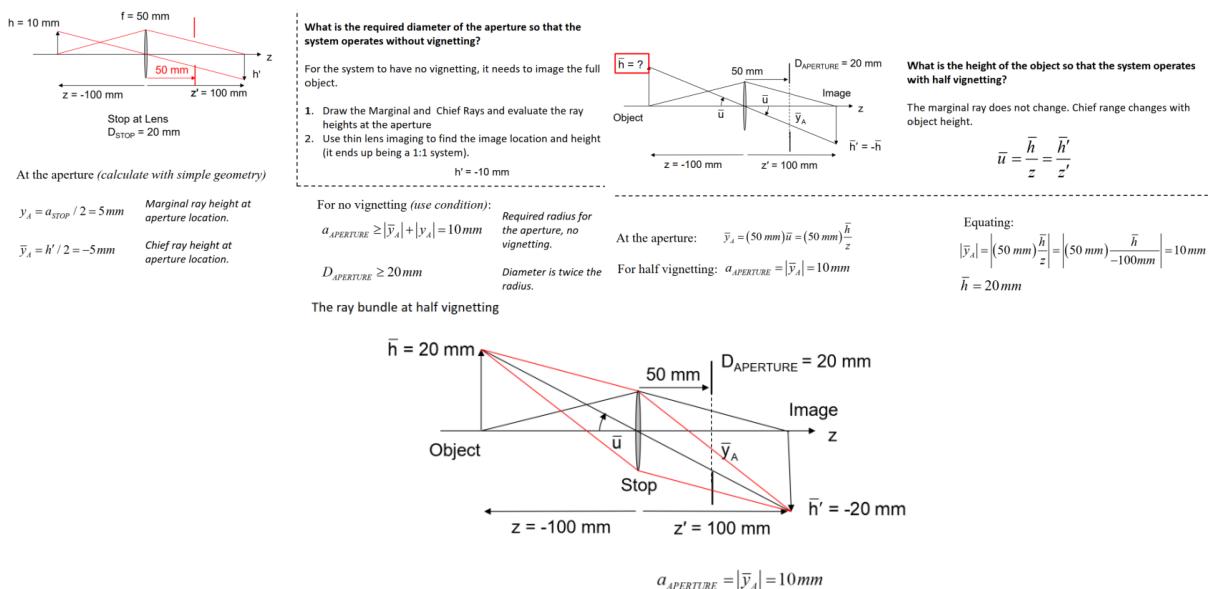
A system with vignetting will have an image that has full irradiance or brightness out to a radius corresponding to the unvignetted FOV limit. The irradiance will then begin to fall off, going to about half at the half-vignetted FOV, and decreasing to zero at the fully vignetted FOV. This fully vignetted FOV is the absolute maximum possible.

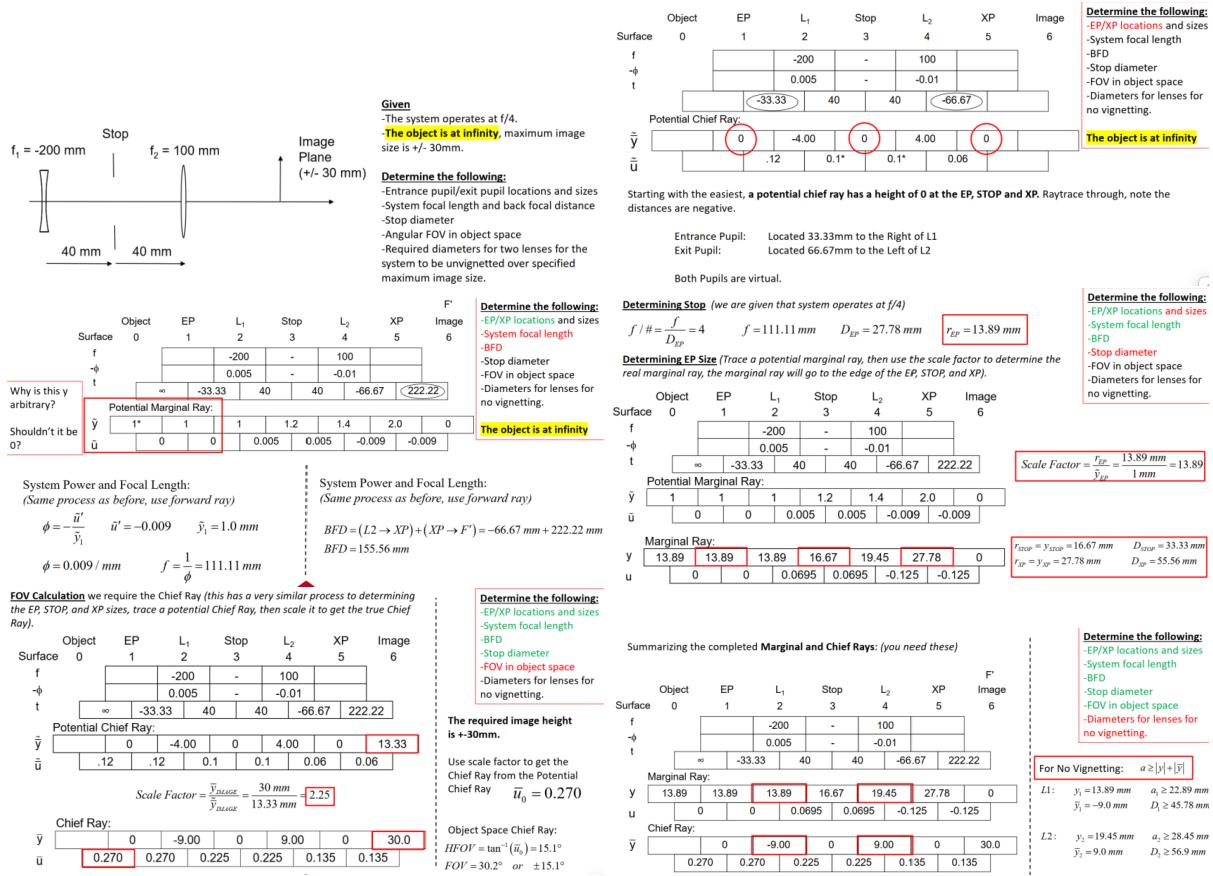


The diameter of the aperture stop is very important design parameter for an optical system as it controls five separate performance aspects of the system:

- The system FOV determined by vignetting.
- The radiometric or photometric speed of the system or its light collection ability.
- The depth of focus and depth of field of the system.
- The amount of aberrations degrading image quality.
- The diffraction-based performance of the system.

### Ejemplo 2.5



**Ejemplo 2.6****Vignetting with paraxial raytrace**

In general,

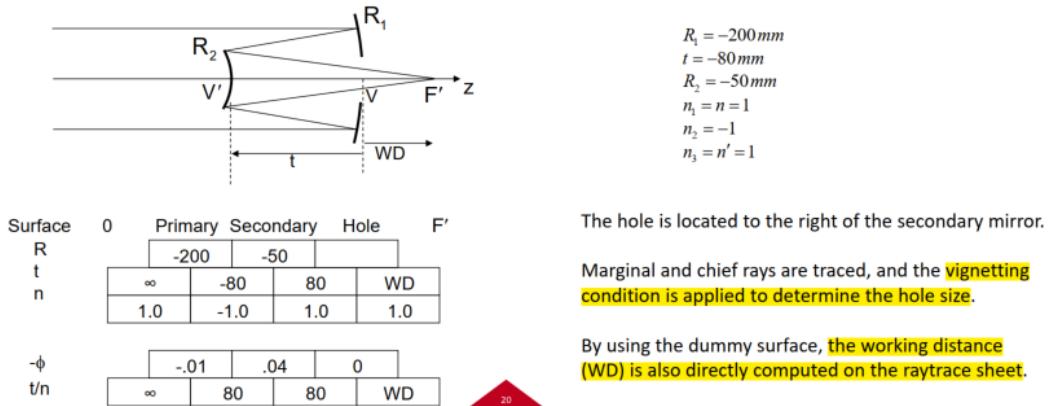
**Key points in solving problems**

- Trace the potential chief ray (CR) to know the locations of the pupils (and image size).
- Trace the potential marginal ray (MR) to determine image location and pupil sizes.
- If the MR comes parallel, then it can be used to obtain the first-order properties.
- The F-number gives us the real size of EP so we can scale the MR.
- The image size allows us to get the real CR.
- The HFOV is determined with the incident angle  $\bar{u}$  at EP in the real CR:  $HFOV = \tan^{-1}(\bar{u})$ .
- The vignetting is found by looking at  $y$ ,  $\bar{y}$  in the real MR and CR and applying the criteria.
- We can arbitrarily define a dummy surface to our convenience.
- EP and XP are dummy surfaces (w location defined) of zero-power.

**2.2.3 Dummy surfaces**

In a raytrace, a zero-power surface can be inserted at any location to examine the ray properties.

An example of its application is the following Cassegrain objective, where we require to find the size of the hole. For that, we place a dummy surface **at the hole**.



## 2.3 Radiative transfer

### 2.3.1 Radiometry

**Radiometry** characterizes the propagation of radiant energy through an optical system. The basic unit is the watt W. Radiometric terminology and units are: There are some assumptions:

Quantity	Symbol	Units	Units description
Energy	$Q$	J	
Flux	$\Phi$	W	Power
Intensity	$I$	W/sr	Power per unit solid angle
Irradiance	$E$	W/m <sup>2</sup>	Incident power per unit area
Exitance	$M$	W/m <sup>2</sup>	Exiting power per unit area
Radiance	$L$	W/m <sup>2</sup> sr	Power per unit projected area per unit solid angle

- The source is **incoherent**, meaning that scenes are collection of independently point sources, no interference.
- Objects and images on-axis and perpendicular to the optical axis, so that the projected area equals the area.

The solid angle  $\Omega$  equals the surface area of the unit sphere in a given vicinity. The units are  $4\pi$  steradians (sr).

$$d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin \theta d\theta d\phi}{r^2} = \sin \theta d\theta d\phi \xrightarrow{\int} \Omega_{\text{sphere}} = 4\pi \sin^2(\theta_0/2) .$$

In optics, it's common to approximate the solid angle of a sphere to the section of a cone:

$$\Omega \approx \frac{\pi r_0^2}{d^2} \approx \pi \sin^2 \theta_0 \approx \pi \theta_0^2 \quad (\text{small angle approximation}).$$

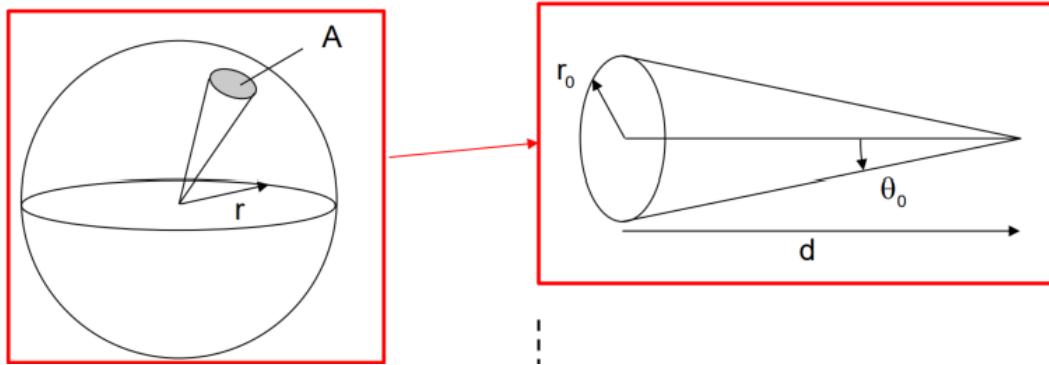


Figure 2.1 The solid angle of a sphere can be approximated to the solid angle of a cone.

### 2.3.2 Radiative transfer

**Radiative transfer** uses first-order geometrical principles to determine the amount of light from an object that reaches an image or detector.

Exitance and irradiance are related by the **reflectance** of the surface  $\rho$ :

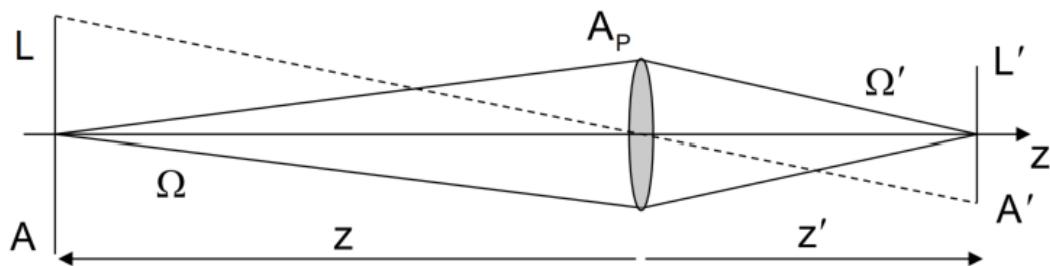
$$M = \rho E. \quad (2.15)$$

For average scenes,  $\rho = 18\%$ . Exposures are often set using this value, that is, we expose the print to that average so that the print reflectance ends up with 18%.

The irradiance of a **Lambertian source** (perfectly diffuse surface) is constant. The intensity falls off with the apparent source size or the **projected area** (**Lambert's law**). The exitance of a Lambertian source is related to its radiance by  $\pi$ .

Lambertian source	$L(\theta, \phi) = \text{constant}$	$I = I_0 \cos \theta$
	$M = \pi L$	$\pi L = \rho E$

We now analyze the optical power from an object that reaches the image in an optical system.



In air, the radiance and the **AΩ product** or **throughput** are conserved, and the flux collected by the lens  $\Phi$  is transferred to the image area  $A'$ :

$$\Phi = L(\text{object area})(\text{solid angle projection in lens}) = LA_p \Omega = LA \frac{\pi D_p^2}{4z^2} \xrightarrow{A' = m^2 A} \Phi = \frac{\pi L A' D_p^2}{4m^2 z^2}.$$

Using gaussian equations and f-number equations, the image plane irradiance  $E'$  is

Camera equation	$E' = \frac{\Phi}{A'} = \frac{\pi L}{4(f/\#W)^2} \rightarrow \pi L (\text{NA})^2, \quad L = \frac{\rho E_0}{\pi}$	$(2.16)$
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## Spectral dependence

Spectral dependence can also be added.

$$\begin{aligned} E_0(\lambda) &= \text{Object irradiance} \\ \rho(\lambda) &= \text{Object reflectance} \quad \longrightarrow \quad L(\lambda) = \frac{M(\lambda)}{\pi} = \frac{\rho(\lambda)E_0(\lambda)}{\pi} \\ L(\lambda) &= \text{Object radiance} \quad E'(\lambda) = \frac{\rho(\lambda)E_0(\lambda)}{4(1-m)^2(f/\#)^2} \end{aligned}$$

This can be integrated over all wavelengths for total irradiance

$$E' = \frac{1}{4(1-m)^2(f/\#)^2} \int_{\lambda_1}^{\lambda_2} \rho(\lambda)E_0(\lambda) d\lambda.$$

## Exposure

Most detectors respond to energy per unit area rather than power per unit area. Multiplying the image irradiance by the exposure time gives the exposure ( $J/m^2$ ):

$$\text{Exposure} \quad H = E' \Delta t. \quad (2.17)$$

The mean solar constant is  $1368 \text{ W/m}^2$  outside the atmosphere of the earth, and the solar irradiance on the surface is about  $1000 \text{ W/m}^2$ .

### 2.3.3 Photometry

**Photometry** is the subset of radiometry that deals with visual measurements, and luminous power is measured in **lumens**  $lm$ . The lumen is a watt weighted to the visual **photopic response**. This peak response occurs at  $555 \text{ nm}$ , where the conversion is  $683 \text{ lm/W}$ . The dark adapted or **scotopic response** peaks at  $507 \text{ nm}$  with  $1700 \text{ lm/W}$ .

(a) Photometric terminology		(b) Luminous photopic efficacy	
		$\lambda (\text{nm})$	$lm/W$
Luminous power	$\Phi_V$ $lm$	400	0.3
Luminous intensity	$I_V$ $lm/sr$	420	2.7
Illuminance	$E_V$ $lm/m^2$	440	15.7
Luminous exitance	$M_V$ $lm/m^2$	460	41.0
Luminance	$L_V$ $lm/m^2 sr$	480	95.0
Exposure	$H_V$ $lm s/m^2$	500	221
candela (cd)	$I_V$ $lm/sr$	520	485
lux (lx)	$E_V$ $lm/m^2$	540	652
foot-candle (fc)	$lm/ft^2$	560	680
	$1fc = 10.76 \text{ lx}$	580	594
foot-lambert (fL)	$L_V$ $\frac{1}{\pi} cd/ft^2$	600	425
nit (nt)	$= cd/m^2$	620	260
	$1fL = 3.426 \text{ nt}$	640	120
lux-second (lx s)	$H_V$ $lm s/m^2$	660	41.7
		680	11.6
		700	2.8
		720	0.7
(c) Typical illuminance levels			
Sunny day	$10^5 \text{ lx}$	Moonlit night	$10^{-1} \text{ lx}$
Inerior	$10^2 \text{ lx}$	Desk lighting	$10^3 \text{ lx}$
		Overcast day	$10^3 \text{ lx}$
		Starry night	$10^{-3} \text{ lx}$

The candela (cd) is the fundamental SI unit for luminous intensity.

### $A\Omega$ product

Recall the flux through a system is

$$\Phi = LA\Omega.$$

The  $A\Omega$  product appears to be the geometric portion, while  $L$  would be related to the source characteristics. In an object or an image plane,

object/pupil plane	$A = \pi\bar{y}^2, \quad \theta = u, \quad A\Omega = \pi^2\bar{y}^2u^2 = \pi^2\chi^2/n^2, \quad \chi = n\bar{y}u$
pupil plane	$A = \pi y^2, \quad \theta = \bar{u}, \quad A\Omega = \pi^2y^2\bar{u}^2 = \pi^2\chi^2/n^2, \quad \chi = ny\bar{u}$

In the general situation when the index is not unity, the **basic throughput**  $n^2A\Omega$  and the **basic radiance**  $L/n^2$  are invariant. Since throughput is based on areas, the basic throughput is proportional to the Lagrange invariant squared:

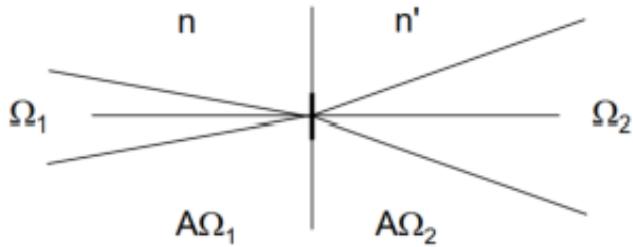
$$n^2A\Omega = \pi^2\chi^2. \quad (2.18)$$

For a lossless optical system, the flux through the system is constant. For  $n = 1$ , we have

$$\Phi = L_1A_1\Omega_1 = L_2A_2\Omega_2 = \dots = \text{constant}.$$

Since  $A\Omega$  is also constant, the radiance  $L$  must also be constant. This allows us to relate different portions of the optical system as the flux is conserved.

However, if the index of refraction is not unity and changes, the radiance is no longer conserved. It will change at each interface as the solid angle will change. We then have that



$$A\Omega_1 \neq A\Omega_2 \wedge \Omega = L_1A\Omega_1 = L_2A\Omega_2 \Rightarrow L_1 \neq L_2.$$

The flux is still constant. In fact,  $L/n^2 = \text{constant}$  as well as  $n^2A\Omega = \text{constant}$ .

## 2.4 Objectives

### 2.4.1 Type of objectives

**Objectives** are lens element combinations used to image distance objects. To classify them, separated group of lens elements are modeled as thin lenses.

- **Simple objective** consists of a positive thin lens.

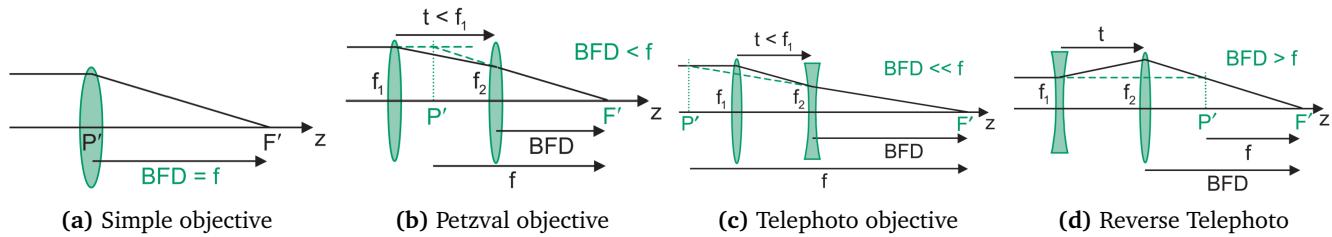


Figure 2.1 Different type of objectives.

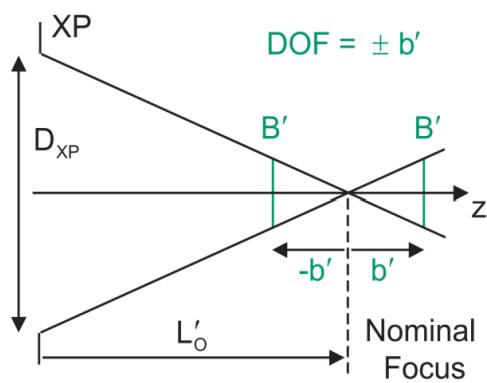
- **Collimator** is a reversed simple objective. It creates a collimated beam from a source at the system focal point.
- **Petzval objective** consists of two separated positive groups of elements. The rear principal plane is located between the groups.
- **Telephoto objective** produces a system focal length longer than the BFD. It consists of a positive element followed by a negative group.
- **Reverse telephoto objective** consists of a negative group followed by a positive group. Used to produce a system with BFD larger than the system focal length.

## 2.4.2 Depth of focus and field

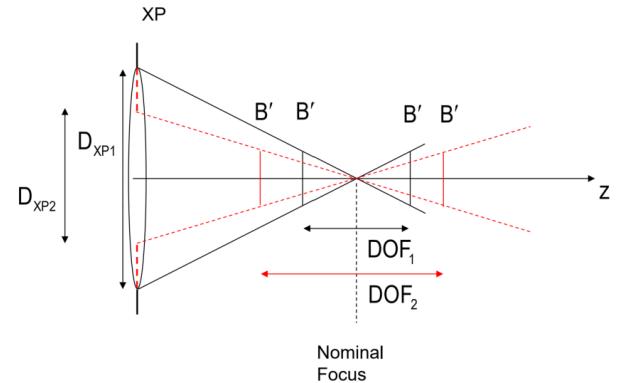
There is often some allowable image blur that defines the performance requirement of an optical system. No diffraction or aberrations are included.

### Depth of focus

The **depth of focus** DOF describes the amount the detector can be shifted from the nominal image position (focal plane) for a fixed object before the resulting blur exceed the blur diameter criterion  $B'$ .



(a) Depth of focus



(b) Depth of focus vs F-number

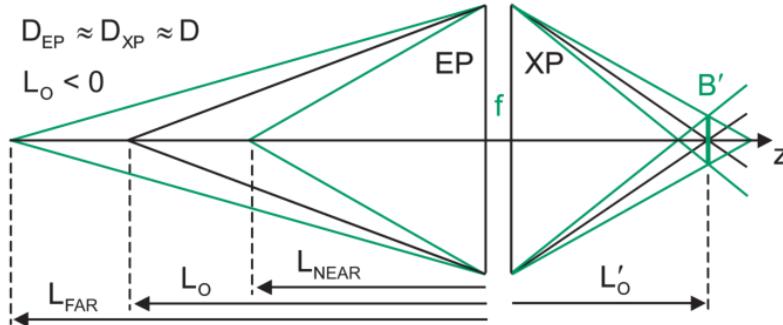
The  $\pm b'$  transverse distance from the nominal point is used to define the DOF as the whole distance is  $2b'$ :

$$b' = \frac{B' L'_0}{D_{XP}} \approx \frac{B' z'}{D_{EP}}, \quad DOF \approx \pm b' \approx \pm B' f / \#W \approx \frac{B'}{2NA}. \quad (2.19)$$

The figure illustrates how the DOF is changed as the diameter of the lens is varied; they are inversely proportional.

## Depth of field

The **depth of field** is the maximum distance, from  $L_{\text{near}}$  to  $L_{\text{far}}$ , the object can move before exceed the acceptable blur  $B'$  at the fixed image plane. The following relations are given



$$L_{\text{far}} \approx \frac{L_0 f D}{f D + L_0 B'}, \quad L_{\text{near}} \approx \frac{L_0 f D}{f D - L_0 B'}. \quad (2.20)$$

All objects positions between these distances will produce images on the detector that have geometrical blurs less than the blur criterion  $B'$ .

### 2.4.3 Hyperfocal distance

When the far point of the depth of field  $L_{\text{far}}$  is at infinity, the optical system is focused at the **hyperfocal distance**  $L_H$ , and all objects from  $L_{\text{near}}$  to infinity meet the image plane blur criterion.

$$L_{\text{far}} = \infty \implies f D + L_H B' = 0.$$

Solving for  $L_H$ ,

$$\text{Hyperfocal distance} \quad L_H = -\frac{f D}{B'} = -\frac{f^2}{(f/\#) B'}. \quad (2.21)$$

Substituting of  $L_H$  in  $L_{\text{near}}$  yields

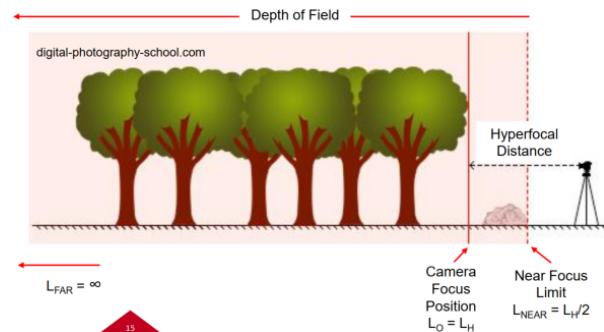
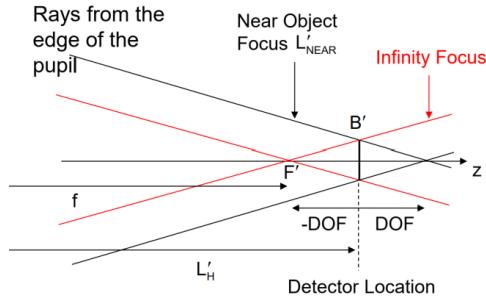
$$L_{\text{near}} \approx -\frac{f D}{2 B'} = \frac{L_H}{2}. \quad (2.22)$$

### Hyperfocal distance and depth of

The relation between the depth of focus and the hyperfocal distance where the detector is placed is, by the thin-lens equations,

$$L'_H \approx f + B' f = f + \text{DOF}. \quad (2.23)$$

- Focusing at the hyperfocal distance ensures that any greater distance than  $L_{\text{near}} = L_H/2$  meets the blur criteria.
- As  $f/\#$  increases (lens stopped down), the hyperfocal distance moves closer to the lens.



### Ejemplo 2.7

You take a picture with a film camera. You want to apply that picture to a larger print. Given the specifications, what is the f/#?

**System Specifications**

Film size:	24 x 36 mm
Print size:	100 x 150 mm
Maximum blur on print:	0.15 mm
Focal length:	38 mm
Near focus:	1200 mm

We image onto film. Notice the blur criteria is for the PRINT.

$$f/\# = f/Dep$$

$$L_H = -\frac{fD}{B'} \quad D = -\frac{L_H B'}{f}$$

$$L_H = 2L_{NEAR} = -2400\text{mm}$$

$$\text{Print Magnification} \approx 4X$$

$$B' = \frac{15\text{mm}}{4} = .038\text{mm}$$

We are given the focal length; we need to find the diameter EP.

Use relationship for hyperfocal distance to isolate D. Need to determine  $L_H$  and  $B'$ .

$L_H$  is easy, use relationship with  $L_{NEAR}$ . Be careful with signs!

Our maximum blur on the PRINT is 4mm, we need to calculate the maximum blur on the FILM.

After plugging in everything, you will find  $D = 2.44\text{mm}$  and  $f/15.5$ .

### Fixed-focus camera

### Ejemplo 2.8

You take a picture with your phone. Given the specifications, what is the f/#?

**System Specifications**

Number of Pixels =	3264 x 2488 (8MP)
Pixel Size =	1.4 μm
Near focus =	1200 mm
Focal length =	4.8 mm

Image blur is twice the pixel size. Why?

$$f/\# = f/D$$

$$L_H = -\frac{fD}{B'} \quad D = -\frac{L_H B'}{f}$$

$$L_H = 2L_{NEAR} = -2400\text{mm}$$

$$B' = 2 * \text{Pixel Size} = 2.8\mu\text{m}$$

Same process as before, the key difference is HOW the blur is calculated.

$$D = 1.4\text{mm}$$

$$f/\# = f/D = f/2.9$$

### Fixed-focus digital camera

#### 2.4.4 Zoom lenses

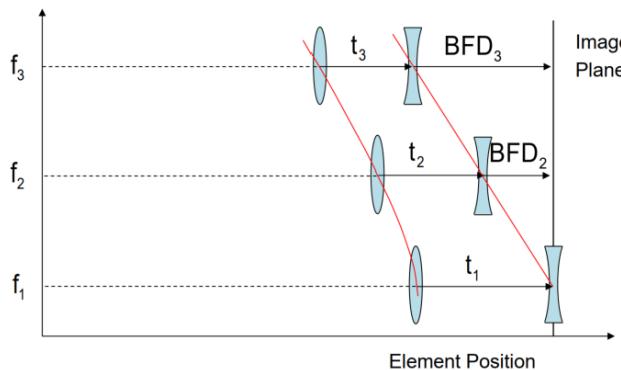
A **zoom lens** is a variable focal length objective with a **fixed** image plane. The simplest system is composed of two groups with powers  $\phi_1$  and  $\phi_2$  where both the focal length  $f$  and the BFD vary with the element spacing  $t$ .

$$\phi = \frac{1}{f} = \phi_1 + \phi_2 - \phi_1 \phi_2 t \quad \text{BFD} = f + d' = f - \frac{\phi_1}{\phi} t.$$

To vary the focal length with a fixed image plane, we:

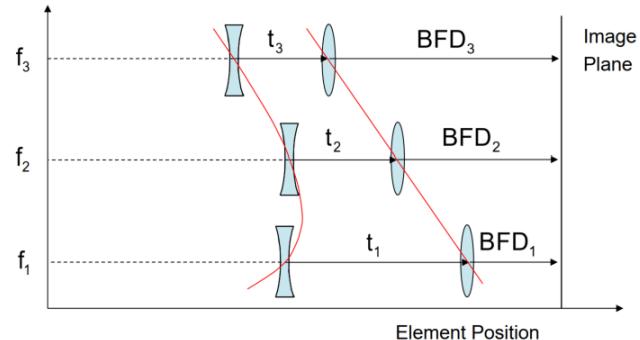
- Move the lens  $L_1$  a distance  $L = t + \text{BFD}$  from the image plane.
- Displace lens  $L_2$  a distance  $t$  from  $L_1$ .

Focal Length



(a) Telephoto zoom (limited by BFD)

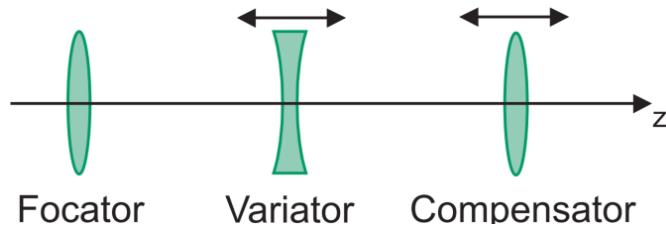
Focal Length



(b) Reverse telephoto zoom (commonly used)

As the separation approaches the sum of the individual focal lengths ( $f_1 + f_2$ ), the system becomes afocal ( $f \rightarrow \infty$ ).

A mechanical cam provides the complicated lens motion required for these **mechanical compensated** zoom lenses. A common three group configuration uses a fixed front element and moving second and third groups.



### Ejemplo 2.9

### Digital camera

You are using a digital camera to take a picture of a house against a distant mountain backdrop. The camera is focused on the house. You have a wide selection of camera lenses and 16:9 (width = 12.48mm, height = 7.02 mm) CCD detectors at your disposal. The CCD detectors available to you have pixels with sizes ranging between 3  $\mu\text{m}$  and 6  $\mu\text{m}$ , however, cameras with pixel sizes of exactly 3  $\mu\text{m}$  and 6  $\mu\text{m}$  are not available. You want to make sure that you can take a high-quality picture of the house and the mountains and therefore, want to achieve a diffraction-limited image based on the pixel pitch of the CCD detector you selected. The location allows you to place the camera anywhere between 10 and 30 meters (not exactly 10 nor 30 m) away from the house. Assume that the lens is a thin lens with the stop at the lens.

- What are the hyperfocal distance and acceptable blur for your camera system?
- Determine the focal length,  $f$ , and the diameter,  $D$ , of the lens.
- How far behind the lens must the detector be located? What is the maximum size (height and width) of the house in meters you would be able to image on the detector?
- What is the horizontal and vertical Field of View of this photographic system (in degrees)?
- What is the f/# of this camera? What is the closest object that will be considered to be in focus?

- f) It is an overcast day (outdoor illuminance=103 lux=103 lm/m<sup>2</sup>). The reflectance of the house is  $r = 0.5$  and the mountains have an average reflectance of  $r=0.18$ . What are the image plane illuminances for the house and the mountains individually?

### Solution

- a) CCD pixel pitch chosen is  $4 \mu m$ . And camera at  $L_H = 20 m$  away from the house.  
 b) The hyperfocal distance is  $L_H = 20 mm$ . The blur is  $B' \approx 4 \mu m$ .

We also fit the diameter of Airy in the pixel size with the average wavelength  $\lambda = 550 nm$  to reach the diffraction-limited condition:

$$2.44\lambda f/\# = B' \rightarrow f/\# = \frac{4}{2.44 \cdot 0.5} = 2.981 \approx 4.$$

- c) The hyperfocal distance can be related to focal length:

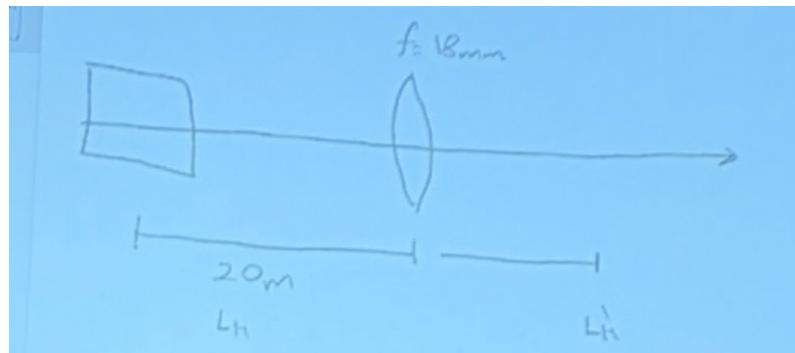
$$|L_H| = \frac{f^2}{B' f/\#} \rightarrow f = \sqrt{L_H B' f/\#} = \sqrt{20 \cdot 4 \mu m \cdot 4} = 17.889 mm.$$

The diameter is then:

$$D = \frac{f}{f/\#} = \frac{17.889 mm}{4} = 4.472 mm.$$

We can think of the thin lens taking an object at  $-20 m$  and imaging at  $L'_H$  (approximately):

$$\frac{1}{L'_H} = \frac{1}{L_H} + \frac{1}{f}$$



But  $L'_H = f + \text{DOF} \approx f$  so that the magnification is:

$$m = \frac{L'_H}{L_H} = \frac{\approx 18 mm}{-20 m} \approx -1 \cdot 10^{-3}.$$

The field of view for each dimension in image space is the triangle formed by  $L'_H$  and the size of one pixel detector:

$$\begin{aligned} \text{HFOV}_H &= \tan^{-1} \frac{7.02}{18 mm} = \pm 1.568^\circ \\ \text{HFOV}_W &= \tan^{-1} \frac{12.48}{18 mm} = \pm 1.569^\circ. \end{aligned}$$

- d) The near distance is half of the hyperfocal distance

$$L_{\text{near}} = \frac{L_H}{2} = 10 mm.$$

e) For the house,

$$E' = \frac{M}{4(f/\#_W)^2} = \frac{\rho E}{4(f/\#_W)^2} = \frac{\rho E}{4(f/\#)^2(1-m)^2}.$$

For the mountain (located at infinity):

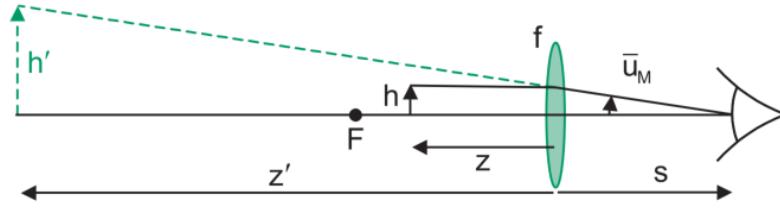
$$E' = \frac{\rho E}{4(f/\#)^2}.$$


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## 2.5 Magnifiers and Telescopes

### 2.5.1 Magnifiers

The largest image magnification possible with the unaided eye occurs when the object is placed at the **near point** of the eye, by convention, 250 mm or 10 in. A **magnifier** is a single lens that provides an enlarged **erect virtual image** of a nearby object for visual observation. The object must be placed within the front focal length of the lens.



The **magnifying power** MP is defined as (stop at the eye):

$$\begin{aligned} \text{MP} &= \frac{\text{Angular size of the image (with lens)}}{\text{Angular size of the object at the near point}} \\ &= \frac{\bar{u}_M}{\bar{u}_U} = \frac{h'/(z' - s)}{h/d_{NP}}, \quad d_{NP} = -250 \text{ mm} \\ &= \frac{250 \text{ mm}(z' - f)}{f(z' - s)} \approx \frac{250 \text{ mm}}{f}. \end{aligned}$$

The approximation relation is the most common definition of the MP. It assumes that the lens is close to the eye and the image is presented to a relaxed eye ( $z' = \infty$ ).

The angular subtense  $\theta$  of the image  $h'$  at the eye is

$$\theta = \frac{h\text{MP}}{250 \text{ mm}}. \quad (2.24)$$

The resolution of the human eye is about 1 *arcmin*, or  $(1/60)^\circ$ . In order to resolve an object of size  $h$ , the required MP is then

$$\text{MP} \geq \frac{0.075 \text{ mm}}{h}.$$

Magnifiers up to about 25X are practical; 10X is common.

### 2.5.2 Telescopes

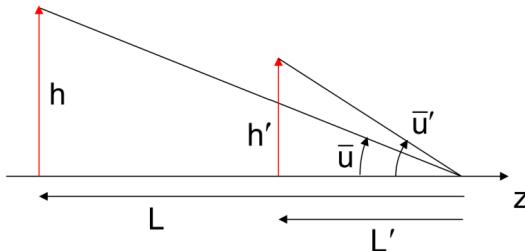
Telescopes are afocal systems used for visual observation of distant objects. The image through the telescope subtends an angle  $\theta'$  different from the angle subtend by the object  $\theta$ . The magnifying power of a telescope is:

$$MP = \frac{\theta'}{\theta} = \begin{cases} |MP| > 1, & \text{Telescope magnifies} \\ |MP| < 1, & \text{Telescope minifies} \end{cases}. \quad (2.25)$$

In Keplerian and Galilean telescope, the lateral magnification  $m$  is given by:

$$\text{Lateral magnification of telescope} \quad m = \frac{1}{MP} = -\frac{f_{\text{EYE}}}{f_{\text{OBJ}}}. \quad (2.26)$$

It is important to notice the reciprocal relation between the magnification and the magnifying power. For instance an image smaller than the object, also have the image much closer, so that the apparent size is much larger.



**Figure 2.1** In this case,  $m < 1$  while also  $MP > 1$ : height and distance are important for these quantities.

### Keplerian telescope

A Keplerian telescope or astronomical telescope consists of an objective that creates an **aerial image** (real image in the air) followed by a magnifier separated by  $f_1 + f_2$ . The system stop is usually at or near the objective lens.

The Keplerian has a negative MP:  $MP < 1$  and the image presented to the eye is inverted and reverted (rotated 180°). The eye should be placed at the real XP to couple the eye to the telescope and see the entire FOV, if not vignetting may occur. The XP position from the last surface is called the **eye relief** ER. The magnification of the telescope related the diameters of EP and XP:

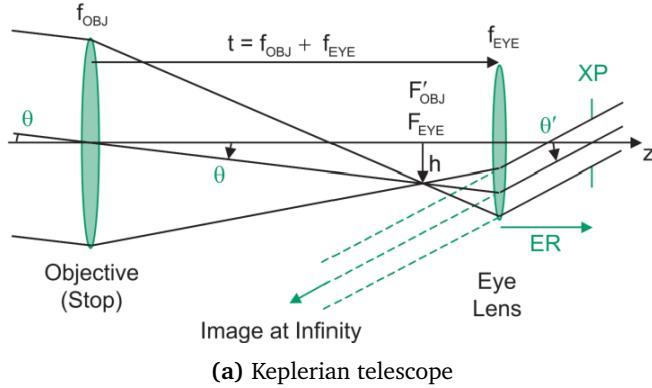
$$ER = z' = (1 - m)f_{\text{EYE}}, \quad D_{\text{XP}} = |m|D_{\text{EP}}. \quad (2.27)$$

The XP of a visual instrument is also known as the **eye circle** or the **Ramsden circle**.

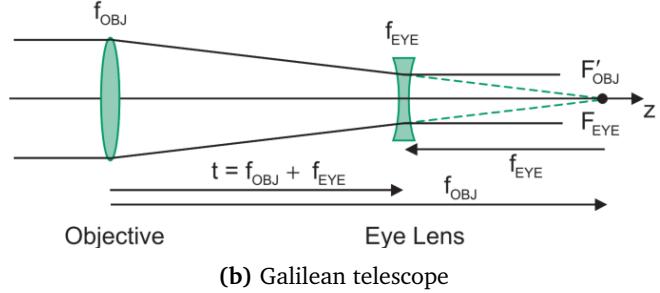
### Galilean telescope

The Galilean telescope uses a positive lens followed by a negative lens to obtain an erect image and a positive MP  $MP > 1$ . In this case, the XP is internal o virtual and not accesible to the eye. The FOV of the system is therefore small. There is no intermediate image plane.

For a given  $|MP|$ , the Galilean telescope is shorter and the FOV smaller than the corresponding Keplerian telescope.



(a) Keplerian telescope



(b) Galilean telescope

We describe several important points:

- Usually in telescope, the objective is the stop so that this lens is also the EP. Keplerian have real XP to the right of the eye lens.
- A **reversed Galilean telescope** provides a minified erect image  $0 < MP < 1$  and the eye is usually the system stop.
- Binoculars** are a pair of parallel telescopes for each eye.
- The specification provided on telescope and binoculars is of the form

$$AXB \implies A = |MP|, \quad \text{and} \quad B = \text{Objective diameter in mm.} \quad (2.28)$$

### Ejemplo 2.10

### Eye relief of a Keplerian telescope

A 5X Keplerian telescope is constructed out of two thin lenses. The separation between the two lenses is 120 mm, and the diameter of the objective lens is 25 mm. The system stop is at the objective. Determine the eye relief and the diameter of the exit pupil for this telescope.

$$D_{EP} = |m| D_{EP} = \frac{D_{EP}}{|MP|} = \frac{f_{EYE}}{f_{OBJ}} D_{EP}$$

$$ER = z' = (1 - m) f_{EYE}$$

#### Exit Pupil Diameter

$$D_{EP} = D_{STOP} = 25 \text{ mm}$$

$$D_{XP} = \frac{D_{EP}}{|MP|} = \frac{25 \text{ mm}}{5} = 5 \text{ mm}$$

#### We can use this same information to find the focal lengths

$$MP = -5 = -\frac{f_{OBJ}}{f_{EYE}} \quad f_{OBJ} = 5f_{EYE} \quad t = f_{OBJ} + f_{EYE} = 6f_{EYE} = 120 \text{ mm}$$

$$f_{EYE} = 20 \text{ mm} \quad f_{OBJ} = 100 \text{ mm}$$

#### Determining Eye Relief Location

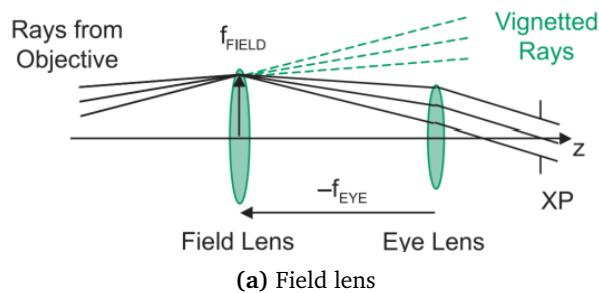
image stop through lens, or use equation

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_{EYE}} \quad z = -(f_{OBJ} + f_{EYE}) = -t = -120 \text{ mm}$$

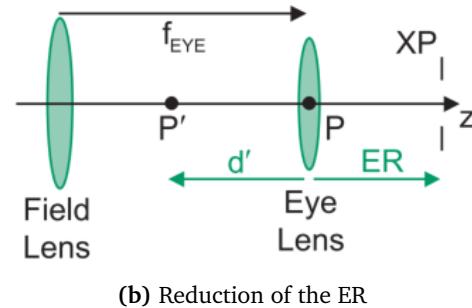
$$z' = ER = 24 \text{ mm}$$

### 2.5.3 Field lenses

The FOV of the Keplerian telescope is limited by vignetting at the eye lens. A **field lens** placed at the intermediate image plane increases the FOV by bending the ray bundle into the aperture of the eye lens.



(a) Field lens



(b) Reduction of the ER

This combination is called an **eyepiece**. If we assume the separation between them is equal to the focal length of the eye lens, then the overall power is:

$$\phi = \phi_{EYE} + \phi_{FIELD} - \phi_{EYE}\phi_{FIELD} \cdot \frac{1}{\phi_{EYE}} = \phi_{EYE}.$$

Thus, the eyepiece maintains its focal length and therefore its MP. The shifts of the principal planes are:

$$d = \frac{\phi_{EYE}}{\phi_{EYE} \phi_{FIELD}} \frac{1}{\phi_{EYE}} = f_{EYE}, \quad \text{and} \quad d' = -\frac{\phi_{FIELD}}{\phi_{EYE}} \frac{1}{\phi_{EYE}} = -\frac{f_{EYE}^2}{f_{FIELD}}. \quad (2.29)$$

To summarize, the inclusion of the field lens:

- Decrease vignetting increasing in the same way the FOV.
- Does not change the MP of the telescope or the size of the XP.
- The front principal plane is unchanged, but the rear principal plane and ER are shifted by  $d'$ .
- The field lens is usually also displaced from the image plane to minimize the inclusion of information of the field lens to the image through defocus.
- MP of the eyepiece is the same as the MP of a magnifier.

#### 2.5.4 Eyepieces

An **eyepiece** or **ocular** is the combination of the field lens and the eye lens. A simple eye piece does not have a field lens. A compound eyepiece has both of them. A field stop can be placed at the intermediate image plane to restrict the system FOV. This aperture serves to correct the vignetting.

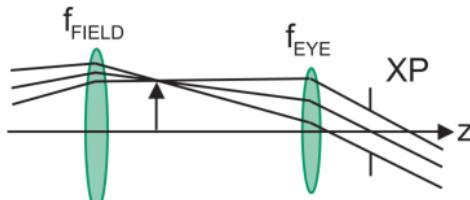
Two special eyepiece configurations displace the field lens from the intermediate image:

- **Huygens eyepiece** has the intermediate image between the two elements.
- **Ramsden eyepiece** has intermediate image in front of the field lens. This configuration has about 50% more ER than the Huygens eyepiece.
- **Kellner eyepiece** replaces the singlet eye lens of the Ramsden eyepiece with a doublet for color correction.

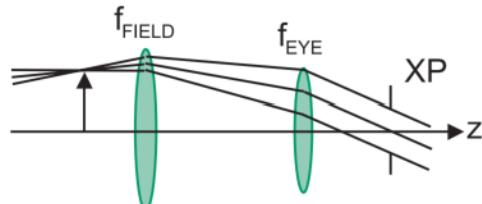
Hand-held instruments should have 15 – 20 mm of eye relief. Microscopes may have as little as 2 – 3 mm. Other systems, such as riflescopes, should have a very long eye relief. The human eye pupil diameter varies from 2 – 8 mm, with a diameter of about 4 mm under ordinary lighting conditions. When overfilled, the eye becomes the system stop.

#### 2.5.5 Mirror-based Telescopes

The imaging properties of conic surfaces are used in the design of **mirror-based telescopes**.

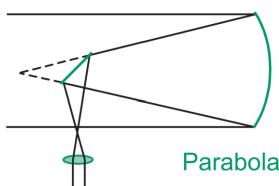


(a) Huygens eyepiece

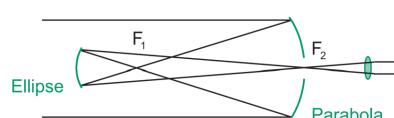


(b) Ramsden eyepiece

- **Newtonian telescope** uses a parabola with a fold flat. It is analogous to a Keplerian refracting telescope.
- **Gregorian telescope** uses a parabola followed by an ellipse to relay the intermediate image. It produces an erect image.
- **Cassegrain telescope** uses a parabola combined with a hyperbolic secondary mirror to reduce the system length. It is equivalent to a telephoto objective. The two conic surfaces correct the spherical aberration.
- **Ritchey-Chretien telescope** is identical to Cassegrain telescope in layout, except that it uses two hyperbolic mirrors to correct coma as well as spherical aberration.



(a) Newtonian



(b) Gregorian



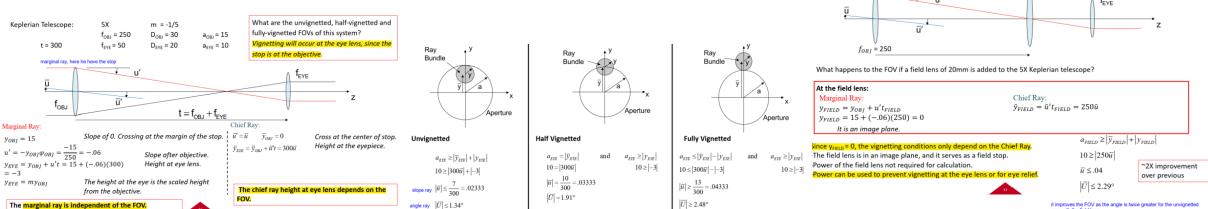
(c) Cassegrain

### Ejemplo 2.11

### Vignetting in a Keplerian telescope

### Ejemplo 2.12

### Vignetting in a Galilean telescope



When looking through a Galilean telescope, it appears that you are looking through a hole well out in front of the telescope. The hole is the image of the objective lens through the negative eye lens. Vignetting at the objective lens usually limits the FOV of a Galilean telescope.

The apparent size of the image of the objective lens (the hole) as viewed from the eye determines the half-vignetted FOV and the chief ray at the eye.

Image of the objective through the eye lens.

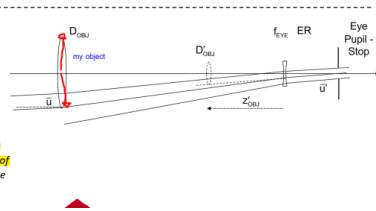
$$\frac{1}{z_{\text{OBJ}}'} = \frac{1}{z_{\text{OBJ}}} + \frac{1}{f_{\text{EYE}}} = -t + \frac{1}{f_{\text{EYE}}}$$

$$z_{\text{OBJ}}' = -33.33 \text{ mm}$$

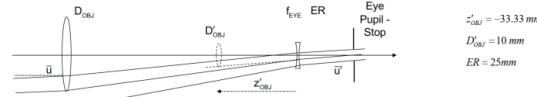
$$m = z_{\text{OBJ}}'/z_{\text{OBJ}} = 0.3333^*$$

$$D_{\text{OBJ}}' = 10 \text{ mm}^*$$

Is this expected? Yes. The objective is in object space. It has a diameter of 30mm. The image of the objective is in image space, it is 10mm. The diameters must be related by the telescope magnification (3X)



The apparent size of the image of the objective lens (the hole) as viewed from the eye determines the half-vignetted FOV and the chief ray at the eye. The chief ray in object space also goes through the edge of the objective lens.



$$i' = \tan(\theta') = \frac{D_{\text{OBJ}}/2}{ER + |z_{\text{OBJ}}'|} = \frac{5 \text{ mm}}{58.33 \text{ mm}}$$

$$\theta' = \pm 4.9^\circ \quad MP = \frac{\theta'}{\theta}$$

$$\theta = \frac{\theta'}{MP} = \pm 1.6^\circ \quad \text{we have FOV in object and image space, and they are related through the magnifying power MP}$$

This is the half-vignetted FOV of the telescope. Note that the FOV depends on the value of the Eye Relief or where the telescope is placed relative to the eye.

## 2.6 Relays and Microscopes

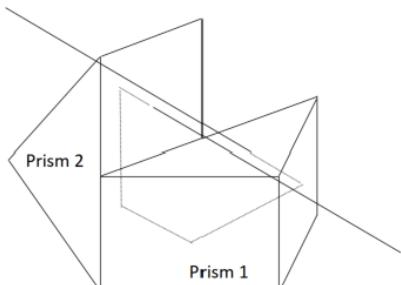
### 2.6.1 Prisms and relay lenses

For many applications, it is important that the image have the same orientation as the object. The commonly used methods for image erection is to use prism, and relay systems.

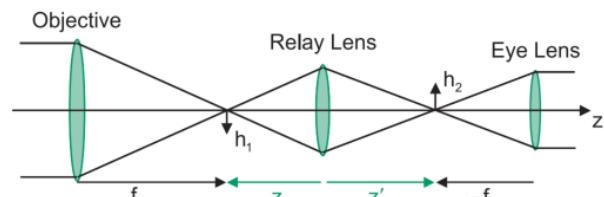
On the one hand, prisms as porro or Pechan-roof prism may help to erect the image. It is important that the ray bundle must be sized so the entrance and exit faces do not vignette the FOV. Unfolding the prism make possible to treat it as a plane parallel plate of glass.

On the other hand, the **relay lens** consists of an additional lens that takes the image of the objective (object for this lens) and image it to the right. This image is the object for the eyepiece. The net MP of the relayed Keplerian telescope in the figure is positive and equals to the product of the magnification of the relay and the MP of the original Keplerian telescope:

$$MP = m_R MP_K = -\frac{z'_R f_{\text{OBJ}}}{z_R f_{\text{EYE}}}, \quad \text{where} \quad m_R = \frac{z'_R}{z_R}. \quad (2.30)$$



(a) Using a prism



(b) Using a relay lens

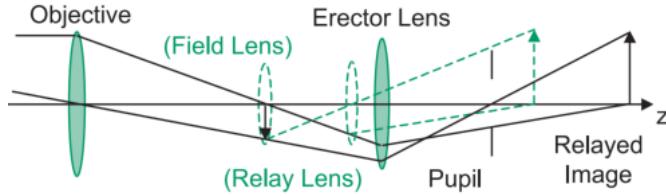
Multiple relay lenses can be used to transfer the imager over long distances, such as in periscopes, and scopes and borescopes. Field lenses can also be added at the intermediate lenses.

The functions of a field lens and a relay lens can be combined into a single **erector lens**. This lens will require a diameter larger than the replaced field or relay lenses. The relayed image and pupil are shifted from their original positions.

### Ejemplo 2.13

A 30X relayed Keplerian telescope is constructed out of three thin lenses in air. The relay lens of the telescope

### Keplerian telescope with relay lens



operates with a magnification of 1.5. The focal length of the objective lens is 400 mm, and the overall telescope length is 500 mm.

- Determine the design of the telescope.
- Assuming that the system stop is located at the objective lens, determine the eye relief of the telescope (distance from the eye lens to the XP).

### Solution

- The magnification of the relay lens and the total magnification are:

$$m_R = \frac{z'_R}{z_R} = -1.5, \quad MP = m_R MP_K = (-1.5)(-\frac{f_1}{f_2}) = 30 \rightarrow f_2 = \frac{400 \cdot 1.5}{30} = 20 \text{ mm.}$$

Therefore, the remaining space is 80 mm which is distributed for the relay lens.

$$-z_R + z'_R = 80 \text{ mm.}$$

Using it with the relay magnification we have that

$$z_R = -32 \text{ mm, and } z'_R = 48 \text{ mm.}$$

The focal length of the relay lens is obtained with the thin-lens equation:

$$\frac{1}{z'_R} = \frac{1}{z_R} + \frac{1}{f_R} \rightarrow f_R = 19.2 \text{ mm.}$$

- The eye relief is located at the image position in image space of the stop. The relay lens takes that object located at  $z = -(32 + 400) = -432 \text{ mm}$ .

$$z' = \frac{1}{\frac{1}{-432} + \frac{1}{19.2}} = 20.093 \text{ mm.}$$

This image is located at  $z = -(20 + 48 - 20.093) = -47.907 \text{ mm}$  to the eye lens and is seen as the object, which is imaged to:

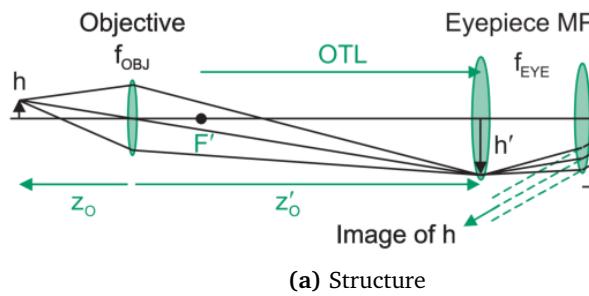
$$z'_{XP} = ER = \frac{1}{\frac{1}{-47.907} + \frac{1}{20}} = 34.333 \text{ mm.}$$

### 2.6.2 Microscopes

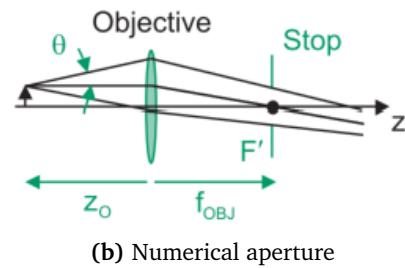
#### Definition

A **microscope** is a sophisticated magnifier consisting of an objective plus an eyepiece. The **visual magnification** is the product of the objective magnification and the eyepiece MP:

$$\text{Visual magnification} \quad m_V = m_{OBJ} MP_{EYE} = \frac{z'_0}{z_0} \frac{250 \text{ mm}}{f_{EYE}}. \quad (2.31)$$



### (a) Structure



**(b)** Numerical aperture

The **optical tube length** OTL of a microscope is defined as the distance from the rear focal point of the objective to the front focal point of the eyepiece (intermediate image). Standard values are 160 mm and 125 mm. The OTL is a Newtonian image distance:

$$m_{\text{OBJ}} = -\frac{\text{OTL}}{f_{\text{OBJ}}}. \quad (2.32)$$

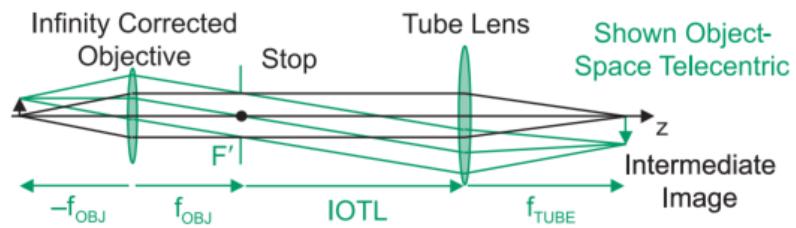
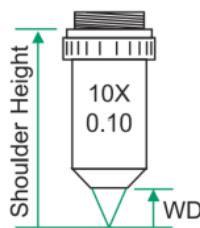
The NA of a microscope objective is defined in object space by the half-angle of the accepted input ray bundle. Along with the objective magnification, the NA is inscribed on the objective barrel:

$$\text{NA} = n \sin \theta. \quad (2.33)$$

Microscope objective are often telecentric in object space. The stop is placed at the rear focal point of the objective so that the magnification does not change with defocus.

## Microscope terminology

- The **working distance** WD is the distance from the object to the first element of the objective; can be less than 1 mm for high-power objectives.
  - The **mechanical tube length** is the separation between the shoulder of the threaded mount of the objective and the end of the tube into which the eyepiece is inserted.
  - A set of **parfocal objectives** have different magnifications, but the same **shoulder height** and the same shoulder-to-intermediate image distance.
  - **Biological objectives** are aberration corrected assuming a cover glass between the object and the objective.
  - Research-graded microscopes are usually designed using **infinity corrected objectives**. The object plane is the front focal plane of the objective, and a collimated beam results for each point.



The magnification of the objective-tube lens combination is

$$m_{\text{OBJ}} = -\frac{f_{\text{TUBE}}}{f_{\text{OBJ}}}. \quad (2.34)$$

If the objective is object-space telecentric and  $f_{\text{TUBE}}$  equals the infinite optical tube length IOTL, the combination is afocal and double telecentric. This is a useful feature when using reticles in the eyepiece.

### Ejemplo 2.14

### Microscope

As an optical engineer, you are asked to design a finite conjugate microscope. The microscope is a combination of the objective lens and the eyepiece, which is also composed of a field lens and an eye lens.

Fortunately, your company has collected over the years all kinds of parts, including:

- Optical tube of any length ranging from 170 – 200 mm.
  - Lenses of any diameter between 6 – 18 mm with any focal length between 50–500 mm.
- Design a microscope with a visual magnification between 50X – 100X. Pick a value, select the focal length of your eye piece, and determine the working distance in object space of the microscope.
  - Your microscope will have to operate with a numerical aperture between 0.15 – 0.25. Select the stop location such that your microscope operates telecentric in object space, select a nominal value for NA, and determine the required stop diameter to produce this NA.
  - Select a lens diameter for your microscope objective and determine the maximum object size  $\pm h$  that you can image with this instrument while unvignetted. If it is not possible with your chosen design, explain why in terms of marginal and chief ray heights.

### Solution

- a) We pick  $m_v = 50X$ , an  $OTL = 200 \text{ mm}$ , and a eye lens with  $f_{\text{EYE}} = 50 \text{ mm}$ .

$$m_v = \frac{OTL}{f_{\text{obj}}} \frac{250}{f_{\text{EYE}}} = -\frac{z'_0}{z_0} \frac{250}{f_{\text{EYE}}}.$$

We solve in the above equation for  $f_{\text{obj}}$ :

$$f_{\text{obj}} = \frac{OTL250}{m_v f_{\text{EYE}}} = \frac{200 \cdot 250}{50 \cdot 50} = 20 \text{ mm}.$$

The image distance is  $z'_0 = 200 + 20 = 220 \text{ mm}$ . We now solve for  $z_0$ :

$$z_0 = WD = -\frac{z'_0}{m_v} \frac{250}{f_{\text{EYE}}} = -22 \text{ mm}.$$

- b) The numerical aperture chosen is  $NA = 0.2$ , which we use to get the slope ray and raytrace to the objective lens:

$$U = \sin^{-1}(NA) = \sin^{-1}(0.2) = 11.54^\circ \rightarrow u = \tan(11.54^\circ) = 0.204.$$

$$y' = y + ut = 0 + 0.204 \cdot 22 = 4.49 \text{ mm}, \quad u' = u - y\phi = 0.204 - 4.49/20 = -0.0205$$

$$y_{\text{stop}} = y' + u'f_{\text{obj}} = 4.08 \text{ mm} \rightarrow D_{\text{stop}} = 8.16 \text{ mm}.$$

- c) We choose a diameter of the lens to be  $D_{\text{obj}} = 10 \text{ mm}$ . Now, we impose the unvignetting condition to get the maximum height of the object:

$$D = 2(y + \bar{y}) = 2(4.49 + \bar{y}) \rightarrow \bar{y} = y_{\text{obj height}} = h = \frac{10 - 2 \cdot 4.49}{2} = \pm 0.51 \text{ mm}.$$

## 2.7 Telecentric systems

### 2.7.1 Telecentricity

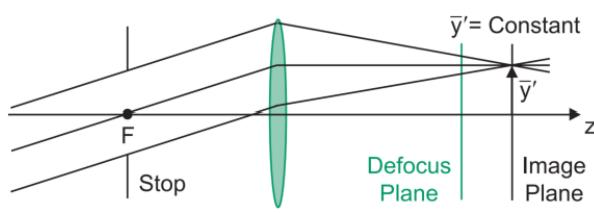
In a **telecentric system**, the EP and/or the XP are located at **infinity**. **Telecentricity** in object or image space requires that the chief ray be parallel to the axis in that space. Consequently, the apparent system magnification is **constant** even if the object or image plane is displaced. The image will be blurred, but of the correct size or magnification.



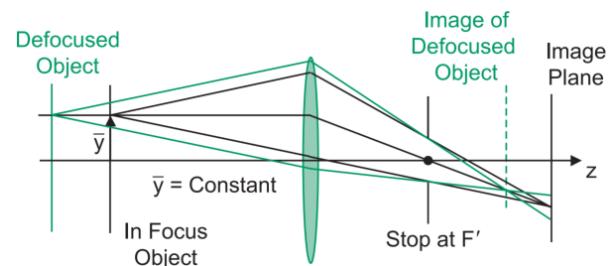
### 2.7.2 Single telecentric systems

When the stop is located at the front focal plane of a focal system, the XP is at infinity, and the system is **image-space telecentric**. Defocus of the image plane or detector will not change the image height.

On the other hand, placing the stop at the rear focal plane puts the EP at infinity and forms an **object-space telecentric** system. The blur from the defocused object is centered about the chief ray and the image height at the nominal image is constant.



(a) Image-space telecentric



(b) Object-space telecentric

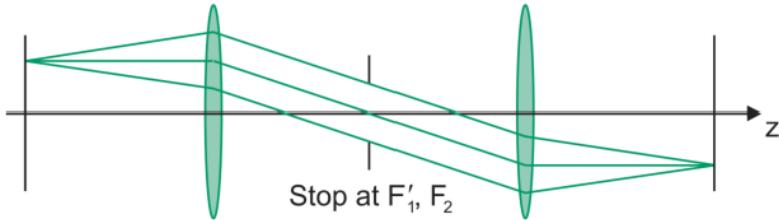
The telecentric system must be unvignetted over the entire object FOV, otherwise the blur spot will no longer be symmetric and the centroid will shift.

Object-space telecentric systems are almost used at finite conjugated. The maximum object size is limited to approximately the radius of the objective lens due to vignetting considerations. This system doesn't have a  $f/\#$  because the EP is at infinity and is infinite in size.

### 2.7.3 Double telecentric systems

An afocal system is made **double telecentric** by placing the system stop at the common focal point. The chief ray is parallel to the axis in object and image space, and both the EP and XP are located at infinity. All double telecentric systems must be afocal.

Since the ray bundle is centered on the chief ray, this condition guarantees that height of the blur forming the image is independent of axial object/image shifts.



### FOV in telecentric systems

Defining the angular FOV relative to the EP or XP is impossible if the system is telecentric in that particular optical space because that pupil is in infinity. The object height or image height can be used instead.

A second method is to measure the angular size of the object relative to the front nodal point N. Angular sizes of the object and image are equal when viewed from the respective nodal points. But for afocal systems (like telecentric) fails as they do not have these points.

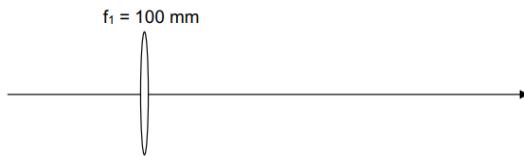
The choice of the method is of little consequence when the object is distant.

### Ejemplo 2.15

### Double telecentric system

A double telecentric system is constructed out of two thin lenses in air. It has a magnification of  $|1/10|$ . The focal length of the first lens of the system is 100 mm. An object of size  $\pm 100 \text{ mm}$  is located 500 mm to the left of the first lens in the system.

- Provide a layout of the system showing the second lens, spacings and the stop.
- What is the focal length of the second lens?
- Where is the image plane (location relative to the second lens element)?
- What is the size and orientation of the image?



### Solution

- Second lens should be  $f_1 + f_2$  to the right of  $L_1$  while the stop in the intermediate image.
- Magnification is

$$m = -\frac{f_2}{f_1} = -\frac{1}{10} = -\frac{f_2}{100 \text{ mm}} \rightarrow f_2 = 10 \text{ mm}.$$

The length between the lenses is therefore  $L = f_1 + f_2 = 110 \text{ mm}$ .

- The longitudinal magnification is

$$\bar{m} = m^2 = 0.01.$$

Assuming the pair of conjugates from the front and rear focal lengths, the depth of the object is:

$$\Delta z = -500 - (-100) = -400 \text{ mm}, \quad \text{and} \quad \Delta z' = \bar{m}\Delta z = -4 \text{ mm}.$$

The object is located 500 mm to the left of the first lens and the image is 6 mm to the right of the second lens.

- d) The size of the image is

$$h' = mh = (-0.1)(\pm 100) = \mp 10 \text{ mm},$$

which is inverted.

### 2.7.4 Imaging with an afocal system

An afocal system consists of two focal systems sharing a common focal point. If the stop is located at this location, the system becomes telecentric. Because the magnification is constant, the cardinal points are not defined.

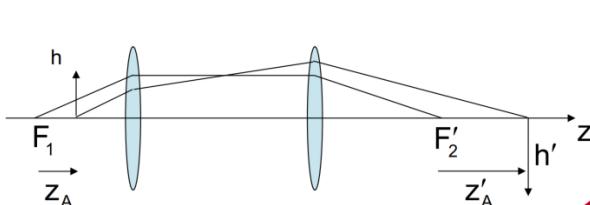
However, we can pick any pair of conjugate planes coupled with the longitudinal magnification:

$$\bar{m} = m^2 = \frac{\Delta z'}{\Delta z} = \frac{z'_A}{z_A}. \quad (2.35)$$

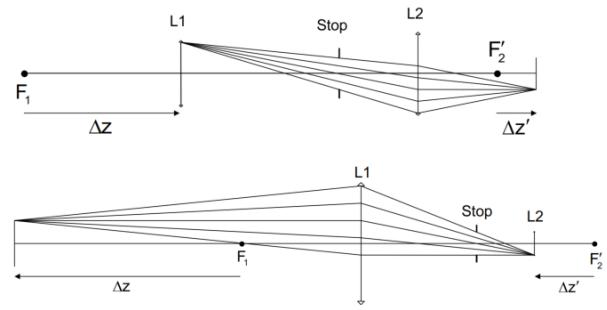
A usual pair is the front focal point of the first system with the rear focal point of the second system. Another is either the object is located at the first lens, or the image is at the second lens; these two situations image as if the other lens as not present.

Once selected, any other pair can be found using the longitudinal magnification. An axial shift in object space results in an image plane shift given by

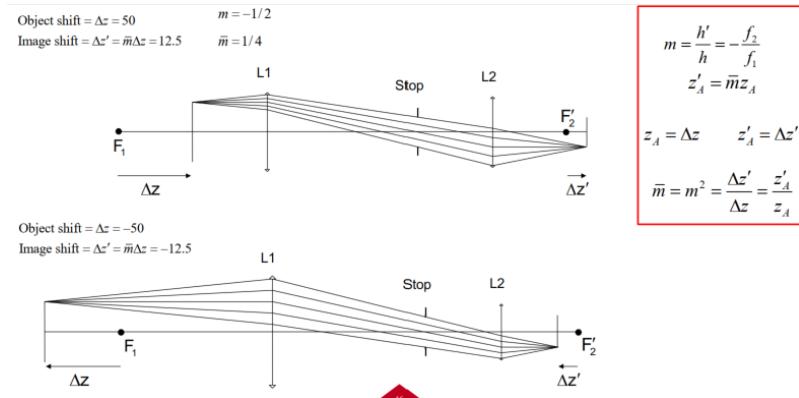
$$z'_A = \bar{m}z_A. \quad (2.36)$$



(a) Focal points conjugate pair



(b) First/second lens conjugate pair



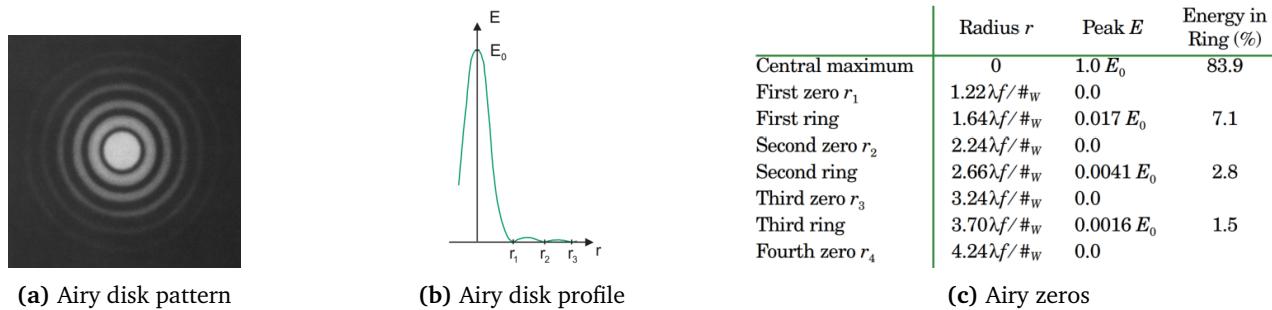
## 2.8 Stop and image quality

### 2.8.1 Diffraction-limited

Because light is a wave, it does not focus to a perfect point image. Diffraction from the aperture limit the size of the image spot. For circular aperture, we get an **Airy disk** pattern.

$$\text{Airy disk equation} \quad E = E_0 \left[ \frac{2J_1(\pi r/\lambda f/\#W)}{\pi r/\lambda f/\#W} \right]^2, \quad (2.37)$$

where  $r$  is the radial coordinate,  $J_1$  is the first-kind Bessel function, and  $f/\#W$  the image space working.



The diameter of the Airy disk to the first zero is:

$$D = 2.44\lambda \cdot f/\#W. \quad (2.38)$$

The **Rayleigh resolution criterion** states that two point objects can be resolved if the peak of one falls on the first zero of the other:

$$\text{Resolution} = 1.22\lambda \cdot f/\#W. \quad (2.39)$$

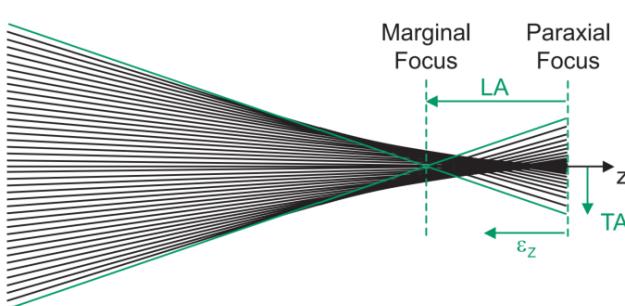
The **angular resolution** is found by dividing by the focal length (or image distance):

$$\text{Angular resolution} = \alpha = 1.22\lambda/D_{EP}. \quad (2.40)$$

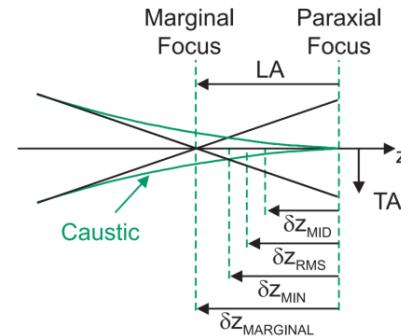
### 2.8.2 Spherical aberration

**Spherical aberration** SA causes the power or focal length of the system to vary with pupil radius. For a singlet, the power of the lens increases quadratically with pupil radius; the focal length decrease quadratically.

The image plane can be shifted from paraxial focus to obtain better image quality in the presence of SA. There are different focus criteria as seen in the figure.



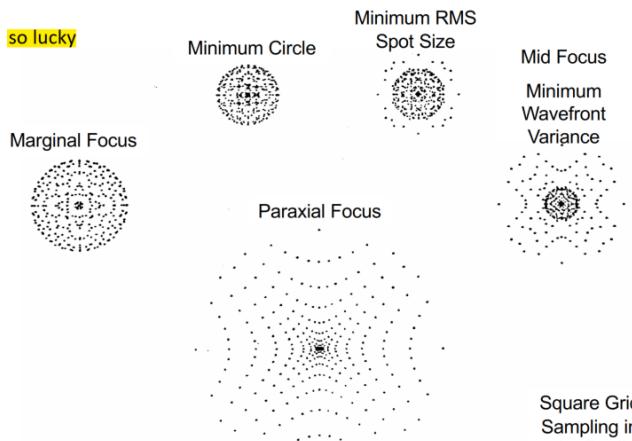
(a) Spherical aberration



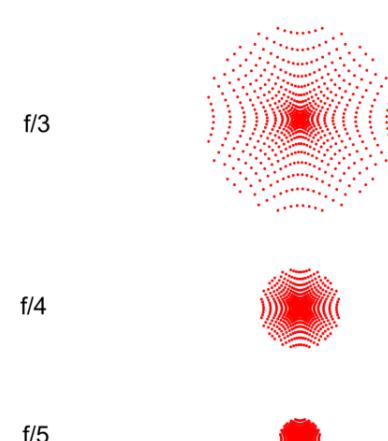
(b) Image plane criteria

**Figure 2.2** Spherical aberration produces different image plane criteria. LA and Ta stand for longitudinal and transverse aberration, respectively.

In first-order geometrical optics, each point on the object plane corresponds to a point on the image plane. However, in real life we are not so lucky.



(a) Spot diagram of focus criteria



(b) Spot size versus F-number

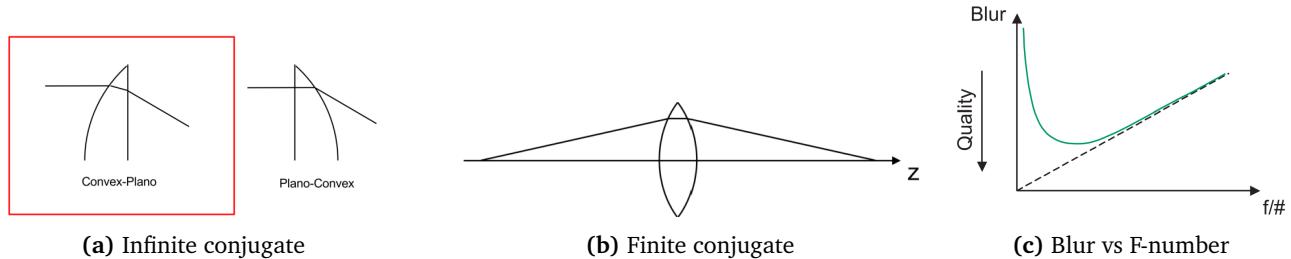
The spot size scales as the cube of the entrance pupil diameter.

### 2.8.3 Lens bending and minimum spherical aberration

**Bending the lens** of orientation does not change the power, but its aberration do change. The minimum SA occurs when the ray is bent the same at both surfaces. This is directly analogous to the angle of minimum deviation for prisms. For an object at infinity and  $n = 1.5$ , the correct lens shape is approximately convex-plano. At finite conjugates, a biconvex lens is used. A trick to further minimize spherical aberration in finite conjugates is to split the biconvex into two plano-convex lenses and then flip each of

the lenses.

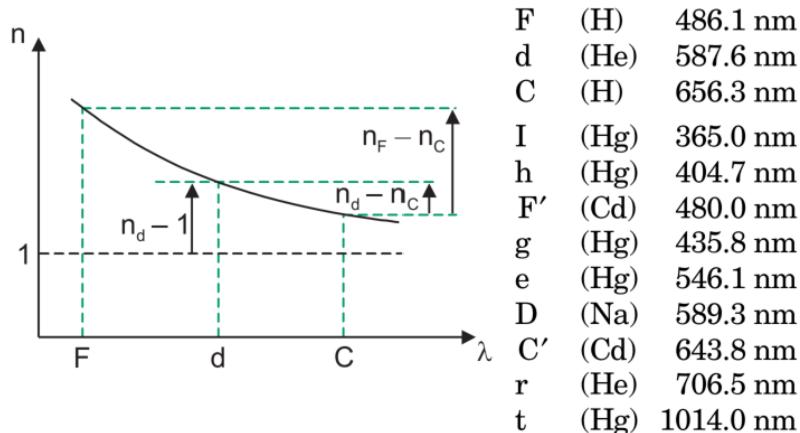
With large apertures, aberrations and depth of field errors are dominant. With small apertures, diffraction dominates with a linear dependence of blur with  $f/\#$ .



## 2.9 Materials

### 2.9.1 Dispersion

Index of refraction is commonly measured and reported at the specific wavelengths of elemental spectral lines. Over the visible spectrum, the **dispersion** of the index for optical glass is about 0.5% (low dispersion) to 1.5% (high dispersion) of the mean value of the index.



**Figure 2.1** For visible applications, the F, d, and C lines are usually used.

We define some useful quantities:

$$\text{Refractivity} = n_d - 1, \quad \text{Principal dispersion} = n_F - n_C, \quad \text{Partial dispersion} = n_d - n_C. \quad (2.41)$$

The **Abbe number** is the single number used to characterize the dispersion of the index of an optical material:

$$\text{Abbe number} \quad \nu = V = \frac{n_d - 1}{n_F - n_C} \quad (2.42)$$

Typical values of the Abbe number for optical glass range from 25 to 65. Low  $\nu$ -values indicate high dispersion.

Relative partial dispersion ratio or P-value gives the fraction of the total index change that occurs between the d and C wavelengths  $n_d - n_C$ :

$$P = P_{d,C} = \frac{n_d - n_C}{n_F - n_C}. \quad (2.43)$$

Due to flattening of the dispersion,  $P_{d,C} < 0.5$ . P-values can also be defined for other sets of wavelengths:

$$\text{Relative partial dispersion ratio} \quad P_{X,Y} = \frac{n_X - n_Y}{n_F - n_C}. \quad (2.44)$$

## 2.9.2 Optical glass

### Glass map

The **glass map** plots index of refraction versus Abbe number. By tradition, the Abbe number increases to the left, so that dispersion increases to the right. The **glass line** is the locus of ordinary optical glasses based on silicon dioxide.

The green line at  $\nu \sim 50 - 55$  separates the glasses into crown glass (low dispersion) and flint glass (high dispersion).

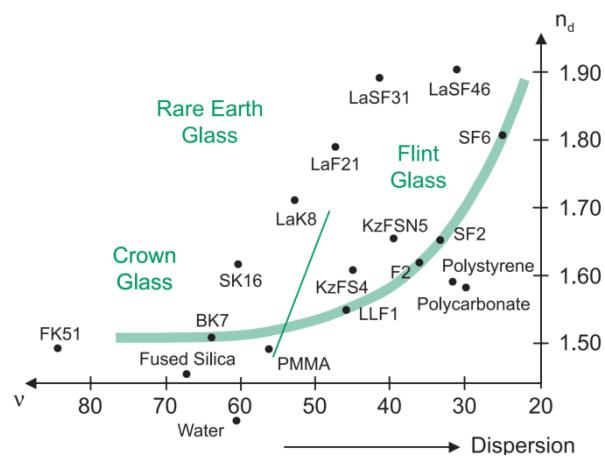
The addition of lead oxide increases the dispersion and the index and moves the glass up the glass line. To increase the index without changing the dispersion, barium oxide is added.

Glass away from the glass line are softer and more difficult to polish. Low index glasses are less dense and generally have better blue transmission.

### Glass code

The six-digit **glass code** specifies the index and the Abbe number:

$$abcdef \implies n_d = 1.abc, \quad \nu = de.f \quad (2.45)$$



(a) Glass map

Material	Code	$n_d$	$n_F$	$n_C$	$\nu$	$P$
N-FK51*	487845	1.48656	1.49056	1.48480	84.5	0.306
N-BK7	517642	1.51680	1.52238	1.51432	64.2	0.308
LLF1	548458	1.54814	1.55655	1.54457	45.8	0.298
N-KzFS4	613445	1.61336	1.62300	1.60922	44.5	0.301
N-F2	620364	1.62005	1.63208	1.61506	36.4	0.294
N-SK16	620603	1.62041	1.62756	1.61727	60.3	0.305
SF2	648339	1.64769	1.66123	1.64210	33.9	0.292
KzFSN5	654396	1.65412	1.66571	1.64920	39.6	0.298
N-LaK8	713538	1.71300	1.72222	1.70897	53.8	0.304
N-LaF21	788475	1.78800	1.79960	1.78301	47.5	0.301
N-SF6	805254	1.80518	1.82783	1.79608	25.4	0.287
N-LaSF31	881410	1.88067	1.89576	1.87429	41.0	0.297
N-LaSF46	901316	1.90138	1.92156	1.89307	31.6	0.292
Fused Silica	458678	1.45847	1.46313	1.45637	67.8	0.311
PMMA	492574	1.492	1.498	1.489	≈55	≈0.33
Polycarbonate	585299	1.585	1.600	1.580	≈30	≈0.25
Polystyrene	590311	1.590	1.604	1.585	≈31	≈0.26
Water	333560	1.333	1.337	1.331	≈60	≈0.33

(b) Glass code

The properties of an individual sample, especially for the plastic material and water, can vary from these catalog values. The measured indices of the actual glass should be used in final design for precision

systems. The listed indices are measured relative to air ( $n \approx 1.0003$ ), and the indices should be corrected for use in vacuum. The glass catalog lists other material properties important for a design such as [thermal expansion coefficient](#), [teperature coefficient of refractive index](#), [internal transmission](#), etc.

## Chapter 3

# Applications

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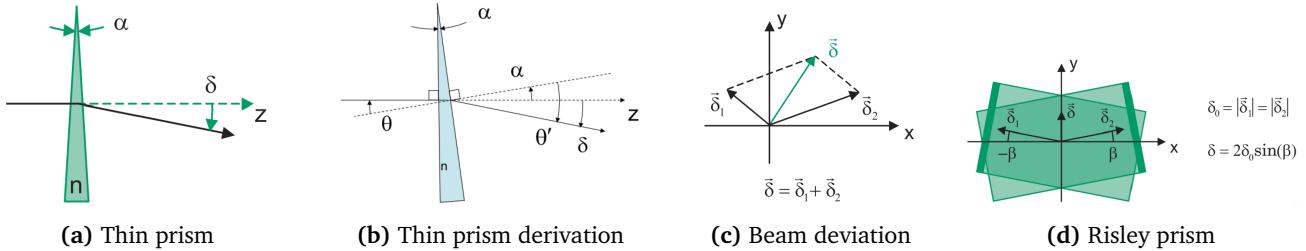
## 3.1 Thin prisms and dispersing prisms

### 3.1.1 Thin prisms

**Thin prisms** introduce small angular beam deviations  $\delta$  that is approximately independent of the incident angle:

$$\text{Thin prism deviation} \quad \delta \approx -(n - 1)\alpha. \quad (3.1)$$

The deviation is measured in prism diopters. A prisms of 1 diopter deviates a beam by 1 cm at 1 m. The beam deviation is parallel to a principal section of the prism and towards the thick end of the prism.



#### Proof of the thin prism deviation

We tip the prism by  $\theta$  so that the front face is perpendicular to the input ray (no refraction):

$$\begin{aligned} \theta &= -\alpha & n\theta &= \theta' \quad (\text{small angle}) \\ \theta' &= \delta - \alpha & -n\alpha &= \delta - \alpha \\ & & \delta &\approx -(n - 1)\alpha \end{aligned} . \quad (3.2)$$

The magnitude and direction of the deviation defines a vector perpendicular to the optical axis (xy plane). The net deviation vector for a series of thin prisms is then the vector sum of the component vectors:

$$\text{Algebra of deviation} \quad \delta = \delta_1 + \delta_2.$$

### 3.1.2 Risley prism

A **Risley prism** consists of a pair of identical, but opposing, thin prisms. The prisms are counter-rotated by  $\pm\beta$  to obtain a variable net deviation in a fixed direction. The Risley prism allows the fine angular alignment for an optical system by adjusting the prism orientation  $\beta$ .

### 3.1.3 Thin prism dispersion

#### Thin prism

The dispersion of a thin prism  $\Delta$  measures the total angular spread from  $C$  to  $F$  light, and the **secondary dispersion**  $\varepsilon$  gives the spread from the  $C$  to  $d$  wavelengths. The results depend on the index  $n_d$ , the Abbe number  $\nu$  and the partial dispersion ratio  $P$  of the glass:

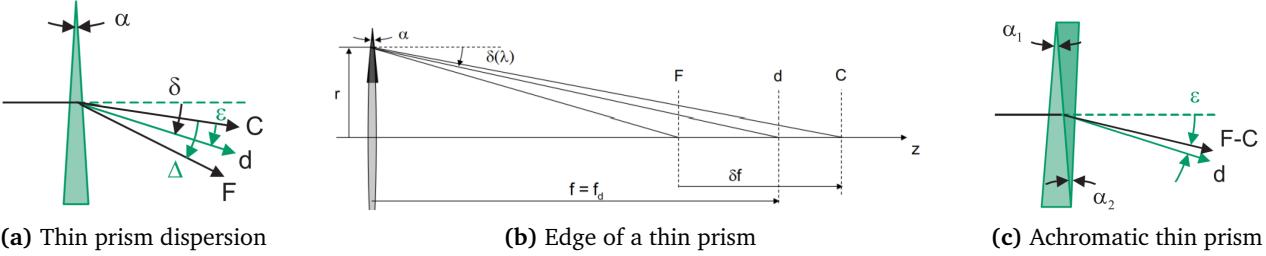
$$\text{Deviation} \quad \delta = -(n_d - 1)\alpha \quad (3.3)$$

$$\text{Dispersion} \quad \Delta = (n_F - 1)(-\alpha) - (n_C - 1)(-\alpha) = -(n_F - n_C)\alpha \quad (3.4)$$

$$\text{Secondary dispersion} \quad \varepsilon = (n_d - 1)(-\alpha) - (n_C - 1)(-\alpha) = -(n_d - n_C)\alpha. \quad (3.5)$$

From above and the definition of the Abbe number, we can also get the following relations

$$\frac{\delta}{\Delta} = \frac{n_d - 1}{n_F - n_C} = \nu \longrightarrow \Delta = \frac{\delta}{\nu}, \quad \text{and} \quad \frac{\varepsilon}{\Delta} = \frac{n_d - n_C}{n_F - n_C} = P \longrightarrow \varepsilon = P \frac{\delta}{\nu}. \quad (3.6)$$



The edge of a thin lens can be considered to be a thin prism. The angle  $\alpha$  depend on the power and diameter of the lens. Each wavelength will be deviated by a different angle. The focal length of the lens is defined for  $d$  light. From the above figure, we have the following relation.

#### How does the difference in focal length is related with the Abbe number?

$$\begin{aligned} \delta(\lambda) &= -(n_\alpha - 1)\alpha, \quad f(\lambda) = -\frac{r}{\delta(\lambda)}, \quad f_d = -\frac{r}{\delta_d} \\ \delta f &= f_C - f_F = -\frac{r}{\delta_C} + \frac{r}{\delta_F} = -r \frac{\delta_F - \delta_C}{\delta_F \delta_C} \approx -r \frac{\delta_F - \delta_C}{\delta_d^2}. \\ \frac{\delta f}{f_d} &= \frac{-r \frac{\delta_F - \delta_C}{\delta_d^2}}{-\frac{r}{\delta_d}} = \frac{\delta_F - \delta_C}{\delta_d} = \frac{-(n_F - 1)\alpha + (n_C - 1)\alpha}{-(n_d - 1)\alpha} = \frac{n_F - n_C}{n_d - 1} = \frac{1}{\nu} \longrightarrow \frac{\delta f}{f_d} = \frac{1}{\nu}. \end{aligned}$$

An inverted prism deviates a ray up and has a negative vertex angle  $\alpha$ .

Deviations and dispersions adds:

$$\text{Deviation } \delta = \sum_i \delta_i, \quad \text{Dispersion } \Delta = \sum_i \Delta_i, \quad \text{Secondary dispersion } \varepsilon = \sum_i \varepsilon_i. \quad (3.7)$$

#### Achromatic thin prism

An **achromatic thin prism** or **achromatic wedge** provides deviation without dispersion. Opposite prisms made from two different glasses ( $(n_{d1}, \nu_1, P_1)$  and  $(n_{d2}, \nu_2, P_2)$ ) are combined to force the dispersion between the  $F$  and  $C$  wavelengths to be zero ( $\Delta = 0$ ):

$$\Delta = \Delta_1 + \Delta_2 = \frac{\delta_1}{\nu_1} + \frac{\delta_2}{\nu_2} = 0 \longrightarrow \delta_2 = -\frac{\nu_2}{\nu_1} \delta_1.$$

A deviation of  $\delta$  is maintained for  $d$  light:

$$\delta = \delta_1 + \delta_2 = \delta_1 - \frac{\nu_2}{\nu_1} \delta_1 = (\nu_1 - \nu_2) \frac{\delta_1}{\nu_1} = -(\nu_1 - \nu_2) \frac{(n_{d1} - 1)\alpha_1}{\nu_1}.$$

Doing  $\alpha_1/\delta$  and  $\alpha_2/\delta$  gives the deviation ratio for each element

$$\text{Achromatic deviation ratio} \quad \frac{\alpha_1}{\delta} = \frac{1}{\nu_2 - \nu_1} \frac{\nu_1}{n_{d1} - 1}, \quad \frac{\alpha_2}{\delta} = -\frac{1}{\nu_2 - \nu_1} \frac{\nu_2}{n_{d2} - 1}. \quad (3.8)$$

The high-dispersion prism is inverted to obtain an opposing deviation. While the F and C wavelengths are corrected, a residual secondary dispersion remains. The residual dispersion is the secondary dispersion  $\varepsilon$ :

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = P_1 \frac{\delta_1}{\nu_1} + P_2 \frac{\delta_2}{\nu_2}.$$

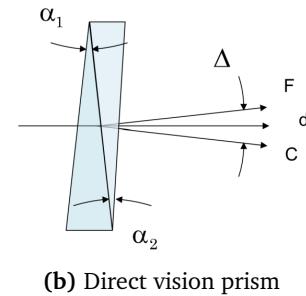
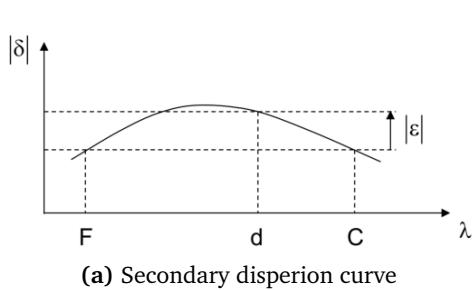
From the achromatic condition  $\delta_2 = -(\nu_2/\nu_1)\delta_1$  we have:

$$\begin{aligned} \nu_1 \delta_2 &= -\nu_2 \delta_1 \\ \nu_1 (\delta - \delta_1) &= -\nu_2 \delta_1 \\ \nu_1 \delta &= (\nu_1 - \nu_2) \delta_1 \\ \frac{\delta_1}{\nu_1} &= \frac{\delta}{\nu_1 - \nu_2}, \quad \frac{\delta_2}{\nu_2} = -\frac{\delta}{\nu_1 - \nu_2}. \end{aligned}$$

Putting these two ratios back in the secondary dispersion and rearranging yields:

$$\text{Secondary dispersion in the thin prism} \quad \frac{\varepsilon}{\delta} = \frac{P_2 - P_1}{\nu_2 - \nu_1} = \frac{\Delta P}{\Delta \nu}. \quad (3.9)$$

For most glass pairs, d light will be bent more than the F and C wavelengths and also  $\frac{\varepsilon}{\delta} > 0$ . The shape of the curve of  $|\delta| - \lambda$  is concave and the maximum dispersion **does not occur** at d light.



The achromatic thin prism has about 40 less secondary dispersion compared to a simple thin prism:

$$\text{Simple thin prism } (\nu \approx [30, 70]) \quad \frac{\Delta P}{\Delta \nu} = \frac{1}{\nu} \approx [1.5, 3]\% \quad \text{vs} \quad \text{Achromatic thin prism} \quad \frac{\Delta P}{\Delta \nu} \approx 0.045\%.$$

### Direct vision prism

A **direct vision prism** uses opposing prisms to provide dispersion without deviation of the d light ( $\delta = \delta_1 + \delta_2 = 0$ ). For a desired dispersion  $\Delta$ , we have

$$\Delta = \Delta_1 + \Delta_2 = \frac{\delta_1}{\nu_1} + \frac{\delta_2}{\nu_2} = -\frac{\nu_1 - \nu_2}{\nu_1 \nu_2} \delta_1.$$

Consequently,

Dispersion in a direct vision prism

$$\frac{\alpha_1}{\Delta} = \left( \frac{\nu_1 \nu_2}{\nu_1 - \nu_2} \right) \left( \frac{1}{n_{d1} - 1} \right)$$

$$\frac{\alpha_2}{\Delta} = - \left( \frac{\nu_1 \nu_2}{\nu_1 - \nu_2} \right) \left( \frac{1}{n_{d2} - 1} \right). \quad (3.10)$$

In order to get sufficient dispersion, it is often necessary to use more than two prisms.

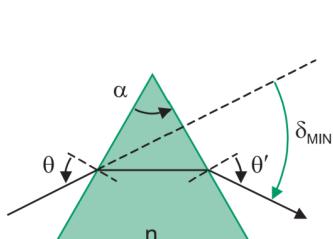
### 3.1.4 Dispersing prism

The total deviation  $\delta$  in the **dispersion prism** is the sum of the deviations at the two surfaces:

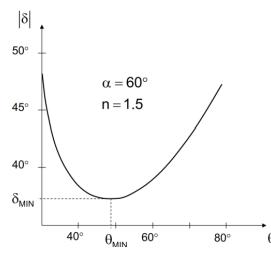
$$\text{Total deviation} \quad \delta = \alpha - \sin^{-1}[\sqrt{n^2 - \sin^2 \theta} \sin \alpha - \cos \alpha \sin \theta] - \theta.$$

There is a minimum deviation angle  $\delta_{\min}$ , at which the ray path through the prism is symmetric  $\theta' = -\theta$ . The ray is bent an equal amount at each surface. The deviation is negative for the orientation of the prism in the figure. The **angle of minimum deviation** is

$$\delta_{\min} = \alpha - 2 \sin^{-1}[n \sin(\alpha/2)]. \quad (3.11)$$



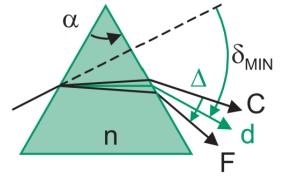
(a) Dispersing prism



(b) Deviation in  $\theta$

For $\alpha = 60^\circ$	
n	$\delta_{\min}$
1.3	-21.1°
1.4	-28.9°
1.5	-37.2°
1.6	-46.3°
1.7	-56.4°
1.8	-68.3°
2.0	-120°

(c) Table for  $\delta_{\min}$



(d) Dispersion of the prism

The measurement of the index depends only on  $\delta_{\min}$  and the prism apex angle  $\alpha$ :

$$n = \frac{\sin[(\alpha - \delta_{\min})/2]}{\sin(\alpha/2)}. \quad (3.12)$$

Minimum deviation provides an extremely accurate method of measuring the index of refraction of a material. Prism spectrometers can obtain accuracies of one part in  $10^6$ .

$$\delta_{\min} = I_2 - I_1. \quad (3.13)$$

- Telescope and collimator mounted in a rotation stage.
- Use the telescope as autocollimator to measure the angle  $\alpha$ .
- Measure the straight through angle  $I_1$  without prism.
- Insert the prism and observe the angle of refracted beam  $\delta$ .
- Rotate the prism to obtain the minimum deviation and measure this angle  $I_2$ .
- Make  $\delta_{\min} = I_2 - I_1$ .
- Calculate the index with the formula above.

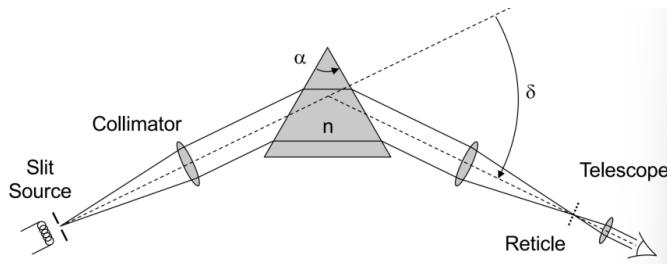
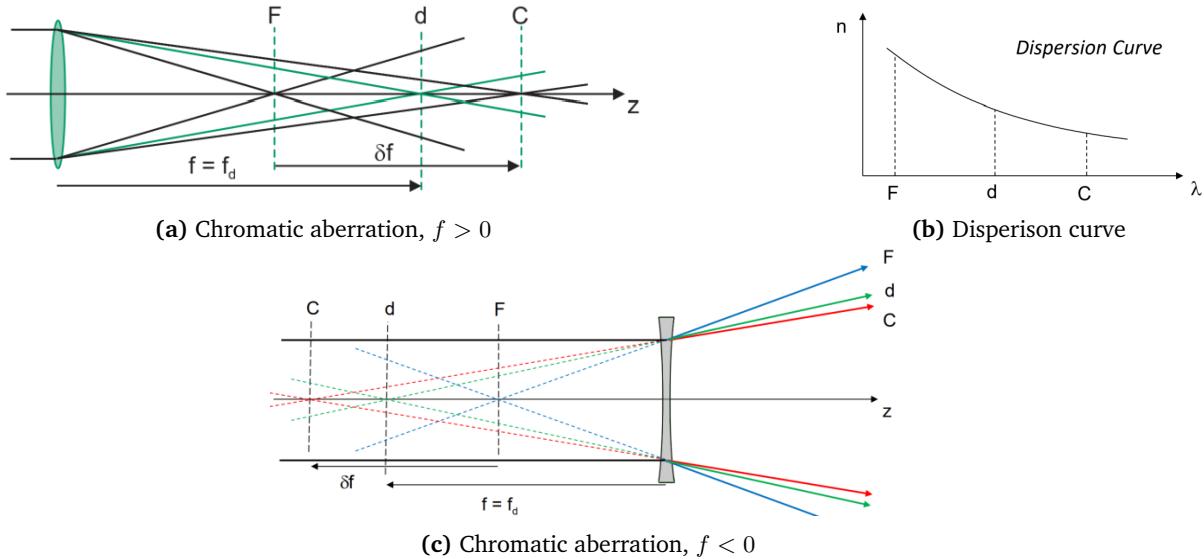


Figure 3.5 Prism spectrometer to measure the index of refraction.

## 3.2 Chromatic effects

### 3.2.1 Chromatic aberration

**Axial chromatic aberration** or **axial color** is a variation of the system focal length with wavelength. This aberration derives from the dispersion of the glass as the index changes with wavelength  $n(\lambda)$ .



Because of the higher index for F light, blue light is bent more and therefore the blue focus is closest to the lens.

#### How much does the focal length change for the F and C wavelengths?

We look at the difference in power between these two wavelengths:

$$\delta\phi_{FC} = \phi_F - \phi_C = (n_F - 1)(C_1 - C_2) - (n_C - 1)(C_1 - C_2) = (n_F - n_C)(C_1 - C_2)$$

$$\delta\phi_{FC} = \underbrace{\frac{n_F - n_C}{n_d - 1}}_{1/\nu} \underbrace{(n_d - 1)(C_1 - C_2)}_{\phi_d} = \frac{\phi_d}{\nu} = \delta\phi_{FC} .$$

Similarly, for the focal length:

$$\delta f_{CF} = f_C - f_F = \frac{1}{\phi_C} - \frac{1}{\phi_F} = \frac{\phi_F - \phi_C}{\phi_C \phi_F} = \frac{\delta\phi}{\phi_C \phi_F} \approx \frac{\delta\phi}{\phi_d^2} = \frac{\phi_d}{\nu \phi_d^2} = \frac{f_d}{\nu} = \delta f_{CF}.$$

The foci of F, d and C are not evenly spaced due to the shape of the dispersion curve. The relative order of the foci is reversed for a negative lens.

+ lens chromatic aberration       $\delta f_{CF} = f_C - f_F, \quad \delta\phi_{FC} = \phi_F - \phi_C, \quad \frac{\delta f_{CF}}{f_d} = \frac{\delta\phi_{FC}}{\phi_d} = \frac{1}{\nu}$       (3.14)

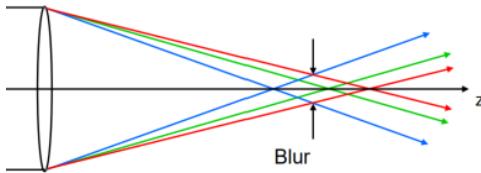
- lens chromatic aberration      same, with  $f_d < 0, \quad \phi_d < 0, \quad \delta f_{CF} < 0, \quad \delta\phi_{FC} < 0$ .      (3.15)

### 3.2.2 Type of chromatic aberrations

#### Longitudinal chromatic aberration

The blur associated with the chromatic aberration of the objective lens limits the performance of an objective. To reduce the blur, a small diameter objective lens is required.

$$\text{Blur} \propto \text{Lens diameter}.$$



(a) Axial longitudinal chromatic aberration



(b) Small diameter to reduce aberration

#### Transverse axial chromatic aberration

**Transverse axial chromatic aberration** measures the image blur size due to axial chromatic aberration. It depends only on the glass and the pupil radius  $r_P$  (stop at the lens): The rays from the edge of the pupil are approximately parallel in the vicinity of the focus ( $f \gg \delta f$ ).

$$\tan U' = -\frac{\text{TA}_{CH}}{\delta f} = -\frac{r_p}{f} \rightarrow \frac{\text{TA}_{CH}}{\delta f} = \frac{r_p}{f} \rightarrow \frac{\text{TA}_{CH}}{r_p} = \frac{\delta f}{f} = \frac{1}{\nu}.$$

Therefore,

$$\text{Transverse axial chromatic aberration} \quad \text{TA}_{CH} = \frac{r_p}{\nu}. \quad (3.16)$$

#### Lateral chromatic aberration

**Lateral chromatic aberration** or **lateral color** is the chromatic aberration of the marginal ray and is caused by dispersion of the chief ray. The edge of the lens behaves like a thin prism. Off-axis image points will exhibit a radial color smear. The blur length increases linearly with the image height. Each color has a different lateral magnification.

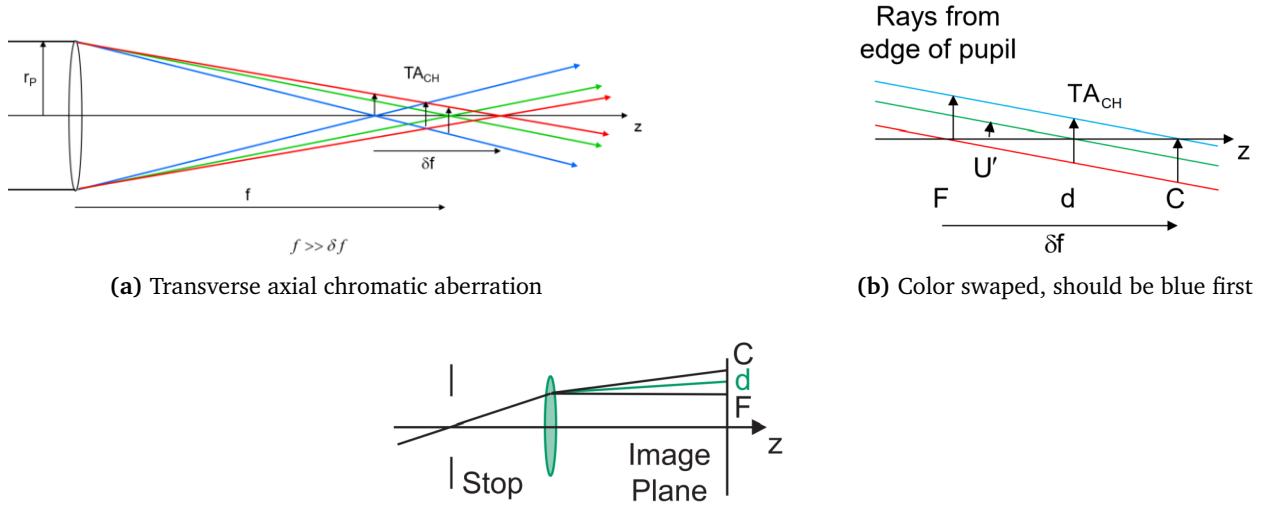
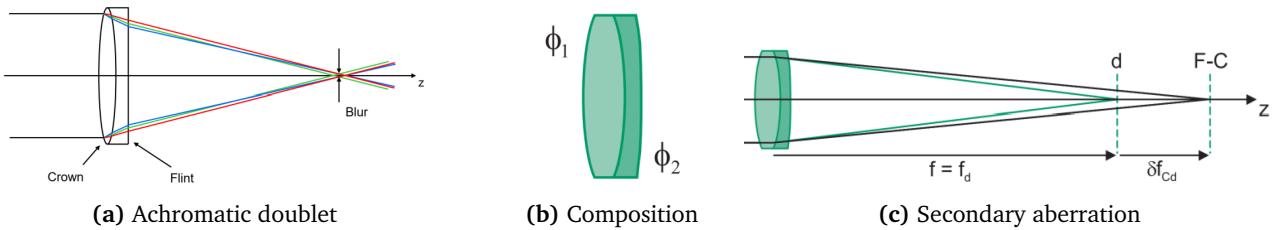


Figure 3.4 Lateral chromatic aberration

### 3.2.3 Achromatic doublet

The thin lens **achromatic doublet** corrects longitudinal chromatic aberration by combining a positive element with a negative one. Two different glasses ( $\nu_1, P_1$ ) and ( $\nu_2, P_2$ ) are used. The nominal powers and focal lengths are for d light.



#### How do we design the individual powers of the achromatic doublet?

Red and blue light are made to focus at the same location ( $\Delta f = 0$ ):

$$\phi = \phi_1 + \phi_2 \implies \delta\phi_{FC} = \delta\phi_{FC1} + \delta\phi_{FC2} = \frac{\phi_1}{\nu_1} + \frac{\phi_2}{\nu_2} = \phi_F - \phi_C = 0 \implies \frac{\phi_1}{\nu_1} = -\frac{\phi_2}{\nu_2}.$$

$$\phi = \phi_2 - \frac{\nu_1}{\nu_2} \phi_2 = \frac{\nu_2 - \nu_1}{\nu_2} \phi_2 \implies \frac{\phi_2}{\phi} = -\frac{\nu_2}{\nu_1 - \nu_2} \wedge \frac{\phi_1}{\phi} = \frac{\nu_1}{\nu_1 - \nu_2}.$$

The doublet design places excess power in the positive element that is cancelled by the negative element. Both contribute equal, but opposite amounts of longitudinal chromatic aberration. Large differences in the Abbe number minimize the excess power and provide better performance.

### 3.2.4 Secondary chromatic aberration

We have ensured the F and C lines are at the same longitudinal distance, but the d line have a different focal position.



**Figure 3.6** Excess power in achromatic doublet

First, we have:

$$\delta\phi_{dC} = \delta\phi_{dC1} + \delta\phi_{dC2}$$

We derive each term:

$$\delta\phi_{dC1} = (n_{d1} - n_{C1})(C_1 - C_2) = \frac{(n_{d1} - n_{C1})}{(n_{F1} - n_{C1})}(n_{F1} - n_{C1})(C_1 - C_2) = P_1 \delta\phi_{FC1}.$$

Similarly,

$$\delta\phi_{dC1} = P_1 \delta\phi_{FC1}, \quad \delta\phi_{dC2} = P_2 \delta\phi_{FC2}, \quad P_1 = \frac{n_{d1} - n_{C1}}{n_{F1} - n_{C1}}, \quad P_2 = \frac{n_{d2} - n_{C2}}{n_{F2} - n_{C2}}.$$

We go back to the initial equation:

$$\begin{aligned} \delta\phi_{dC} &= \delta\phi_{dC1} + \delta\phi_{dC2} \\ &= P_1 \delta\phi_{FC1} + P_2 \delta\phi_{FC2} \\ &= P_1 \frac{\phi_1}{\nu_1} + P_2 \frac{\phi_2}{\nu_2} \\ \delta\phi_{dC} &= \frac{P_1 - P_2}{\nu_1 - \nu_2} \phi \quad \text{Achromat eq: } \frac{\phi_1}{\nu_1} = -\frac{\phi_2}{\nu_2}. \end{aligned}$$

Then, the secondary deviation is:

$$\text{Secondary chromatic aberration} \quad \delta\phi_{dC} = \frac{\Delta P}{\Delta \nu} \phi.$$

Consequently, in order to obtain zero secondary achromatic aberration with a doublet, the partial dispersion ratio  $P$  of the two glasses **must be zero**:

$$\Delta P = 0 \implies \delta f_{Cd} = \frac{\Delta P}{\Delta \nu} f = 0. \quad (3.17)$$

To correct chromatic aberration at additional wavelength, more than two glasses are used.

### 3.2.5 Partial dispersion ratio and Abbe number

On a plot of  $P$  versus  $\nu$ , most glasses lie on a straight line. For a singlet and doublet, we have:

$$\delta f_{CF} \approx \frac{f}{50} \quad \text{vs} \quad \delta f_{Cd} \approx \frac{f}{2200}. \quad (3.18)$$

#### Ejemplo 3.1

#### Design of an achromatic doublet

Design a 160 mm focal length thin-lens achromatic doublet using the following glasses. Provide the focal lengths and indices of refraction of the two thin lenses.

Glass 1: Fused Silica, 458678, Glass 2: SF6, 805254.

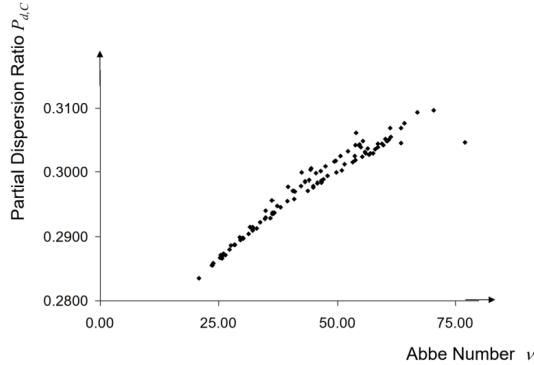


Figure 3.7 Partial dispersion ratio versus Abbe number.

## Solution

Glass 1:  $n_1 = 1.458$ ,  $\nu_1 = 67.8$ , Glass 2:  $n_2 = 1.805$ ,  $\nu_2 = 25.4$ .

$$\frac{1}{f_2} = -\frac{\nu_2}{\nu_1 - \nu_2} \frac{1}{f} = -\frac{25.4}{67.8 - 25.4} \frac{1}{160} = -0.00374 \text{ mm}^{-1} \rightarrow f_2 = -267.380 \text{ mm}$$

$$\frac{1}{f_1} = \frac{\nu_1}{\nu_1 - \nu_2} \frac{1}{f} = \frac{67.8}{67.8 - 25.4} \frac{1}{160} = 0.0010 \text{ mm}^{-1} \rightarrow f_1 = 100 \text{ mm}.$$

## 3.3 Illumination systems

### 3.3.1 Illumination systems and types

The illumination system provides light for the optical system. Important considerations are the amount of light, the uniformity, and the angular spread of the light as seen by the object.

A **projector** is the general term for an imaging system that also provides the illumination for the object.

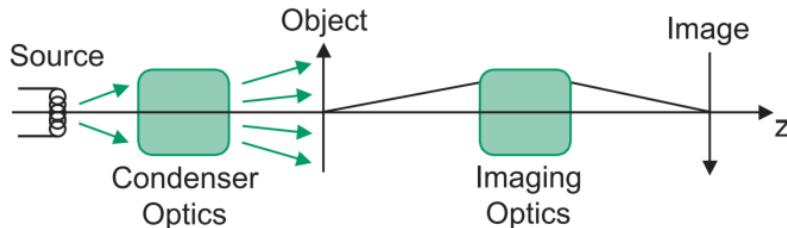


Figure 3.1 Projector as the illumination system.

There are three basic classifications of illumination systems:

- **Diffuse illumination** Light with a large angular spread is incident on the object. There is no attempt to image the source into the imaging system. It provides uniform illumination but is light inefficient.

No source coupling

- **Specular illumination** The light source is imaged by the condenser optics into the EP of the imaging optics. As it is good light efficient, is used for most optical systems with an integral light source.

Source to pupil coupling

- **Critical illumination** The light source is imaged directly onto the object.

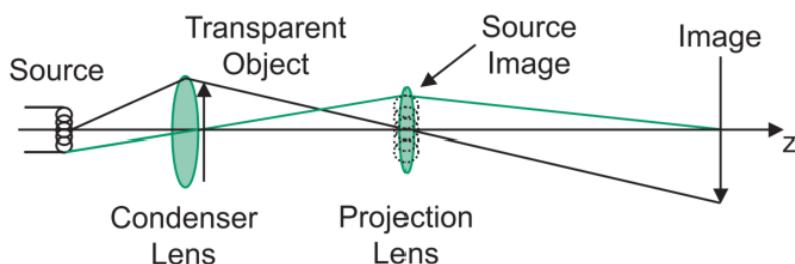
Source to object coupling

### 3.3.2 Specular illumination

Source is coupled into the EP of the imaging system.

Projection condenser system

The most common example is the **projection condenser system**.

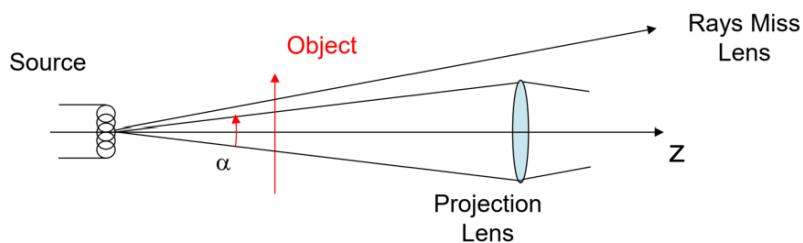


**Figure 3.2** Projection condenser system for specular illumination.

In the projection condenser system, the source is optical conjugated (imaged) at the EP of the projection lens (imaging system).

- The condenser lens serves as a field lens, bending the chief ray of the imaging system back into the projection lens.
- The condenser lens should be as fast as possible ( $f/\#_W$  faster than  $f/1$  on the source side).
- The projection lens diameter must be larger than the size of the source image.
- The marginal ray of the condenser system becomes the chief ray of the imaging system and viceversa.

Without a condenser lens, the light collection angle  $\alpha$  is limited by the projection lens. The amount of source energy that is collected and used is defined by the solid angle given by the angular size of the projection lens.

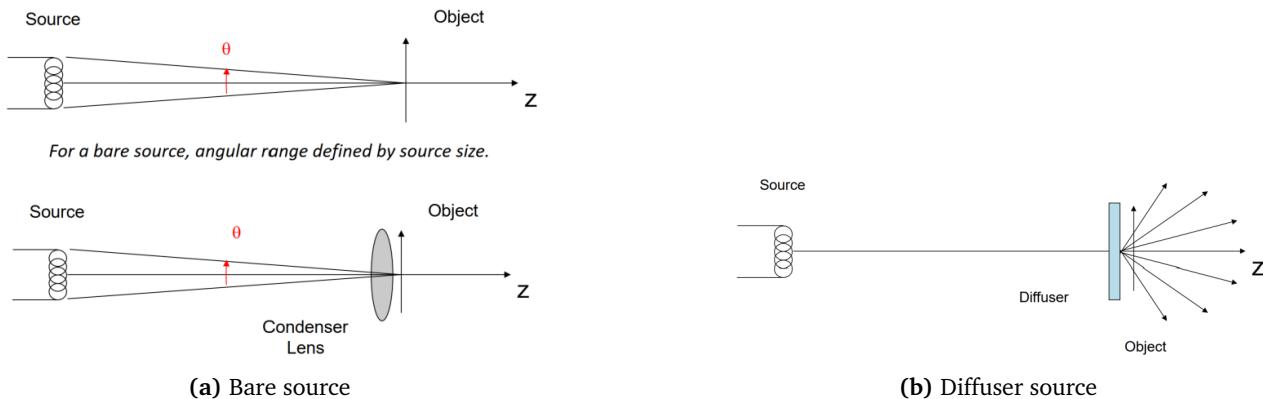


**Figure 3.3** Without the condenser lens, the collection angle is limited by the projection lens.

The inclusion of the condenser makes that each point on the source illuminates all points on the object, resulting in a uniform illumination. The collection angle  $\alpha$  is limited by the condenser lens size.

### Apparent source size

With a bare source, the apparent source size (how large appears to be at the observation point) is limited by the angular size of the source, regardless if there is a condenser lens. However, the inclusion of a **diffuser** makes that all angles are present due to scattering; the source now appears to be large.



### Projection condenser design

- **What drives the choice in projection lens?**

We image the transparent object. How big is the screen? Where is it? What kind of magnification do we need?

- **What drives the physical size of the condenser lens?**

The object size and any separation between the condenser lens and object. Pick the fastest condenser lens you can.

- **What drives the position of the source?**

The image of the source must be at the projection lens.

- **Must be the projection lens diameter be larger than the source image size?**

Yes, this determines the  $f/\#$  of the projection lens.

### Kohler illumination

The **Kohler illumination** is an example of specular illumination, often used in microscopes.

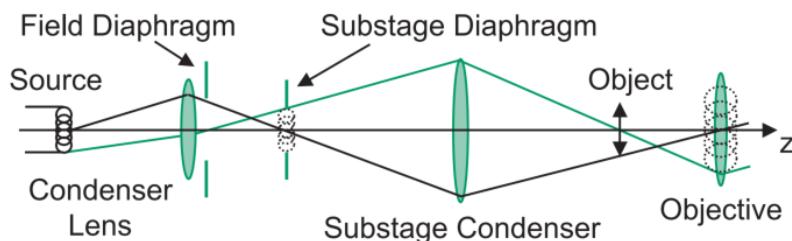


Figure 3.5 Kohler illumination system.

The source produces an intermediate image at the substage diaphragm, and an image at the objective. Similarly, the object is at the field diaphragm and generates an intermediate image after the substage condenser.

- The **substage diaphragm** (at source image) allows the overall light level to be varied.
- The **field diaphragm** (at object image) changes the amount of the object that is illuminated.

### 3.3.3 Critical illumination

Critical illumination images the light directly onto the object. While it is light efficient, is not very used as the image is modulated by the source structure. This requires to use a very uniform source. The FOV of this system is typically small

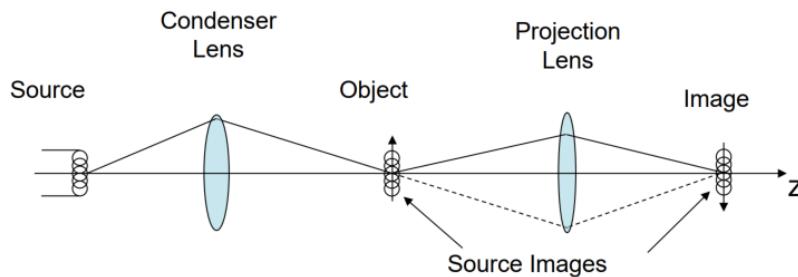


Figure 3.6 Critical illumination system.

### 3.3.4 Diffuse illumination

Diffuse illumination provides light with a large angular spread onto the object. There is no attempt to image the source into the imaging system. It is usually achieved by the insertion of a diffuser into the system. This makes a very uniform illumination but with a light inefficiency.

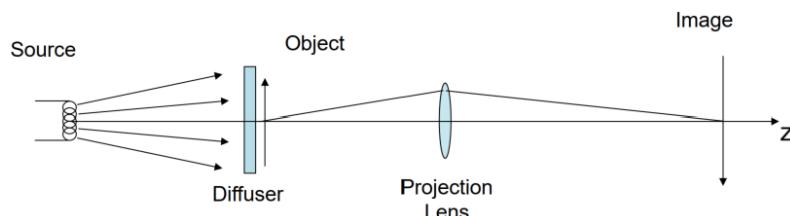


Figure 3.7 Diffuse illumination system.

The diffuser increases the apparent source size, resulting in greater uniformity of illumination. This greater range of illumination angles also provides **scratch suppression** that will hide phase errors on the object, such as scratch or defect in the substrate of the object transparency. Diffusers are commonly made of ground glass or opal glass.

### 3.3.5 Integrating sphere

Works through the ideas of diffuse illumination. Inside the hollow sphere is coated with a highly-reflective diffuse white coating. Light directed into the entry port undergoes many random reflections before escaping through the exit port. Output light is extremely uniform with a brightness that is independent of viewing angle, and a very good approximation to a Lambertian source.

The system is in general light inefficient, and the two ports are usually at 90° to prevent the direct viewing

of the source and the first source reflection.

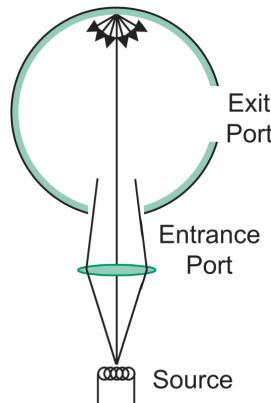
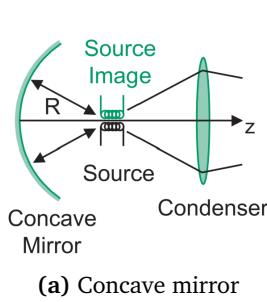


Figure 3.8 Integrating sphere.

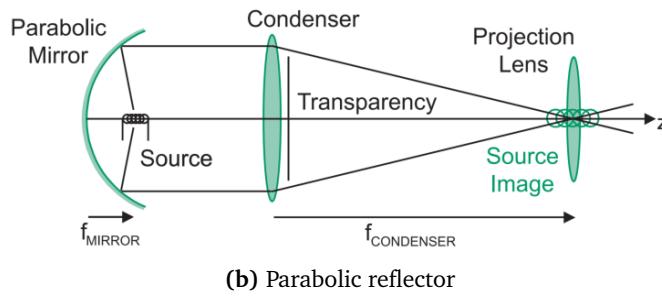
### 3.3.6 Source mirrors

#### Concave source

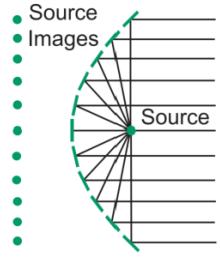
Placing a **concave mirror** behind the source can increase the light level in the projection system. The source is placed at the center of curvature of the mirror. This creates an intermediate image at the source, which becomes the object for the condenser lens. An improvement of less than a factor of two is obtained.



(a) Concave mirror



(b) Parabolic reflector



(c) Faceted reflector

#### Parabolic reflector

Dramatic increases in illumination level occur by placing the source at the focus of the concave mirror. The source image occurs at infinity. The solid angle of the mirror can be more than  $2\pi\text{sr}$ , and the amount of light intercepted and reflected by the mirror can exceed the light directly collected by the condenser by a factor of 10 or more.  $f/\#$  of the condenser lens does not influence the light collection efficiency.

The design of these systems typically ignore the forward light through the condenser. The mirror shape is usually parabolic.

#### Faceted reflectors

To provide a greater level of diffuseness, the surface of the parabola can be segmented into small flat mirrors. A virtual source is formed behind each facet. The details of the **faceted parabolic reflector** are

complicated, but for design purposes it can be modeled as an extended source located at or near the concave mirror. The mirror aperture defines the extent of the extended source.

The number, size and tilt of the facets are designed so that uniform illumination is achieved at the object plane. The total view is limited by the overall aperture of the reflector. This overall aperture is imaged into the projection lens as well as an image of the source, but this one contains much less light.

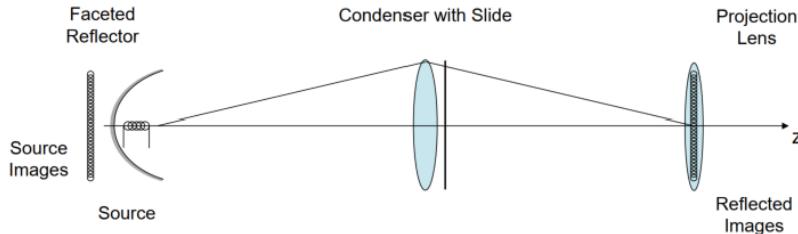


Figure 3.10 System with the facet reflector.

### Elliptical reflector

An elliptical reflector can be used to focus the source into a small aperture. The source is placed at one focus of the ellipse, and a real image is formed at the other focus.

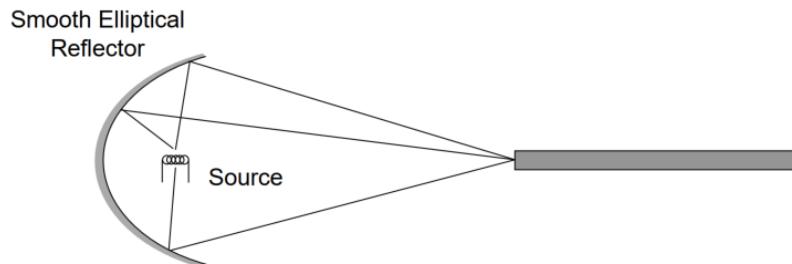
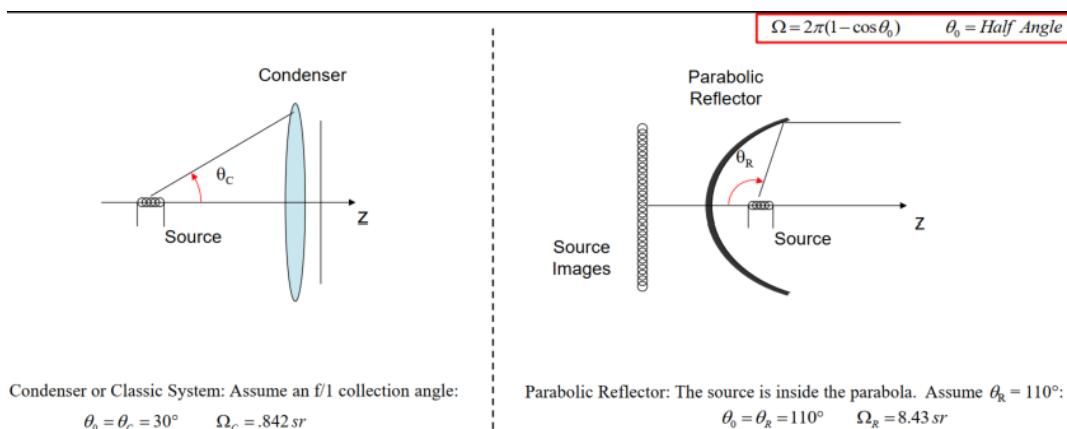


Figure 3.11 Elliptical reflector.

### Light collection efficiency

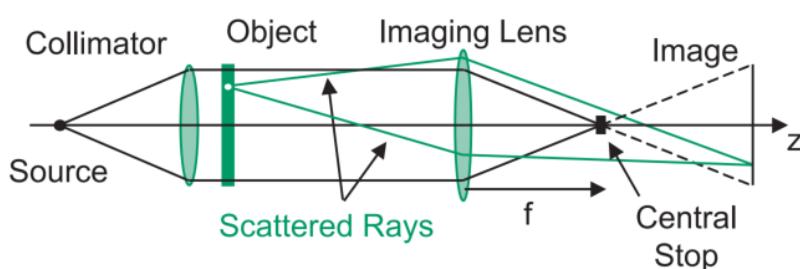


### 3.3.7 Schlieren and dark field systems

Specular or narrow angle illumination can be used to identify features on an object.

#### Schlieren system

In a **schlieren system**, light from a small source is collimated before passing through the object plane. The image of the source is blocked by an opaque disk or knife edge. With no object present, the image appears black. When the object is inserted, any feature will scatter some light past the obscuration. These localized areas in the object will appear bright in the image.



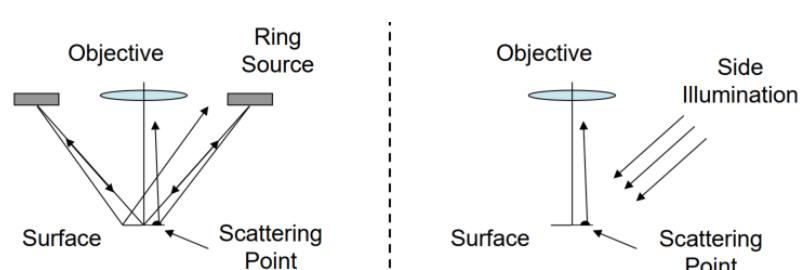
(a) Schlieren system



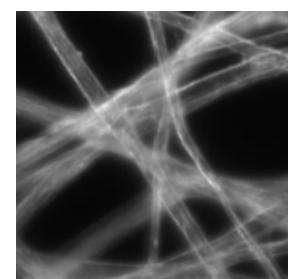
(b) Schlieren example

#### Dark field system

**Dark field illumination** is a variation of this technique using directional lighting. The light source is placed to the side of the objective lens, or in a ring around the lens. If the object is perfectly smooth (a mirror), a specular reflection within the FOV misses the objective, and the image is dark. Features on the surface will scatter light into the objective and appear bright in the image.



(a) Dark field system



(b) Dark field example

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