Problem Set 6

Problem 1

<u>a.</u> $\tilde{a}(t)|\varphi_m\rangle = U^\dagger a U |\varphi_m\rangle =$

$$= e^{iHt/\hbar} a\, e^{-iHt/\hbar} |\varphi_m\rangle =$$

$$= e^{iHt/\hbar} a\, e^{-iE_m t/\hbar} |\varphi_m\rangle =$$

$$= e^{-iE_m t/\hbar} e^{iHt/\hbar} a |\varphi_m\rangle =$$

$$= e^{-iE_m t/\hbar} e^{iHt/\hbar} \sqrt{n}\, |\varphi_{m-1}\rangle =$$

$$= e^{-iE_m t/\hbar} \sqrt{m}\, e^{iE_{m-1}t/\hbar} |\varphi_{m-1}\rangle =$$

$$= e^{-i(E_m - E_{m-1})t/\hbar} \sqrt{m}\, |\varphi_{m-1}\rangle =$$

$$= e^{-i(\cancel{\hbar}\omega)t/\cancel{\hbar}} a|\varphi_m\rangle =$$

$$\tilde{a}(t)|\varphi_m\rangle = e^{-i\omega t} a|\varphi_m\rangle \Rightarrow \boxed{\tilde{a} = e^{-i\omega t} a}$$

Similarly: $\tilde{a}^\dagger(t)|\varphi_m\rangle = U^\dagger(t)\, a^\dagger\, U(t)\, |\varphi_m\rangle =$

$$= e^{iHt/\hbar} a^\dagger e^{-iHt/\hbar} |\varphi_m\rangle =$$

$$= e^{-iE_m t/\hbar} e^{iHt/\hbar} \sqrt{m+1}\, |\varphi_{m+1}\rangle =$$

$$= e^{i(E_{m+1} - E_m)t/\hbar} \sqrt{m+1}\, |\varphi_{m+1}\rangle =$$

$$= e^{i\cancel{\hbar}\omega t/\cancel{\hbar}} \sqrt{m+1}\, |\varphi_{m+1}\rangle =$$

$$= e^{i\omega t} a^\dagger |\varphi_m\rangle \Rightarrow \boxed{\tilde{a}^\dagger = e^{i\omega t} a^\dagger}$$

these are the Heisenberg picture raising and lowering operators

b. $\tilde{x}(t) = u^{+}(t)\, x\, u(t) =$

$$= u^{+}(t) \frac{\sigma}{\sqrt{2}} (a + a^{+})\, u(t) =$$

$$= \frac{\sigma}{\sqrt{2}} \left[u^{+}(t)\, a\, u(t) + u^{+}(t)\, a^{+}\, u(t) \right] =$$

$$= \frac{\sigma}{\sqrt{2}} \left(e^{-i\omega t}\, \tilde{a} + e^{+i\omega t}\, \tilde{a}^{+} \right) =$$

$$= \frac{\sigma}{\sqrt{2}} \left\{ \left[\cos(\omega t) - i \sin(\omega t)\right] \tilde{a} + \left[\cos(\omega t) + i \sin(\omega t)\right] \tilde{a}^{+} \right\} =$$

$$= \frac{\sigma}{\sqrt{2}} \left[\cos(\omega t) \cdot (\tilde{a} + \tilde{a}^{+}) + i \sin(\omega t)\, (\tilde{a}^{+} - \tilde{a}) \right] =$$

$$\boxed{\tilde{x}(t) = \cos(\omega t)\, x + \frac{\sigma^2}{\hbar} \sin(\omega t)\, p}$$

$$\tilde{p}(t) = u^{+}(t)\, p\, u(t) =$$

$$= u^{+}(t) \frac{i\hbar}{\sqrt{2}\,\sigma} (a^{+} - a)\, u(t) =$$

$$\sigma = \sqrt{\frac{\hbar}{m\omega}}$$

$$= \frac{i\hbar}{\sqrt{2}\,\sigma} \left(e^{+i\omega t}\, \tilde{a}^{+} - e^{-i\omega t}\, \tilde{a} \right) =$$

$$= \frac{i\hbar}{\sqrt{2}\,\sigma} \left\{ \left[\cos(\omega t) + i \sin(\omega t)\right] \tilde{a}^{+} - \left[\cos(\omega t) - i \sin(\omega t)\right] \tilde{a} \right\} =$$

$$= \frac{i\hbar}{\sqrt{2}\,\sigma} \left[\cos(\omega t)\, (\tilde{a}^{+} - \tilde{a}) + i \sin(\omega t)\, (\tilde{a}^{+} + a) \right] =$$

$$\boxed{\tilde{p}(t) = \cos(\omega t)\, p - \frac{\hbar}{\sigma^2} \sin(\omega t)\, x}$$

$$\frac{\hbar}{\sigma^2} = \frac{\hbar}{\cancel{\hbar}/m\omega} = m\omega$$

These are the Heisenberg picture position and momentum operators. They have the functional form of the classical equation of motion, where the quantum operators replace the classical state variables.

c. $P\left[U^+\left(\frac{\pi}{2\omega},0\right)|x\rangle\right] = \underbrace{c}_{\text{scalar eigenvalue}}\cdot\, U^+\left(\frac{\pi}{2\omega},0\right)|x\rangle$

$$P\,U^+\left(\frac{\pi}{2\omega}\right)|x\rangle = U^+\left(\frac{\pi}{2\omega}\right) U\left(\frac{\pi}{2\omega}\right) P\, U^+\left(\frac{\pi}{2\omega}\right)|x\rangle =$$

$$= U^+\left(\frac{\pi}{2\omega}\right) \underbrace{U^+\left(-\frac{\pi}{2\omega}\right) P\, U\left(-\frac{\pi}{2\omega}\right)}|x\rangle =$$

$$= U^+\left(\frac{\pi}{2\omega}\right) \tilde{P}\left(-\frac{\pi}{2\omega}\right)|x\rangle =$$

$$= U^+\left(\frac{\pi}{2\omega}\right)\left[\cancelto{0}{\cos\left(-\omega\frac{\pi}{2\omega}\right)} P - \frac{\hbar}{\sigma^2}\sin\left(-\omega\frac{\pi}{2\omega}\right) X\right]|x\rangle$$

$$= U^+\left(\frac{\pi}{2\omega}\right) \frac{\hbar}{\sigma^2}\, X|x\rangle =$$

$P\left[U^+\left(\frac{\pi}{2\omega}\right)|x\rangle\right] = \frac{\hbar x}{\sigma^2}\left[U^+\left(\frac{\pi}{2\omega}\right)|x\rangle\right] \Rightarrow$ eigenvalue of P is $\frac{\hbar x}{\sigma^2}$

Sim.

$$X\,U^+\left(\frac{\pi}{2\omega}\right)|p\rangle = U^+\left(\frac{\pi}{2\omega}\right) U\left(\frac{\pi}{2\omega}\right) X\, U^+\left(\frac{\pi}{2\omega}\right)|p\rangle =$$

$$= U^+\left(\frac{\pi}{2\omega}\right) U^+\left(-\frac{\pi}{2\omega}\right) X\, U\left(-\frac{\pi}{2\omega}\right)|p\rangle =$$

$$= U^+\left(\frac{\pi}{2\omega}\right) \tilde{X}\left(-\frac{\pi}{2\omega}\right)|p\rangle =$$

$$= U^+\left(\frac{\pi}{2\omega}\right)\left[\cancelto{0}{\cos\left(-\omega\frac{\pi}{2\omega}\right)} X + \frac{\sigma^2}{\hbar}\sin\left(-\omega\frac{\pi}{2\omega}\right) P\right]|p\rangle =$$

$X\,U^+\left(\frac{\pi}{2\omega}\right)|p\rangle = -\frac{\sigma^2 p}{\hbar}\, U^+\left(\frac{\pi}{2\omega}\right)|p\rangle \Rightarrow$ eigenvalue of X is $-\frac{\sigma^2 p}{\hbar}$

d. $\psi\left(x, t_q = \frac{q\pi}{2\omega}\right) = \langle x|\psi\left(\frac{q\pi}{2\omega}\right)\rangle = \langle x|U\left(\frac{q\pi}{2\omega}\right)|\psi(0)\rangle$

$\psi(x, t=0) = \langle x|\psi(0)\rangle$ is known

From above:

$$P|p\rangle = p|p\rangle \qquad p = \frac{\hbar x}{\sigma^2} \qquad \left|p = \frac{\hbar x}{\sigma^2}\right\rangle = U^+\left(\frac{\pi}{2\omega}\right)|x\rangle$$

$$X|x\rangle = x|x\rangle \qquad x = -\frac{\sigma^2 p}{\hbar} \qquad \left|x = -\frac{\sigma^2 p}{\hbar}\right\rangle = U^+\left(\frac{\pi}{2\omega}\right)|p\rangle$$

when I switch to momentum, I need to multiply by $\sqrt{\frac{\hbar}{\sigma^2}}$ to preserve normality of the state.

$$\Rightarrow U^+\left(1\cdot\frac{\pi}{2\omega}\right)|x\rangle = \sqrt{\frac{\hbar}{\sigma^2}}\left|p = \frac{\hbar x}{\sigma^2}\right\rangle$$

$$\Rightarrow U^+\left(2\cdot\frac{\pi}{2\omega}\right)|x\rangle = U^+\left(\frac{\pi}{2\omega}\right)\left|p = \frac{\hbar x}{\sigma^2}\right\rangle =$$

$$= \left|x' = -\frac{\cancel{\sigma^2}}{\cancel{\hbar}}\cdot\frac{\cancel{\hbar} x}{\cancel{\sigma^2}}\right\rangle =$$

$$= |-x\rangle$$

$$U^+\left(3\cdot\frac{\pi}{2\omega}\right)|x\rangle = U^+\left(\frac{\pi}{2\omega}\right)|-x\rangle =$$

$$= \sqrt{\frac{\hbar}{\sigma^2}}\left|p = -\frac{\hbar x}{\sigma^2}\right\rangle$$

$$U^+\left(4\cdot\frac{\pi}{2\omega}\right)|x\rangle = U^+\left(\frac{\pi}{2\omega}\right)\left|p = -\frac{\hbar x}{\sigma^2}\right\rangle =$$

$$= \left|x' = -\frac{\cancel{\sigma^2}}{\cancel{\hbar}}\cdot\left(-\frac{\cancel{\hbar} x}{\cancel{\sigma^2}}\right)\right\rangle =$$

$$= |x\rangle$$

So the function is periodic for q. Taking HC of above:

$$\Rightarrow \langle x|U\left(q\cdot\frac{\pi}{2\omega}\right) = \begin{cases} \sqrt{\frac{\hbar}{\sigma^2}}\left\langle p = \frac{\hbar x}{\sigma^2}\right| & q = 4k+1 \\ \langle -x| & q = 4k+2 \\ \sqrt{\frac{\hbar}{\sigma^2}}\left\langle p = -\frac{\hbar x}{\sigma^2}\right| & q = 4k+3 \\ \langle x| & q = 4k+4 \end{cases} \qquad k = 0, 1, 2, 3 \ldots$$

for $q = 1$, $\langle x | U\left(1 \cdot \frac{\pi}{2\omega}\right) | \psi_0 \rangle = \sqrt{\frac{\hbar}{\sigma^2}} \langle p = \frac{\hbar x}{\sigma^2} | \mathbb{1}_x | \psi_0 \rangle =$

$$= \sqrt{\frac{\hbar}{\sigma^2}} \int_{-\infty}^{+\infty} dx \, \langle p | x \rangle \langle x | \psi_0 \rangle =$$

$$= \sqrt{\frac{\hbar}{\sigma^2}} \int_{-\infty}^{+\infty} dx \, e^{-ipx/\hbar} \, \psi(x,0) \Big|_{p = \frac{\hbar x}{\sigma^2}} =$$

$$= \sqrt{\frac{\hbar}{\sigma^2}} \int_{-\infty}^{+\infty} dx \, e^{-i \frac{\cancel{\hbar} x}{\sigma^2} x / \cancel{\hbar}} \, \psi(x,0) =$$

$$= \sqrt{\frac{\hbar}{\sigma^2}} \int_{-\infty}^{+\infty} dx \, e^{-i x^2/\sigma^2} \, \psi(x,0) = \qquad \int_{-\infty}^{+\infty} e^{-\frac{1}{2} i t^2} dt = \sqrt{2\pi} \, e^{-i\frac{\pi}{4}}$$

$$= \sqrt{\frac{\hbar}{\sigma^2}} \frac{1}{\cancel{\sqrt{2\pi}}} \cdot \cancel{\sqrt{2\pi}} \, e^{-i\frac{\pi}{4}} \cdot FT[\psi(x,0)] \Big|_{p = \frac{\hbar x}{\sigma^2}} =$$

$$= \sqrt{\frac{\hbar}{\sigma^2}} \, e^{-i\frac{\pi}{4}} \, FT[\psi(x,0)] \Big|_{p = \frac{\hbar x}{\sigma^2}}$$

$q = 2$, $\langle x | U\left(2 \cdot \frac{\pi}{2}\right) | \psi_0 \rangle = e^{-2 \cdot i \cdot \frac{\pi}{4}} [\psi(-x,0)]$

$q = 3$, $\langle x | U\left(3 \cdot \frac{\pi}{2}\right) | \psi_0 \rangle = e^{-3 i \frac{\pi}{4}} \cdot \sqrt{\frac{\hbar}{\sigma^2}} \, FT[\psi(x,0)] \Big|_{p = -\frac{\hbar x}{\sigma^2}}$

$q = 4$, $\langle x | U\left(4 \cdot \frac{\pi}{2}\right) | \psi_0 \rangle = e^{-4 \cdot i \frac{\pi}{4}} \, \psi(x,0)$

<u>In general</u>:

$$\psi\left(x, \frac{q \cdot \pi}{2\omega}\right) = e^{-i q \frac{\pi}{4}} \cdot \begin{cases} \sqrt{\frac{\hbar}{\sigma^2}} \, FT[\psi(x,0)] \Big|_{p = \frac{\hbar x}{\sigma^2}} & q = 4k+1 \\ \psi(-x,0) & q = 4k+2 \\ \sqrt{\frac{\hbar}{\sigma^2}} \, FT[\psi(x,0)] \Big|_{p = -\frac{\hbar x}{\sigma^2}} & q = 4k+3 \\ \psi(x,0) & q = 4k+4 \end{cases}$$

$k = 0, 1, 2, 3 \ldots$

e. $|\Psi(x,0)\rangle = |\varphi_m(x)\rangle$

From above: $\Psi\left(x, \frac{\pi}{2\omega}\right) = e^{-i\frac{\pi}{4}} \sqrt{\frac{\hbar}{\sigma^2}}\, FT[\Psi(x,0)]\Big|_{p=\frac{\hbar x}{\sigma^2}}$

$\Psi(x,0) = \langle x|\varphi_m(x)\rangle = \varphi_m(x)$

$$\Rightarrow \Psi\left(x, \frac{\pi}{2\omega}\right) = e^{-i\frac{\pi}{4}} \sqrt{\frac{\hbar}{\sigma^2}}\, FT[\varphi_m(x)]\Big|_{p=\frac{\hbar x}{\sigma^2}} \qquad (1)$$

Also $\Psi\left(x, \frac{\pi}{2\omega}\right) = e^{-i\hat{H}t/\hbar}\, \varphi_m(x,0) =$

$= e^{-i\hbar\omega\left(\frac{1}{2}+N\right)\cdot\frac{\pi}{2\omega}\cdot\frac{1}{\hbar}}\, \varphi_m(x,0) =$

$= e^{-i\frac{\pi}{4}}\, e^{-i\frac{\pi}{2}N}\, \varphi_m(x,0) =$

$= e^{-i\frac{\pi}{4}}\, e^{-i\frac{\pi}{2}m}\, \varphi_m(x,0) =$

$= e^{-i\frac{\pi}{4}} \left(e^{-i\frac{\pi}{2}}\right)^m \varphi_m(x,0) =$

$= e^{-i\frac{\pi}{4}} (-1)^m\, \varphi_m(x,0) \qquad (2)$

From (1) and (2): $\boxed{FT[\varphi_m(x)]\Big|_{p=\frac{\hbar x}{\sigma^2}} = \sqrt{\frac{\sigma^2}{\hbar}}\,(-1)^m\, \varphi_m(x,0)}$

So the energy eigenstates are their own scaled Fourier transforms

f. (i) $\Psi(x,0) = e^{ikx}$ This is a plane wave

After $t = \frac{\pi}{2\omega}$, the state will be a delta function

$t = 2\cdot\frac{\pi}{2\omega}$, —— " —— plane wave w/ $-k$

$t = 3\cdot\frac{\pi}{2\omega}$, —— " —— delta function

$t = 4\cdot\frac{\pi}{2\omega}$, —— " —— the same as initially

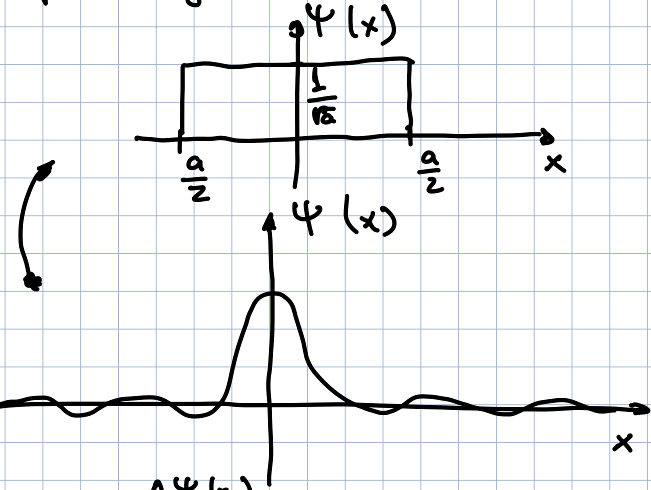
evolution is periodic after

(ii) $FT\left[e^{-\rho|x|} \right] =$ Lorentzian

Similar to above, $\psi(x,t)$ changes periodically between the initial wavefunction and Lorentzian.

(iii) $\psi(x,0) = \begin{cases} \frac{1}{\sqrt{a}} & \text{if } -\frac{a}{2} \leq x \leq \frac{a}{2} \\ 0 & \text{else} \end{cases}$

time evolution



$FT[\psi(x,0)] \propto \text{sinc}(x)$

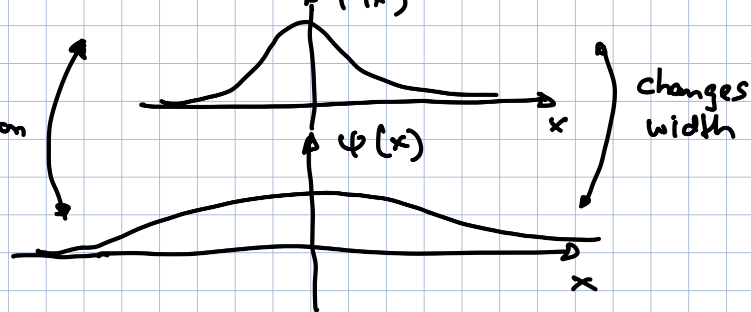
$\propto \frac{\sin(x)}{x}$

(iv) $\psi(x,0) = e^{-\rho^2 x^2}$

Gaussian

$FT[\psi(x,0)] \propto e^{-\frac{x^2}{4\rho^2}}$

time evolution



Problem 4

(a) We did this in class:

$$\alpha_0 = \frac{1}{\sqrt{2}}\left(\frac{\langle x\rangle_0}{\sigma} + \frac{i\langle p\rangle_0 \sigma}{\hbar}\right)$$

$$\langle x\rangle_0 = \sqrt{2}\,\sigma\cdot \mathrm{Re}[\alpha_0] = \underline{0}$$

$$\langle p\rangle_0 = \sqrt{2}\,\frac{\hbar}{\sigma}\,\mathrm{Im}[\alpha_0] = \underline{p_0}$$

(b) In class:

$$\sigma = \sqrt{\frac{\hbar}{m\omega}}$$

$$\underline{\langle H\rangle_0 = \hbar\omega\left(|\alpha_0|^2 + \frac{1}{2}\right)} =$$

$$= \hbar\omega\left(\frac{\langle x\rangle_0^2}{2\sigma^2} + \frac{\langle p\rangle_0^2 \sigma^2}{2\hbar^2} + \frac{1}{2}\right) =$$

$$= \hbar\omega\left(\frac{p_0^2\sigma^2}{2\hbar^2} + \frac{1}{2}\right) =$$

$$= \frac{\hbar\omega}{2} + \cancel{\hbar}\cancel{\omega}\cdot\frac{p_0^2}{2\cancel{\hbar^2}}\,\frac{\cancel{\hbar}}{m\cancel{\omega}} =$$

$$\underline{\langle H\rangle_0 = \frac{\hbar\omega}{2} + \frac{p_0^2}{2m}}$$

(c) $$\alpha(t_1) = \alpha_0 e^{-i\omega t_1}$$

(d) $$[a, x] = \left[a, \frac{\sigma}{\sqrt{2}}(a^+ + a)\right] = \frac{\sigma}{\sqrt{2}}[a, a^+] = \frac{\sigma}{\sqrt{2}}$$

$$[a, T(p_0)] = [a, e^{ip_0x/\hbar}] = [a, x]\,\frac{d\,e^{ip_0x/\hbar}}{dx} =$$

$$= \frac{\sigma}{\sqrt{2}}\; ip_0/\hbar\cdot T(p_0) =$$

$$= \frac{i\sigma p_0}{\sqrt{2}\hbar}T(p_0)$$

(e) $$a[T(p_0)|\psi(t_1)\rangle] = \{[a, T(p_0)] + T(p_0)a\}\,|\psi(t_1)\rangle =$$

$$= \left(\frac{i\sigma p_0}{\sqrt{2}\hbar} + \alpha_0 e^{-i\omega t_1}\right)T(p_0)\,|\psi(t_1)\rangle$$

$\Rightarrow T(p_0)|\Psi(t_1)\rangle$ is a coherent state b/c. is an eigenstate of a w/ eigenvalue $\frac{i\sigma p_0}{\sqrt{2}\hbar} + \alpha_0 e^{-i\omega t_1} = \alpha_1 = \frac{\sigma p_0}{\sqrt{2}\hbar}\cdot\left(i + e^{-i\omega t_1}\right)$

(f) $\langle\hat{H}\rangle_1 = \hbar\omega\left(|\alpha_1|^2 + \frac{1}{2}\right) =$ $\quad |\alpha_0|^2 = \frac{p_0^2\sigma^2}{2\hbar^2}$

$$= \hbar\omega\left[\frac{\sigma^2 p_0^2}{2\hbar}\cdot\left(i + e^{-i\omega t_1}\right)\left(-i + e^{i\omega t_1}\right) + \frac{1}{2}\right] =$$

$$= \hbar\omega\left[\frac{\sigma^2 p_0^2}{2\hbar}\left(1 + ie^{i\omega t_1} - ie^{-i\omega t_1} + 1\right) + \frac{1}{2}\right] =$$

$$= \hbar\omega\left[\frac{\sigma^2 p_0^2}{\hbar}\quad\left(1 + \cos(\omega t_1)\right) + \frac{1}{2}\right] =$$

$$\boxed{\langle\hat{H}\rangle_1 = \frac{\hbar\omega}{2} + \frac{p_0^2}{m}\left[1 + \cos(\omega t_1)\right]}$$

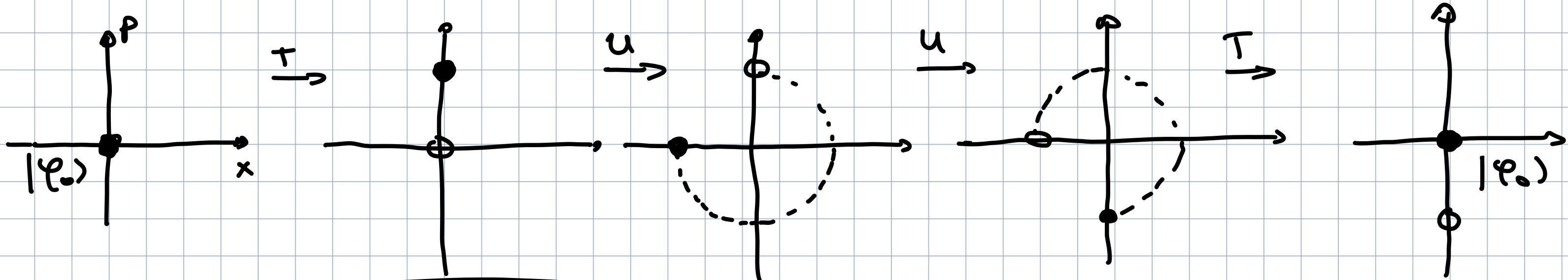
(g) $\langle\hat{H}\rangle$ unchanged $\Rightarrow$ $\langle\hat{H}\rangle_1 = \langle\hat{H}\rangle_0$

$1 + \cos(\omega t_1) = \frac{1}{2}$ $\qquad \cos(\omega t_1) = -\frac{1}{2}$

$\omega t_1 = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$t_1 = \frac{2\pi}{3\omega}$ or $\frac{4\pi}{3\omega}$

(h) choose $t_1 = \frac{4\pi}{3\omega}$

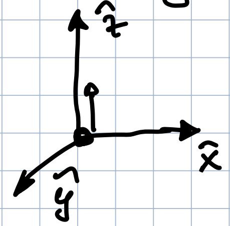


$$\boxed{\hat{Q} = \hat{T}(p_0)\hat{U}_0\hat{U}_0\hat{T}(p_0)}$$

Problem 14

$m_{Rb} = 1.5 \cdot 10^{-25}$ kg $\qquad \omega = 100$ rad/s $\qquad V_0 = 20 \frac{m}{s}$ in $+\hat{x}$

(a)



$V_0 + \delta V$

$z_0 + \underbrace{\Delta z}_{1\,mm}$

$\sigma = \sqrt{\frac{\hbar}{m\omega}}$

$$|\psi(0)\rangle = |\alpha_x\rangle |\alpha_y\rangle |\alpha_z\rangle$$ – 3D coordinates

$$|\alpha_x\rangle = T(m\,\delta v) \cdot |\varphi_0\rangle$$
$$|\alpha_y\rangle = |\varphi_0\rangle$$
$$|\alpha_z\rangle = S(\Delta z) \cdot |\varphi_0\rangle$$

From class: $$\alpha(t) = \frac{1}{\sqrt{2}}\left[\frac{\langle x\rangle(t)}{\sigma} + \frac{i\sigma\langle p\rangle(t)}{\hbar}\right]$$

$$\alpha_x(0) = \frac{1}{\sqrt{2}} \frac{i\sigma\langle p\rangle_0}{\hbar} = \frac{i}{\sqrt{2}} \frac{\sigma \cdot m \cdot \delta v}{\hbar}$$

$$\alpha_z(0) = \frac{1}{\sqrt{2}} \frac{(\Delta z)}{\sigma}$$

in x: max displacement is $\frac{\sigma \cdot m \cdot \delta v}{\hbar} = \frac{\langle x\rangle_{max}}{\sigma}$

$$\Rightarrow \langle x\rangle_{max} = \frac{\sigma^2 m \cdot \delta v}{\hbar} = \frac{\cancel{\hbar}}{\cancel{m}\omega} \frac{\cancel{m} \cdot \delta v}{\cancel{\hbar}} = \frac{\delta v}{\omega}$$

in z: max displacement is $\langle z\rangle_{max} = 1$ mm

(b) circular orbit $\Rightarrow \langle x\rangle_{max} = \langle z\rangle_{max}$

$$\delta v = \omega \cdot (1\,mm) = 100 \frac{rad}{s} \cdot (1\,mm) = \underline{0.1 \frac{m}{s}}$$

(c) we know energy is $E_n = \hbar\omega\left[|\alpha_0|^2 + \frac{1}{2}\right]$ and the resulting Poisson distribution will peak at the highest integer $\hbar\omega$ smaller than $|\alpha_0|^2$.

$$\alpha_x = \frac{i}{\sqrt{2}} \frac{\sqrt{\frac{\hbar}{m\omega}} \cdot m\,\delta v}{\hbar} = \frac{i}{\sqrt{2}} \sqrt{\frac{m}{\hbar\omega}}\,\delta v$$

$$|\alpha_x|^2 = \frac{1}{2} \frac{m}{\hbar\omega} (\delta v)^2 = 71118.9 \rightarrow \boxed{n_x = 71{,}118}$$

$$|\alpha_z|^2 = \frac{1}{2} \frac{\Delta z^2}{\sigma^2} = 71118.9$$

same, makes sense since same energy $\boxed{n_z = 71118}$

$$\boxed{n_y = 0}$$

$$\Rightarrow E = \hbar\omega \left(\frac{3}{2} + n_x + n_y + n_z \right) = \hbar\omega \cdot 142237.5 = 1.5 \cdot 10^{-27}\,\mathrm{J}$$

state after measurement is $|\, n_x = 71118,\ n_y = 71118,\ n_z = 71118 \rangle$

(d) $\langle x \rangle = \langle p_x \rangle = 0$ for any energy measurement after $t=0$

This is because the atom is in an energy eigenstate and not a coherent state.

Problem IV

$M_E = 6 \cdot 10^{24}$ kg $\qquad R_E = 6.4 \cdot 10^6$ m

$G = 6.7 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$

$$V(r) = \begin{cases} -G \dfrac{M_E m}{r} & \text{for } r > R_E \\[2ex] -\dfrac{3}{2} \dfrac{G M_E m}{R_E} + \dfrac{G M_E m r^2}{2 R_E^3} & \text{for } r < R_E \end{cases}$$

(a) $$\frac{1}{2} \cancel{m} w^2 R_E^2 = -V(R_E) = \frac{G M_E \cancel{m}}{2 R_E}$$

$$\Rightarrow w = \sqrt{\frac{G M_e}{R_E^3}} = 0.001238 \ s^{-1}$$

$$T = \frac{2\pi}{w} = 5074 \ s = 84.6 \text{ minutes}$$

$$v = w \cdot R_E = 7925 \ \frac{m}{s} = 28532 \ \frac{km}{h}$$

all these # only have one digit of precision

(b) $$\cancel{\frac{1}{2}} m w^2 R^2 = \cancel{\frac{1}{2}} k_e \frac{e^2}{R}$$

$$w = \sqrt{\frac{k_e e^2}{m R^3}} \boxed{= 2.945 \cdot 10^{15} \frac{rad}{s}}$$

(c) in the QHO, the ground state wavefunction is localized to approx $\Delta x \approx \sigma = \sqrt{\frac{\hbar}{m w}} \boxed{= 4.59 \cdot 10^{-12} \ m}$

(d) $$\lambda = \frac{c}{\nu} = \frac{c}{w/2\pi} \boxed{= 640 \ nm}$$

(e) $$v = w \cdot R \boxed{= 73625 \ \frac{km}{s}}$$

~ 9x higher than in the gravitational problem