OPTI 570, Fall 2025 **Problem Set 3** Prof. C. D. Panda

Due: Thurs, Sept. 11 (Main Campus); Tues, Sept 16 (Online section)

PART I: Complete the following problems from CT Chapter II, complement $\mathbf{H}_{\mathrm{II}}$. These are provided in the PDFs on the course D2L site if you don't have CT yet. Primary relevant reading:
◦ CT II, sections B-D, and Chapter II-complement B.

Problem 2: **(a)** You saw this matrix in Problem Set 1. Solve again for the eigenvalues and eigenvectors, but this time make sure your eigevenvectors (i) have their first element being real and positive and (ii) are normalized to 1. If you remember these results from earlier, feel free to just write them down without all of the math. Then solve part **(b)**. Part **(c)** is not required, but feel free to solve if you would like more practice with doing matrix algebra
Problem 3: This exercise will help you get used to working with projection operators.
Problem 6: You may need to refer to CT Complement B-II-4.
Problem 7: Solve as written.
Problem 9: Hints:

- in part **(b)**-β, you can use results from **(b)**-α and **(a)** to do this part quickly.
- in part **(b)**-γ, you can use results from **(b)**-α. Eq. 51 in CT Comp. B-II will also help. This part takes a lot of playing around with expressions and trying different approaches. You may not immediately see what to do, or know what you're doing, but spend some time on it and give it your best shot; the act of trying out various approaches is worthwhile in itself.

PART II (not in CT): Let the state space $\mathcal{E}$ be spanned by the orthonormal basis $\{\,|u_1\rangle, |u_2\rangle, |u_3\rangle, |u_4\rangle\}$. We will call this the $\{\,|u_n\rangle\}$ basis. Answer questions II-1 through II-3 below regarding $\mathcal{E}$.

II-1. Define a ket $|\psi\rangle \in \mathcal{E}$ as $|\psi\rangle = c\,(\,2|u_1\rangle - i\sqrt{3}|u_2\rangle - 3e^{i\theta}|u_3\rangle + 3|u_4\rangle\,)$, where θ is an arbitrary real scalar. Find c, assuming that it is real and positive, such that $\langle\psi|\psi\rangle = 1$.

II-2. Give the column vector, row vector, or matrix that corresponds to each one of the following items when expressed in the $\{\,|u_n\rangle\}$ representation.

(a) $|u_2\rangle$

(b) $\langle u_3|$

(c) $|u_2\rangle\langle u_3|$

(d) The projector onto $|u_2\rangle$.

(e) $\displaystyle\sum_{m=1}^{m=4}\sum_{n=1}^{n=4} |u_m\rangle\langle u_n|$

II-3. An operator $\hat{Q}$ acts on elements of the $\{\,|u_n\rangle\}$ basis, and is defined by the actions

$$\hat{Q}|u_1\rangle = i|u_4\rangle, \qquad \hat{Q}|u_2\rangle = 2|u_3\rangle, \qquad \hat{Q}|u_3\rangle = 2|u_2\rangle, \qquad \hat{Q}|u_4\rangle = -i|u_1\rangle.$$

Express $\hat{Q}$ as a matrix in the $\{|u_n\rangle\}$ representation.