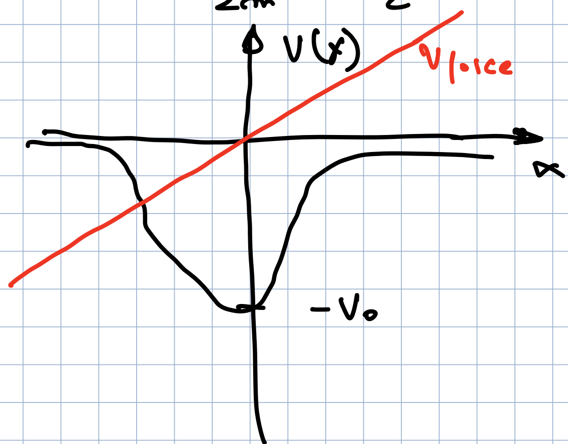
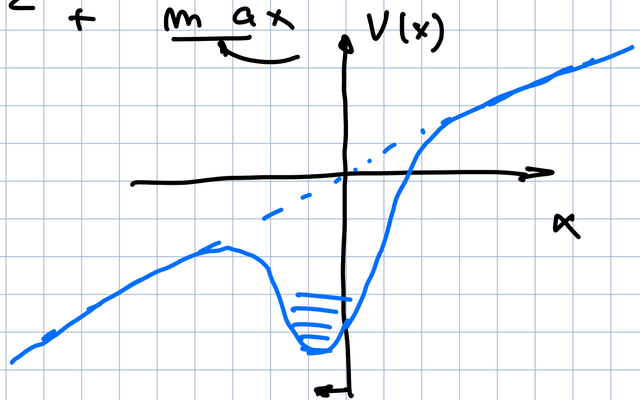
OPTI 570 LECTURE Tu OCT 14

Atoms trapped by light

Gaussian potential in x, add a linear force
(magnetic, electric gradient,
light sheet w/ gradient, gravity)

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \underline{m a x}$$





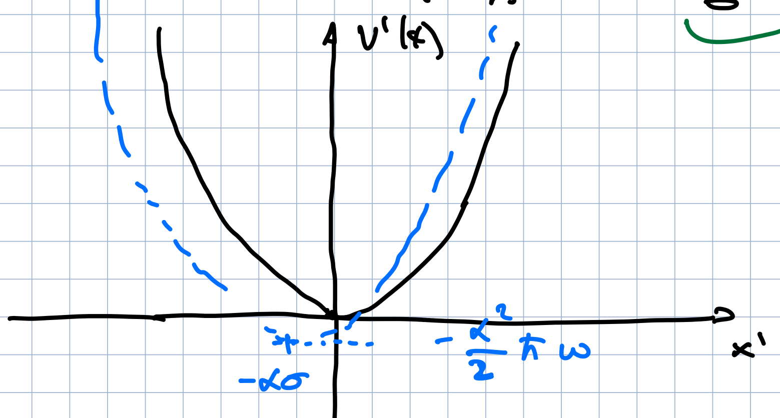
Case I : GHO - approx is still good

$$H = \frac{1}{2}\hbar\omega\left(\frac{\sigma^2 p^2}{\hbar^2} + \frac{x^2}{\sigma^2} + \frac{2max}{\hbar\omega}\right) = \qquad \alpha = \frac{m a \sigma}{\hbar\omega}$$

$$= \frac{1}{2}\hbar\omega\left[\frac{\sigma^2 p^2}{\hbar^2} + \frac{1}{\sigma^2}\left(x^2 + 2\alpha x \sigma\right)\right] =$$

$$= \frac{1}{2}\hbar\omega\left[\frac{\sigma^2 p^2}{\hbar^2} + \frac{1}{\sigma^2}\left(x + \alpha\sigma\right)^2 - \alpha^2\right] =$$

$$= \frac{1}{2}\hbar\omega\left(\frac{\sigma^2 p^2}{\hbar^2} + \frac{x'^2}{\sigma^2}\right) - \frac{\alpha^2}{2}\hbar\omega \qquad x' = x + \alpha\sigma$$

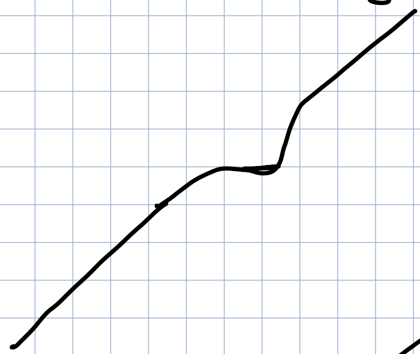


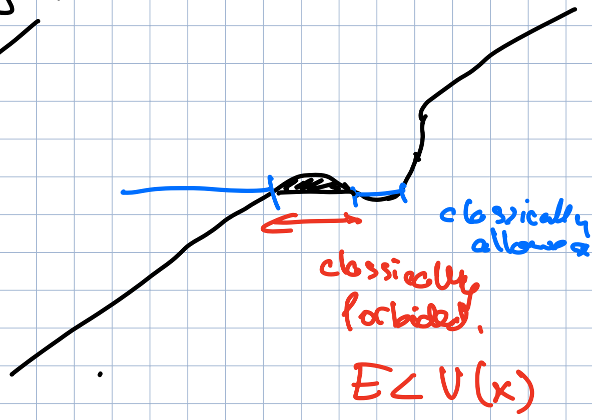
$$E_1 = \frac{1}{2}\hbar\omega - \frac{\alpha^2}{2}\hbar\omega$$

$$E_2 = \frac{3}{2}\hbar\omega - \frac{\alpha^2}{2}\hbar\omega$$

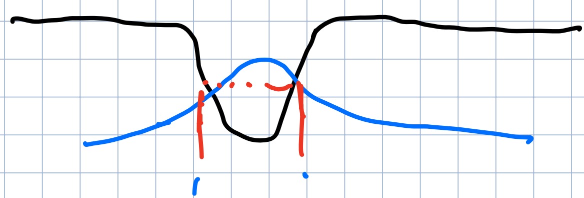
$$\cdots$$

Case II: strong force - cannot use QHO approx.





tunneling is possible - probability that the particle moves from trapped to un-trapped state



2D QHO isotropic

x - direction

$$a_x = \frac{1}{\sqrt{2}}\left(\frac{x}{\sigma} + \frac{i\sigma p_x}{\hbar}\right)$$

$$a_x|\alpha_x\rangle = \alpha_x|\alpha_x\rangle$$

y - direction

$$a_y = \frac{1}{\sqrt{2}}\left(\frac{y}{\sigma} + \frac{i\sigma p_y}{\hbar}\right)$$

$$a_y|\alpha_y\rangle = \alpha_y|\alpha_y\rangle$$

basis: $\{|n_x\rangle\}$ $\{|n_y\rangle\}$

Tenser product space $\mathcal{E}_x \otimes \mathcal{E}_y$

Tensor product basis $\{|n_x\rangle|n_y\rangle\} = \{|n_x, n_y\rangle\}$

$$|\alpha_x, \alpha_y\rangle = |\alpha_x\rangle|\alpha_y\rangle = e^{-\frac{|\alpha_x|^2}{2}} e^{-\frac{|\alpha_y|^2}{2}} \sum_{\substack{n_x=0 \\ n_y=0}}^{\infty} \frac{\alpha_x^{n_x}}{\sqrt{n_x!}} \frac{\alpha_y^{n_y}}{\sqrt{n_y!}} |n_x, n_y\rangle$$

$$|n_x, \alpha_y\rangle = |n_x\rangle \underline{|\alpha_y\rangle} = e^{-\frac{|\alpha_y^2|}{2}} \sum_{n_y=0}^{\infty} \frac{\alpha_y^{n_y}}{\sqrt{n_y!}} |n_x, n_y\rangle$$

$$|2,5\rangle = ? \quad \begin{cases} |n_x = 2, \alpha_y = 5\rangle \\ |\alpha_x = 2, \alpha_y = 5\rangle \end{cases}$$

<u>Dispacement operator</u>

$$D_x(\alpha_x)\, D_y(\alpha_y)\, |n_x = 0, n_y = 0\rangle = |\alpha_x\rangle |\alpha_y\rangle \quad - \text{2D coherent state}$$

<u>Energy eigenvalues</u>

How many states have energy $E = \cancel{\tfrac{3}{2}\hbar\omega}$ $2\hbar\omega$?

$\underline{|n_x = 0, n_y = 1\rangle}$ $\qquad |n_x = 1, n_y = 0\rangle \qquad E = \hbar\omega\left(n_x + \frac{1}{2} + n_y + \frac{1}{2}\right)$

<u>basis</u> : $\qquad n = n_x + n_y$

$$\{|n_x, n_y\rangle\} = \{|0,0\rangle, \underbrace{|1,0\rangle, |0,1\rangle}_{n=1 \text{ manifold}}, \underbrace{|2,0\rangle, |1,1\rangle, |0,2\rangle}_{n=2 \text{ manifold}}, \ldots$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$$

$$|\alpha_x, \alpha_y\rangle = e^{-\frac{|\alpha_x|^2}{2}} e^{-\frac{|\alpha_y|^2}{2}} \cdot \Big(|00\rangle + \alpha_x |10\rangle + \alpha_y |01\rangle + \frac{\alpha_x^2}{\sqrt{2}} |20\rangle + \alpha_x \alpha_y |11\rangle + \frac{\alpha_y^2}{\sqrt{2}} |02\rangle + \ldots \Big)$$

$$|\alpha_x, \alpha_y\rangle \rightarrow \begin{pmatrix} 1 \\ \alpha_x \\ \alpha_y \\ \alpha_x^2/\sqrt{2} \\ \alpha_x \alpha_y \\ \alpha_y^2/\sqrt{2} \\ \vdots \end{pmatrix}$$

$$H \rightarrow \hbar\omega \begin{pmatrix} 1 & & & & & & & & & 0 \\ & 2 & & & & & & & & \\ & & 2 & & & & & & & \\ & & & 3 & & & & & & \\ & & & & 3 & & & & & \\ & 0 & & & & 3 & & & & \\ & & & & & & 4 & & & \\ & & & & & & & 4 & & \\ & & & & & & & & \ddots & \end{pmatrix}$$

$$\hat{B} \equiv i(a_x a_y^\dagger - a_x^\dagger a_y) \qquad ?$$

1. Is B Hermitian?

$$\hat{B}^\dagger = -i\,(a_y a_x^\dagger - a_y^\dagger a_x) =$$

$$= i(a_x a_y^\dagger - a_x^\dagger a_y) =$$

$$= \hat{B}$$ ✓ Yes, Hermitian!

2. $[H, B] = ?$

$$= [\hbar\omega(a_x^\dagger a_x + a_y^\dagger a_y + 1),\ i(a_x a_y^\dagger - a_x^\dagger a_y)] =$$

$$= i\hbar\omega \{ [a_x^\dagger a_x, a_x a_y^\dagger] + [a_x^\dagger a_x, -a_x^\dagger a_y] + [a_y^\dagger a_y, a_x a_y^\dagger] +$$

$$+ [a_y^\dagger a_y, -a_x^\dagger a_y] \} =$$

$$= i\hbar\omega \{ \quad [a_x^\dagger a_x, a_x] a_y^\dagger = [a_x^\dagger, a_x] a_x a_y^\dagger =$$

$$= - a_x a_y^\dagger$$

. . .

$$= i\hbar\omega \{ \cancel{-a_x a_y^\dagger} \cancel{- a_x^\dagger a_y} \cancel{+ a_y^\dagger a_x} \cancel{+ a_y a_x^\dagger} \} =$$

$$[H, B] = 0$$

<u>3.</u> <u>$n=1$</u> $\Rightarrow$ what is the common eigenstate w/ eigenvalue?

$|n_x = 1, n_y = 0\rangle$ and $|n_x = 0, n_y = 1\rangle$

<u>Q</u>: are $|1\ 0\rangle$, $|0, 1\rangle$ eigenstates of B?

$$B = i\,(a_x a_y^\dagger - a_x^\dagger a_y)$$

$$B\,\underline{|1,0\rangle} = i\,(|0,1\rangle - \sqrt{2}\cdot 0\,|2,0\rangle = i\,\underline{|0 1\rangle}$$

$$B|0,1\rangle = i\,(0 - |0,1\rangle = -i\,|1,0\rangle$$

<u>Let</u>: $|\psi\rangle = c_1|1,0\rangle + c_2|0,1\rangle$

$$B|\psi\rangle = i\,c_1|0,1\rangle - i\,c_2|1,0\rangle$$

$$B|\psi\rangle = \lambda|\psi\rangle$$

$$= \lambda c_1 |1,0\rangle + \lambda c_2 |0,1\rangle$$

$\Rightarrow\ i c_1 = \lambda c_2 \qquad -i c_2 = \lambda c_1 \ \Rightarrow\ c_2^2 = -c_1^2$

$\Rightarrow\ c_1 = \frac{1}{\sqrt{2}} \qquad c_2 = \pm\frac{i}{\sqrt{2}}$

$$\underbrace{|\psi_\pm\rangle = \frac{1}{\sqrt{2}}|1,0\rangle \pm \frac{i}{\sqrt{2}}|0,1\rangle \quad \text{w/} \quad \lambda = \pm 1}$$

What is the physical meaning of B?

$$\hat{B} = i(a_x a_y^\dagger - a_x^\dagger a_y)$$

$$= i\left[\frac{1}{\sqrt{2}}\left(\frac{x}{\sigma} + \frac{i p_x \sigma}{\hbar}\right)\frac{1}{\sqrt{2}}\left(\frac{y}{\sigma} - \frac{i p_y \sigma}{\hbar}\right) - \quad\right]$$

$$= (\hat{x}\hat{p}_y - \hat{y}\hat{p}_x)/\hbar$$

$\hbar\hat{B} = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x =$ z component of a 3D vector $\hat{R}\times\hat{P}$

$\hat{L}_z = \hbar\hat{B} \Rightarrow$ operator for orbital angular momentum of the particle about the z axis

eigenvalues? $= \pm\hbar$

$|\psi_+\rangle$ is a state w/ angular momentum about z of $\hbar$

$|\psi_-\rangle$ ——— " ——— of $-\hbar$

Energy: $2\hbar\omega$

$$\psi_\pm(x,y) \propto x e^{-\frac{x^2}{2\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} \pm i y e^{-\frac{y^2}{2\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$= r e^{\pm i\theta} e^{-\frac{r^2}{2\sigma^2}} \qquad r^2 = x^2 + y^2 \qquad \theta = \arctan\left(\frac{y}{x}\right)$$

$$|\psi_\pm(x,y)|^2 \propto r^2 e^{-\frac{r^2}{\sigma^2}}$$