Notes of Quantum Mechanics

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Preface

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Contents

List of Figures

List of Tables

Listings

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Chapter 1

Theory of angular momentum

1.1 Spin 1/2 particle: quantization of the angular momentum

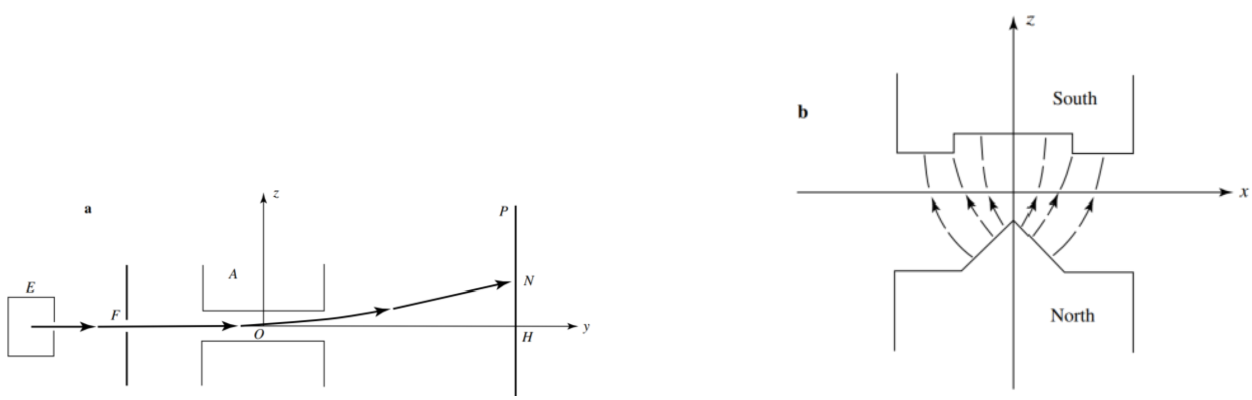
1.1.1 Experimental demonstration

We are going to describe and analyze the Stern-Gerlach experiment, which demonstrated the quantization of the componentes of the angular momentum.

The Stern-Gerlach apparatus

The experiment consists of studying the deflection of a beam of neutral paramgnetic atoms (in this case silver atoms). The leave a furnace E through a small opening and propagate in a straigth line in the high vacuum existing inside the apparatus. Then, the atomic beam traverses the electromagnet A and thus being deflected before reaching the plate P.

This B-field has a plane of symetry yOz that contains the initial direction Oy of the atomic beam. The B-field has no components along Oy, and its larges component is along Oz; it varies strongly with z. Since the B-field has a conserved flux $\nabla \cdot \boldsymbol{B} = 0$, it must also have a component along Ox which varies with the distance x from the plane of symmetry.



(a) Stern-gerlach apparatus

(b) B-field distribution

Figure 1.1

1.1.2 Classical calculations of the deflection

The neutral silver atoms posses a permanent magnetic moment $\boldsymbol{\mu}$ (they are paramgnetic atoms); the resulting forces are derived from the potential energy:

$$W_B = -\boldsymbol{\mu} \cdot \boldsymbol{B}. \tag{1.1}$$

For a given atomic level, the magnetic moment $\boldsymbol{\mu}$ and the angular momentum $\boldsymbol{J}$ are proportional:

$$\boldsymbol{\mu} = \gamma \boldsymbol{J}, \tag{1.2}$$

where γ is the **gyromagnetic ratio** of the level.

1.2 Illustration of the postules in the case of a spin 1/2

1.2.1 Evolution of a spin 1/2 particle in a uniform magnetic field

1.2.2 The interaction Hamiltonian and the Schrodinger equation

Consider a silver atom in a unifrom magnetic field $\boldsymbol{B}_0$, and choose the Oz axis along $\boldsymbol{B}_0$. The classical potential energy of the magnetic moment $\boldsymbol{\mu} = \gamma \boldsymbol{J}$ of this atom is then:

$$W = -\boldsymbol{\mu} \cdot \boldsymbol{B}_0 = -\mu_z B_0 = \underbrace{-\gamma B_0}_{\omega_0} J_z. \tag{1.3}$$

Since we are quantizing onl the internal degrees of freedom of the particle, J_z must be replaced by the operator S_z, and the clasical energy above becomes an operator: it is the Hamiltonian H which describes the evolution of the spin of the atom in the field $\boldsymbol{B}_0$:

$$H = \omega_0 S_z. \tag{1.4}$$

Since H is time-independent, we solve the respective eigenequation. We see that the eigenvectors of H are tose of S_z:

$$H|\pm\rangle = \pm\frac{\hbar\omega_0}{2}|\pm\rangle = E_\pm|\pm\rangle. \tag{1.5}$$

There are theerefore two energy levels, $E_\pm$. Their separation $\hbar\omega_0$ is proportional to the B-field; they define a single Bohr frequency:

$$\nu_{+-} = \frac{1}{\hbar}(E_+ - E_-) = \frac{\omega_0}{2\pi}. \tag{1.6}$$

- If $\boldsymbol{B}_0$ is parallel to the unit vector $\boldsymbol{u}$, the Hmailtonian (1.4) msut be replaced by its general form:

$$\text{General form Hamiltonian} \qquad H = \omega_0 \boldsymbol{S} \cdot \boldsymbol{u}\,. \tag{1.7}$$

- For silver atoms, $\gamma < 0$; ω_0 is therefore positive.

Larmor precession

Consider the spin at $t = 0$ in the state

$$|\psi(0)\rangle = \cos\frac{\theta}{2}e^{-i\phi/2}|+\rangle + \sin\frac{\theta}{2}e^{i\phi/2}|-\rangle. \tag{1.8}$$

We saw that any state can be put in this form. To calculate the state at $t > 0$, we apply the evolution operator:

$$|\psi(t)\rangle = \cos\frac{\theta}{2}e^{-i\phi/2}e^{-iE_+t/\hbar}|+\rangle + \sin\frac{\theta}{2}e^{i\phi/2}e^{-iE_-t/\hbar}|-\rangle = \cos\frac{\theta}{2}e^{-\frac{i(\phi+\omega_0 t)}{2}}|+\rangle + \sin\frac{\theta}{2}e^{\frac{i(\phi+\omega_0 t)}{2}}|-\rangle.$$

The presence of $\boldsymbol{B}_0$ therefore introduces a phase shift between $|+\rangle$ and $|-\rangle$. The direction of $\boldsymbol{u}(t)$ along which the spin component if $+\hbar/2$ with certainty is defined by the polar angles:

$$\begin{aligned} \theta(t) &= \theta \\ \phi(t) &= \phi + \omega_0 t \end{aligned} \tag{1.9}$$

The angle between $\boldsymbol{u}(t)$ and Oz therefore remains constant, but $\boldsymbol{u}(t)$ revolves about Oz at an angular velocity of ω_0. THis effect is called the **Larmor precession**.

It can verified from $|\psi(t)\rangle$ that the probabilities of obtaining $+\hbar/2$ or $-\hbar/2$ in a measurement of this observables are time-indendent. These probabilities are equal, respectively, to $\cos^2\theta/2$ and $\sin^2\theta/2$. The mean value of S_z is also time-independent:

$$\langle\psi(t)|S_z|\psi(t)\rangle = \frac{\hbar}{2}\cos\theta. \tag{1.10}$$

Because S_x and S_y do not comute with H, we have that

$$\langle\psi(t)|S_x|\psi(t)\rangle = \frac{\hbar}{2}\sin\theta\cos(\phi+\omega_0 t), \quad \langle\psi(t)|S_y|\psi(t)\rangle = \frac{\hbar}{2}\sin\theta\sin(\phi+\omega_0 t). \tag{1.11}$$

We again find the Bohr frequencies $\omega_0/2\pi$ of the system. Moreover, the mean values above behave like the components of a classical AM of modulus $\hbar/2$ undergoing Larmos precession.

1.3 Two-level systems

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