OPTI 570 LECTURE Tu DEC 9

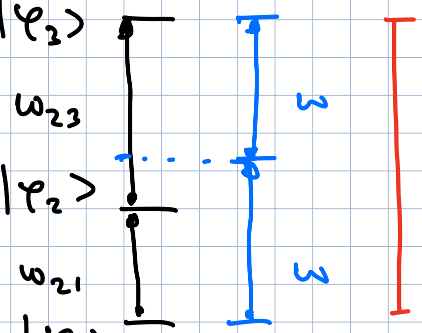
Last time

^{87}Rb atoms that are interacting w/ st light

$|5D_{\frac{5}{2}}, F=4, m_F=4\rangle = |\varphi_3\rangle$

$|5^2P_{\frac{3}{2}}, F=3, m_F=3\rangle = |\varphi_2\rangle$

$|5^2S_{\frac{1}{2}}, F=2, m_F=2\rangle = |\varphi_1\rangle$



$\omega_{31} = \omega_{32} + \omega_{21} \approx 2\omega$

$\omega \neq \omega_{21} \neq \omega_{32}$

atom starts in $|\varphi_1\rangle$

TDPT – prob. of transition from $|\varphi_1\rangle$ to $|\varphi_3\rangle$

$$\boxed{P_3^{(2)}(t) = \left|\frac{\Omega_{21}\Omega_{32}}{4\Delta_{21}}\right|^2 t^2 \operatorname{sinc}^2\left(\frac{\delta \cdot t}{2}\right)}$$

$\delta \equiv 2\omega - \omega_{31}$ "two-photon detuning"

$\delta = 0 \Leftrightarrow \omega = \frac{\omega_{31}}{2}$ maximizes $P_3^{(2)}(t)$

$$P_3^{(2)}(t) = \left|\frac{\Omega_{21}\Omega_{32}}{\Delta_{21}}\right|^2 \left(\frac{t}{2}\right)^2 \qquad \Delta_{21} = \omega - \omega_{21}$$

Q: What is $\lambda^2 P_2^{(2)} = ?$

$$\lambda^2 b_2^{(2)}(t) = \frac{1}{i\hbar}\int_0^t dt'\, e^{i\omega_{32}t'} \underbrace{W_{22}}_{0}(t')\left[\lambda\, \underline{b_2^{(1)}(t)}\right]$$

$$\lambda^2 b_2^{(2)}(t) = 0$$

$$\Rightarrow P_2^{(2)}(t) = P_2^{(1)}(t) = \left|\frac{\Omega_{21}}{\Delta_{21}}\right|^2 \sin^2\left(\frac{\Delta_{21} t}{2}\right)$$

$$\frac{P_3^{(2)}}{P_2^{(1)}} = \left|\frac{\Omega_{21}\cdot\Omega_{32}}{\Delta_{21}}\right|^2 \left(\frac{t}{2}\right)^2 \left|\frac{\Delta_{21}}{\Omega_{21}}\right|^2 \frac{1}{\sin^2(\Delta_{21}t/2)} =$$

$$= \left|\frac{\rho_{32}\,\epsilon_0}{\hbar\,\Delta_{21}}\right|^2 \cdot \frac{1}{\mathrm{sinc}^2\left(\frac{\Delta_{21}t}{2}\right)}$$

$$\frac{\sin^2(\Delta_{21}t/2)}{\Delta_{21}^2\cdot t^2/4}\cdot\frac{t^2}{4} = \frac{t^2}{4}\cdot \mathrm{sinc}^2(\Delta_{21}t/2)$$

$P_3^{(2)}$, $P_2^{(1)} \ll 1$ necessary

but: ration $\gg 1$ possible.

$$I = \frac{1}{2} c\,\varepsilon_0\,|\epsilon_0|^2$$

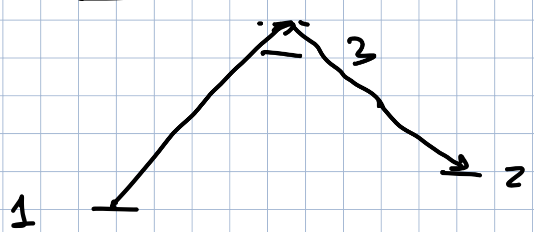
$P_2^{(1)} \propto I$

$P_3^{(2)} \propto I^2$ $\rightarrow$ non-linear in intensity

two-photon process

Large I: two-photon process $(|1\rangle \to |3\rangle)$ could be stronger than single photon absorption $(|1\rangle \to |2\rangle)$

Lambda level structure



Stimulated Raman Transitions

Pulse perturbations

- separate $\hat{W}(t)$ into time independent + time-dependent.

$$\lambda \hat{W}(t) = \lambda\,\hat{W}_i\,f(t)$$

$|\psi\rangle_i$, let $t_0 = -\infty$, evaluate $P^{(1)}_{i\to f}(t=\infty)$

$$\lambda\, b_f^{(1)}(\infty) = \frac{1}{i\hbar}\int_{-\infty}^{\infty} e^{i\omega_{fi}t'}\left(\lambda\,\hat{W}_{fi}\right) f(t')\,dt'$$

$$= \frac{\lambda \hat{W}_{fi}}{i\hbar} \underbrace{\int_{-\infty}^{+\infty} e^{-i(-\omega_{fi})t'} f(t')\, dt'}_{} =$$

$$= \frac{\lambda \hat{W}_{fi}}{i\hbar} FT\{f(t)\}\Big|_{\omega = -\omega_{fi}}$$

$$P^{(1)}_{i\to f}(\infty) = \frac{|\lambda \hat{W}_{fi}|^2}{\hbar^2} \left| FT\{f(t)\}_{\omega=-\omega_{fi}} \right|^2$$

Overview:

Goal in QM:

given $|\Psi(t_0)\rangle$

Find state $|\Psi(t)\rangle$ - dynamics

Schroedinger eq. - time evolution.

$\hat{H}$ - Hamiltonian -

$$|\Psi(t)\rangle = \sum_n \langle u_n | \Psi(t)\rangle e^{-iE_n(t-t_0)/\hbar} |\varphi_0\rangle$$

Learning QM

I. symbols: $|\ \rangle, \langle\ |, \langle\ \rangle, [\,,\,], \frac{\partial}{\partial t}, \int \ldots$

representations & transformations

II. Postulates of QM

	Physical Meaning	Math symbol.
1.	physical states	$|\Psi\rangle \in \mathcal{E}$
2.	physical observables	operators (Hermitian)
3.	States evolve w.r.t. time	SE
4.	Collapse: after meas. state of the system - eigenstate	after eigenstate of operators

5. measure a quantity. - result — eigenstate of the operator

6. probability of each measurement — projector operator

III. Examples & techniques

$H(x, p)$ — 1D → x, p representations.

Specific examples: H_{QHO}, $H_{\text{inf sq well}}$, H_{free}, H_{delta}.

$H(\vec{R}, \vec{P})$ — 2D, 3D

$H(|\vec{R}|, \vec{P})$ - central potentia problem

ex: hydrogen

→ a^{+}, a

$H(\vec{S}, \vec{L}, \dots)$ — AM

spin -

$H = H_0 + W$ — approx. methods

$H(|\vec{R}|, \vec{P}^2, \vec{S}, \vec{L}, \vec{I}, \vec{F} \dots)$ hydrogen

$H(t)$