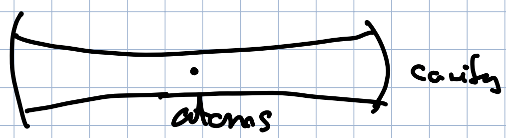
OPTI 570 LECTURE Tu NOV 4

Single-photon transitions in an atom w/ 2 energy levels in an optical cavity, 1 or 0 photons



Atom energy levels

$E_e = \hbar\omega_0$ — $|e\rangle$

$E_g = 0$ — $|g\rangle$

$\hbar\omega_0$

EM field energy levels

$E_1 = \hbar\omega$ — $|1\rangle$

$E_0 = 0$ — $|0\rangle$

$\hbar\omega$

Basis: $\{ |g,0\rangle, |g,1\rangle, |e,0\rangle, |e,1\rangle \}$

energies: 0 $\quad \hbar\omega \quad \hbar\omega_0 \quad \hbar(\omega+\omega_0)$

— $\hbar(\omega_0+\omega)$

$=$; $\hbar\omega, \hbar\omega_0$

— 0

$$|\psi(0)\rangle = |g,1\rangle$$

$$H_0 = \hbar\omega\, |g,1\rangle\langle g,1| + \hbar\omega_0\, |e,0\rangle\langle e,0|$$

$$W = -\bar{d}\cdot\bar{\mathcal{E}}$$

$$W = \frac{1}{2}\hbar\Omega_0\, |e,0\rangle\langle g,1| + \frac{1}{2}\hbar\Omega_0^*\, |g,1\rangle\langle e,0|$$

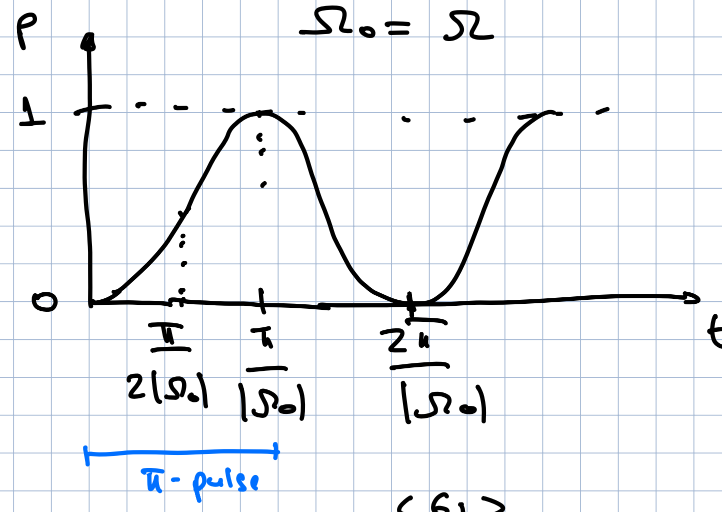
$$\Omega_0 = \rho\,\frac{\mathcal{E}_0}{\hbar}$$

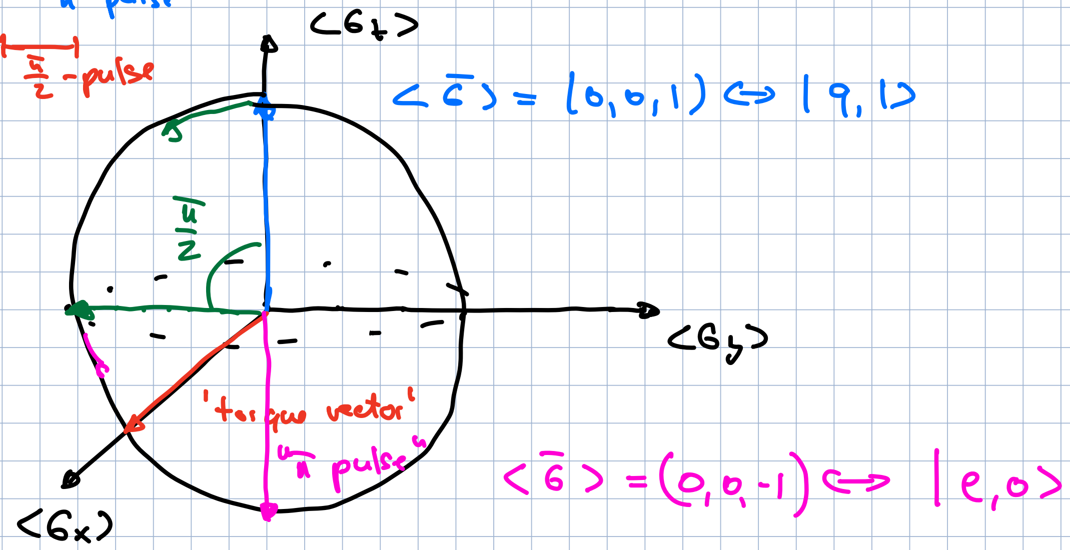
"squiggle" $\rho = -e\,\hat{\varepsilon}\cdot\overline{\langle e|\vec{R}\,|g\rangle}$ matrix element of interaction

$$P_{|g,1\rangle \to |e,0\rangle}(t) = \frac{|\Omega_0|^2}{\Omega^2}\sin^2\left(\frac{\Omega t}{2}\right)$$

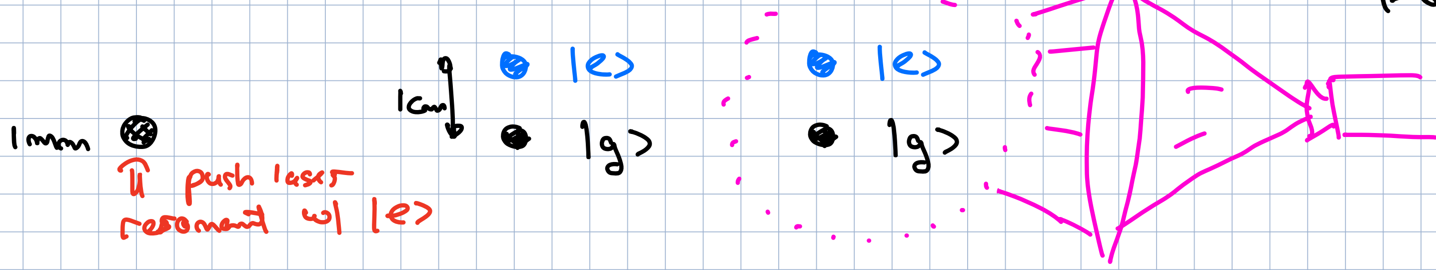
If $\omega = \omega_0$, $\Delta = 0$, on-resonance

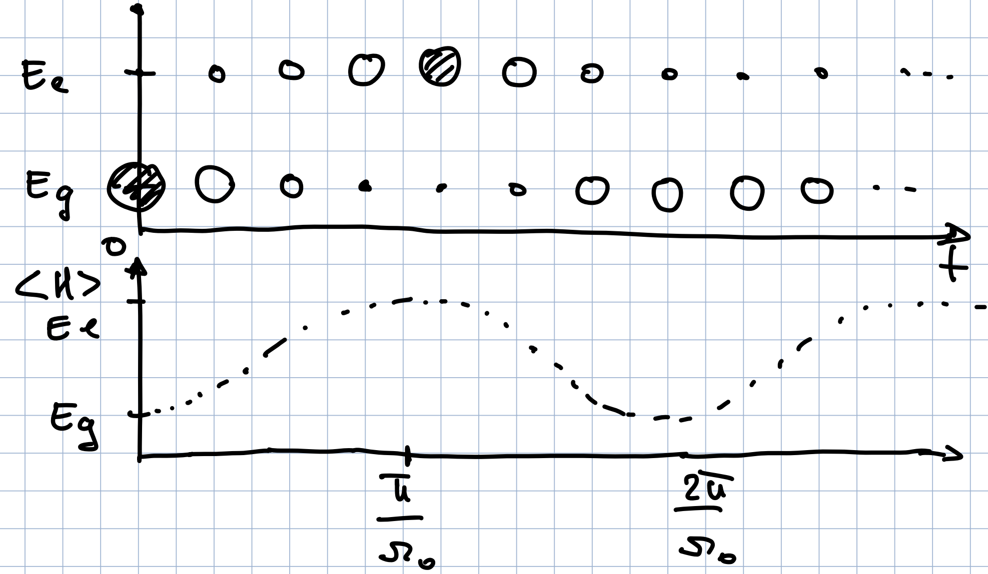
$$\Omega_0 = \Omega$$





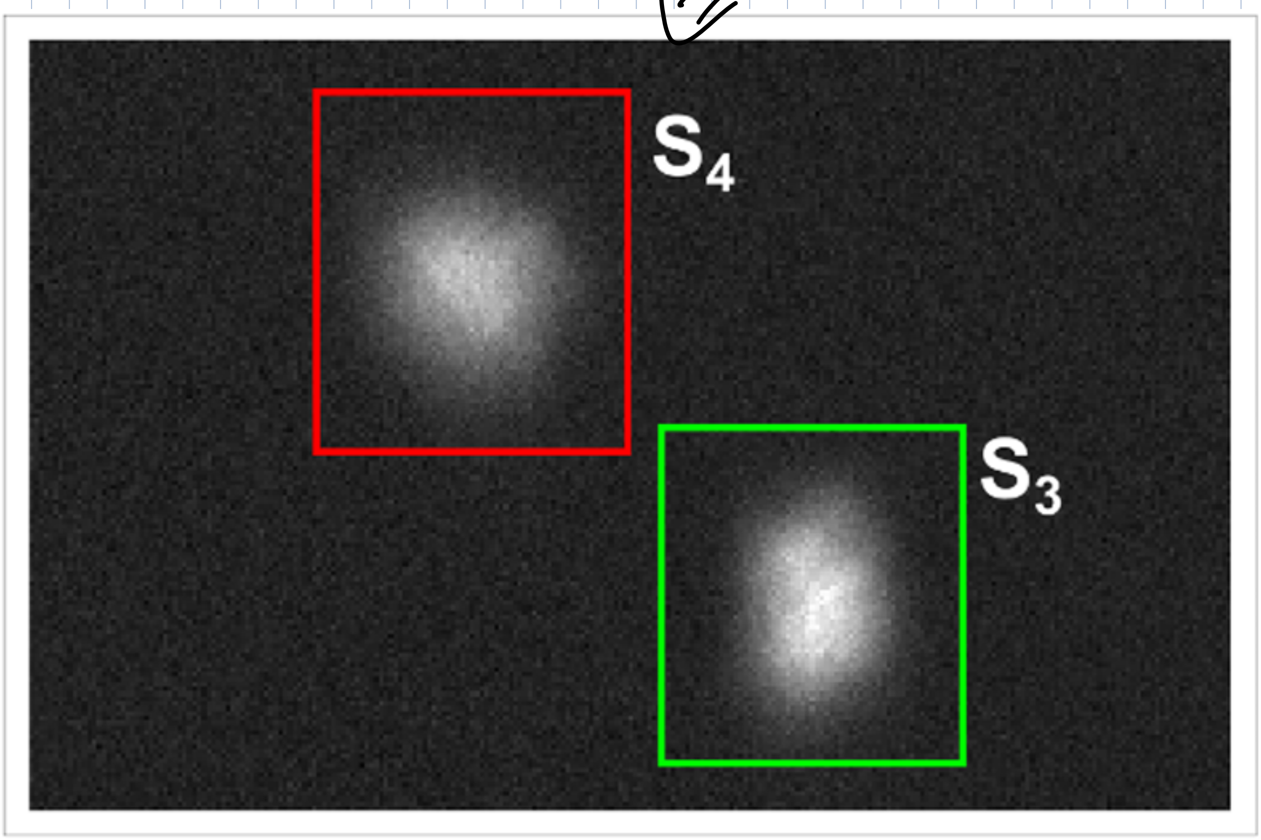
- System: 10^6 atoms Cs, $|g\rangle$ at $t=0$
  $|e\rangle$
- microwave EM field on-resonance w/ atoms: $\Delta = 0$
- measure internal state: - either E_g or E_e
  - fast compared Rabi osc. $\delta t \ll \frac{\pi}{|\Omega_0|}$





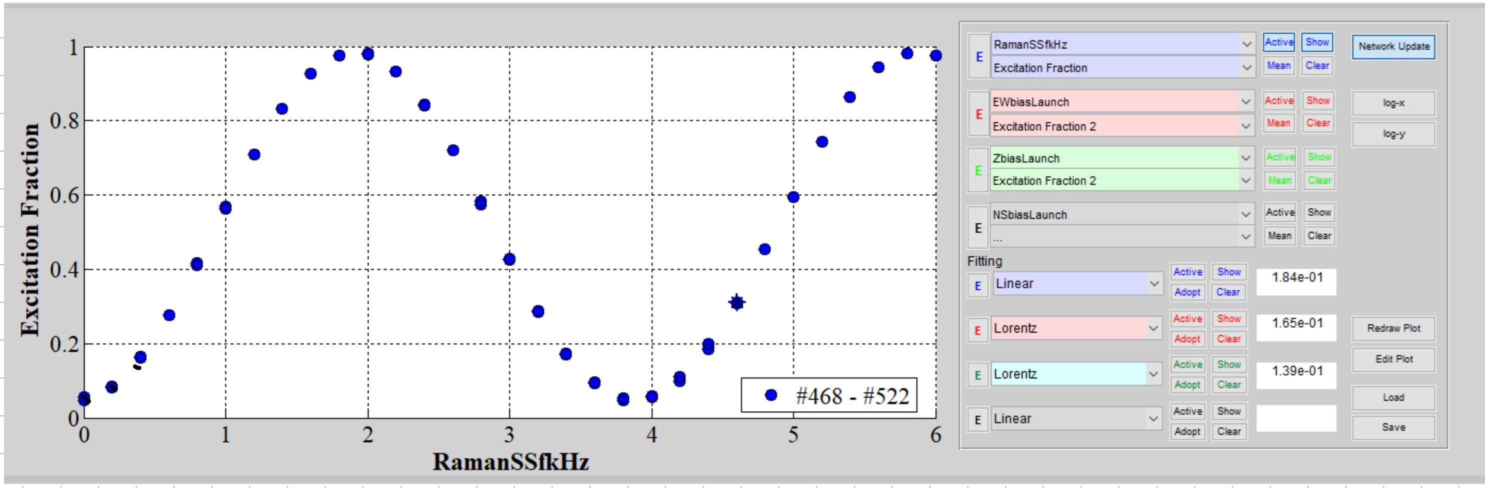
CDP et. al, "Measuring gravitational attraction with a lattice atom interferometer", Nature 631, 515-520 (2024).

$|e\rangle$



$|g\rangle$

$$\text{Excitation fraction} = \frac{S_4}{S_4 + S_3}$$



$s=1$ $\quad \mathcal{E}_{s=1}$

basis $\{ |m_z=+1\rangle, |m_z=0\rangle, |m_z=-1\rangle \}$

What is the Bloch vector for $|s=1, m_z=1\rangle$?

What about spin Pauli matrix?

$\langle \bar{S} \rangle = (\langle S_x \rangle, \langle S_y \rangle, \langle S_z \rangle)$

$|s=1, m_z=1\rangle$, $\quad \langle S_x \rangle = 0, \ \langle S_y \rangle = 0, \ \langle S_z \rangle = \hbar$

$\| \langle \bar{S} \rangle \| = \hbar$

$|s=1, m_z=0\rangle$: $\quad \langle S_x \rangle = 0, \ \langle S_y \rangle = 0, \ \langle S_z \rangle = 0$

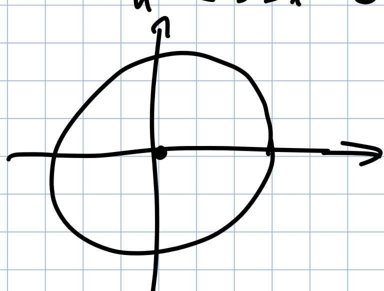
$\| \langle \bar{S} \rangle \| = 0$

ex: $|S=1, m_z=0\rangle$ meas spin along x -

	$\hbar$	0	$-\hbar$
prob:	$\frac{1}{2}$	0	$\frac{1}{2}$

What is $\| \langle \bar{S} \rangle \|$ for $|s=1, m_{\hat{u}}=0\rangle$, $\hat{u}$ is any direction?

$\| \langle \bar{S} \rangle \| = 0$



$$|\psi\rangle = \underline{a_+}\,|z+\rangle + \underline{a_0}\,|z0\rangle + \underline{c_-}\,|z-\rangle$$

– 4 independent quantities

'Spinless Hydrogen' : CT $\underline{\text{VIII}}$

- e^- bound to p^+ by Coulomb interaction.
- $V(r) = -\dfrac{e^2}{4\pi\varepsilon_0}\,\dfrac{1}{\underline{r}}$

$e = 1.6 \cdot 10^{-19}\,C$

$q^2 \equiv \dfrac{e^2}{4\pi\varepsilon_0}$. so $V(r) = -\dfrac{q^2}{r}$

'spinless hydrogen' assumptions : – e^- and p^- – no spin

- $\underline{\text{Find}}$: · energy eigenvalues, eigenstates, eigenfunctions.

$\underline{\text{Strategy}}$:

(1) $H = H_p + H_e + H_{int}$

(2) Reduce to a center of mass problem → rel. motion of p^+ and e^-

(3) p^+ is much more massive than e^-

(4) $\mu = \dfrac{m_e m_p}{m_e + m_p} \simeq m_e$ to <0.1% difference

$\underline{\text{Hamiltonian in pos. rep.}}$

$$H_{\{R\}} = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{q^2}{r}$$

Find solutions

$\hat{H}\,|\psi_k\rangle = E_k\,|\psi_k\rangle$ or. $H_{\{\vec{R}\}}\,\psi_k(\vec{r}) = E_k\,\psi(\vec{r})$

k : set of quantum numbers

Solutions:

CSCO of $\{\hat{H}, \hat{L}^2, \hat{L}_z\}$

(1): $\hat{H}\,|\Psi_{n \ell m_\ell}\rangle = E_n\,|E_{n \ell m_\ell}\rangle$, $\hat{H}\,|n\ \ell\ m_\ell\rangle = E_n|n\,\ell\,m_\ell\rangle$

- n, ℓ, m_ℓ - integers

  n: 'principal' quantum #

  ℓ: e^- OAM magnitude

  m_ℓ: e^- OAM about the $\hat{z}$ axis ("magnetic" q. #)

$$E_n = -\frac{E_I}{n^2}, \qquad n \text{ integer } \geq 1$$

$$E_I = \frac{1}{2}\alpha^2 \mu c^2 \simeq 13{,}6\ \text{eV} \quad (1\ \text{eV} \simeq 1.6 \cdot 10^{-19}\ \text{J})$$

$\mu \approx m_e$

$$\alpha^2 = \frac{e^2}{4\pi \varepsilon_0 \hbar c}$$

(2) $\hat{L}^2\,|n \ell m_\ell\rangle = \hbar^2 \ell(\ell+1)\,|n \ell m_\ell\rangle$

$\hat{L}_z\,|n \ell m_\ell\rangle = \hbar m_\ell\,|n \ell m_\ell\rangle$

for any $\ell \in \{0, 1, 2, \ldots n-1\}$

$m_\ell \in \{\ell, \ell-1, \ell-2, \ldots -\ell\}$

Energy eigenfunctions

$$\Psi_{n \ell m_\ell}(r, \theta, \varphi) = \langle \vec{r}\,|\,n \ell m_\ell\rangle = R_{n\ell}(r)\ Y_\ell^{m_\ell}(\theta, \varphi)$$

"Radial Wavefunction"
Depend on $V(r)$
hydrogen: Ass. Laguerre polynomials

FG p 9.9.

spherical harmonics

$Y_\ell^{m_\ell} = e^{im\varphi} \cdot$ assoc. Legendre Polynomials

$$\int_{\text{all space}} dr\, d\theta\, d\varphi\; r^2 \sin\theta\, |\Psi_{n\ell m}(\vec{r})|^2 = 1$$

$$\int_0^\infty dr\; r^2 \,|R_{nl}(r)|^2 = 1 = \int_0^{2\pi} d\varphi \int_0^\pi d\theta \,\sin\theta \,|Y_l^{m_l}(\theta,\varphi)|^2$$

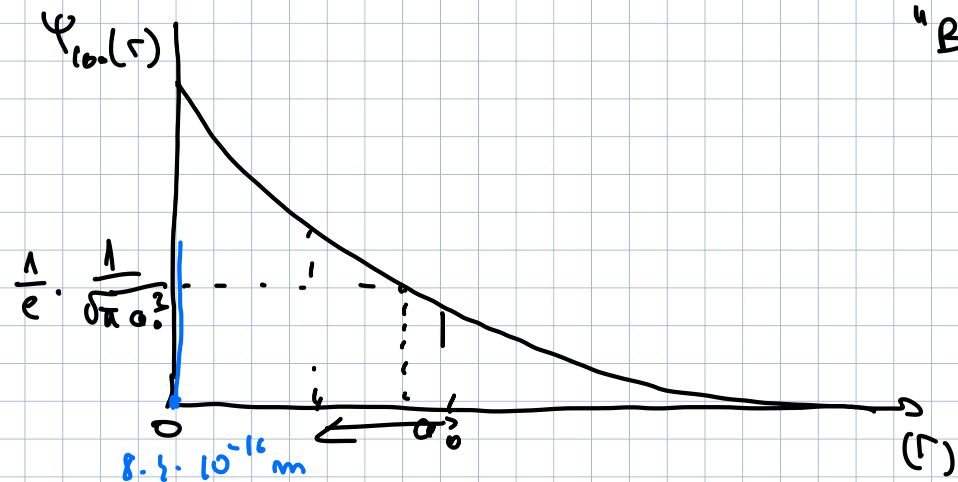
Ground state $\quad |n=1,\, l=0,\, m_l=0\rangle = |1,0,0\rangle$

$$\psi_{100}(\vec r) = R_{10}(r)\cdot Y_0^0(\theta,\varphi) =$$

$$= \left(2a^{-\frac{3}{2}} e^{-\frac{r}{a_0}}\right)\cdot \frac{1}{\sqrt{4\pi}} =$$

$$\psi_{100}(\vec r) = \frac{1}{\sqrt{\pi a_0^3}}\, e^{-\frac{r}{a_0}} \qquad \text{w}\;\; a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2} = 0.5\cdot 10^{-10}\,\text{m}$$

"Bohr Radius"



Q: $\qquad P_{\text{inside the } p^+} = \underline{10^{-14}}$

Energy level diagram