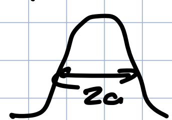
$\boxed{\text{OPTI 570 RECAP Th Sep 18}}$

• Ap 1: $[\hat{p},\hat{H}] = [\hat{p}, \hat{p}^2/2m] + \underline{[\hat{p}, V(\hat{x})]}$

$$\underline{[\hat{p}, V(\hat{x})]} = -i\hbar \frac{dV(\hat{x})}{d\hat{x}} = \underline{-[\hat{x},\hat{p}] \frac{dV(\hat{x})}{d\hat{x}}}$$

$$[\hat{p}, \hat{p}^2/2m] = 0$$

$|\tilde{\varphi}(p)|^2$ has 1/e half-width of a, $\underline{\langle\hat{p}\rangle = p_0}$, $\underline{\langle\hat{x}\rangle = x_0}$



$$\varphi_0(p) = \underline{c e^{-\frac{p^2}{2a^2}}} \quad |p-p_0| \qquad \underline{\langle\hat{p}\rangle_{\varphi_0} = 0} \qquad \langle\hat{x}\rangle = 0$$

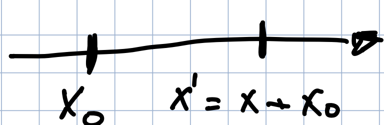
$\tilde{\varphi}(p)$ is even or odd about p_0 $\Rightarrow$ $\underline{\langle\hat{x}\rangle = 0}$, $\underline{\langle\hat{p}\rangle = p_0}$

$$\langle p| T(p_0)|\varphi\rangle = c e^{\underline{-\frac{(p-p_0)^2}{2a}}}$$

$$\langle p| S(x_0)T(p_0)|\varphi\rangle = c \cdot \underline{e^{-ix_0 p/\hbar}}\, e^{-\frac{(p-p_0)^2}{2a}}$$

$$T(p_0)\tilde{\varphi}(p) = \varphi(p-p_0) \qquad \underline{\hat{T}(p_0) = e^{ip_0\hat{x}/\hbar}}$$

$\hat{S}$



$\boxed{x' - x = x_0}$

$\underline{x' - x_0} = \underline{x}$

$$\hat{x}\hat{S}(x_0)|x\rangle = \hat{S}(x_0)(\hat{x} + x_0)|x\rangle =$$

$$= \hat{S}(x_0)(\underline{\hat{x}|x\rangle} + x_0|x\rangle) =$$

$$= \hat{S}(x_0)(\underline{x|x\rangle} + x_0|x\rangle) =$$

$$= \ldots$$

Problem IV

d. $[H, B] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} =$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \neq 0$$

$\Rightarrow \langle B \rangle$ • time - dependent