OPTI 570 RECAP 5

$[\hat{A}, \hat{B}] = 0 \Rightarrow$ at least one common set of eigenvectors for both $\hat{A}$ and $\hat{B}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ 1 eigenvalue, 3 eigenvectors

- CSCO

$$\cancel{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \text{ and } \boxed{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}}$$

commute

$$\Rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \underline{c_1 \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}}$$

$$M \cdot V_1 = E \cdot V_1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \underline{\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}} = \begin{pmatrix} 0 \\ 2/\sqrt{2} \\ 2/\sqrt{2} \end{pmatrix} = \underbrace{\underline{2}}_{\text{eigen value}} \cdot \underline{\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}}$$

$$M \underline{V_1} = \underline{e_1} \underline{V_1}$$

$$\langle \psi | \psi \rangle =$$

Problem 9 - $\boxed{b}$ $\hat{H} = \frac{1}{2m} \hat{p}^2 + V(\hat{x})$

$\boxed{\alpha}$ $[\hat{H}, \hat{P}] = \hat{H}\hat{P} - \hat{P}\hat{H} = \left[\frac{1}{2m}\hat{p}^2 + V(\hat{x})\right]\hat{p} - \hat{p}\left[\frac{1}{2m}\hat{p}^2 + V(\hat{x})\right] =$

$$= \cancel{\frac{1}{2m}\hat{p}^2\hat{p}} + V(\hat{x})\hat{p} - \cancel{\frac{1}{2m}\hat{p}\hat{p}^2} - \hat{p}V(\hat{x}) =$$

$$\boxed{[\hat{H},\hat{p}] = V(\hat{x})\hat{p} - \hat{p}V(\hat{x})}$$

$$[\hat{H},\hat{x}] = \left[\frac{1}{2m}\hat{p}^2 + V(\hat{x}),\hat{x}\right] =$$

$$= \left[\frac{1}{2m}\hat{p}^2,\hat{x}\right] + \underbrace{[V(\hat{x}),\hat{x}]}_{0} =$$

$$= \frac{1}{2m}\left(\hat{p}^2\hat{x} - \hat{x}\hat{p}^2\right) =$$ Taylor exp in powers of $\hat{x}$

$[\hat{x},\hat{p}] = i\hbar$

$\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$

$$= \frac{1}{2m}\left[\hat{p}(\hat{p}\hat{x}) - (\hat{x}\hat{p})\hat{p}\right] =$$

$$= \frac{1}{2m}\left[\hat{p}(\hat{x}\hat{p} - i\hbar) - (i\hbar + \hat{p}\hat{x})\hat{p}\right] =$$

$$= \frac{1}{2m}\left[\cancel{\hat{p}\hat{x}\hat{p}} - i\hbar\hat{p} - i\hbar\hat{p} - \cancel{\hat{p}\hat{x}\hat{p}}\right] =$$

$$= \frac{1}{2m}(-2i\hbar\hat{p}) =$$

$$\boxed{[\hat{H},\hat{x}] = -\frac{i\hbar}{m}\hat{p}}$$