OPTI 570 LECTURE Th Oct 9

ATOM TRAPPING

General: neutral atom in a position dependent EM field

Stark effect: shifts in energy levels of atom from electric fields

Zeeman effect: — 〃 — magnetic fields

potential wells for atoms → force that keeps atoms localized (trapped) around the minimum of well

EM-fiels: laser far-off-resonant from atomic transitions.

- oscillating electric field induces an oscillation dipole moment

$\bar{d}$ in the atom, delayed in phase w.r.t. the driving electric-field

- electric dipole interaction energy:

$$V = -\bar{d}(t) \cdot \underline{\bar{\mathcal{E}}(r,t)}$$ ← electric field

- neglect magnetic component
- for off-resonant laser ⇒ no absorption

$$V(\bar{r}) \propto \frac{I(\bar{r})}{\Delta}$$

Δ: laser detuning from atom resonance.

$\Delta = \omega_{laser} - \omega_{atom}$

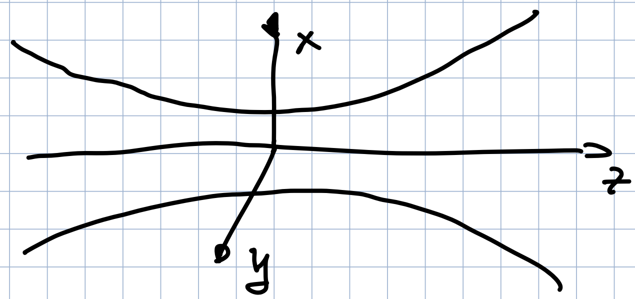
Δ < 0: "red detuning"

- atom will be atracted to high-intensity regions of potential.

Δ > 0: "blue detuning"

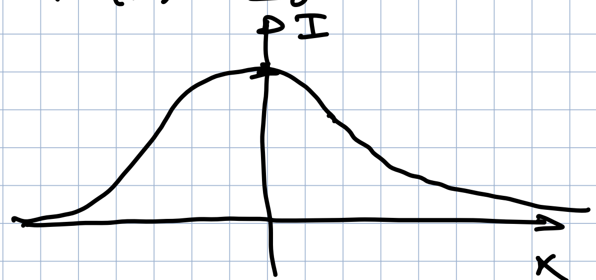
- — 〃 — repelled by — 〃 —

Dipole trap / optical tweezer : Gaussian laser beam travelling along z



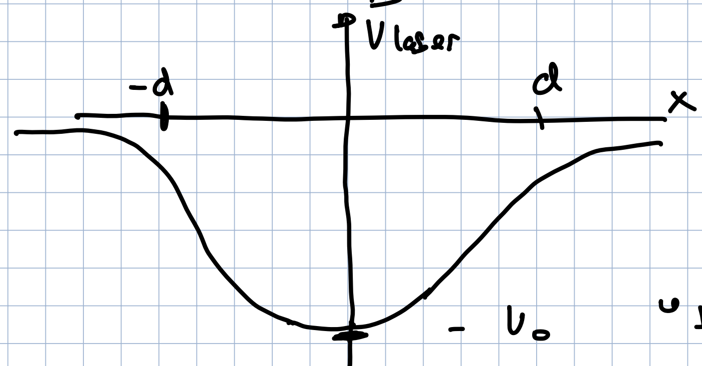
transverse direction: $I(\bar{r})$ vs x where $z = y = 0$

$$I(x) = I_0 \, e^{-\frac{2x^2}{d^2}}$$



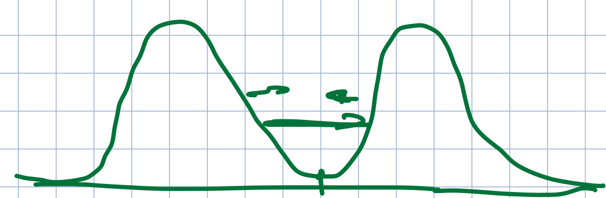
- $\Delta < 0$: laser will create a potential well

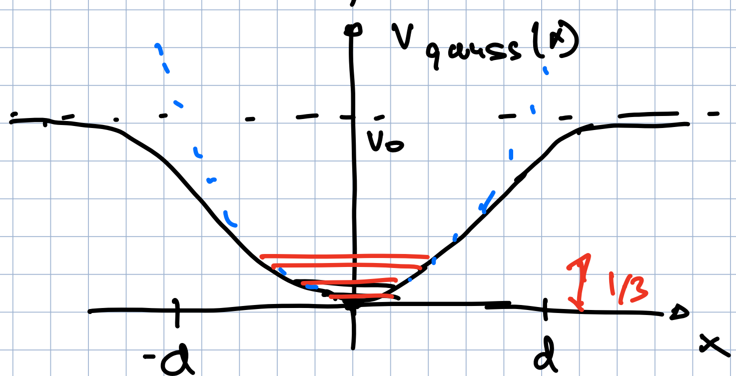
$$V_{laser} \propto \frac{I(x)}{\Delta} = -V_0 \, e^{-\frac{2x^2}{d^2}} \qquad (V_0 > 0)$$



"trap depth"

Blue detuned





$$V_{gauss}(x) = -V_0 \cdot e^{-\frac{2x^2}{d^2}} + V_0 = V_0 \cdot \left(1 - e^{-\frac{2x^2}{d^2}}\right)$$

"trap depth"

need $\hbar\omega \ll U_0$

What is ω?

Taylor expand the potential:

$$V_{gauss}(x) = V_0\left(1-e^{-\frac{2x^2}{d^2}}\right) =$$

$$= V_0\left(1-\left[1-\frac{2x^2}{d^2}+\frac{4x^4}{2d^4}+\dots\right]\right) =$$

$$= V_0\left(\underbrace{\frac{2x^2}{d^2}}_{\text{harmonic}} \underbrace{-\frac{2x^4}{d^4}+\dots}_{\text{anharmonic}}\right)$$

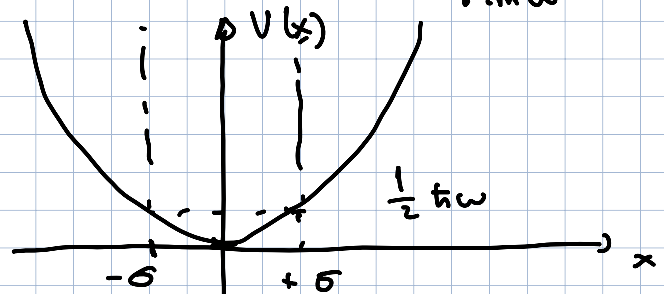
$$V_{gauss}(x) = V_{QHO}(x) + V_{anharmonic}(x)$$

$$V_{QHO}(x) = V_0\frac{2x^2}{d^2} = \frac{1}{2}m\omega^2x^2$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{4V_0}{md^2}}}$$ – for atoms that are low energy

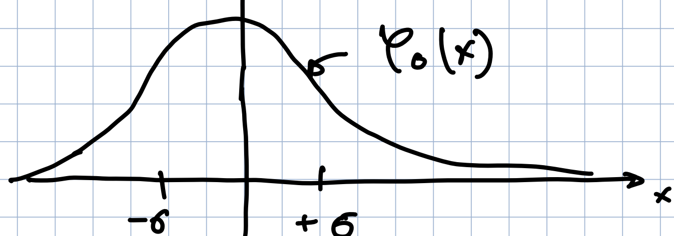
<u>Problem:</u>
- atom in ground state of the atom trap
- laser red-detuned – no absorption.
- QHO approx works well

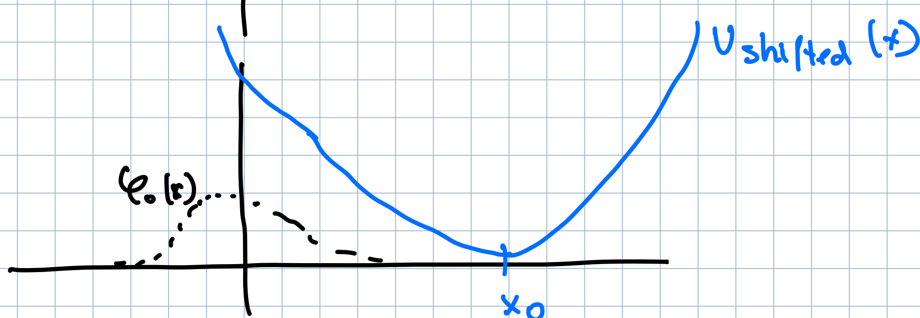
$$\sigma = \sqrt{\frac{\hbar}{m\omega}} \Rightarrow \sigma^2 = \frac{\hbar}{m}\sqrt{\frac{md^2}{4V_0}} = \frac{\hbar d}{2\sqrt{mV_0}}$$



$$\langle x\rangle = 0 \qquad \Delta x = \frac{\sigma}{\sqrt{2}}$$

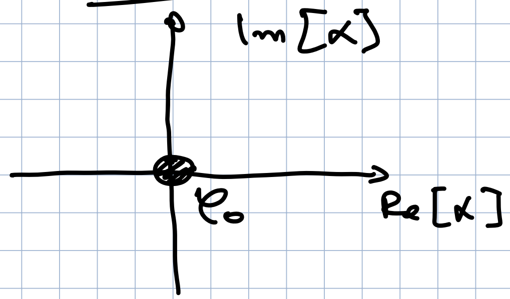
$$\langle p\rangle = 0 \qquad \Delta p = \frac{\hbar}{\sqrt{2}\,\sigma}$$



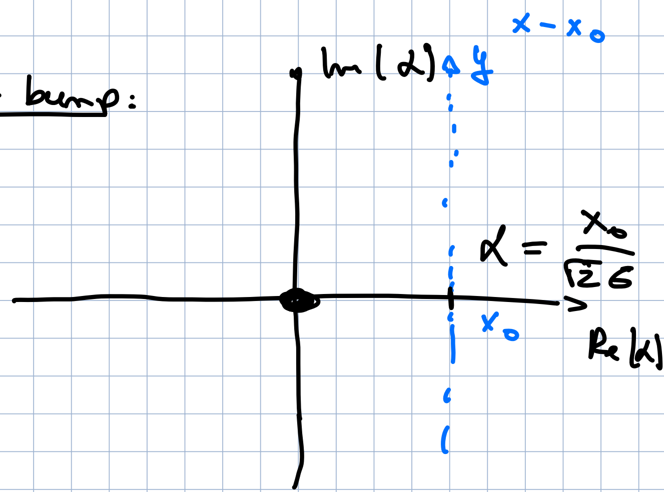


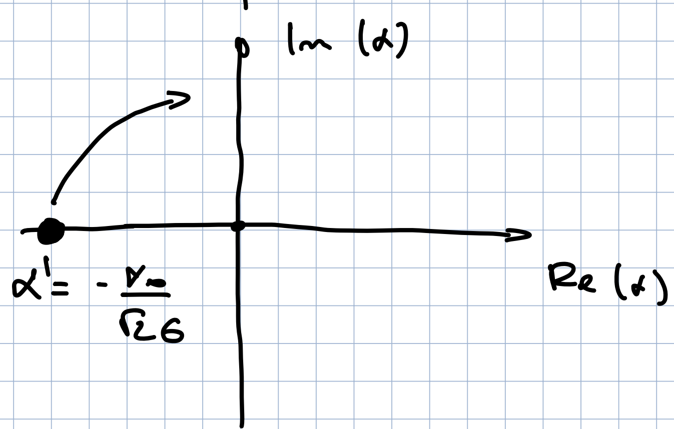
Phase space

initially:



After bump:





Formally: $|\psi(0)\rangle = D(\alpha')|\varphi_0\rangle$ w/ $\alpha' = -\frac{x_0}{\sqrt{2}\sigma}$

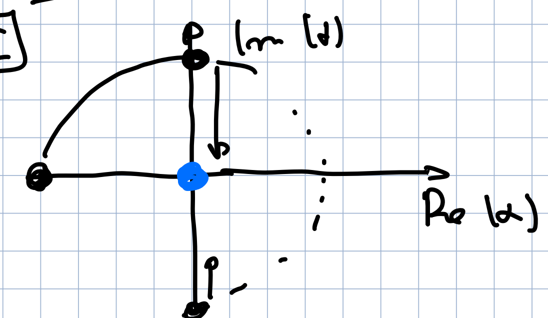
Time evolution: $\alpha'(t) = \alpha'_0 e^{-i\omega t} =$

$$= -\frac{x_0}{\sqrt{2}\sigma} e^{-i\omega t} =$$

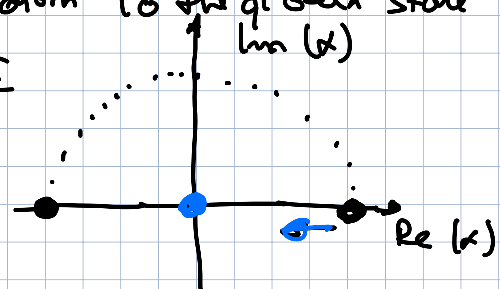
$$= -\frac{x_0}{\sqrt{2}\sigma}\cos(\omega t) + \frac{i x_0}{\sqrt{2}\sigma}\sin(\omega t)$$

Q: how do I return the atom to the ground state?

$\boxed{I}$

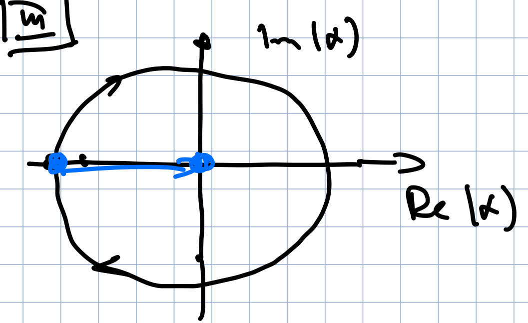


$\underline{II}$



$D(-i\alpha'_0)\cdot U\left(\frac{\pi}{2\omega}\right)D(\alpha'_0)|\varphi_0\rangle$ $\quad$ $D(\alpha'_0)U\left(\frac{\pi}{\omega}\right)D(\alpha'_0)|\varphi_0\rangle$

III



$D(-\alpha_0')\, U\!\left(\frac{2\pi}{\omega}\right) D(\alpha_0') |\varphi_0\rangle$

1 D Optical lattice

counter-propagating laser beams

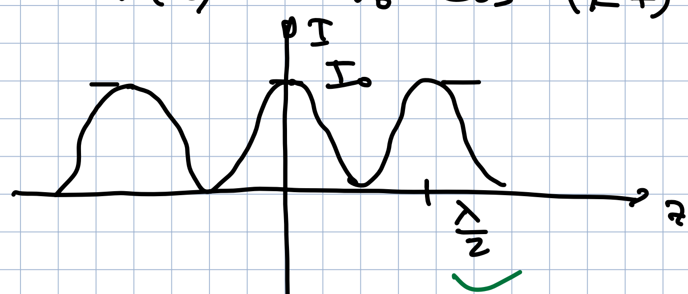
- perfect collimation:

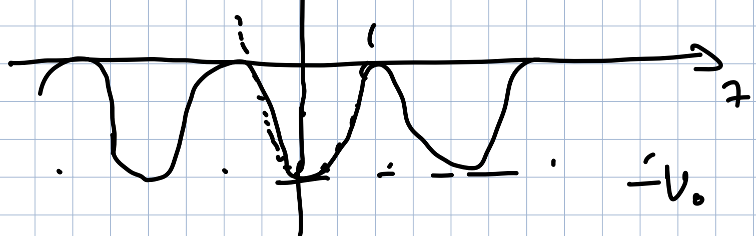
$$I = I_0 \, \underline{e^{-\frac{2x^2}{\sigma^2}}} \; \underline{e^{-\frac{2y^2}{\sigma^2}}} \; \underline{\cos^2(kz)} \qquad k = \frac{2\pi}{\lambda}$$

Along z axis: $\quad I(z) = I_0 \cos^2(kz)$

$$V(z) = \frac{I_0 \cos^2(kz)}{\Delta} \qquad \Delta < 0$$

$$V(z) = -V_0 \cos^2(kz)$$





"optical lattice"

$$V(z) = V_0 - V_0 \left(1 - \frac{1}{2}k^2 z^2 + \frac{1}{4!}k^4 z^4 + \dots\right)^2 =$$

$$= \underbrace{V_0 k^2 z^2}_{V_{\text{harmo}}} + \underline{\qquad\qquad}$$

$$V_0 k^2 z^2 = \frac{1}{2} m \omega_z^2 z^2 \Rightarrow \boxed{\omega_z = \frac{2\pi}{\lambda}\sqrt{\frac{2V_0}{m}}}$$

Example exp system

Cs atoms

- Cs resonance is at 894 nm
- trap light at 1064 nm $\Rightarrow$ red-detuned

$$\omega_{atom} = 2\pi \frac{c}{894\,nm} = 2\pi \cdot \underbrace{(3.35 \cdot 10^{14}\,Hz)}_{335\,THz}$$

$$\omega_{laser} = 2\pi\,(2.82 \cdot 10^{14}\,Hz)$$

$$\Delta = \omega_{laser} - \omega_{atom} = -2\pi\,(\underbrace{5.36 \cdot 10^{13}}_{54\,THz}\,Hz)$$

$P = 10\,W$ laser focused to $d = 20\,\mu m$

linearly polarized, counter-propagating beams

$$I_0 = 4 \cdot I_{laser} = 4 \cdot \frac{2P}{\pi d^2} = 6.37 \cdot 10^{10}\,\frac{W}{m^2}$$

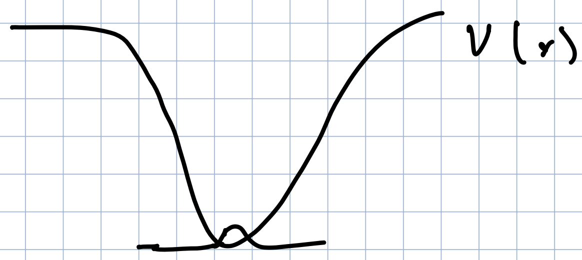
$$V_0 = \underbrace{\left(\frac{\hbar \Gamma^2}{8 I_{sat}}\right)}\frac{I_0}{|\Delta|} = 8.2 \cdot 10^{-26}\,J$$

$$\omega_z = \frac{2\pi}{\lambda}\sqrt{\frac{2V_0}{m}} = \underline{2\pi \cdot 810\,kHz}$$

$$\frac{V_0}{\hbar \omega_z} = 150 \Rightarrow$$ QHO is good approx for gaussian trap as long as $N \ll 50$

$$\sigma_z = \sqrt{\frac{\hbar}{m\omega_z}} = 9.7\,nm \qquad \lambda = 1064\,nm$$

$$\Rightarrow \sigma_z \sim \frac{1}{100}\lambda$$



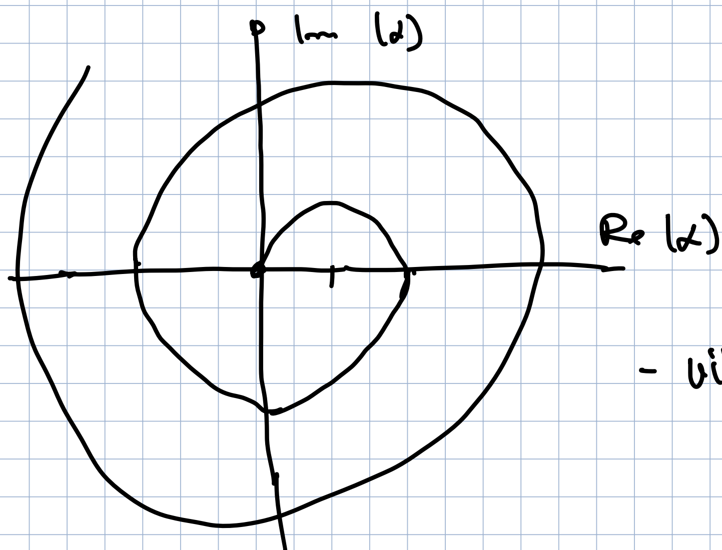
Temp of trapped atom?

$V_0 = k_B \cdot T \Rightarrow \underline{T = 5.9\,mK}$

laser cooling Cs $\Rightarrow$ $\underline{2\mu K}$

$T_{\text{ground state}}: \quad \frac{\hbar \omega_z}{k_B} \sim \underline{40\mu K}$

Effect of trap motion on the atomic state



- vibrations that drive atoms at 2× trap frequency.