Assignment 6

OPTI 570 Quantum Mechanics

University of Arizona

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October 9, 2025

Total time: 5 hours

Problem I

a) On the one hand, the action of $\tilde{a}(t)$ is

$$\begin{aligned}\tilde{a}(t)|\varphi_n\rangle &= U^\dagger(t,0)aU(t,0)|\varphi_n\rangle = U^\dagger(t,0)ae^{-i\left(N+\frac{1}{2}\right)\omega t}|\varphi_n\rangle = e^{-i\left(n+\frac{1}{2}\right)\omega t}U^\dagger(t,0)a|\varphi_n\rangle \\ &= \sqrt{n}e^{-i\left(n+\frac{1}{2}\right)\omega t}U^\dagger(t,0)|\varphi_{n-1}\rangle = \sqrt{n}e^{-i\left(n+\frac{1}{2}\right)\omega t}e^{i\left(n-\frac{1}{2}\right)\omega t}|\varphi_{n-1}\rangle = \sqrt{n}e^{-i\omega t}|\varphi_{n-1}\rangle.\end{aligned}$$

Therefore,

$$\tilde{a}(t)|\varphi_n\rangle = \sqrt{n}e^{-i\omega t}|\varphi_{n-1}\rangle = e^{-i\omega t}a|\varphi_n\rangle.$$

On the other hand, the action of $\tilde{a}^\dagger(t)$ is

$$\begin{aligned}\tilde{a}^\dagger(t)|\varphi_n\rangle &= U^\dagger(t,0)a^\dagger U(t,0)|\varphi_n\rangle = e^{-i\left(n+\frac{1}{2}\right)\omega t}U^\dagger(t,0)a^\dagger|\varphi_n\rangle = \sqrt{n+1}e^{-i\left(n+\frac{1}{2}\right)\omega t}U^\dagger(t,0)|\varphi_{n+1}\rangle \\ &= \sqrt{n+1}e^{-i\left(n+\frac{1}{2}\right)\omega t}e^{i\left(n+\frac{3}{2}\right)\omega t}|\varphi_{n+1}\rangle.\end{aligned}$$

Consequently,

$$\tilde{a}^\dagger(t)|\varphi_n\rangle = \sqrt{n+1}e^{i\omega t}|\varphi_{n+1}\rangle = e^{i\omega t}a^\dagger|\varphi_n\rangle.$$

b) We can compute the operators if we stimulate them with a ket $|\varphi_n\rangle$. We make use of the $a, a^\dagger$ expression for $\tilde{X}$ and $\tilde{P}$.

$$\tilde{X}(t)|\varphi_n\rangle = \frac{\sigma}{\sqrt{2}}\left[U^\dagger a^\dagger U + U^\dagger a U\right]|\varphi_n\rangle.$$

But, we have already computed these operations in the previous part, so we will use it here:

$$\tilde{X}(t)|\varphi_n\rangle = \frac{\sigma}{\sqrt{2}}\left[e^{i\omega t}a^\dagger + e^{-i\omega t}a\right]|\varphi_n\rangle \Longrightarrow \tilde{X}(t) = \frac{\sigma}{\sqrt{2}}\left[e^{i\omega t}a^\dagger + e^{-i\omega t}a\right].$$

In the same manner, we have for $\tilde{P}$:

$$\tilde{P}(t)|\varphi_n\rangle = \frac{1}{\sqrt{2}}\left[U^\dagger a^\dagger U - U^\dagger a U\right]|\varphi_n\rangle = \frac{1}{\sqrt{2}}\left[e^{i\omega t}a^\dagger - e^{-i\omega t}a\right]|\varphi_n\rangle$$

Therefore,

$$\tilde{P}(t) = \frac{1}{\sqrt{2}} \left[e^{i\omega t} a^\dagger - e^{-i\omega t} a \right].$$

They are like trigonometric functions cosine and sine, with the different that the operator $a, a^\dagger$ is in the middle.

c) To show that a ket is an eigenvector of an operator, it will have to satisfy its eigenequation with a constant representing the eigenvalue: $P|\psi\rangle = \lambda|\psi\rangle$.

d) asfa

e) asfafasf

f) asfsa

Problem II

a) asfaf

b) asfasf

c) asfafasf

d) asfa

e) sgag

f) asgag

g) asgagasg

h) asg

Problem III

a) asfaf

b) asfasf

c) asfafasf

d) asfa

Problem IV

Part 1.

a)

Part 2.

b) asfaf

c) asfasf

d) asfafasf

e) asfa