This Problem Set does not need to be turned in, but it does need to be completed for Final Exam practice.

Problem I.
    CT Chapter XIII, Complement E (F in newer textbook editions), Exercise 1, **parts (a) and (b) only.** Part (c) is entirely optional, you may choose to work it for extra practice.

Problem II.
    A particle of mass m is in the ground state of a 1D harmonic oscillator of frequency ω. A second untrapped particle moves along the x axis at a constant velocity v from time $t = -\infty$ to $t = \infty$. The interaction between the two particles leads to a perturbation $W(x,t)$ for the trapped particle that is given (in the position representation) by

$$W(x,t) = \lambda\hbar\omega \exp\left\{-\frac{(x-vt)^2}{2a^2}\right\},$$

 where λ is real and $|\lambda| << 1$, and a is a real scalar with units of length.

(a) What is the approximate probability P_1 of finding the particle in the first excited state once the perturbation has completely passed through the well (i.e., at $t = \infty$)? Calculate to lowest non-zero order in λ, and simplify your answer as much as possible. Your answer should be a function of v and a. HINT: there are two integrals that you need to perform, one over time, and one over position. **It is *much* easier to do the time integral first!** You may use the following integral: For a real, $a \geq 0$,

$$\int_{-\infty}^{\infty} \exp\left\{-\frac{u^2}{2a^2} + bu\right\}du = \sqrt{2\pi} \cdot a \cdot \exp\left\{b^2a^2/2\right\}$$

After doing the time integral, you can even skip the actual integration over position by writing the matrix element needed in a clever way (look at the form of the operator involved).

(b) What is the value of v that maximizes P_1 for arbitrary a, and what is this maximum probability?

(c) For the maximized probability found in (b), what is the value of a that maximizes the transition probability, and what is this maximum probability?

Problem III.
    A particle of mass m is in a 1D quantum harmonic oscillator potential of frequency ω_0 with Hamiltonian H_0. The particle is initially in the ground state of the oscillator, at time $t = -\infty$. A time-dependent perturbation $W(t)$ is later applied that first raises then lowers a Gaussian-shaped bump in the middle of the potential well

$$W(t) = \lambda\,\hbar\omega_0\, \exp\left(-\frac{X^2}{d^2}\right) \exp\left(-\frac{t^2}{2\tau^2}\right)$$

where X is the position operator, d characterizes the size of the bump, τ characterizes the time scale of the perturbation being turned on and off, and $\lambda \ll 1$ is a positive real scalar. The total

Hamiltonian is therefore $H(t) = H_0 + W(t)$, As can be seen, the matrix elements of $W(t)$ reach their maximum values at $t = 0$, and then for times $t \gg \tau$ the perturbation has essentially been removed.

(a) Use the methods of first-order time-dependent perturbation theory to determine the approximate value for the probability that at $t = \infty$ the particle will be found in the **first excited state** of the oscillator. Will this estimated probability change if the perturbation expansion is taken to higher orders, and if so, how? Justify your answer.

(b) Use the methods of first-order time-dependent perturbation theory to determine the approximate value for the probability that at $t = \infty$ the particle will be found in the **second excited state** of the oscillator. Simplify your answer as much as possible.

(c) Find the value of d that maximizes the probability of a transition from the ground state to the second excited state, according to your answer to part (b). Express your answer in terms of $\sigma \equiv \sqrt{\frac{\hbar}{m\omega_0}}$.

(d) Find the value of τ that maximizes the probability of a transition from the ground state to the second excited state, according to your answer to part (b). Express your answer in terms of ω_0.

(e) Using the values of d and τ found above, what is the maximum transition probability to the second excited state?