# OPTI 570 RECAP Th Sep 16

- $\hat{x}$ is Hermitian

$\hat{H}$ is time-independent

$$\hat{U}(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar}$$

$\hat{H}$ is time-dependent, but $[H(t), H(t'))] = 0$

$$\hat{U}(t,t_0) = e^{-i\int_{t_0}^{t} dt'\, \hat{H}(t')/\hbar}$$

$$\hat{p} = -i\hbar \frac{d}{dx}$$

ex: Show $\hat{x}$ is Hermitian $\Leftrightarrow$ for any two $|\varphi\rangle, |\psi\rangle \in \mathcal{E}$

$$\langle\varphi|\hat{x}|\psi\rangle = \langle\varphi|\hat{x}^{+}|\psi\rangle \qquad \hat{x}|\varphi\rangle = x|\varphi\rangle$$

$$\langle\varphi|\hat{x}|\psi\rangle = \int_{-\infty}^{\infty} \varphi^{*}(x)\, x\, \psi(x)\, dx =$$

$$= \underbrace{\int_{-\infty}^{\infty} [x\,\varphi^{*}(x)]}\, \psi(x)\, dx =$$

$$= (\hat{x}|\varphi\rangle)^{+}|\psi\rangle =$$

$$= \langle\varphi|\hat{x}|\psi\rangle$$