OPTI 570 LECTURE Th DEC 4th

Last time

TDPT

Given: $\hat{H}(t) = \hat{H}_0 + \lambda \hat{W}(t)$

- $\hat{H}_0 |\varphi_n\rangle = E_n |\varphi_n\rangle$ – energies and eigenstates known
- $|\lambda| \ll 1$ so that $\lambda \hat{W} \ll H_0$ – weak – pert.
- $|\psi(0)\rangle$

Ideally want: $|\psi(t)\rangle$, but... no exact analytical solution

Instead, we ask: what $P_f(t)$, f is some final state?

Solution: $P_f(t) = |\langle \psi_f(t) | \psi(t)\rangle|^2$ where

$$|\psi(t)\rangle = \sum_n b_n(t) e^{-iE_n(t-t_0)/\hbar} |\varphi_n\rangle$$

$$b_n(t) = \underset{b_n(t_0)}{\underset{\shortparallel}{b_n^{(0)}(t)}} + \lambda b_n^{(1)}(t) + \lambda^2 b_n^{(2)}(t) + \dots$$

$$\lambda^r b_n^{(r)}(t) = \frac{1}{i\hbar} \sum_k \int_0^t dt' e^{i\omega_{nk}t'} [\lambda \hat{W}_{nk}(t')][\lambda^{r-1} b_k^{(r-1)}(t')]$$

Harmonic Perturbations

$$H(t) = H_0 + \lambda \hat{W} \sin(\omega t)$$

$\hat{W}$ ← time-independent

- $\omega > 0$, $t_0 = 0$

Given: $|\psi(0)\rangle = |\varphi_i\rangle$

$|\varphi_f\rangle \neq |\varphi_i\rangle$

$$P^{(1)}_{i\to f}(t) = |\lambda b_f^{(1)}(t)|^2$$

$$\lambda b_f^{(1)}(t) = \frac{\lambda}{i\hbar} \hat{W}_{fi} \int_0^t e^{i\omega_{fi}t'} \sin(\omega t')\, dt' =$$

$$= \frac{\lambda \hat{W}_{fi}}{2i\hbar} \left[A_+(t) - A_-(t) \right]$$

$$A_\pm(t) = -it\, e^{-i(\omega_{fi} \pm \omega)t/2}\, \mathrm{Sinc}\left(\frac{(\omega_{fi} \pm \omega)t}{2} \right) \qquad \mathrm{Sinc}\, x = \frac{\sin x}{x}$$

"Rotating wave" approximation – RWA – 'Resonant' approx

take : $\omega > 0$, $\omega_f > \omega_i$ $(\omega_{fi} > 0)$

$$\omega_{fi} = \omega_f - \omega_i$$

$$|\omega - \omega_{fi}| \ll |\omega_{fi}|, |\omega|$$

$$\frac{\sin\left(\frac{(\omega_{fi} + \omega)t}{2}\right)}{\frac{(\omega_{fi} + \omega)t}{2}} \gg \frac{\sin\left(\frac{(\omega_{fi} - \omega)t}{2}\right)}{\frac{(\omega_{fi} - \omega)t}{2}}$$

$$\underline{A^+} \ll A^-$$

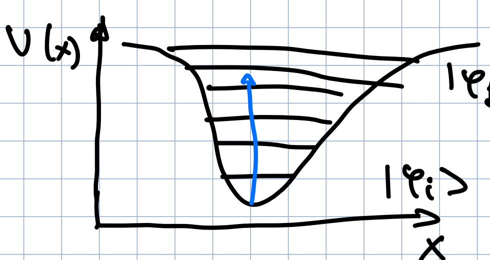
neglect

$$P^{(1)}_{i \to f}(t) = \frac{|\Omega_{fi}|^2}{\Delta_{fi}^2} \sin^2\left(\frac{\Delta_{fi} t}{2}\right)$$

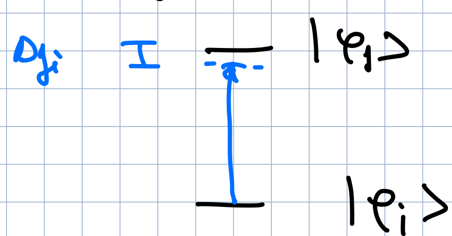
w) $\Delta_{fi} = \omega - \omega_{fi}$

$$\Omega_0 = \frac{\lambda \hat{W}_{fi}}{\hbar}$$

correct when $\Delta \ll \omega_{fi}, \omega$ – RWA

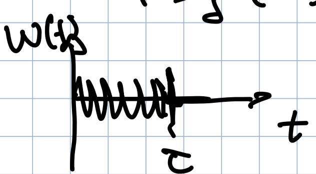


$\rightarrow$



$\underline{\text{RWA}}$: $\Delta_{fi} \ll \omega, \omega_{fi}$

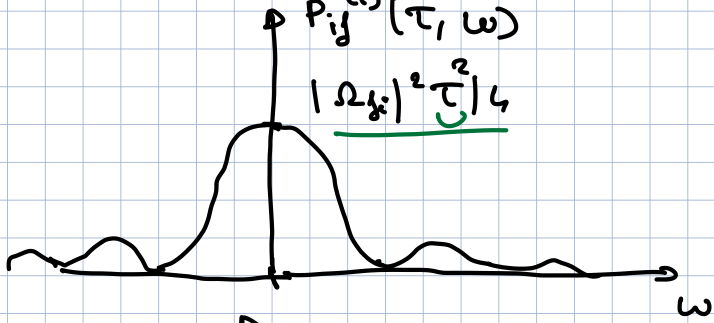
$$P^{(1)}_{i \to f}(t) = \frac{|\Omega_{fi}|^2}{\Delta_{fi}^2} \sin^2\left(\frac{\Delta_{fi} t}{2}\right)$$

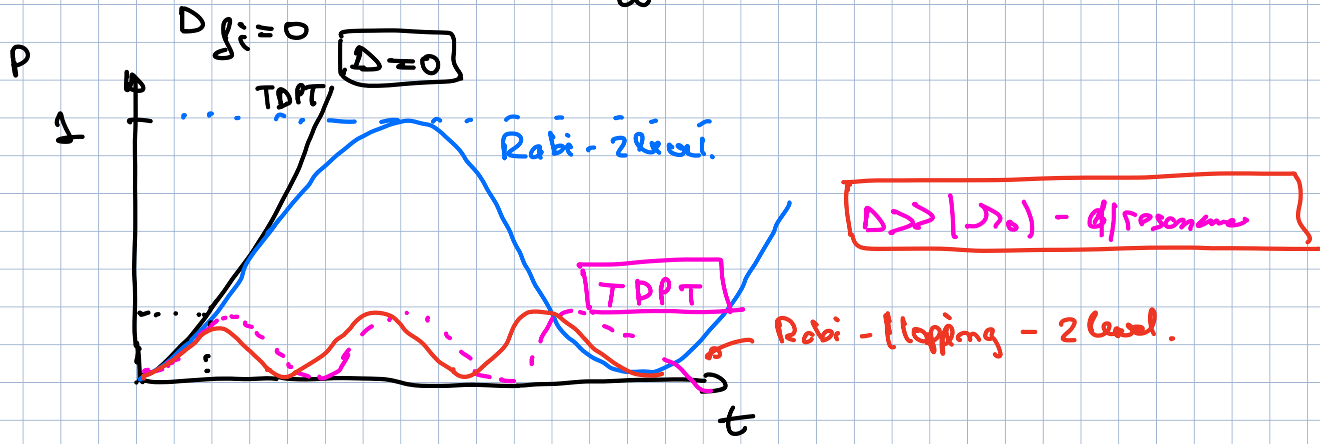


$$\rightarrow P^{(1)}_{i \to f}(\tau, \omega) = \frac{|\Omega_{fi}|^2}{\Delta_{fi}^2} \sin^2\left(\frac{\Delta_{fi} \tau}{2}\right) =$$

$$= \frac{|\Omega_{fi}|^2 \tau^2}{4} \cdot \left(\frac{\sin(\Delta_{fi} \tau / 2)}{\Delta_{fi} \tau / 2} \right)^2$$

$$= \cdots\cdots \cdot \operatorname{sinc}^2\left(\frac{\Delta_{fi}\tau}{2}\right)$$





TDPT:

- near $\Delta \simeq 0$: correct for short times: $\frac{1}{\omega} \ll \tau \ll \frac{\hbar}{|\lambda \tilde{W}_{fi}|}$
  to 1st order..

- $P_{fi} \ll 1$

System: ^{87}Rb, H_0 is full Hyperfine Hamiltonian and no external fields

$$H_0|\varphi_n\rangle = E_n|\varphi_n\rangle$$

n ← all q. # ass. with state

ex: $|\varphi_1\rangle = |5\,^2S_{1/2}, F=2, m_F=2\rangle$

known: $|\psi(0)\rangle = |\varphi_i\rangle$

Shine laser light
- classical EM field
- monochromatic
- polarized
- spatially uniform: $\lambda \gg$ size of atom

Goal: calculate $P_{i\to f}$

Process: $W(t) = \lambda \hat{W}(t) = -\vec{d}\cdot\vec{\mathcal{E}}(t)$

matrix elements: $W_{fi}(t) = -\mathcal{E}_0 \cos(\omega t)\left(-e\,\hat{\epsilon}\,\underbrace{\langle \varphi_f|\vec{R}|\varphi_i\rangle}_{\equiv \rho_{fi}}\right)$

polarization

$= 0$ if $\Delta m \neq 0, \pm 1$

$\Delta l \neq \pm 1$

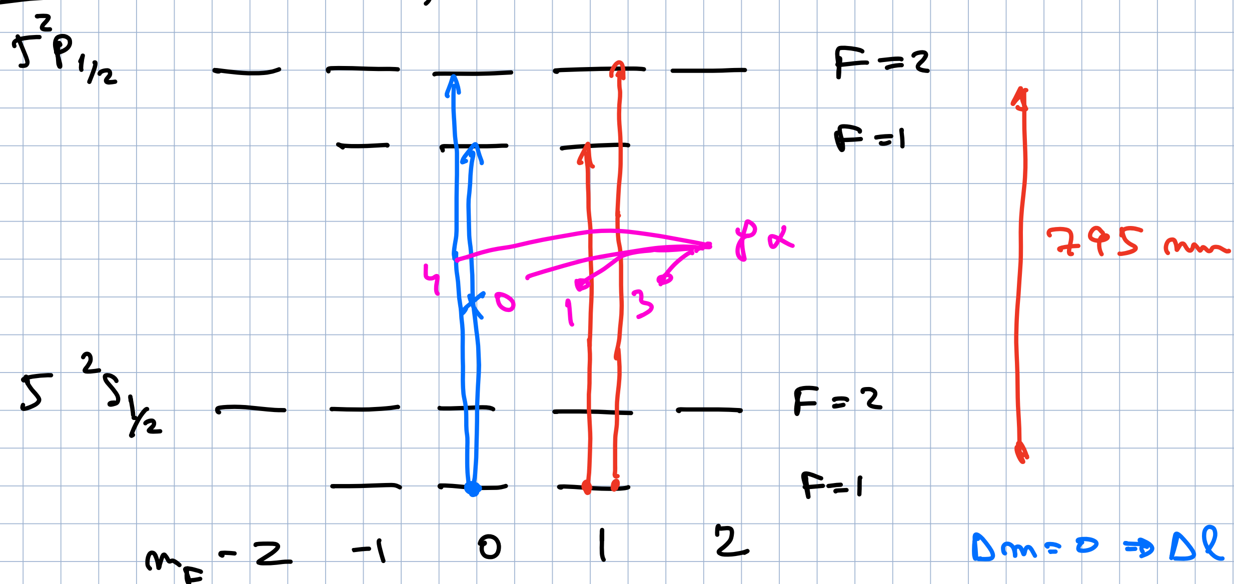
selection rules PS#8

$$\Omega_{fi} = \frac{\rho_{fi}\,\mathcal{E}_0}{\hbar}$$

$$W_{fi}(t) = -\frac{\hbar\,\Omega_{fi}}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

ex1: Rb atoms, $|\varphi_i\rangle = |5\,^2S_{\frac{1}{2}}, F=1, m_F\rangle$

laser: $\lambda = 795$ nm, $\hat{\epsilon} = \hat{z}$



$\Delta m = 0 \Rightarrow \Delta l \neq 0$

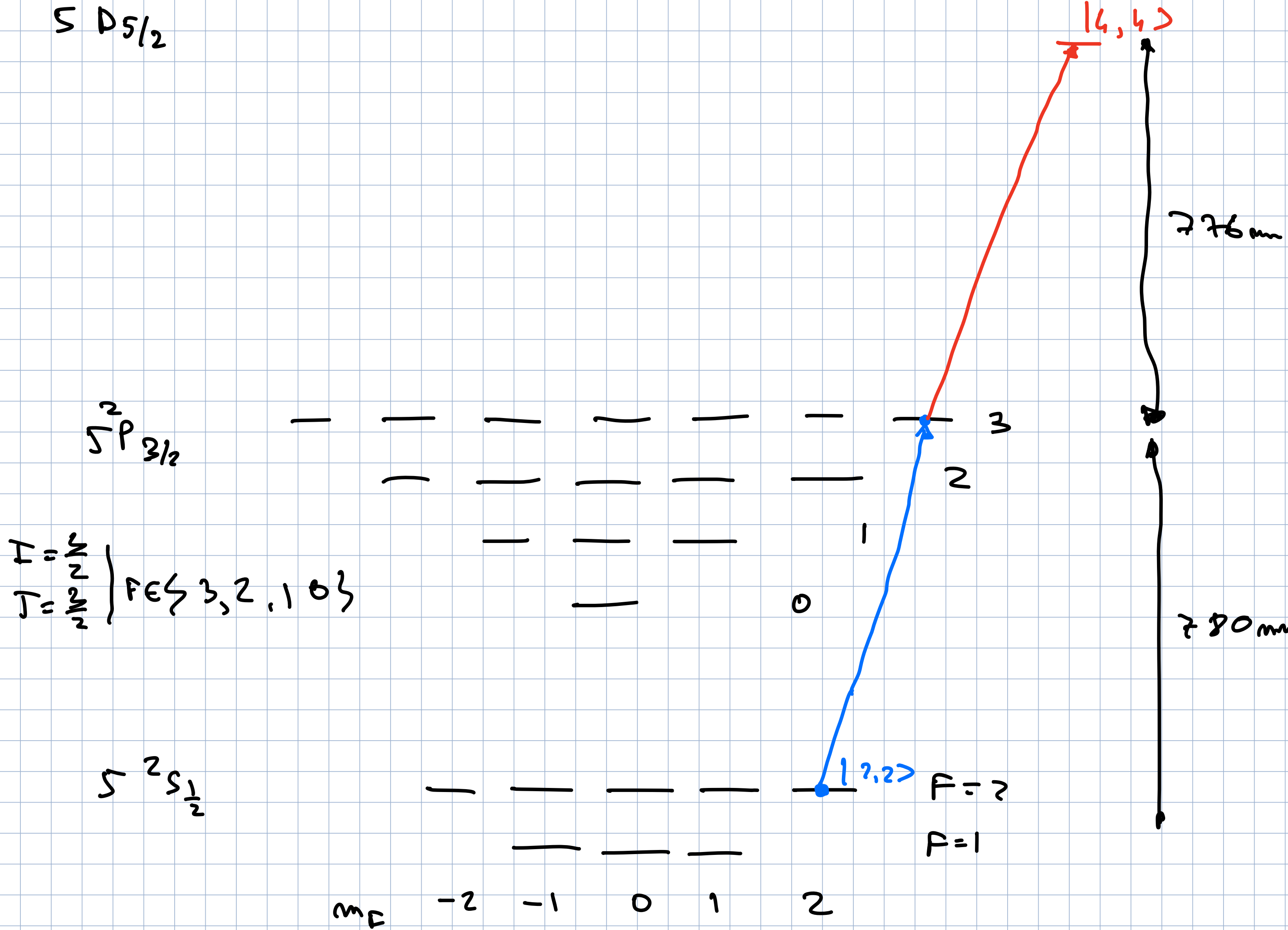
- $$P^{(1)}_{i\to f} = \frac{|\Omega_{fi}|^2}{\Delta_{fi}^2}\sin^2\left(\frac{\Delta_{fi}\,t}{2}\right)$$

ex: $|\psi(0)\rangle = |5\,^2S_{\frac{1}{2}}, F=2, m_F=2\rangle$

$\hat{\epsilon} = \sigma^+$

$m_{F'} = m_F + 1$

$\lambda \simeq 778$ nm



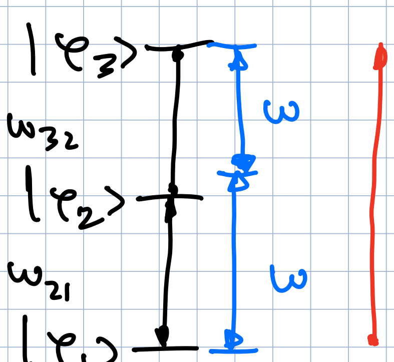
<u>recap</u>: 3-level problem

$$|\varphi_1\rangle = |5^2S_{1/2}, F=2, m_F=2\rangle$$

$$|\varphi_2\rangle = |5^2P_{3/2}, F=3, m_F=3\rangle$$

$$|\varphi_3\rangle = |5^2D_{5/2}, F=4, m_F=4\rangle$$

$$|\psi(0)\rangle = |\varphi_1\rangle$$



$$\omega_{31} = \omega_{32} + \omega_{12} \simeq 2\omega$$

$$\omega \neq \omega_{21} \neq \omega_{32}$$

$$\underline{W_{21}}(t) = -\frac{\hbar\Omega_{21}}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$\underline{W_{32}}(t) = -\frac{\hbar\Omega_{32}}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

use $\{|\varphi_1\rangle, |\varphi_2\rangle, |\varphi_3\rangle\}$ representation

$$H_0 \to \hbar\begin{pmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_3 \end{pmatrix}$$

$$W(t) \to -\frac{\hbar}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)\begin{pmatrix} 0 & \underline{\Omega_{21}^*} & 0 \\ \underline{\Omega_{21}} & 0 & \underline{\Omega_{32}^*} \\ 0 & \underline{\Omega_{32}} & 0 \end{pmatrix}$$

<u>Process</u>: [1] find 0th order coeff.

$$b_1^{(0)}(t) = 1$$

$$b_2^{(0)}(t) = 0$$

$$b_3^{(0)}(t) = 0$$

[2] 1st order

$$\lambda b_1^{(1)} = 0$$

$$\lambda b_3^{(1)} = 0$$

$$\lambda b_2^{(1)} = \frac{1}{i\hbar}\int_0^t e^{i\omega_{21}t'} W_{21}(t')\,dt'$$

$$= \frac{i\Omega_{21}}{2}\int_0^t \left[e^{i(\omega_{21}+\omega)t'} + e^{i(\omega_{21}-\omega)t'}\right] dt'$$

<u>use RWA?</u> ✓

$\omega \sim 2.423 \cdot 10^{15}\ \frac{rad}{s}$

$\omega_{21} \sim 2.417 \cdot 10^{15}\ \frac{rad}{s}$

$\Delta_{21} = \omega - \omega_{21} \simeq 5 \cdot 10^{12}\ \frac{rad}{s}$

$\Delta_{21} \ll \omega, \omega_{21}$

w/ RWA:

$$\boxed{\lambda\, b_2^{(1)}(t) = \frac{\Omega_{21}}{2(\omega_{21}-\omega)} \cdot \left(e^{-i(\omega_{21}-\omega)t} - 1\right)}$$

$$\boxed{P^{(1)}_{1\to 2} = \left|\frac{\Omega_{21}}{\Delta_{21}}\right|^2 \sin^2\left(\frac{\Delta_{21} t}{2}\right)}$$

$$\lambda^2\, b_3^{(2)}(t) = \frac{1}{i\hbar}\int_0^t dt'\, e^{i\omega_{32}t'}\, W_{32}(t')\left[\lambda\, b_2^{(1)}(t')\right]$$

$$\lambda^2\, b_3^{(2)}(t) = \frac{i\,\Omega_{21}\Omega_{32}}{4(\omega_{21}-\omega)}\int_0^t dt'\Big[\underbrace{e^{i(\omega_{32}+\omega_{21})t'} + e^{i(\omega_{32}+\omega)t'} + e^{i(\omega_{21}+\omega_{32}-2\omega)t'}}_{\text{RWA - neglect}} - \underbrace{e^{i(\omega_{31}-\omega)t'}}_{\text{keep}}$$

$$\lambda^2\, b_3^{(2)}(t) \simeq -\frac{i\,\Omega_{21}\Omega_{32}}{4\,\Delta_{21}}\int_0^t dt'\, e^{-i\delta t'}$$

$\delta \equiv 2\omega - \omega_{31}$

"two-photon"
detuning

$$\lambda^2\, b^{(2)}(t) = -\frac{i\,\Omega_{21}\Omega_{33}}{2\,\Delta_{21}\,\delta}\, e^{-i\delta t/2} \sin\left(\frac{\delta t}{2}\right)$$

$$\boxed{P^{(2)}(t) = \left|\frac{\Omega_{21}\Omega_{32}}{4\,\Delta_{21}}\right|^2 t^2\, \mathrm{sinc}^2\left(\frac{\delta t}{2}\right)}$$