OPTI 570 LECTURE Th NOV 13

Determining CG coefficient

1. Identify the two individual AM magnitude Q#s: j_1, j_2

   ex: 2p H w/ e^- spin

   $l = 1, s = \frac{1}{2}$

2. Identify the two bases

   TP basis: $\{ | l=1 \; s=\frac{1}{2} \; m_l \; m_s \rangle \}$

   $m_l \in \{-1, 0, 1\}$, $m_s \in \{-\frac{1}{2}, \frac{1}{2}\}$ $\Rightarrow$ 6 elements

   TAM basis: $\{ | j \; m_j \rangle \}$

   $$\left.\begin{aligned} \max(j) &= l+s = \tfrac{3}{2} \\ \min(j) &= |l-s| = \tfrac{1}{2} \end{aligned}\right\} \quad j \in \{\tfrac{1}{2}, \tfrac{3}{2}\}$$

   $j = \frac{3}{2}$ $\quad m_j \in \{\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\}$ $\Rightarrow$ 6 elements.

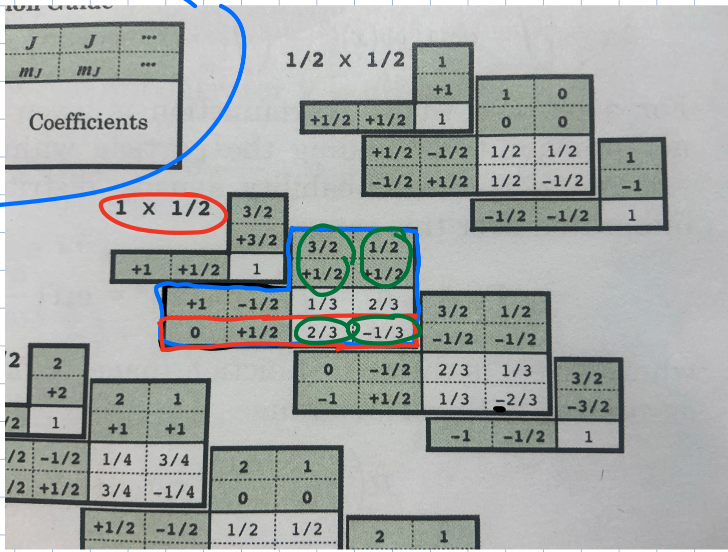
   $j = \frac{1}{2}$ $\quad m_j \in \{\frac{1}{2}, -\frac{1}{2}\}$

3. Identify the TP or TAM state to express in other basis

   ex: $| l=1, s=\frac{1}{2}, m_l=0, m_s=\frac{1}{2} \rangle$ - TP basis $\Rightarrow$ TAM basis

4. Identify non-zero coefficients:

$$|1, \tfrac{1}{2}, \underbrace{0}_{m_l}, \underbrace{\tfrac{1}{2}}_{m_s}\rangle = 0 \, |\tfrac{3}{2}, \underbrace{\tfrac{3}{2}}_{m_j}\rangle + \underline{\quad} \, |\tfrac{3}{2}, \tfrac{1}{2}\rangle + 0 \, |\tfrac{3}{2}, -\tfrac{1}{2}\rangle + 0 \, |\tfrac{3}{2}, -\tfrac{3}{2}\rangle + \underline{\quad} \, |\tfrac{1}{2}, \tfrac{1}{2}\rangle + 0 \, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$



FG p 118-120

$$|1, \tfrac{1}{2}, 0, \tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\, |J = \tfrac{3}{2}, m_j = \tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}\, |J = \tfrac{1}{2}, m_j = \tfrac{1}{2}\rangle$$

· Check: normalization ?

Example: hydrogen in 4d state

$\Rightarrow$ $n = 4$, $l = 2$, $s = \frac{1}{2}$

$2 \times \frac{1}{2}$

TP basis $|l = 2, s = \frac{1}{2}, m_l, m_s\rangle$ or $|2, \frac{1}{2}, m_l, m_s\rangle$

order ?

$m_l + m_s$

$$
\begin{array}{ll}
\frac{5}{2} & \{ |2\ \tfrac{1}{2}\ 2\ \tfrac{1}{2}\rangle \\
\frac{3}{2} & |2\ \tfrac{1}{2}\ 2\ -\tfrac{1}{2}\rangle,\ |2\ \tfrac{1}{2}\ 1\ \tfrac{1}{2}\rangle \\
\frac{1}{2} & |2\ \tfrac{1}{2}\ 1\ -\tfrac{1}{2}\rangle,\ |2\ \tfrac{1}{2}\ 0,\ \tfrac{1}{2}\rangle \\
-\frac{1}{2} & |2\ \tfrac{1}{2}\ 0\ -\tfrac{1}{2}\rangle,\ |2\ \tfrac{1}{2}\ -1\ \tfrac{1}{2}\rangle \\
-\frac{3}{2} & |2\ \tfrac{1}{2}\ -1\ -\tfrac{1}{2}\rangle,\ |2\ \tfrac{1}{2}\ -2\ \tfrac{1}{2}\rangle \\
-\frac{5}{2} & |2\ \tfrac{1}{2}\ -2\ -\tfrac{1}{2}\rangle \}
\end{array}
$$

$l = 2, \ s = \frac{1}{2} \qquad j = \{ \frac{5}{2}, \frac{3}{2} \}$

$m_j = \frac{5}{2} \qquad \{ \ |j = \frac{5}{2}, m_j = \frac{5}{2}\rangle,$

$m_j = \frac{3}{2} \qquad |\frac{5}{2}, \frac{3}{2}\rangle, \ |\frac{3}{2}, \frac{3}{2}\rangle,$

$m_j = \frac{1}{2} \qquad |\frac{5}{2}, \frac{1}{2}\rangle, \ |\frac{3}{2}, \frac{1}{2}\rangle,$

$m_j = -\frac{1}{2} \qquad |\frac{5}{2}, -\frac{1}{2}\rangle, \ |\frac{3}{2}, -\frac{1}{2}\rangle,$

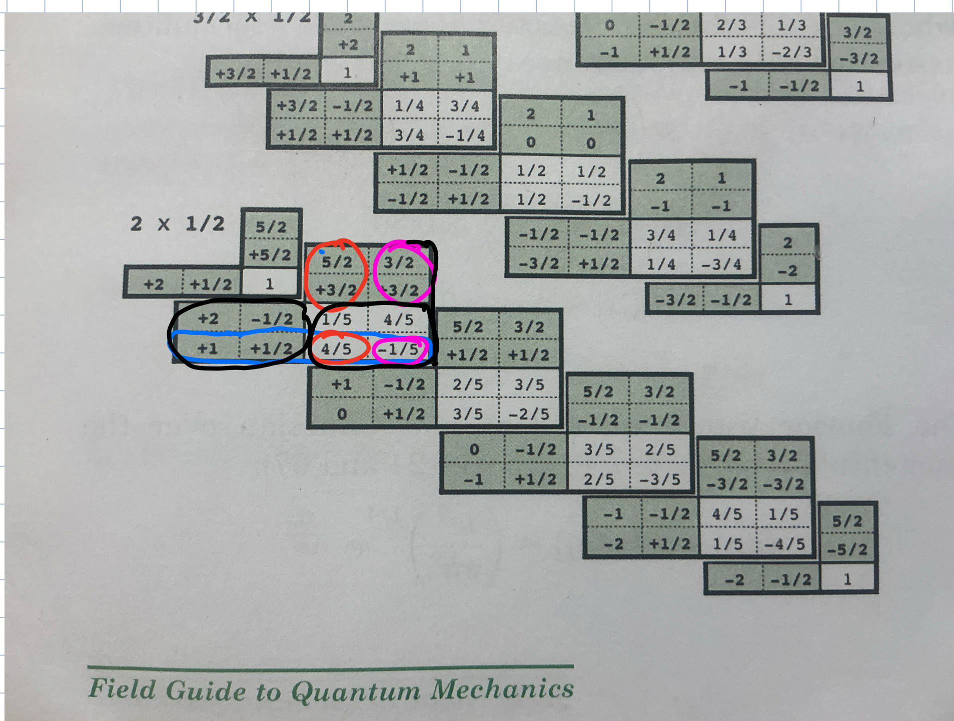
$m_j = -\frac{3}{2} \qquad |\frac{5}{2}, -\frac{3}{2}\rangle, \ |\frac{3}{2}, -\frac{3}{2}\rangle,$

$m_j = -\frac{5}{2} \qquad |\frac{5}{2}, -\frac{5}{2}\rangle \ \}$ 10 elements.

TAM basis w/ standard ordering

Write TP state $|l=2, s=\frac{1}{2}, m_l = 1, m_s = \frac{1}{2}\rangle$ as a superp. of eigenstates of J^2, J_z in the TAM basis

$l = 2, \ s = \frac{1}{2}$



$$|l=2, s=\frac{1}{2}, m_l = 1, m_s = \frac{1}{2}\rangle = \sqrt{\frac{4}{5}}\,|j=\frac{5}{2}, m_j = \frac{3}{2}\rangle - \sqrt{\frac{1}{5}}\,|j=\frac{3}{2}, m_j=\frac{3}{2}\rangle$$

$+$

$|2\ \tfrac{1}{2}\ 1\ \tfrac{1}{2}\rangle$, measure J^2 and J_z. What are the posi outcomes of meas and their prob.?

$|j = \tfrac{5}{2},\ m_j = \tfrac{3}{2}\rangle$

$|j = \tfrac{3}{2},\ m_j = \tfrac{3}{2}\rangle$

eigenvalue of J^2	eigenvalue of J_z	Prob
$\tfrac{5}{2}\cdot\tfrac{7}{2}\hbar^2 = \tfrac{35}{4}\hbar^2$	$\tfrac{3}{2}\hbar$	$\tfrac{4}{5} = 80\%$
$\tfrac{3}{2}\cdot\tfrac{5}{2}\hbar^2 = \tfrac{15}{4}\hbar^2$	$\tfrac{3}{2}\hbar$	$\tfrac{1}{5} = 20\%$

$j\cdot(j+1)\cdot\hbar^2$

Assume that we measure $\tfrac{35}{4}\hbar^2$, $\tfrac{3}{2}\hbar$, what is the z component of e^- spin after the measurement?

unknown

$$|j = \tfrac{5}{2},\ m_j = \tfrac{3}{2}\rangle = \sqrt{\tfrac{1}{5}}\ |2\ \tfrac{1}{2}\ 2\ -\tfrac{1}{2}\rangle\ \sqrt{\tfrac{4}{5}}\ |2\ \tfrac{1}{2}\ 1\ \tfrac{1}{2}\rangle$$

$\Rightarrow$ measure spin along z:

- spin up w/ prob of 80%
- spin down w/ prob of 20%

$$\langle S_z\rangle = 0.8\cdot\frac{\hbar}{2} + 0.2\cdot\left(-\frac{\hbar}{2}\right) = 0.6\,\frac{\hbar}{2}$$

$M \to 10\times 10$

Atomic Structure

AM Source	AM operator	Mag squared and z-operator	q#
single e^- OAM	$\vec{l}$	$\hat{l}^2$, $\hat{l}_z$	l, m_l
total e^- OAM in atom	$\vec{L}$	$\hat{L}^2$, $\hat{L}_z$	L, m_L
single e^- spin	$\vec{s}$	$\hat{s}^2$, $\hat{s}_z$	s, m_s
total e^- spin in atom	$\vec{S}$	$\hat{S}^2$, $\hat{S}_z$	S, m_S
total AM of all e^- in atom	$\vec{J} = \vec{L} + \vec{S}$	$\hat{J}^2$, $\hat{J}_z$	J, m_J
nuclear spin	$\vec{I}$	$\hat{I}^2$, $\hat{I}_z$	I, m_I
total AM in atom	$\vec{F} = \vec{I} + \vec{J}$	$\hat{F}^2$, $\hat{F}_z$	F, m_F

Atomic term symbol:

$$n^{2S+1}L_J$$

$L \rightarrow$ s, p, d, f, ...

ex: $3\,^2S_{1/2}$ in hydrogen

$n = 3$, $L = 0$, $\underline{J = 1/2}$, $S = \frac{1}{2}$

$m_j = \pm\frac{1}{2}$

$\vec{F} = \vec{J} + \vec{I}$

Notation for TP states:

$$|n=2, L=1, m_L=1\rangle \, |s=\tfrac{1}{2}, m_s=-\tfrac{1}{2}\rangle \, |I=\tfrac{1}{2}, m_I=\tfrac{1}{2}\rangle$$

$p^+ =$

$$|2,1,1\rangle \, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \, |\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

corr. bra

$$\langle \tfrac{1}{2}\ \tfrac{1}{2}| \, \langle \tfrac{1}{2}, -\tfrac{1}{2}| \, \langle 2,1,1|$$

Hydrogen atomic structure

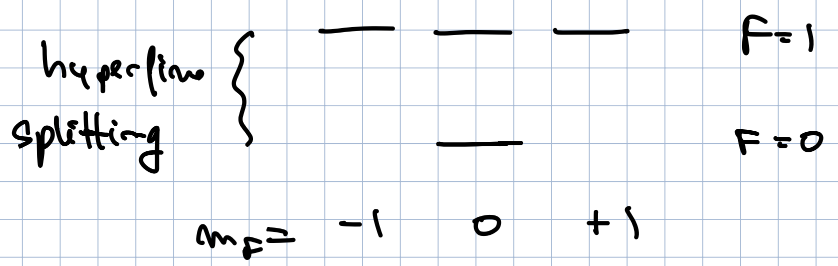
$n=1$

$$\left.\begin{array}{l} L=0 \\ s=\frac{1}{2} \end{array}\right\} \quad J=\frac{1}{2} \quad \Rightarrow \quad 1\,{}^2S_{1/2} \text{ level}$$

add nuclear spin $I=\frac{1}{2}$

$$J=\frac{1}{2} \Rightarrow \left\{\begin{array}{ll} F=1, & m_F=-1,0,1 \\ F=0, & m_F=0 \end{array}\right.$$

Energy level diagram



$n=2$

$L=0$ and $L=1$

- $$\left.\begin{array}{l} L=0 \\ s=\frac{1}{2} \end{array}\right\} \Rightarrow 2\,{}^2S_{1/2} \qquad I=\frac{1}{2}$$

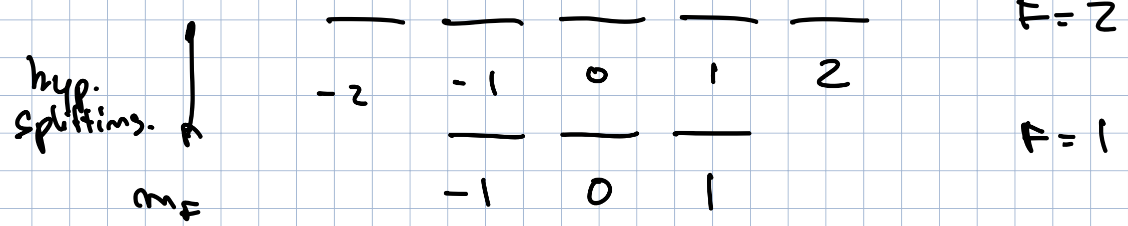
  $F=0$ and $F=1$

- $$\left.\begin{array}{l} L=1 \\ s=\frac{1}{2} \end{array}\right\} \Rightarrow \begin{array}{l} \bullet\ 2\,{}^2P_{1/2} \\ \bullet\ 2\,{}^2P_{3/2} \end{array}$$

add $I=\frac{1}{2}$ $\Rightarrow$ $2\,{}^2P_{1/2} \Rightarrow F=0,1$

$2\,{}^2P_{3/2} \Rightarrow F=1,2$

ex: $2^2P_{3/2}$ hyperfine manifold.



ex: Hydrogenic atoms ^{87}Rb

$$\left.\begin{array}{l} 37\, e^- \\ 37\, p^+ \\ 50\, n^0 \end{array}\right\} \text{nucleons}, \qquad s = \tfrac{1}{2},\ I = \tfrac{3}{2}$$

ground state

36 e^- paired up (Kr ground state)

+1 e^- - hydrogenic atom

$n = 5$

$L = 0, 1, 2, 3, 4$ $\qquad \vec{F} = \vec{J} + \vec{I}$

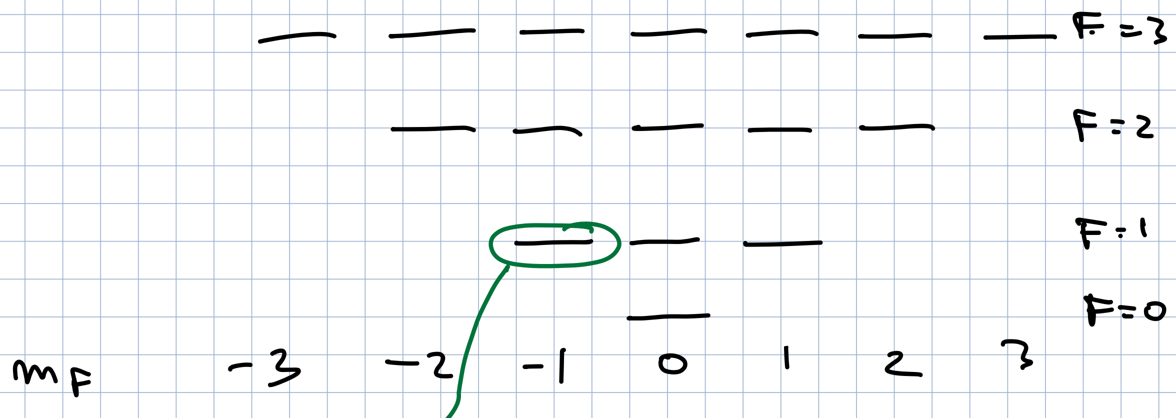
with $I = \frac{3}{2}$

$L = 0 \qquad s = \frac{1}{2} \Rightarrow J = \frac{1}{2} \qquad F = 1, 2$

$L = 1 \qquad s = \frac{1}{2} \Rightarrow \bullet\, J = \frac{3}{2} \iff F = 3, 2, 1, 0$

$\qquad\qquad\qquad\qquad \bullet\, J = \frac{1}{2} \iff F = 1, 2$

ex: $5^2P_{3/2}$ hyperfine manifold.



TAM state: $n = 5$, $l = 1$, $s = \frac{1}{2}$, $J = \frac{3}{2}$, $I = \frac{3}{2}$, $F = 1$, $m_F = -1$

or: $|5^2 P_{3/2}, F = 1, m_F = -1\rangle$ state