

Assignment 6

OPTI 570 Quantum Mechanics

University of Arizona

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Total time: 5 hours

Problem I

a) On the one hand, the action of $\tilde{a}(t)$ is

$$\begin{aligned}\tilde{a}(t)|\varphi_n\rangle &= U^\dagger(t, 0)aU(t, 0)|\varphi_n\rangle = U^\dagger(t, 0)ae^{-i(N+\frac{1}{2})\omega t}|\varphi_n\rangle = e^{-i(n+\frac{1}{2})\omega t}U^\dagger(t, 0)a|\varphi_n\rangle \\ &= \sqrt{n}e^{-i(n+\frac{1}{2})\omega t}U^\dagger(t, 0)|\varphi_{n-1}\rangle = \sqrt{n}e^{-i(n+\frac{1}{2})\omega t}e^{i(n-\frac{1}{2})\omega t}|\varphi_{n-1}\rangle = \sqrt{n}e^{-i\omega t}|\varphi_{n-1}\rangle.\end{aligned}$$

Therefore,

$$\tilde{a}(t)|\varphi_n\rangle = \sqrt{n}e^{-i\omega t}|\varphi_{n-1}\rangle = e^{-i\omega t}a|\varphi_n\rangle.$$

On the other hand, the action of $\tilde{a}^\dagger(t)$ is

$$\begin{aligned}\tilde{a}^\dagger(t)|\varphi_n\rangle &= U^\dagger(t, 0)a^\dagger U(t, 0)|\varphi_n\rangle = e^{-i(n+\frac{1}{2})\omega t}U^\dagger(t, 0)a^\dagger|\varphi_n\rangle = \sqrt{n+1}e^{-i(n+\frac{1}{2})\omega t}U^\dagger(t, 0)|\varphi_{n+1}\rangle \\ &= \sqrt{n+1}e^{-i(n+\frac{1}{2})\omega t}e^{i(n+\frac{3}{2})\omega t}|\varphi_{n+1}\rangle.\end{aligned}$$

Consequently,

$$\tilde{a}^\dagger(t)|\varphi_n\rangle = \sqrt{n+1}e^{i\omega t}|\varphi_{n+1}\rangle = e^{i\omega t}a^\dagger|\varphi_n\rangle.$$

b) We can compute the operators if we stimulate them with a ket $|\varphi_n\rangle$. We make use of the a, a^\dagger expression for \tilde{X} and \tilde{P} .

$$\tilde{X}(t)|\varphi_n\rangle = \frac{1}{\sqrt{2}} \left[U^\dagger a^\dagger U + U^\dagger a U \right] |\varphi_n\rangle.$$

But, we have already computed these operations in the previous part, so we will use it here:

$$\tilde{X}(t)|\varphi_n\rangle = \frac{1}{\sqrt{2}} \left[e^{i\omega t}a^\dagger + e^{-i\omega t}a \right] |\varphi_n\rangle \implies \tilde{X}(t) = \frac{1}{\sqrt{2}} \left[e^{i\omega t}a^\dagger + e^{-i\omega t}a \right].$$

In the same manner, we have for \tilde{P} :

$$\tilde{P}(t)|\varphi_n\rangle = \frac{1}{\sqrt{2}} \left[U^\dagger a^\dagger U - U^\dagger a U \right] |\varphi_n\rangle = \frac{1}{\sqrt{2}} \left[e^{i\omega t}a^\dagger - e^{-i\omega t}a \right] |\varphi_n\rangle$$

Therefore,

$$\tilde{P}(t) = \frac{1}{\sqrt{2}} \left[e^{i\omega t} a^\dagger - e^{-i\omega t} a \right].$$

They are like trigonometric functions cosine and sine, with the different that the operator a, a^\dagger is in the middle.

- c) To show that a ket is an eigenvector of an operator, it will have to satisfy its eigenequation with a constant representing the eigenvalue: $P|\psi\rangle = \lambda|\psi\rangle$.
- d) asfa
- e) asfafasf
- f) asfsa

Problem II

- a) asfaf
- b) asfasf
- c) asfafasf
- d) asfa
- e) sgag
- f) asgag
- g) asgagasg
- h) asg

Problem III

- a) asfaf
- b) asfasf
- c) asfafasf
- d) asfa

Problem IV

Part 1.

- a)

Part 2.

- b) asfaf
- c) asfasf
- d) asfafasf
- e) asfa