OPTI 570 LECTURE 2

State space - object - mass m
- all possible states of system

State - charact. all of the information that is available about object

Quantum state space - all possible states of the system in Quantum mechanics
= Hilbert space
≡ vector space
≡ state vector space

$\mathcal{E}$ - symbol that we use for vector space

- particular system

eg. - potential well : $\mathcal{E}_{\text{inf well}}$
- $\mathcal{E}_{\text{free part.}}$
- $\mathcal{E}_{1\,QHO}$
- $\mathcal{E}_{\text{spin}}$, $\mathcal{E}_{\text{hydrogen}}$

Notation: - elements in state space

$$-\underbrace{|\ldots\rangle}_{\text{"ket"}} \in \mathcal{E}$$

eg: - 1D QHO

$|\psi_3\rangle, |n=3\rangle, |3\rangle, |u_3\rangle \ldots$

- hydrogen atom

$|100\rangle, |n=1, \ell=0, m=0\rangle, |\psi_{100}\rangle \ldots$

- meaning has to defined

Ch II B Ch III A

state space $\neq$ function space

- general - specific

eg. spin HERE NOT HERE

eg: 1DQHO : $\mathcal{E}_{1DQHO}$ $\mathcal{F}_{1DQHO}$

$|\psi\rangle \in \mathcal{E}_{1DQHO}$ $\psi(x) \in \mathcal{F}_{1DQHO}$

1 : 1

- direct relation b/w elements
- "isomorphism"

$\mathcal{E}_{1DQHO} \neq \underline{\mathcal{F}_{x,1DQHO}} \neq \underline{\mathcal{F}_{p,1DQHO}}$

Fourier transforms

$|\psi\rangle \neq \psi(x) \neq \tilde{\psi}(p)$

<u>dimensions?</u> × $\neq$ ✓ $1/\sqrt{\text{length}}$ $\neq$ ✓

$\psi(x) = \underline{\langle x|}\psi\rangle$ $\tilde{\psi}(p) = \langle p|\psi\rangle$

In QM, the state space $\mathcal{E}$ is a vector space:

(1) <u>superposition principle</u> holds

(2) <u>inner product</u> is defined

(1) $|\varphi_n\rangle \in \mathcal{E}$, for $n \geq 0$

$\{\varphi_n\}$ - set of all those states

- create new elements of $\mathcal{E}$ by linearly adding elements already in $\mathcal{E}$

$$|\psi\rangle = \sum_{n=0}^{\infty} \underbrace{c_n}_{\text{weigths} \in \mathbb{C}} |\varphi_n\rangle \in \mathcal{E}$$

ex: $|x\rangle, |y\rangle, |z\rangle$ in 3D space

$c_1|x\rangle + c_2|y\rangle + c_3|z\rangle$ - any position in space

② inner product

- how much vectors overlap

overlap 0 = perpendicular = orthogonal

Let $|\varphi\rangle, |\psi\rangle \in \mathcal{E}$

"inner product of $|\varphi\rangle$ an $|\psi\rangle$: $(|\varphi\rangle, |\psi\rangle)$

- scalar $\in \mathbb{C}$

Dirac notation

new symbol: $\langle\varphi|$, "bra"

$\langle\varphi|\psi\rangle$ - inner product of two vectors.

- scalar $\in \mathbb{C}$

Properties of inner products

- linear in 2nd term

$$\langle\varphi|\lambda_1\psi_1 + \lambda_2\psi_2\rangle = \langle\varphi|\lambda_1\psi_1\rangle + \langle\varphi|\lambda_2\psi_2\rangle = \lambda_1\langle\varphi|\psi_1\rangle + \lambda_2\langle\varphi|\psi_2\rangle$$

- anti-linear in 1st term

$$\langle\lambda_1\varphi_1 + \lambda_2\varphi_2|\psi\rangle = \underline{\lambda_1^*}\langle\varphi_1|\psi\rangle + \underline{\lambda_2^*}\langle\varphi_2|\psi\rangle$$

- $|\psi\rangle \in \mathcal{E}$ - must be normalizable

- $\langle \psi | \psi \rangle > 0$, finite $\in \mathbb{R}$
- when normalized $\langle \psi | \psi \rangle = 1$.

$\langle \varphi |$ - "bra" - what does it mean?

Formally: For every $| \varphi \rangle$ in $\mathcal{E}$, there exists

$\langle \varphi | \in \mathcal{E}^*$ - "dual space"

ex: $\psi(x) = \langle x | \psi \rangle$

$\langle x | \in \mathcal{E}^*$ Q: is there $| x \rangle$ in $\mathcal{E}$?

$| x_0 \rangle = \delta(x - x_0)$

$\Delta x \, \Delta p \gg \frac{\hbar}{2}$

$\Delta x = 0$, $\Delta p = \infty$

$| \psi \rangle \in \mathcal{E} \Rightarrow \langle \psi | \in \mathcal{E}^*$

$\lambda_1 | \psi_1 \rangle + \lambda_2 | \psi_2 \rangle \Rightarrow \lambda_1^* \langle \psi_1 | + \lambda_2^* \langle \psi_2 |$

"antilinear"

Dirac: system of kets, bras, scalars

Operators: Let $| \psi \rangle \in \mathcal{E}$

$\hat{A} | \psi \rangle = c \, | \psi' \rangle \Leftarrow$ new ket.

operator, scalar $\in \mathbb{C}$