Due: Tues, Nov. 18. (Main Campus and Online section)

Problem I.

Suppose that a beam of light is traveling in the z direction. Consider a photon of this beam, and treat the photon as if it were a particle moving through space. Let $|\phi_x\rangle$ be the quantum state corresponding to the photon being linearly polarized along the x-axis, and let $|\phi_y\rangle$ be the orthogonal quantum state corresponding to the photon being linearly polarized along the y-axis. The quantum state corresponding to linear polarization at an angle μ relative to the x-axis is then

$$|\phi_\mu\rangle = \cos(\mu)|\phi_x\rangle + \sin(\mu)|\phi_y\rangle.$$

Similarly, the quantum states corresponding to circular polarizations σ_+ or σ_- are

$$|\phi_\pm\rangle = -\frac{1}{\sqrt{2}}(|\phi_x\rangle \pm i|\phi_y\rangle),$$

(a) For a photon in the state $|\phi_\mu\rangle$ find the probabilities of measuring x, y, σ_+, and σ_- polarization, in terms of the angle μ.

Suppose that a light source produces pairs of photons, where one of the photons travels left along the z axis and and the other right along the z axis. The quantum state for a photon pair can be expressed in a basis of tensor product states of the form $|\phi_j^L\rangle|\phi_k^R\rangle$, where $j, k \in \{x, y\}$ and the single-photon states $|\phi_x^L\rangle$, $|\phi_y^L\rangle$, $|\phi_x^R\rangle$, and $|\phi_y^R\rangle$ correspond to x- or y-polarized photons traveling in the left or right direction. We further assume that the source is rigged to produce photon pairs in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\phi_x^L\rangle|\phi_y^R\rangle - |\phi_y^L\rangle|\phi_x^R\rangle).$$

(b) Suppose that we measure, for each of the two photons, if the polarization is linear along x or y. Write down the two-photon states corresponding to these joint measurement outcomes. In other words, what four possible product states would the system be left in if we measure that the left-going photon has a polarization of j and the right-going photon has a polarization k? (Again, $j, k \in \{x, y\}$.) What is the probability for each of these outcomes? In addition, find the probability of measuring linear polarization along the x or y directions for either the photon moving to the left or to the right, without considering what the polarization of the other photon would be.

(c) Now consider the case where we measure, for each of the two photons, whether the polarization is σ_+ or σ_-. Find again the probability for each of the possible outcomes, both for the two polarizations measured simultaneously, and for one side independent of the other side. *Hint:* To simplify this, you may wish to first write $|\psi\rangle$ as a superposition of tensor product states in terms of the orthogonal σ_+ and σ_- states rather than the linear polarization states.

What do the results of (b) and (c) imply for the polarizations of a pair of photons from a source such as this? Is it reasonable to assign a well-defined polarization state to each photon independently?

Problem II.

CT Chapter X, Complement G_X, exercise 1.

Problem III.

Context: This problem is intended to explore selection rules in light-matter interactions when multiple angular-momentum quantities are involved. We have not yet discussed the origin of the selection rules in this class, so this problem will be treated as an exercise in working with atomic angular momenta.

First, we assume that if an atom is in a state $|\phi\rangle$, and if we shine a weak light field on the atom for a short time, the atom may then be found in a different state $|\phi'\rangle$. We determine the strength of the coupling of the two states with a matrix element $A_{\phi\phi'} = \langle\phi|A|\phi'\rangle$, where A is an operator that represents the action of the light on the state of the atom. If $A_{\phi\phi'} = 0$ then the light can not induce a transition from one state to the other. Take it as a given that the basic model of light-matter interaction tells us the following about A:

(i) A acts only on states $|L, M_L\rangle$. In other words, A does not act on states in the subspaces spanned by the $\{|S, M_S\rangle\}$ basis or the $\{|I, M_I\rangle\}$ basis.

(ii) The matrix element $\langle L', M'_L|A|L, M_L\rangle$ is non-zero *only* if $L' - L = \pm 1$ *and* if $M'_L - M_L = M_\nu$, where $M_\nu = \pm 1, 0$ is related to the polarization of the light field. These conditions encompass what is meant by the *selection rules* governing which transitions are allowed.

The "transition strength" T is proportional to $|\langle\phi'|A|\phi\rangle|^2$. For simplicity we'll set $T = |\langle\phi'|A|\phi\rangle|^2$. Although this is not a standard notation, it allows us to evaluate relative transition strengths, which is one goal of the problem. Finally, we will make the definition $A_{L'M'_L,\, L\, M_L} \equiv \langle L', M'_L|A|L, M_L\rangle$.

The question. Assume that we're interested in determining transition strengths between states of the $5\,^2S_{1/2}, F = 1$ level and those of the $5\,^2P_{1/2}, F' = 1$ and $5\,^2P_{1/2}, F' = 2$ levels of ^{87}Rb, where $I = 3/2$ and $S = 1/2$. (The "primes" just indicated quantum numbers associated with $|\phi'\rangle$.) If the light is linearly polarized along z (ie, $M_\nu = 0$), then we will find transition strengths that are proportional to $|A_{1000}|^2$. For this case, determine the transition strengths for the following initial and final states $|\phi\rangle$ and $|\phi'\rangle$, and express your answers in terms of $|A_{1000}|^2$. (Hint: you should get a different answer for each one of these, with one answer being zero.)

(a) $|\phi\rangle = |5\,^2S_{1/2}, F = 1, M_F = 1\rangle \quad \Rightarrow \quad |\phi'\rangle = |5\,^2P_{1/2}, F' = 1, M'_F = 1\rangle$.

(b) $|\phi\rangle = |5\,^2S_{1/2}, F = 1, M_F = 0\rangle \quad \Rightarrow \quad |\phi'\rangle = |5\,^2P_{1/2}, F' = 1, M'_F = 0\rangle$.

(c) $|\phi\rangle = |5\,^2S_{1/2}, F = 1, M_F = 1\rangle \quad \Rightarrow \quad |\phi'\rangle = |5\,^2P_{1/2}, F' = 2, M'_F = 1\rangle$.

(d) $|\phi\rangle = |5\,^2S_{1/2}, F = 1, M_F = 0\rangle \quad \Rightarrow \quad |\phi'\rangle = |5\,^2P_{1/2}, F' = 2, M'_F = 0\rangle$.