OPTI 570 RECAP 9

$U(t)$ – when is global phase?

$$\begin{pmatrix} \hbar\omega_0 & & \\ & 2\hbar\omega_0 & \\ & & 3\hbar\omega_0 \end{pmatrix}$$

$$|\psi_1\rangle(t) = \underbrace{e^{-iHt/\hbar}}\cdot|\psi_1\rangle(0)$$

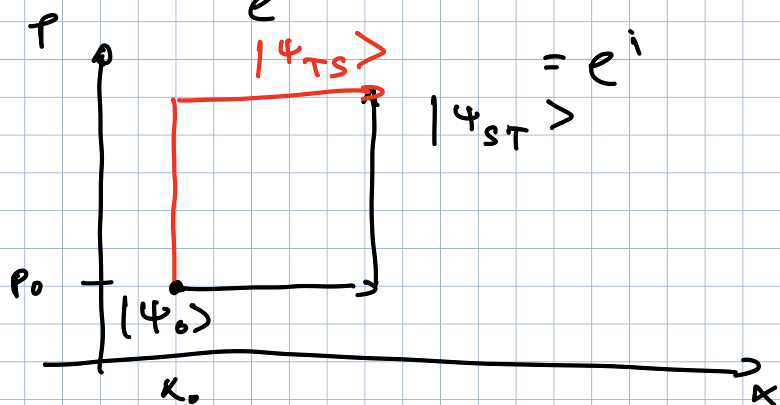
$$|\psi_1\rangle(t_2) = e^{-i\hbar\omega_0 t_2/\hbar}$$

$$|\psi_2\rangle(t_2) = \underbrace{e^{-i2\hbar\omega_0 t_2/\hbar}}$$

$$e^{-i\hbar\omega_0 t_2/\hbar}$$

$$e^{\overset{\text{scalar}}{ic}}$$

global phase factor



$= e^{i}$

Practice Exam - 3C

$$\hat{C} = \int_{-\infty}^{\infty} dx' \, |x'\rangle \underbrace{\langle x' | F \rangle}_{F(x')} \langle x'| \qquad F(x) = \langle x | F \rangle$$

- what does $\hat{C}$ do in pos representation?

$$\mathcal{F} \to \hat{C}(x) \quad \Rightarrow \quad \langle x | \hat{C} \qquad |x\rangle$$

$$\langle x | \hat{C} |\psi\rangle = \int_{-\infty}^{-\infty} dx' \, \underbrace{\langle x | x' \rangle}_{\delta(x-x')} F(x') \langle x' | =$$

$$= F(x) \langle x | = \langle x | F(x) |\psi\rangle$$

- multiplies state by $F(x)$

$\hat{C}^2 = \hat{C}$?

$$\hat{C}^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx' \, dx'' \, |x'\rangle\langle x' | F \rangle \underbrace{\langle x' | x'' \rangle}_{\delta(x'-x'')} \langle x'' | F \rangle \langle F \rangle \langle x''| =$$

$$= \int_{-\infty}^{+\infty} dx' \, |x'\rangle\langle x' | F \rangle \langle x' | F \rangle \langle x' | =$$

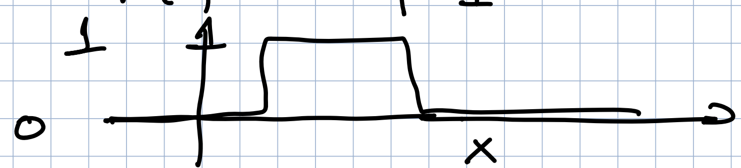
$$= \int_{-\infty}^{+\infty} \underbrace{(\langle x' | F \rangle)^2}_{(F(x))^2} |x'\rangle\langle x' | \, dx'$$

$$\hat{C} = \int_{-\infty}^{+\infty} \langle x' \rangle \underbrace{\langle x' | F \rangle}_{F(x)} dx'$$

$$\hat{C}^2 = \hat{C} \quad \Rightarrow \quad F^2(x) = F(x) \quad \text{for all } x$$

$$\Rightarrow \quad F(x) = \begin{cases} 0 \\ 1 \end{cases}$$

F(x)



$\langle x|$ - pos. repres.

$\langle p|$ - mom. representation.

4 a. $|\psi\rangle = \frac{1}{2}|n=2\rangle + \frac{1}{\sqrt{2}}|n=1\rangle + \frac{1}{2}|n=0\rangle$

$\Rightarrow$ basis of eigenstates of $\hat{B}$

e. $\lambda = 2 \quad |\beta_2\rangle = \frac{1}{\sqrt{2}}(|2\rangle + |-2\rangle)$

$\lambda = -1 \quad |\beta_{-1}\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |-1\rangle)$

$\lambda = -2 : |\beta_{-2}\rangle = \frac{1}{\sqrt{2}}(|2\rangle - |-2\rangle)$

$\lambda = 0 : |\beta_0\rangle = |0\rangle$

Problem 5b

$$\hat{U} = e^{-i\hat{H}t/\hbar}$$

$$\hat{U}_{\{u\}} = \begin{pmatrix} e^{-i\omega t/2} & 0 & 0 \\ 0 & e^{-i\omega t} & 0 \\ 0 & 0 & e^{-i3\omega t} \end{pmatrix}$$

$$\hat{H}_{\{\phi\}} = \frac{\hbar\omega}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -i\sqrt{3} \\ 0 & i\sqrt{3} & 5 \end{pmatrix}$$

$$\hat{H}_{\{\phi\}} = \frac{\hbar\omega}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$\underline{\hbar\omega/2}$ $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad$ $\underline{\hbar\omega}$ $\begin{pmatrix} 0 \\ \sqrt{3}/2 \\ -i/2 \end{pmatrix}$ $\quad$ $\underline{3\hbar\omega}$ $\begin{pmatrix} 0 \\ 1/2 \\ i\sqrt{3}/2 \end{pmatrix}$

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & i/2 \\ 0 & -i/2 & i\sqrt{3}/2 \end{pmatrix}$$

$|M^{\dagger}$

$M|$

$$\hat{U}_{\{u\}} = M^{\dagger} \, \hat{U}_{\{\phi\}} \, M \Rightarrow M \, \hat{U}_{\{u\}} \, M^{\dagger} = \hat{U}_{\{\phi\}}$$

$$\{ |\underline{\phi_1}\rangle, |\underline{\phi_2}\rangle, |\underline{\phi_3}\rangle \}$$

$$\{ |\phi_3\rangle, |\phi_2\rangle, |\phi_1\rangle \}$$