OPTI 570 RECAP Th Nov 12

$J = \frac{3}{2}$, $I = \frac{3}{2}$ $\quad \underline{\vec{F} = \vec{J} + \vec{I}}$ what are pos vals of F?

$max(F) = J + I = 3$

$min(F) = |J - I| = 0$

$F = 3, 2, 1, 0$

$L = 1$, $S = \frac{1}{2} \Rightarrow \vec{J} = \vec{L} + \vec{S}$ $\quad \left.\begin{array}{l} max(J) = \frac{3}{2} \\ min\,|J| = \frac{1}{2} \end{array}\right\}$ $J = \frac{1}{2}, \frac{3}{2}$

Write ^{87}Rb $\underline{|5^2 P_{\frac{3}{2}}, F = 1, m_F = 1\rangle}$ state in the full atomic TP basis.

Quantum #s

$\vec{J} = \vec{L} + \vec{S}$ $\quad \vec{F} = \vec{I} + \vec{J}$

TAM

$\underline{F = 1,\ M_F = 1}$

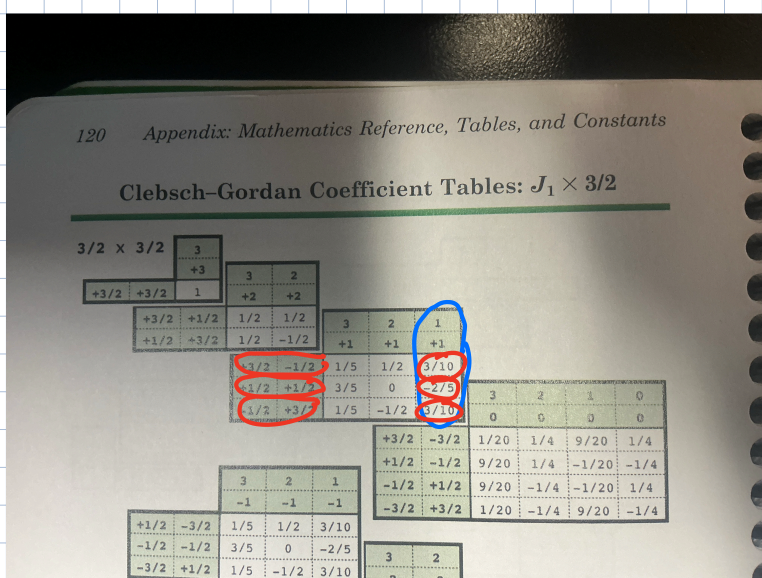
CG: $\underline{\frac{3}{2} \times \frac{3}{2}}$

TP

$\underline{J = \frac{3}{2},\ I = \frac{3}{2}}$

$|F = 1, m_F = 1\rangle = \sqrt{\frac{3}{10}}\ |m_J = \frac{3}{2}, m_I = -\frac{1}{2}\rangle - \sqrt{\frac{2}{5}}\ |m_J = \frac{1}{2}, m_I = \frac{1}{2}\rangle$

$+ \sqrt{\frac{3}{10}}\ |m_J = -\frac{1}{2}, m_I = \frac{3}{2}\rangle$



120 *Appendix: Mathematics Reference, Tables, and Constants*

Clebsch–Gordan Coefficient Tables: $J_1 \times 3/2$

3/2 x 3/2

		3
		+3
+3/2	+3/2	1

		3	2
		+2	+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

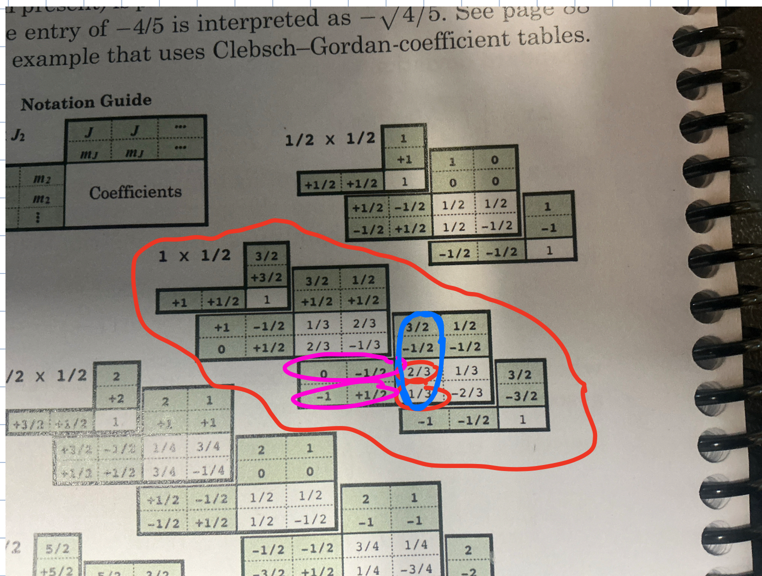
		3	2	1
		+1	+1	+1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

		3	2	1	0
		0	0	0	0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

		3	2	1
		−1	−1	−1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

3	2
−2	−2

$\vec{J} = \vec{L} + \vec{S} \quad {}^2P_{3/2} \Rightarrow L = 1,\ S = \frac{1}{2},\ J = 3/2,\ m_J = -\frac{1}{2}$


e entry of −4/5 is interpreted as $-\sqrt{4/5}$. See page
example that uses Clebsch–Gordan-coefficient tables.

Notation Guide

$1 \times 1/2$

m_1	m_2	$J=3/2$, $m_J=+3/2$
+1	+1/2	1

m_1	m_2	$J=3/2$, $m_J=+1/2$	$J=1/2$, $m_J=+1/2$
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	$J=3/2$, $m_J=-1/2$	$J=1/2$, $m_J=-1/2$
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	$J=3/2$, $m_J=-3/2$
−1	−1/2	1

$1/2 \times 1/2$

m_1	m_2	$J=1$, $m_J=+1$
+1/2	+1/2	1

m_1	m_2	$J=1$, $m_J=0$	$J=0$, $m_J=0$
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	$J=1$, $m_J=-1$
−1/2	−1/2	1

$$|J = \tfrac{3}{2}, m_J = -\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\,|L=1, m_L=0\rangle|S=\tfrac{1}{2}, m_S=-\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\,|L=1, m_L=-1\rangle|S=\tfrac{1}{2}, m_S=\tfrac{1}{2}\rangle$$

$$|m_J = -\tfrac{1}{2}, m_I = \tfrac{3}{2}\rangle = |J=\tfrac{3}{2}, m_J=-\tfrac{1}{2}\rangle|I=\tfrac{3}{2}, m_I=\tfrac{3}{2}\rangle$$

$$|F=1, m_F=1\rangle = \sqrt{\tfrac{3}{10}}\,|L=1, m_L=1\rangle|S=\tfrac{1}{2}, m_S=-\tfrac{1}{2}\rangle|I=\tfrac{3}{2}, m_I=-\tfrac{1}{2}\rangle$$
$$-\sqrt{\tfrac{2}{5}}\cdot\sqrt{\tfrac{1}{3}}\,|1\ 1\rangle|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle$$
$$-\sqrt{\tfrac{2}{5}}\sqrt{\tfrac{2}{3}}\,|1\ 0\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle|\tfrac{3}{2}\ \tfrac{1}{2}\rangle$$
$$+\sqrt{\tfrac{3}{10}}\sqrt{\tfrac{2}{3}}\,|1\ 0\rangle|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle|\tfrac{3}{2}\ \tfrac{1}{2}\rangle$$
$$+\sqrt{\tfrac{3}{10}}\sqrt{\tfrac{1}{3}}\,|1\ -1\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle|\tfrac{3}{2}\ \tfrac{3}{2}\rangle$$

$\Delta L = 0, 1$

$\Delta m_L = 0, \pm 1$

$\Delta S = 0$

$\Delta I = 0$

Q: • What is the probability of measuring spin up?

$$\frac{3}{10} + \frac{4}{15} + \frac{1}{10} = \frac{9+8+3}{30} = \frac{20}{30} = \frac{2}{3}$$

• What is the prob of meas an angular part of Y_1^0?

$L = 1,\ m_L = 0$

$$\frac{4}{15} + \frac{1}{5} = \frac{7}{15}$$

$\langle HF\,1 | W | HF\,2 \rangle$

$\langle \phi | \underset{\sim}{W} | \phi' \rangle$

$= \langle L' M_L' | W | L M_L \rangle$

$\Downarrow$

$\gamma = \langle \phi | \vec{R} \cdot \int Y_x' \, Y_y \, Y_z \, dv | \phi' \rangle$

$| m_J = \frac{3}{2}, m_I = -\frac{1}{2} \rangle = \underbrace{| J = \frac{3}{2}, m_J = \frac{3}{2} \rangle}_{L=1,\ S=\frac{1}{2}} | I = \frac{3}{2}, m_I = -\frac{1}{2} \rangle$

$\vec{J} = \vec{L} + \vec{S}$

$m_L = -1, 0, 1 \quad , m_S = -\frac{1}{2}, \frac{1}{2}$

$m_L + m_S = m_J = \frac{3}{2}$

$m_L = 1$

$m_S = \frac{1}{2}$

$| J = \frac{3}{2}, m_J = \frac{3}{2} \rangle = | L = 1, m_L = 1 \rangle | S = \frac{1}{2}, m_S = \frac{1}{2} \rangle$

$| \phi \rangle = \_\ \_\_ \ldots$