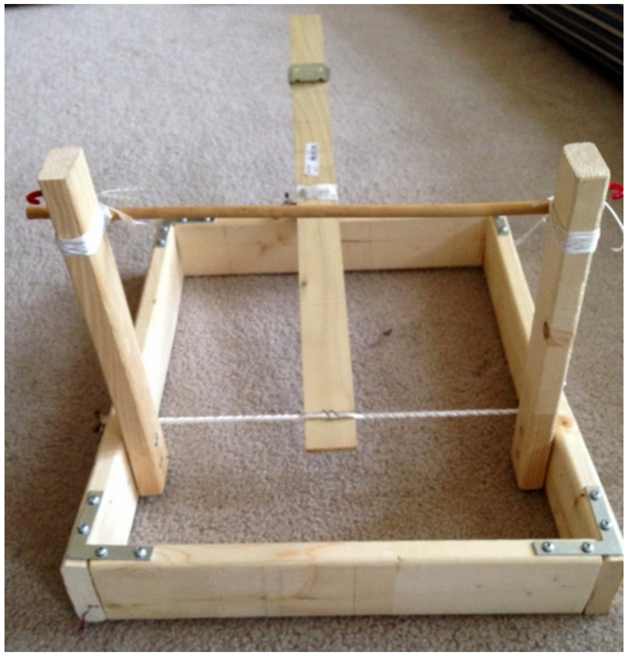

Nicole Rosario's Lab Report

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December 18, 2015

Introduction/Background:

Physics is all around us in daily life and that was no less true thousands of years ago, even if many of the people back then did not understand physics. Catapults were instrumental in war thousands of years ago and variations of catapults are still used in the military today, like catapult slings to launch airplanes from aircraft carriers. People who built catapults thousands of years ago probably didn't pay much or any attention to the physics behind catapults but there is a lot of physics related to how catapults work. The major topics related to how catapults work would be centripetal force and projectiles; all though, not every catapult uses centripetal force, some just used elastic potential energy.

Centripetal force is the force that acts on an object traveling in a circular path and is directed toward the center of the circle, around which the object is traveling. Centripetal force is important for catapults because the net force must be strong enough to cause the object to travel once it exists the circular path, created by the catapult's arm.

$$F_C = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2}$$

A projectiles is an object that is launched through the air along a parabolic path, due to the force of gravity pulling it downward. The launch point and the impact point are not at the same height so the projectile range equation ($x = (v^2 \div g)\sin 2\vartheta$) cannot be used. Instead, the projectile has to be broken into its horizontal (x-direction) and vertical (y-direction). The vertical direction uses the kinematic equations because it is affected by the force of gravity but the horizontal direction just uses $v = d \div t$ because it is not affected by the force of gravity.

Vertical:

1. $d = v_1t + 0.5at^2$
2. $d = v_2t - 0.5at^2$
3. $v_2 = v_1 + at$
4. $v_2^2 = v_1^2 + 2ad$
5. $d = \frac{(v_1 + v_2)}{2} t$

Horizontal:

1. $v = d \div t$

That is a summary of the major physics topics related to catapults but now for an experimental question. What is the relationship between the arm length of a catapult and the distance that the object travels? Do they vary linearly? Do they vary in some other way? Or, is there no relationship at all?

Looking at the centripetal force equation it would make sense that the distance travelled decreases as the arm length increases because the radius, arm length, is in the denominator. This would cause the centripetal force the decrease as the arm length increases which should in turn decrease the distance travelled because it would make sense that, if the angle is kept constant, the largest centripetal force would correspond with the largest distance travelled. Then there are the equations for the projectile which take into account the distance, speed and time in the horizontal and vertical directions as well as the acceleration due to gravity in the vertical direction.

For the experiment the distance travelled will combine all of these equations to either create a relationship between the distance travelled and the arm length, or to create no relationship.

Materials:

- 1 wooden catapult with a firing pin (torsion powered by string)
- 1 object that will be propelled by the catapult (an eraser was used for the experiment because it is fairly light and not easily breakable)
- 1 tape measurer (that can measure to the nearest millimeter)
- Tape (to measure the impact point for each trial)
- 1 stopwatch (use a high accuracy stopwatch if available)

Image 1 - Front-side view of catapult before release

Procedure:

1. Three people are needed for the experiment. One person to launch the catapult, one to record the distance that the object travels, and one to measure the time from the launch to the impact (the launch is when the object leaves the catapult).
2. Create a table that includes the trial number, mass of object, arm length, the distance that the object travelled, and the time that the object travelled for each trial.
3. Measure and record the mass of the object (a new eraser will be used for this experiment).
4. Measure 32cm, 37cm, 42cm, 47cm, and 55cm on the arm and draw a line on the arm and each length.
5. Place an L-bracket at 32 cm (so that the eraser can sit at 32cm)
6. Have an observer stand roughly 1 meter away from the catapult and launch the catapult once so that the observer is close enough to the impact point to accurately see the impact points for all trials at that arm length (all impact points for a particular arm length should be relatively close together).
7. Have the other observed stand where they can see the launch point and impact point.
8. Launch the catapult three times, by pulling quickly on the firing pin. Have one observer place a small piece of tape at each impact point and the other observer measure and record the time from when the object is launched to when it first hits the ground.
9. Measure and record the distance from each impact point (the tape) to the axel of rotation for the arm (the string).
10. Repeat steps 4-9 for 37cm, 42cm, 47cm, and 55cm.
11. Find the average distance travelled for each arm length (3 trials were done for each arm length to reduce experimental error).

Observations:

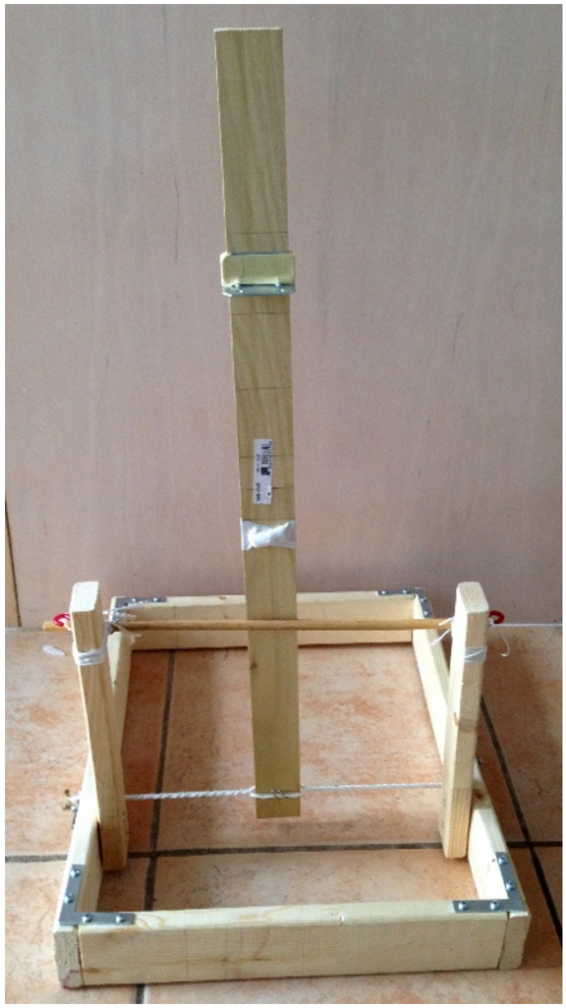


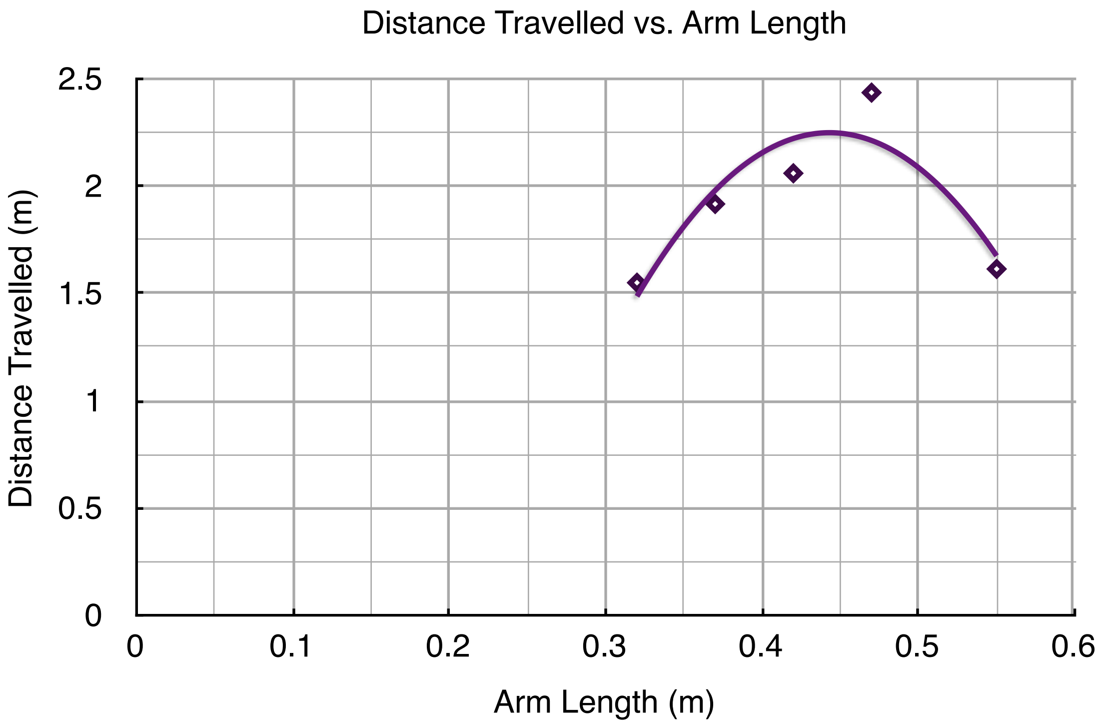
Image 2 - Front view of catapult after release

The measurements in the table were taken by one person releasing the catapult's arm by pulling the firing pin, another person watching for where the object lands, and another person measuring how long the time was between launch and impact. These results are not 100% accurate because multiple factors. First, the eraser did bounce after it landed but the observer was close enough to the point where the eraser first landed so that they could find the impact point, as accurately as possible. Also, the measurements for the arm length were measured by a ruler that can measure to the nearest millimeter so the measurements could be off by $\pm$ 0.5mm or $\pm$ 0.0005m. Second, the measurement for the mass were done on a scale that could measure to the nearest gram so the measurements could be off by $\pm$ 0.5g. To reduce the inaccuracies in the measurement for the mass of the object the measurement was done three times and each time the scale read 27g or 0.027kg. Third, the time was fairly inaccurate because the timer could only measure to the nearest 0.01s and the time was very short so there was some error in the starting and stopping of the stopwatch. The measurements could be off by more than just $\pm$ 0.005s because of the possibility for error in the starting and stopping of the stopwatch. However, to decrease the inaccuracies in the experiment three trials were done per arm length and the time was averaged for each arm length.

In the table the arm length is equal to the radius of the circle because the arm length is measured from the axel of rotation to the point where the objects sits on the arm. The distance travelled is the horizontal distance that the object travels and is measured from the axel of rotation of the arm to the point where the object hits the ground. The time is the time from when the object was launched to when it first hit the ground.

Trial #	Mass of Object (g)	Arm Length (m)	Distance Travelled (m)	Time Travelled (from launch to impact) (s)	Average Distance Travelled for Each Arm Length (m)	Average Time Travelled (s)
1	0.027	0.32	1.531	0.29		
2	0.027	0.32	1.575	0.60		
3	0.027	0.32	1.545	0.51	1.550	0.47
4	0.027	0.37	1.942	0.56		
5	0.027	0.37	1.885	0.64		
6	0.027	0.37	1.923	0.75	1.917	0.65
7	0.027	0.42	2.101	0.89		
8	0.027	0.42	2.021	0.92		
9	0.027	0.42	2.059	0.83	2.060	0.88
10	0.027	0.47	2.393	0.92		
11	0.027	0.47	2.507	0.89		
12	0.027	0.47	2.408	0.89	2.436	0.90
13	0.027	0.55	1.564	0.98		
14	0.027	0.55	1.549	0.97		
15	0.027	0.55	1.729	0.89	1.614	0.95

Table 1 - Relationship between the arm length, measured in meters, and the distance that the object travelled, measured in meters, when the mass of the object is constant. (Time is included for calculations)

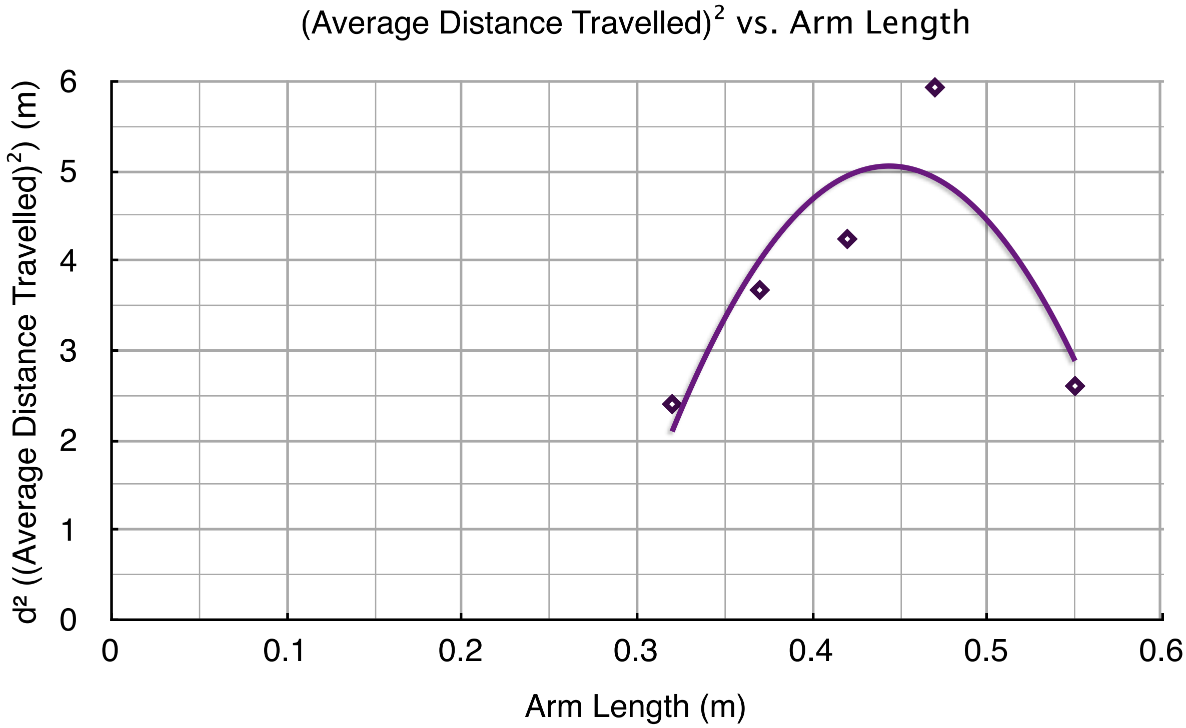


Graph 1 - Distance that the object travelled versus the arm length of catapult.

From this table and graph we can see that for most arm lengths the distance travelled increases as the arm length increases, however for the arm lengths after around 0.45 cm the graph seems to show that the distance decreases as the arm length increases. This may just be experimental error though that does not seem likely since three trials were done at each arm length. Therefore the data seems to show that there is a parabolic relationship between the arm length and the distance travelled but to be sure a graphical analysis approach needs to be taken in which the graph will be straightened out by graphing the distance travelled squared versus the arm length, which is done below.

Mass of Object (kg)	Arm Length (m)	Average Distance Travelled for Each Arm Length (d) (measured in meters)	d^2 ((Average Distance Travelled)^2) (m)
0.027	0.32	1.550	2.403
0.027	0.37	1.917	3.675
0.027	0.42	2.060	4.244
0.027	0.47	2.436	5.934
0.027	0.55	1.614	2.605

Table 2 - Relationship between the arm length, measured in meters, and the average distance that the object travelled squared for each arm length, measured in meters, when the mass of the object is constant.



Graph 2 - Average distance travelled for each arm length squared versus the arm length of catapult

The graphical analysis approach was taken to try to straighten the line but no function would create a straight line out of the data. A quadratic was used because the original function looks like a parabola. The first part of the graph looks linear but then the data point from 55cm arm length turns the data back into a parabola.

Calculations/Connections to Physics:

Note: There will be inaccuracies due to not having a high accuracy stopwatch and the fact that the time is so short, as well as from the other measurements.

For centripetal force, we can solve for the velocity and then solve for centripetal force. The velocity found should be equal to v_H in the projectile equations. To calculate the centripetal find v_1 in the projectile equation and use that as v in the centripetal force equation.

$$F_C = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2}$$

It would make sense that the higher the centripetal force the greater distance that the object would travel because it would have more force to launch it. However, the data found, as shown in the graphs and tables, shows that there is a max at some point between the arm lengths of 47 cm and 55 cm and that the distance traveled varies parabolically with the arm length. Also, centripetal force is not the only factor to take into consideration when looking at the relationship between the arm length and the distance travelled.

For projectiles, they need to be split into their horizontal and vertical components. In the vertical direction the kinematic equations are used because there is acceleration due to gravity but in the horizontal direction there is just the $v = d \div t$ because gravity does not effect the projectile in the horizontal direction. Also, v_1 in the vertical direction is equal to 0 because the object is launched at 90º to the horizontal or parallel to the vertical. v_H should be equal to v in the centripetal force equation because v is constant when the object is traveling in a circular path.

Vertical: ($v_1 = 0$)

1. $d = v_1t + 0.5at^2$
   $d = 0.5at^2$
2. $d = v_2t - 0.5at^2$
3. $v_2 = v_1 + at$
   $v_2 = at$
4. $v_2^2 = v_1^2 + 2ad$
   $v_2^2 = 2ad$
5. $d = ((v_1 + v_2)t) \div 2$
   $d = \frac{v_2t}{2}$

Horizontal:

1. $v_H = d \div t$

Calculations will be done for 2 trials, one before the max and one after the max to show a comparison between the two sections of data.

***Data taken from trial 5 for the first of the two sets of calculations because it is close to the data of the trend line at a given point before the max**

Vertical (Projectile):

$d = 0.37 + 0.038$
$= 0.408m$
$v_1 = 0m/s$
$t_V = t_H$
$a = 9.8m/s^2$
$v_2 = ?$

Note:

- The vertical distance is equal to the arm length plus 3.8cm because the object is launched when the arm is 90º to the horizontal.
- For the vertical, down is positive.
- Time is actually not needed because it can be solved for with the kinematic equations.

Solve for v_2:

$v_2^2 = 2ad$
$= \sqrt{(2)(9.8)(0.408)}$
$= 2.83m/s$

Solve for t:

$d = 0.5at^2$
$t^2 = d \div 0.5a$
$t = \sqrt{(d \div 0.5a)}$
$t = \sqrt{(0.408 \div 0.5(9.8))}$
$t = 0.29s$

Percentage Error Calculation (to show the percent error of the measured time for trial 5 - should be close or greater to that of all the trials):

MV = 0.64s
AV = 0.29s

%Error = ((MV - AV) ÷ AV) x 100
= ((0.64 - 0.29) ÷ 0.29) x 100
= 121%

There was a very high percentage error because the stopwatch used could only measure to the nearest hundredth of a second but mostly because the time interval is so small and there was high error in the start and stop of the stopwatch.

<u>Horizontal (Projectile):</u>
$d = 1.885m$
$t_H = t_V = 0.29s$
$v_H = ?$

$v = d \div t$
$= (1.885) \div (0.29)$
$= 6.50m/s$

<u>Centripetal:</u>
$v = 6.5m/s$
$m = 0.027kg$
$r = 0.37m$
$F_C = ?$

Solve for centripetal force
$F_C = (mv^2) \div r$
$= (0.027)(6.5^2) \div (0.37)$
$= 3.08N$

<u>*Solve for impact velocity:*</u>
$v_2 = 2.83m/s$
$v_H = 6.50m/s$
$v_I = ?$

$v_I = \sqrt{(2.83^2 + 6.50^2)}$
$= 7.09m/s$

$\vartheta = \tan^{-1}(v_2 \div v_H)$
$= \tan^{-1}(2.83 \div 6.50)$
$= 23.53°$

$\therefore v_I = 7.09m/s$ [R 23.53° D]

<u>Note:</u> Right is the direction that the object travelled

***Data taken from trial 14 for the second of the two sets of calculations because it is closest to the data of the trend line at a given point after the max**

<u>Vertical (Projectile):</u>
$d = 0.55 + 0.038$
$= 0.588m$
$v_1 = 0m/s$
$t_V = t_H$
$a = 9.8m/s^2$
$v_2 = ?$

Solve for v_2:
$v_2^2 = 2ad$
$= \sqrt{(2)(9.8)(0.588)}$
$= 3.39m/s$

Solve for t:
$d = 0.5at^2$
$t^2 = d \div 0.5a$
$t = \sqrt{(d \div 0.5a)}$
$t = \sqrt{(0.588 \div 0.5(9.8))}$
$t = 0.35s$

<u>Horizontal (Projectile):</u>
$d = 1.549m$
$t_H = t_V = 0.35s$
$v_H = ?$

$v = d \div t$
$= (1.549) \div (0.35)$
$= 4.43m/s$

<u>Centripetal:</u>
$v = 4.43m/s$
$m = 0.027kg$
$r = 0.55m$
$F_C = ?$

Solve for centripetal force
$F_C = (mv^2) \div r$
$= (0.027)(4.43^2) \div (0.55)$
$= 0.96N$

Solve for impact velocity:
$v_2 = 3.39m/s$
$v_H = 4.43m/s$
$v_I = ?$

$v_I = \sqrt{(3.39^2 + 4.43^2)}$
$= 5.58m/s$

$\vartheta = \tan^{-1}(v_2 \div v_H)$
$= \tan^{-1}(3.39 \div 4.43)$
$= 37.42°$

$\therefore v_I = 5.58m/s$ [R 37.42° D]

	Arm Length (m)	Distance Travelled (m)	Time (s)	Impact Velocity (m/s)	Centripetal Force (N)
Calculations #1 (Trial 5)	0.37	1.885	0.29	7.09m/s [R 23.53° D]	3.08
Calculations #2 (Trial 14)	0.55	1.549	0.35	5.58m/s [R 37.42° D]	0.96

Table 3 - Comparison of the data in the two sets of calculations (from trial 5 and 14)

This data shows the difference between the data from before the maximum point and after the maximum point. The arm length and distance travelled where measured in the experiment but the time, impact velocity and centripetal force where all calculated.

The time is greater after the maximum point which makes sense because in the experiment the trajectory after the maximum point had a larger vertical displacement, meaning that it travelled in the air longer. The impact velocity is lower after the maximum point because the time is greater meaning that the horizontal component of the impact velocity was lower after the maximum point. Lastly, the centripetal force was greater before the maximum point which makes sense because the impact velocity was greater before the maximum point maximum point. However, you would think that a larger centripetal force would result in a larger distance travelled but that is not the case.

Therefore the time increases as the arm length increases; and the distance travelled increases as the arm length increases up to the maximum point and then decreases as the arm length increases.

Conclusions:

Throughout this experiment there is evidence to support that there is a parabolic relationship between the object that a distance travels and the arm length, with a maximum point somewhere between 42 cm and 47 cm. The max point may change for different sizes of catapults (the exact value will definitely change for different size catapults but the ratio between the arm length and the size of the catapult may also change) but this experiment shows that there should be a maximum point for all catapults. Although only one size catapult was used in this experiment there were many trials done for multiple arm lengths so it would make sense that the data would be similar for different sizes of catapults; however, the only way to be certain would be to repeat the experiment with different size catapults and different size masses. This data is also only for torsion based catapults, also known as onager catapults, and the data may vary for different types of catapults such as ballista.