ECON1002: Introductory Macroeconomics

Lecture 11: Economic Growth II

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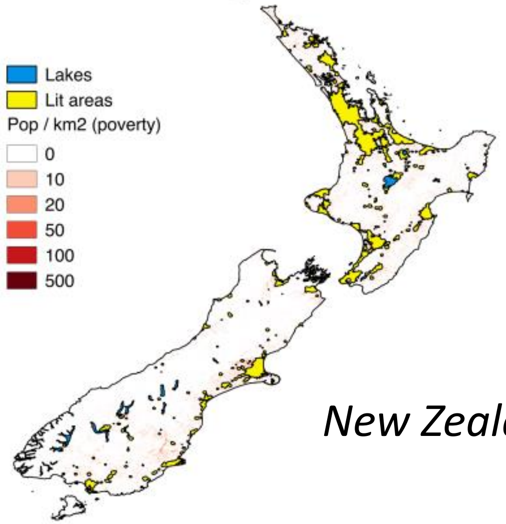
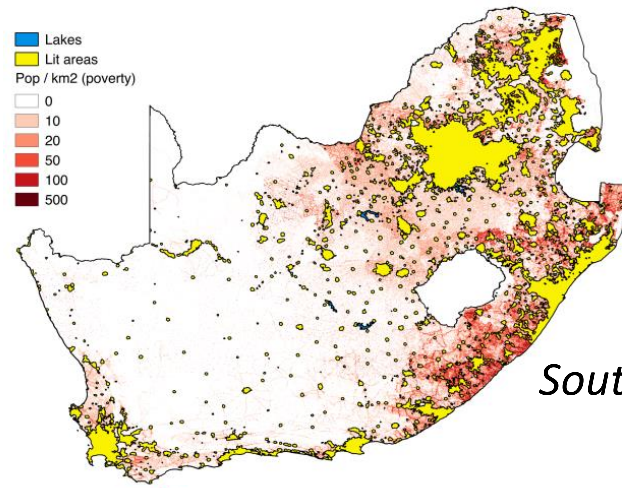
Chapter 12 continued

Last lecture we asked: why are some countries rich and others poor?

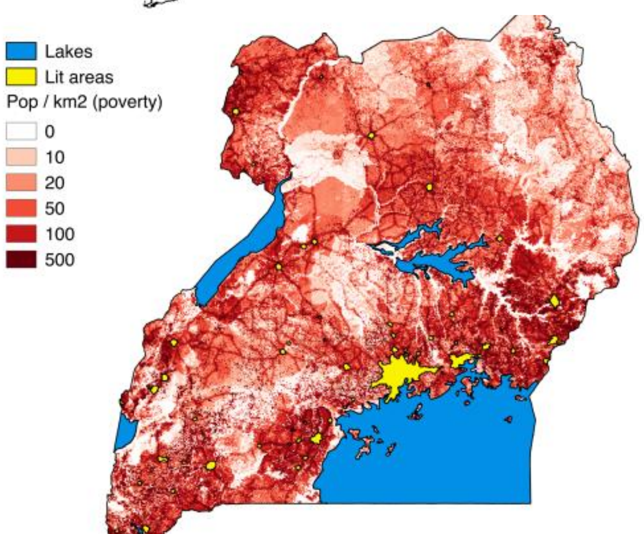
People living in darkness at night



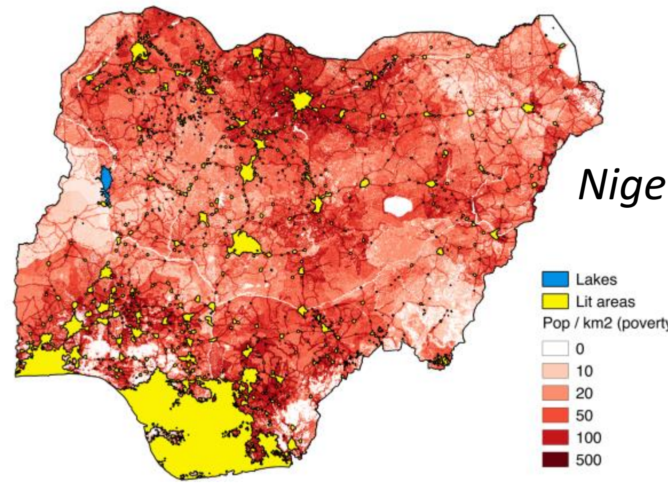
New Zealand



South Africa



Uganda



Nigeria

Source: Smith and Wills (2016)

There are four reasons: labour, land, capital, and technology, which are called the "factors of production"

1. Labour
- The number of people

2. Land
- The amount and quality of land, natural resources, waterways, etc

3. Capital
- Capital is stuff that is accumulated by investment

i. Physical
- The physical tools labour uses

ii. Human
- The talent/education/skills of labour

4. Technology
- Technology is the ability to combine other factors in useful ways

i. Inventions
- Combining capital

ii. Institutions
- Combining labour

iii. Management
- Combining labour and capital

We can represent these factors in a production function, which lets us study useful things like their "marginal product"

Cobb-Douglas Production Function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Marginal Product of Capital

$$MP_k = \frac{\partial Y}{\partial K} = \alpha A_t K_t^{\alpha-1} L^{1-\alpha} = \alpha \frac{Y_t}{K_t}$$

Note: we use the partial derivative, ∂, rather than total derivative, d, because we ignore changes in other factors like labour and technology

Marginal Product of Labour

$$MP_L = \frac{\partial Y}{\partial L} = (1-\alpha) A_t K_t^{\alpha} L^{1-\alpha-1} = \frac{(1-\alpha) A_t K_t^{\alpha} L^{1-\alpha}}{L_t} = (1-\alpha)\frac{Y_t}{L_t}$$

If each factor is used in production, it has to get paid. How much of the income from production does each factor earn?

After the wheat is sold, how income goes to:
- the farmer?
- the tractor owner?
- the farm owner?

Factors earn their marginal product in a perfectly competitive environment (not always the case), because firms maximize profits

To find what a firm will pay their workers, let's study how the firm makes profits

$$\Pi_t = PY_t - rK_t - wL_t$$

Profit equals price x quantity produced, less rent paid to capital, less wages paid to workers

The firm will add workers until they maximize profits, so adding another worker won't increase profit any more

$$\frac{\partial \Pi_t}{\partial L_t} = P\frac{\partial Y_t}{\partial L_t} - 0 - w$$

Take partial derivative w.r.t labour

$$= 0$$

Set marginal profit to zero

$$\frac{\partial Y_t}{\partial L_t} = \frac{w}{P}$$

Rearrange

Marginal product of labour = real wage

..or..

Marginal Benefit = Marginal Cost

Same process for capital

The Cobb-Douglas function means that a fixed share of income goes to labour (1- α) and capital (α).

Combining what we have done on the previous slides, the amount paid to each workers is:

$$MP_L = \frac{\partial Y_t}{\partial L_t} = \frac{w}{P}$$
$$= (1-\alpha)\frac{Y}{L}$$

The amount paid to all workers is:

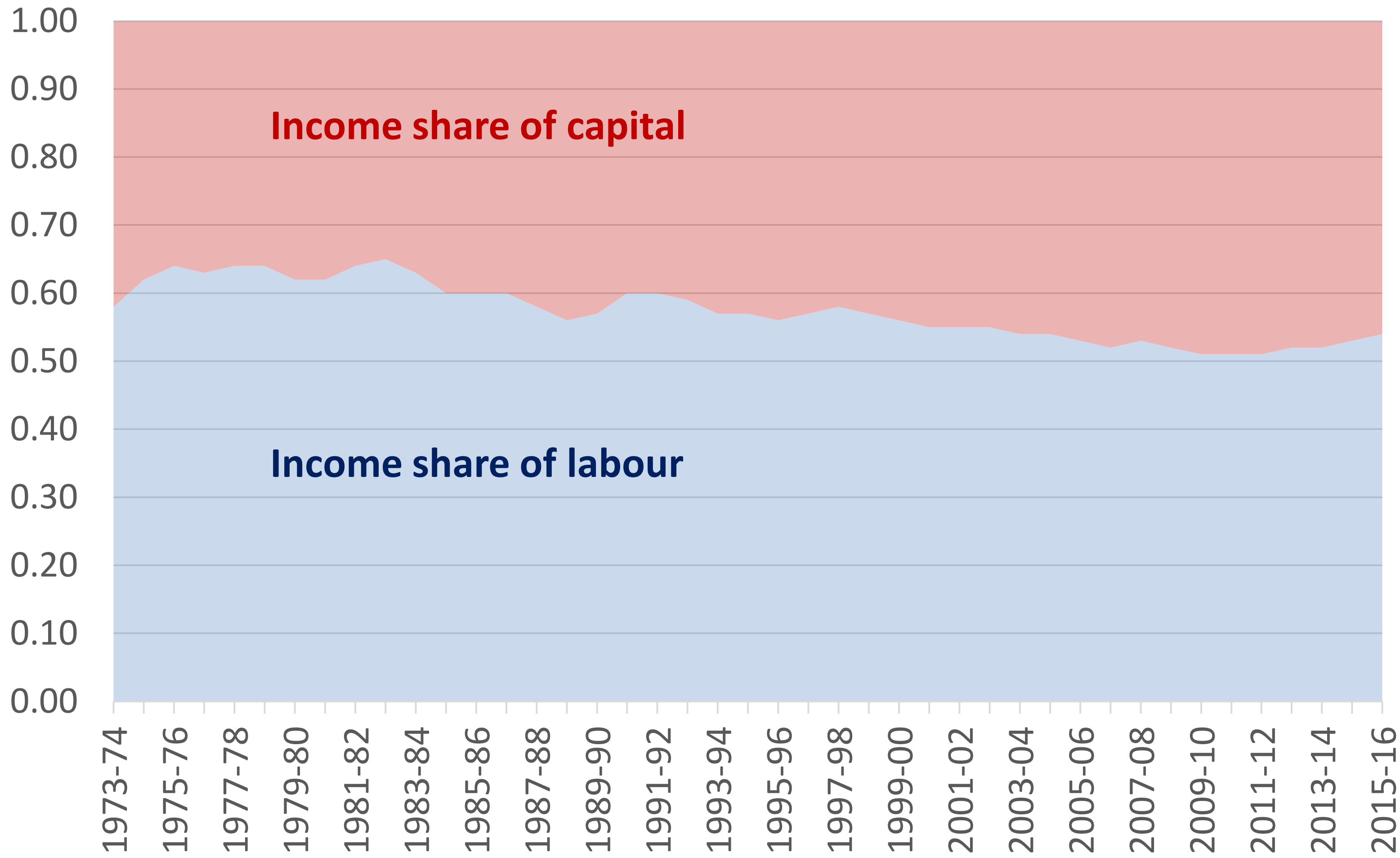
$$(1-\alpha)Y = \frac{wL}{P}$$

Therefore, in the Cobb-Douglas production function:

- (1-α): total share of income that goes to labour
- α: total share of income that goes to capital

The share of Australia's total income going to labour has fallen over the past 40 years, which can lead to higher inequality

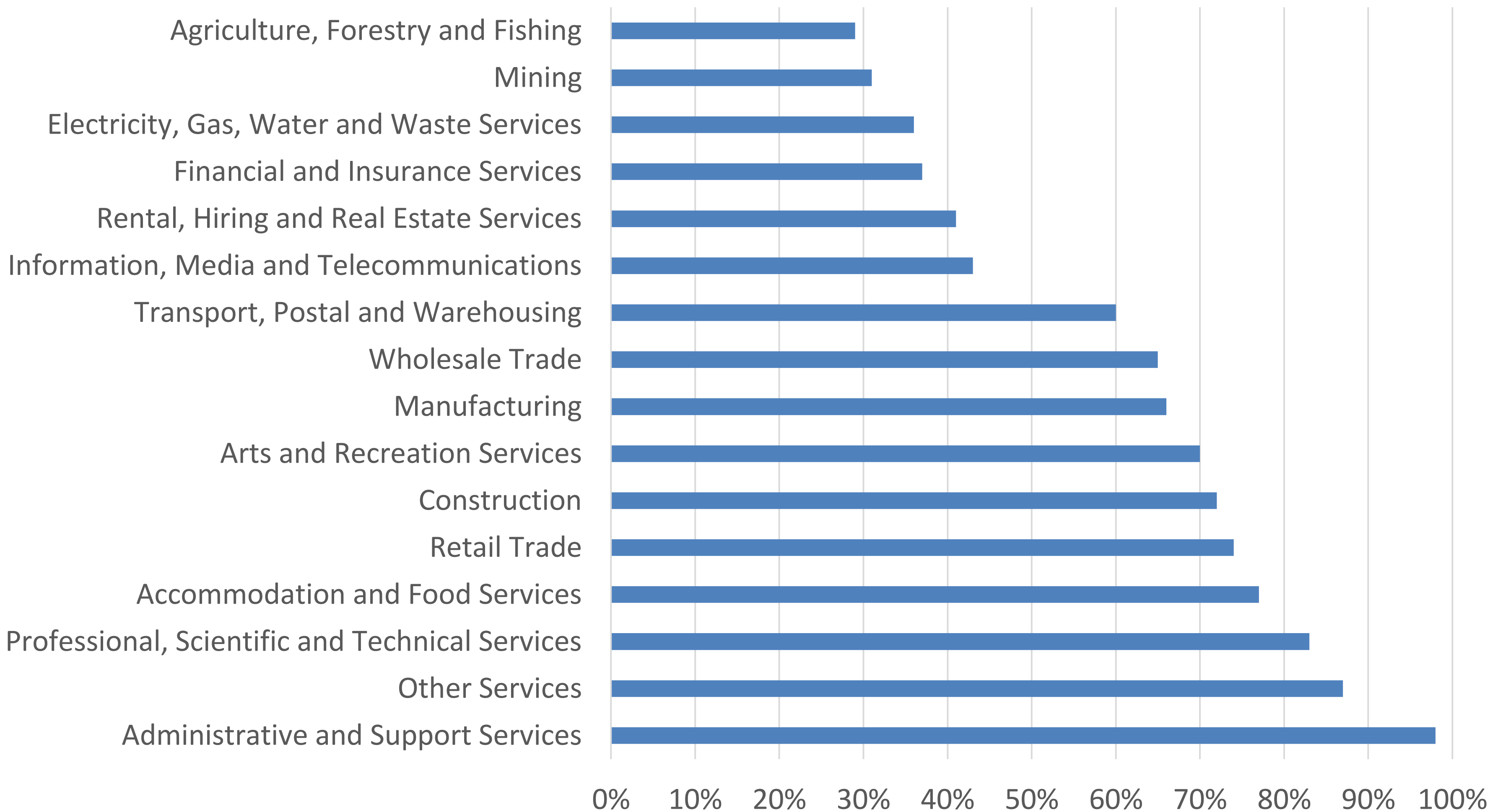
Two period average income share, Australia, 1973-2016, %



Source: ABS

The labour share in income also varies across industries, depending on how labour intensive they are. Farmers don't get much unless they own it

Average labour income share by industry, Australia, 2015-2016, %



Source: ABS

What happens when we increase the amount of factors used in production?

In the Cobb-Douglas function, adding 50% more labour and 50% more capital raises output by 50%. This is called "constant returns to scale".

Example: Constant Returns to Scale

Starting with initial levels of capital and labour:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

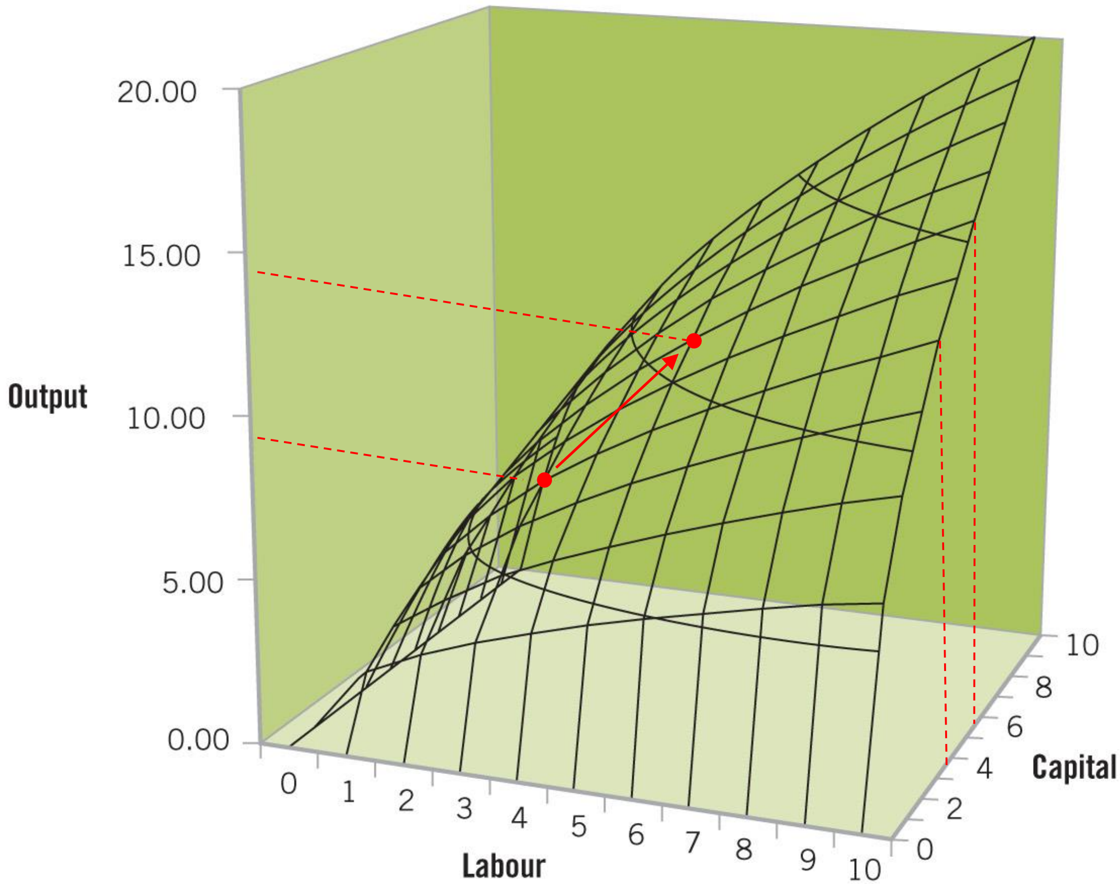
If we add 50% to capital and 50% to labour then…

$$\begin{aligned} Y'_t &= A_t (1.5K_t)^{\alpha} (1.5L_t)^{1-\alpha} \\ &= 1.5A_t (K_t)^{\alpha} (L_t)^{1-\alpha} \\ &= 1.5Y_t \end{aligned}$$

We also add 50% to output.

This can be illustrated by moving diagonally up the surface plot

The Cobb-Douglas production function on a surface plot



If only capital increases by 50%, then output will increase by approximately α x 50%

Example: Returns to a single factor

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Take the partial derivative with respect to capital:

$$\frac{\partial Y_t}{\partial K_t} = A_t \alpha K_t^{\alpha-1} L_t^{1-\alpha}$$

Substitute in the definition of Y

$$= \alpha \frac{Y_t}{K_t}$$

Rearrange

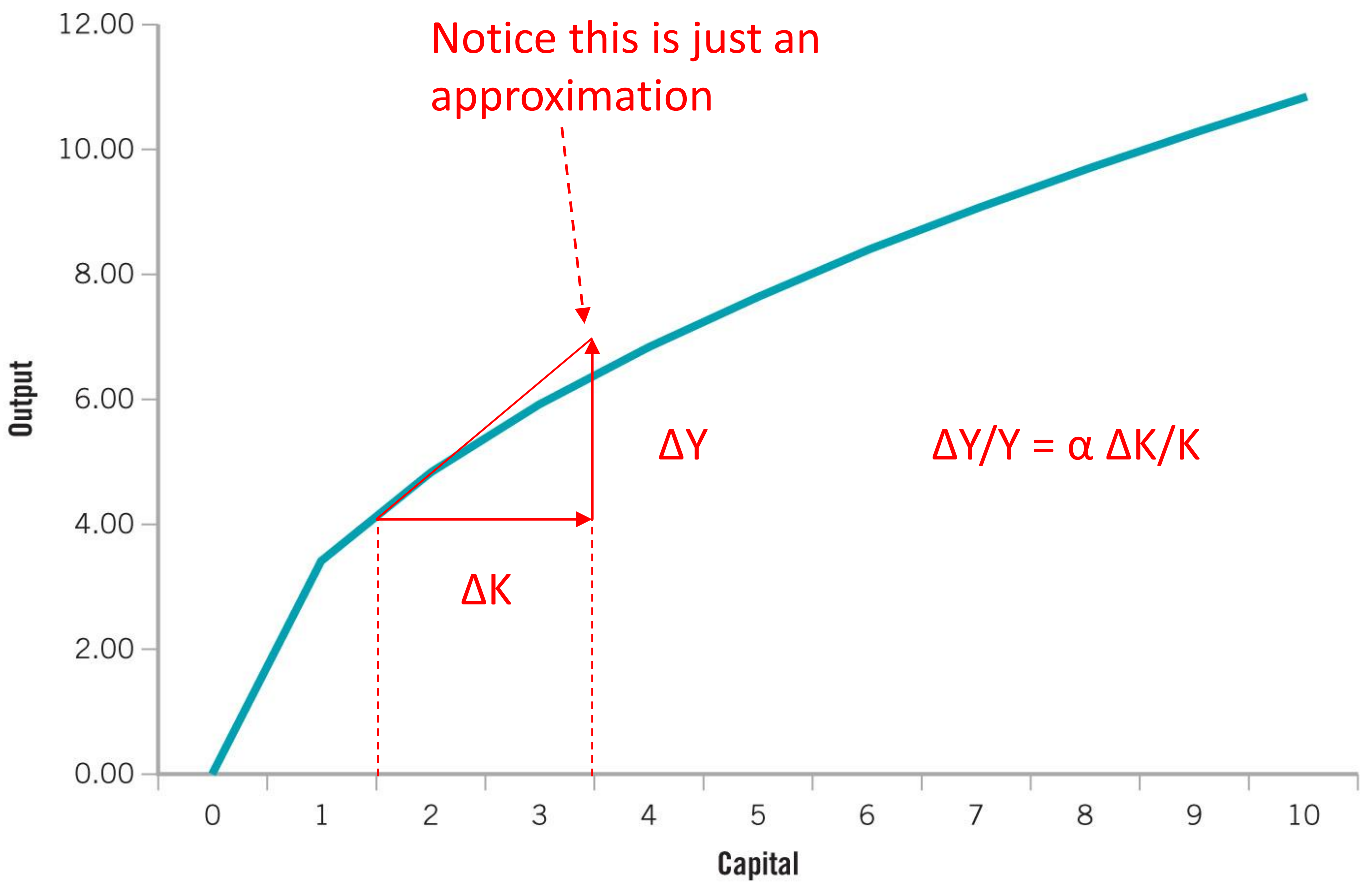
$$\frac{\partial Y_t}{Y_t} = \alpha \frac{\partial K_t}{K_t}$$

Approximate, for small changes in K.

$$\frac{\Delta Y_t}{Y_t} \approx \alpha \frac{\Delta K_t}{K_t}$$

Same process for labour

The returns to an additional unit of capital can be illustrated on a 2 dimensional plot



Productivity is usually taken to mean “labour productivity” or “total factor productivity”

Labour Productivity

- The amount produced by one unit of labour:

$$Labour\ Productivity = A_t K_t^{\alpha}$$

Total Factor Productivity (TFP)

- The amount produced by one unit of all factors – except technology:
- Describes the efficiency with which factors are combined (see how we defined technology)

$$Total\ Factor\ Productivity = A_t$$

An increase in TFP (technology) increases output by the same amount

Example: Returns to TFP

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Take the partial derivative with respect to TFP:

$$\frac{\partial Y_t}{\partial A_t} = K_t^{\alpha} L_t^{1-\alpha}$$

Substitute in the definition of Y

$$= \frac{Y_t}{A_t}$$

Rearrange

$$\frac{\partial Y_t}{Y_t} = \frac{\partial A_t}{A_t}$$

In discrete terms

$$\frac{\Delta Y_t}{Y_t} = \frac{\Delta A_t}{A_t}$$

Growth accounting attempts to estimate where the sources of growth come from

Growth Accounting Framework

Income shares of capital and labour (observed)

TFP/Technology growth (the "Solow residual")

$$\frac{\Delta y_t}{y_{t-1}} = \alpha \frac{\Delta k_t}{k_{t-1}} + (1-\alpha)\frac{\Delta l_t}{l_{t-1}} + \frac{\Delta A_t}{A_{t-1}}$$

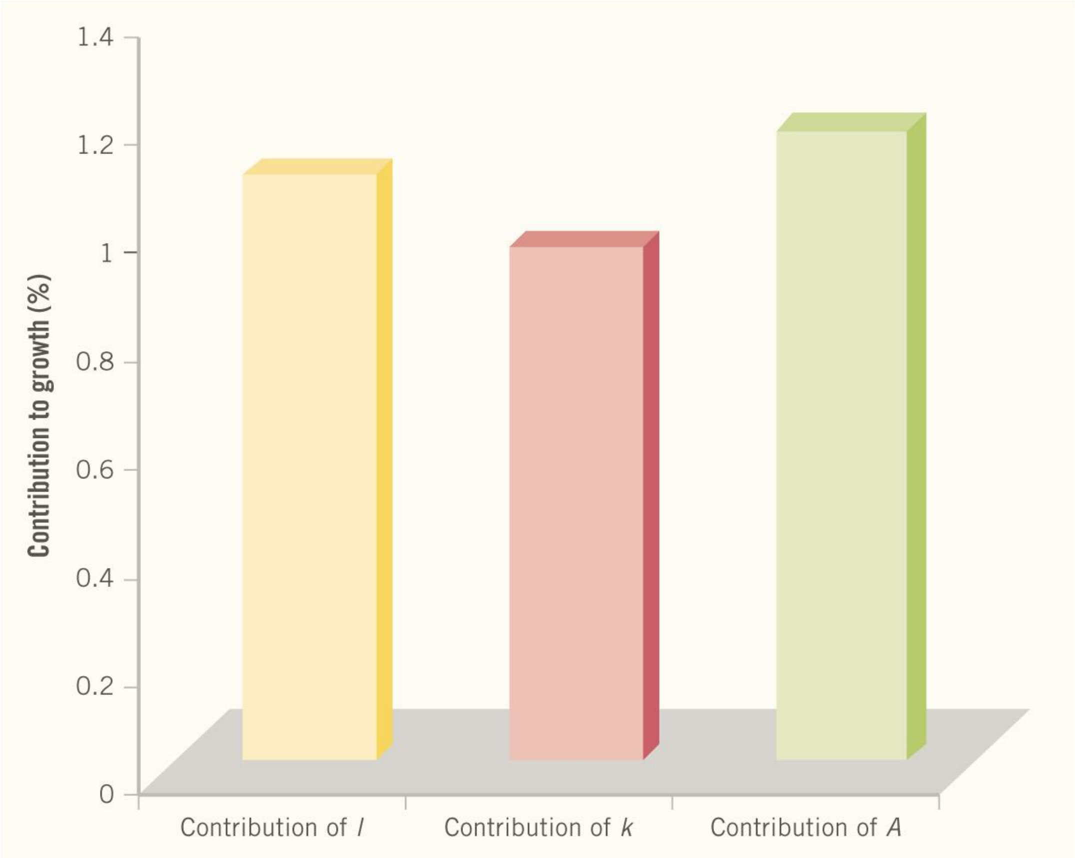
GDP growth (observed)

Capital growth (observed)

Labour growth (observed)

In Australia, growth in TFP and labour have been the most important for overall GDP growth

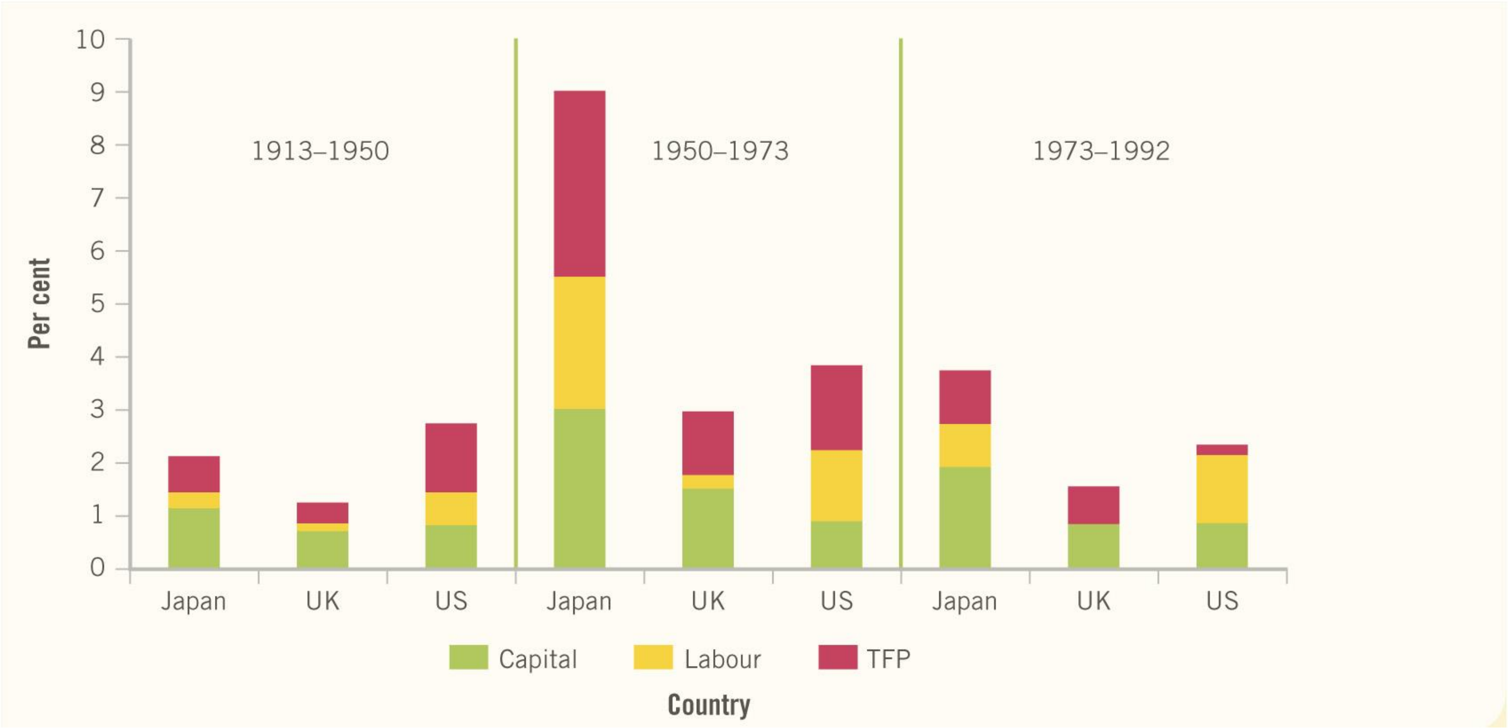
Growth accounting: Contribution to GDP growth, 1976-2009, %



Source: BOF Ch. 10

In contrast, capital construction was crucial in Japan in the post-WWII years

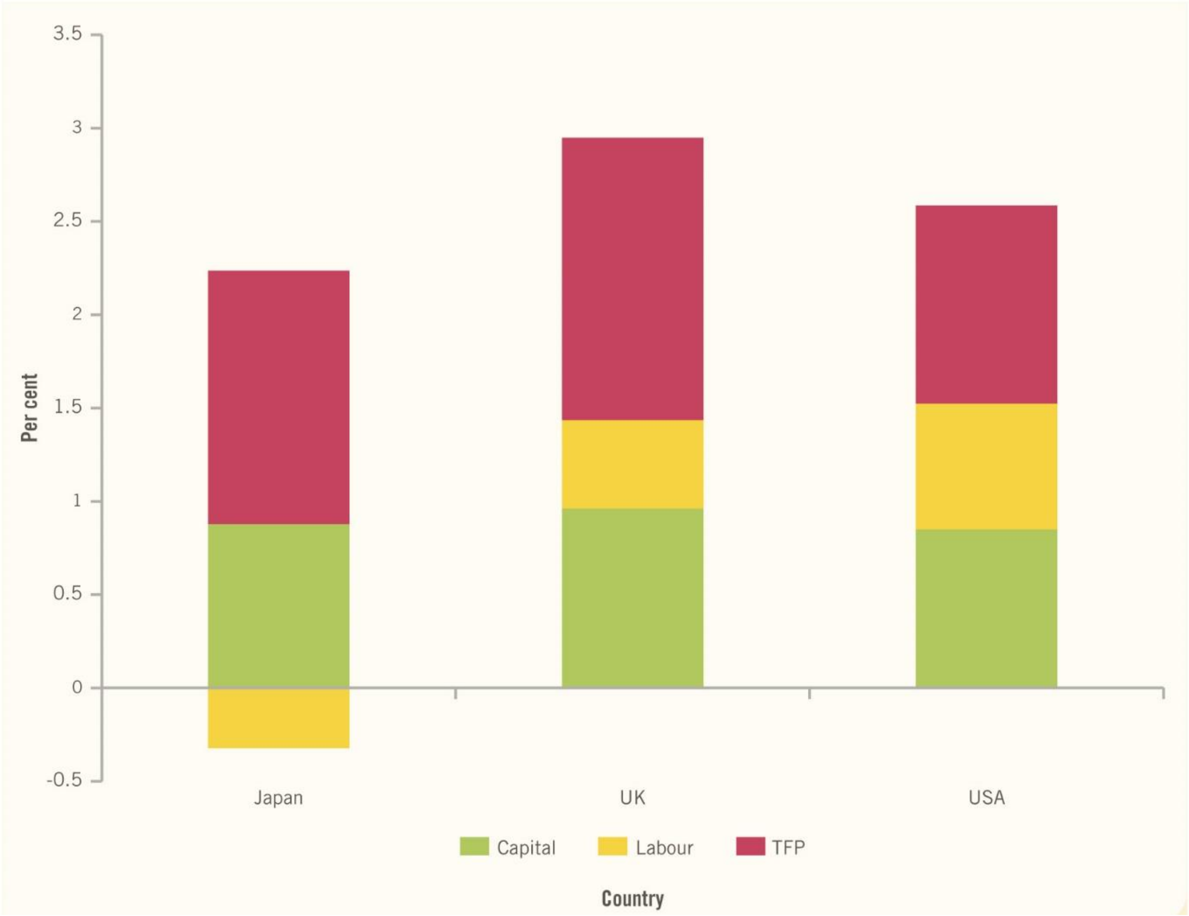
Growth accounting: Contribution to GDP growth, 1913-1992, %



Source: BOF Ch. 10

While in recent years Japan's shrinking population has been a drag on overall growth

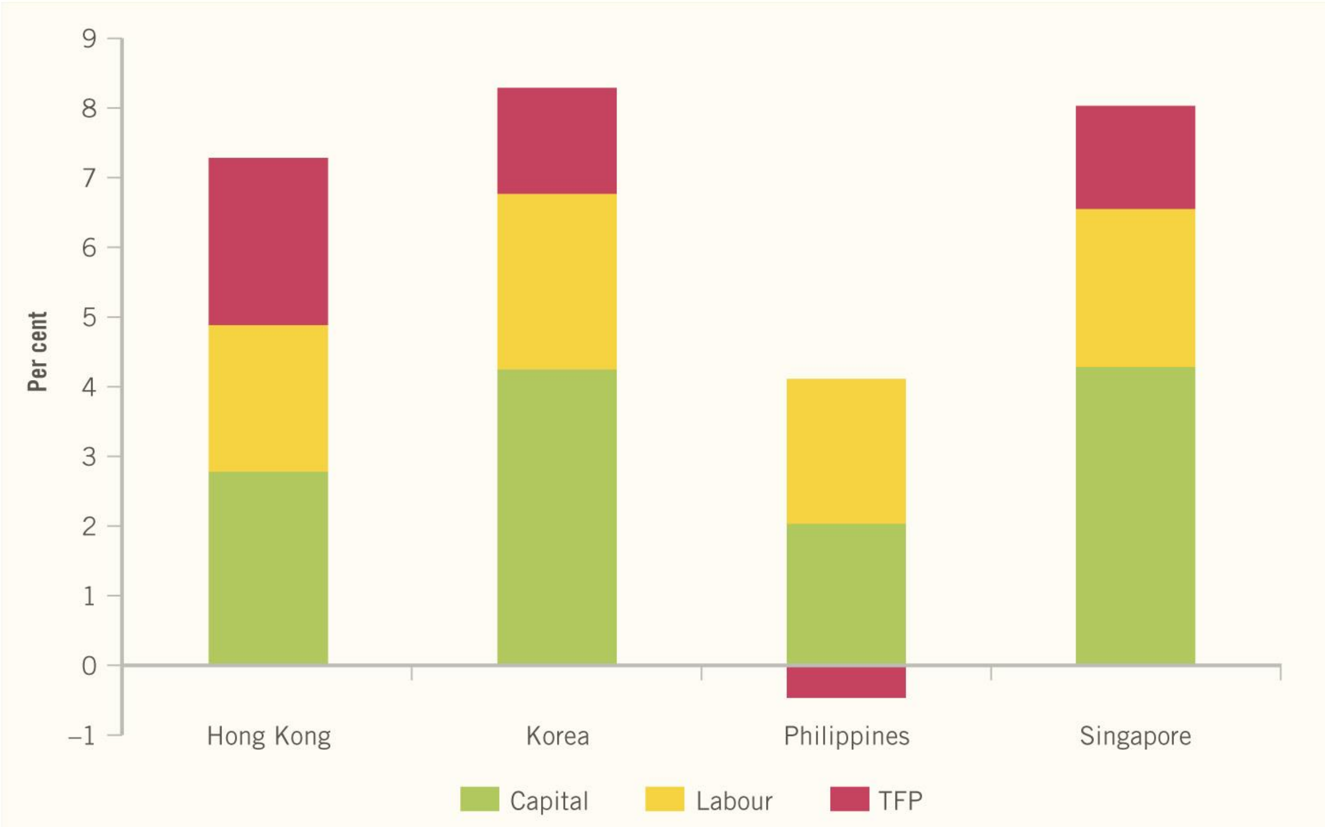
Growth accounting: Contribution to GDP growth, 1985-2010, %



Source: BOF Ch. 10

The growth in the Asian tiger economies from 1960-1994 was mostly driven by capital accumulation

Growth accounting: Contribution to GDP growth, 1960-1994, %



Source: BOF Ch. 10

Summary

- The production function is a representation of the process by which firms combine the primary factors of production, capital and labour, in order to produce output.
- The demand for capital and labour are determined by marginal product of labour and capital, which is subjected to diminishing returns.
- The market for capital is in equilibrium when the rental rate is equal to the marginal product of capital.
- The market for labour is in equilibrium when the real wage rate is equal to the marginal product of labour.
- The Cobb–Douglas production function is often used in economic analysis.
- Growth accounting is the name given to the empirical analysis of the relative contributions made by capital, labour and total factor productivity to a country's rate of economic growth.

Chapter 13

Saving, capital formation and comparative growth

Learning Objectives

13.1 Based on the national income accounting identity, what is the relationship between investment and national saving?

13.2 How can a production function be written in per capita terms?

13.3 What does the graph of a per capita production function look like?

13.4 What is a saving function?

13.5 What is the economy's steady state?

13.6 In what sense do countries converge?

13.7 According to the Solow–Swan model, what is the economy's long-run rate of growth?

13.8 What role does total factor productivity play in promoting long-run growth?

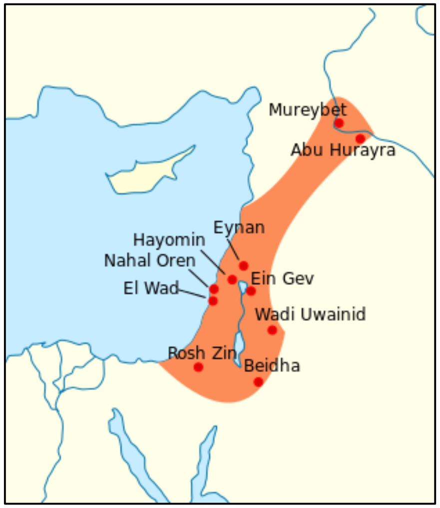
13.9 What is the Solow paradox?

Last lecture we learned that saving surplus, and accumulating capital, was crucial in understanding how economies grow

Natufian storage pit, Nahal Ein Gev II site, Jordan

Natufian culture

- 12,500 – 9,500 BCE
- Some of the earliest agriculture in the world
- Earliest evidence of grain storage



Source: http://www.sci-news.com/archaeology/natufian-village-jordan-valley-03644.html

The Solow-Swan model lets us study how capital explains economic growth. Trevor Swan was one of Australia's finest economists

General 2 factor production function: levels

technology *labour*

$$Y_t = Af\left(K_t, \mathrm{L}_t\right)$$

output *capital*

General 2 factor production function: per capita

Divide each variable above by the labour force:

$$\frac{Y_t}{L_t} = Af\left(\frac{K_t}{L_t}, 1\right)$$

Use lower-case letters to represent per-capita values:

$$y_t = Af\left(k_t, 1\right)$$

Output per worker Capital per worker

Capital per worker changes because of depreciation, more workers, or investment in new capital

The stock of capital per worker only increases with "net investment", after replacing depreciation

Investment in capital

Increase in capital per worker is investment less depreciation less population growth:

$$\Delta k_t = i_t - \delta k_t - n k_t$$

Investment can be split into "net" and "replacement" investment:

$$i_t = ri_t + ni_t$$

"Replacement" investment just makes up for depreciation and population growth:

$$ri_t = (\delta + n) k_t$$

So capital per worker only increases with "net investment":

$$\Delta k_t = ni_t$$

We know from past lectures that investment must be financed by savings in a closed economy

Savings (in a closed economy)

Assume the economy saves a fixed share of output:

$$savings = \theta y$$

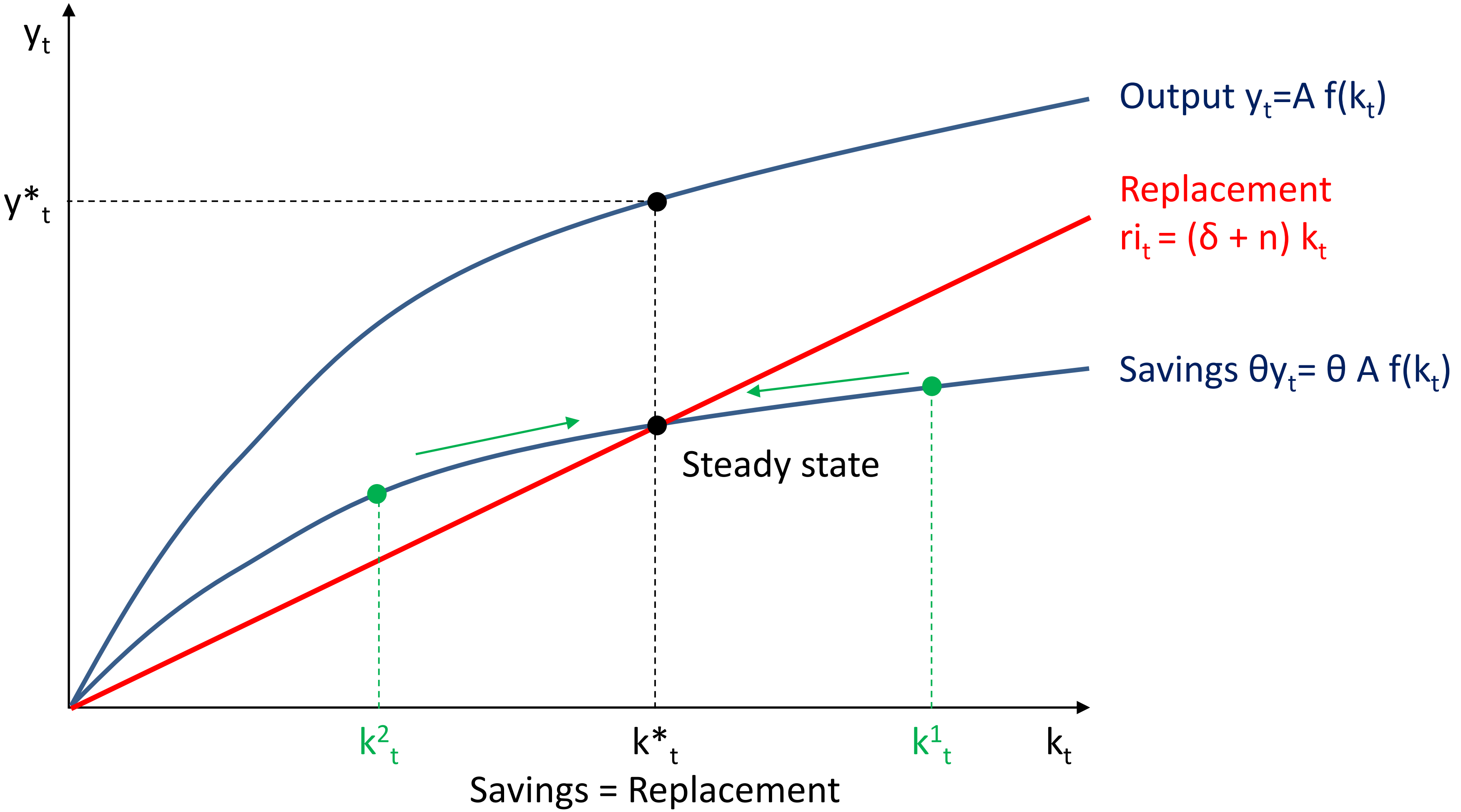
Savings is used to finance investment (both ri and ni):

$$\begin{aligned} \theta y_t &= i_t \\ &= ri_t + ni_t \\ &= (\delta + n)\mathrm{k}_t + \Delta k_t \end{aligned}$$

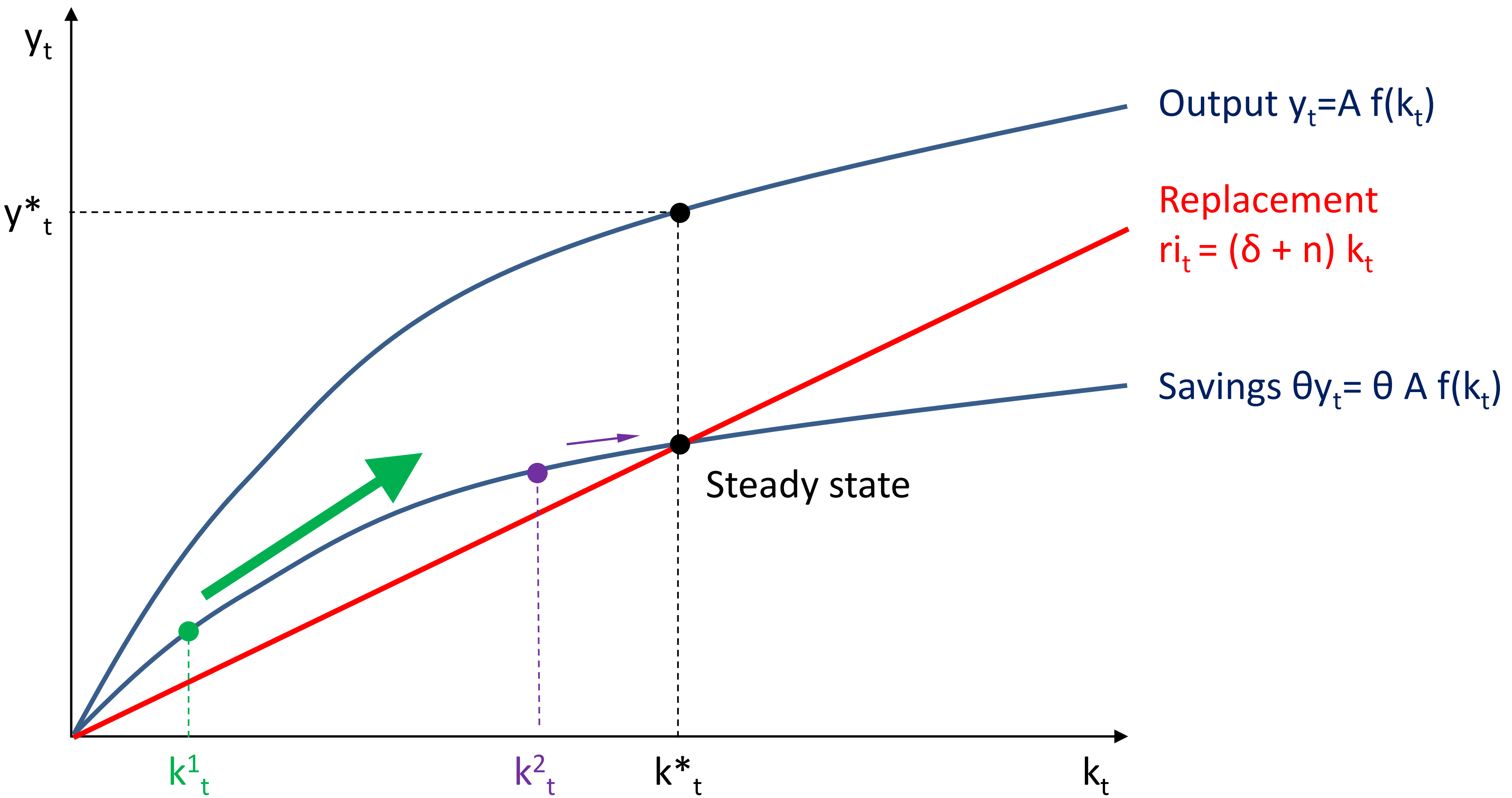
Or, rearranging shows that the change in capital depends on savings less replacement:

$$\Delta k_t = \theta y_t - (\delta + n)\mathrm{k}_t$$

The Solow-Swan diagram illustrates how savings affects GDP. If savings > replacement then the economy grows



The Solow-Swan model predicts that GDP in poor countries will catch-up to rich ones ("convergence"), because they have a higher MPK



There is empirical evidence that in the rich OECD countries, GDP does converge

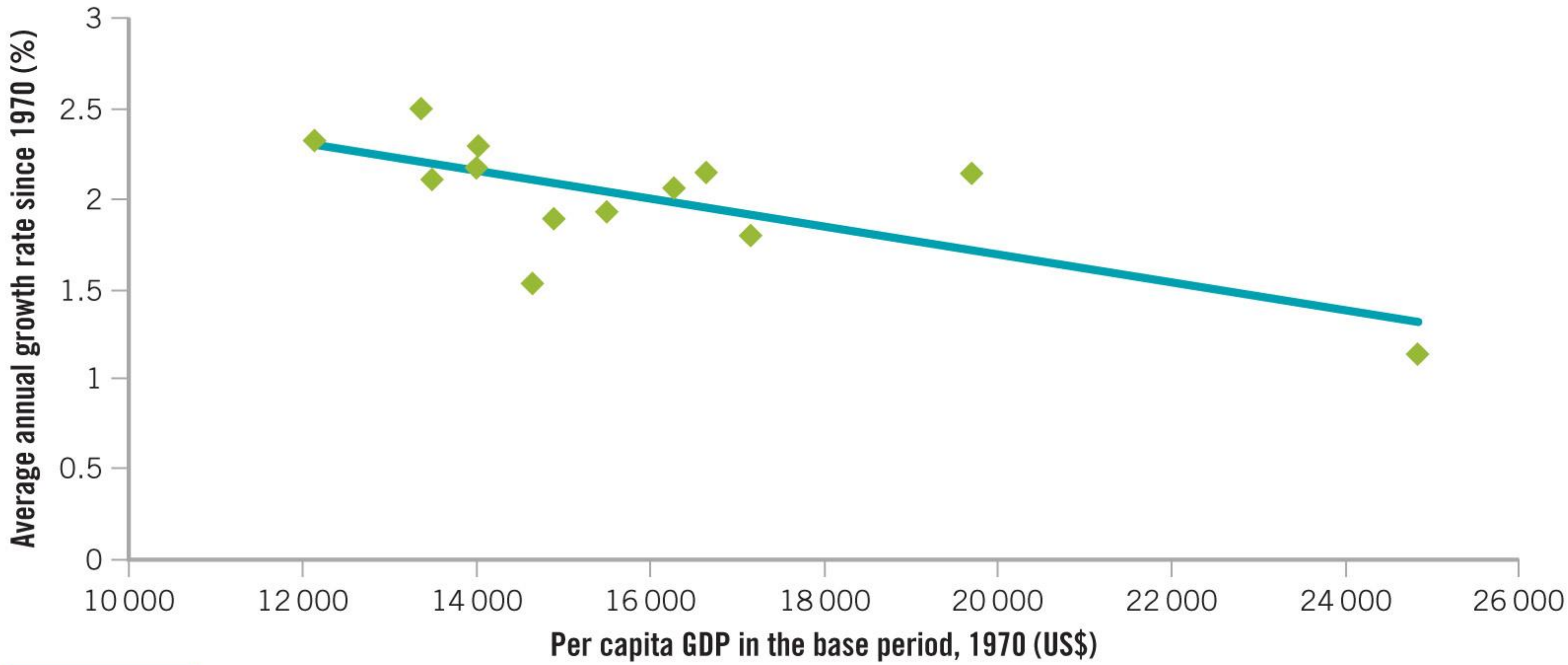


Figure 13.8 **Convergence in high-income OECD countries (1970–2007)** The negative slope of the trend line fitted to these data is consistent with the convergence hypothesis.

However, it doesn't look like poor countries around the world converge…

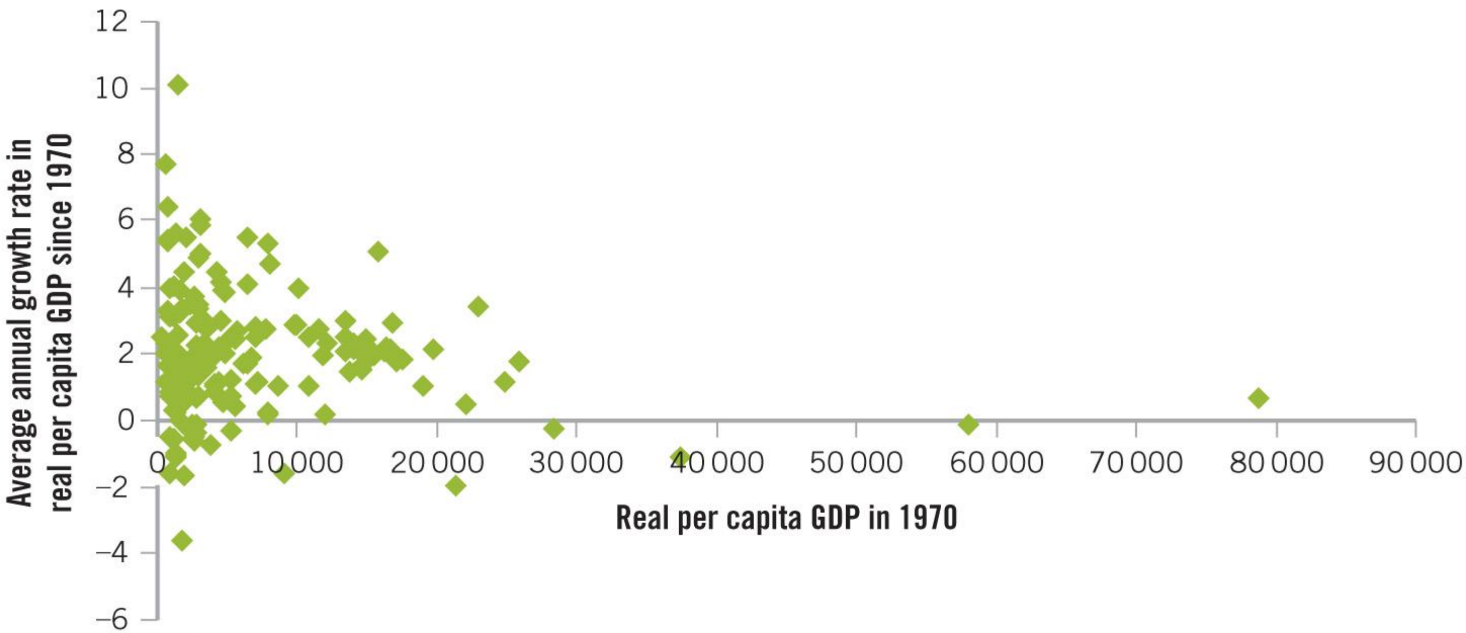


Figure 13.9 **Convergence for the world?** When we take all the world's countries, there seems little evidence in favour of the convergence hypothesis.

Source: Compiled from the Penn World Table 6.3, http://pwt.econ.upenn.edu.

…Unless they are similar and so have similar steady states (eg open economies). This is called "conditional convergence"

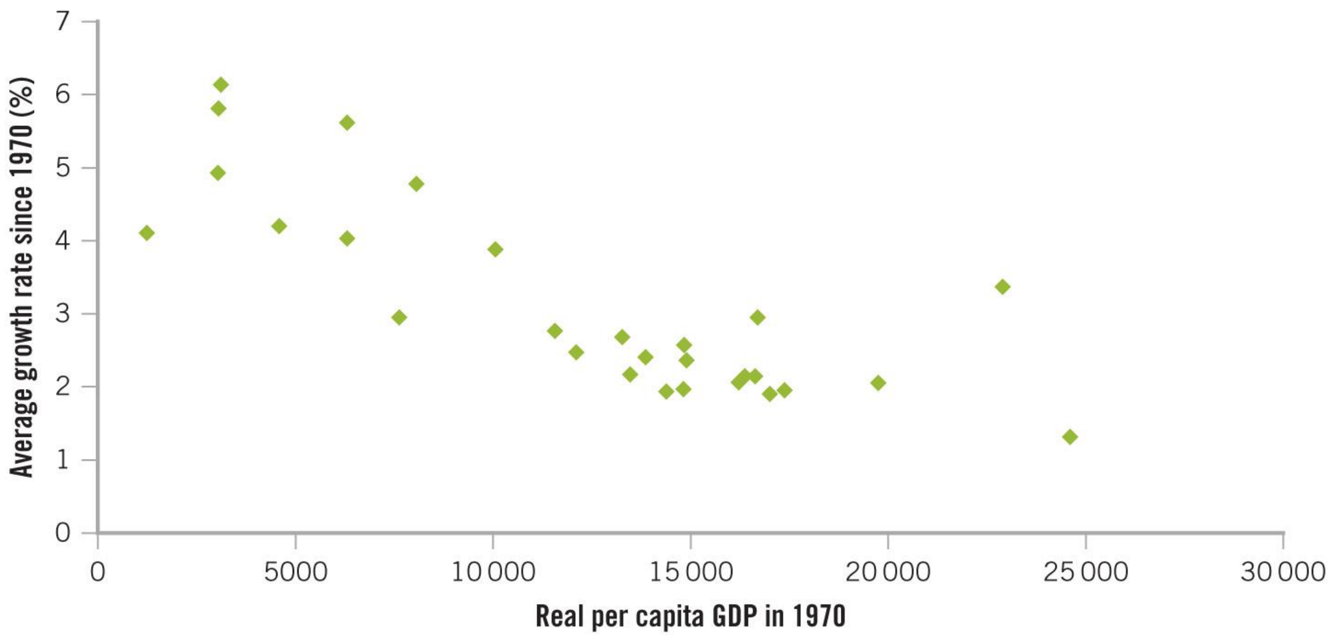
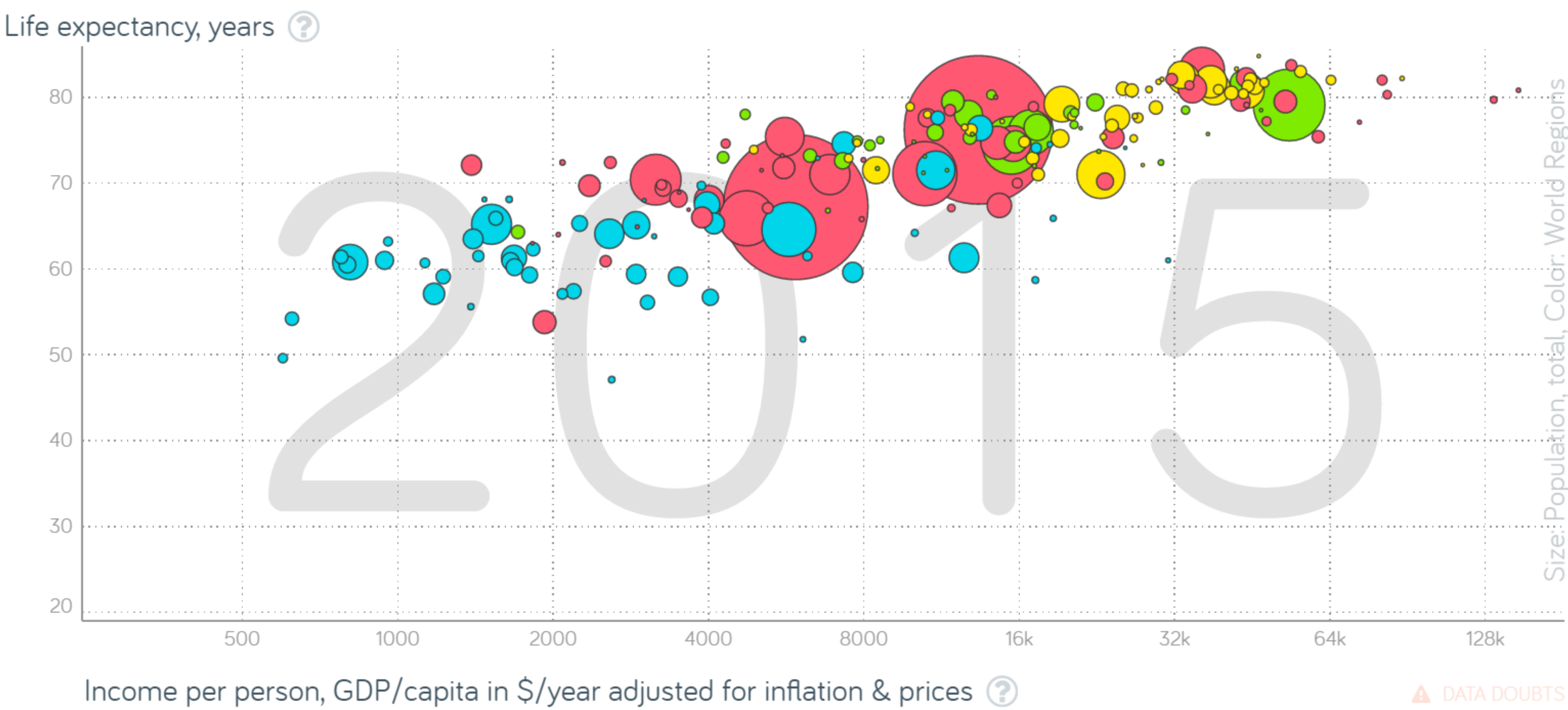


Figure 13.10 **Convergence in the world's open economies** Evidence supporting convergence can be found for economies that are relatively open. Openness facilitates the transfer of technology across countries, making a common steady state more likely than for countries that are relatively closed.

Source: Compiled from the Penn World Table 6.3, http://pwt.econ.upenn.edu.

Convergence also happens with other variables, like life expectancy, as seen with China once it became an open economy

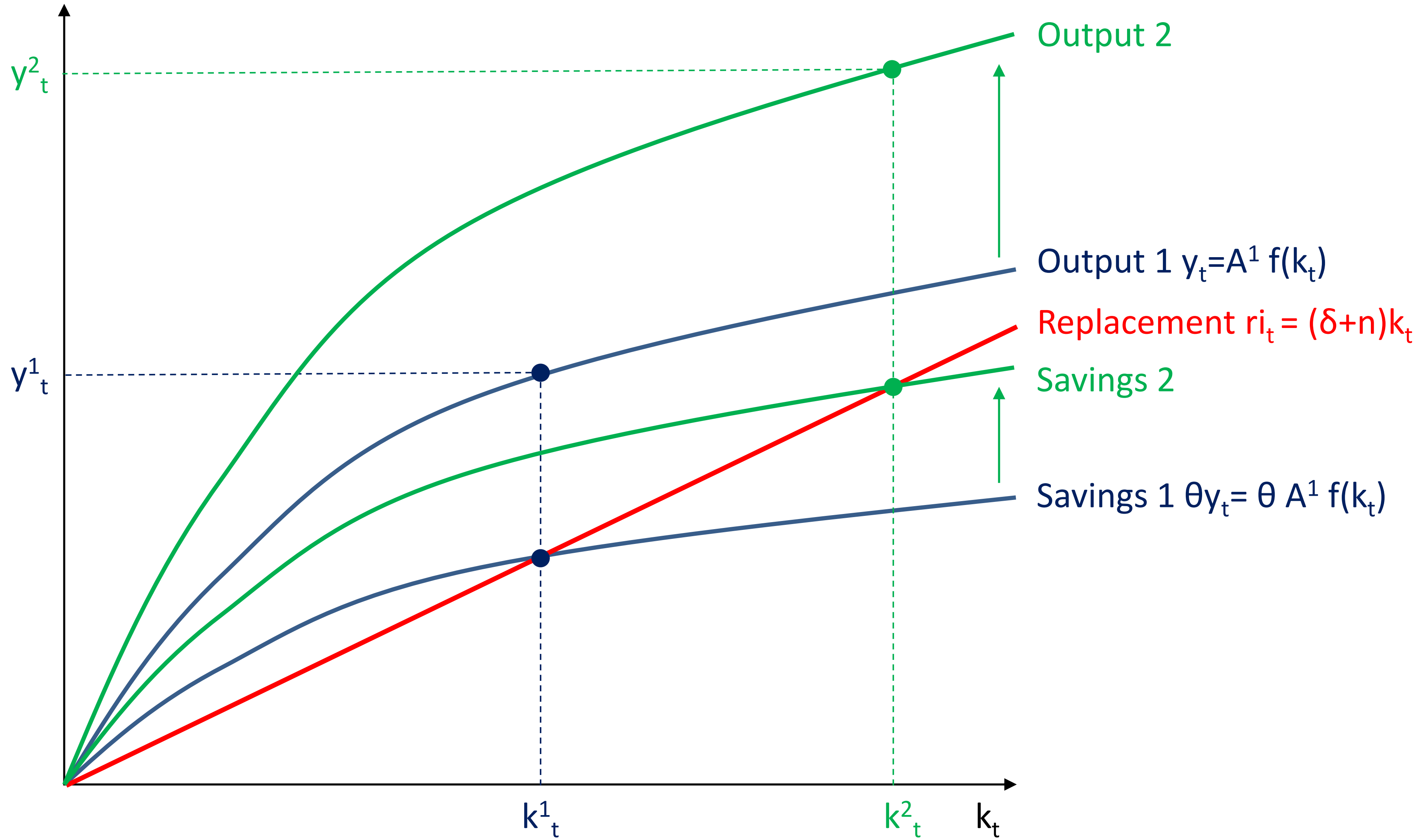
Life expectancy vs GDP/capita, 2015



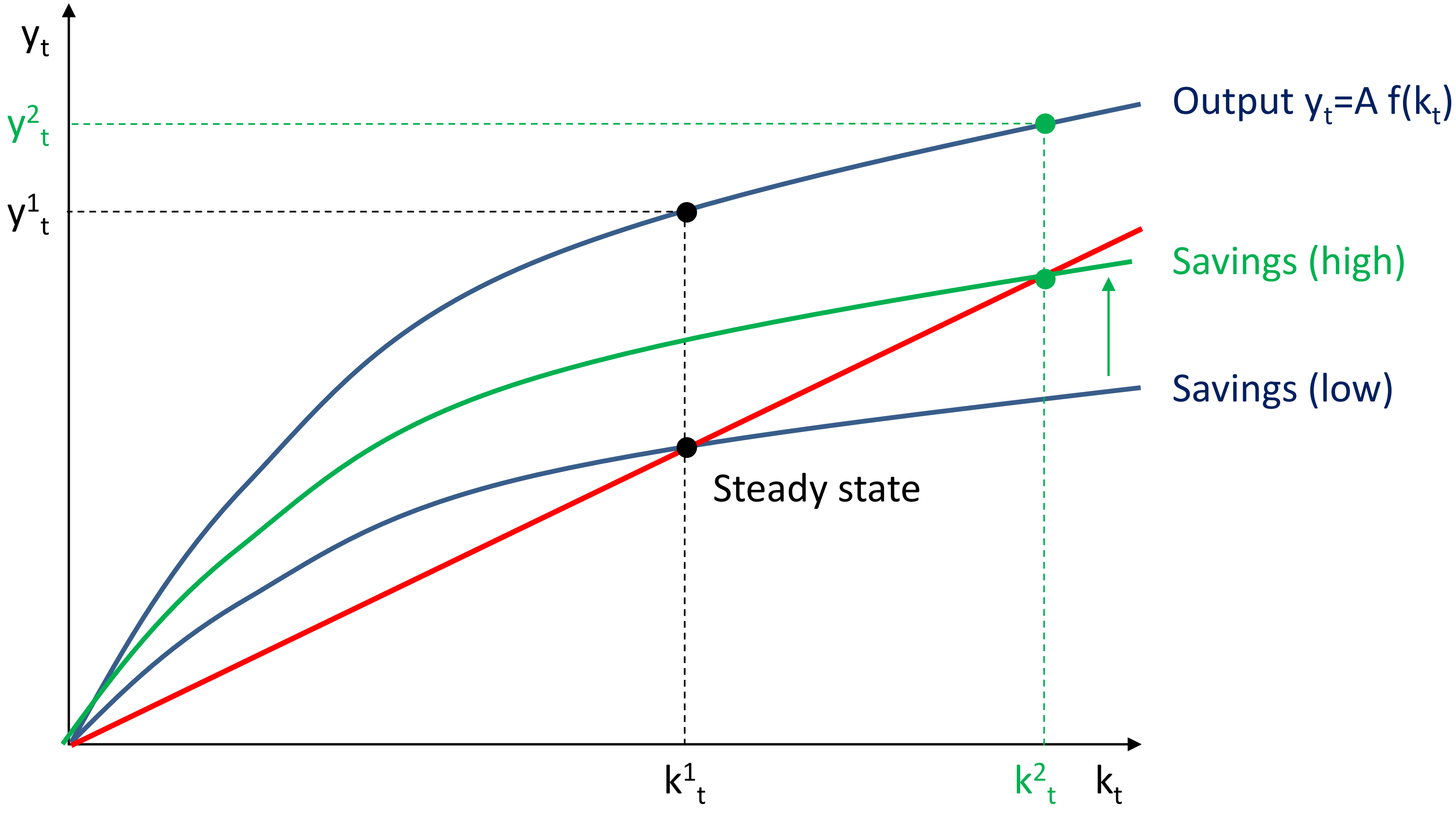
Correlation vs causation: Does GDP cause longer life expectancy, or vice versa?

Source: http://www.gapminder.org/tools/#_chart-type=bubbles

The Solow-Swan model predicts that economies will stop growing when they reach their steady state, unless they increase technology (A)



The Solow-Swan model suggests that increasing savings will increase output in the long run



BUT, Keynes' "Paradox of Thrift" suggests that increasing saving will <u>reduce</u> output. How can we reconcile the two?

The Paradox of Thrift

Savings is income that isn't consumed:

$$S = Y - C$$

However, income is just consumption plus investment:

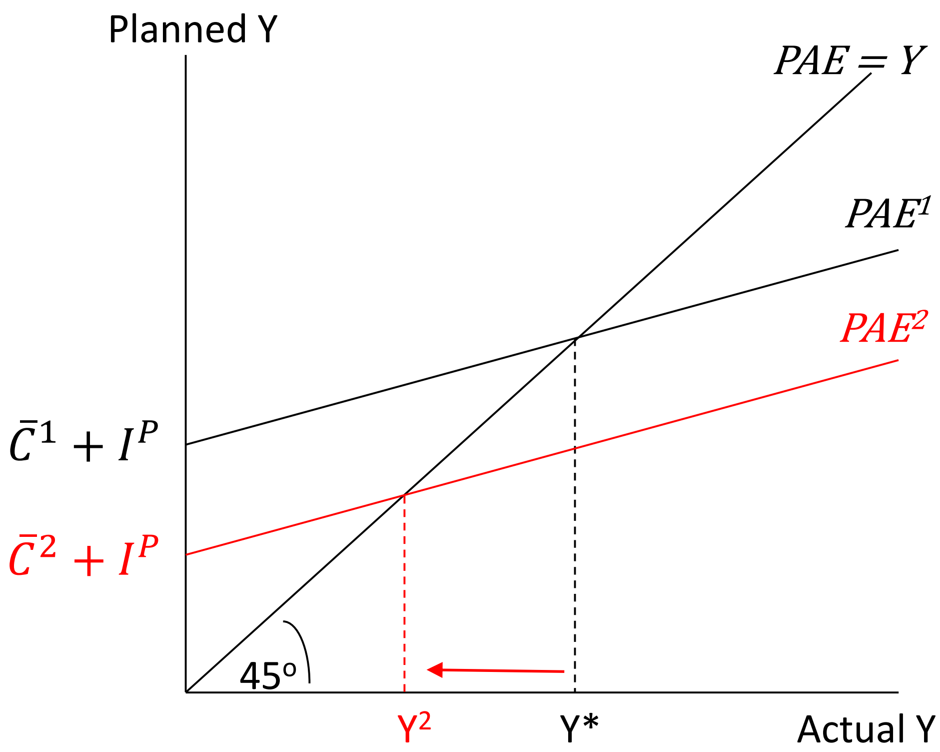
$$Y = C + I^P$$

Therefore, in this closed economy savings must equal investment:

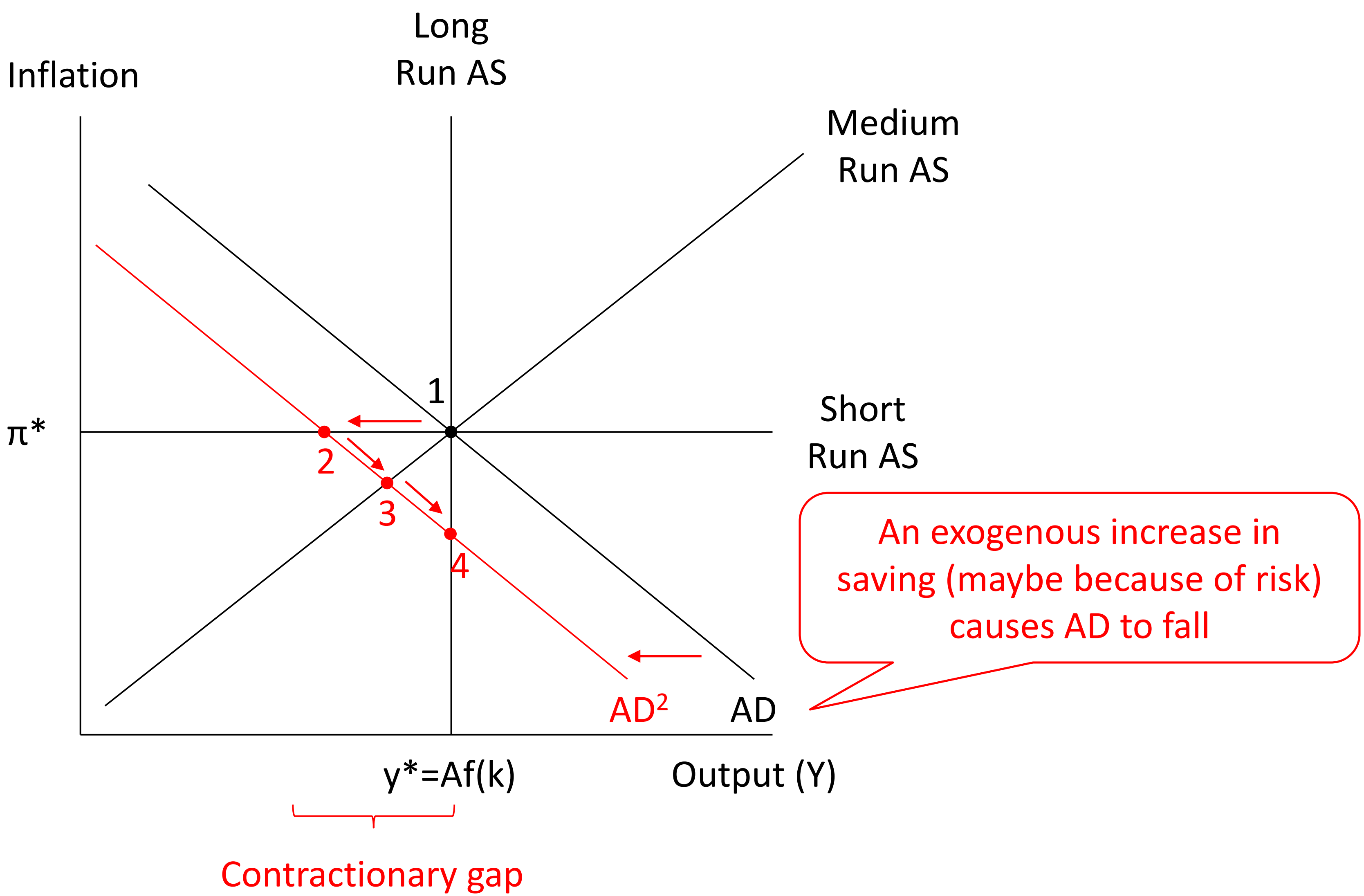
$$S = I^P$$

If $\bar{C}$ falls, both output (Y) and consumption (C) fall by the same amount, so aggregate savings doesn't change because I^P doesn't change.

Illustration

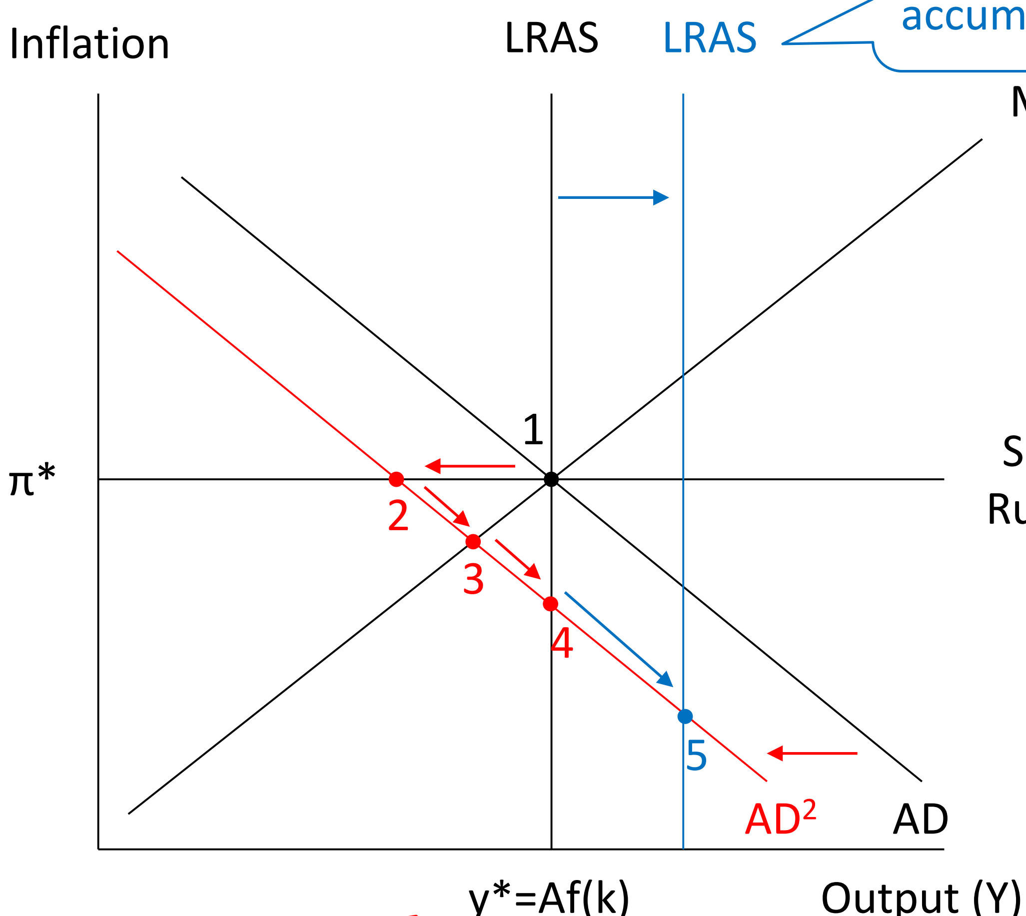


The difference is that the Paradox of Thrift refers to the short run, when supply is determined by demand…



The Solow-Swan refers to the long run, when supply is determined by capital, technology, etc

…and higher savings allows capital to accumulate, which causes LRAS to grow



At point 4, output is the same, but savings is higher…

Summary

- The Solow–Swan model is based on a production function expressed in per capita terms, so that the level of per capita output (or income) depends on total factor productivity and the ratio of capital to labour.

- The Solow–Swan model predicts that there will be no further growth in per capita income once the economy has reached its steady state.

- The law of diminishing marginal productivity of capital means that countries with relatively low per capita capital stocks will grow at a faster rate than countries with high per capita capital stock.

- Countries with similar characteristics tend to converge to the same steady-state capital–labour ratio.