HW2 due 09/18 13.3 35, 52, 67 13.4 24, 30, 42

35. $u=\langle -1,4\rangle$ $v=\langle -4,2\rangle$

$proj_v u = \left(\frac{u\cdot v}{|v|^2}\right)\cdot v = \frac{\langle -1,4\rangle\langle -4,2\rangle}{(\sqrt{16+4})^2}\cdot\langle -4,2\rangle = \frac{4+8}{20}\langle -4,2\rangle = \frac{3}{5}(-4)\vec{i} + \frac{3}{5}2\vec{j} = \frac{-12}{5}\vec{i} + \frac{6}{5}\vec{j}$

$scal_v u = u\cdot\frac{v}{|v|}$

$= \langle -1,4\rangle\cdot\frac{\langle -4,2\rangle}{\sqrt{16+4}} = \frac{4+8}{\sqrt{20}} = \frac{12}{2\sqrt{5}} = 6/\sqrt{5} = \frac{6}{\sqrt{5}}$

52. If they are orthogonal to each other, $v\cdot w=0$

$v\cdot w = \langle 4,-3,7\rangle\langle a,8,3\rangle$

$= 4a + (-3)\cdot 8 + 21 = 0$

$4a - 24 + 21 = 0$

$4a = 3$ $\quad a = \frac{3}{4}$

67. $a(x-x_0)+b(y-y_0)=0$

$\langle a,b\rangle\langle x-x_0, y-y_0\rangle=0$

$\langle 3,-7\rangle\langle x-2, y-6\rangle=0$

$3(x-2)-7(y-6)=0$

$3x-6-7y+42=0$

$3x-7y=-42+6$

$3x-7y=-36$

24. $u=\langle -4,1,1\rangle$ $v=\langle 0,1,-1\rangle$

$u\times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \vec{i}(-1-1)-\vec{j}(4-0)+\vec{k}(-4-0)$
$= -2\vec{i}-4\vec{j}-4\vec{k}$

$v\times u = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -1 \\ -4 & 1 & 1 \end{vmatrix} = \vec{i}(1+1)-(-4)\vec{j}+4\vec{k}$

thus, the answers are $\langle -2,-4,-4\rangle$ and $\langle 2,4,4\rangle$

30. Area of the parallelogram is $|u\times v|$

$u\times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 0 & 2 \\ 1 & 1 & 1 \end{vmatrix} = (0-2)\vec{i}-\vec{j}(-3-2)+\vec{k}(-3)$
$= -2\vec{i}+5\vec{j}-3\vec{k}$
$= \langle -2,5,-3\rangle$

Area $= |u\times v| = \sqrt{(-2)^2+5^2+(-3)^2} = \sqrt{4+25+9} = \sqrt{38}$

42. $u=\langle 1,2,3\rangle$ $v=\langle -2,4,-1\rangle$

$u\times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -2 & 4 & -1 \end{vmatrix} = (-2-12)\vec{i}-\vec{j}(-1+6)+\vec{k}(4+4)$
$= -14\vec{i}-5\vec{j}+8\vec{k}$
$= \langle -14,-5,8\rangle$

thus, the answers are $\langle -14,-5,8\rangle$ and $\langle 14,5,-8\rangle$

extra: 13.3 86, 87, 88 13.4 27, 35, 62, 63, 66, 73, 75

86. $u+v=\langle \sqrt{a}+\sqrt{b}, \sqrt{a}+\sqrt{b}\rangle$ $\quad |u+v| = \sqrt{2(a+b+2\sqrt{ab})}$

$|u|=|v|=\sqrt{a+b}$ $\quad |u|+|v| \geq |u+v|$ according to triangle rule.

$2\sqrt{a+b} \geq \sqrt{2(a+b+2\sqrt{ab})}$

$4(a+b) \geq 2(a+b)+4\sqrt{ab}$

$2(a+b) \geq 4\sqrt{ab}$

$\sqrt{ab} \leq \frac{a+b}{2}$

87. According to the dot product's definition and theorem, $(u+v)\cdot(u+v)$ equals to $|u+v||u+v|\cos\theta$. Since the angle between them is zero, $\cos\theta = 1$

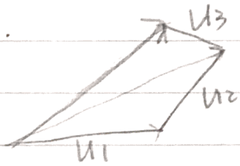
$$|u+v|^2 = (u+v)\cdot(u+v)$$

Assume u and v are in 2 dimensions. thus, $u=\langle u_1, u_2\rangle$ $v=\langle v_1, v_2\rangle$

$$\begin{aligned}(u+v)\cdot(u+v) &= \langle u_1+v_1, u_2+v_2\rangle\cdot\langle u_1+v_1, u_2+v_2\rangle \\ &= (u_1+v_1)^2+(u_2+v_2)^2 \\ &= u_1^2+v_1^2+u_2^2+v_2^2+2u_1v_1+2u_2v_2 \\ &= u_1^2+u_2^2+v_1^2+v_2^2+2(u_1v_1+u_2v_2) \\ &= |u|^2+|v|^2+2u\cdot v\end{aligned}$$

Cauchy-Schwarz inequality.

88.



27. $u=\langle 3,-1,-2\rangle$ $v=\langle 1,3,-2\rangle$

$$u\times v = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 3 & -1 & -2 \\ 1 & 3 & -2 \end{vmatrix} = \vec{i}(2+6)-\vec{j}(-6+2)+\vec{k}(9+1) = 8\vec{i}+4\vec{j}+10\vec{k} = \langle 8,4,10\rangle$$

$$v\times u = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & 3 & -2 \\ 3 & -1 & -2 \end{vmatrix} = \vec{i}(-6-2)-\vec{j}(-2+6)+\vec{k}(-1-9) = \langle -8,-4,-10\rangle$$

35. $A(5,6,2)$ $B(7,16,4)$ $C(6,7,3)$

$\vec{AB} = \langle 7,16,4\rangle - \langle 5,6,2\rangle = \langle 2,10,2\rangle$

$\vec{AC} = \langle 6,7,3\rangle - \langle 5,6,2\rangle = \langle 1,1,1\rangle$

$\frac{1}{2}|\vec{AB}\times\vec{AC}| \Rightarrow \vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 2 & 10 & 2 \\ 1 & 1 & 1 \end{vmatrix} = \vec{i}(10-2)-\vec{j}(2-2)+\vec{k}(2-10) = 8\vec{i}+0\vec{j}-8\vec{k}$

$\frac{1}{2}|\vec{AB}\times\vec{AC}| = \frac{1}{2}\sqrt{64\cdot 2} = \frac{1}{2}\cdot 8\sqrt{2} = 4\sqrt{2}$

62.

13.1 65, 88 13.2 83, 85

HW1 13.1 47, 70, 87, 13.2 24, 67, 70

47(a) $u = \frac{\vec{v}}{|\vec{v}|} = \frac{6\vec{i}-8\vec{j}}{\sqrt{36+64}} = \frac{6\vec{i}-8\vec{j}}{10} = \frac{3}{5}\vec{i} - \frac{4}{5}\vec{j}$ $\langle -\frac{3}{5}, \frac{4}{5} \rangle$

(b) $\sqrt{(\frac{1}{3})^2 + b^2} = 1$ $b^2 = 1 - \frac{1}{9} = \frac{8}{9}$

$b = \pm\frac{2\sqrt{2}}{3}$ $v = \langle \frac{1}{3}, \frac{2\sqrt{2}}{3} \rangle$ $\langle \frac{1}{3}, -\frac{2\sqrt{2}}{3} \rangle$

(c) $\sqrt{a^2 + \frac{a^2}{9}} = 1$ $a^2 + \frac{a^2}{9} = 1$ $a^2 = \frac{10}{9}$

$9a^2 + a^2 = 9$ $a = \pm\frac{\sqrt{10}}{3}$

$10a^2 = 9$ $w = \langle \frac{\sqrt{10}}{3}, \sqrt{10} \rangle$ $\langle -\frac{\sqrt{10}}{3}, -\sqrt{10} \rangle$

70. $2u + 3v = \vec{i}$

$2\langle u_1, u_2 \rangle + 3\langle v_1, v_2 \rangle = \langle \vec{i}, 0 \rangle$

$2u_1 + 3v_1 + 2u_2 + 3v_2 = \langle 1, 0 \rangle$

$2u_1 + 3v_1 = 1$

$3u_2 + 2v_2 = 0$

thus, $\langle \frac{1}{5}, \frac{2}{5} \rangle$ $\langle \frac{1}{5}, \frac{-3}{5} \rangle$

$\langle u_1, u_2 \rangle - \langle v_1, v_2 \rangle = \langle 0, 1 \rangle$

$u_1 - v_1 = 0 \Rightarrow u_1 = v_1$ $2u_1 + 3u_1 = 1$ $u_1 = v_1 = \frac{1}{5}$

$u_2 - v_2 = 1 \Rightarrow v_2 = u_2 - 1$

$3u_2 + 2u_2 - 2 = 0$ $5u_2 = 2$ $u_2 = \frac{2}{5}$ $v_2 = \frac{3}{5}$ $-\frac{3}{5}$

$u = j + v$

$2j + 2v + 3v = i$

$5v = i - 2j = \langle 1, 0 \rangle - 2\langle 0, 1 \rangle$

$5v = \langle 1, -2 \rangle$

$v = \langle \frac{1}{5}, -\frac{2}{5} \rangle$

$u = \langle \frac{1}{5}, -\frac{3}{5} \rangle$

87. if parallel. $u \times v = 0 = \begin{vmatrix} a & 5 \\ 2 & 6 \end{vmatrix}$ $6a - 10 = 0$

$a = \frac{10}{6} = \frac{5}{3}$

if perpendicular $u \cdot v = 0 = \langle a, 5 \rangle \langle 2, 6 \rangle$

$= 2a + 30 = 0$ $a = -15$

24. $(x+2)^2 + (y-2)^2 + z^2 = R^2$ through $(3, 4, 5)$ ($x-1$)

$25 + 4 + 25 = R^2$ $2 \times 3 \times 3 \times 3$

$R = \sqrt{54} = 3\sqrt{6}$ $3\sqrt{3}$ Thus. $(x+2)^2 + (y-2)^2 + z^2 = 3\sqrt{6}$ $\sqrt{33}$

67. $\vec{v} = \langle 2, -4, 4 \rangle$

$|\vec{v}| = \sqrt{4+16 \cdot 2} = 6$ $\frac{\vec{v}}{|\vec{v}|} = u = \frac{2\vec{i} - 4\vec{j} + 4\vec{k}}{6} = \frac{1}{3}\vec{i} - \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$

70. $v = \langle 3, -2, 6 \rangle$

$|\vec{v}| = \sqrt{9+4+36} = 7$

$u = \frac{\vec{v}}{|\vec{v}|} = \frac{3i - 2j + 6k}{7}$ vector $\langle \frac{30}{7}, \frac{-20}{7}, \frac{60}{7} \rangle$ $\langle \frac{-30}{7}, \frac{20}{7}, -\frac{60}{7} \rangle$

65. a. T b. T c. F d. F e. F f. F g. F h. T

88 (a) In one triangle. $|u+v|$ is the length of last side. beside $|u|$ and $|v|$. the sum of two sides' lengths is larger than the third.

(b) when u and v are parllel.

Homework 3 14.2 17, 42, 63 14.4 21, 26, 33

17. $r(t) = \langle t, 3t^2, t^3 \rangle$ $t=1$

tangent vector $= r'(t) = \langle 1, 6t, 3t^2 \rangle = \boxed{\langle 1, 6, 3 \rangle}$

42. $\frac{d}{dt}(u(\sin t))\Big|_{t=0} = u'\sin t \cos t$

$= u'\sin 0 \cos 0 = u'(0) = \boxed{\langle 0, 7, 1 \rangle}$

63. $r(t) = e^{3t}\vec{i} + \frac{1}{1+t^2}\vec{j} - \frac{1}{\sqrt{2t}}\vec{k}$

$\int r(t)\,dt = (\int e^{3t}\,dt)\vec{i} + (\int \frac{1}{1+t^2}\,dt)\vec{j} - (\int \frac{1}{\sqrt{2t}}\,dt)\vec{k} + C$

$= (\int e^{3t}\,dt)\vec{i} + (\int \frac{1}{1+t^2}dt)\vec{j} - \frac{\sqrt{2}}{2}(\int t^{-\frac{1}{2}}\,dt)\vec{k} + C$

$= \frac{1}{3}e^{3t}\vec{i} + \tan^{-1}t\,\vec{j} - (\frac{\sqrt{2}}{2}\,2t^{\frac{1}{2}})\vec{k} + C$

$= \frac{1}{3}e^{3t}\vec{i} + \tan^{-1}t\,\vec{j} - \sqrt{2t}\,\vec{k} + C$

21. $r(t) = \langle \cos^3 t, \sin^3 t \rangle$ for $0 \le t \le \pi/2$

Arc length $l = \int_0^{\frac{\pi}{2}} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\,dt$

$= \int_0^{\frac{\pi}{2}} \sqrt{[3\cos^2 t(-\sin t)]^2 + [3\sin^2 t\cos t]^2}\,dt$

$= \int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t\sin^2 t + 9\sin^4 t\cos^2 t}\,dt$

$= \int_0^{\pi/2} \sqrt{9\cos^2 t\sin^2 t(\cos^2 t + \sin^2 t)}\,dt$

$= \int_0^{\frac{\pi}{2}} 3\cos t\sin t\,dt = 3\int_0^{\frac{\pi}{2}} \cos t\sin t\,dt = 3\,\frac{1}{2}\sin^2 t\Big|_0^{\frac{\pi}{2}}$

$= 3(\frac{1}{2}\cdot 1^2) - 3(\frac{1}{2}\cdot 0) = \boxed{\frac{3}{2}}$

26. $r(t) = \langle e^t\sin t, e^t\cos t, e^t \rangle$ for $0 \le t \le \ln 2$

speed $= |v(t)| = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$

$= \sqrt{(e^t\sin t + e^t\cos t)^2 + (e^t\cos t - e^t\sin t)^2 + (e^t)^2}$

$= \sqrt{e^{2t}\sin^2 t + e^{2t}\cos^2 t + \cancel{2e^{2t}\sin t\cos t} + e^{2t}\cos^2 t + e^{2t}\sin^2 t - \cancel{2e^{2t}\sin t\cos t} + e^{2t}}$

$= \sqrt{2e^{2t}\sin^2 t + 2e^{2t}\cos^2 t\ \ e^{2t}}$

$= \sqrt{2e^{2t} + e^{2t}} = \sqrt{3e^{2t}}$

Arc length $= \int_0^{\ln 2} |v(t)|\,dt = \int_0^{\ln 2} \sqrt{3e^{2t}}\,dt = \sqrt{3}\int_0^{\ln 2} (e^{2t})^{\frac{1}{2}}\,dt = \sqrt{3}\int_0^{\ln 2} e^t\,dt.$

$= \sqrt{3}(e^{\ln 2}) - \sqrt{3}(e^0)$

$= \sqrt{3}(2-1)$

$= \sqrt{3}(1) = \boxed{\sqrt{3}}$

33. $r(t) = \langle 1, \sin t, \cos t \rangle$ for $t \ge 1$

Arc length $l = \int_a^b |v(t)|\,dt$

$|v(t)| = |r'(t)| = \sqrt{0^2 + \cos^2 t + (-\sin t)^2} = 1$

Yes the curve use arc length as a parameter.

15.4 9, 19, 35 15.5 31

Home 14.2 39, 54, 85, 86. 14.4 14, 39.

17\. 39. $\frac{d}{dt}(u\cdot v)|_{t=0}$

$= u'v + uv'$

$= \langle 0,7,1\rangle\langle 0,1,1\rangle + \langle 0,1,1\rangle\langle 1,1,2\rangle$

55 $= 0+7+1+0+1+2$

$= 8+3 = 11$

54\. $r(t) = \langle 3t^{12}-t^2, t^8+t^3, t^{-4}-2\rangle$

$36 \times 11 = 396$

$r'(t) = \langle 36t^{11}-2t, 8t^7+3t^2, -4t^{-5}\rangle$

$56 \times 6 = 336$

0 $r''(t) = \langle 396t^{10}-2, 56t^6+6t, 20t^{-6}\rangle$

$r'''(t) = \langle 3960t^9, 336t^5+6, -120t^{-7}\rangle$

85\. $r(t) = \langle at^2+1, t\rangle$

$r'(t) = \langle 2at, 1\rangle$

if $r(t)$ and $r'(t)$ are orthogonal.

$r(t)\cdot r'(t) = 0$

$\langle at^2+1, t\rangle\langle 2at, 1\rangle = 0$

$(at^2+1)2at + t = 0$

$2a^2t^3+2at+t=0$

$t(2a^2t^2+2a+1)=0$

$t=0$

$2a^2t^2+2a+1=0$

$t = \frac{-2a \pm \sqrt{4a^2-4\cdot 2a^2}}{4a^2}$

DNE

$r(0) = \langle 1, 0\rangle$

86\. $r(t) = \langle \sqrt{t}, 1, t\rangle$

$r'(t) = \langle \frac{1}{2}t^{-\frac{1}{2}}, 0, 1\rangle$

$r(t)\cdot r'(t) = 0$

$\sqrt{t}\cdot\frac{1}{2\sqrt{t}} + 1\cdot 0 + t = 0$

$\frac{1}{2}+t=0$

$t = -\frac{1}{2}$ $\quad r(-\frac{1}{2}) = \langle \sqrt{-\frac{1}{2}}, 1, -\frac{1}{2}\rangle$ DNE.

14\. $r(t) = \langle \cos t + \sin t, \cos t - \sin t\rangle$ for $0 \le t \le 2\pi$

arc length $= \int_0^{2\pi} \sqrt{(-\sin t+\cos t)^2 + (-\sin t - \cos t)^2}$ $= \int \sqrt{\sin^2 t + \cos^2 t - 2\sin t\cos t + \sin^2 t + \cos^2 t + 2\sin t\cos t}$

$= \int_0^{2\pi} \sqrt{\sin^2 t + \cancel{\cos^2 t} + 1 - 2\sin t\cos t}$

$= \int_0^{2\pi} \sqrt{2(1-\cancel{\sin t\cos t})}$ $= \sqrt{2}\int_0^{2\pi} (1-\sin t\cos t)^{\frac{1}{2}}$

$= 6.17852.$

let $u = 2t$ $\quad du = 2dt$

$= \sqrt{2}\int_0^{2\pi} \frac{1}{2}(1-\frac{1}{2}\sin u)^{\frac{1}{2}}\, du$

$x = 1-\frac{1}{2}\sin u$

$dx =$

39\. $r(t) = \langle \cos t^2, \sin t^2\rangle$ for $0 \le t \le \sqrt{\pi}$ $\quad r'(t) = \langle -2t\sin t^2, 2t\cos t^2\rangle$

$r'(t) = \langle -2\cos t\sin t, 2\sin t\cos t\rangle$

$|r'(t)| = \sqrt{4\sin^2 t\cos^2 t + 4\sin^2 t\cos^2 t}$ $= 2\sqrt{2\sin^2 t\cos^2 t}$ $\quad \sqrt{4t^2\sin^2 t^2 + 4t^2\cos^2 t^2}$

$= 2\sqrt{2}\sin t\cos t$ $\quad = \sqrt{4t^2} = 2t$

40\. $r(s) = \langle \cos s, \sin s\rangle$ $0 \le s \le \pi$

Homework 4. 15.3 17.55 15.4 9.19.35 15.5 31

17. $f(x,y)=e^{x^2y}$

$f_x = 2xy \cdot e^{x^2 \cdot y}$

$f_y = x^2 \cdot e^{x^2 \cdot y}$

55. $h(x,y,z)=\cos(x+y+z)$

$h_x = -\sin(x+y+z)$

$h_y = -\sin(x+y+z)$

$h_z = -\sin(x+y+z)$

9. dz/dt where $z = x\sin y$, $x=t^2$, and $y=4t^3$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

$$= \sin y \cdot 2t + x\cos y \cdot 12t^2$$

$$= 2t \cdot \sin y + 12t^4 \cos y = 2t\sin 4t^3 + 12t^4\cos 4t^3$$

19. z_s and z_t, where $z=x^2\sin y$, $x=s-t$, and $y=t^2$

$$z_s = \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$= 2x\sin y \cdot 1 + x^2\cos y \cdot 0$$

$$= 2x\sin y = 2(s-t)\sin t^2$$

$$z_t = \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= 2x\sin y(-1) + x^2\cos y \cdot 2t$$

$$= -2x\sin y + x^2 2t = -2(s-t)\cdot\sin t^2 + (s-t)^2\cos t^2 \cdot 2t$$

$$= 2(s-t)(-\sin t^2 + t(s-t)\cos t^2)$$

35. $x^2-2y^2-1=0$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

$F_x = 2x$ $\qquad \frac{dy}{dx} = -\frac{2x}{-4y}$

$F_y = -4y$ $\qquad = \frac{x}{2y}$

31. $f(x,y)=x^2-4y^2-9$ $P(1,-2)$

$\nabla f = \langle 2x, -8y\rangle$

steepest ascent $= \frac{2x\vec{i} - 8y\vec{j}}{\sqrt{(2x)^2+(-8y)^2}} = \frac{2\vec{i}+16\vec{j}}{\sqrt{4+256}} = \frac{\vec{i}+8\vec{j}}{\sqrt{65}}$

steepest descent $= \frac{-2x\vec{i}+8y\vec{j}}{\sqrt{(-2x)^2+(8y)^2}} = \frac{-\vec{i}-8\vec{j}}{\sqrt{65}}$

31. 1b)

$\nabla f = \langle 2x, -8y \rangle$

$\nabla f \cdot u = 0$

$2x \cdot u_1 - 8y u_2 = 0$

$2x u_1 = 8y u_2$

$u_1 = \frac{8y u_2}{2x}$

$= 4\frac{y}{x} \cdot u_2$

$= -8u_2$

thus, the vector could be $\langle 8, -1 \rangle$

15.3 27, 31, 47. 15.4 43 15.5 21, 59, 69.

27. $f(x,y) = x^{2y}$

$f_x = 2y \cdot x^{2y-1}$

$f_y = 2x^{2y} \ln x$

31. $f(x,y) = \int_x^{y^3} e^{t^2} dt$

$f_x = -e^{x^2}$

$f_y = e^{y^6} \cdot 3y^2$

47. $f(x,y) = \tan^{-1}(x^3y^2)$

$f_x = \frac{3x^2 \cdot y^2}{1+(x^3y^2)^2}$

$f_y = \frac{2y \cdot x^3}{1+(x^3y^2)^2}$

$f_{xy} = f_{yx} = \frac{6yx^2(1+(x^3y^2)^2) - 2yx^3 \cdot 3x^2y^2}{(1+(x^3y^2)^2)^2}$

$= \frac{6x^2y + 6x^5y^3 - 6x^5y^3}{(1+(x^3y^2)^2)}$

$f_{yx} = \frac{2 \cdot 3x^2 \cdot y(1+x^3y^2) - 2yx^3 \cdot (3x^2 \cdot y^2)}{(1+(x^3y^2))^2}$

$= \frac{6x^2y + 6x^5y^3 - 6x^5y^3}{(1+(x^3y^2))^2}$

$f_{xy} = \frac{6yx^2 \cdot (1+x^6y^4) - 4y^3 \cdot x^6 (3x^2y^2)}{(1+x^6y^4)^2} = \frac{6x^2y + 6x^8y^5 - 12x^8y^5}{(1+x^6y^4)^2}$

$f_{yx} = \frac{6x^2y(1+x^6y^4) - 16x^5y^4(2y \cdot x^3)}{(1+x^6y^4)^2} = \frac{6x^2y + 6x^8y^5 - 12x^8y^5}{(1+x^6y^4)^2}$

$f_{xx} = \frac{6xy^2(1+x^6y^4) - 3x^2y^2(6x^5y^4)}{(1+x^6y^4)^2} = \frac{6xy^2 + 6x^7y^6 - 18x^7y^6}{(1+x^6y^4)^2}$

$f_{yy} = \frac{2x^3(1+x^6y^4) - 2yx^3(4y^3x^6)}{(1+x^6y^4)^2} = \frac{2x^3 + 2x^9y^4 - 8y^4x^9}{(1+x^6y^4)^2}$

43. $f(x,y) = x^2y$ where $x = s+t$, $y = s-t$

$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$

$= 2xy \cdot 1 + x^2 \cdot 1 = 2(s^2-t^2) + (s+t)^2$

$= 2xy + x^2 = 2(s+t)(s-t) + s^2 + t^2 + 2st$

$\frac{\partial^2 f}{\partial s^2} = (2s^2 - 2t^2) + (s+t)^2 \frac{\partial}{\partial s}$

$= 4s + 2s + 2t = 6s + 2t$

$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$

$= 2xy \cdot 1 - x^2 \cdot 1$

$= 2s^2 - 2t^2 - s^2 - t^2 - 2st = s^2 - 3t^2 - 2st$

$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t} \cdot \frac{\partial f}{\partial t} = -6t - 2s$

$\frac{\partial f}{\partial s} \cdot \frac{\partial}{\partial t} = \frac{\partial^2 f}{\partial s \partial t}$

$= (2s^2 - 2t^2 + s^2 + t^2 + 2st) \frac{\partial}{\partial t}$

$= -4t + 2t + 2s$

$= -2t + 2s$

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21. $f(x,y) = x^2 - y^2$ $P(-1,-3)$ $\langle \frac{3}{5}, -\frac{4}{5} \rangle$

$\nabla f = \langle 2x, -2y \rangle$

$D_u f = \nabla f \cdot \vec{u}$

$= \langle 2x, -2y \rangle \langle \frac{3}{5}, -\frac{4}{5} \rangle$

$= \langle -2, 6 \rangle \langle \frac{3}{5}, -\frac{4}{5} \rangle$

$= -2 \cdot \frac{3}{5} + 6 \cdot (-\frac{4}{5})$

$= -\frac{6}{5} - \frac{24}{5}$

$= -\frac{30}{5} = -6$

59. $f(x,y,z) = x^2 + 2y^2 + 4z^2 + 10$ $P(1,0,4)$ $\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$

a. $\nabla f = \langle 2x, 4y, 8z \rangle$ at $P(1,0,4)$

$\nabla f = \langle 2, 0, 32 \rangle$

b. $u = \frac{\nabla f}{|\nabla f|} = \frac{2\vec{i} + 0\vec{j} + 32\vec{k}}{\sqrt{2^2 + 32^2}} = \frac{1}{2\sqrt{257}} (2\vec{i} + 32\vec{k}) = \frac{\vec{i}}{\sqrt{257}} + \frac{16\vec{k}}{\sqrt{257}}$

c. rate of change $= \nabla f \cdot \vec{u} = \langle 2, 0, 32 \rangle \langle \frac{1}{\sqrt{257}}, 0, \frac{16}{\sqrt{257}} \rangle$

$= \frac{2}{\sqrt{257}} + \frac{512}{\sqrt{257}} = \frac{514}{\sqrt{257}}$ $= 2\sqrt{257}$

d. $D_u f = \nabla f \cdot \vec{u}$

$= \langle 2, 0, 32 \rangle \langle \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} \rangle$

$= \sqrt{2} + 0 + 32 \cdot \frac{\sqrt{2}}{2} = \sqrt{2} + 16\sqrt{2} = 17\sqrt{2}$

69. $f(x,y) = 12 - 4x^2 - y^2$ $P(1,2,4)$

$\nabla f = \langle -8x, -2y \rangle = \langle -8, -4 \rangle$

$\nabla f \cdot \vec{u} = 0 \Rightarrow -8x - 4y = 0$

$x^2 + y^2 = 1$ $\quad 4y = -8x$

$x^2 + 4x^2 = 1$ $\quad y = -2x$

$5x^2 = 1$ $\quad$ thus, $\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \rangle$ and $\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$

$x = \pm\sqrt{\frac{1}{5}}$