Neural Networks:
Theory and Practice
Lecture I

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Who am I?

Office: 514
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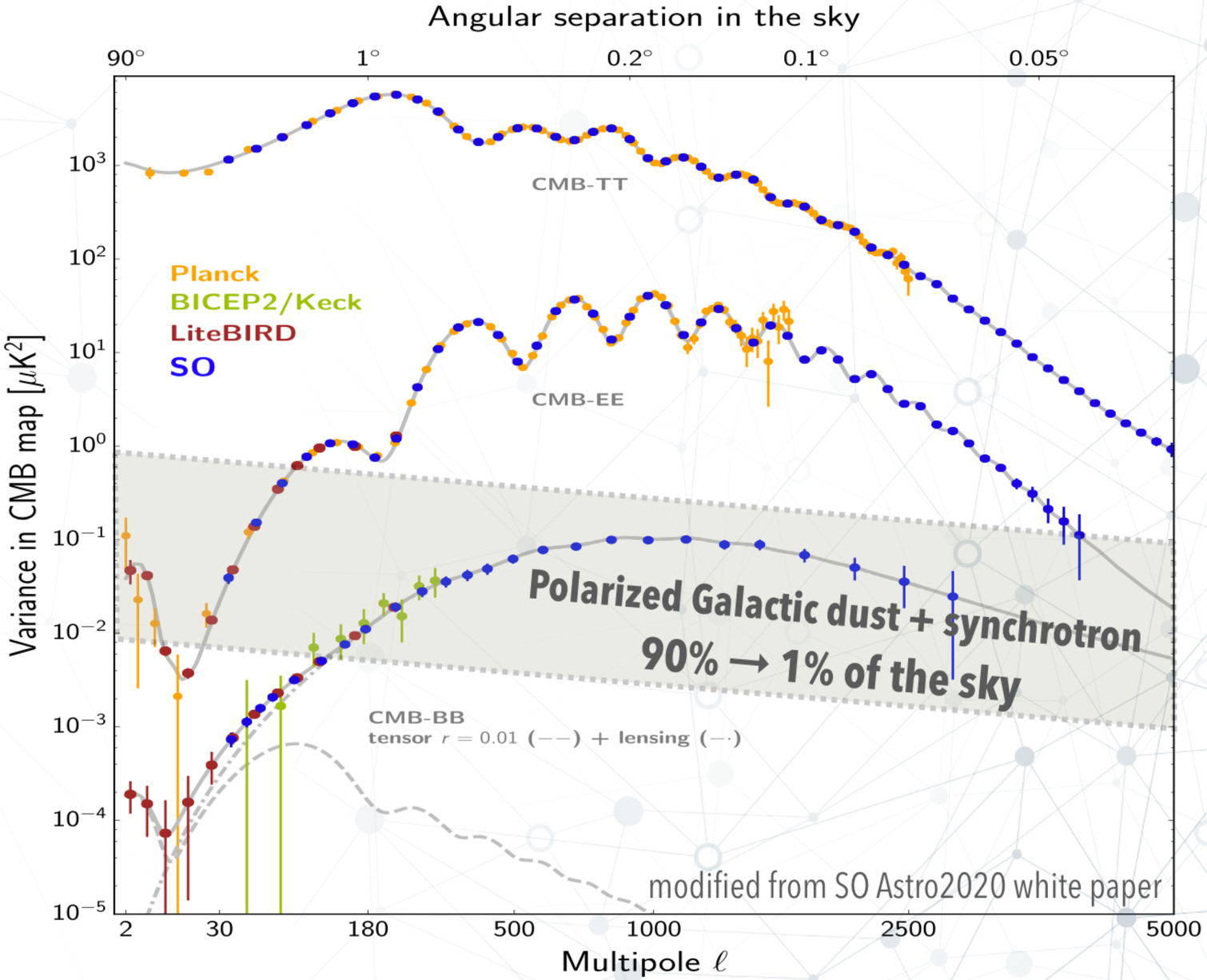
CMB data analysis

planck

POLARBEAR

SIMONS OBSERVATORY

LiteBIRD

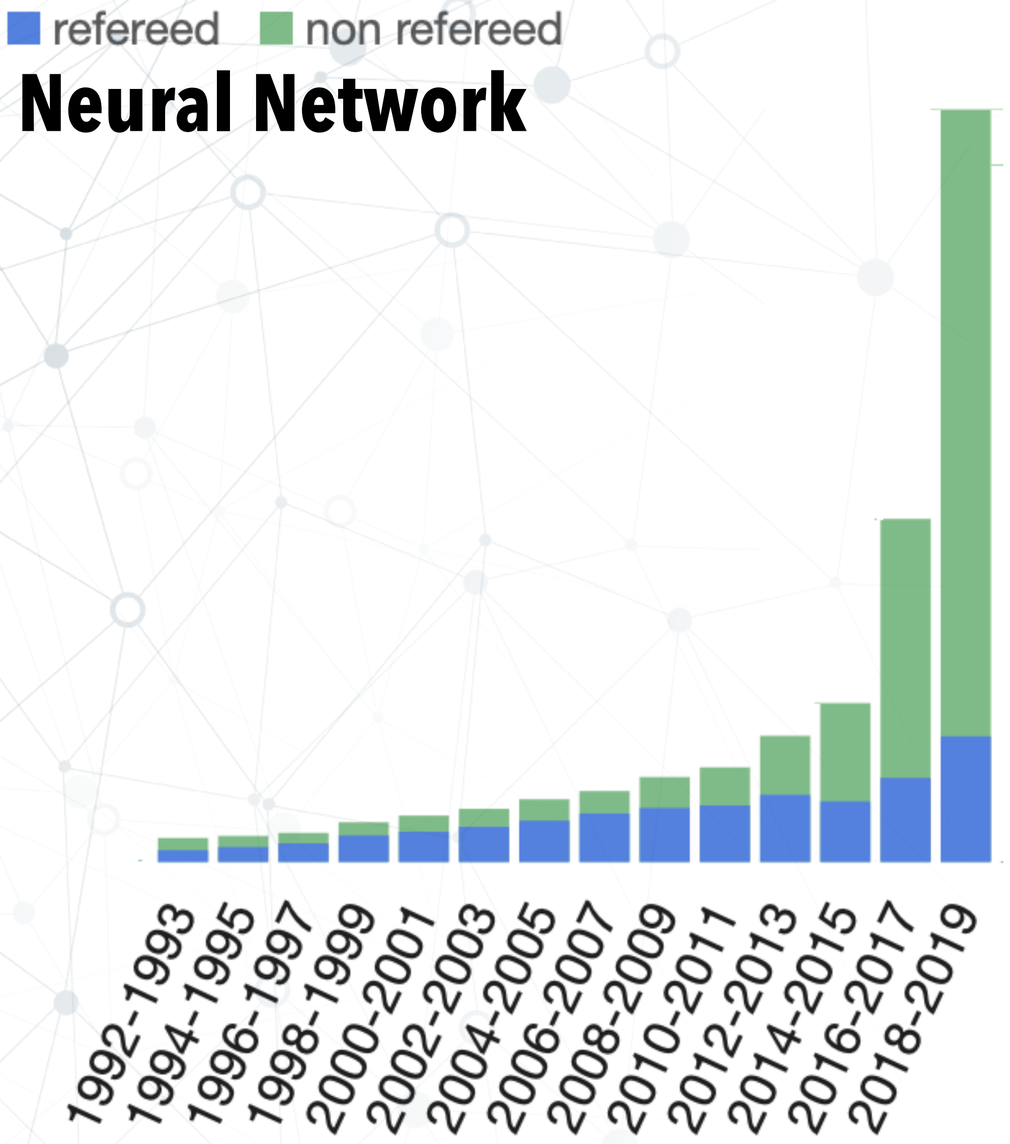


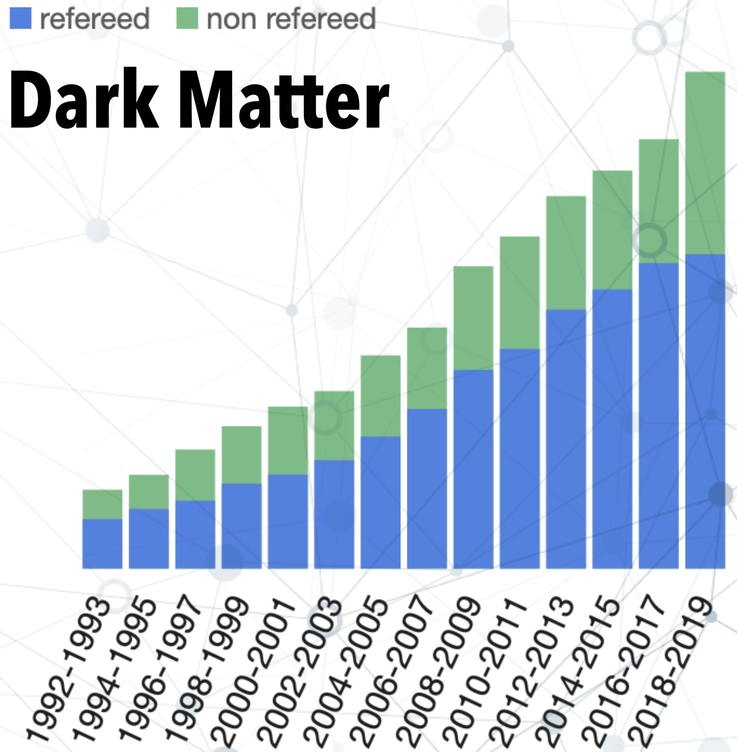
Outline lecture 1

- Introduction: what is a NN?
- Fully connected NNs:
  - Feedforward
  - Loss and cost functions
  - Backpropagation
- **Tutorial 1: coding a simple NN from scratch**
- Libraries for NN development
- Training a NN:
  - Regularization
  - Dropout
  - Batch normalization
  - Mini-batches
- Autoencoders

NN publication metrics

Papers on ADS with “Neural Network " in the text, compared with “Dark Matter”





What is a Neural Network?

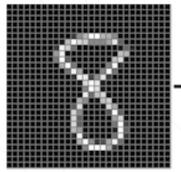
$$y = f(\mathbf{x})$$

- The goal of a **feed-forward Neural Network** is to find a good enough approximation of the function f that maps inputs into outputs

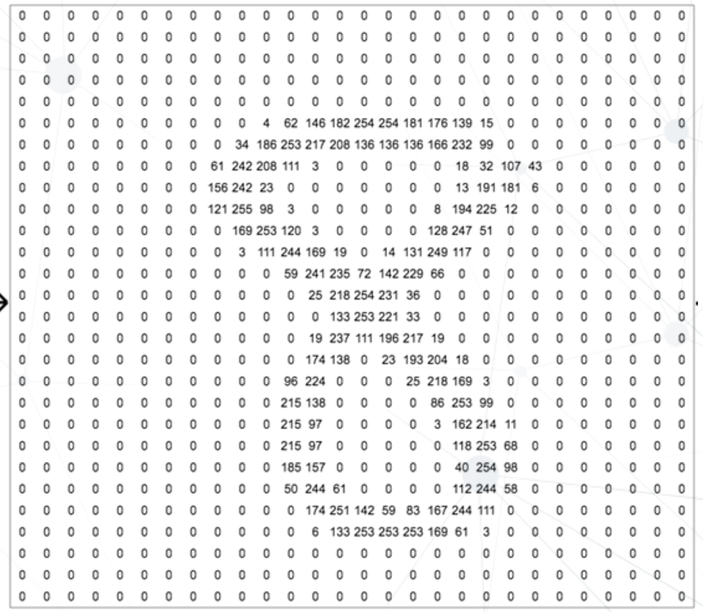
- for a **classifier** $y = f(\boldsymbol{x})$: f maps the input $\boldsymbol{x}$ into the category y
- The Neural Network defines a mapping $\boldsymbol{f^*(x; \theta)}$ and finds the value of the parameter θ that results in the best approximation $\boldsymbol{f^* \sim f}$

What is a Neural Network?

$$f(\text{  }) = 8 \qquad f(\text{  }) = 2$$



28 x 28
784 pixels



$$f : \mathbb{R}^{n \times m} \rightarrow \mathbb{N}$$

What is a Neural Network?

- Neural networks can, in principle, be used every time there is a unique relation between a input and output
- They are a very powerful tool especially when this relation is unknown
- ….but it exists a set of data “large enough” for which the output associated at each input is known (training set)

Supervised Learning

What is a Neural Network?

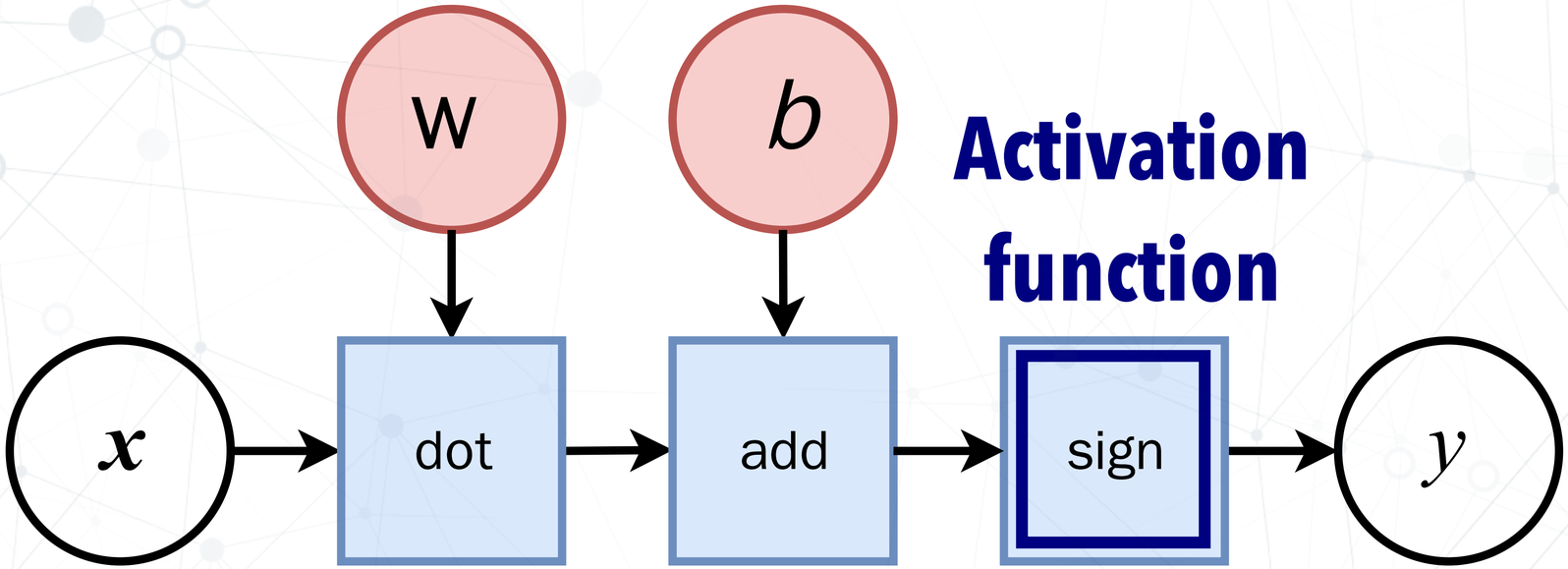
Q: How can a NN approximate very complex unknown functions?

A: By ricursively apply **non-linear** activation functions to a linear combination of input elements

Perceptron

see also previous lectures

- First model of artificial neuron
- Introduced in the late 50s by Frank Rosenbalt



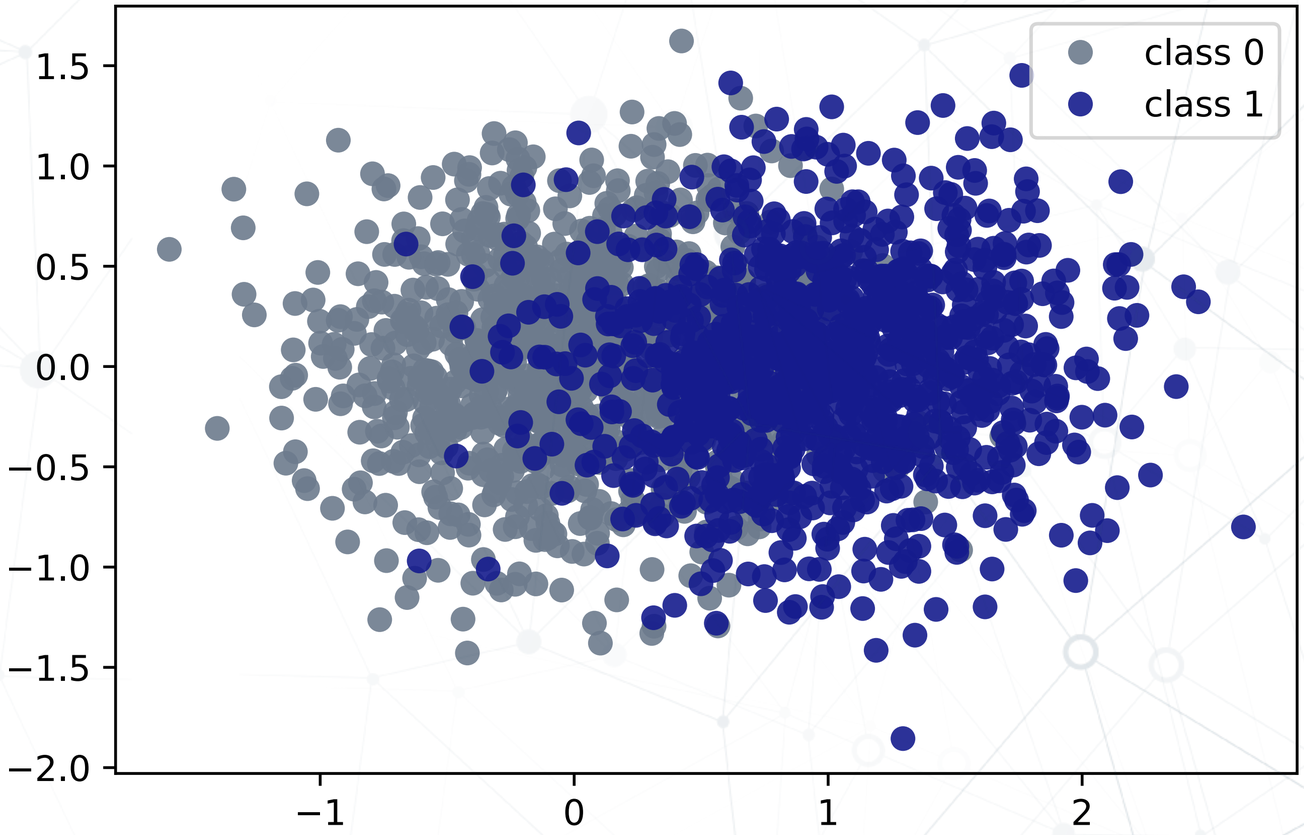
$$sign(\mathbf{x}) = \begin{cases} 1 & \text{if } \sum_i w_i x_i + b \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Sigmoid activation function

see also previous lectures

Training data $(\mathbf{x}, y)$ with:

- $\mathbf{x} \in \mathbb{R}$
- $y \in \{0,1\}$



Class populations are Gaussian, with covariance Σ

$$P(\mathbf{x}|y) = \frac{1}{\sqrt{(2\pi)^p |\Sigma|}} \exp\left(-\frac{1}{2}(\mathbf{x} - \mu_y)^T \Sigma^{-1} (\mathbf{x} - \mu_y)\right)$$

Sigmoid activation function

see also previous lectures

With Bayes' theorem we have:

$$
\begin{aligned}
P(Y=1|\mathbf{x}) &= \frac{P(\mathbf{x}|Y=1)P(Y=1)}{P(\mathbf{x})} \\
&= \frac{P(\mathbf{x}|Y=1)P(Y=1)}{P(\mathbf{x}|Y=0)P(Y=0)+P(\mathbf{x}|Y=1)P(Y=1)} \\
&= \frac{1}{1+\frac{P(\mathbf{x}|Y=0)P(Y=0)}{P(\mathbf{x}|Y=1)P(Y=1)}}.
\end{aligned}
$$

and introducing the **sigmoid function** σ

$$
\sigma(x) = \frac{1}{1+\exp(-x)},
$$

we get:

$$
P(Y=1|\mathbf{x}) = \sigma\left(\log\frac{P(\mathbf{x}|Y=1)}{P(\mathbf{x}|Y=0)} + \log\frac{P(Y=1)}{P(Y=0)}\right).
$$

Sigmoid activation function

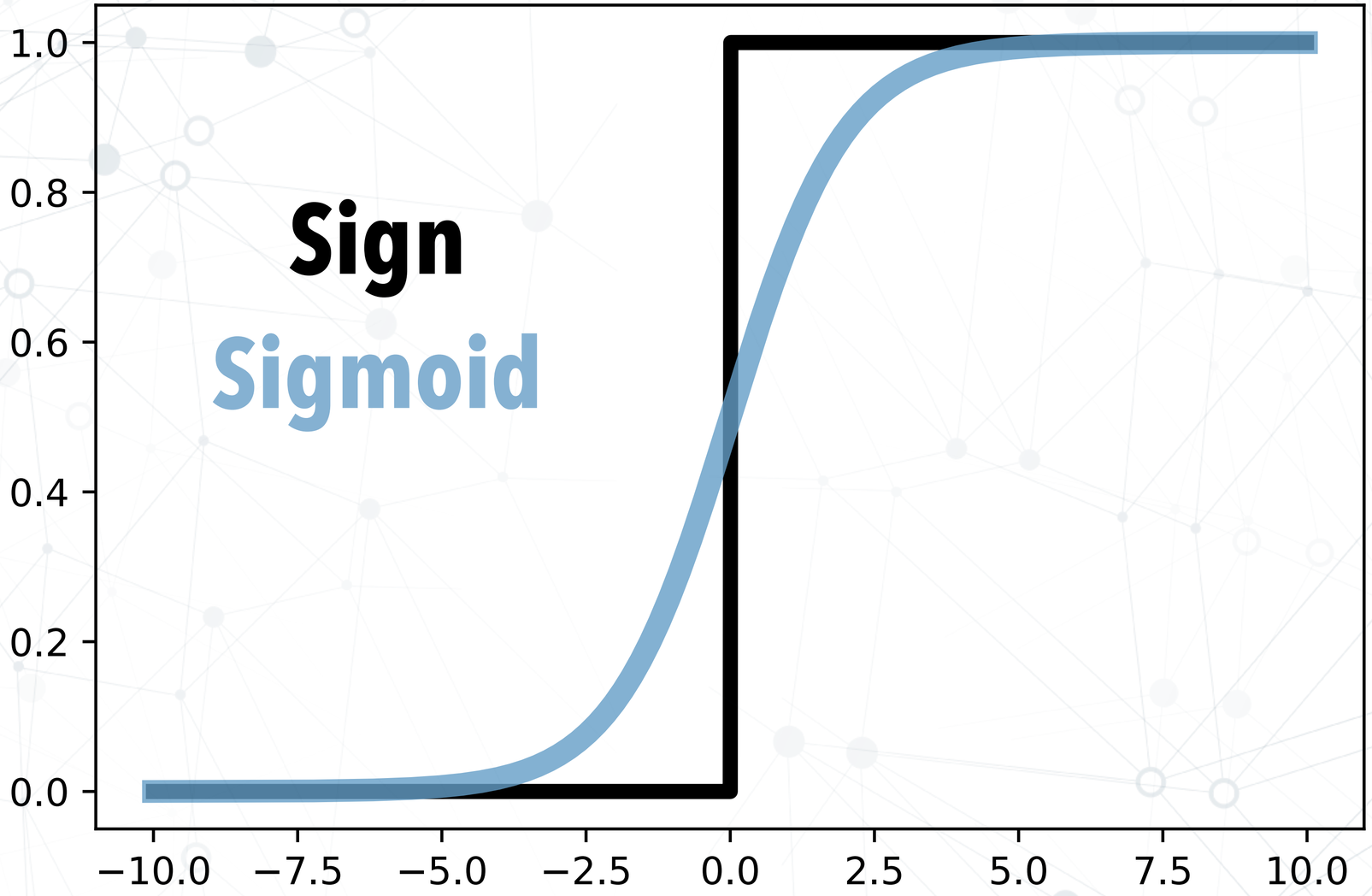
see also previous lectures

$$
\begin{aligned}
& P(Y=1|\mathbf{x}) \\
& = \sigma\left(\log\frac{P(\mathbf{x}|Y=1)}{P(\mathbf{x}|Y=0)} + \underbrace{\log\frac{P(Y=1)}{P(Y=0)}}_{a}\right) \\
& = \sigma\left(\log P(\mathbf{x}|Y=1) - \log P(\mathbf{x}|Y=0) + a\right) \\
& = \sigma\left(-\frac{1}{2}(\mathbf{x}-\mu_1)^T\Sigma^{-1}(\mathbf{x}-\mu_1) + \frac{1}{2}(\mathbf{x}-\mu_0)^T\Sigma^{-1}(\mathbf{x}-\mu_0) + a\right) \\
& = \sigma\left(\underbrace{(\mu_1-\mu_0)^T\Sigma^{-1}}_{\mathbf{w}^T}\mathbf{x} + \underbrace{\frac{1}{2}(\mu_0^T\Sigma^{-1}\mu_0 - \mu_1^T\Sigma^{-1}\mu_1) + a}_{b}\right) \\
& = \sigma\left(\mathbf{w}^T\mathbf{x} + b\right)
\end{aligned}
$$

The probability that data belong to a given class can be exactely written with the **sigmoid function** σ

Sigmoid activation function

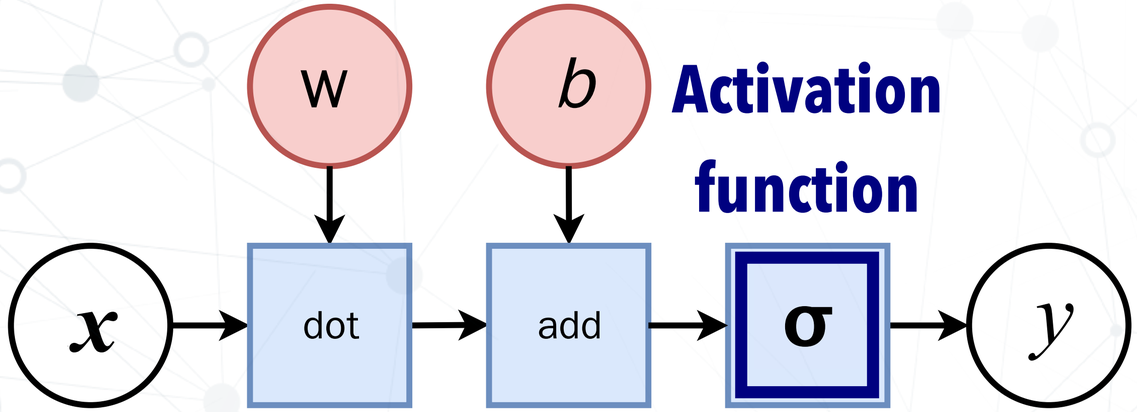
see also previous lectures



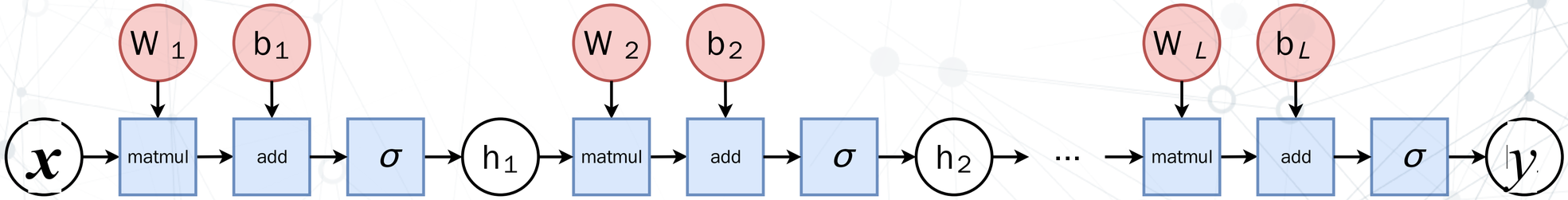
Sigmoid activation function

see also previous lectures

- we can write the same computational graph as for perception but with the sigmoid activation function

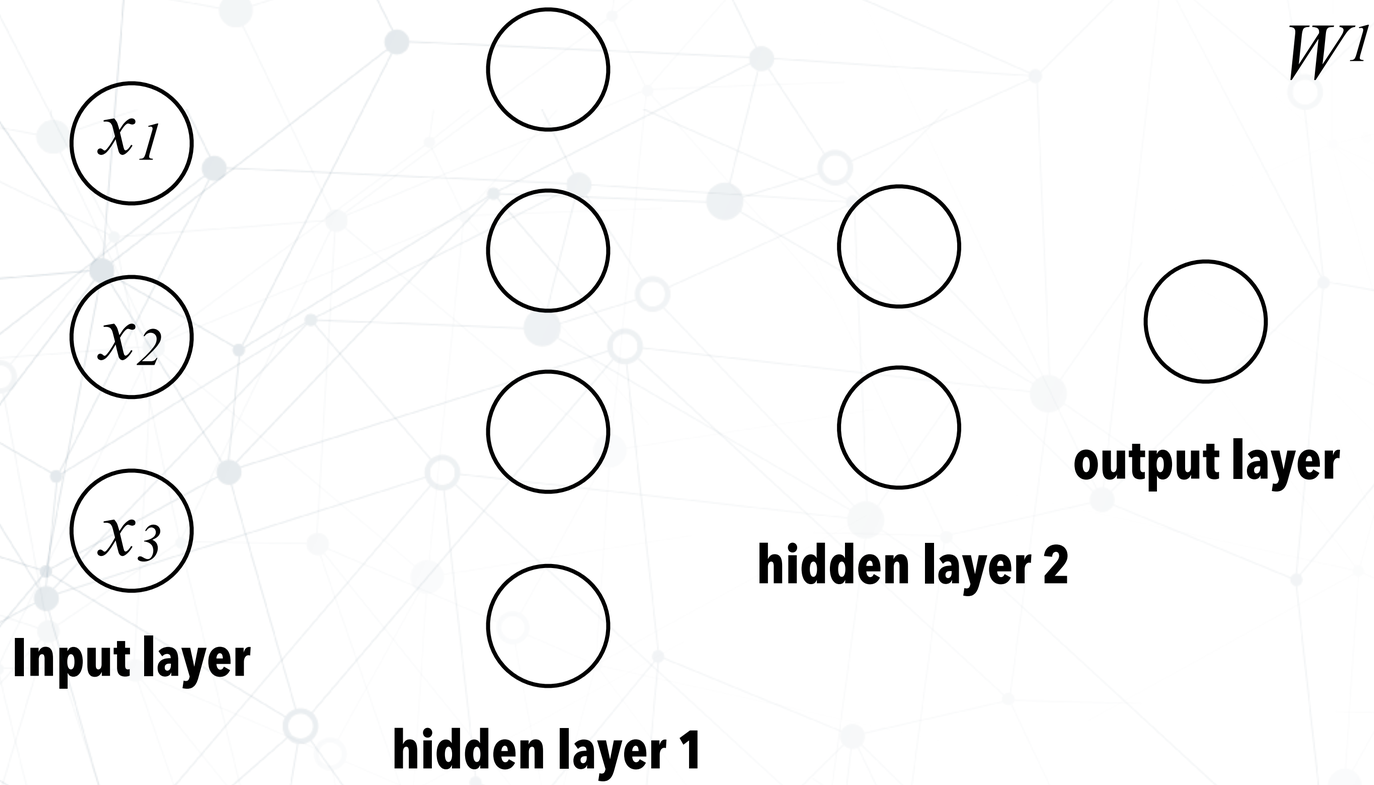


- this represents the **building block of a fully connected feed-forward Neural Network**



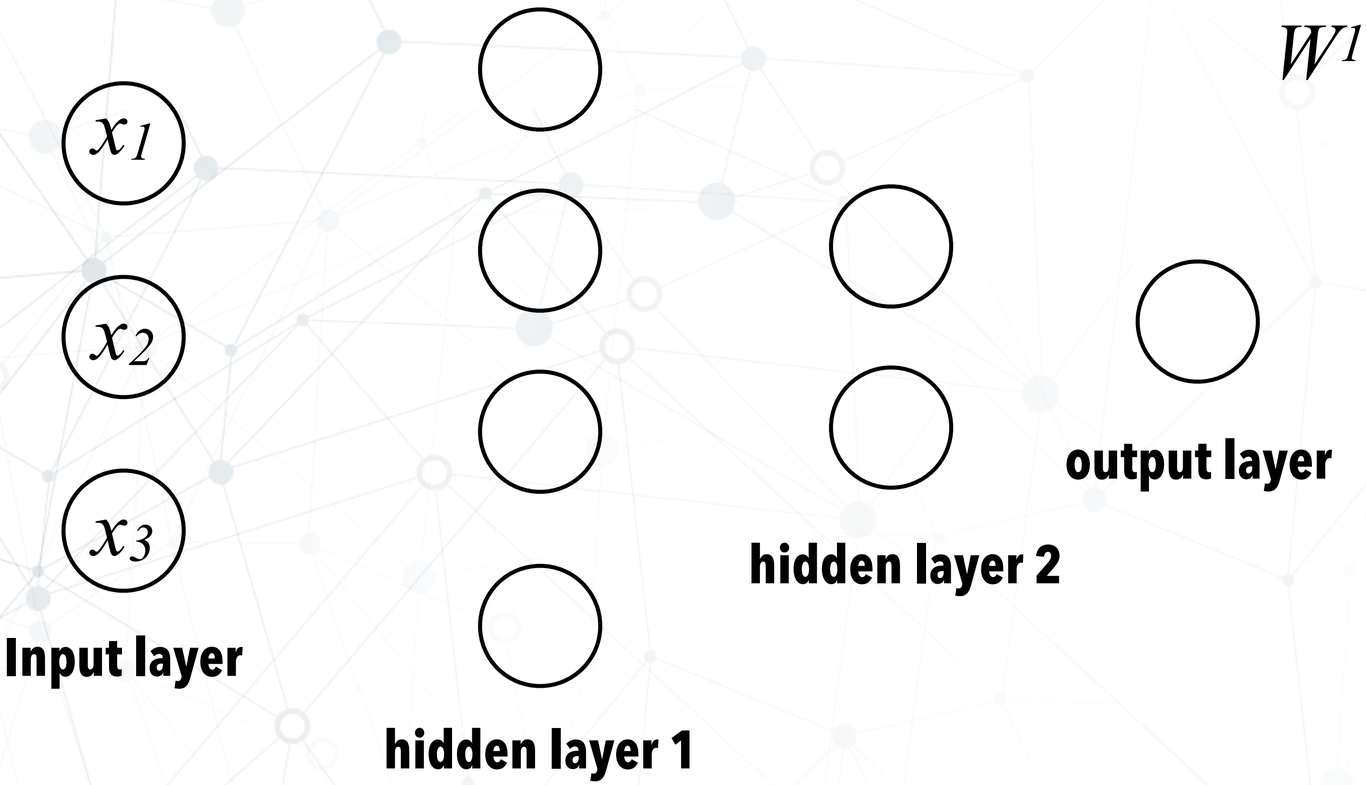
see previous lectures

Multi-layer feedforward NN



$$W^1 = \begin{bmatrix} & \\ & \end{bmatrix} \quad b^1 = \begin{bmatrix} \\ \end{bmatrix}$$

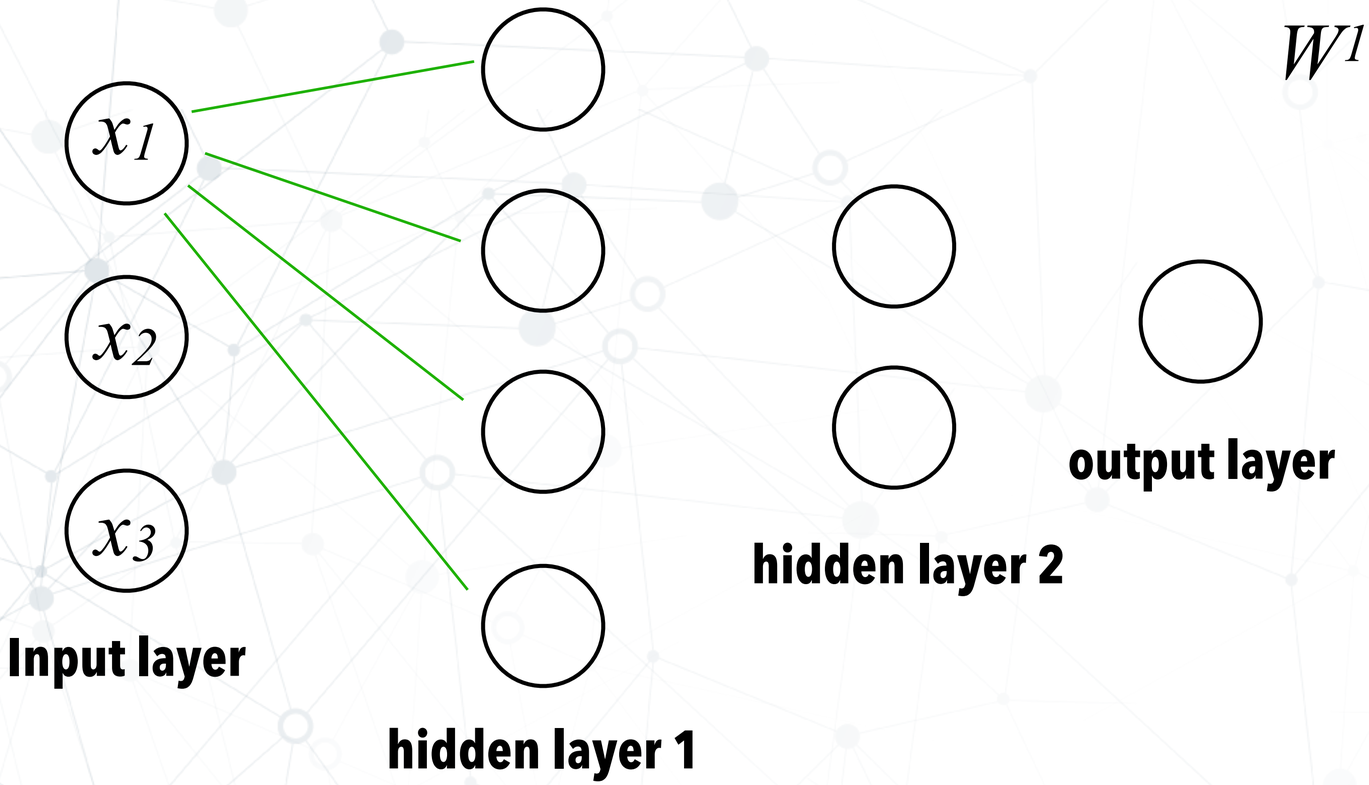
Multi-layer feedforward NN



$$W^1 = \begin{bmatrix} & \\ & \end{bmatrix} \quad b^1 = \begin{bmatrix} \\ \end{bmatrix}$$

- Each line represents a weight w

Multi-layer feedforward NN



$$W^1 = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ & & & \end{bmatrix} \quad b^1 = \begin{bmatrix} \\ \end{bmatrix}$$

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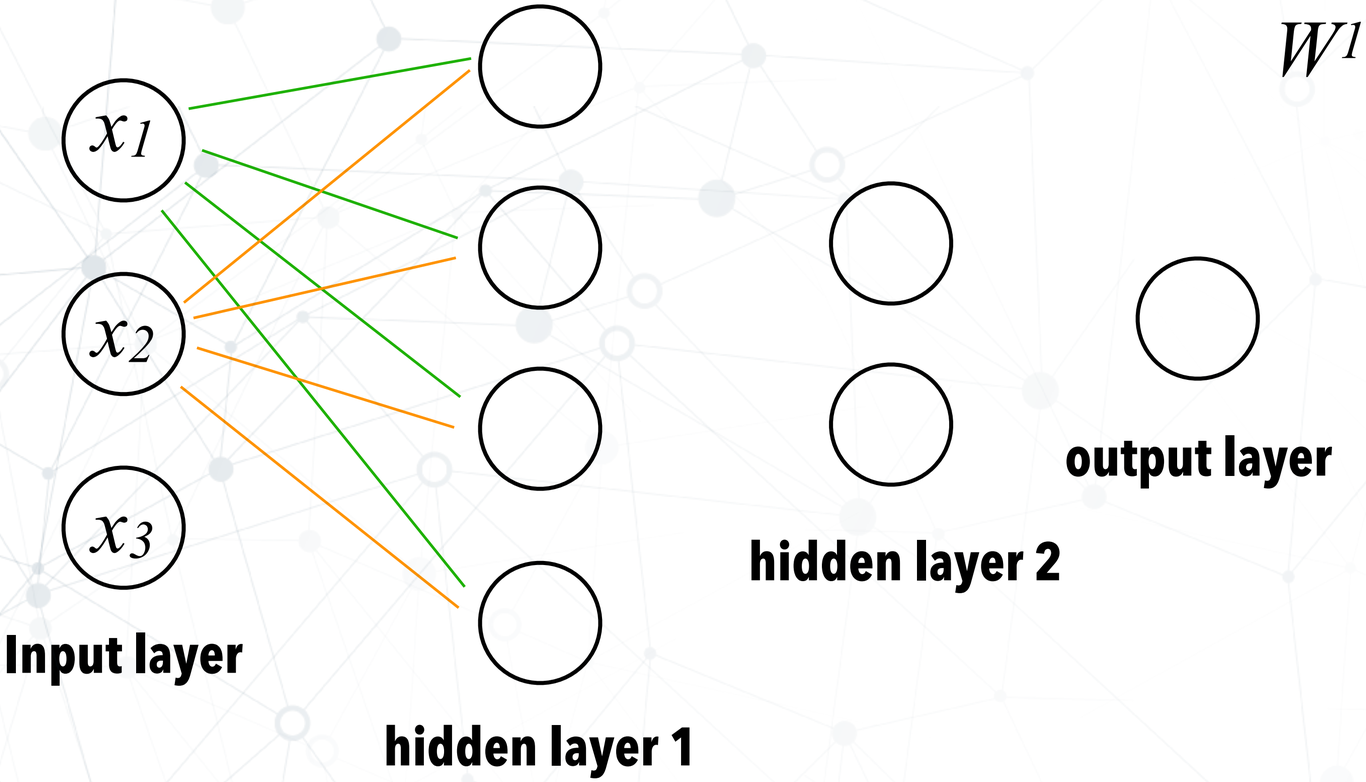
Multi-layer feedforward NN



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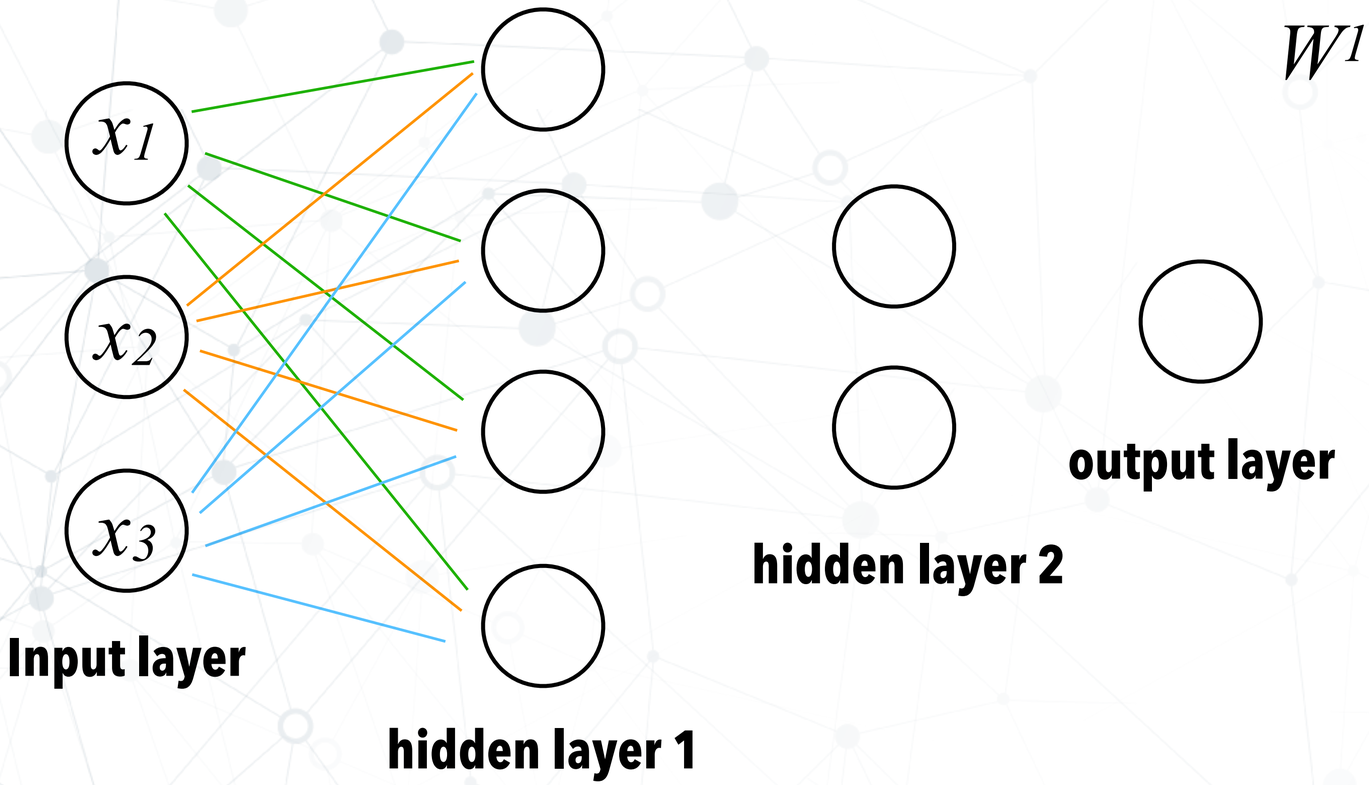
Multi-layer feedforward NN



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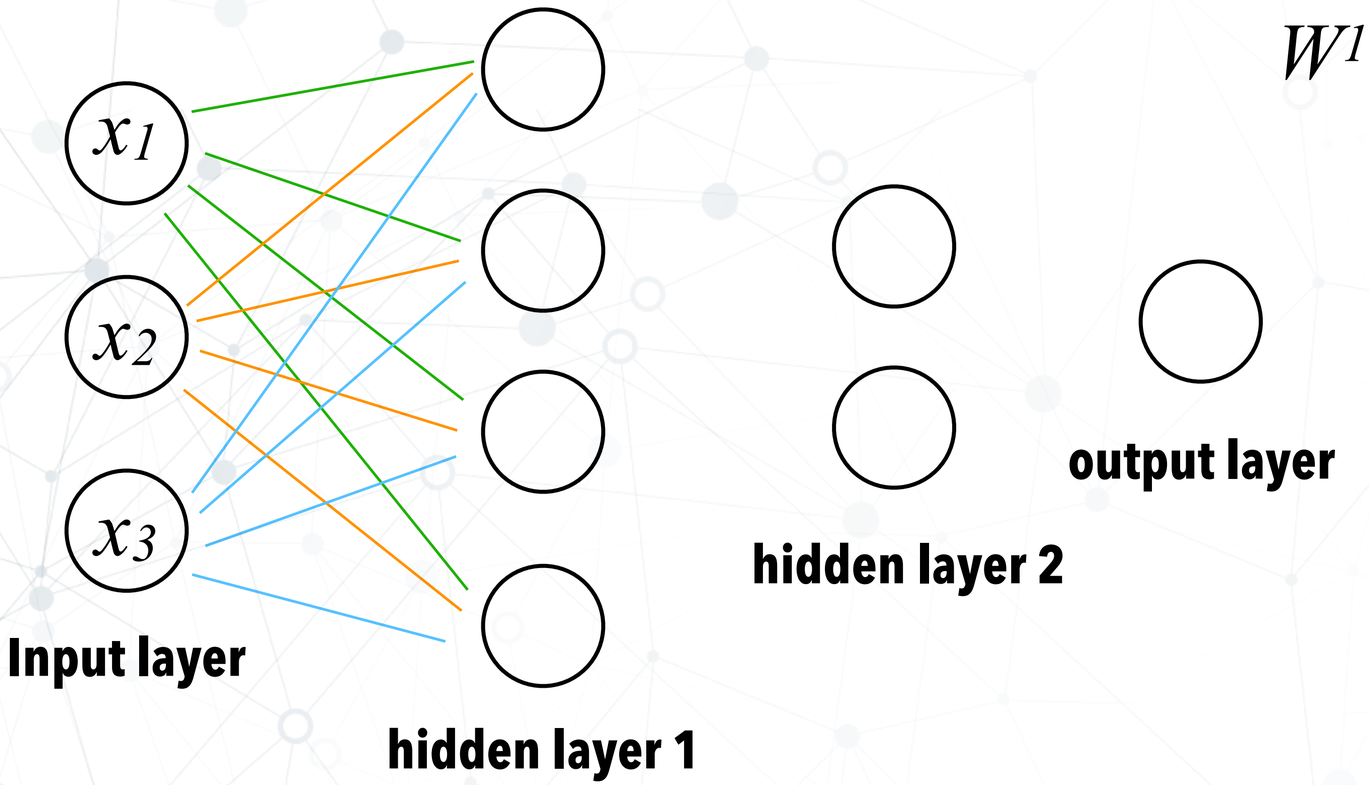
Multi-layer feedforward NN



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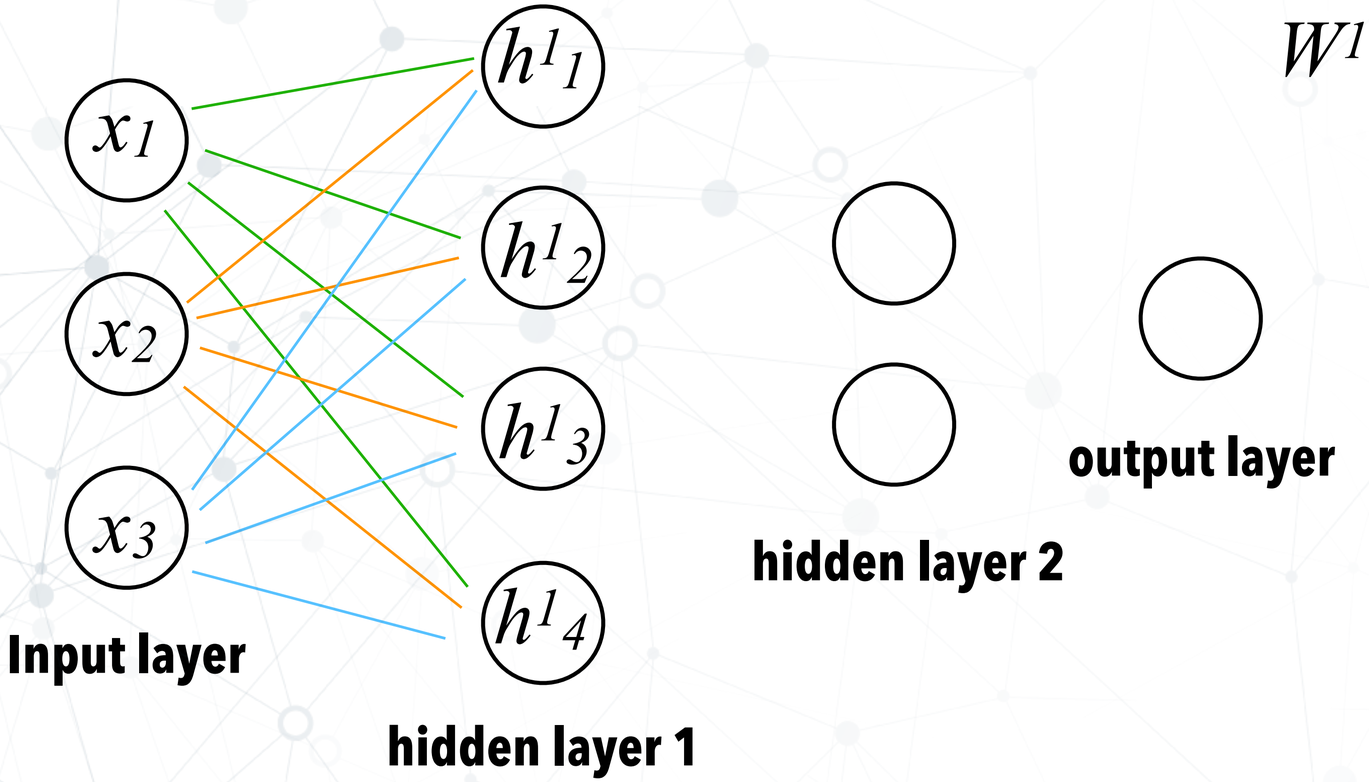
Multi-layer feedforward NN



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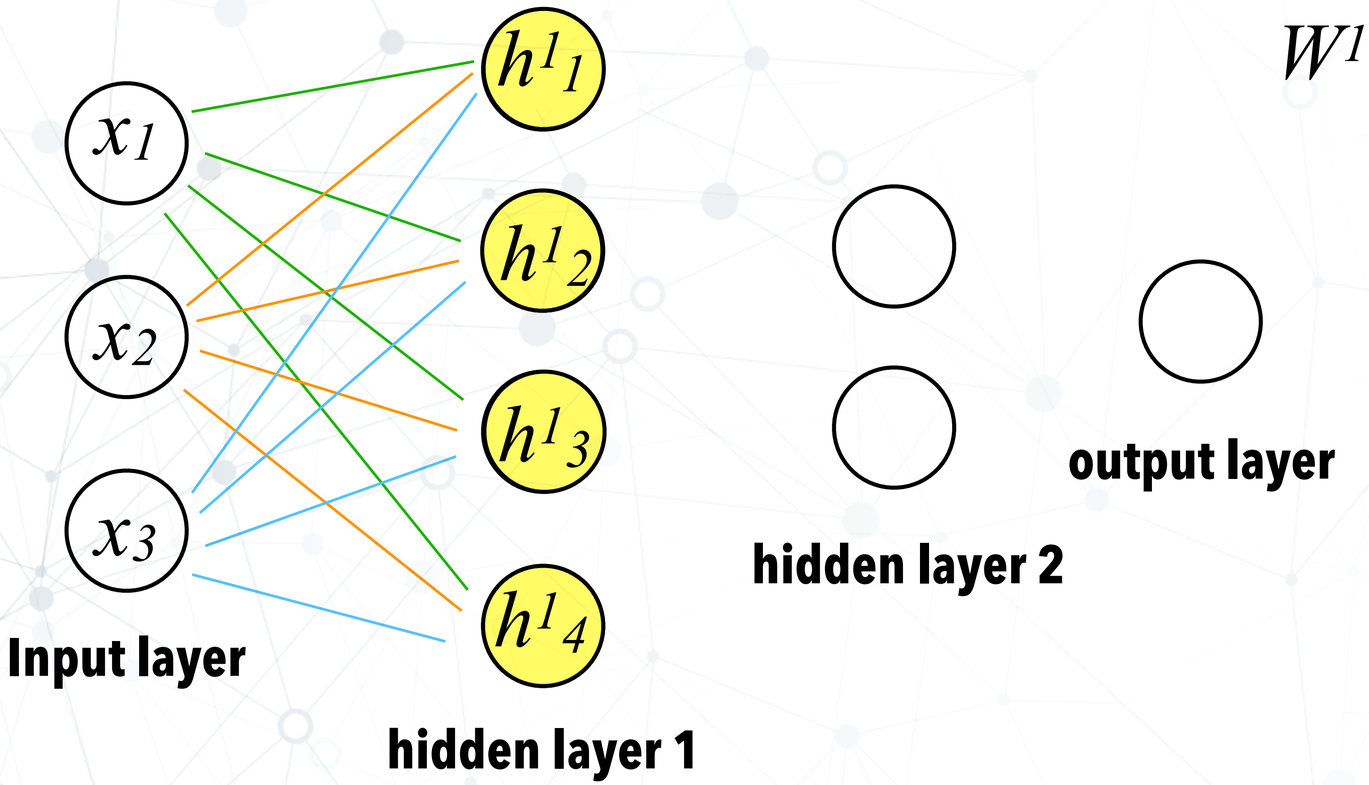
$$\tilde{h}^{[1]} = W^{[1]T} \cdot x + b^{[1]}$$

- Each line represents a weight w
- In each neuron a linear combination of the inputs is computed

Multi-layer feedforward NN



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$$\tilde{h}^{[1]} = W^{[1]T} \cdot x + b^{[1]}$$

$$h^{[1]} = \sigma(\tilde{h}^{[1]})$$

- Each line represents a weight w
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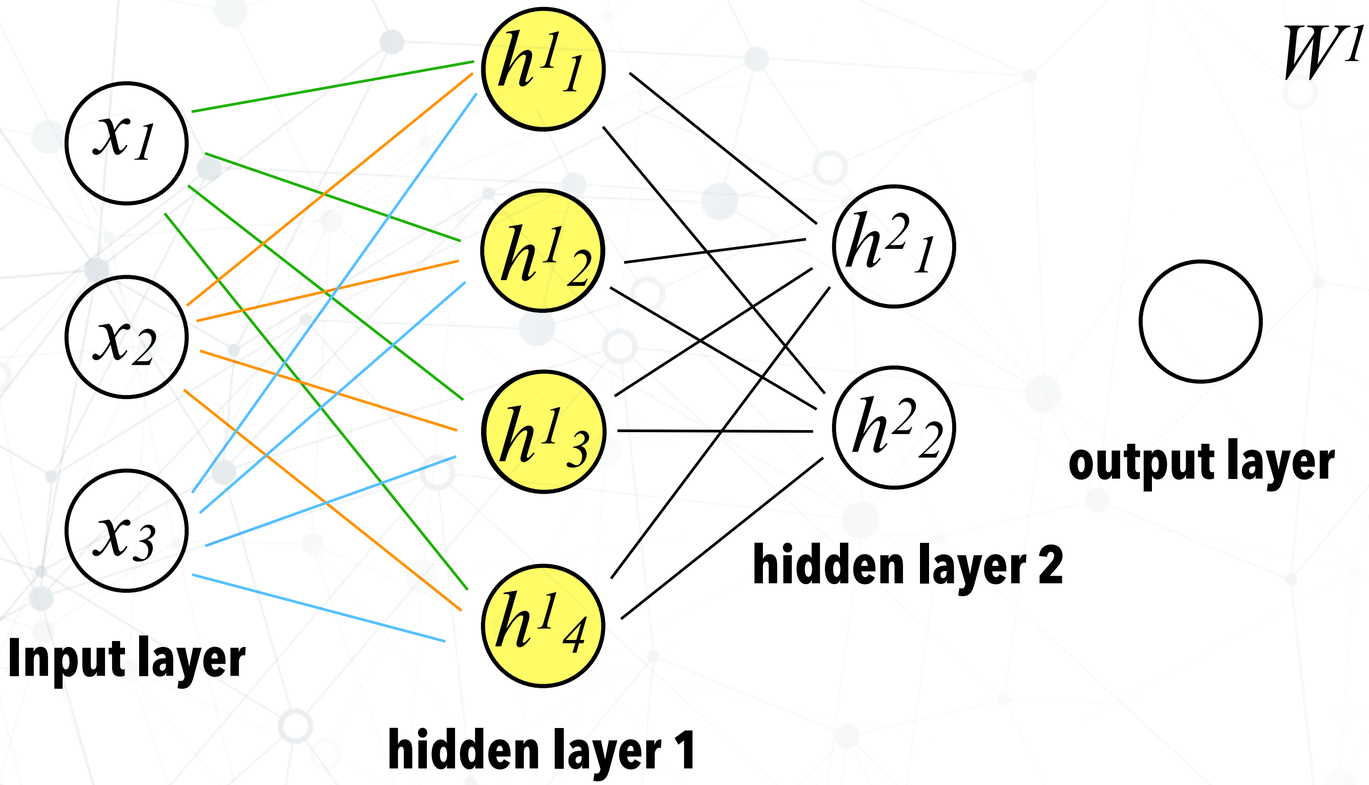
- The result is then activated with a non-linearity and become one of the inputs of the following layer

Multi-layer feedforward NN



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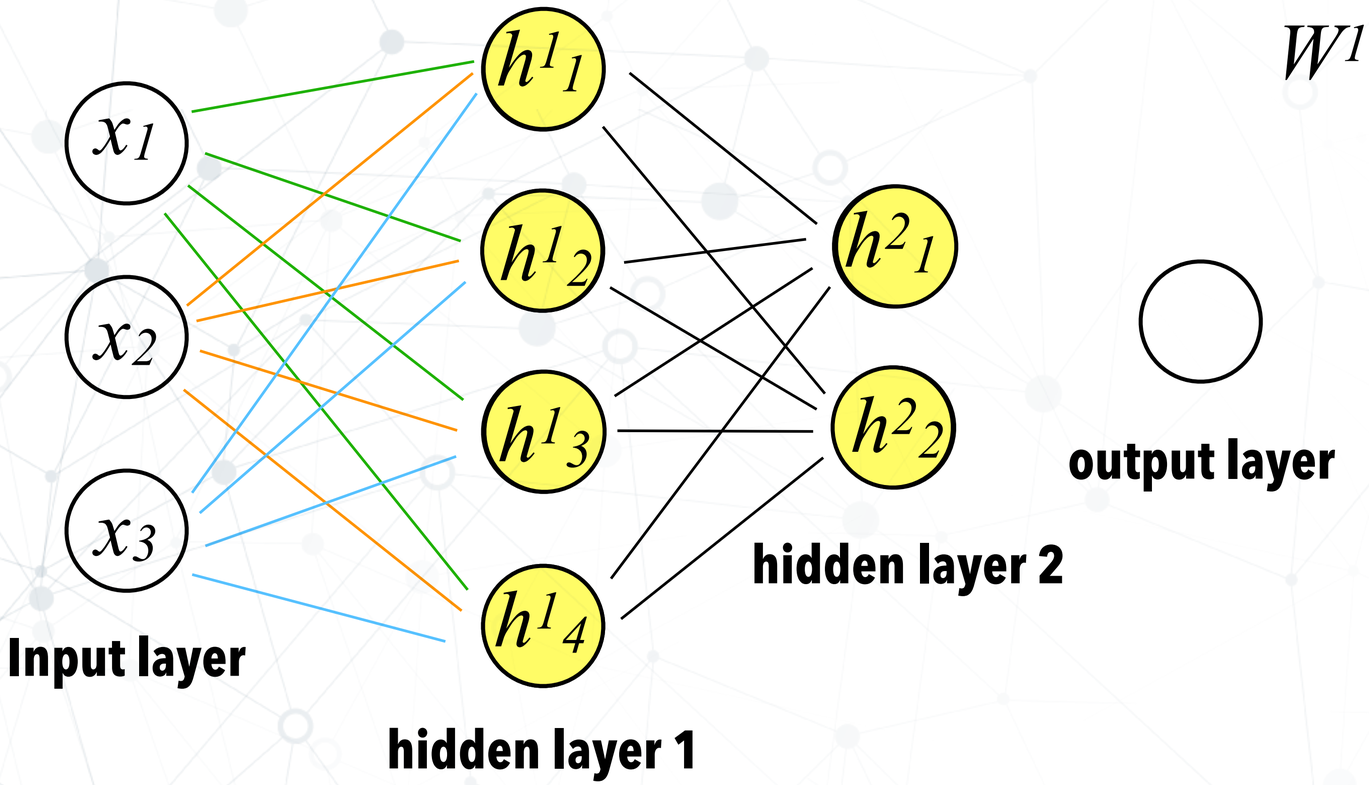
Multi-layer feedforward NN



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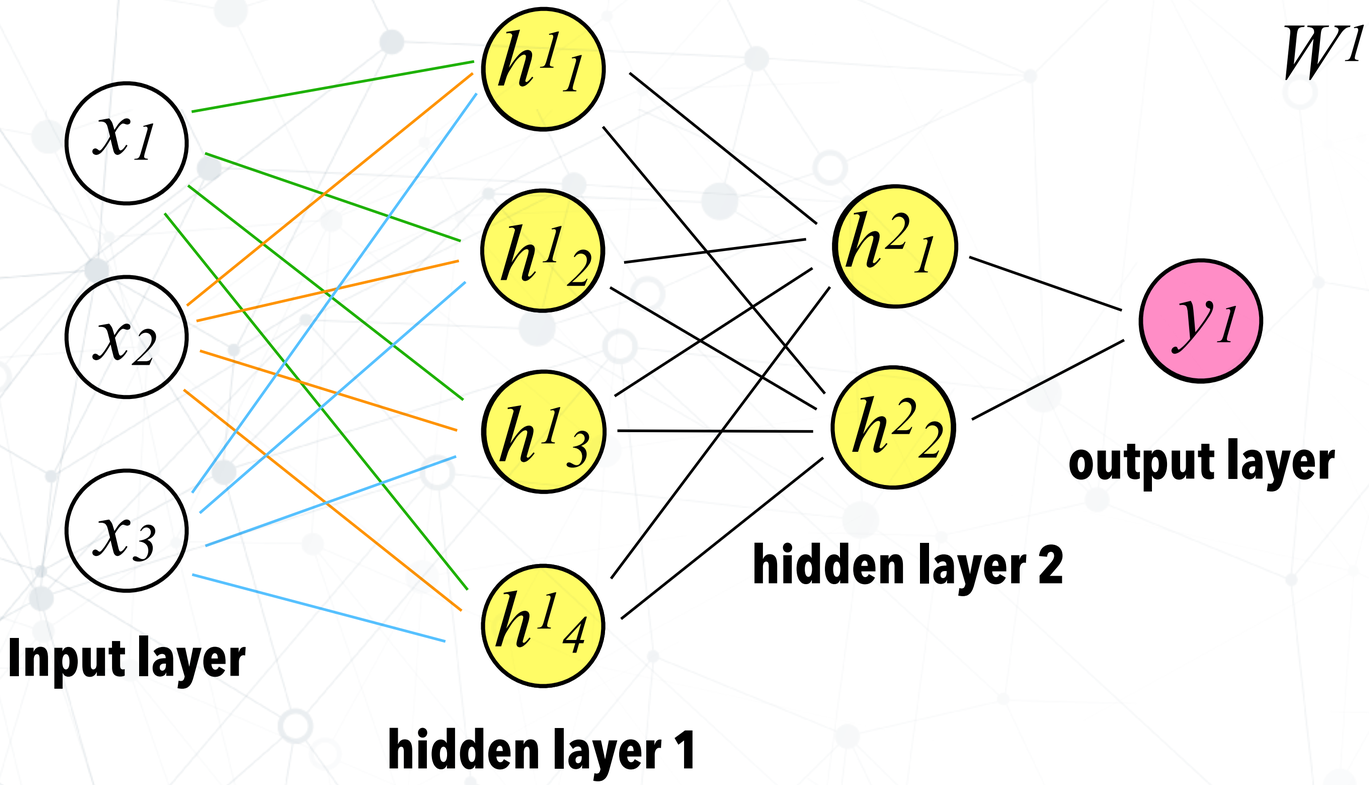
Multi-layer feedforward NN



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$$h^{[2]} = \sigma(\tilde{h}^{[2]})$$

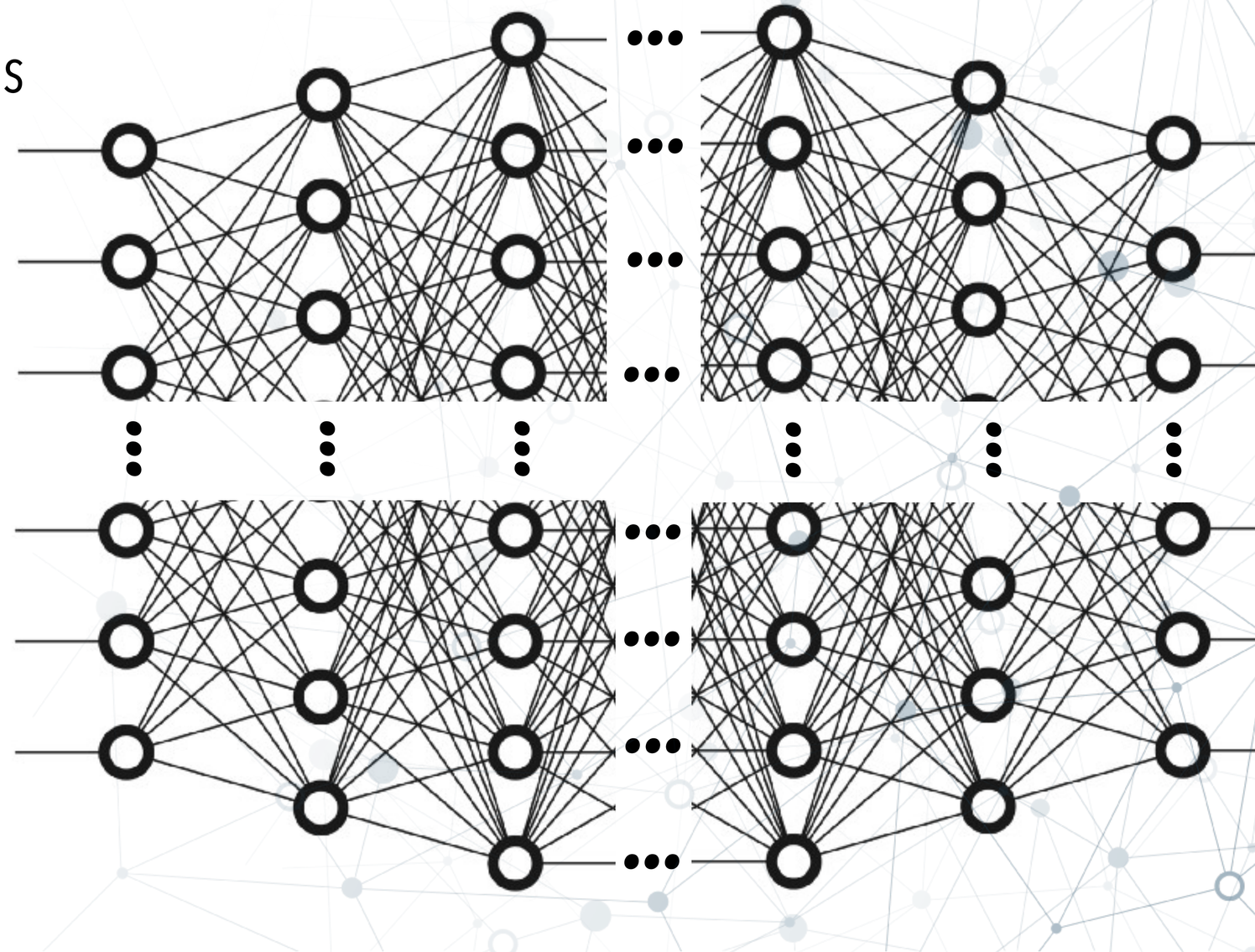
$$\tilde{y} = W^{[3]T} \cdot h^{[2]} + b^{[3]}$$

$$y = \sigma^*(\tilde{y})$$

- Each line represents a weight w
- In each neuron a linear combination of the inputs is computed
- The result is then activated with a non-linearity and become one of the inputs of the following layer

Multi-layer feedforward Neural Network

- This type of NN are called **dense** or **fully connected**
- very deep NNs can have thousands of layers
- and O(10^7) parameters
- The number of layers and neurons in each layer define the architecture of the NN are represent **hyperparameters**



Loss function

- Thourgh the feed-forward propagation the NN produces the output (values of the neurons in the output layer)
- During training the **output is compared with the ground truth**
- This is done by computing a **loss function**

$$\mathscr{L}(\mathbf{y}^{\mathbf{i}}, \hat{\mathbf{y}}^{\mathbf{i}})$$

NN output Ground truth

Cost function

- The **cost function** is the avarage of the loss over the training set

$$\mathcal{J} = \frac{1}{N} \sum_{i=0}^{N} \mathcal{L}(\mathbf{y^i}, \mathbf{\hat{y}^i})$$

- it is a function of all the NN's parameters (weights and biases):

$$\mathcal{J} = \mathcal{J}(\mathbf{w}, \mathbf{b})$$

The goal of training is to find the parameters

$\mathbf{w}$ and $\mathbf{b}$ that minimize $\mathcal{J}$

Gradient descent

see also previous lectures

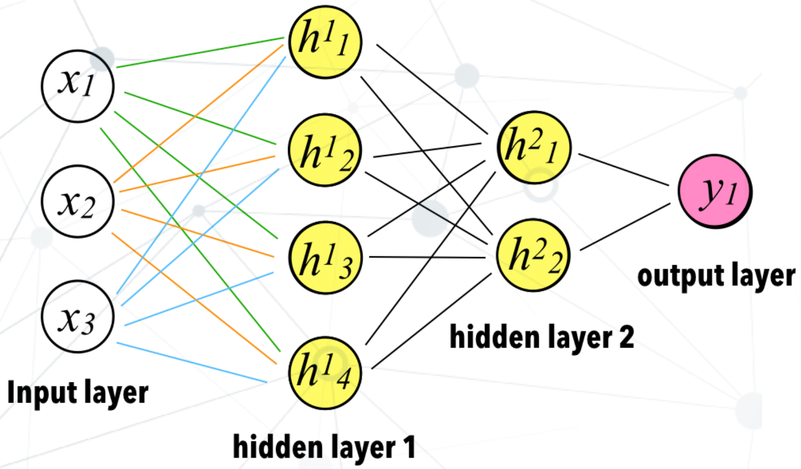
- usually the minimum of the cost function is found with a **gradient descent algorithm**
- at each iteration the parameters are updated as:

$$w := w - \alpha \frac{\partial \mathcal{J}}{\partial w} \qquad b := b - \alpha \frac{\partial \mathcal{J}}{\partial b}$$

- α is the **learining rate** and is an **hyperparameter** of the NN

(hyperparameter are set before training the network and are not optimized during training)

Backpropagation



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$$\tilde{y} = W^{[3]T} \cdot h^{[2]} + b^{[3]}$$
$$y = \sigma^*(\tilde{y})$$

$$\mathscr{L} = \mathscr{L}(y)$$

$$\mathscr{J} = \frac{1}{N} \sum \mathscr{L}$$

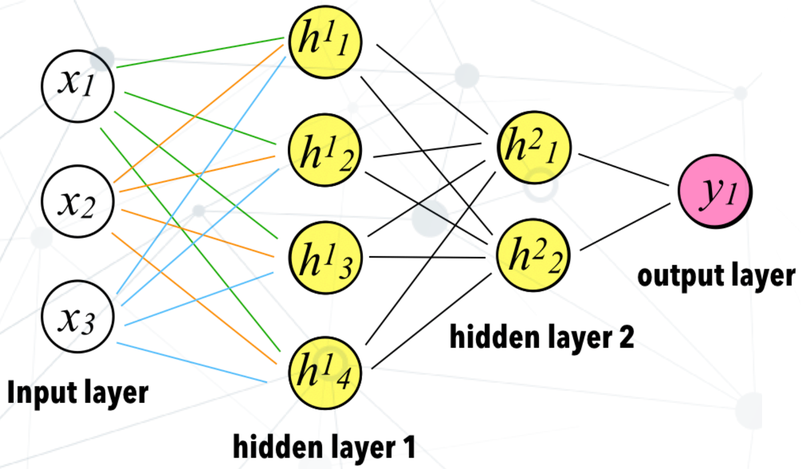
Derivatives are computed with the **chain rule**

see also previous lectures

$$\frac{\partial \mathscr{J}}{\partial W^{[3]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial W^{[3]}}$$

$$\frac{\partial \mathscr{J}}{\partial b^{[3]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial b^{[3]}}$$

Backpropagation



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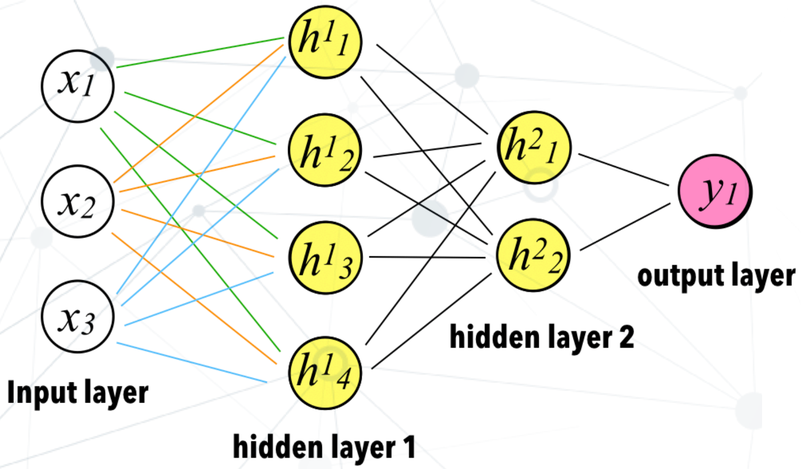
$$\frac{\partial \mathscr{J}}{\partial b^{[2]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial h^{[2]}} \frac{\partial \sigma}{\partial \tilde{h}^{[2]}} \frac{\partial \tilde{h}^{[2]}}{\partial b^{[2]}}$$

Backpropagation



$$\tilde{h}^{[1]} = W^{[1]T} \cdot x + b^{[1]}$$
$$h^{[1]} = \sigma(\tilde{h}^{[1]})$$

$$\tilde{h}^{[2]} = W^{[2]T} \cdot h^{[1]} + b^{[2]}$$
$$h^{[2]} = \sigma(\tilde{h}^{[2]})$$

$$\tilde{y} = W^{[3]T} \cdot h^{[2]} + b^{[3]}$$
$$y = \sigma^*(\tilde{y})$$

$$\mathscr{L} = \mathscr{L}(y)$$

$$\mathscr{J} = \frac{1}{N} \sum \mathscr{L}$$

Derivatives are computed with the **chain rule**

see also previous lectures

$$\frac{\partial \mathscr{J}}{\partial W^{[3]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial W^{[3]}}$$

$$\frac{\partial \mathscr{J}}{\partial W^{[2]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial h^{[2]}} \frac{\partial \sigma}{\partial \tilde{h}^{[2]}} \frac{\partial \tilde{h}^{[2]}}{\partial W^{[2]}}$$

$$\frac{\partial \mathscr{J}}{\partial W^{[1]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial h^{[2]}} \frac{\partial \sigma}{\partial \tilde{h}^{[2]}} \frac{\partial \tilde{h}^{[2]}}{\partial h^{[1]}} \frac{\partial \sigma}{\partial \tilde{h}^{[1]}} \frac{\partial \tilde{h}^{[1]}}{\partial W^{[1]}}$$

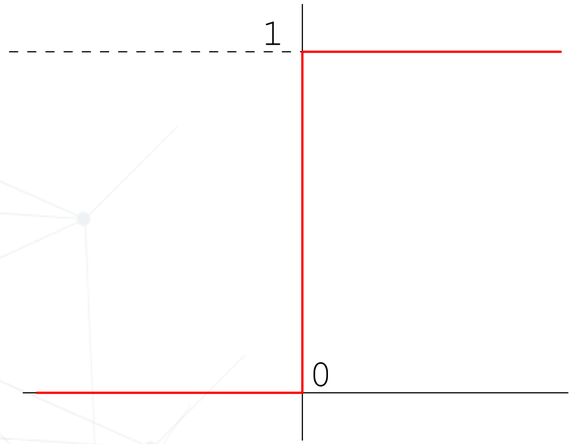
$$\frac{\partial \mathscr{J}}{\partial b^{[3]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial b^{[3]}}$$

$$\frac{\partial \mathscr{J}}{\partial b^{[2]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial h^{[2]}} \frac{\partial \sigma}{\partial \tilde{h}^{[2]}} \frac{\partial \tilde{h}^{[2]}}{\partial b^{[2]}}$$

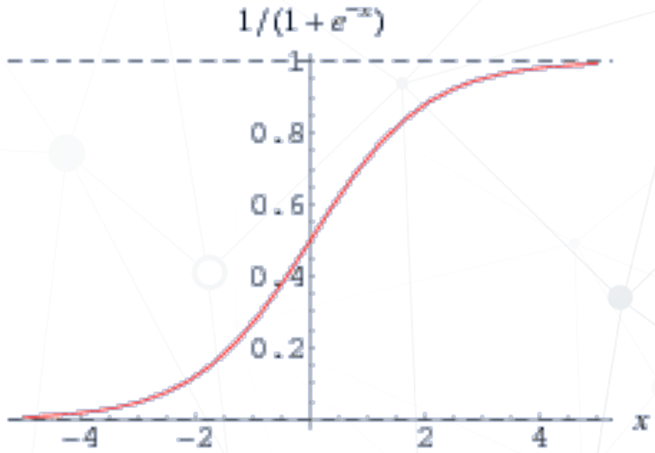
$$\frac{\partial \mathscr{J}}{\partial b^{[1]}} = \frac{\partial \mathscr{J}}{\partial \mathscr{L}} \frac{\partial \mathscr{L}}{\partial y} \frac{\partial \sigma^*}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial h^{[2]}} \frac{\partial \sigma}{\partial \tilde{h}^{[2]}} \frac{\partial \tilde{h}^{[2]}}{\partial h^{[1]}} \frac{\partial \sigma}{\partial \tilde{h}^{[1]}} \frac{\partial \tilde{h}^{[1]}}{\partial b^{[1]}}$$

Activation functions

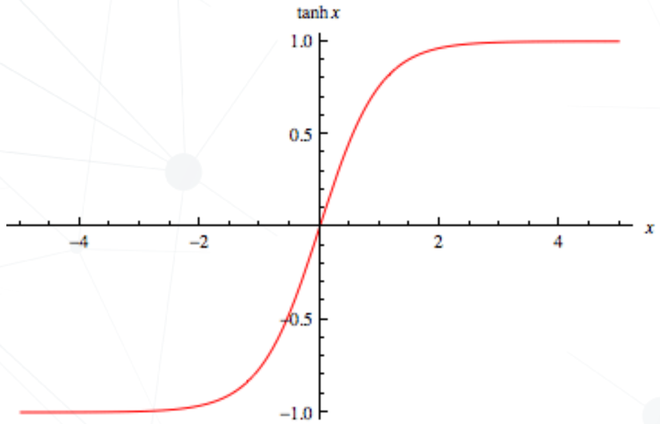
The **ReLU** function is currently the most used, preventing the problem of vanishing gradient during backpropagation



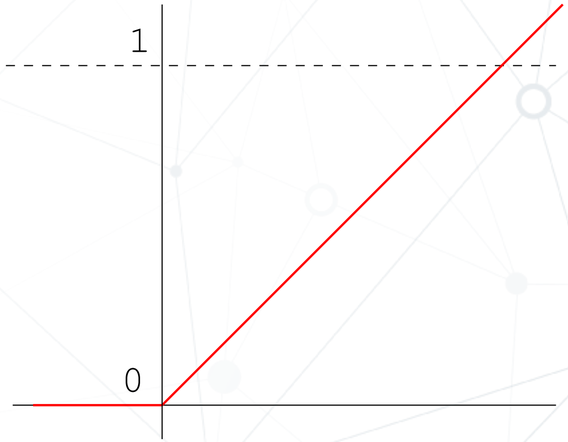
threshold: if $x > 0$ then 1 else 0



logistic function: $\frac{1}{1+e^{-x}}$



hyperbolic tangent: $\frac{e^x - e^{-x}}{e^x + e^{-x}}$



rectified linear (RELU): if $x > 0$ then x else 0

see also previous lectures

Tutorial 1

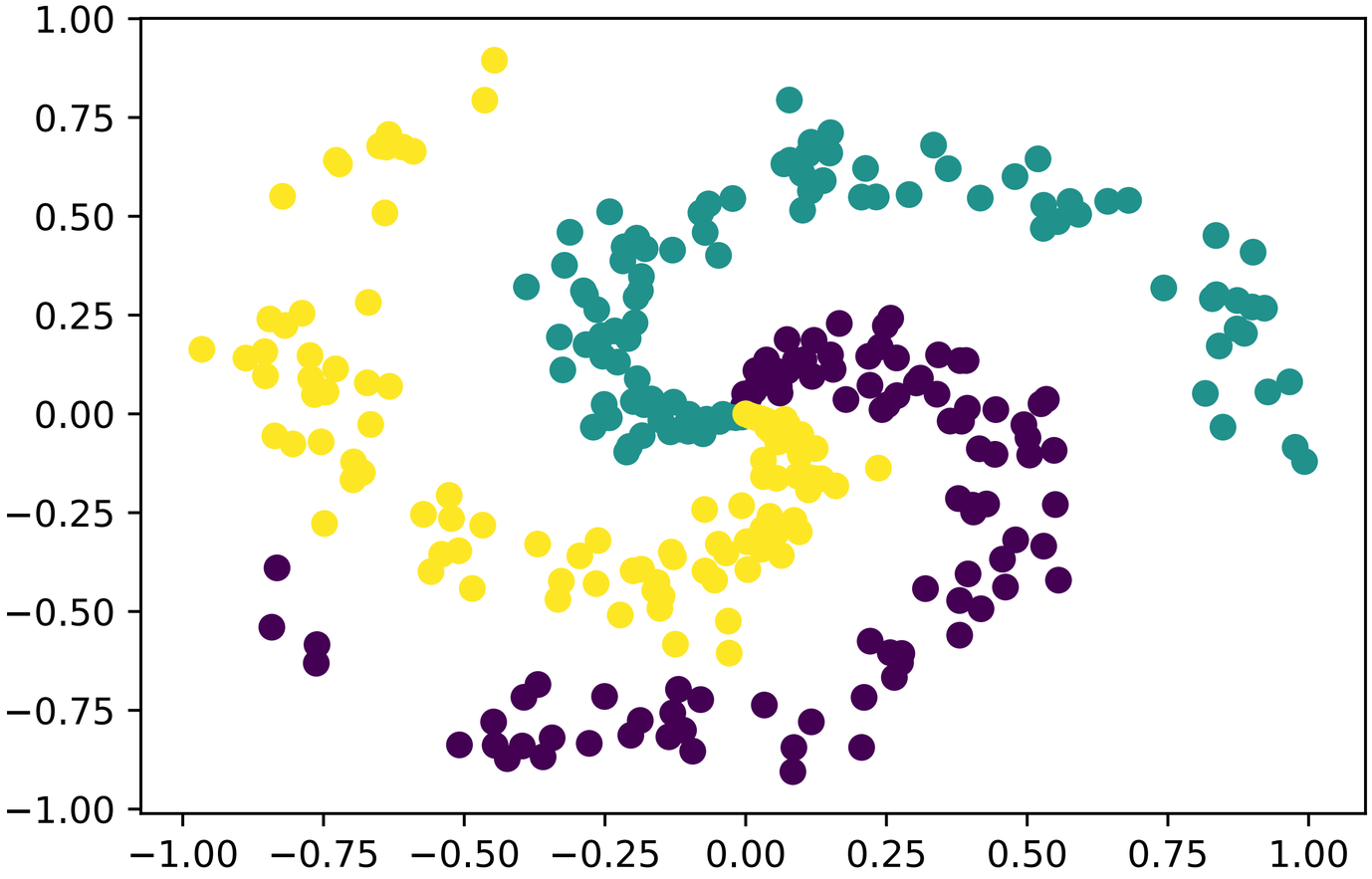
Coding a simple NN from scratch

taken from: http://cs231n.stanford.edu

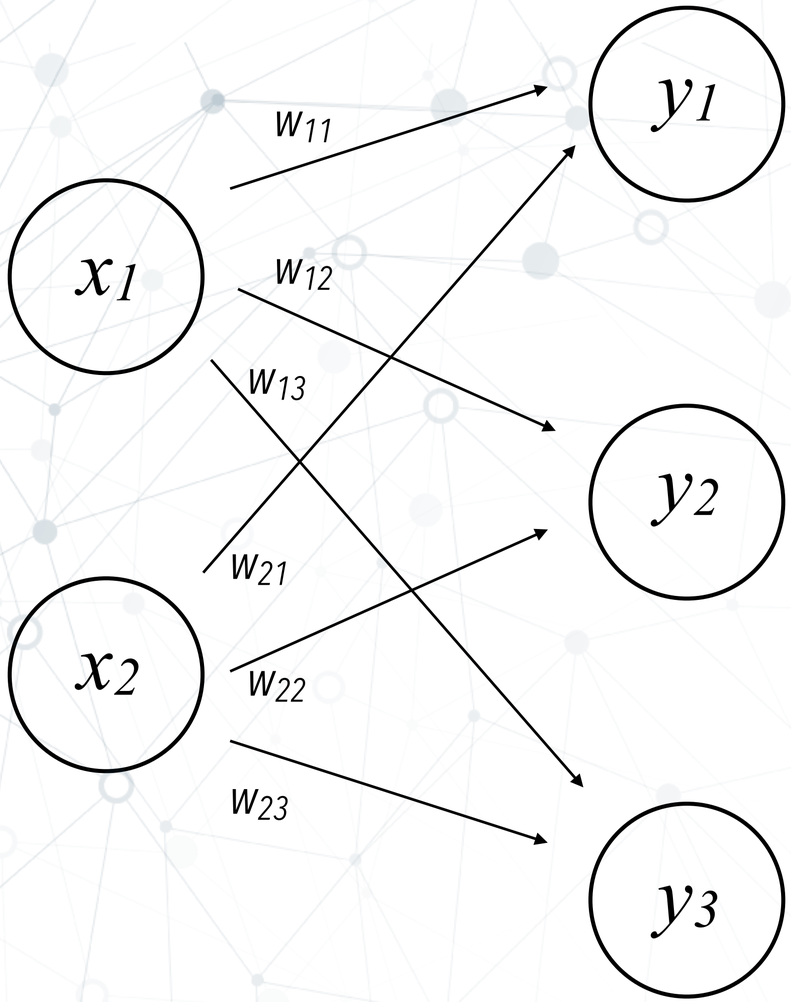
Coding a simple NN from scratch

Toy model: assign each particle to the correct class (arm)

- **step one:** linear classifier
- **step two:** fully connected NN



Coding a simple NN from scratch

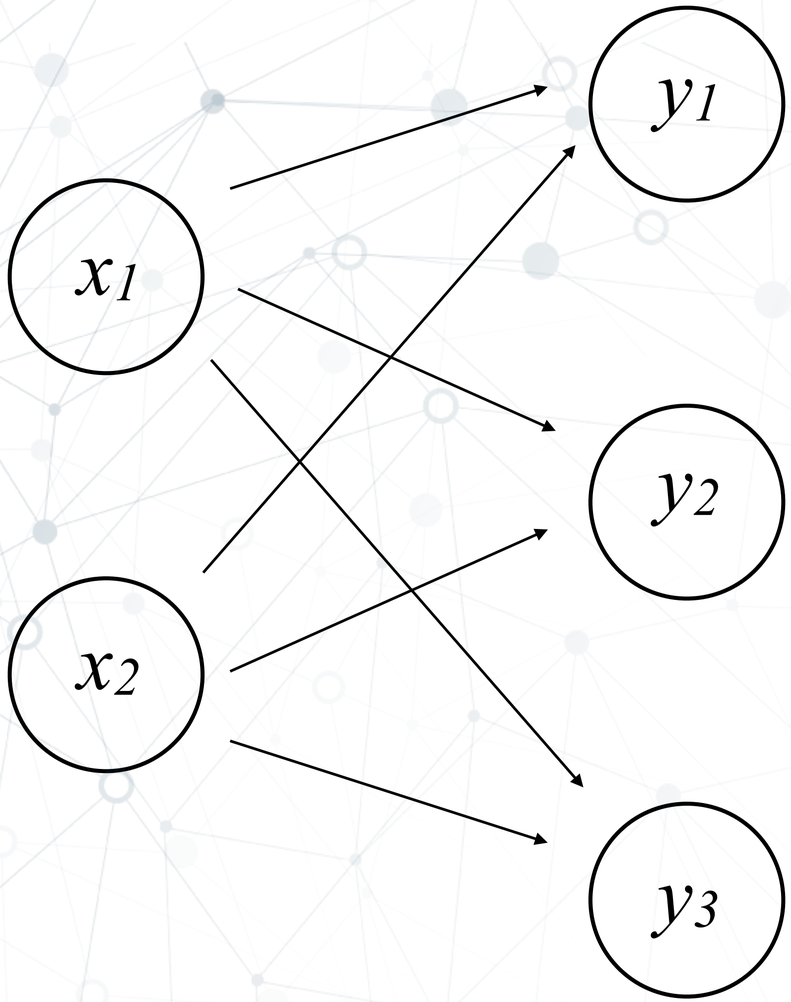


- The input vector x has 2 elements, representing the coordinates of each point
- The NN has just input and output layers, without any non-linear activation funcion

$$y = W^T \cdot x + b$$

$$W : (2, 3)$$

Coding a simple NN from scratch




```
In [1]: D = 2 # dimensionality
        K = 3 # number of classes (arms)
```

Initialize parameters:

```
In [4]: W = np.random.randn(D,K)
        b = np.zeros((1,K))
```

Compute output:

```
In [8]: print('Input shape: ', X.shape)
        num_examples = X.shape[0]
        y = np.dot(X, W) + b
        print('Output shape: ', y.shape)
```

```
Input shape:  (300, 2)
Output shape:  (300, 3)
```

Coding a simple NN from scratch

- The **loss function** is the negative log of the normalized exponential output of the true class (y_t) that can be interpret as a probability

$$\mathscr{L}_i = -\log\left(\frac{e^{y_t}}{\sum_{k=1}^{3} e^{y_k}}\right)_i = -\log\left[p_i(y_t)\right]$$

- The **cost function** is the sum over the number of elements in training set $\mathscr{J} = \frac{1}{N}\sum_{i=0}^{N}\mathscr{L}_i$

Compute loss and cost:

In [9]:
```
exp_y = np.exp(y)
probs = exp_y / np.sum(exp_y, axis=1, keepdims=True)

loss = -np.log(probs[range(num_examples),y_true])
cost = np.sum(loss)/num_examples
```

Coding a simple NN from scratch

- to implement the **backpropagation** we need to compute the derivatives of the loss function w.r.t. the parameters $\boldsymbol{W}$ and $\boldsymbol{b}$
- we use the chain rule and first we compute

$$\mathscr{L}_i = -\log\left(\frac{e^{y_t}}{\sum_{k=1}^{3} e^{y_k}}\right)_i = -\log\left[p_i(y_t)\right] \quad \Rightarrow \quad \frac{\partial \mathscr{L}_i}{\partial y_j} = -\frac{1}{p_i(y_t)} \frac{\partial}{\partial y_j}\left(\frac{e^{y_t}}{\sum_{k=1}^{3} e^{y_k}}\right)_i$$

$$\left(\frac{g}{h}\right)' = \frac{g'h - gh'}{h^2}$$

when $j = t$

$$\frac{\partial}{\partial y_j}\left(\frac{e^{y_t}}{\sum_{k=1}^{3} e^{y_k}}\right) = \frac{e^{y_t}\sum_{k=1}^{3} e^{y_k} - e^{y_t}e^{y_j}}{\left[\sum_{k=1}^{3} e^{y_k}\right]^2} = \frac{e^{y_t}}{\sum_{k=1}^{3} e^{y_k}}\left[1 - \frac{e^{y_j}}{\sum_{k=1}^{3} e^{y_k}}\right] = p(y_t)[1 - p(y_j)]$$

when $j \neq t$

$$\frac{\partial}{\partial y_j}\left(\frac{e^{y_t}}{\sum_{k=1}^{3} e^{y_k}}\right) = -\frac{e^{y_t}e^{y_j}}{\sum_{k=1}^{3} e^{y_k}} = -p(y_t)p(y_j)$$

Coding a simple NN from scratch

therefore: $\frac{\partial \mathcal{L}}{\partial y_j} = -\frac{1}{p(y_t)} \frac{\partial}{\partial y_j} \left(\frac{e^{y_t}}{\sum_{k=1}^{3} e^{y_k}} \right)_i = p(y_j) - 1_{j=t}$

```
In [13]: # derivative w.r.t y
         dy = probs
         dy[range(num_examples),y_true] -= 1
         dy /= num_examples
```

we continue the backpropagation and compute the derivatives w.r.t. the parameters

remind that $y = x \cdot W + b$ and therefore $\frac{\partial \mathcal{L}}{\partial W} = \frac{\partial \mathcal{L}}{\partial y} \frac{\partial y}{\partial W}$ and similarly for b

```
In [12]: dW = np.dot(X.T, dy)
         db = np.sum(dy, axis=0, keepdims=True)
```

Coding a simple NN from scratch

- We have now everything needed to update the value of the parameters

```
In [87]: step_size = 1e-0
         W += -step_size * dW
         b += -step_size * db
```

- and we can train the linear classifier by iterate in order to find the minimum of the cost function

go to notebook

Coding a simple NN from scratch

- We have now everything needed to update the value of the parameters

```
In [87]: step_size = 1e-0
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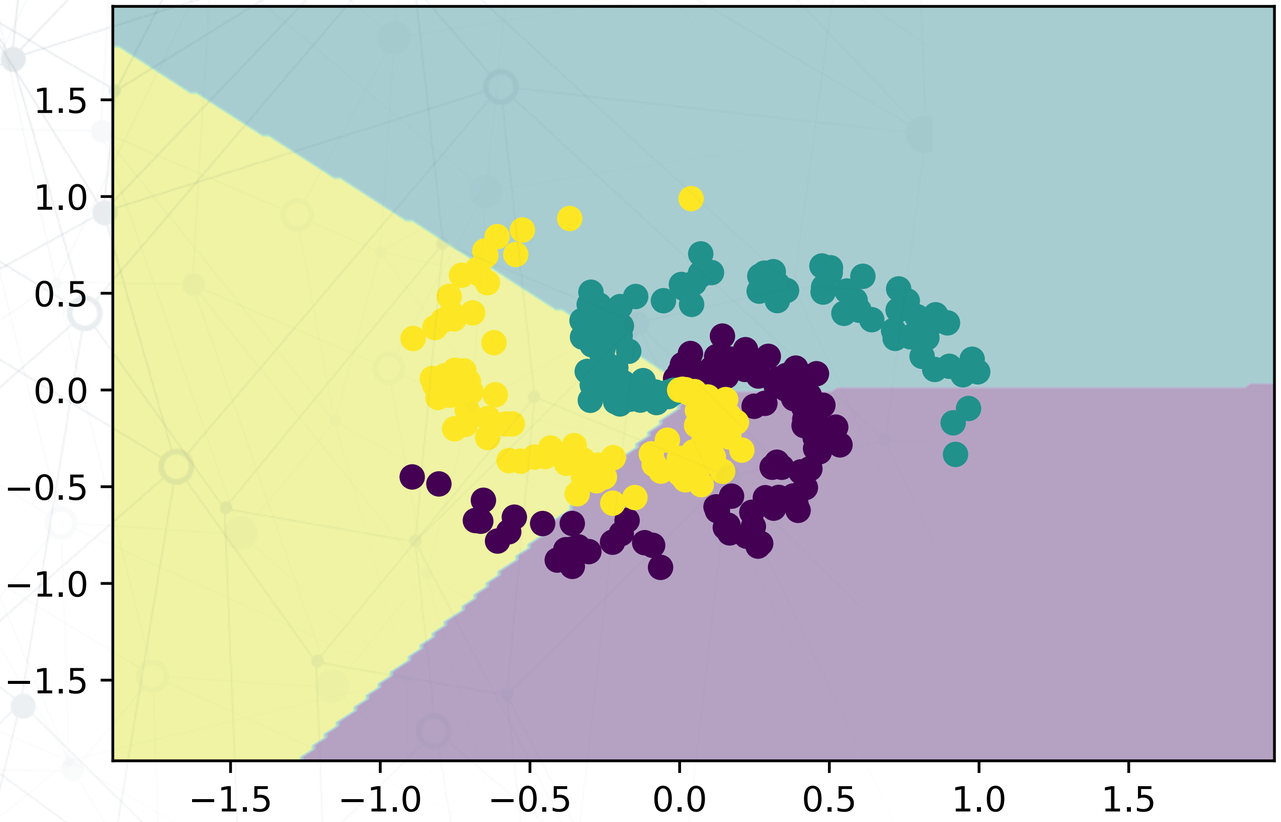
go to notebook



- Not surprisingly, a linear classifier doesn't perform well as our dataset is not linearly separable

Coding a simple NN from scratch

- we add to the NN an hidden layer with 100 neurons activated with a ReLU non-linear function

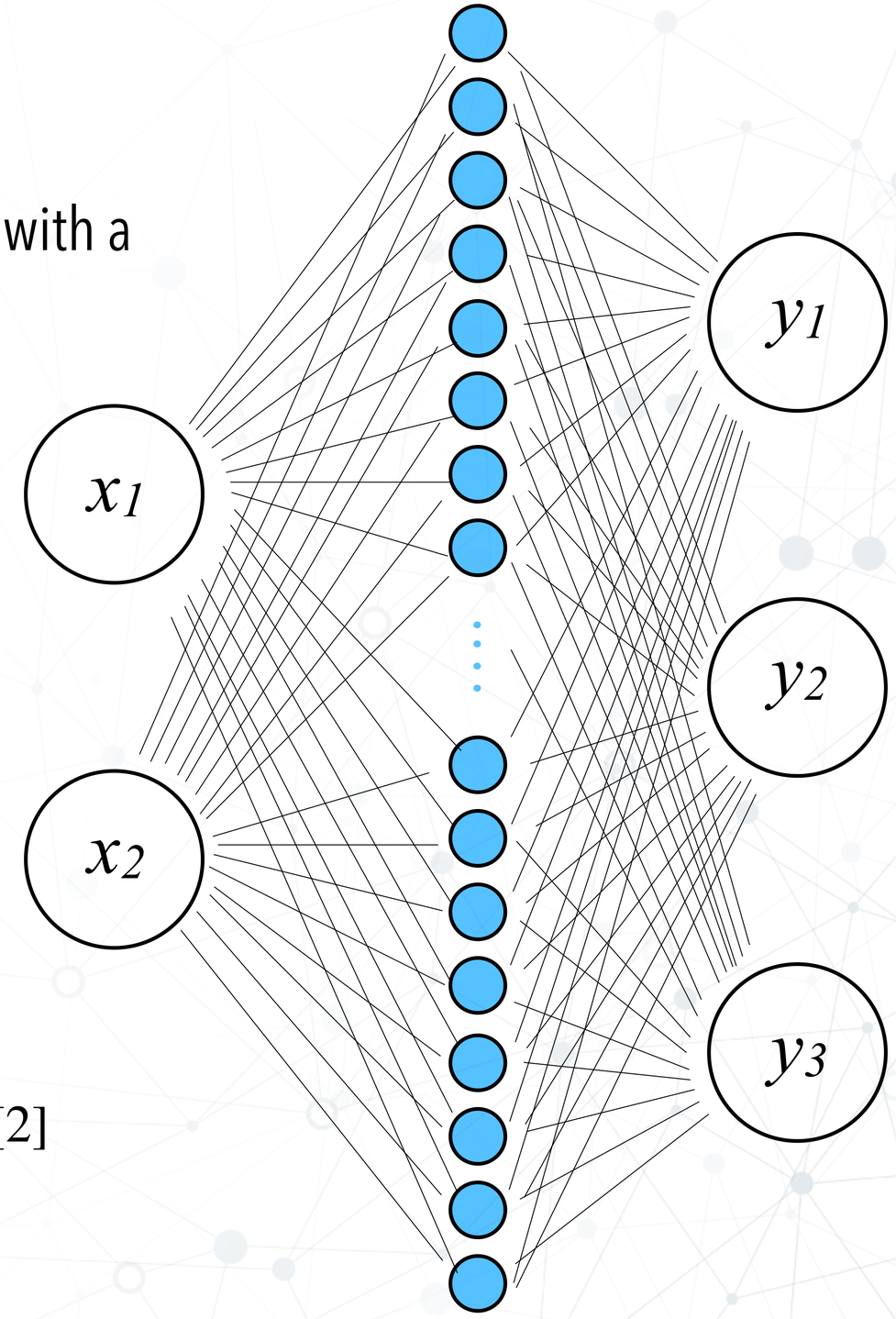
$$W^{[1]} : (2, 100)$$

$$W^{[2]} : (100, 3)$$

$$\tilde{h}^{[1]} = x \cdot W^{[1]} + b^{[1]}$$

$$h = ReLu(\tilde{h}^{[1]})$$

$$y = h \cdot W^{[2]} + b^{[2]}$$



Coding a simple NN from scratch

In [36]:
```
hL = 100 # size of hidden layer
W1 = 0.01 * np.random.randn(D, hL)
b1 = np.zeros((1,hL))
W2 = 0.01 * np.random.randn(hL, K)
b2 = np.zeros((1,K))
```

In [37]:
```
h = np.maximum(0, np.dot(X, W1) + b1) # note, ReLU activation
print('Hidden layer shape: ', h.shape)
y = np.dot(h, W2) + b2
print('Output shape: ', y.shape)
```

```
Hidden layer shape:  (300, 100)
Output shape:  (300, 3)
```

In [38]:
```
# compute the class probabilities
exp_y = np.exp(y)
probs = exp_y / np.sum(exp_y, axis=1, keepdims=True)

# compute loss and cost
loss = -np.log(probs[range(num_examples),y_true])
cost = np.sum(loss)/num_examples
```

Coding a simple NN from scratch

- now we start backpropagation:

$$\tilde{h}^{[1]} = x \cdot W^{[1]} + b^{[1]} \qquad h = ReLu(\tilde{h}^{[1]}) \qquad y = h \cdot W^{[2]} + b^{[2]}$$

In [39]:
```
# derivative w.r.t. y
dy = probs
dy[range(num_examples),y_true] -= 1
dy /= num_examples

# backpropate the gradient to the parameters
# first backprop into parameters W2 and b2
dW2 = np.dot(h.T, dy)
db2 = np.sum(dy, axis=0, keepdims=True)

# next backprop into hidden layer
dh = np.dot(dy, W2.T)

# backprop the ReLU non-linearity
dh[h <= 0] = 0

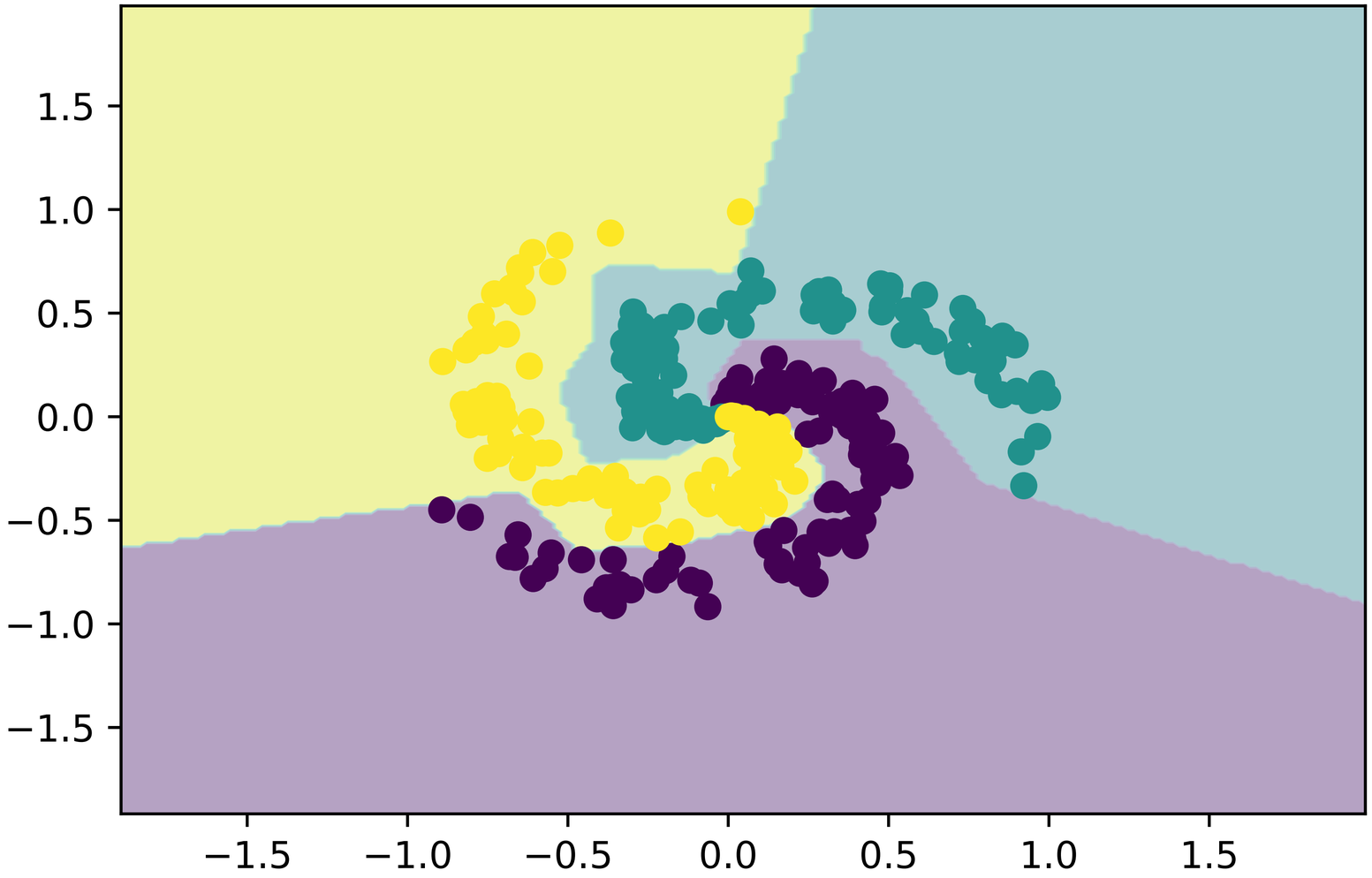
# finally into W,b
dW1 = np.dot(X.T, dh)
db1 = np.sum(dh, axis=0, keepdims=True)

# perform a parameter update
W1 += -step_size * dW1
b1 += -step_size * db1
W2 += -step_size * dW2
b2 += -step_size * db2
```

go to notebook

Coding a simple NN from scratch

- By implementing this simple fully connected NN we reached an accuracy of ~99% on training set



Libraries for NN development

- In the tutorial we have explicitly implemented all the steps of the forward and back propagations, with the calculation of all the needed derivatives
- Doing that for a deep network with hundreds of layers could easily become a hard job

Libraries for NN development

- In the tutorial we have explicitly implemented all the steps of the forward and back propagations, with the calculation of all the needed derivatives
- Doing that for a deep network with hundreds of layers could easily become a hard job

  - There exist several **opensource libraries optimized for tensor computation** which implement the most common kind of layers, activations, loss functions etc, togheter with their backpropagation
  - Most common: **PyTorch** and **Tensorflow+Keras,** developed in python and GPU accelerated

    - **Tensorflow** is an opensource library developed in python and C++ optimized for machine learning. it has been implemented by Google and released in 2015. Can run on both CPU and GPU (https://www.tensorflow.org)
    - **Keras** has an higher level interface specifically used to develop NN and can run with tensorflow backend (https://keras.io)

Keras (+ tensorflow)

Losses

- Usage of loss functions
- Available loss functions
- mean_squared_error
- mean_absolute_error
- mean_absolute_percentage_error
- mean_squared_logarithmic_error
- squared_hinge
- hinge
- categorical_hinge
- logcosh
- categorical_crossentropy
- sparse_categorical_crossentropy
- binary_crossentropy
- kullback_leibler_divergence
- poisson
- cosine_proximity

Metrics

- Usage of metrics
- Arguments
- Returns
- Available metrics
- binary_accuracy
- categorical_accuracy
- sparse_categorical_accuracy
- top_k_categorical_accuracy
- sparse_top_k_categorical_accuracy
- Custom metrics

Optimizers

- Usage of optimizers
- Parameters common to all Keras optimizers
- RMSprop
- Adagrad
- Adadelta
- SGD
- Adam
- Adamax
- Nadam
- TFOptimizer

Activations

- Usage of activations
- Available activations
- elu
- selu
- softplus
- softsign
- relu
- tanh
- sigmoid
- hard_sigmoid
- linear
- softmax
- On "Advanced Activations"

Keras (+ tensorflow)

1) **Define a model**, layer by layer, choosing activation functions
2) **Compile a model**, setting the optimized, loss function and metric
3) **Fit a model** on training set

go to notebook

Keras (+ tensorflow)

1) **Define a model**, layer by layer, choosing activation functions
2) **Compile a model**, setting the optimized, loss function and metric
3) **Fit a model** on training set

Sequential

```
# For a single-input model with 2 classes (binary classification):

model = Sequential()
model.add(Dense(32, activation='relu', input_dim=100))
model.add(Dense(1, activation='sigmoid'))
model.compile(optimizer='rmsprop',
              loss='binary_crossentropy',
              metrics=['accuracy'])

# Generate dummy data
import numpy as np
data = np.random.random((1000, 100))
labels = np.random.randint(2, size=(1000, 1))

# Train the model, iterating on the data in batches of 32 samples
model.fit(data, labels, epochs=10, batch_size=32)
```

Functional

```
# This returns a tensor
inputs = Input(shape=(784,))

# a layer instance is callable on a tensor, and returns a tensor
output_1 = Dense(64, activation='relu')(inputs)
output_2 = Dense(64, activation='relu')(output_1)
predictions = Dense(10, activation='softmax')(output_2)

# This creates a model that includes
# the Input layer and three Dense layers
model = Model(inputs=inputs, outputs=predictions)
model.compile(optimizer='rmsprop',
              loss='categorical_crossentropy',
              metrics=['accuracy'])
model.fit(data, labels)  # starts training
```

go to notebook

Training a NN

- Training a NN is an empirical process, where **tuning of hyper-parameters** (such as #layers, #neurons, learning rate, etc.) is reached trough trails and errors
- usually the available labeled data are divided into **training** (80%), **validation** (10%) and **test** (10%)

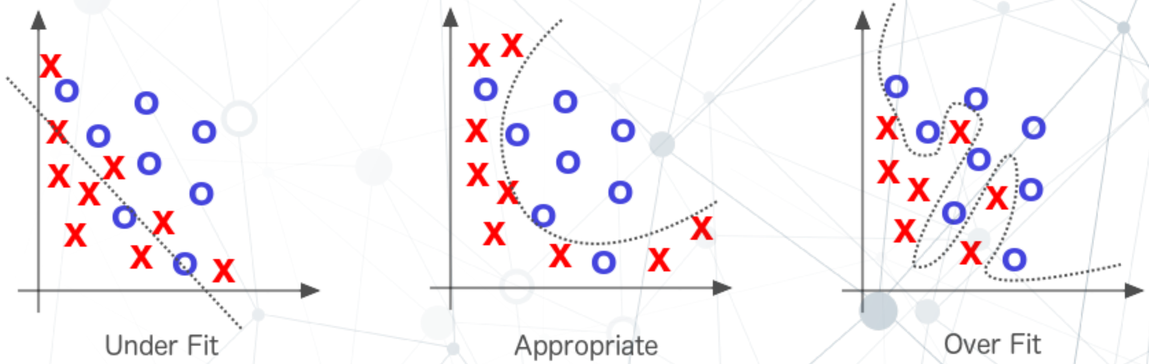
Training: set of data used to optimize NN's weights

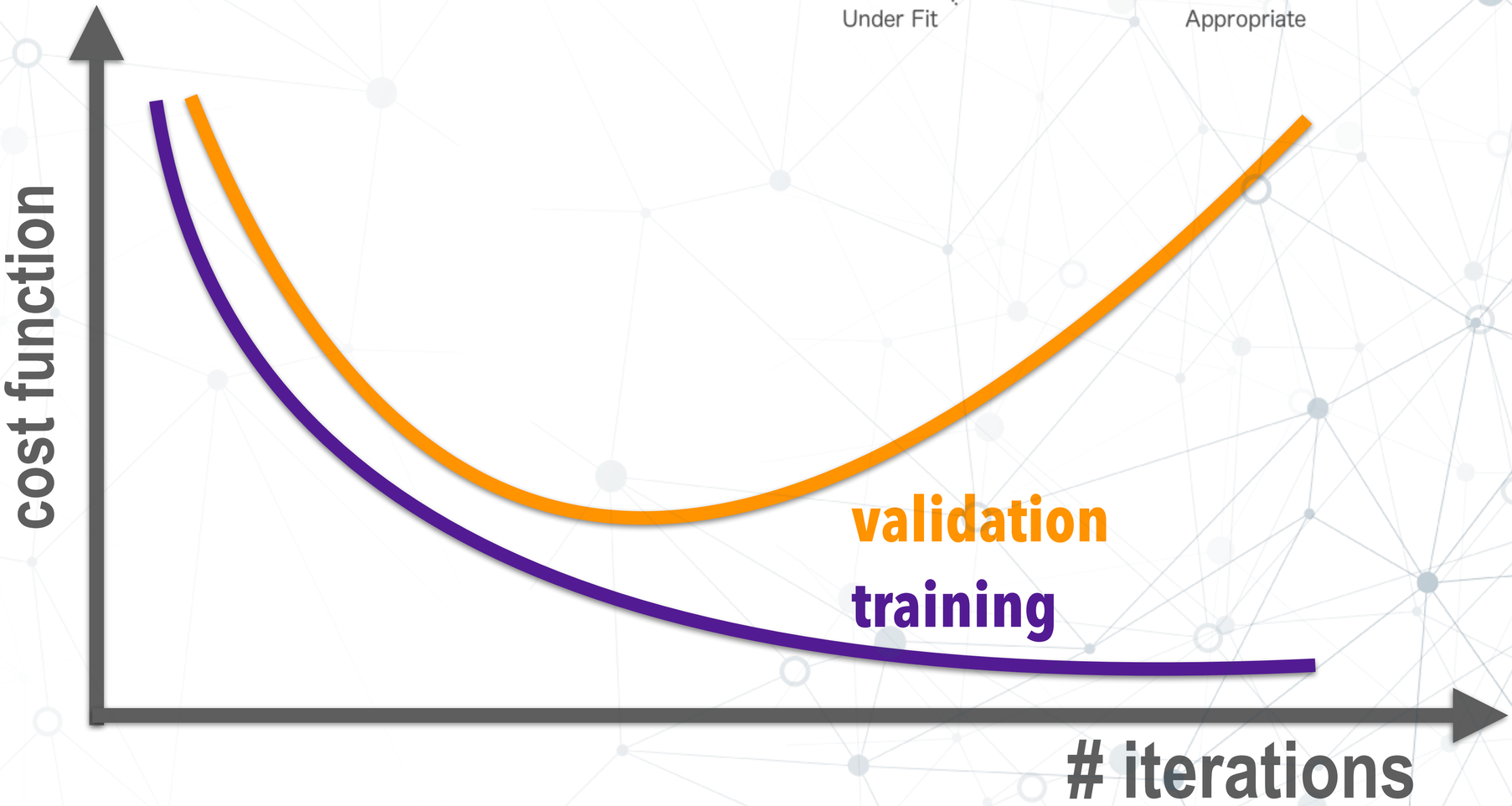
Validation: set of data used to tune NN's hyper-parameters

Test: set of data used to test and asses the final NN's performances once hyperparameters are set

Training a NN: Overfitting

- One of the the typical problem is **overfitting** which doesn't allow the NN to generalize results





Training a NN: regularization

- **Regularization:** a regularization term R is added to the cost function. This term imposes a penalty on the complexity of $\mathscr{L}$. the hyperparameter λ controls the importance of the regularization term

$$\mathscr{J} = \frac{1}{N}\sum_{i=0}^{N} \mathscr{L}_i + \lambda R\left(\mathscr{L}\right)$$

Training a NN: regularization

- **Regularization:** a regularization term R is added to the cost function. This term imposes a penalty on the complexity of $\mathscr{L}$. the hyperparameter λ controls the importance of the regularization term

$$\mathscr{J} = \frac{1}{N}\sum_{i=0}^{N} \mathscr{L}_i + \lambda R(\mathscr{L})$$

- a typical example is the L^2 regularization

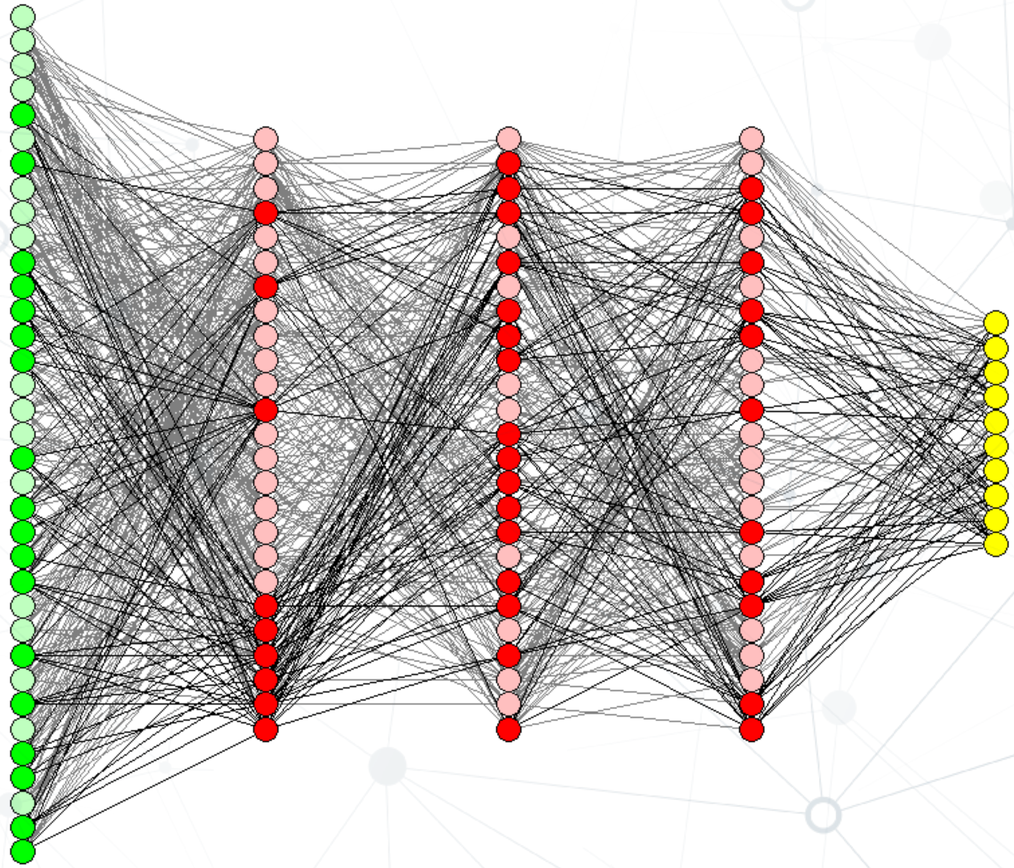
$$\mathscr{J} = \frac{1}{N}\sum_{i=0}^{N} \mathscr{L}_i + \lambda \sum_{l=1}^{L} \|W\|^2$$

sum over the layers

it gives preference to smaller values of the NN's weights

Training a NN: dropout

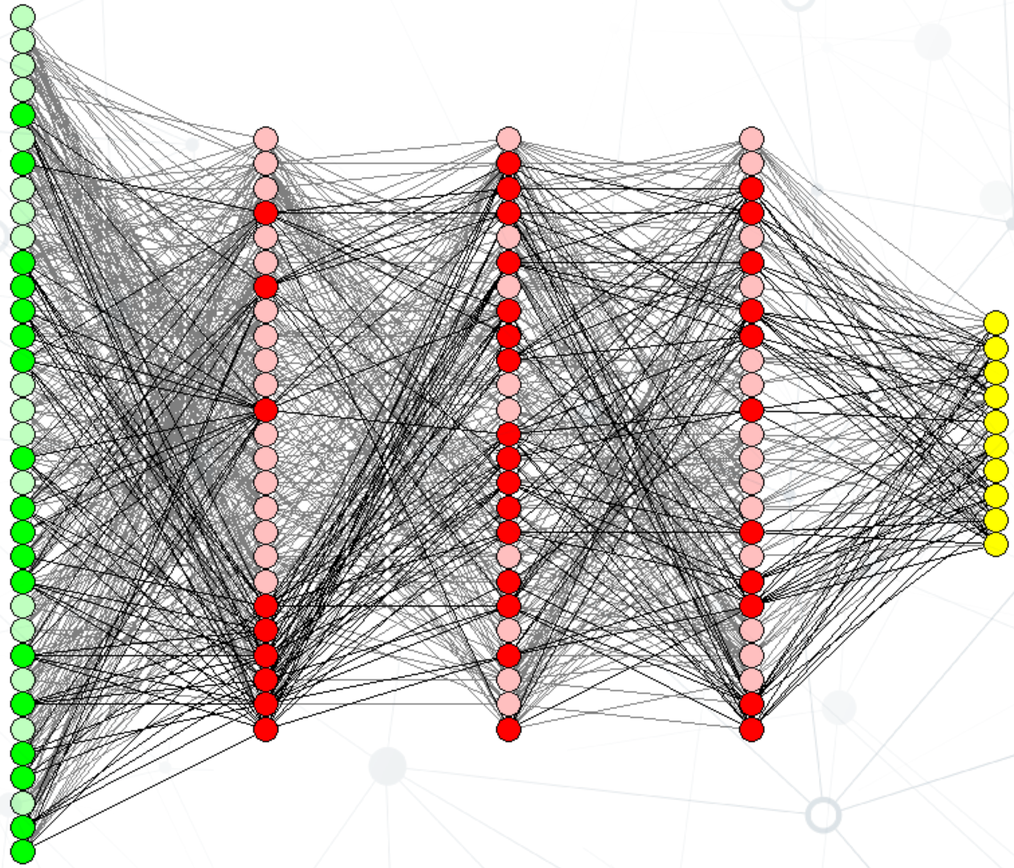
- **Dropout:** during training, at each iteration randomly switch off some neurons during training



- it prevents the NN to rely on the weights of specific neurons
- the number of neurons that are randomly switched off at each iteration is another hyperparameter that needs to be tuned

Training a NN: dropout

- **Dropout:** during training, at each iteration randomly switch off some neurons during training



- it prevents the NN to rely on the weights of specific neurons
- the number of neurons that are randomly switched off at each iteration is another hyperparameter that needs to be tuned

Training a NN: batch normalization

- Inputs of a NN are usually normalized in order to have values in the interval [0, 1] (or [-1, 1]) and small variance
- it has been shown that, in practice, **apply a normalization to each layer of the NN** also improves the speed and efficiency of learning

$$X \xrightarrow[W^{[1]}, b^{[1]}]{} z^{[1]}$$

for each neuron $z_i^{[1]}$ the mean and variance over the training set are computed (μ_i and σ_i^2) and:

$$z_{i,normed}^{[1]} = \frac{z_i - \mu_i}{\sqrt{\sigma_i^2 - \epsilon}} \qquad \tilde{z}_i^{[1]} = \gamma^{[1]} z_{i,normed}^{[1]} + \beta^{[1]}$$

$$X \xrightarrow[W^{[1]}, \cancel{b^{[1]}}]{} z^{[1]} \xrightarrow[\gamma^{[1]}, \beta^{[1]}]{} \tilde{z}^{[1]} \xrightarrow[a]{} h^{[1]} \rightarrow \ldots\ldots$$

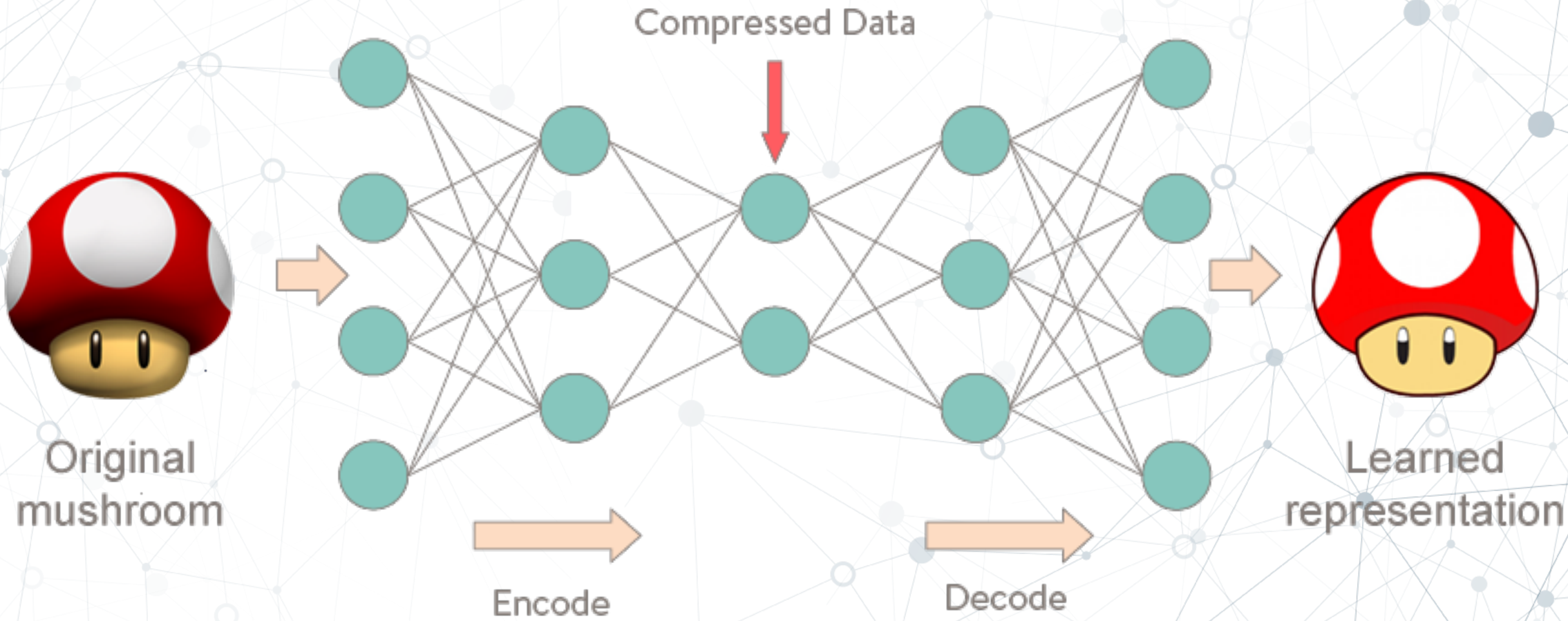
γ and β are learnable parameters of the NN

Training a NN: mini-batches

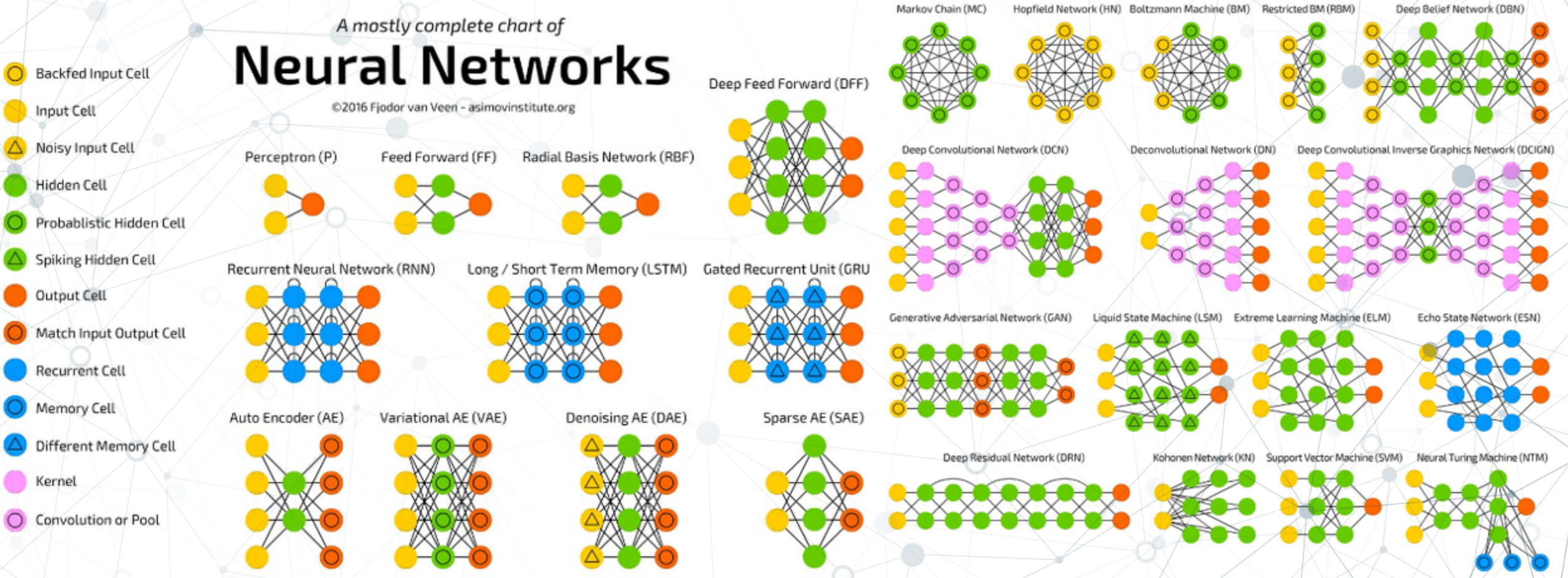
- In our example of spiral arms classification, all the elements of the training set were passed at the same time to the NN, and weights were updated at each iteration
- Usually the training set is instead divided into **mini-batches of few elements** (tipically 32-64-128) . A different batch is passed to the NN at each iteration and weights are updated.
- When all the training set as passed throught the network and **Epoch** has passed
- The number of elements in each mini-batch is another hyperparameter to be tuned

Autoencoders

- NN used to learn efficient rappresentation of data by compressing dimensionality



NN zoo


A mostly complete chart of
Neural Networks
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Backfed Input Cell
Input Cell
Noisy Input Cell
Hidden Cell
Probablistic Hidden Cell
Spiking Hidden Cell
Output Cell
Match Input Output Cell
Recurrent Cell
Memory Cell
Different Memory Cell
Kernel
Convolution or Pool
Perceptron (P)
Feed Forward (FF)
Radial Basis Network (RBF)
Deep Feed Forward (DFF)
Recurrent Neural Network (RNN)
Long / Short Term Memory (LSTM)
Gated Recurrent Unit (GRU)
Auto Encoder (AE)
Variational AE (VAE)
Denoising AE (DAE)
Sparse AE (SAE)
Markov Chain (MC)
Hopfield Network (HN)
Boltzmann Machine (BM)
Restricted BM (RBM)
Deep Belief Network (DBN)
Deep Convolutional Network (DCN)
Deconvolutional Network (DN)
Deep Convolutional Inverse Graphics Network (DCIGN)
Generative Adversarial Network (GAN)
Liquid State Machine (LSM)
Extreme Learning Machine (ELM)
Echo State Network (ESN)
Deep Residual Network (DRN)
Kohonen Network (KN)
Support Vector Machine (SVM)
Neural Turing Machine (NTM)

End of Lecture 1