POLITECNICO MILANO 1863

Financial Engineering

Assignment 6, Group 16

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Alice Vailati - CP: 10683600 - MAT: 222944
Andrea Tarditi - CP: 10728388 - MAT: 251722
Jacopo Stringara - CP: 10687726 - MAT: 222456
Nicolò Toia - CP: 10628899 - MAT: 247208

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1. Introduction and Data

In this Report our aim is to analyze a structured bond issued by XX Bank in a single-curve interest rate modeling setting and neglecting counter party risk. We will consider various type of different sensitivities in order to calibrate and hedge the given position.

Market parameters and Data are provided in the annex here resumed:

Party A Data:

Party A	Bank XX
Payment	Euribor 3m + 2.00%
Payment dates	Quarterly (modified following convention)
Day Count	Act/360

Table 1: Data for Party A

Party B Data:

Party B	I.B.
Payment at start date	X% of principal amount
Payment	Coupon
Payment dates	Quarterly (modified following convention)

Table 2: Data for Party B

First Quarter Coupon	3%
up to the 5th year	€3m +1.10% capped at 4.30%
After 5 years and up to 10 years	€3m +1.10% capped at 4.60%
After 10 years	€3m +1.10% capped at 5.10%

Table 3: Coupon computations

For further data details we refer to the file *MktData_CurveBootstrap_20-2-24.xls* and *Caps_vols_20-2-24.xls*

2. Bootstrap market discounts

In this section we adapted the *Bootstrap* function that we already implemented in order to retrieve the discount factors curve and the zero rates. We chose to use the most liquid assets, the first 4 quoted depo rates, the first 7 futures and the swap rates starting from the 2 years maturity.
In this case, however, our term structure was incomplete. To be more precise, after the 12 year mark swaps were only quoted every 5 or 10 years. Thus, to make the bootstrap work again we chose to use a spline approach to interpolate the mid-market rates. This term structure reflects, real markets term structures, indeed brokers provide more reliable data only for the most liquid instruments.
For ease of interpretation we plotted the discount factors along with the zero rates curve. On the right-hand side y-axis we see the zero rates expressed as a percentage, while on the left hand side y-axis we find the value of the discount factors. On the x-axis we find the date.

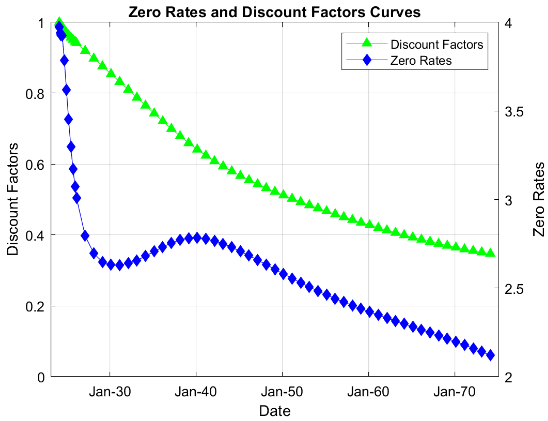


Figure 1: Bootstrap for the Discount factors and Zero rates

The discount factors curve, as expected, has an inverted exponential shape. While it is obvious that investors do not have a crystal ball which can predict the future, the next best thing is the yield rate curve: Shorter-term yields tend to represent what investors believe will happen to central bank policies in the near future. Longer-dated maturities represent investors' best guess at where inflation, growth and interest rates are headed over the medium to long term.
The yield curve tends to flatten when investors expect an economic slowdown, in particular if the curve becomes downward-sloping or inverted. Indeed we can observe that for certain maturities the yield curve still is affected by the rise of rates, in particular the portion of the curve regarding futures' reflects this trend. Moreover the portion of the curve regarding long premium over government paper decreases in steepness. This may be due to the fact that:

- There are higher expectations for inflation.
- Governments in the EU and also in the US, because of the current Geo-Political situation, are dedicating a lot of their resources to rearming themselves.
- Governments are investing a lot of resources into the Green transition.

All these phenomena are expected to bring higher deficits in the future, to lower debt sustainability and therefore to yield a higher premium for long term bonds.

3. Calibrating the spot volatilities

In this section we start from the Flat volatilities quoted in the market and from these calibrate the spot volatilities we will be using in the following.
Before delving into the calibration of the spot vols, let us take a deeper look into the computation of the Cap prices and the model we chose to adopt.
We chose to use the Normal Market Libor Model to derive the caplet formula which is described as follows:

$$\begin{aligned}\mathbb{E}_0[D(T_0,T_{i+1})\cdot\Delta_i\cdot\underbrace{[L_i(T_i)-K]^+}_{\text{Payoff of a Caplet}}] &= \Delta_i\cdot B(T_0,T_{i+1})\mathbb{E}_0^{i+1}[L_i(T_i)-K]^+\\ &= \Delta_i\cdot B(T_0,T_{i+1})\cdot[(L_i(T_0)-K)\cdot N(d^n)-\sigma_i\sqrt{T_i-T_0}\,\phi(d^n)]\end{aligned}$$

This formula is also known as the "Bachelier solution". Moreover we have that:

$$d^n = \frac{L_i(T_0) - K}{\sigma_i \sqrt{T_i - T_0}} \quad \sigma_i^2 = \frac{1}{T_i - T_0} \int_{T_0}^{T_i} \underline{\sigma}_i^2(t)\, dt$$

Of course each Cap can be computed as a sum of Caplets:

$$Cap(T_0, TTM) = \sum_{i=1}^{n} Caplet_i$$

where n is the index of the last caplet whose exercise date is strictly less than the Time of Maturity of the cap. Calibrating the spot vols involves following these steps:

1. Computing the cap market prices using the Flat volatilities quoted in the exchange.
2. Initializing the spot volatilities using the above mentioned Flat vols.
3. Calibrating the spot vols at each expiry to perfectly match the price of the caplet at that expiry.

3.1. Pricing the market caps

As mentioned above, our first step is to price the Caps for all maturities and strikes using the Flat volatilities quoted on the exchange.
Let us take a brief look at the Flat volatilities surface:

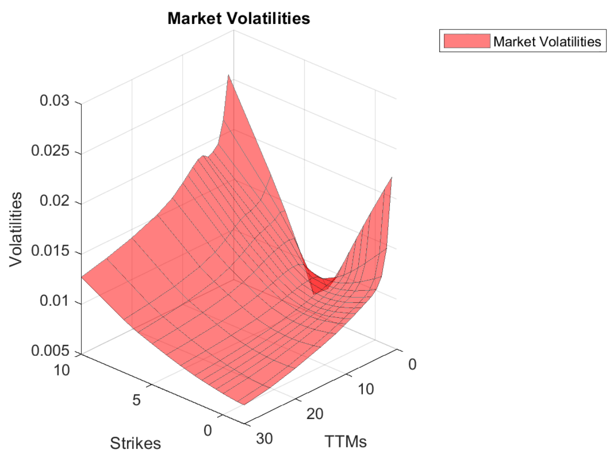


Figure 2: Quoted flat volatilities

As we can clearly see, these quoted volatilities show a clear smiling behaviour. In other words, out-of-the-money and in-the-money option show a higher volatility than at-the-money options even though the underlying asset remains the same. We can also observe that, in general, the volatility tends to decrease as the maturity increases. Let us recall that Flat volatilities are simply obtained with the assumption that the volatility keeps constant across all caplets and expiries. Thus, to obtain our market prices, all we have to do is to plug these quoted volatilities into the Bachelier formula we wrote above:

$$Cap(T_0, TTM) = \sum_{i=1}^{n} Caplet(T_i, \sigma_{TTM}^{flat})$$

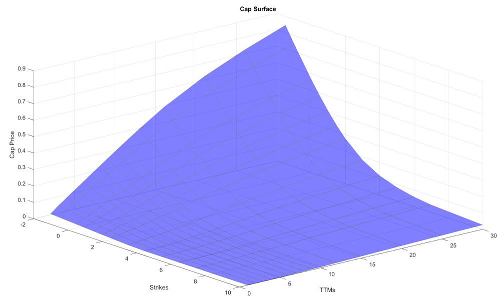


Figure 3: Market cap prices

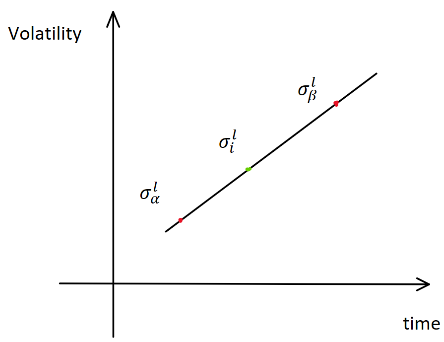
As expected the market prices of the Caps are strictly increasing in the maturity and strictly decreasing in the strikes.

3.2. Calibrating the spot volatilities

To begin calibrating the spot vols we initialized them from the flat ones. Under the one year horizon we assume the spot volatilities to be the same as the flat one.
Now, ideally we would calibrate the spot volatility of each caplet separately. In practice this cannot be done since Caplets are not quoted singularly but rather are all aggregated into Caps. Of course, since each Cap contains the one for the previous time to maturity, we can at least retrieve the aggregate value of the caplets between each quoted maturity as follows:

$$\Delta C = C(T_0, TTM_i) - C(T_0, TTM_{i-1}) = \sum_{i=\alpha}^{\beta} \text{Caplet}(T_i, \sigma_i^{spot})$$

where α and β are the indices of the first and last caplets to be included. At this, point, we can impose that the spot volatilities be such that the ΔC obtained with the spot vols and market flat vols be the same, but this would leave us with an over-determined system of equations. In order to make it exactly determined, we take the further assumption of linearity for the spot volatility between expiry dates:



$$\sigma_i = \sigma_\alpha + \frac{T_i - T_\alpha}{T_\beta - T_\alpha} \cdot (\sigma_\beta - \sigma_\alpha)$$

Where σ_α is the volatility of the previous expiry and σ_β is the volatility at the last expiry of the longer cap. Now, since, as stated above, we have an exactly determined system, we can employ numerical techniques in order to find its roots, or rather its single root σ_β. In practice, this means that the system can be solved by exploiting the Matlab function *fzero*. This procedure can then be repeated for each maturity past the first, which we recall was simply obtained with a flat estimate.
Of course, this procedure can be performed for each quoted strike.

3.3. Calibration Results

Imposing the above described relations, yielded the following results:

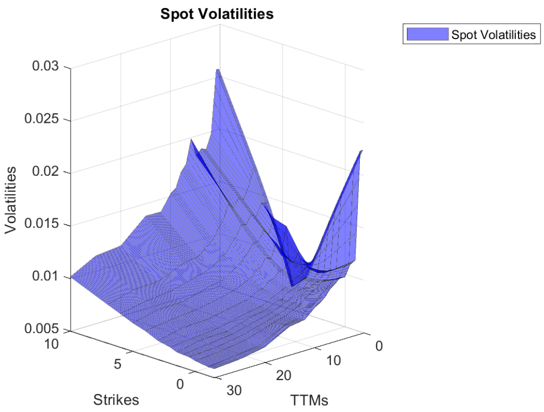


Figure 4: Calibrated spot volatilities

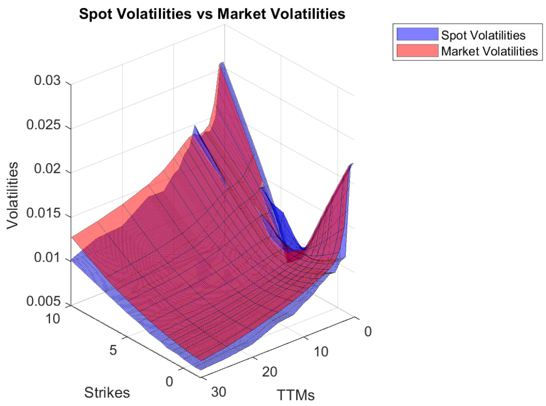


Figure 5: Calibrated vs Market volatilities

We can easily notice that the spot vols for OTM strikes tend to yield an irregular result, this may be a symptom of low liquidity for instruments with strikes distant from the ATM one.
Other than these small inaccuracies, the results were to be expected, the closer we are to the present time the closer the spot and flat volatilities will be. For longer maturities, on the other hand, the flat volatilities must account for all the previous caps volatilities while the spot only account for the involved caplets, thus they tend to be lower.
Moreover, there are different rules of thumb, as mentioned in the following paper, that are important to analyze whenever we take a look at a volatility surface:

- We first need to check how the volatilities behave given a certain strike throughout time.
- Moreover we examine the relationship between the volatility smiles for different strikes at a given maturity.

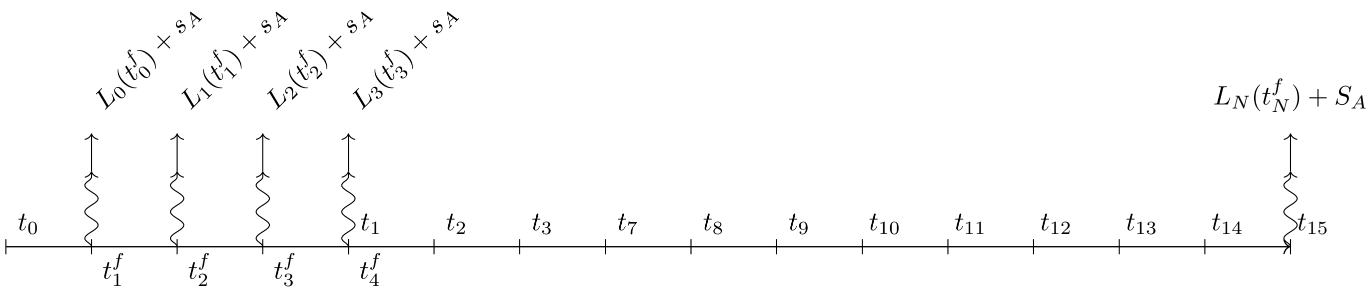
Looking at volatilities across the maturities given a certain strike, which are usually called sticky strike, help with the calculation of Greek letters such as Delta and gamma whilst looking at different strikes given a certain maturity, called sticky Delta, gives us an idea of the general framework we are working with.
From the results we obtained, we can see that, across all strikes the curve is decreasing in time and across different maturities, on the other end, the volatilities tend to present a smile behaviours, especially for lower maturities.

4. Determine Upfront X%

As mentioned in the introduction, we will now be pricing the upfront payment (as a percentage of notional) that party B must pay when entering the certificate contract. Let us now take a look at the two parties payments.

4.1. Party A leg

Party A pays the Euribor 3m plus a flat rate $s_A = 2.0\%$ each quarter.
We can easily visualize this as:



which we can write and then simplify to:

$$NPV_A = \sum_{i=1}^{n} \Delta(t^f_{i-1}, t^f_i)\ (L_0(t^f_{i-1}, t^f_i) + s_A)\ B(t_0, t^f_i) = 1 - B(t_0, t^f_n) + s_A \cdot BPV^f_{0,n}(t_0)$$

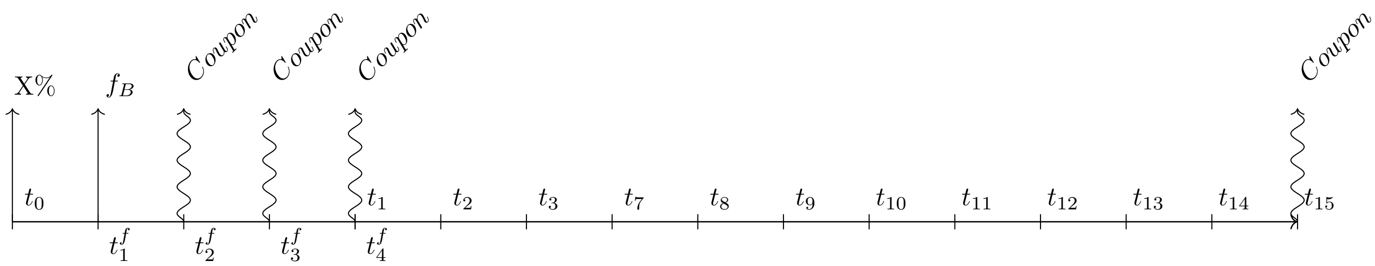
where the dates t^f_i are computed each quarter with the modified following convention.

4.2. Party B leg

Party B pays X% of the notional upfront, then on the first quarter pays a fixed rate $f_B = 3\%$ and subsequently pays the following:

- From the second quarter up to and including the fifth year: $\min(\text{Euribor 3m} + 1.1\%, 4.3\%)$
- From the fifth year up to and including the tenth year: $\min(\text{Euribor 3m} + 1.1\%, 4.6\%)$
- From tenth year up to and including the fifteenth year: $\min(\text{Euribor 3m} + 1.1\%, 4.3\%)$

Visually:



First of all let us remark that the following equation holds:

$$\min(x + q, k) = x + q - max(x - (k - q), 0)$$

We can easily recognize this as being akin to the Libor plus a fixed spread minus a caplet with strike as above. Hence each payoff from the second quarter onwards can be written as:

$$\Delta(T_i, T_{i+1})(L_i(T_i, T_{i+1}) + s_B - [L_i(T_i, T_{i+1}) - (K_j - s_B)]^+)$$

Where the quantity K_j depends on which of the three periods we find ourselves in. Thanks to the fact that the Libor and the fixed rate is the same for all quarters (past the first), just like we did for party A we can easily factorize them into discount factors and a quarterly BPV.

What remain are three sums of caplets:

- The first is a sum of caplets from the second quarter up to (and including) five years with strike $k_1 = 4.3\% - s_B = 3.2\%$. This can be simplfied to be the Cap from t_0 to 5 years with strike k_1.
- The second is a sum of caplets from the first quarter of the sixth year up to 10 years with strike $k_2 = 4.6\% - s_B = 3.5\%$. This can be simplified to be the cap from 5 years up to 10 years with strike k_2.
- The third is a sum of caplets from the first quarter of the tenth year up to 15 years with strike $k_3 = 5.1\% - s_B = 4\%$. This can be simplified to be the cap from 10 years up to 15 years with strike k_3.

Hence the Libor and fixed rate s_B can be simplified to:

$$\sum_{i=2}^{n} \Delta(t^f_{i-1}, t^f_i)\ (L_0(t^f_{i-1}, t^f_i) + s_B)\ B(t_0, t^f_i) = B(t_0, t^f_1) - B(t_0, t^f_n) + s_B \cdot B(t_0, t^f_1)\ BPV^f_{1,n}(t_0)$$

And furthermore, simplifying the caps we obtain:

$$\begin{aligned} NPV_B = f_B\ \Delta(t_0, t^f_1)\ B(t_0, t^f_1) + B(t_0, t^f_1) - B(t_0, t^f_n) + s_B \cdot B(t_0, t^f_1)\ BPV^f_{1,n}(t_0) + \\ - Cap(t_0, t_5, k_1) - Cap(t_5, t_{10}, k_2) - Cap(t_{10}, t_{15}, k_3) + X\% \end{aligned}$$

Hence, if we impose that the NPV of the full certificate be zero, and equate the two legs' payments we arrive at the following form for X:

$$\begin{aligned} X\% = NPV_A - [f_B\ \Delta(t_0, t^f_1)\ B(t_0, t^f_1) + B(t_0, t^f_1) - B(t_0, t^f_n) + s_B \cdot B(t_0, t^f_1)\ BPV^f_{1,n}(t_0) + \\ - Cap(t_0, t_5, k_1) - Cap(t_5, t_{10}, k_2) - Cap(t_{10}, t_{15}, k_3)] \end{aligned}$$

Performing the above computations yields the following results:

Percentage	Euros
18.8665 %	9433259.59

Table 4: Upfront payment as a percentage and as cash payment (Notional of 50 mln €)

5. Delta-bucket sensitivities

We now pass to evaluate the risks connected to the certificate computed above. In this first section, in particular, we try to evaluate the risk connected to changes in the underlying. Usually, in equity markets, one only has to concerns themselves with a single asset. Here, on the other hand, the underlying of our certificate contract is the whole term structure of interest rates. Thus we must understand the sensitivity of our contract's price to each interest rate in the term structure.

To be more exact, instead of a classical Delta we computed the DV01 for each quoted maturity in the market by applying the following procedure:

- Shift the mid-market interest rate corresponding to the given maturity.
- Repeat the bootstrapping procedure.
- Recompute the certificate price X.

Let us stress that we can use the upfront payment as a means of measuring the variation in NPV since:

$$DV01 = \Delta NPV = NPV_{shift} - \cancel{NPV_0} = (\underbrace{NPV_A^{shift} - NPV_{B\setminus X}^{shift}}_{X_{shift}} - X_0) = \Delta X$$

Where X_{shift} is the new certificate upfront payment that balances the two new legs, just like X_0 did for the base case.

The results obtained are the following:

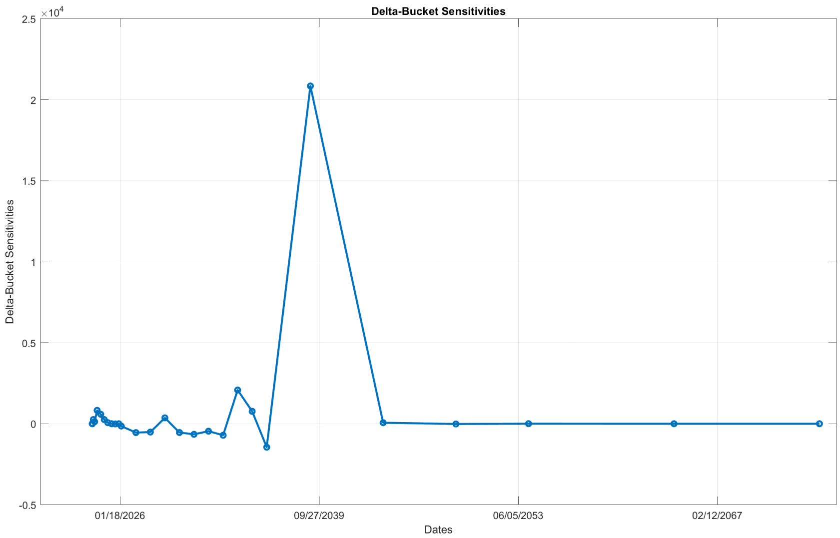


Figure 6: Delta-Buckets in EUR

Let us stress that this is a qualitative graph. Indeed, as mentioned above, we are compute the DV01 for each bucket not the form of the full Delta.

From the graph above we can immediately appreciate that the 15 year maturity has, by far, the largest impact on the price of our certificate. Furthermore, we can also notice other significant peaks of the DV01 around the 3 months, 5 year and 10 year maturities.

These peaks are clearly related to the payoff of the certificate. Indeed, the certificate contains the 5, 10 and 15 year caps, which are sensible respectively to the 5, 10 and 15 year rates. The large spike in the 15 year maturity can be attributed to the influence of the 15 year discount factor in both parties legs. Likewise the 3 months maturity is also represented in party B's leg.

For exact data refer to the table in the appendix.

6. Total Vega

Now, we evaluate another risk factor, the Vega ν. This sensitivity will give us an estimate of how sensitive the price of our certificate is with respect to a 1 bp variation of the flat volatilities.
In order to evaluate the Vega, we simply shift the whole flat volatilities matrix by one basis point, compute the new cap market prices, repeat the calibration on them and finally compute the change in Upfront payment:

$$\nu = X_{shift} - X_0$$

where X_{shift} is the upfront of the certificate obtained using the new, recalibrated, spot vols.
The Total Vega is an indicator that warrants special attention as it represents the amount gained or lost in the event of a one-basis-point increase in volatility.
In our case, the result is the following:

$$\nu = 55831.5765€$$

The Vega, expressed in basis points is $11,1662$ bp. In other words, a parallel shift of 1 bp in the flat volatilities results in a change of 11 bp. From a practical perspective this means, that even in a scenario with a parallel shock of the whole volatility surface our certificate's value will not be significantly impacted. A loss or gain of 50K on a notional of 50 mln, can be considered rather small.

7. Vega-bucket sensitivities

As done above in the Delta case, we now evaluate the value of the Vega buckets. In this case we shock the rows of the flat volatility matrix one at a time, like above, re-evaluate the certificate with the spot volatilities.

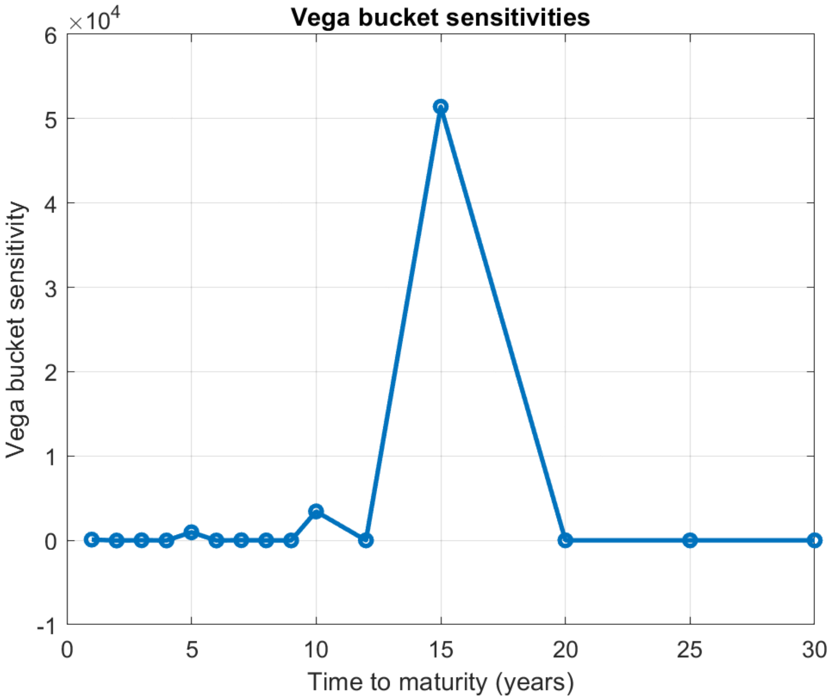


Figure 7: Vega-Buckets in euros

Just like for the DV01 case, we can easily notice three different peaks. The most influential value is clearly the 15 year shift. We can thus conclude that the cap related to this maturity is, by far, the most sensible of the three. This could be due to the fact that the spot volatility for the 15 year maturity is the smallest of the three spot vols involved in the certificate. Hence, a relatively small change of 1 bp could have a bigger impact.
As a final note, let us remark that, as expected a change in later flat volatilities has no impact on the certificate since those caplets do not enter in the certificate's structure.
For exact data refer to the table in the appendix.
For completeness, we all evaluated the Vega buckets matrix for each maturity and each strikes, i.e. evaluating a 1 bp shift of a single element of the flat volatilities matrix.

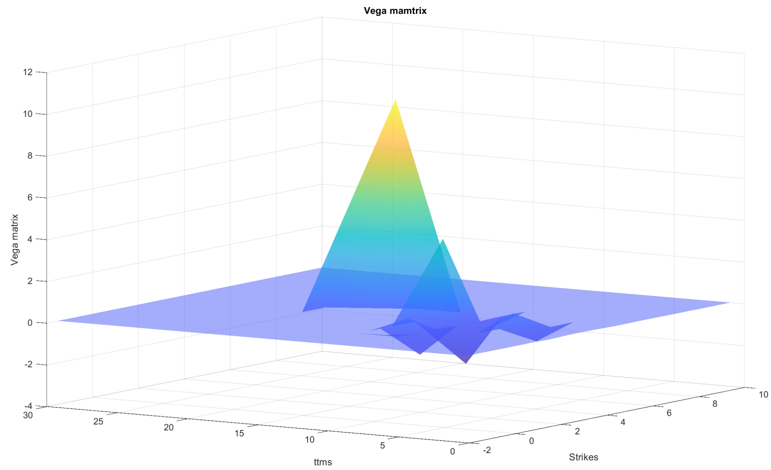


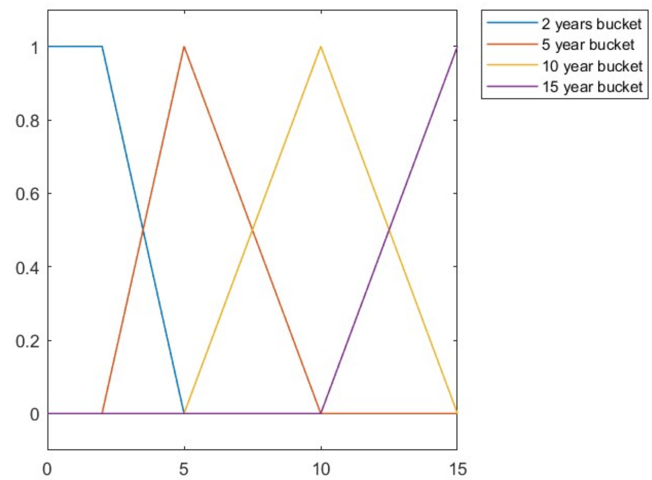
Figure 8: Vega bucket's matrix

As can be seen, the largest peak is found for maturity 15 years and strike 4%, supporting the thesis that the last cap (the one with same maturity and strike) has the largest sensitivity.
Other peaks can be found at the 10 year and 5 year maturities around the 3% and 4% strikes. This is due to how we chose to interpolate the spot volatilities matrix. Indeed, since we chose to interpolate linearly in the times to maturity and with a spline cubic approach in the strikes, the volatilities for the 5 years and 10 years caps in the certificate had to interpolated since their strikes were not quoted in the market.

8. Coarse-grained buckets for the Delta

We will now evaluate the coarse-grained buckets for the Delta sensitivity. These let us aggregate data for different maturities together with other data points with a similar time horizon. This way we can have a bird's eye view of the sensitivity of our portfolio with arbitrary precision. This is particularly useful when high level decision on risk management must be taken by upper management. In our case, we evaluate the 2, 5, 10 and 15 years buckets and we chose the following weights:

$$\begin{cases} w_i^j = \frac{t_i - \hat{t}_{j-1}}{\hat{t}_j - \hat{t}_{j-1}} \text{ if } k_{j-1} \leq i < k_j \\ w_i^j = 1 \text{ if } i = k_j \\ w_i^j = \frac{t_i - \hat{t}_{j-1}}{\hat{t}_j - \hat{t}_{j-1}} \text{ if } k_{j-1} \leq i < k_j \end{cases}$$



These weights, importantly, preserve the property of summing up to one. Aggregating the Delta buckets obtained above yield the following values and graph:

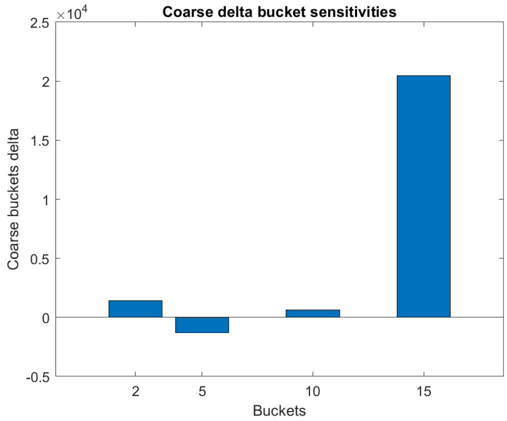


Figure 9: Coarse grained Delta buckets in €

Bucket (years)	DV01 (%)	DV01 (EUR)
2	0.002837	1418.503
5	-0.0026256	-1312.8137
10	0.0012237	611.875
15	0.040945	20472.3201

Table 5: Coarse grained Delta buckets for the Certificate

As we remarked above, the coarse Delta buckets show a high sensibility to the 15 year maturity, indeed the coarse bucket value is rather similar to the simple buckets since all following maturities have sensitivities very close to zero. Aggregating the data for the 2 and 5 year coarse buckets let us see more clearly that their Delta is rather significant even confronted with the 15 year coarse bucket.

9. Coarse buckets Delta hedging

Now, after evaluating the coarse buckets Delta for the certificate in the point above, we proceed to actually hedge our position with respect to the Delta using the coarse buckets.
In order to do this we decided to employ 4 at par Interest Rates swaps with maturity corresponding to each bucket. Indeed, at-par swaps are some of the most liquid assets in the market and thus hedging with them is much easier from a practical point of view.
For each swap we evaluate the coarse-grained Delta with the same buckets as the certificate:

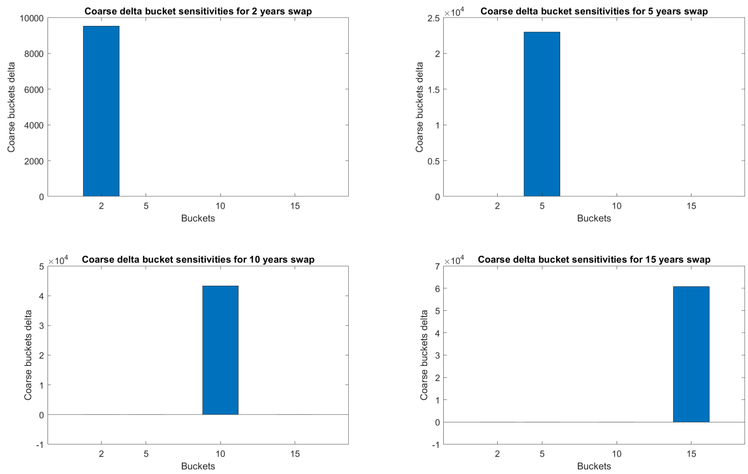


Figure 10: Coarse grained Delta buckets for each swap

As can clearly be seen from the above graphs each swap has its highest sensitivity in the corresponding maturity. Each swap has a positive Delta.

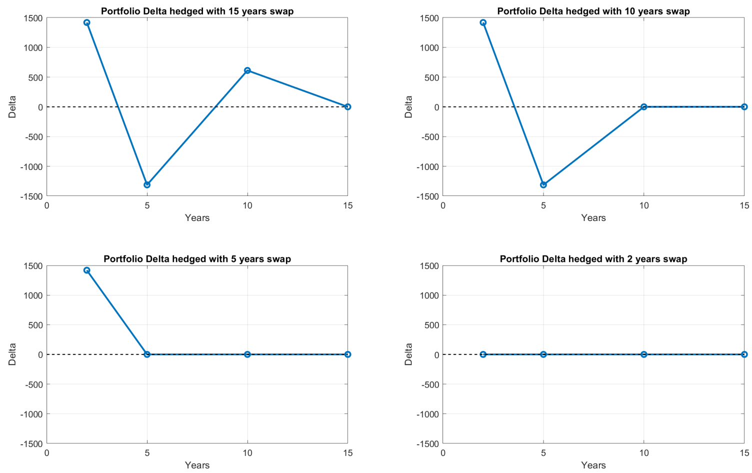


Figure 11: Coarse grained Delta hedging of the certificate

In order to properly hedge our position we start from the last maturity and proceed backwards, as shown in the graph above, ensuring that for each step we preserve the null Delta for the later maturities. To be more precise, the procedure is the following:

- Find the corresponding notion for the swap by taking the ratio between the Delta of the certificate for that bucket and the one of a single swap.
- We update the portfolio exposure with the coarse Delta buckets of the corresponding swap times the above notional.
- Repeat until the position is Delta neutral.

Applying this procedure leads to the following notionals for the corresponding swaps:

Swap Maturity	Notional (EUR)
2	-7438080.1045
5	2853502.6983
10	-707405.7979
15	-16847202.5593

Table 6: Swap notionals for Delta Hedging the Certificate by Maturity

10. Hedge the total Vega of the Portfolio

At this point, we discuss how to hedge the total Vega, i.e. the one obtained by the parallel shift. We chose to hedge the full Vega of our certificate with a single 5 year Cap with ATM strike (i.e. strike equal to the ATM 5y Swap rate). This choice is due to two important factors:

- The 5 year ATM Cap, is one of the most liquid and traded instruments.
- Notionals on Caps in the markets tend to be rather large, so one needs to concentrate our hedging strategy into a single product.

Evaluating the total Vega of the 5 year ATM Cap with the same technique as for the certificate and taking the ratio with the certificate's Vega yields the following notional for the Cap:

$$N_{Cap\ ATM\ 5y} = -220051071.2102€$$

Since both our certificate and the cap have a positive exposure to the Vega, in order to hedge our position we must take a short position on the Cap.

Now, taking this position on the cap, modifies our Delta exposure. Indeed, as above, we proceeded to compute the Coarse buckets Delta of this new portfolio made up of the certificate and the 5 year Cap. Then, following the same technique, we hedged our position against the Delta risk using 4 Swaps. This leads to the following notional for the Swaps:

Swap Maturity	Notional (EUR)
2	14272504.2905
5	97743916.9333
10	-707405.7979
15	-16847202.5593

Table 7: Swap weights for Delta Hedging the Certificate and 5y Cap by Maturity

From an operational point of view, when taking a position on a certificate such as the one we are taking into consideration, the first operation a trader makes is to hedge the Delta risk, for example with swaps as in the point above. Usually, as a second step, one hedges the Vega, for example with the 5 year Cap. This, however, leads to a new exposure on delta that must be hedged in turn. So one, again, hedges the Delta, but this time for the 5 year Cap only.
Let us note that this last operation does not influence the weights of the 10 and 15 years coarse buckets since the 5 year Cap has a null delta in these buckets. Indeed, we can immediately see that the two last notionals for the swaps are the same as in the previous points.

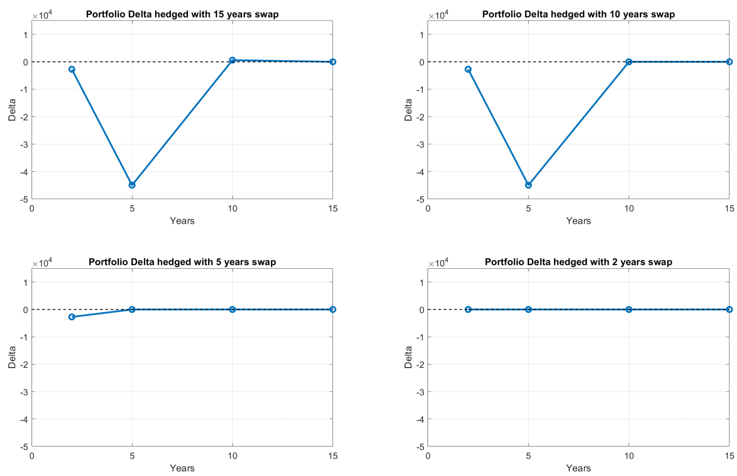


Figure 12: Coarse grained Delta hedging of the portfolio

11. Course-grained buckets for the Vega

In this section, similarly to how we did for the Delta risk, we evaluated the Vega for two coarse-grained buckets: the 5 years and 15 years buckets. This choice of larger buckets, like we mentioned in the previous point, is, in part, due to the higher notionals and liquidity required on Caps in the market.
The coarse grained Vega of the certificate is the following:

5 years	15 years
2742.02992	53128.8455

Table 8: Coarse grained buckets Vega of the Certificate in €

Just like, we did before, we take the 5 year Cap and 15 year Cap with strike equal to the respective at-par swap rates. For both we computed the coarse-grained buckets of the Vega.
Then we start hedging the coarse grained buckets Vega of the certificate by taking the ratio between the sensitivity of the certificate itself and of the Cap with longer maturity. Then we update the bucketed Vega to reflect the new portfolio (i.e. Certificate + 15 year ATM Cap). Then, again, we do the same with the 5 year ATM Cap and the Vega of the portfolio resulting in the following notional for the two Caps:

Cap maturity	Notional (EUR)
5 years	-11125051.8082
15 years	-44442328.5935

Table 9: Notionals for Coarse Buckets Vega hedging with 5y and 15y ATM Cap

As we wanted the Notional values for the two Caps are rather large. This was to be expected, the sensitivity of the Certificate to the Vega, as we have seen in the computation of the Upfront payment, can mostly be attributed to the Caps (e.g. the 0-5 years Cap and the 10-15 years Cap) used to compute the price.
Indeed, on the whole the Notional of the position we must short in order to hedge the Vega of our certificate is rather similar to the Notional of the certificate itself.
As a final note, let us remark that the portfolio given by the notionals above still presents exposure to the Delta risk. In order to hedge this new position we should position as we did above by either shorting or going long on ATM Swaps.

12. Appendix

12.1. Code Optimization

Our initial implementation of the calibration procedure was rather slow. On a closer inspection, done by running the Matlab file using Matlab's built in code profiler, lead us to discover that most of the time was actually spent inside of the *yearfrac* function.
This is mostly due to Matlab's own implementation of the *yearfrac* function and to the fact that for each maturity we recomputed the year fractions and dates.
In order to speed up our computation we devised a few possible solutions:

- To compute all relevant year fractions and date in an overarching data structure to to be used inside of each function. Thus avoid the re-computation cost.
- Using Matlab's suggested format of date-time.
- Saving the calibrated spot vols in order to avoid having to recompute them for different assets.

12.2. More on the use of Caps

- Caps and caplets are options on interest rates. A cap is a series of European call options on interest rates, while a caplet is a single-period option. The payoff of a cap or caplet depends on the level of interest rates relative to a predetermined strike rate.
- Structured Products: Caps are often incorporated into structured financial products to create customized investment opportunities with specific risk-return profiles. These products may combine caps with other derivatives to achieve desired outcomes tailored to investors' preferences.
- Caps are used in this scenario in order to retrieve spot vols from the Market data: this procedure is used since many structured products can be simplified and transformed as a sum of caplets.

12.3. Numerical Results

12.3.1 Delta Buckets Sensitivities

Date	DV01 (%)	DV01 (EUR)
21/02/2024	0	0
27/02/2024	0	0
20/03/2024	0.0005139	256.9481
22/04/2024	0.00025054	125.272
24/06/2024	0.0016511	825.5644
23/09/2024	0.0011724	586.175
20/12/2024	0.00051068	255.3398
20/03/2025	0.00012807	64.0337
23/06/2025	-2.1583e-06	-1.0792
22/09/2025	-2.2609e-05	-11.3046
19/12/2025	-6.4052e-06	-3.2026
20/02/2026	-0.00028823	-144.1141
22/02/2027	-0.0010939	-546.9558
21/02/2028	-0.0010289	-514.4415
20/02/2029	0.00072713	363.5653
20/02/2030	-0.0010829	-541.4452
20/02/2031	-0.0013035	-651.7393
20/02/2032	-0.00092035	-460.1763
21/02/2033	-0.0014195	-709.7708
20/02/2034	0.0041617	2080.8622
20/02/2035	0.0015312	765.5938
20/02/2036	-0.0028933	-1446.6268
21/02/2039	0.041696	20848.0883
22/02/2044	0.00012302	61.5102
22/02/2049	-3.0436e-05	-15.2179
20/02/2054	6.5099e-06	3.255
20/02/2064	-5.6986e-07	-0.28493
20/02/2074	7.1219e-08	0.03561

Table 10: Delta bucket sensitivity of the certificate for each quoted maturity

12.3.2 Vega bucket

Date	Vega (%)	Vega (EUR)
20/02/2025	0.00017802	89.0122
20/02/2026	-4.0066e-05	-20.0331
22/02/2027	3.1219e-05	15.6094
21/02/2028	-2.0423e-05	-10.2115
20/02/2029	0.0019124	956.1952
20/02/2030	-2.8269e-05	-14.1343
20/02/2031	3.8301e-05	19.1504
20/02/2032	-5.5763e-06	-2.7881
21/02/2033	-1.6067e-05	-8.0336
20/02/2034	0.0068606	3430.315
20/02/2036	5.5511e-15	2.7756e-09
21/02/2039	0.10283	51415.7938
22/02/2044	0	0
22/02/2049	0	0
20/02/2054	0	0

Table 11: Vega buckets sensitivities for the certificate for each time to maturity

13. References

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