0. **Exercise** (Variance-covariance method for VaR & ES in linear portfolio):

   Portfolio 0: equally weighted equity portfolio composed by *Adidas*, *Allianz*, *Munich Re* and *L'Orèal* with a notional of 15 mln €.

   First of all, we had to "clean" the given dataset. In particular, we selected the columns related to the companies of our portfolio and then, since some data were missing due to different trading days, we imposed the missed prices equal to the previous ones.

   Regarding the variance-covariance method, we computed the risk factors $\underline{X_t}$ as the daily log-returns of the stocks and we used the assumption of a linearized loss
   $$L(\underline{X_t}) = -V(t)\,[\underline{\omega_t X_t}]$$
   Moreover, we assumed the loss distributed as a t-student with 4 degrees of freedom.

   We report the results:

Daily **VaR** at 99% confidence level	563223.32 €
Daily **ES** at 99% confidence level	787977.23 €

   As we can expect from the definition of Expected Shortfall, it is bigger than the associated VaR.
   <u>Criticality: we tried to apply the plausibility check to have an idea of the VaR of this portfolio, getting 393455.99 € as an output. While the magnitude is the same, the two quantities are not really close</u>.

1. **Case study**: Historical (HS & WHS) Simulation, Bootstrap and PCA

   a) Portfolio 1: *Total* (25K shares), *AXA* (20K Shares), *Sanofi* (20K Shares), *Volkswagen* (10K Shares)

   In the Historical Simulation we took all the daily log-returns from the day of valuation up to 5 years before and evaluated the loss of the portfolio in all of these dates (frozen portfolio hypothesis). We ordered the losses vector from the bigger to the smallest one and then we computed the VaR of the portfolio as the α-quantile of this sorted vector.
   It is worth to comment that this kind of approach is prosecutable only for a linear portfolio of equity stocks, since other instruments such as derivatives and bonds do not maintain the same features along time (their prices depend on the maturity).

   We report the results:

Daily **VaR** at 99% confidence level	95568.99 €
Daily **ES** at 99% confidence level	137635.36 €

   We get a plausibility check for the VaR equal to 92035.63 €, very close to the value that we obtained.

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We were asked to apply also the Statistical Bootstrap method to this portfolio, and substantially we applied the same procedure as above, but this time drawing 200 dates in which we evaluated the loss. So we didn't use the entire 5y data, but only 200 random observations. This kind of approach can be time saving but slightly less precise, since for example if the returns are not identically distributed and independent, drawing with an uniform probability on the past realizations, volatility's clusters are eliminated. Regarding this method, we decided to implement the function in a way such the drawing is with reintroduction.

We report the results:

Daily **VaR** at 99% confidence level	85165.24 €
Daily **ES** at 99% confidence level	108578.77 €

<u>Criticality: the plausibility check returns 93340.71 €, which has evidently the same magnitude of our VaR, but slightly bigger.</u>

b) Portfolio 2: equally weighted equity portfolio *Adidas*, *Airbus*, *BBVA*, *BMW* and *Deutsche Telekom*. We assumed the portfolio value equal to 1€.

In this section we applied the Weighted Historical Simulation method. This kind of approach is very similar to the HS, but in this case we associated a scheme of weights to the observations $\omega_s = C\ \lambda^{t-s}$ where $C = \frac{1-\lambda}{1-\lambda^n}$
By assigning bigger weights to the most recent losses we try to solve one of the main problems of the Historical Simulation, i.e. the choice of the sample size. Indeed choosing a not enough deep dataset we may not consider relevant information from the past values of the risk factors, while on the other side including too many past date we could give too much influence to no more informative data.

We report the results:

Daily **VaR** at 99% confidence level	0.013 €
Daily **ES** at 99% confidence level	0.021 €

<u>Criticality: the plausibility check returns 0.019 €, which has evidently the same magnitude of our VaR, but slightly bigger.</u>

c) Portfolio 3: equally weighted equity portfolio with shares of 18 different companies (n.b. we didn't take exactly the first 18 of the index as we skipped *Adyen*, which didn't have enough data to compute our analysis).

Here we had to apply the PCA methodology, which is very useful when, as in this case, we want to reduce the dimensions of the problem. Substantially we computed the eigenvalues of the dataset's covariance matrix and their associated eigenvectors, we sorted the eigenvalues in decreasing order and we selected the first *k* of them. In this way we could recognized the principal components corresponding to the largest eigenvalues, which capture the most variance in the

data. Afterwards we projected the weights of our portfolio on the principal components, in order to find the so called 'reduced portfolio' for which we computed the 10dd VaR and ES.
In this case we found the daily VaR and ES, and then we rescaled the results with the 'square root of time' scaling rule.

We report the results:

k	1	2	3	4	5
10d VaR 99%	0.0657 €	0.0679 €	0.0678 €	0.0672 €	0.0655 €
10d ES 99%	0.0818 €	0.0840 €	0.0839 €	0.0833 €	0.0817 €

As we can notice, the VaRs do not change significantly as we increase the number of components taken in consideration. This suggests us that a large part of the portfolio loss can be evaluated considering just a small number of factors.
The plausibility check for this portfolio computes the VaR equal to 0.0584 €, which even if there are some differences gives us the exact magnitude of the loss.

2. **Exercise**: Full Monte-Carlo and Delta Normal

FULL EVALUATION MONTE CARLO IMPLEMENTATION

Portfolio 4: long position on stocks of *BMW* for 1,186,680 € , short position on the same number of Call options written on *BMW* (maturity = 18/04/2017, strike 25 €, volatility = 15.4%, dividend yield = 3.1%, interest rate = 0.5%).
We have implemented the function *FullMonteCarloVaR*
In the function we follow a 2y weighted historical simulation approach for the underlying, computing the weights for each date as done in exercise 1. Via the phyton function random.choices these weights has been used as probabilities on the vector of log-returns of the past 2 years. We are now able to simulate the evolution of the stock over the 10 days by sampling 10 log-returns and summing them up. Repeating this procedure for N times (variable numIter in the FullMonteCarloVaR function) we have simulated various scenarios for the underlying. For the computation of the loss we need the initial portfolio value, which is given by the position on the stock (easy to compute from market data) and by a short position on European Call Option, whose price has been computed through the Black-Scholes formula. The portfolio final value is given by the stock position and the derivative position, whose price has been computed with the simulated stocks price after 10 days as starting price of the underlying and considering a time to maturity 10 days shorter
CRITICALITY : The function has been implemented with a for cyle. We know it is not computationally efficient but we have not managed to rewrtite the function in a better way.
We report the result in the following table while we comment it after the Delta Neutral methodology section:

10 Days VaR at 95% Confidence level	816.30 €

QUESTION ON MONTE CARLO

The Full Monte Carlo method can be extremely numerical intensive if we consider a portfolio containing exotic derivatives for which we do not have a closed formula, since in this situation we would require a “nested” Monte Carlo. In fact, we would need to run a double simulation, the first one in order to simulate the underlying stock evolution, the second one to price the derivative. In our case we had to compute exclusively the first simulation, as we used the Black and Scholes closed formula to evaluate the Call Option.

DELTA NORMAL APPROACH

Subsequently we proceeded to compute the 10dd VaR at 95% confidence level using the Delta Normal approach. Since in this case we are considering a non-linear portfolio, we expanded the loss of the derivatives’ portfolio up to the first order:

$$L\left(\underline{X}_t\right) = -\sum_{i=1}^{d} sens_i(t)\, \underline{X}_{t,i}$$

Where $sens_i$ is the delta of the call option. Then we used the WHS at 2 years to evaluate the risk measure.

We report the result:

10 days VaR at 95% confidence level	1039.79 €

Predictably, this result and the previous one obtained via the full MC are close. What is immediately visible is the very small VaR obtained (also considering its entity in percentage of the portfolio value). For us this results can be explained by the fact that the initial stock value (86,53 €) is much greater than the strike (25 €). This leads to a $\Delta = \aleph(d_1)e^{-d(T-t)}$ Since the number of stocks is equal to the number of shares, we are really close to be perfectly Δ-hedged, and so we do not expect a great loss.

QUESTION ON DELTA NORMAL APPROACH

The Delta Normal approach can be improved in several ways. For example we can consider other first order Greek of our derivative, for example the Vega. In addiction we could consider the gamma Greek as well, i.e. expanding the derivatives’ portfolio loss up to the second order.

3. **Case Study**: Pricing in presence of counterparty risk
We were tasked to analyze the following situation: bank XX buys from ISP a 7y Cliquet Option, which is substantially a portfolio composed by one plain vanilla Call Option and 6 forward start Call Options. It is worth to notice that this situation is characterized by an unilateral counterparty risk, ISP indeed receives the price from XX in t=0 and then has exclusively exit cash flows. In order to find the price for the Cliquet Option in presence of counterparty risk, so by the point of view of bank XX, we found the closed formula (procedure in the Assignment4-Group5_ClosedFormula.pdf file attached) and developed a numerical method. In this second way, we simulated the stock in time to evaluated the possible payoffs of the different forward call options.

	PRICE
Closed formula	0.6493558850725897
Without counterparty risk (ISP)	0.6595211147443754
Montecarlo simulation	0.6472196516459078

Moreover, we computed the price at which ISP will try to sell the option just by actualizing the expected cash flows of the coupons, i.e. the payoffs of the calls (with NO counterparty risk). This price is 0.659, that, as expected, is bigger than the 'fair' one at which XX will try to buy.
We tried to vary the volatility of the underlying asset, and we noticed that the price of the Cliquet Option gets bigger as the volatility increases: this makes sense, since if the underlying fluctuates a lot, the possible payoffs, where the strikes are the prices of S observed at previous settlement dates, can be very large.