Financial Risk Laboratory: 4. Capital Requirement (& RWA) associated to a Loan Portfolio

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Overview

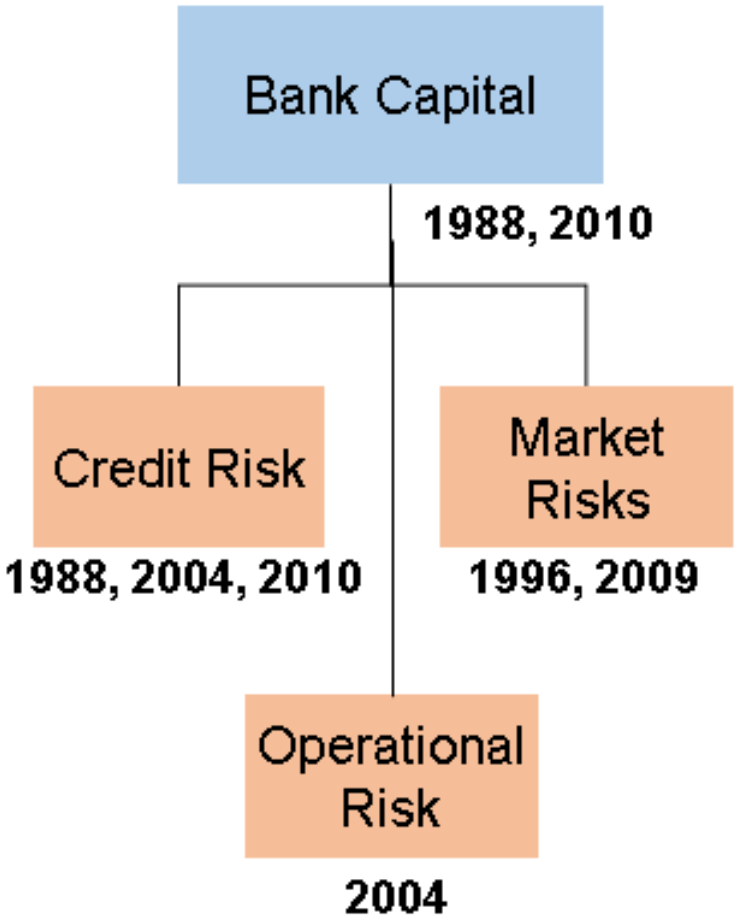
Why Capital Rules?

Bank capital is a cushion protecting bank creditors.
The choice of the level of capital is normally demanded to the market.
But:

- 1970's decline in capital held by banks
- stability of the international banking system
- leveling the international playing field

were the causes of the introduction of Capital Rules.

Banking Risks



- Capital must be able to face three types of risks
  - Credit risk
    - First regulated in 1988, reformed in 2004 and 2010
  - Market risks
    - Regulated in 1996, reformed in 2009
  - Operational risk
    - Regulated in 2004
- Capital ⇨ first regulated in 1988 and significantly revised in 2010

Quality of Bank Capital

Common Equity Tier 1 (CET1) - at least 2/3 of the capital

- Common shares issued by the bank;
- share premium;
- retained earnings.

Additional Tier 1 (AT 1)

- Perpetual - callable subject to supervisory approval;
- full discretion by the bank to cancel distributions/payments

Tier 2

- Original maturity of at least 5 years - callable subject to supervisory approval;
- subordinated to depositors and general creditors;
- there are no step-ups or other incentives to redeem

Capital Ratios

Riskier assets require higher level of capital, hence the concept of *Risk Weight* RW_i associated to each asset i of the bank.

Minimum Capital Requirement (MRC) Formula

$$\frac{RC}{\sum A_i \cdot RW_i} \geq 8\% \tag{1}$$

where the sum is across all assets, RC is the capital (summed across all *tiers*) and A_i is the book value of the $i-th$ asset

Risk Weights and *Risk-Weighted Assets* (RWA)

1 euro of assets requires:

- 0 cents for government bonds $\rightarrow RW_i = 0\%$
- 1.6 cents for a loan to a bank $\rightarrow RW_i = 20\%$
- 4 cents for a real estate mortgage $\rightarrow RW_i = 50\%$
- 8 cents for a corporate loan $\rightarrow RW_i = 100\%$

Regulatory capital must be at least 8% of Risk-Weighted Assets (RWA)

Widespread usage of the RWA concept

- from minimum capital to RWA, i.e. capital times 12.5
- it became very useful after the introduction of capital requirements for market and operational risk

Capital to Cover Losses Caused by Market Risk

After the launch of the 1988 directive, several factors were to be considered:

- the growth in trading activity, particularly in derivatives, on the part of many large banks;
- the increased volatility of the financial markets;
- the securitization of financial assets, which has led many banks to increase their presence in the capital markets.

Such factors drove the introduction in 1996 of the **Minimum Capital Requirement for Market Risk**, additive to the MRC for Credit Risk

Two Approaches

The Standardized Approach (SA)

- Available to all banks
- MRC is set to trading assets/liabilities according to factors and formulas set by the regulator

The Internal Model Approach (IMA)

- Available to large/sophisticated banks
- Subject to supervisory approval
- MRC is a multiple of 10-days, 99%, portfolio Value-at-Risk

The Revolution in Supervisory Policy and the *Regulatory Capture*

The IMA represents a revolution in supervisory policy: for the first time, financial institutions were allowed to determine their own capital requirements based on risk measurements produced in-house (subject to conditions set by the supervisors and to an explicit process of validation).

It represented a move from a detached relationship between the supervisory authority and supervised banks towards a closer relationship based on trust, collaboration, and the exchange of competences and information. It represented a recognition that supervisory authorities and banks, though their objectives are different, share a common interest in the stability of the individual intermediaries and of the financial system.

However, this kind of evolution also gives rise to the risk of *regulatory capture*: the supervisory authorities, after analyzing the banks' models, may impose changes and finally approve them. This means that they become less free in the future to point out limitations and errors in instruments that they themselves helped design. In this respect, their independence might be partially *captured* by the banks, and any problems of inadequacy in the banks' risk management system might not be pointed out or pointed out late.

Pros and Cons

	SA	IMA
Pros	• Simple • Allows comparability across banks	• Risk sensitive • Accounts for the diversification • Gives relevance to good internal risk management practices
Cons	• Not risk sensitive • Does not account for diversification • Opens the way to regulatory arbitrage	• Difficult to compare banks subject to different jurisdictions • Transfer market expectations /consensus into regulatory practice • The *regulatory capture*

Limits of the 1988 Accord

- Even with the 1996 amendment, it did not take into account operational risk;
- Did not sufficiently differentiate between the different positions risks:
  e.g. country risk: only distinction between OECD and non OECD
  e.g. company risk: just one risk-weight at 100%
- Did not recognize the benefits of diversification;
- Only limited recognition to the benefits of guarantees and other *risk mitigation* techniques.

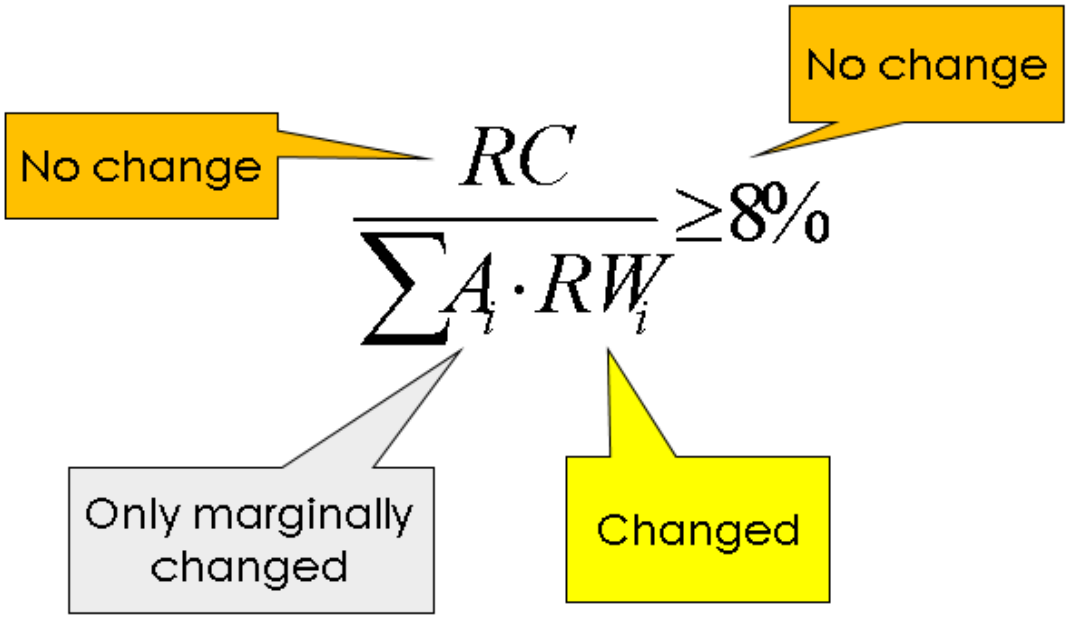
Regulatory Arbitrage (1990's)

- Banks remove from balance sheet those assets with Minimum Required Capital (MRC) $>$ VaR (loan sales, securitisation, credit derivatives) $\rightarrow$ low risk but high required capital $\rightarrow$ spreads are too low to cover capital cost.
- Banks increase those assets with $MRC < VaR$ (ex. real estate commercial mortages) i.e. high risk but low capital requirement.
- **Banks sell low risk assets and vice versa**

Objectives of Basel II

- Narrow the gap between MRC and Economic Capital (EC) via the extension to Credit Risk of the duality Standardized Approach / Internal Model
- Remove regulatory arbitrage incentives
- Introduce Capital Requirement of Operational Risk

Focus of the reform: risk-weights (RW)



Role of Ratings

- Credit risk requirements based on ratings, either external (standardized approach) or internal (IRB foundation and advanced approaches)
- The new capital requirement can lead to higher or lower results depending on the borrower's rating
- assigned by Moody's, S&P's or other agencies and converted into a risk-weight through a **standard table** (SA)
- assigned by the bank itself and converted into a risk weight through a mathematical function (**Internal Ratings-Based** Approach IRB)
- Rating becomes one of the key factors in determining the capital ratios, particularly in the SA

The Standardized Approach

- Use of external ratings Agencies or other "raters" accepted by the authorities E.g. : Moodys', S&P, Fitch
- Better ratings correspond to lower risk-weighted assets
- Risk weights change also according to the type of portfolio... corporate, retail, banks, sovereign, real estate mortgage, commercial mortgage, etc.
- The exposures have to be adjusted according to the guarantees

1988 SA Risk Weights

	AAA	AAA-	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	below	Unrated	Past-due
Corporates	20%					50%			100%						150%				100%	150%
Commercial RE	50 to 100% according to National Supervisors																			150%
Sovereigns	0%					20%			50%			100%						150%	100%	150%
Banks' Country	20%					50%			100%			150%						150%	100%	150%
Banks	20%					50%						100%						150%	50%	150%
Retail	75%																			150%
Residential RE	40%																			100%

Bank' country ⇨ risk weighting based on risk weighting of sovereign in which the bank is incorporated.

Banks ⇨ risk weighting based on the assessment of the individual bank.

Claims on banks of a short original maturity, for example less than six months, would receive a weighting that is one category more favourable than the usual risk weight on the bank's claims

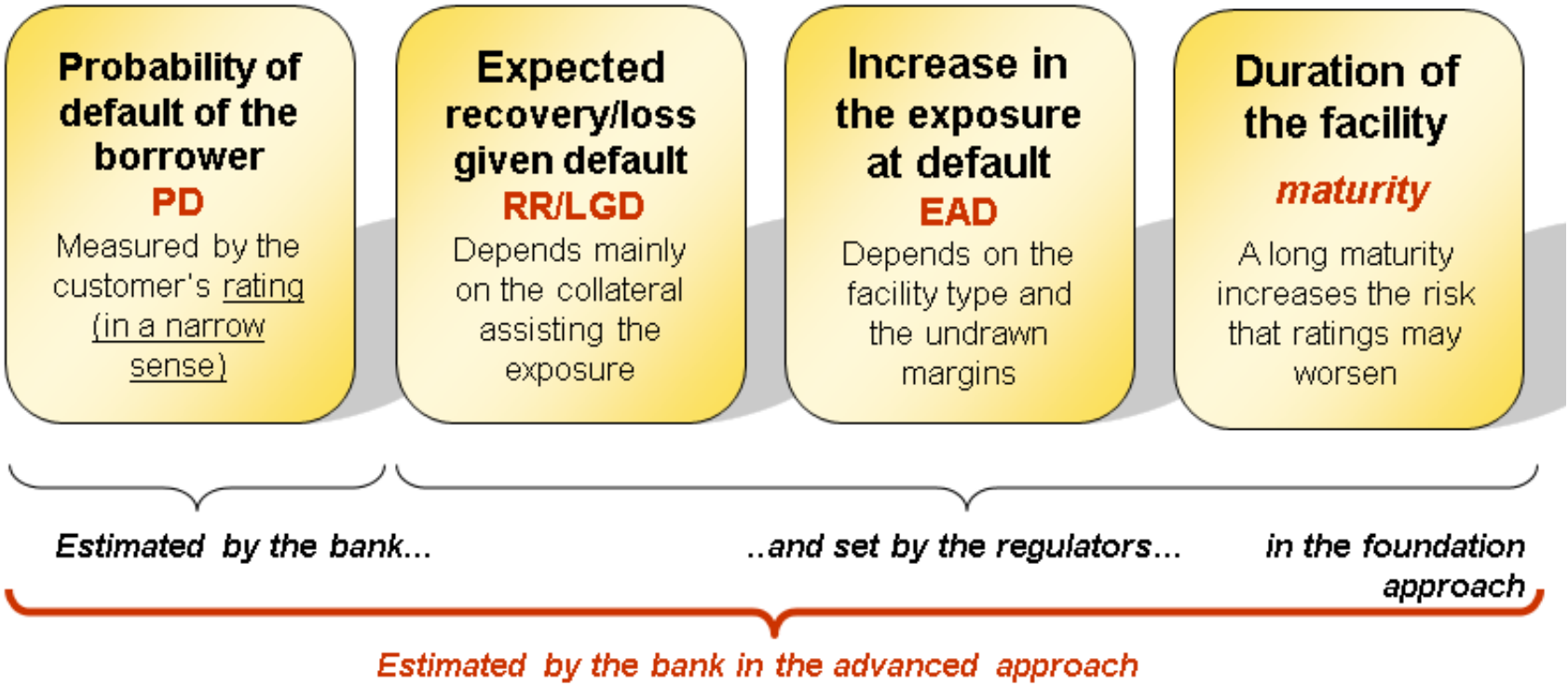
Guarantees

1. Personal guarantees and credit derivatives:
   - accepted if formally accepted by the national regulators - issued by Governments, banks and other financial institutions, private companies with a minimum rating of A-
   - substitution of **guarantor for guaranteed**, only if more favorable
2. Real guarantees (*collateral*) - two alternative approaches:
   - **Simple approach** - allowed for guarantees in cash, gold, qualified debt and equity securities
   - Substitution of the risk-weight of the borrower with that of the instrument (minimum 20%)
   - **Comprehensive approach** - allowed also for listed stocks

Haircuts

1. Personal Guarantees and Credit Derivatives: The amount of the guarantee is reduced (**haircut**) if the guarantee matures earlier than the underlying credit exposure (maturity mismatch) or the guarantee is denominated in a different currency (currency mismatch)
2. Real Guarantees (via the comprehensive approach): the guarantee is reduced through a haircut:
   - Provided by the authorities
   - Estimated in house by the bank using a VaR model.

IRB approaches: foundation vs. advanced



IRB: A Compromise between Risk Sensitivity and Ease in Capital Management

Models developed by the industry measure EC as a tail loss, taking into account portfolio diversification.
According to the SA, MRC is additive (no diversification benefit), this makes management of MRC straightforward.

Portfolio Invariance [BIS 2005]

The model should be portfolio invariant, i.e. the capital required for any given loan should only depend on the risk of that loan and must not depend on the portfolio it is added to. This characteristic has been deemed vital in order to make the new IRB framework applicable to a wider range of countries and institutions. Taking into account the actual portfolio composition when determining capital for each loan - as is done in more advanced credit portfolio models - would have been a too complex task for most banks and supervisors alike. The desire for portfolio invariance, however, makes recognition of institution-specific diversification effects within the framework difficult: diversification effects would depend on how well a new loan fits into an existing portfolio. As a result the B2 Framework was calibrated to well diversified banks.

Asymptotic Single Risk Factor Model (ASRF)

Following the IRB approach, the capital quantified with the *Basel risk weight formula* is based on the **Asymptotic Single Risk Factor Model (ASRF)**

ASRF [BIS 2005]

In the specification process of the Basel II model, it turned out that portfolio invariance of the capital requirements is a property with a strong influence on the structure of the portfolio model. It can be shown that essentially only so-called Asymptotic Single Risk Factor (ASRF) models are portfolio invariant [Gordy 2003].
ASRF models are derived from *ordinary* credit portfolio models by the law of large numbers. When a portfolio consists of a large number of relatively small exposures, idiosyncratic risks associated with individual exposures tend to cancel out one-another and only systematic risks that affect many exposures have a material effect on portfolio losses.
In the ASRF model, all systematic (or system-wide) risks, that affect all borrowers to a certain degree, like industry or regional risks, are modelled with only one (the *single*) systematic risk factor.

ASRF in Practice

The ASRF model is a particular case of a multivariate Merton model for credit portfolio VaR, where we assume a portfolio with exposure to n firms. The standardized Asset Value Return (AVR) across the time horizon of each firm v_i is distributed according to the standard normal distribution. As the realization of the AVR lies below the firm-specific threshold Z_{CCC}, default is triggered (see Eq. 2 - slide pack 2).

Correlation between the AVR of the firms within the credit portfolio is introduced via a common *macroeconomic* factor y, distributed according to a standard normal:

$$v_i = \rho_i \cdot y + \sqrt{1 - \rho_i^2} \cdot \epsilon_i \tag{2}$$

where the std. normal r.v. ϵ_i is a firm-specific idiosyncratic factor and ρ_i is the firm-specific factor load[1] on y.

[1] Also labelled in the literature as w_i

Basel II Treatment of Hypothetical Portfolio Loss

- The capital quantified with the Basel risk weight formula covers every possible future loss up to 99.9% of cases[2] (i.e. 0.1% *worst loss*) includes therefore EL and UL.
- EL, being an expected value, should be considered as a production cost rather than a risk and it should be accounted for in the P&L account and set aside as a reserve
- UL has to be covered with shareholders' capital and is the subject of the MRC formula.

Average and Conditional PDs

- EL attributed to each borrower is derived from the unconditional PD;
- Contribution to the *0.1% worst loss* attributed to each borrower is derived from the **conditional** PD (we denote it by $\bar{PD}$), i.e. the expected default rate conditional to an adverse change of the overall economic condition with probability 0.1%

[2] It is namely based on a Credit Portfolio VaR with 99.9% confidence level

From Average to Conditional PDs

Unconditional PDs (*average* PDs) reflect expected default rates under normal business conditions. Following the IRB approach, these average PDs are estimated by banks.
For the i-th issuer, the *average* PD_i is transformed into *conditional* $\bar{PD}_i$ using a supervisory mapping function:

$$\bar{PD}_i = N\left[\frac{N^{-1}(PD) - \rho_i \cdot N^{-1}(0.001)}{\sqrt{(1-\rho_i^2)}}\right] \tag{3}$$

where:

- ρ_i is the correlation between the standardized asset of the firm and the single factor, so that the standardized asset returns of the i-th and j-th firms have correlation $R = \rho_i \cdot \rho_j$;
- $N^{-1}(0.001) = -3.09$ represents the realization of the single factor y in the ASRF model corresponding to an adverse change of the overall economic condition with probability 0.1%

From Average and Conditional PDs to Risk Weighted Asset

The risk is measured as UL, which reads in term of the average PD and the conditional $\bar{PD}$:

$$K = b\left(\bar{PD} \cdot L\bar{G}D - PD \cdot L\bar{G}D\right) \tag{4}$$

The formula above is the *Basel risk weight formula*, where the symbol K denotes the UL for a credit with 1\$ Exposure at Default[3] and b is a maturity adjustment factor, which takes into account the maturity of the exposure.

Risk Weighted Asset

Coherently with the Basel 1 approach, each individual asset adds to the bank's *Risk Weighted Asset* the following contribution owed to credit risk:

$$RWA = 12.5 \cdot K \cdot E\bar{A}D \tag{5}$$

[3] The symbol $L\bar{G}D$ in (4) represents indeed the *Loss Given Default Rate*

0.1% Worst Loss: Meaning

The term in the RHS of (4): $\bar{PD} \cdot L\bar{GD}$ represents the contribution to the *0.1% worst loss* attributed to a conventional exposure with one-year residual maturity[4] and 1\$ $E\bar{A}D$.
By *portfolio invariance* of the Basel risk weight formula, this represents either the *marginal* and the *standalone* contribution, that is, the MRC is additive.

This is a consequence on the infinite granularity assumption: conditional on the realization $N^{-1}(0.01\%) = -3.09$ of the common factor y in (2) , the variance of the loss distribution driven by the realizations of the idiosyncratic factors ϵ_i goes to zero by the Central Limit Theorem as the number of borrowers n in the portfolio goes to infinity (subject to a finite aggregated portfolio exposure, so that the exposure to each borrower becomes infinitesimal).
Therefore, 99% Portfolio VaR is driven uniquely by the systemic factor and is simply the sum of the conditional expected losses: $\bar{PD} \cdot L\bar{GD}$ for the conventional exposure ($E\bar{A}D = 1\$$ and $b = 1$).

[4] In this case $b = 1$

Correlation in the IRB approach

Following the IRB approach, not all exposures in a bank's loan portfolio are equally sensitive to macroeconomic risks:

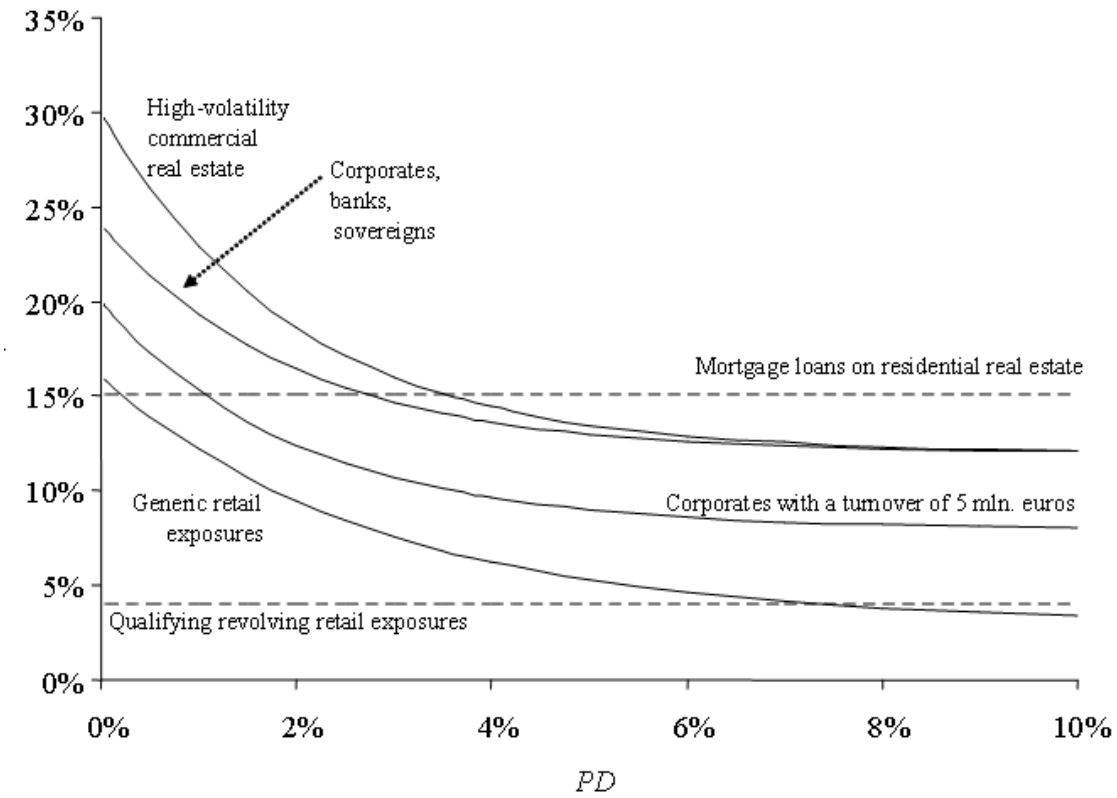
- smaller loans (especially loans to private individuals) are relatively more exposed to risks of individual nature (linked to the e component, i.e. honesty and ability of the individual borrower);
- less reliable borrowers, i.e. those with a higher PD, are such because of idiosyncratic risks, i.e. diversifiable risks.

These two classes of borrowers receive a lower asset correlation. In practice, the BCBS decided not to select a common ρ value, rather different ρ for several *families* (subportfolios) of similar borrowers.

Such correlations are not calculated by the banks, rather set by the *regulatory correlation function*.

The Regulatory Correlation Function

Given the i-th and j-th borrowers, belonging to the same *family* and having $PD_i = PD_j = PD$, the correlation R between their standardized asset returns depends only on their PD as follows:



Within each family, R is a decreasing function of PD.

Regulatiory Correlation Functions (1/2)

Large Corporates[a] and Sovereign

[a] Sales above Euro 50m

$$R_{LC} = R_{min}\frac{1 - e^{-k \cdot PD}}{1 - e^{-k}} + R_{max}\frac{e^{-k \cdot PD}}{1 - e^{-k}} \tag{6}$$

The correlation function is limited between $R_{min} = 0.12$ and $R_{max} = 0.24$ for very high and very low PDs (100% and 0%, respectively). Correlations between these boundaries are modelled by an exponential weighting function that decreases at a pace determined by the so-called "k-factor" $k = 50$

Correlation Functions (2/2)

Other Corporates[a]

[a] Sales below Euro 50m

$$R_{OC} = R_{min}\frac{1-e^{-k\cdot PD}}{1-e^{-k}} + R_{max}\frac{e^{-k\cdot PD}}{1-e^{-k}} - 0.04\left(1-\frac{s-5}{45}\right) \qquad (7)$$

That is, (6) with a size adjustment (to be applied for annual sales s between Euro 5m and Euro 50m, maximal adjustment for size below Euro 5m). R_{min}, R_{max} and k like large corporates.

Retail

Like (6), with $R_{min} = 0.03$ and $R_{max} = 0.16$ and $k = 35$

The Maturity Adjustment b

The time horizon of the IRB risk weight formula is one year[5]. Input PDs are therefore one-year PDs.

The maturity adjustment b is an analytical approximation which accounts for hypothetical losses due to downgrading.

Downgrading risk increases with:

- the creditworthiness of the borrower (lower PD)
- the residual maturity of the exposure (higher M)

[5] The MRC is therefore based on a Credit Portfolio VaR with one year time horizon

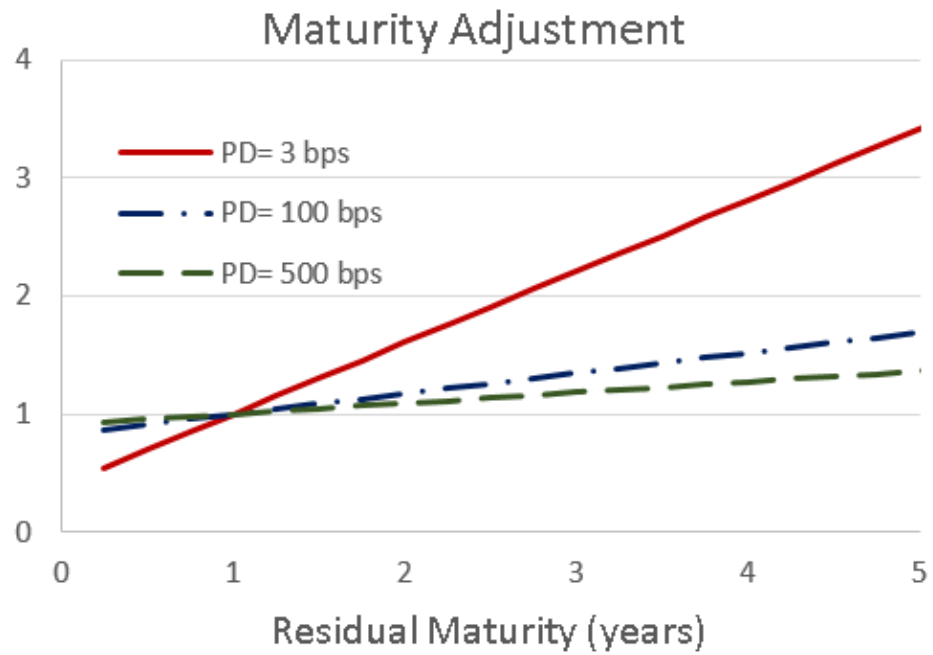
The Maturity Adjustment b: Regulatory Formula

$$b = \frac{1 + (M - 2.5) \cdot [\alpha - \beta \ln PD]^2}{1 - 1.5 \cdot [\alpha - \beta \ln PD]^2} \tag{8}$$

where M is the exposure maturity (in years) and PD is ther annual default probability.

The parameters α and β are set by the regulators as:

- $\alpha = 11.852\%$
- $\beta = 5.478\%$



Internal Ratings

Either following the *Foundation IRB* or the *Advanced IRB*, banks must adopt an *internal rating system*, which must comply with the following requirements:

- Bidimensional system (counterparty + facility)
- The rating scale is composed by of at least 7 classes, plus default
- The rating assignment criteria must be intuitive and well-documented
- Not more than 30% of exposures in a single rating grade
- The rating system is revised annualy
- Ratings are reported monthly to the top management
- Rating for MRC calculation are also used internally (pricing, exposure limits, etc.)

From Internal Rating to PD

Statistical rating models are allowed, not mandatory. Whatever rating model is adopted, each rating class must be associated with a one-year probability of default (PD), based, e.g., on past experience.

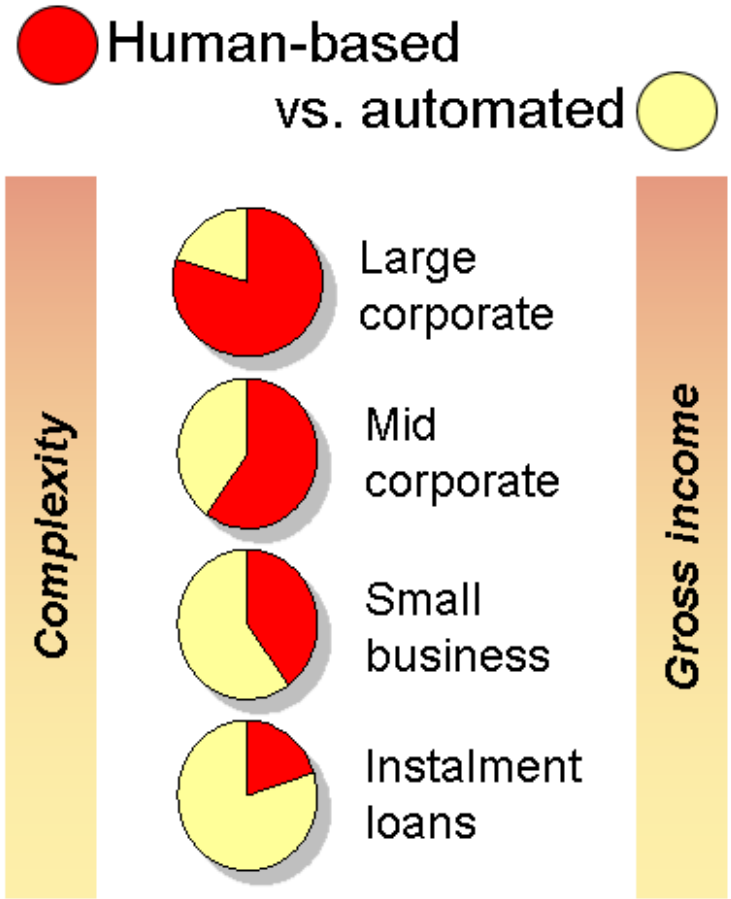
Default occurs if at least one of the following conditions is true:

1. The borrower is unlikely to pay in full (subjective)
2. Payments are delayed by more than 90 days (objective)

For prudential reasons, the PD may never fall below 0.03%

Rating Assignment: Best Practice

- Rating assignment is managed by a mix of statistical algorithms (e.g., discriminant scoring) and human-based procedures, with the latter focusing on qualitative aspects which cannot be objectively measured.
- The balance between automated and labor-intensive techniques varies according to the borrower's complexity and the loan's gross contribution



Loss Given Default

Foundation Approach

$L\bar{G}D$ is **fixed at 45%** with the following exceptions:

- $L\bar{G}D = 75\%$ for subordinated loans.
- $L\bar{G}D = 35 - 45\%$when receivables, real estate or other assets are pledged against the loan.
- $L\bar{G}D = 0\%$ only if some widely marketable financial collateral is present.

Advanced Approach

- The bank uses a system of $L\bar{G}D$ grades (similar to PD grades) based, e.g. on facility types and collateral type.
- $L\bar{G}D$ must be estimated in an economic sense, e.g. accounting for the value of time elapsed between default and recovery.
- $L\bar{G}D$ must be based on at least 7 years of past data.

Exposure at Default $E\bar{A}D$ and Exposure Maturity M

Foundation Approach

A 100-euro committed but undrawn exposure is equivalent to a drawn exposure of:

- 75 euros if not unconditionally and promptly cancelable.
- 0 euros if unconditionally and promptly cancelable.

Maturity M arbitrarily fixed at 2.5 years.

Advanced Approach

- $E\bar{A}D$: The bank must produce accurate estimates for different facility types, based on 7 years of historical data.
- Maturity M: effective time to maturity of each exposure, taking into account intermediate payments.

Advanced IRB: Example

Consider a three-year loan, issued to a company with PD 1%, a turnover of Euro 5 million, with an estimated $L\bar{G}D$ of 45%.

- Correlation: $R = 15.3\% \Rightarrow \rho = \sqrt{R} = 0.391$
- Maturity adjustment: $b = 1.346$
- Conditional PD: $\bar{PD} = N\left[\frac{N^{-1}(0.01)+0.391\cdot 3.09}{\sqrt{1-0.153}}\right] = N\left[-1.21\right] = 11.2\%$
- Risk weight formula:
  $K = b \cdot (\bar{PD} \cdot L\bar{G}D - PD \cdot L\bar{G}D) = 1.346 \cdot (11.2\% - 1\%) \cdot 45\% = 6.2\%$

If we assume $E\bar{A}D$ equal to the loan value, the RWA of a loan of 100,000 euro will be: $12.5 \cdot 6.2\% \cdot 100,000 = 77,500$ euro.

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