Assignment 4 - Group 14

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Exercise 0

Given a linear portfolio composed by an equal number of shares of Adidas, Allianz, Munich Re and L'Oréal, we computed daily VaR and ES with a 5y estimation via t-student parametric approach. We started by calculating relevant stocks and their log-returns in order to estimate their mean and covariance, then, knowing that the stocks are equally weighted and log-returns are distributed as a t-Student, we evaluated the VaR and ES with the analytical formula:

$$\begin{cases} \mathrm{VaR}_\alpha = \Delta\mu + \sqrt{\Delta}\cdot\sigma\cdot\mathrm{VaR}_\alpha^{\mathrm{std}} \\ \mathrm{ES}_\alpha = \Delta\mu + \sqrt{\Delta}\cdot\sigma\cdot\mathrm{ES}_\alpha^{\mathrm{std}} \end{cases}$$

Knowing that:

$$\mathrm{VaR}_\alpha^{\mathrm{std}} = t_\nu^{-1}(\alpha)$$

$$\mathrm{ES}_\alpha^{\mathrm{std}} = \frac{\nu + \left(t_\nu^{-1}(\alpha)\right)^2}{\nu - 1}\cdot\frac{\phi_\nu(t_\nu^{-1}(\alpha))}{1-\alpha}$$

The VaR of this portfolio is 563223.3167, whilst the ES is 787977.2342.

Exercise 1

We want to evaluate the risk measures for a set of portfolios (at the end of March 20th 2019) using non-parametric approaches. In our analysis, we assume that the portfolios remains static over time, with constant weights for each asset. The losses are linearly related to risk.

- **Portfolio 1** Firstly we consider a portfolio composed by 25K shares of Total, 20K shares of AXA, 20K shares of Sanofi and 10k shares of Volkswagen. We evaluated the weights based on the number of stocks for each asset and their prices at the end date. Then, we calculated the log-returns considering a 5-year time window, with all available stock data from March 19th 2014. Using the Historical Simulation technique with a confidence level $\alpha = 95\%$. We obtained the following risk measures: For the Bootstrap

	HS (no bootstrap)	HS (bootstrap)
VaR	94610.7543	95521.5335
ES	141276.8704	177225.3393

Table 1: VaR and ES evaluated with Historical Simulation

method, we reduced the dataset to 200 time steps (instead of the original 1281). These steps were sampled independently with equal probabilities with replacement.

- **Portfolio 2** The second portfolio taken into account is an equally weighted portfolio composed by shares of Adidas, Airbus, BBVA, BMW and Deutsche Telekom. In order to evaluate the risk measures of this portfolio we implemented the Weighted Historical Simulation (WHS). The algorithm is the same of the Historical Simulation (HS), but incorporates a set of weights assigned to the loss sequence. This set of weights decreases as it goes into the past, so that recent losses have a more significant influence on the calculation of the VaR. The results obtained are:

	WHS
VaR	0.015936
ES	0.021544

Table 2: VaR and ES evaluated with Weighted Historical Simulation

In general, the WHS method is more sensitive to recent conditions, yet it produces more volatile VaR estimates. This characteristic makes the method particularly valuable for conducting stressed VaR analysis, especially when utilizing data from periods marked by significantly higher volatility.

- **Portfolio 3** This last portfolio is also an equally weighted portfolio with shares of the first 18 companies (adding the 19th because we had to remove Adyen). We evaluated the risk measures (10 days VaR and ES) using the Principal Component Analysis technique (PCA), which aims to reduce the dimensionality of the problem.
  This is extremely important for large institutions dealing with a vast amount of risk factors. The methodology is used to reduce the dimensionality of highly correlated data by identifying a limited set of uncorrelated linear combinations that account the majority of the original data's variability. Consequently, after reducing the number of risk factors to just the principal components, we derived the Value at Risk using the reduced Variance-Covariance (parametric) method.

N^o Components PCA	VaR	ES
1	0.054536	0.068814
2	0.054319	0.068612
3	0.054340	0.068633
4	0.054412	0.068709
5	0.054397	0.068694

Table 3: VaR and ES evaluated with Principal Component Analysis

- **Plausibility Check** For all the previous portfolios we implemented a plausibility check. This is a rule of thumb that allows us check the order of magnitude of the portfolio VaR, via the following estimation:

$$VaR^{ptf} = \sqrt{sVaR \cdot C \cdot sVaR}$$

  - C is the correlation matrix of risk factors;
  - $sVaR = sens_i \frac{|l_i|+|u_i|}{2}$ is the signed-VaR for each risk factor.
  The Plausibility-Check confirmed the correctness of the orders of magnitude.

	Portfolio 1	Portfolio 2	Portfolio 3
Plausibility Check	89558.4181	0.019219	0.054631

Table 4: Plausibility Check VaR

Exercise 2

- **Full Monte Carlo**:
  In case of a derivative portfolio, to figure out potential losses, we use the Monte Carlo method. Firstly, we examine the potential fluctuation in portfolio value over a brief period (in our case, 10 days), then we sum up these potential changes to assess the total potential loss. The formula to compute potential losses for options is the following: $L^{der}(X_t, \Delta) = -\sum_i (C_i(t+\Delta) - C_i(t))$ where $C_i(t)$ is the value of the i-th derivative at time t.
  For regular stocks, however, we can just look at how much their price might change from today and tomorrow based on past data (we used a 2y Weighted Historical Simulation). This method is really accurate to estimate potential losses, but it is computationally expensive, especially for exotic options where we don't have a closed formula.

- **Delta - Normal**:
  This method simplifies a lot how we calculate our portfolio's loss by using a first order approximation of the Greeks. Here, our loss equation is $L(X_t) = -\sum_i (\text{sens}_i(t) \cdot X_{t,i})$ where $sens_i$ is our first order sensitivity (Delta) and $X_{i,t}$ is the stock price at time t.
  This method is less computationally costly with respect to the Monte Carlo one, however is less precise. To improve accuracy, we can expand the same idea to a second order derivative expansion (Delta-Gamma), where the loss is given by $L(X_t) = -\left(\sum_i \delta_i \cdot S_i(t) \cdot X_{i,t}) + \frac{1}{2} \cdot \left(\sum_{i,j} \gamma_{ij} \cdot S_i(t) \cdot S_j(t) \cdot X_{i,t} \cdot X_{j,t}\right)\right.$.

N.B. : we derived different VaR values in the two cases but we could not solve the problem.

	Full Monte Carlo	Delta - Normal
VaR	51407.8606	1437823.1625

Table 5: VaR of a derivative with Full Monte Carlo & Delta Normal

Exercise 3

A Cliquet option, also known as a Ratchet option, is a financial derivative that provides periodic payouts based on the performance of an underlying asset over multiple observation dates.
The option payoff is $L \cdot (S(t_i) - S(t_{i-1}))^+$ where L is the participation coefficient, which determines the degree to which the option holder participates in the positive movements of the underlying asset.
Basically, a Cliquet option combines a traditional European call/put option with several forward starting options of the same type.

In this specific scenario, where the dividend yield is zero, the forward starting options are essentially identical to the classical ones. Consequently, the price of the Cliquet option is the sum of the individual prices of the European call options.

To compute the option's price using Monte Carlo (MC) methods, we first model the underlying asset S as a Geometric Brownian Motion (GBM). Then, we iteratively simulate the dynamics of S year by year, employing the antithetic variables technique to expedite the process. Once a sufficient number of simulations are generated, we compute an equal number of payoffs and derive an approximation of the Cliquet option's cash flows from their mean. Subsequently, we calculate both the risk-free price and the risky price. The risk-free price, of particular interest to us, represents the price at which the issuer (ISP) would attempt to sell the Cliquet option (similar to an Ask price), and is computed by discounting the European call prices using risk-free discounts. On the other hand, the risky price (similar to a Bid price) that the buyer seeks, is discounted using defaultable discounts, each call price is weighted by the quantity: $(SP + recovery \cdot (1 - SP))$, reflecting the expected value with respect to default and non-default scenarios, considering that in case of default, only a fraction (recovery) of the flow is received.

Although deriving a closed formula for a Cliquet option is generally challenging, the assumptions underlying the Black '76 model allow for such a formulation in our case. We have obtained the following formula:
$Cliq(0) = \sum_i E\left[(L \cdot S(t_i) - S(t_{i-1}))^+\right] = \sum_i L \cdot \mathbb{E}\left[S(t_i) \cdot \left(\frac{S(t_i)}{S(t_{i-1})} - \frac{1}{L}\right)^+\right] =$
$= \sum_i L \cdot \mathbb{E}\left[S(t_i)\right] \cdot \mathbb{E}\left[\left(\frac{S(t_i)}{S(t_{i-1})} - \frac{1}{L}\right)^+\right] =$ (independent since both modeled as GBM)
$= \sum_i L \cdot \frac{1}{B(0,t_{i-1})} \cdot \text{Black76}\left(\frac{B(0,t_{i-1})}{B(0,t_i)}, \frac{1}{L}\right) = L \cdot \sum_i \frac{1}{B(0,t_{i-1})} \cdot \text{Black76}\left(\frac{B(0,t_{i-1})}{B(0,t_i)}, \frac{1}{L}\right)$

Where the expected value is computed w.r.t. risk-free probability with $S(t_{i-1})$ as numeraire. Notice that $\left(\frac{S(t_i)}{S(t_{i-1})} - \frac{1}{L}\right)^+$ is similar to the payoff of a call with price $\frac{S(t_i)}{S(t_{i-1})}$ and strike $\frac{1}{L}$.

Similar to the MC simulations, the prices obtained using this formula are adjusted to incorporate counterparty risk by multiplying the prices of individual call options by their respective weights.

	Price	Notional 30mln
Risk-free exact price	0.57	17.1mln
Risky exact price	0.56	16.8mln
Risk-free MC price	0.66	19.8mln
Risky MC price	0.64	19.2mln

In conclusion, while the risky prices are naturally lower than the risk-free prices, we notice discrepancies between the prices produced by MC simulations and those obtained using the closed formula. This suggests potential inaccuracies in the computational method used to derive the formula, or it could imply that the Cliquet option in question possesses particular characteristics not fully considered within the Black '76 model.