Assignment 1 RM, Group 16

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Hazard Rate Curve

We take into consideration two hazard curves, one for each class of asset. For simplicity's sake we take two piece-wise constant functions. To find the two needed values for each curve we perform a bootstrapping procedure. First of all we linearly interpolate the zero-rates curve in order to reconstruct the discount factor at the needed dates. We also assumed a constant recovery rate $R = 0.4$ (for a face value of 100). Then we find the h_1 that best fits the one year bond by writing the price of the bond as a function of h_1 and the discount factors, and then finding the zeroes of the difference with the observed market price, using MATLAB's *fzero* function. Afterwards we repeat the process to find h_2 by also employing the value of h_1 found above.

Z-Spread

In this section we computed 4 different scalar values for the z-score. We defined the z-score to be the spread (i.e. parallel shift) to be applied to zero-rate curve in order for the defaultable and risk-free bonds to have the same price. Like we did above, we computed the discount factors and applied the shift caused by a constant z-score (i.e. $e^{-z \cdot T}$), then found the zeroes of the difference with the market price.
Let us note that this operations can be parallelized, since for each maturity and firm we have chosen a constant shift.
Furthermore, by looking at the table in the numerical results section, we can also observe that the z-score is always lower than the corresponding hazard rate. Indeed, this was to be expected from theory since we have $\pi > 0$ here.
Lastly let us also remark that the z-score obtained can be seen as a measure of risk. A bit unusually we observe that the z-score of the two year High Yield bond is lower than the one year bond. This is probably due to a combination of the two sources of randomness at play here: the default event and the recovery rate. Indeed for the market the default event is a seamless event, it can simply be seen as an acceleration of the coupon payments.

Market-implied rating transition matrix

In this section we suppose the transition matrix to be time homogeneous. In other words the transition matrix Q only depends on the time step $\tau = T - t$ and can be factorized using the Chapman-Kolmogorov equation. In particular in our case we impose the following:

$$Q(0, 2) = Q(0, 1) \cdot Q(1, 2) = Q^2(\tau = 1)$$

Where the two matrices $Q(0, 1)$ and $Q(1, 2)$ are supposed to be equal since we have assumed time homogeneity. Furthermore we can also leverage the conservation of probability (i.e $\sum_{j=1}^{N} q_{i,j} = 1 \ \forall i$).
We can immediately compute the last column of the matrix as the one-year and two-year unconditional probabilities of default by using the previously computed hazard rate curves. In particular

$$q_{i,3}(t, T) = 1 - e^{-\int_t^T h_i(s)\, ds} \text{ with } i = IG, HY$$

Thus we arrive to a relatively linear system which can be solved analytically to compute the transition matrix for the one-year time horizon.

Numerical results

	h_1	h_2
IG	50.0	125.0
HY	400.0	250.0

Table 1: hazard rates expressed in basis points

	z_1	z_2
IG	30.5	52.9
HY	241.5	197.1

Table 2: z-scores expressed in basis points

	IG	HY	Def
IG	0.7789	0.2161	0.0050
HY	0.4076	0.5532	0.0392
Def	0	0	1

Table 3: Market-implied rating transition matrix