POLITECNICO MILANO 1863

Financial Engineering

Assignment 7, Group 16

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Contents

1. Introduction and Data

Certificate Pricing

In this case study, our aim is to price a certificate issued by Bank XX via several methods.
Market parameters and Data provided in the annex here resumed:

Market Data

Principal Amount	$100 MIO$ €
Issue Date	15/02/2008
Start Date	19/02/2008
Maturity	2 years after the Start Date

Table 1: Market Data

Data for Party A:

Party A	Bank XX
Payment	Euribor 3m + 1.30%
Payment dates	Quarterly (Modified Business Convention)
Day Count	Act/360

Table 2: Data for Party A

Party B Data:

Party B	I.B.
Payment at start date	X% of principal amount
Payment	Coupon
Payment dates	Annually (Following Business Day Convention)
Strike	3200 €
Trigger Level	6%

Table 3: Data for Party B

Coupon Reset Dates	2 Business Days prior to the Coupon Payment Date
First Year Coupon	6%
Last Year Coupon	2%
Day Count	30/360

Table 4: Coupon computations

In addition we have to consider that the Coupon shall be subject to the Early Redemption and Final Coupon Clauses.

Bermudan Swaption Pricing via Hull-White

In this case study our aim is to price a Bermudan Swaption non-call 2y and to find a lower and upper bound for it's price. In the following table we summarize the data for the Bermudan Swaption:

Settlement Date	15/02/2008
Time to Maturity	10 years
Expiry Date	19/02/2018
Strike	5%
alpha	11%
sigma	0.8%

Table 5: Data for the tree

We have also to consider that this one is a 10y Bermudan yearly Payer Swaption Strike 5% non-call 2 which means that it can be exercised every year starting from the second one.

2. Certificate Pricing

In this case study, we aim to price a Structured Bond using the NIG model and subsequently compare these valuations with those obtained through alternative methodologies. We establish the NIG parameters by calibrating the model to the implied volatility surface (refer to Assignment 5 for a detailed discussion). Discount factors are determined through bootstrapping market data.

Sigma	0.1049
Kappa	1.3088
Eta	12.7558

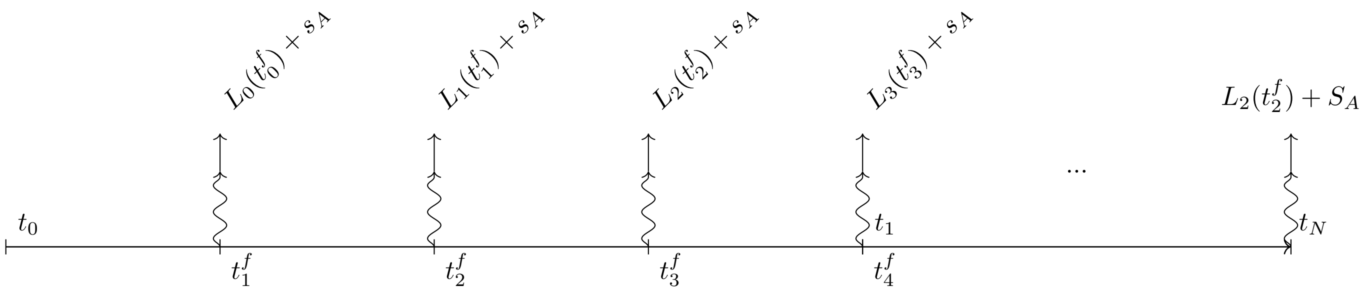
Table 6: NIG calibrated parameters

Given the pay-off outlined in the annex, to accurately price the certificate, we need to establish the upfront payment X as a percentage of the principal amount, which equally matches the two counterparties' legs. Therefore, after calculating the NPV for both party A and party B, we determine X as follows:

$$X = NPV_A - NPV_B$$

Party A leg

Party A pays the Euribor 3m plus a spread of $s_A = 1.3\%$ each quarter. While the payments are certain in the first year, the payments in the second year are subject to the early redemption clause. That is, if the underlying is below the strike, the first coupon is paid, and the contract is extinguished.
We can visualize this as in the following timeline:



Hence, the NPV for party A is given by:

$$NPV_A = principal * (1 - d_{1y} + s_A * BPV_1 y) + principal * (d_{1y} - d_{2y} + s_A * BPV_2 y) * (1 - P);$$

Where d represents the discount factor and P denotes the probability that at the evaluation date the underlying is lower than the Strike.

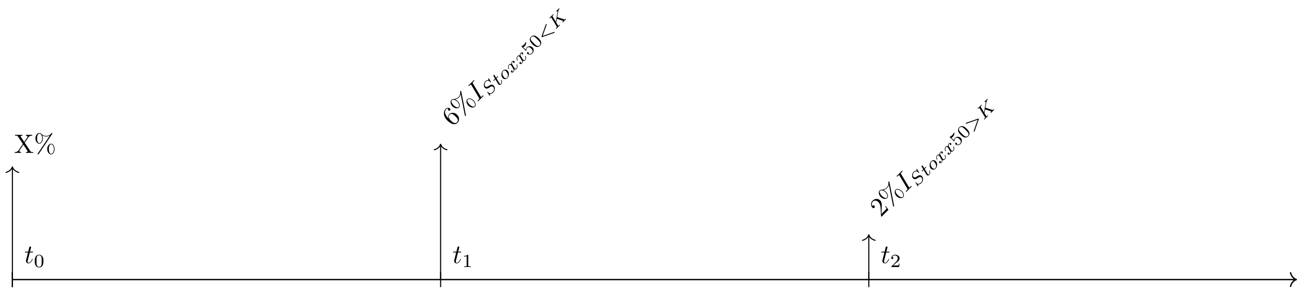
Party B leg

Party B pays X% of the principal amount, subsequently pays the following:

Condition	Payment date	Coupon
S < strike	first year	0.6%
S > strike	second year	0.2%

Table 7: Coupons

Where the condition is verified on the evaluation date, set in the annex as two days before the coupon date one year. If the first coupon is paid, then the contract ends. This particular condition is known as Early Redemption.
We can visualize this as:



The NPV of party B is computed as follow:

$$NPV_B = principal * \delta_{1y} * c_{1y} * d_{1y} * P + principal * \delta_{2y} * c_{2y} * d_{2y} * (1 - P);$$

Where d is the discount factor, P is the probability that at the evaluation date the underlying is lower than the Strike, and δ is the year fraction between the start date and the first coupon, and between the first coupon and the second coupon.

2.1. Value of the upfront X% via Closed Formula

In this case we derived the closed formula to compute the probability of receiving the first coupon. In order to do so we follow the derivation for the call price with the Lewis formula and extract the needed probability.

$$P(S_t < K) = \frac{1}{2} - \frac{1}{\pi} \int_0^{+\inf} \Re \left(\frac{e^{iuk} \Phi_{t0}(u)}{iu} \right)$$

with:

$$\begin{cases} k = \log\left(\frac{S_{t0}}{K}\right) + (r - q)t_0 \\ \Phi(u) = e^{-i \cdot u \cdot \log(\mathcal{L}(\eta))} \cdot \mathcal{L}\left(\frac{u^2 + i \cdot (1 + 2 \cdot \eta) \cdot u}{2}\right) \\ \log(\mathcal{L}(\eta)) = \frac{\Delta t}{k} \cdot \frac{1 - \alpha}{\alpha} \cdot \left[1 - (1 + \frac{\omega k \sigma^2}{1 - \alpha})^{\alpha}\right] \end{cases}$$

Now we proceed computing the integral with a numerical method and obtain the probability of receiving the first coupon. Now we proceed by computing as we have already done in the previous points the NPV for party B and party A.

$\text{NPV}_B = principal \cdot \delta_1 y \cdot c_1 \cdot B(t_0, t_1) \cdot P(S_t < K) + principal \cdot \delta_2 y \cdot c_2 \cdot B(t_0, t_2) \cdot (1 - P(S_t < K))$

$\text{NPV}_A = principal \cdot (1 - B(t_0, t_4^f) + s_A \cdot BPV_1) + principal \cdot (B(t_0, t_4^f) - B(t_0, t_8^f) + s_A * BPV_2) \cdot (1 - P(S_t < K))$

$\text{BPV}_1 = \sum_i = 1^4 \delta_i \cdot B(t_0, t_i)$

$\text{BPV}_2 = \sum_i = 5^8 \delta_i \cdot B(t_0, t_i)$

Finally we compute the upfront:

$X = NPV_A - NPV_B$
and obtain the following results:

Survival Probability	0.3616
X%	2.4949%
X €	249874 €

Table 8: closed formula

2.2. Numerical Approach

As mentioned earlier, we need to determine the probability of receiving the first coupon.
To approach this numerically, one method involves viewing the Early Redemption condition as a "bull-spread" strategy with European Calls.
This involves one call option with a strike equal to the swap rate and another call option with a strike slightly higher (by an amount epsilon). By taking the limit as epsilon approaches zero, we essentially converge towards the payoff of a digital option. Specifically, the price of the digital option can be approximated as:

$$PriceDigital = \frac{Call_K - Call_{K+\epsilon}}{\epsilon}$$

Initially we have selected the Fast Fourier Transform as our numerical method for evaluating the integral required from the Lewis formula to price Calls, utilizing the Normal Inverse Gaussian distribution and its characteristic function.

After doing this, we are able to calculate the survival probability of the certificate until the second year as follows:

$$SurvProb = \frac{PriceDigital}{Discount}$$

where "Discount" is the discount factor obtained via interpolation from the discount curve.
Now, we need to evaluate the NPV of both parties to determine the value of the upfront payment that Bank XX has to make at the start date. Indeed, X is given by:

$$X = NPV_A - NPV_B$$

Using this NIG we obtain the following results:

Survival Probability	0.3566
X%	2.4523%
X €	2452347 €

Table 9: FFT results (NIG)

2.3. Model selection

Utilizing the Fast Fourier Transform (FFT) method for NIG or resorting to the closed formula isn't the sole approach for pricing this certificate. Other models such as Variance Gamma (VG), the Black formula (adjusted or unadjusted), or Monte Carlo methods are viable alternatives. However, it's imperative to highlight that employing the unadjusted Black formula may result in significant errors, notably due to its oversight of digital risk.

Upfront with Variance Gamma

To compute the upfront using the Variance Gamma method, we need to establish a new Laplace exponent since α tends to 0. Indeed, the Laplace exponent for NIG is no longer applicable, as it involves a fraction with alpha in the denominator. Hence, by taking the limit, we compute the new Laplace exponent:

$$\mathcal{L} = -\frac{\Delta t}{\kappa} \cdot \log\left(1 + \kappa \cdot \omega \cdot \sigma^2\right)$$

Then we have to use the same approach as above and found the following results:

X%	2.4353%
X €	2435321€

Table 10: Upfront

Upfront with MonteCarlo

To employ the Monte Carlo method for NIG models, we need to incorporate the parameters previously calibrated in Assignment 5. Indeed, we evaluate the possible outcomes of the underlying corresponding to the forward using an Inverse Gaussian Distribution to simulate.

This approach enables the generation of a large number of diverse paths, facilitating the evaluation of the survival probability of the certificate by examining the frequency of the event $S_T > K$.
We decide to use, as number of simulations, $N = 10^6$ and obtain the following results:

X%	2.511%
X €	2511039€

Table 11: Upfront

2.4. Black model to price the certificate

In the Black case, we need to consider two distinct approaches: one that disregards digital risk (essentially the classical Black formula for Digital), and another that incorporates adjustments for skew.
In both cases, we have to consider as data the EURO STOXX strikes and the volatility surface.
We need to emphasize that the evaluation of NPVs will remain consistent with the other cases. The only divergence lies in the calculation of the digital price, from which we derive the survival probability using the formula:

$$survProb = \frac{PriceDigital}{discountFactor}$$

Here the *discountFactor* is the discount evaluate two days in arrears with respect to the one of the payment of the first Coupon.

Black without skew adjustment

As mentioned above, in this case we are not taking into consideration the digital risk: this will lead us into big discrepancies with the price found via NIG method.

In order to find the price of digital, in this case, we will use the close formula under Black, which is:

$$DigitalPrice_{Black} = Notional * discount_{1y} * cdf(d_2) \qquad d_2 = \frac{\log \frac{F_0}{Strike} + \frac{1}{2} * \sigma_{digital}^2 * T}{\sigma_{digital} * \sqrt{T}} - \sigma_{digital} * \sqrt{T}$$

where cdf is the cumulative distribution function of the standard normal distribution, evaluated at the value d_2.
The results obtained in this case are the following:

X%	1.5388%
X€	1538757 €

Table 12: Upfront

As expected, the disparity between Black without adjustment and NIG is substantial:

$$Error = 37.2537\%$$

We can infer that this model does not accurately price the upfront. This can be easily explained since the simple Black formula is not able to price digital risks. This leads to a mispricing of the digital option, resulting in a lower price.

Black with skew adjustment

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In order to adjust the results obtained through the Black formula for Digital option, we have to evaluate, as first, the skew on the entire surface as follows:

$$skew_i = \frac{vol_{i+1} - vol_{i-1}}{Strike_{i+1} - Strike_{i-1}}$$

where $skew_i$ is the skew evaluate in the i-th point of the volatility surface.
Then, via spline interpolation, we can find the skew in the strike we are interested in and call it $skew_K$.
After this, we have to evaluate also the vega under the Black Model through the following formula:

$$vega = F_0 * d_{1y} * normpdf(d_1) * \sqrt{T} \qquad d_1 = \frac{\log \frac{F_0}{Strike} + \frac{1}{2} * vol^2 * T}{vol * \sqrt{T}}$$

where pdf returns the probability density function of the standard normal distribution, evaluated in d_1.
Then, another time, via spline interpolation, we can find the vega in the strike we are interested in and call it $vega_K$.
At this point we find the price of the digital, taking care of the Digital Risk, as follows:

$$DigitalPrice = DigitalPrice_{Black} - vega_K * skew_K * Notional$$

The results obtained in this case are the following:

X%	2.4975%
X€	2497531 €

Table 13: Upfront

Thanks to the skew adjustment, the error between Black with adjustment and NIG is significantly lower compared to the previous case. Indeed, we achieve:

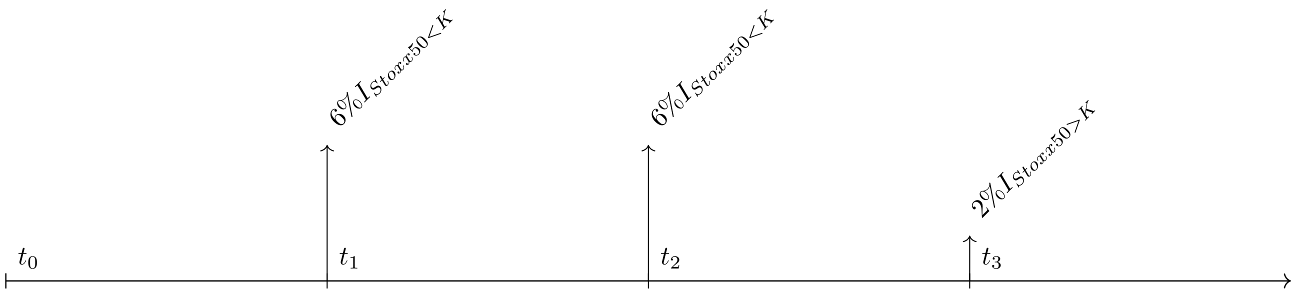
$$Error = 1.8425\%$$

2.5. Bond with a three-year expiry

In this point the structure of the Certificate is different: we are no more considering a two but a three year expiry certificate bond with the following coupons:

Condition	Payment date	Coupon
S_1 < strike	first year	0.6%
S_2 < strike	second year	0.6%
S_2 > strike & S_1 > strike	third year	0.2%

Table 14: Coupons



The Early Redemption option can be exercised at the end of the first two years. This means that it doesn't matter if the first coupon is paid for the survival of the certificate up to the second year.

To price this certificate, we practice a Monte Carlo simulation with the Normal Inverse Gaussian (NIG) model since the FFT method was computationally heavier due to the new formulation of the product.
Specifically, we begin by evaluating the possible outcomes after one year from the start date and then divide the simulations into two vectors: those where the underlying is greater than the strike and those where it is lower. From this point, we conduct another one-year simulation starting from each obtained outcome. At this juncture, we need to consider four possible scenarios:

A) $S_{1y} > K$ and $S_{2y} > K$
B) $S_{1y} > K$ and $S_{2y} < K$
C) $S_{1y} < K$ and $S_{2y} > K$
D) $S_{1y} < K$ and $S_{2y} < K$

Scenario A

Being both of the times lower than the strike value, Party B doesn't have to pay the two middle coupons but only the last one: this means that the certificate survives until the expiry date since there is no Early Redemption.
NPV of Party B:

$$NPV = principal \cdot \delta_{3y} \cdot c_{3y} \cdot d_{3y} \cdot P_A$$

where P_A is the probability of being in the scenario A.

Scenario B

In this case, we don't take the first coupon since at the first evaluation date (two days in arrears with respect to the first payment date) the underlying doesn't fulfil the constraint. We instead take the second coupon since $S_{t_2} < K$ and therefore the certificate ends after the second year due to early redemption.
NPV of Party B:

$$NPV = principal \cdot \delta_{2y} \cdot c_{2y} \cdot d_{2y} \cdot P_B$$

where P_B is the probability of being in the scenario B.

Scenario C

In this scenario we immediately fulfill the constraint $S_{t1} > K$ and therefore obtain the first coupon.
NPV of Party B:

$$NPV = principal \cdot (\delta_{1y} \cdot c_{1y} \cdot d_{1y}) \cdot P_C$$

where P_C is the probability of being in the scenario C.

Scenario D

Due to the fact that both of the times the underlying is below the strike, we get both of the coupons and end the certificate after two years because of Early Redemption.
NPV of Party B:

$$NPV = principal \cdot (\delta_{1y} \cdot c_{1y} \cdot d_{1y} + \delta_{2y} \cdot c_{2y} \cdot d_{2y}) \cdot P_D$$

where P_D is the probability of being in the scenario D.
After evaluating all these scenarios, we can determine the NPV_B as the sum of the four NPVs found in each of the aforementioned scenarios.
Finally, we assess the NPV of Party A. Party A pays the Euribor 3m plus a fixed rate $s_a = 1.3\%$ each quarter for the initial two years (as Early Redemption is not applicable in the one-year case).
Once more, there are two scenarios: if the certificate matures at the conclusion of the second year (as in Scenarios B and D), Party A is exempt from payments in the third year, despite remaining obligated to pay quarterly during the final period.
This can be expressed as:

$$NPV_A = principal \cdot (1 - \delta_{2y} + s_A \cdot BPV_{2y}) + principal \cdot (\delta_{2y} - \delta_{3y} + s_a \cdot BPV_{3y}) \cdot P_A$$

In the end, as usual, we evaluate X as:

$$X = NPV_A - NPV_B$$

The results obtained using this formula are the following:

X%	3.6383%
IC%	[3.6269%, 3.6497%]
X €	3638279€
IC €	[3626854.3698, 3649703.8441]€

Table 15: Upfront X

Some of the previously used methods could be adapted in order to price this type of certificate, in particular we can relay on Monte Carlo approach with NIG or IG methods.

3. Bermudan Swaption Pricing via Hull White

In order to price a Bermudan swaption via Hull white dynamics let's start by describing it's movement. In the Hull White Model we describe the dynamics of the interest rate as:

$$r_t = \phi_t + x_t$$

where x_t follows is a mean-reverting Ornstein–Uhlenbeck process with mean zero:

$$\begin{cases} dx_t = -a \cdot x_t \cdot dt + \sigma \cdot dW_t \\ x_{t0} = x_0 = 0 \end{cases}$$

this model's mean reverting property is given by the fact that the first term of the dx_t dynamics pushes the process towards the mean. In this type of problem we are able to find an analytic solution:

$$x_s = x_t \cdot e^{-a \cdot (s-t)} + \sigma \cdot \int_t^s e^{-a(s-u)dW_u}$$

Our aim is now to discretize the problem in order to implement a numerical approach. In particular, if we go backwards in time, we can discretize the increment in the x process as:

$$\Delta x_{i+1} = -e^{a\ dt}\hat{\mu}x_{i+1} + e^{a\ dt}\hat{\sigma}g_i$$

where we have that $g_i \sim \mathcal{N}(0,1)$, $\hat{\mu} = 1 - e^{-a\ dt}$ and $\hat{\sigma} = \sigma\sqrt{\frac{1-e^{-2a\ dt}}{2a}}$. Having discretized our Ornstein-Uhlenbeck process we can now use this discretization to build a recombining trinomial tree with equally spaced time steps as follows:

$$\begin{cases} \Delta x = \sqrt{3}\hat{\sigma} \\ x = l\Delta x \\ l = -l_{max}, \dots, l_{max} \end{cases}$$

Imposing that the tree reproduces the Hull-White model in mean and variance we obtain the following scheme (scheme A):

$$\begin{cases} p_u = \frac{1}{2}\left(\frac{1}{3} - l\hat{\mu} + l^2\hat{\mu}^2\right) \\ p_m = \frac{2}{3} - l^2\hat{\mu}^2 \\ p_d = \frac{1}{2}\left(\frac{1}{3} + l\hat{\mu} + l^2\hat{\mu}^2\right) \end{cases}$$

This is fine for nodes inside of the tree. However when the liminal values of l_{max} and $l_{min} = -l_{max}$, we get negative values for the probabilities. Hence we have that the index must obey the following constraints:

$$1 - \sqrt{\frac{2}{3}} < l_{max} \cdot \hat{\mu} < \sqrt{\frac{2}{3}}$$

In order to achieve the best accuracy possible we took the lowest possible value for l_{max}.
Furthemore for the two liminal values we must apply two other schemes that keep them within the bounds. In particular for $l = l_{max}$ we apply scheme B:

$$\begin{cases} p_u = \frac{1}{2}\left(\frac{1}{3} + l\hat{\mu} + l^2\hat{\mu}^2\right) \\ p_m = -\frac{1}{3} - 2l\hat{\mu} - l^2\hat{\mu}^2 \\ p_d = \frac{1}{2}\left(\frac{7}{3} + 3l\hat{\mu} + l^2\hat{\mu}^2\right) \end{cases}$$

for $l = l_{max}$ we apply scheme C:

$$\begin{cases} p_u = \frac{1}{2}\left(\frac{7}{3} - 3l\hat{\mu} + l^2\hat{\mu}^2\right) \\ p_m = -\frac{1}{3} + 2l\hat{\mu} - l^2\hat{\mu}^2 \\ p_d = \frac{1}{2}\left(\frac{1}{3} - l\hat{\mu} + l^2\hat{\mu}^2\right) \end{cases}$$

The three schemes' movements can be viewed in the following picture:

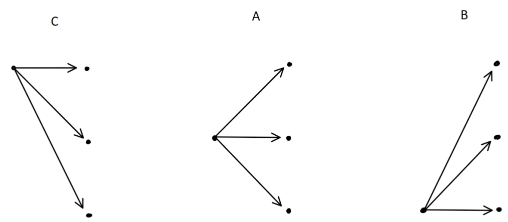


Figure 1: different schemes

3.1. Pricing via a tree

Once we have obtained a full tree for the interest rates movements we can now price our Bermudan option. Our procedure will be the following:

- For each node we take the discretized discounted expected value of the continuation value $\mathbb{E}_{t_i}[D(t_i, t_{i+1})SP^B_{\alpha,\omega}(t_{i+1})]$ according to the node's scheme.
- In reset dates we also compute the Intrinsic value using the discount factors in t_i and the BPV. Where the intrinsic value is computed as: $SP_{\alpha,\omega}(T_\alpha) = BPV(\alpha,\omega) \cdot [S_{\alpha,\omega} - K]^+$
- In reset dates we take the maximum between the intrinsic and continuation value.

In order to do this we must leverage a few useful relations and lemmas:

$$D(t_i, t_{i+1}) = B(t_i, t_{i+1}) \exp\left\{-\frac{1}{2}\hat{\sigma^*}^2 - \frac{\hat{\sigma^*}}{\hat{\sigma}}[e^{-adt}\Delta x_{i+1} + \hat{\mu} x_{i+1}]\right\}$$

$$\hat{\sigma^*} = \frac{\sigma}{a}\sqrt{dt - 2\frac{1-e^{-adt}}{a} + \frac{1-e^{-2adt}}{2a}}$$

$$B(t_i; t_i, t_i+\tau) = B(t_0; t_i, t_i+\tau) \exp\left\{-x_i\frac{\sigma(0,\tau)}{\sigma} - \frac{1}{2}\int_{t_0}^{t_i}[\sigma(u, t_i+\tau)^2 - \sigma(u, t_i)^2]du\right\}$$

which are simply applications of lemma 1 and 2 to the Hull White model.
For each reset date we can compute the Swap rate by computing the discount factor with t_i equal to the reset date and τ such that the second argument spans from the following reset date to the last reset date.
Applying this tecnique to the above mentioned swaption yields the following result:

$$SP^B_{\alpha,\omega}(t_0) = 0.021461$$

This estimate was obtained taking 70 steps between each reset date. Indeed we observed the following behaviour with the swaption prices:

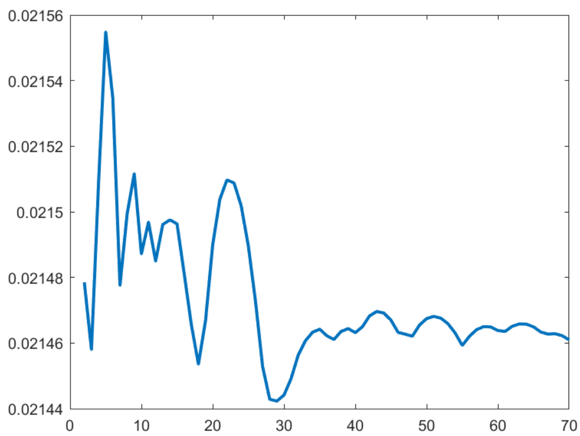


Figure 2: Prices of the Bermudan options as a fucntion of steps between reset dates

3.2. Check of the results with Jamshidian formula

In order to check the accuracy of our tree we try to compare the results obtained via the tree. In particular with the tree constructed above we obtain the following price for the Swaption from 9 years to 10 years:

$$SP_{9y,10y} = 0.0046421$$

We now try to price such a swaption with the Jamshidian approach. First of all we leverage the fact that a Swaption's price is equal to the price of a Put coupon bond option with coupon equal to the swaption's strike times the year fraction and strike equal to 1.
In order to use the Jamshidian approach we must first of all find the value x^* such that:

$$P(x^*, T_\alpha; T_\alpha, T_\omega) \text{ where } P(x_t, t; T_\alpha, T_\omega) = \sum_{i=\alpha}^{\omega-1} c_i B_{\alpha,i+1}(x_t, t)$$

where the values $B_{\alpha,i+1}$ are computed using the relations seen above. Let us remark that such a value x^* always exists. Indeed the ZCB prices are all monotonic funcions, hence their sum the coupon bond must also be monotonic. Therefore the point x^* exists and is unique.
Once we have computed the above value we can now plug it back into the ZCBs to compute the following quantities:

$$K_i = B_{\alpha,i+1}(x^*, T_\alpha)$$

these values can then be plugged into the formulation of the Call Option on the coupon bond instead of K. Indeed since $K = P(x^*)$ and $P(x^*) = \sum_{\alpha}^{\omega-1} B_{\alpha,i+1}(x^*, T_\alpha)$ we can substitute this sum into the Call option and obtain the following:

$$C^P_{\alpha,\omega}(t_0, K) = \sum_{i=\alpha}^{\omega-1} c_i C^{ZC}_{\alpha,i+1}(t_0, K_i)$$

Now we can simply leverage the Put-Call Parity and obtain:

$$P^P_{\alpha,\omega}(t_0, K) = C^P_{\alpha,\omega}(t_0, K) + K \cdot B(t_0, T_\alpha) - P(t_0, T_\alpha, T_\omega)$$

Applying such a tecnnique yields the following value:

$$SP_{\alpha,\omega} = 0.0049533$$

As we can see, the values of the two swaptions are relatively close. Thus, we can put a certain measure of trust into the model of our tree.

3.3. Jamshidian formula for upper and lower bound

We can obtain two rough estimate for the upper and lower bounds of the Swaption. First of all we known that the Bermudan swaption's value must be at least equal or bigger than the maximum value of the possible underlying european swaptions. In our case it must be greater or equal to the maximum of the coterminal european swaptions with expiry in the reset dates and same end date of 10 years.

On the other hand we can also make an estimate of the upper bound by summing the above mentioned coterminal european swaptions.

Computing the prices of the European Swaptions and then summing and taking the maximum yield the following values:

$$maxSP_{\alpha,\omega} = 0.035253$$
$$\sum SP_{\alpha,\omega} = 0.18294$$

As we can clearly observe the lower bound seems unfortunately higher than the value we have obtained for the Bermudan option.

This is probably due to inaccuracies in the implementation of the Jamshidian formula.

We decided to trust the implementation of our tree pricing since we clearly observed a convergence to a finite value.

Our implementation of the Jamshidian formula is not as straight forward as we had hoped and there are probably some inaccuracies in how the integral for the volatility is computed.

4. References

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