CLOSED FORMULA FOR A CLIQUET OPTION

COUPON: $\Rightarrow$ $\mathbb{E}\left[D(t_0,t_i)\,\mathbb{1}_{\{\tau > t_i\}}\,\delta(t_{i-1},t_i)\left[LS(t_i)-S(t_{i-1})\right]^+ \right] =$

$$= B(t_0,t_i)\,P(t_0,t_i)\,\delta(t_{i-1},t_i)\,\mathbb{E}\left[\left(LS(t_i)-S(t_{i-1})\right)^+\right]$$

WE ASSUME INDIPENDENCE BETWEEN DISCOUNTS, TIME OF DEFAULTS AND THE UNDERLYING STOCK

$$\mathbb{E}\left[\left(LS(t_i)-S(t_{i-1})\right)^+\right] = \mathbb{E}\left[\mathbb{E}\left[\left(LS(t_i)-S(t_{i-1})\right)^+ \mid \mathcal{F}_{t_{i-1}}\right]\right]$$

$$= L\,\mathbb{E}\left[\mathbb{E}\left[\left(S(t_i)-\underbrace{\tfrac{1}{L}S(t_{i-1})}_{\tilde{K}}\right)^+ \mid \mathcal{F}_{t_{i-1}}\right]\right] =$$

$$= L\,\mathbb{E}\left[S(t_{i-1})\,\mathcal{N}(d_1) - \tilde{K}e^{-r(t_{i-1})(t_i-t_{i-1})}\mathcal{N}(d_2)\right]$$

BLACK AND SCHOLES PRICE FOR AN EUROPEAN CALL OPTION

$$d_1 = \frac{1}{\sigma\sqrt{t_i-t_{i-1}}}\left\{ \ln\left(S(t_{i-1})/\tilde{K}\right) + \left(r(t_{i-1})+\frac{\sigma^2}{2}\right)(t_i-t_{i-1}) \right\}$$

$$= \frac{1}{\sigma\sqrt{t_i-t_{i-1}}}\left\{ \ln(L) + \left(r(t_{i-1})+\frac{\sigma^2}{2}\right)(t_i-t_{i-1}) \right\}$$

$\Rightarrow$ d_1 AND d_2 DO NOT DEPEND ON $S(t_i)$, SO WE TAKE THEM OUT FROM $\mathbb{E}$

$$= L\left\{\mathcal{N}(d_1) - \frac{1}{L}e^{-r(t_{i-1})(t_i-t_{i-1})}\mathcal{N}(d_2)\right\}\underbrace{\mathbb{E}\left[S_{t_{i-1}}\right]}_{= S_{t_0}e^{r(t_0)(t_{i-1}-t_0)}}$$

DEFAULT PART: $\sum_{J=1}^{7}\pi\left[P(t_0,t_{J-1})-P(t_0,t_J)\right]\sum_{i=J}^{7}B(t_0,t_i)\,\mathbb{E}\left[\left(LS(t_i)-S(t_{i-1})\right)^+\right]$

PRICE CLIQUET OPTION =

$$\sum_{i=1}^{7} S_{t_0}e^{r(t_0)(t_{i-1}-t_0)}L\left\{\mathcal{N}(d_1)-\frac{1}{L}e^{-r(t_{i-1})(t_i-t_{i-1})}\mathcal{N}(d_2)\right\}B(t_0,t_i)\,P(t_0,t_i)\,\delta(t_{i-1},t_i)$$

$$+\ \pi\sum_{J=1}^{7}\sum_{i=J}^{7}\left[P(t_0,t_{J-1})-P(t_0,t_J)\right]B(t_0,t_i)\,S_{t_0}e^{r(t_0)(t_{i-1}-t_0)}L\left\{\mathcal{N}(d_1)-\frac{1}{L}e^{-r(t_{i-1})(t_i-t_{i-1})}\mathcal{N}(d_2)\right\}$$