Financial Risk Laboratory: 3. Credit Portfolio Models

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Overview

Example: Portfolio Risk Assessment under Multiple Scenarios

Importer of steel products

Scenarios	Steel Price	Credit Line Usage	Default Probability
Recession (prob. 20%)	-50%	$9m	20%
Baseline (prob. 60%)	+5%	$5m	2%
Expansion (prob. 20%)	+50%	$8m	0.2%

Oil Producer

Scenarios	Oil Price	Credit Line Usage	Default Probability
Recession (prob. 20%)	25 $US/bl	$9m	40%
Baseline (prob. 60%)	50 $US/bl	$5m	4%
Expansion (prob. 20%)	75 $US/bl	$8m	0.4%

Obligors: (a) importer of semi-finished steel product - (b) shale oil producer.
Product: $ 10m credit line for current account overdrafts for both customers.
When: t is April 1^{st}, 2024 - T is March 31^{st}, 2025

Questions

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- Portfolio Expected Loss (EL) with 1y time horizon
- Portfolio Unexpected Loss (UL) with 99% confidence interval and 1y time horizon

Suggestion

For simplicity, let us measure UL in the simplest way: $UL = VaR - EL$

Portfolio Credit Risk

Expected Loss (EL)

Simple sum of individual ELs.
Given a diversified portfolio composed by N exposures, each with weight w_n ($\sum_{n=1}^{N} w_n = 1$) and EL_n, then the portfolio EL is:

$$EL = \sum_{n=1}^{N} w_n \cdot EL_n \tag{1}$$

Unexpected Loss(UL)

Not trivial: even if we take the simplest solution, i. e. UL = VaR - EL, it is very sensitive to diversification and macro-conditions.
The most critical quantity is the frequency of joint defaults

Our Example - 1: Expected Loss

In our example the portfolio is equally weighted w.r.t. the two obligors:

Obligor	$E\bar{A}D$	PD	$L\bar{G}DR$
Importer of steel products	\$8.1m	5.24%	71%
Oil Producer	\$8.1m	10.48%	71%

Table: Credit risk parameters for the two obligors

EL (\$ million)

$$0.5 \cdot (8.1 \cdot 0.0524 \cdot 0.71) + 0.5 \cdot (8.1 \cdot 0.1048 \cdot 0.71) = 0.45$$

Joint Default Probability

2 def.	1 def.	None
0.55%	14.62%	84.83%

Table: Independent defaults (binomial distribution)

Scenario (prob)	2 def.	1 def.	None
Recession (20%)	8.00%	44.00%	48.00%
Baseline (60%)	0.08%	5.84%	94.08%
Expansion (20%)	0.00%	0.60%	99.40%
Weighted avg. (100%)	1.65%	12.42%	85.93%

Table: We assume independent defaults within each scenario

If we take into account scenarios, two def. probability is 3x.

Our Example - 2: Unexpected Loss

Let us assume we evaluate the UL thanks to VaR with 99% c.l. and $1y$ time horizon, i.e. with $c = 0.99$:

$$UL = VaR_c(L) - EL \tag{2}$$

Independent Defaults

VaR_c is the single-default loss

$$UL = \$8.1m \cdot 0.71 - \$0.45m = \$5.3m \tag{3}$$

Defaults are scenario-dependent

VaR_c is the two-default loss

$$UL = 2 \cdot \$8.1m \cdot 0.71 - \$0.45m = \$11m \tag{4}$$

Credit risk (i.e. UL) doubles with a realistic joint-default probability

Default is Just One among a Set of Credit Events

Credit events = all events that can alter the value of a bond/loan

- Default is only the most significant one: the new value of the bond/loan is equal to the value that can be reasonably recovered (e.g. 30%)
- But all changes in the borrower's *rating class* are credit events, in that they change the value of the bond/loan: the future cash flows from downgraded bond/loans have to be discounted using higher rates (higher spreads) and their present value decreases

Default vs. Migration Risk

Starting from the definition of loss:

Only default ("default only" risk)

- Credit risk can then be modeled by means of binomial models
- Loans can be kept at "book value"

True Credit Risk if the risk horizon is longer than the expiry of the exposures

Any change in value ("migration" risk)

- Two future states are not enough
- Evaluation based on the market value

Migration is material if the risk horizon is shorter than the expiry of the exposures

Creditmetrics™: process

7 steps

1. Market value of exposures
2. Assign a rating grade to each exposure
3. Estimate migration probabilities
4. Estimate recovery rate
5. Market values associated to different hypothetical rating grades across the time horizon
6. Distribution of market values at the end of the risk horizon
7. Portfolio risk

Creditmetrics™: Inputs

- Time (risk) horizon
- Rating system (S&P, Moody's, Fitch, internal)
- Transition matrix
- Recovery rates
- Forward spreads associated to different rating grades

Transition Probabilities

Rating agencies update regularly the assignment to a rating class of a borrower, following their *rating process*.
It has become common to describe the behavior of the rating process with the *transition probabilities*, which are arranged in a **transition matrix**, like the one below.

One-year transition matrix

	Rating at year-end (%)							
Initial rating	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0.00
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0.00
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0.00	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0.00	0.22	1.30	2.38	11.24	64.86	19.79

Source: S&P CreditWeek (15 April 1996)

Figure: One–year transition matrix

Loss Given Default Depends on the Credit Cycle

Category	Senior Secured	Senior Unsecured	Senior Subordinated	Subordinated	Junior Subordinated
Mean	53.80	51.13	38.52	32.74	17.09
Std. dev. (%)	26.86	25.45	23.81	20.18	10.90

Source: Gupton, Finger and Bhatia (1997).

Figure: Recovery Rates for various classes of public debt seniorities

Creditmetrics™incorporates uncertainty across losses given default by assuming RR follows a given distribution (critical parameters: mean and standard deviation)

Losses Due to Migrations are Represented by Credit Spread Changes

One-year* forward zero coupon *rate curve (%)

Maturity	*1 year*	*2 years*	*3 years*	*4 years*
Rating class				
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

Figure: Example of fwd. zero–coupon rates (risk-free + spread) defined on a four–tenors term structure (1y, 2y, 3y, 4y) by rating class

Default & Rating Migration Risk: Single Name

Example: marginal contribution to the hypothetical loss evaluation due to default and rating migration events of a BBB-rated Bond

Distribution of one-year market values of a BBB bond

State at year-end (j)	*Present value in a year's time (FV_j)*	*Probability, p_j (%)*	*$\Delta V_j = FV_j - E(FV)$*
AAA	109.35	0.02	2.28
AA	109.17	0.33	2.10
A	108.64	5.95	1.57
BBB	107.53	86.93	0.46
BB	102.01	5.3	-5.07
B	98.09	1.17	-8.99
CCC	83.63	0.12	-23.45
Default	53.80	0.18	-53.27
Mean, $E(FV)= \Sigma p_j FV_j$	107.07		

Simulated Value Distribution for a Single Exposure

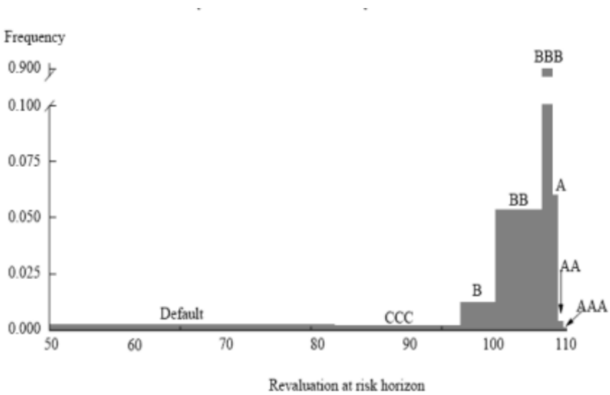

Figure: The simulated values show the loss (and some profits also) distribution at T for a bond BBB-rated at t

Two Issuers - Independent

Let's introduce an elementary portfolio: two issuers (example: rated at time t in the class A and BB respectively.
Following the transition probabilities approach, it is elementary to derive the joint transition probability under the **independence assumption**:

Probability of joint migration of two issuers with ratings A and BB, assuming independence of the relative migration rates

		Issuer A							
		AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*Default*
Issuer BB		*0.09*	*2.27*	*91.05*	*5.52*	*0.74*	*0.26*	*0.01*	*0.06*
AAA	*0.03*	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
AA	*0.14*	0.00	0.00	0.13	0.01	0.00	0.00	0.00	0.00
A	*0.67*	0.00	0.02	0.61	0.40	0.00	0.00	0.00	0.00
BBB	*7.73*	0.01	0.18	7.04	0.43	0.06	0.02	0.00	0.00
BB	*80.53*	0.07	1.83	73.32	4.45	0.60	0.20	0.01	0.05
B	*8.84*	0.01	0.20	8.05	0.49	0.07	0.02	0.00	0.00
CCC	*1.00*	0.00	0.02	0.91	0.06	0.01	0.00	0.00	0.00
Default	*1.06*	0.00	0.02	0.97	0.06	0.01	0.00	0.00	0.00

Figure: Joint distribution of future ratings for two names according to the joint multinomial distribution. The borders of the matrix above represent the individual transition probabilities: Column on the left shows the transition probabilities of a BB–rated issuer. Row on the top shows the transition probabilities of a A–rated issuer.

Default Correlations Driven by Asset Correlations: The Role of Merton

Assumption of independence not realistic: rating changes and defaults are partly the result of common factors (e.g. economic cycle, interest rates, changes in commodity prices, etc.).

- **Factor models** can be based on a modified version of the Merton model, where not only defaults but also migrations depend on changes in the value of corporate assets
- asset value returns (AVR) are standardized over their standard deviation
- estimates the correlation between the asset value returns of the two obligors
- based on that correlation, derives a distribution of joint probabilities

The Distance-to-default Extended to Incorporate Migrations

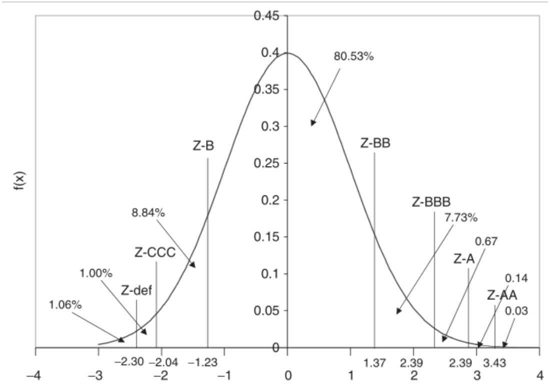

Figure: Example of standardized AVR for a BB-rated exposure

Thresholds for the Standardized AVR

Threshold values for the standardized AVR are based on the cumulative standardized normal distribution, in such a way to match hypothetical realizations of the AVR and migration/default probabilities.

Example: given a BB-rated firm, a negative realization of the AVR could cause the downgrading to B (with probability 8.84%).
The thresholds Z_{CCC} and Z_B along the AVR axis are based on the following relationship:

$$0.0884 = \int_{Z_{CCC}}^{Z_B} f(r_{BB}) \cdot dr_{BB} \tag{5}$$

where $f(x)$ is the standard normal distribution and r_{BB} is the standardized AVR of the BB-rated firm.

How Thresholds are Calculated

Equation (5) is applied recursively, starting from the default probability (PD) for any firm:

$$PD = \int_{-\infty}^{Z_{CCC}} f(r) \cdot dr \tag{6}$$

The table below shows an example with all the thresholds for a BB-rated firm.

State at year end (j)	*Transition probability ($p_{BB \to j}$)*	*Cumulative probability*	*AVRT (Z_j)*
Default	1.06 %	1.06 %	−2.3
CCC	1.00 %	2.06 %	−2.04
B	8.84 %	10.90 %	−1.23
BB	80.53 %	91.43 %	1.37
BBB	7.73 %	99.16 %	2.39
A	0.67 %	99.83 %	2.93
AA	0.14 %	99.97 %	3.43
AAA	0.03 %	100.00 %	

A Multivariate Montecarlo Simulation for Firm Assets Drive the Joint Probabilities

The joint-distribution of losses for a portfolio of N obligor is build by a montecarlo simulation of N normally–multivariate AVRs: $\{r_1, r_2, \ldots, r_N\}$.
To any realization of the AVRs are associated migration/default events by comparison of each realization with the corresponding thresholds.
Below an example with two firms:

Joint probabilities of two obligors (with A and BB ratings)
assuming correlation between asset value returns of 20% - Values %

	Issuer A								
Issuer BB	*AAA*	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*	*Default*	*Total*
AAA	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	*0.03*
AA	0.00	0.01	0.13	0.00	0.00	0.00	0.00	0.00	*0.14*
A	0.00	0.04	0.61	0.01	0.00	0.00	0.00	0.00	*0.67*
BBB	0.02	0.35	7.10	0.20	0.02	0.01	0.00	0.00	*7.69*
BB	0.07	1.79	73.65	4.24	0.56	0.18	0.01	0.04	*80.53*
B	0.00	0.08	7.80	0.79	0.13	0.05	0.00	0.01	*8.87*
CCC	0.00	0.01	0.85	0.11	0.02	0.01	0.00	0.00	*1.00*
Default	0.00	0.01	0.90	0.13	0.02	0.01	0.00	0.00	*1.07*
Total	*0.09*	*2.29*	*91.06*	*5.48*	*0.75*	*0.26*	*0.01*	*0.06*	*100.00*

Source: Gupton, Finger and Bhatia (1997).

Estimation of the Correlation Between AVRs

The most elementary approach is to impose an average correlation ρ among all obligors belonging to the same rating class[1].

Creditmetrics™uses an approach by large building blocks:

- Correlations are first estimated among a large set of industries and countries ("risk factors")
- For each borrower, a set of weights must be specified, expressing his sensitivity to different risk factors and to idiosyncratic risk
- Combining those weights and the risk factor correlations, an estimate of the pairwise correlation of two firms can be obtained

[1] See for reference Schonbucher, 10.4

Creditmetrics™In Practice

In practice: the asset returns are proxied by the return on stock indices. The AVR of a firm is decomposed into one or more systematic components (connected with the dynamics of country- or industry-specific stock indices, e.g., chemical, banking, automotive, etc.), plus an idiosyncratic term which is typical of each individual company:

$$r_j = \beta_{1,j} \cdot I_1 + \beta_{2,j} \cdot I_2 + ... + \beta_{n,j} \cdot I_n + \delta_j \varepsilon_j \tag{7}$$

where $I_1, I_2, ..., I_n$ are common factors (country/industry indices) and ε_j is the specific component for the j-th firm

At the Edge Between a Structural and a Reduced Form Model

- CreditMetrics™is a reduced–form model
- Such class of models, however, does not allow to deal with default/migration correlation
- Default/migration correlation is derived from the correlation of the returns of the firms' assets

Example of Realistic Bank-wide pmf

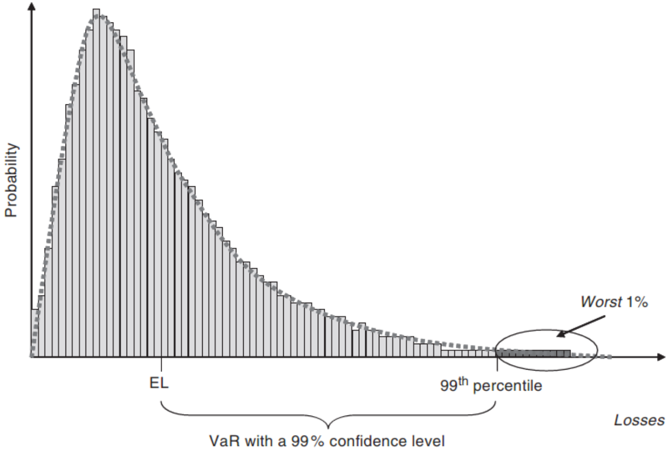

Figure: Source: [Sironi 2007] 14.8

Creditmetrics™: Advantages and Disadvantages

Advantages

- Uses objective and forward looking market data: Interest rate curves and stock indices correlations.
- Evaluates the portfolio market value.
- Takes into account migration risk.

Disadvantages

- Needs a lot of data: forward rates, transition matrices
- Assumes stable transition matrices.
- Proxies correlations with stock indices.
- Maps counterparties to industries and countries in an arbitrary and discretionary way

References

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