Insurance & Econometrics

Solvency II - Project, Group 06

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1. Original text of the project

Consider a simplified insurance company whose assets and liabilities sides are characterized as follows:
ASSETS

- there is a unique fund made of equity (80%) and property (20%), $F_t = EQ_t + PR_t$
- at the beginning (t=0) the value of the fund is equal to the invested premium $F_0 = C_0 = 100,000$
- equity features
  - listed in the regulated markets in the EEA
  - no dividend yields
  - to be simulated with a Risk Neutral GBM ($sigma = 20\%$) and a time varying instantaneous rate r
- property features
  - listed in the regulated markets in the EEA
  - no dividend yield
  - to be simulated with a Risk Neutral GBM ($sigma = 25\%$) and a time varying instantaneous rate r

LIABILITIES

- contract terms
  - whole Life policy
  - benefits
    - in case of lapse, the beneficiary gets the value of the fund at the time of lapse, with 20 euros of penalties applied
    - in case of death, the beneficiary gets the maximum between the invested premium and the value of the fund
  - others
    - Regular Deduction, RD of 2.20%
    - Commissions to the distribution channels, COMM (or trailing) of 1.40%
- model points
  - just 1 model point
  - male with insured aged x=60 at the beginning of the contract
- operating assumptions
  - mortality: rates derived from the life table SI2022 [1]
  - lapse: flat annual rates $l_t = 15\%$
  - expenses: constant unitary (i.e. per policy) cost of 50 euros per year, that grows following the inflation pattern
- economic assumption
  - risk free: rate r derived from the yield curve (EIOPA IT without VA 31.03.24),
  - inflation: flat annual rate of 2%

Other specifications:

- time horizon for the projection: 50 years.
  In case of outstanding portfolio in T=50, let all the people leave the contract with a massive surrender
- the interest rates dynamic is deterministic, while the equity and property ones are stochastic.

QUESTION

1. code a Matlab/Python script to compute the Basic Solvency Capital Requirement via Standard Formula and provide comments on the results obtained. The risks to be considered are:
   - Market Interest
   - Market equity
   - Market property
   - Life mortality
   - Life lapse
   - Life cat
   - Expense
2. Split the BEL value into its main PV components: premiums (=0), death benefits, lapse benefits, expenses, and commissions
3. Replicate the same calculations in an Excel spread sheet using a deterministic projection.
   - Do the results differ from 1? If so, what is the reason behind?
   - For the base case only :
     - calculate the Macaulay duration of the liabilities;
     - calculate the sources of profit for the insurance company, deriving its PVFP

[1] https://demo.istat.it/index_e.php

  - check the magnitude of leakage by verifying the equation MVA = BEL + PVFP (i.e. MVA=BEL+PVFP+LEAK)
  - sense check the PVFP using a proxy calculation, based on the annual profit and the duration of the contract

4. Open questions:
   - what happens to the asset and liabilities when the risk-free rate increases/decreases with a parallel shift of, say, 100bps? Describe the effects for all the BEL components;
   - what happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?

2. Summary tables and results

Results for both deterministic and stochastic approach are presented in the following sections.

2.1. Stochastic Approach

2.1.1 Solvency Capital Requirements & BEL Components

Taking into account the different risks of the study, we have the following results :

Results	MVA(€)	BEL(€)	BOF(€)	Δ BOF(€)	DUR_L
Rates_Base	100000.00	96847.13	3152.87	0.00	5.68
Rates_Up	100000.00	96508.83	3491.17	0.00	5.65
Rates_Down	100000.00	97108.95	2891.05	261.82	5.70
Equity	64600.00	64327.54	272.46	2880.41	5.75
Property	95000.00	92253.95	2746.05	406.82	5.69
Mortality	100000.00	96996.63	3003.37	149.50	5.64
Lapse_Up	100000.00	97475.56	2524.44	628.42	4.03
Lapse_Down	100000.00	96544.50	3455.50	0.00	9.32
Lapse_Mass	100000.00	97984.28	2015.72	1137.14	3.45
Catastrophe	100000.00	96859.29	3140.71	12.15	5.67
Expenses	100000.00	96892.72	3107.28	45.59	5.68

Table 1: Stochastic analysis results

Relying on ΔBOF and the correlation matrix for mareket and life risks we compute the solvency Capital Requirement (SCR):

	Interest	Equity	Property	Market
SCR	261.82	2880.4	406.82	3339

Table 2: Market SCR

	Mortality	Lapse	Catastrophe	Expenses	Life
SCR	149.5	1137.1	12.155	45.586	1175.2

Table 3: Life SCR

Consequently, the Basic Solvency Capital Requirement (BSCR) is given by:

BSCR	3806.85

Table 4: BSCR

Moreover, the Best Estimate Liabilities (BEL) for each components are reported:

Liabilities	Lapse	Death	Expenses	Commissions
Base	81249.03	7455.69	295.03	7847.38
IR Up	81251.21	7135.09	275.00	7847.52
IR Down	81188.08	7766.65	312.64	7841.59
Equity	52464.97	6499.68	295.03	5067.85
Property	77178.07	7326.59	295.03	7454.27
Mortality	80521.84	8392.58	292.53	7789.69
Lapse Up	87513.14	4135.73	205.36	5621.33
Lapse Down	64535.92	18966.92	496.16	12545.50
Lapse Mass	88730.05	4262.00	178.92	4813.31
Catastrophe	81126.37	7600.61	294.66	7837.65
Expenses	81249.03	7455.69	340.61	7847.38

Table 5: PV components

2.2. Deterministic Approach

2.2.1 Solvency Capital Requirements & BEL Components

As for the stochastic method, the deterministic method gives the following results:

Results	MVA(€)	BEL(€)	BOF(€)	Δ BOF(€)	DUR_L
Basic	100000.00	95796.04	4203.96	0.00	5.6136
Rates Up	100000.00	95777.04	4222.96	0.00	5.6130
Rates Down	100000.00	96158.40	3841.60	362.36	5.6402
Equity	64600.00	64115.97	484.03	3719.93	5.7166
Property	95000.00	91202.57	3797.43	406.53	5.6156
Mortality	100000.00	95826.65	4173.35	30.62	5.5728
Lapse Up	100000.00	96976.91	3023.09	1180.87	4.0160
Lapse Down	100000.00	93315.72	6684.28	0.00	9.0348
Lapse Mass	100000.00	97411.60	2588.40	1615.56	3.4020
CAT	100000.00	95801.25	4198.75	5.21	5.6063
Expense	100000.00	95841.62	4158.38	45.59	5.6145

Table 6: Stochastic analysis results

Relying on ΔBOF and the correlation matrix for mareket and life risks we compute the solvency Capital Requirement (SCR):

	Interest	Equity	Property	Market
SCR	362.36	3719.93	406.53	4230.6

Table 7: Market SCR

	Mortality	Lapse	Catastrophe	Expenses	Life
SCR	30.62	1615.56	5.21	45.59	1640.7

Table 8: Life SCR

Consequently, the Basic Solvency Capital Requirement (BSCR) is given by:

BSCR	4905.15

Table 9: BSCR

Moreover, the Best Estimate Liabilities (BEL) for each components are reported:

Liabilities	Lapse	Death	Expenses	Commissions
Basic	81227.10	6428.63	295.03	7845.28
Rates Up	97115.28	6428.63	275.00	7845.28
Rates Down	87090.44	6774.29	312.64	7845.28
Equity	59017.92	6285.83	295.03	5068.05
Property	86799.50	6289.57	295.03	7453.02
Mortality	90404.39	7246.68	292.53	7787.58
Lapse Up	94840.18	3665.21	205.36	5619.63
Lapse Down	77454.72	15750.12	496.16	12543.64
Lapse Mass	94088.56	3701.06	178.92	4812.21
CAT	91230.89	13133.15	294.66	7835.55
Expense	91368.84	6428.63	340.61	7845.28

Table 10: PV components

3. Formulas and calculations

In this case study, our objective is to determine the Basic Solvency Capital Requirement (BSCR). The calculations are executed within the Solvency framework, adhering to the Standard Formula. We've assumed Geometric Brownian Motion dynamics (GBM) to determine the paths of the underlying, and the simulations are carried out using the Monte Carlo method.

3.1. Geometric Brownian Motion

The Geometric Brownian Motion (GBM) is widely employed in financial modeling due to its ability to capture key characteristics of asset price movements, particularly in continuous-time settings. GBM incorporates two essential features: drift, representing the average rate of return, and volatility, depicting the random fluctuations around this average. In our case study, the drift is given by the EIOPA interest rates curve, while the volatility changes based on the underlying: $\sigma = 10\%$ for the property asset and $\sigma = 20\%$ in the equity scenario, indicating greater fluctuation for the equity asset side. The GBM formula is as follows:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where μ represents the drift, σ stands for volatility, and W denotes the Brownian Motion (BM). Brownian Motion presents continuous paths, Gaussian increments, and independent increments, enabling us to simulate the equity and property sides separately and subsequently sum their values in the fund.

3.1.1 Monte Carlo & Martingale Test

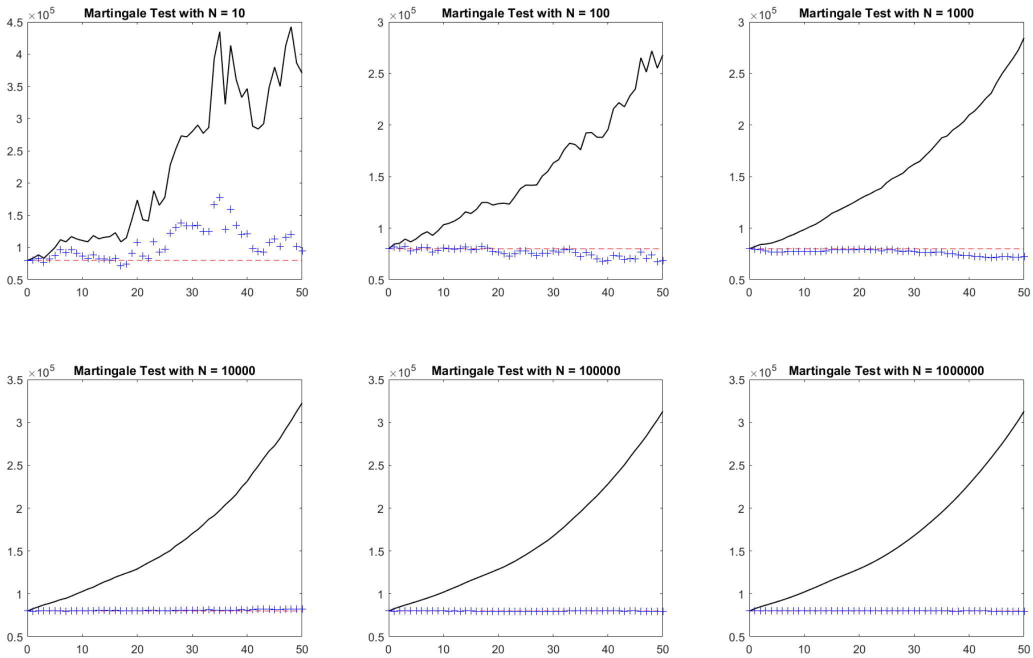
To exploit the potential of the GBM, the Monte Carlo simulation is performed using the solution of the stochastic differential equation (SDE) presented above, which yields:

$$S_{t+\Delta t} = S_t \cdot e^{((r-q-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}g)} \tag{1}$$

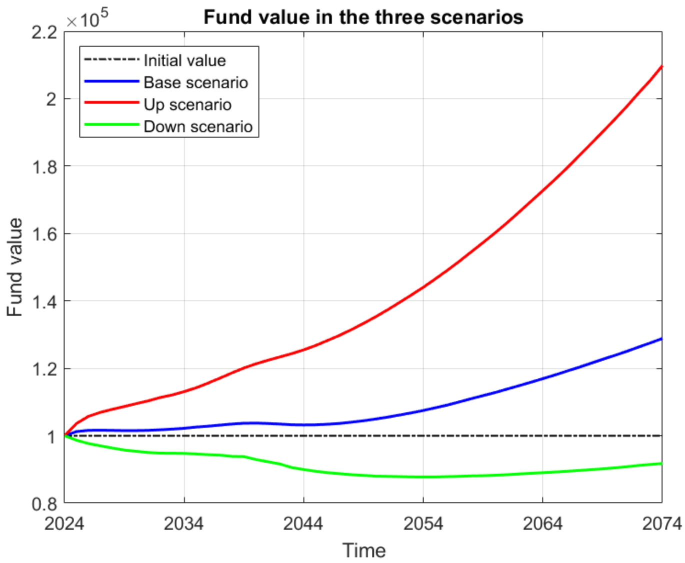
where we discretize the timeline into a grid with a step of $\Delta t = 1$ year, and considering g as a standard normal random variable. Via the same method, both the equity and property assets are simulated, where the standard random variables g_1 and g_2 are independent.
To select an adequate number N of MC simulations, a Martingale test is adopted. This test examines whether the simulated paths exhibit the properties of a martingale process. The main property for a Martingale process is that the expected future value with respect to the filtration in t_0 is equal to the value in t_0 for any t.
The formula for this property can be expressed as:

$$\mathbb{E}[S_t|\mathcal{F}_{t_0}] = S_{t_0}$$

The test exhibits the following results for the equity side considering different values of N:



We can observe that for small N, the martingale test fails, and by setting a threshold of 1% of the value of the underlying, we can accept the martingale behavior from $N = 10^5$. The same test is performed for the property side, yielding similar results. Hence, for accurate computation and moderate computational cost, we decide to set the number of MC simulations at $N = 10^6$.
After setting the hyper-parameter N, we simulate the paths of the fund value for the three rate scenarios:



3.2. Liabilities Formula

Liabilities in insurance represent the reserves insurers set aside to fulfill future obligations to policyholders. Evaluated on a first-order basis, this underscores the insurer's status as a debtor to the policyholder and assigns a quantifiable value to the contractual agreement.
Within this framework, the calculation of liabilities is implemented through several steps:
Initially, the maturity of the contract is set at $T = 50$, and the probability of remaining in the contract at each year i is computed using the death rate and the lapse rate. We set the first probability to be 1 then we cmpute the following probabilities as follow:

$$P_{remain\_contract} = 1 \cdot \prod_{x=2}^{i} (1 - q_{x-1})(1 - l_{x-1}) \quad \forall\, i \geq 2$$

Where q_x is derived from the life table SI2022 and l_x is imposed by the case study as a flat annual rates equal to 15%.

In summary, this formula computes the probability of remaining in the contract at year i by multiplying the probabilities of surviving and not lapsing for each year from the beginning to i .
Successively, the cash flows at each year derived by death and by lapse are computed. The death cash flow is based on the value of the initial capital, the fund, death and lapse rate:

$$\text{death}_{\text{cf}}(i) = (\max(F_0, F_i)) \cdot q_x(i)$$

where F_0 is the initial value of the fund, F_i is the current value and q_x is the respectively probability of dying in that year.
Since the death benefit involves a max function, which is a non-linear operator, we need to consider the full matrix of simulated underlyings. Then, we calculate the mean of the death cash flow and also the mean of the fund value. From this point onwards, all functions will be linear, allowing us to equally work with the mean.

$$\text{Death}_{\text{mean}} = \frac{\sum_{i=1}^{T} \text{death}_{\text{cf}}(i)}{T}$$

$$\text{F}_{\text{mean}} = \frac{\sum_{i=1}^{T} \text{F}_{\text{i}}}{T}$$

To compute the lapse benefits, we rely on the following formulas:

$$\text{lapse}_{\text{cf}}(i) = \begin{cases} (F_{mean} - penalties) \cdot l_t(i) \cdot (1 - q_x(i)) & \text{if } i < T \\ F_{mean} \cdot (1 - q_x(i)) & \text{if } i = T \end{cases}$$

where for $T = 50$ we take into account l_t equal to one since the computations are performed at the end of the year, and the policyholder can only be dead or alive, and in this last scenario ending the contract with probability 1. Therefore, no penalties are paid at the end of the contract.

Another cashflow considered is the one related to the expenses. The vector of future expenses is computed assuming a flat annual inflation rate of 2%.

$$\text{Expenses}_{\text{cf}}(i) = \text{expenses}_0 \times (1 + 0.02)^{i-1} \quad i \geq 1$$

In addition, the commission cash flow is calculated based on the average fund value before the regular deduction is applied, so an adjustment term is added in the formula. Moreover, the commission is a percentage of the Regular Deduction that the company shares with the distribution channel, based on their influence.

$$\text{Commission}_{\text{cf}} = \frac{F_{\text{mean}}}{1 - \text{RD}} \cdot \text{COMM}$$

In conclusion, the total liabilities before discounting, are computed by summing the expected cash flows of lapses, deaths, expenses, commissions and then multiplying them by the probability of remaining in the contract up to time i.

$$V_i = P_{\text{remain\_contract}}(i) \times (\text{lapse}_{\text{cf}}(i) + \text{death}_{\text{cf}}(i) + \text{expenses}(i) + \text{commission})$$

Then it is possible to compute the final liabilities as the sum of the discounted expected cash flows:

$$Liabilities = \sum_{i=1}^{T} V_i \cdot \text{discount}_i$$

Moreover, the Macaulay Duration of liabilities is computed as the sum of the discounted cash flows times t_i, divided by the liabilities:

$$\text{Macaulay Duration} = \frac{\sum_{i=1}^{T} V_i^{discounted} \cdot t_i}{Liabilities}$$

In insurance, Macaulay duration is particularly relevant in the context of liabilities. It represents the average time until the insurer expects to receive the present value of all future cash flows associated with the liabilities, such as premiums, claims, and expenses. This calculation essentially determines the average time it will take for the insurer to recoup the initial capital investment, accounting for the time value of money.

Finally, the different benefits are computed separately for lapses, deaths, expenses, and commissions, firstly obtaining the vectors:

$$Lapse_{\text{benefits}}(i) = P_{\text{remain\_contract}}(i) \cdot \text{lapse}_{\text{cf}}(i)$$

$$Death_{\text{benefits}}(i) = P_{\text{remain\_contract}}(i) \cdot \text{death}_{\text{cf}}(i)$$

$$Expenses_{\text{benefits}}(i) = P_{\text{remain\_contract}}(i) \cdot \text{expenses}(i)$$

$$Commession_{\text{benefits}}(i) = P_{\text{remain\_contract}}(i) \cdot \text{F}(i)$$

Subsequently, we obtain the BEL components by summing up the future discounted benefits:

$$Lapse_{\text{BEL}} = \sum_{i=1}^{T} \text{Lapse}_{\text{benefits}}(i) \cdot \text{discount}(i)$$

$$Death_{\text{BEL}} = \sum_{i=1}^{T} \text{Death}_{\text{benefits}}(i) \cdot \text{discount}(i)$$

$$Expenses_{\text{BEL}} = \sum_{i=1}^{T} \text{Expenses}_{\text{benefits}}(i) \cdot \text{discount}(i)$$

$$Commession_{\text{BEL}} = \sum_{i=1}^{T} \text{Commission}_{\text{benefits}}(i) \cdot \text{discount}(i)$$

3.3. Basic Own Fund & Delta BOF

Own Funds consist of Basic Own Funds and Ancillary Own Funds. Under the Solvency II Directive, the basic own funds are composed of the excess of assets over liabilities and subordinated liabilities. Therefore Basic Own Funds are broadly capital that already exists within the insurer. While the Ancillary Own Funds are Own Fund items other than Basic Own Funds which can be called up to absorb losses.
The calculation of Basic Own Funds is determined by subtracting the total liabilities of the insurer from its available capital resources, as indicated by the formula:

$$BOF = F_0 - Liabilities$$

where F_0 is the initial value of the fund composed by equity and property.
Then we can compute the Delta BOF as follows:

$$\Delta BOF = \max(0, BOF_{\text{Base}} - BOF_{\text{Risk}})$$

where we consider the difference between the basic and a shocked scenario. Since we take the maximum, it is clear that the Delta BOF is greater than zero if the capital of the basic scenario is greater than the one considering a specific risk.

3.4. Solvency Capital Requirement & BSCR

The Solvency Capital Requirement is intended to provide a level of capital that allows an insurer to absorb substantial unexpected losses, thus offering reasonable guarantee to policyholders that payments will be made as scheduled.
The Solvency Capital Requirement (SCR) is determined by the Value at Risk (VaR) of the Basic Own Funds (BOF) of an insurance or reinsurance entity, with a confidence level of 99.50% over a one-year period. It represents the capital necessary for the entity to fulfill its insurance obligations over the next twelve months; or alternatively it is the capital necessary to ensure that extreme events do not occur more than 1 in 200 times.

The SCR can be calculated via Standard Formula or Partial or Full Internal Model, in this report it is computed via the standard formula, which is is defined by EIOPA at European level. The formula outlines capital requirements for different risks using specific formulas and stress parameters, which are then combined using predefined correlation matrices.

In this framework, the SCR can be separated into two modules: one related to market risks and the other corresponding to the life risks. The first module considers three risks: interest, equity and property. Subsequently, the correlation matrix of market SCR is selected based on the value of the ΔBOF derived from the down rate and up rate scenarios.

$$\text{If exposed to IR}_{\text{down}}$$
$$\begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.75 \\ 0.5 & 0.75 & 1 \end{bmatrix}$$

$$\text{If exposed to IR}_{\text{up}}$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.75 \\ 0 & 0.75 & 1 \end{bmatrix}$$

Conversely, the solvency capital requirement of life module considers four risks: mortality, lapse, expenses and catastrophic. Each of these sub-modules is characterized by a specific indicator, and collectively they constitute the correlation matrix for the life SCR.

$$\begin{bmatrix} 1 & 0 & 0.25 & 0.25 \\ 0 & 1 & 0.5 & 0.25 \\ 0.25 & 0.5 & 1 & 0.25 \\ 0.25 & 0.25 & 0.25 & 1 \end{bmatrix}$$

In conclusion, the Basic Solvency Capital Requirement (BSCR) is a cumulative total of all individual risks derived from the different modules. These risks are aggregated using linear correlations to account for diversification and the mitigating effect.

$$BSCR = \sqrt{SCR' \times \text{Correlation\_matrix} \times SCR}$$

Where the correlation matrix for the BSCR is composed by the specific indicator corresponding to the market and the life modules of the solvency capital requirement.

$$\begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix}$$

3.5. Market Interest Risk

The interest rate risk exists for all assets and liabilities that are sensitive to changes in the term structure of interest rates. Taking into account the shifted curve given by the regulator (EIOPA), we obtain the following term structure:

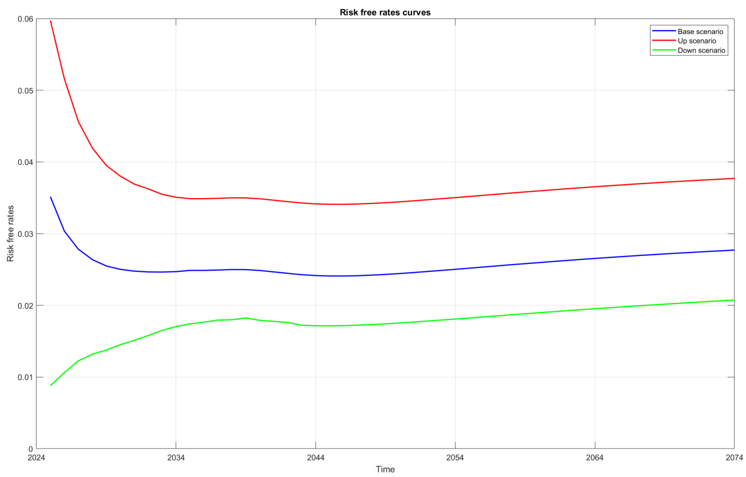


Figure 1: Interest Risk

It is fundamental to find the right capital requirements to consider both an upward and a downward shift in the curve. Following the same computation as above, we can find the SCR for the market interest risk as:

$$SCR_{IR} = \max(\Delta BOF_{IR_{DOWN}}, \Delta BOF_{IR_{UP}})$$

The computations yield the following results:

Scenario	BEL	BOF	ΔBOF	Duration
UP	96508.83	3491.17	0	5.65
DOWN	97108.95	2891.05	261.82	5.70

Table 11: Market Interest Risk

As we can see in the downward shift scenario, the BOF is smaller than in the basic scenario ($BOF_{Basic} = 3152.87$), resulting in a positive SCR. This can be explained by examining the dynamics of the underlying assets. Indeed, since F_0 is the guaranteed capital, there is higher exposure in the event of the policyholder's death when the fund value is lower than F_0. This can also be observed by analyzing the death BEL component, which is greater in the downward scenario compared to the base scenario. Furthermore, as interest rates move down, discount factors move up, influencing the computation of liabilities that involve these discount factors.
Conversely, in the case of an upward shift, our BOF increases, indicating that the insurance company will be better covered than in the basic scenario, and hence, additional capital is not required.

3.6. Market Equity Risk

Equity risk emerges from the magnitude or volatility of market prices for equities. Exposure to equity risk includes all assets and liabilities whose valuation is responsive to fluctuations in equity prices.

The equity risk sub-module must include a risk sub-module for type 1 equities and a risk sub-module for type 2 equities. However, the case study proposed considers only type 1 equity since they are traded in the regulated EEA market. Under Solvency II regulations, the equity risks are simulated by applying a shock to the stock's value in the first year of the contract, that is 39% for type 1 equity. Furthermore, a symmetric adjustment of 5,25% is also taken into account in the first year. The value of the symmetric adjustment is provided by EIOPA (Data from March 2024). In particular, as required by Solvency II regulations, the symmetric adjustment must not be lower than 10% or higher than 10%. We then compute the SCR as follow:

$$S_0^{Sterssed} = S_0 \cdot (1 - 0.39 - 0.0525)$$

$$\begin{aligned} \Delta\mathrm{BOF}_{\mathrm{EQ}} &= \max(0, \mathrm{BOF}_{\mathrm{base}} - \mathrm{BOF}_{\mathrm{EQ}}); \\ \mathrm{SCR}_{\mathrm{EQ}} &= \Delta\mathrm{BOF}_{\mathrm{EQ}}; \end{aligned}$$

Scenario	BEL	BOF	ΔBOF	Duration
Equity	64327.54	272.46	2880.41	5.75

Table 12: Market Equity Risk

Our results highlight the significant exposure of this portfolio to market equity risk. In fact, the ΔBOF showed a significant positive value ($\Delta BOF = 2880.41$), despite just a slight increase in the Macaulay Duration (5.75). As previously explained, this stressed scenario entails an instantaneous shock to the equity value, resulting in a decrease in the fund value (64600.00) and, consequently, a corresponding reduction in liabilities (64327.54). Recalling the computation for the liabilities side when computing the death benefits, we take the maximum between the initial value F_0 and the current value of the fund. In the equity stressed scenario, most of the time the maximum is triggered, and F_0 is taken into account. This causes the liabilities to be relatively higher; for instance, the death BEL component is around 3% greater with respect to the liabilities than its counterpart in the base scenario.
The expenses BEL component is the only one unaffected, as it does not depend on the Market Value Added (MVA). All these factors converge to a significant exposure to equity risk, requiring a consistent level of coverage, hence a significant SCR.

While it might seem contradictory to have lower liabilities despite the requirement for additional safety capital, this scenario is conceivable due to the influence of the new shocked value of the equity on the initial fund value, which has decreased compared to its previous level.

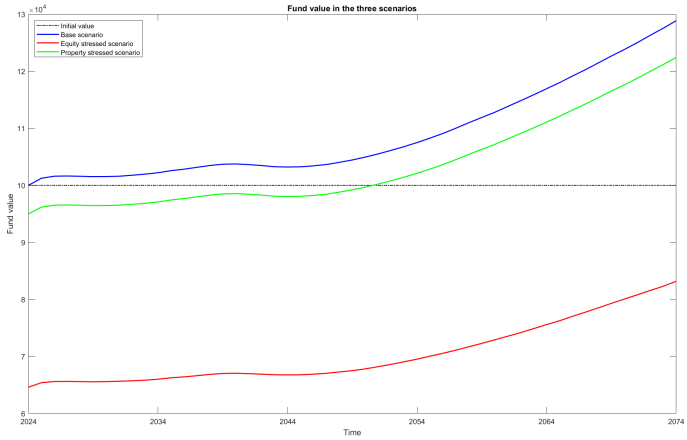


Figure 2: Equity and Property Shock

3.7. Market Property Risk

Property risk emerges due to the sensitivity of assets, liabilities, and financial investments to fluctuations in the level or volatility of property market prices. Under Solvency II regulations, the calculation of the capital charge is associated with an immediate reduction in the market value of assets by 25%. Therefore, the property risk is simulated with a shock of 25% on the first year of the contract. The new value of the property is used to recalculate the Basic Own Fund and to compute the difference from the Base Case. In conclusion, the computation of the solvency capital requirement for property risk is:

$$P_0^{Stressed} = P_0 \cdot (1 - 0.25)$$

$$\begin{aligned} \Delta\text{BOF}_{\text{PR}} &= \max(0, \text{BOF}_{\text{base}} - \text{BOF}_{\text{PR}}); \\ \text{SCR}_{\text{PR}} &= \Delta\text{BOF}_{\text{PR}}; \end{aligned}$$

Scenario	BEL	BOF	ΔBOF	Duration
Property	92253.95	2746.05	406.82	5.69

Table 13: Market Property Risk

Similarly to the equity sub-module, in this scenario, we observe an instantaneous decrease in the fund value (95000.00), with a corresponding decrease in liabilities (92253.95). However, the shock applied in this case is smaller and especially the property asset side comprises only 20% of the entire assets, thus the impact on the ΔBOF is also smaller ($\Delta BOF = 406.82$). Even in property sub-module the duration is almost unvaried (5.69). In conclusion, there is an exposure to the decrease in property value, which is lower than in the previous case but still significant.

3.8. Life Mortality Risk

The mortality risk concerns the potential loss, or unfavorable alteration in the value of insurance liabilities, arising from shifts in the level, trend, or volatility of mortality rates. An elevation in mortality rates corresponds to an augmentation in the value of insurance liabilities. In the calculation of mortality risk an immediate and permanent increase of 15% in the mortality rates is adopted. An important observation is that the new value of the mortality rate must not exceed 100%. This stress factor captures the risk that more policyholders than anticipated die during the policy term. Again we can proceed to analyze the results:

$$\Delta \mathrm{BOF}_{\mathrm{MT}} = \max(0, \mathrm{BOF}_{\mathrm{base}} - \mathrm{BOF}_{\mathrm{MT}});$$
$$\mathrm{SCR}_{\mathrm{MT}} = \Delta \mathrm{BOF}_{\mathrm{MT}};$$

Scenario	BEL	BOF	ΔBOF	Duration
Mortality	96996.63	3003.37	149.50	5.64

Table 14: Life Mortality Risk

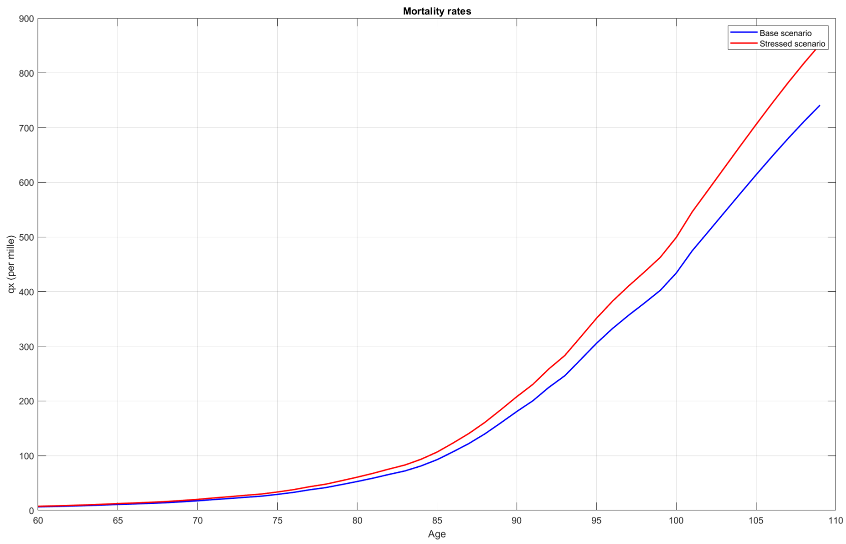


Figure 3: Mortality Shock

In this stressed scenario, the primary adjustment concerns the liabilities, attributed to the heightened mortality rate. Notably, there is an escalation in death BEL components (8392.58) stemming from the aforementioned rise in the probability of mortality. As a consequence, there is a decrease in lapse BEL components (80521.84), indeed in general fewer individuals can terminate the contract, having a higher probability of dying. Conversely, the valuation of the fund, including properties and equity, remains unaltered, as this stress on mortality rates does not affect these computations. However, the final computation of ΔBOF is relatively small, equal to 149.50. Moreover, the duration remains almost constant (5.64). This analysis enables us to affirm that the insurance is slightly exposed to an increase in mortality rate with respect to other risks.

3.9. Life Lapse Risk

Lapse risk refers to the potential for loss or adverse alteration in liabilities resulting from a shift in the anticipated exercise rates of policyholder options. These options include the contractual rights of policyholders to terminate fully or partially, surrender, decrease, restrict, or suspend insurance coverage, or allow the insurance policy to lapse. In our case study, the policyholder can only decide whether or not to terminate the contract at the end of each year.
Under Solvency II regulations, the effects of the Lapse shock are set in three possible scenarios:

- Lapse Up: an instantaneous and permanent increase of 50% is implemented in the assumed lapse rate in all future years, ensuring it does not exceed 100%.
- Lapse Down: an instantaneous and permanent decrease of 50% is applied in the lapse rate in all future years, where the absolute variation does not exceed 20%.
- Lapse Mass: a potential discontinuance of 40% of the insurance policies is adopted only for the first year.

In each stressed scenario, the Basic Own Funds undergo recalculation and it is compared with the baseline case. Subsequently, the Solvency Capital Requirement is determined based on the most significant scenario.

$$\mathrm{SCR_{LP}} = \max(\Delta\mathrm{BOF_{LTU}}, \Delta\mathrm{BOF_{LTD}}, \Delta\mathrm{BOF_{LTM}}) \tag{2}$$

Scenario	BEL	BOF	ΔBOF	Duration
UP	97475.56	2524.44	628.42	4.03
DOWN	96544.50	3455.50	0	9.32
MASS	97984.28	2015.72	1137.14	3.45

Table 15: Life Lapse Risk

Even in this stressed scenario, the primary adjustment concerns the liabilities, as the calculation of the fund remains unaffected by changes in the lapse rate. The liabilities are consequently altered due to fluctuations in lapse benefits, which occur following an increase or decrease in the lapse rate. Similar to the case of mortality risk, this adjustment of lapse benefits is closely associated with alterations in death benefits.

In the "up" scenario, there is an evident increase in lapse BEL component (87513.14) due to the higher lapse rate. Consequently, there is a reduction in death benefits (4135.73), since having a higher probability of lapsing before dying. This leads to a general increase in liabilities, in fact the final BEL is 97475.56. Naturally, there is also a modification in duration: the average policy duration significantly decreases as the scenario of contract lapses is emphasized 4.03. Hence, the ΔBOF in this scenario is positive, yielding a non negligible value of 628.42.

Conversely, in the "down" scenario, there is a decrease in lapse BEL component (64535.92) since the lapse rate decreases. This result in an higher death BEL component (18966.92), since there is a lower probability of lapsing before dying. In conclusion, the values of liabilities decreases to 96544.50 and the direct consequence is a negative variation of the Basic Own Fund (thus, a $\Delta BOF = 0$), and a significant increase in duration (9.32), as the average policy duration is compelled to increase under this stressed scenario. It is remarkable to stress that the SCR is zero as expected, since the complementary up case presents a positive SCR.

Lastly, this scenario emphasizes the possibility of an early exit within the first year. This case leads to an even shorter duration 3.45 compared to the "up" scenario since there is a significant increase in the exits within the first year of the contract. The dynamics of the results parallel those of the first case ($BEL_{\text{lapse}} = 88730.05 \quad \& \quad BEL_{\text{death}} = 4262.00$), but the consequences are more pronounced, resulting in a significant variation of the Basic Own Fund. In fact the ΔBOF is equal to 1137.14, underlining a significant exposure to this scenario.

The analysis of these extreme cases demonstrates a higher exposure to the occurrence of a massive lapse of the contract within the first year, despite the presence of a susceptibility even in the "up" scenario. Conversely, there is no vulnerability to a reduction in the lapse rate scenario. Therefore, for the final calculation, the MASS lapse scenario will be employed.

3.10. Expense Risk

The Expense risk emerges from fluctuations in the costs incurred in servicing insurance and reinsurance agreements. It is stressed by raising future expenses by 10% compared to the best estimate projections and increasing the inflation rate by 1% per year compared to anticipations. The final framework is:

$$Expenses_{UP}(i) = expenses_0 \cdot 1.1 \cdot (1 + 0.03)^{i-1} \quad i \geq 1$$

$$\begin{aligned} \Delta\mathrm{BOF_{EX}} &= \max(0, \mathrm{BOF_{base}} - \mathrm{BOF_{EX}}); \\ \mathrm{SCR_{EX}} &= \Delta\mathrm{BOF_{EX}}; \end{aligned}$$

Scenario	BEL	BOF	ΔBOF	Duration
Mortality	96892.72	3107.28	45.59	5.68

Table 16: Life Expanse Risk

As expected, upon analyzing the BEL components, the only value that deviates from the base scenario is the Expenses, while all other components remain exactly the same. Hence, the Delta BOF and thus the SCR are only affected by the difference in expenses, which, since we are considering an increase in expenses, will result in a higher capital requirement. However, the impact on the SCR life is relatively small, so we can conclude that the insurance company has a small exposure to expenses risk.

3.11. Catastrophe Risk

The Catastrophe (CAT) risk originates from exceptional or irregular events whose impacts are inadequately accounted for in other life underwriting risk sub-modules (for example a pandemic event).
Under Solvency II regulations, the revision shock involves a precise increase of 0.15% in the mortality rate of policyholders over the subsequent year. The calculation yields the following results:

$$\Delta \text{BOF}_{\text{CAT}} = \max(0, \text{BOF}_{\text{base}} - \text{BOF}_{\text{CAT}});$$
$$\text{SCR}_{\text{CAT}} = \Delta \text{BOF}_{\text{CAT}};$$

Scenario	BEL	BOF	ΔBOF	Duration
Catastrophe	96859.29	3140.71	12.15	5.67

Table 17: Life CAT Risk

In this stressed scenario, there is a modification of the liabilities as it simulates a potential increase in the probability of death during the first year of the contract, particularly leading to an increase in death BEL component. Similar to the life mortality risk, the calculation of the fund remains unchanged. Furthermore, it is observed that the Basic Own Fund does not vary significantly, nor does the duration of the contract. Consequently, it can be inferred that there is a small exposure to a potentially catastrophic scenario.

3.12. SCR and BSCR

In conclusion, we can analyze the whole exposure of the portfolio of the fund composed of 80% equity and 20% property. We proceed to compute the aggregate Solvency Capital Requirement (SCR) as described in Section 3.4. The results are as follows:

Category	SCR
Market	3339
Life	1175.2

Table 18: Aggregate SCR

Thus, we can see that the insurance company is more exposed to Market risks than to Life risks, since the SCR_{MKT} is more than double the SCR_{Life}. This can be explained by looking at the composition of the portfolio which is heavily impacted by market and volatility risks.
Finally, we can find the BSCR for the total portfolio:

BSCR	3806.85

Table 19: BSCR

4. Deterministic calculations, comments on results

In the second part of the case study, we proceed to determine the Solvency Capital Requirement under deterministic assumptions. Thus, the value of the fund is simply computed by considering the interest rate curve from EIOPA:

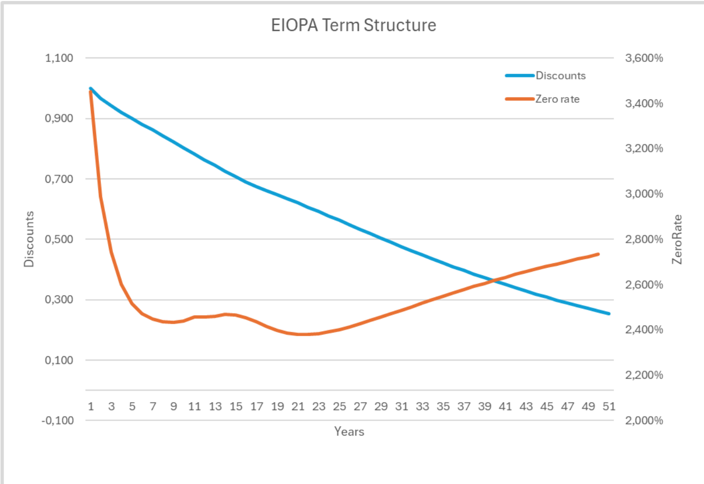


Figure 4: EIOPA TermStructure

Then all calculation are performed keeping the same formulas as above and we can analyze the results.

4.1. Base Scenario and Market Risk

To establish a benchmark and understand the main differences between the stochastic and deterministic scenarios, we first consider the computation for the base case.

Scenario	Lapse	Death	Expenses	Commissions
Deterministic	81227.10	6428.63	295.03	7845.28
Stochastic	81249.03	7455.69	295.03	7847.38

Table 20: PV components

Examining the BEL components for the deterministic and stochastic scenarios, we observe that Expenses are identical as they are independent of the MVA. Moreover, the Commission and Lapse components are comparable; indeed, when taking the mean in the stochastic case for a sufficient number of simulations, the evolution of the fund in the two scenarios must converge. This is also confirmed by the martingale test computed above. Thus, since the probability of lapsing and the mortality life table are the same, the Lapse is not affected by the choice between the two methods (for a sufficient N). Similarly, the commissions, which are a percentage of the fund value at each time step, are not influenced either.
Conversely, the Death component exhibits a significant difference, with the stochastic one being significantly larger than the deterministic one. To explain this discrepancy, we have to recall the formula for the Death benefit:

$$\text{death}(i) = (\max(F_0, F_i)) \cdot q_x(i)$$

We observe that the function is nonlinear. Indeed, the max operator is nonlinear, and so we will get different results when taking the mean of the fund value before the computation (as in the deterministic case) or better after comparing all the possible paths of the fund in the stochastic scenario. In the deterministic scenario, having positive rates in the term structure, the value of the fund will never be less than F_0, unlike the stochastic scenario where some paths will trigger the maximum to take F_0. This brings the death component to be higher in the stochastic case.
Although the base scenario only sets a benchmark to be compared with the following risks, we can see that in the end, the BEL for the deterministic case is clearly lower:

Scenario	BEL
Deterministic	95796.04
Stochastic	96847.13

Table 21: BEL Base

Moreover, the duration in the base case is smaller for the deterministic approach, consistent with the lower level of liabilities.
Now we can analyze the market risk scenarios where the deterministic evolution of the fund are:

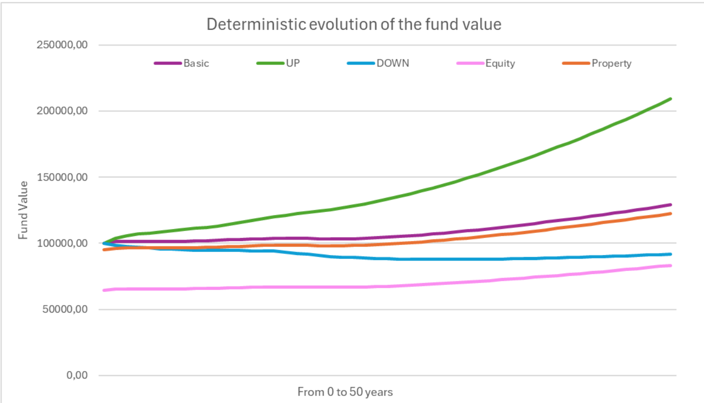


Figure 5: Deterministic Fund Evolution

Similarly to the stochastic case, the downward shift of the interest rates results in a positive SCR, which in the deterministic case is 362.36 (38% bigger than the stochastic one). Regarding equity and property risks, we obtain similar results, with a notable difference in the SCR of the equity risk and a negligible discrepancy in the property risk, as it only decreases the value of the fund by around 5%.
Considering all the market risks together, and again using the adequate correlation matrix, we find the SCR for Market Risk:

Scenario	SCR Market
Deterministic	4230.61
Stochastic	3339

Table 22: SCR Market Risk

4.1.1 PVFP & LEAK & PROXY

In the base case scenario and deterministic method, we compute the Present Value of Future Profit (PVFP), which is an indicator of the sources of profit for the insurance company.

$$\text{Profit}(i) = \frac{(RD - \text{COMM}) \cdot P_{\text{remain}}(i-1)}{1 - RD} \cdot F_i - \text{Expenses}(i) \cdot P_{\text{remain}}(i-1)$$

where we compute the future profit for each year. Then, we discount them using the rates from the EIOPA curve, and finally, all the future discounted profits are summed in order to find the PVFP.

PVFP	4187.99
PROXY	4196.77
LEAK	15.97

Table 23: Profit check

To validate the results, we calculated the LEAK, i.e., the discrepancy between the asset side and the liabilities plus the PVFP.

$$\text{LEAK} = \text{Assets} - (\text{PVFP} + \text{Liabilities})$$

The computations yield a LEAK equal to 15.97, thus we verify that the magnitude of leakage is relatively small. Indeed, by summing the liabilities and the PVFP, we get 9984.03 (almost equal to the MVA = 100000). To further validate our computation, we also compute the PROXY, which is an approximate value of the PVFP. It is calculated as:

$$\text{PROXY} = (RD - \text{COMM}) \cdot \text{Duration} \cdot F_0 - \text{Expenses}(0, \text{Duration})$$

It is notable that the PROXY and the PVFP are comparable, with a difference smaller than 10€. Thus, we can state that our computations are coherent.

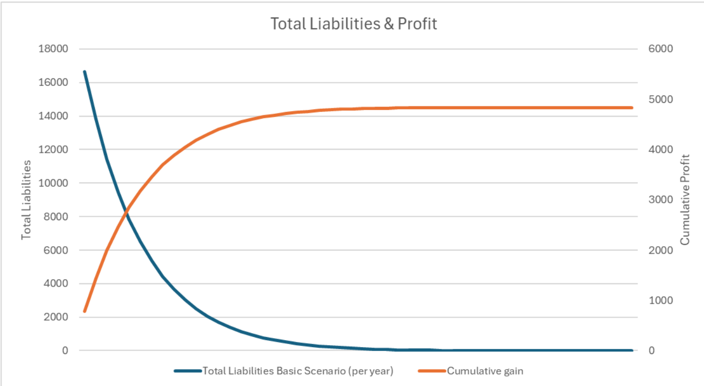


Figure 6: Cumulative Gain & Liabilities

4.2. Life Risk

Following the same intuition as above, we can compute and compare the SCR related to the Life risk. In particular, the results are as follows:

Scenario	Mortality	Lapse	CAT	Expenses
Deterministic	30.62	1615.56	5.21	45.59
Stochastic	149.57	1137.1	12.16	45.59

Table 24: Life SCR Components

As we can see, the value of the expenses remains identical, being independent of external factors. However, for the Mortality and CAT risk, we observe a decrease in the values, in accordance with the base case above. Conversely, the lapse shows a significant increase in the SCR, again due to the discrepancy in death between the two methodologies.
It can be useful to see how the BEL components are distributed to have a better understanding of the output. For instance, comparing the Lapse up and Lapse down scenarios, we can spot some crucial differences:

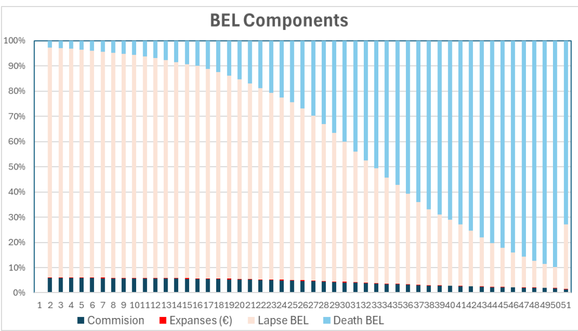


Figure 7: Lapse Up BEL Components

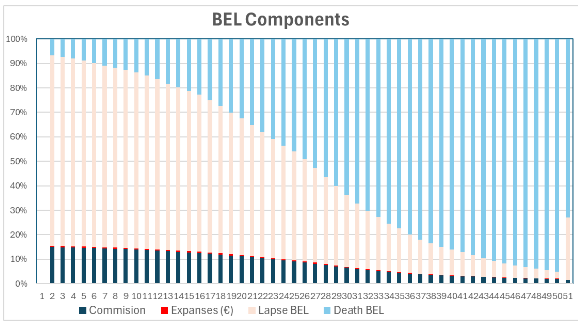


Figure 8: Lapse Down BEL Components

Plotting the distribution of the BEL components at each year, we can see that in the first years, the Lapse risk is the most influential, while close to the end of the contract, the death risk prevails. Hence, the respective BELs are distributed. Expenses and Commissions have a generally lower impact. Analyzing the differences in the two scenarios, we can see how in the lapse up scenario, the risk related to the lapse becomes more relevant in each year of the contract, while in the lapse down scenario, the opposite happens, and the death risk becomes more persistent.

4.3. Basic Solvency Capital Requirement

In conclusion, we can compare the Basic Solvency Capital Requirements for the two scenarios, and as expected, the deterministic BSCR is higher than the stochastic one.

Scenario	BSCR
Deterministic	4905.15
Stochastic	3806.85

Table 25: Basic Solvency Capital Requirements (BSCR)

As discussed earlier, this mismatch in the two scenarios can be explained by the asymmetric payoff of the death benefit, which penalizes the deterministic case. Thus, the insurance company needs a higher SCR in the deterministic scenario, highlighting a greater exposure to this methodology.

5. Open questions

5.1. Question 1

What happens to the asset and liabilities when the risk-free rate increases/decreases with a parallel shift of, say, 100bps? Describe the effects for all the BEL components.

Answer:

The following question requires the analysis of two antithetical scenarios: an increase in the risk-free rate with a parallel shift of 100bps, or a decrease in the risk-free rate, again with a parallel shift of 100bps, while the other assumptions are unvaried. Both scenarios can be analyzed similarly to the calculation of the Solvency Capital Requirement for Market Rate Risk.

In the first scenario, the positive parallel shift of all the terms structure increases the drift of the Fund and decreases the liabilities, consequently leading to an increase in the Basic Own Fund. A closer examination of the specific components of the Best Estimate Liabilities (BEL) will clarify the reduction in liabilities. Indeed, the death component will show a significant decrease since the related discounts decreases if the risk-free rate increases. Conversely, the lapse component and the commission component will remain practically unchanged. The lapse rate for BEL remains nearly constant because the associated cash flows are paid out early, thus remaining unaffected by changes in the rate. Meanwhile, the commission component remains unvaried as the reduction in discount equals the increase in the fund's drift. Finally, the expense component will be reduced as the discount factor decreases. This analysis suggests the hypothesis of a basic solvency capital requirement lower than the one calculated previously in the report.

On the other hand, considering a negative parallel shift of 100bps, the expected dynamics diverge from the aforementioned scenario. Indeed, this change results in an increase in liabilities. In particular, the increase in the death component is crucial as the related discounts increases if the risk-free rate decreases. Additionally, the expense component will also increase. In conclusion, the components related to lapse and commission remain practically unchanged. In this case the basic solvency capital requirement will be higher than the one calculated previously in the report.

The following table confirms the hypothesized results :

Scenario	Lapse BEL	Death BEL	Exepenses BEL	Commissions BEL	BSCR
NO Shift	81249.03	7455.691	295.03	7847.38	3806.85
UP Shift	81249.83	7165.88	279.10	7847.38	3693.17
DOWN Shift	81248.16	7844.42	312.77	7847.38	3944.61

Table 26: Results of "up" parallel shift of 100bps

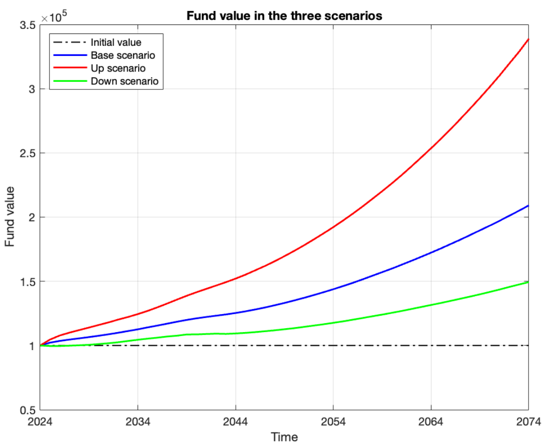


Figure 9: Fund value in up shift

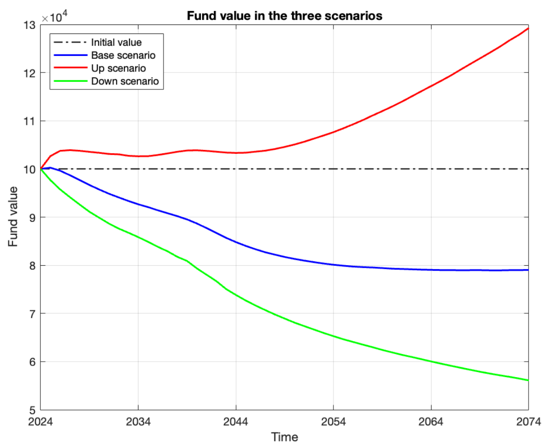


Figure 10: Fund value in down shift

5.2. Question 2

"What happens to the liabilities if the insured age increases? What if there were two model points, one male and one female?;

Answer:

In the first proposed scenario, it is requested to examine a model characterized by one model point whose insured age is strictly greater than 60 years, while the other contract terms remain unvaried. Within this hypothesis, the mortality rate considered will be higher than the one delineated in the report, as the probability of survival decreases with age. Similar to the calculation of the Solvency Capital Requirement for the Life Mortality Risk sub-module, this rise in mortality rate will lead to increased liabilities. Specifically, the death component of the BEL will increase due to the higher probability of death during the contract period. Conversely, the lapse component of the BEL will decrease, thus in general fewer individuals are likely to rescind the contract given the increased probability of death. In analogy to the life mortality risk, the other components of the BEL, namely expenses and commissions, will show only a slight decrease. Additionally, it is important to analyze the change in the Macaulay Duration, which will decrease as in $\text{SCR}_{\text{Mortality}}$ calculation.

On the other hand, the second proposed scenario considers two model points, one male and one female, while the other contract terms remain unvaried. By analyzing the life tables provided by Istat SI2022, it can be seen that the death rate for females is lower than the one referring to males. Consequently, there is a general decrease in the death rate, which results in a dynamic opposite to the one outlined in the previous paragraph. Indeed, there will be a decrease in liabilities due to this modification in q_x. Specifically, the death component of the BEL decreases in line with the dynamics of mortality rates, while conversely, the lapse component of the BEL increases due to a higher probability of lapsing before dying. In this scenario, the other components of the BEL will show slightly higher values. In contrast to the first scenario presented, the Macaulay duration will now be greater.

In summary, the insurance portfolio comprises two policyholders, each with slightly different mortality tables. Consequently, the Solvency Capital Requirement (SCR) will be a weighted average of the contracts within the portfolio. This diversification effect mitigates the overall capital requirement for each component of the SCR. Therefore, the introduction of the female contractor into the portfolio contributes to a more balanced and risk-resilient insurance profile, forecasting the importance of a diversified portfolio also in the insurance sector.

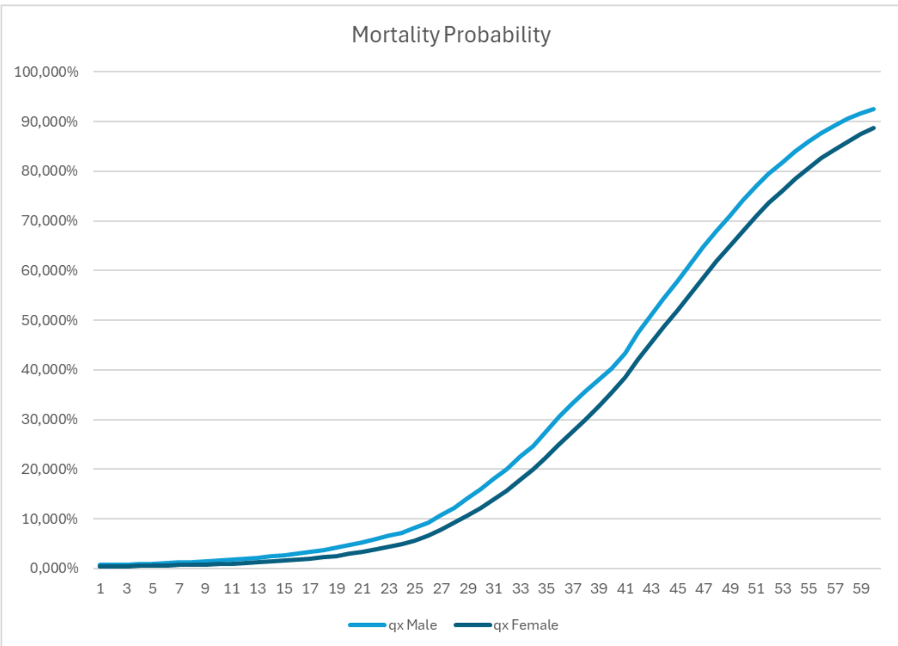


Figure 11: Mortality Table

6. Appendix - Matlab code

6.1. Main code

Listing 1: Main code

```
%%%%%%%%%%%%%%%%%%%%%%      SOLVENCY II Project      %%%%%%%%%%%%%%%%%%%%%%
% Main code for the Insurance Project 2023/2024 - SII Project
% Group 6
%    -> Matteo Ceve
%    -> Taha Rafai
%    -> Marco Ronchetti
%    -> Nicol  Toia

% clear the work sapce
clc;
clear;
close all;

% fix the seed
rng(42); % The answer to life, the universe, and everything

%% Add path to the data folder

addpath('Data');

% start the timer
tic;
%% Data

today = '03-31-2024';    % set the initial date 31-Mar-2024

% Contract maturity
T = 50;

% initial invested capital and fund capital
C0 = 1e5;
F0 = C0;

% Assets

    equity_perc = 0.8;                      % equity percentage
    property_perc = 1 - equity_perc;        % property percentage

    equity = equity_perc * F0;              % initial equity value
    property = property_perc * F0;          % initial property value

    % null dividend yield both equity and property
    dividend = 0;

    % volatilities equity & property
    sigma_equity = 0.20;
    sigma_property = 0.10;

% Laiabilities

    % benefits
        % penalties in case of lapse (    )
        penalties = 20;
```

```
        % death payoff: max(C0, Fund_value)

    % others
        % regular deduction
        RD = 0.022;
        % commisions to the distribution channels
        comm = 0.014;

%Time vector
time=(1:T)';

% age of the insured
age = 60;

% lapse: flat annual rates
l_t = 0.15*ones(1,T)';

% expanses (     )
expenses_0 = 50;

% inflation: flat annual rate of 2%
inflation = 0.02;

% display the data
disp('Data uploaded:');
disp(' ');
disp('Today: 31-Mar-2024');
disp('Contract maturity: 50 years');
disp('Initial invested capital:    100   ,000');
disp('Equity percentage: 80%');
disp('Property percentage: 20%');
disp('Dividend yield: 0%');
disp('Equity volatility: 20%');
disp('Property volatility: 10%');
disp('Penalties in case of lapse:    20   ');
disp('Regular deduction: 2.2%');
disp('Commissions to the distribution channels: 1.4%');
disp('Age of the insured: 60 years');
disp('Lapse rate: 15%');
disp('Expenses (per year):    50   (+ inflation)');
disp('Inflation rate: 2%');
disp(' ');

%% Upload risk free rates from EIOPA_Term_Structures.xlsx and
% mortality probability from LifeTableMale2022.xlsx

% EIOPA risk free rates EU 31-Mar-20204 & ISTAT Life Tables 2022 (male)


rates_base = xlsread ('EIOPA_Term_Structures.xlsx',3  , 'C11:C60');

% up scenario (without VA)
rates_up = xlsread ('EIOPA_Term_Structures.xlsx',5  , 'C11:C60');

% down scenario (without VA)
rates_down = xlsread ('EIOPA_Term_Structures.xlsx',6  , 'C11:C60');

% Upload from excel the mortality rates
```

```
qx = xlsread ("LifeTableMale2022.xlsx",'D63:D112')/1000; % From 60 y-o to 109 y-o

% display the updated data (both rates and mortality rates) in a table
table = table(rates_base, rates_up, rates_down, qx, 'VariableNames', {'Base', 'Up', 'Down'
disp('Risk free rates and mortality rates uploaded:');
disp(' ');
disp(table);
disp(' ');

%% Dates

% check if today is a business date, where today is the settlement date
if ~isbusday(today)
    today = busdate(today);
end

% from the settlement date, find the dates of the next 50 years
dates = datetime(today, 'ConvertFrom', 'datenum') + calyears(1:T)';
% check and move dates to business dates
dates(~isbusday(dates,0)) = busdate(dates(~isbusday(dates,0)), "modifiedfollow", 0);
% convvert to datenum
dates = [datenum(today); datenum(dates)];

% Set the year convention
ACT_365 = 3;

% compute the deltas between t0 and t_i
delta_t0 = yearfrac(dates(1), dates, ACT_365);

% find the time difference for each time step
deltas = delta_t0(2:end) - delta_t0(1:end-1);

% Uncomment the following for deterministic computation
% or as an alternative to have integer values and same step

dt = (0:T)';
delta_t0 = dt;
deltas = delta_t0(2:end) - delta_t0(1:end-1);

% Plot rates curves and mortality rates in sub plots
figure;
%subplot(1,2,1);
plot(dates(2:end), rates_base, 'b', 'LineWidth', 1.5);
hold on;
plot(dates(2:end), rates_up, 'r', 'LineWidth', 1.5);
plot(dates(2:end), rates_down, 'g', 'LineWidth', 1.5);
title('Risk free rates curves');
% set boudaries in x-axis
xlim([dates(1) dates(end)]);
% set the x-axis in datestr onli each 10 years
set(gca, 'XTick', dates(1:10:end));
set(gca, 'XTickLabel', datestr(dates(1:10:end), 'yyyy'));
xlabel('Time');
ylabel('Risk free rates');
legend('Base scenario', 'Up scenario', 'Down scenario');
grid on;
hold off;
```

```
% subplot(1,2,2);
% plot(60:109, qx*1000, 'b', 'LineWidth', 1.5);
% title('Mortality rates');
% xlabel('Age');
% ylabel('qx (per mille)');
% grid on;

%% Forward rates & discounts

% rates shifted by 100bp
% rates_base = rates_base + 0.01;
% rates_up = rates_up + 0.01;
% rates_down = rates_down + 0.01;

% compute the zero rates
rates_base = log(1+rates_base);
rates_up = log(1+rates_up);
rates_down = log(1+rates_down);

% compute the discounts base scenario
discounts = exp(-[1; rates_base].*delta_t0);
fwd_discount = discounts./[1; discounts(1:end-1)];
% compute the forward rates base scenario
fwd_rates_base = -log(fwd_discount);

% compute the discounts up scenario
discounts_up = exp(-[1; rates_up].*delta_t0);
fwd_discount_up = discounts_up./[1; discounts_up(1:end-1)];
% compute the forward rates up scenario
fwd_rates_up = -log(fwd_discount_up);

% compute the discounts down scenario
discounts_down = exp(-[1; rates_down].*delta_t0);
fwd_discount_down = discounts_down./[1; discounts_down(1:end-1)];
% compute the forward rates down scenario
fwd_rates_down = -log(fwd_discount_down);

%% Martingale Test
% perform the martingale test to select an adequate number of simulations for MC

% vector of N simulations to test (number of elements <= 6)
N_sim = [10, 1e2, 1e3, 1e4 ,1e5, 1e6];

% loop over the number of simulations to test Equity
disp(['The number of simulations that pass the Martingale test for equity ' ...
    'are the following:']);
disp(' ');

for i = 1:length(N_sim)
    % perform the martingale test for equity
    martingaleTest = mtgTest(equity ,delta_t0, deltas ,N_sim(i), rates_base, ...
        fwd_rates_base, dividend ,sigma_equity, 0, i, 2);

    if max(martingaleTest) < 0.01*equity
        % display the results
        disp(['N = ', num2str(N_sim(i))]);
    end
end
```

```
% loop over the number of simulations to test Property
disp(' ');
disp(['The number of simulations that pass the Martingale test for property ' ...
    'are the following:']);
disp(' ');

for i = 1:length(N_sim)
    % perform the martingale test for property
    martingaleTest = mtgTest(property ,delta_t0 , deltas ,N_sim(i), rates_base, ...
        fwd_rates_base, dividend ,sigma_property, 0, i, 3);

    if max(martingaleTest) < 0.01*property
        % display the results
        disp(['N = ', num2str(N_sim(i))]);
    end
end

%% Simulate asset and property in the three rates scenario
% simulate the fund value in the three scenarios via Geometric Brownian
% Motion and Monte Carlo simulation

N = 1e6 ; % number of MC simulations

% simulate GBM equity and property base scenario
equity_MC = MC_simulation(equity ,deltas ,N ,fwd_rates_base, dividend ,...
    sigma_equity, RD);
property_MC = MC_simulation(property ,deltas ,N ,fwd_rates_base, dividend ,...
    sigma_property, RD);

% simulate GBM equity and property up scenario
equity_MC_up = MC_simulation(equity ,deltas ,N ,fwd_rates_up, dividend ,...
    sigma_equity, RD);
property_MC_up = MC_simulation(property ,deltas ,N ,fwd_rates_up, dividend ,...
    sigma_property, RD);

% simulate GBM equity and property down scenario
equity_MC_down = MC_simulation(equity ,deltas ,N ,fwd_rates_down, dividend ,...
    sigma_equity, RD);
property_MC_down = MC_simulation(property ,deltas ,N ,fwd_rates_down, dividend ,...
    sigma_property, RD);

% Value of the fund for the three scenarios: base, up and down
Fund_base = equity_MC + property_MC;
Fund_up = equity_MC_up + property_MC_up;
Fund_down = equity_MC_down + property_MC_down;

% create a structure to store the results
Value_Fund = struct("Fund_t0", F0, "Fund_base", Fund_base, "Fund_up", Fund_up, ...
    "Fund_down", Fund_down);

%% Plot the fund value in the three scenarios and the initial value
% set on the x-axis the dates (in datestr) and on the y-axis the fund value

figure;
plot(dates , F0*ones(1,51), '--k', 'LineWidth', 1);
hold on;
plot(dates , mean(Fund_base,1), 'b', 'LineWidth', 1.5);
```

```
plot(dates , mean(Fund_up,1), 'r', 'LineWidth', 1.5);
plot(dates , mean(Fund_down,1), 'g', 'LineWidth', 1.5);
title('Fund value in the three scenarios');
% set boudaries in x-axis
xlim([dates(1) dates(end)]);
% set the x-axis in datestr onli each 10 years
set(gca, 'XTick', dates(1:10:end));
set(gca, 'XTickLabel', datestr(dates(1:10:end), 'yyyy'));
xlabel('Time');
ylabel('Fund value');
% legend on the top left side
legend('Initial value', 'Base scenario', 'Up scenario', 'Down scenario', 'Location', ...
    'NorthWest');
grid on;

%% DETERMINISTIC CHECK
% uncomment to run the deterministic code as a check of robustness

% Fund_base = ones(1e6,51) * 1e5;
% Fund_up = ones(1e6,51)*1e5;
% Fund_down = ones(1e6,51)*1e5;
%
% for i = 2:51
%     Fund_base(:,i) = Fund_base(:,i-1) * (1 - RD) / (fwd_discount(i));
%     Fund_up(:,i) = Fund_up(:,i-1) * (1 - RD) / (fwd_discount_up(i));
%     Fund_down(:,i) = Fund_down(:,i-1) * (1 - RD) / (fwd_discount_down(i));
% end
%
% % display the results
% disp(' ');
% disp('The deterministic value of the fund at the end of the simulation is:');
% disp(['Base scenario: ', num2str(mean(Fund_base(:,end)))]);
% disp(['Up scenario: ', num2str(mean(Fund_up(:,end)))]);
% disp(['Down scenario: ', num2str(mean(Fund_down(:,end)))]);
%
% Value_Fund = struct("Fund_t0", F0, "Fund_base", Fund_base, "Fund_up", Fund_up, "Fund_dow

%% Expenses

% Compute the vector of future expanses assumed a fixed infaltion rate
expenses = ones(T,1);
expenses(1,1) = expenses_0;

for i=2:T
    expenses(i,1) = expenses_0 * (1 + inflation)^(i-1);
end

%% Interest Rate Risk

% Base scenario
% Compute the liabilities, Macaulay duration and BEL components for the base scenario
[Liabilities_base, M_duration_base, Lapse_BEL_base, Death_BEL_base, ...
    Expenses_BEL_base, Commissions_BEL_base] = ...
    Liabilities(C0, Fund_base, discounts, delta_t0, l_t, qx, penalties,...
    expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the base scenario
BOF_base = F0 - Liabilities_base;
```

```
Delta_BOF_base = max(0, BOF_base - BOF_base);

% Up scenario
% Compute the liabilities, Macaulay duration and BEL components for the up scenario
[Liabilities_up, M_duration_up, Lapse_BEL_up, Death_BEL_up, Expenses_BEL_up,...
    Commissions_BEL_up] = ...
    Liabilities(C0, Fund_up, discounts_up, delta_t0, l_t, qx, penalties,...
    expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the up scenario
BOF_up = F0 - Liabilities_up;
Delta_BOF_up = max(0, BOF_base - BOF_up);

% Down scenario
% Compute the liabilities, Macaulay duration and BEL components for the down scenario
[Liabilities_down, M_duration_down, Lapse_BEL_down, Death_BEL_down, Expenses_BEL_down,...
    Commissions_BEL_down] = ...
    Liabilities(C0, Fund_down, discounts_down, delta_t0, l_t, qx, penalties,...
    expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the down scenario
BOF_down = F0 - Liabilities_down;
Delta_BOF_down = max(0, BOF_base - BOF_down);

% Compute the Solvency Capital Requirement (SCR) for the interest rate risk
SCR_IR = max(Delta_BOF_up, Delta_BOF_down);

%% Equity Risk

% shock equity by 39% plus 5.25% symmetry adjastment by Eiopa (consult data
% folder for more details)
equity_shock = 0.39;
symmetry_adj = 0.0525;
% equity stressed
E_stressed = equity*(1 - equity_shock - symmetry_adj);
F0_stressed_E = E_stressed + property;

% simulate the equity value in the stressed scenario
equity_MC_stressed = MC_simulation(E_stressed ,deltas ,N ,fwd_rates_base, ...
    dividend ,sigma_equity, RD);

% Fund value in the equity stressed scenario
F_E_stressed = equity_MC_stressed + property_MC;

%%%%%%%% Uncomment for deterministic calculation %%%%%%%%%%
% F_E_stressed = ones(1e6,51)*F0_stressed_E;
% for i = 2:51
%     F_E_stressed(:,i) = F_E_stressed(:,i-1) * (1 - RD) / (fwd_discount(i));
% end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Compute the liabilities, Macaulay duration and BEL components for the equity stressed sc
[Liabilities_EQ, M_duration_EQ, Lapse_BEL_EQ, Death_BEL_EQ, Expenses_BEL_EQ, ...
    Commissions_BEL_EQ] = ...
        Liabilities(C0, F_E_stressed , discounts, delta_t0, l_t, qx, penalties,...
        expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the equity stressed scenario
```

```
BOF_EQ = F0_stressed_E - Liabilities_EQ;
Delta_BOF_EQ = max(0, BOF_base - BOF_EQ);

% Compute the Solvency Capital Requirement (SCR) for the equity risk
SCR_EQ = Delta_BOF_EQ;

%% Property Risk

% shock property by 25%
property_shock = 0.25;
% property stressed
P_stressed = property*(1 - property_shock);
F0_stressed_P = equity + P_stressed;

% simulate the property value in the stressed scenario
property_MC_stressed = MC_simulation(P_stressed ,deltas ,N ,fwd_rates_base, ...
    dividend ,sigma_property, RD);

% Fund value in the property stressed scenario
F_P_stressed = equity_MC + property_MC_stressed;

%%%%%%%% Uncomment for deterministic calculation %%%%%%%%%%
% F_P_stressed = ones(1e6,51)*F0_stressed_P;
% for i = 2:51
%     F_P_stressed(:,i) = F_P_stressed(:,i-1) * (1 - RD) / (fwd_discount(i));
% end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Compute the liabilities, Macaulay duration and BEL components for
% the property stressed scenario
[Liabilities_PR, M_duration_PR, Lapse_BEL_PR, Death_BEL_PR, Expenses_BEL_PR, ...
    Commissions_BEL_PR] = ...
        Liabilities(C0, F_P_stressed , discounts, delta_t0, l_t, qx, penalties,...
        expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the property stressed scenario
BOF_PR = F0_stressed_P - Liabilities_PR;
Delta_BOF_PR = max(0, BOF_base - BOF_PR);

% Compute the Solvency Capital Requirement (SCR) for the property risk
SCR_PR = Delta_BOF_PR;

%% Mortality Risk

% Shock the mortality rate by 15% (check that the probability of death
% is not greater than 1)
qx_stressed = min(1,qx*(1+0.15));

% Compute the liabilities, Macaulay duration and BEL components for the
% mortality stressed scenario
[Liabilities_MT, M_duration_MT, Lapse_BEL_MT, Death_BEL_MT, Expenses_BEL_MT,...
    Commissions_BEL_MT] = ...
        Liabilities(C0, Fund_base , discounts, delta_t0, l_t, qx_stressed, penalties,...
        expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the mortality stressed scenario
BOF_MT = F0 - Liabilities_MT;
Delta_BOF_MT = max(0, BOF_base - BOF_MT);
```

```
% Compute the Solvency Capital Requirement (SCR) for the mortality risk
SCR_MT = Delta_BOF_MT;

%% Lapse Risk

% UP Scenario shok by 50% the probability of lapse (check that the probability
% of lapse is not greater than 1)
l_t_up = min(1.5*l_t(1),1)*ones(T,1);

% Compute the liabilities, Macaulay duration and BEL components for the lapse up scenario
[Liabilities_LTU, M_duration_LTU, Lapse_BEL_LTU, Death_BEL_LTU, Expenses_BEL_LTU, ...
    Commissions_BEL_LTU] = ...
        Liabilities(C0, Fund_base, discounts, delta_t0, l_t_up, qx, penalties, ...
        expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the lapse up scenario
BOF_LTU = F0 - Liabilities_LTU;
Delta_BOF_LTU = max(0, BOF_base - BOF_LTU);

% DOWN Scenario shok by 50% the probability of lapse (check that the probability
%       of lapse is not smaller than basic scenario minus 20%)
l_t_dw = max(0.5*l_t(1),l_t(1)-0.2)*ones(T,1);

% Compute the liabilities, Macaulay duration and BEL components for
% the lapse down scenario
[Liabilities_LTD, M_duration_LTD, Lapse_BEL_LTD, Death_BEL_LTD, Expenses_BEL_LTD,...
    Commissions_BEL_LTD] = ...
        Liabilities(C0, Fund_base, discounts, delta_t0, l_t_dw, qx, penalties,...
        expenses, RD, comm);


% Compute Basic Own Funds (BOF) and the Delta BOF for the lapse down scenario
BOF_LTD = F0 - Liabilities_LTD;
Delta_BOF_LTD = max(0, BOF_base - BOF_LTD);

% Lapse MASS Risk shock the first year by 40%
l_t_mass = l_t;
l_t_mass(1) = (0.4 + l_t(1));

% Compute the liabilities, Macaulay duration and BEL components for
% the lapse mass scenario
[Liabilities_LTM, M_duration_LTM, Lapse_BEL_LTM, Death_BEL_LTM, Expenses_BEL_LTM,...
    Commissions_BEL_LTM] = ...
        Liabilities(C0, Fund_base, discounts, delta_t0, l_t_mass, qx, penalties,...
        expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the lapse mass scenario
BOF_LTM = F0 - Liabilities_LTM;
Delta_BOF_LTM = max(0, BOF_base - BOF_LTM);

% Compute the Solvency Capital Requirement (SCR) for the lapse risk
SCR_LAP = max(max(Delta_BOF_LTM, Delta_BOF_LTD), Delta_BOF_LTU);

%% Catastrophe Risk

% Shock the mortality rate of the first year by 15 bps
qx_CAT = qx;
```

```
qx_CAT(1) = qx_CAT(1) + 0.0015;

% Compute the liabilities, Macaulay duration and BEL components for
% the catastrophe scenario
[Liabilities_CAT, M_duration_CAT, Lapse_BEL_CAT, Death_BEL_CAT, Expenses_BEL_CAT,...
    Commissions_BEL_CAT] = ...
        Liabilities(C0, Fund_base , discounts, delta_t0, l_t, qx_CAT, penalties,...
        expenses, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the catastrophe scenario
BOF_CAT = F0 - Liabilities_CAT;
Delta_BOF_CAT = max(0, BOF_base - BOF_CAT);

% Compute the Solvency Capital Requirement (SCR) for the catastrophe risk
SCR_CAT = Delta_BOF_CAT;

%% Expense Risk

% Shock the expenses by 10% and the inflation rate by 1%
expenses_stressed = zeros(T,1);
expenses_stressed(1) = expenses_0 * (1 + 0.1);

for i = 2:T
    expenses_stressed(i) = expenses_stressed(i-1) * (1+ inflation + 0.01);
end

% Compute the liabilities, Macaulay duration and BEL components for
% the expense stressed scenario
[Liabilities_EX, M_duration_EX, Lapse_BEL_EX, Death_BEL_EX, Expenses_BEL_EX,...
    Commissions_BEL_EX] = ...
        Liabilities(C0, Fund_base , discounts, delta_t0, l_t, qx, penalties,...
        expenses_stressed, RD, comm);

% Compute Basic Own Funds (BOF) and the Delta BOF for the expense stressed scenario
BOF_EX = F0 - Liabilities_EX;
Delta_BOF_EX = max(0, BOF_base - BOF_EX);

% Compute the Solvency Capital Requirement (SCR) for the expense risk
SCR_EX = Delta_BOF_EX;

%% Solvency Capital Requirement (SCR) and Basic Solvency Capital Requirement (BSCR)

% Aggregate the risks in:
% - Market Risk: Interest Rate Risk, Equity Risk, Property Risk
% - Life Risk: Mortality Risk, Lapse Risk, Catastrophe Risk
% build the SCR vectors

% Market Risk
SCR_market_Risk = [SCR_IR, SCR_EQ, SCR_PR]';
% Life Risk
SCR_life_Risk = [SCR_MT, SCR_LAP, SCR_EX, SCR_CAT]';

% Find the adeguate correlation matrices for the risks

% Choose the right market risk correlatio matrix based on
%       the risk factors we are exposed to (eg. IR_up or IR_down)
if Delta_BOF_down > Delta_BOF_up     % Down scenario
    mkt_corr_matrix = [1 0.5 0.5;
```

```
                        0.5  1  0.75;
                        0.5  0.75  1];
else       % Up scenario
    mkt_corr_matrix = [1  0   0;
                        0  1  0.75;
                        0  0.75  1];
end

% Life correlation matrix
Life_corr_matrix = [1  0  0.25  0.25;
                     0  1  0.5  0.25;
                     0.25  0.5  1  0.25;
                     0.25  0.25  0.25  1];

% BSCR correlation matrix
BSCR_corr_matrix = [1  0.25;
                     0.25  1];

% Calculate the  aggregate SCR

% Market SCR
SCR_Market = sqrt(SCR_market_Risk' * mkt_corr_matrix * SCR_market_Risk);

% Life SCR
SCR_Life = sqrt(SCR_life_Risk' * Life_corr_matrix * SCR_life_Risk);

% build SCR vector
SCR = [SCR_Market, SCR_Life]';

% Calculate the Basic Solvency Capital Requirement (BSCR)
BSCR = sqrt(SCR' * BSCR_corr_matrix * SCR);

%% Save all the results in a structure

% First layer of the struct is DataStructResults
% insiede DataStructResults we have the following fields:
% - Rates scenarios
% - Equity and property shock
% - Mortality shock
% - Lapse shock
% - Catastrophe shock
% - Expense shock
% insiede each field we have the following sub_fields:
% - Results: (which contains the following fields)
            % - Assets (Fund value in t0)
            % - Liabilities
            % - BOF
            % - Delta BOF
            % - Macaulay Duration          -
% - BEL components (Lapse, Death, Expenses, Commissions)

DataStructResults = struct("Rates", struct("Base", struct(), "Up", struct(), "Down", struc
                 "Equity_Property", struct("Equity", struct(), "Property", struct()), ...
                 "Mortality", struct(), ...
                 "Lapse", struct("Up", struct(), "Down", struct(), "Mass", struct()), ...
                 "Catastrophe", struct(), ...
                 "Expenses", struct());
```

```
% Rates scenarios
DataStructResults.Rates.Base.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_base, "BOF", BOF_base, "Delta_BOF", ...
    Delta_BOF_base, "Macaulay_Duration", M_duration_base);

DataStructResults.Rates.Base.BEL = struct("Lapse", Lapse_BEL_base, "Death", ...
    Death_BEL_base, "Expenses", Expenses_BEL_base, ...
    "Commissions", Commissions_BEL_base);

DataStructResults.Rates.Up.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_up, "BOF", BOF_up, "Delta_BOF", ...
    Delta_BOF_up, "Macaulay_Duration", M_duration_up);

DataStructResults.Rates.Up.BEL = struct("Lapse", Lapse_BEL_up, "Death", ...
    Death_BEL_up, "Expenses", Expenses_BEL_up, ...
    "Commissions", Commissions_BEL_up);

DataStructResults.Rates.Down.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_down, "BOF", BOF_down, "Delta_BOF", ...
    Delta_BOF_down, "Macaulay_Duration", M_duration_down);

DataStructResults.Rates.Down.BEL = struct("Lapse", Lapse_BEL_down, "Death", ...
    Death_BEL_down, "Expenses", Expenses_BEL_down, ...
    "Commissions", Commissions_BEL_down);


% Equity and property shock
DataStructResults.Equity_Property.Equity.Results = struct("Assets", ...
    F0_stressed_E, "Liabilities", Liabilities_EQ, "BOF", BOF_EQ, "Delta_BOF", ...
    Delta_BOF_EQ, "Macaulay_Duration", M_duration_EQ);

DataStructResults.Equity_Property.Equity.BEL = struct("Lapse", Lapse_BEL_EQ, ...
    "Death", Death_BEL_EQ, "Expenses", Expenses_BEL_EQ, ...
    "Commissions", Commissions_BEL_EQ);

DataStructResults.Equity_Property.Property.Results = struct("Assets", F0_stressed_P, ...
    "Liabilities", Liabilities_PR, "BOF", BOF_PR, "Delta_BOF", ...
    Delta_BOF_PR, "Macaulay_Duration", M_duration_PR);

DataStructResults.Equity_Property.Property.BEL = struct("Lapse", Lapse_BEL_PR, ...
    "Death", Death_BEL_PR, "Expenses", Expenses_BEL_PR, ...
    "Commissions", Commissions_BEL_PR);

% Mortality shock
DataStructResults.Mortality.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_MT, "BOF", BOF_MT, "Delta_BOF", ...
    Delta_BOF_MT, "Macaulay_Duration", M_duration_MT);

DataStructResults.Mortality.BEL = struct("Lapse", Lapse_BEL_MT, "Death", ...
    Death_BEL_MT, "Expenses", Expenses_BEL_MT, ...
    "Commissions", Commissions_BEL_MT);

% Lapse shock
DataStructResults.Lapse.Up.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_LTU, "BOF", BOF_LTU, "Delta_BOF", ...
    Delta_BOF_LTU, "Macaulay_Duration", M_duration_LTU);

DataStructResults.Lapse.Up.BEL = struct("Lapse", Lapse_BEL_LTU, "Death", ...
```

```
    Death_BEL_LTU, "Expenses", Expenses_BEL_LTU, ...
    "Commissions", Commissions_BEL_LTU);

DataStructResults.Lapse.Down.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_LTD, "BOF", BOF_LTD, "Delta_BOF", ...
    Delta_BOF_LTD, "Macaulay_Duration", M_duration_LTD);

DataStructResults.Lapse.Down.BEL = struct("Lapse", Lapse_BEL_LTD, "Death", ...
    Death_BEL_LTD, "Expenses", Expenses_BEL_LTD, ...
    "Commissions", Commissions_BEL_LTD);

DataStructResults.Lapse.Mass.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_LTM, "BOF", BOF_LTM, "Delta_BOF", ...
    Delta_BOF_LTM, "Macaulay_Duration", M_duration_LTM);

DataStructResults.Lapse.Mass.BEL = struct("Lapse", Lapse_BEL_LTM, "Death", ...
    Death_BEL_LTM, "Expenses", Expenses_BEL_LTM, ...
    "Commissions", Commissions_BEL_LTM);

% Catastrophe shock
DataStructResults.Catastrophe.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_CAT, "BOF", BOF_CAT, "Delta_BOF", ...
    Delta_BOF_CAT, "Macaulay_Duration", M_duration_CAT);

DataStructResults.Catastrophe.BEL = struct("Lapse", Lapse_BEL_CAT, "Death", ...
    Death_BEL_CAT, "Expenses", Expenses_BEL_CAT, ...
    "Commissions", Commissions_BEL_CAT);

% Expense shock
DataStructResults.Expenses.Results = struct("Assets", F0, "Liabilities", ...
    Liabilities_EX, "BOF", BOF_EX, "Delta_BOF", ...
    Delta_BOF_EX, "Macaulay_Duration", M_duration_EX);

DataStructResults.Expenses.BEL = struct("Lapse", Lapse_BEL_EX, "Death", ...
    Death_BEL_EX, "Expenses", Expenses_BEL_EX, ...
    "Commissions", Commissions_BEL_EX);


% Build a structure to store the results of SCR and BSCR
% First layer of the struct is SCR_BSCR
% insiede SCR_BSCR we have the following fields:
% - SCR_MKT_Risk (% insiede each field we have the following sub_fields:
%                interest rate, equity, property)
% - SCR_Life_Risk (% insiede each field we have the following sub_fields:
%            mortality, lapse, catastrophe, expenses)
% - SCR (% insiede each field we have the following sub_fields: market and life)
% - BSCR

SCR_BSCR = struct("SCR_MKT_Risk", struct("Interest", struct(), "Equity", struct(), ...
    "Property", struct()), ...
                  "SCR_Life_Risk", struct("Mortality", struct(), "Lapse", struct(), ...
                  "Catastrophe", struct(), "Expenses", struct()), ...
                  "SCR", struct("Market", struct(), "Life", struct()), ...
                  "BSCR", struct());

SCR_BSCR.SCR_MKT_Risk.Interest = SCR_IR;
SCR_BSCR.SCR_MKT_Risk.Equity = SCR_EQ;
SCR_BSCR.SCR_MKT_Risk.Property = SCR_PR;
```

```
SCR_BSCR.SCR_Life_Risk.Mortality = SCR_MT;
SCR_BSCR.SCR_Life_Risk.Lapse = SCR_LAP;
SCR_BSCR.SCR_Life_Risk.Catastrophe = SCR_CAT;
SCR_BSCR.SCR_Life_Risk.Expenses = SCR_EX;

SCR_BSCR.SCR.Market = SCR_Market;
SCR_BSCR.SCR.Life = SCR_Life;
SCR_BSCR.BSCR = BSCR;


%% Display all the results

disp(' ')
disp('  ---------------------------------------------------------------- SII Project Results ----------
disp(' ')

% Extracting .Results for each field
ratesBaseResults = DataStructResults.Rates.Base.Results;
ratesUpResults = DataStructResults.Rates.Up.Results;
ratesDownResults = DataStructResults.Rates.Down.Results;
equityResults = DataStructResults.Equity_Property.Equity.Results;
propertyResults = DataStructResults.Equity_Property.Property.Results;
mortalityResults = DataStructResults.Mortality.Results;
lapseUpResults = DataStructResults.Lapse.Up.Results;
lapseDownResults = DataStructResults.Lapse.Down.Results;
lapseMassResults = DataStructResults.Lapse.Mass.Results;
catastropheResults = DataStructResults.Catastrophe.Results;
expensesResults = DataStructResults.Expenses.Results;

% Formatting the numbers
formatNumber = @(x) sprintf('%.2f', x);

% Creating a table
tableResults = table({formatNumber(ratesBaseResults.Assets); formatNumber(ratesUpResults.A
                      formatNumber(propertyResults.Assets); formatNumber(mortalityResults.
                      formatNumber(lapseMassResults.Assets); formatNumber(catastropheResul
                     {formatNumber(ratesBaseResults.Liabilities); formatNumber(ratesUpResu
                      formatNumber(propertyResults.Liabilities); formatNumber(mortalityRes
                      formatNumber(lapseMassResults.Liabilities); formatNumber(catastrophe
                     {formatNumber(ratesBaseResults.BOF); formatNumber(ratesUpResults.BOF)
                      formatNumber(propertyResults.BOF); formatNumber(mortalityResults.BOF
                      formatNumber(lapseMassResults.BOF); formatNumber(catastropheResults.
                     {formatNumber(ratesBaseResults.Delta_BOF); formatNumber(ratesUpResult
                      formatNumber(propertyResults.Delta_BOF); formatNumber(mortalityResul
                      formatNumber(lapseMassResults.Delta_BOF); formatNumber(catastropheRe
                     {formatNumber(ratesBaseResults.Macaulay_Duration); formatNumber(rates
                      formatNumber(equityResults.Macaulay_Duration); formatNumber(property
                      formatNumber(lapseUpResults.Macaulay_Duration); formatNumber(lapseDo
                      formatNumber(catastropheResults.Macaulay_Duration); formatNumber(exp
                     'VariableNames', {'Assets', 'Liabilities', 'BOF', 'Delta_BOF', 'Macau
                     'RowNames', {'Rates_Base', 'Rates_Up', 'Rates_Down', 'Equity', 'Prope
                                  'Lapse_Down', 'Lapse_Mass', 'Catastrophe', 'Expenses'});

% Displaying the table
disp(tableResults);
disp(' ')
```

```
%% Disp BEL Components

% Extracting .BEL for each field
ratesBaseBEL = DataStructResults.Rates.Base.BEL;
ratesUpBEL = DataStructResults.Rates.Up.BEL;
ratesDownBEL = DataStructResults.Rates.Down.BEL;
equityBEL = DataStructResults.Equity_Property.Equity.BEL;
propertyBEL = DataStructResults.Equity_Property.Property.BEL;
mortalityBEL = DataStructResults.Mortality.BEL;
lapseUpBEL = DataStructResults.Lapse.Up.BEL;
lapseDownBEL = DataStructResults.Lapse.Down.BEL;
lapseMassBEL = DataStructResults.Lapse.Mass.BEL;
catastropheBEL = DataStructResults.Catastrophe.BEL;
expensesBEL = DataStructResults.Expenses.BEL;

% Creating a table for .BEL
tableBEL = table({formatNumber(ratesBaseBEL.Lapse); formatNumber(ratesUpBEL.Lapse); ...
    formatNumber(ratesDownBEL.Lapse); formatNumber(equityBEL.Lapse); ...
                  formatNumber(propertyBEL.Lapse); formatNumber(mortalityBEL.Lapse); ...
                  formatNumber(lapseUpBEL.Lapse); formatNumber(lapseDownBEL.Lapse); ...
                  formatNumber(lapseMassBEL.Lapse); formatNumber(catastropheBEL.Lapse); ..
                  formatNumber(expensesBEL.Lapse)}, ...
                 {formatNumber(ratesBaseBEL.Death); formatNumber(ratesUpBEL.Death); ...
                 formatNumber(ratesDownBEL.Death); formatNumber(equityBEL.Death); ...
                  formatNumber(propertyBEL.Death); formatNumber(mortalityBEL.Death); ...
                  formatNumber(lapseUpBEL.Death); formatNumber(lapseDownBEL.Death); ...
                  formatNumber(lapseMassBEL.Death); formatNumber(catastropheBEL.Death); ..
                  formatNumber(expensesBEL.Death)}, ...
                 {formatNumber(ratesBaseBEL.Expenses); formatNumber(ratesUpBEL.Expenses);
                 formatNumber(ratesDownBEL.Expenses); formatNumber(equityBEL.Expenses); ..
                  formatNumber(propertyBEL.Expenses); formatNumber(mortalityBEL.Expenses);
                  formatNumber(lapseUpBEL.Expenses); formatNumber(lapseDownBEL.Expenses);
                  formatNumber(lapseMassBEL.Expenses); formatNumber(catastropheBEL.Expense
                  formatNumber(expensesBEL.Expenses)}, ...
                 {formatNumber(ratesBaseBEL.Commissions); formatNumber(ratesUpBEL.Commissi
                 formatNumber(ratesDownBEL.Commissions); formatNumber(equityBEL.Commission
                  formatNumber(propertyBEL.Commissions); formatNumber(mortalityBEL.Commiss
                  formatNumber(lapseUpBEL.Commissions); formatNumber(lapseDownBEL.Commissi
                  formatNumber(lapseMassBEL.Commissions); formatNumber(catastropheBEL.Comm
                  formatNumber(expensesBEL.Commissions)}, ...
                 'VariableNames', {'Lapse', 'Death', 'Expenses', 'Commissions'}, ...
                 'RowNames', {'Rates_Base', 'Rates_Up', 'Rates_Down', 'Equity', 'Property'
                              'Lapse_Down', 'Lapse_Mass', 'Catastrophe', 'Expenses'});

% Displaying the table for .BEL
disp(tableBEL);

%% Displaying the Solvency Capital Requirement (SCR) and Basic Solvency Capital Requiremen

disp(' ')
disp('  ------------------------------------------------------------ SCR & BSCR ----------
disp(' ')

% Display table for SCR_MKT_Risk
table_SCR_MKT_Risk = table(SCR_BSCR.SCR_MKT_Risk.Interest,...
    SCR_BSCR.SCR_MKT_Risk.Equity, SCR_BSCR.SCR_MKT_Risk.Property, ...
    'VariableNames', {'Interest', 'Equity', 'Property'});
```

```
disp("SCR Market Risk:");
disp(table_SCR_MKT_Risk);

% Display table for SCR_Life_Risk
table_SCR_Life_Risk = table(SCR_BSCR.SCR_Life_Risk.Mortality, ...
    SCR_BSCR.SCR_Life_Risk.Lapse, SCR_BSCR.SCR_Life_Risk.Catastrophe, ...
    SCR_BSCR.SCR_Life_Risk.Expenses, ...
    'VariableNames', {'Mortality', 'Lapse', 'Catastrophe', 'Expenses'});

disp("SCR Life Risk:");
disp(table_SCR_Life_Risk);

% Display table for SCR (Market and Life)
table_SCR = table(SCR_BSCR.SCR.Market, SCR_BSCR.SCR.Life, ...
    'VariableNames', {'Market', 'Life'});

disp("SCR (Market and Life):");
disp(table_SCR);

% Display BSCR
disp("BSCR: " + formatNumber(SCR_BSCR.BSCR));
disp(' ')

toc; % stop the timer
```

6.2. GBM Simulation function

Listing 2: MC simulations code

```
function MC_value = MC_simulation(S0 ,deltas ,N ,rates, dividend ,sigma, RD)
% The MC_simulation function run a Monte carlo simulation for an asset with
% initial value S0, that follows a GBM dynamics over time

% INPUTS
% S0            : Initial underlying value
% deltas        : delta time for each year in act 365
% N             : number of MC simulations
% rates         : forward rates
% dividend      : dividend yield (per year)
% sigma         : volatility
% RD            : regular deduction

% OUTPUTS
% MC_value      : matrix of the simulated Monte Carlo GBM value

% number of time steps
n_steps = length(deltas);

% extrapolate N random numbers from a normal distribution (standard normal) for n_steps
g = randn(N, n_steps);

% initialize the matrix of undelying values:
%   -> rows are the paths
%   -> columns are the time step
S = zeros(N, n_steps+1);
S(:,1) = S0*ones(N,1);

% simulate the GBM process
for i = 1:n_steps

    S(:,i+1) = S(:,i) * (1 - RD) .* exp( ( (rates(i+1) - dividend) - 0.5*sigma^2)*...
        deltas(i) + sigma*sqrt(deltas(i))*g(:,i) );

end

% return a matrix of the simulated GBM values
MC_value = S;

end
```

6.3. Liabilities function

Listing 3: Liabilities computation code

```
function [Liabilities, M_duration, Lapse_BEL, Death_BEL, Expenses_BEL, Commissions_BEL] =
            Liabilities(C0, F, discounts, time, lt, qx, penalties, expenses, RD, COMM)
% This function calculates the liabilities of a portfolio of contracts,
% moreover it computes the Macaulay Duration of liabilities
% and the Best Estimate Liabilities components (BEL) such as Lapse Benefits,
% Death Benefits, Expenses and Commissions
%
% INPUTS:
% C0                    : initial fund value
% F                     : matrix of fund values
% discounts             : vector of discounts
% time                  : vector of time
% lt                    : vector of lapse rates
% qx                    : vector of mortality rates
% penalties            : penalties for early lapse
% expenses              : vector of expenses
% RD                    : regular deduction
% COMM                  : commission
%
% OUTPUTS:
% Liabilities           : total liabilities
% M_duration            : Macaulay Duration of liabilities
% Lapse_BEL             : Lapse BEL component
% Death_BEL             : Death BEL component
% Expenses_BEL          : Expenses BEL component
% Commissions_BEL       : Commissions BEL component

% Maturity of the contracts
T = round(time(end));

% compute the probability of remaining in the contract at each year
% cumprod function compute the cumuluative product of the elements of the vector
% we initialize the vector with 1 because the first year the probability of
% remaining in the contract is 1
P_remain_contract = cumprod([1; (1-qx(1:end-1)).*(1-lt(1:end-1))]);

% initialize the vectors
V = zeros(T,1);

% initialize the vectors of BEL components
Lapse_ben=zeros(T,1);
Death_ben=zeros(T,1);
Expenses_ben=zeros(T,1);
Commissions_ben=zeros(T,1);

% loop over the years
for i = 1 : T

    % cash flows at each year for the benefits

    % death benefits, use the full matrix F since max is a non linear operator
    death_cf = (max(C0, F(:,i+1)))*qx(i);

    % after taking the max we can compute the mean for semplicity
    death_cf = mean(death_cf);  % mean of the death benefits
```

```
    F_mean = mean(F(:,i+1));     % mean of the fund value


    % Lapse benefits at each year
    if i == T   % condition because in the last year it's mandatory to lapse
        % the contract (contract is expired)
        lapse_cf = (F_mean)*(1-qx(i));
    else
        lapse_cf = (F_mean-penalties)*lt(i)*(1-qx(i));
    end

    % commission  cash flow at each year
    commision = (F_mean / (1-RD)) * COMM;

    % Total Liabilities at each year pre discounting (cash flows at each year)
    V(i) = P_remain_contract(i) * ( lapse_cf + death_cf + expenses(i) + commision );

    % Benefits at each year
    Lapse_ben(i)         = (P_remain_contract(i) * lapse_cf);
    Death_ben(i)         = (P_remain_contract(i) * death_cf);
    Expenses_ben(i)      = (P_remain_contract(i) * expenses(i));
    Commissions_ben(i)   = (P_remain_contract(i) * commision);
end

% liabilities as the sum of discounted expected cash flows
Liabilities = V' * discounts(2:end);

% Macaulay Duration of liabilities
M_duration = ( (V .* discounts(2:end) )' * time(2:end) ) / Liabilities;

% compute Best Estimate Liabilities components (BEL)
Lapse_BEL       = (Lapse_ben)' * discounts(2:end);
Death_BEL       = (Death_ben)' * discounts(2:end);
Expenses_BEL    = (Expenses_ben)' * discounts(2:end);
Commissions_BEL = (Commissions_ben)' * discounts(2:end);

end
```

6.4. Martingale Test Function

Listing 4: Martingale Test computation code

```
function martingaleTest = mtgTest(underlying ,delta_t0 , deltas ,N ,rates , ...
    fwd_rates , dividend ,sigma , RD, index , fig_number)
% This function performs the Martingale test for the underlying asset:
%   - check if the selected parameters are consistent with the martingale property
%
% INPUTS:
% underlying          : initial value of the underlying asset
% delta_t0            : time intevals between t0 and ti
% deltas              : time steps
% N                   : number of simulations
% rates               : interest rates
% fwd_rates           : forward rates
% dividend            : dividend yield
% sigma               : volatility
% RD                  : regular deduction
% index               : index of the subplot
% fig_number          : number of the figure
%
% OUTPUTS:
% martingaleTest      : vector of the martingale test

% Rates vector
rates = [1; rates];

% Simulate the underlying asset
simulated_UL = MC_simulation(underlying ,deltas ,N ,fwd_rates , dividend ,sigma , RD);
% Compute the mean of the simulated underlying asset
mean_UL = mean(simulated_UL ,1)';
% Compute the flat underlying asset (constant initial value)
flat_UL = underlying * ones(length(delta_t0),1);
% Compute the discounted mean of the simulated underlying asset
DF_UL = exp(-rates.*delta_t0).*mean_UL;

% to check the martingale property we compute the difference between the
% flat underlying asset and the discounted mean of the simulated underlying asset
% hence we accept the martingale property if the difference is smaller than a trheshold
martingaleTest = abs(flat_UL - DF_UL);

% plot Martingale test in a subplot with index i
figure(fig_number)
subplot(2,3,index)
plot(delta_t0 , flat_UL , '--r')
hold on
plot(delta_t0 , mean_UL, 'k','LineWidth',1)
hold on
plot(delta_t0 ,DF_UL, '+b')
% tile with number of simulations
title(['Martingale Test with N = ', num2str(N)])
hold off

end
```

7. References

- The standard formula of Solvency II: a critical discussion, by Matthias Scherer and Gerhard Stahl, Technische Universität München Garching, 19 November 2020
- Solvency Capital Requirement (SCR) Standard Formula, by EIOPA, September 2021
- Solvency II, by EIOPA