Project

Multivariate Pricing

Financial Engineering AY 2023-2024

1. Consider the 2-dimensional multivariate Lévy process defined in proposition 1 of [2] for the forward exponents of two assets. For every asset, the forward exponent $X_i(t)$ is the sum of two independent Lévy processes $X_i(t) = Y_i(t) + a_i\, Z(t)$. Moreover, assume that $Y_i(t)\ and\ Z(t)$ have NIG marginals (i.e. the marginals of section 2.2 of [2] with the tempered stable alpha=1/2). Finally, find a constant p_i such that the processes $\exp(X_i(t) + p_i t)$ are martingales. Consider as forward price for the asset i at time t
$$F_i(t,T) = F_i(0,T)\exp(X_i(t) + p_i t)$$
Where $F_i(0,T)$ is the forward price for the asset i at time t=0.
2. Prove that that the marginal characteristics function of the multivariate process discussed in the previous point is the same characteristic function of a Lévy NIG process (characteristic function in eq. (7) of [2]) if the conditions in (9) are satisfied. Prove that the conditions in (9) are also implying that $\frac{\sigma_1^2}{\theta_1^2 k_1} = \frac{\sigma_2^2}{\theta_2^2 k_2}$=c where c is a constant.
3. Compute the linear correlation between the two processes (i.e. compute the expression in eq. (3) of [2] for the NIG processes). What is the maximum attainable value of correlation (mind the constraints in eq. 10).
4. Consider the dataset of options data on the S&P 500 and the EURO STOXX 50 volatility surface (structs inside OptionData.mat) on the 9 July 2023. The dataset contains expiry dates, bid and ask prices of call and put and strikes for all liquid options. The struct also contains the spot price for both markets.
5. Compute (for both the S&P 500 and EURO STOXX 50 market) discount factors and forward prices from option data following the methodology described in [1]. In particular, you should estimate the discount factor $\bar{B}$ for every maturity with the formula in eq. (5) of [1]. Once you have an estimation of $\bar{B}$ you can obtain the future price for the corresponding maturity using eq. (2) of [1].
6. Calibrate the 2-dimensional process on the two option markets. Notice that the calibration process of the marginal parameters cannot be done independently for the two markets because the constraint $\frac{\sigma_1^2}{\theta_1^2 k_1} = \frac{\sigma_2^2}{\theta_2^2 k_2}$=c should always hold. You should calibrate the two volatility surfaces together and impose the constraint. You should calibrate on all available maturities.
7. Estimate the historical correlation between the two indexes with the *yearly* returns available in the struct SPXSX5Ereturns.mat. Is the correlation attainable with the calibrated parameters of the multivariate Lèvy model?
8. Consider an alternative model for the two assets forward exponents: $X_i(t) = -\frac{1}{2}\sigma_i^2 t + \sigma_i W_i(t)$ with $Corr\left(W_i(t), W_j(t)\right) = \rho$. What is the maximum attainable correlation between the two processes?. Calibrate this Brownian motions model on the two implied volatility surfaces separately (just one parameter σ_i per surface) and select ρ to match the historical correlation.
9. With both models (Lèvy and Brownian) price a derivative with pay-off
$$(S_1(t) - S_1(0))^+ * I_{S_2(t)<0.95*S_2(0)}$$
where $S_1(t)$ and $S_2(t)$ are the spot price of the S&P 500 and of the EUROSTOXX 50 respectively and t is equal to one year.
Does a close formula exist?

10. Comment on the difference in prices between the two models also pointing out what kind of market characteristics each model is reproducing better (e.g. equity skew, correlation…).

Realize a library in Matlab. Optional Python.

[1] Azzone, Michele, and Roberto Baviera. "Synthetic forwards and cost of funding in the equity derivative market." *Finance Research Letters* 41 (2021): 101841.

[2] Ballotta, Laura, and Efrem Bonfiglioli. "Multivariate asset models using Lévy processes and applications." *The European Journal of Finance* 22.13 (2016): 1320-1350.

Delivery address: financial.engineering.polimi@gmail.com.