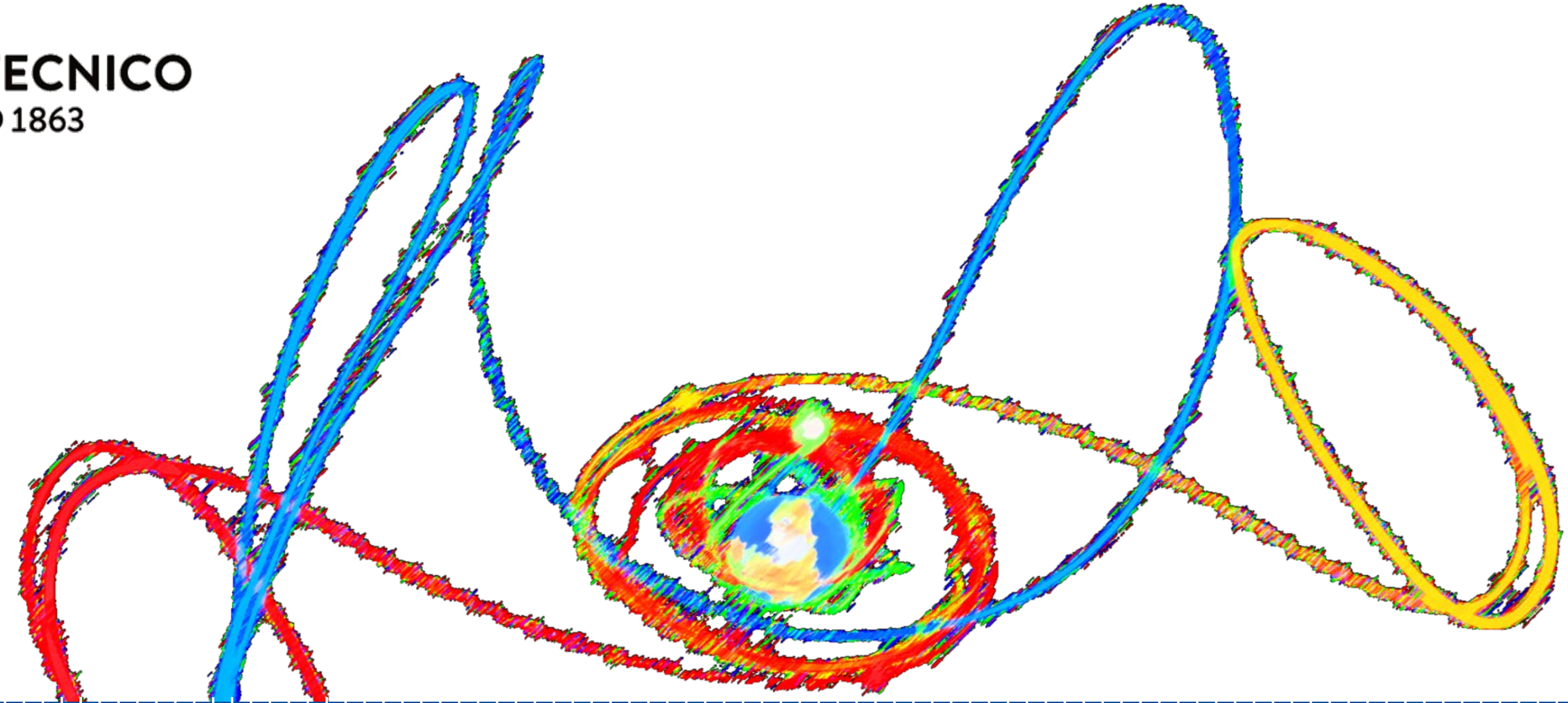

Orbital Mechanics
Lab Chapter 2 – Part 1: Orbit representation

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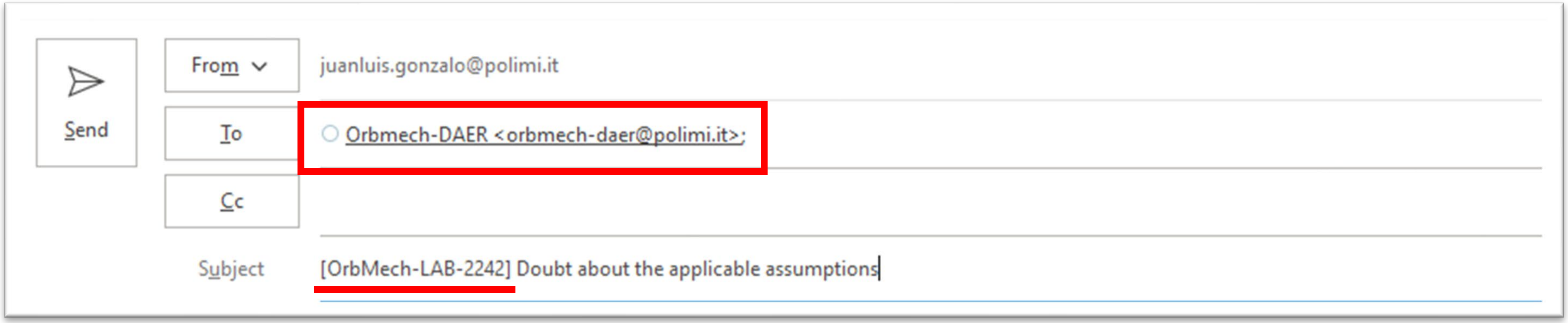
Email questions about the Labs

POLITECNICO MILANO 1863

- **REMINDER:** For email questions about the course (Lectures, Exercises, and Labs), always address them to:
  - orbmech-daer@polimi.it
- For questions about the Labs and Assignments, begin your subject with: `[OrbMech-LAB-xxxx]`, where `xxxx` is your group's ID. You will soon find your group ID in webeep.



- The aim of these rules is to allow an ordered and efficient processing of requests by the teaching staff.
  - Email queries not respecting these rules may be dropped without further notice.

Chapter Contents

Lab – Orbit representation

- **Ground track**
  - Definition, angles, geometrical properties
  - Repeating ground tracks
  - **Exercise 1:** Computation of ground tracks

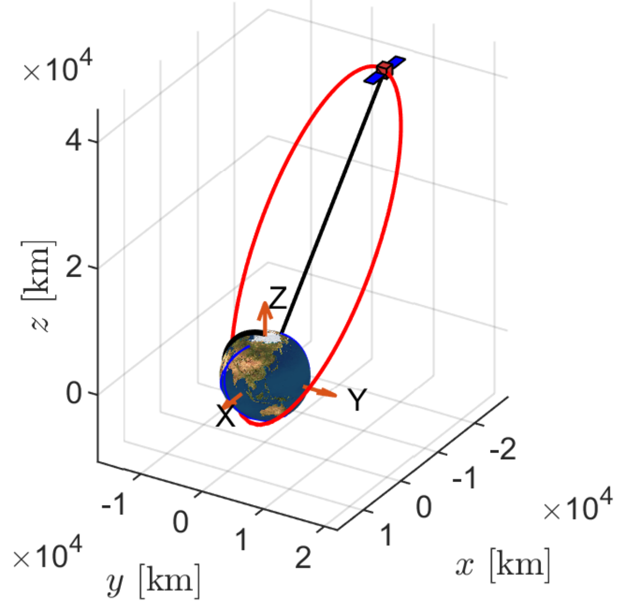
GROUND TRACK

Ground Track

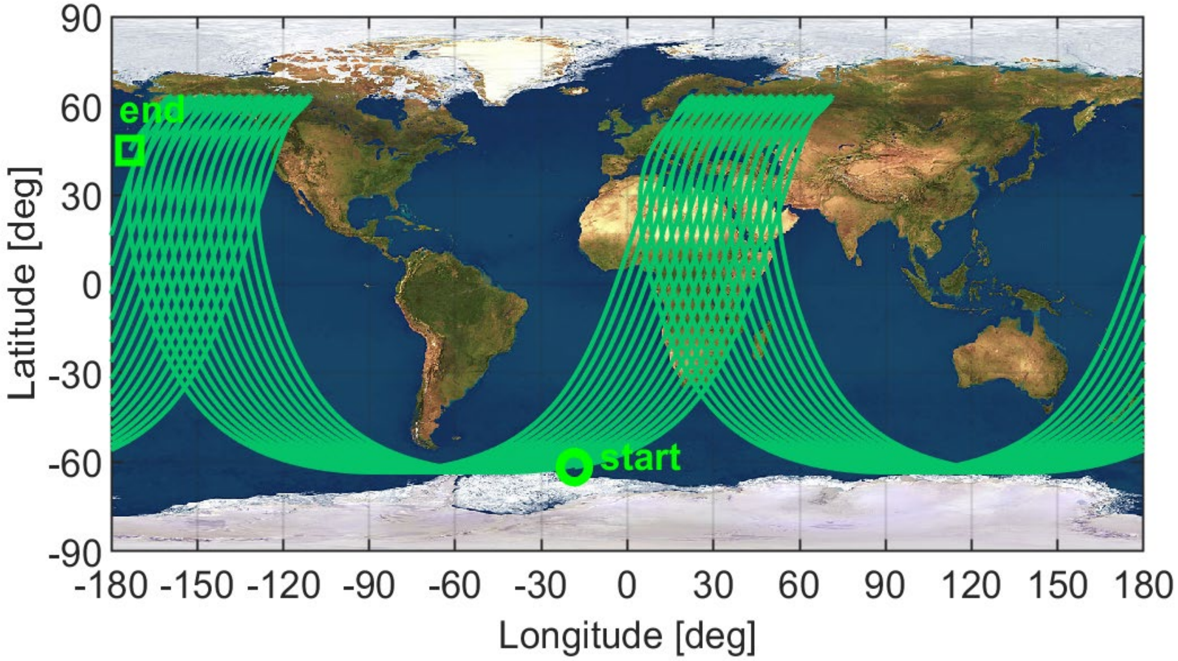
Definition

Ground track: *Projection of a satellite's orbit onto the Earth's surface [1].*

- Neglecting Earth's oblateness, it can be plotted as the trace left on the planet's surface by the line connecting the centre of the Earth and the satellite as it travels its orbit.
- At each time t, the ground track point is located by its **latitude** ϕ and **longitude** λ relative to the **rotating Earth**.



Molniya orbit



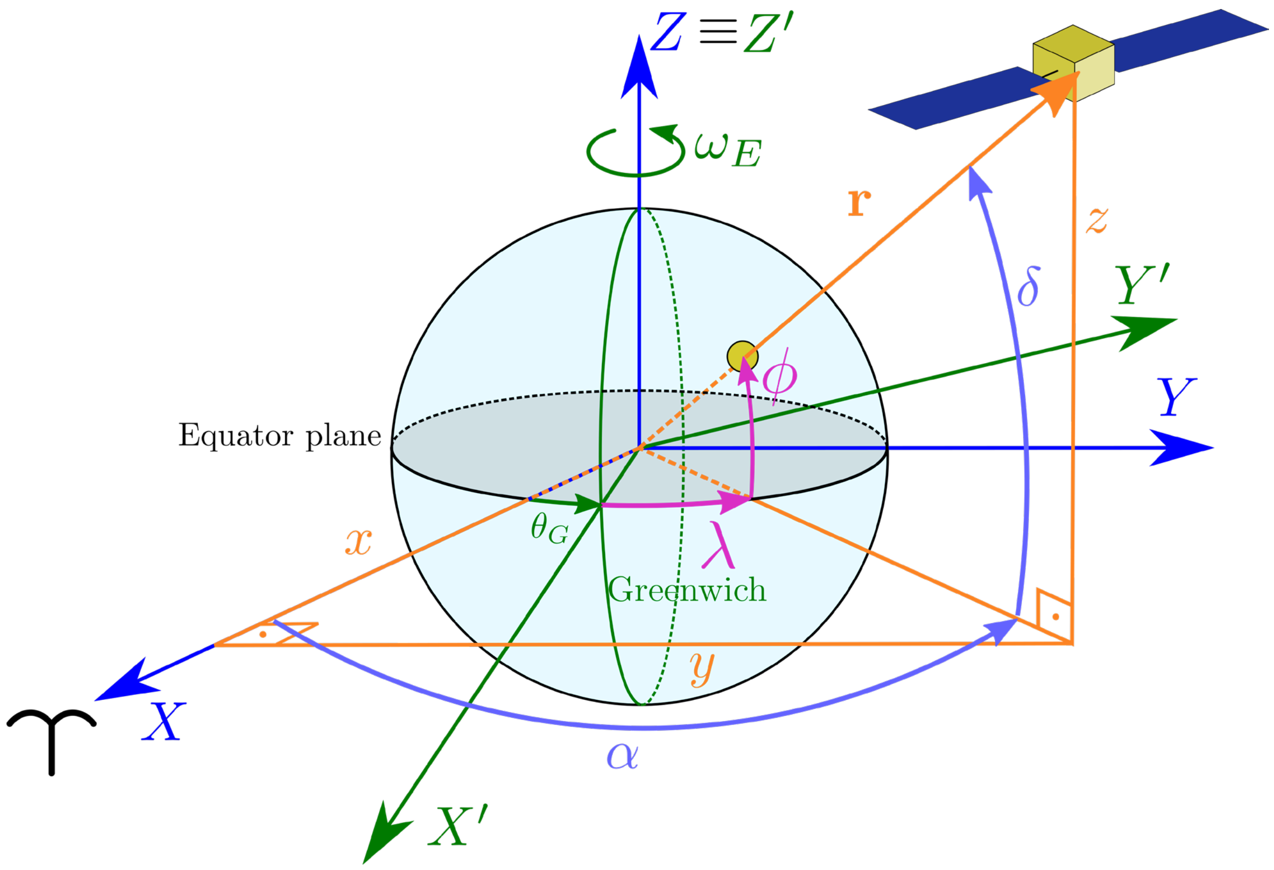
$a = 26600$ km
$e = 0.74$
$i = 63.4$ deg
$\Omega = 50$ deg
$\omega = 280$ deg
$f_0 = 0$ deg ‡
30 orbits

‡ f denotes true anomaly

[1] Curtis, H. D.. *Orbital mechanics for engineering students,* Butterworth-Heinemann , 2014

Ground Track

Angles for the ground track



Declination

$$\delta = \operatorname{asin}\frac{z}{r}$$

Right ascension

$$\alpha = \begin{cases} \operatorname{acos}\dfrac{x}{r\cos\delta} & \dfrac{y}{r} > 0 \\ 2\pi - \operatorname{acos}\dfrac{x}{r\cos\delta} & \dfrac{y}{r} \leq 0 \end{cases}$$

or alternatively

$$\alpha = \operatorname{atan2}(y, x)$$

Longitude

$$\lambda = \alpha - \theta_G$$

Latitude

$$\phi = \delta$$

Ground Track

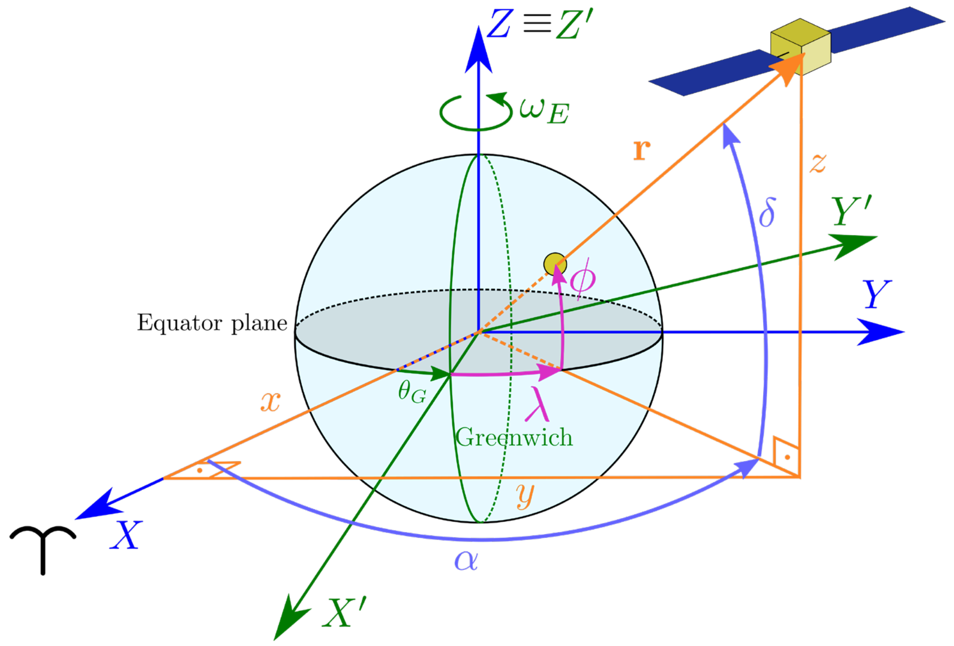
Greenwich Local Sidereal Time (LST)

- The LST of Greenwich can be computed as

$$\theta_G(t) = \theta_G(t_0) + \omega_E \, (t - t_0)$$

It is a function of both the Earth's rotation about its axis (ω_E) and the time of year considered.

The latter dependency is represented by the term $\theta_G(t_0)$, which refers to the Greenwich sidereal time at 0 hours UT.



When not otherwise specified, in the exercises proposed during the Labs it is always possible to define $\theta_G(t_0) = 0$. Instead, if an accurate comparison with a real case was to be made, the correct value of $\theta_G(t_0)$ should be calculated accordingly. Further details on how to compute $\theta_G(t_0)$, given the starting date of the propagation, can be found in **Curtis (pp. 214-216)** [1].

[1] Curtis, H. D.. *Orbital mechanics for engineering students,* Butterworth-Heinemann , 2014

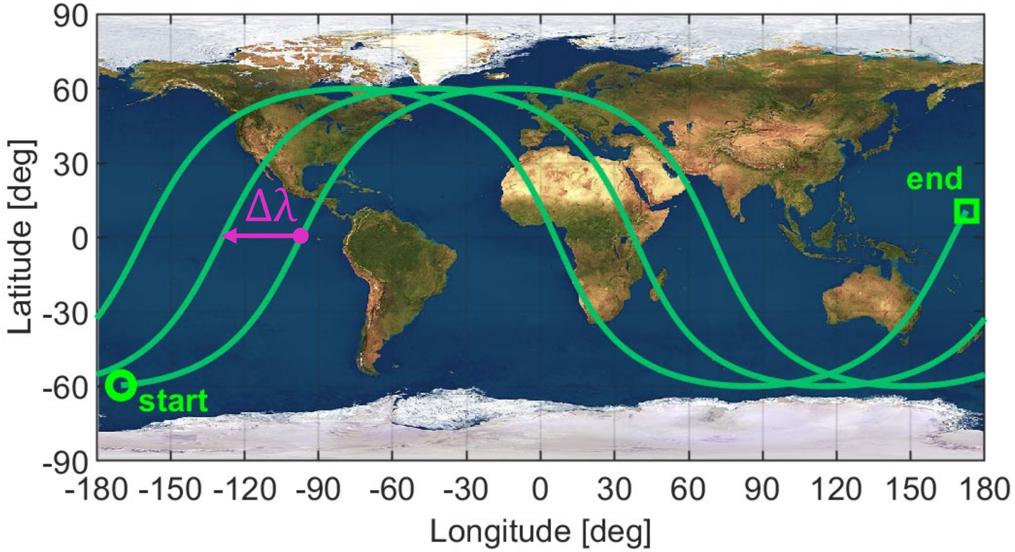
Ground Track

Geometrical properties

- For each orbital revolution, the ground track presents *2 equator crossings*, *one maximum in latitude*, and *one minimum in latitude*.
- For the unperturbed two-body problem, the orbit remains constant. However, **the ground track advances westward** by an angle $\Delta\lambda$ equal to Earth's rotation during one orbital period T of the satellite:

$$\Delta\lambda = T\,\omega_E$$

Earth's rotation velocity (eastwards): $\omega_E = 15.04\ \mathrm{deg/h}$



$a = 8350\ \mathrm{km}$
$e = 0.19760$
$i = 60\ \mathrm{deg}$
$\Omega = 270\ \mathrm{deg}$
$\omega = 45\ \mathrm{deg}$
$f_0 = 230\ \mathrm{deg}$
3.25 orbits

Exercise 1: Computation of ground tracks

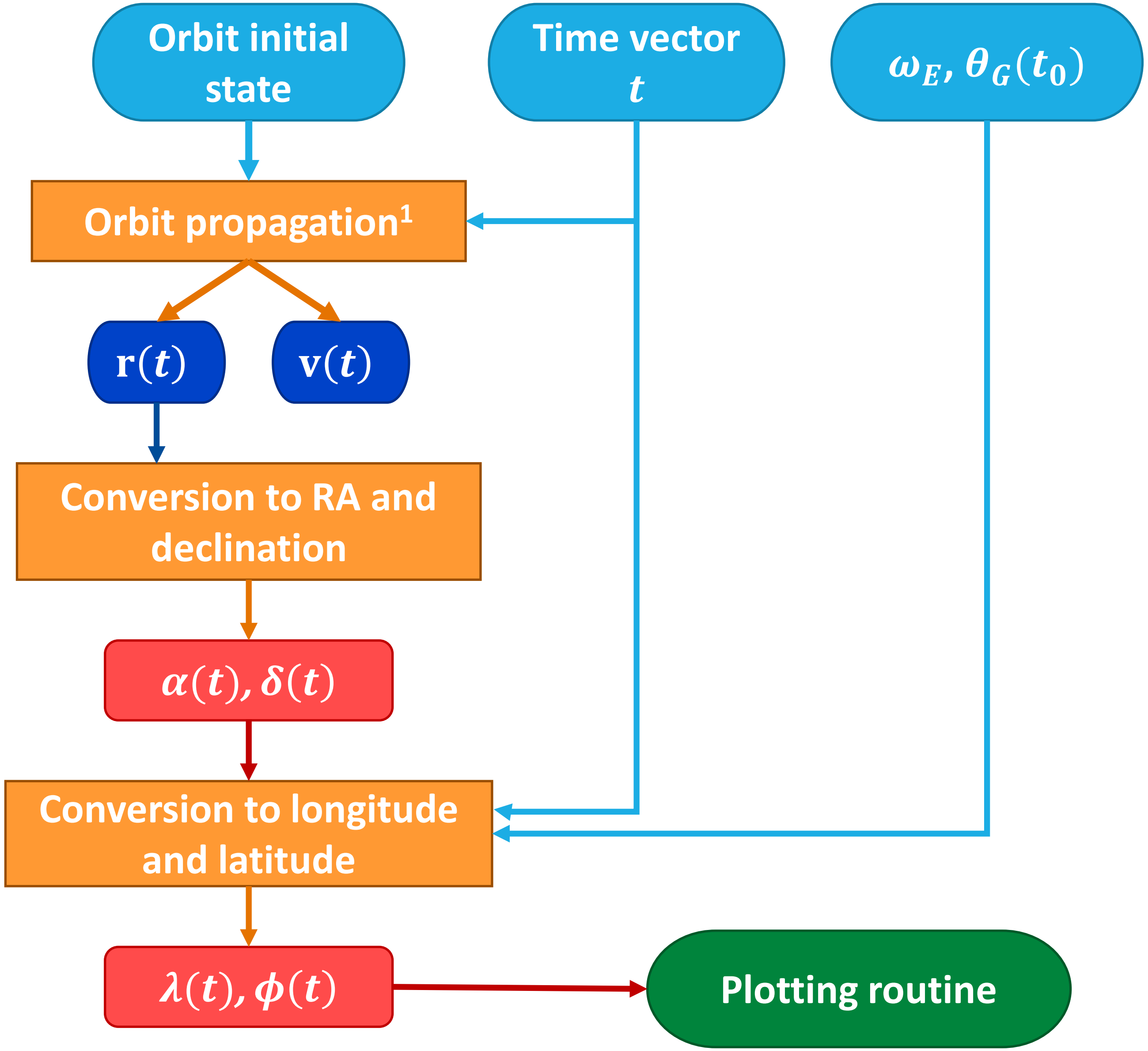
Exercise 1: **Computation of ground tracks**

1. Implement a function `groundTrack` that computes the ground track of an orbit
   - **Inputs:**
     - State of the orbit at the initial time (either in Cartesian or Keplerian elements)
     - Longitude of Greenwich meridian at initial time
     - Vector of times at which the ground track will be computed
     - Other inputs that you consider useful (e.g., ω_E, μ, t_0)
   - **Outputs:**
     - `alpha`: right ascension in Earth Centred Equatorial Inertial frame
     - `delta`: declination in Earth Centred Equatorial Inertial frame
     - `lon`: longitude with respect to rotating Earth (0 deg at Greenwich meridian)
     - `lat`: latitude with respect to rotating Earth

Exercise 1: Computation of ground tracks

Flow diagram

For numerical orbit propagation[1], you can reuse the code from **Chapter 1**.



[1] **Orbit propagation**: prediction of a body's orbital characteristics at a future date given the current orbital characteristics.

Exercise 1: Computation of ground tracks

Exercise 1: Computation of ground tracks

2. Plot the ground track for the following orbits:

   1. $a = 8350$ km, $e = 0.1976$, $i = 60$ deg, $\Omega = 270$ deg, $\omega = 45$ deg, $f_0 = 230$ deg (from [1], Example 4.12)
      In Cartesian coordinates: $\mathbf{r}_0 = [-4578.219, -801.084, -7929.708]$ km, $\mathbf{v}_0 = [0.800, -6.037, 1.385]$ km/s
   2. Molniya orbit with $a = 26600$ km, $e = 0.74$, $i = 63.4$ deg, $\Omega = 50$ deg, $\omega = 280$ deg, $f_0 = 0$ deg
      In Cartesian coordinates: $\mathbf{r}_0 = [3108.128,\ -1040.299,\ -6090.022]$ km, $\mathbf{v}_0 = [5.743,\ 8.055,\ 1.555]$ km/s
   3. Three circular LEO with altitude 800 km, $\Omega = 0$ deg, $\omega = 40$ deg, $f_0 = 0$ deg, and different inclinations:
      - $i = 0$ deg $\quad \mathbf{r}_0 = [5493.312,\ 4609.436,\ 0.000]$ km, $\quad \mathbf{v}_0 = [-4.792,\ 5.711,\ 0.000]$ km/s
      - $i = 30$ deg $\quad \mathbf{r}_0 = [5493.312,\ 3991.889,\ 2304.718]$ km, $\quad \mathbf{v}_0 = [-4.792,\ 4.946,\ 2.856]$ km/s
      - $i = 98$ deg $\quad \mathbf{r}_0 = [5493.312,\ -641.510,\ 4564.578]$ km, $\quad \mathbf{v}_0 = [-4.792,\ -0.795,\ 5.656]$ km/s

Data:

$\mu_\oplus$ and $R_\oplus$ from `astroConstants.m` (identifiers `13`, and `23`, respectively)

$\omega_E = 15.04$ deg/h $\qquad \theta_G(t_0) = 0$ deg

[1] Curtis, H. D.. *Orbital mechanics for engineering students,* Butterworth-Heinemann , 2014

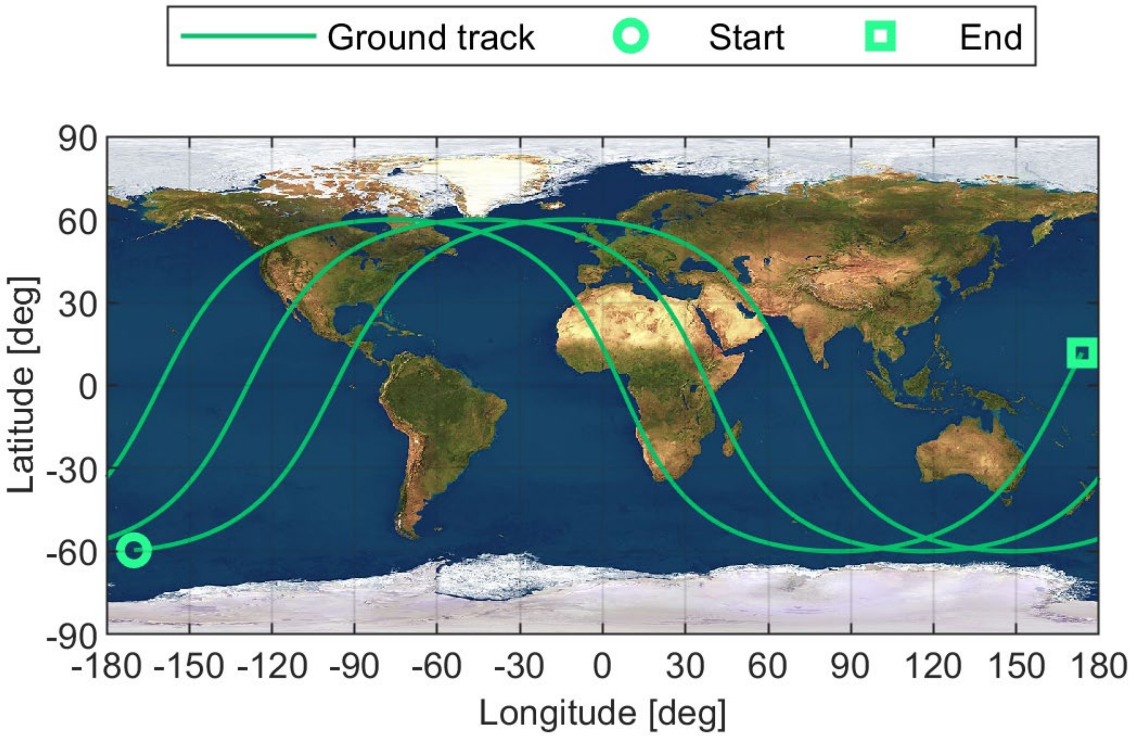
Exercise 1: Computation of ground tracks

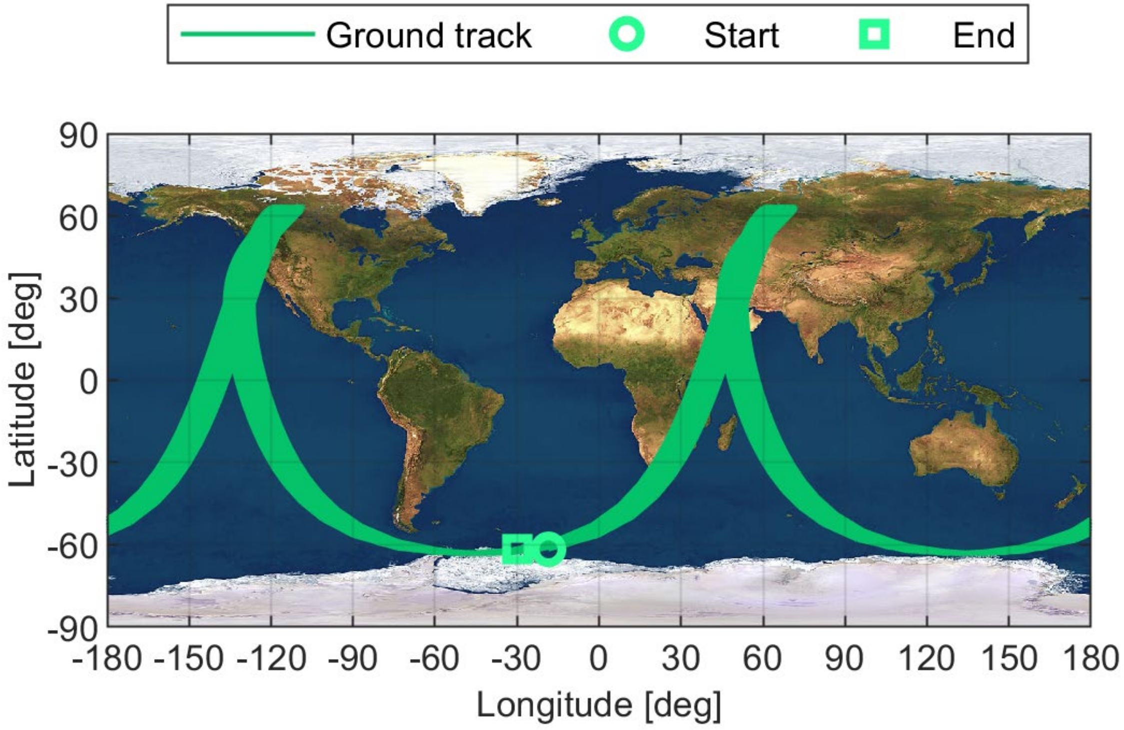
Hints

- **Radians** are more convenient for calculations, while **degrees** are better for graphical representation.
- **Longitude** λ has to be reduced to the 360 deg angular range chosen for the ground tracks (e.g., $[-180, 180]$ deg or $[0, 360]$ deg). Consequently, the ground track plot will be discontinuous in λ at the boundaries of the angular range.
  To avoid having horizontal lines connecting the points before and after the discontinuity:
  - Easiest way is to change the plot style, removing the line connecting the data points and using a marker for each point instead. That is, setting `'LineStyle'` to `'none'` and `'Marker'` to `'.'` (high number of points required to get a continuous path).
  - If instead you want to keep the line connecting the data points, you can introduce `NaN` values in your data arrays separating the points before and after the discontinuity. Matlab does not connect the data points that include `NaN` values.
- If you want, you can add Earth's surface as background, using function `imread` to load the image as a matrix of colored pixels, and function `image` to plot it.
  - Check the **documentation center** for detailed information on how to use them.
- Don't forget to *adjust the limits for the plotting regions*, to *label the axes*, and to *add a legend* if more than one line is represented.

Exercise 1: Computation of ground tracks

Sample solutions





Case 1

$a = 8350$ km, $e = 0.19760$, $i = 60$ deg
$\Omega = 270$ deg, $\omega = 45$ deg, $f_0 = 230$ deg
$\theta_G(t_0) = 0$ deg, 3.25 orbits

Case 2

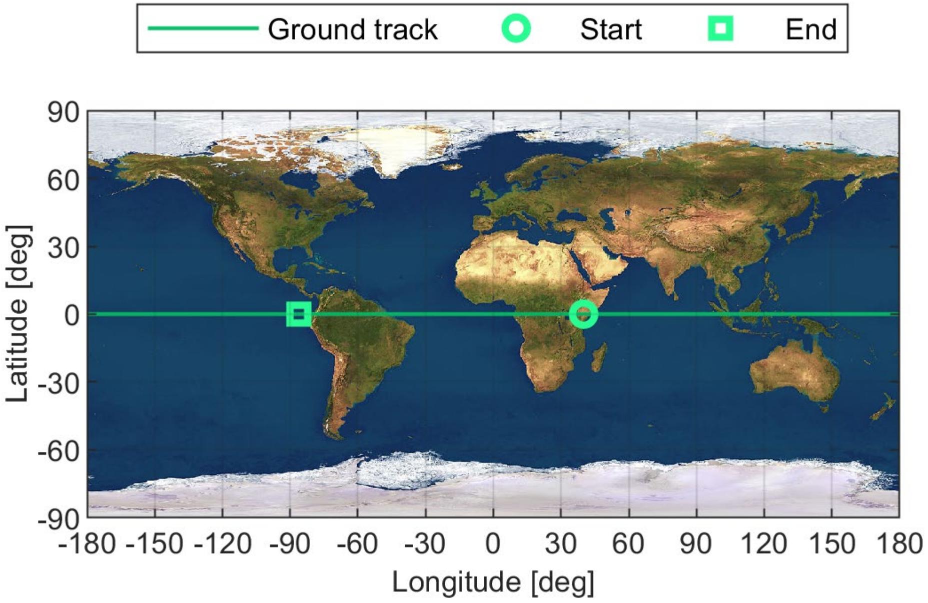
$a = 26600$ km, $e = 0.74$, $i = 63.4$ deg
$\Omega = 50$ deg, $\omega = 280$ deg, $f_0 = 0$ deg
$\theta_G(t_0) = 0$ deg, 30 orbits

Exercise 1: Computation of ground tracks

Sample solutions

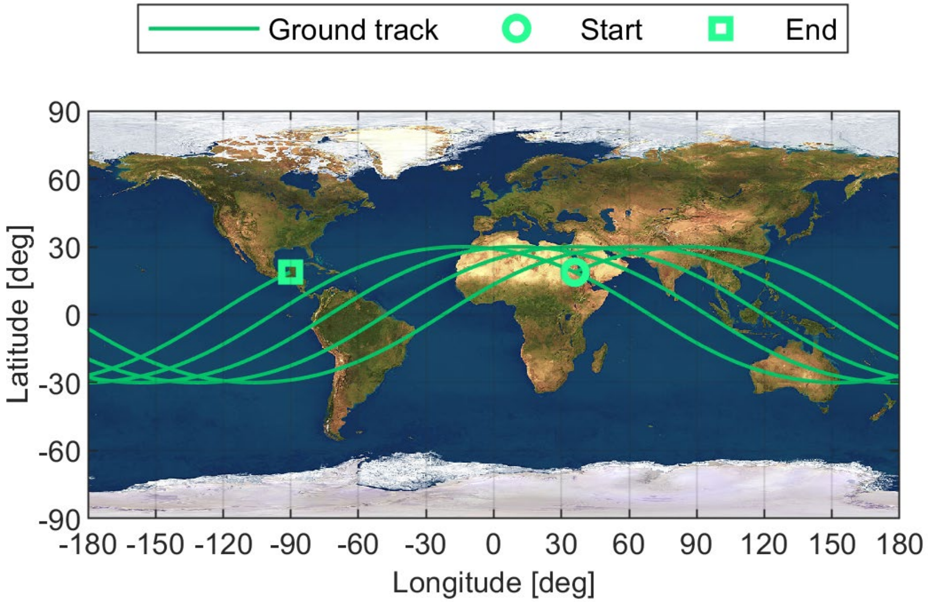
Case 3

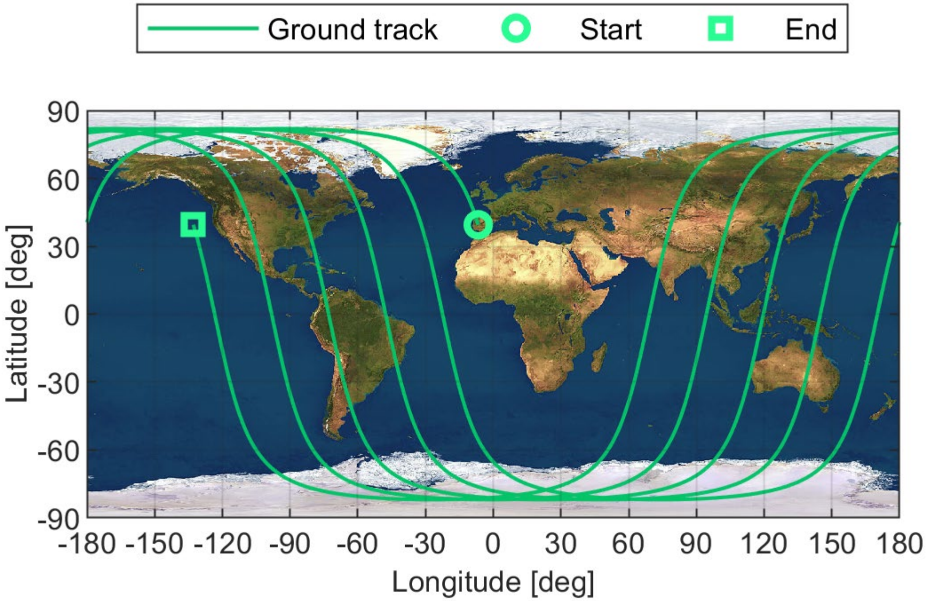
$a = 7171.010$ km, $e = 0$
$\Omega = 0$ deg, $\omega = 40$ deg, $f_0 = 0$ deg
$\theta_G(t_0) = 0$ deg
5 orbits



$i = 0$ deg

$i = 30$ deg





$i = 98$ deg