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Homework 4 Formal logic

1. For each given set of sentences, determine whether it is logically consistent (jointly satisfiable).
   (a) $\neg D$, $(D \vee F)$, $\neg F$
   (b) $(T \to K)$, $\neg K$, $(K \vee \neg T)$
   (c) $\neg(A \to (\neg C \to B))$, $((B \vee C) \wedge A)$
   (d) $(C \to B)$, $(D \vee C)$, $\neg B$, $(D \to B)$

a)

D	F	$\bar D$	$D \vee F$	$\bar F$
0	0	1	0	1
0	1	1	1	0
1	0	0	1	1
1	1	0	1	0

is not logically consistent

b)

T	K	$T \to K$	$\bar K$	$K \vee \bar T$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1

is logically consistent

c) $\overline{A \to (\bar C \to B)} = A \wedge \bar C \wedge \bar B$

A	B	C	$A \wedge \bar C \wedge \bar B$	$(B \vee C) \wedge A$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

is not logically consistent

d)

B	C	D	$C \to B$	$\bar B$	$D \vee C$	$D \to B$
0	0	0	1	1	0	1
0	0	1	1	1	1	0
0	1	0	0	1	1	1
0	1	1	0	1	1	0
1	0	0	1	0	0	1
1	0	1	1	0	1	1
1	1	0	1	0	1	1
1	1	1	1	0	1	1

is not logically consistent

2. Complete the following deductive formal proofs by filling in missing formulae and justifications.

(a)

1	$H \to (R \wedge C)$	Premise
2	$\neg R \vee \neg C$	Premise
3	$\neg(R \wedge C)$	DeM 2
∴	$\neg H$	MT 1, 3

(b)

1	$K \wedge S$	Premise
2	$\neg K$	Premise
3	K	∧E 1
4	$\bot$	¬E 2,3
∴	$\neg S$	X 4

(c)

1	$A \to \neg A$	Premise
⋮		(multiple lines)
∴	$\neg A$	LEM

(d)

1	$(P \wedge Q) \vee (P \wedge R)$	Premise
2	$P \wedge Q$	Assumption
3	P	∧E 2
4	$P \wedge R$	Assumption
5	P	∧E 4
∴	P	∨E 1, 2-3, 4-5

c)

1	$A \to \bar A$	premise
2	A	assume
3	$\bar A$	MP 1,2
4	$\bar A$	assume
5	$\bar A$	R 4
∴	$\bar A$	LEM 2-3, 4-5

3. Symbolize the given arguments with well-formed formulae (WFFs) of propositional logic. For each argument, determine its validity using a truth table. For each *valid* argument, provide a deductive formal proof[1] in Fitch notation. For each *invalid* argument, provide a counterexample valuation.

(a) *If philosophers ponder profound problems, their quandaries quell quotidian quibbles. Either their quandaries don't quell quotidian quibbles or right reasoning reveals reality (or both). Philosophers do ponder profound problems. Therefore, right reasoning reveals reality.*

(b) *If aardvarks are adorable, then either baby baboons don't beat bongos or crocodiles can't consume cute capybaras (or both). Baby baboons beat bongos. Aardvarks aren't adorable unless crocodiles can't consume cute capybaras. Therefore, aardvarks aren't adorable.*

(c) *If discipline doesn't defeat deficiency, then geniuses generally get good grades. If discipline defeats deficiency, then homework has harmed humanity. Therefore, geniuses generally get good grades unless homework has harmed humanity.*

(d) *Crocodiles can consume cute capybaras only if incarcerating iguanas isn't illegal. Mad monkeys make mayhem and dinosaurs do disco dance, unless crocodiles consume cute capybaras. It is known that incarcerating iguanas is illegal. Therefore, dinosaurs do disco dance if and only if mad monkeys make mayhem.*

a) p – philosophers ponder profound problems
q – quandaries quell quotidian quibbles
r – right reasoning reveals reality

p q r	$p \to q$	$\bar{q} \vee r$	p	∴	r
0 0 0	1	1	0	·	0
0 0 1	1	1	0	·	1
0 1 0	1	0	0	·	0
0 1 1	1	1	0	·	1
1 0 0	0	1	1	·	0
1 0 1	0	1	1	·	1
1 1 0	1	0	1	·	0
1 1 1	1	1	1	v	1

is valid

1	$P \to Q$	
2	$\neg Q \vee R$	
3	P	
4	Q	→E 1, 3
5	$\neg Q$	
6	$\bot$	¬E 4, 5
7	$\neg\neg Q$	¬I 5–6
8	R	DS 2, 7

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

b) a – aardvarks ...
b – baby ...
c – crocodiles ...

a b c	$a \to (\bar{b} \vee \bar{c})$	$\bar{a} \vee \bar{c}$	b	∴	$\bar{a}$
0 0 0	1	1	0	·	1
0 0 1	1	1	0	·	1
0 1 0	1	1	1	v	1
0 1 1	1	1	1	v	1
1 0 0	1	1	0	·	0
1 0 1	1	0	0	·	0
1 1 0	1	1	1	x	0
1 1 1	0	0	1	·	0

is invalid

counterexample:

a – true,
b – true,
c – false,

so $a \to (\bar{b} \vee \bar{c})$ – true,
b – true,
$\bar{a} \vee \bar{c}$ – true,

but $\bar{a}$ – false

c) d – discipline ...
g – geniuses ...
h – homework ...

d g h	$\bar{d} \to g$	$d \to h$	∴	$g \vee h$
0 0 0	0	1	·	0
0 0 1	0	1	·	1
0 1 0	1	1	v	1
0 1 1	1	1	v	1
1 0 0	1	0	·	0
1 0 1	1	1	v	1
1 1 0	1	0	·	1
1 1 1	1	1	v	1

is valid

1	$\neg D \to G$	
2	$D \to H$	
3	D	
4	H	→E 2, 3
5	$G \vee H$	∨I 4
6	$\neg D$	
7	G	→E 1, 6
8	$G \vee H$	∨I 7
9	$G \vee H$	LEM 3–5, 6–8

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

d) c – crocodiles ...
I – incarcerating ...
m – mad ...
d – dinosaurs ...

c I m d	$c \to \bar{I}$	$m \wedge d \vee c$	I	∴	$d \leftrightarrow m$
0 0 0 0	1	0	0	·	1
0 0 0 1	1	0	0	·	0
0 0 1 0	1	0	0	·	0
0 0 1 1	1	1	0	·	1
0 1 0 0	1	0	1	·	1
0 1 0 1	1	0	1	·	0
0 1 1 0	1	0	1	·	0
0 1 1 1	1	1	1	v	1
1 0 0 0	1	1	0	·	1
1 0 0 1	1	1	0	·	0
1 0 1 0	1	1	0	·	0
1 0 1 1	1	1	0	·	1
1 1 0 0	0	1	1	·	1
1 1 0 1	0	1	1	·	0
1 1 1 0	0	1	1	·	0
1 1 1 1	0	1	1	·	1

1	$C \to \neg I$	
2	$(M \wedge D) \vee C$	
3	I	
4	$\neg I$	
5	$\bot$	¬E 3, 4
6	$\neg\neg I$	¬I 4–5
7	$\neg C$	MT 1, 6
8	$M \wedge D$	DS 2, 7
9	M	
10	D	∧E 8
11	D	
12	M	∧E 8
13	$D \leftrightarrow M$	↔I 9–10, 11–12

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

is valid

4. For each given argument, construct a deductive proof in Fitch notation using only basic rules.
(a) $\neg\neg A \therefore A$
(b) $((A \to B) \to A) \therefore A$
(c) $(\neg B \to \neg A) \therefore (A \to B)$
(d) $\neg(A \vee B) \therefore (\neg A \wedge \neg B)$
(e) $(\neg A \wedge \neg B) \therefore \neg(A \vee B)$
(f) $(A \to B) \wedge (\neg A \to B) \therefore B$

a)

1	$\neg\neg A$	
2	$\neg A$	
3	$\bot$	¬E 1, 2
4	A	IP 2–3

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

b)

1	$(A \to B) \to A$	
2	$\neg A$	
3	A	
4	$\bot$	¬E 2, 3
5	B	X 4
6	$A \to B$	→I 3–5
7	A	→E 1, 6
8	$\bot$	¬E 2, 7
9	A	IP 2–8

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

c)

1	$\neg B \to \neg A$	
2	A	
3	$\neg B$	
4	$\neg A$	→E 1, 3
5	$\bot$	¬E 2, 4
6	B	IP 3–5
7	$A \to B$	→I 2–6

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

d)

1	$\neg(A \vee B)$	
2	A	
3	$A \vee B$	∨I 2
4	$\bot$	¬E 1, 3
5	$\neg A$	¬I 2–4
6	B	
7	$A \vee B$	∨I 6
8	$\bot$	¬E 1, 7
9	$\neg B$	¬I 6–8
10	$\neg A \wedge \neg B$	∧I 5, 9

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

e)

1	$\neg A \wedge \neg B$	
2	$\neg A$	∧E 1
3	$\neg B$	∧E 1
4	$A \vee B$	
5	A	
6	$\bot$	¬E 2, 5
7	B	
8	$\bot$	¬E 3, 7
9	$\bot$	∨E 4, 5–6, 7–8
10	$\neg(A \vee B)$	¬I 4–9

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

f)

1	$(A \to B) \wedge (\neg A \to B)$	
2	$A \to B$	∧E 1
3	$\neg A \to B$	∧E 1
4	$\neg B$	
5	A	
6	B	→E 2, 5
7	$\bot$	¬E 4, 6
8	$\neg A$	¬I 5–7
9	B	→E 3, 8
10	$\bot$	¬E 4, 9
11	B	IP 4–10

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

5. For each given tautology, construct a deductive proof in Fitch notation.
(a) $(A \to B) \vee (B \to A)$
(b) $A \to (B \to A)$
(c) $(\neg B \to \neg A) \to ((\neg B \to A) \to B)$
(d) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$

a)

1	$\neg((A \to B) \vee (B \to A))$	
2	$\neg(A \to B) \wedge \neg(B \to A)$	DeM 1
3	$\neg(A \to B)$	∧E 2
4	$\neg(B \to A)$	∧E 2
5	A	
6	B	
7	A	R 5
8	$B \to A$	→I 6–7
9	$\bot$	¬E 4, 8
10	$\neg A$	
11	A	
12	$\bot$	¬E 10, 11
13	B	X 12
14	$A \to B$	→I 11–13
15	$\bot$	¬E 3, 14
16	A	IP 10–15
17	$\neg A$	¬I 5–9
18	$\bot$	¬E 16, 17
19	$(A \to B) \vee (B \to A)$	IP 1–18

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

b)

1	A	
2	B	
3	A	R 1
4	$B \to A$	→I 2–3
5	$A \to (B \to A)$	→I 1–4

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

c)

1	$\neg B \to \neg A$	
2	$\neg B \to A$	
3	$\neg B$	
4	A	→E 2, 3
5	$\neg A$	→E 1, 3
6	$\bot$	¬E 4, 5
7	B	IP 3–6
8	$(\neg B \to A) \to B$	→I 2–7
9	$(\neg B \to \neg A) \to ((\neg B \to A) \to B)$	→I 1–8

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

d)

1	$A \to (B \to C)$	
2	$A \to B$	
3	A	
4	B	→E 2, 3
5	$B \to C$	→E 1, 3
6	C	→E 4, 5
7	$A \to C$	→I 3–6
8	$(A \to B) \to (A \to C)$	→I 2–7
9	$(A \to (B \to C)) \to ((A \to B) \to (A \to C))$	→I 1–8

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.