$$Q(G,S) = \frac{1}{2m} \sum_{s \in S} \sum_{i \in s} \sum_{j \in s} \left(A_{ij} - \frac{k_i k_j}{2m}\right)$$

$$= \sum_{s \in S} \left\{ \underbrace{\left[\frac{1}{2m} \sum_{i \in s} \sum_{j \in s} A_{ij}\right]}_{\text{记为 I}} - \underbrace{\left[\frac{1}{2m} \sum_{i \in s} \sum_{j \in s} \frac{k_i k_j}{2m}\right]}_{\text{记为 II}} \right\}$$

$$\text{II} = \frac{1}{2m} \cdot \left(\frac{\sum_i k_i}{2m}\right)^2 = \frac{1}{2m}\left(\frac{\Sigma_{tot}}{2m}\right)^2$$

$\Sigma_{tot} = \sum_{i \in s} k_i$

举例 $k_1^2 + k_1 k_2 + k_1 k_3 + k_2^2 + k_2 k_1 + k_2 k_3 + k_3^2 + k_3 k_1 + k_3 k_2$
$= (k_1 + k_2 + k_3)^2$.

$$\text{I} = \frac{1}{2m}\, 2 \cdot \sum_{i<j} A_{ij} = \frac{\Sigma_{in}}{m}$$

Σ_{in}是 s 内部边的权重之和. 左侧为不考虑自环边情况

因此 $$Q(G,S) = \sum_{s \in S} \frac{\Sigma_{in}}{m} - \left(\frac{\Sigma_{tot}}{2m}\right)^2$$

推导 $\Delta Q(i \to c)$

$$= \left[\frac{\Sigma_{in} + k_{i,in}}{m} - \left(\frac{\Sigma_{tot} + k_i}{2m}\right)^2\right] - \left[\frac{\Sigma_{in}}{m} - \left(\frac{\Sigma_{tot}}{2m}\right)^2 - \left(\frac{k_i}{2m}\right)^2\right]$$

$$= \frac{k_{i,in}}{m} - \frac{k_i \cdot \Sigma_{tot}}{2m^2}$$

$$= \frac{1}{m}\left[k_{i,in} - \frac{k_i \cdot \Sigma_{tot}}{2m}\right]$$

后面例子用的是这个指标.