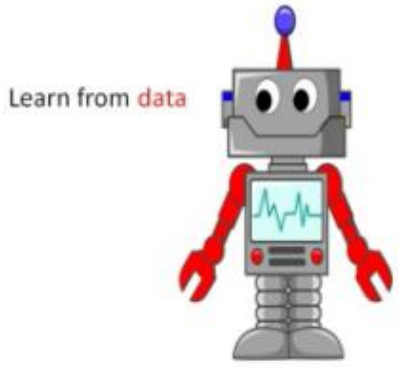
Artificial Intelligence

❑ Propositional Logic in Artificial intelligence

Facts about Propositional Logic
Types of Propositional Logic
Logical Connectives
Truth Table
Precedence of Connectives
Logical Equivalence
Properties of Operators
Limitations of Propositional Logic

❑ Rules of Inference in Artificial intelligence

Inference
Inference Rules
Types of Inference Rules



❑ Propositional Logic in Artificial intelligence

Propositional Logic in Artificial intelligence

Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions. A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

Example

a) It is Sunday.
b) The Sun rises from West (False proposition)
c) 3+3= 7(False proposition)
d) 5 is a prime number.

Facts about Propositional Logic

- Propositional logic is also called Boolean logic as it works on 0 and 1.
- In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol for a representing a proposition, such A, B, C, P, Q, R, etc.
- Propositions can be either true or false, but it cannot be both.
- Propositional logic consists of an object, relations or function, and **logical connectives**.
- These connectives are also called logical operators.
- The propositions and connectives are the basic elements of the propositional logic.
- Connectives can be said as a logical operator which connects two sentences.

Facts about Propositional Logic

- A proposition formula which is always true is called **tautology**, and it is also called a valid sentence.
- A proposition formula which is always false is called **Contradiction**.
- A proposition formula which has both true and false values is called **Contingency.**
- Statements which are questions, commands, or opinions are not propositions such as **"Where is Rohini", "How are you", "What is your name"**, are not propositions.
- The syntax of propositional logic defines the allowable sentences for the knowledge representation.
- A literal in propositional logic is a variable or its negation

Types of Propositional Logic

There are two types of Propositions:

1. Atomic Propositions
2. Compound propositions

Atomic Proposition: Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

- ”2+2 is 4”, it is an atomic proposition as it is a true fact.
- "The Sun is cold" is also a proposition as it is a false fact.

Compound Proposition: Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

- "It is raining today, and the road is slippery."
- “Samar is a doctor, and his clinic is in Karachi."

Logical Connectives

Logical connectives are used to connect two simpler propositions or representing a sentence logically. We can create compound propositions with the help of logical connectives. There are mainly five connectives, which are given as follows:

1. **<u>Negation:</u>**

   A sentence such as ¬ P is called negation of P. A literal can be either Positive literal or negative literal.

   Example:

   P = Today is Sunday.

   ¬ P = Today is not Sunday.

   Q = It is raining.

   ¬ Q = It is not raining.

Logical Connectives

2. **<u>Conjunction:</u>**

A sentence which has ∧ connective such as, **P ∧ Q** is called a conjunction. **Example:**

"Samar is intelligent and hardworking". It can be written as:

P = Samar is intelligent, **Q = Samar is hardworking.**

P∧ Q = Samar is intelligent and hardworking.

3. **<u>Disjunction:</u>**

A sentence which has ∨ connective, such as **P ∨ Q**. is called disjunction, where P and Q are the propositions.

Example:

"Samar is a doctor or an Engineer."

P = Samar is Doctor.

Q= Samar is an Engineer.

P ∨ Q= Samar is a doctor or an Engineer.

Logical Connectives

4. <u>Implication:</u>

A sentence such as $P \rightarrow Q$, is called an implication. Implications are also known as if-then rules. It can be represented as

“If it is raining, **then** the road is slippery.”

Let **P= It is raining**, and **Q= Street is wet**, so

$P \rightarrow Q$ = If it is raining, **then** the road is slippery.

5. <u>Biconditional:</u>

A sentence such as **$P \Leftrightarrow Q$ is a Biconditional sentence,**

Example: “If I am breathing, then I am alive.

P= I am breathing, **Q= I am alive**, it can be represented as:

$P \Leftrightarrow Q$ = If I am breathing, then I am alive.

Summarized Table for PL Connectives

Connective symbols	Word	Technical term	Example
$\wedge$	AND	Conjunction	$A \wedge B$
$\vee$	OR	Disjunction	$A \vee B$
$\rightarrow$	Implies	Implication	$A \rightarrow B$
$\Leftrightarrow$	If and only if	Biconditional	$A \Leftrightarrow B$
$\neg$ or $\sim$	Not	Negation	$\neg A$ or $\neg B$

Truth Table

In propositional logic, we need to know the truth values of propositions in all possible scenarios. We can combine all the possible combination with logical connectives, and the representation of these combinations in a tabular format is called Truth table. Following are the truth table for all logical connectives:

For Negation:

P	¬ P
True	**False**
False	**True**

For Conjunction:

P	Q	P∧ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

Truth Table

For disjunction:

P	Q	P V Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

For Implication:

P	Q	P→ Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

For Biconditional:

P	Q	P⇔ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**True**

Truth Table with Three Propositions

We can build a proposition composing three propositions P, Q, and R. This truth table is made-up of 8 Tuples as we have taken three proposition symbols.

P	Q	R	¬R	P∨Q	P∨Q→¬R
True	True	True	False	True	False
True	True	False	True	True	True
True	False	True	False	True	False
True	False	False	True	True	True
False	True	True	False	True	False
False	True	False	True	True	True
False	False	True	False	False	True
False	False	False	True	False	True

Precedence of Connectives

Just like arithmetic operators, there is a precedence order for propositional connectors or logical operators. This order should be followed while evaluating a propositional problem. Following is the list of the precedence order for operators:

Precedence	Operators
First Precedence	Parenthesis
Second Precedence	Negation
Third Precedence	Conjunction(AND)
Fourth Precedence	Disjunction(OR)
Fifth Precedence	Implication
Six Precedence	Biconditional

For better understanding use parenthesis to make sure of the correct interpretations. Such as ¬R∨ Q, It can be interpreted as (¬R) ∨ Q.

Logical Equivalence

Logical equivalence is one of the features of propositional logic. Two propositions are said to be logically equivalent if and only if the columns in the truth table are identical to each other.

Let's take two propositions A and B, so for logical equivalence, we can write it as A⇔B. In below truth table we can see that column for ¬A∨ B and A→B, are identical hence A is Equivalent to B.

A	B	¬A	¬A∨ B	A→B
T	T	F	**T**	**T**
T	F	F	**F**	**F**
F	T	T	**T**	**T**
F	F	T	**T**	**T**

Properties of Operators

Commutativity:

$P \wedge Q = Q \wedge P$, or

$P \vee Q = Q \vee P$.

Associativity:

$(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$,

$(P \vee Q) \vee R = P \vee (Q \vee R)$

Identity element:

$P \wedge \text{True} = P$,

$P \vee \text{True} = \text{True}$.

Distributive:

$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$.

$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$.

DE Morgan's Law:

$\neg (P \wedge Q) = (\neg P) \vee (\neg Q)$

$\neg (P \vee Q) = (\neg P) \wedge (\neg Q)$.

Double-negation elimination:

$\neg (\neg P) = P$.

Limitations of Propositional Logic

- We cannot represent relations like ALL, some, or none with propositional logic.

  Example:

  All the girls are intelligent.

  Some apples are sweet.

- Propositional logic has limited expressive power.
- In propositional logic, we cannot describe statements in terms of their properties or logical relationships.

❑ Rules of Inference in Artificial intelligence

Inference and Inference Rules

In artificial intelligence, we need intelligent computers which can create new logic from old logic or by evidence, so generating the conclusions from evidence and facts is termed as Inference.

Inference rules are the templates for generating valid arguments.
Inference rules are applied to derive proofs in artificial intelligence, and the **proof** is a sequence of the conclusion that leads to the desired goal.
In inference rules, the implication among all the connectives plays an important role.

Following are some terminologies related to inference rules:

Terminologies

Implication:

It is one of the logical connectives which can be represented as $P \rightarrow Q$.

It is a Boolean expression.

Converse:

The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as $Q \rightarrow P$.

Contrapositive:

The negation of converse is termed as contrapositive, and it can be represented as $\neg Q \rightarrow \neg P$.

Inverse: The negation of implication is called inverse.

It can be represented as $\neg P \rightarrow \neg Q$.

Terminologies

From the above term some of the compound statements are equivalent to each other, which we can prove using truth table:

P	Q	$P \to Q$	$Q \to P$	$\neg Q \to \neg P$	$\neg P \to \neg Q.$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Hence from the above truth table, we can prove that $P \to Q$ is equivalent to $\neg Q \to \neg P$, and $Q \to P$ is equivalent to $\neg P \to \neg Q$.

Types of Inference Rules

1. Modus Ponens:

The Modus Ponens rule is one of the most important rules of inference, and it states that if P and P → Q is true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $$\frac{P \rightarrow Q, \quad P}{\therefore Q}$$

Example:

Statement-1: "If I am sleepy then I go to bed"
==> P→ Q

Statement-2: "I am sleepy" ==> P

Conclusion: "I go to bed." ==> Q.

Hence, we can say that,
if P→ Q is true and P is true then Q will be true.

Proof by Truth Table:

P	Q	$P \rightarrow Q$
0	0	0
0	1	1
1	0	0
1	1	1

Types of Inference Rules

2. Modus Tollens:

The Modus Tollens rule state that if P→ Q is true and ¬ Q is true, then ¬ P will also true. It can be represented as:

Notation for Modus Tollens: $\dfrac{P \rightarrow Q, \quad \sim Q}{\sim P}$

Example:

Statement-1: "If I am sleepy then I go to bed"
==> $P \rightarrow Q$

Statement-2: "I do not go to the bed."
==> $\sim Q$

Statement-3: Which infers that
"I am not sleepy" => $\sim P$

Proof by Truth Table:

P	Q	$\sim P$	$\sim Q$	$P \rightarrow Q$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

Types of Inference Rules

3. Hypothetical Syllogism:

The Hypothetical Syllogism rule state that if P→R is true whenever P→Q is true, and Q→R is true.

Example:
Statement-1: If you have my home key then you can unlock my home. P→Q
Statement-2: If you can unlock my home then you can take my money. Q→R
Conclusion: If you have my home key then you can take my money. P→R

Proof by Truth Table:

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

Types of Inference Rules

4. Disjunctive Syllogism:

The Disjunctive syllogism rule state that if P∨Q is true, and ¬P is true, then Q will be true. It can be represented as:

Notation of Disjunctive syllogism: $\dfrac{P \vee Q, \quad \neg P}{Q}$

Example:

Statement-1: Today is Sunday or Monday. ==>P∨Q

Statement-2: Today is not Sunday. ==> ¬P

Conclusion: Today is Monday. ==> Q

Proof by Truth Table:

P	Q	¬P	P∨Q
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

Types of Inference Rules

5. Addition:

The Addition rule is one the common inference rule, and it states that If P is true, then PvQ will be true.

Notation of Addition: $\dfrac{P}{P \vee Q}$

Example:

Statement: I have a vanilla ice-cream.
==> P
Statement-2: I have Chocolate ice-cream.
==> Q
Conclusion: I have vanilla or chocolate ice-cream.
==> (PvQ)

Proof by Truth Table:

P	Q	$P \vee Q$
0	0	0
1	0	1
0	1	1
1	1	1

Types of Inference Rules

6. Simplification:

The simplification rule state that if P∧ Q is true, then Q or P will also be true. It can be represented as:

Notation of Simplification rule: $\dfrac{P \land Q}{Q}$ Or $\dfrac{P \land Q}{P}$

Proof by Truth Table:

P	Q	$P \land Q$
0	0	0
1	0	0
0	1	0
1	1	1

Types of Inference Rules

7. Resolution:

The Resolution rule state that if P∨Q and ¬ P∧R is true, then Q∨R will also be true. It can be represented as:

Notation of Resolution $\dfrac{P \vee Q, \quad \neg P \wedge R}{Q \vee R}$

Proof by Truth Table:

P	¬ P	Q	R	$P \vee Q$	¬ P∧R	$Q \vee R$
0	1	0	0	0	0	0
0	1	0	1	0	0	1
0	1	1	0	1	1	1 ←
0	1	1	1	1	1	1 ←
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	1	0	1 ←