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STATE VARIABLE ESTIMATION USING ADAPTIVE KALMAN FILTER WITH ROBUST SMOOTHING

F. D. Groutage
R. G. Jacquot
D. E. Smith

Naval Ocean Systems Center, San Diego, CA 92152
University of Wyoming, Laramie, WY 82071
Computer Sciences Corporation, San Diego, CA 92110

Summary

The development of a conventional Kalman filter is based on full knowledge of system parameters, noise statistics and deterministic forcing functions. This work addresses the problem of known system parameters and unknown noise statistics and deterministic forcing functions. Two concepts are investigated: 1) adaptive weight functions for the Kalman filter gain and error covariance matrices, where these weights are functions of sample means and variances of the innovations sequence; and 2) robust smoothing of the estimated state variables. The concepts presented relative to this particular problem address the limited class of linear system dynamics with associated linear measurements. Nonlinear system dynamics with associated linear or nonlinear measurements, however, are not precluded. The concepts apply to those cases where the observations made by a sensor are the variables to be estimated.

An application to a simple linear system is presented; however, primary application would be to the estimation of position, velocity and acceleration for a maneuvering body in three dimensional space based on observed data collected by a remote sensor tracking the maneuvering body. Estimates of the state variables using the adaptive process for the simple linear system during the periods when the system is not being forced are relatively close to those of the conventional Kalman filter for congruent periods, but there is some increase in mean square error because the adaptive estimator is no longer optimal. During periods when the system is being forced a vast improvement, as compared with those estimates of the conventional Kalman filter, is realized with the adaptive gain, covariance weight, and associated robust smoothing procedure. The estimates derived with the adaptive procedure during the periods of system forcing do, however, contain a considerable level of mean-square error. This seems to be a prevailing shortfall of adaptive estimation procedures. The tradeoff is knowledge of the deterministic forcing functions versus high mean-square estimate error in the absence of that knowledge.

Conventional Estimation Process

The conventional Kalman filter works well when operating consistently with the assumptions made in its derivation; however, when the system is subjected to deterministic forcing functions, unbeknownst to the estimation process, the state estimate will diverge from the true state [1], [2]. This is particularly the case when estimating the location of an aircraft undergoing an evasive maneuver.

Figure 1 presents a block diagram of a system model, measurement system, and discrete Kalman filter. The system model could be a discrete representation of a continuous system being observed at discrete times by a measurement system. For this case the system of interest is governed by the stochastic matrix-vector difference equation

$$x(k+1) = Ax(k)+w(k)+Bu(k);\ w(k)\sim N(q(k),Q(k)) \quad (1)$$

with a linear measurement process defined by

$$z(k) = Hx(k) + v(k);\ v(k)\sim N(r(k),R(k)) \quad (2)$$

where $x(k)$ and $w(k)$ are n-vectors; $v(k)$ and $z(k)$ are j-vectors and $u(k)$ an r-vector. All the matrices are of correct size to have the products and sums of (1) and (2) defined. It is assumed that knowledge of the forcing function $u(k)$ exists; thus, the Kalman filter, as illustrated in Figure 1, reflects this knowledge.

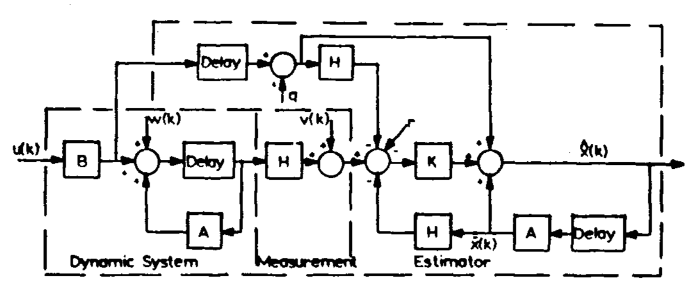

FIGURE 1. SYSTEM MODEL AND CONVENTIONAL KALMAN FILTER

Adaptive Estimation Process

Consider the case where a system is being forced by forcing functions $u(k)$ and $w(k)$ and the output is corrupted by noise to produce measurement $z(k)$. The deterministic forcing function, $u(k)$, often changes rapidly with time. It is desired to formulate estimates of the values of the state variables in a timely manner by using a Kalman filter; however, for this case, knowledge of the system forcing function, $u(k)$, is unknown to the filter. A problem of equal importance is the case where the statistics of the process noise, $w(k)$, are unknown. An excellent treatment of estimation in the presence of unknown noise statistics is presented in Myers and Tapley where empirical estimators of the noise statistics are developed [3].

For the case at hand, state estimation without knowledge of the deterministic forcing functions, several modifications have been incorporated into the estimation process. These include adaptive weighting of the elements of the conventional Kalman gain and covariance matrices, as well as robust statistical smoothing of the estimates made by the adaptive Kalman filter using the modified gain and covariance matrices. Figure 2 illustrates the estimation process with these modifications incorporated. The modified estimator gain matrix $\Lambda\,[K(k)]$ is defined in the next sections

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as well as the robust statistical smoothing procedure.

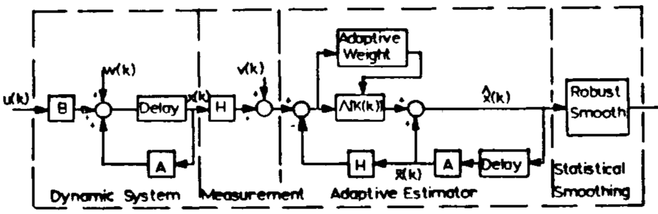

FIGURE 2. MODIFIED ESTIMATOR FOR ADAPTIVE ROBUST ESTIMATION

Robust Estimation of Observed State Variables Using Adaptive Weights for Gain and Error Covariance Matrices

The concepts presented here are empirical in nature as they are based on observations and experimental data. An intuitive perception of these concepts can be obtained by examining the Kalman filtering algorithm for a scalar system with process noise $w(k)$

$$x(k+1) = a\ x(k)+w(k),\ w(k) \sim N(0,Q) \tag{3}$$

and the associated noisy measurements; $v(k)$ is the measurement noise

$$z(k) = x(k) + v(k),\ v(k) \sim N(0,R) \tag{4}$$

The Kalman filter estimation equations for estimating $x(k)$ are

$$\hat{x}(k) = \overline{x}(k) + K(k)\ [z(k)-\overline{x}(k)];\ \overline{x}(k)=a\ \hat{x}(k-1) \tag{5}$$

$$\hat{P}(k) = [1-K(k)]\ \overline{P}(k);\ \overline{P}(k) = a^2\hat{P}(k-1) + Q \tag{6}$$

where the gain is defined as

$$K(k) = \frac{\overline{P}(k)}{\overline{P}(k) + R} \tag{7}$$

Equation (5) for estimating the state variable $x(k)$ contains interesting information concerning the estimation process. Note that the scalar gain, $K(k)$, is bounded from above and below as

$$0 \le K(k) \le 1. \tag{8}$$

For the case when $K(k) = 0$, equation (5) indicates that total faith is placed in the estimation process. In fact, the measurements are ignored and the previous estimate is the updated estimate. Now consider the case where $K(k) = 1$ which indicates that there will be no faith in the estimation process and, in fact, the current measurement is the updated estimate. With these concepts in mind the idea of a pseudogain $\alpha(k)$ is investigated. The pseudogain $\alpha(k)$ is defined as

$$\alpha(k) = 1 + e^{-\beta(k)}K(k) - e^{-\nu\beta(k)} \tag{9}$$

where ν and $\beta(k)$ are parameters to be established and ν can be a constant. The observation residual, $y(k)$, is defined as the difference between the measurement and the propagated state estimate

$$y(k) = z(k) - \overline{x}(k).$$

The recursive sample mean and variance of the observation residual sequence over the sampling interval N_e are $\overline{y}(k)$ and $\hat{\sigma}_y{}^2(k)$. These two quantities are developed in (4).

If the parameter $\beta(k)$ of equation (9) is chosen in the following manner

$$\beta(k) = \gamma\hat{\sigma}_y{}^2(k) \tag{10}$$

then $\alpha(k)$ is a function of the dispersion of the residual sequence $y(k)$. Note that as $\beta(k)$ becomes small the pseudogain $\alpha(k)$ approaches the conventional Kalman filter gain $K(k)$ and as $\beta(k)$ becomes large the pseudogain approaches unity. Thus for small dispersion of the residuals, the gain approaches the optimum Kalman filter gain $K(k)$ while for large dispersion of the residuals no faith is given to the estimation process.

From computer experiments, it was found that an adaptive weighting function for the propagated error covariance $\overline{P}(k)$ was also required [4]. Additionally the concepts of adaptive weighting of the Kalman gain and error covariance matrices can be extended to the vector case. The matrix equivalent of $\alpha(k)$ in Figure 2 is $\Lambda[K(k)]$.

Robust Smoothing

The estimates of the state variables made by the Kalman filter with the modified gain and covariance matrices contain occasional outliers. This is a result of the sampling procedure and the way in which the sample statistics of the residual sequence are utilized to formulate weights for the elements of the gain and covariance matrices. To alleviate the outliers in the state estimates, a robust statistical smoothing concept was incorporated into the estimation procedure. The robust smoother uses a regression procedure in the following manner. Consider a set of n recent estimates of the ith state variable or

$$\lambda = \{\hat{x}_i(k-n-1),\ \hat{x}_i(k-n),\ \hat{x}_i(k-n+1),\ \ldots\hat{x}_i(k)\}. \tag{11}$$

It is desired to find a weighted-least-squares solution for the straight-line regression fit through the n samples of the estimates of the ith state variable, $\hat{x}_i$, over the discrete temporal interval from k-n-1 to k. Details of weighted-least-squares regression theory is presented in [5]. The weight function, w_j, was chosen to be the biweight as defined in [6]. The least-squares solution for the straight-line regression fit through the n samples of the estimates of the ith state variable is used to project n-1 past values of the estimates (as formulated by the adaptive filter) of the ith state variable up to the present discrete time, t = k. The n-1 past values of the estimates of the ith state variable

$$\{\hat{x}_i(k-n-1),\ \hat{x}_i(k-n),\ \ldots,\ \hat{x}_i(k-1)\} \tag{12}$$

which are projected to discrete time t = k define n values of the random variable

$$x_j^\dagger(k).$$

The newly formed random variable, $x_j^\dagger(k)$, is smoothed by using the relationship

$$\hat{x}_i{}'(k) = \frac{\sum_{j=1}^{n} W_j x_j^\dagger(k)}{\sum_{j=1}^{n} W_j} \tag{13}$$

where $\hat{x}_i{}'(k)$ is the smoothed value of the estimated value of the ith state variable as generated by the modified gain and covariance Kalman filter. A new estimate, at discrete time k+1, of the ith state variable is generated, $\hat{x}_i(k+1)$, which is subsequently smoothed by means of the above process; however, the sample space now spans the discrete time interval from time k-n to time k+1. The sample set λ of equation (11) is now defined as

$$\lambda = \{\hat{x}_i(k-n),\ \hat{x}_i(k-n+1,\ \ldots\hat{x}_i(k+1)\} \tag{14}$$

A new weighted-least-squares solution for the straight-line regression fit through the n samples is found and the process repeats as outlined above.

Simulation Results

The system of Figure 3 (with a = 2 and b = 3) was simulated digitally, the output $x_1(t)$ was sampled in time and corrupted by discrete sensor noise, v(k) with zero mean and a variance of 25. The system was driven only by a deterministic forcing function u(t) which was a pulse with a magnitude of 500 units and a 22 second duration.

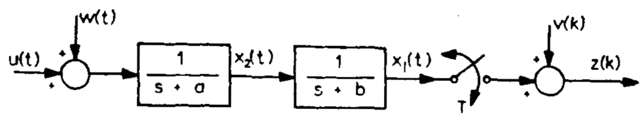

FIGURE 3. LINEAR SYSTEM EXAMPLE

A conventional Kalman filter, without any a priori knowledge of the forcing function u(t) or the time at which the forcing function was applied, was used to process the measurement data z(k). The conventional Kalman filter did not detect the influence of the deterministic forcing functions on the state variables, as illustrated in Figure 4. Since a priori data dictated that there was no process noise, the elements of the Kalman filter gain matrix associated with the observed variables approach zero; thus the estimation process has severed itself from the measurement process and ignores new data brought forth by additional measurements.

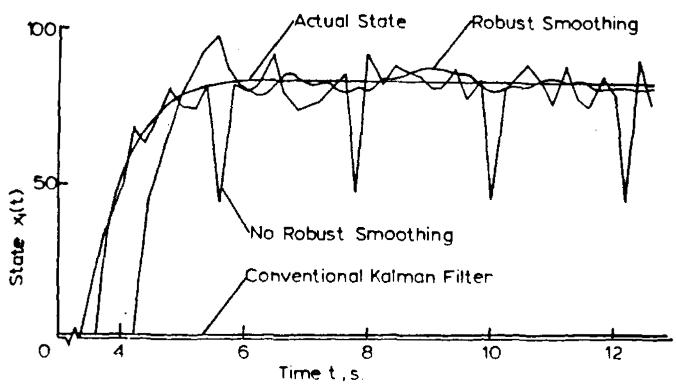

FIGURE 4. COMPARISON OF FILTERING TECHNIQUES

When the elements of the Kalman filter gain and covariance matrices are weighted by the adaptive procedure outlined above (sample statistics of the residual sequence are used to adapt the respective weights), the filter no longer divorces itself from the measurement process. Additional data brought forth by the measurement process are used to update the estimates of the state variables as illustrated in Figure 4. However, since the adaptive procedure uses sample statistics, the estimates contain periodic outliers. The filter will run for a period of time, then monitor the residual sequence to update the adaptive weights. It is this monitoring of the residual sequence to obtain new information which causes the periodic outlier to appear in the estimates. The subsequent processing of the adaptive estimates by a robust smoother reduces the level of mean square error and the periodic outliers.

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