SOME PRACTICE PROBLEMS

1. If $u = (1-2xy+y^2)^{\frac{-1}{2}}$ then prove that
$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$ and $\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[y^2\frac{\partial u}{\partial y}\right] = 0$
2. If $u = \sin(\sqrt{x}+\sqrt{y}+\sqrt{z})$, prove that
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = \frac{1}{2}(\sqrt{x}+\sqrt{y}+\sqrt{z})cos(\sqrt{x}+\sqrt{y}+\sqrt{z})$
3. If $u = \log(\tan x + \tan y + \tan z)$, then show that $\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2.$
4. If $u = e^{xyz}$, prove that $\frac{\partial^3 u}{\partial x\partial y\,\partial x} = (1+3xyz+x^2y^2z^2)e^{xyz}$.
5. If$u = x^3y + e^{xy^2}$, prove that $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$.
6. If $u = \log(x^2+y^2)$, prove that $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$.
7. If $u = x^y$, prove that $\frac{\partial^3 u}{\partial x^2\,\partial y} = \frac{\partial^3 u}{\partial x\partial y\,\partial x}$.
8. If $z = x^y + y^x$, prove that $\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial y\,\partial x}$.
9. If $z(x+y) = (x-y)$, Find $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2$.
10. $u = \tan^{-1}\left(\frac{y}{x}\right)$ Find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.
11. If $z = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, prove that $\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial y\,\partial x} = \frac{x^2-y^2}{x^2+y^2}$.
12. If $u = (x^2+y^2+z^2)^{-\frac{1}{2}}$ $[or\ \frac{1}{u^2} = x^2+y^2+z^2\ or\ u = \frac{1}{r}\ and\ r = \sqrt{x^2+y^2+z^2}]$,
then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.
13. If $u = f\left(\frac{x^2}{y}\right)$, prove that $x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = 0$ and $x^2\frac{\partial^2 u}{\partial x^2} + 3xy\frac{\partial^2 u}{\partial x\,\partial y} + 2y^2\frac{\partial^2 u}{\partial y^2} = 0$
14. If $z = \log(e^x+e^y)$, prove that $rt - s^2 = 0, where, r = \frac{\partial^2 z}{\partial x^2}$, $t = \frac{\partial^2 z}{\partial y^2}$, $s = \frac{\partial^2 z}{\partial x\,\partial y}$.
15. If $u = log(x^3 + y^3 - x^2y - xy^2)$, prove that
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$ and $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 u}{\partial y^2} = -\frac{4}{(x+y)^2}$
16. If $u = f(r)$ & $r^2 = x^2+y^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$.
17. If $u = f(r^2)$ & $r^2 = x^2+y^2+z^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 4r^2f''(r^2) + 6f'(r^2)$
18. If $z = \tan(y+ax) + (y-ax)^{\frac{3}{2}}$, then show that $\frac{\partial^2 z}{\partial x^2} = a^2\frac{\partial^2 z}{\partial y^2}$.
19. If $u = x\log(x+r) - r$, $r^2 = x^2+y^2$, prove that $i)\ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{x+r}$ $ii)\ \frac{\partial^3 u}{\partial x^3} = -\left(\frac{x}{r^3}\right)$

20. If $u\,(x,t) = a\ e^{-g}\ \sin\ (nt - gx)$ where a, g, n are constants, satisfying the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$, prove that $g = \frac{1}{a}\sqrt{\frac{n}{2}}$.

21. If $v = r^n(3cos^2\theta - 1)$ then, find the value of n so that $\frac{\partial}{\partial r}\left(r^2\ \frac{\partial v}{\partial r}\right) + \frac{1}{sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\ \frac{\partial v}{\partial \theta}\right) = 0$

22. If $u = e^{xyz} f\left(\frac{xy}{z}\right)$, prove that $x\frac{\partial u}{\partial x} + z\ \frac{\partial u}{\partial z} = 2xyzu,\quad y\frac{\partial u}{\partial y} + z\ \frac{\partial u}{\partial z} = 2xyzu,$

Hence, show that $x\ \frac{\partial^2 u}{\partial z \partial x} = y \frac{\partial^2 u}{\partial z \partial y}$.

23. If $z = c\,t^{-\frac{1}{2}}\, e^{-\frac{x^2}{4a^2 t}}$, prove that $\frac{\partial z}{\partial t} = a^2\ \frac{\partial^2 z}{\partial x^2}$.

24. If $u = \frac{e^{x+y+z}}{e^x + e^y + e^z}$, show that $u_x + u_y + u_z = 2\,u.$

25. If $u = 3(ax + by + cz)^2 - (x^2 + y^2 + z^2)$ and $a^2 + b^2 + c^2 = 1$,

show that $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 y}{\partial z^2} = 0.$

26. If $x = \cos\theta - r\sin\theta, y = \sin\theta + r\cos\theta$, prove that $\frac{\partial r}{\partial x} = \frac{x}{r}$.

27. If $x = r\cos\theta, y = r\sin\theta$ prove that

$i)\ \frac{\partial x}{\partial r} = \frac{\partial r}{\partial x} \quad ii)\ \frac{\partial x}{\partial \theta} = r^2\ \frac{\partial \theta}{\partial x} \quad iii)\ \left(x\frac{\partial x}{\partial r} + y\frac{\partial y}{\partial r}\right)^2 = x^2 + y^2.$

$iv)\ \frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right] \quad v)\ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$