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Department of Science and Humanities
F.Y. B. Tech. Semester -I (2024-25)
Applied Mathematics-I
IA-II

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BERNOULLI'S EQUATION AND SOLUTION

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References:
https://www.youtube.com/watch?v=NjIMGAIPbzg
https://tutorial.math.lamar.edu/classes/de/bernoulli.aspx
https://salfordphysics.com/gsmcdonald/H-Tutorials/Bernoulli-differential-equations.pdf

A Bernoulli differential equation can be written in the following standard form:

$$\boxed{\frac{dy}{dx} + P(x)y = Q(x)y^n}$$

where n= 1 (the equation is thus nonlinear).

To find the solution, change the dependent variable from y to z, where z =y^{1-n}.This gives a differential equation in x and z that is linear, and can be solved using the integrating factor method.

Note: Dividing the above standard form by y^n gives:

$$\frac{1}{y^n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x)$$

$$\text{i.e. } \frac{1}{(1-n)}\frac{dz}{dx} + P(x)z = Q(x)$$

$$\left(\text{where we have used } \frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}\right).$$

Integrating factor method

An ordinary differential equation (ODE) is first-order if it involves only the first derivative dz/dx. If it is also linear, each term can involve z either through the first derivative or as a single factor of z.

Any such linear first order o.d.e. can be re-arranged to give the following standard form:

$$\boxed{\frac{dz}{dx} + P_1(x)z = Q_1(x)}$$

A linear first-order ODE can be solved using the integrating factor method.

1.Multiply by the integrating factor:

$$\boxed{\mathrm{IF} = e^{\int P_1(x)\,dx}}$$

2.This transforms the equation to:

$$\boxed{\frac{d}{dx}(\mathrm{IF}\cdot z) = \mathrm{IF}\cdot Q_1(x)}$$

3.Integrate both sides:

$$\boxed{\mathrm{IF}\cdot z = \int \mathrm{IF}\cdot Q_1(x)\,dx}$$

Finally, division by the integrating factor (IF) gives z explicitly in terms of x, i.e. **gives the solution to the equation.**