

F.Y. B. Tech SEM-I
Applied Mathematics-I

Practice Problems
Differential Equation of First Order and First Degree

Solve the following.

1. $(x^3 e^x - my^2)dx + mxy dy = 0$

2. $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2y dy = 0$

3. $(y - 2x^3)dx - x(1 - xy)dy = 0$

4. $(x^2 + y^2 + 1)dx - 2xy dy = 0$

5. $y(xy + e^x)dx - e^x dy = 0$

6. $(2xy^2 - y)dx + xdy = 0$

7. $(x + 2y^3)\frac{dy}{dx} = y$

8. $y(x^2y + e^x)dx - e^x dy = 0$

9. $\frac{dy}{dx} = \frac{y}{2y \log y + y - x}$

10. $(y^2 e^{xy^2} + 4x^3)dx + (2xy e^{xy^2} + 3y^2)dy = 0$

11. $x dx + y dy = \frac{a^2(x dx - y dy)}{x^2 + y^2}$

12. $(y^3 - 3x^3y)dy + (x^3 - 3xy^3)dx = 0$

13. $(x\sqrt{x^2 + y^2} - y)dx + (y\sqrt{x^2 + y^2} - x)dy = 0$

14. $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

15. $(y^3 - 2yx^2)dx + (2xy^2 - x^3)dy = 0$

16. $x(x - y)dy + y^2dx = 0$

17. $(x^3 + y^3)dx - xy^2dy = 0$

18. $(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

19. $(y - xy^2)dx - (x + x^2y)dy = 0$

20. $y(2xy + 1)dx + x(1 + 2xy - x^3y^3)dy = 0$

21. $y(1 + xy + x^2y^2 + x^3y^3)dx + x(1 - xy - x^2y^2 + x^3y^3)dy = 0$

22. $(x^2 + y^2 + 2x)dx + 2y dy = 0$

23. $\frac{dy}{dx} + \left(\frac{1-2x}{x^2}\right)y = 1$

Answer: $\left[y = x^2 + ce^{\frac{1}{x}} \cdot x^2\right]$

24. $(1 + y^2)dx = (e^{\tan^{-1}y} - x)dy$

Answer: $\left[xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + c\right]$

25. $x dy - (y - x)dx = 0$

Answer: $[y + x \log x = cx]$

- | | |
|-----------------------------------------------------------------------------|--------------------------------------------------------------------|
| 26. $\frac{dy}{dx} + \frac{y}{1-x} = x^2 - x$ | Answer: $[2y = (1-x)(c^2 - x^2)]$ |
| 27. $x(1-x^2)\frac{dy}{dx} + (2x^2-1)y = x^3$ | Answer: $[y = \tan x + cx\sqrt{1-x^2}]$ |
| 28. $\cos^2 x \frac{dy}{dx} + y = \tan x$ | Answer: $[y = \tan x - 1 + ce^{-\tan x}]$ |
| 29. $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$ | Answer: $[\frac{xy}{1-x} + \frac{x^3}{3} = c]$ |
| 30. $(1+x+xy^2)dy + (y+y^3)dx = 0$ | Answer: $[xy + \tan^{-1}y = c]$ |
| 31. $x \cos y \frac{dy}{dx} - \sin y = x \sin^2 y$ | Answer: $[cosec y + x(\log x + c) = 0]$ |
| 32. $\sec^2 y \frac{dy}{dx} + 2 \tan x \tan y = \sin x$ | Answer: $[\sec^2 x \tan y = \sec x + c]$ |
| 33. $y \frac{dy}{dx} + \frac{4x}{3} - \frac{y^2}{3x} = 0$ | Answer: $[y^2 x^{-\frac{2}{3}} + 2x^{\frac{4}{3}} = c]$ |
| 34. $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ | Answer: $[\tan y = \frac{1}{2}(x^2 - 1) + ce^{-x^2}]$ |
| 35. $\frac{dy}{dx} + (2x \tan^{-1}y - x^3)(1+y^2) = 0$ | Answer: $[\tan^{-1}y = \frac{x^2-1}{2} + ce^{-x^2}]$ |
| 36. $\frac{dy}{dx} + x^3 \sin^2 y + x \sin 2y = x^3$ | Answer: $[\tan y \cdot e^{x^2} = \frac{1}{2}e^{x^2}(x^2 - 1) + c]$ |
| 37. $x \cos y \frac{dy}{dx} - \sin y = x \sin^2 y$ | Answer: $[cosec y + x(\log x + c) = 0]$ |
| 38. $\sec^2 y \frac{dy}{dx} + 2 \tan x \tan y = \sin x$ | Answer: $[\sec^2 x \tan y = \sec x + c]$ |
| 39. $y \frac{dy}{dx} + \frac{4x}{3} - \frac{y^2}{3x} = 0$ | Answer: $[y^2 x^{-\frac{2}{3}} + 2x^{\frac{4}{3}} = c]$ |
| 40. $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ | Answer: $[\tan y = \frac{1}{2}(x^2 - 1) + ce^{-x^2}]$ |
| 41. $\frac{dy}{dx} + (2x \tan^{-1}y - x^3)(1+y^2) = 0$ | Answer: $[\tan^{-1}y = \frac{x^2-1}{2} + ce^{-x^2}]$ |
| 42. $\frac{dy}{dx} + x^3 \sin^2 y + x \sin 2y = x^3$ | Answer: $[\tan y \cdot e^{x^2} = \frac{1}{2}e^{x^2}(x^2 - 1) + c]$ |
| 43. $\frac{dy}{dx}(x^2 y^3 + xy) = 1$ | Answer: $[x(2 - y^2) + (c x e^{-\frac{y^2}{2}}) = 1]$ |
| 44. $\frac{dy}{dx} + y \tan x = y^2 \sec x$ | Answer: $[\frac{1}{y} \cos x = -x + c]$ |
| 45. $\frac{dy}{dx} + y \cos x = y^3 \sin 2x$ | Answer: $[\frac{1}{y^2} = (1 + 2 \sin x) + c e^{2 \sin x}]$ |
| 46. $\frac{dy}{dx} - xy = y^2 e^{-\left(\frac{x^2}{2}\right)} \cdot \log x$ | Answer: $[\frac{1}{y} e^{\frac{x^2}{2}} = x(1 - \log x) + c]$ |
| 47. $x \frac{dy}{dx} + y = x^3 y^c$ | Answer: $\frac{1}{y^5} = \frac{5}{2}x^3 + cx^5$ |
| 48. $(1-x^2)\frac{dy}{dx} + xy = y^3 \sin^{-1}x$ | Answer: $[-2[x \sin^{-1}x + \sqrt{1-x^2}] + c]$ |

49. In a circuit containing inductance L , resistance R , and voltage E , the current I is given by

$$L \frac{di}{dt} + Ri = E. \text{ Find the current } i \text{ at time } t \text{ if at } t = 0, i = 0 \text{ and } L, R, E \text{ are constants.}$$

$$[\text{Ans: } i = \frac{E}{R} (1 - e^{-Rt/L})]$$

50. A constant e. m. f. E volts is applied to a circuit containing a constant resistance R ohms in series

and a constant inductance L henries. The current i at any time t is given by $L \frac{di}{dt} + Ri = E$. If

the initial current is zero. Show that the current builds up to half its theoretical maximum value

$$\text{in } t = \frac{L}{R} \log 2.$$

51. The equation of an L-R circuit is given by $L \frac{di}{dt} + Ri = 10 \sin t$. If $i = 0$ at $t = 0$ express i as a

$$\text{function of } t. \quad [\text{Ans: } i = \frac{10}{\sqrt{R^2 + L^2}} [\sin(t - \phi) + e^{-Rt/L} \sin \phi] \text{ where } \tan \phi = \frac{L}{R}]$$

52. The charge q on the plate of a condenser of capacity C charged through a resistance R by a

steady voltage V satisfies the differential equation $R \frac{dq}{dt} + \frac{q}{C} = V$. If $q = 0$ at $t = 0$, show

that $q = CV(1 - e^{-t/RC})$ Find also the current flowing into the plate.

$$[\text{Ans: } i = \frac{V}{R} \cdot e^{-t/RC}]$$

53. The differential equation of an electrical circuit is $R \frac{dQ}{dt} + \frac{Q}{C} = V$. If $R=20$ ohms, $C=0.01$ farad

and $V=20 e^{-5t}$ and if $Q=0$ when $t=0$, find Q in terms of t .

$$[\text{Ans: } Q e^{5t} = t]$$

54. A resistance of 100 ohms and inductance of 0.5 henries are connected in series with a battery of

20 volts. Find the current at any instant if the relation between L, R, E is $L \frac{di}{dt} + Ri = E$.

$$[\text{Ans: } i = 0.2 (1 - e^{-200t})]$$

55. In a circuit of resistance R , self inductance L , the current I is given by $L \frac{di}{dt} + Ri = E e^{pt}$,

when E and p are constants. Find the current i at time t .

$$[\text{Ans: } i = \frac{E}{L^2 p^2 + R^2} [Lp \sin pt - R \cos pt] + k e^{-Rt/L}]$$

56. In a circuit of resistance R, self inductance L, the current i is given by $L \frac{di}{dt} + Ri = E \cos pt$, Find the current i at any time t if i=0 at t=0.

$$[\text{Ans: } i = \frac{E}{\sqrt{L^2 p^2 + R^2}} [\sin(pt + \varphi) - e^{-Rt/L} \cos \varphi] \text{ where } \varphi = Lp / R]$$

Practice Problems Linear Differential Equation with Constant Coefficient

Solve the following.

$$1. \frac{d^3 y}{dx^3} - 5 \frac{d^2 y}{dx^2} + 8 \frac{dy}{dx} - 4y = 0 \quad [\text{Ans: } y = (c_1 + c_2 x)e^{2x} + c_3 e^x]$$

$$2. \frac{d^3 y}{dx^3} - 6 \frac{d^2 y}{dx^2} + 11 \frac{dy}{dx} - 6y = 0 \quad [\text{Ans: } y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}]$$

$$3. \frac{d^4 y}{dx^4} + k^4 y = 0$$

[Ans:

$$y = e^{(k/\sqrt{2})x} [c_1 \cos(k/\sqrt{2})x + c_2 \sin(k/\sqrt{2})x] + e^{-(k/\sqrt{2})x} [c_3 \cos(k/\sqrt{2})x + c_4 \sin(k/\sqrt{2})x]$$

$$4. \frac{d^4 y}{dx^4} + 6 \frac{d^2 y}{dx^2} + 9y = 0 \quad \text{Ans: } y = (c_1 + c_2 x) \cos \sqrt{3}x + (c_3 + c_4 x) \sin \sqrt{3}x]$$

$$5. \frac{d^4 y}{dx^4} + 2 \frac{d^2 y}{dx^2} + y = 0$$

$$6. \frac{d^4 y}{dx^4} + y = 0$$

$$[\text{Ans: } y = e^{(x/\sqrt{2})} [c_1 \cos(x/\sqrt{2}) + c_2 \sin(x/\sqrt{2})] + e^{-(x/\sqrt{2})} [c_3 \cos(x/\sqrt{2}) + c_4 \sin(x/\sqrt{2})]]$$

$$7. (D^3 - D^2 + D - 1)y = 0$$

$$8. (D^3 - 3D^2 + 4)y = 0$$

$$9. (D^2 - 1)(D - 1)^2 y = 0$$

$$10. (D^4 + 8D^2 + 16)y = 0 \quad [\text{Ans: } y = (c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x]$$

$$11. (D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0 \quad [\text{Ans: } y = e^x [(c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x]]$$

12. $(D^2 + 2D + 1)y = 0$
13. Solve $(D^2 - 2D + 1)y = e^x + 1$
14. Solve $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{-x}$
15. Solve $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = e^x + 1$
16. Solve $\frac{d^2y}{dx^2} - (a + b) \frac{dy}{dx} + ab y = e^{ax} + e^{bx}$
17. Solve $\frac{d^3y}{dx^3} - 4 \frac{dy}{dx} = 2 \cos h^2 2x$
18. Solve $(2D + 1)^2 y = 4e^{-\frac{x}{2}}$
19. Solve $(D^4 + 1)y = \cos h 4x \sin h 3x$
20. Solve $\frac{d^2y}{dx^2} + y = \sin x \sin 2x$
21. Solve $(D^3 + D^2 + D + 1)y = \sin^2 x$
22. Solve $(D^4 + 10D^2 + 9)y = 96 \sin 2x \cos x$

23. Solve $(D^3 + 2D^2 + D)y = x^2 + x$

24. Solve $(D^2 - D - 2)y = 2 \log x + \frac{1}{x} + \frac{1}{x^2}$

25. Solve $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = \sin(e^x)$

26. Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

27. Solve $(D^2 - 4)y = x \sin hx$

Answer: $\left[y = c_1 e^{2x} + c_2 e^{-2x} - \frac{x}{3} \sin hx - \frac{2}{9} \cos hx \right]$

28. Solve $(D^2 - 1)y = x^2 \sin 3x$

Answer: $\left[y = c_1 e^x + c_2 e^{-x} - \frac{1}{10} \left\{ x^2 \sin 3x + \frac{6}{5} x \cos x - \frac{13}{25} \sin 3x \right\} \right]$

29. Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$

Answer: $\left[y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x - \frac{1}{48} (x^4 - 9x^2) \cos x + \frac{x^3}{12} \sin x \right]$

30. Solve $(D^2 + 1)y = x^2 \sin 2x$

31. Solve $(D^2 - 1)y = e^x \sin 3x$

Answer: $\left[y = c_1 e^x + c_2 e^{-x} - \frac{e^x}{117} (6 \cos 3x + 9 \sin 3x) \right]$

32. Solve $([D^3 - D^2 - D + 1])y = \cos hx \sin x$

Answer: $\left[y = (c_1 + c_2 x) e^x + c_3 e^{-x} + \frac{1}{10} e^x (\cos x - 2 \sin x) - \frac{e^{-x}}{50} (3 \cos x - 4 \sin x) \right]$

33. Solve $(D^2 - 1)y = x \sin hx$

Answer: $\left[y = c_1 e^x + c_2 e^{-x} + \frac{x^2}{4} \cos hx - \frac{x}{4} \sin hx \right]$

Hint Put $\left[\sin hx = \frac{e^x - e^{-x}}{2}, \cos hx = \frac{e^x + e^{-x}}{2} \right]$

34. Solve $(D^3 - 3D^2 + 3D - 1)y = x e^x + e^x$

Answer: $\left[y = (c_1 + c_2x + c_3x^2)e^x + e^x \frac{x^4}{24} + e^x \frac{x^3}{6} \right]$

35. Solve $(D^2 - 2D + 1)y = \frac{3e^x}{x^2}$

Answer: $[y = (c_1 + c_2x)e^x - 3e^x \log x]$

36. Solve $(D^2 + 3D + 2)y = e^{-2x} + e^x \cos 2x$

37. Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x \cos 3x$

38. Solve $(D^2 - 3D + 2)y = 2e^x \sin\left(\frac{x}{2}\right)$

39. Solve $(D^3 - 7D - 6)y = e^{2x}(x + 1)$

40. Solve $(D^3 - 2D + 4)y = 3x^2 - 5x + 2$

Answer: $\left[y = c_1 e^{-2x} + e^x(c_2 \cos x + c_3 \sin x) + \frac{1}{4}[3x^2 - 2x + 1] \right]$

Hint: $D^3 - 2D + 4 = D^3 + 2D^2 - 2D^2 - 4D + 2D + 4 = 0 = (D + 2)(D^2 - 2D + 2) = 0$
 $= D = -2, 1 \pm i$

41. Solve $\frac{d^3y}{dt^3} + \frac{dy}{dt} = \cos t + t^3 + 3$

Answer: $\left[y = c_1 + c_2 \cos t + c_3 \sin t + \frac{1}{2}[-t \cos t + \sin t] + \frac{t^3}{3} + t \right]$

42. Solve $(D^2 + 4D + 4)y = x^2$ Answer: $[y = (c_1 + c_2x)e^{-2x} + \frac{1}{4}[x^2 - 2x + \frac{3}{2}]]$

43. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = x - 2x^2$

Answer: $\left[y = e^{-x}(c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x) + \frac{1}{3}\left(-\frac{10}{9} + \frac{11}{3}x - 2x^2\right) \right]$

44. Solve $(D^3 - D^2 + 6D)y = x^2 + \sin x$

Answer: $\left[y = c_1 + c_2 e^{-2x} + c_3 e^{-3x} - \frac{1}{6}\left(\frac{x^3}{3} - \frac{x^2}{6} + \frac{7x}{18}\right) + \frac{1}{50}(\sin x + 7 \cos x) \right]$

45. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^x \cos\left(\frac{x}{2}\right)$ [Answer : $\therefore y = c_1 e^x + c_2 e^{2x} + \frac{8}{5}e^x \left[-2 \sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)\right]$]