Numericals.

→ **LASER:** (CW)

(1) $E_2 = 20.66\,eV$, $E_1 = 18.7\,eV$, $\lambda = ?$

$\Delta E = E_2 - E_1 = h\nu$ (convert ΔE to J $1\,eV = 1.6 \times 10^{-19}\,J$)

$\therefore \Delta E = \frac{hc}{\lambda} \longrightarrow \lambda = \frac{hc}{\Delta E}$

$\therefore \lambda = 6342\,\mathring{A}$

(2) $\lambda = 780\,nm = 780 \times 10^{-9}\,m$, $P = 20\,mW/pulse = 20 \times 10^{-3}\,W/pulse$, $\Delta t \sim 10\,\mu s$.

$n_t = \frac{P_{optical} \times \lambda}{hc}$, $n = n_t \times \Delta t$

$n_t = 7.84 \times 10^{16}$

$n = 7.84 \times 10^{17}$

(3) $P = 100\,W$, diam $= 500\,\mu m = 500 \times 10^{-6}\,m$, $I = ?$

Area of (fibre) beam $= \pi r^2 = 1.96250 \times 10^{-7}\,m^2$.

$\therefore I = \frac{P}{A} = \frac{100 \times 10^{7}}{1.96250} = \frac{10^9}{1.96250} = 509554140.1 = 0.5095 \times 10^9\,W/m^2$

(4) $V = 3.6\,V$, $I = 130\,mA = 130 \times 10^{-3}\,A$, $P_{optical} = 10\,mW = 10 \times 10^{-3}\,W$, $\eta = ?$

$\therefore \eta = \frac{P_{optical}}{V_{operating} \times I_{operating}} = 0.02136$

(5) $\Delta\lambda = 2 \times 10^{-4}\,nm$, $\lambda = 632.8\,nm$, l_{coh}.

$l_{coh} = \frac{\lambda^2}{\Delta\lambda \times 2} \approx 1\,m$

(6) $z_1 = 10\,m$, $d_1 = 10^{-3}\,m$, $z_2 = 35\,m$, $d_2 = 3.5 \times 10^{-3}\,m$

$\therefore \phi = \frac{d_2 - d_1}{z_2 - z_1}$ (in radians)

$\phi = 10^{-4}$ radians.

(7) $\lambda = 694.3nm = 694.3 \times 10^{-9} m.$, $T = 27°C = 300 K$, $\nu = c.$

$$\frac{N_1}{N_2} = e^{h\nu/KT} = e^{hc/\lambda KT} = 1.126 \times 10^{30}.$$

$$\frac{N_2}{N_1} = e^{-h\nu/KT.} = e^{-hc/\lambda KT.} = 8.874 \times 10^{-31}$$

(8) $\lambda = 6000 \mathring{A}$, $A_{21} = 10^6 /s$. B_{21}.

$$B_{21} = \frac{A_{21} \cdot c^3}{8\pi h \nu^3 \mu^3 \to (1)} = 1.297 \times 10^{19} /s.$$

(9) $\lambda = 5000 \mathring{A}.$

$\frac{R_{st}}{R_{sp}} = 1.$, $T = ?$

$$\frac{R_{st}}{R_{sp}} = \frac{1}{e^{h\nu/KT} - 1} \to 1 = \frac{1}{e^{h\nu/KT} - 1} \to e^{h\nu/KT} - 1 = 1 \to e^{h\nu/KT} = 2$$

$$\therefore \frac{h\nu}{KT} = \ln 2 \to T = \frac{h\nu}{K \ln 2} = \frac{hc}{\lambda . K \ln 2} = 41596 K.$$

(10) $r_1 = 100\%.$, $r_2 = 98.9\%.$

i.e. $r_1 = 1$, $r_2 = 0.989.$, $L = 10cm$, $\gamma = 5.34 \times 10^{-4}/cm.$

$\alpha_s = ?$

$$\gamma = \alpha_s + \frac{1}{2L} \ln \frac{1}{(r_1 r_2)}$$

$$\alpha_s = \gamma - \frac{1}{2L} \ln \frac{1}{(r_1 r_2)} = 5.34 \times 10^{-4} - \frac{1}{20} \times 0.0110$$

$$\alpha_s = 5.34 \times 10^{-4} - 5.53 \times 10^{-4}.$$

$$\alpha_s = -1.9 \times 10^{-5} /cm.$$

→ **OPTICAL FIBRE: (CW)**

(1) $n_1 = 1.46$, $n_2 = 1.42$

$NA = \sqrt{n_1^2 - n_2^2} = 0.3394.$

$\sin i_{max} = NA.$

$\therefore i_{max} = \sin^{-1}(NA). = 19.8403°$

(2) $i_{max} = 25°$, $n_2 = ?$, $n_1 = 1.52.$

$\sin i_{max} = \sqrt{n_1^2 - n_2^2}$

$\therefore n_2 = 1.46.$

(3) $i_{max} = 25°$, $\theta_c = 70°$. , $n_1, n_2, \Delta = ?$

$\sin\theta_c = \frac{n_2}{n_1}$; $\sin i_{max} = n_1.\cos\phi_c$, $\Delta = \frac{n_1 - n_2}{n_1}$

$\therefore n_1 = 1.24$, $n_2 = 1.16$, $\Delta = 0.0642$

(4) $NA = 0.3$, $r = 0.05\,mm$, $\lambda = 1.3\,\mu m = 1.3 \times 10^{-6}\,m.$

$V_{no.} = ?$, $N_{max} = ?$ (SI)

$V = \frac{2\pi a}{\lambda}(NA). = 72.46.$

$N_{max} = \frac{V^2}{2} = 2625$ modes.

(5) $r = ?$, $NA = 0.025$, single mode, $\lambda = 850\,nm. = 850 \times 10^{-9}\,m.$

$V = \frac{2\pi r}{\lambda}(NA).$ $V \leq 2.405$

$r = 1.30 \times 10^{-5}\,m$

$r = 13\,\mu m.$

(6) $P_i = 5\,mW = 5 \times 10^{-3}\,W$, $P_o = 0.2\,mW = 0.2 \times 10^{-3}\,W$, $L = 50\,km.$

$\alpha = \frac{10}{L} \cdot \log_{10}\left(\frac{P_i}{P_o}\right)$

$\alpha = 0.28\,dB/km.$

(7) $\alpha = 0.2\, dB/km$, $P_i = 100$, $P_o = 10$.

$$L = \frac{10}{\alpha} \log_{10}\left(\frac{100}{10}\right) = 50\ km.$$

(8) $L = 10km$, $\alpha = 0.2\ dB/km$, $P_i = 5mW$; $5P_{k10} = ?$

(i) $$\alpha = \frac{10}{L} \log_{10}\left(\frac{P_i}{P_o}\right)$$

$\therefore P_o = 3.15\ mW$.

(ii) (a) $$\underset{(5)}{I/p} \longrightarrow \underset{(3.15)}{o/p}$$

(b) $$\underset{(5)}{I/p} \longrightarrow \underset{(3.15)}{o/p} \begin{cases} \longrightarrow A\ (1) \\ \longrightarrow B\ (1) \end{cases}$$

for two connectors, loss $= A + B = 2dB$.

loss before attaching connector $\rightarrow 0.2\ dB/km \times 10 = 2dB$.

$$\therefore \underbrace{I/p \longrightarrow}_{2B} o/p \left.\begin{matrix} \nearrow A \\ \searrow B \end{matrix}\right\} 2dB.$$

Total loss at o/p $= 4dB$

Now, $P_o = \log_{10}\left(\frac{5}{P_o}\right) = 2mW$.

$$\therefore \%\ \text{dec. in loss} = \frac{3.15 - 2}{3.15} \times 100 = 36.5\%$$

(9) $n_1 = 1.48$, $n_2 = 1.45$, $L = 2500m = 2.5km$., step index.

$$T_i = \frac{n_1 L}{c} \Delta\ sec. = 247\ ns.$$

(10) $RI = 1.42 = n_2$, $\Delta = 0.025$, $B = ?$, $L = 2km$, $\tau_m = 1.7ns/km$.

$$B = \frac{0.7}{\tau}\ /s. \quad ; \quad \tau = \sqrt{\tau_i^2 + \tau_m^2}, \quad \tau_i = \frac{n_2 \Delta^2 L}{2c}\ s.$$

$\tau_i = 2.96 ns$. , $\tau_m = 1.7 \times 2 = 3.4\ ns$.

$\therefore \tau = 4.5\ ns$.

$\therefore B = 155$ MBPS.

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ENGINEERING MATERIALS

Numericals (Dielectrics)

(C.W.)

(1) $A = 5cm^2 = 5 \times 10^{-4} m^2$, $d = 1mm = 10^{-3} m$, $Q = 10^{-10} C$, $V = ?$, $\varepsilon_r = 7$

$C_1 = \frac{\varepsilon_0 . A}{d}$ (w/o medium) $= 4.425 \times 10^{-12}$ F

$C_2 = \frac{\varepsilon_0 . \varepsilon_r . A}{d}$ (with medium) $= 3.0975 \times 10^{-11}$ F

$V = \frac{Q}{C} \longrightarrow V_1 = 22.598$ V. $\approx 22.6V$.

$V_2 = 3.228$ V $\approx 3.23V$.

(2) $A = 0.25 cm^2 = 0.25 \times 10^{-4} m^2$, $\varepsilon_r = 2.8$, $d = 5mm = 5 \times 10^{-3} m$, $V = 100$ V.

$Q, E, D, P = ?$

$Q = CV = \frac{A \varepsilon_0 . V}{d}$ (without) $= \frac{A \varepsilon_0 \varepsilon_r . V}{d}$ (with medium)

$= 1.24 \times 10^{-11}$ C

$E = \frac{V}{d} = \frac{100}{5 \times 10^{-3}} = 2 \times 10^4$ V/m.

$D = \varepsilon_0 . \varepsilon_r . E = 4.96 \times 10^{-7} C/m^2$

$P = \varepsilon_0 E (\varepsilon_r - 1) = D - \varepsilon_0 E = 4.96 \times 10^{-7} - 1.77 \times 10^{-7}$

$\therefore P = 3.19 \times 10^{-7} C/m^2$

(3) $E = 1000$ V/m, $P = 4.5 \times 10^{-8} C/m^2$, $\varepsilon_r = ?$, $\chi_r = ?$

$P = \varepsilon_0 E (\varepsilon_r - 1)$

$\therefore \varepsilon_r = \frac{P}{\varepsilon_0 . E} + 1. \longrightarrow \varepsilon_r = 6.08$ F/m

$\chi_r = \varepsilon_r - 1$

$\therefore \chi = 5.08 \; C^2 / Nm^2$

(4) $\alpha_e = ?$, $\varepsilon_r = 1.00043$, $N = 2.7 \times 10^{25} / m^3$

$\alpha_e = \frac{\varepsilon_0 (\varepsilon_r - 1)}{N}$

$\alpha_e = 1.40 \times 10^{-40} Fm^2$

(5) $\alpha_e = \frac{3\varepsilon_0}{N}\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right]$, $\varepsilon_r = 2.87$, $N = 3\times10^{28}/m^3$. , $E = 5000\ V/m$

$\alpha_e = 8.85\times10^{-40} \times 0.3839$

$\alpha_e = 3.4\times10^{-40}\ Fm^2$

$\mu = \alpha_e E = 1.7\times10^{-36}$ C.m.

$P = N\mu = 5.1\times10^{-8}$ C.m^2.

Numericals (semiconductors)

(1) $\mu_e = 0.13\ m^2/Vs$, $\mu_n = 0.05\ m^2/Vs$

$n_i = 10^{16}/cm^3 = 10^{10}\,cm^{-3} = 10^{16}\,m^{-3}$

$\rho_i = \dfrac{1}{n_i e(\mu_e + \mu_h)} = 3472.2\ \Omega m$

(2) $N_D = 10^{16}/cm^3$ phosphorous atoms

↳ no. of donor atoms)

∴ n-type , major charge carriers $n_e = N_D = 10^{22}\,m^{-3}$

min charge carriers,

$n_h = \dfrac{n_i^2}{N_D}$, $n_i = 10^{16}/m^3$, $\mu_e = 0.13$

$\therefore \rho_n = \dfrac{1}{n_e \mu_e e} = 4.8 \times 10^{-3}\,\Omega m$

(3) $\rho_{si} = 2340\ kg/m^3$ at wt $= 28\ g/mol$.

impurity concn (Boron atoms) = 1 ppb.

$D = \dfrac{M}{V}$, $\rho = \dfrac{nM}{NV}$ $\longrightarrow$ $\dfrac{n}{V} = \dfrac{\rho N}{M}$

($M \rightarrow$ mol. wt, $N \rightarrow$ Avogadro's no., $V \rightarrow$ vol. of unit cell)

$N = 6.023 \times 10^{26}/g.mol = 6.023 \times 10^{23}/kg.mole$

no. of Si atoms/m^3 $= \dfrac{\rho N}{M} = \dfrac{2340 \times 6.023 \times 10^{23}}{28 \times 10^{-3}} = 5.03 \times 10^{28}/m^3$

doping concn = 1 ppb = 1 in 10^9 (in Si)

∴ no. of dopants /m^3 = $5.03 \times 10^{19}/m^3$ (ie. ÷ by 10^9)

∴ (i) it becomes p-type

(ii) concn = $5.03 \times 10^{19}/m^3$

(4) $N_C = 4.37 \times 10^{17} cm^{-3}$, $N_V = 8.68 \times 10^{18} cm^{-3}$, $E_g = 1.42 eV$, $T = 300K$.

$n_i = \sqrt{N_C N_V} \cdot exp\left[\frac{-E_g}{2KT}\right]$

$n_i = 2.39 \times 10^6 / dm^3$

(5) $N_D = 10^{16}/cm^3$, $E = 50 V/cm = 5000 V/m$
$\quad \parallel$
$10^{22}/m^3$

$\mu_e = 0.13 m^2/Vs$

$J_{drift} = neV_d = 1.04 \times 10^6 A/m^2$

(6) $P_1 = 3 \times 10^{18}/cm^3$, $P_2 = 5 \times 10^{17}/cm^3$

$x = 1 \mu m = 10^{-6} m$

$D_p = 10 cm^2/s = 10 \times 10^{-4} m^2/s = 10^{-3} m^2/s$.

$J_{drift} = P \cdot D_p \cdot \frac{dp}{dx} = \underbrace{1.6 \times 10^{-19}}_{\text{mag. of charge}} \times 10^{-3} \times 25 \times 10^{29}$

$= 4 \times 10^8 \overset{?}{A}$

$\left\{ \frac{dp}{dx} = \frac{30 \times 10^{23} - 5 \times 10^{23}}{10^{-6}} = 25 \times 10^{29} m \right\}$

(7) $T = 300K$ (room temp), $E_g = 0.66 eV$.

$f(E_c) = \frac{1}{1 + exp\left[\frac{E_c - E_F}{KT}\right]}$; $E_c - E_F = \frac{E_g}{2} = 0.33 eV$.

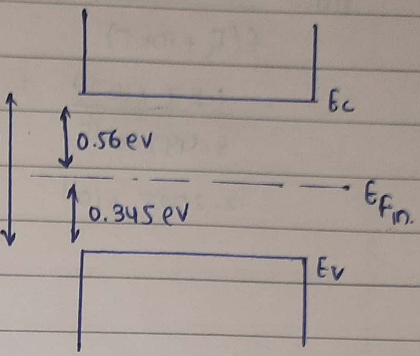
$f(E_c) = \frac{1}{1 + exp\left[\frac{0.33}{0.025}\right]}$

$f(E_c) = 1.85 \times 10^{-6}$

(8) $N_A = 10^{16}/cm^3$

p-type impurity , $n_p = N_A = 10^{22}/m^3$

$n_i = 10^{16}/cc.$



$$E_{Fp} - E_{Fi} = -kT \ln\left[\frac{n_p}{n_i}\right]$$

$$= -0.025 \ln\left[\frac{10^{22}}{10^{16}}\right] = -0.345\,eV$$

$$\therefore f(E_c) = \frac{1}{1+\exp\left[\frac{E_c - E_F}{KT}\right]} = \frac{1}{1+\exp.\left[\frac{0.56+0.345}{0.025}\right]} = \frac{1}{1+\exp(36.2)}$$

$$f(E_c) = 1.9 \times 10^{-16}$$

(9) gap will be $1.12 - (0.56 + 0.345) = -0.215$

$$f(E_v) = \frac{1}{1+\exp\left(\frac{E_v - E_{Fp}}{KT}\right)} = \frac{1}{1+\exp\left(\frac{-0.215}{0.025}\right)}$$

$$f(E_v) = 0.99$$

(10) $T = 300K.$

$$f(E) = \frac{1}{1+\exp\left[\frac{E-E_F}{KT}\right]}$$

if $E = E_c$,

$$f(E_c) = \frac{1}{1+\exp\left[\frac{E_c - E_F}{KT}\right]} = \frac{1}{1+\exp\left[\frac{0.56}{0.025}\right]} = 1.87 \times 10^{-10}$$

In formula,

$E = E_c + 10KT$

$E - E_F = E_c + 10kT - E_F = E_c - E_F + 10kT = 0.56 + 0.25 = 0.81\,eV$

$$f(E_c + 10kT) = \frac{1}{1+\exp\left[\frac{E_c + 10kT - E_k}{KT}\right]} = \frac{1}{1+\exp\left[\frac{0.81}{0.025}\right]} = 8.49 \times 10^{-15}$$

ratio $\dfrac{f(E_c)}{f(E_c + 10kT)}$

$= \dfrac{1.87 \times 10^{-10}}{8.49 \times 10^{-15}} = 0.22025 \times 10^{5}$

$= 2.2025 \times 10^{4}$

QUANTUM MECHANICS

Numericals

(C.W.)

(1) $v = 10^6$ m/s.

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^6} = 7.286\ \text{Å}$$

(2) $m = 0.2$ kg, $v = 100$ m/s

$$\lambda = \frac{h}{mv} = 3.315 \times 10^{-35}\ \text{m}.$$

de Broglie's is applicable for smaller bodies as range is small while for bigger bodies, value of λ is infinitesimally small, hence cannot be calculated.

(3) $E = 1$ MeV, $m = 1.67 \times 10^{-27}$ kg

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$1\ \text{MeV} = 10^6\ \text{eV} = 1.6 \times 10^{-19} \times 10^6\ \text{J} = 1.6 \times 10^{-13}\ \text{J}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}}} = 2.86 \times 10^{-14}\ \text{m}$$

(4) $q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg, $\lambda = 2.86 \times 10^{-14}$ m

$$\lambda = \frac{h}{\sqrt{2mqV}} \rightarrow 2mqV = \frac{h^2}{\lambda^2} \longrightarrow V = \frac{h^2}{\lambda^2 \cdot 2mq}$$

$\therefore V = 1005607.59$

$\therefore V = 1. \times 10^6 = 1$ MV

(5) $E = ?$, e^-, $E = 1 \times 10^3$ eV $= 1.6 \times 10^{-16}$ eV J

$$E = \frac{hc}{\lambda} \longrightarrow \lambda = \frac{hc}{E} = 1.24 \times 10^{-9}\ \text{m}.$$

acc. to info, $\lambda_e = \lambda$ $\therefore \lambda_e = 1.24 \times 10^{-9}$ m.

$$KE = \frac{p^2}{2m} = \frac{(h/\lambda_e)^2}{2m} = \frac{h^2}{2m\lambda_e^2} = 1.57 \times 10^{-19}\ \text{J}.$$

(6) $E_e = E_p$, $m_p = 1800 m_e$, $q_p = q_e = q$.

$\lambda_2 = \frac{h}{\sqrt{2mE}}$, $\lambda_1 = \frac{h}{\sqrt{2mE}}$

$\frac{\lambda_1}{\lambda_2} = \frac{\sqrt{1800 m_e}}{\sqrt{m_e}} = \frac{\sqrt{1800}}{1} = \frac{42.42}{1}$

$\therefore \lambda_1 : \lambda_2 = 42.42 : 1$

(7) $m_\mu = 207 m_e$, $v_\mu = v_e = v$. , $q_\mu = q_e = q$.

$\lambda_1 = \frac{h}{m_e v}$, $\lambda_2 = \frac{h}{207 m_e v}$

$\therefore \frac{\lambda_1}{\lambda_2} = \frac{h/m_e v}{h/207 m_e v} = \frac{207 m_e}{m_e} = \frac{207}{1} = 207 : 1$

(8) $\Delta x = 1nm$, $v = 10^6 m/s$.

uncertainty in mom $= \frac{\Delta p}{p}$

$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.05 \times 10^{-34}}{10^{-9}} = 1.05 \times 10^{-25} kg m/s$

$p = mv = 9.1 \times 10^{-31} \times 10^6 = 9.1 \times 10^{-25} kg m/s$

% unc in mom $= \frac{1.05 \times 10^{-25}}{9.1 \times 10^{-25}} = 0.115 = 11.5\%$

(9) $m = 10g = 10^{-2} kg$, $\Delta x = 50cm = 0.5m$, $v = 20 cm/s = 0.2 m/s$

$\Delta p = \frac{\hbar}{\Delta x}$, $\Delta p = mv$.

$\Delta p = \frac{1.05 \times 10^{-34}}{0.5} = 2.1 \times 10^{-34} kg m/s$

$p = mv = 2 \times 10^{-3} kg m/s$

% unc. in mom $= \frac{2.1 \times 10^{-34}}{2 \times 10^{-3}} = 1.05 \times 10^{-29}\%$.

unacceptable

(10) size of an atom $= 2\times10^{-10}$ m.

$\Delta x = 2\times10^{-10}$ m

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.05\times10^{-34}}{2\times10^{-10}} = 0.525\times10^{-24}$$

$$E = \frac{p^2}{2m}$$

$\Delta p \simeq p_{min}$

$$E = \frac{(0.525\times10^{-24})^2}{2\times10^{-31}\times9.1} = 0.96\ eV$$

(11) $\Delta t = 1ps = 10^{-12}s$

$\Delta E = ?$

$\Delta x . \Delta p \simeq \hbar$

$\Delta E . \Delta t \simeq \hbar$

$$\therefore \Delta E = \frac{\hbar}{\Delta t} = 1.05\times10^{-22} J$$

(12) $p = 2\times10^{-24}$ kgm/s

unc $= 0.05\%$

$$\frac{\Delta p}{p} = 0.05\% \longrightarrow \Delta p = \frac{p\times0.05}{100} = 2\times10^{-24}\times0.05\times10^{-2} = 0.1\times10^{-26} = 10^{-27}$$

kgm/s

$$\Delta x = \frac{\hbar}{\Delta p} = \frac{1.05\times10^{-34}}{10^{-27}} = 1.05\times10^{-7} m$$

(13) $\Delta t = 0.01\mu s = 0.01\times10^{-6}s$

$\Delta\nu = ?$

$$\Delta E.\Delta t = \hbar \longrightarrow \hbar\Delta\nu\Delta t = \hbar \longrightarrow \Delta\nu = \frac{\cancel{h}}{\cancel{h}\,\Delta t\, 2\pi} = \frac{1}{2\pi\times10^{-8}}$$

$\therefore \Delta\nu = 7.96$ MHz

(14) $E = 20\,MeV = 20\times10^{6}\,eV = 20\times10^{6}\times1.6\times10^{-19}\,J = 32\times10^{-13}\,J$

$E = \frac{p^2}{2m} \longrightarrow p = \sqrt{2mE}$

$m_n = 1.67\times10^{-27}\,kg$

$p = \sqrt{1.6\times2\times10^{-27}\times32\times10^{-13}} = 1.012\times10^{-19}\,s$

$\Delta x = 3\times10^{-15}\,m$ (region of 3 nuclei)

$\Delta p = \frac{h}{\Delta x} = \frac{1.05\times10^{-34}}{1.032\times10^{-15}} = 3.5\times10^{-20}$

$\%p = \frac{3.5\times10^{-20}}{1.012\times10^{-19}}$

$\%p = 3.46\times10^{-1} = 0.346\,\%$

(15) $\psi(x) = \sqrt{\frac{\pi}{2}}\,, \quad 0 \leq x \leq 1$

$x_1 = 0.45\,,\ x_2 = 0.55$

$P = \int_{L_1}^{L_2} |\psi(x)|^2 . dx = \int_{0.45}^{0.55} \frac{\pi}{2} x^2 . dx = \frac{\pi}{2}\left|\frac{x^3}{3}\right| = \frac{\pi}{6}\left((0.55)^3 - (0.45)^3\right)$

$P = 0.0394 = 3.94\%$

(16) width $= a$

limits : $a.3a$ to $0.7a$.

$\psi(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$

for ground state $n=1 \longrightarrow \psi(x) = \sqrt{\frac{2}{a}} \sin\frac{\pi x}{a}$

$P = \int_{L_1}^{L_2} |\psi(x)|^2 . dx = \int_{0.3a}^{0.7a} \left(\sqrt{\frac{2}{a}} \cdot \sin\frac{\pi x}{a}\right)^2 . dx = \frac{2}{a}\int_{0.3a}^{0.7a} \sin^2\frac{\pi x}{a} . dx$

$= \frac{2}{a}\,\frac{1}{2}\int_{0.3a}^{0.7a}\left[1 - \cos\frac{2\pi x}{a}\right] dx = \frac{1}{a}\left[\int_{0.3a}^{0.7a} dx - \int_{0.3a}^{0.7a} \cos\frac{2\pi x}{a} . dx\right]$

$= \frac{1}{a}\left[\left|x\right|_{0.3a}^{0.7a} - \frac{a}{2\pi}\left|\sin\frac{2\pi x}{a}\right|_{0.3a}^{0.7a}\right] = \frac{1}{a}\left[0.4a - \frac{a}{2\pi}(\sin 1.4\pi - \sin 0.6\pi)\right]$

$= 0.4 - \frac{1}{2\pi}(0.0766 - 0.0328) = 0.4 = 40\%$

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(17) $\psi(x) = Ae^{-2x}$, interval $x=0$ to $x=1$.

normalization condition,

$$P = \int_0^1 (Ae^{-2x})^2 \cdot dx = 1$$

$$P = A^2 \int_0^1 e^{-4x} \cdot dx = 1 \longrightarrow P = A^2 \left[\frac{e^{-4x}}{-4}\right]_0^1 = 1$$

$$\therefore \frac{A^2}{-4}[e^{-4} - e^0] = 1 \longrightarrow A^2\left(\frac{1-e^{-4}}{4}\right) = 1.$$

$$A^2(0.2454) = 1.$$

$$\therefore A^2 = 1/0.2454$$

$$A = \sqrt{4.07} = 2.02$$

(18) $L = 10\,\mathring{A}$, $n=1$. $= 10^{-9}\,m$.

$$E_n = \frac{n^2h^2}{8mL^2} \quad (n=1)$$

$$E_1 = \frac{h^2}{8mL^2} = 6.03 \times 10^{-20}\,J$$

$$P = \sqrt{2mE} = 3.315 \times 10^{-25}\,kgm/s$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{3.315 \times 10^{-25}} = 2nm$$

(19) $L = 1nm = 10^{-9}m$

first two allowed states $n=1$, $n=2$.

$$E_1 = \frac{h^2}{8mL^2} \qquad E_2 = \frac{4h^2}{8mL^2}$$

$$E_2 - E_1 = \frac{3h^2}{8mL^2} = \frac{3 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 10^{-18}} = 1.81 \times 10^{-19}\,J$$

$$\Delta E = 1.13\,eV$$

(20) $m = 10g\,m$, $L = 1cm$

$\therefore m = 10 \times 10^{-3}\,kg$. $L = 0.01m$

$$E_1 = \frac{h^2}{8mL^2} \qquad E_2 = \frac{4h^2}{8mL^2}$$

$$\Delta E = \frac{3h^2}{8mL^2} = 1.648 \times 10^{-61}$$

$\therefore \Delta E = 1.65 \times 10^{-61}\ J$

$\Delta E = 1.03 \times 10^{-42}\ eV$

QUANTUM MECHANICS

QUBIT OPERATIONS:

$$|0\rangle^{+} \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}^{*T} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^{T} = [1\ 0] = \langle 0|$$

Q. $|q\rangle = \begin{bmatrix} i & -i \\ 0 & 1 \end{bmatrix}$ find $\langle q|$

conjugate (complex) $\begin{bmatrix} -i & i \\ 0 & 1 \end{bmatrix}$

transpose $\begin{bmatrix} -i & 0 \\ i & 1 \end{bmatrix} \equiv \langle q|$

→ **Qubit Add^n / Sub^n:**

$|q_1\rangle \pm |q_2\rangle = ?$

let $|q_1\rangle = |q_2\rangle = |0\rangle$.

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|q_1\rangle = |q_2\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$|q_1\rangle = |1\rangle$, $|q_2\rangle = |0\rangle$.

$$|q_1\rangle - |q_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

→ **qubit Multiplication:**

(i) scalar product: $\langle p|q\rangle$ $\qquad |q\rangle$ and $|p\rangle$
↳ conjugate

$|q\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|p\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$|0\rangle^{+} = [1\ 0]$$

$$\langle p|q\rangle = [1\ 0]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1.$$

P.T.O.

(ii) Vector product:

$|q\rangle \langle p|$

$|q\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|p\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\langle p| = \langle p| = [1 \ 0]$

$\therefore |q\rangle \langle p| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1 \ 0] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(iii) Tensor product:

$|q\rangle |p\rangle$

let $|q\rangle = |p\rangle = |0\rangle$

$|q\rangle|p\rangle = \begin{array}{c} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} q_1 \\ q_2 \end{array} \begin{array}{c} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} p_1 \\ p_2 \end{array} = \begin{array}{c} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} q_1p_1 \\ q_1p_2 \\ q_2p_1 \\ q_2p_2 \end{array}$

→ Probability of computation:

$|\psi\rangle$ can be found in state $|0\rangle$ after computation?

$P(|\psi\rangle = |0\rangle) = |\langle 0|\psi\rangle|^2$

$= [\langle 0| (\alpha |0\rangle + \beta |1\rangle)]^2$

$= |\alpha \langle 0|0\rangle + \beta \langle 0|1\rangle|^2$

$= |\alpha|^2$

Prob. $|\psi\rangle$ in $|1\rangle \longrightarrow |\beta|^2$

total prob. $= |\alpha|^2 + |\beta|^2 = 1.$

in special case $\alpha = \beta$

$2|\alpha|^2 = 1.$

$\alpha = \pm \frac{1}{\sqrt{2}}$ let $\alpha = \frac{1}{\sqrt{2}}$

$|0\rangle$ and $|1\rangle$ combine

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \equiv |+\rangle$

$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \equiv |-\rangle$

Special superposed state

→ Pauli X gate:

$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\sigma_x |0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\sigma_x |1\rangle \rightarrow |0\rangle$

$\sigma_x |+\rangle \rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\sigma_x |+\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\sigma_x \cdot |+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

→ Pauli Z gate:

$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\sigma_z |+\rangle = ?$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = |-\rangle$

$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

→ Hadamard Gate: $H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

How do you produce a superposed state?

check $H = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = |+\rangle$$

↑ superposed state

physical meaning: $|1\rangle$ excited state
$|0\rangle$ ground state

$$H|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\\0\end{bmatrix} = \frac{2}{2}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$$

$\therefore H|1\rangle \rightarrow |-\rangle$

$H|0\rangle \rightarrow |+\rangle$

→ CNOT gate:

2qubit-gate. $|q_1 q_2\rangle$ (q_1: control bit, q_2: data bit.)

operation: if $q_1 = 1$, flip data bit q_2
$q_1 = 0$, don't change data bit q_2

q_1	q_2	
0	0	0
0	1	1
1	0	1
1	1	0

Numericals

(C.W.)

(1) $f = 20\,KHz = 20 \times 10^3\,Hz$, $Y = 11.6 \times 10^{10}\,N/m^2$, $\rho = 7.23 \times 10^3\,kg/m^3$

$$f = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$$

$$\therefore L = \frac{1}{2f}\sqrt{\frac{Y}{\rho}} = 0.1\,m = 10\,cm.$$

(2) $f = ?$, $L = 40\,mm = 0.04\,m$, $\rho = 7.25 \times 10^3\,kg/m^3$, $Y = 115 \times 10^9\,N/m^2$.

$$f = \frac{1}{2L}\sqrt{\frac{Y}{\rho}} = 49.784\,KHz$$ (if $f > 20KHz$, suitable for ultrasonic waves)

(3) $t = 1.8\,mm = 1.8 \times 10^{-3}\,m$, $Y = 8 \times 10^{10}\,N/m^2$, $\rho = 2650\,kg/m^3$

$$f = \frac{1}{2t}\sqrt{\frac{Y}{\rho}} = 1.53\,MHz$$

for $f = 2MHz$.

$$t = \frac{1}{2f}\sqrt{\frac{Y}{\rho}} = 1.37\,mm$$

$\therefore$ change in thickness $t = t_1 - t_2 = 0.43\,mm$

(4) $t_1 = 4\,mm = 4 \times 10^{-3}\,m$, $f_1 = 400\,KHz = 400 \times 10^3\,Hz$.
$t_2 = ?$, $f_2 = 500\,KHz = 500 \times 10^3\,Hz$

$$f_1 = \frac{1}{2t_1}\sqrt{\frac{Y}{\rho}}, \quad f_2 = \frac{1}{2t_2}\sqrt{\frac{Y}{\rho}}$$

$$\therefore \frac{f_1}{f_2} = \frac{t_2}{t_1} \rightarrow t_2 = \frac{f_1 \times t_1}{f_2} = \frac{400 \times \cancel{10^3} \times 4 \times 10^{-3}}{500 \times \cancel{10^3}}$$

$$t_2 = 3.2 \times 10^{-3}\,m = 3.2\,mm.$$

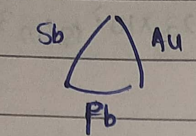
(5) $t_n = 285^\circ c$, $t_c = 0^\circ c$, $T_i = ?$ at $T_c = -30^\circ c$

$T_i = 2T_n - T_c$

$T_i = 2 \times 285 - (-30)$

$T_i = 600^\circ c$

(6)



for Sb-Au thermocouple., calculate emf.

$T_c = 0°c$, $T_n = 100°c$.

$a_{sb-Pb} = 35.58\ \mu V/°c$

$b_{sb-Pb} = 0.146\ \mu V/°c^2$

$a_{Au-Pb} = 2.90\ \mu V/°c$

$b_{Au-Pb} = 0.009\ \mu V/°c$

$a_{Sb-Au} = a_{sb-Pb} - a_{Au-Pb} = 32.68\ \mu V/°c$

$b_{Sb-Au} = b_{Sb-Pb} - b_{Au-Pb} = 0.137\ \mu V/°c$

$e = at + \frac{1}{2}bt^2 = (32.68) \times 100 + \frac{1}{2}(0.137)(100)^2 = 3953\ \mu V.$

(7) $e = 2160\ \mu V$, $t_1 = 0°c$, $t_2 = 250°c$, $t_n = 330°c$

$e = at + \frac{1}{2}bt^2$ —①

at neutral temp, $t_n = \frac{-a}{b} \longrightarrow 330b = -a$ —②

$\therefore 2160 = (-330b)t + \frac{1}{2}bt^2$

$2160 = (-330b)(250) + \frac{1}{2}b(62500)$

$2160 = -51250b$

$\therefore b = -0.042\ \mu V/°c$

put in ②,

$a = -330 \times (-0.042)$

$a = 13.86\ \mu V/°c^2$

(8) $P_{(Fe-Pb)} = 17.5\ \mu V/^\circ c$ @ $0^\circ c$

$= 5\mu V/^\circ c$ @ $125^\circ c$

$P_{(Cd-Pb)} = 3\ \mu V/^\circ c$ @ $0^\circ c$

$= 15\ \mu V/^\circ c$ @ $150^\circ c$

t_n at Fe-Cd junction =? (neutral)

$t_n = \dfrac{-a}{b}$ (neutral)

For Fe-Pb:

$P_1 = a_1 + b_1 t$

$17.5 = a_1 + b_1(0) \rightarrow a_1 = 17.5\ \mu V/^\circ c$

$P_1 = a_1 + b_1 t$

$5 = 17.5 + b_1(125)$

$b_1 = -0.1\ \mu V/^\circ c^2$

For Cd-Pb:

$a_2 = 3\ \mu V/^\circ c$

$15 = 3 + b_1(150)$

$\therefore b_2 = 0.08\ \mu V/^\circ c^2$

For Fe-Cd:

$a_{(Fe-Cd)} = a_1 - a_2 = 14.5\ \mu V/^\circ c$

$b_{(Fe-Cd)} = b_1 - b_2 = -0.18\ \mu V/^\circ c^2$

$\therefore T_n = \dfrac{-a}{b} = \dfrac{-14.5}{-0.18} = 80.55^\circ c$