## SOME PRACTICE PROBLEMS

### Non-Homogeneous Equation

**1.** Test for consistency the following set of equations and obtain the solution if consistent.

**(i)** $3x + 3y + 2z = 1,\ x + 2y = 4,\ 10y + 3z = -2,\ 2x - 3y - z = 5$

**(ii)** $2x - y - z = 2,\ x + 2y + z = 2,\ 4x - 7y - 5z = 2$

**(iii)** $2x_1 + 2x_2 = -11,\ 6x_1 + 20x_2 - 6x_3 = -3,\ 6x_2 - 18x_3 = -1$

**(iv)** $x - 2y + 3t = 0,\ 2x + y + z + t = -4,\ 4x - 3y + z + 7t = 8$

**(v)** $x_1 + x_2 + x_3 = 4,\ 2x_1 + 5x_2 - 2x_3 = 3,\ x_1 + 7x_2 - 7x_3 = 5$

**(vi)** $5x_1 - 3x_2 - 7x_3 + x_4 = 10, -x_1 + 2x_2 + 6x_3 - 3x_4 = -3,$
$x_1 + x_2 + 4x_3 - 5x_4 = 0$

**(vii)** $-x_2 + x_3 = 4,\ 3x_1 - x_2 + x_3 = 6,\ 4x_1 - x_2 + 2x_3 = 7, -x_1 + x_2 - x_3 = 9$

**(viii)** $x + 2y = 1,\ -3x + 2y = -2,\ -x + 6y = 0$

**(ix)** $2x - y + 3z = 9,\ x + y + z = 6,\ x - y + z = 2$

**(x)** $x + y + 4z = 6,\ 3x + 2y - 2z = 9,\ 5x + y + 2z = 13$

**(xi)** $x_1 + x_2 + x_3 = 4,\ 2x_1 + 5x_2 - 2x_3 = 3$

**(xii)** $x + y + z = 6\ ,\ x - y + 2z = 5\ ,\ 3x + y + z = 8\ ,\ 2x - 2y + 3z = 7$

**(xiii)** $2x - y + z = 9\ ,\ 3x - y + z = 6\ ,\ 4x - y + 2z = 7\ , -x + y - z = 4$

**(xiv)** $x_1 - 2x_2 + x_3 - x_4 = 2\ ,\ x_1 + 2x_2 + 2x_4 = 1\ ,\ 4x_2 - x_3 + 3x_4 = -1$

**(xv)** $2x - y + z = 8\ ,\ 3x - y + z = 6\ ,\ 4x - y + 2z = 7\ , -x + y - z = 4$

**(xvi)** $5x + 3y + 7z = 4\ ,\ 3x + 26y + 2z = 9\ ,\ 7x + 2y + 10z = 5$

**(xvii)** $x - 2y + 3t = 2\ ,\ 2x + y + z + t = -4\ ,\ 4x - 3y + z + 7t = 8$

**2.** Show that the system $\begin{aligned} 2x_1 - 3x_2 + 7x_3 &= 5 \\ 3x_1 + x_2 - 3x_3 &= 13 \\ 2x_1 + 19x_2 - 47x_3 &= 32 \end{aligned}$ is inconsistent.

**3.** Investigate for what values of a and b the simultaneous equations $\begin{aligned} 2x - y + 3z &= 2 \\ x + y + 2z &= 2 \\ 5x - y + az &= b \end{aligned}$

will have **(i)** no solution: **(ii)** a unique solution: **(iii)** an infinite number of solutions.

**4.** Determine the values of a & b such that system

$3x - 2y + z = b\ ,\ 5x - 8y + 9z = 3,\ 2x + y + az = -1$ has

(i) no solution (ii) unique solution (iii) infinite number of solutions.

**5.** Investigate for what values of λ and μ the simultaneous equations $\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$

will have **(i)** no solution: **(ii)** a unique solution: **(iii)** an infinite number of solutions.

**6.** Investigate for what values of $\lambda$ and $\mu$ the equations
$2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ has
(i) no solution (ii) unique solution (iii) infinite number of solutions.

**7.** Find the values of λ for which the system of equations
$$\begin{aligned} x + y + 4z &= 1 \\ x + 2y - 2z &= 1 \\ \lambda x + y + z &= 1 \end{aligned}$$
will have **(i)** a unique solutions **(ii)** no solution

**8.** Find values of λ for which the set of equations
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ x_1 + x_2 + x_3 &= \lambda \\ 3x_1 + x_2 + 3x_3 &= \lambda^2 \end{aligned}$$
are consistent and solve equations for those values.

**9.** For what value of $\lambda$ the equations
$$x + y + z = 1,\ x + 2y + 4z = \lambda,\ x + 4y + 10z = \lambda^2$$
have a solution and solve them completely in each case.

**10.** Show that the system of equation
$$\begin{aligned} -2x + y + z &= a \\ x - 2y + z &= b \\ x + y - 2z &= c \end{aligned}$$
have no solution unless $a + b + c = 0$, in which case they have infinitely many solutions. Find these solutions when $a = 1, b = 1, c = -2$.

## Homogeneous Equation

**11.** Find (trivial or non-trivial) solutions of the following linear equations.

**(i)** $x_1 - x_2 + 2x_3 = 0,\ x_1 + 2x_2 + x_3 = 0, 2x_1 + x_2 + 3x_3 = 0$
**(ii)** $x_1 + 2x_2 + 3x_3 + x_4 = 0,\ x_1 + x_2 - x_3 - x_4 = 0, 3x_1 - x_2 + 2x_3 + 3x_4 = 0$
**(iii)** $x_1 - 2x_2 + x_3 = 0, x_1 - 2x_2 - x_3 = 0, 2x_1 - 4x_2 - 5x_3 = 0$
**(iv)** $2x_1 + 3x_2 - x_3 + x_4 = 0, 3x_1 + 2x_2 - 2x_3 + 2x_4 = 0, 5x_1 - 4x_3 + 4x_4 = 0$
**(v)** $x_1 + 2x_2 + 3x_3 = 0$ , $2x_1 + 3x_2 + x_3 = 0$
**(vi)** $4x_1 + 5x_2 + 4x_3 = 0$ , $x_1 + 2x_2 - 2x_3 = 0$
**(vii)** $x_1 + x_2 - x_3 + x_4 = 0$ , $x_1 - x_2 + 2x_3 - x_4 = 0$ , $3x_1 + x_2 + x_4 = 0$
**(viii)** $2x - 2y - 5z = 0$, $4x - y + z = 0$, $3x - 2y + 3z = 0$, $x - 3y + 7z = 0$

**12.** Find the solution of the system given by
$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ x_1 - 2x_2 - x_3 &= 0 \\ 2x_1 - 4x_2 - 5x_3 &= 0 \end{aligned}$$
Also find the relation between column vectors of coefficient matrix.

**13.** Solve the following system of linear equation
$$\begin{aligned} x_1 - 2x_2 - x_3 &= 0 \\ -2x_1 + 4x_2 + 2x_3 &= 0 \\ -3x_1 - x_2 + 7x_3 &= 0 \\ 4x_1 + 3x_2 + 6x_3 &= 0 \end{aligned}$$

**14.** Find k if the system $\begin{matrix} 2x-3y+4z=0 \\ 3x+4y+6z=0 \\ 4x+5y+kz=0 \end{matrix}$ has non trivial solution

**15.** For what values of $\lambda$ the following system of equations possesses a non-trivial solution? Obtain the general solution in each case.

$2x-2y+z=\lambda x$ , $2x-3y+2z=\lambda y$ , $-x+2y=\lambda z$

**16.** If the following system has non – trivial solutions, prove that $a+b+c=0$ or $a=b=c$, Where $ax+by+cz=0, bx+cy+az=0, cx+ay+bz=0$. Find the non – trivial solution when the condition is satisfied.

## Linear Dependence & Independence Of Vectors

**17.** Are the following vectors linearly dependent? If so find the relation between them.

**(i)** $X_1=[1\ 1\ 1\ 3], X_2=[1\ 2\ 3\ 4], X_3=[2\ 3\ 4\ 9]$
**(ii)** $X_1=[2\ 3\ 4-2], X_2=[-1-2-2\ 1], X_3=[1\ 1\ 2-1]$
**(iii)** $X_1=[1\ 2\ 1\ ], X_2=[2\ 1\ 4], X_3=[4\ 5\ 6], X_4=[1\ 8-3]$
**(iv)** $X_1=[1-1\ 1\ ], X_2=[2\ 1\ 1], X_3=[3\ 0\ 2]$
**(v)** $X_1=[1\ 2\ 3\ ], X_2=[2-2\ 6]$
**(vi)** $X_1=[3\ 1-4\ ], X_2=[2\ \ 2-3], X_3=[0-4\ \ 1]$
**(vii)** $X_1=[1\ 1\ 1\ 3], X_2=[1\ 2\ 3\ 4], X_3=[2\ 3\ 4\ 7]$
**(viii)** $X_1=[1\ 1-1\ 1], X_2=[1-1\ 2-1], X_3=[3\ 1\ 0\ 1]$
**(ix)** $X_1=[1-1\ 2\ 0], X_2=[2\ 1\ 1\ 1], X_3=[3-1\ 2-1], X_4=[3\ 0\ 3\ 1]$
**(x)** $[1,2,-1,0]$ , $[1,3,1,2]$ , $[4,2,1,0]$ , $[6,1,0,1]$
**(xi)** $[2,-1,3,2]$ , $[1,3,4,2]$ , $[3,-5,2,2]$
**(xii)** $[1,2,-1,0]$ , $[1,3,1,3]$ , $[4,2,1,-1]$ , $[6,1,0,-5]$
**(xiii)** $[1,\ 3,\ 4,\ 2]$ , $[3,\ -5,\ 2,\ 6]$ , $[2,-1,\ 3,\ 4]$
**(xiv)** $[3,\ 1,\ 1]$ , $[2,\ 0,-1]$, $[4,\ 2,\ 1]$

**18.** Show that the rows of the following matrices are linearly dependent and express any row as a linear combination of other rows.

**(i)** $A=\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$  **(ii)** $A=\begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & 1 & 2 & 1 \\ 4 & 6 & 2 & -4 \\ -6 & 0 & -3 & -4 \end{bmatrix}$