SOME PRACTICE PROBLEMS

1. Express the following matrices as the sum of a symmetric and a skew-symmetric matrix.

(i) $\begin{bmatrix} 2 & -4 & 9 \\ 14 & 7 & 13 \\ 3 & 5 & 11 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 2 & 8 & 6 \\ 0 & 4 & 4 \\ 2 & 10 & 12 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 3a & 2b & 2c \\ b & c & a \\ 3c & 3a & 3b \end{bmatrix}$

(iv) $\begin{bmatrix} 2a & 3b & 2c \\ -b & c & 3a \\ 3c & 3a & 2b \end{bmatrix}$ **(v)** $\begin{bmatrix} 1 & 0 & 5 & 3 \\ -2 & 1 & 6 & 1 \\ 3 & 2 & 7 & 1 \\ 4 & -4 & 2 & 0 \end{bmatrix}$

2. Express the following matrices as the sum of a Hermitian and a skew-Hermitian matrix.

(i) $\begin{bmatrix} 2+i & -i & 3+i \\ 1+i & 3 & 6-2i \\ 3-2i & 6i & 4-3i \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1+i & 2-3i & 2 \\ 3-4i & 4+5i & 1 \\ 5 & 3 & 3-i \end{bmatrix}$

(iii) $\begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$

3. Express the matrix $A = \begin{bmatrix} 2i & -3 & 1-i \\ 0 & 2+3i & 1+i \\ -3i & 3+2i & 2-5i \end{bmatrix}$ as $P + iQ$ where P and Q are Hermitian matrices.

4. Express the Hermitian matrix $A = \begin{bmatrix} 2 & 2+i & -2i \\ 2-i & 3 & i \\ 2i & -i & 1 \end{bmatrix}$ as $P + iQ$ where P is real symmetric and Q is real skew-symmetric matrix.

5. Express the skew-Hermitian matrix $A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$ as $P + iQ$ where P is real skew-symmetric and Q is real symmetric matrix.

6. Express the Hermitian matrix $\begin{bmatrix} 4 & 3-2i & -1+i \\ 3+2i & 2 & 5+4i \\ -1-i & 5-4i & 7 \end{bmatrix}$ as $B + iC$ where B is real symmetric and C is real skew symmetric.

7. Express the skew - Hermitian matrix $\begin{bmatrix} 2i & 3+i & 2-i \\ -3+i & 0 & 6i \\ -2-i & 6i & -2i \end{bmatrix}$ as $P + iQ$ where P is real skew – symmetric and Q is real symmetric.

8. Verify that the matrix A is orthogonal, where $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$ and find A^{-1} .

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9. Show that following matrices are orthogonal.

(i) $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ (ii) $\begin{bmatrix} \cos\emptyset & 0 & \sin\emptyset \\ \sin\theta.\sin\emptyset & \cos\theta & -\sin\theta.\cos\emptyset \\ -\cos\theta.\sin\emptyset & \sin\theta & \cos\theta.\cos\emptyset \end{bmatrix}$

10. Find a, b, c and A^{-1} if the matrix A is orthogonal where A is

(i) $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ (ii) $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$ (iii) $\frac{1}{9}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix}$

11. Are the following matrices orthogonal? If not, can they be converted to an orthogonal matrix? If yes, how?

(i) $A = \begin{bmatrix} 2i & -3 & 1-i \\ 0 & 2+3i & 1+i \\ -3i & 3+2i & 2-5i \end{bmatrix}$ (ii) $B = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

12. Verify that the matrix $A = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is orthogonal and hence find its inverse.

13. Determine the values of $\propto, \beta, \gamma$ when the matrix given by $A = \begin{bmatrix} \alpha & \beta & -\gamma \\ \alpha & -2\beta & 0 \\ \alpha & \beta & \gamma \end{bmatrix}$ is orthogonal.

14. If $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is an orthogonal matrix, determine $l, m,$ and n. Hence find A^{-1}. Also find rank A, rank 3A, rank A^2.

15. Prove that $A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$ is orthogonal and hence find A^{-1} .

16. Prove that following matrices are unitary and hence find A^{-1}.

(i) $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ (ii) $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

(iii) $A = \begin{bmatrix} \frac{2+i}{3} & \frac{2i}{3} \\ \frac{2i}{3} & \frac{2-i}{3} \end{bmatrix}$ (iv) $\begin{bmatrix} \frac{1+i}{2} & \frac{i}{\sqrt{3}} & \frac{3+i}{2\sqrt{15}} \\ \frac{-1}{2} & \frac{1}{\sqrt{3}} & \frac{4+3i}{2\sqrt{15}} \\ \frac{1}{2} & \frac{-i}{\sqrt{3}} & \frac{5i}{2\sqrt{15}} \end{bmatrix}$

17. Find the ranks of the following matrices

(i) $\begin{bmatrix} 1 & 2 & -2 & 3 \\ -1 & -3 & 2 & -2 \\ 0 & -1 & 0 & 1 \\ -1 & -4 & 2 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & -3 & 5 \\ 4 & -6 & 10 \\ -8 & 12 & -20 \\ 6 & -9 & 15 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 6 \end{bmatrix}$ (v) $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ -5 & -12 & -1 & 6 \end{bmatrix}$ (vi) $\begin{bmatrix} 2 & 3 & 1 & 4 \\ 5 & 2 & 3 & 0 \\ 9 & 8 & 0 & 8 \end{bmatrix}$

(vii) $\begin{bmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{bmatrix}$

18. Reduce the following matrices to normal form and find their rank.

$\begin{bmatrix} 1 & -1 & -2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 4 & 3 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 3 & 3 & 1 \\ 1 & 4 & 2 & 0 \\ 0 & -4 & -1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

$\begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 1 & 4 & -5 \\ -1 & -5 & -5 & 7 \end{bmatrix}$

$\begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 8 & 5 & 8 \end{bmatrix}$

$\begin{bmatrix} 2 & -2 & 3 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -2 & 3 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$

$\begin{bmatrix} 3 & -3 & 0 & -1 & -7 \\ 0 & 2 & 2 & 1 & -5 \\ 1 & -2 & -3 & -2 & 1 \\ 0 & 1 & 2 & 1 & -6 \end{bmatrix}$ $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & 1 & -3 & 4 \end{bmatrix}$ $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & 3 \\ 2 & 2 & 0 & 2 & 2 \\ 3 & 3 & 2 & 1 & 1 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix} \quad \begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 15 & 14 & 15 \\ 6 & 24 & 18 & 30 \\ 1 & 4 & 2 & 5 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$$

19. Find the rank of A by reducing it to the normal form, where $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 3 & -1 & 2 & 2 \\ 4 & 1 & 0 & -1 \\ 9 & 1 & 5 & 6 \end{bmatrix}$

    Hence find the rank of A^2.

20. Reduce the following matrices to Echelon Forms and hence find the ranks.

    (i) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$

21. Find the values of P for which the matrix $A = \begin{bmatrix} P & 2 & 2 \\ 2 & P & 2 \\ 2 & 2 & P \end{bmatrix}$ will have (i) rank 1, (ii) rank 2, (iii) rank 3,

22. The rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2. Find the value of λ, where λ is real.

23. Find the rank of $A = \begin{bmatrix} x-1 & x+1 & x \\ -1 & x & 0 \\ 0 & 1 & 1 \end{bmatrix}$ where x is real.