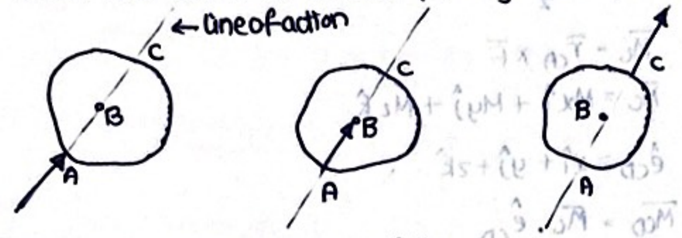
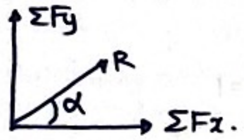
SYSTEM OF FORCES

- coplanar → concurrent
  - → parallel
  - → general (non-concurrent and non-parallel)
- non-coplanar (space forces)

(1) <u>principle of transmissibility of force</u>: a force being a sliding vector will not affect state of a rigid body (whether at rest or in motion) if force acts from a different point along its line of action.
e.g. in train, engine can be located at front pulling other cars with it or at back pushing them forward.



(2) <u>sign conventions</u>: ↑(+) →, ↓(−) ←.

(3)


$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$

$$\boxed{\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x}}$$

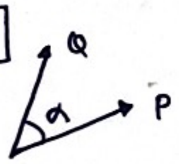
(4) <u>Varignon's Theorem</u>:
algebraic sum of moments of a system of coplanar forces about any point in plane is equal to moment of resultant force of system about same point.

(5) <u>moment of a force</u>: $\boxed{M = F \times d}$ unit: N-m

(6) <u>sign conventions</u>: anticlockwise (+)
clockwise (−)

(7.) $\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}}$

$$\boxed{\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}}$$



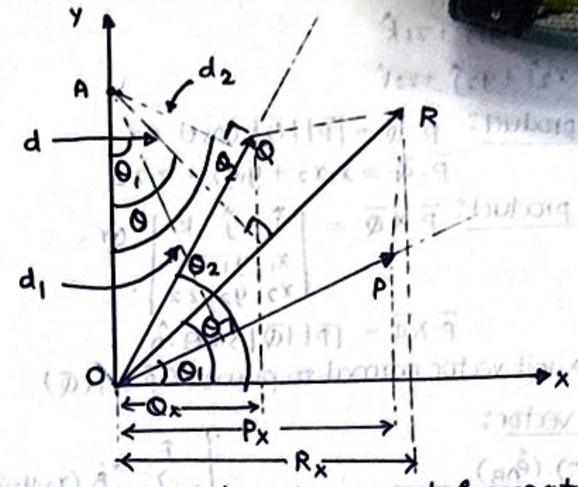
(8) <u>couple</u>: two non-collinear parallel forces of equal magnitude but opp. in directions form couple.
- causes rotation of body.

$\boxed{M = F \times d}$

(9) $\boxed{\Sigma M_O = |R \times d| = |\Sigma F_x \times y| = |\Sigma F_y \times x|}$

module 1 → ch. 2, 7
module 2 → ch. 9, 13
module 3 → ch. 6
module 4 → ch. 3, 4
module 5 → ch. 10, 11, 12

<u>Varignon's Theorem</u>:



Let P and Q be two concurrent forces at O, making angle θ_1 and θ_2 with x-axis, let R be their resultant making angle θ with x-axis.
Let A be a point on y-axis about which we shall find moments of P and Q and also of resultant R.
Let d_1, d_2 and d be moment arm of P, Q and R from moment centre A.
Let x component of forces P, Q and R be P_x, Q_x and R_x respectively.

moment of P about A $= M_A^P = P \times d_1$ —(1)
" Q " $= M_A^Q = Q \times d_2$ —(2)
" R " $= M_A^R$
$= R \times d$.
$= R \times (OA\cos\theta)$
$= OA(R_x)$ —(3)

EM

adding (1) and (2),
$M_A^P + M_A^Q = Pd_1 + Qd_2$
or sum of moments
$\Sigma M_A^F = +(P \times OA\cos\theta_1) + (Q \cdot \times OA\cos\theta_2)$
$= OA \cdot P_x + OA\, Q_x$. ($P_x = P\cos\theta$)
                                        ($Q_x = Q\cos\theta$)
$= OA(P_x + Q_x)$

$\Sigma M_A^F = OA(R_x)$. —(4)

($P_x + Q_x = R_x$, since the resultant of forces in x direction is sum of components of forces in x direction)

comparing (4) with (3),
$\Sigma M_A^F = M_A^R$
Hence proved.

Non-coplanar forces:

(1) $\bar{P} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\bar{Q} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

dot product: $\bar{P}\cdot\bar{Q} = |\bar{P}||\bar{Q}|\cos\theta$ or

$\bar{P}\cdot\bar{Q} = x_1x_2 + y_1y_2 + z_1z_2$

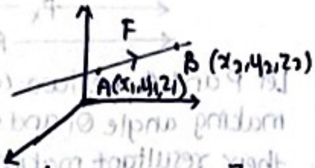
cross product: $\bar{P}\times\bar{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$ or.

$\bar{P}\times\bar{Q} = |\bar{P}||\bar{Q}|\sin\theta\cdot\hat{n}$

($\hat{n} \rightarrow$ unit vector normal to plane of $\bar{P}$ and $\bar{Q}$)

(2) Force vector:

$F = (F)\cdot(\hat{e}_{AB})$



$\boxed{F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}}$

$$F = (F)\left[\frac{(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}\right]$$

(where $\hat{e}_{AB} \rightarrow$ unit vector in direction of AB)

(3) magnitude of force and direction angles:

$|F|$ or $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$F_x = F\cos\theta_x$, $F_y = F\cos\theta_y$, $F_z = F\cos\theta_z$

By cosine rule,

$\cos\theta_x^2 + \cos\theta_y^2 + \cos\theta_z^2 = 1$

(4) moment vector:

$\bar{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\left.\begin{array}{l}\bar{r}_{CA} = (x_1-x_3)\hat{i} + (y_1-y_3)\hat{j} + (z_1-z_3)\hat{k} \\ \quad = x\hat{i} + y\hat{j} + z\hat{k}\end{array}\right\}$ Position vector:

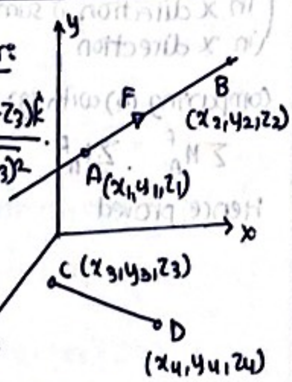
$\left.\begin{array}{l}\bar{M}_C = \bar{r}_{CA}\times\bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \\ \bar{M}_C = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}\end{array}\right\}$ moment vector:

(5) vector component of force along given line:

$\bar{F} = (F)(\hat{e}_{AB})$ — unit vector:

$$\hat{e}_{CD} = \frac{(x_4-x_3)\hat{i} + (y_4-y_3)\hat{j} + (z_4-z_3)\hat{k}}{\sqrt{(x_4-x_3)^2 + (y_4-y_3)^2 + (z_4-z_3)^2}}$$

$\hat{e}_{CD} = x\hat{i} + y\hat{j} + z\hat{k}$

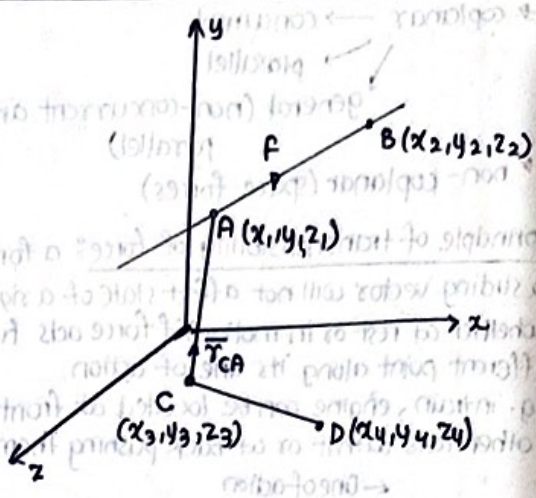


$F_{CD} = \bar{F}\cdot\hat{e}_{CD} \rightarrow$ scalar component:

$F_{CD} = F_x\cdot x + F_y\cdot y + F_z\cdot z$

$\bar{F}_{CD} = (F_{CD})\cdot(\hat{e}_{CD}) \longrightarrow$ vector component:

(6) moment of a force about a given line:



$\bar{M}_C = \bar{r}_{CA}\times\bar{F}$

$\bar{M}_C = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$

$\hat{e}_{CD} = x\hat{i} + y\hat{j} + z\hat{k}$

$M_{CD} = \bar{M}_C\cdot\hat{e}_{CD}$

$M_{CD} = M_x\cdot x + M_y\cdot y + M_z\cdot z$

$\bar{M}_{CD} = (M_{CD})(\hat{e}_{CD})$

(7) $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

(8)

—(3)

KINEMATICS OF A PARTICLE

(1) equations of motion: (uniform accln motion)

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

(2) average velocity: $V_{av} = \frac{\Delta x}{\Delta t}$

instantaneous velocity: $V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

average speed: $s = \frac{\text{distance travelled}}{\text{time interval}}$

(3) average accln: $a_{av} = \frac{\Delta v}{\Delta t}$

instantaneous accln: $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

(4) (uniform velocity motion) $v = \frac{s}{t}$

(5) (variable accln motion) $v = \frac{dx}{dt}$, $a = \frac{dv}{dt}$

$\frac{a}{v} = \frac{dv}{dx} \to a = v \cdot \frac{dv}{dx}$

(6) For a uniformly accelerated rectilinear moving particle, distance covered in nth second is,

$s^{nth} = u + a \times n - \frac{a}{2} = u + a\left(n - \frac{1}{2}\right)$

(7) motion curves:

(i) x-t curve:

v = slope of x-t curve

(ii) v-t curve:

a = slope of v-t curve

$x_f = x_i + (\text{area under v-t curve})_{t_i - t_f}$

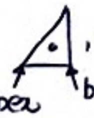
(iii) a-t curve:

$v_f = v_i + (\text{area under curve a-t})_{t_i - t_f}$

$x_f = x_i + v_i \times t + (\text{auc a-t})_{t_i - t_f} \times (t - t_c)$ [centroid]

(iv) v-x curve:

$a = v \times$ slope of v-x curve

* area under concave parabolic curve (up) $= \frac{1}{3} \times b \times h$
* area under concave (down) par. curve $= \frac{2}{3} \times b \times h$
* for a Δ,  , centroid from base $= \frac{\text{base}}{3}$

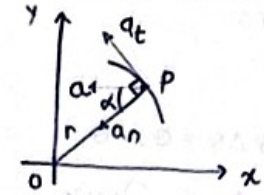
apex base

from apex $= \frac{2 \times \text{base}}{3}$

(consider from ~~left~~ ends)

* km/hr → m/s, multiply by $\frac{5}{18}$

(8) curvilinear motion:



for x-y coor system: $r = x\hat{i} + y\hat{j} + z\hat{k}$

$v = \frac{dr}{dt}$

$v = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} + \left(\frac{dz}{dt}\right)\hat{k}$

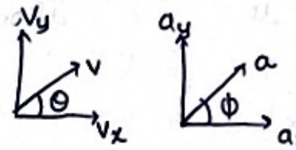
$v = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

$a = \frac{dv}{dt} = \frac{d^2r}{dt^2} = (a_x)\hat{i} + (a_y)\hat{j} + (a_z)\hat{k}$

$v = \sqrt{v_x^2 + v_y^2} \to \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

$a = \sqrt{a_x^2 + a_y^2} \to \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

(magnitude and angle)



for 't-n' coor system:

$a_t = \frac{dv}{dt}$; $a_n = \frac{v^2}{\rho}$ ($v \to$ vel. at given time) ($\rho \to$ rad. of curvature)

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ (if $f(y) = x$)

↓ at that instant.

* constant speed, $a_t = 0$

Total accln: $a = \sqrt{a_t^2 + a_n^2}$

$\alpha = \tan^{-1}\left(\frac{a_t}{a_n}\right)$

also, $\rho = \frac{\left[v_x^2 + v_y^2\right]^{3/2}}{v_x \cdot a_y - v_y \cdot a_x}$, $\rho = \left[\frac{v^3}{a_x \cdot v_y - a_y \cdot v_x}\right]$

($a_t \to$ tangential component of accln)

($a_n \to$ normal component of accln)

* for uniform speed curvillinear motion, $a_t = 0$
* for speed changing at uniform rate,

$v = u + a_t \cdot t$

$s = u \cdot t + \frac{1}{2} a_t \cdot t^2$

$v^2 = u^2 + 2a_t \cdot s$

* $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = \sqrt{x^2 + y^2 + z^2}$

$\bar{v} = \frac{d\bar{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$\bar{a} = \frac{d\bar{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$\tan\theta_r = y/x$, $\tan\theta_v = \frac{v_y}{v_x}$, $\tan\theta_a = \frac{a_y}{a_x}$

(9) Instantaneous Centre Method (ICR):

* 1 revolution = 2π radians = 360
* $1\,rpm = \frac{2\pi}{60}$ rad/s

(i) angular displacement: θ or $\Delta\theta = \theta_2 - \theta_1$

angular velocity: $\omega = \frac{d\theta}{dt}$ ↺ +ve ↻ -ve

angular accln: $\alpha = \frac{d\omega}{dt}$

uniform angular vel. motion → $\omega = \frac{\theta}{t}$

uniform angular accln motion → $\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

variable angular accln motion: $\omega = \frac{d\theta}{dt}$

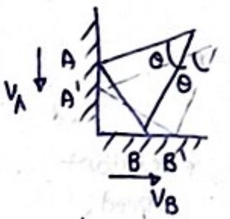
$\alpha = \frac{d\omega}{dt} \rightarrow \alpha = \omega \cdot \frac{d\omega}{d\theta}$

(ii) $\boxed{V = r\omega}$

$a_n = r\omega^2$, $a_t = r\alpha$

(iii) ICR: defined as point about which a general plane moving body rotates at any given instant

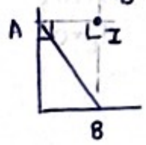
(A) LADDER:



[Translational motion.]

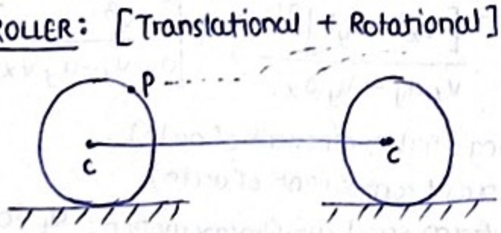
$\boxed{V_A = (IA) \times \omega}$

$\boxed{V_B = (IB) \times \omega}$

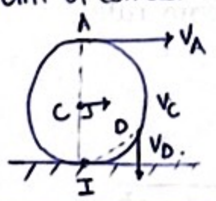


steps:
(i) locate ICR
(ii) find ω, V_A, V_B.

(B) ROLLER: [Translational + Rotational]

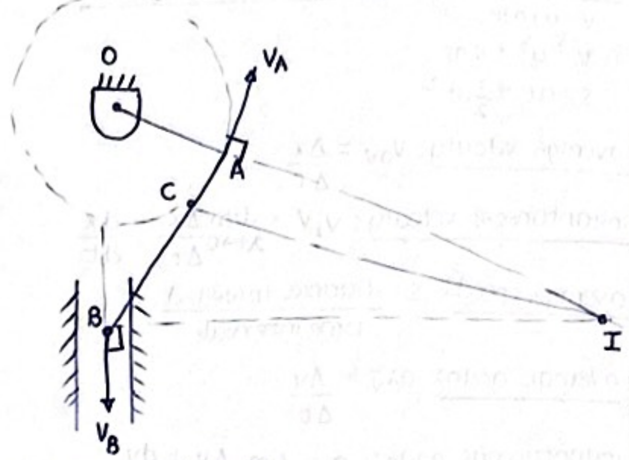


point of contact is I.



$\boxed{V_D = (ID) \times \omega}$

(C) 2 LINKAGES: [Translational + Rotational]



OA: rotational motion.

$\boxed{V_O = 0}$

$\boxed{V_A = (OA)\,\omega}$

AB: $\underbrace{\text{rotational + translational}}_{\text{plane motion}}$.

$\boxed{V_A = (IA)\,\omega}$

$\boxed{V_B = (IB)\,\omega}$

if c is any point on the linkage,

$\boxed{V_C = (IC)\,\omega}$

* vel. of IR (instantaneous rotation) point is zero.
* location of ICR point varies instant to instant
* cosine rule: 2 lengths, 1 angle

sine rule: 1 length, 2 angles

$\sqrt{A^2 + B^2 + 2AB\cos\theta}$