K. J. Somaiya College of Engineering, Mumbai-77
(A Constituent college of Somaiya Vidyavihar University)

F.Y. B. Tech SEM-I (2023-24)
Applied Mathematics-I
Eigenvalues and Eigenvectors

Some Practice Problems

Q: For the following matrices:

i. **Find Characteristic equation.**

ii. **Find Eigenvalues and Eigenvectors.**

iii. **Prove that eigenvectors are linearly independent.**

iv. **Verify Cayley-Hamilton Theorem. Hence, find A^{-1} and A^{4} if exists.**

v. **Check weather the matrix is diagonalisable. If yes, find the transforming matrix M and the diagonal matrix D./ Check whether the given matrix is similar to diagonal matrix. If similar to diagonal matrix, express in form of $D = M^{-1}AM$.**

vi. **Find the minimal polynomial and check weather the matrix is derogatory.**

1. $\begin{bmatrix} 3 & -9 & -12 \\ 1 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

5. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

6. $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

7. $\begin{bmatrix} 2 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$

9. $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$

10. $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

13. $\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

14. $\begin{bmatrix} 4 & 1 & -1 \\ 6 & 3 & -5 \\ 6 & 2 & -2 \end{bmatrix}$

15. $\begin{bmatrix} 3 & -2 & 3 \\ 10 & -3 & 5 \\ 5 & -4 & 7 \end{bmatrix}$

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Function of a square matrix

1. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ then find f(A) = $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ and eigenvalues of f(A).

2. If $A = \begin{bmatrix} sin\theta & cos\theta \\ -cos\theta & sin\theta \end{bmatrix}$ then find f(A) = A^{-1} using Cayley-Hamilton Theorem. Also find eigenvalues of f(A).

3. If $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$ then find f(A) = $2A^5 - 3A^4 + A^2 - 4I$ and eigenvalues of f(A).

4. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ then find f(A) = $e^{A\pi/2}$ and eigenvalues of f(A).

5. Show that $\cos O_{3\times3} = I_{3\times3}$ where $\cos O_{3\times3}$ *and* $I_{3\times3}$ are respectively the zero matrix and the identity matrix of are order 3.

6. If $A = \begin{bmatrix} -3 & 2 \\ 4 & 3 \end{bmatrix}$ then for any positive integer n, prove that $A^{2n+1} = A$

7. If $A = \begin{bmatrix} \theta & \theta \\ \theta & \theta \end{bmatrix}$ then prove that $e^A = e^\theta \begin{bmatrix} cosh\theta & sinh\theta \\ sinh\theta & cosh\theta \end{bmatrix}$

8. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$. Hence find A^{50}.

9. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, that find f(A) = $A^9 - 6A^8 + 10A^7 - 3A^6 + A + I$ and eigenvalues of f(A).

10. If $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, that find f(A) = A^{100} and eigenvalues of f(A).