# SOME PRACTICE PROBLEMS

1. If $u = \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right)^n$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ .

2. If $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$ , prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ .

3. If $= (x/y)^{(y/x)}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ .

4. Verify Euler's Theorem

    i)      $u = 3x^2yz + 5xy^2z + 4xyz^2$

    ii)    $u = \frac{x}{y} + \frac{y}{x}$

    iii)    $u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$

    iv)    $u = \frac{x^2+y^2}{x+y}$

    v)    $u = x^2\tan^{-1}\left(\frac{y}{x}\right)$

    vi)    $u = \frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$

    vii)    $u = \frac{x(x^3-y^3)}{x^3+y^3}$

    viii)    $u = \frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}$

5. If $u = \log(x^2 + y^2) + \frac{x^2+y^2}{x+y} - 2\log(x + y)$ , Find $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ .

6. If $u = x^3\sin^{-1}\left(\frac{\sqrt{y}+\sqrt{x}}{\sqrt{y}-\sqrt{x}}\right)$ , prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3x^3\sin^{-1}\left(\frac{\sqrt{y}+\sqrt{x}}{\sqrt{y}-\sqrt{x}}\right)$

7. If $u = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2+y^2}$ , prove that $x\frac{\partial u}{\partial y} + y\frac{\partial u}{\partial x} + 2u = 0$.

8. If $u = \frac{f(\theta)}{r}$ where $x = r\cos\theta$, $y = r\sin\theta$ , show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + u = 0$.

9. If $u = f(v)$, where v is a homogenous function of x, y of degree n, then prove that,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\,v\,f'(v). \text{ Hence deduce that if } u = \log v, \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n.$$

10. If $u = \frac{x^2 y^2}{x^2 + y^2} + \cos\left(\frac{xy}{x^2 + y^2}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2x^2 y^2}{x^2 + y^2}$ .

11. If $u = \frac{x+y}{x^2 + y^2}$ Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

12. $u = \sin^{-1}\left(\frac{x}{y}\right) + \cos^{-1}\left(\frac{y}{z}\right) + \tan^{-1}\left(\frac{z}{x}\right)$ Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ .

13. If $u = xy\, f\left(\frac{y}{x}\right) + yz\, g\left(\frac{y}{z}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2u$.

14. If $u = x^2 f\left(\frac{y}{x}\right) + y^2 g\left(\frac{x}{y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$.

15. If $u = \frac{x^3 y^3}{x^3 + y^3}$ Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

16. If $u = \frac{x - y - z}{x^2 + y^2 + z^2}$ Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + u = 0$.

17. If $u = \frac{x^2 y + y^2 x}{x^2 + y^2}$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$.

18. If $= x^3 e^{-(y/x)}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 6u$.

19. If If $u = \log\left(\frac{\sqrt{x^2 + y^2}}{x + y}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$.

20. If $y = x\cos u$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

21. If $u = x^2 \sin^{-1}\frac{y}{x} - y^2 \cos^{-1}\frac{x}{y}$, find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u$.

22. If $u = x \sin^{-1}\frac{y}{x} + y \tan^{-1}\frac{y}{x}$, find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

23. If $u = x^3 \sin^{-1}\frac{y}{x} + x^4 \tan^{-1}\frac{y}{x}$, find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

 $at\ x = 1, y = 1$.

24. If $u = x^n f\left(\frac{y}{x}\right) + y^n f\left(\frac{x}{y}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n^2 u$.

25. If $u = \frac{x^4 + y^4}{x^2 y^2}$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

 $at\ x = 1, y = 2$.

26. If find value of $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ $at\ x = 1,\ y = 1$.

 when $z = x^6 \tan^{-1}\left(\frac{x^2 + y^2}{x^2 + xy}\right) + \frac{x^4 + y^4}{x^2 y^2}$.

27. If $u = \frac{(x^2 + y^2)^m}{2m(2m - 1)} + xf\left(\frac{y}{x}\right) + yg\left(\frac{x}{y}\right)$ find value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

28. If $u = x^3\left(\tan^{-1}\frac{y}{x} + \frac{y}{x} e^{-y/x}\right) + y^{-3}\left(sin^{-1}\frac{y}{x} + \frac{x}{y} log\frac{x}{y}\right)$, prove that

 $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 9u$.

29. If $u = \sin^{-1}\left(\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20} \tan u$.

**SOMAIYA**
VIDYAVIHAR UNIVERSITY
K J Somaiya College of Engineering

Somaiya Vidyavihar University
K. J. Somaiya College of Engineering, Mumbai -77
Applied Mathematics - I

Somaiya
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30. If If $u = \frac{1}{3} \log \left( \frac{x^3 + y^3}{x^2 + y^2} \right)$ ,find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ .

31. If $u = log(x^3 + y^3 - x^2 y - xy^2)$, prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3 \text{ and } x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -3.$$

32. If $u = \tan^{-1} \left( \frac{x^3 + y^3}{2x + 3y} \right)$, prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u.$

33. If $u = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 sin u cos 3u = sin 2u(1 - 4 sin^2 u) = sin 4u - sin 2u.$$

34. If $u = sin^{-1} \left( \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}} \right)$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{400} (\tan^2 u - 19).$$

35. If $u = \sinh^{-1} \left( \frac{x^3 + y^3}{x^2 + y^2} \right)$, prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = - \tanh^3 u.$

36. If $u = e^{\frac{x}{y}} + log(x^3 + y^3 - x^2 y + xy^2)$ *then find*

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} .$$

37. If $u = \log r$ & $r^2 = x^2 + y,^2$ prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + 1 = 0.$

38. If $u = \sin^{-1}(x^2 + y^2)^{\frac{1}{5}}$, show that $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{2}{25} \tan u(2 \tan^2 u - 3).$

39. If $u = \sin^{-1} \left( \frac{x + y}{\sqrt{x} + \sqrt{y}} \right)$, prove that

$(i) \ x u_x + y u_y = \frac{\tan u}{2}$      $(ii) \ x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = - \frac{\sin u \cos 2u}{4 \cos^3 u}.$

40. If $u = \log \frac{x + y}{\sqrt{x^2 + y^2}} + sin^{-1} \frac{x + y}{\sqrt{x} + \sqrt{y}}$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = - \frac{sin w \cos 2w}{4 cos^3 w} \ , \text{where } w = sin^{-1} \left( \frac{x + y}{\sqrt{x} + \sqrt{y}} \right)$$

41. If $u = \sec^{-1} \left( \frac{x^2 + y^2}{x - y} \right)$, Find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$

42. If $u = log \left( \frac{x^3 + y^3}{x^2 + y^2} \right)$, prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -1.$