K. J. Somaiya College of Engineering, Mumbai 77

(A Constituent college of Somaiya Vidyavihar University)

F.Y. Btech SEM-I APPLIED MATHEMATICS-I

Practice Problems

Type-1 : Hyperbolic Functions

1. If $\tanh x = 2/3$, find the value of x and then $cosh\ 2x$.

2. Solve the equation for real values of x, $17\cosh x + 18\sinh x = 1$.

3. If $6\sinh x + 2\ cosh\ x + 7 = 0$, find $tanh\ x$.

4. If $cosh^{-1}a + cosh^{-1}b = cosh^{-1}x$, then prove that $a\sqrt{b^2-1} + b\sqrt{a^2-1} = \sqrt{x^2-1}$.

5. If $cosh^6 x = a\cosh 6x + b\cosh 4x + c\cosh 2x + d$, Prove that $25a - 5b + 3c - 4d = 0$

6. Prove that $cosh^7 x = \frac{1}{64}[\cosh 7x + 7\cosh 5x + 21\cosh 3x + 35\cosh x]$

7. If $\cos\alpha\cosh\beta = x/2$, $\sin\alpha\sinh\beta = y/2$, show that

(i) $\sec(\alpha - i\,\beta) + \sec(\alpha + i\,\beta) = \frac{4x}{x^2+y^2}$ **(ii)** $\sec(\alpha - i\,\beta) - sec(\alpha + i\,\beta) = \frac{-4iy}{x^2+y^2}$

8. Prove that $cosech\ x + \coth x = \coth\frac{x}{2}$

9. Prove that $(\cosh x + sinh\ x)^n = \cosh nx + \sinh nx$

10. Prove that $\left(\frac{\cosh x + \sinh x}{\cosh x - \sinh x}\right)^n = \cosh 2nx + \sinh 2nx$

11. If $\log\tan x = y$, prove that $\cosh ny = \frac{1}{2}[tan^n x + cot^n x]$ and

$\sinh(n+1)y + \sinh(n-1)\,y = 2\sinh ny\ \ cosec\ 2x$

12. Prove that $\frac{1}{1-\frac{1}{1-\frac{1}{1+sinh^2 x}}} = -sinh^2 x$

13. If $\cosh u = \sec\theta$, *prove that*

(i) $\sinh u = \tan\theta$ **(ii)** $\tanh u = \sin\theta$ **(iii)** $u = \log\left[tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$

Type -2: Separation into real and Imaginary parts

1. Separate into real and imaginary parts.

(i) $\cosh(x+iy)$ **(ii)** $\cos(x+iy)$ **(iii)** $\coth(x+iy)$

(iv) $\operatorname{sech}(x+iy)$ **(v)** $\coth i(x+iy)$ **(vi)** $\tan(x+iy)$

(vii) $\cot(x+iy)$

2. Separate into real and imaginary parts $tan^{-1}(\alpha + i\,\beta)$

3. Separate into real and imaginary parts $sin^{-1}(e^{i\,\theta})$

4. If A $+i\,B = C\tan(x+iy)$, prove that $tan2x = \frac{2CA}{C^2-A^2-B^2}$

5. If $\cos(\theta + i\,\Phi) = r(\cos\alpha + i\sin\alpha)$, prove that $r^2 = \frac{1}{2}[\cosh 2\,\Phi + \cos 2\,\theta]$ &

$\tan\alpha = -\tan\theta\tanh\Phi$

6. If $\cos(\alpha + i\,\beta) = x + i\,y$, Prove that $\frac{x^2}{cosh^2\beta} + \frac{y^2}{sinh^2\beta} = 1,\ \ \frac{x^2}{cos^2\alpha} - \frac{y^2}{sin^2\alpha} = 1$

7. If $\sinh(a + i\,b) = x + i\,y$, prove that $x^2 cosech^2 a + y^2 sech^2 a = 1$

$and\ \ y^2 cosec^2 b - x^2 sec^2 b = 1$

8. If $\sin\,(x + iy) = \cos\alpha + i\sin\alpha$, Prove that

(i) $\cosh 2y - \cos 2x = 2$ **(ii)** $y = \frac{1}{2}\,log\frac{\cos(x-\alpha)}{\cos(x+\alpha)}$

(iii) $\sin\alpha = \pm\,cos^2 x = \pm\,sinh^2 y$

9. If $\cosh(\theta + i\,\Phi) = e^{i\,\alpha}$, prove that $sin^2\alpha = sin^4\Phi = sinh^4\theta$

10. If $\cos(u + i\,v) = x + i\,y$ Prove that, $(1 + x)^2 + y^2 = (\cosh v + \cos u)^2$ and

$(1 - x)^2 + y^2 = (\cosh v - \cos u\,)^2$

11. If $tan(\alpha + i\,\beta) = x + i\,y$, prove that $x^2 + y^2 + 2x\cot 2\alpha = 1,\ \ x^2 + y^2 - 2y\coth 2\beta + 1 = 0$

12. If $\tan\left(\frac{\pi}{3} + i\,\alpha\right) = x + i\,y$, prove that, $x^2 + y^2 - \frac{2x}{\sqrt{3}} - 1 = 0$

13. If $cot(\alpha + i\,\beta) = x + i\,y$, prove that $x^2 + y^2 - 2x\cot 2\alpha = 1,\ \ x^2 + y^2 + 2y\coth 2\beta + 1 = 0$

14. If $tanh\left(\alpha + \frac{i\,\pi}{6}\right) = x + i\,y$, prove that, $x^2 + y^2 + \frac{2y}{\sqrt{3}} = 1$

15. If $\coth(\alpha + i\,\pi/8) = x + i\,y$, prove that $x^2 + y^2 + 2y = 1$

16. If $\sinh(x + i\,y) = e^{i\,\pi/3}$, prove that

(i) $3cos^2 y - sin^2 y = 4sin^2 y\,cos^2 y$ **(ii)** $3sinh^2 x + cosh^2 x = 4sinh^2 x cosh^2 x$

17. If $x + i\,y = 2\cosh\left(\alpha + \frac{i\,\pi}{3}\right)$, prove that $3x^2 - y^2 = 3$

18. If $\cot(u + i\,v) = cosec(x + i\,y)$, prove that $\coth y\sinh 2v = \cot x\sin 2u$

19. Show that $tan\left(\frac{u+i\,v}{2}\right) = \frac{\sin u + i\sinh v}{\cos u + \cosh v}$

20. If $\sin^{-1}(\alpha + i\,\beta) = x + i\,y$, show that $\sin^2 x$ and $\cos h^2 y$ are the roots of the equation

$\lambda^2 - (\alpha^2 + \beta^2 + 1)\lambda + \alpha^2 = 0$

Type – 3: Inverse hyperbolic functions

1. Prove that **(i)** $tanh\big(log\sqrt{3}\big) = 1/2$ **(ii)** $\tanh\big(log\sqrt{5}\big) = 2/3$.

2. Prove that **(i)** $cosech^{-1}x = log\left[\frac{1+\sqrt{1+x^2}}{x}\right]$ **(ii)** $tanh^{-1}x = cosh^{-1}\frac{1}{\sqrt{1-x^2}}$

(iii) $coth^{-1}x = \frac{1}{2}log\left(\frac{x+1}{x-1}\right)$

3. Prove that **(i)** $tanh^{-1}\cos\theta = cosh^{-1}cosec\,\theta$ **(ii)** $sinh^{-1}tan\theta = log(\sec\theta + \tan\theta)$

4. Separate into real and imaginary parts.

(i) $sin^{-1}(3i/4)$ **(ii)** $cosh^{-1}(i\,x)$ **(iii)** $cos^{-1}\left(\frac{16\,i}{63}\right)$

5. Prove that $cosh^{-1}(3i/4) = \log 2 + i\,\pi/2$

6. Prove that $cos^{-1}(\sec\theta) = i\, log(\sec\theta + \tan\theta)$

7. Prove that $cos^{-1}i\,x = \frac{\pi}{2} - i\, log\left(x + \sqrt{x^2+1}\right)$

8. If $\tan z = \frac{i}{2}(1-i)$, prove that $z = \frac{1}{2}tan^{-1}2 + \frac{i}{4}\log 5$.

9. If $sinh^{-1}(x+i\,y) + sinh^{-1}(x-i\,y) = sinh^{-1}a$, prove that\

   $2(x^2+y^2)\sqrt{a^2+1} = a^2 - 2x^2 + 2y^2$

10. Find all the roots of the equation $\cos z = 2$.

11. If $cos\left(\frac{\pi}{4}+ia\right).co\,h\left(b+\frac{i\pi}{4}\right) = 1$ where a,b arereal, prove that $2b = log\left(2+\sqrt{3}\right)$

12. If $\tan(x+i\,y) = i$ and x, y are real , prove that x is indeterminate and y is infinite.

13. If $tan\left(\frac{\pi}{4}+i\,v\right) = re^{i\,\theta}$, show that,

    (i) $r = 1.$ **(ii)** $tan\theta = \sinh 2v.$ **(iii)** $\tanh v = tan\frac{\theta}{2}$

Type -4 Logarithmic functions

1. Express the following in the form of $a + ib$.

   (i) $\log(-i)$ **(ii)** $\log(1+i)$

2. Find the general value of $\text{Log}\left(1+i\sqrt{3}\right) + \text{Log}\left(1-i\sqrt{3}\right)$

3. Prove that $\log(1+i\tan\alpha) = \log\sec\alpha + i\alpha$

4. Prove that $\log\left(1+e^{i\,\theta}\right) = \log[2\cos(\theta/2)] + i\,\theta/2$

5. Prove that $\log\left(\frac{1}{1+e^{i\,\theta}}\right) = \log\left(\frac{1}{2}\sec\frac{\theta}{2}\right) - i\frac{\theta}{2}$

6. Prove that $\log\left(e^{i\,\alpha}+e^{i\,\beta}\right) = \log\left\{2\cos\left(\frac{\alpha-\beta}{2}\right)\right\} + i\frac{(\alpha+\beta)}{2}$

7. Prove that $\log\cos(x+i\,y) = \frac{1}{2}\log\left(\frac{\cosh 2y + \cos 2x}{2}\right) - i\tan^{-1}(\tan x\tanh y)$

8. Prove that $\log\left\{\frac{\cos(x-i\,y)}{\cos(x+i\,y)}\right\} = 2\,i\tan^{-1}(\tan x\tanh y)$

9. If $\log\sin(x+iy) = a + i\,b$, prove that

   (i) $2e^{2a} = \cosh 2y - \cos 2x$ **(ii)** $\tan b = \cot x\tanh y$

10. If $\log[\log(x+i\,y)] = p + iq$ prove that $y = x\tan\left[\tan(q).\log\sqrt{x^2+y^2}\right]$.

11. If $p\log(a+ib) = (x+iy)\log m$ prove that $\frac{y}{x} = \frac{2\tan^{-1}(b/a)}{\log(a^2+b^2)}$

12. Prove that $\log(x+iy) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{y}{x}$ Hence, deduce that

    If $(a_1+i\,b_1)(a_2+ib_2)\ldots(a_n+ib_n) = A + i\,B$ then

    (i) $\left(a_1{}^2+b_1{}^2\right)\left(a_2{}^2+b_2{}^2\right)\ldots\left(a_n{}^2+b_n{}^2\right) = A^2 + B^2$

(ii) $\tan^{-1}(b_1/a_1) + \tan^{-1}(b_2/a_2) + \cdots + \tan^{-1}(b_n/a_n) = \tan^{-1}(B/A)$.

13. Show that $i \log\left(\frac{x-i}{x+i}\right) = \pi - 2\tan^{-1} x$

14. Prove that $\mathrm{Log}\left[\frac{(a-b)+i(a+b)}{(a+b)+i(a-b)}\right] = i\left(2n\,\pi + \tan^{-1}\frac{2\,ab}{a^2-b^2}\right)$

15. Prove that $\sin \log_e\left(i^{-i}\right) = 1$

16. Prove that $\sin\left[i \log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2\,ab}{a^2+b^2}$

17. Separate into real and imaginary part $\log_{(1-i)}(1+i)$

18. Show that $\log_i i = \frac{4n+1}{4m+1}$ when n, m are integers.