Problem Set #2

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Your solutions are required to be typeset using LaTeX, and to be turned in electronically using Canvas. As discussed on the course syllabus, there are two deadlines for this homework. The first deadline is 8pm on Friday, February 22. The second deadline is 8pm on Sunday, February 24. Submissions received before the first deadline will not be charged any slip days. No submissions will be allowed after the second deadline. See the syllabus for further details regarding the slip day policy.

1. Let k be a nonnegative integer and let $G = (V, E)$ be a connected graph where each edge e in E has an associated weight $w(e)$ in $\{0, 1\}$. Present a polynomial-time algorithm to determine whether G has a spanning tree of weight k.

2. Problem 16–4, page 448.

3. Let $G = (V, E)$ be an undirected graph. Let $\mathcal{I}$ denote the set of all subsets X of V such that there exists a matching of G that matches all of the vertices in X. In class, we claimed that $(V, \mathcal{I})$ is a (matching) matroid. We observed that $\mathcal{I}$ is nonempty since the empty set of vertices belongs to $\mathcal{I}$. Furthermore, the hereditary property holds for $(V, \mathcal{I})$ since any matching that matches all of the vertices in X also matches all of the vertices in any subset of X. Prove that the exchange property holds for $(V, \mathcal{I})$.

4. Let $M = (S, \mathcal{I})$ be a matroid, let X be a basis of M, let x be an element of $S \setminus X$, let Y denote $X + x$, and let C denote the set of all y in Y such that $Y - y$ belongs to $\mathcal{I}$. The parts below establish that C is the unique circuit of M that is contained in Y.

   (a) Prove that C does not belong to $\mathcal{I}$.

   (b) Prove that $C - y$ belongs to $\mathcal{I}$ for all y in C.

   (c) Let Z be a subset of Y that is not contained in $\mathcal{I}$. Prove that C is contained in Z.

5. Problem 17–5, parts (d), (e), (f), and (g), page 476.