Problem Set #4

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Your solutions are required to be typeset using LaTeX, and to be turned in electronically using Canvas. As discussed on the course syllabus, there are two deadlines for this homework. The first deadline is 8pm on Friday, April 5. The second deadline is 8pm on Sunday, April 7. Submissions received before the first deadline will not be charged any slip days. No submissions will be allowed after the second deadline. See the syllabus for further details regarding the slip day policy.

1. Let $G = (V, E)$ be a bipartite graph and let (X, Y) be a given bipartition of V such that $E \subseteq X \times Y$. Assume that each vertex v in V has an associated positive integer weight $w(v)$. For any subset U of V, we define $w(U)$ as $\sum_{u \in U} w(u)$, and we define $\Gamma(U)$ as the set of all vertices outside U that are adjacent to some vertex in U. We say that a subset U of V is *feasible* if $\Gamma(X \cap U) \subseteq U$. We define the *value* of a subset U of V, denoted $v(U)$, as $w(X \cap U) - w(Y \cap U)$. The goal of this problem is to use network flow techniques to compute a maximum-value feasible subset of V.

   (a) We say that a subset U of V is *good* if $U \oplus X$ is feasible. Prove that U is a maximum-value feasible subset of V if and only if $U \oplus X$ is a minimum-weight good subset of V.

   (b) Give a polynomial-time algorithm to compute a maximum-value feasible subset of V. Hints: (1) Use G to construct an associated flow network G'; (2) Use the result of part (a) to derive an optimal solution from a minimum cut of G'.

2. Exercise 26.5–4, page 760.

3. Exercise 26.5–5, page 760.

4. Problem 26–6, parts (a) through (f), page 763. You may also turn in a solution to part (g), which will be graded as a bonus question.