

Algorithms & Data Structures for Big Data

classmate

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Page

Algorithm :- Set of clear, unambiguous instructions to solve a problem in finite amount of time.

Conditions :- a) input b) output c) finiteness
d) definiteness e) effectiveness

Data structures :- A systematic scheme of organizing the related data.

Program :- Converting an algorithm to an executable language.

09/08/2023

Performance of algorithm :-

Space - Memory required for the computation.

Time - CPU time considered, not the compile time.

Space → Variable → instance size, reference variables, recursive function

Fixed → code, fixed variables, constants.

Total space = Fixed space + Variable space

Ex:- 1) Algorithm addElement (a,b,c)
begin

return (a+b+c)

end

→ Only fixed space

2) Algorithm sum (a, n) {

total = 0

ans:-

n=1

for i = 1 to n f n = 1 i = 1

total = 1

total = total + a[i] = n

i = 1

a = variable = n

return total

total space $\geq (n+3)$

[] [] [] [] []

n+3

3) Algorithm Rsum(a, m) {

if ($n \leq 0$) {

return 0;

else {

return (Rsum(a, n-1) + a[n]);

}

return address

Space:- 3 spaces

a = 1 unit (base address)

n = 1

return address = 1.

Total no. of function calls = $n+1$.

$\boxed{\text{Total space} \geq 3n+3}$

11/08/2023

Time complexity :-

Only execution time is considered. Compile time excluded.

Ans

c=0

total = 0

c=1

for (i=1 to n) {

c=2

 total = total + a[i] - n

c=3

}

c++ / i > n

return total

c++

2n+3

2) Algorithm Rsum(a, n) { a: {1, 2, 3, 4, 5} n=5

if ($n \leq 0$) { } = 2 Rsum(1, 2, 3, 4) + 5

return 0 = 1

else { } = 2

return (Rsum(@n-1) + a[n]). Rsum(1, 2, 3) + 9

}

a = {1, 2, 3, 4} n = 4

Rsum(1, 2, 3, 4) + 4

RS

$$T(n) = 2 \text{ when } n \leq 0.$$

$$\boxed{T(n) = 2 + T(n-1)} \text{ when } n > 0$$

$$= 2 + 2 + T(n-2)$$

$$= 2 + 2 + 2 + T(n-3)$$

$$T(k) = 2 * k + T(n-k)$$

$$= 2n + T(n-n)$$

$$= 2n + T(0)$$

$$\boxed{T(n) = 2n + 2}$$

=

Substitution method.

$$T(n) = 2$$

$$T(0) = 2$$

$$n - k = 0$$

$$n = k$$

~~③ Algorithm addMatrix (a, b, m, n, c)~~

~~for (i=1 to m) { — c++ — m~~

~~for (j=1 to n) { — c++ — (n+1)m~~

~~c[i,j] = a[i,j] + b[i,j] — c++ — m*n~~

~~y~~

~~return c. — c++ — 1.~~

~~y~~

~~mm + m + n + 2~~

~~↓ ↓ ↓ b~~
~~a~~
~~[2 3]~~
~~[1 4]~~

~~③ Algorithm addMatrix (a, b, m, n, c) {~~

~~for (i=1 to m) { — c++ — m~~

~~for (j=1 to n) { — c++ — mn~~

~~c[i,j] = a[i,j] + b[i,j] — c++ — m*n~~

~~y c++ / when j > n (= m)~~

~~y c++ / when i > m = 1~~

~~return c. — c++ — 1~~

~~y~~

~~2mn + 2m + 2~~

~~if (m == n)~~

~~2n^2 + 2n + 2~~

~~↓ ↓~~
~~1 2~~
~~3 4~~
~~5 6~~

~~2 + 2 + 2~~

~~3 * 2 m * n~~

Asymptotic notation :-

Big oh (O) :- Upper bound time complexity. Worst case value.

Omega (Ω) :- Best case value. Time complexity in lower bound value.

Theta (Θ) :- Average time complexity.

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General solution for large classes of recur recursion

$$T(n) = aT(n/b) + d(n)$$

Homogeneous solution:-

$$T(n) = O(n^{\log_b a})$$

n = no. of elements

a = subproblems

n/b = elements in subproblem

Particular solution:-

case 1: when $a > d(b)$

$$T(n) = O(n^{\log_b a})$$

$d(n) = \text{Time to create subproblem} + \text{merge the solution}$

case 2: when $a < d(b)$

$$T(n) = O(n^{\log_b d(b)})$$

case 3: when $a = d(b)$

$$T(n) = O(\log_b n * n^{\log_b d(b)})$$

Ex:- 1) $\text{HT}[T(n) = HT(n/2) + n]$

$$a=4 \quad b=2 \quad d(n)=n \Rightarrow d(b)=b=2$$

$$\therefore a > d(b)$$

$$O(n^{\log_2 4})$$

$$T(n) \Rightarrow O(n^2) = \text{both homogeneous \& particular solution}$$

$$2. T(n) = 4T(n/2) + n^2$$

$$a=4 \quad b=2 \quad d(n)=n^2$$

$$\Rightarrow d(b) = b^2 = 4$$

$$a=b$$

$$T(n) = O(\log n * n^{\log_b d(b)})$$

$$= O(n^2 \log n)$$

$$\left(\frac{n^{\log_2 4}}{n^2} \right)$$

$$3. T(n) = 4T(n/2) + 2$$

$$a=4 \quad b=2 \quad d(n)=2 \Rightarrow d(b)=2$$

$$a > d(b)$$

$$T(n) = O(n^{\log_2 4})$$

$$= O(n^2)$$

$$\log_2 2$$

$$2 = 4^{1/2}$$

$$2 = 4^{\frac{1}{2}}$$

Stacks :-

$$\log_2 8$$

→ LIFO data structure

$$8 = 2^{\frac{15}{2}} = 3$$

→ Last in First out

→ Function calls

→ Evaluation expression

$$\log_4 2$$

→ Conversion of postfix/infix expression $2 \times 4^3 \Rightarrow k = 1/2$

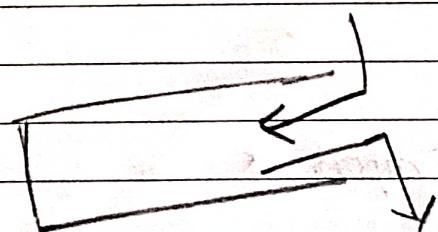
→ Balancing of symbols

→ Webpages, editors

16/8/2023

"self" tells the `__init__` is an instance method.

"this" represents the object.



1	2	3	4
---	---	---	---

3!

1
2
3

$$2 \times 3$$

$$= 6$$

```
class simpleStack:  
    def __init__(self):  
        self.data = []  
        self.count = 0  
    def getElementCount(self):  
        return self.count  
    def isStackEmpty(self):  
        return self.count == 0  
    def stackPush(self, element):  
        self.data.append(element)  
        self.count += 1  
    def stackPop(self):  
        if not self.isEmpty():  
            self.count -= 1  
            return self.data.pop()  
        else:  
            return None  
    def stackPeek(self):  
        if not self.isEmpty():  
            return self.data[-1]  
        else:  
            return None  
    def printElements(self):
```

testSimpleStack.py :-

```
from simplestack import *  
  
def testEmptyStack():  
    s1 = simpleStack()  
    assert(s1.getElementCount() == 0)  
    assert(s1.isStackEmpty() == True)
```

def def testPush():

s2 = simpleStack()

s2.stackPush(10)

assert (s2.getElementCount() == 1)

def testPop():

s3 = simpleStack()

s3.stackPush(30)

s3.stackPush(20)

assert (s3.stackPop() == 30)

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from simplestack import *

def testSymbols(data, stack):

lefty = '{[('

righty = ')])'

for ch in data:

if ch in lefty:

stack.stackPush(ch)

elif ch in righty:

if stack.isEmpty():

return False

if righty.index(ch) != lefty.index(stack.stackPop()):

return False

return (stack.isEmpty())

test

def testAppC():

stack = simpleStack()

f_read = open('l1.txt').open(" ", "r")

data = f_read.read()

f_read.close()

assert (testSymbols(data, stack) == True)

testAPP()

Queue:

→ First in First out principle (FIFO)

Class Queue:

```
def __init__(self):  
    self.data = []  
    self.count = 0  
    self.front = 0.
```

```
def getElementCount(self):  
    return self.count
```

```
def isQueueEmpty(self):  
    return self.count == 0
```

```
def enqueue(self, element):  
    self.data.append(element)  
    self.count += 1.
```

```
def deque(self):  
    if not self.isEmpty():  
        count -= 1  
        self.count -= 1  
        return self.data.remove(front)  
        front += 1  
        return self.data.pop(0)  
    else:  
        return None.
```

23/08/23

Time complexity → enqueue - O(1)
deque - O(n)

Dynamic Queue:-

a. C] * —

class flexqueue:

defaultSize = 2

def __init__(self):

self.data = [None] * flexQueue.defaultSize

self.front = 0

self.count = 0

def length(self):

return self.count

def isEmpty(self):

return self.count == 0

def getFirst(self):

if not self.isEmpty():

return self.data[self.front]

else

return None

def enqueue(self, ele):

if self.count == len(self.data):

self.resize(2 * len(self.data))

idx = (self.front + self.count) // len(self.data)

self.data[idx] = ele

self.count += 1

def dequeue(self):

if not self.isEmpty():

ele = self.data[self.front]

self.count -= 1

self.data[self.front] = None

self.front = (self.front++) // len(self.data)

if 0 < self.size < len(self.data) // 4:

self.resize(len(self.data) // 3)

else:

return None

```

def reverse(self, cap):
    oldData = self.data
    walk = self.front
    self.data = [None] * cap
    for k in range(len(oldData)):
        self.data[k] = oldData[walk]
        walk = (walk + 1) % len(oldData)
    self.front = 0

```

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linked list :-

class slist:

class Node:

def __init__(self, ele):

self.data = ele

self.next = None

def __init__(self):

self.head = None

self.tail = None

self.count = 0

def isEmpty(self):

return self.count == 0

def listCount(self):

return self.count

def addAtHead(self, ele):

if not self.isEmpty():

new_node = self.Node(ele)

if not self.isEmpty():

new_node.next = self.head

self.head = new_node

else:

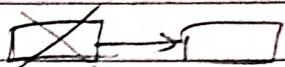
self.head = self.tail = new_node

self.count += 1

```

def addAtTail(self, ele):
    def __init__(self):
        new_node = self.Node(ele)
        if not self.isEmpty():
            self.tail.next = new_node
            self.tail = new_node
        else:
            self.head = self.tail = new_node
            self.count += 1

```



```

def deleteAtHead(self):
    if not self.isEmpty():
        temp = self.head.next
        self.head = temp
    else:
        if self.count > 1:
            temp = self.head.next
            del(self.head)
            self.head = temp
            self.count -= 1
        elif self.count == 1:
            self.head = self.tail = None
            self.count = 0
    else:
        return None

```

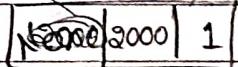
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```

def deleteAtHead(self):
    if not self.isEmpty():
        data = self.head.data
        self.head = self.head.next
        if self.head == None:
            self.tail = None
    else:
        self.count -= 1
    else:
        return data

```



```

def deleteAtTail(self):
    if not self.isEmpty():
        data = self.tail.data
        self.tail
        temp = self.head
        while (temp.next != None):
            temp = temp.next
        self.tail = temp
        temp.next = None

```

```

def deleteAtTail(self):
    if not self.isEmpty():
        if self.count != 1:
            last = self.tail
            cur = self.head
            while cur.next != last:
                cur = cur.next
            self.tail = cur
            cur.next = None
        else:
            self.head = self.tail = None
            self.count -= 1
    else:
        return None

```

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```

def isMember(self, key):
    if not self.isEmpty():
        cur = self.head
        while cur != None:
            if cur.data == key:
                break
            cur = cur.next
        return cur != None
    else:
        return False

```

~~def addGiven~~



~~def addAfterNode(self, ele, pos):~~ ~~newNode =~~

~~if not self.isEmpty():~~

~~temp = self.head~~

~~while (temp.data != pos):~~

~~temp = temp.next~~

~~temp = temp.next~~

~~cur = temp.next~~

~~temp.next = newNode~~

~~newNode.next = cur~~

~~self.count += 1~~

~~def addAfterNode(self, ele, value):~~

~~newNode = self._Node(ele)~~

~~if self.isEmpty():~~

~~temp = self.head~~

~~while (temp.data != value):~~

~~temp = temp.next~~

~~cur = temp.next~~

~~temp.next = None~~

~~if not temp.next == None:~~

~~cur = temp.next~~

~~temp.next = newNode~~

~~newNode.next = cur~~

~~else:~~

~~: (temp.next = newNode~~

~~self.tail = newNode~~

~~self.count += 1~~

~~else:~~

~~return None~~

```

def addAtPos(self, ele, pos):
    if not self.isEmpty():
        temp = self.head
        while self.iteration == 0
            while (self.iteration < pos):
                temp = temp.next
                self.iteration += 1
                if temp == None:
                    cur = temp.next
                    if newNode == self._newNode(ele):
                        if cur == None:
                            temp.next = newNode
                            newNode.next = cur
                        else:
                            temp.next = newNode
                            self.tail = newNode
                            self.count += 1
                    else:
                        return None
                else:
                    return None
            else:
                q1.enqueue(ele)
                q2.enqueue(q1.dequeue())
                ele = q1.dequeue()
                while (q2.count > 0):
                    q1.enqueue(q2.dequeue())
                    return ele

```

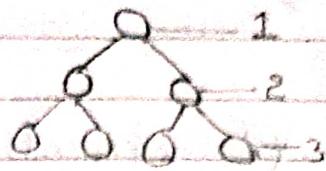
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```

def stackUsingQueues(self):
    q1 = Queue()
    q2 = Queue()
    def push(self, ele):
        q1.enqueue(ele)
    def pop():
        while (q1.count > 1):
            q2.enqueue(q1.dequeue())
            ele = q1.dequeue()
        while (q2.count > 0):
            q1.enqueue(q2.dequeue())
        return ele

```

Tree data structure



→ Non-linear data structure

→ For file system

→ Root node → Starting node

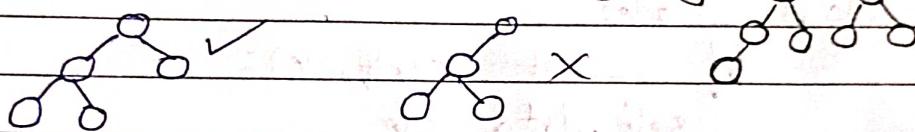
→ Degree - No. of children nodes for a given node

Binary tree

i) No. of nodes at any level $i = 2^{i-1} = 2^0 = 2^1 - 1$

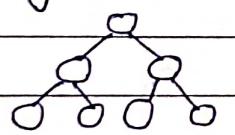
ii) Total No. of nodes at any given depth $k = 2^k - 1 = 2^3 - 1$

Complete binary tree :-



Nodes are filled from left to right then top to bottom

Full binary tree :-



All leaf nodes are filled. There should not be any unfilled slots at any level.

* node at i

(50)

* parent node at $\frac{i}{2}$



* left child at $2i$



* right child at $2i + 1$

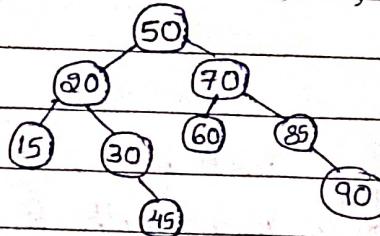
1	2	3	4
40	50	60	55 65

$$\frac{2^2}{2} = 1$$

Binary Search Tree :-

- 1) All elements should be unique.
- 2) Left child should be less than parent.
- 3) Right child should be greater than parent.

Ex:- 50, 20, 70, 30, 60, 85, 45, 15, 90



class BST:

 class Node:

 def __init__(self, ele):

 self.data = ele

 self.left = None

 self.right = None

 def __init__(self):

 self.root = None

 self.count = 0

 def isEmpty(self):

 return self.root == None

 def getCount(self):

 return self.count

 def addNode(self, ele):

 cur = parent = self.root

 while (cur != None and cur.data != ele):

 parent = cur

 if (ele < cur.data):

 cur = cur.left

 else:

 cur = cur.right

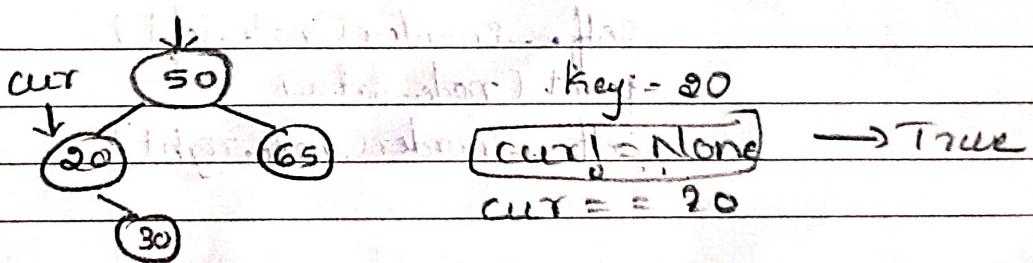
 if (cur == None):

 new_node = self.Node(ele)

```

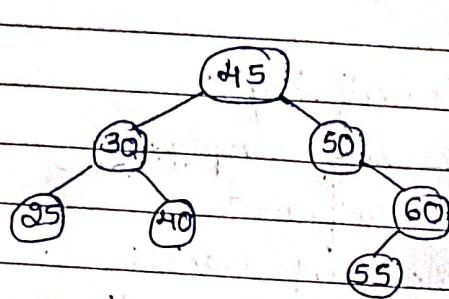
if parent == None:
    self.root = new_node
elif ele < parent.data:
    parent.left = new_node
elif ele > parent.data:
    parent.right = new_node
self.count += 1
def searchNode(self, key):
    if not self.isEmpty():
        cur = self.root
        while (cur != None):
            if cur.data == key:
                break
            else:
                if key < cur.data:
                    cur = cur.left
                else:
                    cur = cur.right
        return cur != None, cur == None
    else:
        return False, False

```



BST Traversals :-

- 1) Inorder - Left \rightarrow Parent \rightarrow Right
- 2) Preorder - Parent \rightarrow Left \rightarrow Right
- 3) Postorder - Left \rightarrow Right \rightarrow Parent
- 4) Level order



Postorder: 25, 40, 30, 55, 60, 50, 45

Preorder: 45, 30, 25, 40, 50, 60, 55

Inorder: 25, 30, 40, 45, 50, 55, 60

Levelorder: 45, 30, 50, 25, 40, 60, 55

```

def Inorder(self):
    if not self.isEmpty():
        self.inorder(self.root)
  
```

```

def inorder(self, node):
    if node != None:
        self.inorder(node.left)
        print(node.data)
        self.inorder(node.right)
  
```

```

def PreOrder(self):
    if not self.isEmpty():
        self.preorder(self.root)
  
```

```

def preorder(self, node):
    if node != None:
        print(node.data)
        self.preorder(node.left)
        self.preorder(node.right)
  
```

def PostOrder(self):

if not self.isEmpty():

self.postOrder(self.root)

def -postOrder(self, node):

if node != None:

self.-postOrder(node.left)

self.-postOrder(node.right)

print(node.data)

def LevelOrder(self):

if not self.isEmpty():

self.levelOrder(self.root)

def -levelOrder(self, node):

while(node != None):

if (node.left != None):

queue.enqueue(node.left)

if (node.right != None):

queue.enqueue(node.right)

queue.enqueue(node)

^{print} enqueue(queue.dequeue())

def -levelOrder(self, node):

queue.enqueue(node)

while(node != None):

ele = queue.dequeue()

print(ele)

if (node.left != None):

queue.enqueue(node.left)

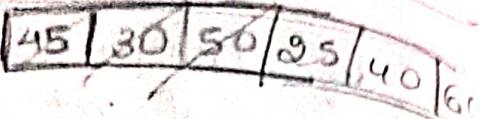
if (node.right != None):

queue.enqueue(node.right)

```

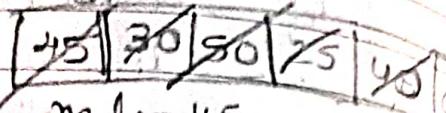
def level_order(Cself):
    if not self.isEmpty():
        q1 = deque()
        q1.enqueue(self.root)
        while not q1.isEmpty():
            node = q1.dequeue()
            print(node.data)
            if node.left:
                q1.enqueue(node.left)
            if node.right:
                q1.enqueue(node.right)

```



15, 30, 50

Q1



node = 45
node = 30

45, 30, 50, 25, 40, 60

→ Implementation of tree traversal using an iterative approach.

Delete Node :-

```

def deleteNode(self, key):
    if not self.isEmpty():
        self.root = self._nodeDelete_(self.root, key)

def _nodeDelete_(self, node, key):
    if node == None:
        root = return None
    elif key < node.data:
        node.left = self._nodeDelete_(node.left, key)
    elif key > node.data:
        node.right = self._nodeDelete_(node.right, key)
    else:
        if node.left and node.right:
            temp = self._findMin_(node.right)
            node.data = temp.data
            node.right = self._nodeDelete_(node.right, temp.data)
        else:
            node = node.left
    return node

```

```

else:
    if node.left == None:
        node = node.right
    self.count -= 1
    return node

```

```

def findMin_(self, node):
    if node.left == None:
        return node
    else:
        return self.findMin_(node.left)

```

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```

def getLeafCount_(self):
    if not self.isEmpty():
        q1 = flexQueue()
        q1.enqueue(self.root)
        while not q1.isEmpty():
            node = q1.dequeue()
            if (node.right == node.left == None):
                count += 1
            if node.left:
                q1.enqueue(node.left)
            if node.right:
                q1.enqueue(node.right)
        return count

```

```

def getLeafCount_(self):
    if not self.isEmpty():
        return self._leafCount_(self.root)
    else:
        return 0

```

```

def _leafCount_(self, node):
    if node:
        if self._isLeafNode_(node):
            return 1
        else:
            return self._leafCount_(node.left) + self._leafCount_(node.right)

```

return 0

```
def isLeafNode(node):
    if node.left == None and node.right == None:
        return True
    else:
        return False
```

25/09/2023

Bubble sort :-

```
def bubbleSort(input):
    for i in range(len(input)-2, 0, -1):
        for j in range(len(input)-1, i, -1):
            if input[j] < input[j-1]:
                input[j], input[j-1] = input[j-1], input[j]
```

Time complexity = $O(n^2)$

Improve bubble sort to reduce the number of sorts.

```
def testSorting(input)
    for edx in range(0, len(input) - 1):
        assert input[edx] <= input[edx + 1]
```

test son testSorting(input)

Selection sort:-

Less no. of swapping

```
def selectionSort(input):
    for itr in range(0, len(input) - 1):
        pos = itr
```

```

for idx in range(i+1, len(input)-1):
    if input[idx] < input[pos]:
        pos = idx
    if input[i+1], input[pos] = input[pos], input[i+1]

```

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Insertion sort:-

```

def insertionSort(arr, input):
    for i in range(1, len(input)):
        idx = i - 1
        key = input[i]
        while idx >= 0 and key < input[idx]:
            input[idx + 1] = input[idx]
            idx -= 1
        input[idx + 1] = key

```

Time complexity = $O(n^2)$

no. of comparison is very less.

insertion sort is better than selection & bubble sort.

29/09/2023

Quick sort :-

→ Divide & conquer principle

0	1	2	3	4	5	6	7	8	9	10	pivot = 35
35	20	45	15	98	9	69	95	16	30	60	
↑ up	↑ up	↑ up					↑ down	↑ down			

$\text{input}[up] \leq \text{pivot}$ $\text{input}[down] > \text{pivot}$
 $up++;$ $down--;$

$\text{input}[up] = 45 \neq 35$ $\text{input}[down] \neq 35$

If all elements are not compared & can't increment
decrement up & down

swap up & down

35

35	30	30	15	98	9	69	75	16	45	60
	↑						↑			

up

down

Pivot = 35

up = 30

down = 15

Swap pivot with down if all elements are traversed

Depends on the partition, the time complexity will be
 → either $O(n^2)$ or $O(n \log n)$

def quickSort(input):

start = 0

end = len(input) - 1

quicksort(input, start, end)

return input

def _quicksort(input, start, end):

if start < end: # if start = end, only one element

mid = partition(input, start, end)

_quicksort(input, start, mid - 1)

_quicksort(input, mid + 1, end)

def _partition(input, start, end):

up = start

down = end

pivot = input[start]

while up <= down:

while up <= down and input[up] <= pivot:

up += 1

while up <= down and input[down] > pivot:

down -= 1

input[up], input[down] = input[down], input[up]

input[start], input[down] = input[down], input[start]

return down

Balanced partition :- (or best case) :-

$$T(n) = 2T(n/2) + O(n)$$

partition takes $O(n)$ time, maximum time for number of comparison needed is $n+1$.

$$a = 2 \quad b = 2 \quad d(n) = n$$

$$d(b) = b = 2$$

$$a = d(b)$$

$$\text{Homogeneous solution} = O(n^{\log_b a})$$

$$= O(n)$$

$$\text{Particular sol. solution} = O(n^{\log_b d(b)} \cdot \log n)$$

$$= O(\log n \cdot n)$$

$$= O(n \log n)$$

Unbalanced partition :- (worst case)

$$T(n) = T(n-1) + O(n)$$

$$= T(n-1-1) + n-1+n \quad T(n-1) = T(n-1-1) + (n-1)+n$$

$$= T(n-1-1) + n-1-1+n-1+n = T(n-2) + n+n-1$$

$$= T(n-2) + (n-2)+(n-1)+n = T(n-3) + n+(n-1)+(n-2)$$

$$= \vdots \quad = \vdots$$

$$= T(n-n) + (n-n) + \dots + 1 + O$$

$$T(n) = O(n^2)$$

$$= T(n-n) + n+n-1+n-2+\dots+n-n$$

$$= \frac{n(n-1)}{2}$$

$$= n + \frac{n^2 - n}{2}$$

$$= O(n^2)$$

Merge sort :-

- Divide & conquer method
- $T(n) = O(n \log n)$
- This is not an in-place algorithm. That means we will be using an additional temporary array.

```
def mergeSort(input):
    if len(input) == 0 or len(input) == 1:
        return input
    else:
        mid = len(input)/2
        a = mergeSort(input[:mid])
        b = mergeSort(input[mid:])
        return merge(a, b)
```

```
def merge(a, b):
    tempList = []
    while len(a) != 0 and len(b) != 0:
        if a[0] < b[0]:
            tempList.append(a[0])
            a.pop(0)
        else:
            tempList.append(b[0])
            b.pop(0)
    if len(a) == 0:
        tempList = tempList + b
    else:
        tempList = tempList + a
    return tempList
```

This merge function leads to $O(n^2)$ time complexity

→ Implement merge function with time complexity $O(n \log n)$.

Time complexity.

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$$a = 2, b = 2, d(b) = 2$$

$$a = b$$

$$T(n) = O(n^{\log_2 2} \cdot \log n)$$

$$\underline{T(n) = O(n \log n)} \rightarrow \text{Particular solution}$$

Homogeneous solution = $O(n)$

Heap:-

→ Complete binary tree

→ Maximum heap: The parent value should always greater or equal to its children.

→ Minimum heap: The parent value should always smaller or equal to its children.

→ Heap sort :- Efficient way of in-place sorting algorithm with time complexity $O(n \log n)$.

→ Works very well even if the dataset is very huge.

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class maxHeap:

def __init__(self, list=[[]]):

self.data = list

self.buildHeap()

def isEmpty(self):

return len(self.data) == 0

def parent(self, idx):

return (idx - 1) // 2

def lchild(self, idx):

return 2 * idx + 1

def rchild(self, idx):

return 2 * idx + 2

```

def swap(self, i, j):
    self.data[i], self.data[j] = self.data[j], self.data[i]

def buildHeap(self):
    length = len(self.data)
    start = (length - 2) // 2
    for idx in range(start, -1, -1):
        self._downHeap(self, idx, length)

def _downHeap(self, idx, length):
    if self.lchild(idx) < length:
        left = self.lchild(idx)
        bigChild = left
        if self.rchild(idx) < length:
            right = self.rchild(idx)
            if self.data[right] > self.data[left]:
                bigChild = right
        if self.data[bigChild] > self.data[idx]:
            self._swap(bigChild, idx)
            self._downHeap(bigChild, length)

```

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```

def addElement(self, key):
    self.data.append(key)
    self._upHeap(len(self.data) - 1)

```

```

def _upHeap(self, j):
    parent = self.parent(j)
    if j > 0 and self.data[j] > self.data[parent]:
        self._swap(j, parent)
        self._upHeap(parent)

```

Time required to move biggest item from lowest level to the highest level (up-heap) = $O(\log n)$

Time required to move smallest item from highest level to the lowest level (down-heap) = $O(\log n)$

1) Priority queue implementation using normal queue :-

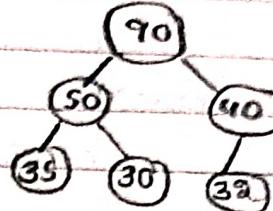
Adding n elements = $O(n)$

Deleting n elements = $O(n^2)$

2) Sorted queue

$O(n^2)$ — Addition

$O(n)$ — Deletion



90	50	40	35	30	32
----	----	----	----	----	----

3) Heap datastructure

$O(n \log n)$ — Addition

$O(n \log n)$ — Deletion

32	50	40	35	30	90
----	----	----	----	----	----

50	32	40	35	30	90
----	----	----	----	----	----

Building a heap $\rightarrow O(n \log n)$

30	32	40	35	50	90
----	----	----	----	----	----

```
def testMaxHeap (self):
```

```
    if not self.isEmpty():
```

```
        for idx in range(len(self.data)-1, 0, -1):
```

```
            assert (self.data[idx] <= self.data[self.parent(idx)])
```

30	32	35	40	50	90
----	----	----	----	----	----

```
def heapSort (self):
```

```
    if not self.isEmpty():
```

```
        for idx in range(len(self.data)-1, 0, -1):
```

```
            self._swap (0, idx)
```

```
            self._downHeap (0, idx)
```

Time complexity :-

heap sort — for loop = $O(n-1) = O(n)$

down heap = $O(\log n)$

Total = $O(n \log n)$

Hashing :-

Mechanism which allows insertion/deletion/search operations with constant average time.

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Array based :-

→ Collision is a common process in this

→ To overcome collision, we will do rehashing

Linear probing $\rightarrow H_R(x) = H(x) + i$, $i = 0, 1, 2, \dots, n-1$

Quadratic probing $\rightarrow H_R(x) = H(x) + i^2$. This is used to avoid only one particular area being thickly populated

→ Takes constant average time.

→ Searching → Iterate through hashmap and stop if a blank cell is seen in which was never occupied no element was stored before.

Linked list based :-

→ Hashing & compression function is there

→ While adding a node, add at the head. external linking

→ Efficiency wise it is better than the array based

→ No limit on the no. of elements stored.

Hash function = hash code generation + compression

Hash code generation :-

1) Bit code method - sum of arr[i] (temp01 = t + e + m + p + 0 + 1)

2) Polynomial method - $t\alpha^{n-1} + e\alpha^{n-2} + m\alpha^{n-3} + p\alpha^{n-4} + \dots$ (33, 37, 39)

3) Cyclic Shift method - bit shift

Compression method :-

1) Division method - hashcode % N, table size is prime no.

2) MAD method - Multiplication, Addition, Division method

Cyclic shift :-

$$a = \begin{smallmatrix} 4 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 7 & \leftarrow s \end{smallmatrix}$$

def cyclic_shift(input):

 hashcode = 0

 for ch in input:

 hashcode = hashcode \ll 5

 hashcode = hashcode + ch

$$27 / 2^5$$

$$a \ll 1$$

$$11011 / 2$$

$$\begin{smallmatrix} 4 & 3 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{smallmatrix}$$

$$16 + 8 + 4 + 2 = 30$$

$$16 + 4 =$$

MAD method :-

p, a, b

p \geq 1 (a prime no.).

a $\in (1, (p-1))$

b $\in (0, (p+1))$

hashcode = 0.

$$MAD = [(a * p + b) \% p] \% N.$$

→ Duplicates are not stored.

→ Dictionaries are implemented using hash map in the backend.

class extHash:

 class node:

 def __init__(self, id, name):

 self.id = id

 self.name = name

 self.next = None

 tableSize = 11

 def __init__(self):

 self.hashtable = [None] * extHash.tableSize

 self.p = 13

 self.a = np.random.randint(1, p-1)

 self.b = np.random.randint(0, p+1)

```

def hashCode__(self, element):
    hashCode = 0
    for ch in element:
        hashCode = hashCode << 5
        hashCode = hashCode + ord(ch)
    return hashCode

```

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```

def compression_(self, hcode):
    return ((C.C.hcode * self.a + self.b) + self.p) % extHashTable.size

```

```

def hash_(self, element):
    hashCode = hashCode_(element)
    tableIndex = compression_(hashCode)
    return tableIndex

```

```

def isMember(self, id):
    bIndex = hash_(id)
    node = self.hashTable[bIndex]
    while node:
        if node.id == id:
            break
        else:
            node = node.next
    return node != None

```

```

def addElement(self, id, name):
    if not self.isMember(id):
        bIndex = hash_(id)
        oldAdd = self.hashTable[bIndex]
        node = self._node_(id, name)
        node.next = oldAdd
        self.hashTable[bIndex] = node

```

```

def deleteElement(self, id):

```

```

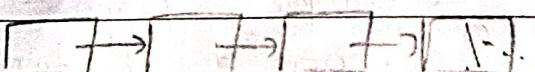
if self.isMember(id):
    bIndex = hash(id)
    node = self.hashtable[bIndex]
    while node.id == id:
        if node.id == id:
            prev = None
            while node:
                if node.id == id:
                    prev.name = node.name
                    prev.if (prev == None):
                        self.hashtable[bIndex] = node.next
                    else:
                        prev.next = node.next
                else:
                    prev = node
                    node = node.next
            return ele

```

```

def deleteElement(self, id):
    bIndex = self.hash_(id)
    if self.hashtable[bIndex]:
        if self.hashtable[bIndex].id == id:
            self.hashtable[bIndex] = self.hashtable[bIndex].next
        else:
            node = self.hashtable[bIndex]
            while node.next != None:
                if node.next.id == id:
                    node.next = (node.next).next
                else:
                    node = node.next

```



```

def printHashTable(self):
    for idx in range(0, extHash.tableSize):
        node = self.hashTable[idx]
        while node:
            print(node.id)
            node = node.next
        print("\n")

```

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Graphs:-

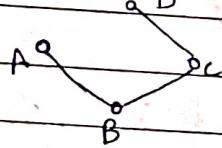
$$\rightarrow G = (V, E)$$

\rightarrow Set of vertices (nodes) : $V = \{v_1, v_2, v_3, \dots, v_n\}$

\rightarrow Set of edges : $E = \{e_1, e_2, e_3, \dots, e_m\}$

Types of graphs:-

- Undirected graph - pair of vertices in a edge is unordered
 $(A, B) = (B, A)$



- Directed graph - each edge is a directed pair of vertices

\rightarrow Self edge / loop edge - start & end vertex is same (v_i, v_i)

\rightarrow We say "a path exists from v_p to v_n " if there is a list of vertices $[v_0, v_1, v_2, \dots, v_n]$ such that $(v_p, v_{i+1}) \in E$ for all $0 \leq i \leq n$

\rightarrow A cycle is a path that begins and ends at the same node.

\rightarrow Connected graph - An undirected graph is connected if for all pairs of vertices $u \neq v$, there exists a path from u to v .

\rightarrow Complete graph - An undirected graph is complete or fully connected, if for all pairs of vertices $u \neq v$ there exists an edge from u to v .

Representation of graph :-

i) Adjacency matrix :-

- Let $G = (V, E)$ be a graph with n vertices.
- The adjacency matrix of G is a 2-D n by n array.
- If the edge (v_i, v_j) is in $E(G)$, $\text{adj mat}[i][j] = 1$.
- If there is no edge in $E(G)$ for (v_i, v_j) , $\text{adj mat}[i][j] = 0$.
- Adjacency matrix for an undirected graph is symmetric.
- Adjacency matrix for a digraph need not be symmetric.

ii) Adjacency list :-

- Each row in adjacency matrix is represented as list.

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Implementation of adjacency list representation of graph

```
import sys
```

```
class Graphs:
```

```
    class vertex:
```

```
        def __init__(self, id):
```

```
            self.id = id
```

```
            self.adjacentNeighbour = {}
```

```
        def isNeighbour_(self, id):
```

```
            return (id in self.adjacentNeighbour)
```

```
        def addNeighbour_(self, id, wt=0):
```

```
            if not self.isNeighbour_(id):
```

```
                self.adjacentNeighbour[id] = wt
```

```
        def listNeighbours_(self):
```

```
            return self.adjacentNeighbour.keys()
```

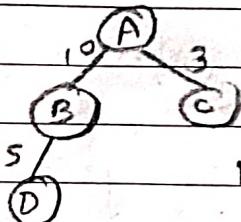
```
        def edgeCost_(self, id):
```

```
            if self.isNeighbour_(id):
```

```
                return self.adjacentNeighbour[id]
```

```
            else:
```

```
                return sys.maxsize
```



```
{ A:
```

```
{ B: 10, C: 34,
```

```
      B:
```

```
{ A: 10, D: 54 }
```

→ dictionary keyword

```
def init(self):
    self.vertexCount = 0
    self.adjacencyList = {}

def getVertexCount(self):
    return self.count

def addVertex(self, id):
    if not id in self.adjacencyList:
        newVertex = self.Vertex(id)
        self.adjacencyList[id] = newVertex
        self.vertexCount += 1

def addEdge(self, from, to, wt=0):
    if not from in self.adjacencyList:
        self.addVertex(from)
    if not to in self.adjacencyList:
        self.addVertex(to)

    this is only for self.adjacencyList[from]. addNeighbour(to, wt)
    undirected graph                         self.adjacencyList[to]. addNeighbour(from, wt)

def getEdgeCost(self, from, to):
    if from in self.adjacencyList and to in self.adjacencyList:
        return self.adjacencyList[from]. edgeCost(to)

def getVertexList(self):
    return list(self.adjacencyList.keys())

def neighborList(self, id):
    if id in self.adjacencyList:
        return list(self.adjacencyList[id]. listNeighbours())
```

Spanning tree :-

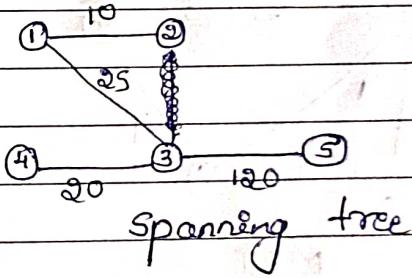
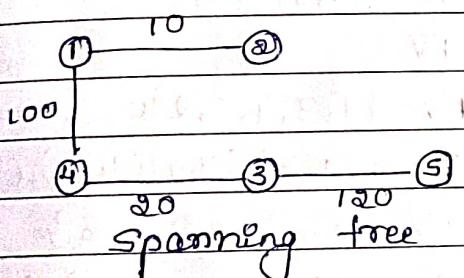
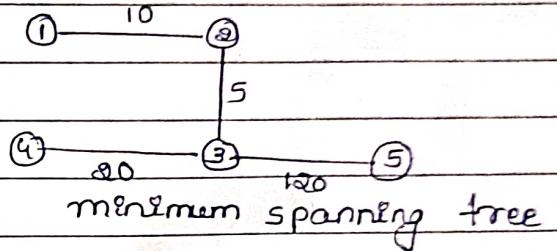
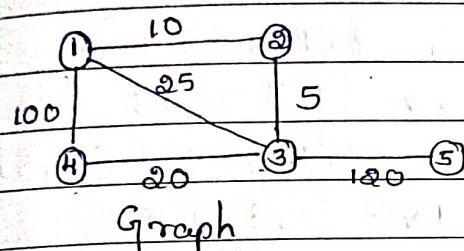
Spanning tree is a acyclic connected subgraph containing all the vertices of a given sub-graph.

spanning tree should have maximum $(n-1)$ edges, where 'n' is the number of nodes.

Minimum spanning tree :-

Spanning tree is any

A spanning tree whose total cost of selected edges is minimum.



Greedy algorithm :- In later steps, we can't change decision made in previous stages!

Prem's algorithm :-

Let TV be set of vertices and T be set of edges

To start with let TV contain an one vertex and T is empty.

$$TV = \{ \text{any one vertex} \}$$

$$T = \{ \}$$

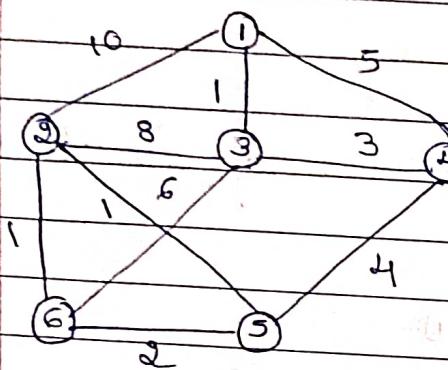
while $\{ T \text{ contains less than } n-1 \text{ edges} \}$ {

Let (u, v) be least cost edge such that vertex 'u' is

in 'TV' and 'v' is not in 'TV'

add vertex 'v' to TV and edge (v, u) to T
if no such edge break

Print (not a minimum spanning tree if T contains less than $(n-1)$ edges)



$$\textcircled{1} \quad TV = \{1\}$$

$$T = \{\}$$

$$\textcircled{2} \quad TV = \{1, 3\}$$

$$T = \{(1, 3)\}$$

$$\textcircled{3} \quad TV = \{1, 3, 4\}$$

$$T = \{(1, 3), (3, 4)\}$$

$$\textcircled{4} \quad TV = \{1, 3, 4, 5\}$$

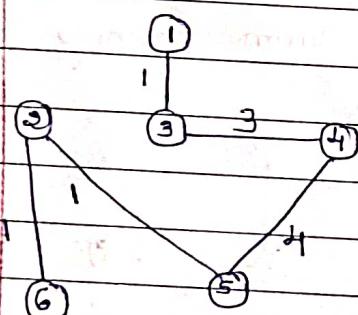
$$T = \{(1, 3), (3, 4), (4, 5)\}$$

$$\textcircled{5} \quad TV = \{1, 3, 4, 5, 2\}$$

$$T = \{1, 3, 4, 5, 2\}$$

$$\textcircled{6} \quad TV = \{1, 3, 4, 5, 2, 6\}$$

$$T = \{(1, 3), (3, 4), (4, 5), (5, 2), (2, 6)\}$$



MST

Kruskal's algorithm :-

Let T be set edges, E is set representing set of all edges of given graph.

To start with T is empty

$$T = \{\}$$

while { T contains less than $n-1$ edges and E is not empty }
choose least cost edge (u, v) from E . Remove (u, v) from E
if (u, v) doesn't form cycle in T
Add edge (u, v) to T

else

Discard edge (v_6, v_5)

3

print ("not a minimum spanning tree if T contains less than $(n-1)$ edges")

$$\textcircled{1} \quad T = \{ \}$$

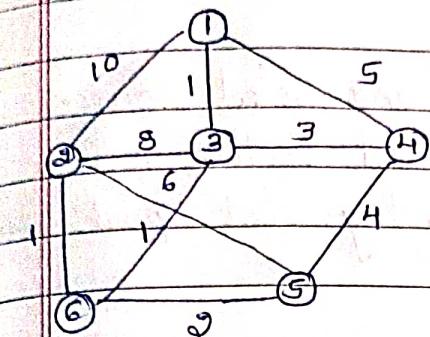
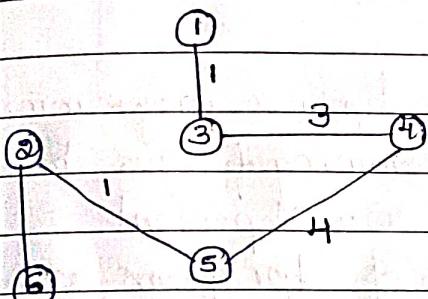
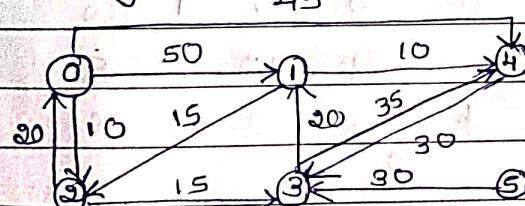
$$\textcircled{2} \quad T = \{(1, 3)\}$$

$$\textcircled{3} \quad T = \{(1, 3), (2, 5)\}$$

$$\textcircled{4} \quad T = \{(1, 3), (2, 5), (2, 6)\}$$

$$\textcircled{5} \quad T = \{(1, 3), (2, 5), (2, 6), (3, 4)\}$$

$$\textcircled{6} \quad T = \{(1, 3), (2, 5), (2, 6), (3, 4), (4, 5)\}$$

 \rightarrow MST.Dijkstra's algorithm :- \rightarrow Single source all destination \rightarrow Single source shortest path

		cost[]				
		0	1	2	3	4
0	0	50	10	1000	45	1000
1	1000	0	15	1000	10	1000
2	20	1000	0	15	1000	1000
3	0	20	1000	0	35	1000
4	1000	1000	1000	30	0	1000
5	1000	1000	1000	30	1000	0

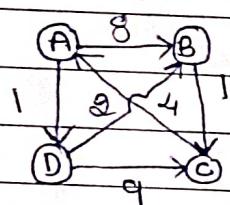
	0	1	2	3	4	5
0	0	50	10	1000	45	1000
0, 1	0	50	10	35	45	1000
0, 1, 2	0	45	10	25	45	1000
0, 1, 2, 3	0	45	10	25	45	1000
0, 1, 2, 3, 4	0	45	10	25	45	1000

- 1) Initialize distance matrix vector and select source
- 2) Select the vertex which is nearest to the source
- 3) If new path cost is less than existing cost update 'Distance vector' with new value.

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All pair shortest path algorithm :-

(Floyd Warshall algorithm)



- Example for a dynamic programming
- Dynamic programming follows bottom up
- Subprograms are dependent
- All results of subprograms are stored.

$$\text{cost}(i, j) = \text{cost}(i, k) + \text{cost}(k, j)$$

Let intermediate vertex = k = A

A	B	C	D
A	0	8	∞
B	∞	0	1
C	4	∞	0
D	∞	2	9

A	B	C	D
A	0	8	∞
B	∞	0	1
C	4	12	0
D	∞	2	9

$$\text{dist}(A, A) = \text{dist}(A, A) + \text{dist}(A, A) = 0 + 0 = 0$$

$$\text{dist}(A, B) = \text{dist}(A, A) + \text{dist}(A, B) = 0 + 8 = 8$$

$$\text{dist}(A, C) = \text{dist}(A, A) + \text{dist}(A, C) = 0 + \infty = \infty$$

$$\text{dist}(A, D) = \text{dist}(A, A) + \text{dist}(A, D) = 0 + 1 = 1$$

$$\text{dist}(B, A) = \text{dist}(B, A) + \text{dist}(A, A) = \infty + 0 = \infty$$

$$\text{dist}(B, B) = \text{dist}(B, A) + \text{dist}(A, B) = \infty + 8$$

$$\text{dist}(B, C) = \text{dist}(B, A) + \text{dist}(A, C) = \infty + \infty = \infty$$

$$\text{dist}(B, D) = \text{dist}(B, A) + \text{dist}(A, D) = \infty + 1 = \infty$$

$$\text{dist}(C, A) = \text{dist}(C, A) + \text{dist}(A, A) = 4 + 0 = 4$$

$$\text{dist}(C, B) = \text{dist}(C, A) + \text{dist}(A, B) = 4 + 8 = 12$$

$$\text{dist}(C, C) = 0$$

$$\text{dist}(C, D) = \text{dist}(C, A) + \text{dist}(A, D) = 4 + 1 = 5$$

$$\text{dist}(D, A) = \text{dist}(D, A) + \text{dist}(A, A) = \infty$$

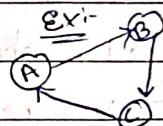
$$\text{dist}(D, B) = \text{dist}(D, A) + \text{dist}(A, B) = \infty$$

$$\text{dist}(D, C) = \text{dist}(D, A) + \text{dist}(A, C) = \infty$$

$$\text{dist}(D, D) = \infty$$

Final answer :-

	A	B	C	D
A	0	3	4	1
B	5	0	1	6
C	4	7	0	5
D	7	2	3	0



```
def allpairshortestPath(graph):
```

```
    nodes = len(graph)
```

```
    dist = graph
```

```
    for k in range(nodes):
```

```
        for i in range(nodes):
```

```
            for j in range(nodes):
```

```
                dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j])
```

```
    return dist
```

Time complexity $\rightarrow O(n^3)$

Binary Search :-

- It works on divide & conquer method.
- Input should be a sorted array.
- Time complexity $\rightarrow O(\log n)$

08/11/2023

Brute Force :-

- Pattern matching
- It will return at the beginning of the first occurrence of the text/pattern
- Time complexity = $O(n*m)$

Boyer-Moore algorithm :-

- Number of iteration less.
- Number of comparison less
- Requires additional datastructure dictionary where key is the character and value is the index value of it.
- Time complexity = $O(n+m)$

10/11/2023

Knuth-Morris-Pratt :-

- Keeps track how much pattern is matched.
- It works well only if overlap is present