AML5153 | Applied Probability and Statistics | Lab Final Exam

1. The number of times that a person contracts a cold in a given year is a Poisson random variable with parameter $\lambda = 5$. Suppose that a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 2$ for 70 percent of the population. For the remaining population, the drug has no appreciable effect on colds.

   (a) As a member of a pharmaceutical decision making body, you are interested in knowing how likely is it that the drug is beneficial for an individual if he/she tries the drug for a year and has 1 cold in that time. Calculate this probability.

   Hint: apply Bayes' theorem to $P(\text{Drug beneficial} \mid 1 \text{ cold})$.

   (b) You will approve the drug for production if 10 people who tried the drug for a year and have 1 cold in that time each, the probability that the drug is beneficial for at least 8 of them is greater than 98%. Will you approve the drug or not?

2. Suppose a random number of K customers shop at a supermarket in a day. Let $X_1, X_2, \ldots, X_K$ represent the random number of items purchased independently by the 1st, 2nd, ..., Kth customer. The total number of items sold in a day is a random number Y such that:

$$Y = X_1 + X_2 + \cdots + X_K.$$

   Suppose that on an average 30 customers arrive per day. Each individual customer is

- 20% likely to be in the age group 20-40 (encoded as 0) who buys on an average 10 items based on a Poisson distribution.
- 35% likely to be in the age group 40-60 (encoded as 1) who buys on an average 20 items based on a Poisson distribution.
- 45% likely to be in the age group 60 and above (encoded as 2) who buys on an average 5 items based on a Poisson distribution.

If you are the supermarket owner and want to increase the *expected* total number of items sold in a day. You have two options:

- increase the *expected* number of customers across all age groups by 10% by spending on external advertisement;
- increase the *expected* number of items purchased by customers in the 20-60 age group by 15% by spending on internal (in-shop) advertisement.

(a) Do both options result in an increase in the expected total number of items sold in a day?
Useful functions: `rpois()`, `sample()`, `sum()`, `rep()`, `mean()`.

(b) Which option results in the greatest increase in the expected total number of items sold in a day?

(c) Which option is least risky?
Hint: standard deviation