# AML5153 | Applied Probability and Statistics | Lab Final Exam

1. Consider a court system that has 12 members (called jurors) who will decide on a case. It takes at least 9 juror votes to convict a defendant (that is, the person standing trial). If there are less than 9 votes, the defendant is acquitted. Suppose that the probability that a jurors votes that an actually guilty person is innocent is 0.2 and the probability that a juror votes that an actually innocent person is guilty is 0.1. Each juror acts independently.

   Assuming that 75% of the defendants are actually guilty, we are interested in the probability that the court renders a correct decision. To this end, consider the following approach:

$$
\begin{aligned}
P\left(\text{Court correct decision}\right) &= P\left(\underbrace{\text{Court correct decision } \cap \text{ Person guilty}} \cup \underbrace{\text{Court correct decision } \cap \text{ Person not guilty}}\right)\\
&= \underbrace{P\left(\text{Court correct decision}\,|\,\text{Person guilty}\right)}_{\text{Replace by one of 1-4 below}} \times P\left(\text{Person guilty}\right)\\
&+ \underbrace{P\left(\text{Court correct decision}\,|\,\text{Person not guilty}\right)}_{\text{Replace by one of 1-4 below}} \times P\left(\text{Person not guilty}\right).
\end{aligned}
$$

   Now choose two appropriate conditional probabilities from the list below to replace the conditional probabilities in the expression above:

1. $P\,(\text{Court decides to convict}\ |\ \text{Person not guilty})$
2. $P\,(\text{Court decides to convict}\ |\ \text{Person guilty})$
3. $P\,(\text{Court decides to acquit}\ |\ \text{Person not guilty})$
4. $P\,(\text{Court decides to acquit}\ |\ \text{Person guilty})$

Even if you are not sure which two conditional probabilities will be chosen, calculate all the four above. After that, calculate the probability that the court comes to a correct decision.

2. Suppose a random number of $K$ customers shop at a supermarket in a day. Let $X_1, X_2, \ldots, X_K$ represent the random number of items purchased independently by the 1st, 2nd, ..., $K$th customer. The total number of items sold in a day is a random number $Y$ such that:

$$Y = X_1 + X_2 + \cdots + X_K.$$

Suppose that on an average 30 customers arrive per day. Each individual customer is

- 20% likely to be in the age group 20-40 (encoded as 0) who buys on an average 10 items based on a Poisson distribution.
- 35% likely to be in the age group 40-60 (encoded as 1) who buys on an average 20 items based on a Poisson distribution.
- 45% likely to be in the age group 60 and above (encoded as 2) who buys on an average 5 items based on a Poisson distribution.

Calculate the probability that the total number of items sold in a day exceeds 350 using simulation. After that, plot a histogram showing the distribution of the total number of items sold in a day.

Useful functions: `rpois()`, `sample()`, `sum()`, `mean()`.