



AML5153 | Applied Probability and Statistics | Lab Final Exam

1. Consider a court system that has 12 members (called jurors) who will decide on a case. It takes at least 9 juror votes to convict a defendant (that is, the person standing trial). If there are less than 9 votes, the defendant is acquitted. Suppose that the probability that a jurors votes that an actually guilty person is innocent is 0.2 and the probability that a juror votes that an actually innocent person is guilty is 0.1. Each juror acts independently. Assuming that 75% of the defendants are actually guilty, we are interested in the probability that the court renders a correct decision. To this end, consider the following approach:

$$\begin{aligned} P(\text{Court correct decision}) &= P\left(\underbrace{\text{Court correct decision} \cap \text{Person guilty}} \cup \underbrace{\text{Court correct decision} \cap \text{Person not guilty}}\right) \\ &= \underbrace{P(\text{Court correct decision} \mid \text{Person guilty})}_{\text{Replace by one of 1-4 below}} \times P(\text{Person guilty}) \\ &\quad + \underbrace{P(\text{Court correct decision} \mid \text{Person not guilty})}_{\text{Replace by one of 1-4 below}} \times P(\text{Person not guilty}). \end{aligned}$$

Now choose two appropriate conditional probabilities from the list below to replace the conditional probabilities in the expression above:

1. $P(\text{Court decides to convict} \mid \text{Person not guilty})$
2. $P(\text{Court decides to convict} \mid \text{Person guilty})$
3. $P(\text{Court decides to acquit} \mid \text{Person not guilty})$
4. $P(\text{Court decides to acquit} \mid \text{Person guilty})$

Even if you are not sure which two conditional probabilities will be chosen, calculate all the four above. After that, calculate the probability that the court comes to a correct decision.

2. Suppose a web server receives a random number of requests every minute from humans and bots. On an average there are 15 human requests per minute and 5 bot requests per minute. All requests are assumed to be independent. A network security software that you recently installed on the server can malfunction and cause an error in the following two ways:
 1. there is a 9% chance that the software erroneously detects a human request as a bot request (false positive rate);
 2. there is a 5% chance that the software erroneously detects a bot request as a human request (false negative rate).
 - (a) Which is of a more serious concern here, the false positive or false negative rate? Briefly explain why.
 - (b) You will review the network security software if there is more than an 80% chance that there is at least one error per minute. Will you review or not?
 - (c) Plot the probability mass function of the number of errors per minute.

Useful functions: `rpois()`, `sample()`, `sum()`, `rep()`, `table()`.