

## COMP557 Optimisation - Programming Assignment 1

This report describes the mixed-integer linear programming model that was created to maximize Soltari's logistics growth. It focuses on facility openings, production scheduling, and shipping routing in order to maximize revenue within a set transportation and resource budget.

### Decision Variables :

1. **Facility opening decisions:** For each possible factory location (like A, B, C, etc.), we decide yes or no whether to open it.  
Binary variable  $y_f \in \{0,1\}$  for each facility  $f$  in  $F$ , where  $y_f = 1$  if facility  $f$  is opened and 0 otherwise.
2. **Production amounts:** For each open factory and each product (Alpha, Beta, Gamma), we decide exactly how many units of that product to make there.  
Continuous variable  $x_{\{f,p\}} \geq 0$  for the number of units of product  $p$  in  $P$  produced at facility  $f$  in  $F$ .
3. **Shipping amounts:** For each open factory, each product, and each customer location it's connected to, we decide how many units of that product to send there.  
Continuous variable  $z_{\{f,d,p\}} \geq 0$  for the number of units of product  $p$  shipped from facility  $f$  in  $F$  to demand location  $d$  in  $D$  (only if connected via edge  $(f,d)$  in  $E$ ).  
 $E$  is the set of feasible shipping edges, which represents all possible direct connections (routes) between facility locations (in  $F$ ) and demand locations (in  $D$ ) where shipping is allowed.

### Objective Function :

Maximize total money made from sales:

Add up, for every customer location and every product, the number of units actually delivered there, times the selling price for that product at that location. But we can't count more units than the customer wants (demand cap), since extras don't sell and just waste money. The aim is to get as much revenue as possible without blowing the budget.

$$\max \left( \sum_{d \in D} \sum_{p \in P} \pi_{d,p} w_{d,p} \right)$$

### Constraints :

- **Factory production limits:** If we open a factory, the total units made there (all products combined) has to be at least the minimum required (for efficiency) and no more than the maximum allowed (for environmental rules). If we don't open it, zero production there.

$L_f * y_f \leq \sum_{p \in P} x_{\{f,p\}} \leq U_f y_f \quad \forall f \in F$  ; Lower bound  $L_f$  (efficiency) and upper bound  $U_f$  (regulations).

- **Supply matches demand flow:** Everything we produce at a factory for a product must exactly equal everything we ship out from that factory for that product, no leftovers or magic creation.

$$x_{\{f,p\}} = \sum_{d:(f,d) \in E} z_{\{f,d,p\}} \quad \forall f \in F, p \in P ;$$

- **No shipping where impossible:** We can only ship between factories and customers if there's a direct route on the map (distance given). No connection, then zero shipments.
- **Don't over-deliver:** Total units shipped to a customer for a product can't exceed what they want (demand).

$$w_{d,p} = \sum_{f:(f,d) \in E} z_{\{f,d,p\}} \leq D_{d,p} \quad \forall d \in D, p \in P ;$$

- **Budget limit:** The grand total cost (resources bought for production at each factory, based on what the product needs times local prices) plus (shipping costs, which are distance times units times product shipping rate), can't go over the 500,000 budget. Everything must fit under that one pot of money.  
Total resource costs + shipping costs  $\leq B = 500,000$
- **Non-negativity:** All amounts (production, shipping) have to be zero or positive, no negative units must be present.

$$x_{\{f,p\}} \geq 0$$

$$z_{\{f,d,p\}} \geq 0$$

$$w_{\{d,p\}} \geq 0$$

### Commentary on the Optimal Solution

Gurobi delivers optimal revenue of 149,507.67 for the Section 3 instance. Opens A, B, C, E (skips D due to high min bound and distant edges); total production 2,676.97 units, maxing A/B/C for scale while E runs partial (267 > min 180) to avoid waste.

The solution opens low-min-bound facilities with strong network access, maxing capacities for scale while running partial on others to hit minima without waste. Production favors low-resource products like Gamma for broad coverage and high-price spots for Alpha/Beta, minimizing costs by matching requirements to cheap sites. Shipments target short, high-margin routes for full/partial demand satisfaction, skipping distant/low-ROI locations.

In the end, the plan smartly juggles production limits (tied to yes/no facility choices), matching output to shipments, and order ceilings, all while maxing out the budget. It settles for partial deliveries where it makes sense, zeroing in on profitable routes.

**Output :**

**Total Optimal revenue** = 149507.67

**Opened Facilities** = A,B,C,E

**Production (Units per Facility/Product)**

Facility	Total Units	Alpha	Beta	Gamma
A	800.00	475.00	215.00	110.00
B	1000.00	0	0	1000.00
C	600.00	110.00	490.00	0
E	266.97	0	0	266.97

**Shipments to Demand Locations (Total Units Received)**

Demand	Alpha (Demand/Price)	Beta (Demand/Price)	Gamma (Demand/Price)
1	0 (205/65)	0 (380/50)	310 (590/42)
2	475 (475/60)	215 (215/45)	800 (800/55)
3	0 (600/48)	490 (490/70)	0 (270/38)
4	110 (900/80)	0 (675/65)	267 (450/42)
5	(No shipments)	(No shipments)	(No shipments)
6	(No shipments)	(No shipments)	(No shipments)