Intro to Financial Engineering
Markowitz Portfolio Optimization

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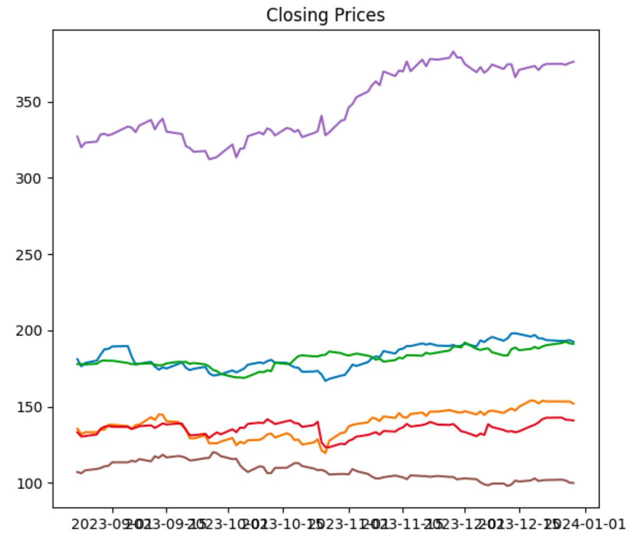
Dataset

We have choosen 6 assets, since we were obtaining a more well-defined Markowitz bullet. The database for 10 assets has been commented in code. The assets used are: AAPL, AMAZN, GLD, GOOG, MSFT, XOM. Their closing prices and pct_change (returns) *are shown below*



	Asset	Risk	Return
0	AAPL	0.000686	0.01
1	AMZN	0.001285	0.02
2	GLD	0.000809	0.01
3	GOOG	0.000633	0.017321
4	MSFT	0.00157	0.014142
5	XOM	-0.000778	0.014142

Return

$$K(n) = \frac{S(n) - S(n-1)}{S(n-1)}$$

Log return

$$k(n) = ln \frac{S(n)}{S(n-1)}$$

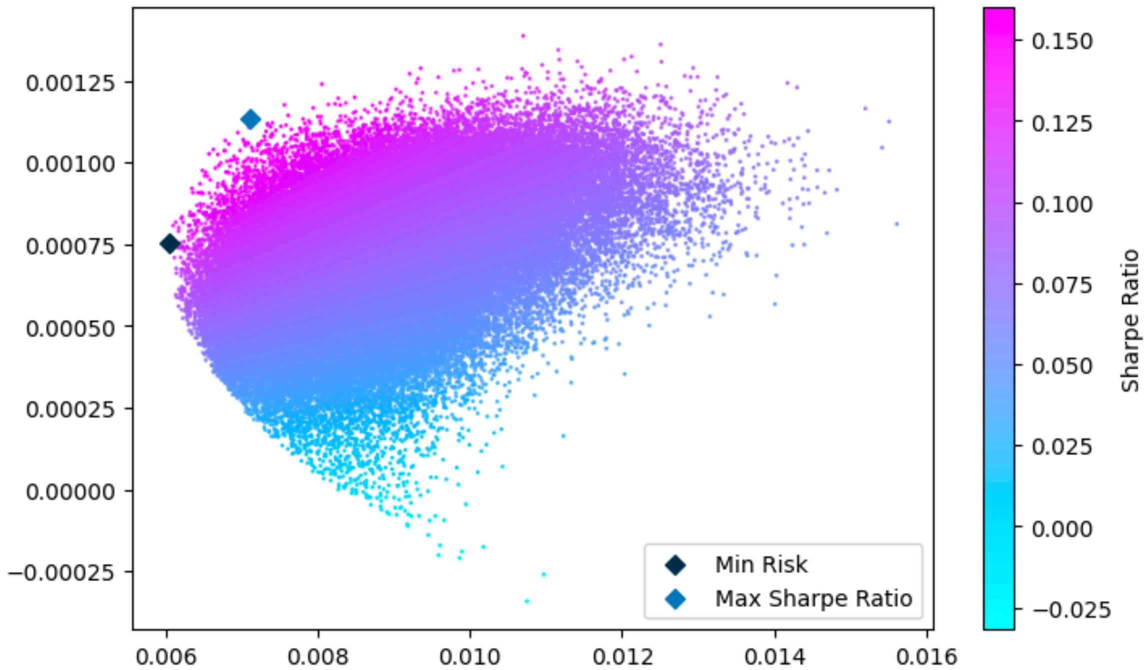
Markowitz Bullet & Efficient Frontier

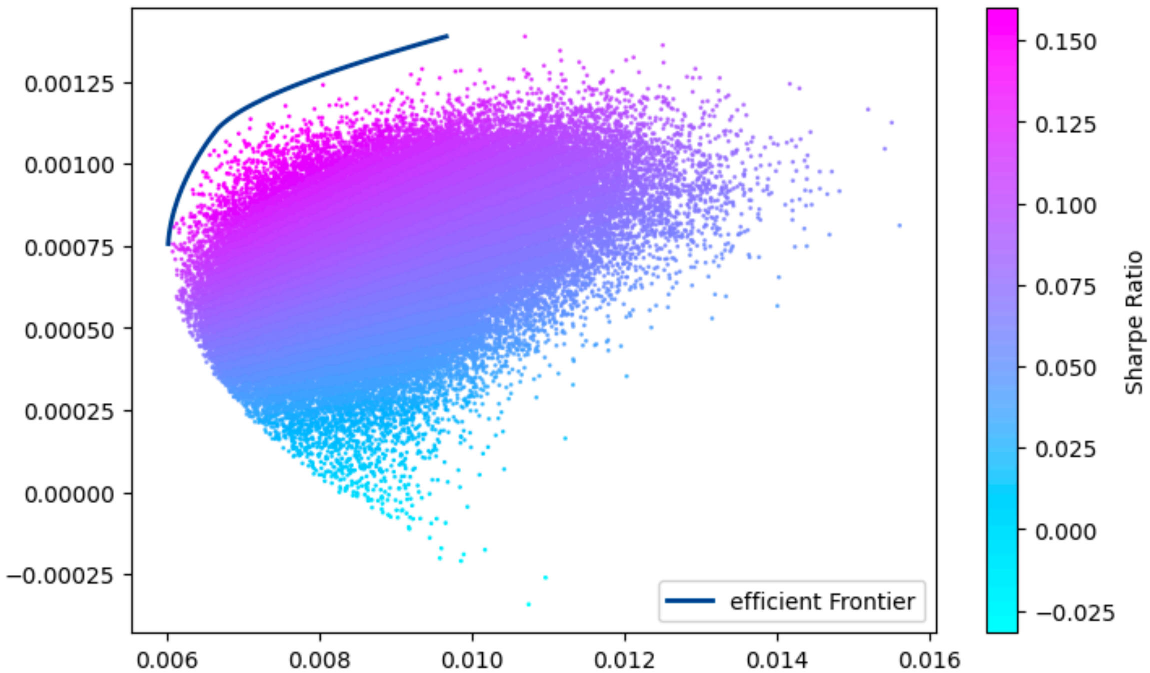
10000 random protfolios have been chosen, normalized, their mean and variances calculated and scatter plot to construct the Markowitz bullet. Let w be the weight vector. Then

$$\mu_v = mw^T \qquad \sigma_v^2 = wCw^T \qquad S.Ratio = \frac{\mu_v}{\sigma_v} \qquad \sum_i w_i = 1$$

Efficient Frontier is defined as set of portfolio at given risk tolerance offering max return or vice-versa. It boils down to solving the following optimization problem

$$min \quad wCw^T \ \ s.t. \ mw^T = \mu \ and \ \sum_i w_i = 1$$





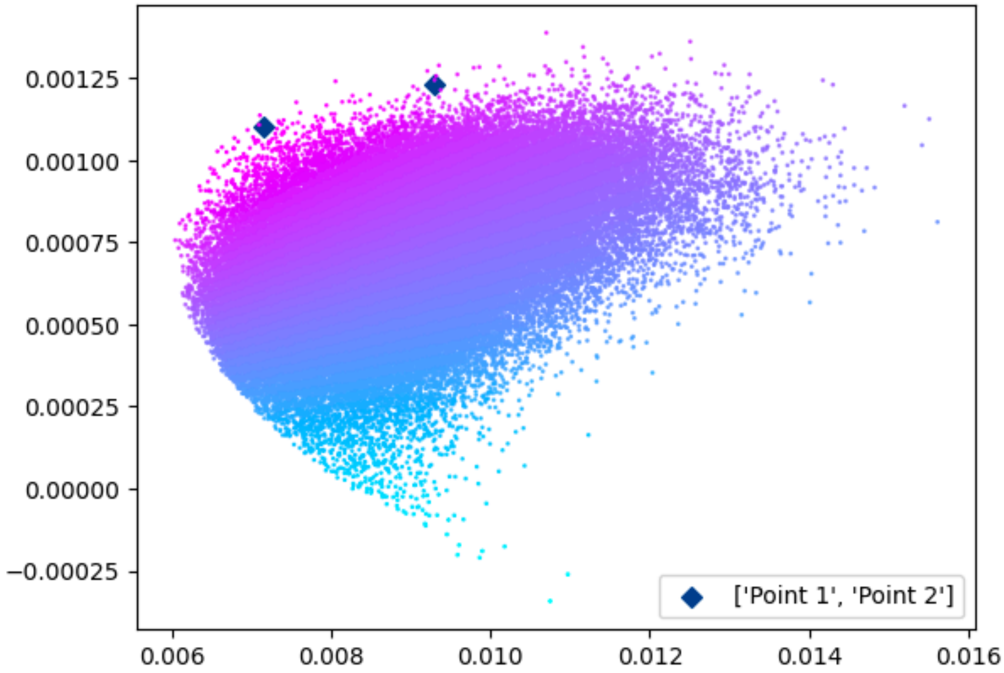
2 Points on frontier with max return, given risk tolernace

Two points have been randomly chosen on the frontier having some risk associated. Finding the portolios with maximum return is equivalent to the following two constraint optimization problems

$$max \quad mw^T \; s.t. \quad wCw^T = \sigma^2 \; and \; \sum_i w_i = 1$$

OR

$$min \quad -mw^T \; s.t. \quad wCw^T = \sigma^2 \; and \; \sum_i w_i = 1$$



The following results have been observed:

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Point 1 Risk Tolerance: 0.007157130812666459
Optimal Weights:
AAPL=0.13   AMZN=0.054  GLD=0.444   GOOG=0.0    MSFT=0.372  XOM=0.0
Return at this portfolio: 0.0011020587899146475

==================================================
Point 2 Risk Tolerance: 0.009289084093612164
Optimal Weights:
AAPL=0.098  AMZN=0.219  GLD=0.249   GOOG=0.0    MSFT=0.434  XOM=0.0
Return at this portfolio: 0.0012317064134817596
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